C. Anthony Anderson

Russell on Order in Time

After a brief flirtation with instants of time as primitive entities, Russell proceeded to construct instants out of classes of events. His most vigorous and rigorous analysis of the construction appears in a little-discussed paper "On Order in Time" (*Proceedings of the Cambridge Philosophical Society*, vol. 32 [1936], pp. 216–28; hereafter: OT). Small wonder that the paper is little discussed. It utilizes the intricate logical notation of *Principia Mathematica* throughout, and if you attempt to work through the details, you will discover that Russell's powers of deduction had not waned. (He was sixty-three.) That such a mind thought the matter worthy of such concentration is a sufficient justification for reopening the case. In the present essay I explain what Russell accomplished in "On Order in Time," with some consideration of similar ideas in the earlier works *Our Knowledge of the External World* and *The Analysis of Matter*. I find some flaws and suggest repairs and improvements. Proceeding from the precise and uncontroversial to the vague and perennially debatable, I consider philosophical objections, some due to Russell's earlier self, to constructing instants out of events. Finally, I take on the most general question in the immediate neighborhood: "Why construct anything out of anything?" Russell's enthusiasm for the epistemic power of logical constructions diminished over the years. But I think his own appraisal of the upshot of his paper underestimates what can be established and overestimates what has been established:

> It is shown that the existence of instants requires hypotheses which there is no reason to suppose true. (OT, p. 216)

The Postulates

In *Our Knowledge of the External World* (KEW) Russell had constructed the time series of instants on the basis of the following primitive concepts, definitions, and postulates. (I have formalized these for comparison with OT.)

Primitives: $S(x,y)$ – "x and y are events and the times at which x exists coincide (in part or whole) with the times at which y exists." $P(x,y)$ – "x and y are events and x temporally wholly precedes y, i.e., every time at which x exists is temporally precedent to any time at which y exists." I have added the English paraphrases as an aid to intuition. Russell just says of the first relation that it holds between two events if they overlap or are contemporaries or are (at least partially) simultaneous. And $P(x,y)$ if x and y are events and x wholly precedes y. The notions of an event and of a time are not really permissible in the explanation of these relations—both are defined (in KEW). The relations are known by acquaintance in experience.

DEFINITION: $Evt(x) \rightarrow (\exists y)S(x,y)$.

"x is an event" means "x overlaps something or other."

POSTULATES:

I. (a) $Evt(x) \supset \sim P(x,x)$.

"If x is an event, then x does not wholly precede x."

(b) $Evt(x) \supset .Evt(y) \supset .Evt(z) \supset .P(x,y) \supset .P(y,z) \supset .P(x,z)$.[1]

"If x, y, and z are events and x wholly precedes y and y wholly precedes z, then x wholly precedes z."

(c) $Evt(x) \supset .Evt(y) \supset .P(x,y) \supset \sim S(x,y)$.

"If x and y are events and x wholly precedes y, then x does not overlap y."

(d) $Evt(x) \supset .Evt(y) \supset .\sim S(x,y) \supset .P(x,y) \vee P(y,x)$.

"If x and y are events and x does not overlap y, then either x wholly precedes y or y wholly precedes x."

To these four, which give the fundamental properties of overlapping and precedence (with an exception to be noted later), Russell adds the definition:

DEFINITION: $Init\,(x,y) \rightarrow S(x,y).\sim(\exists z)[P(z,x).S(z,y)]$.

"x is an initial contemporary of y" means "x overlaps y but there is no z that wholly precedes x and that overlaps y." In terms of this, he adopts the postulate:

II. $Evt(x) \supset Evt(y) \supset .S(z,x) \supset .P(z,y) \supset (\exists w)[Init(w,x).P(w,y)]$.

"If x and y are events and a contemporary of x, say z, wholly precedes y, then an *initial* contemporary of x, w, wholly precedes y." This postulate guarantees, given the others, that events shall have first instants. I'm agin it. The Big Bang, I am told, was an event that likely did not have a first instant—the initial singularity is outside of the space-time manifold.

Finally, to ensure that the instants of the construction are compact (there is another between any two), Russell adopts:

$$\text{III. } Evt(x) \supset . \ Evt(y) \supset .P(x,y)$$
$$\supset (\exists z)(\exists w)[Evt(z).P(x,z).S(z,w).P(w,y)].$$

"If x and y are events and x wholly precedes y, then there is an event z wholly preceded by x and overlapping a w wholly preceding y."

