4π in the Sky

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based on work with G. D'Amico, A. Lawrence, and L. Sorbo, arXiv:1607.06105, arXiv:1404.2912, arXiv:1101.0026, arXiv:0811.1989, arXiv:0810.5346 + …

THEORIES OF INFLATION AND CONFORMAL TRANSFORMATIONS

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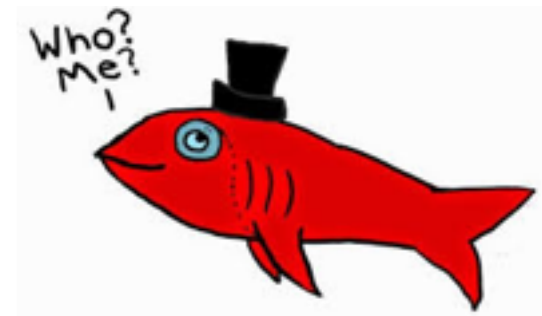
Large field inflation: bottom up (from the top down!)

- Many different inflation candidates now thanks to some older and a lot of new interesting work.

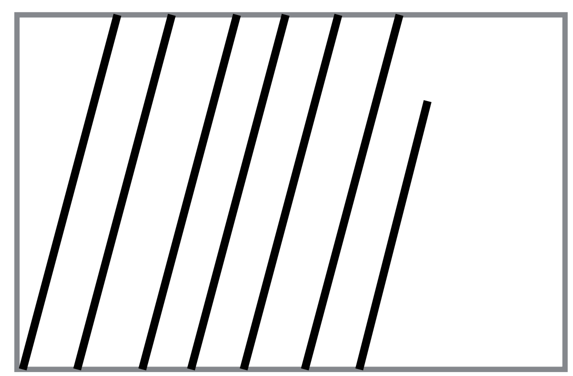


- pNGBs ("axions"), (with... shift symmetry?)
- Instead of focusing on how, let's try to figure out why
- A tempting conjecture: an "equivalence class" of inflatons: explicit models are detail-dependent, but share the same underlying dynamics???

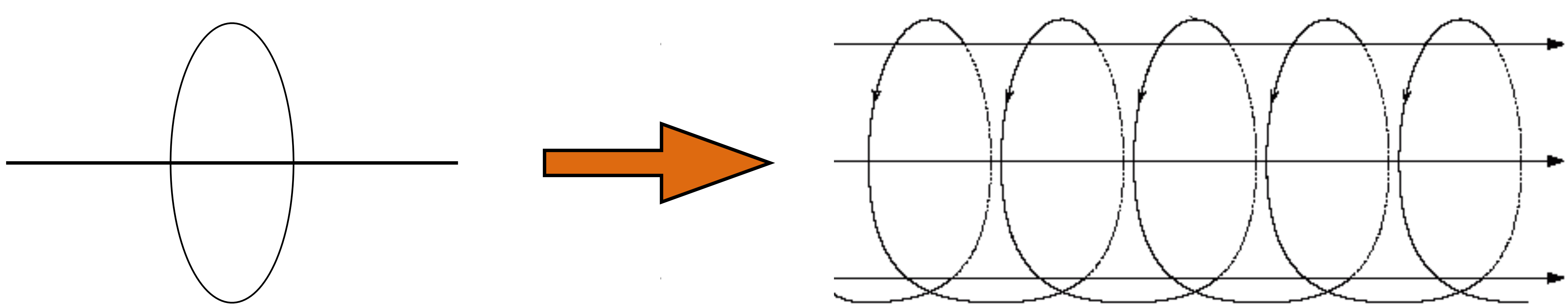
Monodromy

Silverstein, Westphal; McAllister, Silverstein, Westphal; NK, Sorbo; NK, Lawrence, Sorbo; …

Idea: try to get large field ranges in small compact spaces which do not violate the reliability of low energy EFT - so you want to move a lot without colliding with your path in phase space



A simple physical example: a particle in a magnetic field, with a general velocity vector (in QM a very complex system, many many Landau levels…)



A pedestrian view: what is the point here?

NK, Lawrence

- A caricature: below the string scale, string theory is a QFT + corrections

- Inflation is below string scale - so string constructions - if they work - must give consistent QFTs of inflation with corrections included

- If inflation is high scale single field there is a lightest inflaton and a MASS GAP in the spectrum of QFT; one can integrate out everything at and above the mass of the next lightest particle - which sets the cutoff

- **!!!!!!!!!! DECOUPLING !!!!!!!!!!**

- Thus: THERE MUST BE A COVARIANT EFT OF INFLATION WHICH GETS AROUND THE USUAL HEADACHES

- All that matters about the constructions: 1) they exist 2) they compute the inflaton mass and the cutoff - the rest is just EFT

Pedestrian constructions

NK, Lawrence, Sorbo

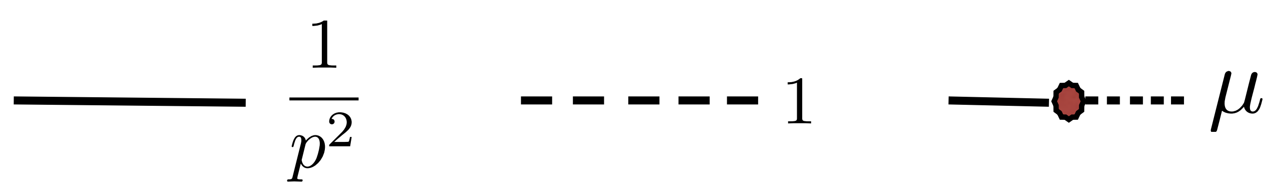
$$\mathcal{S}_{bulk} = \int d^4x\sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots\Big)$$

Di Vecchia and Veneziano; Quevedo and Trugenberger; Dvali and Vilenkin; NK & Sorbo; NK, Lawrence & Sorbo.

