

Universal Cradle

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1. Overview

This report outlines the work completed on the Universal Cradle undergraduate research project conducted at the University of Minnesota Duluth under the guidance of Dr. Tarek AlGeddawy. In summary, a carrying cradle was designed for integration with Dr. AlGeddawy's Changeable Learning Factory (CLF) project. This cradle was designed to fit on pallets in a conveyor-loop assembly (Figure 1), and built to hold a variety of geometrically-variable parts as they make their way through the assembly system. The design process involved research, experimental testing, and physical prototyping. This report outlines the purpose of the project, the design constraints involved, the research concerning existing literature, the initial design concept, the experiments conducted and the physical model built to create a universal pallet tailored for integration with Dr. AlGeddawy's work.

2. Purpose

The purpose of this research project was to design and physically develop a carrying cradle, operating as a universal secured pallet. The universal pallet is designed to produce the necessary reaction forces to fix a given set of prismatic parts with wide geometrical variability, doing so with minimal changeover time. These cradles are installed on top of a set of pallets in an integrated robotic assembly system with integrated reentrant material handling. A conveyor-belt loop is used to transfer pallets along a number of robotic assembly and disassembly stations, forcing the pallets to reenter the assembly line after each cycle. Designing a pallet for specific parts with little or no variability is relatively simple, but for parts with many design variants, a different pallet is usually used for each design, decreasing both economic and engineering effectiveness.

The proposed pallet-adapter design is the result of a developmental process involving the design, validation and production of a universal cradle to support product assembly of a wide variety of product models. The main goal of the project was to increase the sustainability of assembly systems by reducing the required number of pallets, and creating a cradle that secures products for assembly as well as disassembly. This will theoretically reduce equipment requirements and use of resources, as well as increasing recycling and material recovery.

3. Design Constraints

The proposed universal cradle was designed specifically to complement the research project of Dr. Tarek AlGeddawy in the field of sustainable manufacturing systems. The project involves building a changeable assembly/disassembly system with mobile stations and robots, in addition to an autonomous self-recognition system for detection of assembly configuration and layout. Other students will be working on the theory of operation and controls for the system as well as supporting work on the installation of a multiple-gripper scenario for the robotic assembly cells in the near future.

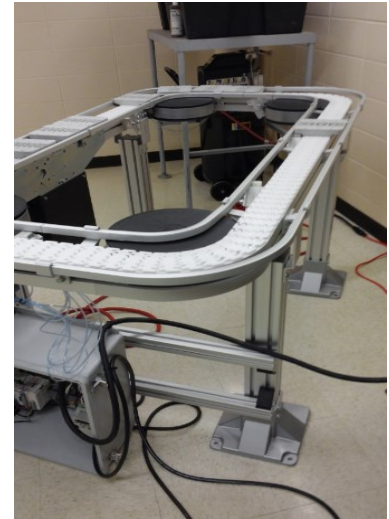


Figure 1: Conveyor loop preliminary set-up

This scenario provided some design constraints atypical for that of an average fixturing project. With the addition of robotic stations to the system, the pallet must be designed for easy access and manipulation for a set of geometrically-variable objects without sacrificing the ability to keep them firmly positioned on the pallet. Also, because the pallet is designed to physically move along a conveyor system, challenges are presented that make integration of mechanical or pneumatic fixture-vices difficult. This is also partially due to the conception of mobile stations and robots. Lastly, unit cost was an important design consideration due to the requirement for multiple pallets called for by Dr. AlGeddawy's project. Due to these particular constraints, a unique design was conceptualized with the help of a little research.

4. Literature Review

Scholarly articles were an invaluable resource for the success of this project. To begin, research was conducted on the general concept of fixturing objects for manufacturing processes. From there, focus was narrowed to works more closely related to the specific project. To begin, the article *Flexible Fixture Design and Automation: Review, Issues and Future Directions* by Bi and Zhang provided a broad overview of the subject matter. This work was useful because it provided a general description of how fixturing an object is done in an industrial setting, as well as providing some insight on the general processes and terminology used in the field (Bi & Zhang, 2001).

Another source of information came in the form of the paper *Locating and Clamping of Complex Geometry Workpieces with Skewed Holes in Multiple-Constraint Conditions*, written by Tadic et al. The discussion was focused on modular toolkits and the fixturing of a specific part using the 3-2-1 method of fixturing. This paper was useful because the 3-2-1 method of fixturing was adapted to form the basis of this project (Tadic, Bogdanovic, Jeremic, Todorovic, Luzanin, Budak & Vukelic, 2013).

As mentioned previously, a more focused approach was used to collect the other sources after the more generalized research was completed. In *Planning for Modular and Hybrid Fixtures* by Wallack and Canny, a process description for the creation of a heuristic algorithm used for designing fixtures for prismatic objects was found. This was extremely useful because their

fixture-vise system was very similar to the original conception of this project. Also, Wallack and Canny's algorithm inspired the creation of my own optimization algorithm described later in this document (Wallack & Canny, 1996).

Wallace and Canny expanded on their work in the paper *Modular Fixture Design for Generalized Polyhedra*. This work was useful because it provided greater detail on how the boundaries were calculated for use in the iterative solution of their algorithm. This work also expanded the study from just prismatic shapes to circular ones as well (Wallack & Canny, 1997).

In his final contribution *Generic Fixture Design Algorithms for Minimal Modular Fixture Toolkits*, Wallack describes some interesting fixture-vise configurations. Multiple pin fixture vices are combined to create a chuck for more variable pin placement (Wallack, 1996). These written works listed above formed the informational basis of this project.

5. Design Conception

Taking the problem statement and design constraints into account, an initial solution was conceptualized based on information collected from the review of the literature. A unique design was proposed to face the challenges brought by this particular project. Because the conveyor assembly from Dr. AlGeddawy's system came with its own set of pallets, an adapter design was proposed that would allow the pallets to fix a wide variety of part geometries. This would be accomplished by creating a hole-pattern on the adapter plate for jiggling-pins to be inserted. The clearance between the pins and the holes would be designed to create sliding contact, allowing for the quick insertion and re-arrangement of the pins, either manually or autonomously with robots. This solution brings universality because the pins can be easily re-arranged to allow for direct contact with a wide variety of part geometries. Under normal fixturing circumstances, a set of vices would be used in tandem with the jiggling pins. This particular project however, requires less complexity because no machining forces are produced in Dr. AlGeddawy's conveyor system.

A set of assumptions made by this design must be experimentally verified before any physical model could be created. The first assumption is that a total of four jiggling pins are required to create force closure on any given object. This was indicated by the research, but must be verified. The idea is that the vibration and centripetal forces created by moving the parts along the conveyor system will produce a resultant force vector. The theory is that for a given direction of this force, three reaction forces would be required from the pins to create force closure. If the direction of the resultant force was reversed, again three reactions would be required, but in different locations from the first set. The assumption made is that if four pins were used in total, they could together account for any direction of the resultant force created by the conveyor system. Three of the four pins would be 'active' for a given direction of the external force – they would create reaction forces to keep the object fixed in place. If the direction of this force was changed, then a different grouping of the three pins would become 'active.' The second assumption made was that each object has its own optimal pin arrangement determined by its geometry. The optimal pin arrangement would be where the set of reaction forces are minimized. It was hypothesized that this arrangement would be where the pins are located as far from the centroid of the part as possible, while still remaining in direct contact. Theory states that to create

force closure, the sum of the forces in x, in y, and the sum of the moments all have to equal zero. Since a moment is the result of a force crossed on a distance (often called the lever-arm), it makes sense that to minimize the reaction forces, the largest lever-arms possible would be required to create the optimal arrangement. These assumptions were experimentally verified, as discussed in the preceding sections.