The proof of compactness using postulates II and III is neat, but we shouldn't get carried away. Russell didn't. He listed II and III as *assumptions* in KEW, but his statements concerning their truth is cautious. *Provided* II is true, he says, events will have first instants. *If* III, then the series of instants is compact. But if our project is epistemological, and it is, these last postulates cannot be taken as data. Things brighten in OT, where we find:

DEFINITION: $S(x,y) \rightarrow \ \sim P(x,y). \sim P(y,x)$.

"x overlaps y" means "x does not wholly precede y and y does not wholly precede x."

I(c) and I(d) become logical truths. II and III are dropped *as postulates* and Russell adopts:

A. $\sim P(x,x)$.
B. $P(x,y) \supset .P(y,z) \supset P(x,z)$.

from which I(a) and I(b) follow. The construction also requires:

C. $R(x,y) \supset .R(y,z) \supset .R(x,z)$

where:

DEFINITION: $R(x,y) \rightarrow (\exists z)[S(x,z).P(z,y)]$.

("$R(x,y)$" may be read "x begins to exist before y begins to exist.")

Quine has complained, in his review of OT, that Russell's definition of overlapping ($S(x,y)$) misses the mark if the type of individuals includes nonevents.[2] For in that case two individuals, if nonevents, will overlap if neither temporally precedes the other (which they won't). Quine suggests defining events as entities in the field of P and confining S to events. Russell would respond, I think, that all his individuals are events. Everything else is constructed. Quine's suggestion won't do anyway since it rules out a priori, what should be left open, namely, that there should be one great event that persists from the beginning of time to the end.

Well, things are getting nice and tidy. We are left with only A, B, and C and a definition. Postulate C, if you don't cheat and use the informal reading, is not self-evident. It really reads:

$$(\exists u)[S(x,u).P(u,y)] \supset .(\exists v)[S(y,v).P(v,z)]$$
$$\supset (\exists w)[S(x,w).P(w,z)].$$

Now write out S in terms of its definition and contemplate the result. The thing is not self-evident. (It is evident if you think about it.) Imagine, then, my delight at finding that *both* B and C follow from:

D. $Q(x,y) \supset .P(y,z) \supset P(x,z)$,

where

DEFINITION: $Q(x,y) \rightarrow (\exists z)[P(x,z).S(z,y)]$.

(Read "$Q(x,y)$" as "x ends before y ends.") This reads, with no cheating, "If x wholly precedes something which overlaps y, and y wholly precedes z, then x wholly precedes z." Now *that* really *is* self-evident. And imagine my disappointment at finding out that Norbert Wiener has already discovered this in 1914 (as Russell at one time knew—Wiener is cited in KEW, 2nd ed., p. 97, note).[3]

Enough tinkering with the postulates. We should adopt A and D; the rest that is required for the construction follows.

Instants as Classes of Events

According to Russell, an instant is a class of events such that (1) everything in the class overlaps everything else in the class, and (2) nothing outside the class overlaps everything in the class. Using *Principia* notation, the simplest expression of this is that a class of events α is an instant if and only if:

$$\alpha = \hat{x}(y)[y \in \alpha \supset S(x,y)].$$

"The class α consists of exactly those events x such that if y is anything in α, then x overlaps y."

Russell credits the idea of this *sort* of construction to Whitehead.[4] But Whitehead had constructed instants as "enclosure series,"[5] roughly as infinite sequences of events, each containing another that converges to . . . well, they may not converge *to* anything. Whitehead just took the infinite series of Chinese-box-like events to *be* the instants. To this Russell objects in *The Analysis of Matter* (AMa):

> Let us begin with the absence of a lower limit or minimum. Here we are confronted with a question of fact, which might conceivably be decided against Dr. Whitehead, but could not conceivably be decided in his favour. The events which we can perceive all have a certain duration, i.e. they are simultaneous with events which are not simultaneous with each other. Not only are they all, in this sense, finite, but they are all above an assignable limit. I do not know what is the shortest perceptible event, but this is the sort of question which a psychological laboratory could answer. We have not, therefore, direct empirical evidence that there is no minimum to events. Nor can we have indirect empirical evidence, since a process which proceeds by very small finite differences is sensibly indistinguishable from a continuous process, as the cinema shows. *Per contra*, there might be empirical evidence, as in the quantum the-

ory, that events could not have less than a certain minimum spatio-temporal extent. . . .

