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Prediction And Simulation Of Origami Through Boolean Satisfiability

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Abstract

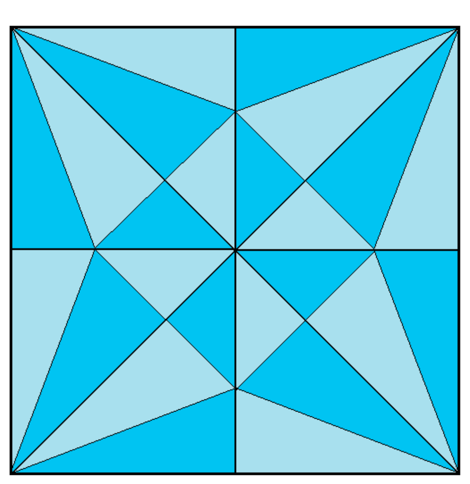
Origami is the ancient Japanese art of folding paper into 3 dimensional shapes. Over the past century the advent of computer technology and use of mathematics have allowed the art to greatly expand beyond producing simple animals or flowers. Here I introduce the art as a mathematical problem and discuss the use of fundamental logic theory to formulate a specific Origami model as a computer simulation problem. The complexity of predicting a model from an initial plane and then simulating its folding steps are delineated, and the use of techniques such as Craig Interpolation and minimal logical synthesis are brought up to reduce the said complexity. Beyond simply predicting and designing Origami models and procedures, this work can be applied into a variety of problems, like protein folding, manufacturing designs or multi-dimensional folding.

Introduction to Origami

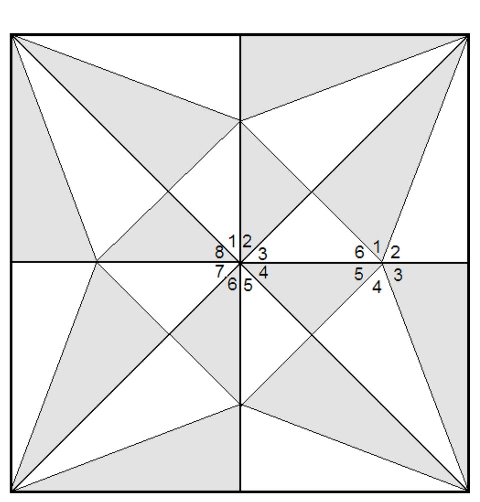
Traditional Origami starts off from a flat square of paper and involves folding a model without making cuts to the paper. Modern origami started to develop in the 1950s when Akira Yoshizawa developed a notation system for origami diagramming. The development of such notation allowed a basis for sharing blueprints of models called Crease patterns - these are 2D paper designs with all the folds for a model marked on them.

Over the past century various mathematical rules have been established for verifying a possible origami model through its crease pattern such as:

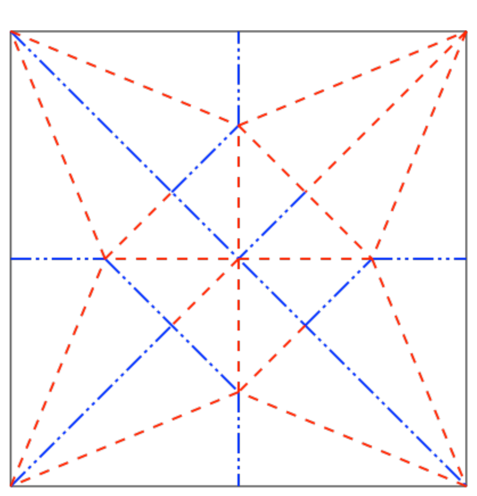
- Two-colorability



- Even-Odd angles sum to 180 degrees



- Difference in folds at vertices

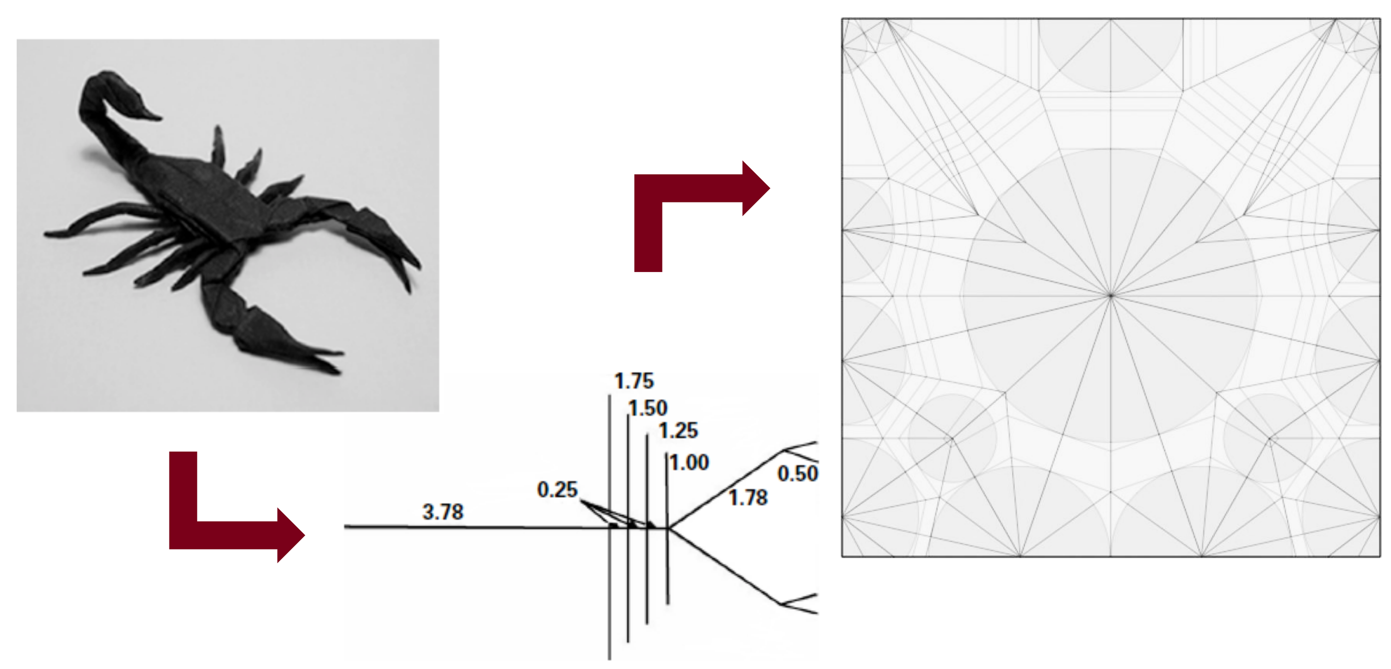


- A sheet cannot penetrate a fold, and cannot stretch or curve unless subjected to external treatment

