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Particle creation in the expanding universe

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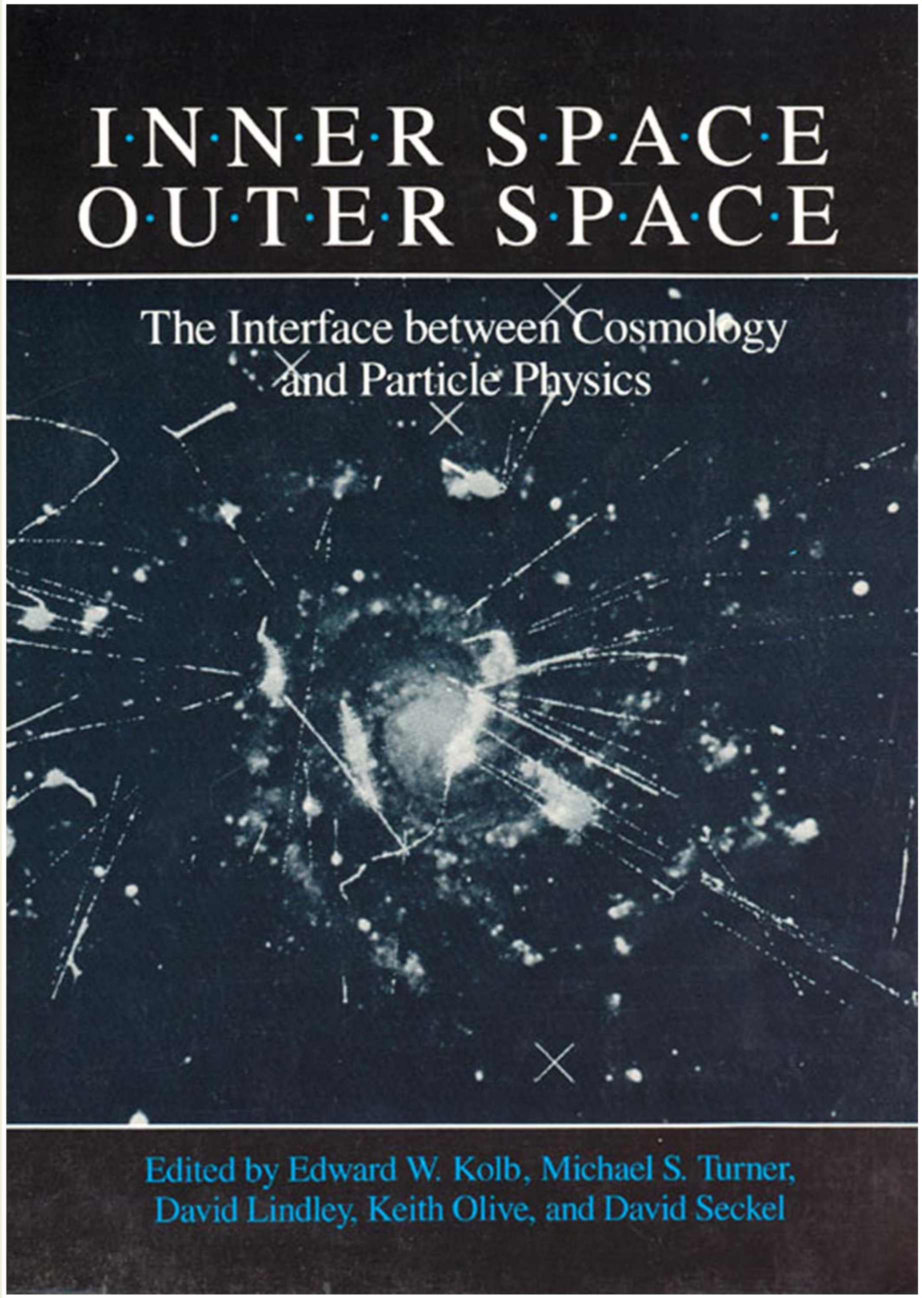
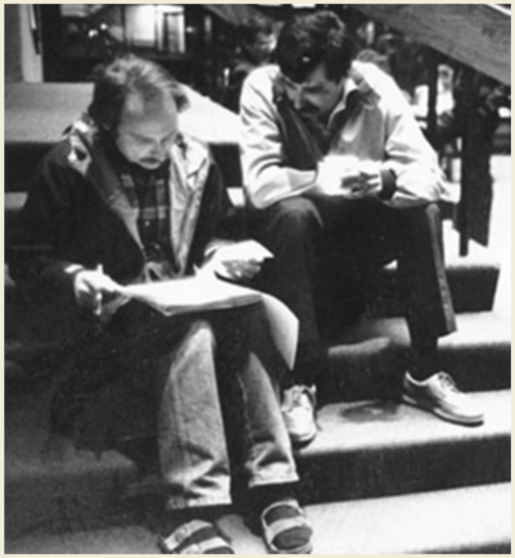
Inner Space/Outer Space May 1984



Primordial Keith Olive
Inner Space/Outer Space May 1984



Particle creation in the expanding universe

Two mechanisms

Thermal Particles from the Primordial Soup

Schrödinger's Alarming Phenomenon

Physica VI, no 9 October 1939

THE PROPER VIBRATIONS
OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einsteins theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (Einstein- and De Sitter-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

For *light*: when referred to the customary *co-moving* coordinates, an *arbitrary* wave process exhibits essentially the same succession of states as without expansion. Briefly, the wave function shares the general dilatation. Hence all *wave lengths* increase proportionally to the radius of curvature. — The *time rate* of events is slowed down. It is, in every moment, proportional to R^{-1}. Moreover all *intensities* are affected by a common factor such as to make the total energy of an arbitrary wave process proportional to R^{-1}.

For the *material particle* the broad results are these: a strictly monochromatic process (i.e. a proper vibration) again shares the

— 899 —

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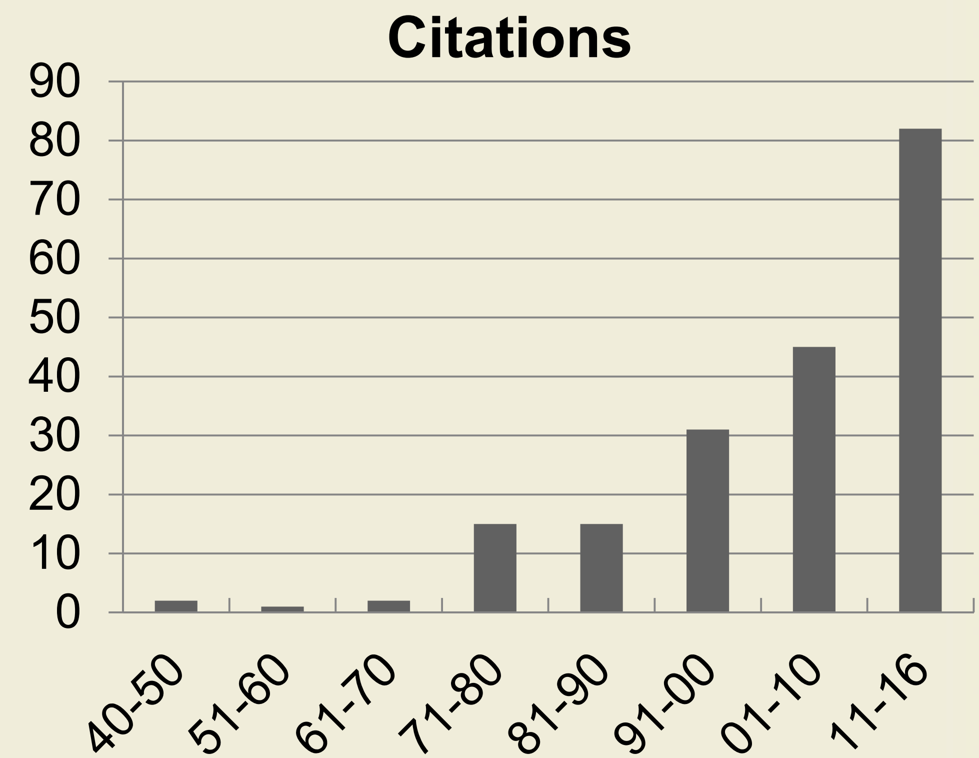

common dilatation, so that its wave length λ is proportional to R, as before. From the changing λ the changing *frequency* is calculated by de Broglies formula. This implies different frequencies to be affected by different factors. Therefore an arbitrary wave function can no longer be said to simply share the common dilatation. But since de Broglie's dispersion formula persists, the familiar connection (momentum $= h/\lambda$) between linear group velocity (= particle velocity) and wave length is also preserved, which causes the former or more precisely the momentum, to decrease proportional to R^{-1}. As regards the amplitudes, the most reliable information about them, valid for any particle wave function whatsoever, is this, that the *normalisation* is rigorously conserved during the expansion.