I conclude that there is at present no means of knowing whether events have a minimum or not; that there never can be conclusive evidence against their having a minimum; but that conceivably evidence may hereafter be found in favour of a minimum. It remains to consider the question of a maximum. (pp. 292, 293).

Russell is a bit harsh here. It might turn out that our best theory of the external world postulates the infinite divisibility of events. In such a case, probability anyway would be with Whitehead. Still, if we are building from the bottom, it is better not to have such assumptions at the beginning (I think we shall need the assumption that there are infinitely many events *later*).

In Russell's construction there is no need to assume that events are nested one within the other without end. They need only overlap—we collect them together into "maximal" overlapping classes and call the results "instants." The relation of *temporal precedence* for instants thus constructed is defined:

DEFINITION: $T(\alpha,\beta) \rightarrow (\exists x)(\exists y)[x\epsilon\alpha . y\epsilon\beta . P(x,y)] . In(\alpha) . In(\beta)$.

One instant α temporally precedes another β if and only if some $x\epsilon\alpha$ and some $y\epsilon\beta$ are such that $P(x,y)$, i.e., x wholly precedes y.[6] But Russell was worried that still there might not *be* any such maximal classes of events. So there might not be any instants.

Russell also defines, which may be independently interesting, the *duration* of an event as the class of all the events that overlap the event. And he proves that durations can be arranged in a series according to the following plan: if two durations do not begin together, put first the one with the earlier beginning; if they begin together, put first the one with the earlier end. I have not been able to figure out why anyone would want to prove such a thing.

The Existence of Instants

Russell begins and ends OT with discouraging comments about the existence of instants. On the first page, he says that their existence "requires hypotheses which there is no reason to suppose true." And on the last page he says, "But in the absence of such possibilities[7] [to be discussed later in this section] I do not know of any way of proving the existence of instants anywhere if it is possible that all events existing at the beginning of some event (or at the end) continue during a period when others begin and cease (or have previously existed during such a period)" (p. 228). And about the conditions that he proves sufficient for the existence of instants, Russell says in the middle of the essay, "There is, however, no reason, either logical or empirical, for supposing these assumptions to be true" (p. 219).

But there is a clear, distinct, and sufficient reason for the existence of instants. Russell himself noted it and proved informally in *The Analysis of Matter* (chapter 27) that there are instants. But first, the condition he develops in OT:

$$\dot{\exists}\,!S \mathbin{\dot{-}} S|P|S$$

is a sufficient condition for the existence of at least one instant. This guarantees that there are two events a and b, such that the last instant of a is the first instant of b. Yes, but what does the condition mean? $S|P|S$ is the *relative product* of S, P, and S, in that order, so that $x(S|P|S)y$ if and only if for some u, v, x overlaps u and u precedes v and v overlaps y. The condition states that for some x and y, x overlaps y but not-$x(S|P|S)y$.

And Russell develops a condition sufficient for *every* event having a *first* instant. Adopt this definition:

DEFINITION: $U(x,y) \rightarrow (\exists z)[Init(z,x).P(z,y)]$.

Then the sufficient condition is:

$$R(x,y) \supset U(x,y),$$

where R is defined earlier. This is the condition that is taken in KEW as postulate II.

A condition sufficient for every event having a last instant is given thus:

DEFINITION: $Fin(x,y) \rightarrow S(x,y). \sim(\exists z)[S(z,y).P(x,z)]$.

"x is a final contemporary of y" means "x overlaps y but nothing overlapping y is wholly preceded by x."

Then the sufficient condition for events having last instants is as follows: anything wholly preceding something overlapping the event, wholly precedes a final contemporary of the event. Given:

DEFINITION: $V(x,y) \rightarrow (\exists z)[Fin(z,x).P(y,z)]$,

the desired condition is

$$Q(y,x) \supset V(x,y),$$

where Q is defined earlier. This one is, I suppose, as reasonable as the other.

So there you have it and I must agree that these conditions are neither evident to the senses, self-evident, nor likely to be deducible from such by the rules of reason.

But *the* condition only mentioned by Russell in OT and actually used by him in AMa is that the set of events overlapping a given event can be well ordered. The principle that any set can be well ordered is better known nowadays as an equivalent of the axiom of choice. My favorite version is this:

(AC_1) For any relation there is a function that is a subset of it and that has the same domain.

