Dark Olives (and other Dark Middle Eastern Delights)

- Wilsonian Dark Matter in string derived Z' model
- Time permitting: The equivalence postulate approach to quantum gravity

OliveFest, University of Minnesota, Minneapolis, 17 May 2017

HAPPY FESTDAY KEITH!

TPI–MINN–99/32
UMN–TH–1806–99
hep-ph/9906345
June 1999

Probing the Desert with Ultra–Energetic Neutrinos from the Sun and the Earth

Alon E. Faraggi, Keith A. Olive and Maxim Pospelov

Department of Physics, University of Minnesota, Minneapolis, MN 55455, USA

Abstract

Realistic superstring models generically give rise to exotic matter states, which arise due to the "Wilson–line" breaking of the non–Abelian unifying gauge symmetry. Often such states are protected by a gauge or local discrete symmetry and therefore may be stable or meta–stable. We study the possibility of a flux of high energy neutrinos coming from the sun and the earth due to the annihilation of such exotic string states. We also discuss the expected flux for other heavy stable particles – like the gluino LSP. We comment that the detection of ultra–energetic neutrinos from the sun and the earth imposes model independent constraints on the high energy cutoff, as for example in the recently entertained TeV scale Kaluza–Klein theories. We therefore propose that improved experimental resolution of the energy of the muons in neutrino detectors together with their correlation with neutrinos from the sun and the center of the earth will serve as a probe of the desert in Gravity Unified Theories.

Realistic free fermionic models

‘Phenomenology of the Standard Model and Unification’

- Minimal Superstring Standard Model — NPB 335 (1990) 347 (with Nanopoulos & Yuan)
- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Moduli fixing — NPB 728 (2005) 83
- Classification & Exophobia — 2003 – $\cdots$ (with Nooij, Assel, Christodoulides, Kounnas, Rizos & Sonmez)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Low scale Z' in free fermionic models:

- $\frac{3}{2}U(1)_{B-L} - 2U(1)_R \in SO(10)$ @ $1TeV$ MPL A6 (1991) 61

  (with Nanopoulos)

- But $m_t = m_{\nu_\tau}$ & $1TeV\ Z' \Rightarrow\ m_{\nu_\tau} \approx 10MeV$ PLB 245 (1990) 435

  $E_6 \to SO(10) \times U(1)_A \implies U(1)_A$ is anomalous!

  $\implies U(1)_A \notin$ low scale $U(1)_{Z'}$

- 1996-2013, Pati, AEF, Guzzi, Mehta, Athanasopoulos, $U(1) \notin E_6$

- On the other hand ....(AEF, Viraf Mehta, PRD88 (2013) 025006)

$$\sin^2\theta_W(M_Z)\ ,\ \alpha_s(M_Z) \implies U(1)_{Z'} \in E_6$$

- Z' string derived model, (with Rizos) NPB 895 (2015) 233

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \quad \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\ & \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Free Fermionic Models: $Z_2 \times Z_2$ orbifolds with discrete Wilson lines

Exotic matter :

In realistic string models

Unifying gauge group $\Rightarrow$ broken by "Wilson lines".

$\Rightarrow$ non–GUT physical states.

$\Rightarrow$ Meta-stable heavy string relics

$\longrightarrow$ Dark Mater NPB 477 (1996) 65

(with Coriano and Chang)

UHECR candidates NPB 614 (2001) 233

(with Coriano and Plümacher)

Exotics classified by: $SO(10) \rightarrow \mathrm{subgroup} \quad b(\bar\psi^{1\cdots5}_{\frac{1}{2}})$:

notation $[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)]_{(\ Q_Y\ ,\ Q_{Z'}\ ,\ Q_{\mathrm{e.m.}}\ )}$

$SO(6) \times SO(4)$ **type states**

$[(3, \frac{1}{2}); (1, 0)]_{(\ 1/6\ ,\ 1/2\ ,\ 1/6\ )} \quad ; \quad [(\bar{3}, -\frac{1}{2}); (1, 0)]_{(\ -1/6\ ,\ -1/2\ ,\ -1/6)}$

$[(1, 0); (2, 0)]_{(\ 0\ ,\ 0\ ,\ \pm 1/2\ )}$

$[(1, 0); (1, \pm 1)]_{(\ \pm 1/2\ ,\ \mp 1/2\ ,\ \pm 1/2\ )}$ $[(1, \pm 3/2); (1, 0)]_{(\ \pm 1/2\ ,\ \pm 1/2\ ,\ \pm 1/2\ )}$