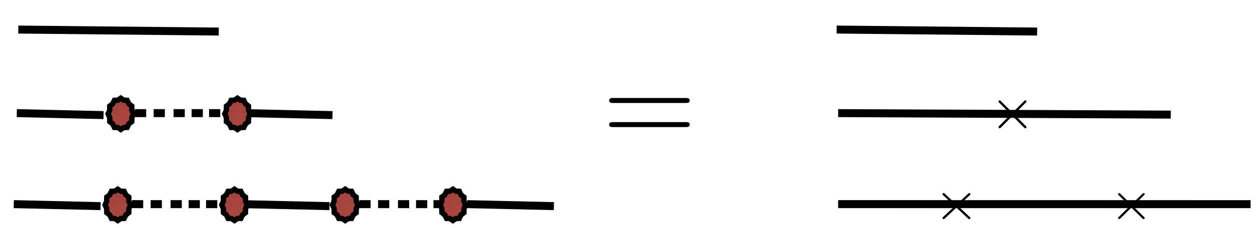
Action invariant under (perturbative) shift symmetry (red herring!)

under $\phi \to \phi + c$, $\mathcal{L} \to \mathcal{L} + c\,\mu\,\varepsilon^{\mu\nu\varrho\lambda}F_{\mu\nu\varrho\lambda}/24$

Dynamics: the scalar propagator, 4-form propagator and scalar-form vertex:



So the physical scalar propagates as a massive field… with a radiatively stable mass



$$\frac{1}{p^2} + \frac{1}{p^2}\mu^2\frac{1}{p^2} + \frac{1}{p^2}\mu^2\frac{1}{p^2}\mu^2\frac{1}{p^2} + \ldots = \frac{1}{p^2-\mu^2}$$

Dynamics a la Landau

$$S_{class} = \int d^4x\sqrt{g}\left(m_{pl}^2 R - \frac{1}{48}F^2 - \frac{1}{2}(\partial\varphi)^2 + \frac{\mu}{24}\varphi^* F\right)$$

$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]}$ $\delta A_{\mu\nu\lambda} = \partial_{[\mu} \Lambda_{\nu\lambda]}$ $\varphi \equiv \varphi + 2\pi f_\varphi$

U(1) gauge symmetry +
discrete gauge symmetry

$H_{tree} = \frac{1}{2}p_\phi^2 + \frac{1}{2}\left(p_A + \mu\phi\right)^2 + \ grav.$ $p_A = ne^2$

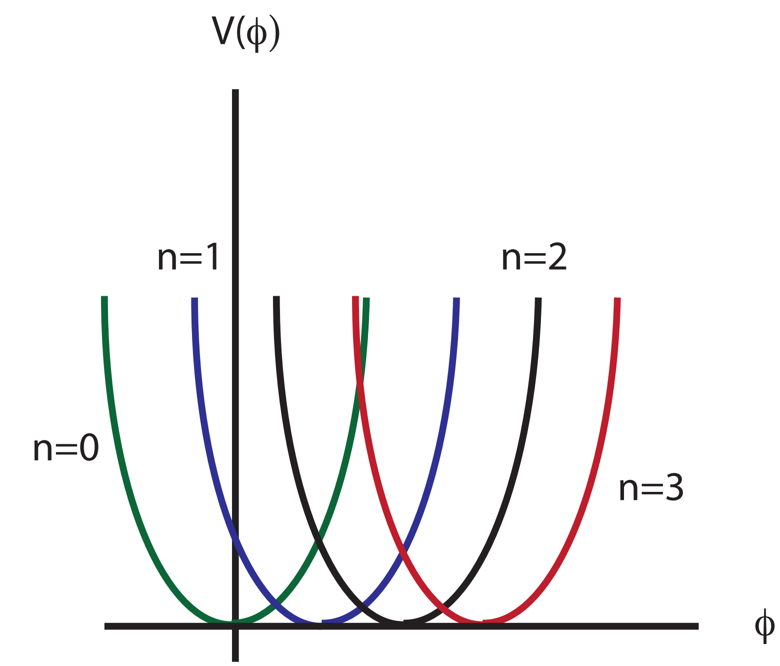
Compactness of U(1)

Dim. red. to 0+1:
charged particle
in B-field on torus.
k = magnetic flux quantum
= LL degeneracy

$\longrightarrow$ $\mu f_\phi = e^2$

$\varphi \to \varphi + 2\pi k f_\phi\,; \quad n \to n - k$

discrete gauge symmetry in phase space



Déjà vu! It really IS just a particle in a magnetic field!

The theory has TWO sets of gauge symmetries: the nonlinearly realized U(1) of the massive 4-form and the discrete gauge symmetry of the scalar!

Er... Why is this???

Dynamical mode is a massive scalar field? What is really going on here?

What do we see: a 4-form gauge field + a 'random' massless (pseudo)scalar who mix up and yield… a massive theory??? SSB! But… there is no "Higgs"? - i.e. no "radial" mode - just a "phase"?

Rings a bell! A massive U(1) gauge theory - eg BCS superconductor!

Electrons in a material: a free gas - sometimes interactions with the lattice generate a state with long range correlations breaking QM phase shift invariance. This symmetry is gauged - it's U(1) charge. The resulting Goldstone eaten by the U(1) field which becomes massive.

Microscopics challenging - but effective theory straightforward: Londons' eq!

Fritz

$$\vec{A} = \frac{\vec{J}}{m^2}$$

Heinz

Phases of a massive U(1): an insight of Julia and Toulouse

- Conventional UV embedding is by way of a Higgs mechanism
- But for U(1) there are other options...!!! JULIA & TOULOUSE
- EXAMPLE: consider a gauge theory with defects - eg a U(1) theory with vortices (= strings), go to the phase containing MANY defects and let them couple strongly. LET THEM CONDENSE!
- ASK: fluctuate this system. What is the EFT of fluctuations???
- Wisdom after the fact: superfluids! In the presence of many vortices, the superfluid velocity is not irrotational; superfluid current: $\vec{j} \simeq \vec{v}$
- Julia-Toulouse: take that current and DECLARE IT to be a gauge field potential - and add a gauge field strength for it to boot!

Table II. — *Hydrodynamic variables for* He_4.

p	Number of components: Space only	Number of components: Space + time	No vortex	Many vortices
0	1	1	Variable φ	
1	3	4	Superfluid velocity v, $\mathrm{d}x = \mathrm{d}\varphi$	Variables A_μ

The Scoop on Julia-Toulouse

- TAKE A GAUGE THEORY WITH DEFECTS, MAKE MANY CONDENSE; WHAT IS THE NEW EFT OF THE SYSTEM?
- The rule: TAKE THE *DUAL* OF THE OLD GAUGE FIELD STRENGTH - which in the presence of the defects obeys

$$^*F \sim J$$

- NOT closed any more - DECLARE IT TO BE THE GAUGE POTENTIAL for the new phase
- THE DUALITY TRANSFORMATIONS KNOW THIS!
- (look up the magnetic field scalar! potential in Jackson's book! (in the section on magnetostatics))
- NOTE: you need to know that the condensate EXISTS and the excitations are gapped!!! For this you must have a microscopic theory (or some wisdom after the fact: superconductors exist...)

