Essays on Monetary Unions and Insurance

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Dedication

To Krista, with all my love, forever.

Abstract

This thesis consists of two essays, linked by the theme of the provision of incentives for insurance arrangements. The first examines a prominent feature of the U.S. social safety net. Many of the welfare benefits provided by the U.S. government take the form of in-kind transfers, such as SNAP (Food Stamps), and these in-kind benefits are provided both to people who are active participants in the labor market and those who are not, such as retirees and recipients of long-term disability insurance. I show that, under very general conditions, consumption margins for agents that do not work should not be distorted in an ex ante efficient allocation in a standard dynamic Mirleesian framework. However, I then show that by adding home production to the model, and rationale for consumption distortions arises for agents that do not work, thus rationalizing the use of in-kind welfare benefits for retirees and the disabled from an efficiency perspective. The second essay turns to the crisis in the Eurozone, asking why would low debt members of a monetary union choose to bailout members with high debt? I propose a novel mechanism that explains this behavior. The central bank will try to use surprise inflation to devalue the nominal debt of member countries who are net borrowers, as this will tend to decrease consumption inequality between households in debtor countries and creditor countries. Creditor countries can forestall this costly surprise inflation by bailing out debtor countries, as this reduces between-country inequality and dampens the central bank's desire to redistribute. If countries accumulate debt in response bad shocks, the bailouts mimic a risk sharing arrangement between member countries. The model then also offers a new theory of why countries would choose to form a monetary union: a common monetary policy provides the incentives required for risk sharing.

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Chapter 1

Introduction

The design of risk sharing arrangements for private agents frequently poses a simple problem. Insurance against risk ex ante requires that lucky agents take action on behalf of unlucky agents ex post, after uncertainty has been resolved. This necessitates some mechanism to align the interests of lucky agents with those of unlucky agents. For example, suppose that workers face risk to their labor productivity and hence their ability to generate income for themselves. Insurance against this risk requires that high productivity workers earn extra income that can be transferred to low productivity workers. An income tax system that can provide this insurance thus must give high productivity workers a reason to earn this extra income. This thesis is comprised of two essays that consider this basic problem in two different contexts.

The first considers the use of in-kind welfare benefits in the optimal design of social insurance programs. It takes as its point of departure an observation and conjecture of Currie and Gahvari (2007). These authors document that a significant portion of in-kind welfare benefits in the United States are paid out to households that do not work. They contend that this indicates that the primary purpose of these in-kind benefits is not to incentivize labor supply. My work builds on theirs in two ways. First, I confirm their conjecture. In a standard model of insurance against labor productivity shocks, it is not optimal to distort consumption decision margins for agents that do not supply labor, which is exactly what in-kind welfare benefits do. However, I then show that, if goods are used in home production, it is optimal to distort decision margins of agents that are out of the labor force. Whether goods used in home production should be subsidized or taxed relative to other goods is determined by the degree of their complementarity with labor in home production.

The second essay turns to monetary unions. With the German bailouts of Greece during the Eurozone debt crisis in mind, I ask why, if countries form a monetary union, would countries with low sovereign debt choose to offer bailouts to countries with high sovereign debt. I show that, if the monetary union's central bank cannot commit to monetary policy, then it will be tempted to use inflation to redistribute from households in low debt countries to households in high debt countries, as this devalues the payments the high debt households implicitly owe low debt households. Low debt countries can forestall this use of inflation by redistributing to high debt countries directly through bailouts. This initial result treats the debt position of countries as exogenous. I then endogenize the choice of nominal debt by allowing the governments of countries to borrow or save in sovereign debt markets in response to income shocks. In general, countries that receive favorable income shocks will save, and countries that receive bad income shocks will borrow. Countries with good shocks thus become the low debt countries that make a bailout payment to high debt, and thus bad shock, countries. The bailouts mimic a risk sharing arrangement between countries from an ex ante perspective. Given that the rationale for bailouts is the central bank's temptation to use the monetary union's shared inflation rate for redistribution, the monetary union can be seen as a way for countries to credibly commit to insure each other against shocks.

The rest of the thesis is organized as follows. Chapter 2 contains the work on in-kind welfare benefits. Chapter 3 addresses monetary unions. Chapter 4 offers concluding remarks.

Chapter 2

In-Kind Social Insurance Benefits and Home Production

2.1 Introduction

In-kind welfare benefits constitute a significant portion of the U.S. social safety net. In 2012, government spending on Medicare, Medicaid, SNAP (Food Stamps), and primary and secondary education constituted 10% of GDP according to the Treasury Department's Greenbook, and this number has only risen with the full implementation of the Affordable Care Act. Such welfare programs effectively subsidize the goods in which benefits are received relative to other goods. Atkinson and Stiglitz (1976) and later Saez (2002) argue that there is an efficiency reason to use such subsidies to provide labor supply incentives if there are non-separabilities in preferences and private information about individual preferences and labor productivity. However, as documented in Currie and Gahvari (2007), a significant fraction of these in-kind welfare benefits are paid out to households that do not supply labor. The single largest in-kind program in the U.S. is Medicare, which is targeted at retirees. Many enrolled in Medicaid are also recipients of long-term disability insurance payments. In 2012, according to the USDA, 10% of heads of household receiving SNAP benefits were over age 60. In light of these facts, Currie and Gahvari contend that it is difficult to see the goal of these programs as incentivizing labor supply. This chapter shows that Currie and Gahvari's conjecture is correct in the sense that, in a standard Mirrleesian framework, there is no efficiency reason to distort margins between goods when agents do not supply labor, even in the presence of non-separabilities. However, I then show that by adding home production to the standard Mirrleesian framework there is an efficiency reason for such distortions, even when agents are not working in the market.

2.2 A Simple Mirrleesian Model

The purpose of this section is to go over a two productivity type, two good Mirrleesian model to examine when it is optimal to distort the consumption margins of the agents. In particular, I will show that, assuming that only the local downward incentive constraint binds, the consumption margins of the high type are never distorted in the optimal allocation. It may be optimal to distort the consumption margins of the low type. However, if the optimal allocation has the low type not working, then consumption margins for the low type will not be distorted in the optimal allocation either. This result is generalized in the next section of this chapter. Analysis of this simple model provides intuition for the broader result that follows in section 2.3. In particular, I will show that the rationale for distorting the consumption margin of low types is to take advantage of differing degrees of complementarity of different goods with leisure. By offering an allocation for low types that "subsidizes" goods that are substitutes for leisure and "taxes" goods that are complements to leisure, the planner can increase the utility of honest low types more than the utility of high types claiming to be low types because a dishonest high type consumes more leisure than an honest low type. These distortions will thus slacken the incenitve constraint, allowing the planner to provide higher expected utility. However, in the case of an optimal allocation where the low type does not work, the low type and the dishonest high type consume the same amount of leisure and this motivation for distorting the consumption margins disappears.

2.2.1 Model

There is a single time period and a continuum of agents. Each agent receives a productivity type $\theta \in \{\theta_l, \theta_h\}$, where $\theta_h > \theta_l > 0$. The probability that an agent is of type θ_l is given by $\pi \in (0, 1)$. Productivity types are private information of agents. There is a single final good y which may transformed one for one into 2 different consumption goods c_1 and c_2. An agent of type θ may transform labor l into a quantity y of the final good according to $y = \theta l$. The labor supply of agents is not publicly observable; however, consumption and output are. Agents have type-independent preferences over consumption and labor represented by the utility function $U(c_1, c_2, l)$. U is assumed to be increasing in c_1 and c_2 and decreasing in l. Appealing to the revelation principle, substituting labor for output divided by productivity in the utility function, and assuming that only the incentive constraint of the high productivity type is binding, the problem of an information constrained utilitarian social planner in this environment is to choose type-specific consumption and output $c_1^\theta, c_2^\theta, y^\theta$ to solve

$$\max_{c_1^{\theta_l}, c_2^{\theta_l}, y^{\theta_l}, c_1^{\theta_h}, c_2^{\theta_h}, y^{\theta_h}} \quad \pi U\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right) + (1-\pi) U\left(c_1^{\theta_h}, c_2^{\theta_h}, \frac{y^{\theta_h}}{\theta^h}\right)$$

$$\text{s.t.} \qquad \pi[c_1^{\theta_l} + c_2^{\theta_l}] + (1-\pi)[c_1^{\theta_h} + c_2^{\theta_h}] = \pi y^{\theta_l} + (1-\pi) y_t^{\theta_h}$$

$$U\left(c_1^{\theta_h}, c_2^{\theta_h}, \frac{y^{\theta_h}}{\theta^h}\right) \geq U\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)$$

Let λ and μ denote the Lagrange multipliers on the resource constraint and the incentive constraint, respectively. The first order conditions for consumption goods $i = 1, 2$ for low and high θ types are given by

$$c_i^{\theta_l}: \qquad \pi U_{c_i}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right) - \pi\lambda - \mu U_{c_i}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right) = 0 \tag{2.1}$$

$$c_i^{\theta_h}: \quad (1-\pi) U_{c_i}\left(c_1^{\theta_h}, c_2^{\theta_h}, \frac{y^{\theta_h}}{\theta_h}\right) - (1-\pi)\lambda + \mu U_{c_i}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right) = 0 \tag{2.2}$$

(2.1) and (2.2) imply that marginal rates of substitution between goods in the optimal allocation will satisfy

$$\frac{U_{c_1}(\theta_l)}{U_{c_2}(\theta_l)} = \frac{1 - \frac{\mu}{\pi}\frac{U_{c_2}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{U_{c_2}(\theta_l)}}{1 - \frac{\mu}{\pi}\frac{U_{c_1}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{U_{c_1}(\theta_l)}} \tag{2.3}$$

$$\frac{U_{c_1}(\theta_h)}{U_{c_2}(\theta_h)} = \frac{1 + \frac{\mu}{1-\pi}}{1 + \frac{\mu}{1-\pi}} = 1 \tag{2.4}$$

Here, I use the notation

$$U_{c_i}(\theta) = U_{c_i}\left(c_1^{\theta}, c_2^{\theta}, \frac{y^{\theta}}{\theta}\right)$$

for $\theta = \theta_h, \theta_l$. It follows from (2.4) that the consumption margin of the high type is never distorted in the optimal allocation (the marginal rate of substitution is set equal to the marginal rate of transformation, which in this case is 1). (2.3) implies that the consumption margins of the low type will be distorted whenever

$$\frac{U_{c_1}(\theta_l)}{U_{c_2}(\theta_l)} = \frac{U_{c_1}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right)}{U_{c_2}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right)} \neq \frac{U_{c_1}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{U_{c_2}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)} \tag{2.5}$$

There will be a positive distortion to the consumption margin of the low type whenever

$$\frac{U_{c_1}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right)}{U_{c_2}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right)} < \frac{U_{c_1}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{U_{c_2}\left(c_1^{\theta_l}, c_2^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)} \tag{2.6}$$

If $y_l > 0$, then labor supply of an honest low type, $\frac{y_l}{\theta_l}$, will be higher than the labor supply of a high type who reports a low value of θ, $\frac{y_l}{\theta_h}$. Thus (2.6) implies that if an increase in labor supply tends to decrease the marginal utility of good 2 less than the marginal utility of good 1, then the optimal allocation will feature a positive distortion. More succinctly, the planner should tax good 1 relative to good 2 if good 1 is more complementary to leisure than good 2. This will harm a lying high type more than it will harm an honest low type because the lying high type supplies less labor and therefore enjoys more leisure. The distortion thus makes it easier to provide incentives. This sort of motive is exactly what the weak separability condition of Atkinson and Stiglitz (1976) rules out, as separability ensures that no goods are relatively more complementary to leisure than others. Notice, though, that even without assuming weak separability, if $y_l = 0$, then (2.5) cannot hold. This implies that, if in the optimal allocation low types supply no labor, then their consumption margin should not be distorted either. If low types don't work, then a true low type and a lying high type will consume the same amount of leisure, and there is no leverage to be gained against incentives by taxing complements to leisure.

2.3 A General Mirleesian Life Cycle Model

Section 2.2 presented a simple, static model to provide some intuition for why, in an optimal allocation, consumption margins should not be distorted for agents that do not work. This section extends that result to a richer dynamic model that can address periods of labor market inactivity over the life cycle such as retirement or temporary or permanent disability. In particular, I will show that, in the absence of very strong time inseparability in preferences, in any period in which an agent does not work, his consumption margins should not be distorted.

2.3.1 Environment

Time is discrete and finite, indexed by $t = 0, \ldots, T$. The economy is populated by a continuum of ex ante identical agents with mass 1 who work to produce output and consume. In each period t, each agent receives a labor productivity shock θ_t. An agent with labor productivity θ_t can use labor input l_t to produce output y_t according to the linear technology $y_t = \theta_t l_t$. θ_t follows a Markov process with conditional distribution $F_t(\theta_t|\theta_{t-1})$ (all agents have a common seed value θ_{-1} for the process) and support $[\underline{\theta}, \overline{\theta}]$ that does not vary with time. θ_t is private information of the agent; however, output and consumption are publicly observable. In each period, output y_t can be transformed one-for-one into N consumption goods $\{c_{jt}\}_{j=1}^{N}$. Output can also be transfered across periods at a fixed interest rate R. Agents' preferences over labor and consumption are given by

$$\mathbf{E}\left[U(c_{10}, \ldots, c_{N0}, l_0, \ldots, c_{1t}, \ldots, c_{Nt}, l_t, \ldots, c_{1T}, \ldots, c_{NT}, l_T)\right]$$

where U is strictly increasing in all consumption goods, strictly decreasing in all labor supplies, and is twice continuously differentiable.

2.3.2 Planner's Problem

I will set up the Planner's problem in terms of choosing a direct revelation mechanism to minimize the time 0 resource cost of providing a given utility level $\underline{U}$. Let θ^t denote a t-length history of labor productivity shocks and $\Theta^t = [\underline{\theta}, \overline{\theta}]^t$ be the set of all possible t-length histories. An allocation rule is a set of functions $\{\{c_{jt}\}_{j=1}^N, y_t\}_{t=0}^T$ where $c_{jt} : \Theta^t \to \mathbb{R}_+$ and $y_t : \Theta^t \to \mathbb{R}_+$ are Borel measurable. A reporting strategy is a set of measurable functions $\{\sigma_t\}_{t=0}^T$ where $\sigma_t : \Theta^t \to [\underline{\theta}, \overline{\theta}]$. Any reporting strategy implies a set of reported history functions $\{\sigma^t\}_{t=0}^T$ where $\sigma^t : \Theta^t \to \Theta^t$ is given by $\sigma^t(\theta^t) = (\sigma_0(\theta_0^t), \ldots, \sigma_t(\theta_t^t))$. In what follows, I will ignore time subscripts where it will not cause confusion. Define an agent's utility from an allocation rule $\{\{c_j\}, y\}$ as

$$U(\{\{c_j\}, y\}) =$$
$$\mathbf{E}\left[U(c_{10}(\theta^0), \ldots, c_{N0}(\theta^0), \frac{y_0(\theta^0)}{\theta_0}, \ldots, c_{1T}(\theta^T), \ldots, c_{NT}(\theta^T), \frac{y_T(\theta^T)}{\theta_T})|\theta_{-1}\right]$$

and an agent's utility from a reporting strategy σ given an allocation rule $\{\{c_j\}, y\}$ as

$$U^\sigma(\{\{c_j\}, y\}) = \mathbf{E}\Bigg[U\Bigg(c_{10}(\sigma^0(\theta^0)), \ldots, c_{N0}(\sigma^0(\theta^0)), \frac{y_0(\sigma^0(\theta^0))}{\theta_0}, \ldots$$
$$c_{1T}(\sigma^T(\theta^T)), \ldots c_{NT}(\sigma^T(\theta^T)), \frac{y_T(\sigma^T(\theta^T))}{\theta_T}\Bigg) \mid \theta_{-1}\Bigg]$$

An allocation rule is incentive compatible if

$$U(\{\{c_j\}, y\}) \geq U^\sigma(\{\{c_j\}, y\}) \; \forall \; \sigma$$

Note that if σ is the truthful reporting strategy (i.e. $\sigma_t(\theta^t) = \theta_t$), then $U(\{\{c_j\}, y\}) = U^\sigma(\{\{c_j\}, y\})$, thus this requires the optimality of truth telling. Finally, an allocation rule satisfies promise keeping if

$$U(\{\{c_j\}, y\}) \geq \underline{U}$$

I can now formulate the planner's problem, which is given by:

$$\min_{\{\{c_j\},y\}} \quad \sum_{t=0}^{T} R^{-t} \int_{\underline{\theta}}^{\overline{\theta}} \dots \int_{\underline{\theta}}^{\overline{\theta}} \left(\sum_{j=1}^{N} c_{jt}(\theta^t) - y_t(\theta^t) \right) dF_t(\theta_t|\theta_{t-1}) \dots dF_0(\theta_0|\theta_{-1})$$

$$\text{s.t. } U\left(\{\{c_j\}, y\}\right) \geq \underline{U}$$

$$U\left(\{\{c_j\}, y\}\right) \geq U^{\sigma}\left(\{\{c_j\}, y\}\right) \; \forall \, \sigma \tag{PP}$$

Assumption 1. *The expected utility function U is weakly time separable, i.e. $\exists$ real-valued functions $\{G_t\}_{t=0}^{T}$ and V s.t.*

$$U(c_{10}, \dots, c_{N0}, l_0, \dots, c_{1T}, \dots, c_{NT}, l_T) =$$
$$V(G_0(c_{10}, \dots, c_{N0}, l_0), \dots, G_T(c_{1T}, \dots, c_{NT}, l_T))$$

Notice that Assumption 1 is satisfied by recursive prefernces such as Epstein Zin as well as by more conventional utility functions that are additively separable in time. I can now state the main result of section 2.3.

Theorem 1. *Suppose Assumption 1 holds and $\{\{c_j^*\}, y^*\}$ is a solution to (PP). Then for almost every history θ^t s.t. $y_t^*(\theta^t) = 0$ consumption margins are not distorted in θ^t, i.e.*

$$\frac{G_{t,i}(\theta^t)}{G_{t,j}(\theta^t)} = 1 \; \forall \, i, j \in \{1, \dots, N\}$$

The proof of Theorem 1 can be found in the appendix. This confirms the conjecture of Currie and Gahvari (2007): a standard Mirrleesian model does not provide an efficiency reason for distortions to consumption margins for agents that do not work.

2.4 A Simple Mirrleesian Model with Home Production

The issue with the standard Mirrleesian life cycle model of the previous section is that, in the absence of very strong assumptions of time inseparability, it can't deliver distortions to consumption margins during periods in which an agent is not working. This implies it cannot justify the in-kind benefits received by retirees and people on long-term disability under the U.S. social insurance system. To reiterate the analysis of section 2.2, the reason for this is that consumption margins should only be distorted when honest agents and dishonest agents consume different amounts of leisure, in order to take advantage of differing degrees of complementarity. In this section, I will show that introducing home production in a simple Mirrleesian model can break down this result. The home production model is isomorphic to a Mirrleesian model with type dependent preferences. MRS can thus vary across types as in Saez (2002), even when agents do not work. This is true even if the underlying preferences of agents are additively separable, as I show in an example in section 2.4.2.

2.4.1 Model

There are again two productivity types: θ_h and θ_l, with $\theta_h > \theta_l > 0$. There is a single market output good y. An agents with type θ produces y with market labor l_m^θ according to $y = \theta l_m$. y can be transformed one for one into two consumption goods: c_f and c_i. c_f is consumed directly by agents. Agents combine home labor l_h^θ and c_i to produce a home good c_h according to $c_h = f(c_i, \theta l_h^\theta)$. Preferences are represented by the utility function $v(c_f, c_h, l_m, l_h)$. Market output y, and consumption of the market goods c_f and c_i are observable to the planner. Home production and productivity are not. It is convenient to define a type dependent indirect utility functions over market quantities

$$u^\theta(c_f, c_i, l_m) = \max_{l_h} \quad v(c_f, f(c_i, \theta l_h), l_m, l_h) \tag{2.7}$$

It follows from the envelope theorem that the marginal rate of substitution between c_f and c_i for the indirect utility function is given by

$$\frac{u^\theta_{c_f}}{u^\theta_{c_i}} = \frac{v_{c_f}}{v_{c_h} f_{c_i}} \tag{2.8}$$

The utilitarian planner's problem can be formulated in terms of the indirect utility functions. Applying the revelation priniciple, substituting market output divided by productivity for market labor, and assuming that only the incentive constraint of the high type is binding, the planner's problem is given by

$$\max_{c_f^{\theta_l}, c_i^{\theta_l}, y^{\theta_l}, c_f^{\theta_h}, c_i^{\theta_h}, y^{\theta_h}} \quad \pi u^{\theta_l}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_l}\right) + (1-\pi)u^{\theta^h}\left(c_f^{\theta_h}, c_i^{\theta_h}, \frac{y^{\theta_h}}{\theta^h}\right)$$

$$\text{s.t.} \qquad \pi[c_f^{\theta_l} + c_i^{\theta_l}] + (1-\pi)[c_f^{\theta_h} + c_i^{\theta_h}] = \pi y^{\theta_l} + (1-\pi)y^{\theta_h}$$

$$u^{\theta_h}\left(c_f^{\theta_h}, c_i^{\theta_h}, \frac{y^{\theta_h}}{\theta_h}\right) \geq u^{\theta_h}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right) \tag{2.9}$$

Very similarly to the model in section 2.2, the MRS between c_f and c_i for the high θ type in the optimal allocation will satisfy

$$\frac{u_{c_{1t}}^{\theta_h}(\theta_h)}{u_{c_{2t}}^{\theta_h}(\theta_h)} = 1 \tag{2.10}$$

and for the low θ type will satisfy

$$\frac{u_{c_f}^{\theta_l}(\theta_l)}{u_{c_i}^{\theta_l}(\theta_l)} = \frac{1 - \frac{\mu}{\pi}\frac{u_{c_i}^{\theta^h}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{u_{c_i}^{\theta_l}(\theta_l)}}{1 - \frac{\mu}{\pi}\frac{u_{c_f}^{\theta^h}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{u_{c_f}^{\theta_l}(\theta_l)}} \tag{2.11}$$

where μ is the Lagrange multiplier on the incentive constraint. Again, (2.10) implies the consumption margin will not be distorted for high types in an optimal allocation, and, by (2.11), that of the low type will be distorted if and only if

$$\frac{u_{c_f}^{\theta_l}(\theta_l)}{u_{c_i}^{\theta_l}(\theta_l)} \neq \frac{u_{c_f}^{\theta_h}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)}{u_{c_i}^{\theta_h}\left(c_f^{\theta_l}, c_i^{\theta_l}, \frac{y^{\theta_l}}{\theta_h}\right)} \tag{2.12}$$

Notice that the functions on the left and the right hand side of (2.12) are no longer identical, as they were in (2.5). This implies that the inequality can hold even if the arguments of the functions on the left and right hand side are identical. The result of section 2.2 no longer necessarily goes through. In what follows, I will give an example where it explicitly does not.