$SU(5) \times U(1)$ **type states**

$[(1, \pm 3/4); (1, \pm 1/2)]_{(\ \pm 1/2\ ,\ \pm 1/4\ ,\ \pm 1/2\ )}$

$SU(3) \times SU(2) \times U(1)^2$ **type states**

$[(3, \frac{1}{4}); (1, \frac{1}{2})]_{(\ -1/3\ ,\ -1/4\ ,\ -1/3\ )} \quad ; \quad [(\bar{3}, -\frac{1}{4}); (1, \frac{1}{2})]_{(\ 1/3\ ,\ 1/4\ ,\ 1/3\ )}$

$[(1, \pm\frac{3}{4}); (2, \pm\frac{1}{2})]_{(\ \pm 1/2\ ,\ \pm 1/4\ ,\ (1,0)\ ;\ (0,-1)\ )}$

$[(1, \pm\frac{3}{4}); (1, \mp\frac{1}{2})]_{(\ 0\ ,\ \pm 5/4\ ,\ 0\ )}$

CCR, SSR, NPB1996.

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6, \qquad N = 4 \text{ Vacua} \\
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1 \\
\alpha &= \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \qquad \& \quad SO(10) \to SO(6) \times SO(4) \times \cdots \\
\beta &= \{\bar{\psi}^{1,\cdots,5} \equiv \tfrac{1}{2}, \cdots\} \qquad \& \quad SO(10) \to SU(5) \times U(1) \times \cdots
\end{aligned}
$$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2	α
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$	$\pm$
b_1												$\pm$	$\pm$
b_2												-1	$\pm$
α													

A priori 66 independent coefficients $\rightarrow$ 2^{66} distinct vacua

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0,1$$

sectors B^i_{pqrs} $\longrightarrow$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1,0,-1)$

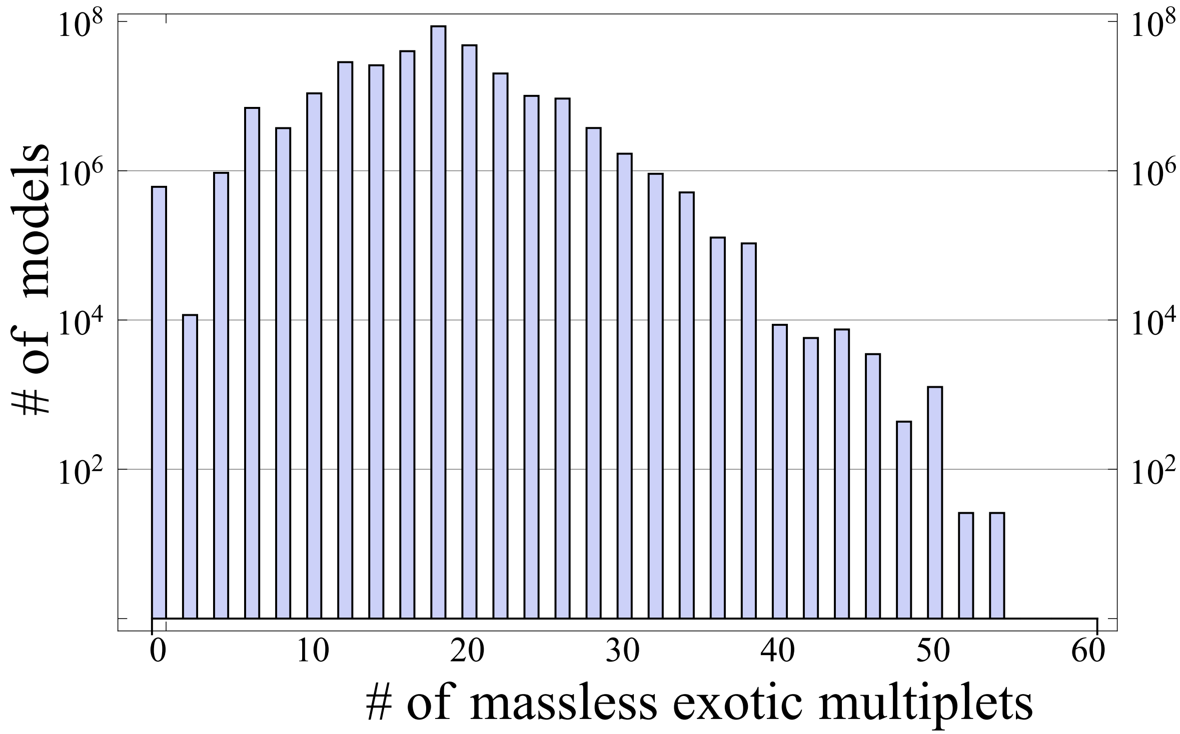
$B^i_{pqrs} + x$ $\longrightarrow$ 10 of $SO(10)$ with multiplicity $(1,0)$

