The Impact of Problem-based Learning on Non-Science Undergraduate Students' Attitudes towards Mathematics in an Egyptian Classroom

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Dedication

This thesis is dedicated to Mama, for all the goodness she has done in this world.

Abstract

While Problem-based Learning (PBL) has been established in the literature in different contexts, there remains few studies on how PBL has an impact on students' attitude towards mathematics and their conceptual understanding of it in Egyptian classrooms. This study was conducted in an international university in Egypt, and the participants were non-science undergraduate students who took a course called "Fun with Problem-Solving" as a requirement core class. The study shows that students' attitude towards mathematics developed throughout the course, and this was tested using the Fennema-Sherman Mathematics Attitude Scale, where students had a pretest and posttest. While the sample size was small, there was statistical significance in the change of the means of how students perceived mathematics as a male domain, and how teachers perceived students' achievements. This notion was coupled with students' development of conceptual understanding, which was tracked throughout the semester by mapping students' work with the Lesh Translation Model.

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Glossary of Abbreviations and Acronyms

AUC ....American University
....in Cairo

(C) .... Confidence in Mathematics

LTM .... Lesh Translation Model

(M) ....Mathematics as a Male Domain

MANOVA..Multivariate Analysis of Variance

MEAs .... Model-Eliciting Activities

PBL .... Problem-Based Learning

STEM..Science, Technology, Engineering, Mathematics

(T) ....Teacher's Perception

(U) ....Usability of Mathematics

USAID ....United States Agency for International Development

Chapter One: Introduction

1.1 Background

In parallel with the development of complex civilizations in the river valleys of the Nile in Egypt and the Tigris in Babylonia, the transfer of knowledge through storytelling and imitating became an increasingly intricate process. In turn, this growing complexity gave rise to the earliest school systems. Over the centuries and until the present day, the complex organization of structured education continues to generate intense debate, particularly in terms of the relevance of academic subjects to real life. Since our ancestors' time, the most significant goals of education have been focused on helping students become lifelong learners, providing them with the skills and knowledge to solve problems relevant to both themselves and to the community to which they belong (Schlett, Doll, Dahmen, Polacsek, Federkeil, Fischer, & Butzlaff, 2010). One tool that could facilitate this vital problem-solving skill is mathematics. Mathematics is considered an effective vehicle/medium for acquiring problem-solving skills since it involves many practical applications (Boaler, 2008). Engaging students in actively solving problems in a meaningful and relatable setting may improve their general attitude towards learning, thus facilitating the acquisition of problem-solving skills (Bransford & Donovan, 2004; Von Glaserfeld, 2002). When students are engaged in these favorable settings, Problem-Based learning could serve as a means to help enhance students' attitudes towards learning as they work on solving real-life problems using mathematical tools (Lesh & Zawojewski, 2007).

Latterell and Wilson (2013) interviewed liberal arts undergraduate students in an attempt to unpack their views on the value of mathematics in their everyday lives. Less than half the participants thought of mathematics as an approach to help solve everyday problems, while less than 10% viewed mathematics as a vehicle to discover and delve into the world. Latterell and Wilson (2013) propose that such perceptions of mathematics indicate the need for more concerted efforts to address the methods of teaching mathematics and validating its purpose/usefulness outside the classroom. There are different schools of opinion as to why we should learn mathematics and appreciate its purpose; in this regard, Padula (2011) lays out the different schools of thought in her paper “Analyzing the mathematical experience: posing the 'what is mathematics?’.” While the paper discusses the nature of mathematics from a philosophical perspective, it suggests that mathematics could be thought of as a language that features abstract elements, which bear a vital relationship to daily life. This perception validates the significance of mathematics in terms of its practical uses, such as calculating the time needed to travel somewhere or how much money one has spent in a specific situation. However, mathematics in the classroom context is often perceived merely as a set of procedural facts; accordingly, many university-level professors require their students to master these basic operations before embarking on more advanced courses or concepts (Latterell & Wilson, 2013). This would beg the question of what we should really be teaching our students in a mathematics class. Boaler (2000) discusses the actual elements of mathematics in a class driven by project work and emergent mathematical problems, suggesting that the ways in which students can relate this material to their everyday lives is a kind of extended mathematics. Boaler believes that this approach should be encouraged despite any institutional obstacles in reconciling students' constructed knowledge with mathematics as a required school subject. Building on the same argument, students who have developed negative attitudes towards mathematics should be given the chance to see mathematics in a different perspective, with the belief that this fresh perspective may help students recognize the value of mathematics as a practical problem-solving tool. However, the question remains whether providing students with such an opportunity would change students' attitude towards mathematics (in terms of how usable mathematics is in everyday life among other factors affecting attitude), and whether relating mathematics to non-science undergraduate students would develop their conceptual understanding of mathematics.

Since this study will be conducted at a university based in Egypt, one must mention that the Egyptian mathematics classes, whether at university or pre-university level, are teacher-centered in their delivery and assessment (Ministry of Education, 2008). Mathematics courses in Egypt are taught using a lecture format, and students are expected to memorize procedures and applications. Moreover, examinations are devoid of context; students are assessed through tests that merely reflect their ability to memorize and apply procedural rules, rather than their understanding of mathematical concepts (Asabere-Ameyaw, Raheem, & Anamuah-Mensah, 2012). This disconnect from contextual experiences hinders student understanding of how mathematics education applies to real life, contributing to the perception that mathematics education is not useful for daily practicalities (Newburghl, 2008).

Pedagogical considerations also contribute to the current state of pedagogic malfunction. Even though Egypt was the first civilization in the world to create a schooling system, Egypt's educational system is currently considered among the worst in the world (Schwab, 2010). Mathematics classes that are designed solely for students to achieve high grades on tests may lead to disengagement, and contribute to the formation of negative attitudes towards mathematics courses in general; this is especially true for students who struggle to master mathematical processes by rote learning (Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014). Researching students' attitudes towards mathematics is important because negative attitudes towards mathematics tend to correlate with low mathematical achievement (Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014). This is quite evident in the Egyptian context, and will be illustrated clearly in the following section.

Research suggests that Egyptian students are not satisfied with what universities offer them, and sense that there is a gap between what they learn in university and with what they believe they need to learn in order to succeed in their careers, lives, and future professions (Al-Harthi, 2011). Many students view mathematics as an academic formality due to the concerns mentioned above regarding students' experiences in mathematics classes (Nagi & Nagi, 2011). Given that mathematics has many practical applications, there is an urgent need to identify specific ways to change students' attitudes towards mathematics and subsequently, their achievement in mathematics.

The perception that mathematics that is not useful in real life, and the resulting lack of motivation contribute to students' mischaracterization of themselves as having learning difficulties (Brown, Colins, & Duguid, 1989). The negative attitudes that students develop towards mathematics during their formal school years can inhibit their success and motivation in pursuing mathematics throughout their lifetime (Colins, 1997; Hannula, 2002; Ma, 1997; Ma & Xu, 2004). In the Egyptian context, there are many more students majoring in the arts than the sciences and mathematics (Assaad & Barsoum, 2007). Students' choice of major, and consequently their choice of career, can change if these students' attitudes towards mathematics are improved (Conklin et al., 2013).

Despite research findings that confirm that mathematics instruction in Egypt is not delivered in such a way that students can connect with it at a pragmatic level it to their everyday life, there is little research examining the influence of student-centered mathematics instruction on non-science students' attitudes and conceptual understanding towards mathematics in-Egyptian undergraduate classrooms. The aim of this study is to investigate the potential attitudes of students when exposed to an alternative approach in teaching methodology.

1.2 Problem Statement

This study was conducted in the context of an undergraduate-level college mathematics course at an Egyptian university. Data will examine how students in a freshman level core curriculum course titled “Fun with Problem Solving” utilize mathematical concepts, procedures, and rules to tackle real-life problems. Thus, this study is important due to its focus on non-science undergraduate students, assuming that some students enrolled in non-science majors in Egypt have already developed negative attitudes towards mathematics throughout their elementary and secondary school years. Using mixed quantitative and qualitative data, the following Research Questions will be addressed:

1. What is the impact of a Problem-Based Learning approach on the attitude of non-science undergraduate students towards mathematics in an Egyptian university?

2. What is the impact of a Problem-Based Learning approach on students’ conceptual understanding of mathematics in an Egyptian university?

1.3 Conceptual Framework

The proposed conceptual framework guiding this study encompasses the following educational theories and approaches: Situated Learning Theory, Model-Eliciting Activities (MEAs), and the Lesh-Translation Model. A brief overview will introduce each of the former three concepts, as they will all be merged in one combined framework, before they are revisited again in the Literature Review.

Situated Learning Theory. Situated Learning is among the theories that discourage lecture-based teaching in favor of student-focused teaching (Lave & Wenger, 1991). Using this approach, students are able to discuss course topics with peers and with the teacher (Lave & Wenger, 1991). The situated learning theory can be used to create an environment where students can relate content to relevant contexts, giving learners the opportunity to engage in problem-based thinking with their peers (Lave & Wenger, 1991), in addition to developing better conceptual understanding in real life situations (Koschmann, 2011). Furthermore, students gain interest in mathematics when presented with subjects that they can personally relate to (Julie, Holtman, & Mbekwa, 2011; Kilpatrick et al., 2001).

An important element in Situated Learning is the social aspect, in which students from varying experience levels work together (Matusov, Bell & Rogoff, 1994). Within this social construct, students' comprehension levels evolve from beginner to expert (Lave & Wenger, 1991). In order for the students to collaborate, become engaged, and utilize the material, the problems presented to them have to be linked to their own lives. Such a setting will give students the opportunity to relate what they learn in class to their real lives, while allowing for social interactions that can reflect positively on their comprehension (Lave & Wenger, 1991). A student-centered learning environment can create a more meaningful learning experience than that of a traditional, teacher-focused environment (Lynch & Livingstone, 2000). In contrast, teacher-centered practices do not typically allow time for meaningful sharing and discussions among students (Lynch & Livingstone, 2000), depriving them of the opportunity to learn from each other. In learning environments based on Lave and Wenger's situated learning theory, learning is grounded in nature and context, presenting a beneficial alternative to traditional teaching. Such an environment enhances students' engagement and attitude, which in turn heightens their conceptual understanding of the taught subject matter (Krasny, Tidball, & Sriskandarajah, 2009).

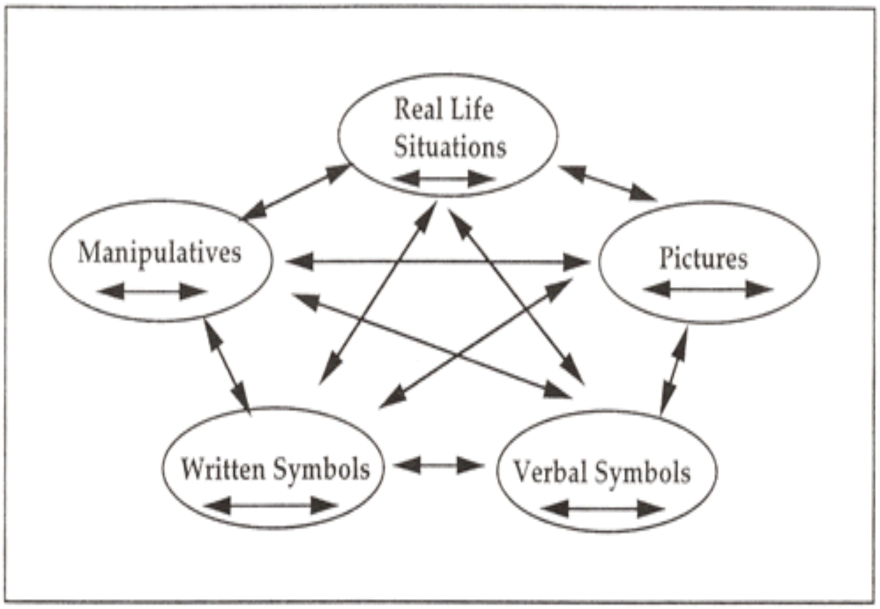
Problem-based Learning: Why? In a learning environment which supports problem-based learning (PBL), students work in groups to solve open-ended real-world problems (Prince & Felder, 2007). In such an environment, students are engaged in developing solutions based on what they know and what they need to know, as well as exploring new knowledge to evaluate their solutions accordingly, thus giving students the responsibility of defining the knowledge and skills they need to come up with a logical and sound solution. While it may be challenging to orchestrate a classroom with PBL settings, it can be well-planned if the activities provided to the students are tested and researched (Prince & Felder, 2007). PBL works well when teachers are prepared with the activities and are familiar with the content that can drive the possible solutions.

Model-Eliciting Activities (MEAs). Model-Eliciting Activities (MEAs) are realistic and meaningful problems that reflect the real world, and as a result, establish a more authentic assessment of the students' abilities. Accordingly, they can be considered a means to provide a learning environment in which Situated Learning and PBL are considered. One of the characteristics of MEAs is that it provides students with multiple reiterative learning cycles. This repetition opens students up to multiple opportunities to evaluate, modify, and adjust their mathematical models (Lesh & Doerr, 2003). MEAs help students reflect on their way of thinking as they solve a problem; this is because in using MEAs, students have to describe their procedure clearly so that their peers can learn from their thinking process. In this sense, MEAs are “thought-revealing” and “model-eliciting” (Ashmann, Zawojewski, & Bowman, 2006). There are several ways to assess students’ conceptual understanding of content while and after solving an MEA, but for the purpose of this study, the Lesh Translation Model will be used. While the the Lesh Translation Model can be used as a guideline to prepare and design activities for students, it can also be a means of evaluating students’ work, and identify the level of their conceptual understanding, especially in mathematics (Lesh & Doerr, 2003).

The Lesh Translation Model. The Lesh Translation Model is a means of identifying students’ conceptual understanding of mathematical concepts by giving students the chance to use multiple representations to express their understanding. According to the Lesh Translation Model, there has to be a reciprocal relationship between real life situations and different learning elements (e.g. pictures, written symbols, and verbal communication); and the links between these different elements should be formed naturally (that is, by the students) Figure 1 explains the different connections between the learning elements as per the Lesh Translation Model.

Figure 1

The Lesh Translation Model



The Lesh Translation Model can offer insights into the development of students' conceptual understanding (Chapin, Anderson, O'Connor, & Ebrary, 2003; Lesh & Doerr, 2003). The ability to communicate mathematics in writing can also help educators identify students' comprehension of different mathematical concepts (Santos & Semana, 2015). Previous studies suggest that students who interpret images, or describe mathematical concepts in their own words express better conceptual understanding of mathematics than those who cannot perform these tasks (Chapin, Anderson, O'Connor, & Ebrary, 2003; Martin & Gourley-Delaney, 2014; Santos & Semana, 2015).

The Framework. The present study addresses the effect of applying a PBL teaching approach on students' attitudes and achievement levels. It is important to clarify that achievement will be measured by students' conceptual understanding of mathematics, using the Lesh Translation Model. I hypothesize that the situated learning theory serves as the umbrella that encompasses a PBL teaching approach, which in turn features Model-Eliciting Activities that enhance students' performance when combined together. Applying practices that adhere to the Situated Learning Theory creates a learning challenge/experience that is relevant to the students' contexts, making learning more interesting to the students. This improves students' attitudes and engagement. More engaged students with positive attitudes actively participate in the classroom and are motivated to seek more knowledge, which in turn leads to higher achievement.

Due to its hands-on nature, PBL allows students to develop the conceptual understanding and higher level thinking in mathematics. Furthermore, PBL liberates the teacher from the traditional information transmission role; instead, the instructor acts as a facilitator inside the classroom. Thus, the PBL approach can maximize student participation, compounding the benefits of active participation generated from the improved attitudes and engagement levels through the Situated Learning theory practices.

Also assigned to students are Model-Eliciting Activities (MEAs). These follow the PBL approach of the Situated Learning theory as they align with concepts of relevance and realism instrumental to the theory's practices. Finally, the benefits of all these practices are fully realized when delivered within the framework of what scholars and practitioners refer to as a good learning environment.

PBL was deliberately chosen for the purposes of this study since all the participants involved will be non-science majors with presumed partial knowledge of the mathematical concepts presented. This will offer the opportunity to explore whether students can learn new content through alternative learning theories that may affect their attitudes towards mathematics and, consequently, their performance. To accomplish the purpose of this study, the traditional means of assessing students through exams will be excluded. Rather, work by Richard Lesh on how students' conceptual understanding can be represented through various methods, can provide us with the necessary framework for assessing students' learning of mathematics without relying on routine means of assessment.

The next chapter will provide a literature review about the impact of Problem-based Learning on non-science undergraduate students' attitudes towards mathematics in the Egyptian classroom. Chapter three will discuss the methodology, data source, data collection, and how the respective data will be analyzed. Chapters four, five, and six will describe the findings, which will be discussed in chapter seven, which will also serve as a conclusion to this paper.

Chapter Two: Literature Review

2.1 Introduction

The interplay between the different variables in this study is complex; nonetheless, it is safe to say that an extensive body of literature supports PBL as an effective teaching approach in terms of its positive effects in improving students' attitudes towards mathematics. Different variables can include students' previous experience in mathematics classes, the nature of the problems presented to them, their existing attitude towards mathematics, or how they perceive mathematics as a subject. Existing literature that addresses the relationship between PBL and students' attitudes towards learning mathematics shows that PBL can be very effective in enhancing students' attitudes towards mathematics, provided that it is delivered appropriately and within an equally appropriate context.

2.2 Achievement as a Key Indicator of Effective PBL Instruction and Changed Attitude Towards Mathematics.

Attempts to establish the target effectiveness of the educational practice of choice is affected by a plethora of factors. The aim of this literature review is to shed light on the governing factors in the relationship between achievement towards a subject, and attitude towards it; keeping in mind many critical aspects raising several important questions: Since achievement on tests is usually the yardstick used when seeking to evaluate the effectiveness of teaching practices, should we really be using achievement as an indicator? How do other crucial factors come into play such as the learning environment, for example? What is the link between achievement and engagement and how does it affect the learners' attitude towards mathematics? Lastly, could achievement be a direct sign of the learners' true mastery of the conceptual knowledge in question?

The following section of the literature review aims at answering the above questions to establish a clear understanding of the dynamics governing such educational practices in order to justify the choice of PBL as a proposed approach for mathematical educational reform in Egypt.

2.3 Achievement as Conceptual Knowledge

There is on-going debate on the ultimate aim of mathematical instruction, and whether the focus should be conceptual understanding, procedural knowledge, or both. Rittle-Johnson, Schneider, and Star (2015) believed that the relationship between conceptual understanding and procedural knowledge is bidirectional, where one leads to the other. There is extensive research on achievement in mathematics, but methods of measuring achievement are not always explicitly mentioned in relation to procedural knowledge versus conceptual understanding. For the purposes of this paper, developing conceptual understanding will be considered as an 'achievement', rather than the usage of standardized tests, the conventional way to measure achievement in Egypt (Sayed, & Manṣūr, 2000). The tool of choice for such assessment is Lesh's Translation Model, in which students' representations of the activities and problems will serve as a measure of their conceptual understanding, and hence their achievement in mathematics.

2.4 PBL and Achievement

Within the literature that examines PBL in mathematics classrooms, there is consensus on the advantages for both students and teachers. When students learn by creating and solving real-life problems that they can relate to, teachers can also better observe students' mental processes through examining the representation of their own work (Beneke & Ostrosky, 2008). Such observations are then used to refine future PBL instruction (Loyens, Kirschner, & Paas, 2011; Mandeville & Stoner, 2015; Sahin, 2010). PBL consistently leads to enhanced performance for all levels of learners. It is important to note, however, that the definition of 'achievement' varies from one study to another. An analysis of the reasons why PBL can improve achievement, including the sub-variables, is discussed in the following section.

In their study addressing the bridge between beliefs in education and students' own feelings about what they are learning, Di Martino and Zan (2011) argue that high school students can become higher achievers in mathematics when engaged in more progressive, problem-based learning environments. Furthermore, Erickson (1999) asserted that high school students from different backgrounds, such as students learning mathematics in languages other than their native tongue, and others with different abilities in mathematics, gained considerable mathematical processing skills when exposed to the PBL approach in mathematics classes. Another study was conducted to compare the results of biomechanics students engaged in PBL activities as opposed to those who were taught using traditional methods. In this study, Mandeville and Stoner (2015) found that the results for students engaged in PBL surpassed those of their counterparts taught in traditional lectures. Tandogan and Orhan (2007) noted that PBL improved seventh-grade students' attitudes toward mathematics and science with parallel progress in their performance in these subjects.

Han, Capraro, and Capraro (2015) observed how attitude and achievement are interrelated in their research. Their study included over 1500 high school students with different achievement levels classified as *Low*, *Medium*, or *High Achievers* using the Texas Assessment of Knowledge and Skills (TAKS). The study yielded outstanding results, especially with long-time low achievers, who showed significant improvement with the use of PBL activities. Butler and Christofili (2014) conducted a study at the community college level in Portland, which targeted first term students. The students were enrolled in developmental education (DE), comprising courses in reading, writing, mathematics, and college survival and success (CSS), all of which utilized PBL as a learning technique. The study, which gathered data across four terms, indicated increasingly positive attitudes and results by the end of the fourth term, indicating that the students' progress was due to redesigned PBL activities rather than increased familiarity.

Moreover, Cotič and Zuljan (2009) developed mathematics lessons driven by learning activities involving problem posing and solving. In the study context, the Slovenian undergraduate students performed well with computational problems, but struggled to solve mathematical problems that required deeper knowledge of mathematics. Accordingly, some study participants were engaged in PBL activities while others were exposed to traditional teaching. The latter group showed more success, especially when problems increased in rigor and complexity, as opposed to the control group that was taught via traditional methods. PBL-taught students showed better understanding of key concepts and a greater ability to apply strategies learnt.

