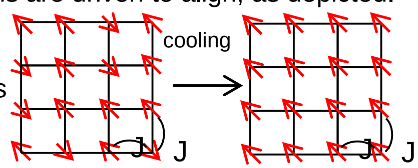
Metropolis Monte Carlo Simulation for the 2D Ising Model

Joe Eix
University of Minnesota School of Physics and Astronomy

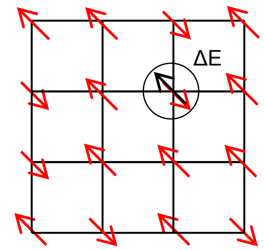
2D Ising Model Description

- The electrons present in crystals behave as tiny magnets due to a quantum mechanical property called spin. It is the interaction between the spins that causes magnetism.
- In the Ising model, the spins are restricted to point up or down, and interact via exchange interactions. We study the Ising model on a square lattice as depicted.
- The model is described by $H = J\sum_{\langle i,j\rangle} S_i \cdot S_j$, where H is the Hamiltonian, J is the exchange interaction, S_i is the spin at site i, and <i,j> symbolizes the sum is only over nearest-neighbors.[1]
- If the exchange J is negative, spins are driven to align, as depicted.
- This ordering only occurs below a critical temperature, T_c, since thermal energy in the lattice allows spins to overcome exchange interactions.



Goal and Methods

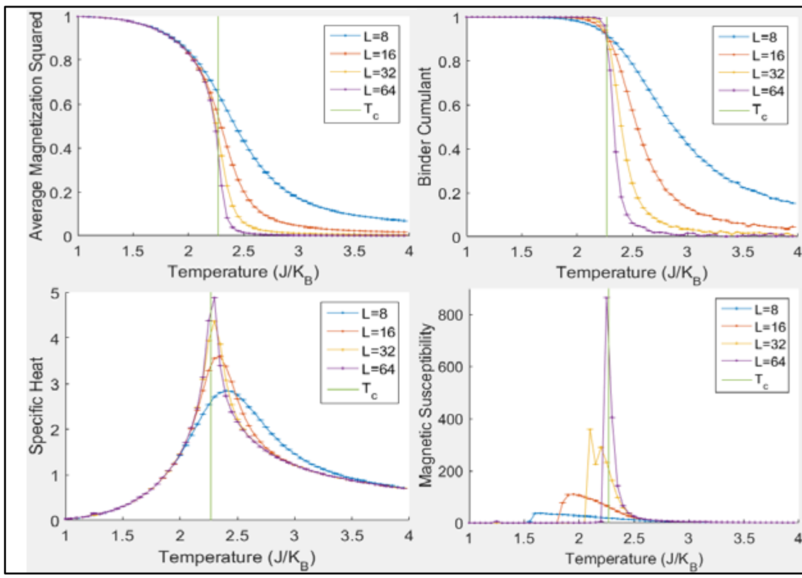
- **Goal:** A Metropolis Monte Carlo simulation to find T_c. This will be modified for use in more complicated magnetic models.
- **Method:** Choose a single spin and flip it, accepting the flip with a probability of $e^{-\frac{\Delta E}{k_B T}}$, where ΔE is the energy change of the flip.[3]
- For a lattice LxL, L^2 flip attempts is one step. After 100 steps, system observables are measured: average square magnetization, total energy, specific heat, magnetic susceptibility, and the Binder cumulant.
- Run this simulation over 60 temperatures from 1.0-4.0 in units of J/k_B.
- Repeat entire simulation for system size L's of 8, 16, 32, 64.
- Analyze statistical uncertainty using jackknife and binning analysis.



Discussion

- For each lattice size, 2^{18} measurements were taken. The figure shows key observables with respect to temperature for all lattice sizes. Superimposed is the analytical T_c line at 2.269 J/k_B.[2]
- T_c was found by the Binder cumulant, as the curves for system size should cross the same point at T_c. The measured T_c value is 2.275 ± 0.025 J/k_B. This value is within acceptable uncertainty. Uncertainty is primarily due to spacing between temperature measurements.

Results



Closing Notes

- The simulation is functioning correctly, so it will be modified for analysis of more complex magnetic systems.

References

1. P. M. Chaikin, T.C. Lubensky. "Principles of Condensed Matter Physics." Cambridge Univ. Press, (2000).
2. L. Onsager. Phys. Rev. 65, 117-149 (1944).
3. A. W. Sandvik. "Monte Carlo simulation in classical statistical physics." Dept. of Physics, Boston Univ. (2016).

Acknowledgements

I would like to thank Professors Rafael Fernandes and Peter Orth for their guidance in this project.