Julia, Toulouse; Quevedo, Trugenberger; NK, Lawrence

4-form massive gauge theory

- Follow the known rules of massive QED

$$\mathcal{L} = -\frac{1}{48}F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho} - \frac{m^2}{12}\left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^2 - \frac{1}{2\xi}\left(\partial^\mu A_{\mu\nu\lambda} - \frac{\xi m^2}{2}b_{\nu\lambda}\right)^2$$

- Londons' eq: $A_{\mu\nu\lambda} \sim \frac{J_{\mu\nu\lambda}}{m^2}$: current of membranes!
- At high momenta:

$$\langle A_{\mu\nu\lambda}(p)A_{\mu'\nu'\lambda'}(-p)\rangle = \epsilon_{\mu\nu\lambda\rho}\epsilon_{\mu'\nu'\lambda'\rho'}\left(\frac{\frac{\xi}{2}\eta^{\rho\rho'}}{p^2 - \frac{\xi m^2}{2}} + \frac{\left(1-\frac{\xi}{2}\right)p^\rho p^{\rho'}}{(p^2-m^2)(p^2-\frac{\xi m^2}{2})}\right)$$

- The mass term is innocuous as momenta get big so the theory is well behaved in the UV; gauge symmetry protects it. Further it has ONLY one propagating DOF - a scalar!
- To see this dualize! Ignore the GF and use 1st order formulation;

$$\mathcal{L} = -\frac{1}{48}F^2_{\mu\nu\lambda\sigma} - \frac{m^2}{12}(A_{\mu\nu\lambda} - h_{\mu\nu\lambda})^2 + \frac{m}{6}\phi\epsilon^{\mu\nu\lambda\rho}\partial_\mu h_{\nu\lambda\rho}$$

- The constraint enforces h = db; integrate out h

More on massive 4-form:

- The resulting action is, with F = dA,

$$\mathcal{L}_{\phi,A} = -\frac{1}{48}F^2_{\mu\nu\lambda\sigma} - \frac{1}{2}(\partial_\mu\phi)^2 + \frac{m}{24}\phi\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho}$$

- Now we can integrate out F; adding $\frac{Q}{24}\epsilon_{\mu\nu\lambda\sigma}(F^{\mu\nu\lambda\sigma} - 4\partial^\mu A^{\nu\lambda\sigma})$

$$\mathcal{L}_{\phi,A} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}(\phi + \frac{Q}{m})^2 + \frac{1}{6}\,\epsilon_{\mu\nu\lambda\sigma}Q\,\partial^\mu A^{\nu\lambda\sigma}$$

- NOTE: the dualizations show $\phi = -\frac{24}{m}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu\lambda\sigma}$
- LET'S READ THIS RIGHT:
- we started with a magnetic dual of F, the Q - this is a 0-form FIELD STRENGTH! In a condensate this will realize a DISCRETE GAUGE SYMMETRY $Q \to Q + ne^2$ $\phi \to \phi - ne^2/m$ when we complete the theory with a gauge field ϕ
- The gauge field potential is $(\partial\phi)^2$ and the gauging of Q is $(m\phi + Q)^2$
- All along we have dQ = 0 locally

Gauge theory of inflation

- Gauge symmetry is nonlinearly realized, and there is U(1) symmetry of the 4-form and the discrete gauge symmetry of the dual
- THEY CANNOT BE BROKEN BY GRAVITY, OR ANYTHING ELSE… THEY ARE REDUNDANCIES BUILT INTO THE THEORY
- Mass = charge - hence stably small!
- Corrections must come in the form that preserves these symmetries

$$\delta\mathcal{L}_1 = c_n \frac{F^n}{M^{2n-4}} \qquad \delta\mathcal{L}_2 = d_n \frac{m^{2n}}{M^{4n-4}} A^{2n} \qquad \delta\mathcal{L}_3 = e_{k,n} \frac{m^{2k} A^{2k} F^n}{M^{4k+2n-4}}$$

- The m^{2n} in the latter term follows from Goldstone boson Equivalence Thm!
- After dualizing: (NOTE: THE KEY IS THE POWERS OF m!)

$$F_{\mu\nu\lambda\sigma} \sim (m\phi + Q)\epsilon_{\mu\nu\lambda\sigma} \qquad m^2 A^2 \sim (\partial\phi)^2$$

- These are just canonical transformations!

Inflaton is the gauge flux!

- Note:

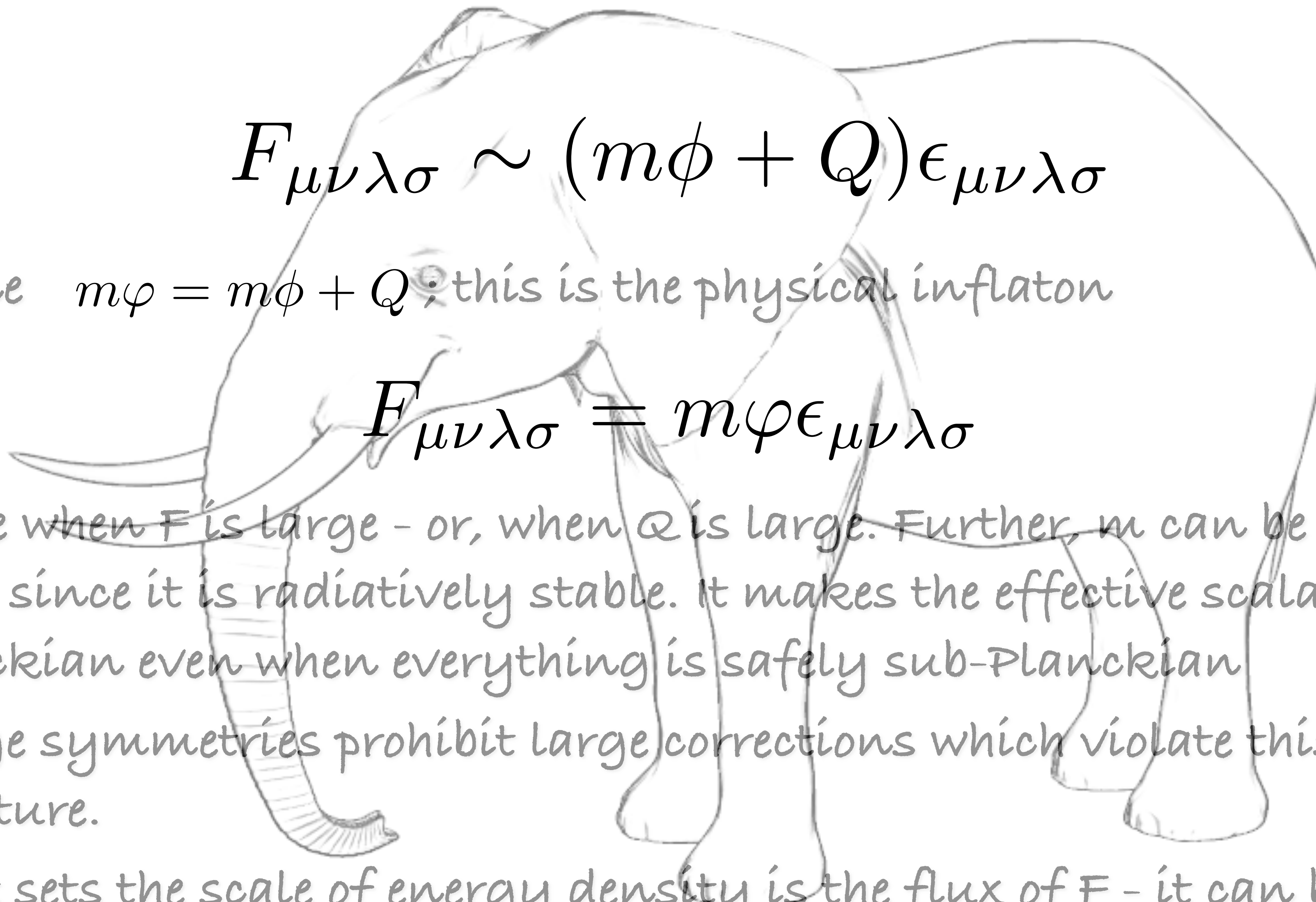
$$F_{\mu\nu\lambda\sigma} \sim (m\phi + Q)\epsilon_{\mu\nu\lambda\sigma}$$