$x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\}$ $\qquad x -$ map $\leftrightarrow$ spinor–vector map

Algebraic formulas for $S = \sum_{i=1}^{3} S_+^{(i)} - S_-^{(i)}$ and $V = \sum_{i=1}^{3} V^{(i)}$

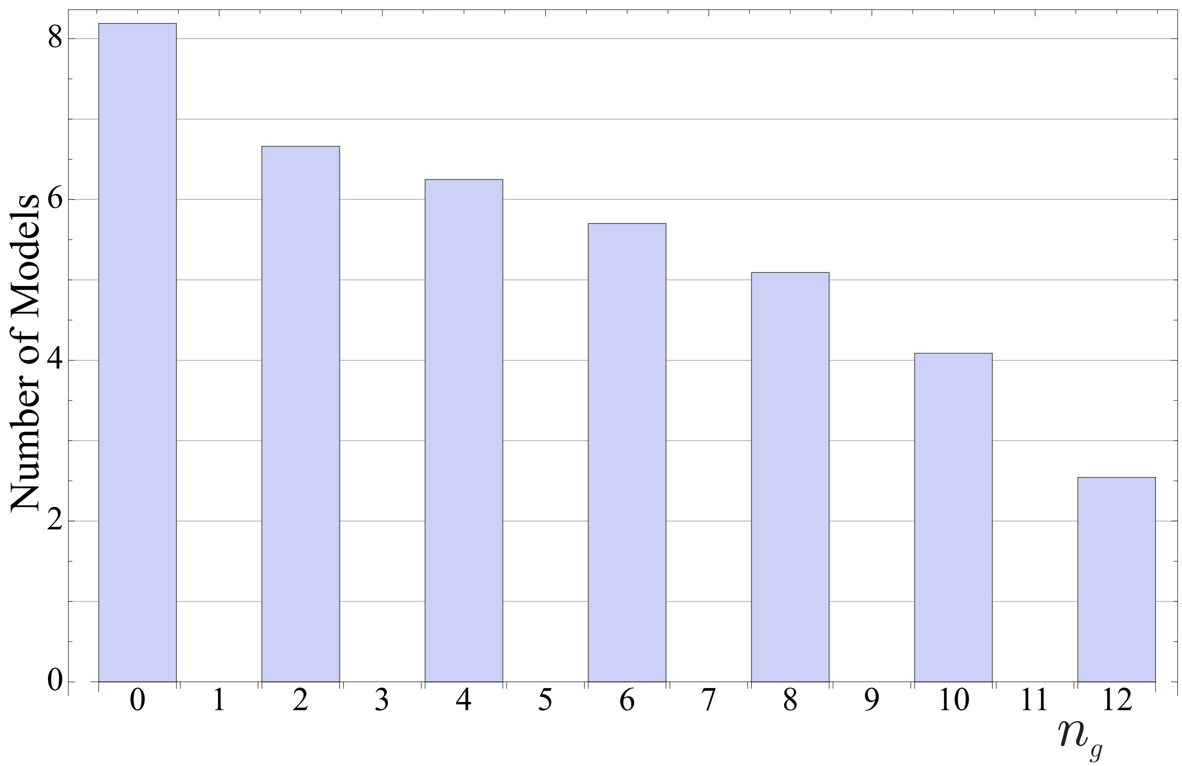
Pati–Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10^{11} vacua



Number of 3-generation models versus total number of exotic multiplets flipped $SU(5)$ class: with Sonmez, Rizos