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time. Waves travelling in one direction can be rigorously separated from those travelling in the opposite direction. The results for D'Alemberts equation (light) and Gordons equation (material particles), which have been used throughout in this paper for the sake of simplicity, are given in sect. 5 and 6 respectively. I have confirmed the results with Diracs equation, but reserve it to a subsequent paper.

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Even in an expanding universe, a particle's wavefunction can be decomposed into "proper vibrations" (positive & negative frequency modes):

$$\Psi(t) = \frac{\alpha}{\sqrt{2\omega}}\, e^{-i\omega t} + \frac{\beta}{\sqrt{2\omega}}\, e^{+i\omega t}$$

Particle occupancy number $\propto |\beta|^2$

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The expansion of the universe creates particles!

This alarms me [ed. why?], so I wrote a paper.

$e^{2\pi i\nu t}$ will re-assume (or approximately re-assume) the form $Ae^{2\pi i\nu' t}$ — and *not* $Ae^{2\pi i\nu' t} + Be^{-2\pi i\nu' t}$ — whenever $R(t)$, after an intermediate period of arbitrary variation, returns to constancy (or to approximate constancy). I can see no reason whatsoever for $f(t)$ to behave rigorously in this way, and indeed I do not think it does. There will thus be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called „the alarming phenomena". They are certainly very slight, though, in two cases, viz. 1) when R varies slowly 2) when it is a linear function of time (see the following sections).

A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitters universe I know three of them. Besides the static one, for which P. O. Müller (l.c.) has redently given the proper vibrations, there is an expanding form with infinite R and an expanding form with finite R *). A proper vibration of one frame will not transform into a proper vibration of the other frame, for the separation of variables is destroyed by the transformation.

---

*) From De Sitters line-element in static form

$$ds^2 = -R_0^2[d\chi^2 + \sin^2\chi(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)] + R_0^2 \cos^2\chi\, dt^2$$

the transformation of Lemaître (J. Math. and Phys. M.I.T., 4, 188, 1925) and Robertson (Phil. Mag. 5, 835, 1928)

$$\bar{r} = R_0 \operatorname{tg} \chi\, e^{-t} \quad \bar{t} = t + \lg \cos \chi$$

gives the expanding *flat* form

$$ds^2 = -e^{2\bar{t}}[d\bar{r}^2 + \bar{r}^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)] + R_0^2 d\bar{t}^2.$$

The following transformation

$$\operatorname{tg} \chi' = \frac{\operatorname{tg} \chi}{\operatorname{Cos} t} \quad \operatorname{Sin} t' = \operatorname{Sin} t \cos \chi$$

or

$$\sin \chi = \sin \chi' \operatorname{Cos} t' \quad \operatorname{Tg} t = \operatorname{Tg} t'(\cos \chi')^{-1}$$

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$$ds^2 = -R_0^2(\operatorname{Cos} t')^2[d\chi'^2 + \sin^2\chi'(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] + R_0^2 dt'^2.$$

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Schrödinger's two favorite phrases:

1. alarming phenomenon
2. mutual adulteration

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Schrödinger was alarmed by creation of a <u>*single*</u> particle

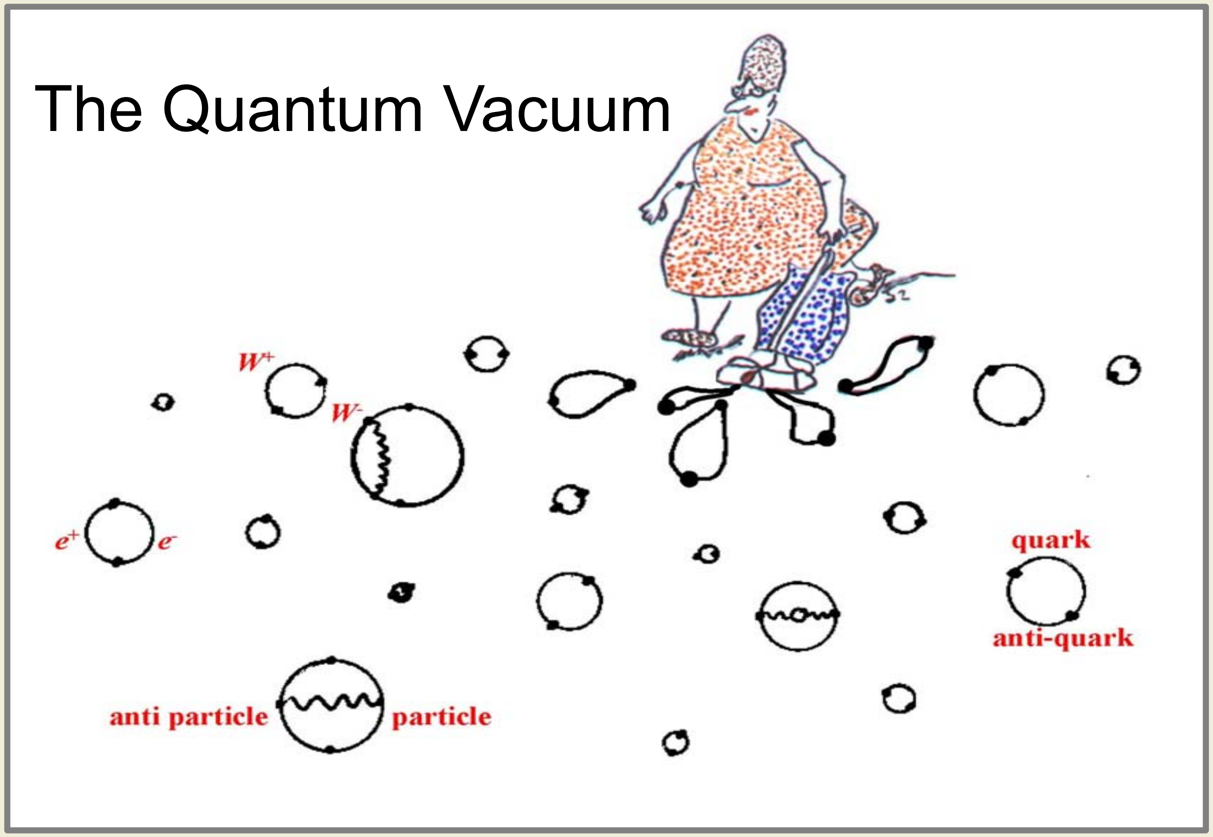
per Hubble time $(H_0^{-1} \sim 10^{10}\ \text{yr})$

