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DAVID MALAMENT

Observationally Indistinguishable Space-times

In his paper "Indistinguishable Space-times and the Fundamental Group"[1] Clark Glymour poses a criterion for the observational indistinguishability of space-time models and presents two sets of examples from the subclass of Robertson-Walker models. The underlying idea is quite intuitive.

In some space-time models studied in relativity theory any particular observer can receive signals from, and hence directly acquire information about, only a limited region of space-time. This happens, for instance, in a rapidly expanding universe in which galaxies that might try to signal one another are actually receding from one another at velocities approaching that of light. It may turn out in these cases that the information from that limited region of space-time which any one observer can have access to is compatible with quite different overall space-time structures. Two space-times are observationally indistinguishable under Glymour's criterion if, for precisely these reasons, no observer in either space-time would have grounds for deciding which of the two, if either, was his. No observer would be able to discriminate observationally between the two even if he did nothing but sit and record signals beamed at him from all directions all day long, even if the signals themselves coded all the spatio-temporal information that the sender had to offer, and even if the observer lived eternally.

Glymour is proposing a reason why the spatio-temporal structure of the universe might be underdetermined by all observational data that we could ever, even just in principle, obtain. Some claims of underdetermination in science are of a very general sort, to the effect that no body of evidence will ever force a particular scientific hypothesis upon us

NOTE: Most of the ideas in this paper arose in conversation with Robert Geroch and Clark Glymour. I have not hesitated to incorporate their many contributions. I am grateful to both.

if only we are prepared to make sufficiently sweeping revisions in relevant theory. Along these lines one might argue, and some have argued, that we can always ascribe more than one topology to space-time if we are imaginative in the invocation of phantom effects such as the duplication of all events at a distance. Such a claim may or may not be irrefutable. But it is certainly quite different from the claim that we have a choice in the ascription of topology *even without giving up any of our trustiest theories*. This is the possibility that Glymour is suggesting, if only tentatively. He makes no argument to the effect that the space-time structure of our universe, i.e., the real one, is in fact one of a pair of observationally indistinguishable space-times. The point is rather that if it were an element of such a pair, then it would be underdetermined by all observational evidence. We would then be unable to determine its global structure even if we adamantly insisted on holding on to our best theories and exploiting them fully in the attempt.

Rather than comment on the details of Glymour's proofs, I want to try to make the geometric ideas in his paper more perspicuous by considering several very simple, easily visualized examples of observationally indistinguishable space-times in two and three dimensions. In doing so I shall establish a few simple results concerning the invariance of global properties of space-times under the relation of observational indistinguishability. I shall also discuss several other relations concerning observational indistinguishability, some weaker and some stronger. The upshot of my remarks will be that the cosmologist's predicament is even worse than one thought at first. Observational underdetermination of one sort or another is more the rule than the exception.

I

Let me first rehearse a few definitions.[2] An *n-dimensional space-time* (for $n \geq 2$) is taken to be a connected, smooth, n-dimensional differentiable manifold (without boundary), endowed with a smooth, nondegenerate pseudo-Riemannian metric of Lorentz signature $(+, -, \ldots, -)$. The metric associates with each point a light cone (in the tangent space at that point). It is assumed that space-times are *temporally oriented*, i.e., that they are further endowed with a continuous, nonvanishing vector field which assigns a timelike vector to every point. The vector field distinguishes a "future lobe" in the light cone at each point.

Given two points x and y, we say y is to the *timelike future* of x and

write $x << y$ if there is a piecewise smooth curve from x to y whose tangent vector (or vectors) at each point lies inside the future lobe of the light cone at that point—in short, if there is a *future-directed timelike curve* from x to y. If in the definition tangent vectors are permitted to be on the boundary as well as in the interior of the light cone, y is said to be the *causal future* of x, and we write $x < y$. In this case the connecting curve is called a *future-directed causal curve*. The relation $x < y$ is usually interpreted to mean that it is possible for a signal to travel from x to y; $x << y$ is interpreted to mean that it is possible for a heavier than light particle to make the trip. Associated with each relation is its respective past and future sets: $I^-(z) = \{y: y << z\}$, $I^+(z) = \{y: z << y\}$, $J^-(z) = \{y: y < z\}$, and $J^+(z) = \{y: z < y\}$. The I sets are open (in the space-time manifold topology) and are for this reason somewhat easier to work with than the J sets, which are in general neither open nor closed. The set $I^-(z)$ is called the *observational past of* z; it consists of those points in space-time which can possibly send a (slower-than-light) signal to z.

We can associate with each *observer* his space-time trajectory or cosmic world-line which is itself, necessarily, a future-directed timelike curve. If σ is such a world-line, the *observational past of* σ is just the union: $I^-[\sigma] = \cup \{I^-(x): x \in \sigma\}$. The idea that an observer live "eternally" is captured in the condition that his associated world-line be *future-inextendible*, i.e., that as a curve in the space-time manifold, it be extended as far as possible into the future. Such curves, by definition, have no "future end point."[3] That an observer *have* lived eternally could be captured, symmetrically, in the condition that his world-line be *past inextendible*. But as far as capacity for observation is concerned, no advantage comes through this kind of longevity. If x and y are successive points on a world-line, then $x << y$ and $I^-(x) \subset I^-(y)$ by the transitivity of the relation $<<$.

We now have all the components for Glymour's definition of observational indistinguishability:

Definition: Two space-times M and M' are *observationally indistinguishable* if for every future-directed, future-inextendible, timelike curve σ in M there is a curve σ' of the same type in M' such that $I^-[\sigma]$ and $I^-[\sigma']$ are isometric; *and*, correspondingly, with the roles of M and M' interchanged.

