

1 **Traffic Flow Variation and Network Structure**

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1

## Abstract

2 This study defines and detects competitive and complementary links in a complex network and  
3 constructs theories illustrating how the variation of traffic flow is interconnected with network  
4 structure. To test the hypotheses, we extract a grid-like sub-network containing 140 traffic links  
5 from the Minneapolis - St. Paul highway system. We reveal a real-world traffic network comprises  
6 both competitive and complementary links, and there is a negative network dependency between a  
7 competitive link pair and a positive network dependency between a complementary link pair. We  
8 validate a robust linear relationship between standard deviation of flow in a link and its number  
9 of competitive links, its link correlation with competitive links, and its network dependency with  
10 both competitive and complementary links. The results indicate the number of competitive links  
11 in a traffic network is negatively correlated with the variation of traffic flow in congested regimes  
12 as drivers are able to take alternative paths. The results also signify that the more the traffic flow of  
13 a link is correlated to the traffic flow of its competitive links, the more the flow variation is in the  
14 link. Considering the network dependency, however, it is corroborated that the more the network  
15 dependency between a link and its competitive links, the more the flow variation in the link. This  
16 is also true for complementary links.

17 **Keywords:** Traffic flow variations; Reliability; Competitive links; Weight matrix; Network  
18 structure

## 1 INTRODUCTION

2 In investigating traffic system data, an increase in the reliability of traffic links boosts the con-  
3 fidence of system users that the average traffic conditions, such as traffic flow and travel time,  
4 represent the typical traffic conditions (1). This interrelationship derives from the statistical ex-  
5 pression that the higher the variability of the data, the lower the system reliability becomes. As a  
6 result, the road network reliability marries the statistical concept of traffic conditions variability,  
7 which has become a substantial component regulating commute route, departure time, and trip  
8 linking decisions (2).

9 Traffic flow variability deteriorates the road network reliability by perplexing the task of  
10 monitoring and controlling the operation of road networks and doubling travel uncertainty (3). The  
11 road network reliability requires understanding the changing dynamics of traffic flow variation.  
12 Monitoring and alleviating traffic flow variability, hence, has caught the attention of operating  
13 agencies for the sake of avoiding unexpected delays and ensuring a smooth travel under normal  
14 traffic flow fluctuations (4). In parallel, a well-established literature has gradually been documented  
15 that contemplates the variation of traffic conditions as a function of spatial and temporal traffic  
16 network circumstances including incidents, road construction, weather variations, departure time,  
17 fluctuations in demand, and inadequate capacity (5, 6, 7, 8, 9).

18 Concretely speaking, three distinct components cause variations in traffic conditions (10):

- 19 1. Regular condition-dependent variations, which are predictable and a function of time-  
20 of-day, day-of-week, and seasonality,
- 21 2. Irregular condition-dependent variations, which are unpredictable with an irregular stochas-  
22 tic incident source, and
- 23 3. Random variations which, unlike the irregular condition-dependent variations, are not  
24 noticeable for an extended period of time and only affect a single trip.

25 Contrary to irregular condition-dependent variations, regular condition-dependent varia-  
26 tions are expected by drivers, and consequently they perform the necessary adjustments to offset  
27 the added costs. Interpreting these variations at the link-level has manifold practical applications,  
28 including real time dynamic control of traffic systems, congestion and incident detection, and ramp  
29 metering, to name a few.

30 Despite efforts to explain the variation of traffic flows and network reliability, little is known  
31 about the association between traffic flow variations and network structure. This deficiency stems  
32 from the lack of knowledge about the network dependency between traffic links in a complex  
33 network. A real-world traffic network consists of links both in series and in parallel, which are  
34 spatially correlated and either compete with or complement each other. The complementary nature  
35 indicates that vehicles observed upstream at one time interval will be observed downstream at  
36 a later time interval. The competitive nature, however, demonstrates that competitive links bear  
37 a significant proportion of diverted vehicles, when one of them is saturated or closed (11, 12).  
38 We add to the current knowledge of traffic flow variations by building theories describing how  
39 the variation of traffic flow in a link is interconnected with the network structure and its link

1 interdependencies. In particular, we contribute to the literature of network reliability by fulfilling  
2 the following objectives:

- 3 • Is the variability of traffic flow in a traffic link associated with the number of competitive  
4 links?
- 5 • Is the variability of traffic flow in a traffic link associated with link dependency?
- 6 • Is the variability of traffic flow in a traffic link associated with network dependency?