2.5 PBL, Attitude, and Achievement

PBL is deemed effective in improving students’ attitudes towards mathematics with students typically preferring PBL to traditional methods of teaching (Köğce, Yıldız, Aydın, & Altındağ, 2009). According to Lave and Wenger (1991), Problem-Based Learning activities can be used to contextualize material for students, creating situated learning environments. The emphasis on context in situated learning allows students to master content by understanding the relationship between content, context, and the practical application of these elements in real life. When high school students were engaged in more hands-on, relevant activities, they performed better on exams, as well as positively changing their attitudes towards mathematics (Khan, Mehmood, Abbasi, & Khalil-Ur-Rehman, 2011).The association of relevant experiences with classroom problem solving can help students construct knowledge that makes sense to them, thereby enhancing their attitudes towards mathematics and enabling them to master math concepts (Lave & Wenger, 1991).

In their study on PBL and its effectiveness in raising students’ achievement in mathematics, Cotič and Zulian (2009) found that the chosen PBL challenges should be anchored to the students’ own lives in order to tap all their skills and abilities. In another study, Butler and Christofili (2014) redesigned the PBL activities to become more realistic and relevant, meaning that they were rendered more relatable. The favorable results generated by this study indicated significant improvement in both attitudes and comprehension of concepts.

Research suggests that there is a correlation between students' attitude towards mathematics and their achievement (Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014). According to their study, Azar and Mahmoudi (2014) asserted that when students' attitudes towards mathematics improve, their achievement in mathematics also improves. Negative attitudes towards mathematics cause anxiety, making it difficult for students to feel comfortable studying this subject, an impediment that in turn reflects a correspondingly lackluster performance on exams (Di Martino & Zan, 2011).

However, it is important to note that 'achievement' can be measured in various ways. For instance, Kalchman (2011) believed that success in mathematics requires that students understand the importance and utility of mathematics in everyday life. In Ma's (1997) meta-analysis, achievement is measured based on performance on standardized testing; however, Gordon (2013) argued that standardized exams do not necessarily reflect mathematical ability and mastery. Moreover, self-efficacy, which correlates with attitude (Azar & Mahmoudi, 2014), seems to be a stronger contributor to academic success in mathematics than anxiety and testing (Norwich, 1987).

2.6 Engagement and Achievement

Engagement, motivation, and active participation are all terms used interchangeably across the literature. Farooq & Shah (2008) highlighted the role of active participation, stating that more positive attitudes correlate with the level of engagement inside the classroom. This observation highlights the merits of the situated learning theory in terms of its ability to engage students inside the classroom context. This is achieved by creating meaningful learning experiences for the students and delivering content that is relatable to their daily lives. Thus, increased levels of engagement contribute to higher achievements as the students actively seek knowledge related to the subject on hand to solve relatable problems (Mandeville & Stoner, 2015; Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014).

Accordingly, it is crucial to view achievement as a main criterion in examining conceptual understanding of mathematics. This perception should focus on analyzing how PBL affects students' engagement and attitude, and how this translates to achievement. In order to create a favorable teaching environment by combining the teaching practice with the above conditions, the learning environment needs to be unconventional, with an emphasis on how the environment affects the learners’ attitudes an achievement levels.

2.7 The Learning Environment

Learning environments influence the relationship between attitude and achievement. Park and Choi (2013) discussed the differences between non-traditional and traditional learning environments. In their study, a group of students from South Korea participated in an active learning environment as opposed to others who were taught in a traditional classroom. Students who were grouped together and asked to work on different problems felt more empowered and developed more positive attitudes towards mathematics.

The same study noted that the physical placement or seating (including the method thereof) of students situated inside the classroom plays a major role in their motivation and participation inside the class. In traditional classrooms students felt disadvantaged by their respective positions within the seating plan, with the students sitting closer to the instructor feeling more empowered. On the other hand, students in active learning environments feel equally engaged. Another study, conducted in Ghana, made a case against traditional classroom settings; Asante (2012) found that the school environment and instruction method influenced the relationship between attitude and performance.

Furthermore, students who do show intent and interest in pursuing progressive STEM professions are more likely to risk being labeled as ‘nerds’, for example. For this reason, social support for those groups and, in particular, young women, is essential (Rice, Barth, Guadagno, Smith, & McCallum, 2013). Better learning results and an improvement in the teacher-student relation could be attained by the educators’ receptiveness to student input and their readiness to work out problems individually with students. An instructor who is willing to go the extra mile, and adjust his or her teaching to accommodate students’ individual learning obstacles, while keeping an open mind to feedback, criticism, and inspiration, embodies the defining characteristics of a good teacher: Being “caring” and “open-minded” were noted to achieve the best positive relationship and, thus results, with the students. Teacher confirmation includes teacher/student communication that validates the students' contributions, making them feel like valuable individuals in the community (Hsu, 2014).

Affirmation practices, which include reacting to students' inquiries and remarks, exhibiting enthusiasm towards the students' contributions, and utilizing intelligent teaching styles, are all conformational behaviors in the teacher's repertoire. Those methods have been shown to diminish anxiety on the part of the students and foster an intellectual learning environment for college teachers (Ellis 2000, 2004). The more affirmation students receive from their educator, the less they dread their involvement in class. With a reduced anxiety threshold, students are better able to grasp the course material (Preiss, Wheeless, & Allen 1990). Moreover, they are also inclined to develop a positive attitude towards both their teacher and the course (Chesebro & McCroskey, 2001). Teacher confirmation also resulted in better performance, lower stress levels, and higher motivation (Goodboy & Myers, 2008; Schrodt, Turman, & Soliz, 2006).

Similarly, teacher's perception can positively affect students' attitudes towards learning. In a study conducted in a Mid-Atlantic university, 232 students participated in a communication course, with a focus on the importance of the instructor-student and student-student relationship in the classroom. Students in this study disclosed their view of relational connections in the classroom and their apparent interest in learning. The results suggest that student-instructor and student-student relationships are directly related to the extent of their connectedness in the classroom. Only the student-teacher relationship directly foresaw anticipation, eagerness to learn, and subjective learning (Rice, Barth, Guadagno, Smith, & McCallum, 2013).

As previously mentioned, Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar (2014) asserted that motivation inside the classroom leads to higher achievements in mathematics. However, the authors contributed motivation to the presence of a healthy learning environment. Other authors related higher motivation and participation inside the classroom to attitude. This is an integral part of creating a healthy learning environment, one that is physically, emotionally, and intellectually safe, allowing the students to participate and express themselves.

2.8 Model – Eliciting Activities (MEAs)

MEAs have also been considered a means of improving students' attitudes toward mathematics by helping them acquire the problem-solving skills necessary for success in mathematics (Ashmann, Zawojewski, & Bowman, 2006). Based on real-world problems, MEAs enhance students' perceived usability of content, as well as their engagement with it, fostering more frequent and more likely learning of necessary mathematical skills (Magiera & Zawojewski, 2011). When students' mathematics skills improve, their mathematic achievements are likely to show a parallel improvement. Moreover, students' attitudes towards mathematics are influenced by mathematics achievement (Martin, Cirino, Barnes, Ewing-Cobbs, Fuchs, Stuebing & Fletcher, 2012). Students' attitudes towards mathematics are also a predictor of students' perceptions regarding the difficulty of mathematics. This means that students who improve their mathematics skills using

MEAs will likely experience fewer difficulties with mathematics than peers who are taught with traditional methods (Martin, Cirino, Barnes, Ewing-Cobbs, Fuchs, Stuebing & Fletcher, 2012).

Ashmann, Zawojewski, and Bowman (2006) described a successful activity as one in which mathematical thinking is not separated from mathematical concepts and skills, thus capturing students' curiosity by allowing for more than one reasonable solution. This latter factor is seen as the essence of professional work in integrated mathematics fields, such as engineering. Teachers' roles with MEAs should therefore focus on developing tasks and activities that are likely to promote the development of students' understandings of concepts and procedures in a way that also fosters students' abilities to solve problems, to reason, and to communicate in a scientific and mathematical manner.

During MEAs, students have several opportunities to expand their understanding and experience successes, increasing the likelihood of developing a proactive approach to problem solving and the content presented in the problem by developing skills and strategies to solve real-world problems that they can relate to (Diefes-Dux, Hjalmarson, Miller, and Lesh, 2008; Hestenes, 1987; Hjalmarson, 2008; Johnson & Lesh, 2003; Nersessian, 2009; Redish & Smith, 2008). MEAs also offer students a chance to step back from traditional education using institutional books and national exams. Moreover, MEAs can also provide students with opportunities to discover the applicability of mathematics and/or science concepts that may be embedded in a given task as they grapple with real-world problems. This is particularly useful since the latter are often neither inherently simple nor decontextualized, rendering them similar to problems in traditionally-taught teacher-centered classes. Accordingly, using MEAs help instructors avoid the pitfalls of following a fragmentation approach to mathematics instruction, characteristic of educational institutions and the selection of knowledge in school curricula (Britzman, 1991). Through MEAs, students are given the opportunity to practice metacognition (Dark, 2003) and to reflect on their work as a means of assessing their own understanding, while developing: analogy making, abstraction, visualization and simulative imagining, deductive reasoning, inductive reasoning, and inferential reasoning (Nersessian, 2008).

Chapter Three: Research Methodology and Methods

3.1 Introduction

The proposed study examines the effect of PBL activities within a situated learning environment on students' attitudes towards mathematics. The study was carried out in the context of a private university in Cairo, Egypt. The aim of the "Fun with Problem Solving" course at the American University in Cairo is to foster a conceptual understanding of various aspects in mathematics.

First, the research design and data collection are discussed, as there are different means of data collection for different modes of data. Second, the research context will be explained, as the research questions are context-specific. Third, the participant sample will be discussed to justify the reason why participants were selected for interviews. Next, the analysis of the quantitative and qualitative findings will be discussed. Finally, the purpose of each mode of data collected will be discussed, and discuss the purpose it serves.

3.2 Research Design and Data Collection

This research aims to answer two questions: 1. What is the impact of a Problem-Based Learning approach on the attitude of non-science undergraduate students towards mathematics in an Egyptian university? 2. What is the impact of a Problem-Based Learning approach on students' conceptual understanding of mathematics in an Egyptian university?

To answer the research questions a convergent parallel mixed design (Creswell,

2007) will be used. This is a design in which different sets of data are collected and analyzed independently, the results of which are subsequently aligned. The aim of using this research design is to: 1) identify similarities between the datasets as a means of triangulation; 2) identify differences between the datasets or disconfirmation; and, 3) identify a combination of similarities or differences to expand on the research and obtain more data to discuss any discrepancies in the findings (Creswell, 2009; Morgan, 1998). Moreover, the integration of independent data can address any biases that might occur with each separate research design. While quantitative data can be used as a means of nomothetic generalizations, transcending context and time (Reale, 2014), the impact of a single course on non-science undergraduate students' attitudes towards mathematics cannot be reduced to descriptive statistics. Similarly, relying solely on qualitative data will not offer the necessary pre-post comparative dimensions that quantitative data can provide.

3.3 Examining the Research Context

The University. This study took place in a course taught at The American University in Cairo (AUC), a private university located in Cairo's suburbs. All of the student participants in this course were required to take core curriculum courses, such as philosophical thinking, scientific thinking, and humanities, as well as various science and mathematics courses. Despite the availability of several kinds of scholarships offered as well as different forms of financial aid, the high tuition fees of AUC mean that most of the students at the university come from high socioeconomic status (SES) families. While public universities, such as Cairo University, can accommodate up to 240,000 undergraduates (“CU on spot,” 2010), AUC hosts around 5,000 students ("Student enrolment statistics," 2012). The language of instruction at AUC is English, and students must meet English proficiency requirements for admission. According to several university ranking agencies, AUC ranks highly in Egypt, with some of its programs achieving top ranking in Africa and at a global level (Rashed, 2014). Class size is usually fewer than twenty students per class, with a student to faculty ratio of 12:1. The tuition and fees at the American University in Cairo can reach up to $19,000 per year, which is considered very high in comparison to the almost free tuition offered in the Egyptian public universities.

Participant Sample. Twenty-one non-science-major students enrolled in the Fun with Problem Solving course at the American University of Cairo, 17 of which are female. However, only 17 students responded to the pre- and post- survey given to them, 14 of which are female. This course is a freshman level science, offered for non-science students. The students’ majors were Business Administration (12 students), Mass Communication (4 students), Political Science (3 students), and Psychology (2 students). I was the instructor of the course, and the students signed consent forms to approve their participation in the study. The consent form explicitly stated that students’ grades will not be affected whether they participated or not, and their identities will remain anonymous throughout. Despite being non-science students, the students were still required to take a science course as part of their core curriculum. There are several course offerings to choose from, and the participants willingly joined this course and signed consent forms to participate in this research. The students were grouped throughout the semester in 5 groups of 4 (and one group of 5), but for the final project, the students were divided into seven groups of three students. The groups remained the same until the final project, in which the groups changed.

The Class and Course Structure. During the course, the students were taught mathematics in a PBL environment. Throughout the course's activities, the class was arranged according to five groupings, except for the final project in which they were divided into 7 groups. These groups were chosen based on students' majors, where different majors were grouped together, in an attempt to diversify the groups as much as possible. However, by the final project, the groups have changed to get students to work with others they had not worked with previously, and the students were divided into more groups.

At the beginning of the course, students were asked to work on simpler MEAs, allowing them to become familiar with the open-ended nature of the problems. Basically, the simpler MEAs were designed for middle and high school students, and addressed topics such as arithmetic means, and measurement and data. Such activities were designed to promote students' problem-solving skills, while also raising their awareness on how different mathematical concepts could address different mathematical problems. The mathematical concepts this course addressed were primarily statistical reasoning and algebraic thinking. Moreover, students were asked to develop mathematical models based on their prior knowledge, and to solve relatable model-eliciting activities specifically designed to them.

The activities presented to the students were open-ended, and did not prescribe mathematical concepts/models for the students to use. Different groups were expected to use different methods to tackle the challenging assignments given by the instructor, as the activities were open-ended and the students were not told what concepts they need to use, leaving their approach to solve the problems open to their own interpretation. This was done by shifting the instructor's focus from dispensing information to acting as a facilitator of learning by asking open-ended questions and mentoring the students. The goal of this method was to provide the students with more room for exploration and creativity by making them take charge of their own learning.

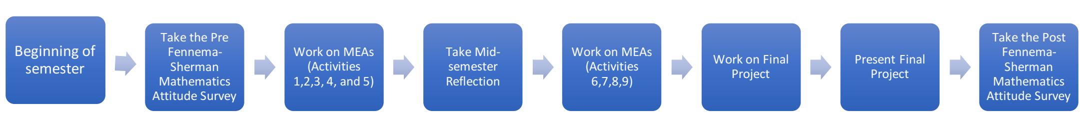
The students were assessed based on their thinking process and their demonstration of mathematical concepts mastery as per the Lesh Translation Model. Students' responses were mapped out using the Lesh Translation Model, and the frequency of translations from one representation to the other was taken into consideration, as well as the quality of the translation as per a rubric designed specifically for this study. The rubric describes how each of the translations can be mapped out to itself, or other translations, and how the strength of the arrows that connect the different translations can be evaluated. Each of the activities the students worked on had several data inputs, such as written work, verbal interactions, and presentations; and all the data inputs were used to map out their performance using Lesh Translation Model, which was used to assess students' progress throughout the semester.

This was an attendance-based class, dispensing the need of including conventional midterm and final exams. However, students were given reflective questions, to which they had to write about what they learned from each activity. The students were informed that these reflections would not affect their grades, giving them the opportunity to express themselves in a more creative and spontaneous manner.

Having practiced in several PBL sessions, where the students engaged in "simpler" MEAs, the students were given the opportunity to pick a problem they encountered in their everyday lives called the "Grand Challenge". The students were asked to research, solve, and present solutions to their respective Grand Challenges by the end of the semester. Solving relatable, Egypt-centric problems using mathematics gave students the opportunity to experience the relevance of mathematics in their day to day lives. Moreover, the course exposed students to real data about problems in Egypt and empowered them to believe that such problems were indeed solvable. The order of the activities involved in this course are described as per the sequence below.

Figure 2

Breakdown of Semester Activities



3.4 Data Collection and Analyses

The following table (1) describes how the research questions are addressed by indicating the data sources, the type of data, the method of analysis for the respective data, and how the different types of data were triangulated, and each of these elements will be described in details in the following sections.

Table 1

Description of Data Sources, Types, Methods of Analysis, and Triangulation

Research Questions	Data Source	Type of Data	Method of Analysis	Triangulation
RQ 1: *What is the impact of a Problem-Based Learning approach on the attitude of non-science undergraduate students towards mathematics in an Egyptian university?*	Survey (pre- and post-)	Quantitative	*Time* Measure subscales, pre- and post- Measure total, pre- and post-	Identify similarities between the datasets, as a means of triangulation; Identify differences between the datasets or disconfirmation; Identify a combination of similarities or differences to expand on the research and

				obtain more data to discuss any findings' discrepancies
			Scale Measure contrast between subscales	
	Mid-semester reflection	Qualitative	Open coding Axial coding	
RQ 2: What is the impact of a Problem-Based Learning approach on students' conceptual	Students' artefacts (audio conversations and written	Qualitative	Lesh Translation Model	Align students' perception of their understanding towards mathematics with their progress in

understanding of mathematics in an Egyptian university?	work)			conceptual understanding as per the Lesh Translation Model
	Mid-semester reflection	Qualitative	Open coding Axial coding	

RQ1: Students' attitudes towards math. Data collection was conducted in two ways: at a quantitative level, the Fennema-Sherman (1976) mathematics attitude scale survey (Appendix A) was administered twice to the students, the first prior to the start of the semester, and the second after the semester ended (pre- and post-). The Fennema-Sherman mathematics attitude scale is a well-established instrument that measures students' attitudes through four different subscales, namely: confidence in mathematics, usability of mathematics, teacher perceptions towards students' performance, and perceiving mathematics as a male domain. This instrument was established in 1976, and has been a popular instrument to measure students' attitude towards mathematics in various studies ever since. The survey consists of 47 questions, which address the four subscales. The usability (U) subscale describes how students perceive mathematics as a usable field, one that would help them in everyday situations, as well as in their careers. The confidence (C) subscale describes how the students are confident in solving mathematics problems, and the teacher perception (T) subscale describes how students

perceive their teachers' perception of their abilities in mathematics. Mathematics as a male domain subscale (M) investigates whether students perceive mathematics as a male field, and questions how students perceives mathematics as a male domain.

Qualitatively, a mid-semester reflection task was assigned in which the students were required to answer five questions: What do you like about this course?; What do you hate about this course?; What mathematics concepts do you think you have learned in this course?; What mathematics concepts that were addressed in this course did you think you did not fully understand?; and, What mathematics concepts did you wish you had learnt in this course? The focus in this qualitative measurement was on the first two questions.

3.5 Quantitative Data

Following data collection, an analysis was carried out for each measurement method. Results from the Fennema-Sherman Scale were quantitatively analyzed using a matched pair t-test of each of the subscales pre- and post-, and the total of all scales pre- and post-. This method aimed to observe the difference in the means between the subscales as a subject of time: usability in mathematics, students' confidence in mathematics, perception of mathematics between mathematics students as being a male dominant field, teacher perception of student ability, as well as overall attitude towards mathematics. Reliability of quantitative data analysis relied on ensuring that each matched pair t-test for every scale was carried out independently for survey results from the one the students took in the beginning of the semester, and the one they took at the end of the semester. Another measure of contrast was conducted between the different subscales. This was used to give a clear idea of which scale provided significantly unique results.

3.6 Qualitative Data

Mid-semester reflections during the qualitative data collection phase were analyzed with a primary focus on the first two of the five questions, namely; what do you like about the course? And, what do you hate about the course? Open coding was done, where each response was labelled according to its content, after which axial coding was done to categorize the responses under more relevant themes.

The first research question was subjected to independent quantitative and qualitative analysis. As a means of triangulation, similarities between the datasets were identified. Disconfirmation between the datasets, and differences between the datasets, were identified. As a final step, a combination of similarities or differences was put forth. This was to allow expansion of the research, as well as for more data collection to properly address any discrepancies in the findings.

RQ 2: Students' Conceptual Understanding of Mathematics. Data collection for students' conceptual understanding of mathematics was performed qualitatively. Students' artefacts were collected throughout the semester. Students' artefacts were mainly students' written work, as well as written reflections, and presentations they prepared to present their solutions to the activities throughout the semester. The artefacts also included students' work out of the problems. Activities carried out at the beginning and middle of the semester, as well as their final projects were collected for the research, and an activity from the beginning of the semester, one from the middle of the semester, and one at the end of the semester were considered as data for the purpose of this study. Students' written work was collected, as well as audio recordings of students' verbal interactions while working on these activities. Audio recordings of students' verbal interactions were recorded using recorders, and they were recorded from the beginning of each session, for all activities. For the purpose of this study, only three activities were considered for the recordings. Mid-semester reflections included the same set of five questions previously mentioned, with an emphasis on the answers to the last three questions; namely, what mathematics concepts do you think you have learned in this course?; What mathematics concepts that were addressed in this course do you think you have not fully understood?; And, what mathematics concepts do you wish you can learn in this course?

Qualitative analysis of this data qualitatively consisted of two parts. Regarding the students' artefacts and work was described, and assessment of their presentations was conducted using the Lesh Translation Model. Students' artefacts were mapped according to the Lesh Translation model by evaluating written work, and listening to audio recordings of the students while working on their activities. Students' audio recordings were coded and analyzed, resulting in different themes.