Indeed, it suffices to confine our attention to binary relations. The relation, R, say, is any definite set of ordered pairs $<x,y>$. The function, F, let's say, is to be a subset of R with just one such pair $<x,y_1>$ for each x such that a pair $<x,y>$ belongs to R. If R is a definite such multitude, then *surely* there *is* such a subcollection as F! This is, as Gödel remarked,[8] almost as evident as any of the other principles of set theory. And Russell says in AMa that he had been persuaded of its truth (in another formulation) by Frank P. Ramsey and Henry M. Sheffer. He then proceeds to give (in AMa) a perfectly cogent, if slightly informal, proof that there are instants (pp. 299–302). Sometime between 1926 and 1936 Russell changed his mind about the axiom of choice. In his reviews of Ramsey's essays[9] Russell seems optimistic about the extensionalized and simplified theory of types advocated by Ramsey. But by the time of *My Philosophical Development* he advocates intensionalism again. The R and F in AC_1 are *not*, for Russell, sets of ordered pairs; they are *attributes*—relations-in-intension. (Strictly, talk of functions, for Russell, reduces to talk of propositional functions.) Or, worse, perhaps they are expressions. But in any case, the thing loses its appearance of truth in either its intensionalistic or nominalistic versions. (Not that it looks false on the attribute reading. One just can't tell.) This is, I suppose, why Russell rejected the axiom of choice (or its equivalent) and the proof of the existence of instants based on it.

But we need not follow him in this. If we don't, and I won't, instants come easily. Take one of the least evident equivalents of (AC_1), *Zorn's lemma*:

(Z) If A is a nonempty set of sets and if for any chain B that is a subset of A, UB, the union of B, is an element of A, then A contains a maximal element.

A *chain* of sets is a collection B such that for any sets C and D belonging to B, either C is a subset of D or D is a subset of C. A *maximal* element of a set of sets A is an element of A that is not a subset of any other element of A.

THEOREM. If z is any event, then there exists an instant containing z, i.e., a set $\alpha = \hat{x}(y)[y \in \alpha \supset S(x,y)]$, such that $z \in \alpha$.

Proof: Let α be the set of all sets A containing z and such that every element of A overlaps every other element of A. This is a subset of the power set of $\mathscr{E}$, the set of events, and hence exists. α is nonempty since $\{z\} \in \alpha$. Let B be a subchain of α. Consider UB. Since z belongs to every element of B, z belongs to UB. Let x,y be two elements of UB. Then at worst for some sets C and D, say, $x \in C$ and $y \in D$, where C and D are elements of the chain B. Then $C \subseteq D$ or $D \subseteq C$, and x and y belong to some element of α and hence overlap. Thus UB belongs to α and, by Zorn's lemma, α has a maximal element M. Now M contains z and is an instant—for anything in it overlaps everything else in it. And if some event x overlapped everything in M and was not in M, then $M \cup \{x\}$ would be an element of α. But then M would not be maximal.

Q.E.D.

Of course, this proof is in Zermelo-Fraenkel set theory, something that Russell did not accept. But we might. So the axiom of choice, which implies Zorn's lemma (that would be the epistemic order), implies the existence of an instant containing any given event. Since events exist, instants exist. That is about as close to a proof as anything ever gets in philosophy.

Let me sum up. Russell's construction can be based on some pretty evident axioms about S and P, and the existence of instants can then be proved using the axiom of choice. From an extensional point of view the latter looks quite evident, though Russell himself rejects it. Further, Russell has given some interesting and correct sufficient conditions for events having first and last instants. These might be independently important in the detailed development of the theory. So far, pretty good.

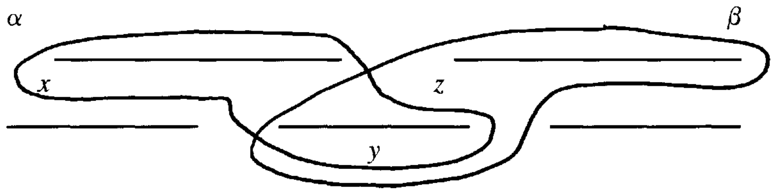
Compactness

When you think about it, instants *are* quite strange. They do not appear often in everyday talk at all. They are of use to the scientist who needs, apparently, instantaneous slices of reality matched with real numbers in order to formulate mathematical laws of nature. Given this reflection, the Russellian conquests so far begin to pale a bit. If instants of time are to be of much, perhaps any, use, they must form a *compact* series; there must be an instant between any two instants. In KEW Russell adopts postulates II and III, which entail the compactness of the series of instants (see Wiener, notes this chapter, for details). But these postulates are seriously a posteriori. I agree with the Russell of OT that there are no obvious reasons for accepting either as epistemically basic. So we should seek more evident sufficient conditions for compactness. True to form, Russell does exactly that. In OT he claims to prove that the two conditions:

(a) No event lasts only for an instant,
(b) Any two overlapping events have at least one instant in common,

guarantee compactness. The second of these follows from Zorn's lemma. Just go through the construction of the theorem proved earlier but begin with *two* overlapping events x and y instead of one event z. This would leave us with only (a), which, though debatable, is certainly plausible.

Unfortunately, there's an error in Russell's proof.[10] The two conditions are not sufficient for compactness. In the accompanying diagram, let *being to the left of* represent temporal precedence. And let temporal overlapping be represented by a line segment being over or under (or identical with) another.



Then the event y lasts only for two instants α (the set consisting of x and y) and β (consisting of y and z). And if the diagram is imagined to continue in the same way, all events last for two instants and any two overlapping events have an instant in common. But the series of instants is not compact (please verify that the postulates A and D obtain). Alas, the best condition I can think of that entails compactness is just:

E. $P(x,y) \supset (\exists z)[P(x,z).P(z,y)]$,

i.e., there is an event between x and y if x wholly precedes y. As far as I can see, Russell's two postulates from KEW (which Wiener, by the way, also uses) are no better epistemically than this. Worse, I think.

Well, then, what about *continuity?* If we get this far, I see no obstacle to using the ideas of Russell's construction of real numbers to construct a continuum of hyperinstants—segments of instants so far constructed. Then we will have what we need. Or almost.

Should Instants be Constructed out of Events?

I want now to consider some detailed objections to the project. I reserve general objections to the very idea of construction until the next section.

Recurrence

In another little-noticed paper "Is Position in Time and Space Absolute or Relative?" (*Mind*, n.s., no. 39 [1901], pp. 293–317), Russell attempts to refute his future theory. He claims that it is "difficult, if not impossible, to free the relational theory from contradiction" (p. 293).

In this paper he takes an event to be just a compound consisting of a quality together with a time. Events are incapable of recurring. Qualities are inherently capable of recurring. Therein lies the problem for the relational theory. If you begin with qualities, which can recur, how can you obtain events, which cannot recur, with using times? I quote:

> Whatever can, in ordinary language, recur or persist, is not an event; but it is difficult to find anything logically incapable of recurrence or persistence, except by including temporal position in the definition. When we think of the

> things that occur in time – pleasure, toothache, sunshine, etc. – we find that all of them persist and recur. In order to find something which does not do so, we shall be forced to render our events more complex. The death of Caesar or the birth of Christ, it may be said, were unique: they happened once, but will never happen again. Now it is no doubt probable that nothing exactly similar to these events will recur; but, unless the date is included in the event, it is impossible to maintain that there would a logical contradiction in the occurrence, of a precisely similar event. (p. 295)

A possible answer occurs to Russell. Why not take time to be the entire state of the universe? Or, as we might put it, take instants to be maximal sets of contemporaneous events. Sound familiar? But early Russell has objections to later Russell:

> Perhaps it may be said that the whole state of the universe has the required uniqueness: we may be told that it is logically impossible for the universe to be twice in the same state. But let us examine this opinion. In the first place, it receives no countenance from science, which, though it admits such recurrence to be improbable, regards it as by no means impossible. In the second place, the present state of the universe is a complex, of which it is admitted that every part may recur. But if every part may recur, it seems to follow that the whole may recur. In the third place, this theory, when developed so as to meet the second objection, becomes really indistinguishable from that of absolute position. There is no longer an unanalyzable relation of simultaneity: there are a series of states of the universe, each of which, as a whole and only as a whole, has to each other a simple relation of before and after; an event is any part of a state of the universe, and is simultaneous with any other part of the same state, simultaneity meaning merely the being parts of some one state; before and after do not hold between events directly, but only by correlation. Thus the theory in question, except for the fact that it is no longer simple, is merely the absolute theory with states of the whole universe identified with moments. The reasons against such an identification are, first, that events seem to have an order which does not, in its very meaning, involve reference to the whole universe, and secondly, that immediate inspection seems to show that recurrence of the whole state of the universe is not logically absurd. (pp. 295–296)

The first of these objections to constructing moments of time is easily answered on Russell's OT theory. Events, in the OT sense, stand in the relation of precedence to one another independently of any reference to moments of time – events in this new sense do not involve instants in their very structure. It's the other way around.