7 Answering these questions bolsters the design and evaluation of operation strategies and  
8 transportation planning. To test the hypotheses, we extract a sub-network of the Minneapolis -  
9 St. Paul highway system. The selected sub-network comprises 140 traffic links with a grid-like  
10 configuration that enables us to understand whether and to what extent the competitive traffic links  
11 impact the traffic flow variation at the link-level.

## 12 **COMPETITIVE AND COMPLEMENTARY LINKS**

- 13 • **Competitive links:** Two links are competitive, if an increase in the cost of one increases  
14 the flow of the other.
- 15 • **Complementary links:** Two links are complementary, if an increase in the cost of one  
16 decreases the flow of the other.

## 17 **DATA AND NETWORK CONFIGURATION**

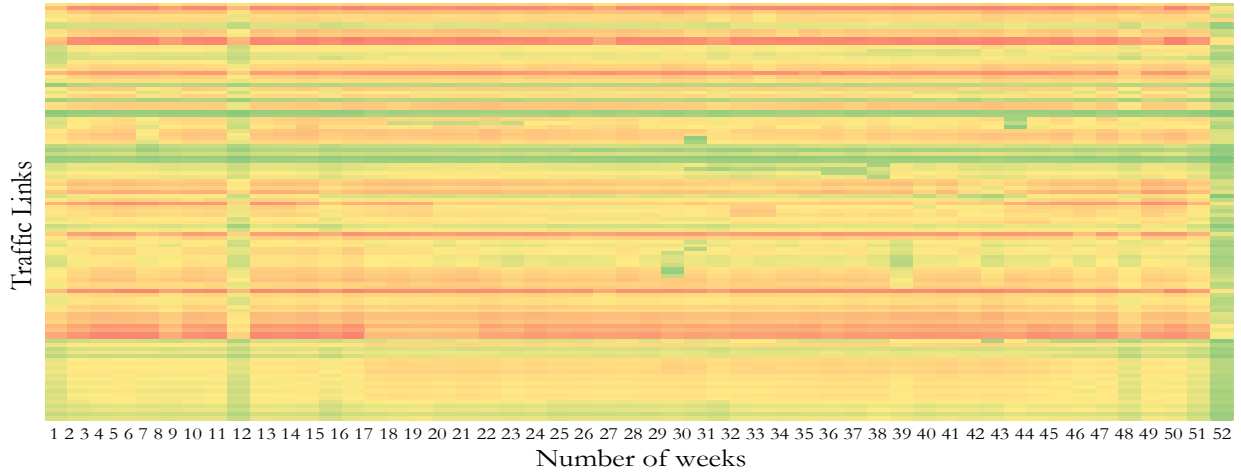
18 We extracted the data from the Minnesota Department of Transportation's Intelligent Roadway  
19 Information System (IRIS), which is an open source advanced traffic management system. The  
20 system contains loop and virtual detectors collecting and recording traffic volume every 30 sec-  
21 onds throughout the Minneapolis - St. Paul highway network. For the purpose of analysis, we  
22 selected a grid-like network topology that includes both competitive and complementary links.  
23 This topology enables us to clearly decompose the role of competitive and complementary links in  
24 a reliability assessment. The selected topology consists of 140 traffic links, which are located in  
25 major highways in the western suburbs, specifically I-494, I-94, I-394, US 169, TH 212, TH 100,  
26 and TH 62 for the East-West and South-North directions.

27 We are of the opinion that the association between network structure and traffic flow vari-  
28 ations is a function of time-of-day and day-of-week. We therefore randomly culled Tuesday as  
29 a day-of-week. We then extracted the traffic flow of selected 140 traffic links over 2015 for the  
30 following time thresholds at a 30-second time interval:

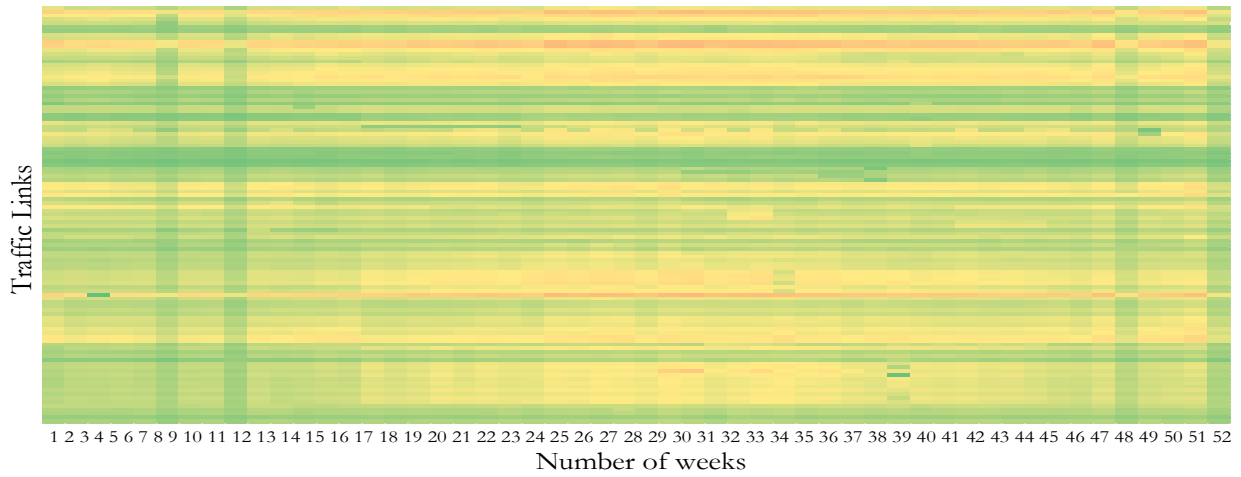
- 31 • Morning rush hour: From 7:30-8:30 AM
- 32 • Morning non-rush hour: From 10:30-11:30 AM
- 33 • Evening rush hour: From 4:30-5:30 PM

1 In the morning rush hour, traffic flow fluctuates between 529 vehicles per hour and 6,388  
2 vehicles per hour, with an average value of 3,107 vehicles per hour over 140 traffic links in 2015.  
3 In the morning non-rush hour, traffic flow ranges from between 182 vehicles per hour and 4,377  
4 vehicles per hour, with an average value of 2,024 vehicles per hour over 140 traffic links in 2015.  
5 Akin to the morning rush hour, traffic network experiences a high level of congestion in the evening  
6 rush hour between 4:30 PM and 5:30 PM. Traffic flow in this time thressed fluctuates between 119  
7 vehicles per hour and 6,999 vehicles per hour, with an average value of 3,545 vehicles per hour.

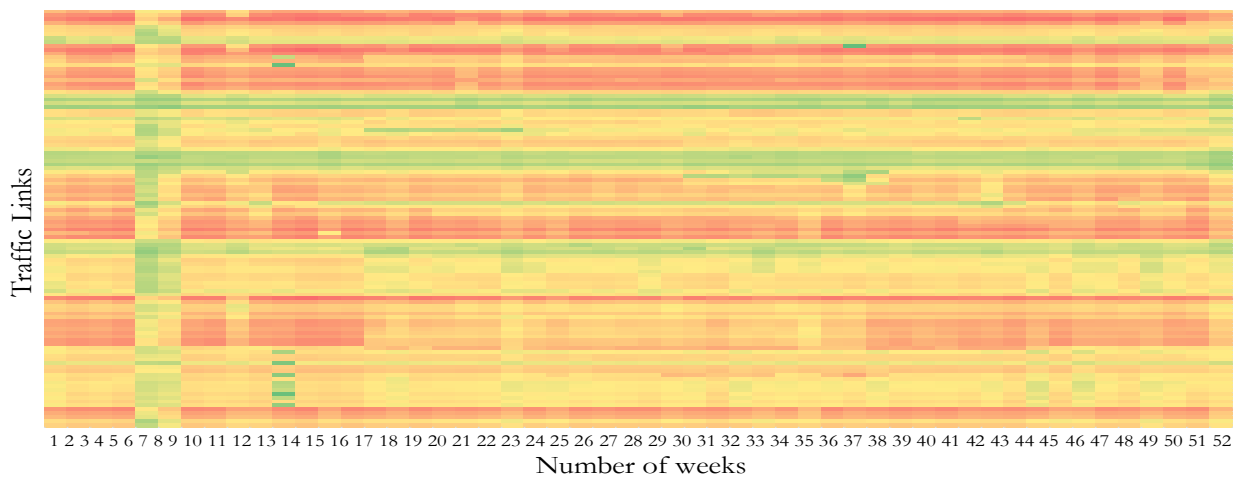
8 To give the reader a sense of traffic flow variations in each traffic link over a year, we draw  
9 the profile of traffic flow in 140 links for all Tuesdays of 2015 in Figure 1. To depict this figure,  
10 we smoothed the traffic flow of each link over an hour. This resulted in 52 observations for each  
11 traffic link as we extracted the traffic flow of each link over all Tuesdays of 2015. This figure  
12 also schematically compares the traffic flow between selected times of day. The color spectrum  
13 changes from green, representing the lowest 10 percentile of traffic flow values, to red, representing  
14 the highest 10 percentile of traffic flow values. As traffic flow increases in value, the green color  
15 approaches yellow, which indicates the 50 percentile of the values. With higher traffic flow, the  
16 yellow spectrum then becomes red. Looking at Figure 1, it is found that the traffic flow in the  
17 evening rush hour is significantly greater than morning rush hour, and of course non-rush hour.  
18 Looking at horizontal lines, it is inferred that the variation of traffic flow over a year is remarkable  
19 in some traffic links, particularly in rush hours.