As for mid-semester reflections of the three last questions from the list of five questions, both open coding and axial coding were implemented. Students' reflections were aligned with their progress in representing mathematics, in an attempt to validate their perception of how their progress developed

3.7 Structure of Class Assessments

In this course, students worked in groups and were accordingly assessed based on their discussions, written work, presentations, and active collaborative participation in the group. Accordingly, students were asked to work together to come up with clear and well-articulated solutions to the problems they were given. Students were evaluated, as a group on a weekly basis, on different aspects including on measures of conceptual understanding, procedural knowledge, group work, and presentation skills.

Students were given a weekly activity, usually MEAs, requiring them to present a mathematical solution to the given problems. These activities were open-ended, and could be solved using different mathematical concepts. The role of the instructor here was to guide the math concepts to be discussed in class by using students' arguments and presentations of solutions to introduce different mathematical concepts. The purpose of this discussion was to spur students' acknowledgment of using mathematics to solve real-life problems. Since these students came from non-science majors, they might not have necessarily known that their solutions were based on actual mathematical concepts, even if the rationale behind their solutions could be articulated using mathematics.

3.8 Ethical Considerations

Smith (1996) suggested four principles that researchers in education should take into consideration: 1) Demonstrate respect for each person as an individual; 2)

Communicate honestly and truthfully; 3) Enhance the self-esteem of others; and, 4) Help build fair and compassionate social and cultural systems that promote the common good of all persons. (p. 17). The suggested considerations are central to the academic and professional standards of the American University in Cairo, and these standards were taken into consideration in a strict and professional manner. The students were presented with a syllabus at the beginning of the semester citing students' responsibilities as students, as well as the instructor's academic and professional responsibilities towards them. The students understood the grading procedures, and were fully aware of how grades were distributed. Students who put forth the same efforts in the classroom received the same grades, whether or not they participated in the study.

Bournot-Trites and Belanger (2005) cited ethical considerations for researchers in education. Among the questions raised in their work, Bournot-Trites and Belanger (2005) argued that teacher-researchers might face the challenge of weighing whether the students will benefit more from the course, or whether their presence is considered merely as a means of data collection. To address this issue, this study adhered to the strict academic standards of the American University in Cairo, where students' progress and achievement is objectively measured without prejudice. While students may understand and acknowledge that they were research participants, they were also aware of their responsibility as students who strive for high grades. Students' grades were not affected by their participation in the study. Students' progress throughout the course and their conceptual understanding of the course deliverables were the instructor's main priority during the course. Students were given the opportunity to express any course-related concerns, which were not considered research data.

3.9 Limitations of the Study

Since the data will be obtained from a limited number of individuals, the findings cannot be generalized to a larger population. Repeating the study with another set of students is recommended to obtain more reliable results and consequently a more reliable analysis of the relationship between teaching style and students' attitudes towards the taught material. Also, the changes in the students' attitudes towards mathematics as a result of this course need to be compared to the changes occurring in response to other courses. This would serve to eliminate other factors that might have influenced the data obtained. Although choosing a convergent parallel mixed design would account for the limitation of relying solely on the qualitative data from a small sample, this method can also be limiting when trying to triangulate between different data (i.e. quantitative and qualitative), and can also be challenging in case of contradictions within the data (Creswell & Plano Clark, 2007).

Given the dual role of the researcher/instructor in this study, a certain amount of bias is inevitable (Russell & Kelly, 2002). Students understood that their instructor played this dual role, but were mainly interacting with the instructor as an instructor, and did not put into consideration how their work would have an impact on the research. Students were natural and perceived their participation in the class as learners rather than research participants. However, one way to make bias less prevalent is to collect multiple data sources and attempt to triangulate the findings. Collected concurrently, the data involved were both qualitative and quantitative, with data analysis of both sets conducted separately using parallel mixed methods design (Creswell & Plano Clark, 2007; 2011).

Chapter Four: Quantitative Findings of Students' Attitudes

The Fennema-Sherman Mathematics Attitude Scale measures student attitudes towards mathematics. The survey consists of four subscales, each designed to measure the following: confidence (C); math as a male domain (M); teacher perception (T); and, usefulness of mathematics (U), with a measurement scale out of five. Confidence (C) as a domain evaluates how participants feels when engaging in mathematics problems, and how confident they perceive themselves when doing such tasks. Usability of mathematics (U) is a field that is concerned with participants' feelings about the practical use of mathematics in their day-to-day life, and if it is generally applicable in a non-academic format. Math-as-a-male (M) domain focuses on gender concerns that mathematics is believed to be a generally male-driven field that tends to exclude women. The key aspect when considering this domain is that it transfers to other aspects such as choice of major (and, therefore, career); as such, this domain carries fundamental implications in the social dynamics governing the lives of women at work and in the domestic sphere. Mathematics-as-a-male-domain also shows that the teacher may have the ability to challenge cultural aspects characteristic of patriarchal societies by providing a favorable environment for women to excel in mathematics-based subjects. The potential of assessing social elements and teacher perception may offer a broader view of mathematics teachers as key societal drivers within a community.

The final aspect considered is teacher perception (T) relating to participants' performance in mathematics. The important point to consider is that teacher perception varies considerably as there are many dynamics that may shape students' perceptions of their teachers such as age, gender, and sense of humor, among others. These variables are significant, since no two classrooms are the same in the same way, and no two instructors are identical in terms of teaching methods and classroom environment. The confidence and usability of mathematics domains are also important to consider when it comes to students lacking in mathematics practice. This inexperience may affect how students view their own abilities, causing the students to steer clear of using mathematics in non-academic contexts. In essence, this undesirable outcome raises the issue of whether students with a non-mathematics-based field will ever have the capability and will to use mathematics in a broader range of real-life activities.

A paired t-test was conducted between each of the sum of the subscales, pre- and post, in addition to the total sum of all the subscales. Descriptive statistics (mean, median, standard deviation, range, skewness, kurtosis, standard error) are described in Table 2 for each subscale, pre- and post-. The purpose of this table is to show the differences between the different data, whether as a subject of scale, or a subject of time.

Table 2

Descriptive statistics of Fennema-Sherman Mathematics Attitude Subscales

Pre-Confidence

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
	17	43.29	10.27	46	44.53	4.45	15	53	38	-1.79	2.04	2.49

1

Pre-Math as a Male Domain

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	45.59	4.82	45	45.47	4.45	38	55	17	0.42	-1.09	1.17

Pre-Usability of Math

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	49.29	5.96	51	49.6	5.93	35	59	24	-0.54	-0.03	1.44

Pre-Teacher Perception

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	42.29	8.56	44	42.87	4.45	21	55	34	-1.16	0.61	2.08

Pre-Sum

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	180.47	22.92	182	182.47	17.79	117	214	97	-1.23	1.3	5.56

Post-Confidence

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	43.41	11.49	45	44.4	5.93	13	59	46	-1.24	0.94	2.79

Post-Math as a Male Domain

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	47.94	4.78	48	48	5.93	40	55	15	-0.05	-1.43	1.16

Post-Teacher Perception

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	45.24	6.31	46	45.07	2.97	33	60	27	0.01	0.31	1.53

Post-Usability

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	50.94	6.2	51	51.13	7.41	39	60	21	-0.23	-1.09	1.5

Post-Sum

VARS	N	MEAN	SD	MEDIAN	TRIMMED	MAD	MIN	MAX	RANGE	SKEW	KURTOSIS	SE
1	17	187.53	20.77	191	188.13	11.86	135	231	96	-0.45	0.81	5.04

Most of the subscales did not violate normality, as most of their median, mode, and mean were close to each other. However, the absolute value of the skewness in four of the t-tests was more than 1, suggesting a violation of normality. A closer look at these tests show that the skewness is negative and the standard deviation is relatively high, with some outliers towards the lower end of the tested subscale.

Matched pair t-tests were conducted between the subscales pre- and post- to indicate whether there was a change in how students responded to the survey before the semester started and after it ended. A summary of the t-test results between the difference subscales, pre- and post- are shown below in table 3.

Table 3

Summary of t-test Results Between Different Subscales, pre- and post-

T-test	**P-value**
Confidence	0.9104
Usability of Mathematics	0.0675
Mathematics-as-a-Male-Domain	0.008253**
Teacher Perception	0.0378*

Total Attitude	0.014 57*

**p<0.05 **p < 0.01*

According to table 3, there was no statistical significance in students' confidence in mathematics from the beginning of the semester to end of the semester, nor how students perceived usability of mathematics. However, there was a statistical significance of students' perception of mathematics as a male domain, and how they perceived teacher's perception towards their achievement in mathematics. The (M) and (T) subscales might have contributed to students' overall attitude towards mathematics, which had a statistically significant increase from the beginning of the semester to the end of the semester. The following sections take a closer look at the individual subscales, which upon further analysis, using effect size, give more insight as to how significant the p-values are.

4.1 Confidence in Math

Results from a matched pair t-test indicated that students' confidence in mathematics, according to the Fenema-Sherman Attitude Scale, before the semester started (M = 43.29, SD = 10.27, N = 17) was not appreciably different to students' confidence in mathematics after the semester ended (M = 43.41, SD = 211.49, N = 17), t(16) = -0.11438, The mean of the differences is negligible, and the p value is 0.9104; which is not statistically significant where alpha = 0.05. The 95% confidence interval around difference between the group means was relatively precise (-2.298120 to 2.062826). The effect size was 0.010, which is considered extremely low, suggesting that there is no statistical significance between the mean of the pre- and post-; in fact, they are almost identical. That is, students' confidence in mathematics did not change whatsoever as a result of the course.

4.2 Usability

Results from a matched pair t-test indicated that students' perception of mathematics usability before the semester started ($M = 49.29$, $SD = 5.96$, $N = 17$) was not significantly lower than students' attitude towards usability after the semester ended ($M = 50.94$, $SD = 6.2$, $N = 17$), $t(16) = -1.961$, $p < .1$, two-tailed. The mean of the differences is 2.941176, and the p value is 0.0675; which is not statistically significant where alpha = 0.05. The 95% confidence interval around difference between the group means was relatively precise (-3.4275898 to 0.1334721).

The p value is slightly higher than alpha = 0.05. While this could be considered significant, as $n=17$ only, the effect size is only 0.270, suggesting that the change in means might not necessarily be as significant as what the p-value suggests. However, we can say it is significant for alpha = 0.1, and would merit further investigation.

4.3 Teacher Perception

Results from a matched pair t-test indicated that students' attitudes towards mathematics, based on their teachers' perception towards their abilities, before the semester started ($M = 42.29$, $SD = 8.56$, $N = 17$) was significantly lower than students' total attitude after the semester ended ($M = 45.24$, $SD = 6.31$, $N = 17$), $t(16) = -2.2643$, $p < .05$, two-tailed. The mean of the differences is 2.941176, and the p value is 0.0378;

which is statistically significant where alpha = 0.05. The 95% confidence interval around difference between the group means was relatively precise (-5.6948378 to -0.1875152). The effect size is 0.391, which is relatively higher than the previous individual subscales, but is approaching a medium effect size (which is around 0.5). Accordingly, further investigation might be needed, but the results can be explained in light of Egypt's perception as a communal/collective society, in which interpersonal relationships can be important for the students, potentially contributing to students' aptitude for learning and performance.

4.4 Math as a Male Domain

Results from a matched pair t-test indicated that students' perception of mathematics as a male domain before the semester started (M = 45.59, SD = 4.82, N = 17) was significantly lower than students' total attitude after the semester ended (M = 47.94, SD = 4.78, N = 17), $t(16) = -3.013$, $p < .005$, two-tailed. The mean of the differences is 2.352941, and the p value is 0.008253; which is statistically significant where alpha = 0.05. The 95% confidence interval around difference between the group means was relatively precise (-4.0084523 to -0.6974301). The effect size is 0.490, which can be considered a medium effect size with N=17, and shows that the difference in the means did change considerably towards the higher end of the scale. This means that the students did indeed perceive mathematics as a male domain before the class started, but this notion seems to have changed as the course progressed. Students markedly changed their perception that mathematics is a male domain, enabling them to recognize the usability of mathematics as a real-life skill that is accessible to both genders.

4.5 Total Attitude

Results from a matched pair t-test indicated that students' total attitude towards mathematics, according to the Fenema-Sherman Attitude Scale, before the semester started ($M = 180.47$, $SD = 22.92$, $N = 17$) was significantly lower than students' total attitude after the semester ended ($M = 187.53$, $SD = 20.77$, $N = 17$), $t(16) = 2.7387$, $p < .05$, two-tailed. The mean of the differences is 7.058824, and the p value is 0.01457; which is statistically significant where alpha = 0.05. The 95% confidence interval around difference between the groups' means was not relatively precise (1.594908 to 12.522740), with an effect size of 0.322. While these results could be considered significant given that n=17, and the relatively short duration of the course (12 weeks and 3 hours of class time per week), the effect size is small, indicating that the results may not be as significant as the p-value suggests.

4.5 Further Analysis

There is evidence that the data is reliable according to Cronbach's Alpha, which is shown in Table 4. Basically, the table shows how the different subscales, pre- and post-, were answered consistently by the students. The analysis was conducted for all subscales, resulting in 10 Cronbach Alpha for each of the subscales, pre and post.

Table 4

Reliability of Data according to Cronbach's Alpha

Items	Cronbach Alpha
All Items	0.8944
Pre- Confidence Excluded	0.8704
Pre- Math-as-a-male Domain Excluded	0.9047
Pre- Teachers' Perception Excluded	0.8719
Pre- Usability Excluded	0.8904
Pre- Total Excluded	0.8736
Post- Confidence Excluded	0.8633
Post- Math-as-a-male Domain Excluded	0.9067
Post- Teachers' Perception Excluded	0.8826
Post- Usability Excluded	0.8899
Post- Total Excluded	0.867

The results presented by the Cronbach Alpha table attest to the reliability of the results in the pre-and post-evaluation of the subscales. The purpose of the table is to assess the consistencies and inconsistencies of the results per individual participant. The values presented here show that the participants displayed similar modes of answering each question, indicating that the answers were not generated in a random fashion. The results are significant in that the subscales indicate that the students' responses were consistent throughout.

A separate MANOVA test was conducted to determine the significance of the study, and whether results varied over time, or between scales. MANOVA output indicates a main effect for scale, but no interaction between time and scale; therefore, a contrast analysis for scale was conducted to determine which scales differed. Table 5 shows how the significance of time and scale, as variables of the survey analysis, and the lack of significance between their interaction.

Table 5

Type III Repeated Measures MANOVA Tests: Pillai test Statistic

	df	*test stat*	*approx f*	*num df*	*den df*	*pr(>f)*
Intercept	1	0.98758	1271.94		1	16 < 2.2e-16 ***
Time	1	0.34947	8.60	1	16	0.009774 **
Scale	1	0.64728	8.56	1	14	0.001774 **
Time:Scale	1	0.23348	1.42	3	14	0.278267
*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*						

Table 5 suggests that the survey results change as subject of time, and as a subject of subscale, the contrast amongst the subscales is important to consider as one specific subscale may perceived differently than the others. According to Table 6, there was a statistical significance between the subscales. While the t-tests showed that (M) and (T) scales were statistically significant as a subject of time, the MANOVA tests describes how the subscales vary.

Table 6

Type III Repeated Measures MANOVA Tests: Pillai test statistic

Contrast	*Df*	*approx f*	*pr(>f)*
M vs U	(1, 16)	1.079	0.314
C vs T	(1, 16)	3.284	0.177
M&U vs. C&T	(1, 16)	22.5672	0.000652 ***
c = confidence m = math-as-male t = teacher perception u = usability			
*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*			

Contrast results indicated that (M) subscale cannot be distinguished from the (U) subscale, and (C) cannot be distinguished from the (T) subscale. Matched pairs t-test indicated that change from pretest to posttest was not statistically significant for (C) and (U), but was statistically significant for (M) and (T). Since there was a statistical siginifance in the contrast analysis of (M) and (U) versus (T) and (C), suggesting that there was a statistically significant change in subscales (M) and (T). This may suggest that students' attitudes as a whole is affected negatively by how their teachers perceive them, even if they have confidence, find mathematics useful, and do not view it as a male-domain, reiterating the idea that the teacher fulfils a pivotal role in the classroom with regards to the attitude students bring into a classroom. Thus, it is important to assess the contrast between the subscales given the possibility of strengthening the significance of a certain subscale by using another. The results show significance of teacher perception when considering the attitudes of students towards mathematics. When the subscales of (U), (C), and (M) are assessed versus (T), it becomes clear that the teacher plays a notable role in how the students perceive mathematics. In the case of usability, there is slightly less significance compared to the other subscales, but in the post evaluation the results suggest that through the teacher's approach the students were able to recognize and value the usability of mathematics. The indication here is that teachers in the classroom exert power by triggering the social potential within students, as evident in the confidence segment of the results**.** Student motivation, therefore, is driven by techniques and innovative methods used by teachers to increase interest in the subject.

4.6 Conclusion

According to the p-value of the matched pairs t test, subscales (M) and (T) indicated statistical significance as a subject of time, contributing to the statistical significance of the overall attitude (calculated as the sum of the subscales) towards mathematics as a subject of time. While (M) and (T) had a relatively higher effect size than (C), (U), and overall attitude, the small sample size ($N=17$) could have contributed to the small effect size of the subscales, suggesting that further investigation and larger sample sizes are needed. However, the contrast analysis did suggest that (T) is considerably lower than (M), (U), and (C) as a subscale, suggesting that further research should address the impact teachers' perception has on students' attitude towards mathematics. Moreover, further study is needed to address the change in female students' perception of mathematics as a male domain as a result of problem-based learning in mathematics, as the majority of the participants were female. The (M) subscale changed the most throughout the semester, with a statistically significant p-value and a relatively higher effect size, suggesting that the difference in the means should be taken into consideration for further investigation.

Chapter Five: Qualitative Findings on Students' Attitudes

As mentioned earlier, the Fennema-Sherman Mathematics Attitude Scales offers a numerically-based approach in summarizing students' responses regarding course perception. In this section of the study, open-ended reflection opportunities were given to the students. They were asked five questions related to different features and their sentiments towards the course. The questions were simple and gave students the chance to evaluate the course without any constraints. The five questions were:

1. What do you like about the course?
2. What do you hate about the course?
3. What mathematics concepts do you think you have learned in this course?
4. What mathematics concepts that were addressed in this course you do not think you have fully understood?
5. What mathematics concepts you wish you can learn in this course?

The simple nature of these five questions primed the students for expressing many insightful thoughts and opinions about the course. For the purpose of this chapter, only the first two questions will be considered and analyzed. The other questions will be address the research question 2, and will be addressed in Chapter 6. The answers in the reflections reveal more depth and a wider scope for unpacking the perception of students towards mathematics through the course experience. Based on the students' answers, these insights fall into academic and non-academic factors, including detailed subcategories, that the students believed have been influenced by the course. Table 7 shows the different themes that emerged from the students' answers, which are described under the "*Axial Coding"* column.

Table 7

Coding of Qualitative Data

Open Coding	Axial Coding	Category
Nature of concepts	Content	Academic
Used nature of the problems presented		
Nature of thinking skills needed for this course		
Applicability of concepts		
Frequency of reflections needed to be written	Course requirements	
Lack of a perfect final answer		
Flexibility of the professor	Relationships	Non-Academic
Instructor-student relationship		
Nature of the course itself	Classroom setting	
Environment of the course		

Time of class

5.1 Academic Factors

Nature of Concepts Used. Students were intrigued by the use of different mathematical concepts throughout the class. This means that students' feelings of surprise at their own capacity for enjoying basic mathematics, albeit in a different light, suggest that the kind of mathematics they were exposed to throughout their formal education has fallen short of engaging them to develop positive attitudes towards this subject. The comments below reflect how a number of students perceived the nature of concepts used throughout the class. Students expressed through the mid-semester reflection exercise the following:

Student 1: "I liked that I learned new concepts in mathematics that I didn't think I'd need"

Student 2: "I liked that there were more conceptual ideas beyond step-by-step procedures in mathematics"

Student 3: "I liked that I was able to see mathematics in a different light. All my other courses were focusing on certain concepts without any relevance"

Student 4: "I felt engaged when solving the problems because they were fun"

The assumption that mathematics is not a “needed” subject as much as it is merely an academic requirement appeared to apply to the students attending this course. Accordingly, their attitude towards mathematics changed because they came to perceive it as both enjoyable and relevant. Moreover, students appreciated the fact that they did not have to go through the problems in a methodical fashion, exploring instead different ways in which they can solve mathematics. A number of students described how much they enjoyed doing mathematics, with nearly a quarter of the 21 students mentioning that the fun aspect was due to the way mathematics was presented to them. This indicates that course design is essential in encouraging students to develop a positive attitude towards mathematics, as evident in the reflections of these students.

Nature of the Problems Presented. One of the main reasons for students to develop negative attitudes towards a certain subject stems from assigning problems that are not within their Zone of Proximal Development (Siyepu, 2013). That is, students can feel overwhelmed when asked to solve problems that they find too challenging, and this is something they expressed in their reflections.