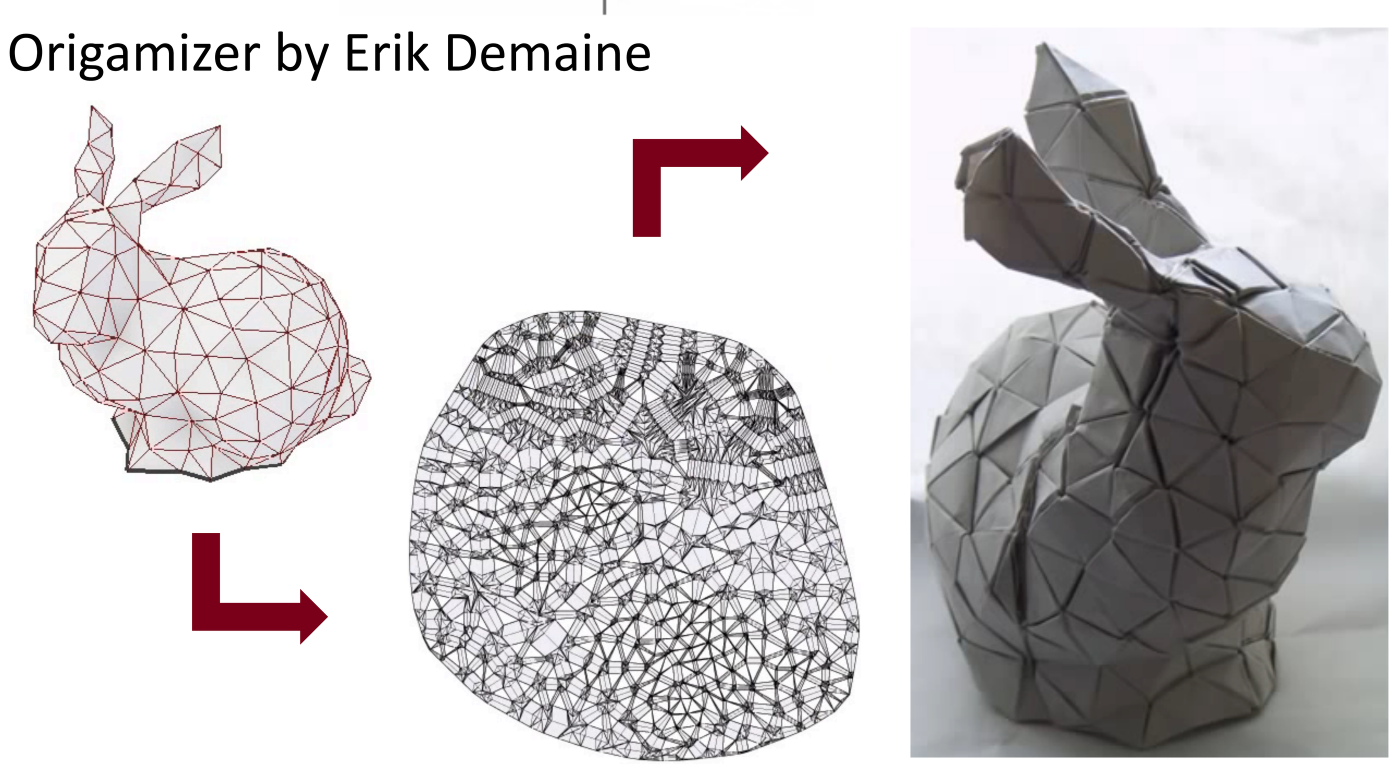
How Boolean Logic Fits in

Boolean algebra is a field of algebra where variables are restricted to either True or False values (or 1 and 0 in binary). Since the folds on a Crease pattern are either mountain or valley folds, origami maps to Boolean logic. This allows the use of algorithms and procedures for Boolean logic to be implemented on origami, seen in works like:

- Treemaker by Robert J. Lang



- Origamizer by Erik Demaine

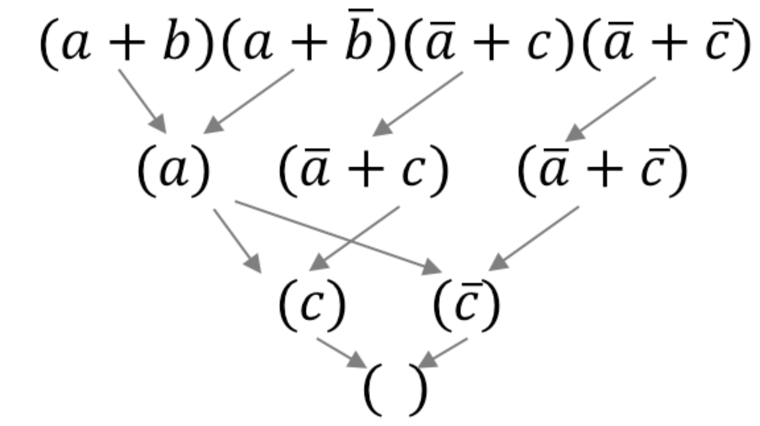


These works illustrate how computer technology can be used to design crease patterns for nearly any desirable shape.

Hypothesis

Through the use of Boolean satisfiability, any crease pattern can be instantiated as a Boolean expression from which some or all possible 3D models it can folded to can be predicted, and a set of steps on folding these can be obtained from a resolution proof. Once a certain model is picked, these steps can be simulated in a 3D software like Google's SketchUp to verify the final state of the model.

Boolean Satisfiability asks whether a given Boolean formula has an assignment to all its variables that renders the expression true or satisfiable (abbreviated as SAT). In Artificial Intelligence and Computation Theory it is common to verify entailment from an expression by testing unsatisfiabtlity. State-of-the-art SAT solvers like MiniSAT and ABC even provide a proof for unsatisfiability if given a Boolean expression that is not satisfiable, in the form of a directed acyclic graph as shown below.



A resolution proof for UNSAT

Thus predicting and simulating an origami model would be rendered into writing its crease pattern into a data structure as a Boolean expression, testing for the opposite of the expression and then utilizing the pivots of the resolution proof to determine the order of folds – similar to the process of determining interpolants as discussed in the linked paper.

Applications

Origami has a vast spectrum of applications in today's society besides art, from huge telescope lens brought into orbit in space down to contractible robots to patrol the human body for medical purposes. There would be several applications of my work in domains beyond art such as:

- Protein Folding: A protein chain could be approximated by a very long thin rectangular crease pattern, and then through simulation shaped into a predicted complex shape.

- Machine Design and Manufacturing: Paper has certain limitations for folding due to non-zero thickness and non-stretchability. Using a plane obtained from a more versatile material could allow for higher dimensional design.

References

Erik D. Demaine and Tomohiro Tachi, "Origamizer: A Practical Algorithm for Folding Any Polyhedron", in *Proceedings of the 33rd International Symposium on Computational Geometry (SoCG 2017)*, Brisbane, Australia, July 4–7, 2017, 34:1–34:15.

Backes, John D., and Marc D. Riedel. "Reduction of interpolants for logic synthesis." *Proceedings of the International Conference on Computer-Aided Design*. IEEE Press, 2010.

Cipra, Barry A. "In the fold: Origami meets mathematics." *SIAM news* 34.8 (2001): 1-4.