per Hubble volume $(H_0^{-3} \sim 10^{57}\ \text{km}^3)$

with Hubble energy $(H_0 \sim 10^{-33}\,\text{eV})$

Schrödinger's Alarming Phenomenon

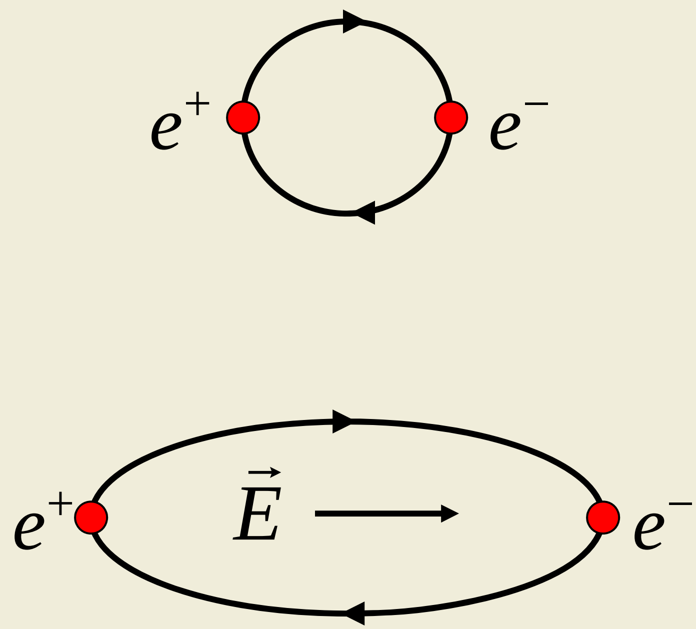
How to understand mutual adulteration (particle creation)?



External Fields Can Disturb The Quantum Vacuum

Disturbing the Quantum Vacuum

Changing Electric Field $\longrightarrow$ Particle creation

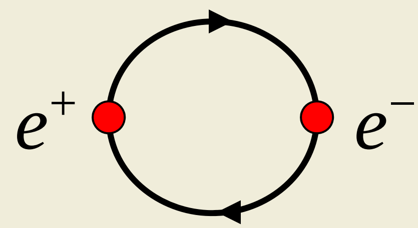


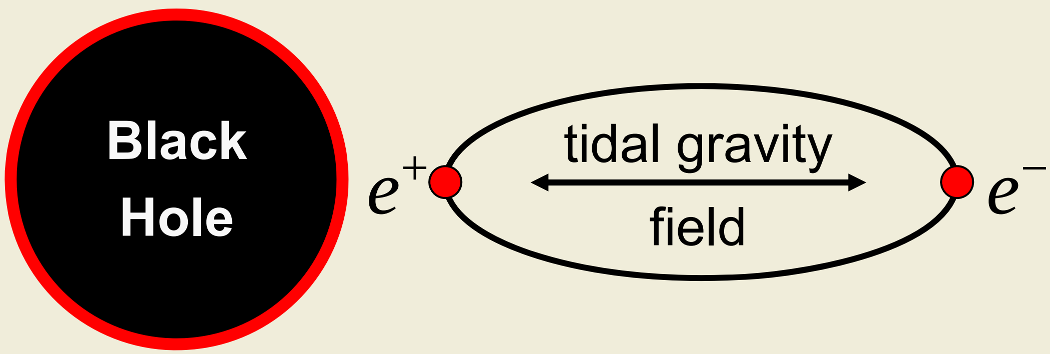
Particle creation if energy gained in acceleration from electric field over a Compton wavelength exceeds the particle's rest mass.

Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)

Disturbing the Quantum Vacuum

Tidal gravitational field $\longrightarrow$ Particle creation



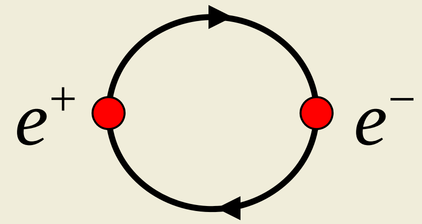


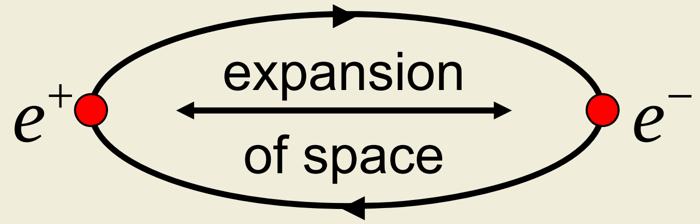
Particle creation if energy gained in acceleration from gravitational field over a Compton wavelength exceeds the particle's rest mass.

Hawking (1974); Bekenstein (1972)

Disturbing the Quantum Vacuum

Expanding universe ⟶ Particle creation



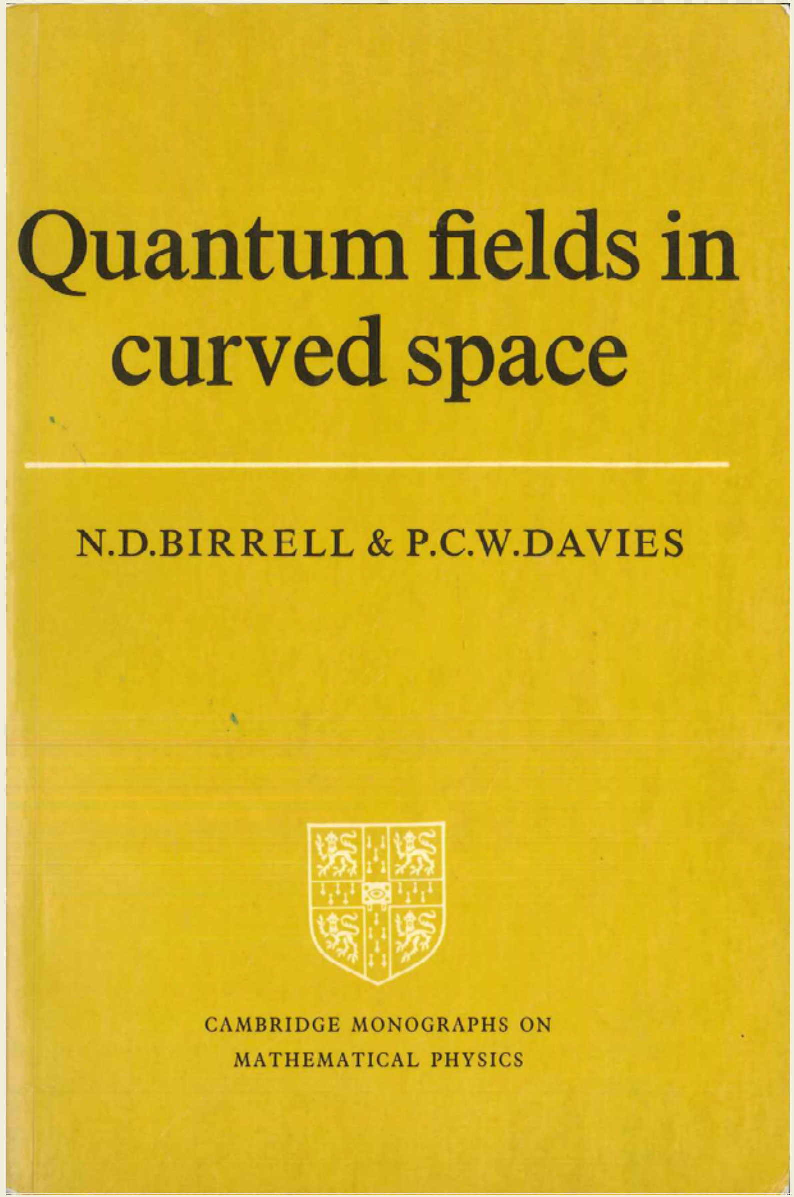


Particle creation if energy gained in expansion over a Compton wavelength exceeds the particle's rest mass.