2.4.2 Example for Home Production Model

Suppose that the utility function $v(c_f, c_h, l_m, l_h)$ has the form

$$v(c_f, c_h, l_m, l_h) = \log(c_f) + \log(c_h) - \alpha(l_m + l_h)$$

where $\alpha > 0$ is a constant, and the home production function f has the form

$$f(c_i, \theta l_h) = \left\{ \gamma c_i^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(\theta l_h)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}$$

where $\gamma \in (0,1)$ and $\sigma > 0$. With these preferences, for a given quantity of c_i, an agent of type θ will choose time spent in home production to satisfy the first order condition

$$\frac{(1-\gamma)(\theta l_h)^{\frac{\sigma-1}{\sigma}}}{\gamma c_i^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(\theta l_h)^{\frac{\sigma-1}{\sigma}}} \times \frac{1}{l_h} = \alpha \tag{2.13}$$

This first order condition leads to the following proposition:

Proposition 1. *Let $l_h(\theta)$ denote the solution to (2.13). Then $\theta \times l_h(\theta)$ is increasing in θ for $\theta > 0$.*

Proof. Let $\theta_1 > \theta_2 > 0$, and let $\tilde{l}_h(\theta_1)$ be such that $\theta_1 \tilde{l}_h(\theta_1) = \theta_2 l_h(\theta_2)$. Define the function

$$\Phi(\theta l_H) = \frac{(1-\gamma)(\theta l_h)^{\frac{\sigma-1}{\sigma}}}{\gamma c_i^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(\theta l_h)^{\frac{\sigma-1}{\sigma}}}$$

Notice that the left hand side of (2.13) can be written as

$$\Phi(\theta l_H) \times \frac{1}{l_H}$$

Next, note that, because $\theta_1 > \theta_2$, it follows that $\tilde{l}_h(\theta_1) < l_h(\theta_2)$. This implies that

$$\Phi(\theta_1 \tilde{l}_h(\theta_1)) \frac{1}{\tilde{l}_h(\theta_1)} = \Phi(\theta_2 l_h(\theta_2)) \frac{1}{\tilde{l}_h(\theta_1)} > \Phi(\theta_2 l_h(\theta_2)) \frac{1}{l_h(\theta_2)} = \alpha$$

Here, the first equality follows from the definition of $\tilde{l}_H(\theta_1)$ and the second equality from the definition of $l_h(\theta)$. Note next that the left hand side of (2.13) is decreasing in l_H (this follows immediately from the concavity of the natural log and the CES production function f). Then given that

$$\Phi(\theta_1 \tilde{l}_h(\theta_1)) \frac{1}{\tilde{l}_h(\theta_1)} > \alpha$$

it must be the case that $l_h(\theta_1) > \tilde{l}_h(\theta_1)$, hence

$$\theta_1 l_h(\theta_1) > \theta_1 \tilde{l}_h(\theta_1) = \theta_2 l_h(\theta_2)$$

Then because θ_1 and θ_2 were arbitrary, this completes the proof. □

Proposition 1 establishes that, in the optimal allocation that solves the planner's problem (2.9), a high type who claims to be a low type will choose to supply more effective labor in home production than an honest low type. As in the standard Mirrleesian model of section 2.2, the consumption margin between final goods c_f and and intermediate consumption goods c_i for the low type in the optimal allocation will satisfy

$$\frac{u^{\theta_l}_{c_f}(\theta_l)}{u^{\theta_l}_{c_i}(\theta_l)} \gtrless 1$$

if and only if

$$\frac{u^{\theta_l}_{c_f}(\theta_l)}{u^{\theta_l}_{c_i}(\theta_l)} \lessgtr \frac{u^{\theta_h}_{c_f}\left(c^{\theta_l}_f, c^{\theta_l}_i, \frac{y^{\theta_l}}{\theta_h}\right)}{u^{\theta_h}_{c_i}\left(c^{\theta_l}_f, c^{\theta_l}_i, \frac{y^{\theta_l}}{\theta_h}\right)}$$

Then, by (2.8), this will hold if and only

$$\frac{\gamma(c^{\theta_l}_i)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(\theta_l l_h(\theta_l))^{\frac{\sigma-1}{\sigma}}}{c^{\theta_l}_f} \times (c^{\theta_l}_i)^{\frac{1}{\sigma}} \lessgtr$$

$$\frac{\gamma(c^{\theta_l}_i)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(\theta_h l_h(\theta_h))^{\frac{\sigma-1}{\sigma}}}{c^{\theta_l}_f} \times (c^{\theta_l}_i)^{\frac{1}{\sigma}}$$

Finally, as per Proposition 1, $\theta_h l_h(\theta_h) > \theta_l l_h(\theta_l)$, hence this will be satisfied if and only if $\sigma \gtrless 1$. Summing up

$$\frac{u^{\theta_l}_{c_f}(\theta_l)}{u^{\theta_l}_{c_i}(\theta_l)} \gtrless 1 \Leftrightarrow \sigma \gtrless 1 \tag{2.14}$$

The parameter σ is the elasticity of technical substitution between goods and effective labor in home production. Goods are substitutes for labor in production if $\sigma > 1$ and complements if $\sigma < 1$. The interpretation of this condition is that high types in essence have more labor to use in home production because of their higher productivity. A tax on goods for home production for low types will thus be more painful for high types than for low types if goods are complements to labor in production and is an efficient way to provide incentives. A subsidy on goods for home production for low types will be more beneficial for low types than for dishonest high types if goods are substitutes for labor in production, and thus provides a cheap way of increasing the utility of low types. Notice, though, that the analysis above in no way depends on $y_m^{\theta_l}$. The sign distortion to the MRS of low types is determined entirely by σ. Hence even if $y_m^{\theta_l} = 0$ in the optimal allocation and the low type does not work in the market, there will be either marginal taxation or subsidization of intermediate goods for home production relative to final goods. A Mirrleesian model with home production can thus provide an efficiency reason for in-kind welfare benefits during retirement and periods of disability. This is nice for two reasons. First, there is a substantial literature following Aguiar and Hurst (2005) that argues that some form of home production that uses labor and goods as input is a significant explanatory factor in the behavior of life cycle consumption patterns. It is thus of independent interest to examine optimal taxation and social insurance design in a model that includes this feature. Second, in-kind welfare benefits in the U.S. seem to subsidize goods that are used in home production (unprepared food through SNAP, home energy through energy assistance programs, housing through Section 8) relative to goods that are consumed directly, which is the margin the Mirrleesian model with home production predicts should be distorted.

Chapter 3

Bailouts, Inflation, and Risk Sharing in Monetary Unions

3.1 Introduction

Since 2008, thinking on the Euro has been inseparably linked to the ongoing Greek debt crisis and the various bailout programs implemented by the Troika. Greece is not unique among countries that have suffered sovereign debt crises in receiving foreign aid; however, as argued and documented by Bulow and Rogoff (2015), the scale of this aid is. Moreover, the governments of other Eurozone members (especially Germany) directly participated in these bailouts, providing rescue funds, as well as the bond purchases and political muscle required for domestic participation in the PSI write off. These governments also acted indirectly through the European Commission and the European Central Bank, all of which reinforces the idea that there is something special about the role of the Euro in these bailouts. Although there is an intuitive appeal to connect the Euro and the Greek bailouts, the exact nature of this relationship has been elusive. Why would a country like Germany, with a stable economy and low sovereign debt, agree to rescue Greece just because the two share a currency? This chapter explores a novel mechanism that rationalizes bailouts of high debt members of a monetary union by low debt members. The incentive for bailouts stems from a commitment problem in monetary policy. I construct a model where the central bank will be tempted ex post to use surprise inflation to devalue sovereign debt, as this will redistribute from low debt countries to high debt countries. Fiscal authorities in low debt countries will rationally choose to engage in redistribution through bailouts as this will forestall redistribution through inflation and the distortions that such redistribution entails. Fiscal authorities in the model issue debt to smooth shocks to their economies. This implies that countries with high debt are countries that have been unlucky, and low debt countries have been lucky. Bailouts redistribute from the lucky to the unlucky, so they function as a sort of risk sharing between the union members and improve the welfare of households from an ex ante perspective. This risk sharing raises the specter of moral hazard. Countries might over-borrow if they anticipate receiving a bailout, leading to higher nominal debt and consequently higher inflation. However, in equilibrium, although countries will borrow more relative to an environment in which bailouts are not allowed, they will increase their borrowing less than one for one with the size of the bailout. Intuitively, there is a permanent income-like response. If a fiscal authority believes it will be bailed out in the future, it will want to increase consumption in its country in both the current period and in the future. It accomplishes the former by increasing its borrowing; it accomplishes the latter by limiting that borrowing increase. This implies that bailouts will work to reduce inequality between countries, which tempers the central bank's incentive to inflate. Interestingly, the opportunity for risk sharing provides a new theory as to why countries might choose to form a monetary union in the first place. The monetary union acts as a commitment mechanism that allows countries to engage in a beneficial risk sharing arrangement. The incentives required for countries to make a positive payment after a good shock are provided by the threat of surprise inflation if they renege. The same forces that give rise to risk sharing in the monetary union, namely nominally denominated debt and lack of commitment in monetary policy, also create a well-known free-rider problem. In such an environment, fiscal authorities tend to over-issue nominal debt because they do not internalize the costs of the surprise inflation this imposes on other members of the union. Countries will necessarily trade off the costs of a free-rider problem and the benefits of risk sharing. I present an example to show that it is possible for risk sharing to dominate this trade off. Given that the benefit to forming a monetary union is access to risk sharing, countries stand to gain more from forming a monetary union if the shocks they face are less correlated. This is a counterpoint to the classic analysis of the effects of cross-country correlation of shocks, wherein countries with highly correlated shocks have similar monetary policy needs and face the lowest cost of sharing policy.

The rest of the chapter is organized as follows. Section 2 contains a literature review. Section 3 presents the base line model in 2 steps. First, it introduces a static model in which countries are exogenously creditors or debtors. I use the static model to show that if debtor countries owe creditors a sufficiently large payment, creditors will agree to bailout debtors by forgiving some of their bonds. I then embed the static model into a dynamic model in which countries' asset positions are determined endogenously and show that bailouts improve the welfare of households in the monetary union ex ante through risk sharing and a decrease in inflation. In Section 4, the model is extended to discuss the results on optimal currency areas. Section 5 concludes.

3.2 Literature Review

The analysis contained in this chapter contributes to several strands of literature. The first is recent work on sovereign debt and asset markets in monetary unions. Chari and Kehoe (2007, 2008) consider the free-rider problem that arises in a monetary union when countries issue nominal debt and the central bank cannot credibly promise to avoid surprise inflation. Their analysis focuses on the possibility of solving this free rider problem through restrictions on fiscal policy, such as the debt constraints of Maastricht Treaty. Here, I show how the same factors that give rise to their free-rider problem also provide incentives for risk sharing between countries. Aguiar et al. (2013) also discuss an environment in which member countries of a monetary union issue nominal debt and there is a commitment problem in monetary policy. In their environment, countries face the risk of a debt crisis. If lenders believe the central bank will devalue the union's currency in the event of a crisis to prevent a default, they will not run on the bonds of high debt members and a crisis will not occur. The union needs a sufficiently large number of high debt members for the central bank to agree to inflate in the event of a crisis. However, if there are too many high debt members the central bank will also inflate even if there isn't a crisis. There is thus an interior solution for the optimal debt distribution from the perspective high debt members. I also consider the ability of the debt distribution to act as a disciplinary tool in a monetary union. In their work, it provides the incentives for action in the face of a crisis and tempered policy in normal times; in mine, it enforces payments in a risk sharing arrangement. Farhi and Werning (2012) show that monetary unions give rise to an externality in asset markets. In a monetary union, private agents tend to under insure themselves, as they do not internalize how by insuring themselves, they decrease the heterogeneity in the monetary policy needs of the members of the union. The purchase of insurance augments monetary policy's ability to smooth asymmetric shocks across countries. This externalilty can be addressed by fiscal policy coordination at the union level. I see their work as extremely complementary to mine. They show how a monetary union creates a demand for more insurance, and I show how a monetary union creates the incentives required for more insurance to be provided. The paper most closely related to my work here is that of Chari et al. (2016a). They also consider the possibility that members of a monetary union might engage in bailouts to avert redistributive action. They show that the threat of action by the central bank leads countries to anticipate higher bailouts and issue more real debt, a result that also obtains in my environment. However, their analysis assumes linear utility and does not focus on ex ante welfare, and thus abstracts from the consumption smoothing and risk sharing effects that are key to my welfare results.

This chapter also makes a contribution to the optimal currency area literature, in particular the strand that focuses on the correlation of income shocks of the potential members of the monetary union. The classical analysis of this issue originated in Friedman (1958) and was further elaborated on in Mundell (1961). This work takes as given that there are shocks which require that action be taken by the monetary authority. If countries experience drastically different shocks, then they will have drastically different policy needs and hence forming a monetary union will be costly. This implies that countries whose shocks are more highly correlated benefit the most from forming a monetary union. Two lines of contemporary work rebut this point. The first, following Frankel and Rose (1998) considers the potential endogeneity of the criteria laid out in the Friedman-Mundell logic. This literature uses structural models to estimate the effects of fixed exchange rate regimes (usually the Euro) on the correlation of macroeconomic variables of the countries involved. It does not seek to counter the essence of the Friedman-Mundell argument, but augment it by showing that, if countries form a monetary union, their economies will integrate and shocks to which monetary policy should respond will become more positively correlated. Countries that might not seem like an optimal currency area ex ante become an optimal currency area ex post. The second counter is that Friedman-Mundell assumes commitment to monetary policy. If there are shocks to which monetary policy ought not respond ex ante but will ex post because of a temptation for surprise inflation, as in Barro and Gordon (1983), then countries with less correlated shocks stand to gain from forming a monetary union, as the monetary authority in a union simply will not be able to respond to all of the individual shocks, temptation or not. Friedman (1973) and Alesina and Barro (2001) discuss this effect in the context of dollarization, in which a country effectively adopts the currency of another whose central bank does not weight its welfare and will not respond to its temptation shocks. Chari et al. (2016b) extend this analysis to an environment in which the central cares about all countries. For an excellent survey of the both the classical and contemporary literature on optimal currency areas, see Dellas and Tavlas (2009). I see my work as a third alternative for a counterweight to the original Friedman-Mundell logic. Less correlated shocks allow for more risk sharing, which is the benefit of monetary unions I seek to highlight here.

3.3 Model

In this section, I introduce a model of a monetary union with bailouts. I proceed with this exposition in two steps. First, I present a static model of a monetary union in which the nominal asset positions of countries are determined exogenously. I use this static model to show how bailouts and the inflation rate are determined given nominal asset positions. Next, I embed this static model as the second period of a dynamic model in which the asset positions of countries are determined endogenously. I then use the dynamic model to derive the first key result of the current chapter, that allowing for bailouts improves the ex ante welfare of the members of the monetary union.

3.3.1 Static Model

There is a continuum of countries of measure 1, indexed by $i \in [0,1]$ that form a monetary union. Each country is populated by a fiscal authority and a representative household. I follow Aguiar et al. (2013) and assume that the representative household has preferences over a single consumption good c and the union-wide inflation rate π represented by a separable utility function

$$u(c) - \psi(\pi)$$

where $u : \mathbb{R}_{++} \to \mathbb{R}$ and $\psi : \mathbb{R}_{+} \to \mathbb{R}$. The fiscal authority acts to maximize the utility of the representative household in its country. The function ψ is meant to capture the distorting effects of inflation. The inflation rate π is set by the monetary union's central bank. The inflation rate represents the growth rate of the price level between the time period of the static model and some previous period, not included in the static model. Normalizing the price level in this previous period to one, the price level in the static model is just the gross inflation rate $1 + \pi$.

All countries receive a common, deterministic endowment of the consumption good y. Fraction $\frac{1}{2}$ of these countries are debtor countries, and the remaining fraction are creditor countries. The fiscal authority in each debtor country must make a nominal payment equal to B, and the fiscal authority in each creditor country is owed a nominal payment equal to B.[1] Fiscal authorities in debtor countries finance their payments with a lump sum tax on the endowments of their respective households, and the payments received by the fiscal authorities in creditor countries are rebated to their households through a lump sum transfer. The timing of the model is as follows:

1. Countries receive their endowments y.

2. Fiscal authorities in creditor countries collectively offer a bailout to debtor countries. As in Chari et al. (2016a), this bailout takes the form of an offer of debt forgiveness of some amount Δ. Each creditor country will receive a nominal payment of $B - \Delta$ and each debtor country will make a nominal payment equal to $B - \Delta$.

[1] I think of B as being positive, and indeed B is assumed to be positive in the statement of the results of the static model. However, I will will allow B to be negative in order to extend the domain of the functions that constitute an equilibrium of the static model.

3. Debtor countries collectively choose to accept or reject the bailout. If debtor countries reject the bailout, then $\Delta = 0$ and no debt is forgiven.

4. The central bank sets the inflation rate π.

5. Nominal payments are made and consumption takes place.

The consumption of households in creditor countries, denoted c_H, is given by

$$c_H = y + \frac{B - \Delta}{1 + \pi}$$

and the consumption of households in debtor countries, denoted c_L, is given by

$$c_L = y - \frac{B - \Delta}{1 + \pi}$$

In the next section, I will define and characterize the equilibrium of the static model.

3.3.2 Equilibrium of the Static Model

Equilibrium in the static model will consist of two objects: an inflation rate and a bailout. I begin with the problem of the central bank, which determines the inflation rate. The central bank chooses the inflation rate after the bailout Δ has been determined.[2] The central bank acts to maximize the utilitarian welfare of the households in the monetary union. The central bank's choice of the inflation rate is restricted by non-negativity constraints for consumption and a no-deflation constraint $\pi \geq 0$. Implicitly, deflation is assumed to be sufficiently costly that the central bank would never choose to engage in it. The central bank's problem is then

$$\max_{\pi} \; \frac{1}{2}\left[u\left(y + \frac{B - \Delta}{1 + \pi}\right) - \psi(\pi)\right] + \frac{1}{2}\left[u\left(y - \frac{B - \Delta}{1 + \pi}\right) - \psi(\pi)\right]$$

$$\begin{aligned} \text{s.t.} \qquad & \pi \geq 0 \\ & c_H, c_L \geq 0 \end{aligned} \tag{3.1}$$

[2] This formulation of the central bank's problem implicitly assumes that the central bank cannot commit to monetary policy before bailouts are determined, and hence does not try to discipline the choice of Δ

where $c_H = y + \frac{B-\Delta}{1+\pi}$ and $c_L = y - \frac{B-\Delta}{1+\pi}$. Let $\pi(B,\Delta)$ denote the solution to the central bank's problem.

Assuming that the non-negativity constraints of this problem don't bind, the central bank's first order condition is given by

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}\left[u'\left(y-\frac{B-\Delta}{1+\pi}\right)-u'\left(y+\frac{B-\Delta}{1+\pi}\right)\right]=\psi'(\pi) \tag{3.2}$$

The right-hand side of (3.2) is the marginal cost of inflation from the perspective of the central bank, which is just the marginal disutility of inflation. The left-hand side represents the marginal benefit of inflation, which is its marginal effect on reducing inequality in the utility of consumption between debtor and creditor countries. As inflation rises, the real values of payments of debtors to creditors $\frac{B-\Delta}{1+\pi}$ falls and consumption is redistributed from creditors to debtors. Equation (3.2) makes it clear that the central bank's policy will tradeoff the benefits of redistribution and the distorting costs of inflation.

