

PROTECTION OF SCALAR MASSES FROM HIGHER DIMENSIONS ?

Based on :

W. Buchmuller, M.Dierigl, E.D & J. Schweizer,
arXiv:1611.03798 [hep-th]

may 18, 2017

Olivefest

« Astroparticle Physics Looking Forward »

Outline

- 1) Higher-dimensional completions of the Standard Model
- 2) Magnetic compactifications
- 3) Effective field theory
- 4) Quantum corrections, Wilson lines as goldstone bosons
- 5) Conclusions

1) Higher-dimensional completions of the Standard Model

- Structure of Standard Model points towards **Grand Unification** (matter content, unification of gauge couplings)
- Strong theoretical arguments for **supersymmetry** at high scales (gravity, string theory)
- **Extra dimensions** unavoidable in string theory



Extra dims. address:

- Unification of all forces
- Holographic solutions to the hierarchy problem
- New ways to **break SUSY**
- New models of **inflation**

- Sometimes **higher-dim. symmetries** protect quantum corrections in a way invisible from 4d.

Ex: Internal comp. of a gauge field protected by **gauge symmetry**

$$\delta m_0^2 \sim (\text{loop}) \times \frac{1}{R^2}$$

- **Compactification scale** $M_c = R^{-1}$ usually defines the GUT/unification scale.
- Scale of supersymmetry breaking M_{SUSY} usually much smaller.

2) Magnetic compactifications



Consider a 6-dim. theory : $x_0 x_1 x_2 x_3 x_5 x_6$

An internal magnetic field $F_{56} = B = f$

- break SUSY, due to the magnetic moment coupling

$$H = -\mu \mathbf{B} = -\frac{q}{m} \mathbf{S} \mathbf{B}$$

- Turns KK states k_1, k_2 into **Landau levels** n , mass

$$\delta M^2 = (2n + 1) |qB| + 2qB \Sigma_{56}$$

where Σ_{45} is the **internal helicity** of particles.



- An internal magnetic field is **quantized**

$$\int_{T^2} F = 2\pi N \quad \longrightarrow \quad f = \frac{N}{2\pi R^2} \sim M_{\text{GUT}}^2 ; N = \text{integer}$$

- Each Landau level is **N times degenerate**.
- Precisely **N chiral fermion zero modes** (index theorem).

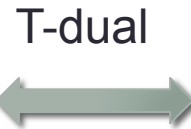
Magnetized models : Bachas (1995)...Cremades, Ibanez, Marchesano...

- Starting with a SUSY 6d theory, usually said that the effect of the magnetic field is to add a **D-term** Fayet-Iliopoulos (FI) term in 4d

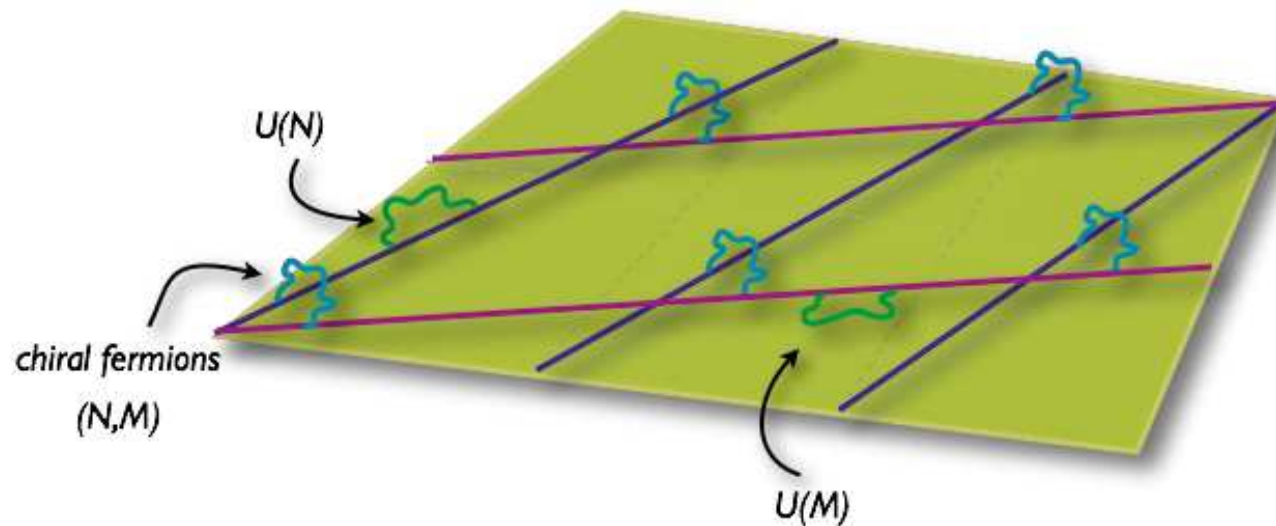
$$D = f \quad \longrightarrow \quad V = \frac{1}{2} D^2 = \frac{1}{2} f^2 \sim M_{\text{GUT}}^4$$

