Natural Low-Scale Inflation and the Relaxion

Jason L. Evans

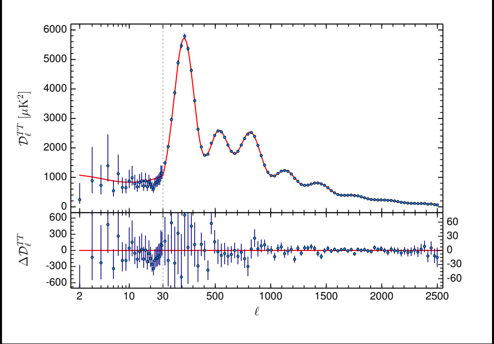
Korea Institute for Advanced Study

JLE, Gherghetta, Nagata, Peloso arXiv:1704.03695

Seeds of the CMB Anisotropy

- $\Delta T/T \sim 10^{-5}$ from quantum fluctuations
- Fluctuations related to the power spectrum

$$\langle \mathcal{R}\mathcal{R} \rangle = \int A_{\mathcal{R}} \, d \ln k \qquad A_{\mathcal{R}} = \left(\frac{H}{\dot{\phi}} \right)^2 \frac{H^2}{(2\pi)^2}$$



- CMB determines $A_{\mathcal{R}}$
- $D_\ell^{TT} \propto \int \frac{dk}{k} A_{\mathcal{R}}(k) \Delta^2_{T\ell}(k)$
- EE,ET modes help verify

Inflation Parameters

- Normalization of D_ℓ^{XY} determined by power spectrum

$$(A_\mathcal{R})^{1/2} = \left(\frac{H}{\dot\phi}\right)\frac{H}{(2\pi)} \simeq 5\times 10^{-5}\left(\frac{H}{10^{13}\,\mathrm{GeV}}\right)\left(\frac{2.6\times 10^{-3}}{\epsilon}\right)^{1/2}$$

- Experimental constraints on large ϵ becoming marginal
  - ϵ already borderline for explaining n_s

$$r \lesssim 0.07\ (95\%) \qquad \rightarrow \qquad \epsilon \lesssim 4\times 10^{-3}$$

- Spectral tilt requires largish slopes for either ϵ or η

$$n_s - 1 = 2\eta - 6\epsilon \simeq -0.03$$

- ϵ related to slope of potential

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_P^2\frac{V_{\phi\phi}}{V}.$$

Axions and the Hubble Scale

- Breaking of $U(1)_{PQ}$ lead to strings and domain walls
- If $U(1)_{PQ}$ breaks before inflation strings inflate away
- $\theta_a \sim 1$ axion can be dark matter

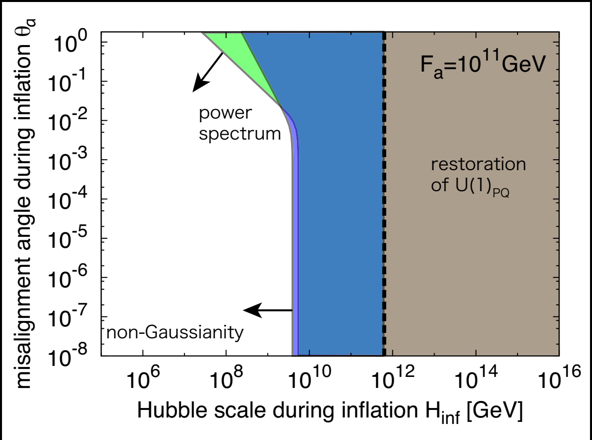
$$\Omega_a h^2 = 0.12 \left[\theta_a^2 + \left(\frac{H_I}{2\pi F_a}\right)^2\right] \left(\frac{F_a}{7 \times 10^{11}\ GeV}\right)^{1.19}$$