Schrödinger's Alarming Phenomenon (1939)

Schrödinger's Alarming Phenomenon

Technical Details:



Schrödinger's Alarming Phenomenon

Free quantum scalar field χ of mass M in Minkowski space:

$$S = \int d^4x \left[\tfrac{1}{2} \partial^\mu \chi \partial_\mu \chi - \tfrac{1}{2} M^2 \chi^2 \right]$$

Mode expand and make plane-wave ansatz:

$$\chi(x,t) = \sum_k \hat{a}_k \, e^{ik\cdot x} \chi_k(t) + \hat{a}_k^\dagger \, e^{-ik\cdot x} \chi_k^*(t)$$

Equation of motion (Klein-Gordon equation):

$$\ddot{\chi}_k(t) + \omega_k^2 \chi_k(t) = 0 \qquad \omega_k^2 = \left|\vec{k}\right|^2 + M^2$$

Choose pure outgoing (+ frequency) solution

$$\chi_k(t) = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t}$$

other solution: $\chi_k(t) = \dfrac{1}{\sqrt{2\omega_k}} e^{+i\omega_k t}$

Schrödinger's Alarming Phenomenon

Couple scalar field χ to gravity:

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \tfrac{1}{2}M^2\chi^2 - \xi R\chi^2\right]$$

Equation of motion:

$$\ddot{\chi}_k(t) + \frac{\dot{a}(t)}{a(t)}\dot{\chi}_k(t) + \left[\frac{1}{a^2(t)}\left|\vec{k}\right|^2 - (1-6\xi)\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{\ddot{a}(t)}{a(t)}\right] + M^2\right]\chi_k(t) = 0$$

If express in terms of conformal time η :

$$\chi_k''(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0 \qquad \omega_k^2(\eta) = \left|\vec{k}\right|^2 - (1-6\xi)\frac{a''(\eta)}{a(\eta)} + a^2(\eta)M^2$$

wave equation with time-dependent mass that depends on evolution in time of the scale factor

Schrödinger's Alarming Phenomenon

Solutions to wave equation are adulterated:

$$\chi_k''(\eta)+\omega_k^2(\eta)\chi_k(\eta)=0 \qquad \omega_k^2(\eta)=\left|\vec{k}\right|^2-(1-6\xi)\frac{a''(\eta)}{a(\eta)}+a^2(\eta)M^2$$

Pure outgoing (+ frequency) solution $\chi_k(\eta)=\dfrac{1}{\sqrt{2\omega_k(\eta)}}e^{-i\int \omega_k(\eta)d\eta}$

is a good solution if $\left|\dfrac{\omega_k'(\eta)}{\omega_k^2(\eta)}\right|^2 \ll 1$ and $\left|\dfrac{\omega_k''(\eta)}{\omega_k^3(\eta)}\right|^2 \ll 1$

Abrupt-ish changes in $a(\eta)$ leads to abrupt changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to *Schrödinger's Alarming Phenomenon* of particle creation in the expanding universe.

Schrödinger's Alarming Phenomenon

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{+i\int \omega_k(\eta) d\eta}$$

If start with only outgoing waves, $\beta_k(\eta) = 0$,
generate incoming waves, $\beta_k(\eta) \neq 0$.

Comoving number density of particles at late time is

$$n = \frac{1}{(2\pi)^3} \int d^3\vec{k} \left|\beta_k(\eta)\right|^2$$

Schrödinger's Alarming Phenomenon

- Expansion of the Universe leads to time dependence of the coupling of fields to gravity

- Expansion of the universe leads to creation of all species of particles so long as there is a "time" dependence to $\omega_k(\eta)$

$$\omega_k^2(\eta) = \left|\vec{k}\right|^2 - (1-6\xi)\frac{a''(\eta)}{a(\eta)} + a^2(\eta) M^2$$ note: $\frac{a''(\eta)}{a(\eta)}$ can be positive

- Adulteration efficacy depends on abruptness of change in $\omega_k(\eta)$

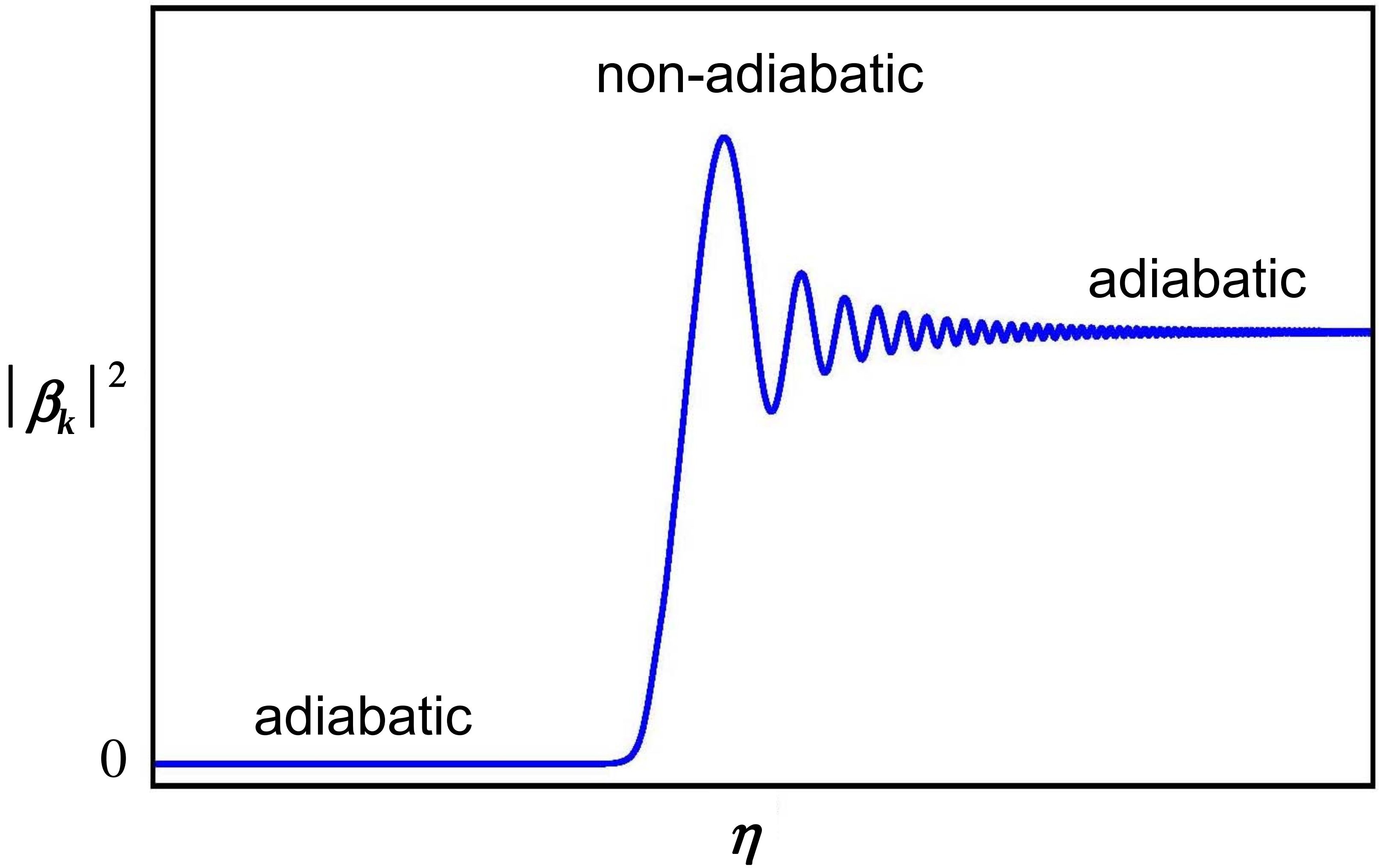
$$\left|\frac{\omega_k'(\eta)}{\omega_k^2(\eta)}\right|^2$$ and $$\left|\frac{\omega_k''(\eta)}{\omega_k^3(\eta)}\right|^2$$