- Define $m\varphi = m\phi + Q$; this is the physical inflaton

$$F_{\mu\nu\lambda\sigma} = m\varphi\epsilon_{\mu\nu\lambda\sigma}$$

- Large when F is large - or, when Q is large. Further, m can be dialed by hand since it is radiatively stable. It makes the effective scalar super-Planckian even when everything is safely sub-Planckian
- Gauge symmetries prohibit large corrections which violate this structure.
- What sets the scale of energy density is the flux of F - it can be huge as long as its energy density is below the cutoff

Inflaton is the gauge flux!



- Note:

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The observational input

- Planck+BICEP+… : the primordial tensors are small!

$$r < 0.1$$

- Problem: inflation IS NOT a weakly coupled quadratic ($r \neq 0.16$)!!!
- Silverstein et al: constructions include corrections from heavy fields which display "flattening": $V \sim \phi^p, p < 2$
- Since there is a gap in QFT, there must be a description of this in EFT
- STRONG COUPLING! Take large field vevs - when they are large enough the theory goes to strong coupling and eventually breaks down!
- One would worry about strong coupling giving large corrections to the inflaton potential and destroying flatness!
- YET WE HAVE GAUGE REDUNDANCIES: STRONG OR WEAK COUPLING, THEY CANNOT BE VIOLATED!

EFT of strongly coupled inflaton I

- Georgi and Manohar developed Naive Dimensional Analysis (NDA) to study heavy quarks in the 80's; We use the exact same techniques here!
- The idea: take the theory to strong coupling but below the cutoff M
- Impose naturalness: all operators are equally important thanks to strong coupling. Easiest to work with scalars!
- Then can normalize the operators correctly by including loop factors:

$$\phi \to \frac{4\pi\phi}{M}, \qquad \partial, m \to \frac{\partial}{M}, \frac{m}{M}$$

$$Q \propto m\phi \quad \text{by gauge symmetry :} \quad Q \to \frac{4\pi Q}{M^2}$$

$$\text{overall normalization : } \mathcal{L} \to \frac{M^4}{(4\pi)^2}\mathcal{L}_{dimensionless}$$

restore combinatorial factors to reproduce Feynman diagrams

$$\left(4! \times 3! \simeq (4\pi)^2\right)$$

EFT of strongly coupled inflaton II

- NOTE: large vevs = high energies and strong coupling - the fields experiencing such interactions in principle are "fluffy" - with large form factors - as opposed to weak coupling, where form factors are small
- A large hierarchy between the M and m allows soft fields to drive inflation - they are "hard" enough over many efolds
- NDA shows that at large field vevs - but below the cutoff - the EFT of the monodromy inflation is (with all operators important!)

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(m\phi+Q)^2 - \sum_{n>2} c'_n \frac{(m\phi+Q)^n}{n!(\frac{M^2}{4\pi})^{n-2}}$$
$$-\sum_{n>1} c''_n \frac{(\partial_\mu\phi)^{2n}}{2^n n!(\frac{M^2}{4\pi})^{2n-2}} - \sum_{k\geq 1, l\geq 1} c'''_{k,l} \frac{(m\phi+Q)^l}{2^k k! l! (\frac{M^2}{4\pi})^{2k+l-2}} (\partial_\mu\phi)^{2k}$$

- Keep in mind that $F_{\mu\nu\lambda\sigma} \sim (m\phi+Q)\epsilon_{\mu\nu\lambda\sigma}\,,\ m^2 A^2 \sim (\partial\phi)^2$

EFT of strongly coupled inflaton III

- Much less mess than it seems! Define $\varphi = \phi + Q/m$

$$\mathcal{L} = -\frac{M^4}{16\pi^2}\mathcal{K}\Big(\frac{4\pi m\varphi}{M^2}, \frac{16\pi^2 X}{M^4}\Big) - \frac{M^4}{16\pi^2}\mathcal{V}_{eff}(\frac{4\pi m\varphi}{M^2}), \quad X = (\partial\varphi)^2$$

- Mukhanov, Garriga et all: "k-inflation": perturbative potential+large corrections, without and with derivatives!
- BUT GAUGE SYMMETRIES STILL CONTROL THE EFT!

$$\frac{M^4}{16\pi^2}\frac{1}{n!}\Big(\frac{4\pi m\varphi}{M^2}\Big)^n, \qquad \frac{M^4}{16\pi^2}\frac{1}{2^n n!}\Big(\frac{16\pi^2(\partial_\mu\phi)^2}{M^2}\Big)^n$$

- plus cross terms
- This means, the EFT of inflation involves actions like

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_{eff}(\frac{4\pi m\varphi}{M^2})(\partial_\mu\varphi)^2 - \frac{M^4}{16\pi^2}\mathcal{V}_{eff}(\frac{4\pi m\varphi}{M^2}) + \text{higher derivatives}\,,$$

- This is where flattening is hidden!!!