The condition that $I^-[\sigma]$ be isometric to $I^-[\sigma']$ formalizes the condition that the portion of M which σ can possibly see over the course of his eternal lifetime is, "space-time-wise," identical with that portion of M' which σ' can possibly see over the course of his lifetime. The definition is mathematically well formed since both $I^{-}[\sigma]$ and $I^{-}[\sigma']$ are pseudo-Riemannian manifolds (without boundary) in their own right.

The simplest example of a space-time which admits no observationally indistinguishable counterpart is Minkowski space-time. Here the observational past of every future-inextendible timelike curve σ is the entire manifold; equivalently, $\mathrm{Bnd}(I^{-}[\sigma]) = \phi$. The set $\mathrm{Bnd}(I^{-}[\sigma])$, the boundary of $I^{-}[\sigma]$, may be termed the *observational horizon of* σ (the expression "event horizon" is more common).

One need not look far to find a space-time in which *all* observers have observational horizons. The light cones in Minkowski space-time are all fixed at 45°. Consider the two-dimensional plane in standard t, x coordinates with a metric whose associated light cones, while situated at 45° for $t = 0$, rapidly narrow to the vertical as t increases in absolute value. For example, although others would serve just as well, let the metric be $ds^2 = dt^2 - (\cosh^2 t)\, dx^2$. (Recall that $\cosh t = \tfrac{1}{2}(e^t + e^{-t})$). Because the cones collapse, null geodesics (trajectories of light rays) will be confined to a region of space-time of bounded x-width (see Figure 1). For the particular metric cited, they will be confined to a region of x-width π. Correspondingly, the observational past of every future-inextendible timelike curve will be confined to a region of x-width 2π.

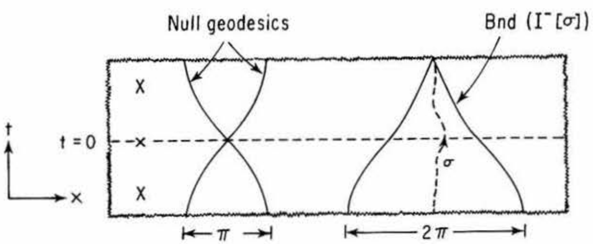


Figure 1. The covering space of two-dimensional De Sitter space-time, i.e., the t, x plane with metric $ds^2 = dt^2 - (\cosh^2 t)\, dx^2$. Light cones narrow rapidly to the vertical as $|t| \to \infty$. Every future-extendible timelike curve σ has an observational horizon of x-width 2π. Two-dimensional De Sitter space-time arises by identifying all points $(t, x + 2n\pi)$ for integers n. By introducing coordinates $\bar{t} = \sinh t$, $\bar{x} = (\cosh t)(\cos x)$, $\bar{y} = (\cosh t)(\sin x)$, it assumes the familiar form of a hyperboloid of one sheet $-\bar{t}^2 + \bar{x}^2 + \bar{y}^2 = 1$ in $\mathbf{R}^3$ with metric $ds^2 = -d\bar{t}^2 + d\bar{x}^2 + d\bar{y}^2$.

Now let M be the two-dimensional space-time just described. Let M' be the result of cutting a vertical strip in M of x-width 2π and identifying opposite sides. It is clear that M and M' must be observationally indistinguishable from one another. Since no observer in either M or M' can see beyond his 2π horizons, none will be able to determine which of the two, if either, is his. (Incidentally, M' is the two-dimensional version of De Sitter space-time.)

This first example exhibits the general features of Glymour's construction. He considers a subclass of space-times which he calls *standard*. These, like M and M', are manifolds topologically of form $\mathbf{R} \times V$ carrying Robertson-Walker metrics $ds^2 = dt^2 - R(t)^2 d\sigma^2$, where $d\sigma^2$ is a smooth, complete Riemannian metric of constant curvature on V and is independent of t. Within the class he finds space-times $\mathbf{R} \times V$ which admit nice families of isometries, and forms new space-times $\mathbf{R} \times V'$ by taking quotient manifolds under them. The essential requirement on the isometries is that they move points sufficiently far so that the observational past of every point is disjoint from the observational pasts of all of its image points. In the present example V is $\mathbf{R}$, V' is S^1, and the isometry in question is just the translation: $(t, x) \to (t, x + 2\pi)$.

All the examples of observationally indistinguishable space-time pairs which Glymour generates with this construction are such that either one is a covering space of the other or they share a common covering space. But examples can easily be given in which this is not the case. The building blocks for one are vertical slabs of two types, A and B, both cut from the plane. In standard t, x coordinates, both may be taken to be the set $\{(t, x)\colon 0 < x < 2\pi\}$. Slabs A will carry the De Sitter metric from the first example: $ds^2 = dt^2 - (\cosh^2 t)\, dx^2$. Slabs B will carry the metric $ds^2 = dt^2 - (\cosh^2 t)(1 + x(2\pi - x))\, dx^2$. This metric shares the property that its associated light cones collapse to the vertical as t increases in absolute value. It approaches the metric of A smoothly along its borders so that when the two slabs are glued together (with an appropriate common boundary line inserted), the resulting double slab carries a smooth metric. The B metric also has an extra wiggle factor inserted which further narrows and then restores the cones in moving from $x = 0$ to $x = 2\pi$ for any fixed value of t. One could equally well use any other smooth wiggle factor. The point is simply to distinguish the two slabs metrically.