- During inflation, quantum fluctuations arise in axion
  - If axions contribute to CDM, get isocurvature perturbations
  - Low-scale inflation suppresses isocurvature perturbations

$$\left[\theta_a^2 + \left(\frac{H_I}{2\pi F_a}\right)^2\right] \left(\frac{H_I}{2\pi F_a}\right)^2 \left(\frac{F_a}{10^{12}\ GeV}\right)^{2.38} < 3.6 \times 10^{-11}$$

Axion Parameter Space

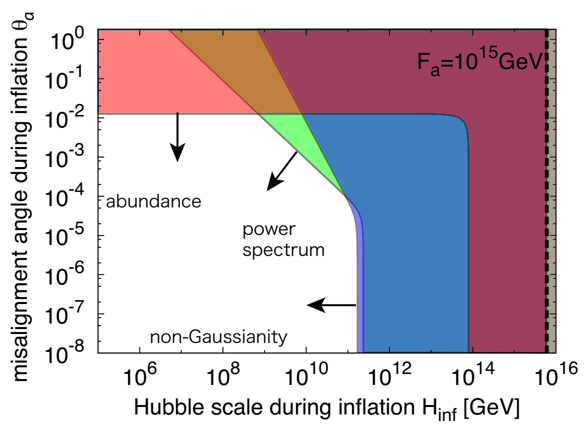
- PQ breaking before inflation with $F_a = 10^{11}$ GeV



Kawasaki,Nakayama

Axion Parameter Space

- PQ breaking before inflation prefers with $F_a = 10^{15}$ GeV



Kawasaki,Nakayama

Low-Scale Inflation

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_P^2 \frac{V_{\phi\phi}}{V}.$$

- Difficulties of low scale inflation

$$\epsilon = 8.6 \times 10^{-17}\left(\frac{H}{10^7\ \mathrm{GeV}}\right)^2$$

- For low-scale inflation we know η

$$n_s - 1 = 2\eta - 6\epsilon \simeq 2\eta \simeq -0.03 \qquad \eta \simeq -5 \times 10^{-3}$$

- Need potential very flat with large 2nd derivative
  - For relaxion, needs to be technically natural

D-term Inflation

- D-term with FI term

$$V_D = \frac{g^2}{2}\left[|\phi_+| - |\phi_-| - \xi\right]^2$$

- Inflaton couples directly to $U(1)$ charged particles

$$W = \kappa T \phi_+ \phi_-$$

- $\langle \phi_\pm \rangle = 0$ during inflation

$$|\kappa T| > \sqrt{g^2 \xi}$$

- Potential perfectly flat at tree level

$$V_D = \frac{g^2}{2}\xi^2$$

Coleman-Weinberg Potential

- During inflation $\phi_\pm$ massive
  - integrating $\phi_\pm$ generates potential for T
- Potential to one-loop for $|\kappa T| > \sqrt{g^2\xi}$

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\left[\frac{|\kappa T|^2}{Q^2}\right]\right)$$

- Slow roll parameters

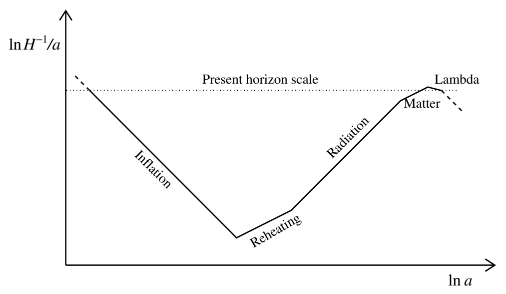
$$\epsilon = \frac{g^2}{16\pi^2}\frac{1}{N_{CMB}} \qquad \eta = -\frac{1}{2N_{CMB}}$$

- For small g, ϵ small enough

$$A_{\mathcal{R}}^{1/2} = 5\times 10^{-5}\left(\frac{N_{CMB}}{40}\right)\left(\frac{H_I}{10^7\ \mathrm{GeV}}\right)\left(\frac{7.3\times 10^{-7}}{g}\right)$$