6. Experimental Verification

6.1. Finite Element Analysis

To validate the initial assumptions made during the design's conception, a series of SolidWorks design studies were created. Using the program's simulation feature, static studies were generated for the purpose of analyzing a variety of shapes. Stress, strain and displacement plots were calculated for each shape after an external loading was applied. The results of these studies agreed with the initial assumptions made. By creating two separate studies for each shape, the assumption concerning the requirement of three point contacts for a given force direction was given credence. To show this, both studies were given the same parameters; their connections, fixtures, and meshes were the same. The one variable altered between the two studies was the external load. While this external force was placed at the same location in both studies, the direction of the applied load in study2 was equal and opposite that of the load in study1. When the stress and strain plots for both studies are examined, it can be seen in figure 2 that only three out of the four pins supply a reaction force for a given direction of the external load. It is also obvious from figure 2 that the specific three active pins vary between the two studies.

The second hypotheses regarding optimal pin location was also supported by the finite element analyses. A design study was created that used the stress results, calculated by the two static studies, as inputs for an optimization simulation minimizing the reaction forces of the pins. The variable x (or θ depending on the study) was used to describe the location of the pins relative to an initial location. By changing the value of this variable, the design study was able to isolate an optimal pin arrangement that minimizes the maximum stresses calculated from the analysis of

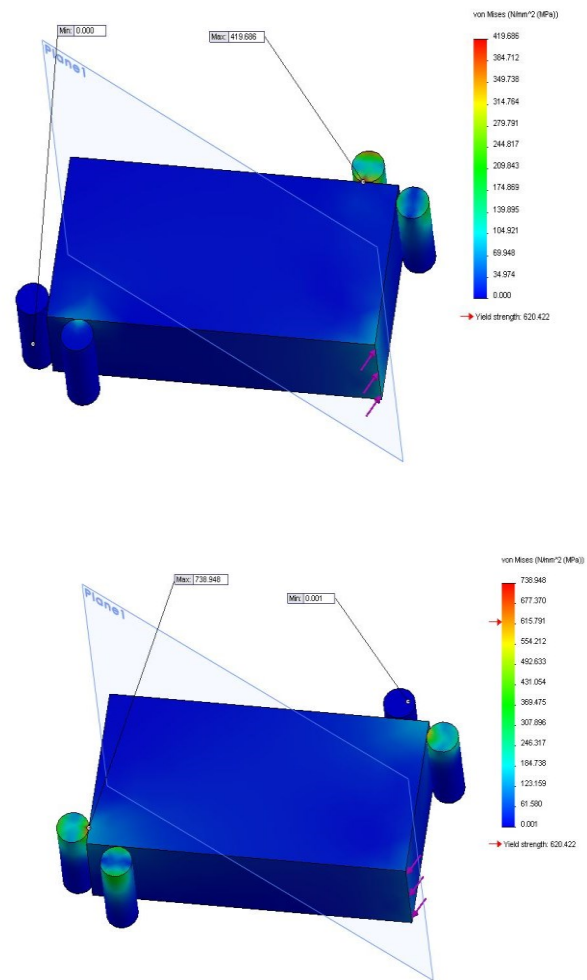


Figure 2: comparison of pin loading for both directions of the applied force on the optimal solution for a rectangular body

the static studies. As predicted, the optimal arrangement for the pins was where they were positioned at the maximum distance away from the centroid of the object (or near the edges of a prismatic, symmetrical body as shown in figure 2). This ‘edge’ arrangement produces the smallest reaction forces required to keep the given object in a static state because the lever-arms that produce the reaction moments of the pins are as great as possible in this orientation. These results concurred for three entirely different geometries and they, as well as stress, strain and displacement comparisons between two different pin arrangements for a rectangular body can be found in appendix A. In summary, the results of the finite element analyses, conducted using Solid Works simulation, proved to support the validity of the assumptions made during the conception of the proposed solution. These results also paved the way for the creation of an optimization algorithm, used as a secondary means to verify the hypotheses experimentally.

6.2. Pin Optimization Algorithm

As a secondary means of verification, a series of algorithms based fundamentally on static mechanics were written to mathematically identify the magnitudes of all reaction forces for a specific arrangement of pins on a given object. An iterative solution was required; therefore a computer was used to process the results. To allow this, the algorithm was written in Matlab using the program editor feature. A Matlab script was created for the analysis of a rectangular body, an L-shaped body, and for a triangular body. The geometries defining these objects were one of the known parameters used as an input for the program. This parameter-set takes the form of a matrix, “P” that holds 20 sample locations distributed uniformly around the perimeter of the given object. The program then calls a function ‘combinator’ to create another matrix, “Com” that holds every permutation without repetition for a set of 20 integers, choosing 4 at a time. This matrix is used to assign position variables to 4 pins simulated as point contacts along the perimeter of the object. For the other input, an external force is applied orthogonal to its lever-arm at a location along the object’s perimeter, and farthest away from its centroid. The magnitude of this simulated resultant force was arbitrarily chosen to be 10N. With these two pieces of information, the pin optimization algorithm can then find the rest of the variables required by the force-closure equations, which form the basis of this mathematical model (sum of the forces in x, sum of forces in y, and sum of the moments about the centroid). Figure 4 shows the code implementing these three equations while Figure 3 shows a sketch detailing the process for a rectangular body. The first three pins of the total four are chosen as the ‘active’ pins (as discussed previously) for the first portion of the program. Using the three statics equations mentioned above, the three unknown reaction forces are solved for. Next, the direction of the external force is reversed and the reactions are calculated again. This time however, pins 1, 2 and 4 are the ‘active pins’ used as variables in the formula. A set of 6 reaction forces are found for each iteration of the algorithm. If each of these

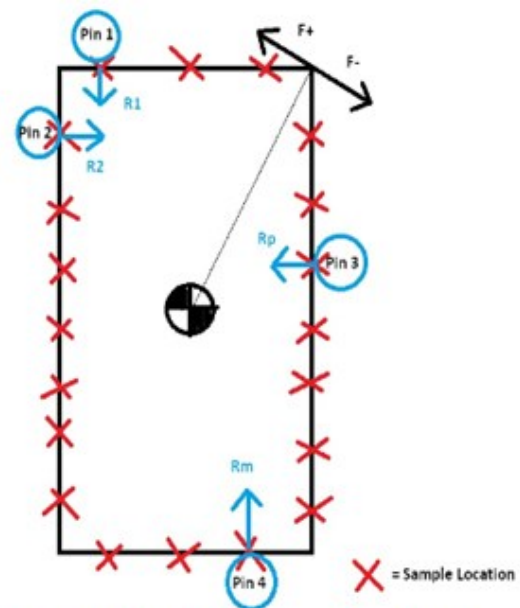


Figure 3: Sketch detailing algorithm set-up for a rectangular body

Figure 4 shows the code implementing these three equations while Figure 3 shows a sketch detailing the process for a rectangular body. The first three pins of the total four are chosen as the ‘active’ pins (as discussed previously) for the first portion of the program. Using the three statics equations mentioned above, the three unknown reaction forces are solved for. Next, the direction of the external force is reversed and the reactions are calculated again. This time however, pins 1, 2 and 4 are the ‘active pins’ used as variables in the formula. A set of 6 reaction forces are found for each iteration of the algorithm. If each of these


```

sum of Fx:
Fxp=R1p*C1+R2p*C2+R3*C3+F*CFp;
Fxm=R1m*C1+R2m*C2+R4*C4+F*CFm;

sum of Fy
Fyp=R1p*S1+R2p*S2+R3*S3+F*SFp;
Fym=R1m*S1+R2m*S2+R4*S4+F*SFm;

sum of M about G
Mp=R1p*C1*d1+s1+R1p*S1*d1*c1+R2p*C2*d2*s2+R2p*S2*d2*c2+R3*C3*d3
*s3+R3*S3*d3*c3+F*CFp*dF*sF+F*SFp*dF*cF;
Mm=R1m*C1*d1*s1+R1m*S1*d1*c1+R2m*C2*d2*s2+R2m*S2*d2*c2+R4*C4*d4
*s4+R4*S4*d4*c4+F*CFm*dF*sF+F*SFm*dF*cF;
    
```

Figure 4: The force closure equations of static mechanics; sum of the forces in x, in y, and sum of the moments about the centroid

reaction forces are found to be greater than zero, the given pin arrangement was successful in its ability to create force closure for the object in question. The reaction forces and the Cartesian coordinates of the pins are then outputted as an

additional row in the matrix, "Reaction." Making use of a 'for' loop, this process is then repeated for every pin arrangement possible given the number of sample points chosen (which turns out to be a total of 116,280 iterations for a sample size of 20 locations).