Widely studied in string theory :

Internal magnetic fields



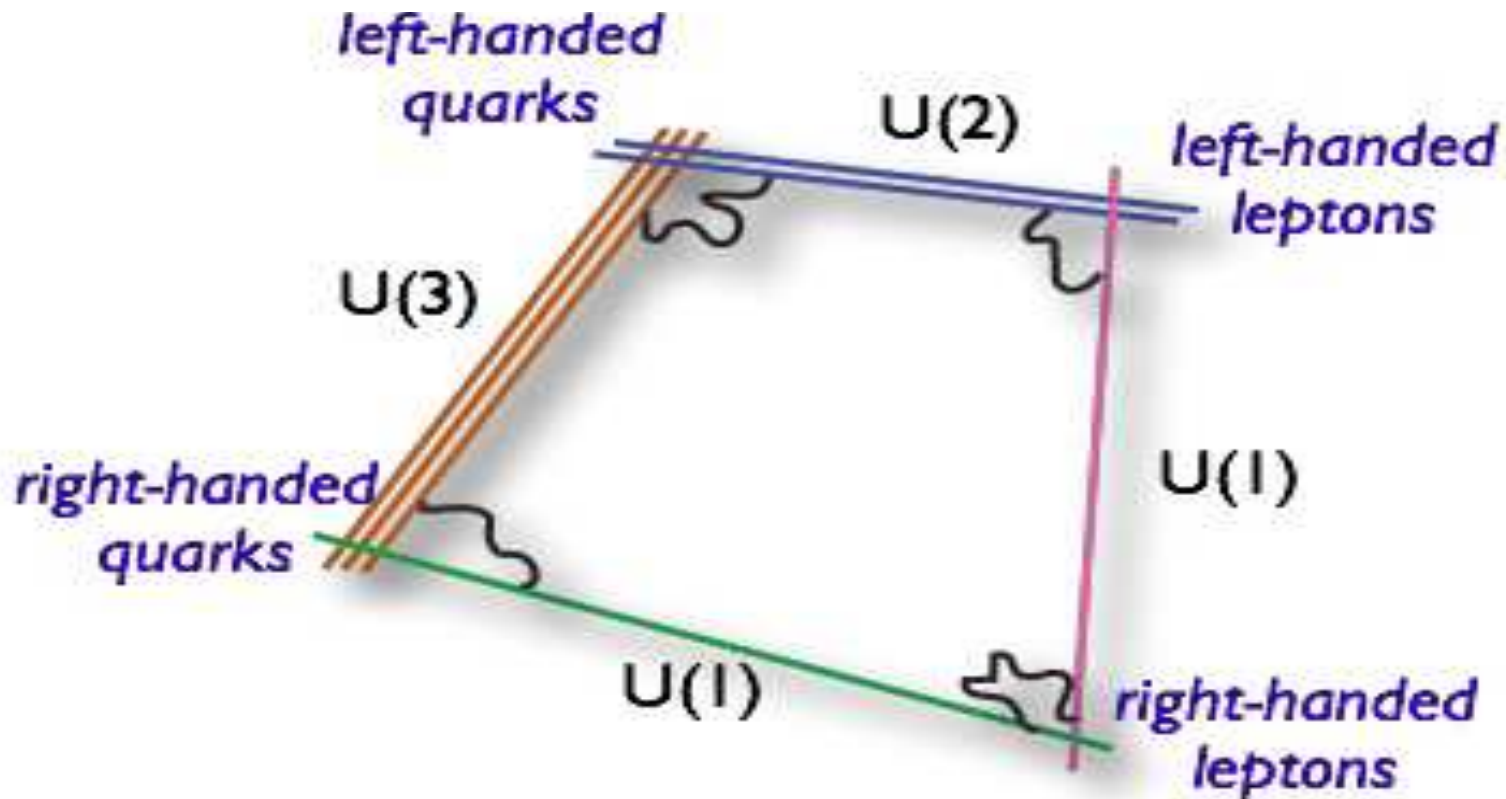
intersecting branes



Elegant **geometrical interpretations** :