Next, I turn to the decision of debtor countries to accept or reject a bailout offer Δ. When making this decision, debtor countries will anticipate the actions of the central bank. Their welfare from accepting a bailout of Δ can then be written as a function

$$U_L(B,\Delta) = u\left(y-\frac{B-\Delta}{1+\pi(B,\Delta)}\right)-\psi(\pi(B,\Delta))$$

Debtor countries will accept the bailout if and only if[3]

$$U_L(B,\Delta) \geq U_L(B,0)$$

Finally, I turn to the creditor countries' choice of the bailout. Creditor countries are not allowed to use the bailout to increase the nominal repayment of debtor countries, hence they will face a non-negativity constraint $\Delta \geq 0$.[4] Creditors will make this choice anticipating the decision of debtor countries and the central bank's policy. They

[3] I show later that this participation constraint of debtor countries never binds in the solution to the creditors' bailout problem. However, I have to entertain the possibility that debtors would reject bailouts because, in general, bailouts decrease the inflation rate and could potentially increase the real value of the repayment debtors must make. Also, note that this formulation of the participation constraint implicitly assumes that debtor countries cannot commit to reject bailouts to try to discipline creditors' offers.

[4] Proposition 3 implies that debtor countries would reject any bailout $\Delta < 0$

will choose Δ to solve

$$\max_{\Delta} \quad u\left(y + \frac{B-\Delta}{1+\pi(B,\Delta)}\right) - \psi(\pi(B,\Delta))$$

$$\text{s.t.} \qquad U_D(B,\Delta) \geq U_D(B,0)$$
$$\Delta \geq 0 \tag{3.3}$$

Let $\Delta(B)$ denote the solution to this problem. With the central bank's problem (3.1) and the bailout problem (3.3) stated, I can define equilibrium for the static model.

Definition. *An equilibrium of the static model is a pair of functions $\Delta : \mathbb{R} \to \mathbb{R}_+$ and $\pi : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+$ such that*

i.) *$\pi(B,\Delta)$ solves the central bank's problem (3.1) $\forall$ levels of nominal bonds B and bailouts Δ*

ii.) *$\Delta(B)$ solves the bailout problem (3.3) for all values of B given $\pi(B,\Delta)$*

I will begin the characterization of the equilibrium of the static model with some standard assumptions on the utility functions u and ψ. In particular, I will assume that u is strictly increasing, strictly concave, with a convex derivative; that u satisfies the Inada conditions; and that ψ is strictly increasing and convex, with a first derivative such that $\psi'(0) = 0$. I summarize these assumptions below

Assumption 2. *The functions u and ψ satisfy*

i.) *u is C^3, $u' > 0, u'' < 0, u''' \geq 0$*

ii.) $\lim_{c\to 0} u'(c) = \infty$ *and* $\lim_{c\to\infty} u'(c) = 0.$

iii.) *ψ is C^2, $\psi'(\pi) \geq 0$, with equality if and only if $\pi = 0$,$\psi'' \geq 0$*

With these assumption in place, I can state the first result of the static model

Proposition 2. *Suppose that u and ψ satisfy the conditions of assumption 1. Then*

i.) *There is a unique function $\pi(B,\Delta)$ that solves the central bank's problem.*

ii.) $\forall\ B > 0$ *and* $\Delta \in [0, B]$ $\frac{\partial \pi}{\partial \Delta} \leq 0$ *with equality only if* $\Delta = B$*, i.e. inflation is decreasing in the size of the bailout*

iii.) *If* $\Delta \in [0, B)$*, the real value of debt repayment* $\frac{B-\Delta}{1+\pi(B,\Delta)}$ *is decreasing in* Δ*.*

The proof of proposition 1 can be found in the appendix, but *ii.*) is readily apparent from the central bank's first order condition (3.2). The marginal benefit of inflation for the central bank consists of two terms: the difference in marginal utilities

$$u'\left(y - \frac{B-\Delta}{1+\pi}\right) - u'\left(y + \frac{B-\Delta}{1+\pi}\right)$$

which captures the need for redistribution, and the term

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}$$

which determines the magnitude of the redistribution that inflation achieves and is proportional to $B - \Delta$. An increase in the size of the bailout will thus decrease both the need for redistribution by reducing the spread in marginal utilities, as well as the efficacy of inflation as a tool for redistribution, lowering the marginal benefit of inflation. The central bank then responds by decreasing the inflation rate. This result highlights the motive for creditors to engage in bailouts: redistribution through bailouts prevents redistribution through inflation. Part *iii.*) shows the cost of bailouts for creditor countries, namely the real value of the repayment they receive decreases as the size of the bailout increases. The next proposition establishes some properties of the solution to the bailout problem $\Delta(B)$

Proposition 3. *Suppose that* u *and* ψ *satisfy the conditions of assumption 1. Then*

i.) *The solution to the bailout problem* (3.3) *satisfies* $\Delta(B) \leq B$

ii.) *If* $\Delta \in [0, B]$*, the participation constraint of debtor countries holds with equality* $\Leftrightarrow \Delta = 0$.

Proposition 2 implies that debtor countries will never reject bailouts. Creditor countries will use bailouts to tradeoff the loss of consumption through decreases in the real value of the payment they receive and decreases in the distortions from inflation. Notice that the costs of distortionary taxation potentially required to finance bailouts are absent from this margin. The model implicitly assumes that these costs are small relative to the distortions associated with inflation. This will be true if bailouts take the form of debt forgiveness, which is the interpretation I take here. Such bailouts do not require additional tax revenue, and hence further tax distortions, to finance. Moreover, given that bailouts in the form of debt forgiveness constitute a substantial portion of the Second Economic Adjustment Program for Greece, restricting attention to bailouts in the form of debt forgiveness still allows the model to address a significant amount of what was observed during the Eurozone debt crisis.

To obtain a sharper characterization of the solution to the bailout problem $\Delta(B)$, I will make further assumptions on the the functions u and ψ.

Assumption 3. *The functions u and ψ satisfy*

i.) $\psi'(\pi)(1+\pi)$ *is concave*

ii.) u''' *is non-increasing.*

Condition *i.*) restricts the marginal cost of inflation not to increase too quickly. It will be satisfied by a linear disutility of inflation, as in Aguiar et al. (2013). Assumption 2 implies the final result for the static model

Proposition 4. *Suppose that u and ψ satisfy the conditions of assumptions 1 and 2. Then the equilibrium bailout function $\Delta(B)$ is unique and has the form*

$$\Delta(B) = \begin{cases} 0 & B \leq \bar{B} \\ B - \bar{B} & B > \bar{B} \end{cases}$$

for some $\bar{B} \in (0, \infty)$

Proposition 3 establishes that if the initial nominal payment owed by debtor countries to creditor countries B is sufficiently large, then creditor countries will find it optimal to make a strictly positive bailout. Moreover, there is an optimal value of the nominal payment that debtors make to creditors, $\bar{B}$. Creditor countries will reduce the nominal payment to $\bar{B}$ whenever $B > \bar{B}$, and the non-negativity constraint for bailouts will bind whenever $B < \bar{B}$. The role of assumption 2 in obtaining this result is to guarantee that, as the creditor countries reduce the real repayment they receive by making bailouts, these reductions have an ever-diminishing effect on the inflation rate, hence the marginal benefit of the inflation reduction must decrease until it is eventually equal to 0. This implies the existence of an optimal repayment from the perspective of creditors. I now move onto the analysis of a dynamic model in which nominal debt and repayments are determined endogenously. The dynamic model will have two periods: a first period in which countries either buy or sell nominal bonds in response to income shocks, and a second period, in which bailouts and inflation are determined as in the static model.

3.3.3 Dynamic Model

The environment for the dynamic model is as follows. There are two time periods indexed by $t = 1, 2$. There is a continuum of countries of mass 1, indexed by $i \in [0, 1]$, that form a monetary union. Each country consists of a representative household and a fiscal authority.

The representative household in country i has preferences over consumption of a single good in periods 1 and 2, as well as the inflation rate between periods 1 and 2, represented by the expected utility function

$$\mathbf{E}[u(c_{1i}) + \beta[u(c_{2i}) - \psi(\pi)]]$$

where c_{1i} and c_{2i} a consumption in periods 1 and 2, respectively, and π is the union-wide inflation rate.

The fiscal authority in each country is benevolent and seeks to maximize the utility of its country's representative household. Fiscal authority's buy or sell nominal bonds in a competitive market in period 1, financing purchases through a lump sum tax on households and distributing the revenues of bond sales to households through a lump sum transfer. The monetary union is a closed economy, so the market for bonds must clear in equilibrium. In period 2, fiscal authorities engage in bailouts, repay nominal bonds and collect payments, financing payments through lump sum taxation and distributing payments collected through lump sum transfers.

There is also a central bank, which conducts monetary policy by setting the inflation rate π in period 2. The central bank cannot commit to monetary policy.

In period 1, all countries receive a stochastic endowment of the period 1 consumption good y_{1i}, which can take one of two values: $y_H = y + \epsilon$ or $y_L = y - \epsilon$ for some $\epsilon > 0$.

Realizations of y_{1i} are iid across countries, and the distribution of the shocks is given by

$$y_{1i} = \begin{cases} y_H = y + \epsilon & \text{Prob. } \frac{1}{2} \\ y_L = y - \epsilon & \text{Prob. } \frac{1}{2} \end{cases}$$

In period 2, all countries receive a common, deterministic endowment y. Timing for the dynamic model is as follows:

Period 1

1. Countries realize their endowment shocks.
2. The bond market opens. The price of nominal bonds is q, and fiscal authorities choose their bond positions.
3. Period 1 consumption occurs. Consumption in country i is given by

$$c_{1i} = \begin{cases} y + \epsilon - qb_i & y_{1i} = y_H \\ y - \epsilon + qb_i & y_{1i} = y_L \end{cases}$$

Period 2

1. Countries receive their endowment y.
2. High endowment countries collectively offer a bailout to a low endowment countries. This bailout takes the form of an offer to forgive some amount $\Delta \geq 0$ of the face value of the debt of all countries that received a low endowment shock.
3. Low endowment countries then collectively choose to accept or reject the bailout. If the bailout is rejected, then $\Delta = 0$.
4. The central bank sets the inflation rate π, taking the bailout and the bond positions of countries as given.
5. Bond payments are made and received.
6. Period 2 consumption occurs.

$$c_{2i} = \begin{cases} y + \frac{b_i - \Delta}{1+\pi} & y_{1i} = y_H \\ y - \frac{b_i - \Delta}{1+\pi} & y_{1i} = y_L \end{cases}$$

Note that I write consumption as if countries with high endowments purchase bonds and countries with low endowments sell bonds. This will be the case in equilibrium, but here it is a notational convenience. I allow b_i to be negative for either value of the shock. This would correspond to a sale of bonds by high endowment countries and a purchase of bonds by low endowment countries. The formulation for period 2 consumption normalizes the price level in period 1 to 1, so that the price level in period 2 is the gross inflation rate $1+\pi$. I follow Chari and Kehoe (2016) in the requirement that the value of Δ is common for all high and low endowment countries, regardless of an individual country's choice of its nominal bond position. They provide a justification for this assumption which, in essence, states that such symmetric treatment is optimal if monitoring sovereign debt is costly and imperfect. This seems reasonable in the context of this model, considering the ease with which the Greek government concealed the true value of its debt in the run-up to the Eurozone crisis.

3.3.4 Bailouts and Inflation in the Dynamic Model

In a symmetric equilibrium, the fiscal authorities in all countries that receive a high endowment shock in the first period will purchase an identical amount of nominal bonds B_H and the fiscal authorities in all countries that received a low endowment shock will sell an amount of bonds B_L. Since the monetary union is a closed economy, it follows that $B_H = B_L = B$. Given a bailout of Δ and a value of B, the central bank will choose π to solve

$$\max_{\pi} \ \frac{1}{2}\left[u\left(y+\frac{B-\Delta}{1+\pi}\right)-\psi(\pi)\right]+\frac{1}{2}\left[u\left(y-\frac{B-\Delta}{1+\pi}\right)-\psi(\pi)\right]$$

$$\text{s.t.} \qquad \pi \geq 0$$

$$c_H, c_L \geq 0$$

where $c_H = y + \frac{B-\Delta}{1+\pi}$ and $c_L = y - \frac{B-\Delta}{1+\pi}$. This is identical to the central bank's problem in the static model (3.1), and its solution will coincide with that of the static model, i.e. the central bank will set $\pi = \pi(B,\Delta)$ in response to a bailout of Δ for a given value of bond holdings B. Given that low endowment countries and high endowment countries will anticipate the same policy response from the central bank as in the static model, the bailout chosen will be the same as the bailout chosen in the static model, i.e. for a given value of bond holdings B, the value of the bailout in the dynamic model will be $\Delta(B)$.

3.3.5 Equilibrium of the Dynamic Model

In the first period, fiscal authorities will take the price of nominal bonds q as given and they will anticipate B, the quantity of nominal bonds purchased by the representative fiscal authority in high endowment countries and the quantity of nominal bonds sold by the representative fiscal authority in low endowment countries. Given the value of B, they will also anticipate the value of the bailout $\Delta(B)$ and the value of the inflation rate $\pi(B, \Delta(B))$.

If country i receives the high endowment shock $y_H = y + \epsilon$, the fiscal authority will choose b_i to solve

$$\max_{b_i} \quad u(y + \epsilon - qb_i) + \beta \left[u\left(y + \frac{b_i - \Delta(B)}{1 + \pi(B, \Delta(B))} \right) - \psi(\pi(B, \Delta(B))) \right] \tag{3.4}$$

Note that the solution to this problem depends only on q and B, and not on i. Denote this solution as $b_h(q, B)$.Similarly, if country i receives the low endowment shock $y_L = y - \epsilon$, the fiscal authority will choose b_i to solve

$$\max_{b_i} \quad u(y - \epsilon + qb_i) + \beta \left[u\left(y - \frac{b_i - \Delta(B)}{1 + \pi(B, \Delta(B))} \right) - \psi(\pi(B, \Delta(B))) \right] \tag{3.5}$$

Again, the solution of this problem will depend only on B and q, and not on i. Denote the solution to this problem as $b_L(q, B)$. With the functions b_H and b_L defined, I can define equilibrium for the dynamic model.

Definition. *An equilibrium of the dynamic model is a set of functions $b_H : \mathbb{R}_{++} \times \mathbb{R} \to \mathbb{R}$, $b_L : \mathbb{R}_{++} \times \mathbb{R} \to \mathbb{R}$, $\pi : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+$, and $\Delta : \mathbb{R} \to \mathbb{R}_+$ as well as values for the price of nominal bonds q^* and the representative nominal bond position B^* such that*

i.) $\pi(B, \Delta)$ *and* $\Delta(B)$ *are an equilibrium of the static model*

ii.) Given π *and* Δ, $b_H(q, B)$ *solves* (3.4) *and* $b_L(q, B)$ *solves* (3.5)

iii.) q^* *and* B^* *satisfy the fixed point condition* $b_H(q^*, B^*) = b_L(q^*, B^*) = B^*$

In the setup of the dynamic model and the definition of equilibrium, I glossed over where the objective function and constraints of the bailout problem come from, and why they do not change when individual countries deviate from the equilibrium path, choosing $b_i \neq B$. I show in the appendix that the bailout problem derives from a model in which bailouts are determined through majority voting. In the voting model, the bailout is chosen to maximize the utility of the high endowment fiscal authority that buys the median quantity of nominal bonds in the first period, and the bailout will not be rejected if the low endowment country that sells the median quantity of bonds in the first period prefers the offered bailout to no bailout. In a symmetric equilibrium, the representative fiscal authority's bond position B is the median of the distribution of both high and low endowment countries, and individual deviations from the equilibrium path have no effect on the value of the median.

In an equilibrium of the dynamic model, fiscal authorities issue nominal bonds treating the price of nominal bonds and the inflation rate as fixed. It is thus possible to restate their problems as choosing a quantity of real bonds to buy or sell. In this setup, they will choose $\hat{b}_i = \frac{b_i}{1+\pi(B,\Delta(B))}$, taking as given B and the price of real bonds $\hat{q} = q(1+\pi(B,\Delta(B)))$. This formulation is convenient for proving some of the results on the dynamic model, so I will go over these alternative problems and an alternative definition of equilibrium in terms of real bonds and the price of real bonds.

For any price of real bonds $\hat{q}$ and a value of B, fiscal authorities in countries that receive the high endowment shock y_H will choose a quantity of real bonds to buy $\hat{b}_i$ to solve

$$\max_{\hat{b}_i} \quad u(y+\epsilon-\hat{q}\hat{b}_i)+\beta\left[u\left(y+\hat{b}_i-\hat{\Delta}(B)\right)-\psi(\pi(B,\Delta(B)))\right] \tag{3.6}$$

where $\hat{\Delta}(B) = \frac{\Delta(B)}{1+\pi(B,\Delta(B))}$. Fiscal authorities in countries that receive the low endowment shock y_L will choose a quantity of real bonds to sell $\hat{b}_i$ to solve

$$\max_{\hat{b}_i} \quad u(y-\epsilon+\hat{q}\hat{b}_i)+\beta\left[u\left(y-\hat{b}_i+\hat{\Delta}(B)\right)-\psi(\pi(B,\Delta(B)))\right] \tag{3.7}$$

Let $\hat{b}_H(\hat{q},B)$ and $\hat{b}_H(\hat{q},B)$ denote the solutions to (3.6) and (3.7) respectively.

Definition. *A real-valued equilibrium of the dynamic model is a set of functions* $\hat{b}_H : \mathbb{R}_{++}\times\mathbb{R}\to\mathbb{R}$, $\hat{b}_L : \mathbb{R}_{++}\times\mathbb{R}\to\mathbb{R}$, $\pi : \mathbb{R}\times\mathbb{R}_+\to\mathbb{R}_+$, *and* $\Delta : \mathbb{R}\to\mathbb{R}_+$ *as well as values for the price of real bonds* $\hat{q}^*$ *and the representative nominal bond position* B^* *such that* *i.)* $\pi(B,\Delta)$ *and* $\Delta(B)$ *are an equilibrium of the static model*

ii.) *Given* π *and* Δ, $\hat{b}_H(\hat{q},B)$ *solves* (3.6) *and* $\hat{b}_L(\hat{q},B)$ *solves* (3.7)

iii.) $\hat{q}^*$ *and* B^* *satisfy the fixed point condition* $\hat{b}_H(\hat{q}^*,B^*) = \hat{b}_L(\hat{q}^*,B^*) = \frac{B^*}{1+\pi(B^*,\Delta(B^*))}$

To characterize the real-valued equilibrium of the dynamic model, I will make one more assumption.

Assumption 4. *For any value of* B, *the function* $\hat{b}_H(\hat{q},B)$ *is strictly decreasing in* $\hat{q}$ *and the function* $\hat{b}_L(\hat{q},B)$ *is strictly increasing in* $\hat{q}$

Assumption 3 just requires that the demand for real bonds $\hat{b}_H$ and the supply of real bonds $\hat{b}_L$ are downward sloping in the price of real bonds and upward sloping in the price of real bonds, respectively. With assumption 3 in place, the next proposition follows.

Proposition 5. *Suppose that assumptions 1, 2, and 3 are satisfied. Then*

i.) $\exists$ *a unique real-valued equilibrium of the dynamic model*

ii.) *The price of real bonds is given by* $\hat{q}^* = \beta$

iii.) $\hat{b}_H(\hat{q}^*,B^*) = \hat{b}_L(\hat{q}^*,B^*) = \frac{1}{1+\beta}[\epsilon + \hat{\Delta}(B^*)]$

where $\hat{\Delta}(B^*) = \frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))}$

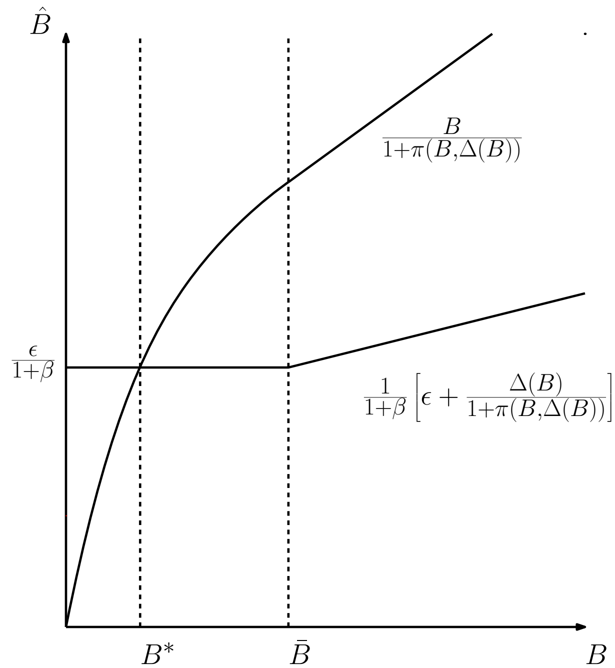

Figure 3.1: Equilibrium in the Dynamic Model

Figure 3.1 shows a plot of the functions

$$\hat{b}_H(\beta, B) = \hat{b}_L(\beta, B) = \frac{1}{1+\beta}\left[\epsilon + \frac{\Delta(B)}{1+\pi(B, \Delta(B))}\right]$$

and

$$\frac{B}{1+\pi(B, \Delta(B))}$$

The horizontal coordinate of the intersection corresponds to the equilibrium value of the representative fiscal authority's nominal bond position B, and the vertcial coordinate is its real value in equilibrium. As per Proposition 4, $\hat{q}^* = \beta$, hence the fixed point condition implies that the equilibrium value of B^* for the dynamic model is determined by the intersection of these curves. $\bar{B}$ is the critical value of B such that

$$\Delta(B) = \begin{cases} 0 & B \leq \bar{B} \\ B - \bar{B} & B > \bar{B} \end{cases}$$

The kinks in the two curves at $B = \bar{B}$ represent the switch from no bailouts in the second period to positive bailouts in the second period. The inflation rate becomes fixed for $B > \bar{B}$, hence $\frac{B}{1+\pi(B,\Delta(B))}$ grows at a constant rate. The bailout encourages low endowment countries to borrow more and high endowment countries to save more, so the curve $\hat{b}_H(\beta, B) = \hat{b}_L(\beta, B)$ starts to increase for $B > \bar{B}$. Notice that the value of B^* in Figure 3.1 is less than $\bar{B}$, which implies that there are no bailouts in equilibrium. As ϵ increases, the curve

$$\frac{1}{1+\beta}\left[\epsilon + \frac{\Delta(B)}{1+\pi(B,\Delta(B))}\right]$$

shifts up, and the value of B^* increases. For ϵ sufficiently large enough, $B^* > \bar{B}$, and there are strictly positive bailouts in equilibrium. This effect can be seen in Figure 3.2.