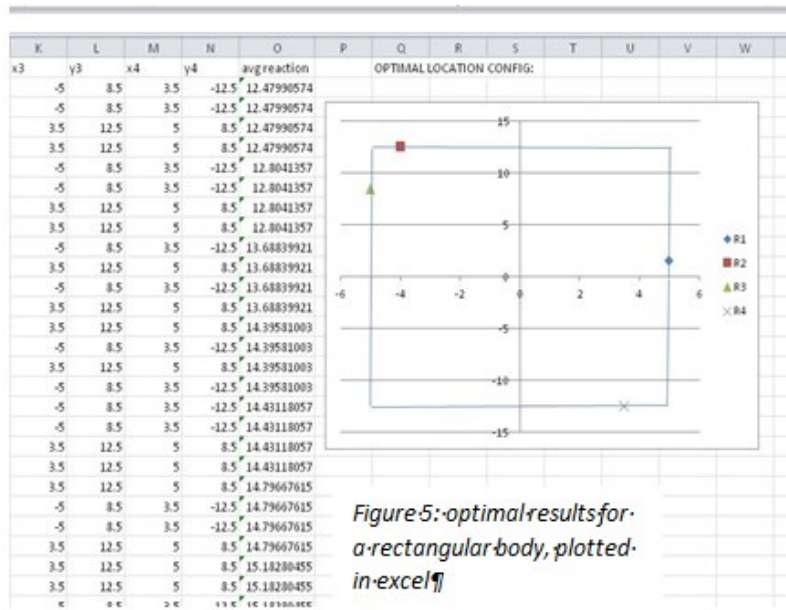


Figure 5: optimal results for a rectangular body, plotted in excel

The results from the pin optimization algorithm agree with the findings from the finite element analyses designed in SolidWorks. This was shown by transferring the solution set from the output matrix ("Reaction") in Matlab, to a Microsoft excel spreadsheet. The six reactions along with the (x, y) locations for each pin arrangement were sorted by the value of the average reaction force calculated per row from least to greatest. The set of reactions that generated the smallest average reaction force was chosen as the optimal

solution. The arrangement of this solution is shown in figure 5. By comparing this to the rectangular bodies shown in figure 2, it can be seen that the optimal arrangement from the finite element analysis closely resembles the result generated by the pin optimization algorithm for the given geometry. Not only was the algorithm useful for validating some of the initial assumptions made at the conception of this project, it also supported a means to project the possible jiggling zones for a given object. This can be accomplished by first isolating each pin-arrangement solution while proceeding to graph the resultant Cartesian coordinates for each iteration of the solution-set. This should result in a discrete range of feasible pin-placement zones distributed around the perimeter of each object. This information can then be used to determine possible pin locations required to create force closure on any given part. The Matlab scripts written for a rectangle, L-shape and triangle can be found in appendix B.

7. Physical Model

After the initial design was given experimental credibility, a physical prototype was built. A



Figure 6: 3-D printed test shapes

SolidWorks model for the adapter plate was generated along with three geometrically variable parts, created for use in testing the model. The SolidWorks part files were then converted into STL files, simply by saving them as a different file-type. The STL files for the

adapter plate and the three test-shapes were then processed by the software package: Catalyst. This program converted the STL files into XMB files specific to the University's in-house 3-D printer. The four parts in total were then 3-D printed, as shown in figures 5 and 6.

While the test-parts were satisfactory, it became readily apparent that the adapter prototype would need more work. The tolerances associated with the hole diameters were too dimensionally in-accurate from the 3-D print to create the required fit between the hole pattern and the jiggling pins. Also, the hole pattern itself was too simple and grid-like to be truly universal. Another challenge with the 3-D printed prototype was with the model material it was made from.

As shown in figure 7, the pegs designed to attach the adapter-plate to the pallets were too structurally weak; they just snapped right off. Also, the support material was rather difficult to remove given the large and complex geometry of the adapter plate. Due to these unsatisfactory qualities, the adapter was re-designed.



Figure 7: 3-D printed adapter-plate; shown here with a broken peg and clinging support material

The adapter plate was re-designed to be machined from an aluminum block cut to the correct dimensions using the school's water jet machine. The hole pattern was given more complexity

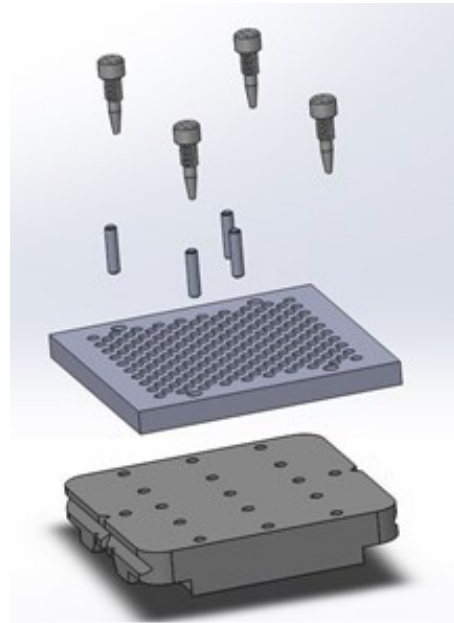


Figure 8: Exploded view of universal pallet assembly in SolidWorks

and the holes were toleranced to specifically create a sliding contact fit. After the part was designed in SolidWorks, Mastercam was used to generate the toolpaths for CNC machining. G code was then generated for the CNC machine to read the toolpath instructions. The pegs joining the adapter to the pallet were also re-worked. They were made into a separate piece by taking $\frac{3}{4}$ "-16 bolts and turning their ends down using a lathe. These bolts were then screwed into the adapter plate, creating the assembly shown in figure 8. Part drawings for the adapter plate as well as the peg screws can be found in appendix C. The final adapter plate was tested using the 3-D printed test parts created earlier. The newly designed pallet was able to successfully fix the test parts using $\frac{1}{4}$ " diameter jiggling pins. The final design of the adapter plate is shown in figure 9.

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Figure 9: Final universal pallet design

8. Results/Conclusion

In conclusion, a universal cradle was to be designed for integration with the Dr. AlGeddawy's sustainable manufacturing project. Design challenges were presented that made this research project unique. Research was conducted and a collection of informative sources were found. Experiments were conducted to test the validity of the initial design conceptions. These experiments involved using finite element analyses in SolidWorks and creating an optimization algorithm to find optimal pin arrangements for a variety of two-dimensional shapes. Finally, a physical prototype was built and tested using an iterative design process and a variety of manufacturing techniques.

I believe this project to be a great success. Not only did I personally learn a lot more about the art of fixturing and about engineering in general, a physical addition to Dr. AlGeddawy's work was successfully implemented. The prototype generated by this project was able to fulfill its goal in creating force closure for a variety of geometric objects. The design could be further improved by decreasing its weight and by tailoring the hole pattern on the plate to the specific part-set used on the future assembly system in Dr. AlGeddawy's on-going project. This hole pattern could be found experimentally by adapting the algorithm designed in Matlab for the specific shapes to be

fixed by the universal pallet. This concludes the summary of the Universal Pallet research project.