- chiral fermions live at the intersection of branes
- Number of generations: intersection numbers
- Yukawa couplings : governed by areas

Among the most successful quasi-realistic Standard Model realizations in String Theory



Why be interested in **field theory approach** to magnetic compactifications ? Several reasons:

- If **broken SUSY**, most of quantum corrections not calculable in string theory
- Subtlety: there is **no mass gap** in the spectrum : soft masses given by the FI term of the same order ($1/R^2$) as the masses of Landau levels



one needs an **effective theory for the whole tower**.
Truncation to « zero modes » **inconsistent**.

3) Effective field theory

- Abelian 6d SUSY theory compactified on a torus.
N=2 SUSY in 4d before the magnetic flux;
4d multiplets: **vector** (V, ϕ)
charged hyper (Q, \tilde{Q})
- 6d effective action in superfields: (Marcus, Sagnotti, Siegel ;
Arkani-Hamed, Gregoire, Wacker)

$$\begin{aligned}
 S_6 = \int d^6x \left\{ \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left(\partial V \bar{\partial} V + \phi \bar{\phi} + \sqrt{2} V (\bar{\partial} \phi + \partial \bar{\phi}) \right) \right. \\
 \left. + \int d^2\theta \tilde{Q} (\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left(\bar{Q} e^{2gqV} Q + \tilde{Q} e^{-2gqV} \tilde{Q} \right) \right\} \\
 \partial = \partial_5 - i\partial_6, \quad \phi|_{\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}} (A_6 + iA_5)
 \end{aligned}$$

ϕ are **internal components of gauge fields** =
Wilson lines

Mode expansions with **flux**:

$$\phi_0|_{\theta=\bar{\theta}=0} = \frac{f}{2\sqrt{2}} (x_5 - ix_6) + \varphi, \quad \varphi = \frac{1}{\sqrt{2}} (a_6 + ia_5)$$

$$Q(x_M) = \sum_{n,j} Q_{n,j}(x_\mu) \psi_{n,j}(x_m) = \sum_{n,j} Q_{n,j}(x_\mu) \frac{1}{\sqrt{n!}} (a^\dagger)^n \psi_{0,j}(x_m),$$

$$\bar{Q}(x_M) = \sum_{n,j} \bar{Q}_{n,j}(x_\mu) \bar{\psi}_{n,j}(x_m) = \sum_{n,j} \bar{Q}_{n,j}(x_\mu) \frac{1}{\sqrt{n!}} (a)^n \bar{\psi}_{0,j}(x_m).$$

where (**harmonic oscillator algebra**)

$$a = \sqrt{\frac{1}{-2qgf}} (iD_5 - D_6)$$

$$a^\dagger = \sqrt{\frac{1}{-2qgf}} (iD_5 + D_6)$$

The final 4d effective action for Landau levels is

FI term



$$\begin{aligned}
 S_4^* = \int d^4x \left[\int d^4\theta \left(\bar{\varphi}\varphi + \sum_{n,j} (\bar{Q}_{n,j} e^{2ggV_0} Q_{n,j} + \bar{\tilde{Q}}_{n,j} e^{-2ggV_0} \tilde{Q}_{n,j}) + 2fV_0 \right) \right. \\
 + \int d^2\theta \left(\frac{1}{4} \mathcal{W}_0^\alpha \mathcal{W}_{\alpha,0} \right. \\
 \left. \left. + \sum_{n,j} \left(-i\sqrt{-2qgf(n+1)} \tilde{Q}_{n+1,j} Q_{n,j} + \sqrt{2}qg \tilde{Q}_{n,j} \varphi Q_{n,j} \right) \right) + \text{h.c.} \right]
 \end{aligned}$$

Coupled mass terms



- **SUSY broken** like in the FI model, with an infinite number of fields. Truncation to a finite number **inconsistent**.

