space defined on a four-dimensional manifold; what has physical significance is the quotient space of the constraint hypersurface within this function space over the mappings associated with the full gauge group of the theory. Even on a three-dimensional Cauchy hypersurface it appears risky to think primarily of a diagonalized configuration space (i.e., of a sharply defined three-dimensional metric g_{mn} on a three-dimensional spacelike hypersurface). Although the constraints restrict to some extent the range of the canonically conjugate variables, their uncertainty is unbounded, sufficiently so that the assumed sharpness of the 3-metric does not propagate at all.

Perhaps it is irrelevant whether we think of well-defined world-points with a fuzzy light cone, or conversely, of a sharp light cone, with considerable uncertainty as to which world-points lie on it. Most likely, both of these viewpoints are too naïve. Suppose we attempt a physical measurement, by means of instruments that had better not intrude too crudely on the physical situation, lest their large masses and stresses (if they are to contain any rigid components) modify the gravitational field far beyond the minimal effects required by the uncertainty relations. In elaborating what such an instrument measures we must discuss in detail not only which components of the fields are to be observed, but also in which space-time region these observations are to take place.

Perhaps it is just as well if I conclude my introductory remarks on this uncertain note, with all the technical and nontechnical connotations of "uncertain" you can imagine. It is this uncertainty that makes the whole field of quantum gravitation attractive to me.

WESLEY C. SALMON

The Curvature of Physical Space

If one were seriously to entertain, even in a highly programmatic fashion, the thesis "there is nothing in the world except empty curved space. Matter, charge, electromagnetism, and other fields are only manifestations of the bending of space,"[1] it would seem highly germane to examine the nature of this curvature, which is to serve as "a kind of magic building material out of which everything in the physical world is made."[2] Such an examination has been carried out in depth by Adolf Grünbaum in "General Relativity, Geometrodynamics, and Ontology," a chapter that appears for the first time in the new edition of his *Philosophical Problems of Space and Time*.[3] The present discussion is intended primarily as an addendum to that chapter[4]—although, I should hasten to add, not necessarily one that he would endorse.

1. Metrical Amorphousness

The question I shall be addressing can be phrased, "Does physical space possess intrinsic curvature?" This way of putting it is liable to serious misunderstanding on account of the term "intrinsic," for it would be natural to call the Gaussian curvature of a surface "intrinsic" because, as Gauss showed in his *theorema egregium*, it can be defined on the basis of the metric of the surface itself, without reference to any kind of embedding space. The mean curvature, in contrast, is not intrinsic in this sense. It is entirely uncontroversial to state that, *in this sense*, any Riemannian space—not just a two-dimensional surface—possesses an intrinsic curvature (possibly identically zero) which is given by the type (0, 4) covariant

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curvature tensor R_{jmhk}. For present purposes I shall, however, deliberately avoid use of the term "intrinsic" in this sense, and shall use the term ***internal curvature*** to characterize those types of curvature that can be defined in terms of the metric tensor g_{hj}—or more generally, those types of curvature that do not depend upon an embedding space. This use of the word "internal" is nothing more than a terminological stipulation made for purposes of this particular discussion. I hope it will prove convenient and not misleading or confusing.[5]

In line with the foregoing stipulation, when I now ask whether physical space has intrinsic curvature, I am asking a question that is similar and closely related to one discussed by Grünbaum in various of his works, namely, "Does physical space possess an intrinsic metric?"[6] In the context of Riemannian geometry, this question is motivated by the fact that the salient geometrical properties of a space—such as its Euclidean or non-Euclidean character, or the type of internal curvature it possesses—are determined entirely by congruence relations to which its metric gives rise. The converse is, of course, not true. The internal curvature—e.g., the Gaussian curvature for a two-dimensional surface—does not determine a metric uniquely (even up to a constant factor k). This fact does nothing to undermine Grünbaum's claim about the nonintrinsicality of the Riemannian curvature. Although a given curvature tensor R_{jmhk} of type (0, 4) does not determine a unique metric tensor g_{hj}, it does determine a unique class of metric tensors. Given two metric tensors from two such distinct classes, they do determine distinct curvature tensors.