References

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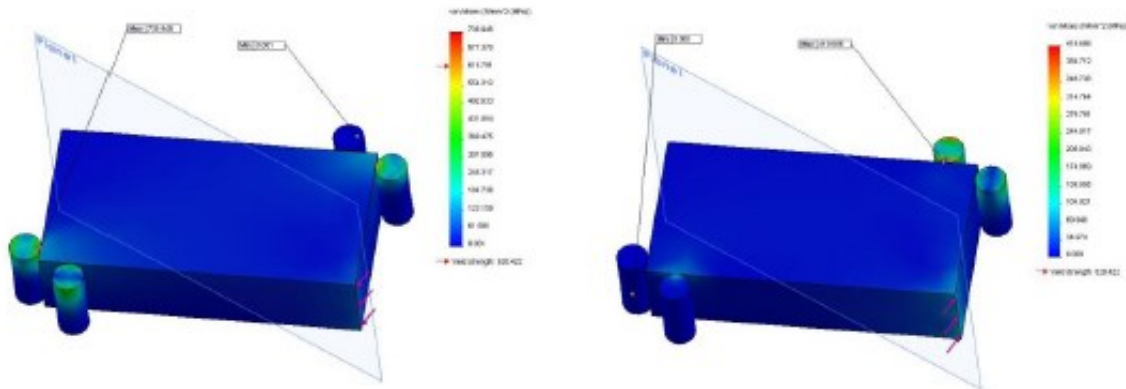
Wallack, Aaron S., & Canny, John F. (1996). Modular fixture design for generalized Polyhedra. *Proceedings of IEEE International Conference on Robotics and Automation (Minneapolis, April 22-28)*, 1, 830-837.

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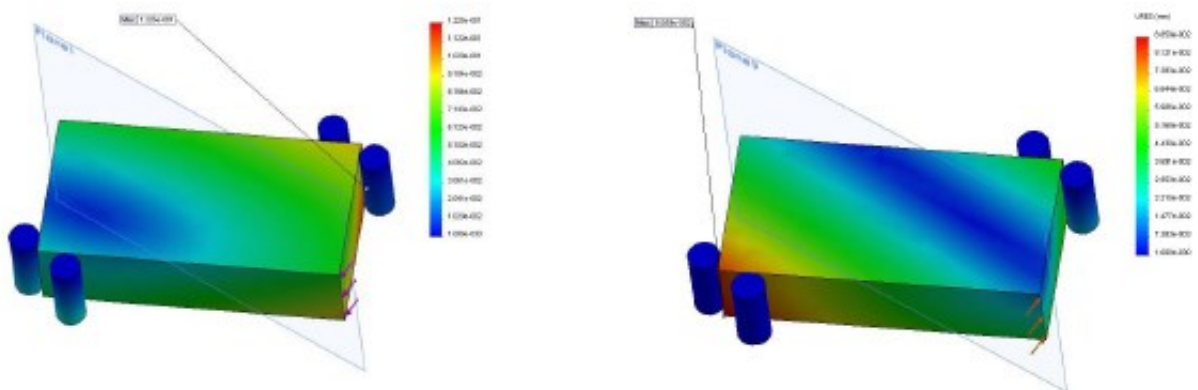
Wallack, Aaron S. (1996). Generic fixture design algorithms for minimal modular fixture toolkits. *Proceedings of IEEE International Conference on Robotics and Automation (Minneapolis, April 22-28)*, 1, 487-494.

Appendix A SolidWorks finite element analysis

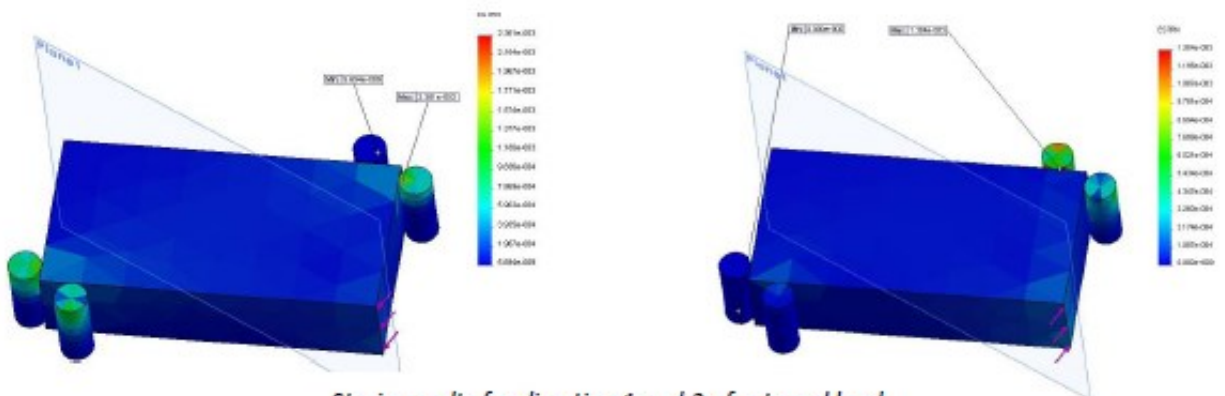
I. Stress, displacement, and strain plots for optimal pin configuration on a rectangular body