(a) Morning rush hour: 7:30-8:30 AM



(b) Non-rush hour: 10:30-11:30 AM



(c) Evening rush hour: 4:30-5:30 PM

**FIGURE 1** : Profile of traffic flow in 140 links for all Tuesdays of 2015

1 **ANALYSIS OF RESULTS**

2 Following the definition of competitive and complementary traffic links and detecting them in the  
 3 Minneapolis - St. Paul highway system as described in (11), this section tests the three hypotheses  
 4 postulated earlier in the paper. To measure flow variation in a traffic link, we simply calculate the  
 5 standard deviation of traffic flow throughout 2015 for each link. The traffic flow is smoothed over  
 6 an hour for all three time intervals. For example, for the morning rush hour, we aggregated traffic  
 7 flow of links between 7:30 AM and 8:30 AM for all Tuesdays of 2015. This resulted in 52 flow  
 8 observations for each link. We then calculated standard deviation of traffic flow of each link over  
 9 a year. The following subsections represent the results of analysis for each hypothesis.

10 **Variability and Competitive Links in Numbers**

11 The first hypothesis asserts that the variability of traffic flow in a link is negatively correlated with  
 12 the number of competitive links. In other words, a traffic link with more associated competitive  
 13 links experiences less traffic flow fluctuation. The reason is more alternative paths are available  
 14 for drivers to take, which limits the variability of traffic flow in the link. We regress the traffic flow  
 15 variation in each link against the number of competitive links associated with the study link, which  
 16 is obtained from the three-dimensional temporal detrending algorithm. Table 1 outlines the results  
 17 of the regression analysis for all three time intervals.

**TABLE 1** : Results of regressing traffic flow variations against number of competitive links

Model	Time of Day	Variable	Coefficient	t-test	P-value	Adjusted $R^2$
1	Morning	No. Competitive Links	-2.34	-1.68	0.09	0.02
		<i>Constant</i>	485.16	6.53	0.00	
2	Non Rush	No. Competitive Links	0.19	0.14	0.88	0.00
		<i>Constant</i>	254.05	4.44	0.00	
3	Evening	No. Competitive Links	-3.29	-2.08	0.00	0.03
		<i>Constant</i>	617.62	6.90	0.00	

18 The results indicate a negative correlation between the variability of traffic flow in a link  
 19 and its number of competitive links in both morning and evening rush hours, as congestion causes  
 20 traffic flow to switch to competitive paths. This correlation is stronger in evening rush hours. We  
 21 speculate that traffic demand on Tuesday evenings is significantly higher than Tuesday mornings,  
 22 resulting in a stronger correlation between the number of competitive links and the variability of  
 23 traffic flow. Comparing the average traffic flow of both Tuesday evenings and Tuesday mornings  
 24 in the selected sub-network, we corroborate our hypothesis. The average traffic flow over all links  
 25 on Tuesday evenings equals 3542 vehicles per hour, while this number equals 3105 vehicles per  
 26 hour for Tuesday mornings. However, we have not found a significant correlation between the  
 27 variability of traffic flow in a link and its number of competitive links during the non-rush hour  
 28 period. Unlike the rush hour period, in which the high level of congestion causes traffic flow to  
 29 switch to the competitive paths, the low level of traffic congestion during non-rush hour makes  
 30 switching to the competitive paths unnecessary. As a result, the variation of traffic flow in a link is  
 31 no longer a function of competitive links.