In his well-known "theory of equivalent descriptions," Reichenbach maintains that, by employing different definitions of congruence, one and the same physical space can be described equivalently by the use of alternative geometries.[7] Different choices of coordinating definitions of congruence lead to different metrics, and these different metrics are such as to lead to different curvatures (or geometries). Reichenbach is not making merely an epistemological claim when he points to the existence of equivalent descriptions. He is arguing instead that since the descriptions are genuinely equivalent, they describe the same aspects of the same reality.[8] Given that the resulting descriptions are equivalent—including the fact that they are either both true or both false—a geometry involved in one but not the other of these descriptions cannot, in itself, represent an intrinsic characteristic of the space. This leads Reichenbach to his thesis of the *relativity of geometry*.[9]

Grünbaum has argued for the extrinsicality of the metric, and in consequence, the curvature, on the basis of what he has called "the metrical amorphousness of space." Appealing to an argument similar to one advanced by Riemann (which he calls RMH for Riemann's metrical hypothesis),[10] he maintains that the congruence or incongruence of two intervals in a continuous homogeneous manifold cannot be an intrinsic property of those intervals. Since I plan to use a similar argument below, it will be well to state it explicitly. I shall not present the argument in the same manner as Grünbaum, but I intend to offer a schematic restatement of his argument, not an alternative argument. Again, *he* might not regard it as a mere reformulation.

Although Riemann was obviously unaware of Cantor's theory of the cardinality of the linear continuum, he did seem to recognize that any closed linear interval is isomorphic to any other closed linear interval. In the jargon of set theory, any simply ordered, dense, denumerable set containing its end points is of the same order type as any other simply ordered, dense, denumerable set with end points. Even though the linear continuum is no longer considered denumerable, Riemann's basic notion is not invalidated, for it is easily proved that any closed, continuous, linear interval is of the same order type as any other closed, continuous, linear interval. Indeed, Cantor's proof that any two line segments ("regardless of length") have equal cardinality proceeds by establishing an *order-preserving* one-to-one correspondence between the points of the two intervals. Hence, if one regards a linear continuum merely as a point set that is ordered by a simple ordering relation, it follows that any closed interval of any linear continuum is isomorphic to any other closed interval of any linear continuum.

In dealing with "Zeno's metrical paradox of extension," Grünbaum takes pains to show how it is possible without contradiction to regard a linear continuum of finite nonzero length as an aggregate of points whose unit sets have "length" or measure zero.[11] This is done, essentially, by assigning coordinate numbers to the points on the line, and identifying the length (measure) of a nondegenerate closed interval with the absolute value of the difference of the coordinates of the end points.[12] Given the arbitrariness of the coordinatization of the line, it is evidently possible to assign coordinates to that line so as to yield *any* desired positive value as the length of *any* segment of that line. Since the coordinate number assigned to a point on the line obviously does not represent any intrinsic property of the point, we may say that the length of a segment or closed linear interval is an extrinsic property. And since the line can also be coordinatized in such a way that *any two* nonoverlapping segments receive the same length, we may add that equality or inequality of length is an extrinsic relation between them.[13]

We could say that two line segments are nonisomorphic only if we could invoke some further property or relation of the segments or of their constituent points such that no property-and-relation-preserving one-to-one correspondence between their members exists when the new property or relation is taken into account. This is Grünbaum's reason for insisting emphatically that continuity by itself is *not* sufficient to guarantee metrical amorphousness. Homogeneity is required in order to exclude possible further properties or relations that would destroy the isomorphism of any closed interval with any other nonoverlapping closed interval.[14] Although it may be hard to certify that, as points on the line, the elements have no property or relation that would render one segment nonisomorphic to another nonoverlapping segment, the obvious arbitrariness of the coordinatization lends strong prima facie plausibility to the supposition that no such properties or relations exist. The absence of any reasonable suggestions as to what properties or relations might render two nonoverlapping segments intrinsically equal or unequal in length lends stronger presumptive evidence to the claim that, as geometrical intervals on a line, any two nonoverlapping segments are isomorphic to each other, and that this isomorphism holds with respect to *all* of the *intrinsic* spatial properties and relations among the elements.[15]

The same considerations apply whether we are dealing with segments of a single one-dimensional continuum, or with all sorts of finite closed intervals on one-dimensional curves (which do not intersect themselves) in a continuous manifold of higher dimension. Given the fact that any segment of any curve is isomorphic to any other nonoverlapping segment of any curve, we have extremely wide latitude in the choice of a congruence relation and a metric. This exhibits a facet of the metric amorphousness of physical space. The full force of this metric amorphousness is revealed by the fact that a Riemannian space is coordinatized by regions, and that any coordinate region of an n-dimensional Riemannian space is isomorphic to any other coordinate region of any n-dimensional Riemannian space. The argument to support this latter claim is essentially the

same as that designed to show the isomorphism of nonoverlapping segments in the one-dimensional line. This isomorphism among coordinate neighborhoods of equal dimension will play a major role in the subsequent discussion.