N_{CMB} and the History of the Universe

- N_{CMB} depends on expansion of universe after inflation
  - Lower $\rho_{reh} \rightarrow$ decrease radiation domination
  - Slower expansion during RD less e-folds needed



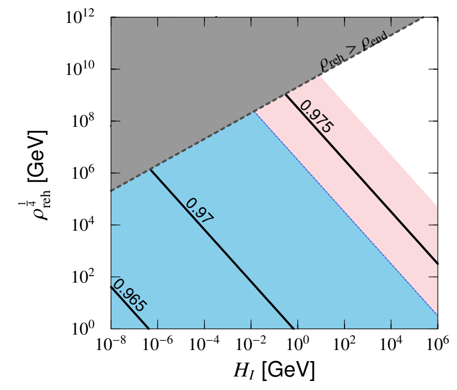
Liddle, Leach

- Instantaneous reheat/low-scale inflation

$$N_{CMB} = 39 + \frac{1}{3}\ln\left(\frac{H_I}{10^5\ \text{GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{reh}^{1/4}}{100\ \text{GeV}}\right)$$

The Spectral Tilt

- Reduced N_{CMB} puts n_s in $1-\sigma$ lines



– Spectral tilt

$$n_s - 1 = 2\eta = -0.026\left(\frac{39}{N_{CMB}}\right)$$

– Number of e-folds

$$N_{CMB} = 39 + \frac{1}{3}\ln\left(\frac{H_I}{10^5\ \mathrm{GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{reh}^{1/4}}{100\ \mathrm{GeV}}\right)$$

Why Dynamical D-Terms

- Field independent FI terms hard to realize in SUGRA
- Field dependent gravity generated FI tend to be large

$$\sqrt{\xi} \sim \langle \phi_\xi \rangle \sim M_P$$

- Dynamically generated FI terms
  - Dynamical scale can be arbitrary

$$\sqrt{\xi} \sim \langle \phi_\xi \rangle \sim \Lambda$$

- Dynamical D-terms provide U(1) breaking during inflation

Reheating After Inflation

- Majority of energy stored in ϕ_+ not inflaton T
- ϕ_+ has large vev (10^{16} GeV) and small mass (10^3 GeV)
- Generic couplings to ϕ_+ give large masses
  - Small couplings give small masses, but decays after BBN
  - Supersymmetry ristricts allowed couplings

$$-\mathcal{L} \supset |\lambda|^2 |\phi_+|^2 |\varphi|^2 + \lambda \phi_+ \bar{\psi}\psi$$

- Couple ϕ_+ through dynamical sector to scalar
  - This allows ϕ_+ to couple like a singlet

$$\Delta W = \kappa_1 R M_- \phi_+ + \kappa_2 R H_u H_d + m_R R \bar{R}$$

- Trilinear coupling allows ϕ_+ decay
  - $\bar{R}$ will cancel any vev of ϕ_+ keeping everything light

$$\Delta V \supset |\kappa_1 M_- \phi_+ + \kappa_2 H_u H_d + m_R \bar{R}|^2$$

- Can these couplings be detected?

Relaxions and the Hubble Scale

$$-\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \frac{1}{2}(g\phi)^2 + \Lambda^4(H,T)\cos\left(\frac{\phi}{f}\right)$$

- Three important parts: m_H^2, m_ϕ^2, and barrier
- Instanton potential for axion relaxion

$$-\mathcal{L} \supset y\langle H\rangle e^{i\frac{\phi}{f}}\langle \bar{q}_L q_R\rangle + y^\dagger\langle H^\dagger\rangle e^{-i\frac{\phi}{f}}\langle \bar{q}_R q_L\rangle = 2y\langle H\rangle\langle \bar{q}_L q_R\rangle \cos\left(\frac{\phi}{f}\right)$$

- Classical rolling restricts size of H

$$H_I < (gM^2)^{1/3} \simeq \left(\frac{m_\pi^2 f_\pi^2}{f}\right)^{1/3} = 6\times 10^{-5}\ \mathrm{GeV}\ \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