## 1 Variability and Link Dependency

2 The second hypothesis declares that the variability of traffic flow in a link is negatively correlated  
3 with the link dependency in the network. The magnitude of correlation, depends on when the  
4 competitive link dependency or complementary link dependency is taken into the consideration.  
5 The link dependency is captured in two steps:

6 • **Step 1:** We correlate the traffic flow of a link with other links.

7 • **Step 2:** To calculate competitive (complementary) link dependency, we sum the correla-  
8 tions between the study link and its competitive (complementary) pair.

9 The competitive and complementary links dependencies are formally defined by Equation  
10 1 and Equation 2, respectively.

$$L_i^- = \sum_{j=1}^J Cor(\tilde{q}_i, \tilde{q}_j) \quad (1)$$

$$L_i^+ = \sum_{k=1}^K Cor(\tilde{q}_i, \tilde{q}_k) \quad (2)$$

11 Where:

12  $L_i^-$  ( $L_i^+$ ) = Competitive (complementary) link dependency

13  $\tilde{q}_i$  = Vector of traffic flow in link  $i$

14  $J$  ( $K$ ) = Number of competitive (complementary) links to link  $i$

15 The formula helps explains the variation in the flow of traffic. This variation ranges from  
16 0 to 1, which shows the completely independent and completely dependent links, respectively.  
17 Obviously, the complementary link dependency is higher than competitive link dependency as the  
18 former includes upstream and downstream links, in which the same vehicles are observed multiple  
19 times in a certain time period. To test the association between traffic flow variation and link depen-  
20 dency, we regress the link dependency derived from Equation 1 and Equation 2 against the standard  
21 deviation of traffic flow over 2015. To juxtapose between the competitive and complementary links  
22 dependencies, we develop three different models for each time-of-day interval: (1) A model with  
23 only competitive link dependency, (2) A model with only complementary link dependency, and (3)  
24 A model including both competitive and complementary links dependencies. Table Hypothesis2  
25 depicts the results of the regression analysis for all three time intervals.



**TABLE 2** : Results of regressing traffic flow variations against link dependency

Model	Time of Day	Variable	Coefficient	t-test	P-value	Adjusted $R^2$
1	Morning	$L_i^-$	-8.72	-4.84	0.00	0.17
		Constant	558.06	13.28	0.00	
2	Morning	$L_i^+$	-2.06	-1.40	0.16	0.01
		Constant	451.47	6.94	0.00	
3	Morning	$L_i^-$	-8.54	-4.59	0.00	0.17
		$L_i^+$	-0.58	-0.42	0.67	
		Constant	579.40	8.81	0.00	
4	Non Rush	$L_i^-$	-2.26	-1.32	0.18	0.00
		Constant	210.88	5.25	0.00	
5	Non Rush	$L_i^+$	1.04	1.03	0.30	0.00
		Constant	203.08	3.49	0.00	
6	Non Rush	$L_i^-$	-2.02	-1.16	0.24	0.00
		$L_i^+$	0.84	0.83	0.40	
		Constant	168.22	2.57	0.01	
7	Evening	$L_i^-$	-10.81	-4.09	0.00	0.13
		Constant	644.59	11.98	0.00	
8	Evening	$L_i^+$	2.95	1.47	0.14	0.01
		Constant	326.93	4.34	0.00	
9	Evening	$L_i^-$	-11.04	-4.22	0.00	0.15
		$L_i^+$	3.34	1.79	0.07	
		Constant	527.11	6.23	0.00	

1 The results demonstrate that there is a significant negative correlation between traffic flow  
2 variation in a link and its competitive links dependency in both morning and evening rush hours.  
3 The coefficient of the competitive links dependency in the evening rush hour is greater than the  
4 morning rush hour. However, we have not found a significant correlation between the comple-  
5 mentary link dependency and traffic flow variation. Akin to the association between the number  
6 of competitive links and traffic flow variation during the non-rush hour period, there is not a sig-  
7 nificant correlation between the competitive link dependency and traffic flow variation in non-rush  
8 hour. This echoes the fact that the competitive nature of traffic links becomes meaningful when the  
9 traffic network witnesses the high level of congestion.