production suppressed if $M \gtrsim H$

particles "produced" when $\left|\vec{k}\right|/a \simeq H$ $(\xi = 0)$

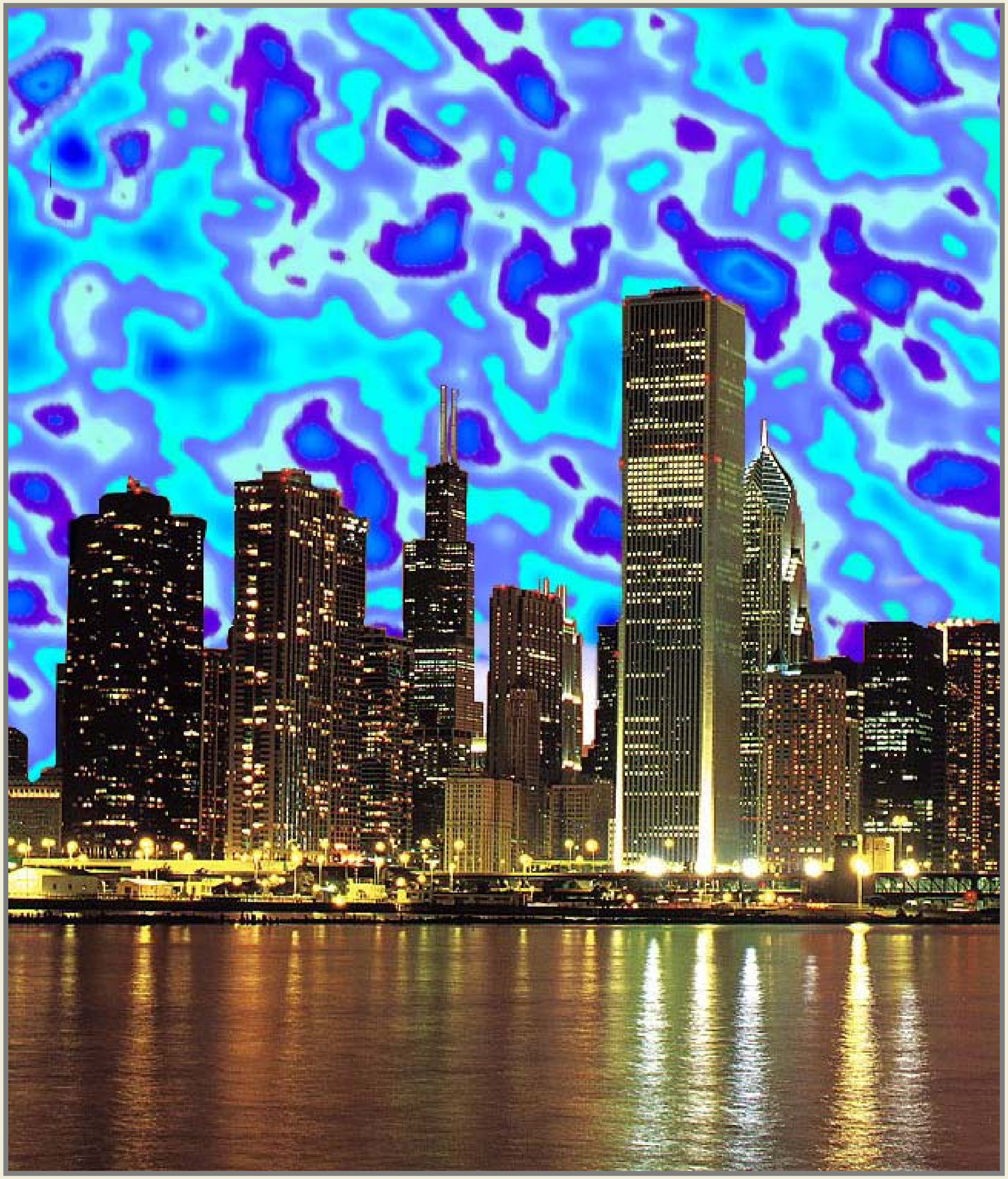
- Many subtleties glossed over

Schrödinger's Alarming Phenomenon



A pattern of vacuum quantum fluctuations

density perturbations and gravitational waves (gravitons)



$$\hbar \to 0$$

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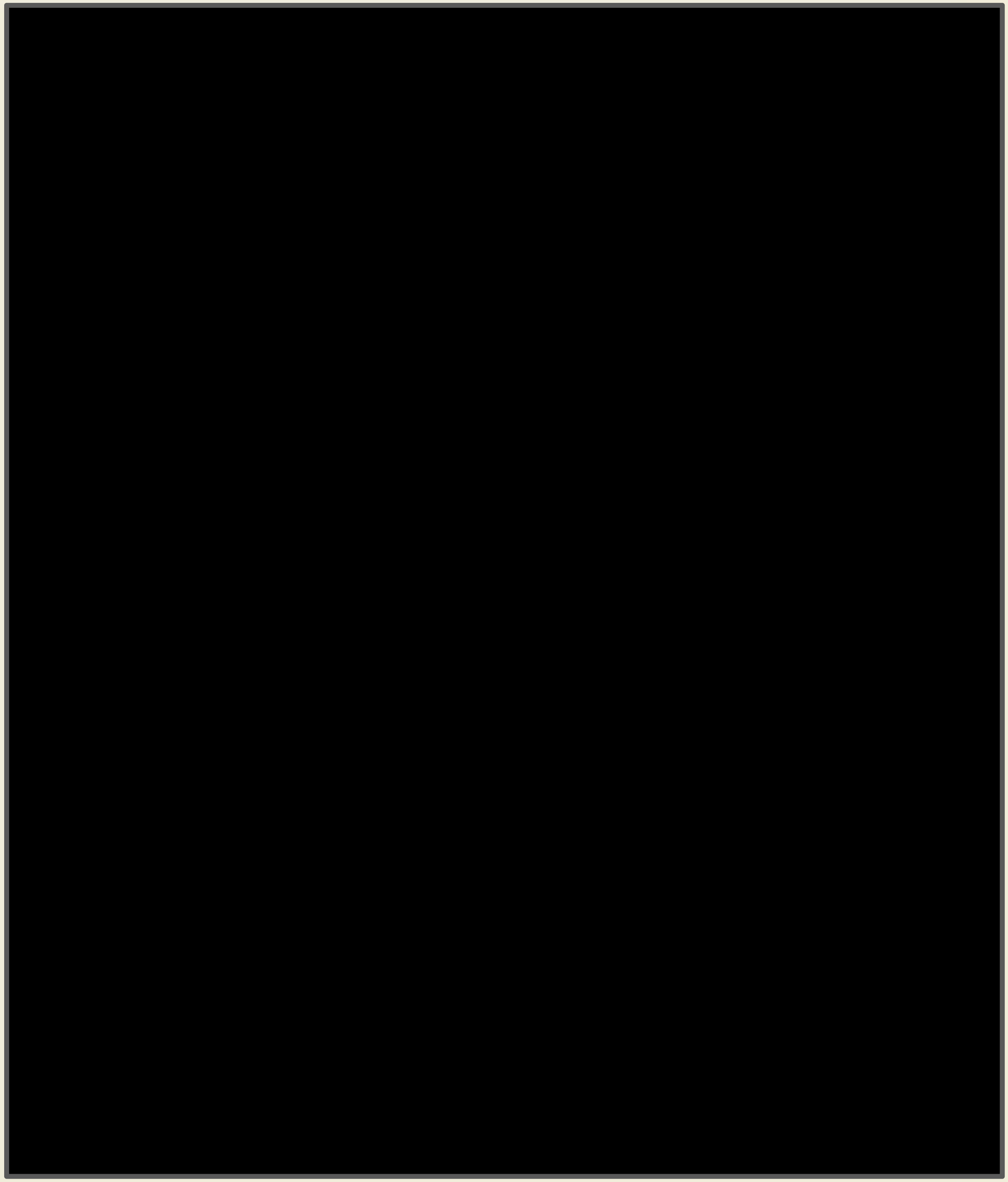


Some Details

- Scalars: big difference between minimally-coupled ($\xi = 0$) and conformally-coupled ($\xi = 1/6$) scalars

$$\omega_k^2(\eta) = \left|\vec{k}\right|^2 - (1 - 6\xi)\frac{a''(\eta)}{a(\eta)} + a^2(\eta) M^2$$

If $\xi = 0$ tachyonic during inflation at $k/a\,H \sim 1$

If $\xi = 1/6$ nonadiabatic at end or just after end of inflation

Should probably view ξ as a free parameter?