EFT of strongly coupled inflaton IV

- At a given inflaton value $\mathcal{Z}_{eff} \simeq \mathcal{Z}_0 + \mathcal{Z}_1 \frac{4\pi m \Delta\varphi}{M^2}\,, \quad \mathcal{V}_{eff} \simeq \mathcal{V}_0 + \mathcal{V}_1 \frac{4\pi m \Delta\varphi}{M^2}$
- Shift the field to remove the constant from the potential $\hat{\varphi} = \Delta\varphi + \frac{M^2}{4\pi m}\frac{\mathcal{V}_0}{\mathcal{V}_1}$
- Now if in Z the constant dominates, canonically normalize the field

$$\chi = \hat{\varphi}/\sqrt{\hat{\mathcal{Z}}}$$

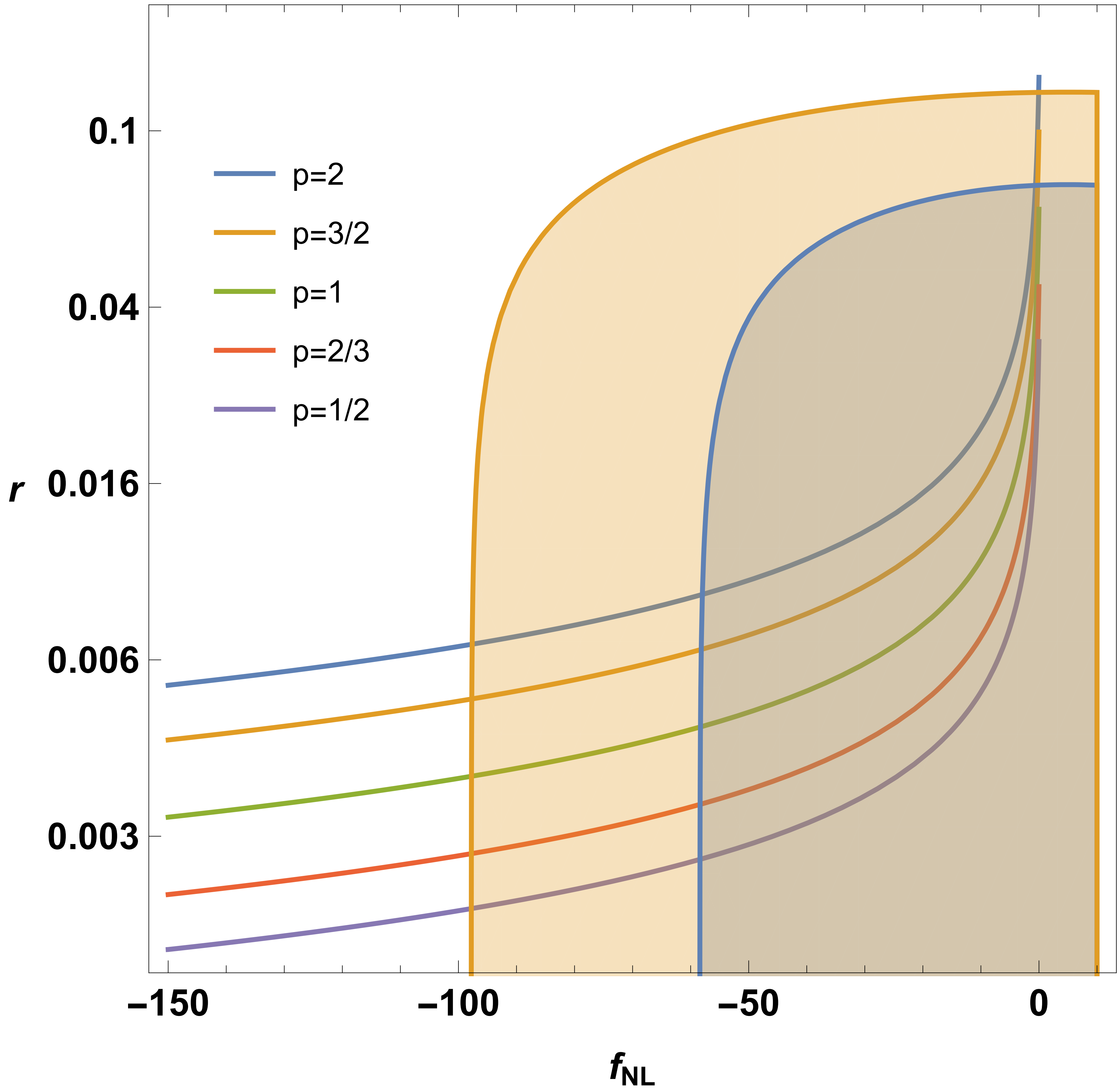
- The effective theory is:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \chi)^2 - \frac{\mathcal{V}_1}{\sqrt{\hat{\mathcal{Z}}}}\frac{M^2 m}{4\pi}\chi + \text{corrections}$$