Now we form space-times M and M' by taking two nonisomorphic $-\omega + \omega$ sequences of A and B slabs and gluing them together. One such sequence might be . . . *ABABBA* . . . . Any sequences will do if at least one token of each slab type occurs in each. The observational past of any observer in either *M* or *M′* will be restricted to an *A* slab, a *B* slab, or an *AB* or *BA* double slab (see Figure 2). In each case, his observational past is compatible with both space-time structures. Hence *M* and *M′* are observationally indistinguishable. But by our initial choice of sequences, they are not isometric. As they stand, *M* and *M′* are homeomorphic (i.e., they have the same topology, that of the Euclidean plane). But with a simple variation on the slab theme we could distinguish *M* and *M′* topologically as well. We would need to distinguish only the component *A* and *B* slabs topologically.

Next I want to consider the condition of "observational indistinguishability after finite time." As Glymour's definition is formulated, space-times can be observationally distinguishable from each other without an observer in either one necessarily being able to distinguish between them *at any time* during the course of his life. It is sufficient that the composite, lifelong, integrated knowledge of one observer distinguish between them. A weaker condition of observational indistinguishability which Glymour considers insists that observational distinction between space-times be made *within* the lifetime of some observer.

Definition: Two space-times *M* and *M′* are *observationally indistinguishable after finite time* if for every point x in *M* there is a point x' in *M′* such that $I^{-}(x)$ and $I^{-}(x')$ are isometric; *and*, correspondingly, with the roles of *M* and *M′* are interchanged.

This seems the more natural way to formulate the condition.

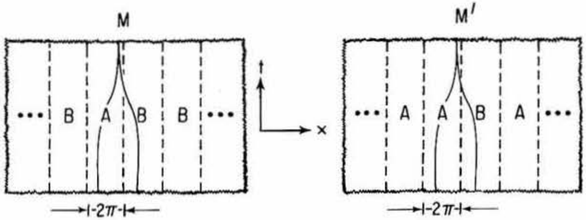


Figure 2. *M* and *M′* are observationally indistinguishable, although they do not share a common (metric-preserving) covering space. The observational past of a typical observer in *M* is indicated together with his counterpart in *M′*.

The two notions of observational indistinguishability are certainly not equivalent. Consider, for example, simple two-dimensional Minkowski space-time and "truncated" Minkowski space-time consisting of that portion of the former below the x-axis. The observational pasts of all points in both space-times are isometric, so the two are certainly observationally indistinguishable after finite time. But they are not observationally indistinguishable. As noted above, Minkowski space-time has no observationally indistinguishable counterpart.

There seems to be something rather unsatisfactory about truncated Minkowski space-time. Perhaps it violates our sense that the universe should satisfy what Leibniz called the "principle of plenitude." However compelling the metaphysics, a condition of *inextendibility* is often imposed on space-times. It is the condition that it not be possible to embed the space-time isometrically in another without the two being isometric. Clearly truncated Minkowski space-time *is* extendible.

If we restrict our attention to inextendible space-times, then it becomes more difficult to show that space-time pairs can be observationally indistinguishable after finite time while not observationally indistinguishable. In fact, as Glymour points out, within the class of inextendible standard space-times the two conditions are equivalent. But if standardness is not also demanded, examples showing the difference in strength are still available.

One such is found by elaborating the slab construction from the second example. Consider this time vertical "half slabs" of two types, *A* and *B*. Each is respectively that portion of its earlier counterpart falling beneath the x-axis. Let $\mathcal{S}$ be the set of all finite sequences of *A* and *B* slabs and consider $-\omega + \omega$ sequences in $\mathcal{S}$, i.e., sequences of the form $\ldots S_{-2}S_{-1}S_{0}S_{+1}S_{+2}\ldots$, which include all elements of $\mathcal{S}$. Take two, in particular, which are distinct in the sense that their underlying composite sequences of *A*'s and *B*'s are not isomorphic. Glue all these slabs together nicely and finally glue to both of them "on top" the upper half of two-dimensional Minkowski space-time. The resulting mosaics are inextendible space-times (see Figure 3). Given any point in either *M* or *M′*, its observational past intersects with only finitely many adjacent slabs and so has an isometric counterpart in the other space-time. Thus *M* and *M′* are observationally indistinguishable after finite time. But any future-inextendible timelike curve in either space-time will include in its observational past the entire manifold; hence it will include the entire finite

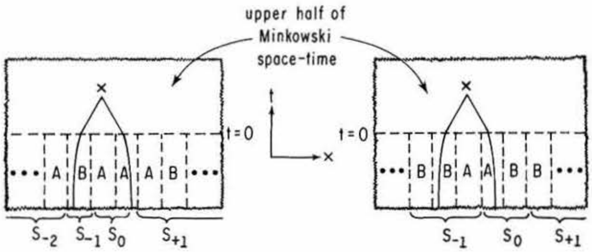


Figure 3. M and M' are observationally indistinguishable after finite time, but not observationally indistinguishable. The observational past of a typical point x in M is indicated together with its counterpart x' in M'.

sequence of A and B slabs which, by our initial choice, finds no isometric counterpart in the other space-time. Thus M and M' are not observationally indistinguishable.

In one sense, even the condition of observational indistinguishability after finite time is overly stringent. Suppose we have two space-times M and M', and suppose for every point x in M there is a corresponding point x' in M' such that $I^{-}(x)$ and $I^{-}(x')$ are isometric. Then no observer in M at any point in his life will be in a position to determine which of the two space-times, if either, is his. Yet M and M' need not necessarily be observationally indistinguishable after finite time because there might be an observer in M' who could at some time distinguish between them.

As far as the epistemological situation of the M-observer is concerned, it makes no difference what the M'-observer can or cannot determine. For this reason it is worth considering a new condition of observational indistinguishability which, unlike the first two, is not symmetric.