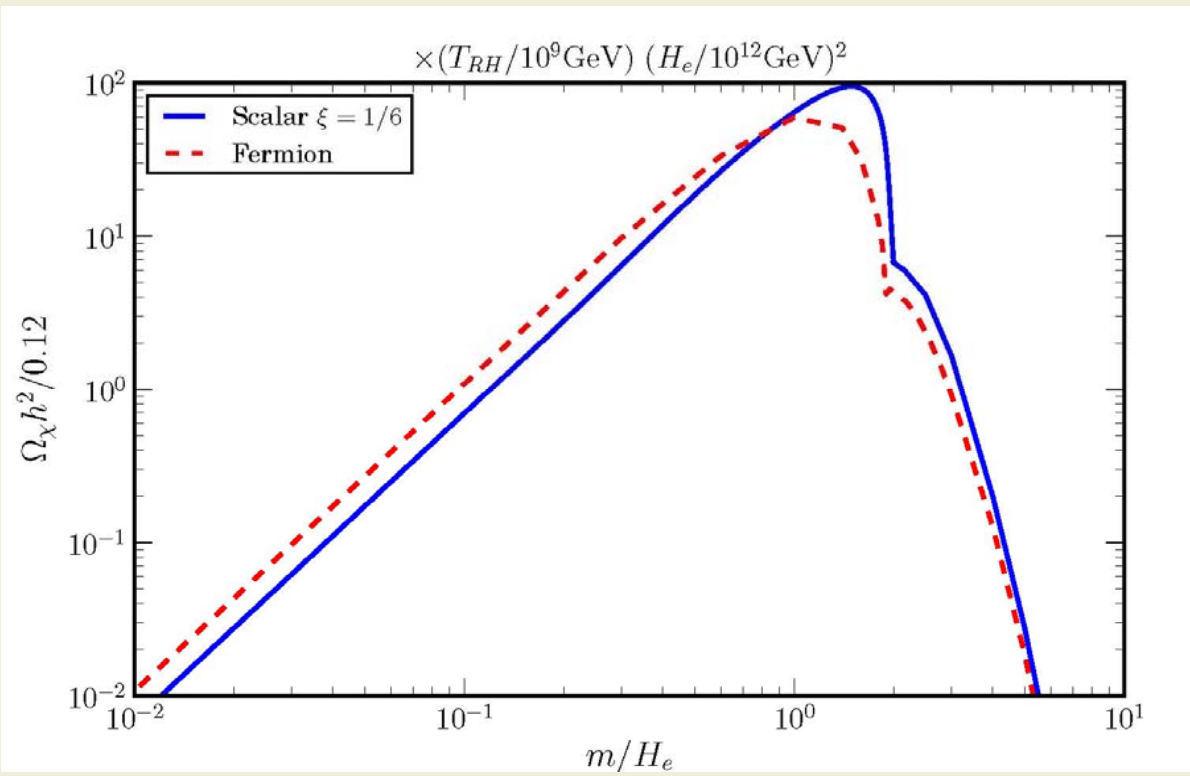
- Fermions: naturally similar to conformally-coupled scalars

- Vectors: in the works

- Crucial parameter is ratio M/H

WIMPzillas

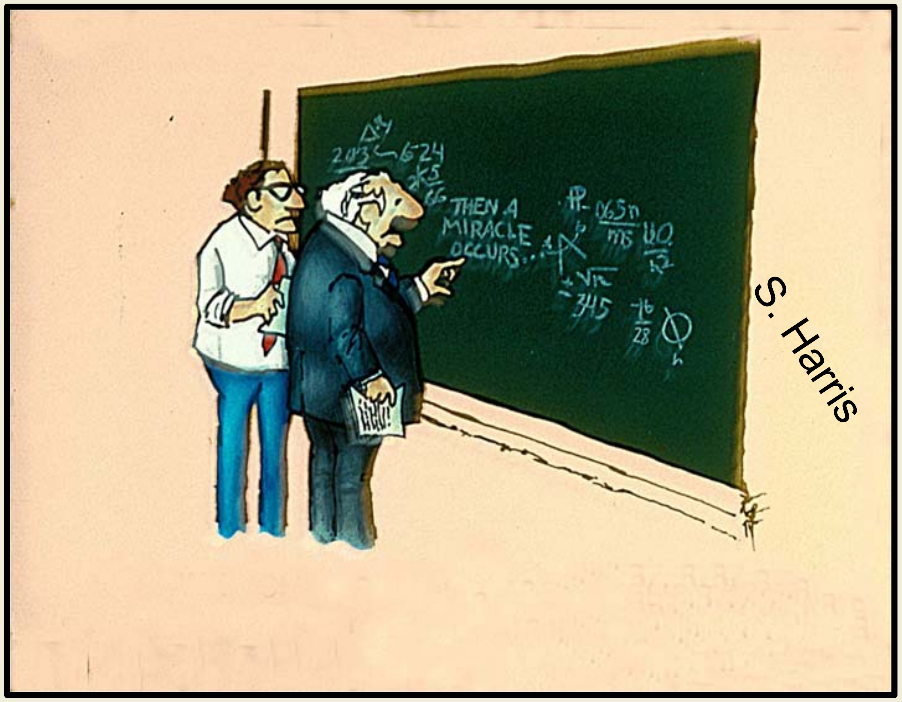
- Schrödinger's Alarming Phenomenon applies to all particles*
- All particles* produced during inflation
- Could dark matter be produced during inflation?



- Inflation signifies a new mass scale.
- H_e, expansion rate at end of inflation, comparable to inflaton mass.
- Might expect other particles with mass comparable to inflaton mass.
- If one is stable, natural candidate for dark matter (WIMPzilla miracle).

* So long as conformal symmetry violated.