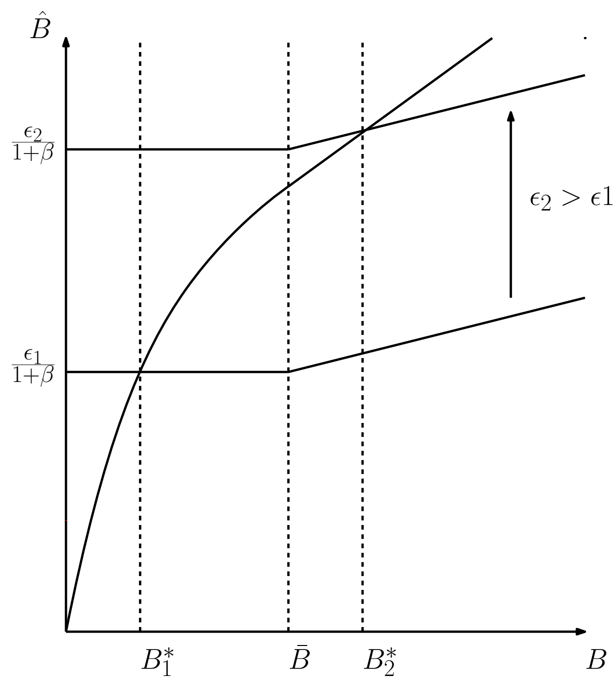

Figure 3.2: Effects of ϵ on Equlibrium

ϵ is the standard deviation of the period 1 endowment shocks, and thus a measure of the risk faced by households in the monetary union. The graphical analysis above suggests bailouts become active as the risk faced by households increases. This offers a preview of the risk sharing benefits of bailouts, which will be discussed in detail in the next section.

3.3.6 Welfare Effects of Bailouts

In this section, I will discuss how bailouts affect the expected utility of households in the monetary union. To do this, I will compare the equilibrium of the dynamic model to the equilibrium of a version of the dynamic model in which fiscal authorities cannot make bailouts. The environment and timing of this model are identical to the dynamic model; The only difference is that there is no bailout stage in the second period.As in the dynamic model with bailouts, in equilibrium all high endowment countries will purchase the same amount of bonds nominal bonds B_H, all low endowment countries will sell the same amount of bonds B_L, and the market for bonds will clear, so $B_H = B_L = B$. The central bank will choose the inflation rate π to solve

$$\max_{\pi} \ \frac{1}{2}\left[u\left(y + \frac{B}{1+\pi}\right) - \psi(\pi)\right] + \frac{1}{2}\left[u\left(y - \frac{B}{1+\pi}\right) - \psi(\pi)\right]$$

$$\text{s.t.} \qquad \pi \geq 0$$

$$c_H, c_L \geq 0$$

which implies that it will set the inflation rate to $\pi(B, 0)$.In the first period, individual fiscal authorities will treat the representative bond position B as given, and hence will treat the inflation rate as independent of their choice of how many bonds to buy or sell. This implies that, as in the model with bailouts, the problem of fiscal authorities can be formulated in terms of buying and selling real bonds. In particular, fiscal authorities in high endowment countries will choose to purchase a quantity real bonds $\hat{b}_i$ to solve

$$\max_{\hat{b}_i} \quad u(y + \epsilon - \hat{q}\hat{b}_i) + \beta\left[u\left(y + \hat{b}_i\right) - \psi(\pi(B, 0))\right] \tag{3.8}$$

Denote the solution to this as $\hat{b}_H^{NB}(\hat{q}, B)$. Similarly, low endowment countries will choose to sell a quantity of real bonds $\hat{b}_i$ to solve

$$\max_{\hat{b}_i} \quad u(y + \epsilon + \hat{q}\hat{b}_i) + \beta\left[u\left(y - \hat{b}_i\right) - \psi(\pi(B, 0))\right] \tag{3.9}$$

Denote the solution to this as $\hat{b}_L^{NB}(\hat{q}, B)$. With these problems stated, I can define equilibrium for the model without bailouts.

Definition. *A real-valued equilibrium of the dynamic model without bailouts is a set of functions $\hat{b}_H^{NB}(\hat{q}, B)$, $\hat{b}_L^{NB}(\hat{q}, B)$, and $\pi(B, \Delta)$, as well as values for the price of real bonds $\hat{q}^*_{NB}$ and the representative bond position B^*_{NB} such that*

i.) $\pi(B, \Delta)$ *is part of an equilibrium of the static model*

ii.) Given π, $\hat{b}_H^{NB}(\hat{q}, B)$ *solves* (3.8) *and* $\hat{b}_L^{NB}(\hat{q}, B)$ *solves* (3.9)

iii.) $\hat{q}^*_{NB}$ *and* B^*_{NB} *satisfy the fixed point condition* $\hat{b}_H^{NB}(\hat{q}^*_{NB}, B^*_{NB}) = \hat{b}_L^{NB}(\hat{q}^*_{NB}, B^*_{NB}) = \frac{B^*_{NB}}{1+\pi(B^*_{NB},0)}$

As with the model with bailouts, I will assume that demand for real bonds is decreasing in the price of real bonds, and the supply of bonds is increasing in the price of real bonds. Formally

Assumption 5. *For any value of* $B \geq 0$, *the function* $\hat{b}_H^{NB}(\hat{q}, B)$ *is strictly decreasing in* $\hat{q}$ *and the function* $\hat{b}_L^{NB}(\hat{q}, B)$ *is strictly increasing in* $\hat{q}$

Assumption 4 guarantees that the results of proposition 4 will carry through to the model without bailouts, i.e. the equilibrium is unique, the price of real bonds is equal to β, and in equilibrium

$$\hat{b}_H^{NB}(\hat{q}^*_{NB}, B^*_{NB}) = \hat{b}_L^{NB}(\hat{q}^*_{NB}, B^*_{NB}) = \frac{\epsilon}{1+\beta}$$

Let

$$\begin{aligned}
c^*_H &= y + \frac{1}{1+\beta}\left(\epsilon - \beta\frac{\Delta(B^*)}{1+\pi(B^*, \Delta(B^*))}\right) \\
c^*_L &= y - \frac{1}{1+\beta}\left(\epsilon - \beta\frac{\Delta(B^*)}{1+\pi(B^*, \Delta(B^*))}\right) \\
c^*_{H,NB} &= y + \frac{1}{1+\beta}\epsilon \\
c^*_{L,NB} &= y - \frac{1}{1+\beta}\epsilon
\end{aligned}$$

i.e. c_H is the equilibrium value of consumption in high endowment countries in the model with bailouts (consumption is the same in both periods, as the real interest rate is equal to the natural rate in equilibrium), c_H^{NB} is the equilibrium value of consumption in high endowment countries in the model without bailouts, etc. Then the expected utility of households in the equilibrium of the dynamic model with bailouts is given by

$$U^* = (1+\beta)\left[\frac{1}{2}u(c_H^*) + \frac{1}{2}u(c_L^*)\right] - \beta\psi(\pi(B^*, \Delta(B^*)))$$

and the expected utility of households of in the equilibrium of the dynamic model without bailouts is given by

$$U_{NB}^* = (1+\beta)\left[\frac{1}{2}u(c_{H,NB}^*) + \frac{1}{2}u(c_{L,NB}^*)\right] - \beta\psi(\pi(B_{NB}^*, 0))$$

This leads to the following

Proposition 6. *Suppose that assumptions 1-4 hold, and that in the equilibrium of the dynamic model with bailouts* $\Delta(B^*) > 0$*. Then*

i.) $\pi(B^*, \Delta(B^*)) < \pi(B_{NB}^*, 0)$

ii.) $U^* > U_{NB}^*$

Proposition 5 states that the ex ante welfare of households will be higher in an equilibrium with bailouts than in an equilibrium without bailouts, and that inflation will be lower in an equilibrium with bailouts than in an equilibrium without bailouts.

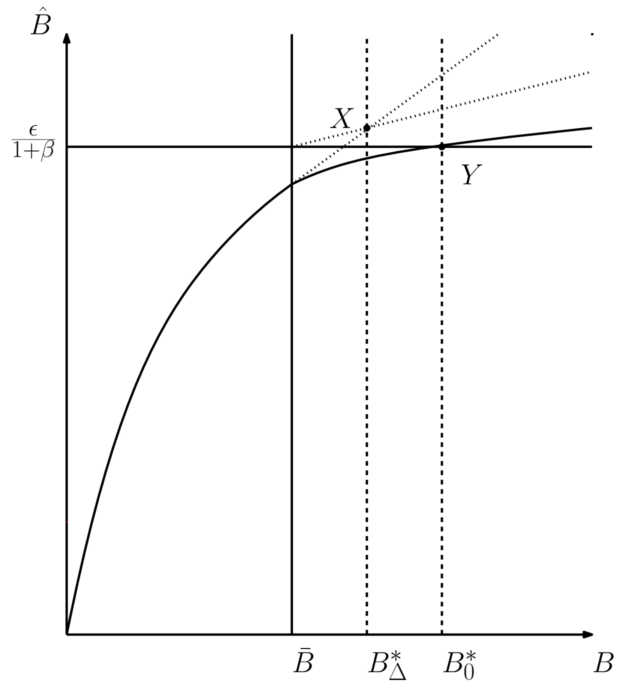

Figure 3.3: Equilibrium in the Dynamic Model Without Bailouts

To gain some intuition for this result, consider Figure 3.3, which shows a graph of the fixed point condition for an equilibrium without bailouts. This equilibrium is point Y. The dotted extensions of the curves correspond to the fixed point condition for an equilibrium with bailouts. Hence without bailouts, the equilibrium would occur at point X. Notice that, relative to the equilibrium with bailouts, the equilibrium without bailouts features a lower value of real bonds. It appears as if prohibiting bailouts is solving a moral hazard problem for low endowment countries. However, the vertical axis of the graph represents the real value of bonds, $\hat{b}$, not the real value of the repayment that low endowment countries will make in the second period, $\hat{b} - \hat{\Delta}$. When low endowment countries anticipate a positive bailout in the second period, they act as in a permanent income model, changing their debt issuance in a way that lets households consume more in periods 1 and periods 2 to achieve intertemporal consumption smoothing. This implies that the value of real debt issuance $\hat{b}$ increases less than one for one with the size of the bailout $\hat{\Delta}$, and the real repayment low endowment countries make in equilibrium will be lower with the bailout. Since low endowment countries will be able to issue more real debt and make smaller real repayments, this implies that there will be less risk in consumption ex ante. Furthermore, a smaller real repayment from low endowment countries to high endowment countries implies less inequality in consumption in the second period. The central bank reacts to less inequality with a lower inflation rate. The fact that the inflation rate is lower in the equilibrium with bailouts can also be seen in the graph, as the equilibrium nominal value of bonds B_{Δ} is lower than B_0, the equilibrium nominal value of bonds without bailouts, but the equilibrium real value of bonds is higher.

3.4 Optimal Currency Areas

The analysis of the dynamic model shows that bailouts act as a form of risk sharing between countries and improve the ex ante welfare of households, and that fiscal authorities in high endowment countries agree to bailouts because bailouts forestall surprise inflation by the central bank. This implies that there is a benefit to forming a monetary union in an environment in which central banks cannot commit to monetary policy: the threat of surprise inflation provides the incentives required for risk sharing. Monetary unions in such environments are subject to a free-rider problem in fiscal policy, as shown in Chari and Kehoe (2007, 2008). Fiscal authorities tend to over-issue nominal bonds, as they do not fully internalize the effect of their decision on the incentive of the central bank to create surprise inflation.[5] This presents a new decision margin for the theory of optimal currency areas. On one hand, the monetary union acts as a commitment device that makes risk sharing possible; on the other, countries expose themselves to a potentially costly externality.

In this section, I first present a model without monetary unions to use as a benchmark. I then go over an example of the dynamic model with quadratic utility to show that it is possible for the benefits of risk-sharing to exceed the costs of the free-rider problem if the variance of endowment shocks is sufficiently large. I then alter the dynamic model slightly to allow for correlation between the endowment shocks of countries and show that forming a monetary union is more attractive when endowment shocks are less correlated. This implies that the tradeoff between risk sharing and the free rider problem

[5] For a discussion of the free-rider problem in the context of this model, see the appendix.

acts to offset the classic Friedman-Mundell optimal currency area criterion, that the countries that stand to gain the most from forming a monetary union are those that have the most highly correlated business cycles.

3.4.1 A Model Without Monetary Unions

The environment for the model without monetary unions is identical to that of the dynamic model with two differences. First, the continuum of countries are no longer a closed economy. Instead, they trade bonds on the world market at a fixed real interest rate equal to $\frac{1}{\beta}$. I make this change to abstract from any general equilibrium effects caused by changes in the real interest rate. Second, countries now have individual central banks that set country-specific inflation rates between periods 1 and 2. Each central bank faces the same commitment problem as the monetary union's central bank did in the dynamic model. The timing for the model without monetary unions is as follows:[6]

Period 1

1. Countries receive their random endowments
2. Fiscal authorities buy or sell nominal bonds b_i on the world market at a price $q(b_i)$
3. Period 1 consumption occurs

Period 2

1. Countries receive their deterministic endowment
2. Central banks set country specific inflation rates π_i
3. Bonds payments are made
4. Period 2 Consumption occurs

[6] Notice that there is no bailout problem in this timing. It is easy to show that high endowment countries would never offer a bailout to low endowment without a monetary union.

In period 2, given a bond position b_i in country i, the central bank chooses the inflation rate π to solve

$$\max_{\pi} \ u\left(y - \frac{b_i}{1+\pi}\right) - \psi(\pi)$$

$$\begin{aligned} \text{s.t.} \qquad & \pi \geq 0 \\ & y - \frac{b_i}{1+\pi} \geq 0 \end{aligned} \tag{3.10}$$

Denote the solution of this problem as $\pi(b_i)$. Notice that the solution to this problem is only country-specific through the value of b_i, hence in equilibrium only the argument of the inflation function π will vary across countries, not the function itself.
In period 1, countries and the world market will forecast the actions of central banks. This implies that the price schedule for nominal bonds will be given by

$$q(b_i) = \frac{\beta}{1+\pi(b_i)}$$

and the fiscal authority in country i will choose b_i to solve

$$\max_{b_i} \ u\left(y_{1i} + \frac{\beta b_i}{1+\pi(b_i)}\right) + \beta\left[u\left(y - \frac{b_i}{1+\pi(b_i)}\right) - \psi(\pi(b_i))\right] \tag{3.11}$$

where

$$y_{1i} = \begin{cases} y+\epsilon & y_{1i} = y_H \\ y-\epsilon & y_{1i} = y_L \end{cases}$$

In a symmetric equilibrium, all high endowment countries will choose the same value of b_i, denoted as b_H, and all low endowment countries will choose the same value of b_i, denoted as b_L.

Definition. *An equilibrium of the model without a monetary union is a pair of functions* $\pi : \mathbb{R} \to \mathbb{R}_+$ *and* $q : \mathbb{R} \to \mathbb{R}_{++}$ *and values* b_H, b_L *such that*

i.) $\pi(b)$ *solves the central bank's problem* (3.10).

ii.) $q(b) = \frac{\beta}{1+\pi(b_i)}$

iii.) Given π, b_H *solves the fiscal authority's problem* (3.11) *for* $y_{1i} = y_H = y+\epsilon$*, and* b_L *solves the fiscal authority's problem* (3.11) *for* $y_{1i} = y_L = y - \epsilon$

3.4.2 Quadratic Utility

To offer a comparison of outcomes with and without a monetary union, I will present an example with a quadratic utility function for consumption that can be solved in closed form, i.e.

$$u(c) = -[\bar{y} - c]^2$$

where $\bar{y}$ is a parameter assumed to be sufficiently large that c is always smaller than y. In this environment, there are three reasons why a monetary union might be preferable to no monetary union from an ex ante perspective. The first is that in a monetary union, the inflation rate is deterministic. This is generally not the case if the countries don't form a monetary union. If ψ is strictly convex, forming a monetary union might be preferable because it eliminates risk in the inflation rate. The second potential benefit is that, in a monetary union, the central bank takes into account that inflation has a negative effect on the wealth of high endowment countries. This offers discipline on inflation that is absent if there is no monetary union. A low endowment country will not have a counterweight to its negative asset position to reduce surprise inflation if it is not a union member. Finally, joining the monetary union gives countries access to risk sharing through bailouts. My analysis will focus on the third incentive. To eliminate the first, I will assume linear disutility from inflation

$$\psi(\pi) = \hat{\psi} \times (1 + \pi)$$

for some value $\hat{\psi} > 0$. Although this linear disutility of inflation function satisfies assumption 2 and an optimal level of nominal repayment exist for the monetary union version of the model, linear disutility from inflation and quadratic utility from consumption will not satisfy all of the conditions of assumption 1 that guarantee an interior solution to the central bank's problem. I will have to impose other restrictions on model parameters to guarantee an interior solution for the central bank's problem in the equilibrium of the monetary union model. More detail on this can be found in the appendix, where I go over the solution to the quadratic utility model. To eliminate the second incentive, I focus on a subset of the model's parameter space where country's would optimally choose not to form a monetary union if they were restricted not to make bailouts in the second period. Again, I provide details on how to choose such parameters in the appendix.

When an interior solution exists, the ex ante utility of being in a monetary union with bailouts has a simple quadratic structure. In particular, it is given by the expression

$$V_{\Delta}^{MU}(\epsilon) = \begin{cases} -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon}{1+\beta}\right)^2 & \epsilon \leq \epsilon^* \\ -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon^*}{1+\beta}\right)^2 & \epsilon > \epsilon^* \end{cases} \tag{3.12}$$

for some critical value ϵ^*. $\epsilon > 0$, recall, is the endowment shock parameter

$$y_H = y + \epsilon$$
$$y_L = y - \epsilon$$

and is equal to the standard deviation of the endowment shocks. ϵ^* is the minimum value of ϵ such that there will be bailouts in equilibrium. It is depicted graphically in Figure 3.4.

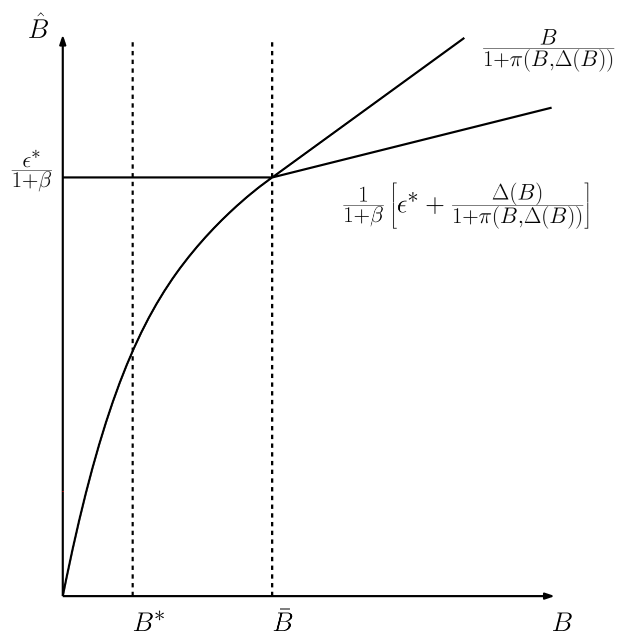

Figure 3.4: Critical Value ϵ^*

When $\epsilon < \epsilon^*$, no risk sharing occurs. As ϵ increases, risk goes up and ex ante welfare V_{Δ}^{MU} decreases. Once ϵ passes ϵ^*, bailouts and hence risk sharing kick in, and V_{Δ}^{MU}

ceases to decline. This compares with the function

$$V^{MU}(\epsilon) = -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon}{1+\beta}\right)^2 \tag{3.13}$$

which gives ex ante welfare if bailouts are not allowed. The two functions coincide for $\epsilon < \epsilon^*$, at which point they diverge as V^{MU} continues to decline.