Definition: If M and M' are space-times, *M is weakly observationally indistinguishable from M'* if for every point x in M there is a point x' in M' such that $I^{-}(x)$ and $I^{-}(x)$ are isometric.

Quite trivially, M can be weakly observationally indistinguishable from M' without the two being observationally indistinguishable after finite time. For example, take M as in the very first example—the t, x plane with metric $ds^2 = dt^2 - (\cosh^2 t)\, dx^2$; and take M' to be either one of the two space-times in the second example, the ones built from vertical A and

B slabs. The observational past of every point in M, indeed the observational past of every future inextendible timelike curve in M, finds an isometric counterpart in any of the A slabs of M'. But obviously no observer in M' who ever catches a glimpse of the B portion of his space-time could think himself to be in M.

This third notion of observational indistinguishability seems a straightforward rendering of conditions under which observers could not determine the spatio-temporal structure of the universe. Yet, and this is what is most interesting, the condition of weak observational indistinguishability is so widespread in the class of space-times as to be of epidemic proportions.

There are some space-times that are not weakly indistinguishable from any other. These include space-times M in which the observational past of some point is the entire manifold, i.e., $I^{-}(x) = M$ for some x. The simplest such example is two-dimensional Minkowski space-time "rolled up" along the t-axis (i.e., for some $k > 0$, the points (t, x) and $(t + nk, x)$ are identified for all integers n). A more interesting example is Gödel space-time. But only these quite bizarre space-times seem to escape having counterparts from which they are weakly observationally indistinguishable. (There is a theorem lurking here.)

Let me give a geometrically intuitive argument sketch which, while falling short of a proof, suggests why this should be so. To keep things simple, let us restrict attention (much more than we have to) to space-times that are decent in their "causal structure" and have no closed or almost closed future-directed timelike curves. To be specific, let us consider only "strongly causal" space-times.[4]

Let M be one such space-time and let $\{x_i\}$ be a countable sequence of points in M, the union of whose observational pasts covers all of M, i.e., $\cup \{I^{-}(x_i)\} = M$.[5] Using a "clothesline construction" we can string out these $I^{-}(x_i)$ with appropriately chosen "space-time filler" to form a new space-time M'. (The causality assumption here disallows the possibility of the $I^{-}(x_i)$ folding back on themselves.) In other words, we can find a space-time M' in which all the $I^{-}(x_i)$ can be isometrically embedded. The space-time filler with which M' is constructed can be chosen quite arbitrarily, subject only to the constraint, of course, that it be smooth on the boundaries of the $I^{-}(x_i)$. Exercising this freedom we can so choose the filler as to guarantee that M' not be isometric to M. But clearly M must be weakly observationally indistinguishable from M'. Any point

$x \in M$ will be in some $I^-(x_i)$. Since $I^-(x) \subset I^-(x_i)$, $I^-(x)$ will find an isometric counterpart in the $I^-(x_i)$ portion of the clothesline.

M' as it stands suffers from being (very) extendible. But it follows from an argument of Robert Geroch[6] that every space-time has an inextendible extension. We can choose one for M', say M'', and we have enough freedom in doing so to ensure that M'' not be isometric to M. At least if M is itself inextendible, the $I^-(x_i)$ excised from it will remain unaffected by the extension from M' to M''. So for every point $x \in M$, $I^-(x)$ will still find an isometric counterpart in M''.

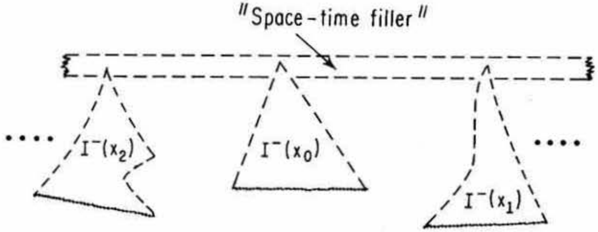


Figure 4. "Clothesline construction" by which a space-time is formed in which all the past observation sets $I^-(x_i)$ can be isometrically embedded.

This argument sketch shows that every strongly causal space-time is weakly observationally indistinguishable from some other space-time, and that if the first is inextendible, the second may be taken to be inextendible as well. If greater care is exercised in hanging the clothesline used in the construction, the argument goes through under much weaker "causality" assumptions. Indeed, it is sufficient that for some point x, $I^-(x) \neq M$.

II

To get a better feeling for the three observational indistinguishability relations it will help to consider several global properties of space-times and see whether any are preserved under them, i.e., whether it is the case that given two space-times, the first observationally indistinguishable (respectively observationally indistinguishable after finite time, weakly observationally indistinguishable) from the second, the property obtains in the first space-time only if it obtains in the second. The question is of interest because even in the presence of observational indistinguishability

there is the possibility that an observer can make *some* determinations concerning the global spatio-temporal structure of the universe and so at least delimit the range of open choices.

The accompanying tabulation lists a sampling of commonly studied global properties and indicates their invariance under the three relations. The arrows indicate, of course, that if a property is not preserved under a strong sense of observational indistinguishability, it is not preserved under weaker senses (and correspondingly when it is preserved). Counterexamples for (1)–(5) and (8) are appended. The other causality properties are of some special interest because, as indicated in the table, they diverge from their respective negations when it comes to preservation under weak observational indistinguishability.