Stress results for direction 1 and 2 of external load

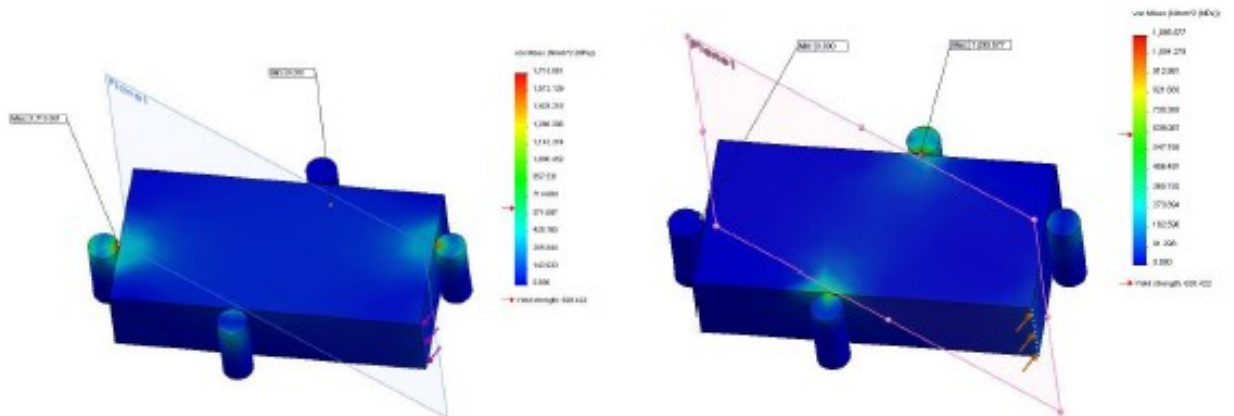


Displacement results for direction 1 and 2 of external load

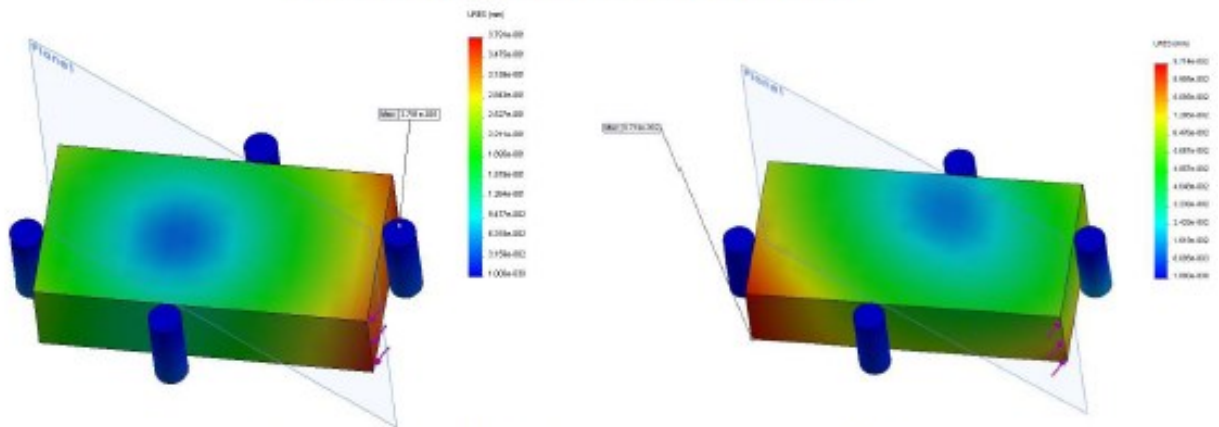


Strain results for direction 1 and 2 of external load

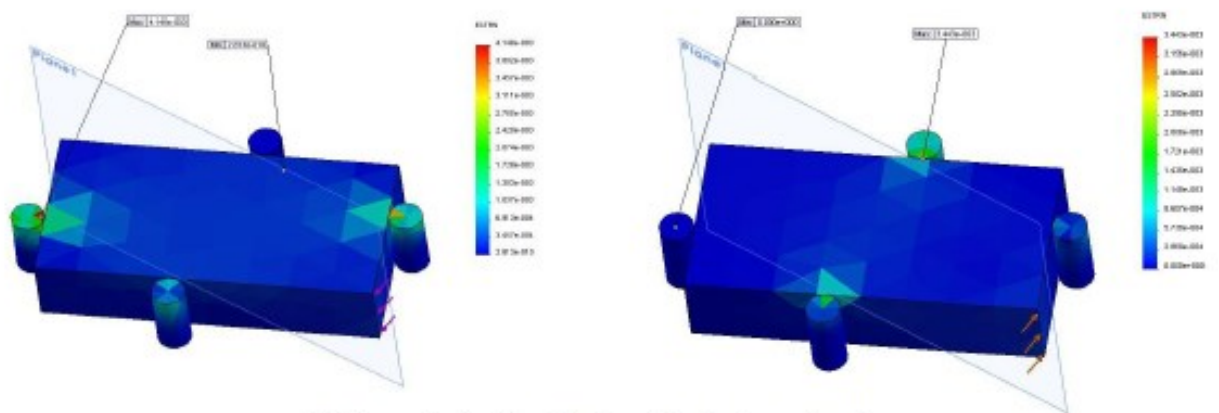
II. Stress, strain and displacement plots for less-optimal pin configuration on a rectangular body



Stress results for direction 1 and 2 of external load

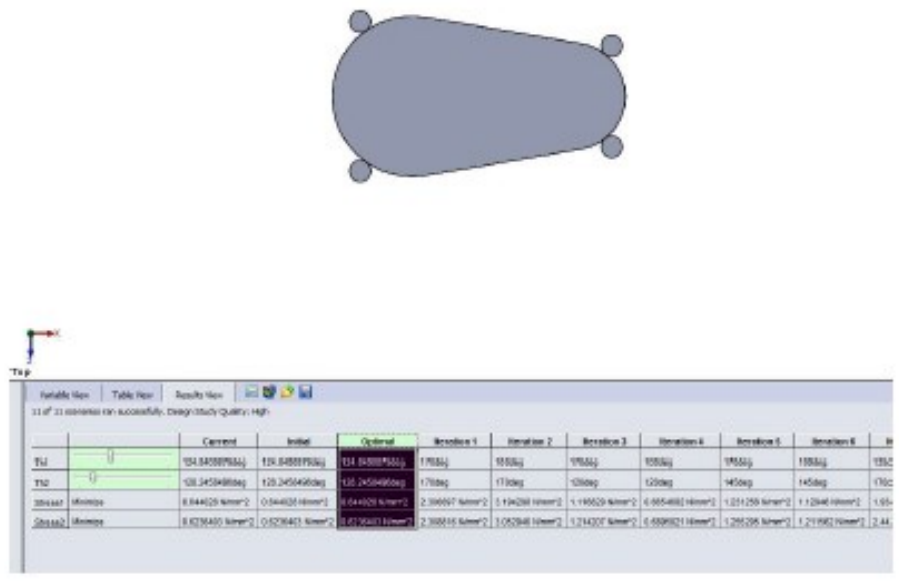
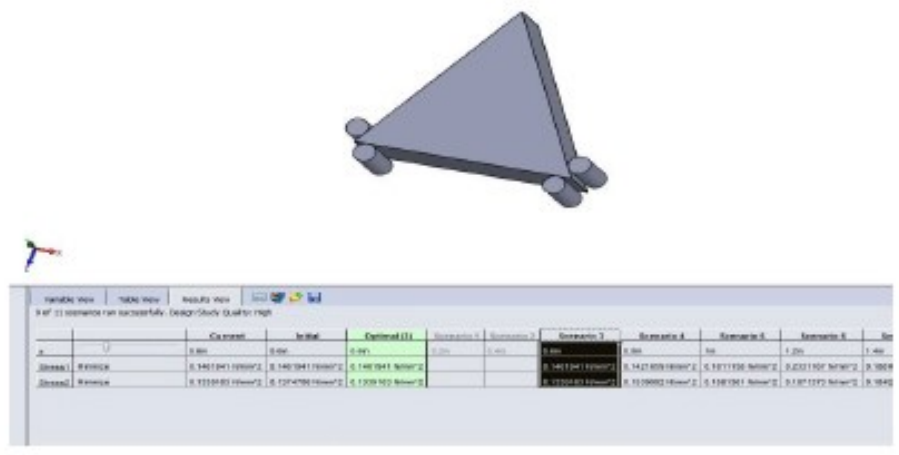
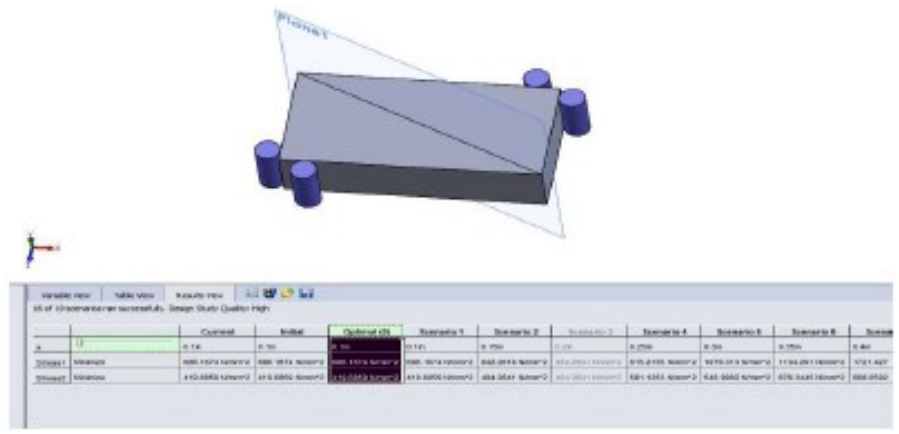