If countries don't form a monetary union, then countries that receive a high endowment shock will choose to save, which does not induce the central bank to inflate. In a model with a general utility function, low endowment countries will restrict their borrowing relative to the monetary union case to try to discipline the central bank's choice of the inflation rate (this is discussed in more detail in the appendix section on the free-rider problem). In the case of the quadratic utility model, low endowment countries borrow as much as possible without inducing the central bank to inflate. Denote this value as b^*. The ex ante utility of countries that don't form a monetary union is given by

$$V^A(\epsilon) = -(1+\beta)[\bar{y}-y]^2 - \frac{2+\beta}{2}\epsilon^2 - \frac{\beta(1+\beta)}{2}b^{*2} + \beta\epsilon b^* \tag{3.14}$$

In the appendix, I show how to choose model parameters $\hat{\psi}$, β, $\bar{y}$, and y such that there is an interval $[\underline{\epsilon}, \bar{\epsilon}]$ such that ϵ^* is in the interval, $V^A > V^{MU}$ everywhere in the interval, and there is a value $\tilde{\epsilon} \in [\underline{\epsilon}, \bar{\epsilon}]$ such that $V^{MU}_{\Delta} > V^A$ if $\epsilon > \tilde{\epsilon}$. This implies that, for these parameters, countries would never choose to form a monetary union if they couldn't make bailouts. However, if there is enough risk , the monetary union allows countries to credibly commit to risk sharing, and the benefits of risk sharing outweigh the costs of the free-rider problem. This is depicted in Figure 3.5.

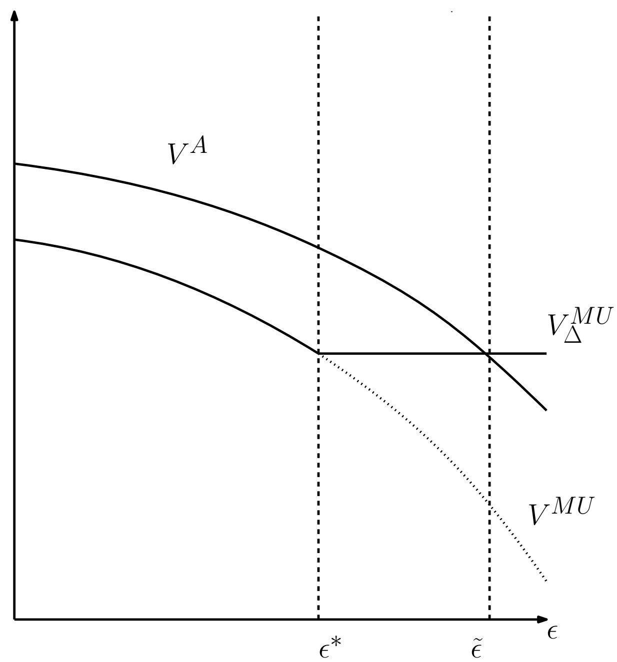

Figure 3.5: Bailouts Make Monetary Union Optimal

3.4.3 Correlation of Shocks

The previous section established the existence of utility functions u and ψ, discount factors β, and values of the shock ϵ such that countries prefer to form a monetary union to gain access to bailouts. In this section I will assume that this is the case. I alter the standard set up slightly to allow for correlation between the income shocks of countries. Countries still buy and sell bonds on the world market at a real interest rate equal to the natural rate $\frac{1}{\beta}$. There is still a continuum of countries of mass 1. Fraction $\frac{1}{2}$ are type 1 countries, the remainder are type 2 countries. Countries know their type ex ante. There are four possible states of the world, denoted θ_{HH}, θ_{HL}, θ_{LH}, and θ_{LL}, one of which will be realized. If the state θ_{HH} is realized, both types of countries receive the high endowment shock y_H; if the state θ_{HL} is realized, type 1 countries receive the high endowment shock y_H, and type 2 countries receive the low endowment shock y_L; if state θ_{LH} is realized, type 1 countries receive the low endowment shock y_L, and type 2 countries receive the high endowment shock y_H; if the state θ_{LL} is realized, both countries receive the low endowment shock y_L. The probability of states is given by

$\Pr\{\theta_{HL}\} = \Pr\{\theta_{LH}\} = p\frac{1}{2}$ and $\Pr\{\theta_{HH}\} = \Pr\{\theta_{LL}\} = (1-p)\frac{1}{2}$.

For any individual country, the unconditional distribution of their income shock is the same as in the standard model: regardless of whether country i is a type 1 or type 2 country, y_{1i} is equal to y_H with probability $\frac{1}{2}$ and y_{1i} is equal to y_L with probability $\frac{1}{2}$. Since the values of the shocks in other countries are irrelevant if countries do not form a monetary union, the ex ante welfare of not forming a monetary union is the same as in the standard set up. Denote this value as V^A.

Now consider what will happen if the countries form a monetary union that allows bailouts. Let $V^{MU}_{\Delta,H}$ denote the ex post welfare of a household in a high endowment country in a monetary union with bailouts for the same utility functions, discount factor, and shocks in the standard model. Similarly, let $V^{MU}_{\Delta,L}$ denote the ex post welfare of a household in a low endowment country in a monetary union with bailouts for the same utility functions, discount factor, and shocks in the standard model. If state θ_{HL} is realized, $\frac{1}{2}$ of all countries will have a high endowment (type 1 countries), and $\frac{1}{2}$ of all countries will have a low endowment (type 2 countries). Then because these countries form a monetary union, and the monetary union outcome with bailouts is unique, households in type 1 countries' welfare will be given by $V^{MU}_{\Delta,H}$ and households in type 2 countries' welfare will be given by $V^{MU}_{\Delta,L}$. Similarly, if state θ_{LH} is realized, the welfare of households in type 1 countries will be $V^{MU}_{\Delta,L}$ and the welfare of households in type 2 countries will be given by $V^{MU}_{\Delta,H}$. Let $\Pr\{\cdot|\ \theta_{HL}, \theta_{LH}\}$ denote probabilities conditional on state θ_{HL} or state θ_{LH} being realized. By construction, $\Pr\{\theta_{HL}|\ \theta_{HL}, \theta_{LH}\} = \Pr\{\theta_{LH}|\ \theta_{HL}, \theta_{LH}\} = \frac{1}{2}$. This implies that the expected utility of households in type 1 or type 2 countries conditional on θ_{HL} or θ_{LH} being realized is given by

$$\frac{1}{2}V^{MU}_{\Delta,H} + \frac{1}{2}V^{MU}_{\Delta,L}$$

which is just V^{MU}_{Δ}, the ex ante welfare of households in the monetary union with bailouts in the standard model. By assumption $V^{MU}_{\Delta} > V^A$. If state θ_{HH} is realized, at the natural rate, all fiscal authorities in the monetary union will save to equalize consumption in periods 1 and 2. Since fiscal authorities save, the no deflation constraint for the central bank will bind, and the inflation rate will be equal to 0. This is exactly the same as the outcome for an individual country that is not in a monetary union and receives the high endowment. Denote the welfare of households in this state V^A_H.

If the state θ_{LL} is realized, all countries will borrow the same amount to equalize consumption in both periods, and there will be no bailouts.[7] Denote the welfare of households in this state as $\underline{V}_L^A$. Note that this does not coincide with the outcome without a monetary union if a country receives a low endowment shock. Recall that the fiscal authority in a monetary union issues nominal debt to solve

$$\max_b \; u\left(y - \epsilon + \frac{\beta b}{1+\pi(b)}\right) + \beta\left[u\left(y - \frac{b}{1+\pi(b)}\right) - \psi(\pi(b))\right]$$

and the first order condition for this problem is

$$\begin{aligned} &\frac{\beta}{1+\pi(b)}\left(u'\left(y - \epsilon + \frac{\beta b}{1+\pi(b)}\right) - u'\left(y - \frac{b}{1+\pi(b)}\right)\right) \\ -\pi'(b)&\frac{\beta b}{(1+\pi(b))^2}\left(u'\left(y - \epsilon + \frac{\beta b}{1+\pi(b)}\right) - u'\left(y + \frac{b}{1+\pi(b)}\right)\right) - \beta\psi'(\pi(b))\pi'(b) = 0 \end{aligned}$$

The inflation function $\pi(b)$ coincides with the value of inflation that will prevail in the monetary union if all countries issue the same nominal debt b, as there will be no bailouts in this case. If the fiscal authority outside of the monetary union borrows to equalize consumption, as in the monetary union case, the left hand side of this equation will be

$$-\beta\psi'(\pi(b))\pi'(b)$$

This will be negative, as $\pi' > 0$,[8] hence the level of debt in the monetary union is not optimal for countries that don't form a monetary union. This implies that it must be the case that the ex post utility of households in a country that receives a low endowment shock and is not a member of a monetary union $V_L^A > \underline{V}_L^A$.

Let $\Pr\{\cdot|\, \theta_{HH}, \theta_{LL}\}$ denote probabilities conditional on state θ_{HH} or θ_{LL} being realized. Note that $\Pr\{\theta_{HH}|\; \theta_{HH}, \theta_{LL}\} = \Pr\{\theta_{LL}|\; \theta_{HH}, \theta_{LL}\} = \frac{1}{2}$. The expected utility of

[7] I do not provide a formal proof of this in the thesis, but the argument for this is along the following lines: If one set of countries has low debt in the second period and offers to bailout the remaining countries with high debt, the bailout they offer will be smaller than the bailout required to equalize consumption in the two sets of countries, hence consumption in the countries that make bailouts must be higher than consumption in countries that receive bailouts. In period 1, countries that anticipate making a bailout in period 2 treat the bailout as a loss of period 2 income. Since all countries have the same income in period 1, countries making a bailout have a lower present value of income and will choose to consume less than countries that receive a bailout in both periods, which is a contradiction.

[8] See the Appendix section on the free-rider problem

households in both types of country in a monetary union, conditional on a realization of state θ_{HH} or θ_{LL} is given by

$$\tilde{V} = \frac{1}{2}V_H^A + \frac{1}{2}\underline{V}_L^A < \frac{1}{2}V_H^A + \frac{1}{2}V_L^A = V^A$$

The unconditional expected utility of households in both types of countries in a monetary union is

$$V_\rho^{MU}(p) = (\Pr\{\theta_{HL}\} + \Pr\{\theta_{LH}\})V_\Delta^{MU} + (\Pr\{\theta_{HH}\} + \Pr\{\theta_{LL}\})\tilde{V} = pV_\Delta^{MU} + (1-p)\tilde{V}$$

As $V_\Delta^{MU} > V^A > \tilde{V}$, $V_\rho^{MU}(p)$ is strictly increasing in p, $V_\rho^{MU}(1) > V^A$, and $V_\rho^{MU}(0) < V^A$. This implies that there is some critical value of p, denoted as p^* such that if $p < p^*$, $V_\rho^{MU}(p) < V^A$, and countries prefer not to form a monetary union; and if $p > p^*$, $V_\rho^{MU}(p) > V^A$, and countries do prefer to form a monetary union. The correlation of the endowment shocks of type 1 and type 2 countries is given by

$$\rho_{12} = 1 - 2p$$

so p is a perfect proxy for the correlation of shocks across countries. In this light, the previous results imply that the value of forming a monetary union is decreasing in the correlation of shocks between type 1 and type 2 countries; and if the value the correlation isn't too high, countries would choose to form a monetary union. The intuition for the result on correlation is simple. The benefit to forming a monetary union in this environment is access to risk-sharing through bailouts. If shocks are too correlated across countries, risk sharing opportunities are scarce, and countries would be better off not forming a monetary union. This contrasts with the analysis of Friedman (1958) and Mundell (1961). In this classical treatment, monetary policy is an important tool for smoothing business cycle shocks. Countries with high correlations of shocks stand to gain the most from forming a monetary union, as such countries tend to have similar monetary policy needs and hence lose the least from forgoing policy independence. The analysis of this paper does not seek to upend their result, merely to augment it by considering another margin (the tradeoff between risk sharing and free-riding) that they do not. In this sense, I see my result as a counter-weight to the Friedman-Mundell logic rather than a contradiction of it.

3.5 Conclusion

The bailouts the Greek government received during the Eurozone crisis are not a singular event in history. Many sovereigns received some form of foreign assistance during similar debt crises. The bailout of Greece does, however, stand out in its scale. This chapter rationalizes this observation by showing that members of a monetary union with low debt, such as Germany, have an incentive to bailout members with high debt, such as Greece, that would not be present if the countries did not share monetary policy: low debt countries can prevent the central bank from acting to devalue the union's currency by engaging in bailouts. It then shows that these bailouts are effectively a form of risk sharing and improve the welfare of households in member countries from an ex ante perspective. This presents a new margin for consideration in the theory of optimal currency areas. Countries can use a monetary union as a commitment mechanism to engage in risk sharing, but by forming a monetary union the countries expose themselves to a costly free-rider problem in fiscal policy. I show through an example that it is possible for the benefits of risk sharing to outweigh the costs of the policy externality. I also demonstrate that this is more likely to occur if shocks are less correlated across countries, as lower correlation gives rise to more opportunities for risk sharing, which contrasts with the classical analysis of the effects of correlation of shocks in the optimal currency area literature. Overall, I take a more optimistic tone than others on the future of the Euro. The Eurozone crisis, although costly, demonstrates that the Euro can facilitate risk sharing between the members of the EMU and might thus be worth saving.

Chapter 4

Conclusion and Discussion

This thesis has addressed the provision of incentives for risk sharing two contexts. First, I showed that in-kind benefits for agents that do not work are not necessary to incentivize labor market participation in a standard model of labor productivity risk. However, if the standard model is augmented with home production, such a scheme may be beneficial. Next I showed that a monetary union provides a rationale for low debt countries to offer bailouts to high debt countries, as such bailouts temper the incentive of the union's central bank to inflate the union's currency to redistribute from low debt countries to high debt countries. If countries accumulate debt in response to adverse shocks, then these bailouts will effectively function as insurance payments, as they flow from lucky countries to unlucky. The monetary union can thus be viewed as a mechanism that provides the incentives required for countries to engage in this risk sharing arrangement.

Bibliography

Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath (2013), "Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises."

Aguiar, Mark and Erik Hurst (2005), "Lifecycle Prices and Production." NBER Working Papers 11601, National Bureau of Economic Research, Inc, URL https://ideas.repec.org/p/nbr/nberwo/11601.html.

Alesina, Alberto and Robert Barro (2001), "Dollarization." *American Economic Review, Papers and Proceedings*, 91, 381–85.

Atkinson, A. B. and J. E. Stiglitz (1976), "The design of tax structure: Direct versus indirect taxation." *Journal of Public Economics*, 6, 55–75, URL https://ideas.repec.org/a/eee/pubeco/v6y1976i1-2p55-75.html.

Barro, Robert J. and David B. Gordon (1983), "Rules, discretion and reputation in a model of monetary policy." *Journal of Monetary Economics*, 12, 101–121.

Bulow, Jeremy and Kenneth Rogoff (2015), "The modern greek tragedy." *Vox EU*.

Chari, VV, Alessandro Dovis, and Patrick Kehoe (2016a), "Journey down the slippery slope to the european crisis: A theorist's guide."

Chari, V.V., Alessandro Dovis, and Patrick Kehoe (2016b), "Rethinking optimal currency areas."

Chari, V.V. and Patrick Kehoe (2007), "On the need for fiscal constraints in a monetary union." *Journal of Monetary Economics*, 2399–2408.

Chari, V.V. and Patrick Kehoe (2008), "Time inconsistency and free-riding in a monetary union." *Journal of Money, Banking, and Credit*, 1329–1356.

Chari, V.V. and Patrick Kehoe (2016), "Bailouts, time inconsistency, and optimal regulation: A macroeconomic view." *American Economic Review*, Forthcoming.

Currie, Janet and Firouz Gahvari (2007), "Transfers in Cash and In Kind: Theory Meets the Data." NBER Working Papers 13557, National Bureau of Economic Research, Inc, URL https://ideas.repec.org/p/nbr/nberwo/13557.html.

Dellas, Harris and George S. Tavlas (2009), "An Optimum-Currency-Area Odyssey." *Journal of International Money and Finance*, 28, 1117–1137.

Farhi, Emmanuel and Ivan Werning (2012), "Fiscal Unions." NBER Working Papers 18280, National Bureau of Economic Research, Inc.

Frankel, Jeffrey and Andrew Rose (1998), "The endogeneity of the optimal currency area criteria." *Economic Journal*, 1009–1025.

Friedman, Milton (1958), "The case for flexible exchange rates." In *Essays in Positive Economics*, University of Chicago Press, Chicago.

Friedman, Milton (1973), "Monetary policy in developing countries." In *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovitz* (Paul A. David and Melvin W. Reder, eds.), 265–278, Academic Press, New York.

Mundell, Robert A. (1961), "A theory of optimum currency areas." *American Economic Review*, 657–665.

Saez, Emmanuel (2002), "The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes." *Journal of Public Economics*, 83, 217–230, URL https://ideas.repec.org/a/eee/pubeco/v83y2002i2p217-230.html.

Appendix A

Appendix to Chapter 2

A.1 Omitted Proofs

Proof of Theorem 1. Let $\{\{c_j^*\}, y^*\}$ be an allocation rule that solves (PP). For simplicity and clarity, I will consider a single history θ^t with positive mass and $y_t^*(\theta^t) = 0$. This argument can easily be extended to a set of such histories with positive mass by applying the following argument to all histories in the set. Consider the following minimization problem

$$\min_{\{c_j\}} \quad \sum_{j=1}^{N} c_j$$

$$\text{s.t. } G_t(c_1, \ldots, c_N, 0) = G_t(c_{1t}^*(\theta^t), \ldots, c_{Nt}^*(\theta^t), 0) \tag{P1}$$

Let $\{c_j\}$ be any consumption bundle that satisfies the constraints of (P1). Define an allocation rule $\{\{\hat{c}_j\}, \hat{y}\}$ as follows

$$\hat{y}(\theta^s) = y^*(\theta^s)$$

$$\hat{c}_{js}(\theta^s) = \begin{cases} c_{js}^*(\theta^s) & \theta^s \neq \theta^t \\ c_j & \theta^s = \theta^t \end{cases}$$

Notice that if we can show that the allocation rule $\{\{\hat{c}_j\}, \hat{y}\}$ satisfies the constraints of (PP), it would follow that $\{c_{jt}^*(\theta^t)\}$ must solve (P1) (otherwise, it would be possible to construct an alternative allocation to the solution to (PP) that was incentive compatible and satisfied voluntary participation, and used strictly less resources in history θ^t, and hence lowered the planner's objective, namely $\{\{\hat{c}_j\}, \hat{y}\}$). To show this, let σ be any reporting strategy, $\tilde{\theta}^T$ be any history of length T, and $\tilde{\theta}^s$ denote an s-length sub-history of $\tilde{\theta}^T$. Observe that, by the construction of $\{\{\hat{c}_j\}, \hat{y}\}$

$$G_s\left(\hat{c}_{1s}(\sigma^s(\tilde{\theta}^s)), \ldots, \hat{c}_{Ns}(\sigma^s(\tilde{\theta}^s)), \frac{\hat{y}_s(\sigma^s(\tilde{\theta}^s))}{\tilde{\theta}_s}\right)$$

$$= \begin{cases} G_s\left(c^*_{1s}(\sigma^s(\tilde{\theta}^s)), \ldots, c^*_{Ns}(\sigma^s(\tilde{\theta}^s)), \frac{y^*_s(\sigma^s(\tilde{\theta}^s))}{\tilde{\theta}_s}\right) & \sigma^s(\tilde{\theta}^s) \neq \theta^t \\ G_t\left(c_1, \ldots, c_N, \frac{y^*(\sigma^t(\tilde{\theta}^t))}{\tilde{\theta}_t}\right) & \sigma^s(\tilde{\theta}^s) = \theta^t \end{cases}$$

and for this second case, we have

$$\begin{aligned} G_t\left(c_1, \ldots, c_N, \frac{y^*_t(\sigma^t(\tilde{\theta}^t))}{\tilde{\theta}_t}\right) &= G_t(c_1, \ldots, c_N, 0) \\ =& G_t(c^*_{1t}(\theta^t), \ldots, c^*_{Nt}(\theta^t), 0) \\ =& G_t\left(c^*_{1t}(\sigma^t(\tilde{\theta}^t)), \ldots, c^*_{Nt}(\sigma^t(\tilde{\theta}^t)), \frac{y^*_t(\sigma^t(\tilde{\theta}^t))}{\tilde{\theta}_t}\right) \end{aligned}$$

Here, the second line follows from the fact that $\{c_j\}$ satisfies the constraint of (P1) and the third line from the fact that $y^*_t(\theta^t) = 0$. Hence, $\forall\ s$

$$\begin{aligned} &G_s\left(\hat{c}_{1s}(\sigma^s(\tilde{\theta}^s)), \ldots, \hat{c}_{Ns}(\sigma^s(\tilde{\theta}^s)), \frac{\hat{y}_s(\sigma^s(\tilde{\theta}^s))}{\tilde{\theta}_s}\right) \\ =& G_s\left(c^*_{1s}(\sigma^s(\tilde{\theta}^s)), \ldots, c^*_{Ns}(\sigma^s(\tilde{\theta}^s)), \frac{y^*_s(\sigma^s(\tilde{\theta}^s))}{\tilde{\theta}_s}\right) \end{aligned}$$