Property	OI?	OI after finite time?	Weak OI?
1. temporal orientability	No	------------------	→
2. spatial orientability	No	------------------	→
3. orientability	No	------------------	→
4. inextendibility	No	------------------	→
5. noncompactness	No	------------------	→
6. causality	←------	Yes	No
noncausality	←------	------------------	Yes
7. strong causality	←------	Yes	No
nonstrong causality	←------	------------------	Yes
8. existence of a global time function	No	------------------	→
9. existence of a Cauchy surface	←------	Yes	No
nonexistence of a Cauchy surface	←------	------------------	Yes

Causality is the condition that there *not* be a closed, future-directed causal curve. If causality is violated in a space-time M, the entire violating curve will be in the observational past of some point x—any point, in fact, which lies to the future of some point on the curve. If now $I^-(x)$ is isometric to $I^-(x')$ for some point x' in a space-time M', the image of the closed curve under the isometry will itself be a closed curve in $I^-(x')$. It follows that noncausality is preserved under weak observational indistinguishability and causality is preserved under observational indistinguishability after finite time (by the symmetry of the relation).

On the other hand, the following simple example shows why there must be a 'No' in the third column of line 6 in the table. Take M to be two-dimensional Minkowski space-time. Construct M' by first cutting two slits in M, say $A = \{(t, x): t = 0 \;\&\; 0 \leq x \leq 1\}$ and $B = \{(t, x): t = 1 \;\&\; 0 \leq x \leq 1\}$, and then identifying the lower edge of slit B, excluding the corner points (1, 0) and (1, 1), with the upper edge of slit A, excluding (0, 0) and (0, 1). (See Figure 5.) M is certainly weakly observationally indistinguishable from M' since every point x in M has as an observationally indistinguishable counterpart every point x' in M' lying, say, beneath slit A. But causality is badly violated in M'; one sample closed timelike curve is indicated in the figure. M' as it stands is extendible, but it can be rendered inextendible by further identifying the upper edge of slit B with the lower edge of slit A, again excluding corner points. In a sense the points (0, 0), (0, 1), (1, 0), and (1, 1) are "missing," but there is no way they can be replaced to extend M'.

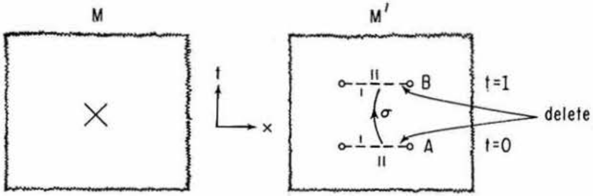


Figure 5. M is weakly observationally indistinguishable from M' although M' is not causal (σ is a sample closed causal curve). The lower edge of slit B is identified with the upper edge of slit A and the upper edge of slit B is identified with the lower edge of slit A (excluding end points). M' is inextendible.

The situation seems to be this: if causality is violated in a space-time, some observer will know about it; if on the other hand it is not violated, no observer will ever know for sure one way or the other. This does not follow from the one example, but it could be established with another clothesline construction. In addition to the other space-time segments $I^-(x_i)$, an additional causality-violating segment would have to be added to the line.

With respect to preservation under the different notions of observational indistinguishability, strong causality is quite similar to causality.

The claims made in the table are verified along the lines just indicated. The condition that there exist a Cauchy surface is a bit different.

If M is a space-time, a set $S \subset M$ is a *Cauchy surface* if S is *achronal* (i.e., for no points x and y in S is $x << y$) and if for every point z in M, every (past- and future-) inextendible timelike curve through z intersects S. For example, the surfaces t = constant are Cauchy in Minkowski space-time. A simple example of a space-time that does not admit any Cauchy surface is (the covering space of) the two-dimensional version of anti-De Sitter space-time. It can be represented as the t, x plane with metric $ds^2 = dt^2 - (\cosh^{-2} x)\, dx^2$. At $x = 0$ the metric reduces to Minkowski form and the associated null cones are at 45°. But as x increases in absolute value, the cones flatten and approach the horizontal asymptotically. (See Figure 6.) In contrast, remember that with the De Sitter metric (i.e., $ds^2 = dt^2 - (\cosh^2 t)\, dx^2$) the cones narrowed to the vertical as t increased in absolute value. No surface t = constant will be Cauchy in anti-De Sitter space-time because there are inextendible curves through many points which "come in from or go out to spatial infinity" without hitting the surface. The same is in fact true of all achronal sets.

The condition that there exist a Cauchy surface is of great interest because of its usual interpretation as a condition of Laplacian determinism and the possibility of cosmic prediction.[7] If a set is Cauchy, no signal propagating causal influence can reach any point in space-time without that signal registering itself, before or after, on the set. In the absence of such a set—in anti-De Sitter space-time, for example—causal influence can "come in from infinity" without registration. For this reason even a

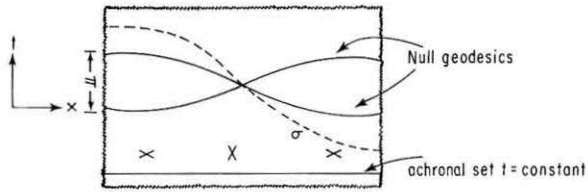


Figure 6. The covering space of two-dimensional anti-De Sitter space-time, i.e., the t, x plane with metric $ds^2 = dt^2 - (\cosh^{-2} x)\, dx^2$, which admits no Cauchy surface. Light cones rapidly flatten to the horizonal as $|x| \to \infty$. An inextendible timelike curve σ coming in from and going out to "infinity" is indicated along with a sample t = constant slice which it fails to hit.

complete specification of "initial data" on a t = constant surface in anti-De Sitter space-time will not uniquely determine past and future spatio-temporal evolution.