- We also worked out the **non-abelian case**: $SU(2)$ gauge group in 6d with $N=2$ vector multiplet, flux in the generator T_3 .
- In this case, there is always a **tachyon** (recombination mode) $\Phi_{+,0}$ which can **restore SUSY** by taking a vev (tachyon condensation)
 - ➔ Nielsen-Olesen instability
- The flux give mass to the W^\pm gauge bosons and **breaks** $SU(2) \rightarrow U(1)$

- There is an induced Fayet-Iliopoulos term for the $U(1)$
- In the true vacuum $U(1)$ will also be broken
- Interesting subtleties with the **Stueckelberg mechanism** for Landau levels

4) Quantum corrections, Wilson lines as goldstone bosons

Interested in Higgs = internal component of the gauge field. Without magnetic flux, **6d gauge symmetry could protect only partially its mass** ($\delta m_0^2 \sim (\text{loop}) \times \frac{1}{R^2}$)

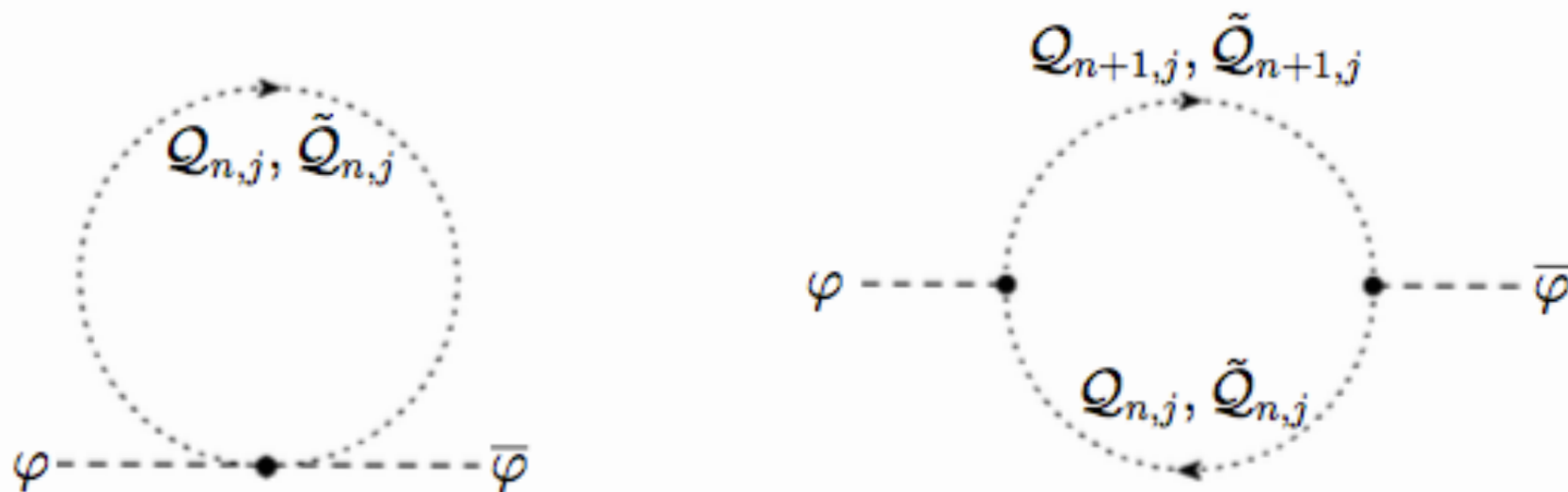
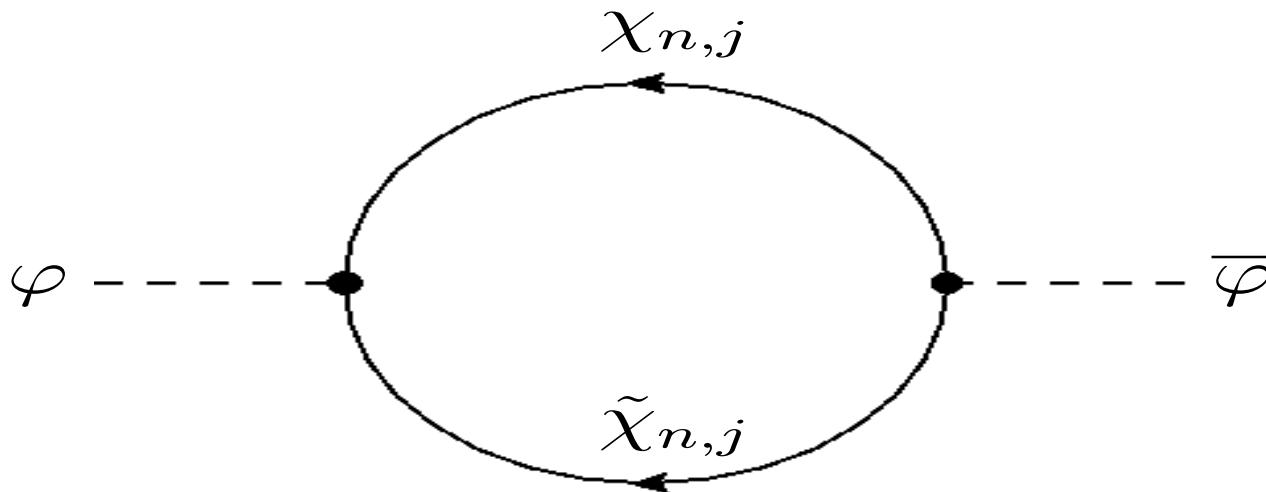