#### 10 **Variability and Network Dependency**

11 As alluded to in the preceding section, there is a spatial or network correlation between traffic links  
12 of a network, which shows up after appropriately removing the temporal dependency between traf-  
13 fic links. Not only does this “network dependency” between two links vary according to the sign,  
14 but it also varies according to the magnitude. The positivity and negativity network dependency  
15 identifies competitive and complementary nature of traffic links. However, the magnitude of de-  
16 pendency determines the criticality of the link as it speaks for the extent to which the links of a  
17 network are dependent on the existence of the link in question.

1 The third hypothesis articulates that the variability of traffic flow in a link is positively cor-  
2 related with the network dependency in a traffic network. Concretely speaking, the more network  
3 dependency exhibits the more vulnerability, and consequently the more traffic flow variation. The  
4 network dependency is affected by the number of competitive and complementary links associated  
5 with the link in question as corroborated earlier in the first hypothesis testing. We therefore capture  
6 this effect using Equation 3 and Equation 4.

$$N_i^- = \frac{\sum_{j=1}^J |\hat{s}(l_i, l_j)|}{j} \quad (3)$$

$$N_i^+ = \frac{\sum_{k=1}^K \hat{s}(l_i, l_k)}{k} \quad (4)$$

7 Similar to link dependency, we intend to discriminate between the competitive and com-  
8plementary network dependencies. Hence, we develop three different models for each time-of-day  
9 interval: (1) A model with only competitive network dependency, (2) A model with only com-  
10plementary network dependency, and (3) A model including both competitive and complementary  
11 network dependencies. Table 3 outlines the results of the regression analysis for all three time  
12 intervals.

**TABLE 3** : Results of regressing traffic flow variations against network dependency

Model	Time of Day	Variable	Coefficient	t-test	P-value	Adjusted $R^2$
1	Morning	$N_i^-$	26.64	4.75	0.00	0.17
		<i>Constant</i>	187.85	4.90	0.00	
2	Morning	$N_i^+$	58.94	3.28	0.00	0.08
		<i>Constant</i>	38.54	0.38	0.69	
3	Morning	$N_i^-$	22.71	3.48	0.00	0.17
		$N_i^+$	23.5013	1.18	0.23	
		<i>Constant</i>	84.67	0.88	0.37	
4	Non Rush	$N_i^-$	39.09	2.65	0.00	0.05
		<i>Constant</i>	147.44	3.31	0.00	
5	Non Rush	$N_i^+$	69.38	3.43	0.00	0.09
		<i>Constant</i>	-94.94	-0.90	0.36	
6	Non Rush	$N_i^-$	27.72	1.87	0.06	0.11
		$N_i^+$	58.88	2.83	0.00	
		<i>Constant</i>	-122.18	-1.17	0.24	
7	Evening	$N_i^-$	59.7260	7.79	0.00	0.35
		<i>Constant</i>	69.6136	1.42	0.15	
8	Evening	$N_i^+$	67.1910	2.53	0.01	0.04
		<i>Constant</i>	66.2842	0.45	0.65	
9	Evening	$N_i^-$	58.6040	7.86	0.00	0.39
		$N_i^+$	58.0396	2.74	0.00	
		<i>Constant</i>	-241.744	-1.96	0.05	

1           The results corroborate our hypothesis. Both competitive and complementary network  
2 dependencies are positively correlated with traffic flow variation. The competitive dependency  
3 plays a more significant role than the complementary dependency during the rush hour period.  
4 This role is flipped during the non-rush hour period. Looking at Model 9, it is inferred that the  
5 complementary dependency is more significant than the competitive dependency. Comparing the  
6 goodness-of-fit of the models, we further find that the network dependency describes traffic flow  
7 variation more accurately, when network experiences the higher level of congestion.

## 8 **SUMMARY AND CONCLUSIONS**

9 The increasing need for monitoring and improving the reliability of transport systems has fueled  
10 the interest of researchers and practitioners to examine the variability of traffic conditions. Under-  
11 standing the variability of traffic conditions assists operating agencies to calibrate their capabilities  
12 through the lens of intelligent transportation systems. It is not surprising then that a burgeoning  
13 literature has documented investigating the variation of traffic conditions as a function of temporal  
14 and spatial network characteristics. There is still little evidence, however, linking the variation of  
15 traffic flow with the structure of network. We revealed there is a structural way to deal with traffic  
16 networks, which facilitates using network peculiarities to explain the variation of flow in a scale  
17 free or independent manner.