The WIMP "Miracle"



S. Harris

The WIMP “Miracle”

mir·a·cle
\ˈmir-i-kəl \
noun

1 : an extraordinary event manifesting divine intervention in human affairs

The WIMP "Miracle"

Miracle

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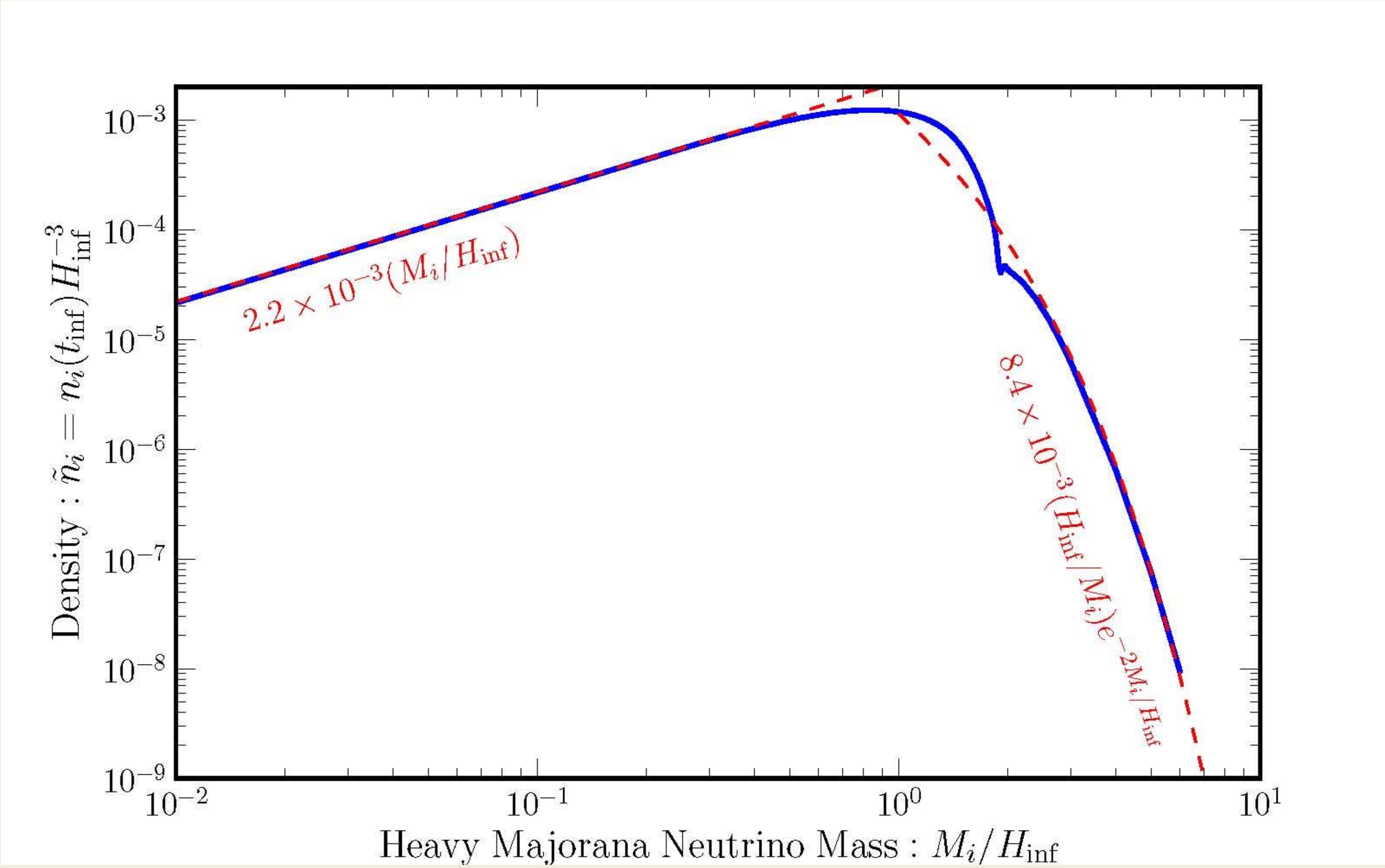
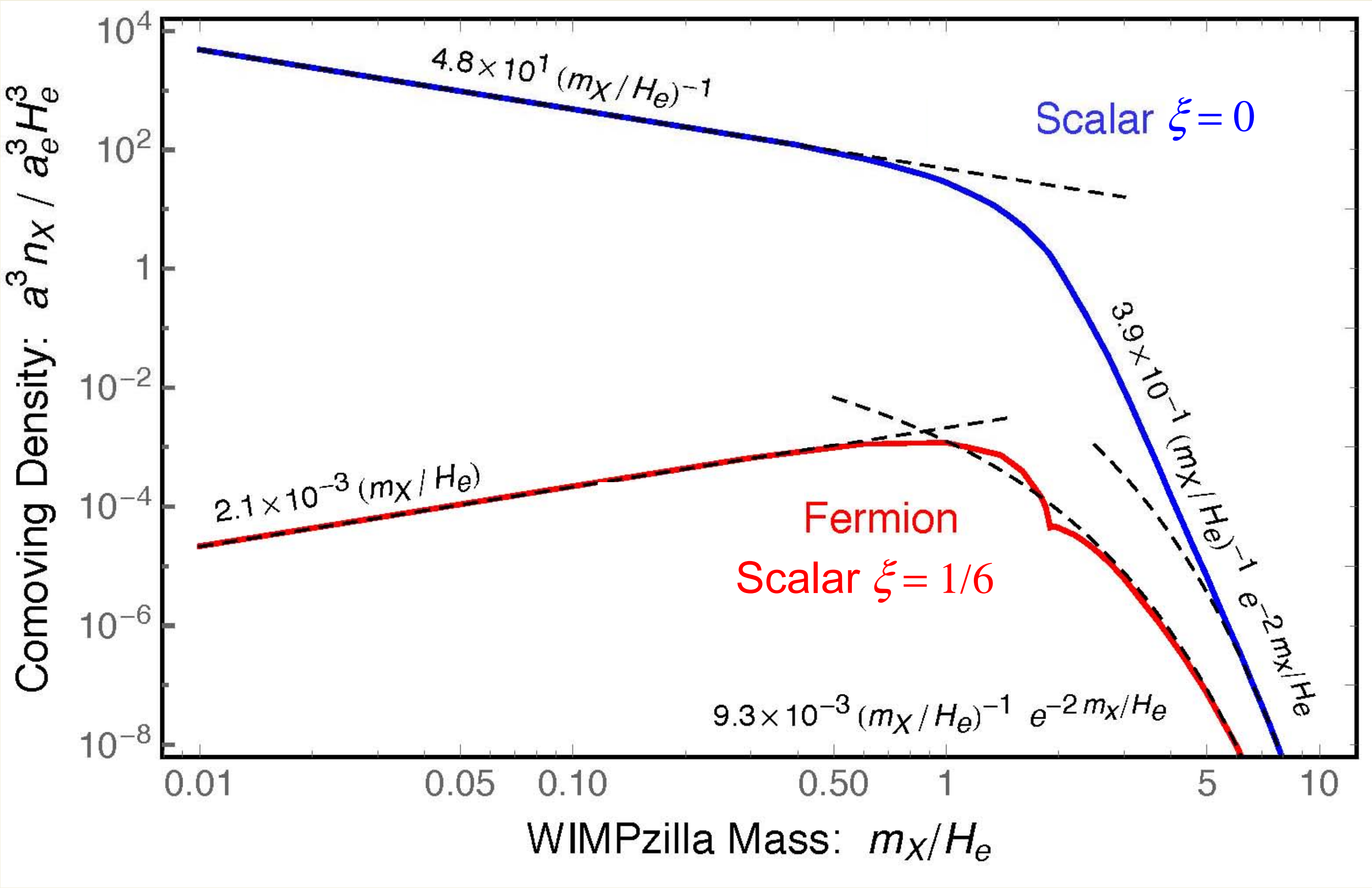
… often used to give an impression of great and unusual value in a trivial context …

WIMPzillas as dark matter



WIMPzillas as dark matter



WIMPzillas and the Higgs Portal

- Original WIMPzilla idea assumed that zillas have only gravitational interactions

- Would only be produced gravitationally—no “thermal” production

- This is naïve: even if zillas SM singlet, they should couple to SM through Higgs portal:

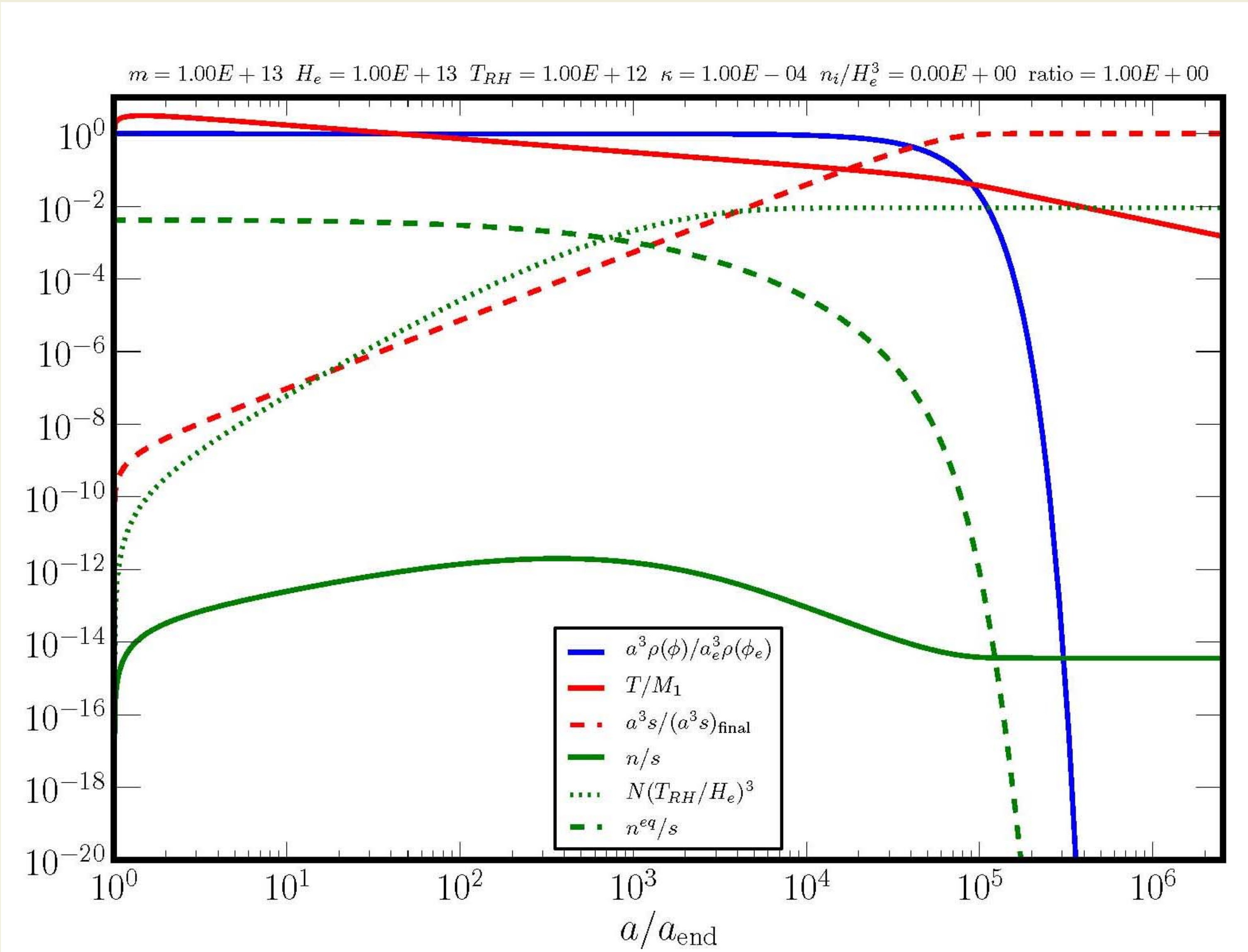
  Fermion zilla: dimension-5 $\dfrac{\kappa}{M_{Pl}}\bar{\chi}\chi H^{\dagger}H$

  Scalar zilla: dimension-4 $\kappa\chi^{*}\chi H^{\dagger}H$

- Opens window to “thermal” production freeze-in after inflation

Between end of inflation and reheating

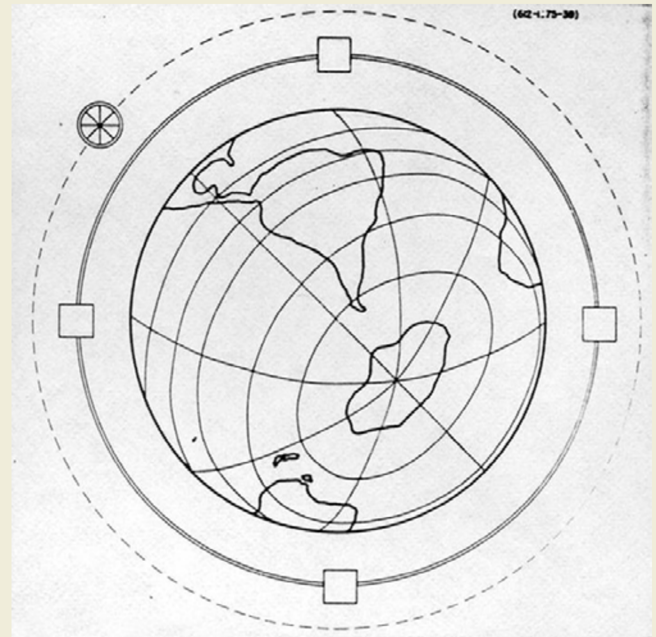
Reheat temperature is *not* maximum temperature after inflation



Giudice, et al.
Kolb & Long

Won't see zillas in direct detection experiments
(target with $A = 10^{12}$ would be good!)

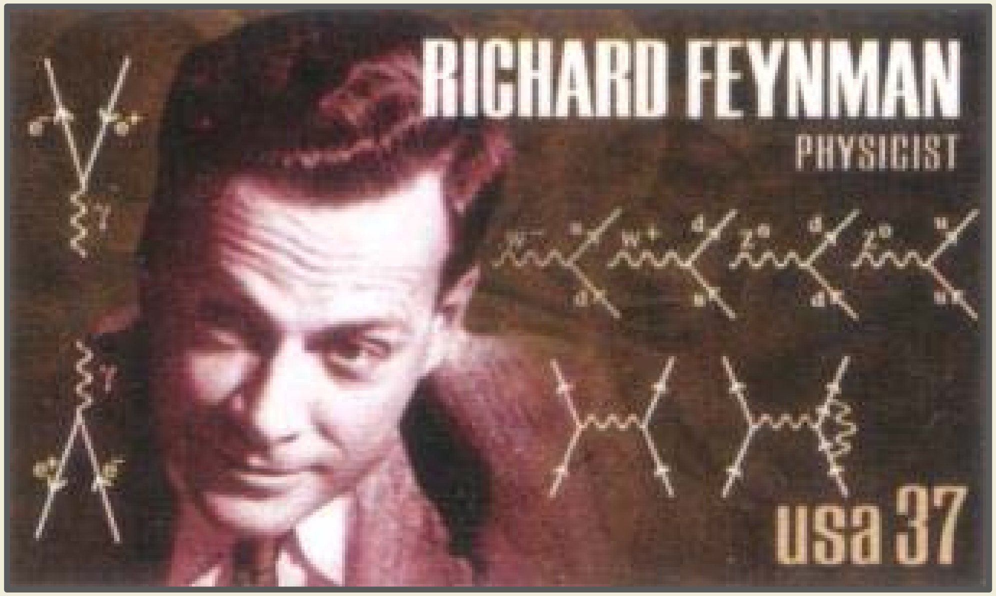
For accelerators, need LHCzilla



Very rare very energetic annihilations today (e.g., GC)

Signatures of primordial isocurvature perturbations

Gravitational Particle Production: seeds of structure, gravitons, dark matter?



Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry.

Richard Feynman

Why gravitational production of dark matter?

1. Based upon beautiful physics

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5. Annoys John Ellis

Olivefest 2017

Particle creation in the expanding universe

Rocky Kolb, The University of Chicago

Collaborators: Dan Chung, Patrick Crotty, Gian Giudice, Andrew Long, Toni Riotto, Alexei Starobinsky, Igor Tkachev