Thus

$$U\left(\hat{c}_{10}(\sigma^0(\tilde{\theta}^0)),\ldots,\hat{c}_{N0}(\sigma^0(\tilde{\theta}^0)),\frac{\hat{y}_0(\sigma^0(\tilde{\theta}^0))}{\tilde{\theta}_0},\ldots\right.$$
$$\left.\hat{c}_{1T}(\sigma^T(\tilde{\theta}^T)),\ldots,\hat{c}_{NT}(\sigma^T(\tilde{\theta}^T)),\frac{\hat{y}_T(\sigma^T(\tilde{\theta}^T))}{\tilde{\theta}_T}\right) =$$

$$V\left(G_0\left(\hat{c}_{10}(\sigma^0(\tilde{\theta}^0)),\ldots,\hat{c}_{N0}(\sigma^0(\tilde{\theta}^0)),\frac{\hat{y}_0(\sigma^0(\tilde{\theta}^0))}{\tilde{\theta}_0}\right),\ldots\right.$$
$$\left.G_T\left(\hat{c}_{1T}(\sigma^T(\tilde{\theta}^T)),\ldots,\hat{c}_{NT}(\sigma^T(\tilde{\theta}^T)),\frac{\hat{y}_T(\sigma^T(\tilde{\theta}^T))}{\tilde{\theta}_T}\right)\right) =$$

$$V\left(G_0\left(c^*_{10}(\sigma^0(\tilde{\theta}^0)),\ldots,c^*_{N0}(\sigma^0(\tilde{\theta}^0)),\frac{y^*_0(\sigma^0(\tilde{\theta}^0))}{\tilde{\theta}_0}\right),\ldots\right.$$
$$\left.G_T\left(c^*_{1T}(\sigma^T(\tilde{\theta}^T)),\ldots,c^*_{NT}(\sigma^T(\tilde{\theta}^T)),\frac{y^*_T(\sigma^T(\tilde{\theta}^T))}{\tilde{\theta}_T}\right)\right) =$$

$$U\left(c^*_{10}(\sigma^0(\tilde{\theta}^0)),\ldots,c^*_{N0}(\sigma^0(\tilde{\theta}^0)),\frac{y^*_0(\sigma^0(\tilde{\theta}^0))}{\tilde{\theta}_0},\ldots\right.$$
$$\left.c^*_{1T}(\sigma^T(\tilde{\theta}^T)),\ldots,c^*_{NT}(\sigma^T(\tilde{\theta}^T)),\frac{y^*_T(\sigma^T(\tilde{\theta}^T))}{\tilde{\theta}_T}\right)$$

Since the choice of $\tilde{\theta}^T$ was arbitrary, this relationship holds in expectation, implying

$$U^\sigma(\{\{\hat{c}_j\},\hat{y}\}) = U^\sigma(\{\{c^*_j\},y\}) \tag{A.1}$$

and since the choice of reporting strategies σ was arbitrary, this holds for all σ. In particular, it holds for truth-telling, which implies

$$U(\{\{\hat{c}_j\},\hat{y}\}) = U(\{\{c^*_j\},y^*\}) \tag{A.2}$$

Then, the fact that $\{\{c^*_j\},y^*\}$ is a solution to (PP) and (A.2) implies that

$$U(\{\{\hat{c}_j\},\hat{y}\}) = U(\{\{c^*_j\},y^*\}) \geq \underline{U}$$

and hence $\{\{\hat{c}_j\}, \hat{y}\}$ satisfies promise keeping. Furthermore, since $\{\{c_j^*\}, y^*\}$ is a solution to (PP), (A.1) and (A.2) imply that

$$U(\{\{\hat{c}_j\}, \hat{y}\}) = U(\{\{c_j^*\}, y^*\}) \geq U^\sigma(\{\{c_j^*\}, y\}) = U^\sigma(\{\{\hat{c}_j\}, \hat{y}\}) \; \forall \, \sigma$$

and hence $\{\{\hat{c}_j\}, \hat{y}\}$ satisfies incentive compatibility, thus $\{c_{jt}^*(\theta^t)\}$ must solve (P1). Then, since $\{c_{jt}^*(\theta^t)\}$ is a solution to (P1), $\{c_{jt}^*(\theta^t)\}$ must satisfy the first order conditions of (P1). These first order conditions are given by

$$1 = \lambda G_{tj}(c_1, \ldots, c_N, 0)$$

where λ is the Lagrange multiplier associated with the equality constraint in (P1). This in turn implies

$$\frac{G_{ti}(c_1, \ldots, c_N, 0)}{G_{tj}(c_1, \ldots, c_N, 0)} = 1 \; \forall \, i, j \in \{1, \ldots, N\}$$

Substituting $\{c_{jt}^*(\theta^t)\}$ above completes the proof. $\square$

Appendix B

Appendix to Chapter 3

B.1 Omitted Proofs

Before going through the proofs for the static model, I will prove two useful lemmas.

Lemma 1. *Suppose that u and ψ satisfy the conditions of assumption 1. Then*

i.) The non-negativity constraints do not bind in any solution to the central bank's problem, and the no-deflation constraint does not bind if $B - \Delta \neq 0$

ii.) The objective function of the central bank is strictly concave in π if $B - \Delta \neq 0$ and $c_H = y + \frac{B-\Delta}{1+\pi} > 0$ and $c_L = y - \frac{B-\Delta}{1+\pi} > 0$

iii.) If $B - \Delta \neq 0$, a unique solution to the central bank's first order condition $\pi(B, \Delta) > 0$ always exists

iv.) if $B - \Delta = 0$, $\pi(B, \Delta) = 0$

Proof. *i.*) follows immediately from the fact that u satisfies the Inada conditions and is strictly concave, and that $\psi'(0) = 0$.
For *ii.*), taking the first derivative of the central bank's objective function with respect to π yields

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}\left[u'\left(y - \frac{B-\Delta}{1+\pi}\right) - u'\left(y + \frac{B-\Delta}{1+\pi}\right)\right] - \psi'(\pi)$$

Differentiating again with respect to π yields

$$-\frac{B-\Delta}{(1+\pi)^3}\left[u'\left(y-\frac{B-\Delta}{1+\pi}\right)-u'\left(y+\frac{B-\Delta}{1+\pi}\right)\right]+$$
$$\frac{1}{2}\left(\frac{B-\Delta}{(1+\pi)^2}\right)^2\left[u''\left(y-\frac{B-\Delta}{1+\pi}\right)+u''\left(y+\frac{B-\Delta}{1+\pi}\right)\right]-\psi''(\pi)$$

which is negative for any value of $B-\Delta \neq 0$ if $c_H, c_L > 0$ because u' is decreasing, $u'' < 0$ and ψ is convex under conditions of assumption 1.
Finally, for *iii.*), the first order condition for the central bank, ignoring the non-negativity and no-deflation constraints as they don't bind, is given by

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}\left[u'\left(y-\frac{B-\Delta}{1+\pi}\right)-u'\left(y+\frac{B-\Delta}{1+\pi}\right)\right]-\psi'(\pi)=0$$

As π decreases, either $c_H = y + \frac{B-\Delta}{1+\pi}$ or $c_L = y - \frac{B-\Delta}{1+\pi}$ goes to zero, depending on the sign of $B-\Delta$, hence the first term approaches infinity because u satisfies the Inada conditions, and the term $-\psi'(\pi)$ increases towards 0 because ψ is convex. As π increases toward infinity, the first term approaches zero, and the second term is bounded away from zero, because ψ' is increasing. The concavity property established in *ii.*) implies that this function is strictly decreasing in π. Hence a unique solution $\pi > 0$ to this first order condition exists by the intermediate value theorem.
iv.) is obvious, neither country's consumption depends on π if $B-\Delta = 0$ and ψ is increasing □

Note that it follows immediately from lemma 1 that the solution to the central bank's problem $\pi(B,\Delta)$ is unique. Note also that the central bank's first order condition holds with equality at $\pi = 0$ for $B-\Delta = 0$. This implies that, although the non-deflation constraint holds with equality at $B-\Delta = 0$, it does not bind, hence there is no discontinuity in π at $B-\Delta = 0$. The next lemma establishes some useful properties of this function.

Lemma 2. *Let $\pi(B,\Delta)$ be the unique solution to central bank's problem. Then $\pi(B,\Delta) = \pi(B-\Delta, 0)$ and $\pi(B,0) = \pi(-B,0)$.*

Proof. For the first property, note that the value of the central bank's first order condition satisfies

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}\left[u'\left(y-\frac{B-\Delta}{1+\pi}\right)-u'\left(y+\frac{B-\Delta}{1+\pi}\right)\right]-\psi'(\pi)=$$
$$\frac{1}{2}\frac{(B-\Delta)-0}{(1+\pi)^2}\left[u'\left(y-\frac{(B-\Delta)-0}{1+\pi}\right)-u'\left(y+\frac{(B-\Delta)-0}{1+\pi}\right)\right]-\psi'(\pi)$$

i.e. the value of the derivative of the central bank's objective is the same for a nominal repayment of B and a transfer of Δ and a nominal repayment of $B-\Delta$ and a transfer of 0, hence the first property follows from result *iii.*) in lemma 1.
For the second property, note that the value of the central bank's first order condition at any value π satisfies

$$\frac{1}{2}\frac{B}{(1+\pi)^2}\left[u'\left(y-\frac{B}{1+\pi}\right)-u'\left(y+\frac{B}{1+\pi}\right)\right]-\psi'(\pi)=$$
$$\frac{1}{2}\frac{-B}{(1+\pi)^2}\left[u'\left(y-\frac{-B}{1+\pi}\right)-u'\left(y+\frac{-B}{1+\pi}\right)\right]-\psi'(\pi)$$

i.e. the value of the derivative of the central bank's objective is the same for B and $-B$, hence the second property follows from result *iii.*) in lemma 1. □

With these lemmas established, I will move onto the proofs of the propositions for the static model.

Proof of Proposition 1. Uniqueness of $\pi(B,\Delta)$ was established in the discussion following the proof of lemma 1. For the result that $\frac{\partial\pi}{\partial\Delta}<0$, note that it follows from lemma 1 that the function $\pi(B,\Delta)$ is implicitly defined by the first order condition of the central bank

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^2}\left[u'\left(y-\frac{B-\Delta}{1+\pi}\right)-u'\left(y+\frac{B-\Delta}{1+\pi}\right)\right]-\psi'(\pi)=0$$

Differentiating this equation with respect to Δ and using the identities $c_H=y+\frac{B-\Delta}{1+\pi}$ and $c_L=y-\frac{B-\Delta}{1+\pi}$, we have

$$\frac{1}{2}\frac{B-\Delta}{(1+\pi)^3}\times[u''(c_L)-u''(c_H)]-\frac{1}{2}\frac{1}{(1+\pi)^2}[u'(c_L)-u'(c_H)]+$$
$$\frac{\partial\pi}{\partial\Delta}\times\left[-\frac{B-\Delta}{(1+\pi)^3}\times[u'(c_L)-u'(c_H)]+\right.$$
$$\left.\frac{1}{2}\left(\frac{B-\Delta}{(1+\pi)^2}\right)^2\times[u''(c_L)+u''(c_H)]-\psi''(\pi)\right]=0$$

Rearranging and solving for $\frac{\partial \pi}{\partial \Delta}$ yields

$$\frac{\partial \pi}{\partial \Delta} = -\frac{\frac{1}{2}\frac{B-\Delta}{(1+\pi)^3} \times [u''(c_L) - u''(c_H)] - \frac{1}{2}\frac{1}{(1+\pi)^2}[u'(c_L) - u'(c_H)]}{-\frac{B-\Delta}{(1+\pi)^3} \times [u'(c_L) - u'(c_H)] + \frac{1}{2}\left(\frac{B-\Delta}{(1+\pi)^2}\right)^2 \times [u''(c_L) + u''(c_H)] - \psi''(\pi)}$$

The denominator of this fraction is just the second derivative of the central bank's objective function, which is negative by result *ii.*) of lemma 1. The numerator is negative whenever $B-\Delta > 0$ because, under the conditions of assumption 1, u'' is non-decreasing and u' is decreasing and $B - \Delta > 0$ implies $c_H > c_L$. Therefore $\frac{\partial \pi}{\partial \Delta} < 0$.
For the third property, consider any value $B > 0$ and values $0 \leq \Delta_1 < \Delta_2 \leq B$. Define a value $\hat{\pi}$ such that

$$\frac{B - \Delta_1}{1 + \pi(B, \Delta_1)} = \frac{B - \Delta_2}{1 + \hat{\pi}}$$

Notice that because $\Delta_1 < \Delta_2$, $\hat{\pi} < \pi(B, \Delta_1)$. Now, if $\hat{\pi} < 0$, then $\pi(B, \Delta_2) > \hat{\pi}$ and $\frac{B-\Delta_1}{1+\pi(B,\Delta_1)} > \frac{B-\Delta_2}{1+\pi(B,\Delta_2)}$. If $\hat{\pi} \geq 0$, then it follows from the definitions of $\hat{\pi}$ and the fact that $\hat{\pi} < \pi(B, \Delta_1)$ that

$$\begin{aligned} &\frac{1}{2}\frac{B - \Delta_2}{(1 + \hat{\pi})^2}\left[u'\left(y - \frac{B - \Delta_2}{1 + \hat{\pi}}\right) - u'\left(y + \frac{B - \Delta_2}{1 + \hat{\pi}}\right)\right] - \psi'(\hat{\pi}) > \\ &\frac{1}{2}\frac{B - \Delta_1}{(1 + \pi(B, \Delta_1))^2}\left[u'\left(y - \frac{B - \Delta_1}{1 + \pi(B, \Delta_1)}\right) - u'\left(y + \frac{B - \Delta_1}{1 + \pi(B, \Delta_1)}\right)\right] \\ &- \psi'(\pi(B, \Delta_1)) = 0 \end{aligned}$$

Then concavity of the objective function of the central bank implies that $\pi(B, \Delta_2) > \hat{\pi}$ and $\frac{B-\Delta_1}{1+\pi(B,\Delta_1)} > \frac{B-\Delta_2}{1+\pi(B,\Delta_2)}$. In both cases, it must be the case that $\frac{B-\Delta_1}{1+\pi(B,\Delta_1)} > \frac{B-\Delta_2}{1+\pi(B,\Delta_2)}$. Since the choices of B, Δ_1, and Δ_2 were arbitrary, this implies that $\frac{B-\Delta}{1+\pi(B,\Delta)}$ is decreasing in Δ.

□

Note that, because the no-deflation constraint does not bind for $B - \Delta = 0$, the formula for $\frac{\partial \pi}{\partial \Delta}$ derived in this proof is valid for the case $B = \Delta$. In particular, $\frac{\partial \pi}{\partial \Delta} = 0$ whenever $B = \Delta$

Proof of Proposition 2. Notice that it follows immediately from part *iii.*) of propostion 1 that, if $B > 0$, then $U_L(B, B) > U_L(B, 0)$, i.e. $\Delta = B$ satisfies the constraints of the bailout problem (3.3). Furthermore, for any value Δ such that $\Delta > B$, lemma 1 implies that $\pi(B,\Delta) > 0$. This, combined with result *iv.)* from lemma 1 implies that

$$u(y)-\psi(0) = u\left(y+\frac{B-B}{1+\pi(B,B)}\right)-\psi(\pi(B,B)) > u\left(y-\frac{B-\Delta}{1+\pi(B,\Delta)}\right)-\psi(\pi(B,\Delta))$$

i.e. it is always feasible for creditor countries to set $\Delta = B$ in the bailout problem, and this yields strictly higher utility than any $\Delta > B$, hence it must be the case that $\Delta(B) \leq B$.

Finally, note that, because $\frac{B-\Delta}{1+\pi(B,\Delta)}$ is decreasing in Δ, and that, as per proposition 1, $\frac{\partial\pi}{\partial\Delta} < 0$ whenever $\Delta \in (0,B)$, the function $U_L(B,\Delta)$ is increasing in Δ on this interval. Since $\Delta(B) \in [0,B]$, by the non-negativity constraint and the previous result, it follows that $U_L(B,\Delta(B)) = U_L(B,0) \Leftrightarrow \Delta(B) = 0$, i.e. the participation constraint of debtor countries holds with equality in the solution to the bailout problem if and only the optimal bailout is 0. □

Proof of Proposition 3. Let $\tilde{B} = B - \Delta$. It follows from lemma 2 that

$$u\left(y+\frac{B-\Delta}{1+\pi(B,\Delta)}\right)-\psi(\pi(B,\Delta)) = u\left(y+\frac{\tilde{B}}{1+\pi(\tilde{B},0)}\right)-\psi(\pi(\tilde{B},0)) \qquad \text{(B.1)}$$

Notice that the right hand side of (B.1) is a function only of $\tilde{B}$, and in particular does not depend on Δ. Call this function $U_{HN}(\tilde{B})$.

Claim 1. *$\exists$ a unique value $\bar{B} \in (0,\infty)$ of $\tilde{B}$ that maximizes U_{HN}.*

Proof of Claim 1. Define a function

$$\hat{B}(\tilde{B}) = \frac{\tilde{B}}{1+\pi(\tilde{B},0)}$$

i.e. $\hat{B}(\tilde{B})$ is the real value of debt repayment associated with a nominal repayment of $\tilde{B}$.

Claim 2. *$\hat{B}(\tilde{B})$ is strictly increasing whenever $\tilde{B} \geq 0$ and strictly decreasing whenever $\tilde{B} < 0$.*

Proof of Claim 2. Suppose that $\tilde{B}_1 > \tilde{B}_2 \geq 0$. Notice that, by lemma 2,

$$\hat{B}(\tilde{B}_2) = \frac{\tilde{B}_2}{1+\pi(\tilde{B}_2,0)} = \frac{\tilde{B}_1-(\tilde{B}_1-\tilde{B}_2)}{1+\pi(\tilde{B}_1,\tilde{B}_1-\tilde{B}_2)}$$

It then follows from proposition 1 that $\hat{B}(\tilde{B}_1) > \hat{B}(\tilde{B}_2)$. That $\hat{B}(\tilde{B})$ is decreasing when $\tilde{B} < 0$ follows immediately from the fact that $\hat{B}(\tilde{B})$ is strictly increasing whenever $\tilde{B} \geq 0$ and the symmetry of π implied by lemma 2. $\square$

Claim 2 establishes that the function $\hat{B}(\tilde{B})$ has a well defined inverse $\hat{B}^{-1}$. Next, define a function

$$\hat{\pi}(\hat{b}) = \pi(\hat{B}^{-1}(\hat{b}), 0)$$

$\hat{\pi}$ gives the inflation rate associated with any real repayment $\hat{b}$. Next, define a function

$$U_{HR}(\hat{b}) = u(y + \hat{b}) - \psi(\hat{\pi}(\hat{b}))$$

$U_{HR}(\hat{b})$ is the utility of receiving real repayment of $\hat{b}$, given the value of inflation rate that the central bank sets for this real repayment $\hat{\pi}$. Then, since $\hat{B}$ is invertible, $\bar{B}$ will maximize U_{HN} if and only if $\hat{B}(\bar{B})$ maximizes U_{HR}. This implies that to prove Claim 1, it suffices to show that there is a unique $\hat{b} \in (0, y)$ that maximizes U_{HR} (the upper bound follows from the non-negativity constraint on consumption in the central bank's problem). The derivative of U_{HR} is given by

$$U'_{HR}(\hat{b}) = u(y + \hat{b}) - \psi'(\hat{\pi}(\hat{b})) \times \frac{d\hat{\pi}}{d\hat{b}} \tag{B.2}$$

It follows from the first order condition of the central bank that $\hat{\pi}$ satisfies the following identity

$$\frac{1}{2}\hat{b}[u'(y - \hat{b}) - u'(y + \hat{b})] = \psi'(\hat{\pi}(\hat{b})) \times [1 + \hat{\pi}(\hat{b})] \tag{B.3}$$

(B.3) immediately implies that $\hat{\pi}(0) = 0$, and that, as $\hat{b} \to y$, $\hat{\pi}(\hat{b}) \to \infty$, because u satisfies the Inada conditions by assumption 1. Let $\mathcal{H}(\pi) = \psi'(\pi)[1 + \pi]$. Notice that, because ψ is convex by assumption 1, $\mathcal{H}$ is increasing, and that, by assumption 2, $\mathcal{H}$ is concave. Differentiating both sides of (B.3) with repsect to $\hat{b}$, we have

$$\frac{1}{2}[u'(y - \hat{b}) - u'(y + \hat{b})] - \frac{1}{2}\hat{b}[u''(y - \hat{b}) + u''(y + \hat{b})] = \mathcal{H}'(\hat{\pi}(\hat{b})) \times \frac{d\hat{\pi}}{d\hat{b}} \tag{B.4}$$

The left hand side of (B.4) is positive if $\hat{b} > 0$ and negative if $\hat{b} < 0$ because u is strictly concave by assumption 1. $\mathcal{H}'$ is positive, so this implies that $\frac{d\hat{\pi}}{d\hat{b}}$ is positive if $\hat{b} > 0$ and negative if $\hat{b} < 0$. Furthermore, this implies that the function U_{HR} is always increasing when $\hat{b} < 0$, hence any $\hat{b} < 0$ can't be a maximum, and I can restrict attention to positive values of $\hat{b}$. Note that both items in the term $\psi'(\hat{\pi}(\hat{b})) \times \frac{d\hat{\pi}}{d\hat{b}}$ are positive whenever $\hat{b} > 0$, as ψ is strictly increasing by assumption 1. Differentiating both sides of (B.4) with respect to $\hat{b}$, we have

$$-\frac{1}{2}[u''(y-\hat{b}) + u''(y+\hat{b})] - \frac{1}{2}\hat{b}[u''(y-\hat{b}) + u''(y+\hat{b})]$$
$$-\frac{1}{2}[u'''(y+\hat{b}) - u'''(y-\hat{b})] = \mathcal{H}''(\hat{\pi}(\hat{b})) \times \frac{d\hat{\pi}}{d\hat{b}} + \mathcal{H}'(\hat{\pi}(\hat{b})) \times \frac{d^2\hat{\pi}}{d\hat{b}^2} \tag{B.5}$$