Student 3: “They can be really challenging sometimes”

Student 4: “The problems take a lot of time to solve”

Student 5: “Pushes me to think hard”

Student 6: “I would like to know the right concept to be use in each activity”

Four students said that the nature of the problems in this course was something they did not enjoy or like. For instance, one student said that the problems can be “very challenging sometimes”, which made him feel uncomfortable. He felt that finding the problems challenging did not make him feel good about himself, and that he was upset that he was not on the same level as others. Another student complained that the problems took a lot of time to solve, a complaint based on the fact that she had to come up with many different ways to come up with the final solution. In a similar vein, another student expressed concern that she did not like the requirement to think so hard, especially given the timing of the course at the end of the day, another aspect marring the students’ enjoyment of the course. Since the difficulties experienced by the students may have warped their mindset regarding the problems, creating less ambitious milestones in the problems may elicit positive feedback for the students to proceed with the task of solving the bigger problems. This dilemma gives rise to the challenging task of designing problems that appeal to both excelling and struggling students at the same time. The format of including milestones may encourage motivation in the students, thus avoiding the frustration of feeling left behind in the tasks. An example of the type of support expected from teachers to students is mentioned in one reflection where the student “would like to know the right concept to be use in each activity.” The idea here is that before the activity begins, the main guidelines and mathematical subjects should be clarified, so that the students can solve the problems a little more easily. However, this idea may clash with other aspects of the course that require problem solving abilities and critical thinking to follow patterns independently achieved by the students. The way the course is designed may consider a group or person to person basis where the teacher assesses the necessary support needed for each case in order not to segregate certain students based on ability.

Nature of Thinking Skills Needed for this Course. This course is called "Fun with Problem Solving". When discussing a suitable title for the course with the department, they favored giving it a catchy title. As a result, the students who joined this course had expectations that it would be fun, and would also address problem-solving skills. While they knew from the outset that this was a mathematics course, and sometimes even mistook its name for "Fun with Numbers", students taking this course acknowledged the need for skills different than those conventionally linked to mathematics.

> *Student 2: "I liked that I used critical thinking more than math concepts"*
>
> *Student 4: "I had to think outside the box a lot, which is not something common in other courses"*
>
> *Student 7: "I had to think of innovative ways to solve problems, which was a lot of fun"*
>
> *Student 8: "I was exposed to different ways to solve a problem, which was something I am not used to, but I liked that a lot"*

One student acknowledged the need for critical thinking. The fact that they said they enjoyed the class just because they were using these different skills suggests that they were not given such opportunities elsewhere. One student even stated that he liked that he “had to think outside the box a lot, which is something not common in other courses”. This suggests that this student is able, willing, and positively inclined towards thinking independently, despite never having been given enough opportunity to do so. Another student expressed how much fun she has when she has to think of “innovative ways to solve problems”, also suggesting that content is not what usually puts people off from a subject like mathematics; rather, it is the method employed in teaching this subject.

Another example of how students perceived problem-solving was given when a student stated that she liked how she was “exposed to different ways to solve a problem”. In Egyptian classrooms, students are told there is only one answer to a problem, and only one way to reach this solution (Hargreaves, 1997). Four students mentioned the positive feelings they developed towards mathematics based on the skills they thought the course helped them develop. The enjoyment provided in this course shows that the way the course is designed is the core reason for student progress rather than the problems themselves.

Applicability of Concepts. As stated in the literature review, students are more likely to enjoy mathematics if they were given the opportunity to see how applicable it is in their everyday life (Azar & Mahmoudi, 2014). As research on Egyptian classrooms shows, mathematics in formal education, especially high school, is based on rote learning. Students are expected to memorize certain formulas and different patterns of questions, and are asked to solve these problems as is, devoid of any meaning or relevance. This might cause stress among students, eventually leading students to develop a negative attitude towards mathematics. Students expressed their relief from having to study mathematics the way they were used to throughout their formal education years, and some of them even said that they started to “enjoy” mathematics.

> St*udent 1: “I was glad that I was able to use maths concepts to solve real world problems”*
>
> *Student 6: “I started thinking more mathematically. For example, I think of time and money as maths problems rather than take them for granted”*
>
> *Student 9: “Now I can think of maths when I cross the street”*
>
> *Student 10: “I liked how we use maths to evaluate a short film”*
>
> *Student 11: “It was nice to see that maths can be used in different situations”*
>
> *Student 12: “I liked the feeling of achievement I got when I solved problems that were related to various mathematical concepts”*
>
> *Student 13: “I felt more comfortable with using maths because I saw how it can be used in real life”*
>
> *Student 14: “The problems presented in class were not the usual boring maths concepts used in other courses”*
>
> *Student 15: “Now I understand the importance of maths in life because it is connected to everything in everyday life”*

Student 16: "It gave me mathematical perspective to real life"

Students enjoyed the opportunity of applying mathematical concepts to solve real-life situations. They expressed how they were able to see different situations from a mathematical perspective, even contexts portrayed through the medium of cinema. Moreover, they were able to think of different aspects of everyday life (money, time, crossing the street) as mathematics problems. Moreover, students expected mathematics to be boring, and were surprised that it can actually be engaging provided it is meaningfully connected to their everyday experiences. Significantly, one student mentioned the word "comfortable", which suggests that mathematics is indeed a source of anxiety for many students; it also implies that working with it to solve everyday problems provided another lens that views mathematics in a less stressful light. Another student used the word "achievement", which suggests that students might have not enjoyed mathematics because they were previously judged as underachievers incapable of ever shining in this area. This 'labeling' may plant the idea that mathematics is too difficult for them to enjoy. Ten students expressed feelings that they liked the course because they were given the chance to see how mathematics can be applicable rather than abstract or redundant.

The interesting outcome in the results that was quite apparent in both the reflection segment and the survey is that the reflections placed significance on the real world applications; survey results, however, show that there is no significance in the usability of math. This discrepancy shows that there is a slight difference between how the students responded to the survey and the thoughts expressed in the reflection segments of the study. The p-value of the usability of the math subscale was 0.06753, suggesting that there was a potential significance in the course's ability to encourage students to use mathematics beyond the classroom. This combination supports students' reflections that reveal they recognize the value of solving problems with a real world application. As seen in the reflections, the perspective of mathematics has shifted with some students expressing the belief that they will or have actually used mathematics beyond the classroom.

Frequency of Reflections Needed to be Written. Reflection is important for students to develop metacognitive skills. Accordingly, students in this course are asked to write reflections after each completed activity. These reflections are read by the instructor, and are taken into consideration in mapping out future classes. However, some students expressed their dislike of the frequency of these reflections.

Student 6: "we have to write a lot of reflections"

Student 7: "I don't see what's the use of reflections"

Student 12: "I would have liked if we saw what others write in their reflections"

Overall, students considered it a burden to write down reflections. This difficulty may be due to several reasons: 1. Students view reflections as just an extra workload; 2. They cannot see the point, and have only a sketchy idea of what metacognitive skills are; 3. They do not feel that these reflections are taken seriously by the instructor, as they are never discussed in class. Moreover, some students were curious as to how others reflected, citing that it would be interesting if others can share how they felt, and the experiences they had. Three students said that reflection assignments were the least liked component in the course.

Students who have experienced mathematics as an absolutist field may find refuge in the fact that it can be considered as a perfect field with perfect solutions and right answers (Rippin, Booth, Bowie, & Jordan, 2002). This means that they actually appreciate the fact that there is a final answer that suits the question perfectly, and makes absolute sense. However, mathematics can also be considered a medium to develop other skills, and does not necessarily perfect respective concepts (Bybee, 2013). While research suggests classrooms in which uncertainty is accepted and acknowledged can foster learning and critical thinking (Jordan, Cheng, Schallert, Song, Lee, & Park, 2014), some students can find it unsettling not to be given a perfect model answer, especially if they were taught that there is always a perfect answer in mathematics classes. Such discomfort was expressed by the following students:

> *Student 17: "I am a business student, and I am used to final answers. I didn't like that I wasn't given the perfect answer in the end"*
>
> *Student 18: "There is no precision in the answers given, and the professor tells us it's ok to have answers different than other groups"*

Student 19: "I wish my work can be evaluated on its quality rather than be always accepted as is"

Three students did not like the uncertainty accompanying the problems they were given. One of them is a business student, who "is used to final answers". Accordingly, she was uncomfortable with questions that were open-ended, and could be solved in different ways. Another student wanted more feedback on whether her answer is right or wrong, as she could not see the sense in having more than one answer that could address the same question.

5.2 Non-academic Factors

Flexibility of the Professor. More often than not, university students find courses stressful and demotivating due to the nature of instruction in the course (Roorda, Koomen, Spilt, & Oort, 2011). They usually study for the grades, and are not given much say on how they should be assessed, or how the policies will affect them. Accordingly, they do not perceive the class as a learning environment, but rather an obligation to satisfy the pressure they receive from their parents or the society (Mata, Monteiro, & Peixoto, 2012). Accordingly, it is no surprise that some students feel more ownership towards their learning if they were given a say on how assessments will take place, and what kind of policies dictate the class, and they expressed that in their reflections.

Student 13: "I liked that we were learning for the sake of learning, because everyone will get an A as long as they do the work. It made me more comfortable because I don't have to stress out about the grades"

Student 20: "I liked that the professor took our opinion on how we're graded and the policies of the class such as attendance"

Student 21: "I liked that he never gave us more work than what we need to finish inside the classroom"

Students appreciated the lack of stress in the course, as they did not have to worry about how they will be evaluated, a sentiment shared by the students throughout. Moreover, they expressed how they were listened to when they requested doing all the work in the class, rather than being assigned a bulk of homework and miscellaneous assignments. Hands-on classwork was seen as providing students with the opportunity to think about learning rather than obsessing about the grade. Three students attributed course enjoyment to their empowerment by the professor, and how they did not have to feel stressed out about either the content or the workload.

Nature of the Course Itself. The nature of the learning environment can affect students' attitude towards the course or the subject itself (Shan, Li, Shi, Wang, & Cai, 2014). Accordingly, it is important to provide students with a safe and healthy learning environment to foster a positive attitude towards a given subject. In typical Egyptian classrooms, students are expected to sit facing the instructor, passively receiving information without participating actively in developing and constructing the content of the course. This is why any opportunity for change can spark students' interest, encouraging them to participate actively in a learning environment that focuses on effort and engagement.

> *Student 3: "I like the group work"*
>
> *Student 5: "Made me work with others that I wouldn't have interacted with otherwise"*
>
> *Student 9: "I like that I am not judged on how good I am in math, but I was given the opportunity to make mistakes and it was ok"*
>
> *Student 14: "I liked that I don't have to worry about grades or assignments as long as I attend and do all the work in the class"*
>
> *Student 18: "The workload is light"*

Students in Egyptian classrooms are not used to working in groups; instead, the emphasis is on individual work where students sit quietly at a desk facing the front. It is no surprise, therefore, that students should think of group work as something novel, and claim it is a good reason to like the course. This point was mentioned by another student who stated how much she loved working with others she would not have ordinarily worked with had it not been for the classroom setting and switching of groups for different activities.

Another theme that emerged was the notion that students felt comfortable making mistakes, and that the demands of the courses did not stress them out, reflecting the low anxiety levels in this particular learning environment. One student mentioned how she did not want to be judged on how she performed; she also expressed appreciation of the opportunities to learn from her mistakes without worrying whether these mistakes would affect her grade. She expressed how learning made more sense that way, and that she looked forward to this class as a result.

While students had to work on a different activity every class, they said they considered the workload as generally light, especially that they could finish a specific task in the next class session if they ran out of time, and that they were not expected to do any work at home. Another theme that emerged was students' strong approval of the grading system, one that did not generate feelings of anxiety over grades. Five students out of the 20 expressed how the classroom environment shaped their positive feelings towards the course.

The nature of the group work can also provide the students with the support needed to contribute to the activities in the classroom. Some students found the work challenging but doable, while others actively struggled with the work; however, the concept of working in a group on mathematical problems may have created a favorable environment to encourage students. In addition to supporting students facing difficulties, the group dynamic offers a bigger picture in terms of knowledge through the concepts available to each student, while the general idea of sharing the new and familiar concepts builds their problem-solving abilities. Also, the idea of collectively sharing in the struggle and realizing that a colleague is facing issues with the same concepts can bolster the motivation and confidence of less able students. The collaborative nature of group work engaged in a challenging task creates a shared learning experience that does not leave any student behind. Switching groups with each activity also ensures whole class inclusion, since a single group facing difficulties may have continued to struggle throughout the course were it not for the constant group member turnover. The variables of students within a group and the regular group switching created an environment where students had to work with different peers each time, providing a healthy alternative in terms of collaborative problem-solving involving progress as well as setbacks.

Instructor-student Relationship. According to research, support from the teacher, and the teacher's perception of students' performance, can play a major role in the development of students' attitudes. Since Egypt can be considered as a host of communal/collective societies, rapport between a teacher and a learner can foster a positive attitude from students towards the subject taught (Frisby & Martin, 2010; Hartmann, 2008). Such rapport can make the students feel safer and more comfortable in expressing their thoughts and concerns in a spontaneous manner.

Student 2: "The class is friendly. Professor is nice and jokes"

Student 5: "The doctor is understanding and doesn't stress us out"

Student 6: "I like that I feel I am heard in the course. The doctor really cares for me"

Student 10: "The professor"

Student 15: "The age gap is very small between us and the doctor, which makes it a fun place for us to be"

Student 17: "The professor speaks our language and is good role model"

Student 20: "I like how the doctor offered to help in other courses through his course"

Student 21: "I feel like it is safe to say anything in this course"

Students felt comfortable, and enjoyed the course because they thought that their instructor is friendly, pleasant, and understanding of their needs. Some students even stressed that one of the main reasons they like the course is because of the person who teaches it, and that they felt that the generational gap between the instructor and the students is very small, making the learning space relevant and comfortable for them. Moreover, they appreciate how their instructor does not put a lot of pressure on them, even suggesting that they could bring their coursework from other classes and try to make learning as fun and relevant to them as possible.

Students also expressed how they liked that they felt safe in speaking their mind during classes, and were even given the opportunity to challenge their instructor at times. The students were sometimes surprised that someone aged only 30 can teach them at the university level, claiming that narrow age gap made them look upon their instructor as a "role model".

One student commented that he felt he was "heard in the class", which suggests that this is not something he is used to. Students in Egyptian classrooms are not expected to challenge the teacher, and more often than not are treated with in an oppressive and authoritarian manner (Naguib, 2006), often leading to anxiety and discomfort in the classroom, and, in turn, build a negative attitude towards the subject taught in this class (Burić, 2015). Eight out of 20 students cited how important it is for them to feel comfortable with their instructor as an indicator of their positive feelings towards the course.

The pivotal role of the student-teacher relationship is thoroughly supported throughout the study, since the reflections reiterate the importance of feeling comfortable in class from the students' point of view. The previous section also considers the teacher perception as a significant factor that changed dramatically between the pre-and-post course activities. In fact, the supportive role of the teacher is a key factor in providing the environment necessary for the students to perceive mathematics positively complemented with the nature and structure of the course and the inclusion of all students in aspects such as difficulty. Other aspects forging a bond between the students and the teacher are features such as age, language, and caring approach, as mentioned in the reflections. The young age of the professor, ability to speak the same language, support in other courses, and the creation of a platform for free commentary contributed to students' appreciation of the course.

As for the second reflection question, all of the students' responses fell under one of the following categories:

- Classroom environment
- Professor's relationship with the student

Environment of the Course. One of the main factors that can have an impact on the degree to which students may like or dislike a certain subject is the learning environment. It is not realistic to expect all students in one class to respond positively to the conditions and format of a specific learning environment. The latter could include the campus location in relation to the distance from students' respective homes, and the timing at which this specific class is held (3:30 pm), given that traffic congestion in Cairo escalates after during midafternoon, entailing commuting distances of up to three hours.

With respect to course design/activities, the following comments encapsulate some of the students' concerns:

Student 4: "Not all students work the whole time"

Student 8: "Sometimes I am not sure what I am learning because the class is too loud and we don't have time to finish everything and get feedback"

Student 10: "The class is late after a very long day"

Student 12: "I live far and the class keeps me until late"

Student 15: "If this class was in the morning, I'd solve much better"

Student 19: "It is unfair that people are not judged by their performance. It doesn't motivate me to do better than others"

Since this course requires high mental capacity, the fact that it is held at the end of the day made some students uncomfortable. In fact, one student claimed that “if this class was in the morning” she would have solved much better.

One of the setbacks of having group work as the main setting of learning is that some students may feel think that others are not doing the same amount of work as they are. Moreover, students in Egypt have always competed for grades (Hargreaves, 1997); the fact that others may receive the same results for less work completed can be demoralizing, emphasizing stronger students’ resentment towards those perceived to be taking a ‘free ride’ at the expense of their peers. One student claimed that it was unfair to be “judged” similarly to others, even if not all students exert the same effort. Three students disliked the course because of its learning environment with one stating that the reason the learning environment was inappropriate in terms of noise levels, impeding her ability to focus and get things done within the time allowed.

Although the nature of the group work encouraged a lot of students, this segment of the results indicated that the group dynamic did not match their vision of the ideal learning environment. The individuality of the student is something many looked for when assessing their grades, since it does not cause a differentiating factor between them and their peers. The aforementioned importance of grades is a pivotal aspect in the learning environment, one that many students regard as the definitive indicator of learning and progress. This means they view a final concrete outcome with regards to their grade or a grading rubric as an essential component in tracking progress. While this may detract somewhat from the essence of the course, it still provides the feedback necessary to develop the course in a way that transitions the students into a course with a different model.

5.3 Auxiliary factors

Student-student Discrepancies. On separate occasions in the reflection, some students with a more mathematical background such as engineering expressed more confidence with the nature and concepts in the course, declaring that they did not have a problem with this material. Majors such as engineering, economics, and business entail regular use of mathematical concepts; therefore, there is more recent access to the information needed for problem solving. This point was raised by one student who stated that “many of the activities require previous knowledge in maths that some students already have from their majors”, pointing out that it was difficult to “catch up” with some of these students. This elaboration indicates the possible need for a more inclusive approach to the course in all aspects of problem-solving beyond mathematics. Certain considerations may be necessary for other majors in fields that may not require number and values, but do involve coming up with solutions to a certain problem. This detail may be an indicator of why the subscale of confidence did not increase in the post segment of the surveys.

Chapter Six: Development of Students' Conceptual Understanding

6.1 The Description of activities

The nature of the course kept the problems challenging yet solvable. After receiving information on the problem assigned to them, the students were required to find a possible answer to a specific question; a task that involved working in separate groups and experimenting with different formats to come up with a deterministic answer to this problem. These formats included formulas or even the use of a ruler, without being limited to a specific device or medium. The medium usually chosen was a picture or a word problem serving as a kind of springboard from which students could extract information. Evaluation of these activities was done through a series of questions that were asked by the instructor to each group independently, where the questions differed from one group to the other based on the students' work.

Kieran (2004) identified the different fields used outside the algebraic scope such as problem-solving, identifying structure and change, making generalizations and relationships, as well as making predictions and inferences. The above mentioned questions were designed to create an expansive thought process in order to generate ideas that help to achieve the answer through each group's chosen method. The only evaluation criterion given to the students regarding the problems is that answers were required to be logical solutions repeatable to other problems with the same format. The ideas also had to make sense to the group as well as to the entire class, creating an environment where everyone was free to critique each other's methods and to reach a consensus on the most suitable one. This approach also examines students' levels of understanding leading to this representation, as well as the generalizations and relations they managed to draw on through this understanding. If a specific idea did not work, they would be asked to reconsider it; however, this did not happen during problem-solving, since there was on-going follow up by the instructor to the groups.

Activity 1. Activity 1 was a simple problem of evaluating a picture provided. The activity, called "White Party", provided the students with preliminary brief information about a nightclub in Sahel, an imaginary city in Egypt. The problem focused on concerns by the club management regarding over capacity and justifying the potential need to install a larger sound system to cater to the clientele of the club. Students were initially asked to think of all the factors and information needed to enable them to estimate the total number of customers frequenting the club. The second question entailed an explanation of why these factors were needed. After they had sifted through this information, the students were then given a bird's eye view of the club taken through a drone. On a scale of 10 meters, this picture featured an abstract shape with numerous white dots. Figure 3 indicates the information for the problem. They were then requested to estimate the number of attendees based on the picture provided.

Figure 3

Photograph Used in Activity 1

The reasons for the structure of this activity is to make it relatable to the students by providing them with information and a problem about places they know are real. It gives substance to the idea that this issue might actually be a real problem that the management have actually faced. The details included, such as the information provided in a memo, the location of the city, and the club's name lend an extra veneer of realism. In this way, students get to use mathematics in the real word by engaging in the tasks related to Activity 1, attempting to solve for density, and calculating the total area so that they can extract the final answer.

Activity 5. Activity 5 had a different format provided through a structure that does not rely on pictures. This featured an optimization question that provided information with regards to a hotel's details such as operating cost, number of rooms, rate per room, and the change in fare per unoccupied room. The details, provided were given in the context of a brief story, which served as concrete clues to finding out the optimum number of rooms for the purpose of maximizing profit for the hotel owner, while deducting the servicing cost of the rooms.

The ability to extract information, in addition to correlating it to a given formula, is the ideal method of answering the question. However different methods such as trial and error may have also yielded an answer, but the determining factor for the problem is that they needed a solid format they can apply when the information such as servicing costs and room rates changes.

Final Project. In the final project the groups were given the freedom to choose a real-life problem they think is an issue in their society and attempt to find a possible solution for it, if applicable. A series of questions were provided in order to process the problem. From determining the problem or issue the students were then required to establish certain aspects such as the parties affected by the problem, evidence of the problem and listing their sources, possible solutions to the problem, and locating data for the solution and various other components.