The second objection, that it appears to be logically possible for the entire state of the universe to recur, should just be allowed as correct. And in *Inquiry into Meaning and Truth*, much later, Russell did admit this (pp. 126–27). Perhaps this is not quite directly incompatible with his construction, but it's pretty close. The relation of temporal precedence T is irreflexive. *Instants* cannot therefore recur. But isn't it possible for the time series to close on itself?

One possible line is this. We should admit that we are not entirely justified in taking temporal precedence to be irreflexive. An instant might precede itself if the loop closes somehow beyond our ken. But temporal precedence is *locally* irreflexive, and we can just regard our construction as dealing with a nonrepetitive stretch.

Another approach would be to attempt to avoid the consequence that T is irreflexive by altering the postulates. Oddly, it doesn't help to drop the postulate that P is irreflexive. If α is an instant, then $\sim T(\alpha,\alpha)$ holds independently of this property of P. This is actually momentarily slightly puzzling. It looks as if logic has taken the upper hand and demanded that the series of instants be ordered by an irreflexive relation. Suppose $T(\alpha,\alpha)$. Then $(\exists x)(\exists y)[x\epsilon\alpha . y\epsilon\alpha . P(x,y)]$. If α is an instant, for all $x,y\epsilon\alpha$, $S(x,y)$, i.e., $\sim P(x,y) . \sim P(y,x)$—which contradicts the last clause of the quantified sentence just noted. The catch is that the definition of $S(x,y)$ is no longer adequate if we have allowed that we might have $P(x,x)$. The correction along this line is to restore S as a separate primitive and to revert to a longer list of axioms in the manner of KEW. But probably I am seriously taxing your tolerance for technicalities.

One way then, and perhaps the simplest to state, is just to narrow the scope of the construction. If someone complains that this amounts to limiting the application of the temporal series to apply where it applies, we reply: Yes, it applies where it applies and not in another place. So what?

Relativity

It will be said that the construction is a mistake because the clear teaching of the general theory of relativity is that time is *not* a single linear continuum. Russell's reply, or one of them, is that since the construction concerns psychological time (AMa, p. 294), all the events concerned are in your head. Since local time is a single linear continuum even according to the theory of relativity, all is well.

I fear that I am ill qualified to do combat with the issues here, given my quite superficial understanding of relativity. Still, Russell's reply seems odd on any theory. Surely he supposes that the postulates governing events apply also to (local) events that are not experienced? Indeed, in OT he mentions physics as the discipline to which his construction is relevant.

But the reply seems to be along the right lines. If the theory of relativity has been confirmed by experiments and observations, and it has, then these processes and the events involved in them will have to be supposed to obey some simple postulates about order in time. And the resulting theory had better be consistent with the (local) validity of those temporal assumptions. And I gather that it is.[11]

In chapters 28 and 29 of *The Analysis of Matter*, Russell applies his method of construction directly to space-time using a five-termed relation of *compresence* between space-time volumes. This involves new complications and, for me, new puzzles. A complete analysis would require another paper (by somebody else). But, as presently advised, relativity does not seem to invalidate Russell's linear construction locally applied.

Epistemology

Some will complain that Russell's postulates are insufficient or dubious. The objection, hinted at earlier, is that the assumptions that are really obvious or subject to very little doubt, A and D (on my reconstruction), are not sufficient to endow instants with many of the properties required for their principal use—in the theory and application of science. The postulate (or postulates) that guarantees compactness, for example, is not clearly and distinctly true. Certainly we do not experience events between any two events we experience.

Some will say that we must simply drop the dubious epistemological project implied by the objection and revert to a modest branch of natural science: naturalized epistemology. Perhaps there are hints of this in Russell himself—in *Human Knowledge, Its Scope and Limits*. (I think they are only hints.) I would argue against such a line. Rational reconstruction, with the emphasis on the "rational," is the name of the game. If we are to replace the rotting planks of the ship by something more solid, there had better be some objective sense of "more solid." As far as I can see, this pretty much constrains the principles of rationality to be necessary (not contingent) and normative (and not merely descriptive). Of course I cannot argue this large issue in such an essay as this. Russell saw the task (usually) as one of rational reconstruction—and not simply as the job of describing what we take ourselves to know.