If physical space is, in fact, metrically amorphous, then obviously it can, with equal legitimacy, be endowed with metrics that differ from each other nontrivially, even to the extent of giving rise to different curvatures. In that case, neither the metric nor the curvature based thereon can be held to represent intrinsic properties of the space. It is tempting to argue for the converse proposition: if a given manifold can, *with equal legitimacy*, be metrized by means of two different metric tensors, g_{hj} and $\bar{g}_{hj}$, which are associated, respectively, with two distinct curvature tensors, R_{jmhk} and $\bar{R}_{jmhk}$, then the curvature tensor R_{jmhk} does not reflect an intrinsic characteristic of that manifold. Appealing as this principle is, it must be treated with caution, as is shown by Grünbaum's discussion of the logical relations between alternative metrizability and metrical amorphousness.[16] However, for spaces composed of homogeneous elements—the most likely case for physical space—van Fraassen has offered a plausible account of alternative metrizability and metrical amorphousness that equates the two concepts.[17] The success of his program hinges upon our ability to recognize the difference between trivial variants of the same metric and pairs of metrics that differ significantly from each other. I have offered a different account of alternative metrizability which is also designed to exclude all cases in which the alternative metrizability rests solely upon the existence of trivial variants of a single metric.[18]

Even if we refuse to admit that, with suitable explicitly stated caveats, alternative metrizability of a space *entails* metrical amorphousness, it still seems reasonable to construe alternative metrizability as usually or frequently symptomatic of metrical amorphousness. Since I am not attempting to repeat in full detail the arguments for the metrical amorphousness of physical space, and the consequent extrinsicality of the Riemannian curvature given by the type (0, 4) curvature tensor, I shall simply accept the conclusion that alternative metrizability is, in this case, indicative of the relevant sort of amorphousness. I am thus inclined to agree with Grünbaum et al. that such extrinsic curvature does not seem to constitute a fundamental property of empty space that qualifies it as a "magic building material" from which everything else in the physical world is to be constructed. The main purpose of this discussion is to show that considerations of the same type furnish equally strong grounds for claiming that another type of curvature is likewise extrinsic.

2. The Mixed Curvature Tensor

Clark Glymour has quite properly pointed out that there is a type (1, 3) mixed curvature tensor $K_{h}{}^{j}{}_{mk}$ that can be defined on a differentiable manifold endowed with an affine connection, even if it does not possess a metric.[19] The existence of such a curvature tensor is not controversial. This shows that there is a type of curvature that does not depend upon a metric; consequently, it does not follow immediately from the thesis of the *metrical* amorphousness of space that space lacks intrinsic curvature of *this* type. At the same time, to show that curvature represented by the type (1, 3) tensor may exist independently of a metric does not show that this type of curvature is indeed an intrinsic property of space. Glymour's consideration shows simply that the intrinsicality of this type of curvature is still an open question.

Glymour's argument does nothing to vindicate the original geometrodynamic program of constructing everything in the physical world out of curved empty space; at best, it provides a temporary reprieve. For the question now becomes: is the curvature represented by the mixed tensor a genuinely intrinsic property of space, or is it extrinsic in precisely the same sense as the metric is extrinsic to the Riemannian manifold? The question can be rephrased: is space as amorphous with respect to the curvature tensor furnished by the affine connection as it is with respect to the metric tensor? This is the crucial question, but neither Glymour nor Grünbaum has addressed it.[20]

The answer to this question could be furnished, I believe, by means of the following consideration. We asked above whether a Riemannian manifold, endowed with a particular metric, could with equal legitimacy be described by means of a different metric—one that leads to a different curvature and a different geometry. In a completely parallel fashion, we can now ask whether a differentiable manifold, endowed with a particular affine connection yielding a particular mixed curvature tensor $K_{h}{}^{j}{}_{mk}$, could with equal legitimacy be endowed with a different affine connection that would yield a different curvature tensor $\bar{K}_{h}{}^{j}{}_{mk}$. If so, we can provide a pair of equivalent descriptions of the same manifold embodying different curvatures. We could then argue, along the same lines as Reichenbach

did with respect to the metric, that this type of curvature is also nonintrinsic, for it can vary nontrivially among equally legitimate descriptions of one and the same manifold. This amounts to an argument, similar to that based on alternative metrizability, which might be said to rest upon alternative connectability.