It turns out that the existence of a Cauchy surface is equivalent to the condition of *global hyperbolicity*.[8] A space-time satisfies this condition if it is causal *and* if for all points x and y, the set $J^+(x) \cap J^-(y)$ is either empty or compact. (Note that this is not true in anti-De Sitter space-time.) Now suppose M is weakly observationally indistinguishable from M' and M fails to be globally hyperbolic. If causality fails in M then, as we know, it must fail in M' too. Suppose then that for some x, y in M, $J^+(x) \cap J^-(y)$ is neither empty nor compact. Suppose further that z is some point to the future of y, i.e., $y << z$. Then $J^+(x) \cap J^-(y)$ is contained in $I^-(z)$. (Fact: $w < y$ & $y << z \rightarrow w << z$). If z' is in M' and $\phi: I^-(z) \rightarrow I^-(z')$ is an isometry, then $J^+[\phi(x)] \cap J^-[\phi(y)] = \phi[J^+(x) \cap J^-(y)]$ will be a set in M' neither empty nor compact. Hence M' is not globally hyperbolic. Thus the nonexistence of a Cauchy surface is preserved under weak observational indistinguishability. (The same argument, of course, establishes that the existence of a Cauchy surface is preserved under observational indistinguishability after finite time.)

The following example shows, in contrast, that the existence of a Cauchy surface is not necessarily preserved under weak observational indistinguishability. Once again take M to be two-dimensional Minkowski space-time. For M' we in effect glue together the lower half of M with the upper half of two-dimensional anti-De Sitter space-time. More specifically, M' is the t, x plane with the metric $ds^2 = dt^2 - dx^2$ in the region $t \leq 0$, and the metric $ds^2 = dt^2 - (\cosh^{-2} x)\, dx^2$ in the region $t \geq 1$. For the buffer strip $0 < t < 1$ choose any metric whatsoever that smoothly connects the other two. M' is clearly an inextendible space-time without a Cauchy surface. Equally clearly, however, M is weakly observationally indistinguishable from M'. Every point in M has as a counterpart every point in M' with coordinate $t \leq 0$.

The comments made before about causality can now be paraphrased. In particular, it seems that if a space-time has a Cauchy surface, none of its native observers will ever know for sure whether it does or not!

III

The predicament of the cosmologist attempting to determine the global space-time structure of his universe has been cast as a serious one. However, an objection could be made. It is important and should be considered. The notion of weak observational indistinguishability was introduced on the suggestion that Glymour's two conditions are unnecessarily stringent. According to the objection they are not stringent enough.

The different conditions of observational indistinguishability are posed solely in terms of spatio-temporal structure. They do not mention the things and processes which populate space-time. But, the objection runs, two cosmological models are only truly observationally indistinguishable if neither underlying space-time structure *nor* its contents (stars, galaxies, background radiation, or whatever) distinguish them to any observer.

In response, the several definitions of observational indistinguishability can be extended in a straightforward way to include the physical goings-on within space-time. Let us suppose that the fundamental furniture of the universe consists of a number of *matter-fields* (e.g., an electromagnetic field) which are mathematically represented by tensor fields defined on the underlying space-time manifold, and whose dynamical histories are constrained by (partial differential) field equations. This is no more than the framework within which relativity is in fact studied. In this context, a *cosmological model* may be construed as an ordered $(n+1)$ tuple $(M, F_1, \ldots, F_n)$ whose first element is a space-time and whose remaining elements are tensor fields of the appropriate type on M, satisfying appropriate field equations. We say that two *cosmological models* $(M, F_1, \ldots, F_n)$ and $(M', F'_1, \ldots, F'_n)$ *are observationally indistinguishable* in any of the three senses defined if M and M' are observationally indistinguishable in that sense *and* if the isometries between past observational sets called for in the definitions also preserve the values of respective matter fields, i.e., for any such isometry ϕ, $\phi_*(F_i) = F'_i$ for $i = 1, \ldots, n$.

In reply to the present objection it can now be argued that observational indistinguishability between cosmological models, if not in the narrower sense between space-times, really *is* a sufficient condition for the empirical underdetermination of space-time structure. The reply seems a strong one. If one accepts the idealization of a cosmological model in the first place, then specification of the values of the various matter fields in a region of space-time completely specifies what there is to be observed—background radiation, quasars, or whatever. If, for example, the cosmological model $(M, F_1, \ldots, F_n)$ is weakly observationally indistinguishable from $(M'_1, F'_1, \ldots, F'_n)$, then nothing any observer in the first model could "see" at any time in his life, no matter how discerning his instruments, would ever distinguish between the two models. It seems clear, too, that no physical theory such as any observer could ever extrapolate from the matter-fields as he sees them could possibly cut the ice between them. To the extent that he is entitled to adopt the would-be ice-cutting theory, so is his observational counterpart. By the very definition of weak observational indistinguishability it would seem that any theory supported by the observational evidence available to any one observer in $(M, F_1, \ldots, F_n)$ would have to be neutral, as between two cosmological models.

Suppose we grant now that observational indistinguishability between cosmological models (in any of the three senses) is a sufficient condition for the empirical underdetermination of space-time structure. There remains the question of existence. Suppose $(M, F_1, \ldots, F_n)$ is a cosmological model and that M as a space-time is observationally indistinguishable (in one of the senses) from some other space-time. We can ask whether there necessarily exists a cosmological model $(M', F'_1, \ldots, F'_n)$ which is observationally indistinguishable (in the same sense) from $(M, F_1, \ldots, F_n)$. At least with respect to the sense of weak observational indistinguishability the answer seems to be clearly yes! The same clothesline construction that served to generate space-times can be used to generate cosmological models as well. Instead of linking the space-times $I^-(x_i)$ we link the cosmological models

$$(I^-(x_i), F_1\big|_{I-(x_i)}, \ldots, F_n\big|_{I-(x_i)}),$$

connecting them with space-time and matter-fields filler.

The cosmologist's epistemological predicament, it thus appears, is not at all relieved by bringing into the picture the matter-fields that populate space-time.