Displacement results for direction 1 and 2 of external load



Strain results for direction 1 and 2 of external load

III. Optimal solution results for a variety of shapes



Appendix B

Pin optimization algorithm

I. Matlab script for rectangular body

```
%pin placement optimization

%function definiton:
%matrix P is the Nx3 position matrix for N samplings of possible pin
locations around the given object defined by the geometry listed in
the f matrix.
%matrix Com holds every permutation w/o repetition of the N
configurations, choosing 4.
%After a given force input, this program is designed to calculate the
reaction forces of the pins using static mechanics.
%A comprehensive list of the reaction forces is the desired output.

%geometry of object
f1=1;
f2=2;
f3=3;
f4=4;
f=[f1;f2;f3;f4];
P=[-4,-12.5,1;-1.5,-12.5,1;1,-12.5,1;3.5,-12.5,1;-4,12.5,3;-
1.5,12.5,3;1,12.5,3;3.5,12.5,3;-5,-9,2;-5,-5.5,2;-5,-2,2;-5,1.5,2;-
5,5,2;-5,8.5,2;5,-9,4;5,-5.5,4;5,-2,4;5,1.5,4;5,5,4;5,8.5,4];
%permutation function
Com= combinator(20,4,'p');
%input force (N)
F=10;
%pick F to act where distance from center of gravity(G) is the
greatest
xF=5;
yF=12.5;
k=1;
j=1;
for counter = 1:116280
    %xi, yi, zi
    x1=P(Com(counter,1),1);
    x2=P(Com(counter,2),1);
    x3=P(Com(counter,3),1);
    x4=P(Com(counter,4),1);
    y1=P(Com(counter,1),2);
    y2=P(Com(counter,2),2);
    y3=P(Com(counter,3),2);
    y4=P(Com(counter,4),2);
    z1=P(Com(counter,1),3);
    z2=P(Com(counter,2),3);
    z3=P(Com(counter,3),3);
    z4=P(Com(counter,4),3);
    X=[x1;x2;x3;x4];
```

```
Y=[y1;y2;y3;y4];
Z=[z1;z2;z3;z4];
%initialization of theta 1-4
theta1=0;
theta2=0;
theta3=0;
theta4=0;
TH=[theta1;theta2;theta3;theta4];
i=1;
for i = 1:4
    if f(Z(i,1),1)==1
        TH(i,1)=pi/2;
        i=i+1;
    elseif f(Z(i,1),1)==2
        TH(i,1)=0;
        i=i+1;
    elseif f(Z(i,1),1)==3
        TH(i,1)=(3/2)*pi;
        i=i+1;
    elseif f(Z(i,1),1)==4
        TH(i,1)=pi;
        i=i+1;
    else
        disp('error')
    end
end

%reinizitalization of theta
theta1=TH(1,1);
theta2=TH(2,1);
theta3=TH(3,1);
theta4=TH(4,1);
%theta of F
thetaD=atan2(yF,xF);
thetaF=thetaD+pi/2;
%phi
phi1=atan2(-y1,-x1);
phi2=atan2(-y2,-x2);
phi3=atan2(-y3,-x3);
phi4=atan2(-y4,-x4);
phiF=atan2(-yF,-xF);
%radii
dF=(xF^2+yF^2)^(1/2);
d1=(x1^2+y1^2)^(1/2);
d2=(x2^2+y2^2)^(1/2);
d3=(x3^2+y3^2)^(1/2);
d4=(x4^2+y4^2)^(1/2);
%simplification of sines and cosines
C1=cos(theta1);
C2=cos(theta2);
C3=cos(theta3);
C4=cos(theta4);
```

```
CF=cos(thetaF);
S1=sin(theta1);
S2=sin(theta2);
S3=sin(theta3);
S4=sin(theta4);
SF=sin(thetaF);
c1=cos(phi1);
c2=cos(phi2);
c3=cos(phi3);
c4=cos(phi4);
cF=cos(phiF);
s1=sin(phi1);
s2=sin(phi2);
s3=sin(phi3);
s4=sin(phi4);
sF=sin(phiF);
% sum of Fx
% Fxp=R1p*C1+R2p*C2+R3*C3+F*CFp;
% Fxm=R1m*C1+R2m*C2+R4*C4+F*CFm;
% sum of Fy
% Fyp=R1p*S1+R2p*S2+R3*S3+F*SFp;
% Fym=R1m*S1+R2m*S2+R4*S4+F*SFm;
% sum of M
%
Mp=R1p*C1*d1*s1+R1p*S1*d1*c1+R2p*C2*d2*s2+R2p*S2*d2*c2+R3*C3*d3*s3+R3*
S3*d3*c3+F*CFp*dF*sF+F*SFp*dF*cF;
%
Mm=R1m*C1*d1*s1+R1m*S1*d1*c1+R2m*C2*d2*s2+R2m*S2*d2*c2+R4*C4*d4*s4+R4*
S4*d4*c4+F*CFm*dF*sF+F*SFm*dF*cF;

%solve
C=C3;
S=S3;
d=d3;
s=s3;
c=c3;
B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
r=inv(A)*B;
R1p=r(1,1);
R2p=r(2,1);
Rp=r(3,1);
if R1p>0 && R2p>0 && Rp>0
    thetaF=thetaF+pi;
    CF=cos(thetaF);
    SF=sin(thetaF);
    C=C4;
    S=S4;
    d=d4;
    s=s4;
    c=c4;
```

```

        B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
        A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
        r=inv(A)*B;
        R1m=r(1,1);
        R2m=r(2,1);
        Rm=r(3,1);
        if R1m>0 && R2m>0 && Rm>0

Reaction(j,:)= [R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
        j=j+1;
        if R1p+R1m+R2p+R2m+Rp+Rm<2000

Solution(k,:)= [R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
        k=k+1;
        else
        end

        else
        end

    else
    end
end

```

II. Matlab script for L shaped body

```

%pin placement optimization for L

%function definiton:
%matrix P is the Nx3 position matrix for N samplings of possible pin
locations around the given object defined by the geometry listed in
the f matrix.
%matrix Com holds every permutation w/o repetition of the N
configurations, choosing 4.
%After a given force input, this program is designed to calculate the
reaction forces of the pins using static mechanics.
%A comprehensive list of the reaction forces is the desired output.

%geometry of object
f1=1;
f2=2;
f3=3;
f4=4;
f5=5;
f6=6;
f=[f1;f2;f3;f4;f5;f6];
P=[0,4,1;0,8,1;0,12,1;0,16,1;0,20,1;0,24,1;2.25,25,2;4.5,25,2;2.25,0,6
;4.5,0,6;6.75,0,6;9.25,0,6;10,2.25,5;10,4.5,5;7.75,5,4;9.5,5,4;5,9.75,
3;5,14.5,3;5,19.25,3;5,24,3];
%permutation function
Com= combinator(20,4,'p');


```



```
F=10;
%pick F to act where distance from center of gravity(G) is the
greatest
xF=5;
yF=25;
xG=3.33;
yG=10.83;
k=1;
j=1;
for counter = 1:116280
    %xi, yi, zi
    x1=P(Com(counter,1),1);
    x2=P(Com(counter,2),1);
    x3=P(Com(counter,3),1);
    x4=P(Com(counter,4),1);
    y1=P(Com(counter,1),2);
    y2=P(Com(counter,2),2);
    y3=P(Com(counter,3),2);
    y4=P(Com(counter,4),2);
    z1=P(Com(counter,1),3);
    z2=P(Com(counter,2),3);
    z3=P(Com(counter,3),3);
    z4=P(Com(counter,4),3);
    X=[x1;x2;x3;x4];
    Y=[y1;y2;y3;y4];
    Z=[z1;z2;z3;z4];
    %initialization of theta 1-4
    theta1=0;
    theta2=0;
    theta3=0;
    theta4=0;
    TH=[theta1;theta2;theta3;theta4];
    i=1;
    for i = 1:4
        if f(Z(i,1),1)==1
            TH(i,1)=0;
            i=i+1;
        elseif f(Z(i,1),1)==2
            TH(i,1)=(3/2)*pi;
            i=i+1;
        elseif f(Z(i,1),1)==3
            TH(i,1)=pi;
            i=i+1;
        elseif f(Z(i,1),1)==4
            TH(i,1)=(3/2)*pi;
            i=i+1;
        elseif f(Z(i,1),1)==5
            TH(i,1)=pi;
            i=i+1;
        elseif f(Z(i,1),1)==6
            TH(i,1)=pi/2;
            i=i+1;
        end
    end
end
```

```
        else
            disp('error')
        end
    end
end

%reinitalization of theta
theta1=TH(1,1);
theta2=TH(2,1);
theta3=TH(3,1);
theta4=TH(4,1);
%theta of F
thetaD=atan2(yF-yG,xF-xG);
thetaF=thetaD+pi/2;
%phi
phi1=atan2(yG-y1,xG-x1);
phi2=atan2(yG-y2,xG-x2);
phi3=atan2(yG-y3,xG-x3);
phi4=atan2(yG-y4,xG-x4);
phiF=atan2(yG-yF,xG-xF);
%radii
dF=((xF-xG)^2+(yF-yG)^2)^(1/2);
d1=((x1-xG)^2+(y1-yG)^2)^(1/2);
d2=((x2-xG)^2+(y2-yG)^2)^(1/2);
d3=((x3-xG)^2+(y3-yG)^2)^(1/2);
d4=((x4-xG)^2+(y4-yG)^2)^(1/2);
%simplification of sines and cosines
C1=cos(theta1);
C2=cos(theta2);
C3=cos(theta3);
C4=cos(theta4);
CF=cos(thetaF);
S1=sin(theta1);
S2=sin(theta2);
S3=sin(theta3);
S4=sin(theta4);
SF=sin(thetaF);
c1=cos(phi1);
c2=cos(phi2);
c3=cos(phi3);
c4=cos(phi4);
cF=cos(phiF);
s1=sin(phi1);
s2=sin(phi2);
s3=sin(phi3);
s4=sin(phi4);
sF=sin(phiF);
% sum of Fx
% Fxp=R1p*C1+R2p*C2+R3*C3+F*CFp;
% Fxm=R1m*C1+R2m*C2+R4*C4+F*CFm;
% sum of Fy
% Fyp=R1p*S1+R2p*S2+R3*S3+F*SFp;
% Fym=R1m*S1+R2m*S2+R4*S4+F*SFm;
```

```

% %sum of M
%
Mp=R1p*C1*d1*s1+R1p*S1*d1*c1+R2p*C2*d2*s2+R2p*S2*d2*c2+R3*C3*d3*s3+R3*
S3*d3*c3+F*CFp*dF*sF+F*SFp*dF*cF;
%
Mm=R1m*C1*d1*s1+R1m*S1*d1*c1+R2m*C2*d2*s2+R2m*S2*d2*c2+R4*C4*d4*s4+R4*
S4*d4*c4+F*CFm*dF*sF+F*SFm*dF*cF;

%solve
C=C3;
S=S3;
d=d3;
s=s3;
c=c3;
B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
r=inv(A)*B;
R1p=r(1,1);
R2p=r(2,1);
Rp=r(3,1);
if R1p>0 && R2p>0 && Rp>0
    thetaF=thetaF+pi;
    CF=cos(thetaF);
    SF=sin(thetaF);
    C=C4;
    S=S4;
    d=d4;
    s=s4;
    c=c4;
    B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
    A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
    r=inv(A)*B;
    R1m=r(1,1);
    R2m=r(2,1);
    Rm=r(3,1);
    if R1m>0 && R2m>0 && Rm>0

Reaction(j,:)= [R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
    j=j+1;
    if R1p+R1m+R2p+R2m+Rp+Rm<2000

Solution(k,:)= [R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
    k=k+1;
    else
    end

    else
    end

else
end