3. Affine Amorphousness

I shall now attempt to make a case for the view that the curvature associated with the type (1, 3) mixed tensor is extrinsic—in other words, that differentiable manifolds are amorphous, not only metrically, but also with respect to their affine connections. Let an n-dimensional differentiable manifold X_n be given. This manifold can, by definition, be covered by a finite number of overlapping coordinate neighborhoods; in each of these neighborhoods, every point can be assigned coordinates by means of a biunique continuous mapping of the points of the neighborhood onto n-tuples of real numbers. These n-tuples constitute an open subset of the n-dimensional space of real numbers R_n. Since R_n is obviously isomorphic to the Euclidean n-space E_n, the coordinatization of any coordinate neighborhood of our differentiable manifold X_n establishes an isomorphism between that coordinate neighborhood and an open n-dimensional region of E_n. This is, of course, a local isomorphism between a region of X_n and a region of E_n; it is not possible in general to extend this isomorphism to the entire manifold X_n, for it is not possible in general to cover the entire manifold with any single system of coordinates.

The question I am raising is, however, a local question. I am attempting to clarify the relationship between the curvature associated with the type (0, 4) covariant curvature tensor R_{jmhk} and that associated with the type (1, 3) mixed curvature tensor $K_{m}{}^{j}{}_{hk}$. The two types of tensors are defined at each point of their respective manifolds. The question of the metrical amorphousness of space is a local matter; it does not involve the global topological characteristics of the space. In raising the question of the intrinsicality of the curvature represented by the mixed curvature tensor, I am putting aside the global considerations in precisely the same fashion. In dealing with the nature of the affine connection and the curvature based thereon, it will therefore be sufficient to restrict attention to one coordinate neighborhood.

To state this point explicitly is to give the whole show away. As already remarked, the coordinate neighborhood of X_n is isomorphic to a region of

Euclidean n-space. If I am correct in saying that the fundamental basis for claiming that Riemannian space is metrically amorphous is the isomorphism of any finite closed interval to any other, then it would seem plausible to maintain that the isomorphism of any coordinate region of X_n to some region of Euclidean n-space has a fundamental bearing upon the question of whether the differentiable manifold is amorphous with respect to the curvature based upon the affine connection. It shows that any differentiable manifold that can be endowed with any affine connection whatever may be endowed locally with a connection whose components are identically zero in some given coordinate system. Obviously, the curvature tensor based upon this connection will also have components that vanish identically, and this property holds in *all* admissible coordinate systems.

The affine connection is not a tensor; under special circumstances it may therefore vanish with respect to some sets of coordinates but not with respect to others. For instance, while it vanishes identically for a Euclidean space with Cartesian coordinates, it does not vanish for the same space referred to curvilinear coordinates. But the related curvature is tensorial, and if it vanishes in one coordinate system it will vanish in all. This means that any coordinate region of any differentiable manifold may legitimately be provided with a set of coordinates and an affine connection such that the type (1, 3) mixed curvature tensor vanishes. It is easy to see an important analogy here between the two types of curvature. Given even a non-Euclidean space, such as the surface of a sphere or the surface of a torus, it is possible, on account of metrical amorphousness, to remetrize an arbitrary region (provided it is not too large) in such a way that the region becomes Euclidean and its Riemannian curvature vanishes throughout that region. In a completely analogous way, any coordinate region of a differentiable manifold can be coordinatized and endowed with an affine connection such that its mixed curvature tensor vanishes throughout that region. This shows, I believe, that the *presence* of nonvanishing curvature of the type indicated by the type (1, 3) mixed curvature tensor cannot be an intrinsic local property of a coordinate neighborhood of a differentiable manifold.