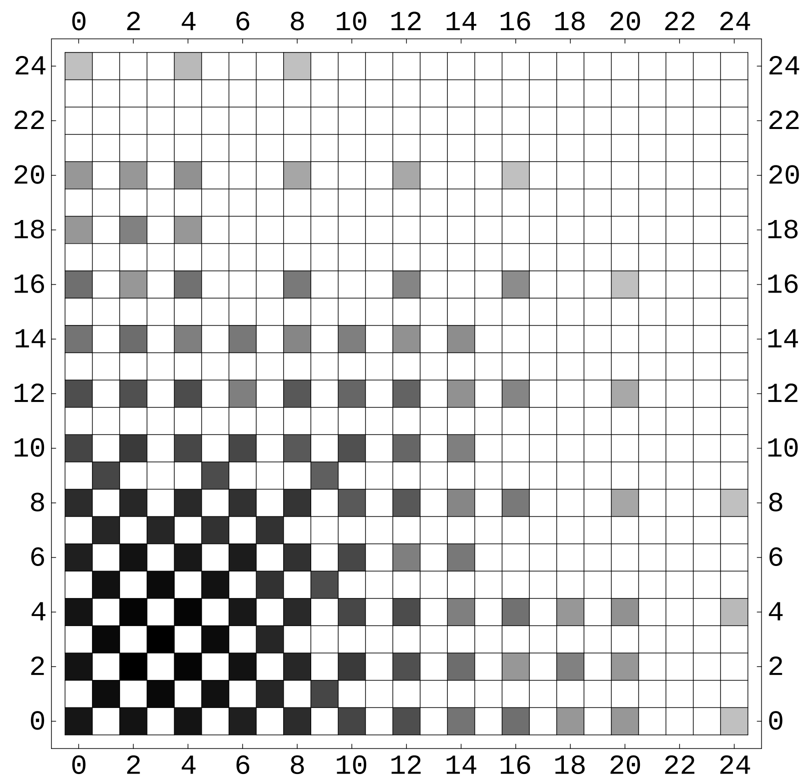
RESULTS: of random search of over 10^{12} vacua



Number of exophobic models versus the number of generations

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;\; <-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

light Z' heterotic–string model $c\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right] = \exp[i\pi(v_i|v_j)]$:

$$
(v_i|v_j) \;=\; \begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{array}
\!\!\begin{array}{c}
\begin{array}{ccccccccccccc} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \end{array} \\
\left(\begin{array}{ccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}\right)
\end{array}
$$

Observable gauge group: $SO(6) \times SO(4) \times U(1)_{1,2,3}$

$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$ is anomaly free

sector	field	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_\zeta$
$S+b_1$	$\bar{F}_{1R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	1/2	0	0	1/2
$S+b_1+e_3+e_5$	F_{1R}	$(\mathbf{4},\mathbf{1},\mathbf{2})$	1/2	0	0	1/2
$S+b_2$	F_{1L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	1/2	0	1/2
$S+b_2+e_1+e_2+e_5$	F_{2L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	1/2	0	1/2
$S+b_2+e_1$	$\bar{F}_{2R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	1/2	0	1/2
$S+b_2+e_2+e_5$	$\bar{F}_{3R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	1/2	0	1/2
$S+b_3+e_1+e_2$	F_{3L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	0	1/2	1/2
$S+b_3+e_2$	$\bar{F}_{4R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	0	1/2	1/2
$S+b_3+x$	h_1	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	−1/2	0	−1
$S+b_2+x+e_5$	h_2	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	0	−1/2	−1
$S+b_2+x+e_1+e_2$	h_3	$(\mathbf{1},\mathbf{2},\mathbf{2})$	−1/2	0	−1/2	−1
$S+b_3+x+e_1$	D_4	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	−1/2	0	−1
	χ_1^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1/2	1	+2
	χ_1^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1/2	−1	0
	$\zeta_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1/2	0	0
	$\bar{\zeta}_a, a=2,3$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	1/2	0	0
$S+b_2+x+e_1+e_5$	D_5	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	0	−1/2	−1
	χ_2^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1	1/2	+2
	χ_2^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1	1/2	0
	$\zeta_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	0	−1/2	0
	$\bar{\zeta}_a, a=4,5$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	0	1/2	0
$S+b_2+x+e_2$	D_6	$(\mathbf{6},\mathbf{1},\mathbf{1})$	−1/2	0	−1/2	−1
	χ_3^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	1	1/2	+2
	χ_3^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1	1/2	0
	$\zeta_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	0	−1/2	0
	$\bar{\zeta}_a, a=6,7$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	0	1/2	0
$S+b_1+x+e_3$	$\bar{D}_6$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\chi}_4^+$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1	−1/2	−1/2	−2
	$\bar{\chi}_4^-$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	−1/2	−1/2	0
	$\zeta_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	−1/2	0
	$\bar{\zeta}_a, a=8,9$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	1/2	0
$S+b_1+x+e_5$	D_7	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1
	χ_5^+	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	1/2	1/2	+2
	χ_5^-	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1	1/2	1/2	0
	$\zeta_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	−1/2	0
	$\bar{\zeta}_a, a=10,11$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	1/2	0
$S+b_3+x+e_2+e_3$	ζ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1/2	−1/2	0	0
	$\bar{\zeta}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	−1/2	1/2	0	0
$S+b_1+x+e_3+e_4+e_6$	ϕ_1	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\phi}_1$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1
$S+b_1+x+e_4+e_5+e_6$	ϕ_2	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	1/2	1/2	+1
	$\bar{\phi}_2$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	−1/2	−1/2	−1