```

end

III. Matlab script for triangular body

```
%pin placement optimization for triangle

%function definiton:
%matrix P is the Nx3 position matrix for N samplings of possible pin
locations around the given object defined by the geometry listed in
the f matrix.
%matrix Com holds every permutation w/o repetition of the N
configurations, choosing 4.
%After a given force input, this program is designed to calculate the
reaction forces of the pins using static mechanics.
%A comprehensive list of the reaction forces is the desired output.

%geometry of object
f1=1;
f2=2;
f3=3;
f=[f1;f2;f3];
P=[-.85526,2.6,1;-1.71053,5.2,1;-2.56579,7.8,1;-3.42105,10.4,1;-
4.27632,13,1;-5.13158,15.6,1;-
5.98684,18.2,1;8.076923,2.25,2;6.153846,4.5,2;4.230769,6.75,2;2.307692
,9,2;0.384615,11.25,2;-1.53846,13.5,2;-3.46154,15.75,2;-
5.38462,18,2;1.8,0,3;3.6,0,3;5.4,0,3;7.2,0,3;9,0,3];
%permutation function
Com=combinator(20,4,'p');
%input force (N)
F=10;
%pick F to act where distance from center of gravity(G) is the
greatest
xF=-6.25;
yF=19;
xG=1.25;
yG=6.33;
k=1;
j=1;
for counter = 1:116280
    %xi,yi,zi
    x1=P(Com(counter,1),1);
    x2=P(Com(counter,2),1);
    x3=P(Com(counter,3),1);
    x4=P(Com(counter,4),1);
    y1=P(Com(counter,1),2);
    y2=P(Com(counter,2),2);
    y3=P(Com(counter,3),2);
    y4=P(Com(counter,4),2);
    z1=P(Com(counter,1),3);
    z2=P(Com(counter,2),3);
    z3=P(Com(counter,3),3);
    z4=P(Com(counter,4),3);
```



```
X=[x1;x2;x3;x4];
Y=[y1;y2;y3;y4];
Z=[z1;z2;z3;z4];
%initialization of theta 1-4
theta1=0;
theta2=0;
theta3=0;
theta4=0;
TH=[theta1;theta2;theta3;theta4];
i=1;
syms X
for i = 1:4
    if f(Z(i,1),1)==1
        slope1=diff(-3.04*X);
        TH(i,1)=atan(slope1)+pi/2;
        i=i+1;
    elseif f(Z(i,1),1)==2
        slope2=diff(-1.17*X+11.7);
        TH(i,1)=atan(slope2)+3*pi/2;
        i=i+1;
    elseif f(Z(i,1),1)==3
        TH(i,1)=pi/2;
        i=i+1;
    else
        disp('error')
    end
end

%reinitalization of theta
theta1=TH(1,1);
theta2=TH(2,1);
theta3=TH(3,1);
theta4=TH(4,1);
%theta of F
thetaD=atan2(yF-yG,xF-xG);
thetaF=thetaD+pi/2;
%phi
phi1=atan2(yG-y1,xG-x1);
phi2=atan2(yG-y2,xG-x2);
phi3=atan2(yG-y3,xG-x3);
phi4=atan2(yG-y4,xG-x4);
phiF=atan2(yG-yF,xG-xF);
%radii
dF=((xF-xG)^2+(yF-yG)^2)^(1/2);
d1=((x1-xG)^2+(y1-yG)^2)^(1/2);
d2=((x2-xG)^2+(y2-yG)^2)^(1/2);
d3=((x3-xG)^2+(y3-yG)^2)^(1/2);
d4=((x4-xG)^2+(y4-yG)^2)^(1/2);
%simplification of sines and cosines
C1=cos(theta1);
C2=cos(theta2);
C3=cos(theta3);
```

```

C4=cos(theta4);
CF=cos(thetaF);
S1=sin(theta1);
S2=sin(theta2);
S3=sin(theta3);
S4=sin(theta4);
SF=sin(thetaF);
c1=cos(phi1);
c2=cos(phi2);
c3=cos(phi3);
c4=cos(phi4);
cF=cos(phiF);
s1=sin(phi1);
s2=sin(phi2);
s3=sin(phi3);
s4=sin(phi4);
sF=sin(phiF);
% %sum of Fx
% Fxp=R1p*C1+R2p*C2+R3*C3+F*CFp;
% Fxm=R1m*C1+R2m*C2+R4*C4+F*CFm;
% %sum of Fy
% Fyp=R1p*S1+R2p*S2+R3*S3+F*SFp;
% Fym=R1m*S1+R2m*S2+R4*S4+F*SFm;
% %sum of M
%
Mp=R1p*C1*d1*s1+R1p*S1*d1*c1+R2p*C2*d2*s2+R2p*S2*d2*c2+R3*C3*d3*s3+R3*
S3*d3*c3+F*CFp*dF*sF+F*SFp*dF*cF;
%
Mm=R1m*C1*d1*s1+R1m*S1*d1*c1+R2m*C2*d2*s2+R2m*S2*d2*c2+R4*C4*d4*s4+R4*
S4*d4*c4+F*CFm*dF*sF+F*SFm*dF*cF;

%solve
C=C3;
S=S3;
d=d3;
s=s3;
c=c3;
B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
r=inv(A)*B;
R1p=r(1,1);
R2p=r(2,1);
Rp=r(3,1);
if R1p>0 && R2p>0 && Rp>0
    thetaF=thetaF+pi;
    CF=cos(thetaF);
    SF=sin(thetaF);
    C=C4;
    S=S4;
    d=d4;
    s=s4;