Relaxions and the Hubble Scale

$$-\mathcal{L} \supset (\mathcal{D}(H,\phi)/m_H^2)|H|^2 + \frac{1}{2}(g\phi)^2 + \Lambda^4(H,T,\phi)\cos\left(\frac{\phi}{f}\right) \quad + m_T^2|T|^2$$

- Four important parts: m_H^2, m_ϕ^2, m_T^2, and barrier
- $\Lambda(H,T,\phi)$ for two-field model

$$\Lambda(H,T,\phi) = \left(m_N - g_S\phi - g_T\sigma + \frac{\lambda}{M_L}H_u H_d\right)$$

- Supersymmetric two-field model a little better

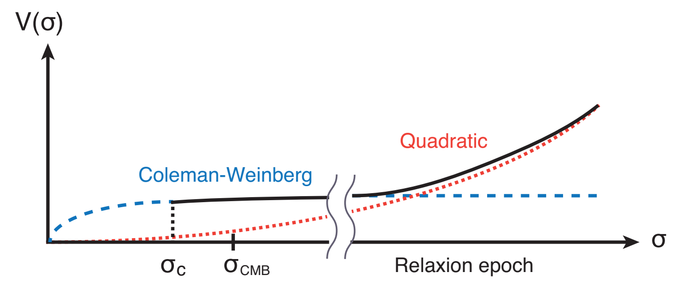
$$H_I < (m_S m_{SUSY} f_\phi)^{1/3} \simeq 4.6\ \mathrm{GeV}\ \left(\frac{m_S}{10^{-7}\ \mathrm{GeV}}\right)^{1/3}\left(\frac{f_\phi}{10^5\ \mathrm{GeV}}\right)^{1/3}\left(\frac{m_{SUSY}}{10^5\ \mathrm{GeV}}\right)^{1/3}$$

Inflaton as the Second Field

- The inflaton can play the roll of the amplitudon ($T = \tau + i\sigma$)

$$W_{S,T} = \frac{m_S}{2} S^2 + \frac{m_T}{2} T^2 \qquad W_{inf} = \kappa T \phi_+ \phi_-$$

- D-term and relaxion have very different energies

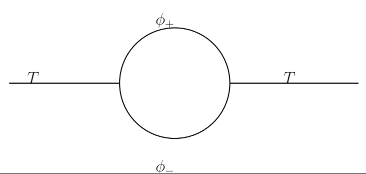


Shift Symmetry Breaking of Inflaton Sector

- Inflaton has relatively large shift symmetry breaking

$$W = \kappa T \phi_+ \phi_- \qquad \kappa \gtrsim 10^{-2}$$

- Loop correction transmit shift symmetry breaking to Kähler



$$K \supset \frac{|\kappa|^2}{16\pi^2}|T|^2$$

- SUGRA corrections to scalar potential generate mass for T

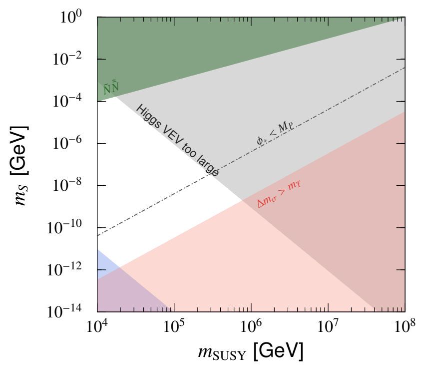
$$V \supset e^{\frac{K}{M_P}}|F_S|^2 + \ldots \qquad \rightarrow \qquad V \supset \frac{|\kappa|^2}{16\pi^2}\frac{|F_S|^2}{M_P^2}|T|^2$$

- Kähler corrections give lower bound on m_T

$$m_T \gtrsim \frac{\kappa}{4\pi}\frac{|F_S|}{M_P} = \frac{\kappa}{4\pi}\frac{m_{SUSY} f}{M_P}$$