Table 1: *Observable twisted matter spectrum and* $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^3$ *quantum numbers.*

The chiral spectrum is self–dual under spinor–vector duality

maintains the E_6 embedding $\Rightarrow$ $U(1)_\zeta$ is anomaly free

Exophobic, three generations $+$ heavy and light higgs $+ \lambda_t \sim 1$

$$U(1)_{Z'} = \frac{1}{5}\, U(1)_C - \frac{1}{5}\, U(1)_L - U_\zeta$$

$Z' \Rightarrow$ Additional matter at the Z' breaking scale

Exotic $SO(10)$ singlets with non–standard $U(1)_\zeta$ charges

$\Rightarrow$ Natural Wilsonian dark–matter candidates

"Wilsonian" singlet dark matter   Delle Rose, AEF, Marzo, Rizos, 1704.02579

$[(1,0);(1,0)]_{(\ 0\ ,\ \pm 1\ ,\ 0\ )}$ $SO(10)$ singlets E_6 exotics

Several cases to consider :

1. $M >> M_{Z'}$ without inflation $\Rightarrow M \leq 10^5$ GeV

2. $M >> M_{Z'}$ with inflation and $T_R > M_{Z'}$ $\Rightarrow M > T_R \left[25 + \frac{1}{2}\ln\left(\frac{M}{T_R}\right)\right]$

3. $M << M_{Z'}$ without inflation $\Rightarrow M < 3$ keV

4. $M << M_{Z'}$ with inflation

$$M \begin{cases} > \ T_R \left[25 + \frac{1}{2}\ln\left(\frac{M^5}{M_{Z'}^4 T_R}\right)\right], & T_R < M \\ < \ \frac{M_{Z'}^4}{T_R^3} 6.9 \times 10^{-25} \left(\frac{g_*}{200}\right)^{1.5} \frac{1}{N_{Z'} g_{\rm eff}^2}, & T_R > M \end{cases}$$

UMN-TH-1720-98-TPI-MINN-98/15
UFIFT-HEP-97-18, DFPD98/TH/41
September 1998, hep-th/9809127

The Equivalence Postulate of Quantum Mechanics

Alon E. Faraggi[1] and Marco Matone[2]

[1] *Department of Physics*
University of Minnesota, Minneapolis MN 55455, USA
e-mail: faraggi@mnhepo.hep.umn.edu
[2] *Department of Physics "G. Galilei" – Istituto Nazionale di Fisica Nucleare*
University of Padova, Via Marzolo, 8 – 35131 Padova, Italy
e-mail: matone@padova.infn.it