```

```
    c=c4;
    B=[-F*CF;-F*SF;-(-F*CF*dF*sF+F*SF*dF*cF)];
    A=[C1,C2,C;S1,S2,S;(-C1*d1*s1+S1*d1*c1),(-C2*d2*s2+S2*d2*c2),(-
C*d*s+S*d*c)];
    r=inv(A)*B;
    R1m=r(1,1);
    R2m=r(2,1);
    Rm=r(3,1);
    if R1m>0 && R2m>0 && Rm>0

Reaction(j,:)=[R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
    j=j+1;
    if R1p+R1m+R2p+R2m+Rp+Rm<2000

Solution(k,:)=[R1p,R1m,R2p,R2m,Rp,Rm,x1,y1,x2,y2,x3,y3,x4,y4];
    k=k+1;
    else
    end

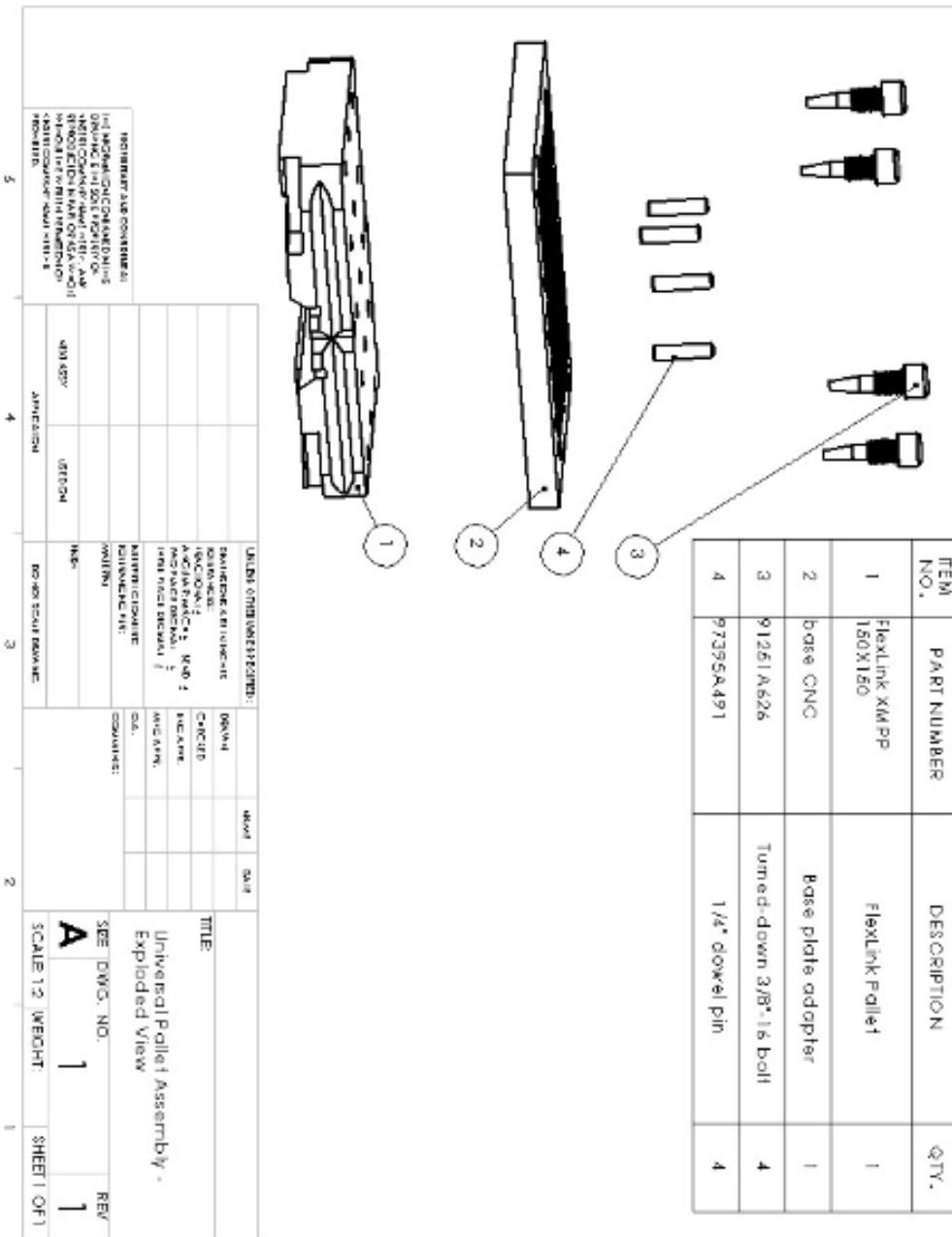
    else
    end

    else
    end
end
```

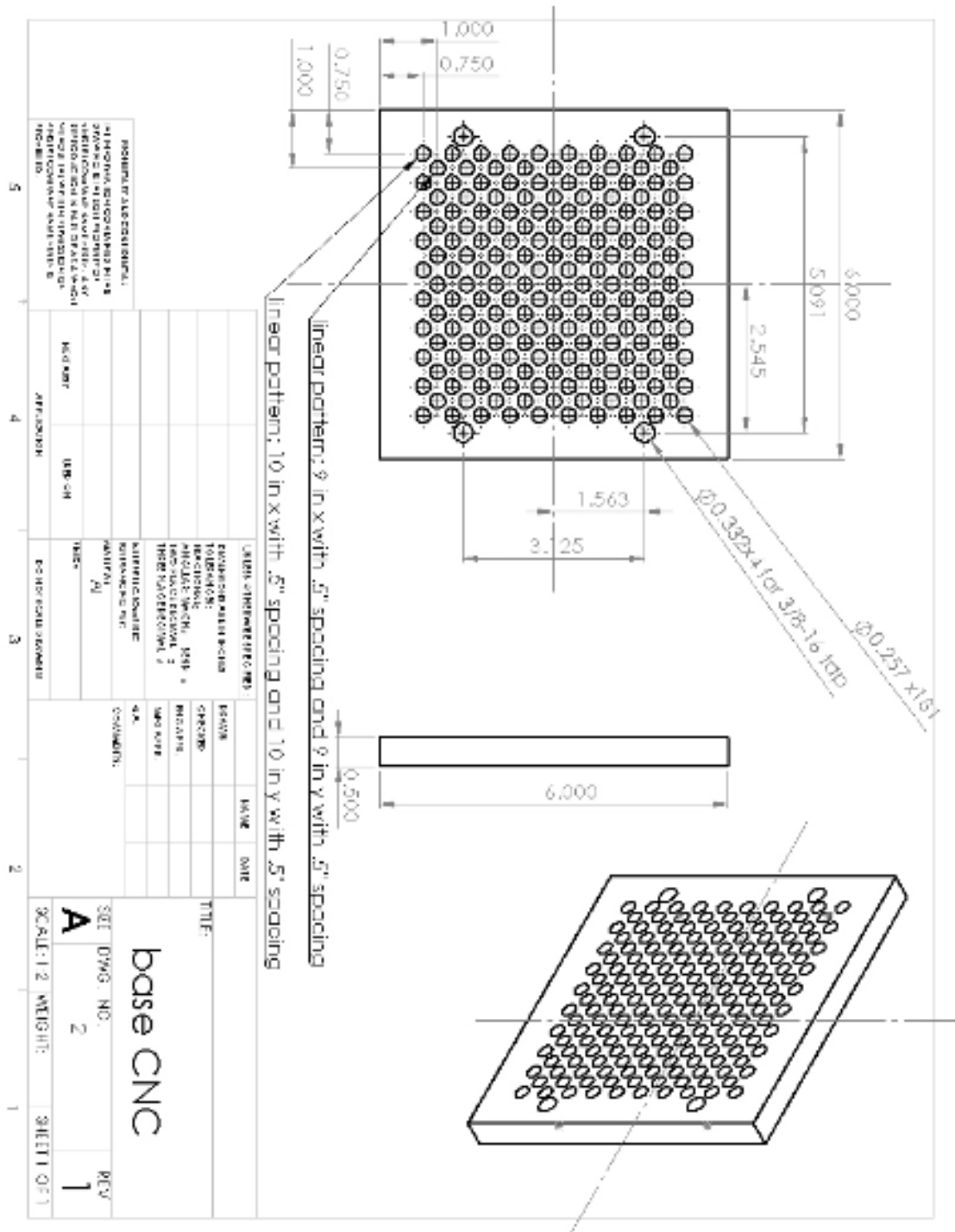
Appendix C

SolidWorks drawings for physical parts

I. Exploded view of assembly and BOM



III. Adapter plate



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