Since u is strictly concave by assumption 1 and u''' is non-increasing by assumption 2, the left hand side of (B.5) is non-negative if $\hat{b} > 0$. Since $\mathcal{H}$ is strictly increasing by assumption 1 and concave by assumption 2, and $\hat{\pi}$ is strictly increasing, non-negativity of the left hand side requires $\frac{d^2\hat{\pi}}{d\hat{b}^2}$ to be non-negative if $\hat{b} > 0$, i.e. $\hat{\pi}$ is convex when $\hat{b}$ is positive. Then because the function $\hat{\pi}$ is convex and non-decreasing, the term $\psi'(\hat{\pi}(\hat{b})) \times \frac{d\hat{\pi}}{d\hat{b}}$ is non-decreasing when $\hat{b} > 0$, because ψ is convex by assumption 1 and both terms are positive. By assumption 1 u is strictly concave, which implies U'_{HR} is strictly decreasing on the interval $(0, y)$. Notice that $U'_{HR}(0) > 0$, which implies that $U'_{HR} > 0$ for sufficiently small values of $\hat{b}$. Furthermore, as $\hat{b} \to y$, $\hat{\pi}(\hat{b}) \to \infty$ and $y + \hat{b}$ is bounded. This, combined with the fact that ψ is convex by assumption 1, implies that $U_{HR} \to -\infty$ as $\hat{b} \to y$, hence U_{HR} must be decreasing at some point, which implies that U'_{HR} switches signs. The intermediate value theorem implies that $\exists\, \hat{b}^* > 0$ such that $U'_{HR}(\hat{b}^*) = 0$. Monotonicity of U'_{HR} implies $\hat{b}^*$ is unique, and U'_{HR} switches from positive to negative at $\hat{b}^*$, which implies that $(\hat{b}^*)$ is a maximum. Furthermore, it must be a global maximum as U_{HR} is increasing $\forall\, \hat{b} \in (0, \hat{b}^*)$ (because $U'_{HR} > 0$) and decreasing $\forall\, \hat{b} \in (\hat{b}^*, y)$ (because $U'_{HR} < 0$). $\bar{B} = \hat{B}^{-1}(\hat{b}^*)$ is the value of nominal debt for claim 1. $\square$

It follows from claim 1 and its proof that U_{HN} is strictly increasing if $\tilde{B} \in [0, \bar{B})$ and strictly decreasing if $\tilde{B} \in (\bar{B}, \infty)$. Then since

$$u\left(y + \frac{B - \Delta}{1 + \pi(B, \Delta)}\right) - \psi(\pi(B, \Delta)) = U_{HN}(B - \Delta)$$

it follows that the objective of the bailout problem (3.3) is increasing in Δ whenever $\Delta < B - \bar{B}$ and is decreasing otherwise. Given that the participation constraint of debtor countries will be satisfied for any $\Delta \geq 0$ as per proposition 2, this implies that the solution to the bailout problem has the form

$$\Delta(B) = \begin{cases} 0 & B \leq \bar{B} \\ B - \bar{B} & B > \bar{B} \end{cases}$$

Then given that $\bar{B}$ is unique, $\Delta(B)$ is also unique. $\square$

Proof of Proposition 4. First, uniqueness of the functions π and Δ follow from propositions 1 and 3. Uniqueness of $\hat{b}_H$ and $\hat{b}_L$ follow trivially from the strict concavity of u. To show that $\hat{q}^*$ and B^* are unique, I begin the proof with the following claim:

Claim 3. *Suppose that $\hat{b}_H(\hat{q}, B)$ and $\hat{b}_L(\hat{q}, B)$solve (3.6) and (3.6), respectively. Then $\hat{b}_H(\hat{q}, B) = \hat{b}_L(\hat{q}, B) \ \Leftrightarrow \ \hat{q} = \beta$*

Proof of Claim 3. For $\hat{q} = \beta$, both high endowment and low endowment countries will choose $\hat{b}_i$ to equate consumption in periods 1 and 2. This implies

$$\hat{b}_H(\beta, B) = \hat{b}_L(\beta, B) = \frac{1}{1+\beta}[\epsilon + \hat{\Delta}(B)]$$

where

$$\hat{\Delta}(B) = \frac{\Delta(B)}{1 + \pi(B, \Delta(B))}$$

Then because $\hat{b}_H$ and $\hat{b}_L$ are strictly decreasing in $\hat{q}$ and strictly increasing in $\hat{q}$, respectively, $\hat{q} = \beta$ is the only value of $\hat{q}$ such that $\hat{b}_H(\hat{q}, B) = \hat{b}_L(\hat{q}, B)$. $\square$

This implies that in any real-valued equilibrium of the dynamic model, $\hat{q}^* = \beta$. This immediately implies that

$$\hat{b}_H(\hat{q}^*, B^*) = \hat{b}_L(\hat{q}^*, B^*) = \frac{1}{1+\beta}[\epsilon + \hat{\Delta}(B^*)]$$

To see that the real valued equilibrium is unique, note that the above argument implies that the equilibrium price of real bonds $\hat{q}^*$. The functions π and Δ are unique, by propositions 1 and 3 respectively, and the functions $\hat{b}_H$ and $\hat{b}_L$ are uniquely defined by the strict concavity of u. All that remains to be established is that B^* is unique. To see this, note that, given the functional form for Δ derived in proposition 3,

$$\hat{b}_H(\beta, B) = \hat{b}_L(\beta, B) = \frac{\epsilon}{1+\beta} + \frac{1}{1+\beta} \times \frac{\max\{0, B - \bar{B}\}}{1 + \pi(\max\{0, B - \bar{B}\}, \min\{B, \bar{B}\})}$$

This implies that the fixed point condition for equilibrium requires

$$\frac{\epsilon}{1+\beta}+\frac{1}{1+\beta}\times\frac{\max\{0,B-\bar{B}\}}{1+\pi(\max\{0,B-\bar{B}\},\min\{B,\bar{B}\})}=$$

$$\frac{B}{1+\pi(\max\{0,B-\bar{B}\},\min\{B,\bar{B}\})} \tag{B.6}$$

Notice that the left-hand side of (B.6) is positive and the right-hand side is equal to 0 at $B=0$. For $B<\bar{B}$ the left hand side is constant and the right-hand side is increasing (this is implied by proposition 2 and lemma 2). For $B>\bar{B}$, the left hand side increases with a constant slope equal to $\frac{1}{(1+\beta)(1+\pi(\bar{B},0))}$ and the right-hand side increases with a constant slope equal to $\frac{1}{1+\pi(\bar{B},0)}$. This implies that the derivative of the right-hand side of (B.6) is always lager than that of the left-hand side. Combined with the fact that the left-hand side is larger than the right-hand side at $B=0$ and that the right-hand side increases without bound as $B\to\infty$, this immediately implies that there is a unique $B^*>0$ such that (B.6) is satisfied. □

Proof of Proposition 5. Recall from proposition 4 that the equilibrium real interest rate in the dynamic model with bailouts is equal to the natural rate $\frac{1}{\beta}$. By a similar argument to the one presented in the proof of proposition 4, this will also hold in the equilibrium of the dynamic model without bailouts. This implies that in both both models, fiscal authorities will choose asset positions that equalize consumption in periods 1 and 2. Let c_H^* denote the equilibrium value of consumption for high endowment countries with bailouts, $c_{H,NB}^*$ denote the equilibrium value of consumption for high endowment countries without bailouts, c_L^* denote the equilibrium value of consumption for low endowment countries with bailouts, and $c_{L,NB}^*$ denote the equilibrium value of consumption for low endowment countries with without bailouts. These values will be given by

$$c_H^* = y + \frac{1}{1+\beta}\left(\epsilon - \beta\frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))}\right)$$
$$c_L^* = y - \frac{1}{1+\beta}\left(\epsilon - \beta\frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))}\right)$$
$$c_{H,NB}^* = y + \frac{1}{1+\beta}\epsilon$$
$$c_{L,NB}^* = y - \frac{1}{1+\beta}\epsilon$$

Households' expected utility from consumption with bailouts is given by

$$U_C^* = (1+\beta)\left[\frac{1}{2}u(c_H^*) + \frac{1}{2}u(c_L^*)\right]$$

households' expected utility from consumption without bailouts is given by

$$U_{C,NB}^* = (1+\beta)\left[\frac{1}{2}u(c_{H,NB}^*) + \frac{1}{2}u(c_{L,NB}^*)\right]$$

Notice that the expected value of consumption is equal to y with and without bailouts. If $\Delta(B^*) > 0$, then the variance of consumption with bailouts is strictly less than the variance of consumption without bailouts. As u is strictly concave by assumption 1, it follows immediately that $U_C^* > U_{C,NB}$. Recall from proposition 4 that, with bailouts, the equilibrium real value of bonds is given by

$$\hat{B}^* = \frac{B^*}{1+\pi(B^*,\Delta(B^*))} = \frac{1}{1+\beta}\left[\epsilon + \frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))}\right]$$

and the equilibrium real value of the repayment from low endowment countries to high endowment countries will be given by

$$\hat{B}_\Delta^* = \hat{B}^* - \frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))} = \frac{1}{1+\beta}\left[\epsilon - \beta\frac{\Delta(B^*)}{1+\pi(B^*,\Delta(B^*))}\right]$$

By a similar argument to that presented in the proof of proposition 4, without bailouts, the equilibrium real value of debt, which is equal to the real value of the repayment made from low endowment to high endowment countries, is equal to

$$\hat{B}_{NB}^* = \frac{B_{NB}^*}{1+\pi(B_{NB}^*,0)} = \frac{1}{1+\beta}\epsilon$$

Notice that if $\Delta(B^*) > 0$, then $\hat{B}^*_\Delta < \hat{B}^*_{NB}$. Furthermore, the first order condition of the central bank's problem implies that

$$\frac{1}{2}\hat{B}^*_\Delta\left[u'(y-\hat{B}^*_\Delta) - u'(y+\hat{B}^*_\Delta)\right] = \psi'(\pi(B^*,\Delta(B^*)))[1+\pi(B^*,\Delta(B^*))] \tag{B.7}$$

and

$$\frac{1}{2}\hat{B}^*_{NB}\left[u'(y-\hat{B}^*_{NB}) - u'(y+\hat{B}^*_{NB})\right] = \psi'(\pi(B^*_{NB},0))[1+\pi(B^*_{NB},0)] \tag{B.8}$$

Then because $\hat{B}^*_\Delta < \hat{B}^*_{NB}$, the left-hand side of (B.7) is strictly less than the left-hand side of (B.8), because u' is strictly decreasing, by assumption 1. This implies that $\pi(B^*,\Delta(B^*)) < \pi(B^*_{NB},0)$, since the function $\psi'(\pi)(1+\pi)$ is strictly increasing, because ψ is convex and strictly increasing by assumption 1. This establishes *i.*). *ii.*) follows immediately from the fact that expected utility from consumption is higher with bailouts and $\pi(B^*,\Delta(B^*)) < \pi(B^*_{NB},0)$.

□

B.2 Median Voter Model

There are two time periods, $t = 1, 2$. There is a continuum of countries indexed by $i \in [0,1]$ that form a monetary union. Each country i is populated by a single representative household and a fiscal authority. The household has preferences over expected consumption of a single final good in periods 1 and 2, and the union-wide inflation rate π represented by

$$\mathbf{E}[u(c_{1i}) + \beta[u(c_{2i}) - \psi(\pi)]]$$

where u and ψ satisfy the conditions of assumptions 1 and 2. The fiscal authorities are benevolent and seek to maximize the welfare of households. In the first period, each country receives a random endowment of the period 1 consumption good, y_{1i}. y_{1i} takes one of two values with equal probability: $y_H = y+\epsilon$ and $y_L = y-\epsilon$. Endowment shocks are iid across countries. In the second period, all countries receive a common endowment of the period 2 consumption good y. Let $\{y_{1i}\}$ be the distribution of endowment shocks in period 1. After the distribution of shocks is realized, fiscal authorities trade nominal bonds in a perfectly competitive market at a price $q(\{y_{1i}\})$. Let $\{b_j(\{y_{1i}\})\}_{y_{1j}=y_H}$ and $\{b_j(\{y_{1i}\})\}_{y_{1j}=y_L}$ denote the distribution of bond positions high and low endowment countries, respectively. The monetary union is a closed economy, so the bond market must clear in equilibrium. For notational convenience, I will write the problem of high endowment fiscal authorities as choosing a quantity of bonds to sell and low endowment fiscal authorities as choosing a quantity of bonds to buy, but I don't impose any non-negativity constraints, so both types of countries could be borrowers or lenders in the bond market.

There is a single central bank that sets the inflation rate π between periods 1 and 2 without commitment. In period 2, fiscal authorities in high endowment countries will offer a bailout to fiscal authorities in low endowment countries. Bailouts will be determined by representatives voted on by the fiscal authorities in high endowment and low endowment countries. In particular, there are 4 political candidates, denoted P_{1H}, P_{2H}, P_{1L}, and P_{2L}. Two of these politicians, P_{1H} and P_{2H}, are candidates to represent high endowment countries. Both candidates make a binding campaign promises to offer low endowment countries a bailout, which takes the form of an offer to forgive some amount $\Delta \geq 0$ of the face value of the nominal bonds of all low endowment countries. The winning candidate receives some non-pecuniary benefit $U > 0$, and the losing candidate receives nothing. The winning candidate is decided by majority voting, and all fiscal authorities are assumed to vote sincerely. After the winning candidate makes the bailout proposal, candidates P_{1L} and P_{2L} make binding campaign promises to accept or reject the bailout offer. If the bailout offer is rejected, then the value of the bailout is set equal to zero. Again, the winning candidate receives a non-pecuniary benefit U, the losing candidate receives nothing, the winner is chosen by majority voting, this time by fiscal authorities in low endowment countries, and fiscal authorities vote sincerely. It is assumed that politicians and fiscal authorities in high endowment countries rationally anticipate election outcomes in low endowment countries. The central bank sets the inflation rate after bailouts are determined, taking as given the distribution of nominal bond holdings of fiscal authorities and the value of the bailout Δ as given. Its problem is given by

$$\max_{\pi} \int_{y_{1i}=y_H} u\left(y + \frac{b_i - \Delta}{1+\pi}\right) - \psi(\pi)di + \int_{y_{1i}=y_L} u\left(y - \frac{b_i - \Delta}{1+\pi}\right) - \psi(\pi)di$$

$$\text{s.t.} \qquad \pi \geq 0$$

$$c_i \geq 0 \; \forall \; i \tag{B.9}$$

Denote the solution to this problem by $\pi(\{b_j(\{y_{1i}\})\}_{y_{1j}=y_H}, \{b_j(\{y_{1i}\}_{y_{1j}=y_L}, \Delta)$, and for brevity, write this as $\pi(B_H, B_L, \Delta)$, understanding that capital B's denote distributions. The period 2 utility of the household in a high endowment country with bond position b_i from a bailout of Δ is for bond distributions $\{b_j(\{y_{1i}\})\}_{y_{1j}=y_H}$ and $(\{b_j(\{y_{1i}\})\}_{y_{1j}=y_L}$ given by (again, using the abbreviated notation)

$$U_H(b_i, B_H, B_L, \Delta) = u\left(y + \frac{b_i - \Delta}{1 + \pi(B_H, B_L, \Delta)}\right) - \psi(\pi(B_H, B_L, \Delta))$$

Let $b_H^M(B_H)$ denote the median of the support of the distribution of B_H. Consider two alternative values of the bailout Δ_1 and Δ_2. Let $M_H(\Delta_1)$ denote that measure of countries in the distribution B_H such that $U_H(b_i, B_H, B_L, \Delta_1) \geq U_H(b_i, B_H, B_L, \Delta_2)$ and let $M_H(\Delta_2)$ denote the measure of countries such that $U_H(b_i, B_H, B_L, \Delta_2) \geq U_H(b_i, B_H, B_L, \Delta_1)$.

Claim 4. *$M_H(\Delta_1) \geq M_H(\Delta_2)$ if and only if*

$$U_H(b_H^M(B_H), B_H, B_L, \Delta_1) \geq U_H(b_H^M(B_H), B_H, B_L, \Delta_2)$$

Proof. Suppose that $U_H(b_H^M(B_H), B_H, B_L, \Delta_1) \geq U_H(b_H^M(B_H), B_H, B_L, \Delta_2)$. I consider 2 cases. First, suppose that $\pi(B_H, B_L, \Delta_1) > \pi(B_H, B_L, \Delta_2)$. If this is true, then the consumption of the household of the a fiscal authority holding $b_H^M(B_H)$ must be higher with a bailout equal to Δ_1 than with a bailout of Δ_2. The difference in the consumption of an arbitrary household whose fiscal authority's bond position is b_i under Δ_1 and under

Δ_2 is given by

$$\left(y + \frac{b_i - \Delta_1}{1 + \pi(B_H, B_L, \Delta_1)}\right) - \left(y + \frac{b_i - \Delta_2}{1 + \pi(B_H, B_L, \Delta_2)}\right) =$$
$$b_i \left(\frac{1}{1 + \pi(B_H, B_L, \Delta_1)} - \frac{1}{1 + \pi(B_H, B_L, \Delta_2)}\right) -$$
$$\left(\frac{\Delta_1}{1 + \pi(B_H, B_L, \Delta_1)} - \frac{\Delta_2}{1 + \pi(B_H, B_L, \Delta_2)}\right)$$

Notice that only the first term depends on b_i and that, given that$\pi(B_H, B_L, \Delta_1) > \pi(B_H, B_L, \Delta_2)$, this term is decreasing in b_i. This implies that, if consumption for the household of the fiscal authority with the median bond holding increases, then so must the consumption of households in countries whose fiscal authorities' bond holdings are lower than the median. Furthermore, these households gain more consumption than the median bond holder's household. Given that these households also have higher marginal utilities of consumption, they receive a larger utility gain than the median bond holder's household, hence $M_H(\Delta_1) \geq M_H(\Delta_2)$. For the second case, suppose that $\pi(B_H, B_L, \Delta_1) < \pi(B_H, B_L, \Delta_2)$. Again, the difference in consumption for a household whose fiscal authority has bond holdings b_i will be

$$b_i \left(\frac{1}{1 + \pi(B_H, B_L, \Delta_1)} - \frac{1}{1 + \pi(B_H, B_L, \Delta_2)}\right) -$$
$$\left(\frac{\Delta_1}{1 + \pi(B_H, B_L, \Delta_1)} - \frac{\Delta_2}{1 + \pi(B_H, B_L, \Delta_2)}\right)$$

where, again, only the first term depends on b_i, but is now increasing in b_i. This implies that if the household of the fiscal authority gains consumption under Δ_1, households of all fiscal authorities with bond holdings larger than the median will also gain consumption, and hence will also gain utility under Δ_1, as inflation will be lower and consumption will be higher. If the household of the median bond holder loses consumption under Δ_1, households whose fiscal authorities hold more than the median bond holding will either lose less consumption than the household of the median bond holder or will gain consumption. Since these households have lower marginal utilities of consumption than the median bond holder's household, they will lose at most as much utility from consumption as the household of the median bond holder. Then given that the household of the median bond holder has a net utility gain under Δ_1, so will all households whose fiscal authorities hold more bonds than the median bond holder. In either case, all households whose fiscal authorities hold more bonds than the median must have higher utility under Δ_1, hence $M_H(\Delta_1) \geq M_H(\Delta_2)$. This establishes necessity. The proof of sufficiency is essentially identical. □

The period 2 utility of the household in a low endowment country with bond position b_i from a bailout of Δ for bond distributions $\{b_j(\{y_{1i}\})\}_{y_{1j}=y_H}$ and $\{b_j(\{y_{1i}\})\}_{y_{1j}=y_L}$ is given by

$$U_L(b_i, B_H, B_L, \Delta) = u\left(y - \frac{b_i - \Delta}{1 + \pi(B_H, B_L, \Delta)}\right) - \psi(\pi(B_H, B_L, \Delta))$$

Let $b_L^M(B_L)$ denote the median of the support of the distribution of B_L. Consider two alternative values of the bailout Δ_1 and Δ_2. Let $M_L(\Delta_1)$ denote that measure of countries in the distribution B_L such that $U_L(b_i, B_H, B_L, \Delta_1) \geq U_L(b_i, B_H, B_L, \Delta_2)$ and let $M_L(\Delta_2)$ denote the measure of countries such that $U_L(b_i, B_H, B_L, \Delta_2) \geq U_L(b_i, B_H, B_L, \Delta_1)$.

Claim 5. $M_L(\Delta_1) \geq M_L(\Delta_2)$ *if and only if*

$$U_L(b_L^M(B_L), B_H, B_L, \Delta_1) \geq U_L(b_L^M(B_L), B_H, B_L, \Delta_2)$$

The proof of Claim 5 is identical to that of Claim 4, with the signs switched.
Claims 4 and 5 establish that the election system for determining bailouts is a median voter model. Low endowment country candidates will promise to accept bailouts if and only if $U_L(b_L^M(B_L), B_H, B_L, \Delta) \geq U_L(b_L^M(B_L), B_H, B_L, \Delta)$. High endowment country candidates will anticipate this behavior and will choose bailouts to solve

$$\max_{\Delta} \; U_H(b_H^M(B_H), B_H, B_L, \Delta)$$

$$\begin{aligned} U_L(b_L^M(B_L), B_H, B_L, \Delta) \geq U_L(b_L^M(B_L), B_H, B_L, \Delta) \\ \Delta \geq 0 \end{aligned} \tag{B.10}$$

Before moving on with the definition of equilibrium notice that, if the distribution of bond holdings is such that all high endowment countries buy nominal bonds B and all low endowment countries sell nominal bonds B, or the distribution differs from such a distribution only on a set of measure zero, as in the case of an individual deviation, the central bank's problem (B.9) coincides with the central bank's problem in the static model (3.1), and hence the bailout problem (B.10) will coincide with the bailout problem of the static model (3.3). Denote the solution of this problem $\Delta(B_H, B_L)$.