Abstract

The removal of the peculiar degeneration arising in the classical concepts of *rest frame* and *time parameterization* is at the heart of the recently formulated Equivalence Principle (EP). The latter, stating that all physical systems can be connected by a coordinate transformation to the free one with vanishing energy, univocally leads to the Quantum Stationary HJ Equation (QSHJE). This is a third–order non–linear differential equation which provides a trajectory representation of Quantum Mechanics (QM). The trajectories depend on the Planck length through hidden variables which arise as initial conditions. The formulation has manifest p–q duality, a consequence of the involutive nature of the Legendre transformation and of its recently observed relation with second–order linear differential equations. This reflects in an intrinsic ψ^D–ψ duality between linearly independent solutions of the Schrödinger equation. Unlike Bohm's theory, there is a non–trivial action even for bound states and no pilot–wave guide is present. A basic property of the formulation is that no use of any axiomatic interpretation of the wave–function is made. For example, tunnelling is a direct consequence of the quantum potential which differs from the Bohmian one and plays the role of particle's self–energy. Furthermore, the QSHJE is defined only if the ratio ψ^D/ψ is a local homeomorphism of the extended real line into itself. This is an important feature as the $L^2(\mathbb{R})$ condition, which in the Copenhagen formulation is a consequence of the axiomatic interpretation of the wave–function, directly follows as a basic theorem which only uses the geometrical gluing conditions of ψ^D/ψ at $q = \pm\infty$ as implied by the EP. As a result, the EP itself implies a dynamical equation that does not require any further assumption and reproduces both tunnelling and energy quantization. Several features of the formulation show how the Copenhagen interpretation hides the underlying nature of QM. Finally, the non–stationary higher dimensional quantum HJ equation and the relativistic extension are derived.

Basic identity:

Identity

$$\left(\frac{\partial S_0}{\partial q}\right)^2 = \frac{\beta^2}{2}\left(\{\mathrm{e}^{\frac{i2S_0}{\beta}}; q\} - \{S_0; q\}\right)$$

$$\alpha^2(\nabla S_0)\cdot(\nabla S_0) = \frac{\Delta(Re^{\alpha S_0})}{Re^{\alpha S_0}} - \frac{\Delta R}{R} - \alpha\left(2\frac{\nabla R\cdot\nabla S_0}{R} + \Delta S_0\right),$$

$$\alpha^2(\partial S)\cdot(\partial S) = \frac{\partial^2(Re^{\alpha S})}{Re^{\alpha S}} - \frac{\partial^2 R}{R} - \alpha\left(2\frac{\partial R\cdot\partial S}{R} + \partial^2 S\right),$$

$$\alpha^2(\partial S - eA)\cdot(\partial S - eA) = \frac{D^2(Re^{\alpha S})}{Re^{\alpha S}} - \frac{\partial^2 R}{R} - \frac{\alpha}{R^2}\partial\cdot\left(R^2(\partial S - eA)\right),$$

$$D^\mu = \partial^\mu - \alpha e A^\mu$$

Extend:

$$\alpha^2(\partial_\mu S)(\partial^\mu S) = \frac{\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\partial^\mu\left(R\mathrm{e}^{\alpha S}\right)}{R\mathrm{e}^{\alpha S}} - \frac{\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}\partial^\mu R\right)}{R} - \frac{\alpha}{R^2}\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}R^2\partial^\mu S\right),$$

$$\alpha^2 G_{\mu\nu\eta\rho}\frac{\delta S}{\delta g_{\mu\nu}}\frac{\delta S}{\delta g_{\eta\rho}} = \frac{1}{R\mathrm{e}^{\alpha S}}G_{\mu\nu\eta\rho}\frac{\delta^2\left(R\mathrm{e}^{\alpha S}\right)}{\delta g_{\mu\nu}\delta g_{\eta\rho}} - G_{\mu\nu\eta\rho}\frac{1}{R}\frac{\delta^2\left(R\right)}{\delta g_{\mu\nu}\delta g_{\eta\rho}} - \frac{\alpha}{R^2}G_{\mu\nu\eta\rho}\frac{\delta}{\delta g_{\mu\nu}}\left(R^2\frac{\delta S}{\delta g_{\eta\rho}}\right),$$

Conclusions

- DATA $\longrightarrow$ UNIFICATION
- STRINGS $\longrightarrow$ GAUGE & GRAVITY UNIFICATION
- Free fermionic models $\longrightarrow$ A Fertile Crescent
- Free fermionic models $\longleftrightarrow$ $Z_2 \times Z_2$ orbifolds
- Question: Does the $Z_2 \times Z_2$ orbifold structure play a role in nature?
- Dark Matter is in the Dark. A low scale Z' will help.
- String Phenomenology $\longrightarrow$ Physics of the third millenium

  e.g. Aristarchus to Copernicus

HAPPY

FESTDAY

KEITH!