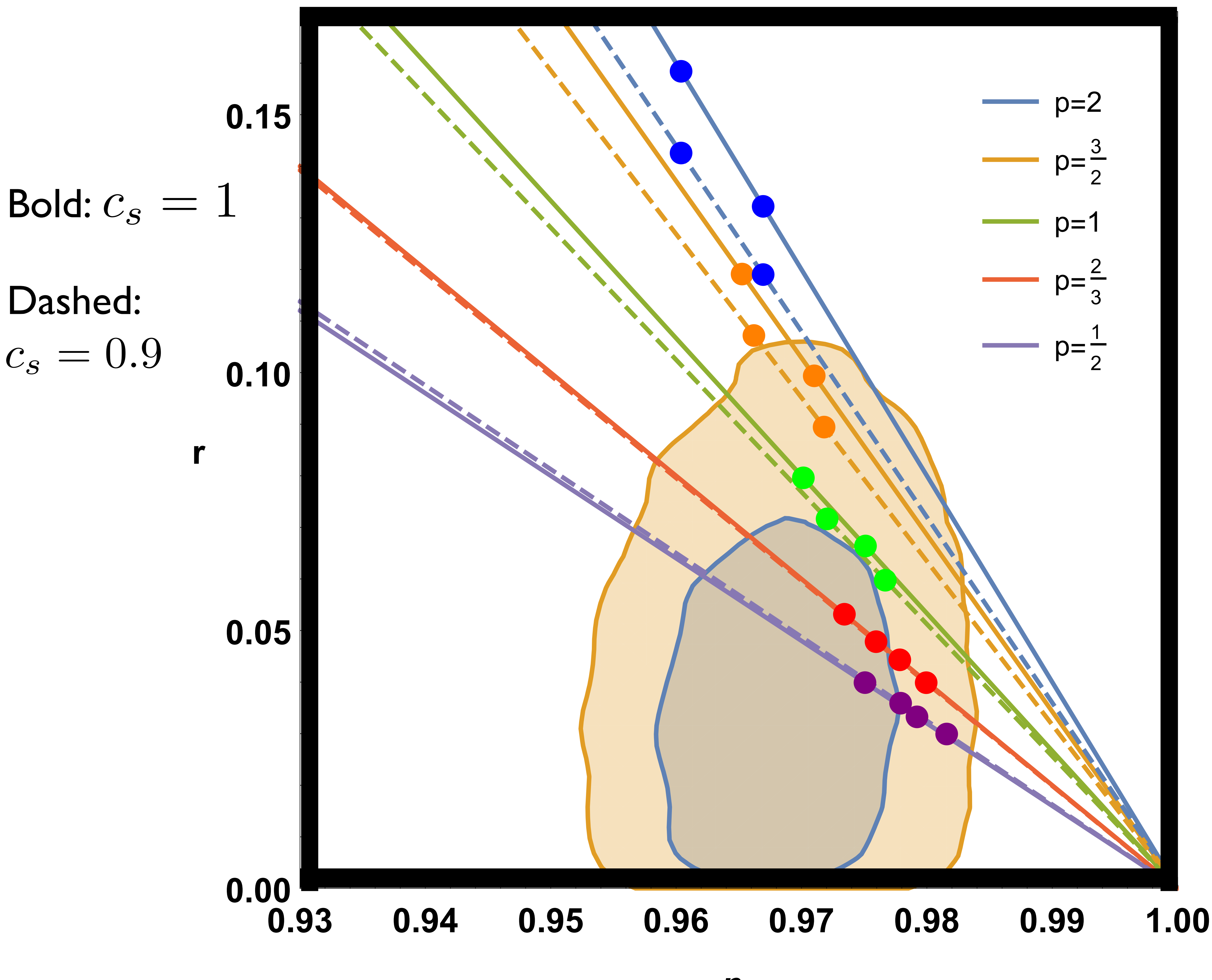
- Ie linear potential!
- If the linear term dominated we'd get p=2/3 (less natural, still...)
- Other powers may occur if approximant functions are splines
- Important point: canonical fields are also normalized as $\frac{4\pi m \chi}{M^2}$
- This is guaranteed by gauge symmetries!

EFT of strongly coupled inflaton V

- As long as $\frac{4\pi m M_{Pl}}{M^2} \ll 1$ the potential stays FLAT!!! - i.e. below the cutoff ($< M^4$) even when $\chi \gg M_{Pl}$
- We only need ~60 efolds... benefiting all the while from $16\pi^2 \simeq 158$
- Not the whole story - flattening & irrelevant operators with derivatives
- Flattening increases spectral index
- Higher derivatives generate non-gaussianities
- So the stronger coupling reduces r but it increases n_s and f_{NL}
- This means that coupling cannot be excessively strong
- THIS ALL IMPLIES A LOWER BOUND ON r !!! (TBD)
- The strongly coupled EFT of monodromy either yields an observable prediction for tensors, OR too large non-gaussianities - it is on the edge, very falsifiable...



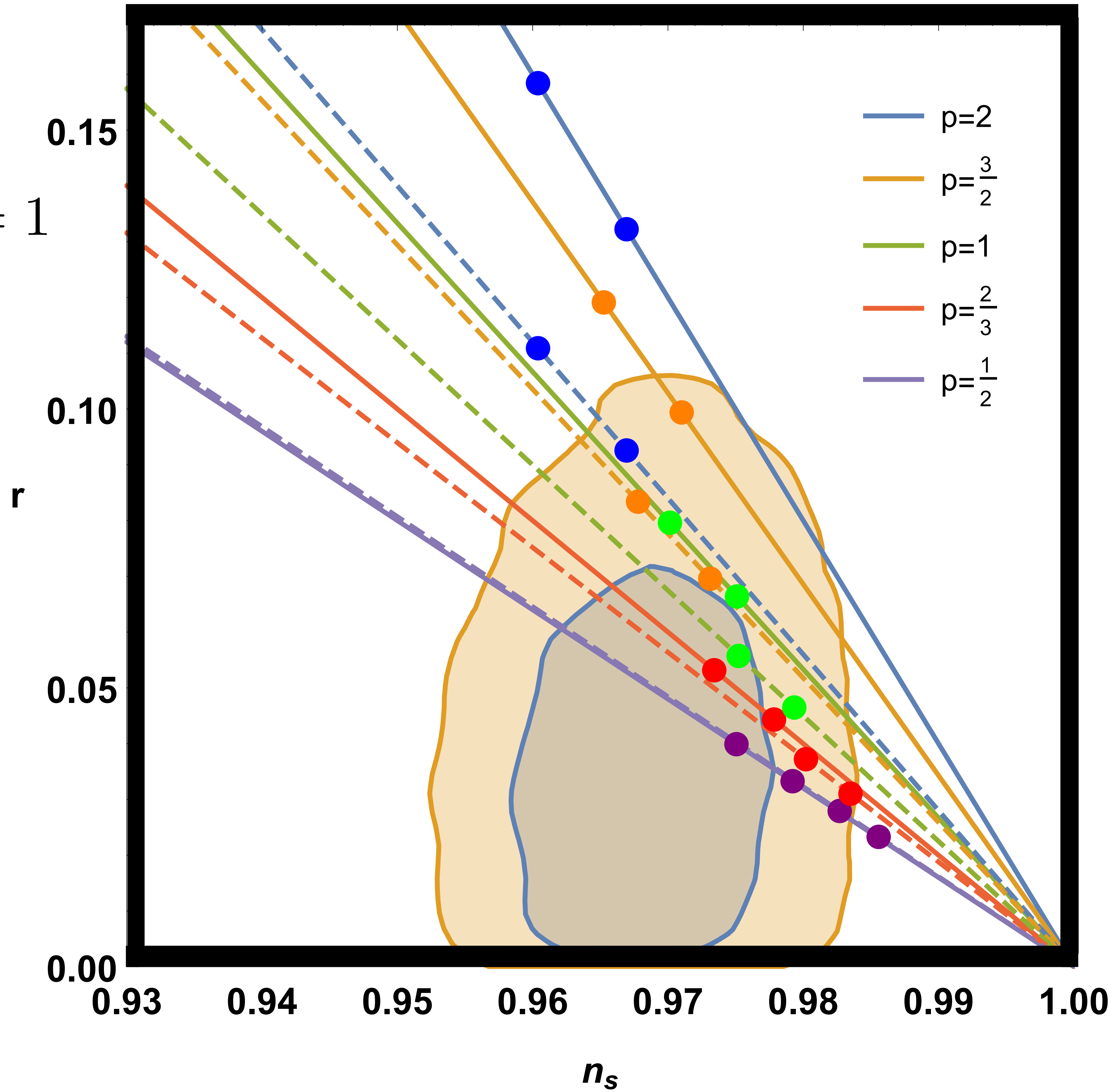
p=2
p=3/2
p=1
p=2/3
p=1/2
r
0.1
0.04
0.016
0.006
0.003
−150
−100
−50
0
f_{NL}