Constraint Summary

- $\zeta = 10^{-8} \quad r_{TS} = 0.1 \quad r_\Lambda = 1 \quad r_{\rm SUSY} = 1.$



- Parameters

$$g_S = \zeta \frac{m_S}{f_\phi} \quad g_T = \zeta \frac{m_T}{f_\sigma}$$

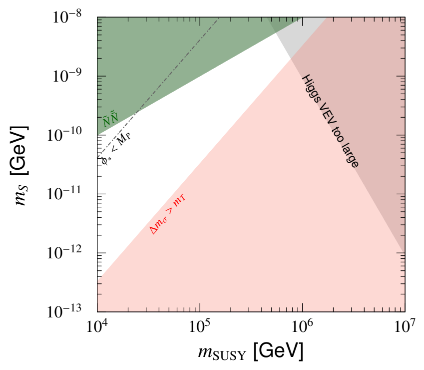
$$f \equiv f_\phi = f_\sigma \quad r_{TS} \equiv \frac{m_T}{m_S}$$

$$r_\Lambda \equiv \frac{\Lambda_N}{f} \quad r_{\rm SUSY} \equiv \frac{m_{\rm SUSY}}{f}$$

$$M_L = m_{\rm SUSY} \,,$$

Constraint Summary

- $\zeta = 10^{-14} \quad r_{TS} = 0.1 \quad r_\Lambda = 1 \quad r_{\rm SUSY} = 1.$



- Parameters

$$g_S = \zeta \frac{m_S}{f_\phi} \quad g_T = \zeta \frac{m_T}{f_\sigma}$$

$$f \equiv f_\phi = f_\sigma \quad r_{TS} \equiv \frac{m_T}{m_S}$$

$$r_\Lambda \equiv \frac{\Lambda_N}{f} \quad r_{\rm SUSY} \equiv \frac{m_{\rm SUSY}}{f}$$

$$M_L = m_{\rm SUSY} \ ,$$

Conclusions

- CMB fluctuations relate potential slope to Hubble
- Constraints on r push us towards low-scale inflation
- Relaxion and axions prefer very low-scale inflation
- Very low-scale D-term Inflation possible
  - ϵ and η of correct size (from loops)
  - Low-scale inflation reduces N_{CMB} giving correct n_s
- Cosmic strings can be solved through dynamical D-terms
- Reheating is difficult but possible
- Inflaton and amplitudon can be combined

String Formation: Two Sectors of U(1) Breaking

- Hidden sector breaking of U(1)

$$V = \frac{g^2}{2}\left(|\phi_+|^2 - \xi\right)^2 + |\lambda T|^2|\phi_+|^2 + C\phi_+ + C^\dagger\phi_+^\dagger \quad \supset \quad |C|v_+\cos(\theta+\theta_c)$$

- Superpotential connects phase of two sectors

$$V_F \supset |\lambda T\phi_+ + \lambda_- T M_-|^2 \quad \supset \quad \lambda\lambda_+|T|^2 M_- v_+ \cos(\theta)$$

- Quantum fluctuations could still form strings ($\phi_+ = v_r e^{i\alpha}$)

$$\langle\alpha^2\rangle = \frac{H^3}{12\pi^2 m v_r^2} < 1$$

- Quantum fluctuation at end of inflation

$$H_I < 1\times 10^9\ \mathrm{TeV} \times \left(\frac{|\kappa_+|}{10^{-12}}\right)\left(\frac{|M_+|}{10^{16}\ \mathrm{GeV}}\right)\left(\frac{\kappa}{10^{-2}}\right)^{-\frac{1}{2}}\left(\frac{A_s}{2.1\times 10^{-9}}\right)^{-\frac{1}{4}}\left(\frac{1-n_s}{0.03}\right)^{-\frac{1}{2}} .$$