Students' work on the projects were mapped out on the Lesh Translation Model, and arrows between different representations and translations were formed. As mentioned earlier, the Lesh Translation Model was used as a tool to assess how students develop connections between the representations and translations, and these were considered an indicator of conceptual understanding in mathematics. Below is a table (8) that describes how the arrows in the modified Lesh Translation Model (Figure 4) were formed. There are 3 levels in which the relationship between different representations (or with themselves) can be formed. To develop these arrows, students' written work was collected, and their conversations while working were recorded. Moreover, students had to present their solutions for every activity, and these presentations were taken into consideration in the following table.

Figure 4

Modified Lesh Translation Model

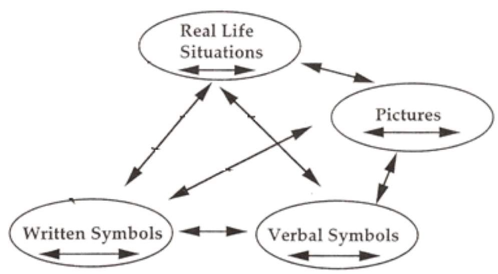


Table 8

Rubric of Arrows in the Lesh Translation Model

Header				
Arrow from	Arrow to	No line	Dotted line	Solid line
Real-life	Real-life	The students did not make a new connection to real life from the existing one	The students changed the real-life situation a bit to make it more relevant to them	The students took the real-life situation given and critiqued it to come up with a more relevant real life situation
Real-life	Verbal symbols	The real-life situation they were given was not communicated verbally and was not relevant to the students' answer	The students perceived the real-life situation given in a trivial manner, and did not reflect on the sophistication of the situation in their mathematical expressions	The students perceived the real-life situation in a solid manner, and talked about this situation can be perceived in a mathematical manner
Real-life	Written symbols	The real-life situation they were given was not communicated using mathematical symbols and was not relevant to the students' answer	The students perceived the real-life situation given in a trivial manner, and did not reflect on the sophistication of the situation in their written mathematical expressions. The written symbols used are trivial and basic (e.g. use of simple arithmetic operations)	The students perceived the real-life situation in a solid manner, and created a mathematical model that explains the real-life situation. The written symbols used are considered complex mathematical symbols (e.g. use of variables and equations)

Real-life	Pictures	The real-life situation they were given was not communicated using pictures and was not relevant to the students' answer	The students perceived the real-life situation given in a trivial manner, and the images they used to represent real life situations were not very reflective of the actual situation	The students represented the real-life situation using pictures, or used images such as tables, graphs, or drawings to represent how they understood the real-life situation given, or to represent the solution to given problem
Verbal symbols	Real-life	Students' verbal symbols were not used to change the parameters of the real-life situation given	Students used verbal symbols to question and discuss the real-life situation, but they didn't really do anything to change the parameters of the situation given	Students came up with a solution, based on their discussion, that requires a change in the real-life situation, and were critical on how this situation was presented to them. Their solution requires a change in parameters, or they believe that the situation was not realistic for them
Verbal symbols	Verbal symbols	Students' discussion did not lead them to change their idea on how they perceive a problem	Students' discussion gave them the opportunity to critically think about their solution to the problem, but they did not come up with a solution that reflects how their perception towards the problem changed	Students used mathematical verbal symbols to critically think about the problem, and come up with a solution that requires a change of how the problem is communicated and perceived

Verbal symbols	Written symbols	Students' verbal communication and use of symbols did not influence their written representation of the problem or the solution	Students' verbal communication and use of symbols allowed the students to explore different written symbols for the problem, but were not utilized in coming up with a solution for the given problem. The written symbols used were trivial and basic (e.g. use of simple arithmetic operations)	Students' written work was directly influenced and affected by students' use of verbal symbols in communicating with each other. Accordingly, students used complex mathematical symbols (e.g. use of variables and equations)
Verbal symbols	Pictures	Students' verbal communication and use of symbols did not influence how they can perceive or develop images such as graphs, tables, or pictures of the given problem or its solution	Students' verbal communication and use of symbols provided the students with an opportunity to question the given images such as graphs, tables, or pictures, but had little or no impact on how these images can represent the given problem or solution	Students' verbal communication and use of symbols had a direct and explicit impact on how students developed images such as graphs, tables, or pictures to represent the given problem or its solution
Pictures	Real life	Students did not make an explicit relationship between the given picture with a real-life situation	Students used the images given, whether they are pictures, tables, graphs, to make a connection to a real-life situation, but did not use the picture to describe or explain the real-life situation	Students used the images given to come up with an interpretation or a description to a real-life situation, and used these images to come up with a solution to the given problem

Pictures	Verbal symbols	The pictures presented did not initiate the students' use of verbal symbols	Students described pictures, such as images, tables, and graphs using mathematical verbal symbols, but these symbols were trivial and did not reflect students' extensive use of mathematical concepts	Students used given pictures, such as images, tables, and graphs, to come describe mathematical concepts verbally, and they used these pictures to interpret advanced mathematical concepts
Pictures	Written symbols	Students did not translate the pictures or images given to written symbols	Students translated the pictures and images given to mathematical written symbols, but the mathematical symbols used were basic and trivial (e.g. use of simple arithmetic operations)	Students translated the pictures and images given to complex mathematical symbols (e.g. use of variables and equations)
Pictures	Pictures	Students did not use the pictures and images to reflect on what they mean	Students reflected on how the pictures and images represent the problem, but did not critically think about what these pictures or images represent	Students were critical about how the images or pictures represent the given problem, and altered the pictures and images given based on what was needed to solve the problem
Written symbols	Real-life	Students' written mathematical symbols were not translated into real-life situations	Students' written mathematical symbols were used to question the real-life situation, but were not used to change the parameters of the real-life situation	Students' written mathematical symbols were used to question and alter the real-life situation to create new parameters

Written symbols	Verbal symbols	Students' written mathematical symbols were not translated to verbal symbols	Students' written mathematical symbols were translated to verbal symbols without using language that fully reflect mathematical conceptual understanding	Students' written mathematical symbols were used in students' verbal representations, and fully reflected students' mathematical conceptual understanding through using mathematical language
Written symbols	Written symbols	Students did not come up with new written symbols based on a reflection on the existing written symbols	Students reflected on existing written mathematical symbols, but no major changes took places accordingly	Students made corrections to their written work in a reflective manner, and changed their mathematical written symbols accordingly
Written symbols	Pictures	Students' written mathematical symbols were not translated to pictures	Students attempted to translate the written mathematical symbols to pictures and images, but they were misrepresented or did not fully reflect the concepts presented by the written mathematical symbols	Students' written symbols were reflected and represented in the form of pictures, images, graphs, or tables; where mathematical concepts were used correctly and presented clearly

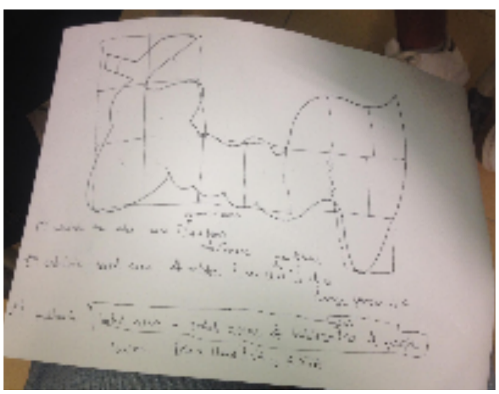
6.2 Description of Groups' Solutions

Activity 1 solutions according to the LTM.

Group 1. The first group used the scale-related information provided in the picture by determining that the area under the one scaled square is 100 meters squared while its length was estimated at 10 meters. From this scaled segment, the group split the outline of the photo into smaller sections that were equal in size to the scaled segment in order to determine the area of the space. The issue they encountered at this point was those areas near the outline that either did not fully fit the scaled segment or were slightly larger. This impediment required the students to make educated guesses with regards to the abstract areas. Figure 5 shows how the students in group 1 used the image given to them as a means to solve the given problem. Figure 5 shows how the students divided the image given to come up with a written solution.

Figure 5

Zone Breakdown of Area by Group 1 in Activity 1



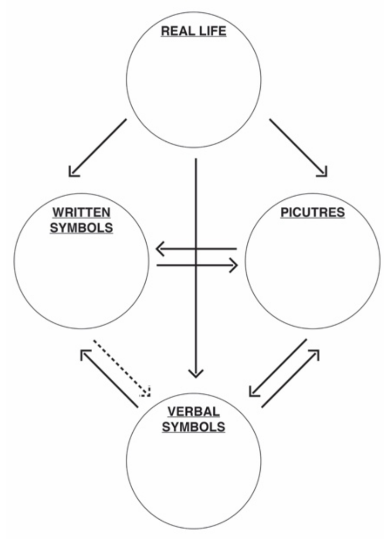
The students made a series of assumptions regarding the number of people that can fit inside an area of 1 meter squared as well as deducting spaces and personnel for servicing areas. The idea of deducting spaces for the services can be considered a good idea; through their communication process, students considered the details of the club's space and their specific allocation as they worked towards the solution to the problem.

The Lesh model for the first group shows many connections one since they related many components to the real aspects of the party. They provided reciprocating verbal, written, and picture communication, as well as the solution. One of the key factors marking this group regarding Activity 1 is that they relied extensively on a party they went to, thus enabling them to assess the details familiar to them. The solutions they provided included the necessary explanation through solutions as well as by providing a diagram illustrating how to measure a total area through the use of smaller scaled segments that was not part of the original problem. Their verbal solution included mentioning aspects such as the need to divide the area into smaller segments to be able to solve for the total area.

The results presented in the Lesh solution of Group 1 shows extensive use of the different representations, since they were able to relate to the written, picture, and verbal fronts to the real-life possibility of the problem. Interestingly enough, the students were able to relate the real-life application to several details that aided with the solving of the activity. Details such as the number of people deducted from the servicing area shows the real-world application to the activity and the higher potential of making sounder assumptions with regards to the problem. Figure 6 shows the Lesh model for group 1's work in activity 1.

Figure 6

Group 1 Solution According to LTM

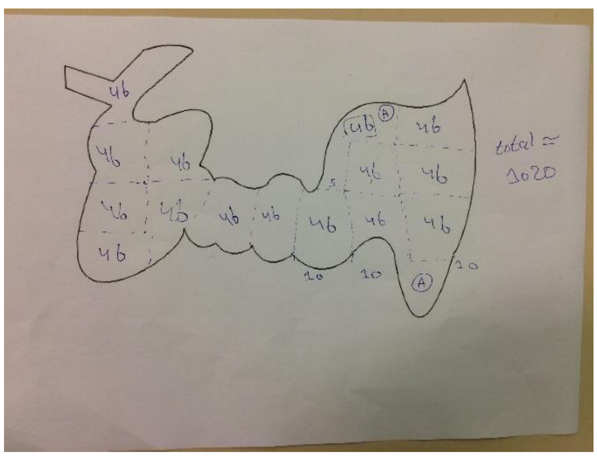


Group 2. One of the interesting approaches that Group 2 followed in the preliminary part of the problem that was different to that of Group 1, is that Group 2 chose to consider the number of attendance by using the number of tickets sold. While this would have been a logical point to establish numbers upon, this information was not provided in the activity itself. The real aspect of the activity became more realistic with this detail taken into account, demonstrating how the students engaged in out-of-the-box thinking before facing limited information that was provided later. Another detail they considered that was not mentioned by Group 1 was the area of the tables themselves, which they regarded as space that will not include individuals. They did this by calculating the total area and then deducting the area of the tables. They did, however, provide an assumption of two individuals to every 1 meter squared compared to the assumption of 1 person per meter squared of Group 1. The unique detail included in the problem here was the tables, thus altering the solution of the group.

Through the preliminary part of the discussion the verbal aspect of Group 2 was extensive, from which discussion originated all the points they wished to elaborate on. From the verbal solution, they began formulating the formulas and establishing the details of the question into written responses. The basis for the question was the use of pictures for establish the number of scaled segments there were to the picture given. They determined that there were 14 sections compared to Group 1's figure of 15. They also provided other pictures to elaborate on their concept of total area. Figure 7 shows how divided the area given.

Figure 7

Group 2's Use of Pictures

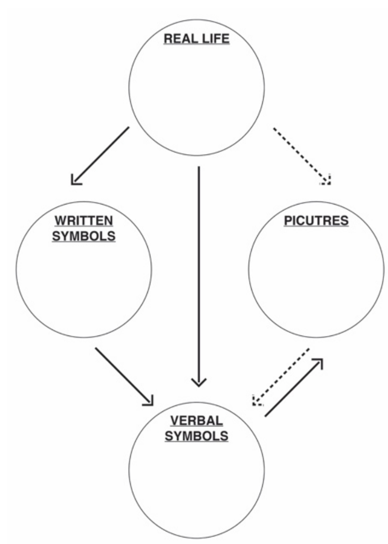


The real-life aspect they mentioned was the table addition to the space, a detail attributed to other parties they were familiar with.

Group 2's mapped out work shows the use of written and verbal approaches to the problem. Through their verbal communication and discussion of the problem, they were able to extract the bulk of their information. There was no tie, however, between the written and picture portion of their solution; rather, they extracted their information mostly from the verbal approach prior to presenting their points orally, so that it made sense to them and the rest of the class. Figure 8 shows how group 2's work was mapped out using the Lesh Translation Model, with the respective connections between the representations.

Figure 8

Group 2 Solution According to LTM



Group 3. The idea used in Group 3 was slightly different since they used an additional concept of average in their solution of the problem. In the preliminary section of the problem they thought of details such as security, involving no additional personnel entering the space without a ticket. Another detail included was the ability to forecast the number of attendees based on the parties the club previously hosted, from which figure they could propose a round figure on how many people would be attending the current

party. Group 3 seemed to focus on information documented by the venue and from there they could come up with answers to the problem; this is why when it came to solving the activity, they presented a different way of accounting for personnel. Their solution to the problem involved splitting the picture into smaller sections based on the scale. The additional aspect setting them apart from their peers is that they counted the number of people under each section. They did this for four square sections and found the average number per section to be 35; from there, they multiplied the number of sections to the average number of people per section to give a total number of attendees. This model of considering density of personnel is different, since it utilized the information provided in the problem through the picture, unlike the other groups which made an assumption about the number of people.

This group's solution did not include any picture form, but rather a series of formulas and explanations of the information they gathered.

Group 3's use of verbal symbols:

> *"We divided the pictures into squares. There were 14 squares, and so we multiplied 14 by 100, that is the area of one square. Firstly, we calculated the are of the entire venue, by assuming the venue is divided into separate squares. The first square, which is the tables area, is 10 meters wide, when assuming it's a square, then it's also 10 meters in length. Thus, 10x10 equals 100m^2, so the tables' area is 100 m^2 alone, when the calculating the area of the rest of the tables, by assuming they're also squares, with 10m in width and length, they equal 700 m^2, and thus the entire venue equals 750 m^2. We subtracted the area of the*

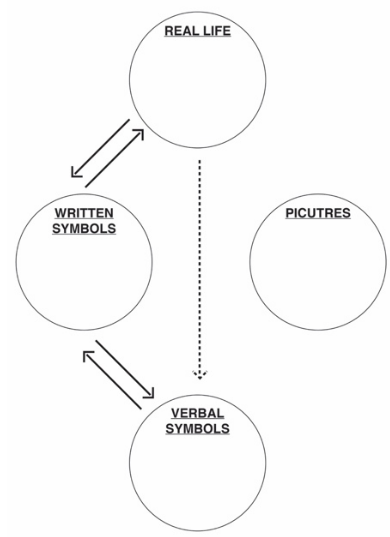
tables from the total area, and we assumed that 4 people will sit on each tabled."

They thoroughly assessed the concepts of the realistic aspects of the problem through details that they were acquainted with, and from there progressed to gather information from the problem. Their use of words was more related to the use of a perspective that made sense to them rather than calculations in the problem. Their use of the written dynamic was more word intense than formula intense, since their approach was simpler and more directly correlated to the information provided in the question.

The logic behind this problem was rigorously presented in written form so that it made sense to the students and all others considering the solution after this. Reasons for the latter were then followed with brief formulae to match the reasoning provided in writing. The students used the information from the picture to establish the scale and average per scale; however, there was no picture presented in the solutions of the students themselves. The verbal discussion was extensive in order to establish the logical points to discuss. The missing factor from Group 3's solution was the use of a diagram or picture in the solution. Figure 9 shows how group 3's work was mapped out using Lesh Translation Model.

Figure 9

Group 3 Solution According to LTM

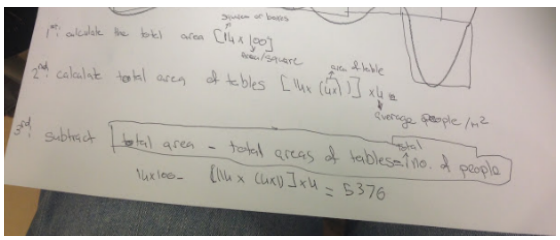


Group 4. The group used proportionality to focus their solution for this activity. They initially used the concept of scales, but then relied on another method using proportionality through a ruler and an established unit of measure. The unit of measure used was one of the student's fingers through which they reconstructed a proportion of the general length and width to match a certain number of fingers. Through the number of fingers, they created a rectangle that would encompass the space with the extensions in the shapes to later fill the voids. Through this proportionality to the area of the given picture, they used a ruler to more concisely provide values to the sections of the picture, the number of personnel affiliated to each section; through the measurement and calculation of the length and width of each section, they correlated each section with a value that they would conclusively add to each other.

The solution provided used written, verbal, and picture form, since they wanted to provide a full understanding of the answer they provided. In the solution they used the picture provided to them and also expanded on this visual by creating a rectangle to enable measuring the personnel through the proportional correlation they formed. The written explanation was used where they explained the process of their reasoning, supported through the use of formulas and calculations to finalize the concepts they developed. This is shown in figure 10 below.

Figure 10

Group 4's Use of Written Symbols

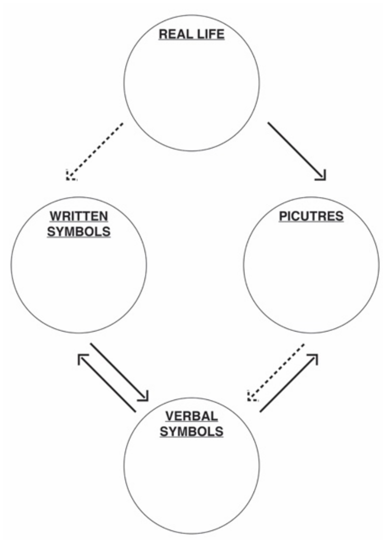


The verbal segment used counting to establish the proportionality between the students' fingers, ruler measurement, and the drone pictures in order to provide a more concise number of the people in the venue. Attributing to the real life component of the problem was mostly addressed through picture and verbal forms of the solution.

The solution of the group derived from both the real life domain and the picture and verbal domain, but was considered weak in the written part of the solution. The verbal solution was pivotal in establishing the format and response for the written and picture domains of the Lesh model. However, the picture was neither established with the written form, nor was it present in presenting the written aspect of their solution. Figure 11 shows how group 4's work was mapped out using the Lesh Translation Model.

Figure 11

Group 4 Solution According to LTM



Group 5. In the presentation of the problem, the group attempted to use the scale system but then decided to switch to a much simpler method. They split the model into four segments and each member of the group independently counted the number of dots found in the picture. While this method is not repeatable to variant cases, it split the work in a way that made sense to the students, enabling them to proceed accurately with the information they acquired.

The solution they used was to discuss the real problem by agreeing on it verbally, and then utilizing the picture to acquire the information they needed, before communicating in to the whole class. While the problem would have been more complex and more difficult for a larger photo, this solution seemed to fulfill the criteria assigned to the students without using any mathematical concepts. The concepts used considered counting and addition to sum the different areas that each student counted. The solution included an explanation and a brief presentation of the areas they split the picture into and a list of the values of the areas counted.

The group in this case did not account for the written segment as there was no complex process or mathematical concept to explain; rather, data was shown through the picture and was verbally presented to the class. Despite evident attribution in a written format, there was no expansion to the problem beyond the brief details given to assess the complexity of the problem itself. Figure 12 shows all their work for this activity.

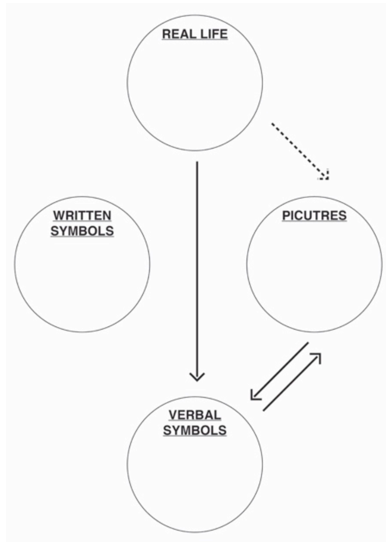
Figure 12

Group 5's Use of Pictures

Students did not attempt to perceive the problem from different perspectives, it showed by the lack of a written response associated with the thought process involving the students' response. Figure 13 shows the connections for group 5's work in activity 1 as per the Lesh Translation Model.

Figure 13

Group 5 Solution According to LTM

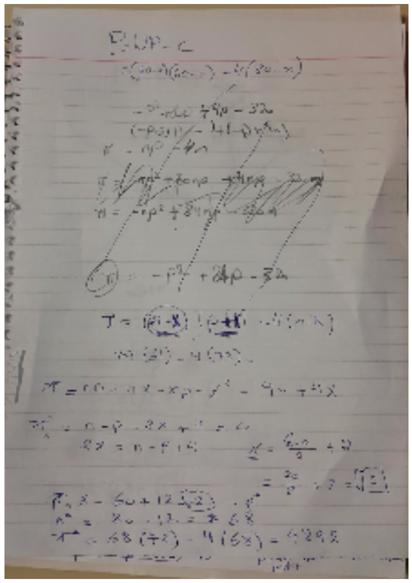


Activity 5 Solutions according to the LTM.