One further point should be added. Because of the transitivity of the isomorphism relation, the foregoing argument shows that any coordinate neighborhood of a differentiable manifold X_n is isomorphic to any other coordinate neighborhood of any differentiable manifold of the same dimension n. Thus, if any coordinate neighborhood of such a manifold can be endowed with an affine connection which gives rise to a nonvanishing curvature, then any other coordinate neighborhood of equal dimensionality can be endowed with the same connection and the same nonvanishing curvature. Given the obvious fact that some spaces are *so endowed with a metric or affine connection* that they are flat,[21] we may thus conclude that *absence* of nonvanishing curvature (i.e., the presence of zero curvature) of the sort associated with the type (1, 3), mixed curvature tensor is not an intrinsic local property of a differentiable manifold. In other words, a coordinate neighborhood of a differentiable manifold is neither intrinsically flat nor intrinsically nonflat.

4. Parallelism

I do not wish to rest the argument there, however, for I believe it can be made more compelling by considering the nature and function of the affine connection. Let us, therefore, look at the grounds for introducing such connections. In order to deal with certain kinds of physical and geometrical problems, we introduce tensors of various types, including vectors as special cases. At each point of our differentiable manifold X_n we construct a series of vector spaces—type (r, s) tensors at a given point constituting the members of an n^{r+s}-dimensional tangent vector space. The vectors or tensors that are elements of these tangent vector spaces are *not* elements of the differentiable manifold X_n; they are members of abstract vector spaces associated with the points p of X_n.[22] Such algebraic operations as addition, multiplication, and contraction are performed on the elements of the vector spaces associated with one and the same point p of X_n. At this stage of the analysis, the vectors that are elements of these various vector spaces have no physical or metrical significance; they are simply elements of an abstract mathematical structure.

There are many circumstances in which we must deal with relationships among vectors or tensors located at different points of X_n. For this purpose, we introduce vector or tensor fields. For the present discussion it will be sufficient to confine attention to contravariant vectors, i.e., type (1, 0) tensors. A contravariant vector field is simply a collection of uniquely specified contravariant vectors, each of which is associated with a distinct point of a region of X_n. The vector field thus consists of members of the various tangent spaces of contravariant vectors associated with the different points of the region of X_n over which the vector field is defined, one member being selected from each such tangent vector space. Of particular importance is the so-called *parallel vector field*.

We may, indeed, define the concept of parallel displacement. The general idea is this. We say loosely that a vector can be moved about in a Euclidean space, and as long as it retains the same length and direction it is still the same vector.[23] In the very special case of Euclidean space and Cartesian coordinates, this condition is equivalent to saying that its components are always the same, regardless of its point of application. A better way to say the same thing is to say that we can define a parallel vector field consisting of one vector at each point of the space, and that relative to the Cartesian coordinates, each of these vectors has precisely the same components. If this same Euclidean space is recoordinatized with curvilinear coordinates, the *same* parallel vector field is defined in a different way. Given a contravariant vector X^j at a point p with coordinates x^k, a vector $X^j + dX^j$ at a neighboring point q with coordinates $x^k + dx^k$ results from parallel displacement or is a member of the same parallel vector field as X^j if its components satisfy the equations

$$dX^j + \{{}_h{}^j{}_k\}\, X^h\, dx^k = 0 \tag{1}$$

where $\{{}_h{}^j{}_k\}$ is a Christoffel symbol of the second kind. Vectors related in this way are said to have the same magnitude and direction. It is of crucial importance to be clear about what is going on here. Given a particular vector at p, which is a member of a tangent space at p, we associate with it a unique member of the corresponding tangent space at q. Indeed, by means of the Christoffel symbol we establish an isomorphism between the members of the tangent space at p and those of the tangent space at the neighboring point q. It is in this sense that we specify vectors at q which uniquely correspond with vectors at p.

An analogous procedure can be carried out, not only in Riemannian spaces in general, but also in differentiable manifolds that do not possess a metric, provided they are endowed with an affine connection. To handle this more general situation, one defines parallel displacement of a vector from point p to point q along a curve C (defined by a parameter t) by the condition

$$X^j(t) = X^j_{(0)} - \int_{C\,t_0}^{t} \Gamma_h{}^j{}_k\, X^h\, \frac{dx^k}{dt}\, dt, \tag{2}$$

where $\Gamma_h{}^j{}_k$ is the affine connection, and t_0 corresponds to the point p. This equation serves to define the differential dX, enabling us to write (2) in differential form:

$$\frac{dX^j}{dt} + \Gamma_h{}^j{}_k\, X^h\, \frac{dx^k}{dt} = 0 \qquad \text{along } C(t) \tag{3}$$

For arbitrary points p and q, the specification of a vector at q as parallel to a given vector at p may not be unique, but may depend upon the choice of the curve $C(t)$. For a point q in the neighborhood of p, however, a unique vector $X^j + dX^j$ corresponding to the vector X^j at p is given by the condition

$$dX^j = -\,\Gamma_h{}^j{}_k\, X^h\, dx^k \tag{4}$$

Thus $\Gamma_h{}^j{}_k$ provides a unique local mapping of the tangent space $T_n(p)$ onto $T_n(q)$.