A better reply is this. The evident postulates are to be taken as given. *Additional* assumptions, such as are required to ensure the compactness or even continuity of the series of instants, are taken as hypotheses that are confirmed according as their consequences are verified or not. Of course, the verification will be through the application of the constructed theory of instants as used in science. Or, if this is not the correct analysis of scientific inference, the additional assumptions must be found to be part of the best explanation of the observed facts.

If we proceed in that way, you may say, why not just take *instants* as primitive, postulate *their* properties, deduce, and confirm (or whatever one does in nondeductive inference)? Partly this involves a more general question: why construct anything out of anything if inductive inference is ultimately going to be used? I discuss this general matter later. The remainder of the objection is answered thus: events are better known than instants. Some of the assumptions about events are quite evident; the remainder are as well confirmed – by the same evidence – as the corresponding assumptions about instants. The postulates about instants just contemplated would *all* involve the unobserved and the nonevident.

In summary, then, the problem of recurrence is handled by narrowing the scope of the analysis to nonrecurring times, relativity is appeased by considering only local time, and uncertainty is minimized by leaving dubious postulates to be made probable by a posteriori methods. A great deal of contraction! Well, what did you expect? We are seeking the hard center.

General Considerations on Constructionalism

The Russellian approach to the Problem of the External World is not very popular in some quarters. Indeed, constructional systems generally, except in certain parts of mathematics, are quite unpopular. I want to consider, last of all, some extremely general criticisms of the idea of attempting to "construct" some things out of others.

Arbitrariness

One can extract from Paul Benacerraf's seductive paper, "What Numbers Could Not Be,"[12] a criticism of constructions generally. It is roughly this: there are many constructions, any of which fulfills whatever role we require the constructed entity to play. These different constructions will sometimes involve necessarily distinct entities; e.g., in the case of numbers, $2 = \{\{\varnothing\}\}$ or $2 = \{\varnothing, \{\varnothing\}\}$, hence they cannot all be correct. But we have and can have no nonarbitrary way of choosing one of them. Hence, we must choose none. Or, perhaps, all the constructions are incorrect.

Whatever may be the case with numbers, it is not obviously true that in all cases there are alternative constructions that are just as good. Whitehead's construction is epistemically inferior. And sheer isomorphism is not sufficient to ensure identity of goodness. Suppose we take the first instant at which an event exists to be the set of all events that precede it. Incorrect. For then it would be possible for the first instant at which an event exists to exist even if the event itself never exists. (Actually, this observation bodes ill for the construction of hyperinstants mentioned earlier. Perhaps the best plan is just to postulate more events and properties thereof as needed.)

Suppose that the worst comes and we cannot find arguments based on any reasonable criteria for ruling out the remaining competing constructions. Then it still may be that any of the constructions is better than the reconstructed theory. So one distributes reasonable belief equally among the constructions. Just because one cannot choose, nonarbitrarily, between four men, say, all equally strong, but stronger than Jones, does not imply that none is better than him if one wants some lifting done.

But, you may ask, *are* any of the reconstructed theories really any better than the original? This brings me to: the next point.

Rationality

I have already confessed my allegiance to old-fashioned epistemology. Now I would like to do something more to justify (epistemically, of course—we are not merely describing here) that faith. Some have argued[13] that we are no better off *after* the constructing is done than before. How, exactly, is the claim to be made out that we are more justified in believing in instants, say, as collections of events, than in instants sui genera?

Let T_1 and T_2 be theories consisting of sets of propositions (not sentences) and closed under entailment. And suppose that with respect to all the known relevant observational facts, they both entail or fail to entail, explain or fail to explain (N.B.), the same propositions. Still T_1 might be more probable than T_2. I speak here of comparative probability—the idea of assigning numerical probabilities to arbitrary propositions seems highly doubtful (to me). If T_1 is the theory of events together with the set theory needed for Russell's construction and T_2 is this *plus* an independent theory of instants, then T_1 is a priori more probable than T_2. For T_1 consists of propositions true in every possible world in which all the propositions of T_2 are true, but not conversely. So T_1 has less chance of being wrong. If the two theories otherwise have the same explanatory power (and aren't we dealing with a case where that is so?), then T_1 is to be preferred. Of course, T_1 still may not be the only theory left in the field.