Group 1. The group decided to formulate a series of formulas to create a correlation between the information provided in the problem. They initially provided a negative correlation between the number of rooms and the rates provided, and then they proceeded to create a formula to establish a model for the hotel owner. The group also produced a graph showing the amount of profit potentially generated for Mr. Kareem, the hotel owner. The graph was represented by the equation they formulated, and through the insertion of different values for the different variables they were able to arrive at their answer. The discussion considered the equation and graph, but there was no written explanation to the problem. The written symbols group 1 used are expressed in figure 14.

Figure 14

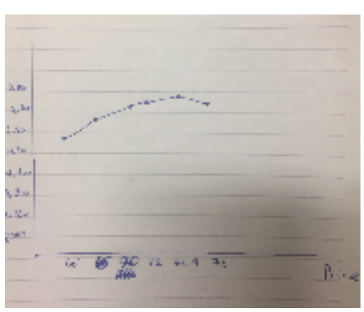
Group 1's use of written symbols



This group used various forms to present the information they interpreted from the problem. They used verbal symbols to discuss information they extracted from the problem, and presented in an equation in addition to drawing it in a graph. They presented pictures by drawing both a table and a graph, which aided in finding the prime point. The picture and written form complemented one another as shown by the formulas presented in the graph (as shows in figure 15); further elaboration on the problem was given through the extracted graph.

Figure 15

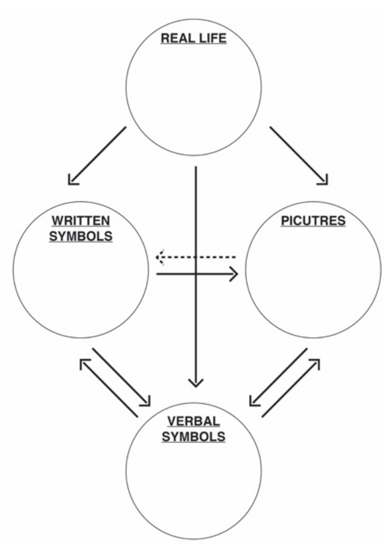
Group 1's Use of Pictures



The verbal communication is the form that aided both the written and picture methods as the problem was discussed first and then the formulation of a solution began through the group discussion. The figure below (16) shows how group 1's solution to activity 5 was mapped out on the Lesh Translation Model.

Figure 16

Group 1 Solution According to LTM

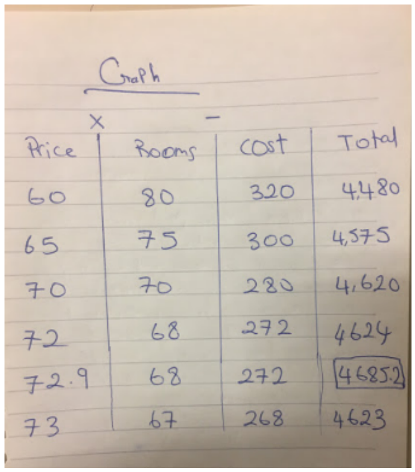


Group 2. The group in this case formulated a response based on constructing a table that evaluated the information presented to them. They set a table (figure 17) with the price, rate, number of rooms, cost and total profit. From there they were able to estimate the exact point at which the profit rate rose and then dropped. From the table they determined this to be at 68 rooms; they then proceeded to graph each point on a coordinated graph to present a brief representation of a parabolic graph. The graph they drew was the profit versus the rate determined which essentially correlated to the number

of rooms occupied since there was a direct relationship.

Figure 17

Group 2's Use of Pictures



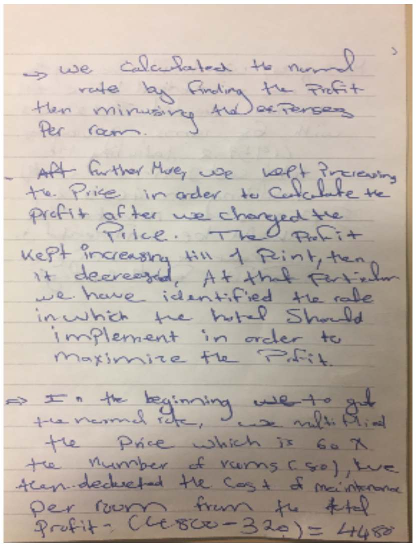
Graph

Price	Rooms	Cost	Total
60	80	320	4,480
65	75	300	4,575
70	70	280	4,620
72	68	272	4624
72.9	68	272	$\boxed{4685.2}$
73	67	268	4623

The information presented by this group shows a core establishment of their process and logic from their verbal communication to one another. From their verbal response they were able to extract written relationships and therefore represented their method in a series of bullet point paragraphs to elaborate on how they extracted and made use of their information. They then proceeded to conduct a table and graph to represent their written findings and formula into an image that would clearly represent the relationship they found. However, they established a formula without any variables, instead they used the exact figure to obtain an answer. From their verbal discussion they moved on to the written response before proceeding with a picture response in order to fully represent their solution.

The group here verbally discussed the essential points of the questions that, in turn, informed all of their response types. From their verbal solution, they were then able to correlate it into a written response to establish a logical response to what they wished to present (shown in figure 18).

Figure 18


⇒ we calculated the normal rate by finding the Profit then minusing the expenses per room.

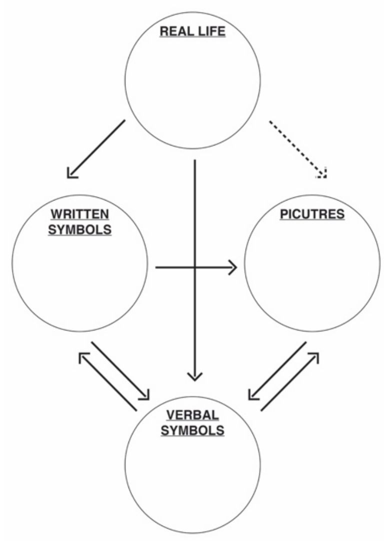
~~Aft~~ further More, we kept increasing the Price in order to Calculate the profit after we changed the Price. The Profit kept increasing till 1 Point, then it decreased, At that Particular we have identified the rate in which the hotel Should implement in order to maximize the Profit.

⇒ In the beginning ~~we~~ to get the normal rate, we multiplied the Price which is 60 X the number of rooms (80), we then deducted the Cost of maintenance per room from the total profit. (4800 − 320) = 4480

The written formula was then represented in an image (initially, a table) to present the information in a concise form so that they would confirm their answer. From the table, they then proceeded to graph the table's information, demonstrating the establishment of multiple images to present an answer in different formats; that is, the table confirmed the answer while the graph simply accentuated the answer in a more coherent form. Figure 19 shows how group 2's activity 5 was mapped out on the Lesh Translation Model.

Figure 19

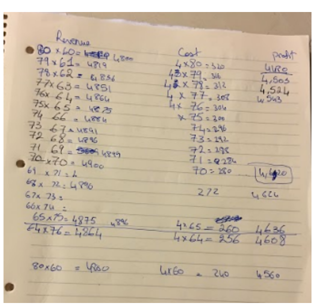
Group2 Solution According to LTM



Group 3. Group 3 initially decided to use the trial and error method by listing out all possible the values as they proceeded with the number of rooms. They calculated the possible revenue in each scenario, and the possible cost of each scenario with regards to the number of rooms (as shown in figure 20).

Figure 20

Group 3's Use of Tables



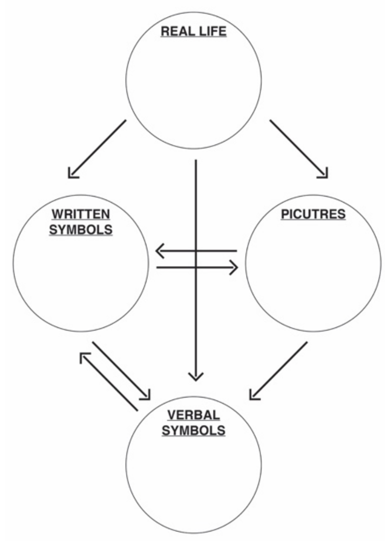
From the values they produced, they then calculated the profit by deducting the cost from the revenue. The increase in value of profit followed by the drop of the value after peaking was the indicator for profit, and from that value they were able to indicate the number of rooms representing the ideal value to maximize profits each night.

Based on their verbal solution, they decided to proceed with the trial and error method, from which method they began to produce a table to physically represent the concept they were calculating. They then discovered a pattern from the values they were producing, and from there they were able to acquire an answer. They presented written solutions through the use of an equation with variables and unknowns, and from that equation they included the formula for the trial and error calculations.

Their solution demonstrated the ability to extract information from the question in order to establish a solution. The answers they produced included a predominantly picture and verbal segment, indicating that the trial and error method was their method of choice. They also created brief formulae to allow correct production of the correct values for revenue and cost, so that they may finally acquire the profit. The Lesh model applied by this group used the picture, verbal, and written form, albeit with more emphasis on the table of the trial and error reasoning they used. Figure 21 is the Lesh Translation Model mapping of group 3's activity 5.

Figure 21

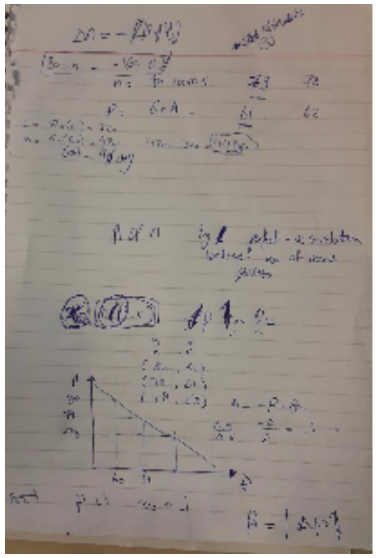
Group 3 Solution According to LTM



Group 4. This group broke down the problem into manageable chunks to facilitate finding a solution. They initially wrote down all the values and variables and set a corresponding operator to each so that they were able to make sense of all of them. They then proceeded with a trial and error method to determine the maximum value for their problem was, proceeding to further optimize their solution without using calculus. They continued to present their data using a coordinate graph using the values they calculated during for in the trial and error in order to see the physical representation of the peak point in the graph (as shown in figure 22). Then finally, they presented their data in the form of a letter to the hotel owner, Mr. Kareem, determining the rate he should maintain is $68 per room.

Figure 22

Group 4's Use of Pictures

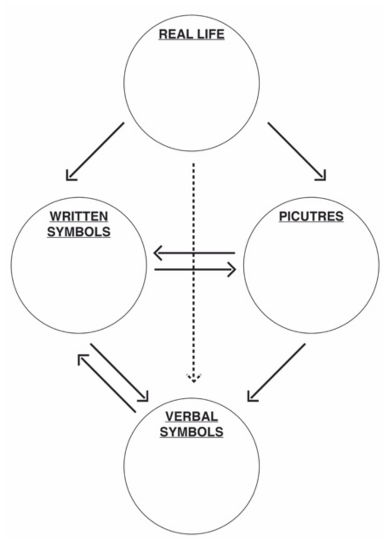


The representations used in this group included the written, picture, and verbal solutions. Through the information extracted, they were able to represent the data in the written solution of trial and error, and continued to optimize their picture solution by providing a graph that represented their solution in a more elaborate way. The peak of the graph is apparent in the graph and shows the value extracted in the trial and error segment of their solution. From their verbal communication they were able to create an input-output format to their problem solving, and through the breakdown of each section of their solution, they were able to close in on the answer and finally represent it physically.

The solution provided by the group includes the entire segments necessary to present the written, verbal, and picture forms of a solution. They provided a detailed solution in their picture solution by providing both a table and a graph to represent it, giving them the ability to actually see the parabolic shape represented. The verbal segment communicated both the written and picture form, and this shows that all segments of the Lesh model were used. Figure 23 shows group 4's solution to activity 5 as per the Lesh Translation Model.

Figure 23

Group 4 Solution According to LTM

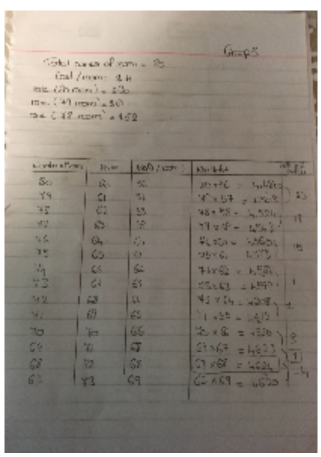


Group 5. The solution provided by this group was simple, consisting of written notes on the information they received. They then proceeded to create a table based on the number of rooms occupied, rate per room, profit per room, and total profit. Unlike their counterparts, instead of calculating the cost independently in the table, they immediately deducted the $4 service charge from the rate per room to give a finalized value of profit per room. When multiplied by the number of occupied rooms, it presented the final value of profit, indicated in their answer to be at 68 occupied rooms. They then proceeded to draw a graph that initiated the value of 68 and then presented the negative slope with the decrease of profit with the increase of the number of occupied rooms.

The solutions here included written use of the information provided. Instead of using any unknowns or variables, however, they simply restated all the problems values. They proceeded to take these values and presented them through a table in which they calculated the profits of the hotel per night through the immediate deduction of cost from the rate per room. When the pattern emerged through the table (shown in figure 24), they then proceeded to plot a graph to show the negative correlation between the increase of room number and profits beyond the point of 68 rooms. This was all thoroughly explained through their explanatory communication to one another, allowing them to establish the written and picture solutions finally presented.

Figure 24

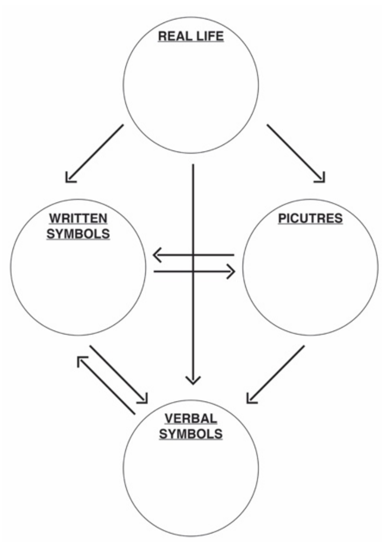
Group 5's Use of Tables



The group showed they fulfilled the Lesh translation model, since they explained all their procedures verbally and then continued to present the information and calculation in writing. They also presented a table and graph to represent their findings. The verbal segment was also considered when they explained all aspects of their solution through the use of the writing and visual aids. Figure 25 shows group 5's solution to activity 5 as per the Lesh Translation Model.

Figure 25

Group 5 Solution According to LTM



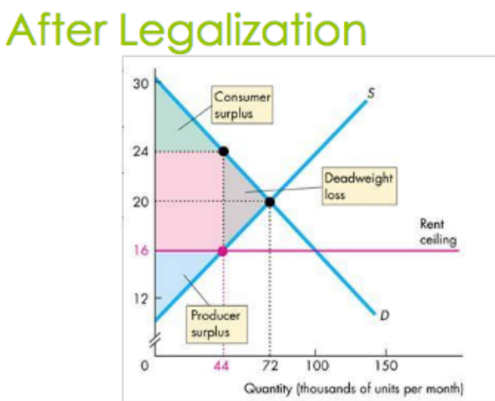
Students' final project according to the LTM.

Group 1. The entire classroom was given the freedom to assess a problem they found in the real world. Deciding on the legalization of hash, Group 1 assessed the concept and found that the illegality of hash poses a problem for the Egyptian economy by creating a dynamic for the unofficial economy to thrive. In addition, it applies pressure on the government to marshal massive resources to fight its consumption. This was tracked through a series of questions they answered to evaluate the problem.

In this activity, students used various representations to express their solution, and the process in which it was developed. They were able to come up with a solution based on their discussions, and they were able to relate their solution to a table, and a graph (shown in figure 26).

Figure 26

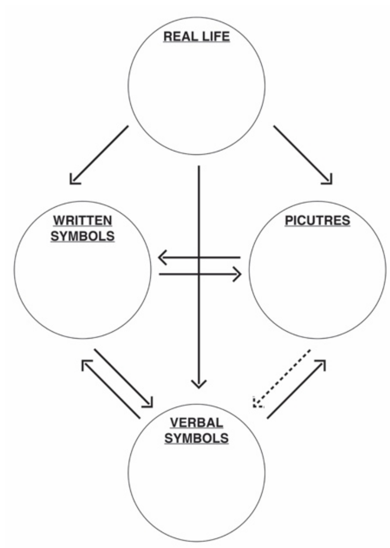
Group 1's Use of Pictures



The students in this group were relating the mathematical concepts with their real lives, and described, using numbers and figures, how their lives were affected by the economy. The students also provided mathematical solutions that were communicated verbally, both between themselves and to the whole class, making connections between the graphs they provided and what the verbal symbols they used. Figure 27 shows how group 1 students' work was mapped out using the Lesh Translation Model, and it shows how the different elements of the Lesh Translation Model were translated onto each other. This group was able to make different connections and translations, suggesting that their work addressed the mathematics concepts from different perspectives.

Figure 27

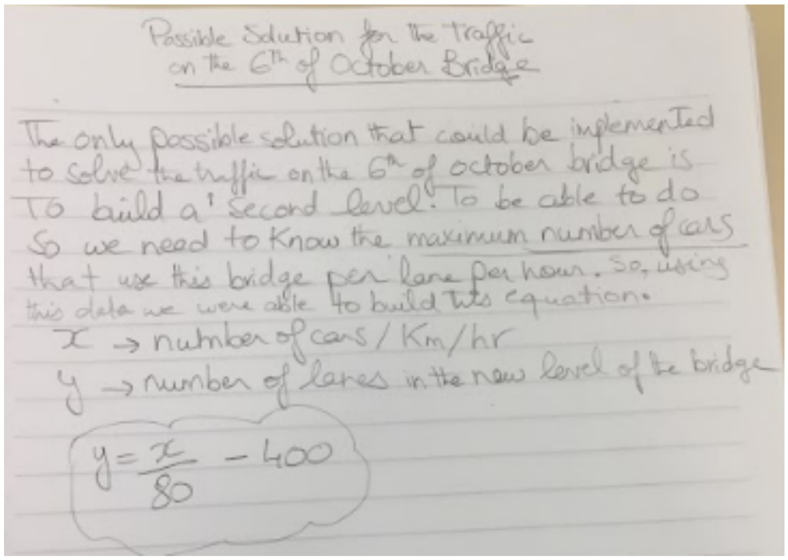
Group 1 Solution According to LTM



Group 2. This group decided to take the issue of traffic in Tahrir Square in Cairo and evaluate the core problem and the possible solutions it requires. They believe that rush-hour becomes severely congested due to many public ministries and private sector companies in the vicinity, generating a flow of traffic through Tahrir Square at specific hours. The idea suggested by the group is that the working hours of the five ministries in the area should be shifted away from each other over 30 min intervals, so that the leaving hours for each ministry are paced 30 minutes apart from the next one. The rationale behind this plan is that shifted working hours will significantly reduce congestion in the area. The solution provided by the group shown through the Lesh model, demonstrates that they had an extensive explanation of the problem in a written format, as well as extracting a possible formula to account for rush-hour traffic between 8 - 9 am and 3 - 4 pm (shown in figure 28).

Figure 28

Group 2's Use of Written Symbols

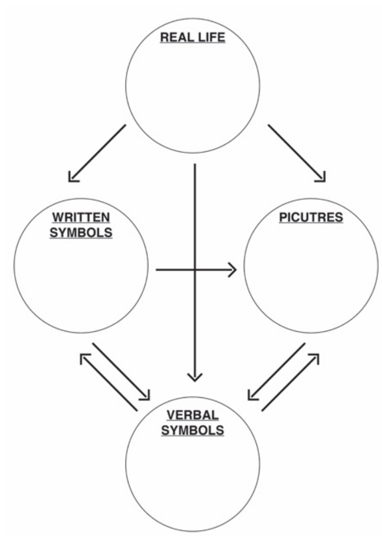


From discussing the problem and presenting a linear formula, the students then graphed a potential decreasing pattern of the traffic, based on their formula and consideration of the ministry exit times only. While it does not consider other elements, this answer is thoroughly evaluated and the limitations of the solution are presented by the students. From the answers they presented, it became possible to extract realistic solutions such as regulating pedestrian crossings and bus hours to reduce the flow of traffic.

Group 2 students were able to use different representations to discuss and communicate their solutions, with an emphasis on the verbal symbols. Accordingly, their discussions were used to develop the solution to the problem through different representations, and their final presentation expressed their understanding of the mathematics of a real life situation such as traffic in a busy street. Figure 29 is the Lesh Translation Model map of group 2 students' work.

Figure 29

Group 2 Solution According to LTM

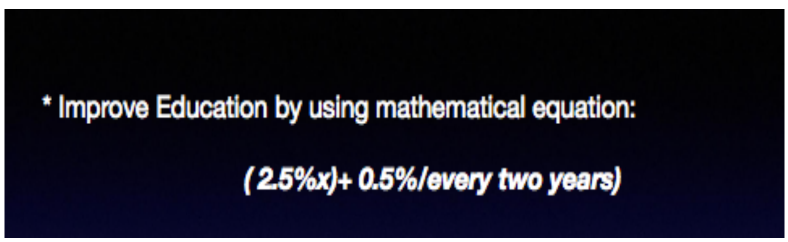


Group 3. The issue discussed by Group 3 was education in Egypt. They evaluated the deteriorating standard of education in Egypt and the possible reasons for this decline. They compared education in Egypt to countries such as Ireland and the U.S. in order to evaluate how Egypt stands amongst other nations. The difference they focused on was the amount of money invested by government and its percentage in relation to the country's GDP. Given Egypt's disappointingly low global ranking, the solution provided by the group through the written format was that Egypt needs to prioritize funds allocated to education in order to catch up with other developing nations. The group then established a direct correlation between the amount invested by the Egyptian government

in education and the scope for improvement. The formula and concept may not represent an absolute solution, but limitations to the solutions were also provided by the group. The group used a histogram to represent the amount of money spent by parents on private tutoring and how much that value is growing due to the deteriorating standard of schools. The discussion through the verbal communication helped to initially develop the problem, coming up with a formula to correlate spending to the standard of education (shown in figure 30).