18           In particular, this study established a connection between traffic network structure and traf-  
19 fic flow variation at the link level on a real-world traffic network, which has the potential of being  
20 used for constructing and controlling road networks, modeling of network travel time reliability,  
21 and boosting reliability of travel in a network. The selected network was a grid-like sub-network  
22 containing 140 competitive and complementary links, which extracted from the Minneapolis - St.  
23 Paul highway system. This research tested how and to what extent network structure explains the  
24 variation of flow in a traffic link. The paper validated a robust linear relationship between standard  
25 deviation of flow in a link and its number of competitive links, its link correlation with competitive  
26 links, and its network dependence with both competitive and complementary links. Through use of  
27 computational results of different time-of-day intervals, this paper confirmed that well-established  
28 relationships between the structure of the network and traffic flow variation are a function of traffic  
29 regimes. We encapsulate the key findings:

- 30           • A real-world traffic network is comprised of both competitive and complementary links.  
31           The network dependency, which is a function of physics of the network, forms a negative  
32           correlation between a competitive link pair and a positive correlation between a comple-  
33           mentary link pair. The magnitude of network dependency is associated with time-of-day,  
34           in which there is a stronger competitive network dependency in congested regimes and a  
35           stronger complementary network dependency in uncongested regimes.
- 36           • The existence of competitive links enables drivers to switch to alternative paths, when  
37           they experience the high level of congestion. Immediately, we revealed the number of  
38           competitive links in a traffic network is negatively correlated with the variation of traffic  
39           flow in congested regimes as drivers are able to take alternative paths. In uncongested  
40           regimes, however, this relationship is insignificant as there is no congestion effect to shift  
41           or stall traffic.

- 1 • The sum of correlations between the traffic flows of each competitive link pair explains  
2 the variation of flow in traffic links. The results signified that there is a negative cor-  
3 relation between the link correlation and the flow variation at the link level. In other  
4 words, the more the traffic flow of a link is correlated to the traffic flow of its competitive  
5 links, the more the flow variation is in the link. This association is insignificant in the  
6 uncongested traffic regimes.
- 7 • The network dependency introduced in the current empirical study identifies the critical-  
8 ity of a link, which is positively correlated with the variation of traffic flow at the link  
9 level. The more the link is physically dependent to its competitive links, the more the  
10 flow variation in the link. This is also true for complementary links. However, the com-  
11 petitive network dependency has remarkably more potential to explain the traffic flow  
12 variation in congested regimes. In uncongested regimes, the potential of competitive net-  
13 work dependency is defeated by complementary network dependency. The reason is the  
14 nature of competitive links becomes trivial in uncongested traffic regimes.

15 As the theories and methodologies introduced in this paper contemplated how the variation  
16 of traffic flow is interconnected with the structure of network, this provides insights and a solid  
17 foundation for future research avenues. The following suggestions are made for further research:

- 18 • This study is devoted to explain how traffic flow variations at the link level are benefiting  
19 from the structure of network. There is, however, much scope to broaden the range of  
20 implementation of these theories and methodologies on other traffic conditions including  
21 travel time and traffic speed.
- 22 • Although this research employed the network dependency with theoretical assertions for  
23 explaining the variation of traffic flow, it is envisaged that the proposed method to extract  
24 network dependency could find a wide range of applications in traffic forecasting during  
25 predictable and unpredictable incidents. Hence, a research avenue is open for further  
26 validation in follow-up traffic forecasting studies.
- 27 • For testing the hypotheses postulated in this research, Tuesday was randomly selected as  
28 a representative of weekdays. This study also targeted to unravel whether there is a sig-  
29 nificant difference between the network dependency in morning rush hour, evening rush  
30 hour, and non-rush hour. Future research may benefit from the methodology introduced  
31 in this research and broadens the conclusions more, if not all, days and times.
- 32 • This study took the long-term flow variation into consideration, as it analyzed the stan-  
33 dard deviation of traffic flow in each link over a year. A research avenue is open for using  
34 testing how the short-term traffic flow variation is explained by physics of network and  
35 its link interdependencies.

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