Appendix

Counterexamples (see tabulation page 71)

(1) Spatio-temporal orientability conditions are not preserved under observational indistinguishability.

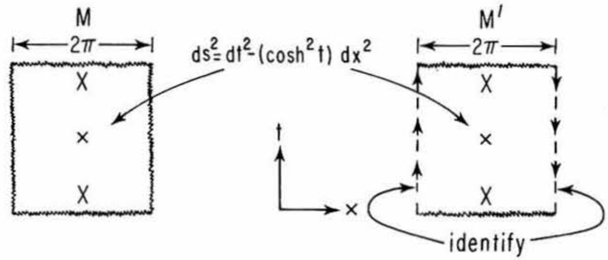
Definition: A space-time is:

(a) *temporally orientable* if it admits a continuous, nonvanishing timelike vector field;

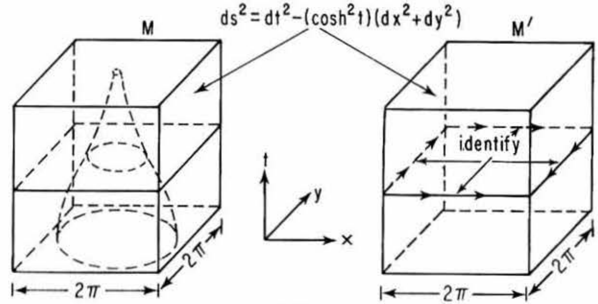
(b) *spatially orientable* if it admits three (or in general $n - 1$) continuous spacelike vector fields whose vectors are at every point linearly independent;

(c) *orientable* if it is both temporally and spatially orientable, or neither.

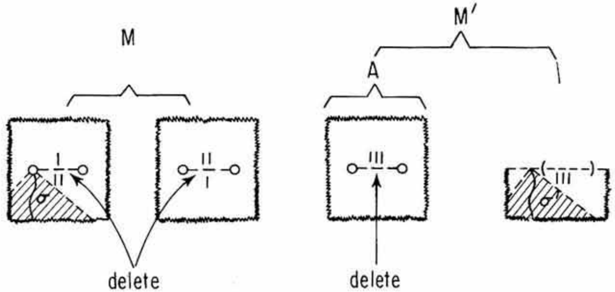
For (a), take M to be the covering space of two-dimensional De Sitter space-time (Figure 1) and form M' by first cutting a vertical strip of width 2π in M, twisting, and then identifying opposite sides. M' is topologically a Möbius strip. M and M' are observationally indistinguishable, but only M is temporally orientable (see figure).[9]



For (b) and (c), take M to be a three-dimensional version of the previous M. The observational past of every future-inextendible curve will be confined to a vertical square cylinder with sides of width 2π. Form M' by cutting out such a cylinder from M and then cross-identifying the t = constant surfaces, turning them into Möbius strips. M and M' are observationally indistinguishable; but while M is spatially orientable and orientable, M' is neither (see figure).



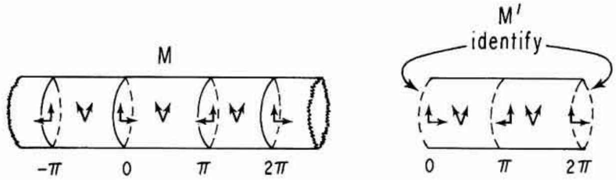
(2) The condition of inextendibility (see page 67) is not preserved under observational indistinguishability. The construction resembles that in Figure 5. Start with two copies of two-dimensional Minkowski space-time and excise from each the slit $I = \{(t, x): t = 0 \ \& \ 0 \leqslant x \leqslant 1\}$. Now identify the upper edge of each slit with the lower edge of the other (excluding end points). This will be M. It is inextendible. For M', start again with two copies of two-dimensional Minkowski space-time. From one cut the same slit $I = \{(t, x): t = 0 \ \& \ 0 \leqslant x \leqslant 1\}$. From the other cut away the closed upper half, leaving the set $\{(t, x): t < 0\}$. Now identify the upper edge of I with a corresponding section of unit width from the edge of the second space-time (see figure).



M, but not M', is inextendible. But they are observationally indistinguishable. To see this it suffices to check the few possibilities. Any future-inextendible timelike curve in M will find a counterpart in the "A section" of M'. But any future-inextendible curve on M' will also find a counterpart in M. In particular the curve σ' which runs off the edge of M' in its truncated portion finds a counterpart σ in M which runs to the "hole" (0, 0).

(3) The condition of noncompactness is not preserved under observational indistinguishability.

For M we take a two-dimensional "horizontal cylinder space-time" with metric $ds^2 = (\cos x)\, dxdt + (\sin^2 x)\,(dt^2 - dx^2)$. At $x = n\pi$ the light cones are tangent to the horizontal, pointing to the right for even n and to the left for odd n. At $x = (n + 1/2)\pi$ the cones are at 45° to the horizontal (see figure). The observational past of every future-inextendible curve in M is confined to a region of x-width 2π. M' is formed from M by identifying



points $x = 2n\pi$. Clearly M, but not M', is noncompact. But M and M' are observationally indistinguishable.

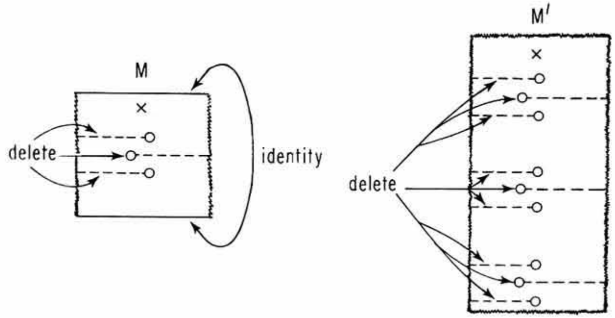
(4) The condition that there exist a global time function is not preserved under observational indistinguishability.