Since the function of the affine connections is to establish isomorphisms between the members of vector spaces located at neighboring points of our differentiable manifold, it is natural to ask what restrictions are to be imposed. Given any two vector spaces of equal dimension, there is obviously a vast array of possible isomorphisms to choose from. In a Euclidean space (where the metric is given) we want all of the members of a parallel vector field to have the same magnitude and direction. In the absence of a metric, however, no sense can be attached to the question: are the members of the two spaces that are correlated by a given isomorphism *really* parallel (equal) to each other? As entities that are correlated with one another by the isomorphism of (4), they are parallel *by definition*, regardless of the particular isomorphism involved.

As equation (4) shows, the parallel vector field is defined in terms of relations between the respective components of the vectors at the two neighborhing points p and q. Given the continuity conditions imposed in characterizing a differentiable manifold, it is natural to impose continuity requirements upon the components of the vector under parallel displacement. This is, of course, built into the definition of the affine connection. In addition, the connection must have certain properties with respect to coordinate transformations. This is obvious from the following consideration. If a particular vector X at a point p is chosen, it will have certain components X^j relative to a system of coordinates x^j. If a coordinate transformation to a system $\bar{x}^k$ is performed, the *same* point p will have a different set of coordinates, and the *same* vector X will have components $\overline{X}^k$ that are different functions of its new coordinates. The same is true of the vector $X + dX$ located at the point q, which is said to belong to the same parallel vector field as X. The condition that must be satisfied by the affine connection is that parallelism is invariant under coordinate transformations. Two vectors which are parallel with respect to one set of coordinates must also be parallel with respect to any other set of coordinates. This fact is exhibited clearly when we state the defining relation in terms of the absolute differential

$$DX^j = 0 \qquad \text{(along } C\text{)} \tag{5}$$

which is equivalent to equation (3). This property of the affine connection is secured by demanding that it obey the transformation law

$$\overline{\Gamma}_{m}{}^{j}{}_{p} = \frac{\partial \bar{x}^j}{\partial x^n} \frac{\partial x^h}{\partial \bar{x}^m} \frac{\partial x^k}{\partial \bar{x}^p} \Gamma_h{}^n{}_k - \frac{\partial^2 \bar{x}^j}{\partial x^h \partial x^k} \frac{\partial x^h}{\partial \bar{x}^m} \frac{\partial x^k}{\partial \bar{x}^p} \tag{6}$$

Moreover, *any* set of 3-index symbols that satisfy this law qualify as an affine connection. This law, which embodies the necessary invariance conditions, constitutes the only restriction on the affine connection. Looking at this transformation law, one would hardly be tempted to suppose that it determines a unique affine connection. This impression is correct.

5. A Simple Example

Let us consider the situation concretely by reference to a simple example. Suppose we have two two-dimensional differentiable manifolds X_2 and Y_2; we confine attention to a coordinate neighborhood of each. Thus, in the region of X_2 which we are examining, we have a coordinate system x^1, $\bar{x}^2$, and in our region of Y_2 we have a coordinate system y^1, y^2. We endow X_2 with an affine connection whose components are, relative to the coordinate system $(\bar{x}^1, \bar{x}^2)$, identically zero. That this is a legitimate set of connection components is evident from the fact that it is the appropriate connection to use in Euclidean 2-space referred to Cartesian coordinates. We may introduce a new set of coordinates (x^1, x^2) according to the transformation

$$\begin{aligned} \bar{x}^1 &= x^1 \cos x^2 \\ \bar{x}^2 &= x^1 \sin x^2 \end{aligned} \tag{7}$$

which is recognized immediately as a transformation from Cartesian to polar coordinates. (We assume that the pole is not in the coordinate

neighborhood we are discussing.) With this new system of coordinates is associated a new affine connection, which is identical with the Christoffel symbol of the second kind in this simple example:

$$\begin{aligned} \Gamma_2{}^1{}_2 &= -x^1 \\ \Gamma_2{}^2{}_1 &= 1/x^1 = \Gamma_1{}^2{}_2 \end{aligned} \tag{8}$$

all other components being zero. Routine calculation shows that the curvature tensor $K_h{}^j{}_{mk}$ based upon this connection, like that based on the connection all of whose components vanish, is identically zero.