Figure 30

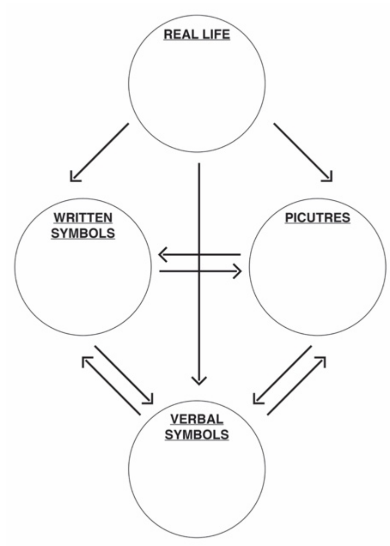
Group 3's Use of Written Symbols



Students in this group were able to develop the solution to the problem through various representations, making connections between the different elements of the Lesh Translation Model as they came up with their presentation. They used verbal symbols to come up with several solutions, going back and forth as they discussed what would work and what wouldn't, and they were able to translate their verbal symbols to written symbols and graphs, making their answer elaborate and coherent. Figure 31 shows group 3's mapped out work as per the Lesh Translation Model.

Figure 31

Group 3 Solution According to LTM



Group 4. The issue of population was evaluated by this group. The group wanted to tap the possible solutions to limit the rate of overpopulation, and personal congestion in certain areas. Rather than relying on a lot of verbal communication by the group, the solution was represented by a series of written explanations and formulas (as shown in figure 32).

Figure 32

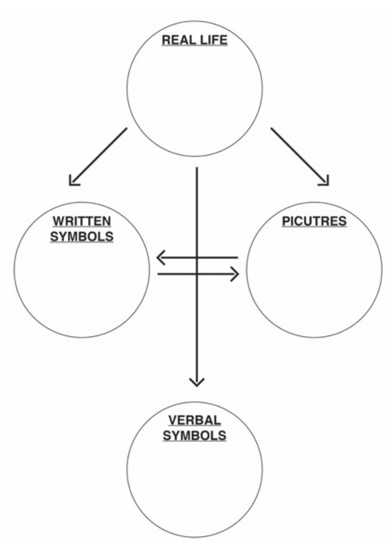
Group 4's Use of Written Symbols

- (0.04* PG) + (0.9* Carrying capacity of desert)+ (Cost for services/Economic budget)= -(over-crowdedness+ unemployment+ limited resources)
- It is a negative relationship between the inputs and the outputs.

The solution provided aimed to utilize spatial resources to cope with the overpopulation problem, with the rational that utilizing space can reduce crowding in the Nile Valley by expanding into desert areas. They recommended planting areas in the desert and relocating numbers of the population from urban areas in order to exploit all of Egypt's potential space. The group did not verbally communicate well, they did, however, present information in a written format that thoroughly explained the solution as well as maps, equations, and information graphs to focus details of the solutions. Figure 33 is a mapped out work of group 4's final project as per the Lesh Translation Model.

Figure 33

Group 4 Solution According to LTM

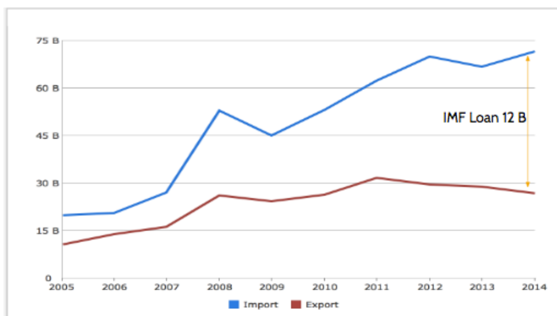


Group 5. The issue of the US dollar versus the Egyptian Pound and shortage of USD in Egypt was the problem this group presented. They evaluated the numerous components of this problem and the most suitable way to fix it.

The written and picture form-based solution presented by Group 5 demonstrated that the verbal element was not given the most significance. The written solution, however, thoroughly explained the solution, from which point the students were able to extract the written information and equations necessary to solve the problem. The presentation of the solutions employed the use of graphs (figure 34) to illustrate the difference between Egyptian imports and exports as well as the potential ease that an IMF loan can possibly create to address the resulting deficit.

Figure 34

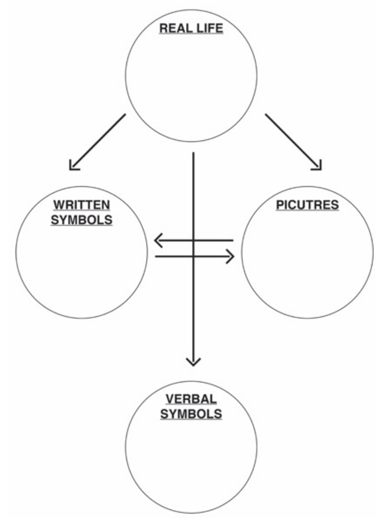
Group 5's Use of Graphs



Students' verbal symbols were not used to come up with other representations. However, students in group 5 communicated well with each other, and used verbal symbols that were presented to the classroom in a well-articulated manner. The group's presentation indicated awareness of the limitations of the solution, especially since the group clearly showed they understood that the USD deficit is not the sole issue plaguing the Egyptian economy. To round up, they presented other auxiliary solutions such as the invigoration of tourism in order to pump more USD for the government. Group 5's mapped out work, according to the Lesh Translation Model, is shown in figure 35.

Figure 35

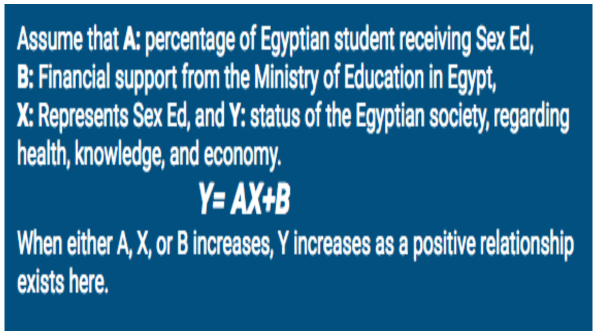
Group 5 Solution According to LTM



Group 6. Group 6 considered the issue of sexual education in Egypt and the need to spread awareness of this issue. The issue considered the implications accompanying the problem, including rising divorce rates, overpopulation, and health concerns. The group correlated the population/segment of Egyptian students receiving sex-education, financial support by the Ministry of Education, sex-education, and societal attitudes to these issues in general. The solution presented in the written segment of the Lesh model is representative of the information they provided in terms of considering the limiting factors, while failing to establish a solid relationship between sex-education and the general improvement of the Egyptian society. They presented a graph (figure 36) to display the solution and initial problem through a simpler and more audience-friendly format.

Figure 36

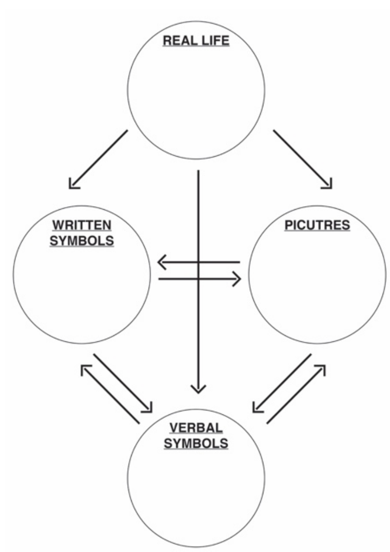
Group 6's Use of Written Symbols



The verbal communication was the core element, through which they were able to extract all the information before sifting through the data acquired from sources and deciding which information they needed to use. Figure 37 shows the connections between the different representations as per the Lesh Translation Model.

Figure 37

Group 6 Solution According to LTM

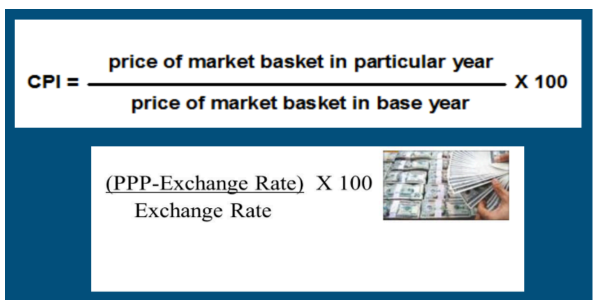


Group 7. The issue of the US dollar and ensuing price hikes was also considered by Group 7. However, the solutions presented by the group were noticeably different in

the method they proceeded with. From the issue of inflation, they communicated the potential solutions in a written format while considering a picture form through graphs and visual statistics. The formulas they extracted were from already existing sources, demonstrating the expansion of the potential to learn from other sources and other concepts presented to them. The issue was presented by a detailed solution using written and picture forms to complement each other, such that each point was understood; the potential measurement of the solution itself was concluded in a final equation (figure 38).

Figure 38

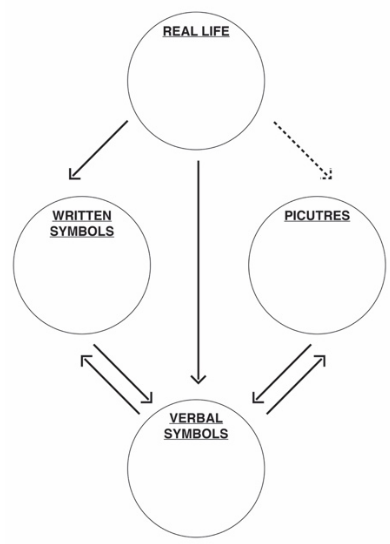
Group 2's Use of Written Symbols



They also extracted an exponential relationship in their graphs, which is different to the answer-solutions provided by the other groups. Figure 39 shows the connections between the different elements in the Lesh Translation Model according to group 7's work on the final project.

Figure 39

Group 7 Solution According to LTM



Discussion on Students' Work

Applicability of Concepts. Formal education in Egypt does not always give students the opportunity to see mathematics as an applied field; instead, students have to memorize certain questions and answers, a requirement that can develop negative attitudes in students. Accordingly, providing students with the opportunity to see how mathematical concepts can be applied, especially in their everyday life, can contribute to students' positive attitude towards mathematics, provided it is presented to them in an interactive and problem-based manner. The notion that such a student needed a course at the undergraduate level in order to fill this perceived gap suggests that he was never exposed to the necessary learning environment to tap into and foster his mathematical thinking capability. Moreover, grade-school mathematics can be considered an applied concept by design (Muschla, Muschla, & Muschla-Berry, 2015), as confirmed by the fact that it took most of this undergraduate years of study to finally learn to see things from a mathematical perspective; an endeavor that is worth exploring further.

Relating mathematical concepts to real life situations is an important element in the Lesh Translation Model, which can actually be considered a means to develop necessary conceptual understanding in mathematics. Accordingly, providing the students with problem-based learning opportunities to create the link between different mathematical representations of mathematics with real life situations can foster their metacognitive skills. This is because such opportunities allow students to reflect on how they can learn mathematics through applying the skills and connections (Bol, Campbell, Perez, & Yen, 2016). If students can developed these skills throughout the course, then their conceptual understanding of mathematics will have improved, raising the need to cross-reference this progress with an analysis of their work throughout the course through the activities they presented. The Lesh model was a highly effective tool in tracking the thought process and solution development of the students, painting a significant picture of the group intricacies involved in producing mathematical answers through various activities.

Algebraic Thinking. Algebraic thinking can be described as a developed skill given that it is a component of analysis and pattern-finding rather than essentially problem solving (Radford, & Peirce, 2006). The practice of establishing patterns draws out skills in the form of algebraic thinking by creating a sense of ease to approaching problems and finding immediate solutions, which is what some of the students expressed in their reflections.

The course provided the students with the dynamic to establish the necessary conditions to develop their algebraic thinking skills. These conditions included giving the students room to investigate problems freely, a departure from the typical mathematics classroom. As a result, the students perceived the activities to be fruitful, educational, and engaging, as shown in their reflection responses presented in Chapter 5. Many of the students learned new techniques of establishing problems, equation creation, plotting graphs, and many other mathematical concepts and strategies. This shows that the environment created cultivates a potential setting for students to truly enjoy learning and problem solving within a mathematics context, previously considered by the students as a subject of non-interest. Students' solutions referenced to all the concepts they learned, and from the activities presented along with other tasks in the course, their mathematics skills were able to progress, especially for students with non-mathematical backgrounds. The answers of the students varied tremendously from one another in both activities, as the Lesh Translation Model maps show more connections between translations, and more solid arrows between the translations were mapped out. While some of the methods were similar in scope, the way each group went about it differed in format ranging or assumption value to the order of calculation (that is, initial use of either calculations or a picture).

Graphing. Representing mathematics through graphs and pictures is one of the elements in the Lesh Translation Model. Unfortunately, not all students are able to represent data through graphs. Despite their assumed knowledge of graphing and familiarity with different terms such as slope, students did not appear well-versed in the ability to visualize mathematics using graphs. The representation of graphs is an essential component since it represents many elements of the real world, with numerous applications in science and engineering such as data analysis and information gathering (Delgado & Lucero, 2015).

While many students felt comfortable with graphing, some of them had the opportunity to develop this skill during this course. However, out of the students who felt they needed to develop the skill, some became more confused. The dynamic of graphing and using tables was overwhelmingly evident in Activity 1 where a number of students successfully formulated necessary correlations through written and verbal communication.

Statistical Reasoning. This is a key concept that has become essential in the developmental skills of younger learners. After taking root, the importance of statistical reasoning has progressed to incorporate undergraduate classes due to the increasingly important notion to foster statistically literate individuals. Real world applications of the concept consider all realms of mathematics, since statistics represents a large span of mathematics in both the business and natural world (Garfield, Ben-Zvi, Chance, Medina, Roseth, & Zieffler, 2008).

In the final project activity, the students chose an issue that presented a problem in their society. From there they needed to extract information derived from various sources regarding the problem, including collecting information leading to establishing a solution. The students in all of the activities were able to establish the correlation needed from amongst the information they extracted and the solution they needed to find; however, in several cases they lacked the necessary knowledge to formulate the necessary calculations for the desired solution. The responses showed that the students knew how to select data based on its importance, as well as the different types of data to use and the significance of each.

Verbal Communication. Due to the group format of the classroom, the students initiated all of their solutions with verbal communication amongst each other. While some showed more animation than others, the general trend that emerged was that both the written and picture aspects of a solution would be formulated after a brief discussion within the group. This was more apparent in Activity 5 in which all the students used more problem-related writing than the visual segment. This can be seen in the minor difference between Activity 1 and Activity 5, since some of the groups immediately went to the picture offered in Activity 1 to pursue a solution to the problem. The latter involved a counting process for each dot, density per scale, or multiple scales in order to find the average. Each method was unique and independent in its own way, a clear manifestation of the student dynamic in an environment that embraces creative problem solving.

Chapter Seven: Discussion

While issues of applying PBL in different mathematics classrooms across the globe have been addressed and discussed, this study addresses issues in an understudied context (an undergraduate course in an Egyptian university in this case. Results of this study suggest that the subscales of mathematics-as-a-male-domain and teacher perception appeared to a significant degree in the results presented in the survey. This means that the attitudes students had towards mathematics had undergone a considerable change from the beginning of the course till the end. This change was dwelt upon at length in the reflections of the students where they mentioned self-awareness of their own change in attitude towards the course; the main reason cited was their relationship with their instructor as well as the nature of the teacher. The implications of the results shown by the two segments of the results magnify the great importance of the teacher in terms of the development of students. While matched pairs t-tests in the (M) and (T) scales resulted in statistically significant change, effect size was not particularly strong, as the effect size for both (M) and (T) subscales were of medium size. The sample size had an impact on how the effect size turned out to be, indicating that this study should perhaps be conducted with a bigger population to show more substantial statistical significance. However, contrast analysis did show that there was a significant change in (M) and (T), indicating that these subscales are worthy to investigate in further studies.

The results revealed that math-as-a-male domain and teacher perception emerged as the two salient subscales marking the students' changed perception. Considering maths-as-a-male-domain as a significant subscale underscores the already patriarchal nature of the Egyptian society. This is reflected in the general lack of female students in Engineering and mathematics-based subjects, a finding that may be attributed to the fact that these majors are considered inherently ‘unfeminine.’ This creates a sharp divide between the untapped potential of many female students and general perceptions towards this subject, severely inhibiting the future opportunities that the youth may receive both in education and in the professional realm. This is one of the reasons why many fields in Egypt, such as construction, are heavily dominated by males, due to the traditional perspective of the masculinity of the job, which may originally stem from the deeply ingrained view of mathematics as a male-driven subject.

One interesting aspect that emerges is that many female students joining the realm of mathematics-based science and engineering generally choose to major in architectural engineering as it is considered a less intense, thus more ‘female-friendly’ scientific area. The perception that changed in the results may create an empowering factor in terms of enabling a favorable environment for women to truly believe that they can rightfully pursue careers and educational paths hitherto regarded as ‘male preserves’. The nature of the course created a sense of equity in the groups, and may have established a domain in which women were able to speak and pursue concepts that they had previously believed both irrelevant and inappropriate to their interests. Although the results of the survey present the significance of the change in the mathematics-as-a-male-domain subscale, this concept is not explicitly referred to in the reflections of the students, indicating that students are not consciously aware of the gendered view of mathematics. Or, we may take it that associating mathematics with men is such a deeply-embedded belief it is simply taken for granted in academic decision-making affecting women in Egyptian society.

According to research by Van Loo and Rydell (2014), females are repeatedly intimidated by males with regards to math-related interactions. Their research examined how females' performance in mathematics can be affected if they watch videos of men behaving dominantly towards women. Women exposed to videos of a dominant male working on a mathematical problem performed much less competently in this area than those who watched videos of a man and a woman of equal authority working on the same task. According to this study, females may be expected to perform satisfactorily provided they are not exposed to intimidation and stereotyping inside the classroom.

The other subscale highly affecting the students and representing one of the core challenges facing education in Egypt is teacher perception. In the typical Egyptian classroom, both students and teachers tend to be deprived of the mutually rewarding learner-instructor relationship that fosters a successful learning outcome; this could be due to the way teachers present themselves as administrators of conventional methods of teaching that do not appeal to students. Another compelling factor is the low remuneration received by teachers at state institutions, an injustice that manifests itself by low value classroom instruction and a reliance on the more rewarding activity of private tutoring. This imbalance has given rise to the alarming spread of low quality education in schools, in parallel with a disproportionate investment of time and money by students' families in private lessons. In a research conducted in a south eastern city in the United States (Rice, Barth, Guadagno, Smith, & McCallum, 2013), 1,552 people participated in a study to measure perceived social support from parents, teachers, and friends as well as respondents' perceived ability and attitudes toward mathematics and science. Examining the social support respondents received from their guardians, instructors, and colleges, along with their perceived state of mind towards both mathematics and science, it was concluded that students who received more support from people around them revealed more positive states of mind perceptions of their capacities in these subjects. Structural equation modeling showed that social support leads to both an immediate impact on mathematical and scientific abilities, and an indirect impact on the ways students perceive these study areas. It could be assumed that students who receive social support in terms of mathematics and science have an improved state of mind towards the two subjects and, in turn, in their belief in their own capabilities in these areas.

Underscoring the importance of students' perceptions of their teachers, the results presented from the study showed that the subscale of teacher perception has a large significance, indicating that students' attitude towards mathematics is strongly influenced by it. The results in the survey showed that the students rely heavily on their teacher for support and development. This was also present in their responses to the reflection in which they considered the teacher to be an essential factor in helping the class become more appealing. The two attributing non-academic factors that complement the subscale of teacher perception are the flexibility of the professor and the student-instructor relationship. The flexibility that the students appreciated mostly alluded to the fact that their opinions were heard in the classroom and that the concept of grade was eliminated, provided, of course, that the students did the classwork. Although the relationship factor that the students considered significant varied, it was based on the rapport that the teacher attempted to develop with the students, along with specific personality traits of the professor. The aspect of the teacher is essential to consider for this study as the varying results signified the importance of the teacher in the classroom, creating a healthy template that can be repeatable for other teachers in different subjects.Teachers play an important role in successful implementation of PBL to present different preparations compared to those under traditional systems (Tomul & Savasci, 2012). Future investigative research into problem-based learning in mathematics needs to focus particularly on the area of algebra.

The reflections presented by the students also revealed their thirst for mathematical capacity, including an increased desire to gain more familiarity with specific concepts. A large portion of the students chose similar concepts that they wished to expand upon, indicating that the reason may be due to the classroom itself. The nature of the classroom, the problems presented, the thinking skills and the concepts are some of the academic factors that triggered the interest of the students. The nature of the course followed quite an unorthodox approach in comparison to conventional Egyptian classrooms in certain details such as group work, classwork-based grading, and open-problem solving. The feedback from the responses with regards to these concepts was mostly positive; negative responses alluded to certain perceived elements of low appeal to the students such as the difficulty level of the problems and, in one case, the noise levels generated by the collaborative group work. While the environment of the classroom was quite unique, one question that needs to be explored is whether or not this template is valid for other classrooms and different subjects.