Definition: A smooth map t: $M \rightarrow \mathbf{R}$ is a *global time function* on the space-time M if for all x, y in M, $x < y \ \& \ x \neq y \rightarrow t(x) < t(y)$.

The condition that there exist a global time function is equivalent to the condition of "stable causality." [10]

For M start with two-dimensional rolled up Minkowski space-time and then make excisions (as in the figure) which just prevent null geodesics, which are aligned at 45°, from circumnavigating the manifold. This space-time is causal (and strongly causal), but does not admit a global time function. Any real valued function on M which increases along causal curves will be discontinuous somewhere.

(As it stands the space-time is extendible (we can replace the excisions). But without changing its cone structure or causal properties, we can render it inextendible by multiplying its metric by a conformal factor ϕ^2 which appropriately goes to ∞ as the slits are approached [see figure].) M'



is taken by "unrolling" M. M and M' are observationally indistinguishable since no observational past of any future-inextendible curve in either extends beyond the excision barriers. But only M' admits a global time function.

Notes

1. See also Clark Glymour, "Topology, Cosmology and Convention," *Synthèse* 24 (1972): 195–218.

2. A comprehensive treatment of work on the global structure of (relativistic) space-times is given in: S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge: Cambridge University Press, 1973). See also Roger Penrose, *Techniques of Differential Topology in Relativity* (Philadelphia: Society for Industrial and Applied Mathematics, 1972). More accessible than either is Robert Geroch, "Space-Time Structure from a Global Viewpoint," in B. K. Sachs, ed., *General Relativity and Cosmology* (New York: Academic Press, 1971).

3. A future end point need not be a point on the curve. The definition is this: If M is a space-time, I a connected subset of R, and $\sigma: I \to M$ a future-directed causal curve, a point x is the *future end point* of σ if for every neighborhood O of x there is a $t_0 \in I$ such that $\sigma(t) \in$ O for all $t \in I$ where $t > t_0$, i.e., σ enters and remains in every neighborhood of x.

4. A space-time is *strongly causal* if, given any point x and any neighborhood O of x, there is always a subneighborhood $O' \subset O$ of x such that no future-directed timelike curve which leaves O′ ever returns to it.

5. A countable cover of this form can be found in *any* space-time M, strongly causal or not. Since M is without boundary, for every y in M there is an x in M such that $y << x$, i.e., $y \in I^-(x)$. So the set $\{I^-(x): x \in M\}$ is an open cover of M. But M has a countable basis for its topology (Robert Geroch, "Spinor Structure of Space-Times in General Relativity I," *Journal of Mathematical Physics* 9 (1968): 1739–1744.) So by the Lindelöf Theorem there is a countable subset of $\{I^-(x): x \in M\}$ which covers M.

6. Robert Geroch, "Limits of Spacetimes," *Communications in Mathematical Physics* 13 (1969): 180–193.

7. See John Earman, "Laplacian Determinism in Classical Physics" (to appear) and Robert Geroch's paper in this volume.

8. Robert Geroch, "Domain of Dependence," *Journal of Mathematical Physics* 11 (1970): 437–449. (A somewhat different but equivalent definition of global hyperbolicity is used.)

9. There is a problem of how to define observational indistinguishability in a nontemporally orientable space-time (the definition given presupposed temporal orientation). But under any plausible candidate, M and M' in the example would come out observationally indistinguishable. One could associate with every inextendible timelike curve σ all the points that are connected with some point on the curve by another timelike curve. (In a temporally oriented space-time this would be the union $I^+[\sigma] \cup I^-[\sigma]$.) *Even these* sets in M and M' would find isometric counterparts in the other.

10. A space-time is *stably causal* if there are no closed causal curves *and* if there are no closed causal curves with respect to any metric close to the original. (This can be made precise by putting an appropriate topology on the set of all metrics on the space-time manifold.) Note that in the space-time M of the following example the slightest flattening of the light cones would allow timelike curves to scoot around the barriers. The equivalence is proven in S. W. Hawking, "The Existence of Cosmic Time Functions," *Proceedings of the Royal Society A*, 308 (1968): 433–435.

ROBERT GEROCH

Prediction in General Relativity

1. Introduction

There are at least two contexts within which one might place a discussion of the possibilities for making predictions in physics. In the first, one is concerned only with the actual physical world: one imagines that he has somehow learned what some physical system is like now, and one wishes to determine what that system will be like in the future. In the second, one is concerned only with the internal structure of some particular physical theory: one wishes to state and prove, within the mathematical formalism of the theory, theorems that can be interpreted physically in terms of possibilities for making predictions.

Of the two, the second context certainly seems to be the simpler and the more direct. Indeed, it is perhaps not even clear what the first context means. One's only guide in making a prediction in the physical world is one's past experiences in the relationship between the present and the future. But it is precisely the collection of these experiences, systematized and formalized, which makes up what is called a physical theory. That is to say, one seems to be led naturally from the first context to the second. One would perhaps even be tempted to conclude that the two contexts are essentially the same thing, were it not for the fact that it seems never to be the case in practice that one's past experiences lead in any sense uniquely to a physical theory; one must, at some point, make a choice from among several competing theories in order to discuss prediction. Thus one might divide a discussion of prediction in physics into two parts: (1) the choice of a physical theory and (2) the establishment and interpretation of certain theorems within the mathematical formalism of that theory.

Consider, as an example, Newtonian mechanics. Suppose that we wish to describe within this theory our solar system, which we idealize as

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