Bold: $c_s = 1$
Dashed: $c_s = 0.9$
p=2
p=$\frac{3}{2}$
p=1
p=$\frac{2}{3}$
p=$\frac{1}{2}$
r
0.15
0.10
0.05
0.00
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1.00
n_s

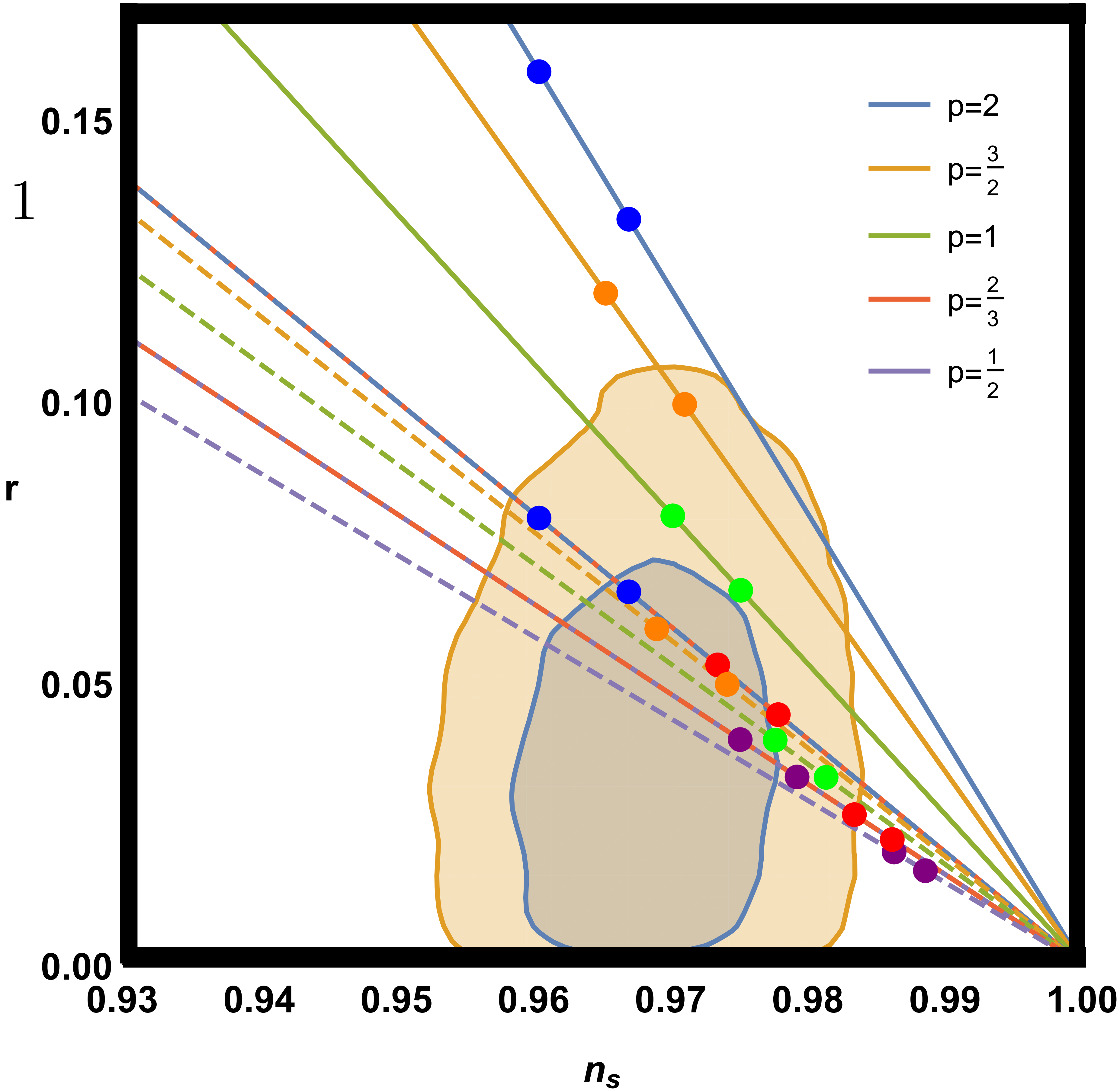
Bold: $c_s = 1$

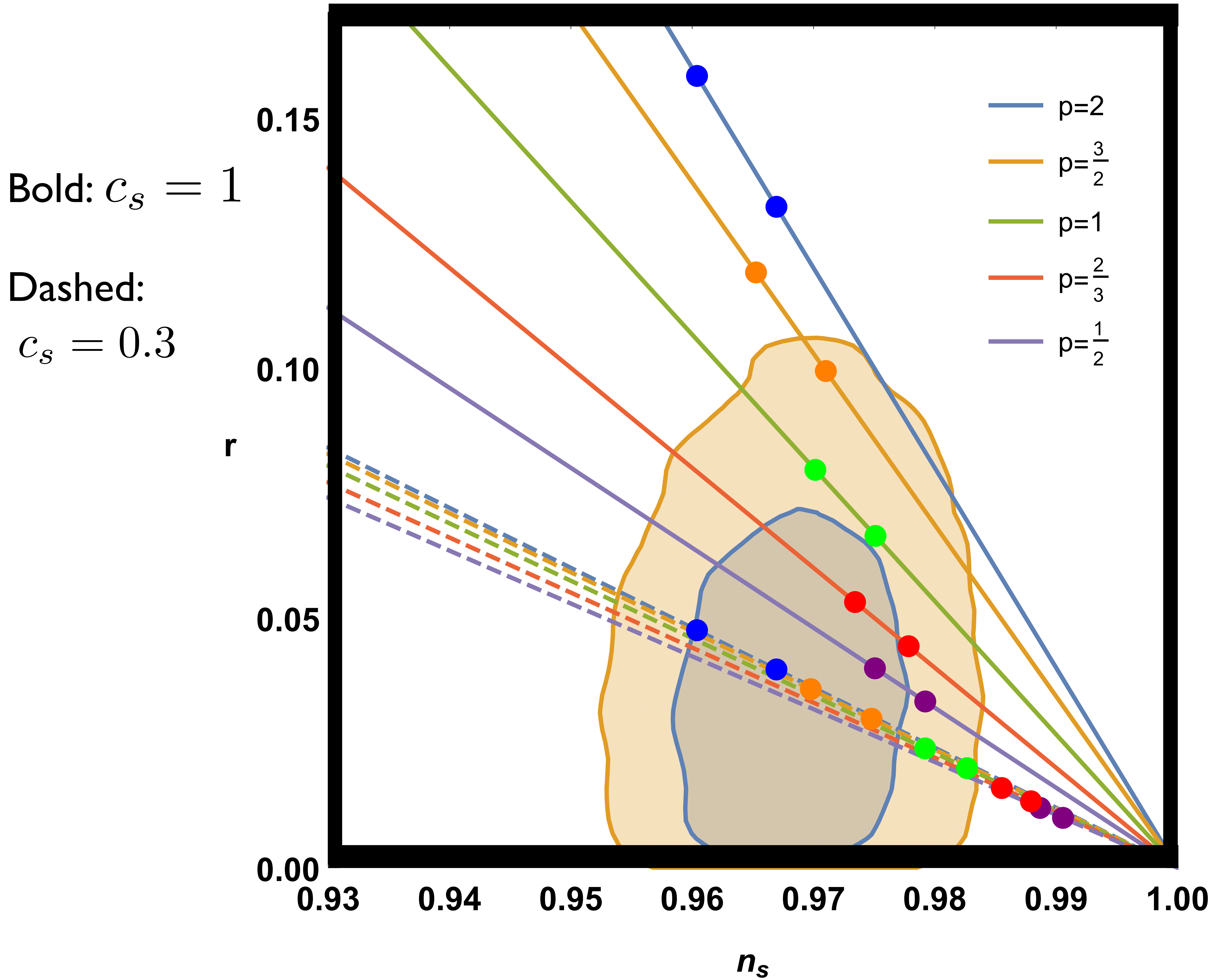
Dashed: $c_s = 0.7$



Bold: $c_s = 1$

Dashed: $c_s = 0.5$





Bold: $c_s = 1$
Dashed: $c_s = 0.3$
p=2
p=$\frac{3}{2}$
p=1
p=$\frac{2}{3}$
p=$\frac{1}{2}$
r
n_s
0.00
0.05
0.10
0.15
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1.00

Summary

- Monodromy QFT accommodates the issue of UV sensitivity of inflation nicely.
- Hidden gauge symmetries: a key controlling mechanism behind monodromy QFT. They protect EFT from itself, and from gravity.
- Gauge symmetries also explain why the large field vevs are fine: they are dual gauge field strengths which count the sources! Large field = many sources.
- UV constructions: needed to understand the ORIGIN of the mass gap - i.e. the smallness of m - analogous to BCS theory!
- The ideas are predictive: experiments already constrain the theory. In a natural theory, will see either tensors or nGs in the next round of CMB experiments. If not the theory is tuned/ unnatural.

Some massive vector U(1) gauge theory

- Gauge symmetry is nonlinearly realized - Julia & Toulouse!!!

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2(A_\mu - \partial_\mu\phi)^2 + (A_\mu - \partial_\mu\phi)J^\mu - \frac{1}{2\alpha}(\partial\cdot A - \alpha m^2\phi)^2$$

- Gauge symmetry protects the theory - it is renormalizable! The key:

$$\Delta_{\mu\nu} = \frac{i}{p^2+m^2}\left(\eta_{\mu\nu} + (1-\alpha)\frac{p_\mu p_\nu}{p^2+\alpha m^2}\right), \qquad \Delta_\varphi = -\frac{i}{p^2+\alpha m^2}$$

- Can send m to zero without messing the theory up! (the scalar is free in QFT and is "modded out" by BCs; but it seems to gravitate (Deser et al).
- Even though it seems to have a global symmetry... this is vacuous since the scalar period $\sim m/e$ so only fluctuations which renormalize gravity
- This limit is dual to a 2-form gauge theory but with ZERO charges - and NO sources - this is consistent with the WGC; if a string is introduced it becomes light when m is zero, implying that the dual U(1) must be completed at zero mass - introducing massless charges