Finally, after observing the realizations of shock $\{y_{1i}\}$, if country j receives a high endowment shock, the fiscal authority in country j chooses how many bonds to buy $b_j(\{y_{1i}\})$ to maximize the welfare of the representative household, taking the price of bonds, the bond positions of other countries, the bailout function Δ, and the central bank's inflation function π as given

$$\max_{b_j} \; u(y - q(\{y_{1i}\})b_j) + \beta U_H(b_j, B_{H,i\neq j}(\{y_{1i}\}), B_L(\{y_{1i}\}), \Delta(B_{H,i\neq j}(\{y_{1i}\}), B_L(\{y_{1i}\})))$$

where I use the notation

$$B_L(\{y_{1i}\}) = \{b_k(\{y_{1i}\})\}_{y_{1k}=y_L}$$

and

$$B_{H,i\neq j}(\{y_{1i}\}) = \{b_k(\{y_{1i}\})\}_{y_{1k}=y_H, k\neq j}$$

If country j receives a high endowment shock, the fiscal authority in country j chooses how many bonds to sell $b_j(\{y_{1i}\})$to maximize the welfare of the representative household, taking the price of bonds, the bond positions of other countries, the bailout function Δ, and the central bank's inflation function π as given

$$\max_{b_j} \; u(y + q(\{y_{1i}\})b_j) + \beta U_L(b_j, B_H(\{y_{1i}\}), B_{L,i\neq j}(\{y_{1i}\}), \Delta(B_H(\{y_{1i}\}), B_{L,i\neq j}(\{y_{1i}\})))$$

Then

Definition. *An equilibrium of the median voter model is a set of functions* $q(\{y_{1i}\})$, $\{b_j(\{y_{1i}\})\}$, $\Delta(B_H, B_L)$, *and* $\pi(B_H, B_L, \Delta)$ *such that*

i.) b_j *solve the problems of fiscal authority* j *for all distributions of endowments* $\{y_{1i}\}$, *taking all other functions as given*

ii.) Δ *solves the bailout problem* (B.10) *taking* π *as given*

iii.) π *solves the central bank's problem* (B.9)

iv.) The bond market clears for all distributions of endowments $\{y_{1i}\}$

$$\int_{y_{1j}=y_H} b_j(\{y_{1i}\})dj = \int_{y_{1j}=y_L} b_j(\{y_{1i}\})dj$$

Clearly, in a symmetric equilibrium of the median voter model, the central bank's problem and bailout problem will coincide with the bailout problem of the static model along the equilibrium path, as the bond holdings of the median fiscal authority will equal the bond holdings of all fiscal authorities, and the solution of both problems in the median voter model will be unaffected by individual deviations from the equilibrium path by fiscal authorities, as such deviations do not affect the value of the median of the bond holding distribution or the objective of the central bank, which implies that, along the equilibrium path, the problems of individual fiscal authorities in the median voter model will coincide with the problems of fiscal authorities in the dynamic model, i.e. they will take bailouts, inflation, and the representative fiscal authority's bond holdings as given when making their individual decisions.

B.3 Free-Rider Problem

The first order condition for a fiscal authority's problem without a monetary union (3.11) is given by

$$\begin{aligned}\frac{\beta}{1+\pi(b_i)}\left(u'\left(y_{1i}+\frac{\beta b_i}{1+\pi(b_i)}\right)-u'\left(y-\frac{b_i}{1+\pi(b_i)}\right)\right)\\ -\pi'(b_i)\frac{\beta b_i}{(1+\pi(b_i))^2}\left(u'\left(y_{1i}+\frac{\beta b_i}{1+\pi(b_i)}\right)-u'\left(y-\frac{b_i}{1+\pi(b_i)}\right)\right)-\beta\psi'(\pi(b_i))\pi'(b_i)=0\end{aligned} \tag{B.11}$$

Claim 6. *If $b_i > 0$, then $\pi'(b_i) > 0$*

Proof. If $b_i > 0$, the solution of the central bank's problem will be interior. From the central bank's first order condition, $\pi(b_i)$ satisfies

$$\frac{b_i}{(1+\pi(b_i))^2}u'\left(y-\frac{b_i}{1+\pi(b_i)}\right)-\psi'(\pi(b_i))=0$$

Differentiating with respect to b_i implies

$$\begin{aligned}\frac{1}{(1+\pi(b_i))^2}u'\left(y-\frac{b_i}{1+\pi(b_i)}\right)-\frac{1}{(1+\pi(b_i))^3}u''\left(y-\frac{b_i}{1+\pi(b_i)}\right)\\ -\pi'(b_i)\left[\frac{2b_i}{(1+\pi(b_i))^3}u'\left(y-\frac{b_i}{1+\pi(b_i)}\right)-\left(\frac{b_i}{(1+\pi(b_i))^2}\right)^2u''\left(y-\frac{b_i}{1+\pi(b_i)}\right)\right]\\ -\pi'(b_i)\psi''(\pi(b_i))=0\end{aligned}$$

Rearranging and solving for π' yields

$$\pi'(b_i) = \frac{\frac{1}{(1+\pi(b_i))^2} u'\left(y - \frac{b_i}{1+\pi(b_i)}\right) - \frac{1}{(1+\pi(b_i))^3} u''\left(y - \frac{b_i}{1+\pi(b_i)}\right)}{\frac{2b_i}{(1+\pi(b_i))^3} u'\left(y - \frac{b_i}{1+\pi(b_i)}\right) - \left(\frac{b_i}{(1+\pi(b_i))^2}\right)^2 u''\left(y - \frac{b_i}{1+\pi(b_i)}\right) + \psi''(\pi(b_i))}$$

which is positive if $b_i > 0$ because $u' > 0$, $u'' > 0$, and $\psi' \geq 0$ by assumption 1. $\square$

Suppose that the fiscal authority chose b_i acting as if π did not depend on b_i, as fiscal authorities do in a monetary union. In this case, they would choose b_i such that

$$u'\left(y_{1i} + \frac{\beta b_i}{1+\pi(b_i)}\right) - u'\left(y_{1i} + \frac{b_i}{1+\pi(b_i)}\right) = 0 \tag{B.12}$$

At such a value of b_i, the left-hand side of (B.11) would be equal to

$$-\beta\psi'(\pi(b_i))\pi'(b_i)$$

If the fiscal authority is borrowing $b_i > 0$, then its central bank will set a strictly positive inflation rate and π' will be positive (issuing more bonds leads to even higher inflation) by Claim 6, which implies that this term will be negative. The fiscal authority could improve welfare by decreasing b_i. This is the essence of the free-rider problem in the monetary union. If fiscal authorities ignore the effect of their debt issuance on the inflation rate, as they do in the dynamic model of the monetary union, then debt and inflation tend to be inefficiently high.

B.4 Solution of Quadratic Model

Monetary Union Outcomes

First, I will solve for $\hat{b}_\Delta$, the optimal value of the real repayment from low endowment countries to high endowment countries in period 2, assuming that it occurs at an interior solution for inflation. For an arbitrary real repayment $\hat{b}$ the first order condition of the central bank's problem for an interior solution is

$$\begin{aligned} & \frac{1}{2}\hat{b}[u'(y-\hat{b}) - u'(y-\hat{b})] = \hat{\psi}(1+\pi) \\ \Rightarrow \quad & \frac{1}{2}\hat{b}[2(\bar{y} - y + \hat{b}) - 2(\bar{y} - y - \hat{b})] = \hat{\psi}(1+\pi) \\ \Rightarrow \quad & 2\hat{b}^2 = \hat{\psi}(1+\pi) \end{aligned}$$

This implies that the utility of a high endowment country from a real repayment $\hat{b}$ is given by

$$-[\bar{y} - y - \hat{b}]^2 - 2\hat{b}^2$$

where the second term is just the disutility from inflation $\hat{\psi}(1 + \pi)$ implied by the solution of the central bank's problem. The first order condition for maximizing this function is

$$2[\bar{y} - y - \hat{b}] - 4\hat{b} = 0$$

which implies that

$$\hat{b}_{\Delta} = \frac{1}{3}[\bar{y} - y]$$

and to guarantee that this corresponds to an interior solution for inflation requires the parameter restriction

$$2\frac{\hat{b}_{\Delta}^2}{\hat{\psi}} = \frac{\frac{2}{3}[\bar{y} - y]^2}{\hat{\psi}} > 1$$

Notice that this restriction implies that in any equilibrium in which there are positive bailouts the solution for inflation will be interior. I will make one further parameter restriction to ensure the solution for inflation will be interior in any equilibrium in which there are no bailouts. If there are no bailouts, the real value of the repayment will be given by

$$\hat{b} = \frac{\epsilon}{1 + \beta}$$

so for an interior solution, it must be the case that

$$2\frac{\left(\frac{\epsilon}{1+\beta}\right)^2}{\hat{\psi}} > 1$$

which implies

$$\epsilon > (1 + \beta)\left(\frac{\hat{\psi}}{2}\right)^{\frac{1}{2}}$$

Turning to the first period, recall that in equilibrium, the real interest rate is equal to the natural rate, so fiscal authorities will borrow to to set consumption in period 1 equal to consumption in period 2. Note that there will be bailouts in equilibrium only if real value of bonds required to equal consumption in the two periods assuming no bailouts $\frac{\epsilon}{1+\beta}$ is larger than the optimal real repayment $\hat{b}_\Delta$ (This is true for any utility specification. See Figure 3.2). Let ϵ^* be the value of ϵ such that

$$\frac{\epsilon^*}{1+\beta} = \hat{b}_\Delta$$

The equilibrium of the dynamic model will have positive bailouts if $\epsilon > \epsilon^*$ and zero bailouts otherwise. If bailouts are zero, in equilibrium real bonds will be equal to

$$\hat{b} = \frac{\epsilon}{1+\beta}$$

In both periods consumption of low endowment countries will be equal to

$$y - \hat{b} = y - \frac{\epsilon}{1+\beta}$$

In both periods consumption of high endowment countries will be

$$y + \hat{b} = y + \frac{\epsilon}{1+\beta}$$

and the disutility of inflation will be

$$\hat{\psi}(1+\pi) = 2\hat{b}^2 = 2\left(\frac{\epsilon}{1+\beta}\right)^2$$

If there are positive bailouts, then the real repayment in the second period will be equal to the optimal real repayment $\hat{b}_\Delta$. This implies that consumption in both periods for low endowment countries will be equal to

$$y - \hat{b}_\Delta = y - \frac{\epsilon^*}{1+\beta}$$

and consumption in both periods of high endowment countries will be equal to

$$y - \hat{b}_\Delta = y - \frac{\epsilon^*}{1+\beta}$$

The disutility of inflation will be given by

$$\hat{\psi}(1+\pi) = 2\hat{b}_\Delta^2 = 2\left(\frac{\epsilon^*}{1+\beta}\right)^2$$

Substituting these solutions into the utility function, the expected utility of households in the monetary union is

$$V_\Delta^{MU}(\epsilon) = \begin{cases} -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon}{1+\beta}\right)^2 & \epsilon < \epsilon^* \\ -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon^*}{1+\beta}\right)^2 & \epsilon \geq \epsilon^* \end{cases}$$

If bailouts are not allowed, then the expected utility of households in the monetary union is

$$V^{MU}(\epsilon) = -(1+\beta)[\bar{y}-y]^2 - (1+3\beta)\left(\frac{\epsilon}{1+\beta}\right)^2$$

Outcomes Without Monetary Union

Note first that, if countries do not form a monetary union, fiscal authorities in high endowment countries will be able equalize consumption in periods one and two by saving. Since they save, the no-deflation constraint of the central bank will bind and disutility from inflation will be at its minimum. Given that the real interest rate is equal to the natural rate, this is obviously the best policy for fiscal authorities in high endowment countries. This implies consumption in both periods in high endowment countries will given by

$$y + \frac{\epsilon}{1+\beta}$$

and the disutility from inflation in high endowment countries will be equal to $\hat{\psi}$ (recall $\psi(\pi) = \hat{\psi}(1+\pi)$, so $\psi(0) = \hat{\psi}$).

Now, consider the problem of low endowment countries. Suppose that the fiscal authority wants to sell a value of real bonds equal to $\hat{b}$. If this level of real borrowing corresponds to an interior solution for the central bank's problem, the first order condition for the central bank implies that

$$\hat{\psi}(1+\pi) = \hat{b}u'(y-\hat{b}) = 2\hat{b}[\bar{y}-y+\hat{b}]$$

Since the price of real bonds must equal β, if the fiscal authority chooses a value of $\hat{b}$ such that the solution for inflation is interior, $\hat{b}$ must solve

$$\max_{\hat{b}} \quad -[\bar{y}-y+\epsilon-\beta\hat{b}]^2 - \beta[\bar{y}-y+\hat{b}]^2 - 2\hat{b}[\bar{y}-y+\hat{b}]$$

The first order condition for this problem is

$$2\beta[\bar{y}-y+\epsilon-\beta\hat{b}] - 2\beta[\bar{y}-y+\hat{b}] - 2\beta[\bar{y}-y+\hat{b}] - 2\beta\hat{b} = 0$$

Solving for $\hat{b}$ yields

$$\hat{b} = \frac{1}{2+\beta}[\epsilon-\bar{y}+y]$$

If I impose the parameter restriction $\bar{y} > y+\epsilon$, i.e. the upper-bound on consumption in the quadratic utility function is sufficiently large compared to y, this value is negative, which implies that it can't correspond to an interior solution of the central bank's problem. This implies that the fiscal authority in low endowment countries will never issue enough debt to induce inflation from the central bank. Instead, they will issue as much debt as possible such that the central bank will choose zero inflation. The central bank's first order condition implies that this corresponds to a real value of debt equal to

$$\hat{b}^* = \frac{-[\bar{y} - y] + \sqrt{(\bar{y} - y)^2 + 8\hat{\psi}}}{4}$$

$\hat{b}^* > 0$, and the parameter restriction imposed in the monetary union problem will imply that $\hat{b}^* < \frac{\epsilon}{1+\beta}$ (To see this, note that any value of the real repayment low endowment countries make $\hat{b}$ that induces inflation in the monetary union will also induce inflation for low endowment countries without the monetary union. This is because the central bank doesn't weigh the loss inflation causes to creditors when choosing the inflation rate in the no union case.). Note that the disutility from inflation for low endowment countries will also be equal to $\hat{\psi}$. These solutions imply that the expected utility of countries without a monetary union is

$$V^A(\epsilon) = -(1+\beta)[\bar{y} - y]^2 - \frac{2+\beta}{2}\epsilon^2 - \frac{\beta(1+\beta)}{2}b^{*2} + \beta\epsilon b^* - \hat{\psi}$$

Notice that, because $0 < \hat{b}^* < \frac{\epsilon}{1+\beta}$

$$V_L^A(\epsilon) < V^A(\epsilon) < V_H^A(\epsilon)$$

where

$$V_L^A(\epsilon) = -(1+\beta)[\bar{y} - y]^2 - \frac{2+\beta}{2}\epsilon^2 - \hat{\psi}$$

and

$$V_H^A(\epsilon) = -(1+\beta)[\bar{y} - y]^2 - (1+\beta)\epsilon^2 - \hat{\psi}$$

V_L^A gives expected utility under the assumption that low endowment countries set $\hat{b} = 0$, and V_H^A gives expected utility if low endowment countries could equalize consumption in periods 1 and 2 (which would require setting $\hat{b} = \frac{\epsilon}{1+\beta}$) without inducing inflation from the central bank. Note that, thus far, I have ignored the non-negativity constraints on consumption. All values of consumption will be strictly non-negative if the fourth parameter restriction $y - \epsilon > 0$ is satisfied.

Choosing Parameters

Consider the problem of choosing model parameters $\hat{\psi}$, y, $\bar{y}$, and β such that $\exists$ an interval $[\underline{\epsilon}, \overline{\epsilon}]$ such that $\epsilon^* \in [\underline{\epsilon}, \overline{\epsilon}]$, $\exists\ \tilde{\epsilon} \in [\underline{\epsilon}, \overline{\epsilon}]$ such that $\forall\ \epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$, the following conditions are satisfied

i.) $V^A(\epsilon) > V^{MU}(\epsilon)$ (No monetary union is preferable to a monetary union without bailouts)

ii.) $V^A(\epsilon) > V^{MU}_{\Delta}(\epsilon)$ if $\epsilon < \tilde{\epsilon}$ and $V^A(\epsilon) < V^{MU}_{\Delta}(\epsilon)$ if $\epsilon > \tilde{\epsilon}$ (A monetary union with bailouts is preferable to no monetary union if ϵ is large enough)

iii.) The parameter restrictions are satisfied

I will give a procedure that accomplishes this. First, $\tilde{\epsilon}$ must be a value of ϵ such that $V^A(\epsilon) = V^{MU}_{\Delta}(\epsilon)$, and if $V^A(\epsilon) > V^{MU}(\epsilon)\ \forall\ \epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$, then it must be the case that $V^A(\tilde{\epsilon}) = V^{MU}(\epsilon^*)$. Since V^A and V^A_H are decreasing, the value of $\tilde{\epsilon}$ must be less than the value $\tilde{\epsilon}_H$ such that $V^A_H(\tilde{\epsilon}_H) = V^{MU}(\epsilon^*)$. Finally, since $\hat{\psi} > 0$, $\tilde{\epsilon}_H < \overline{\epsilon}$ where $V^A_H(\overline{\epsilon}) + \hat{\psi} = V^{MU}(\epsilon^*)$. $\overline{\epsilon}$ then solves

$$-\frac{2+\beta}{2}\overline{\epsilon}^2 = -\frac{1+3\beta}{1+\beta}\epsilon^{*2} = -\frac{1+3\beta}{1+\beta}\left(\frac{1}{3}[\bar{y}-y](1+\beta)\right)^2$$

This implies

$$\overline{\epsilon} = \sqrt{\frac{2(1+\beta)(1+3\beta)}{2+\beta}} \times \frac{1}{3} \times [\bar{y}-y]$$

The third parameter restriction requires that $\epsilon < \bar{y} - y\ \forall\ \epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$. From the above equation, this will hold as long as $\frac{(1+\beta)(1+3\beta)}{2+\beta} < 9$, so fix a value of $\beta \in (0,1)$ such that this is satisfied (this holds at $\beta = 0$, so it will hold for sufficiently small β). Note that $\frac{2+\beta}{2} < \frac{1+3\beta}{1+\beta}$ for any $\beta \in (0,1)$, so $\epsilon^* < \overline{\epsilon}$. Note that

$$V^A(\epsilon) - V^{MU}(\epsilon) > V^A_L(\epsilon) - V^{MU}(\epsilon) = \frac{3\beta - \beta^2}{2\beta}\epsilon^2 - \hat{\psi}$$

The value in this expression will be positive if $\epsilon > \underline{\epsilon}_1(\hat{\psi}) = \left(\frac{2}{3-\beta}\hat{\psi}\right)^{\frac{1}{2}}$. Recall that the second parameter restriction is that

$$\epsilon > (1+\beta)\left(\frac{\hat{\psi}}{2}\right)^{\frac{1}{2}}$$

This parameter restriction will be satisfied if $\epsilon > \underline{\epsilon}_2(\hat{\psi}) = \left((1+\beta)^2\frac{\hat{\psi}}{2}\right)^{\frac{1}{2}}$. The first parameter restriction is that $\frac{\frac{2}{3}[\bar{y}-y]^2}{\hat{\psi}} > 1$. Fix any value for $\bar{y}-y$ greater than zero. Choose a value of $\hat{\psi}$ such that $\epsilon_1(\hat{\psi}) < \frac{1}{3}[\bar{y}-y](1+\beta)$, $\epsilon_2(\hat{\psi}) < \frac{1}{3}[\bar{y}-y](1+\beta)$, and $\frac{\frac{2}{3}[\bar{y}-y]^2}{\hat{\psi}} > 1$ (this is always possible for a sufficiently small value of $\hat{\psi}$). Set $\underline{\epsilon} = \max\{\epsilon_1(\hat{\psi}), \epsilon_2(\hat{\psi})\}$. This guarantees that $V^A > V^{MU}$ on $[\underline{\epsilon}, \bar{\epsilon}]$, that $\epsilon^* \in [\underline{\epsilon}, \bar{\epsilon}]$, and that the first parameter restriction is satisfied. Notice that, because $V^A > V^{MU}$ and that $V^A(\tilde{\epsilon}) = V^{MU}(\epsilon^*)$, it follows that $\tilde{\epsilon} > \epsilon^*$, which implies $\tilde{\epsilon} \in [\underline{\epsilon}, \bar{\epsilon}]$. Choose $y > \bar{\epsilon}$. This guarantees that the fourth parameter restriction $y - \epsilon > 0$ will be satisfied. Finally, choose $\bar{y}$ to match the fixed value of $\bar{y} - y$.