We may note at the same time that the affine connection is sufficiently lacking in intrinsicality that it would have been entirely possible to assign the identically vanishing affine connection to our unbarred system of coordinates (x^1, x^2), or the nonvanishing affine connection to the barred coordinates $(\bar{x}^1, \bar{x}^2)$.[24] The differentiable manifold X_2 has no intrinsic characteristics that render one set of coordinates intrinsically Cartesian and another intrinsically polar.[25] Of course, this example, though it has some heuristic value, does not make the real point I am driving at. For although we have seen that the choice between the two affine connections does not rest upon any intrinsic property of the manifold X_2, the two connections yield the same curvature—namely, identically zero—and hence the space is flat under either description. This feature is, however, peculiar to this particularly simple example; it does not arise from any intrinsic characteristic of our differentiable manifold which demands an affine connection that will render it flat.

Let us therefore look at our other differentiable manifold Y_2. In the neighborhood under consideration, we supply coordinates (y^1, y^2). We then endow it with an affine connection whose components are

$$\begin{aligned} \Gamma_2{}^1{}_2 &= -\sin x^1 \cos x^1 \\ \Gamma_1{}^2{}_2 &= \Gamma_2{}^2{}_1 = \operatorname{ctn} x^1, \end{aligned} \tag{9}$$

all others being zero, which are nothing but the Christoffel symbols of the second kind associated with the usual metric $ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ for the surface of a sphere of unit radius. When we calculate the components of the curvature tensor $K_h{}^j{}_{mk}$ we find that they do not vanish identically. In particular,[26]

$$K_1{}^2{}_{12} = 1 \tag{10}$$

Our neighborhood of Y_2 is not flat.

Since, however, our coordinate neighborhood of Y_2 is isomorphic to our coordinate neighborhood of X_2, there is no reason why we could not endow X_2 with the same affine connection we imposed upon Y_2, and conversely. Although the simple examples of affine connections I have discussed are all Christoffel symbols associated with familiar metrics, it is clear that the situation would be no different in principle if we were to deal with nonsymmetric connections that are not related to any metric at all. One could, in fact, write down an arbitrary set of differentiable functions as connection components, attach them to some definite coordinate system x^j, and *stipulate* that the components of that connection for any other coordinate system $\bar{x}^k$ be given by equation (6). The connection is *that* arbitrary, and *that* insensitive to selection of an original coordinate system to which it is to be attached.

6. Physical Manifolds

There may be some feeling that this discussion, thus far, has rested too heavily upon mathematical considerations, to the neglect of relevant physical factors. I have, it is true, emphasized such aspects as the isomorphisms among coordinate neighborhoods of equal dimension, and the alternative metrics and affine connections that are abstractly possible in such regions of differentiable manifolds. This emphasis is *not* the result of a mistaken notion that the problem under discussion is one of pure mathematics; rather, it comes from an attempt to compare the extrinsicality of the curvature exhibited by the type (0, 4) curvature tensor with that of the type (1, 3) curvature tensor. In constructing this comparison, I have tried to focus upon those features of physical space that underly the potent arguments of Reichenbach and Grünbaum in support of the thesis of extrinsicality of the metric (and associated curvature). The crux of the argument seems to me to hinge upon the kinds of isomorphisms I have mentioned. If such arguments for extrinsicality are to be defeated, it is necessary to show what intrinsic properties of the physical manifold, over and above its structure as a differentiable manifold of given dimension, can be invoked to undermine the alleged isomorphisms of segment with segment, or neighborhood with neighborhood. With regard to the metric and the curvature associated with it, I have explicitly stated my agreement with Reichenbach, Grünbaum, et al. The only question remaining is whether a continuous physical manifold (of space or space-time) might be said to possess an intrinsic affine structure that determines a unique affine

connection, and consequently, an intrinsic curvature of the sort represented by the type (1, 3) tensor.