Julia - Toulouse revisited

- Take a compact scalar - a phase. Consider its variation around the moving vortices

$$\delta\phi \sim \vec{v} \cdot \vec{\delta}x$$

- Since the fluid is NOT irrotational, $\vec{v} \neq \vec{\nabla}\phi$; instead $\vec{j} \simeq \vec{v}$ contains nontrivial physical information; JT declare it to be the new variable and use it to describe the EFT of the new phase. This is London's eq and adding kinetic terms implies that the new gauge theory is a U(1) massive vector.
- Since we are dealing with a system of vortices (strings) it is natural to start with a 3-form. As the system is not perturbatively connected to the usual 3-form vacuum, it is also natural to assume that the 3-form is massive. So we start with

$$\mathcal{L}_H = -\frac{1}{12}H^2_{\mu\nu\lambda} - \frac{m^2}{4}(B_{\mu\nu} - \frac{1}{m}\tilde{F}_{\mu\nu})^2 + \ldots$$

- We then attempt to link it with the JT proposal

Julia - Toulouse cont.

- Consider massive U(1) dualities! Start with a massive 2-form

$$\mathcal{L}_H = -\frac{1}{12}H^2_{\mu\nu\lambda} - \frac{m^2}{4}(B_{\mu\nu} - \frac{1}{m}\tilde{F}_{\mu\nu})^2 + \ldots$$

- Now dualize! Write

$$\mathcal{L}_{BF} = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{12}H^2_{\mu\nu\lambda} - \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}B^{\mu\nu}F^{\lambda\sigma} + \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\tilde{A}_\mu\,\partial_\nu F_{\lambda\sigma} + \ldots$$

- Integrating out F gives massive 2-form; integrating out A gives

$$\mathcal{L}_{BF} = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{12}H^2_{\mu\nu\lambda} - \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}B^{\mu\nu}F^{\lambda\sigma} + \ldots$$

- Integrate BF term by parts, add $\phi\epsilon^{\mu\nu\lambda\sigma}\partial_\mu H_{\nu\lambda\sigma}$ and integrate out H

$$\mathcal{L}_{JT} = -\frac{1}{4}F^2_{\mu\nu} - \frac{m^2}{2}(A_\mu - \partial_\mu\phi)^2 + \ldots$$

- De facto JT ansatz:

$$A'_\mu = \frac{1}{6m}\epsilon_{\mu\nu\lambda\sigma}H^{\nu\lambda\sigma}$$