In their reflection, the students' feedback attributed their academic development to the nature of the course in the details of the activities, especially in terms of the innovative presentation and practice. The nature and accessibility of the concepts set forth, the problems assigned, and the thinking skills involved were the prominent factors discussed by the students in the reflections. Although some of the students have studied these concepts before, the fact that they were presented to them in a new and more dynamic light made them appreciate the material all the more. The type of problems and methods of solving them in the classroom encouraged the students to seek more academic knowledge and pursue new concepts in mathematics.

When the Lesh model is closely examined, it can be understood that its definition of representation shares some similarities with that of Janvier (1987b). Lesh defined representation as external (and therefore observable) embodiments of students' conceptualizations for internal representations. According to Lesh (1979), conceptual understanding relies on students having content-based experiences in each of the representational modes. Shapiro and Williams (2011) built on Lesh' view by adding that understanding in mathematics could be defined as the ability to represent a mathematical idea in multiple ways and to make connections between different representational modes. Lesh (1979) suggested a model for multiple representations of mathematical concepts. This model has five modes of representations: (1) real-world situations; (2) manipulative; (3) pictorial; (4) spoken symbols; and, (5) written symbols. However, for the purpose of this study, I will be looking only at real-world situations, pictures, spoken symbols, and written symbols.

Tomul and Savasci (2012) pointed out that the Lesh model should not be interpreted as a hierarchical model for representation similar to Bruner's model. They particularly emphasized the relationship among the modes of representations and the transformation within one single mode of representation. Lesh indicated that translation requires establishing a relationship (or mapping) from one representational system to another, preserving structural characteristics and meaning in much the same way as in translating from one written language to another. Translation disabilities are significant factors influencing both mathematical learning and problem-solving performance; strengthening or remediating these abilities can facilitate the acquisition and use of elementary mathematical ideas. According to Lesh, in some cases a translation within representational systems can take a plural form; this means that students may begin to solve a problem by making a translation from one representational system to another, and then may map from this representational system to another system; therefore, they can combine more than two representational systems in one problem setting.

This study supported that the Lesh Multiple Representations Theory is based on the premise that students learn the mathematical concepts and build new ones by establishing a meaningful relationship between the new and the old concepts; they can achieve this only by dealing with a variety of representational modes of the concepts, and communicating with these modes of representations. Although it was not the intention to seek evidence supporting the theories considering multiple representations, it is consistent with the learning experiences in the current study (Agee, 2009). Agee (2009) suggests that conceptual learning takes place when students identify a relationship between everyday situations and concrete and abstract representations of a mathematical concept. Lesh, Post, and Behr (1987b) describe how this learning can take place in a process of multiple representations in which students make different relationships among modes of representations. It was possible to observe this process throughout this research study, as students made translations within and between representational modes in the algebra unit. The findings of this study confirm Lesh` theory that multiple representations of mathematical objects are crucial for the learning process of students.

The Lesh Translation Model suggests that elementary mathematical ideas can be represented in five different modes: manipulative, pictures, real-life contexts, verbal symbols, and written symbols. It stresses that understanding is reflected in the ability to represent mathematical ideas in multiple ways, including the ability to make connections among the different embodiments; further, it emphasizes that translations within and between various modes of representation make ideas meaningful to students. Thus, the

Lesh Translation Model extends Bruner's theory by adding to his three modes of representation real-life context and verbal symbols. To show the Lesh Model's emphasis on interactions within and among representations, we can take as an example the arrows connecting the different modes that depict translations between modes as well as the internal arrows depicting translations within modes. The model suggests that the development of a deep understanding of mathematical ideas requires experience both in different modes and in connections between and within these modes of representation. It should be pointed out that translation requires a reinterpretation of an idea from one mode of representation to another (Tabach, 2011). This movement and its associated intellectual activity reflect a dynamic view of instruction and learning.

Although not addressed in this chapter, the Lesh Translation Model can be an effective tool for developing assessment activities. In attempting to achieve the required alignment between instruction and assessment, assessment tasks can be constructed around the translations within and between modes of representation. This allows teachers to assess understanding beyond procedural skill (Trilling & Fadel, 2009). The power of the Lesh Translation Model can be observed in the following multiple-use Models: one for curriculum development; one for classroom curriculum decisions; and another for assessment.

Working with real problems through PBL dispels the idea that learning equates with memorization of concepts transmitted from textbooks. Rather, PBL involves processing the information acquired through the research, and adding new and meaningful understandings, to enable expansion of the investigated knowledge. It also implies abandoning the linear understanding of concepts in favor of gaining knowledge in a process based on various dimensions and variables that both teachers and students should take into account: these dimensions and variables include space, time, access to information sources, research of correct information, all of which are put together in an organized and well-argued way in addition to engaging in certain social skills in terms of shared communication, active listening, and group organization. As discussed in the literature review, PBL has also been implemented in different at all educational stages, with students of differing capacities and ages in almost all the academic disciplines. Provided the teacher is motivated and possesses thorough knowledge of his or her techniques, PBL obtains the best results from all those involved in the teaching and learning process. This is because presenting solid and stimulating content takes students unawares at an affective level, prompting the development of skills needed to develop and thrive in today's complex world. Moreover, PBL can contribute to the curricular organization in addition to defining the teaching (and other) strategies to be applied, whenever the learning objectives demand a deeper understanding. Looking at all the steps in this approach, there seems to be a sufficient basis to believe that PBL is an effective strategy that all teachers should include in their didactic repertoire for new educational practices in the 21st century.

One important aspect of PBL is its capability to support higher order thinking skills among average or highly skilled students. In the PBL system, the students are capable of linking their real life experiences to syllabus-based knowledge, unlike previous learning contexts blocking their ability to reconcile between study areas and real life. Students' improved performance in algebra or lower thinking questions compared to higher thinking questions demonstrates that PBL has the ability to improve the knowledge acquisition and its applications in real life (Tomul & Savasci, 2012).

This information takes us to the essential point of how the problems were solved in the classroom. The non-academic factors encouraging the beneficial nature of the group work created a pathway for students to solve problems through the collective support of the teacher and peers, rather than on their own. The written, verbal, and picture formats were widely used in most solutions in order to present the logic and process that the students wished to elaborate on. One of the main aspects to consider in the results and, therefore, the implications of the dynamic of the Lesh model, was that the cornerstone of solutions was the verbal communication. That is, the group dynamic of the classroom and the activities created a setting in which all the ideas generated and flow of information targeting the problem essentially sprang from this interaction. Had the class placed more emphasis on the independent format, the verbal segment would have not had the same consideration in the solutions. From the verbal communication and the interactive discussions, the students were able to formulate the answers needed, elaborating further on their written responses before expanding to picture response or vice versa. The interactive verbal explanations were instrumental to a sound grasp of the reasoning and logic in approaching the problem, from which they proceeded to answer the problems as a group. Significantly, this method proved beneficial despite its relatively rare presence in conventional classrooms. This counters the general belief regarding the necessity of independent answers and processes in order to facilitate individuality in answers as well as the grading process itself. Ironically, the emphasis on academic independent effort is at odds with current needs of the professional realm; the latter fully endorses team development and collaboration for the sake of business, as more ideas flourish and different ways to problem solving can be established in a group.

It is recognized that teaching and learning are highly dynamic multi-layered processes. The rapid development of scientific and technological knowledge, as well as changes in the economic, social, political and environmental spheres require professionals to have a multidimensional vision of the reality in which they find themselves. In this sense, the educational process needs to advance the introduction of innovative teaching and learning methods that accompany the evolution of knowledge in the training of professionals adept in transferring theoretical knowledge to practice. Thus, PBL presents itself as an innovative method of learning used in several educational institutions of different levels, achieving notable results in learning and developing professional skills. As it is a model that is neither fixed nor closed, PBL can be adapted to the different realities and needs of courses and study contents. Another important issue to be considered, and referred to in the literature on innovative teaching methods, is the need for institutional back-up in terms of incentives, pedagogical support, and infrastructure. Further, educators must recognize that the gains making PBL an effective method, at all levels of education, hinge on its ability to achieve educational objectives broader than those attained by traditional teaching methodologies. In addition to producing knowledge, and developing skills and attitudes, PBL can help students reach success in their academic and professional careers. PBL is also a method that enriches the work of the teachers by encouraging them to follow the process of investigation developed by the students and to track the ways in which they arrive at the solution to the problems. Thus, PBL contributes to the development of continuing teacher education, as teachers are encouraged to consider other ways to improve their pedagogical practice in the face of new learning challenges.

To summarize, PBL is an effective method for presenting important learning outcomes, observed by several teachers who use it in their classes as a method of learning, whether in university courses or in middle school. The positive results mentioned by all of these educators are indicative of the real benefits of this method. Students who do not do well in traditional teaching systems can come into their own in a PBL setting, demonstrating better results in their learning because they have become more active and committed. Within the PBL context, it is the students rather than the teacher who are dominant players in the realm of knowledge; they present their results with greater certainty, since they are the fruits of a process of student-generated investigation and reflection as opposed to simply churning out ready answers to questions given by the teacher. In exercising their problem-solving skills and developing a critical analysis of the scenario for understanding and solving problems, students develop the capacity for interrelation and cooperation in group work as they seek information and evaluate its importance to the task at hand. Finally, students develop the capacity for both self-evaluation and evaluation of their peers' performance within the group setting.

While all the data for this study was collected from students, it may be useful if future research could combine data from both students and their teachers, given teachers' impact on shaping students' representation preferences. The teaching strategies and representation types used inside algebra classrooms by teachers as well as the representations conceptualized by the students appear to be areas worthy of future study. During the reflections, some students claimed that they prefer to use an educational mode of representation to solve mathematical problems as it seems to be more mathematical and generally consistent with their familiar learning background. Studies examining the reasons for such a belief, including the extent of the teacher's role in forming such a notion, may offer a deeper level of investigation after this study. Multiple representation-based instructions can be fully replicated in small groups as students deal with representational modes requiring interactive discussions with group members. Besides, the replications of this study can be conducted with a random sample so that the results could be generalized over a wider population. Another method of gaining a deeper understanding of the theory of multiple representations could be qualitative case studies. Since focusing solely on students` interaction with external representations can be insufficient to conceptualize the entire process of learning in multiple representations, there is a need to analyze the `inside` of young learners' minds, including their creative processes in representational contexts, and internal representations of students. In the present study interviews were conducted with a qualitative and quantitative data collection purpose; however, there is a need for structured task-based interviews to shed light on the ways in which students create new representations, use the representational modes in problem solving, and demonstrate these representations in mathematical situations. It would also be interesting to examine more closely students' behaviors when they are dealing with multiple representations in mathematics. Although multiple representation-based approaches can be implemented to every topic in mathematics, further studies could also be conducted beyond the course. Possible studies in this area could look more closely at gender issues in representation preferences, for example. It was beyond the scope of this study to investigate the gender differences in students` preferences regarding the representational modes. However, it would be interesting to seek evidence determining whether or not girls are, for example, more likely to prefer visual representations than boys. Another research-worthy area might be students` responses and behaviors within a problem-solving group setting that is devoid of teacher guidance.

Nowadays, many attempts can be observed to improve mathematics instruction. Multiple representation-based instructions for conceptual understanding represent such an endeavor that was successfully implemented. Given the benefits of using this method, mathematics educators should recognize the value of making connections between concepts in delivering mathematics instruction to students Indeed, new instructional methodologies such as multiple representation-based instructions can empower students to become better mathematics learner.

Limitations of the study

Generally speaking, in any research there are certain limitations according to the nature of the areas under investigation, the methodology chosen, the techniques used to collect the data, and analysis of the results, among others. Despite being considerable, these limitations affected neither the depth nor the progress and completion of the research (Suleman & Hussain, 2014).

The standard of education in Egypt has diminished tremendously in recent decades. This decline is mainly due to thought-numbing curricula, overcrowded classrooms, and a 'canned' type of instruction that emphasizes rote learning (Sobhy, 2012). Under such stifling conditions, students find it hard to become engaged in school subjects, beyond the single goal of achieving the highest grades possible. In fact, the score-driven nature of the Egyptian educational system is ultimately manifested in the final grade requirement set annually by the Office of University Assortment, a government organ that determines the minimum grades required to gain admission into the various colleges in the country's universities. This system best serves those students who by some miracle have managed to achieve impossibly high examination scores, on which basis they are deemed worthy of joining the elite schools, namely, the Medical and Engineering colleges of Egypt's premier universities. 'Lesser' students can also pursue an undergraduate degree, but face stringent limitations in terms of the university and the college willing to accept them.

This scenario applies to all students wishing to enroll in a public university, but who have not achieved the minimum grades set by, as previously mentioned, the Assortment Office, which change annually depending on a quota. The hardships facing comparatively 'low-scoring' students include the necessity of relocating to attend distantly-situated universities, away from their families, in order to spend four or five years studying subject matter that was technically imposed on them. The other option, obviously available to the very few families that can afford it, is for students to join a private university, reaching an acceptable grade in their freshman year enabling them to determine their major. All of these factors contribute to the current state of substandard education, generating other issues in the society such as the large numbers of unemployable graduates or school drop-outs, many of whom express their frustration with the system by either withdrawing from the mainstream society or, worse still, embarking on a life of crime.

The dismal picture painted above represents the glaring discrepancy between what education *is* and what it can potentially *become* with the correct means of engagement and support. In the study presented on the 'Fun with Problem Solving" class, the students gave plentiful feedback, the significance of which stems directly from the essence of the course itself. The results presented in the survey administered at the close of the semester show that the students' perception of mathematics and the course in general underwent a change since the start and at the end of the course. To develop a framework for reform in education in Egypt, a lot of research has to be conducted, and building on current literature has to be taken into consideration. However, what tends to happen in reform is the borrowing of policy without contextualizing or localizing; thus presenting solutions that can work elsewhere but not necessarily in a given setting without the necessary contextualization.

Policy Borrowing in Education

The current situation of education in Egypt is in dire need of reformation. As the literature review suggests, Problem-Based Learning has been used in several contexts, usually yielding positive results. This makes all the more sense to assume that PBL would smoothly integrate and succeed in the Egyptian context as well. However, research is needed to develop evidence that any reform efforts would guarantee favorable outcomes. The definition of 'favorable' in this instance is highly fluid, with no standardized expectations across different countries or places.

Policy borrowing across nations has been used in an attempt to raise educational systems. 'Borrowing' as a term has always been used with caution, with commentators preferring a variety of other terms such as 'copying' and 'assimilation', as these verbal options seem more descriptive of the process of implementation of policies (Phillips & Ochs, 2003). Unfortunately, there is a lamentable lack of research culture in Egypt in preparation for policy implementation, which consequently stifles opportunities for substantial.

It is imperative to make a distinction between policy borrowing and policy learning before any attempts to conduct research in the field of education. Policy borrowing aims at looking for best practices to transfer them across educational systems (Raffe, 2011). This is seen as a way of utilizing ideas that have shown success in other contexts, in hopes of establishing successful educational systems in communities that are lagging behind or otherwise low achieving (Raffe, 2011) However, such an approach might not be effective if the accompanying pitfalls are not taken into consideration. The dangers of policy borrowing in education include: a) Assumptions that adopting the same 'successful' policies from an international educational system will lead to the same results; b) Failure to take into consideration all dimensions of the policy, whether economically, socially, or culturally; c) Disregard of evidence that a policy might not work, with policymakers picking and choosing policies that match their opinions (Levin, 2011). Ideally, policy learning takes into consideration the context, culture, national and local histories, and an understanding of the origins of the respective policies before implementing them in a new setting (Lingard, 2011). Policy learning seems to be the logical choice in attempting reform within the Egyptian education model.

A number of institutions have addressed policy reform in Egypt. Unfortunately, these efforts have attempted to borrow policy explicitly, without contextualizing them in the new settings. These efforts are clearly represented in government leaders' references to the Japanese model as well as United States Agency for International Development (USAID) Science, Technology, Engineering, and Mathematics (STEM) Education initiatives and projects. Also of note is Raffe's (2011) argument for policy learning rather than policy borrowing, which outlines the criteria for improvement, the implementation of the interventions, and the correlation between the improvements and the interventions. Raffe also puts forth several ideas to challenge certain claims in the Mckinsey Report (2010), one of the most comprehensive reports on global school reform ever published (Mourshed, Chijioke, & Barber, 2010). Raffe criticized McKinsey's failure to mention other improvement criteria outside of international assessments. Furthermore, Raffe noted that informants of such interventions are stakeholders in the perceived success, casting doubt on their authenticity, credibility, and acceptance from the point of view of some researchers. Raffe made a case for studying non-improving systems since the latter have probably used the same interventions, a point which was neglected by McKinsey. Based on its omission to include a comparison between improving and non-improving systems, the study presented a weak argument for attributing the improvement to the interventions (Raffe, 2011). To avert these pitfalls, Raffe advocated an alternative approach - policy learning. Policy learning calls for contextualization of international policies through using the experiences of others to enrich policy analysis and help policy makers gain insight into the strengths and weaknesses of their own systems (Lingard, 2011; Raffe, 2011). Another argument raised by policy learning promotes examination of not only the successful systems, but also the ailing ones to form a clearer vision of what might go wrong. Another important point is learning from the past rather than relying solely on cross-national learning. Finally, Raffe pleaded for a uniform and appropriate structure of governance to spark greater learning potential (Raffe, 2011). Currently, all reform attempts are based on the idea that if a certain system worked somewhere else, then it must also work in Egypt, without any efforts for contextualization.

Given the above, future policies must cater for the context specific to Egypt. Regardless of any empirical evidence demonstrating positive results in places with demographics and cultures similar to Egypt, policy borrowing should be avoided at all costs in favor of policy learning.

This research was conducted to investigate a practice that can be instigate a policy change in how mathematics is taught in Egypt's classrooms, at all levels. As mentioned, there are limitations and considerations that can prevent this study from being generalizable. With necessary contextualization and consideration, this study can serve as a necessary stepping stone to establish a concrete base in which reform in education, especially mathematics education, in Egypt can take place.

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Appendices

Appendix A: Instrument Used for Quantitative Data

Survey

A strongly agree E strongly disagree

1.	I am sure that I can learn math.	A	B	C	D	E
2.	My teachers have been interested in my progress in math.	A	B	C	D	E
3.	Knowing mathematics will help me earn a living.	A	B	C	D	E
4.	I don't think I could do advanced math.	A	B	C	D	E
5.	Math will not be important to me in my life's work.	A	B	C	D	E
6.	Males are not naturally better than females in math.	A	B	C	D	E
7.	Getting a teacher to take me seriously in math is a problem.	A	B	C	D	E
8.	Math is hard for me.	A	B	C	D	E
9.	It's hard to believe a female could be a genius in mathematics.	A	B	C	D	E
10.	I'll need mathematics for my future work.	A	B	C	D	E
11.	When a woman has to solve a math problem, she should ask a man for help.	A	B	C	D	E
12.	I am sure of myself when I do math.	A	B	C	D	E
13.	I don't expect to use much math when I get out of school.	A	B	C	D	E
14.	I would talk to my math teachers about a career that uses math.	A	B	C	D	E
15.	Women can do just as well as men in math.	A	B	C	D	E
16.	It's hard to get math teachers to respect me.	A	B	C	D	E
17.	Math is a worthwhile, necessary subject.	A	B	C	D	E
18.	I would have more faith in the answer for a math problem solved by a man than a woman.	A	B	C	D	E
19.	I'm not the type to do well in math.	A	B	C	D	E
20.	My teachers have encouraged me to study more math.	A	B	C	D	E
21.	Taking math is a waste of time.	A	B	C	D	E
22.	I have a hard time getting teachers to talk seriously with me about math.	A	B	C	D	E
23.	Math has been my worst subject.	A	B	C	D	E

24.	Women who enjoy studying math are a little strange.	A	B	C	D	E
25.	I think I could handle more difficult math.	A	B	C	D	E
26.	My teachers think advanced math will be a waste of time for me.	A	B	C	D	E
27.	I will use mathematics in many ways as an adult.	A	B	C	D	E
28.	Females are as good as males in geometry.	A	B	C	D	E
29.	I see mathematics as something I won't use very often when I get out of high school.	A	B	C	D	E
30.	I feel that math teachers ignore me when I try to talk about something serious.	A	B	C	D	E
31.	Women certainly are smart enough to do well in math.	A	B	C	D	E
32.	Most subjects I can handle OK, but I just can't do a good job with math.	A	B	C	D	E
33.	I can get good grades in math.	A	B	C	D	E
34.	I'll need a good understanding of math for my future work.	A	B	C	D	E
35.	My teachers want me to take all the math I can.	A	B	C	D	E
36.	I would expect a woman mathematician to be a forceful type of person.	A	B	C	D	E
37.	I know I can do well in math.	A	B	C	D	E
38.	Studying math is just as good for women as for men.	A	B	C	D	E
39.	Doing well in math is not important for my future.	A	B	C	D	E
40.	My teachers would not take me seriously if I told them I was interested in a career in science and mathematics.	A	B	C	D	E
41.	I am sure I could do advanced work in math.	A	B	C	D	E
42.	Math is not important for my life.	A	B	C	D	E
43.	I'm no good in math.	A	B	C	D	E
44.	I study math because I know how useful it is.	A	B	C	D	E
45.	Math teachers have made me feel I have the ability to go on in mathematics.	A	B	C	D	E
46.	I would trust a female just as much as I would trust a male to solve important math problems.	A	B	C	D	E
47.	My teachers think I'm the kind of person who could do well in math.	A	B	C	D	E