Figure 3: Bosonic contributions to the Wilson line mass with flux.

Each contribution **quadratically divergent**:
 sum over the whole charged tower of Landau levels is
 however **exactly zero** !

$$\begin{aligned}
 \delta m_b^2 &= -4q^2 g^2 |N| \sum_n \int \frac{d^4 k}{(2\pi)^4} \left(\frac{n}{k^2 + \alpha(n + \frac{1}{2})} - \frac{n+1}{k^2 + \alpha(n + \frac{3}{2})} \right) \\
 &= -\frac{q^2 g^2}{4\pi^2} |N| \sum_n \int_0^\infty dt \frac{1}{t^2} \left(n e^{-\alpha(n + \frac{1}{2})t} - (n+1) e^{-\alpha(n + \frac{3}{2})t} \right) \\
 &= -\frac{q^2 g^2}{4\pi^2} |N| \int_0^\infty dt \frac{1}{t^2} \left(\frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t} - 1)^2} - \frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t} - 1)^2} \right) = 0
 \end{aligned}$$

The same is true for the fermionic contribution



Reminder: without the flux, scalar and fermion loops give separately

$$\delta m_0^2 \sim (\text{loop}) \times \frac{1}{R^2}$$

We checked also that the quartic coupling is zero.

Is there's a **symmetry reason** ?

Action of charged matter fields invariant under translations

$$S_6 = \int d^6x \left(-D_M \bar{Q} D^M Q \right), \quad D_M Q = (\partial_M + i q g A_M) Q$$

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta A_n = \epsilon^m \partial_m A_n$$

Symmetries for constant Wilson line background

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta a_n = 0$$

Flux background breaks the symmetries **spontaneously**

$$D_m Q = \left(\partial_m + i q g \left(a_m + \frac{f}{2} \epsilon_{mn} x_n \right) \right) Q, \quad \langle A_m \rangle = \frac{f}{2} \epsilon_{mn} x_n$$

Translational symmetries now **non-linearly realized**
with Wilson lines as **Goldstone bosons**

$$\delta Q = \epsilon^m \partial_m Q, \quad \delta a_n = \epsilon^m \frac{f}{2} \epsilon_{nm}$$

- Need **realistic examples** with **pseudo-Goldstone bosons**

$$\delta m_0^2 \ll \frac{1}{R^2}$$

maybe from gravitational or higher-loop corrections.

Conclusions, Perspectives

- ◆ Strong theoretical arguments for **SUSY** at high scales: gravity, string theory
- ◆ Energy scale of **grand unification** $M_{GUT} \sim 10^{16}$ GeV
Scale of **SUSY breaking** M_{SUSY} ?
- ◆ **Magnetized** compactifications : high-scale SUSY breaking

$$M_{SUSY} \sim M_{GUT} \sim R^{-1}$$

- ◆ Hope for a **higher-dim. protection** of scalar masses: Higgs mass, inflation.
- ◆ Various **applications** possible: moduli stabilization, inflation, string and field theory orbifold GUT's.

Thank you for the collaboration and

**Happy Birthday,
Keith !**