Aesthetics

A final word on theory construction and acceptance. William James was right when he urged that it is entirely sensible to accept theories on the basis of nonrational criteria.[14] This is, as I construe it, not so shocking as it sounds. We have all been instructed by philosophers of science that theories are badly underdetermined by the data. I take this to mean that given the rules of deductive logic, the formulas of probability theory, the necessary truths of epistemology, and the facts of experience, alternative theories that are incompatible are still justified to the same degree. We are then free to turn to other marks of goodness. I would urge that aesthetic criteria may be applied. Indeed, it is clear that they already are. Economy, cited as a good-making feature of theories, is basically an aesthetic matter when applied to ideology—i.e., to the primitive concepts adopted. I suggest that if we give the Beautiful its rightful place as Judge next after the Good and the True, Russell's theory of instants defeats its nearest competitors.

Notes

1. Our use of dots as punctuation in logical formulas follows Alonzo Church, *Introduction to Mathematical Logic I* (Princeton, N.J.: Princeton University Press, 1956); see p. 75. Roughly, the

dot replaces a left bracket whose mate is as far right as possible consistent with the formula being well formed.

2. W. V. O. Quine, Review of "On Order in Time," *Journal of Symbolic Logic*, 1 (June 1936), pp. 72-73.

3. N. Wiener, "A Contribution to the Theory of Relative Position," *Proceedings of the Cambridge Philosophical Society*, 17 (1914), pp. 441-49.

4. In *Our Knowledge of the External World* (Chicago and London: Allen & Unwin, 1914). See the preface and Lecture IV, p. 91.

5. A. N. Whitehead, *The Concept of Nature* (Cambridge: Cambridge University Press, 1920), especially chapter 4.

6. It is then not excessively difficult to prove, using A and D (and as Russell does in OT using A, B, and C) and a modicum of set theory, that instants of time form a simply ordered series. Formally one uses the definitions:

DEFINITION: $In(\alpha) \rightarrow \alpha = \hat{x}(y)[y \in \alpha \supset S(x,y)]$.
DEFINITION: $T(\alpha, \beta) \rightarrow (\exists x)(\exists y)[x \in \alpha . y \in \beta P(x, y).].In(\alpha).In(\beta)$.

It follows, using A and D, that:

(i) $\sim T(\alpha, \alpha) - T$ is irreflexive.
(ii) $T(\alpha, \beta) \supset .T(\beta, \gamma) \supset T(\alpha, \gamma) - T$ is transitive.
(iii) $In(\alpha) \supset .In(\beta) \supset .T(\alpha, \beta) \vee (\alpha = \beta) \vee T(\beta, \alpha) - T$ is connected in its field. That is to say, T is a *simple ordering*. I omit the proof; Russell's is entirely valid.

7. The possibilities mentioned here both require the use of the axiom of choice or its equivalent, the well-ordering principle.

8. Kurt Gödel, "What Is Cantor's Continuum Problem?" *American Mathematical Monthly*, 54 (1947), pp. 515-25.

9. Reviews of *Foundations of Mathematics and other Logical Essays* by F. P. Ramsey. *Mind*, n.s., 49 (Oct. 1931), pp. 476-82, and in *Philosophy*, 7 (1932), pp. 84-86.

10. The error occurs on line 10 from the bottom, p. 219 (line 7 from the bottom of p. 351 of the reprint of OT in *Logic and Knowledge*, ed. Robert C. Marsh [London: Allen & Unwin, 1956]). The stated further condition is not sufficient as claimed.

11. See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (San Francisco: Freeman, 1973), p. 19, section 1.3, cited in Major L. Johnson, Jr., "Events as Recurrables," in *Analysis and Metaphysics*, ed. K. Lehrer (D. Reidel, Dordrecht, 1975).

12. Paul Benacerraf, "What Numbers Could Not Be," *Philosophical Review*, 74 (1965), pp. 47-73.

13. For example, Ernest Nagel, "Russell's Philosophy of Science," in *The Philosophy of Bertrand Russell* ed. P. A. Schilpp (New York: Tudor, 1944).

14. William James, "The Will to Believe," in *The Will to Believe and other Essays in Popular Philosophy* (New York: Dover, 1956).