If one were to accept the Reichenbach-Grünbaum argument, then, it seems to me he would be hard put to imagine what sort of intrinsic structure of space (or space-time) could support the claim that the affine connection is uniquely determined. We recall that the function of the affine connection is to establish a biunique correspondence between vectors in a tangent space at point p and those in another tangent space at the neighboring point q. It is difficult to see what intrinsic property of the underlying manifold could determine which isomorphism is the correct one to represent parallel displacement of vectors. What conceivable intrinsic property could it be, and in what way could it compel the choice of an isomorphism? At this stage of the discussion, it seems to me that the burden of proof (or the burden of suggestion, at least) shifts to the proponent of intrinsicality.

One way to use the concept of parallel displacement is in the definition of an *autoparallel* curve as a curve whose tangent vectors are parallel to one another. Such autoparallel curves, or *paths* as they are sometimes called, bear striking resemblance to the stright lines of Euclidean spaces and the geodesic curves of Riemannian spaces. In a physical theory that employs an affinely connected differentiable manifold, the autoparallel curves may be interpreted as trajectories of gravitational test particles. At this juncture it is essential to remember that gravitational test particles are as extrinsic to a spatial or spatio-temporal manifold as Einstein's rods and clocks.[27] Just as one can argue for the metrical amorphousness or alternative metrizability of physical space on the basis of alternative admissible coordinating definitions of congruence, so also can one argue for "affine amorphousness" or "alternative connectability" on grounds of the possibility of alternative affine connections.[28]

When we endow our physical manifold with one affine connection, we may notice that gravitational test particles have autoparallel curves as trajectories, while with another affine connection we find that the trajectories of these test particles are not autoparallel. Such an observation would be entirely analogous to the commonplace that certain kinds of solid bodies remain self-congruent wherever they are located if our space is endowed with one metric, while under a different metric these same solid objects change their size as they are transported from place to place.

And just as we can ask (1) whether physical space has some intrinsic structure that determines whether the amount of space occupied by the measuring rod in one place is equal to the amount of space occupied by the same rod in another place, so also must we ask (2) whether physical space has an intrinsic structure that determines whether the tangent vectors of the particle trajectory are *really* parallel or not.[29] If, on the one hand, one accepts the Reichenbach-Grünbaum negative answer to question (1), it is difficult to see how he could then go on to answer question (2) affirmatively. At this point, it seems to me, the burden of proof becomes acute. If, on the other hand, one wants to avoid the embarrassment of trying to give an affirmative answer to one of these questions while giving a negative answer to the other, it becomes necessary either to refute the powerful arguments that have been advanced for the negative answer to (1), or else admit that the affine structure of physical space is no more intrinsic than its metric structure.

7. Conclusions

The considerations advanced in this discussion seem to me to lend strong support—albeit inductive support—to the view that curvature is not an intrinsic local property of physical space, whether that curvature be of the kind associated with the type (0, 4) tensor or that associated with the type (1, 3) tensor. Thus Glymour's observation that there exists a curvature tensor which is independent of any metric does nothing to show that there is a kind of curvature which is intrinsic, and which could therefore be employed by geometrodynamicists as a "magic building material" from which to construct "the furniture of the world."[30]

Appendix
Alternative Metrizability of Space

The concept of metrical amorphousness, and the related concepts of alternative metrizability and intrinsicality of metrics, are difficult to define in the generality needed to cover wide varieties of *mathematically* interesting spaces, as is attested by the volume and complexity of recent literature on the subject.[31] As Gerald J. Massey has written, it would have been reasonable to interpret some of Grünbaum's earlier writings as maintaining that the concepts of metrical amorphousness and alternative metrizability are interchangeable.[32] In his recent detailed investigation of

intrinsic metrics, however, Grünbaum explicitly rejects this equivalence by exhibiting a manifold which possesses a nontrivial intrinsic metric—thus disqualifying it from metrical amorphousness—but which possesses at least two nontrivially distinct intrinsic metrics.[33] At the same time, van Fraassen has offered an account of alternative metrizability *for a restricted class of spaces* which seems to allow for the equivalence of metrical amorphousness and alternative metrizability within the range of his discussion.[34]

In this note, I shall not attempt to lay down necessary and sufficient conditions for the general applicability of the concepts mentioned above; rather, I shall attempt to enunciate a sufficient condition which seems to me to capture their import as applied to the sorts of manifolds that have figured prominently in discussions of the structure of physical space, physical time, and physical space-time. For this purpose, it seems to me, sufficient generality is achieved if we can explain the applicability of these concepts to differentiable manifolds—roughly, spaces of dimension one or greater that satisfy certain continuity requirements. The foundation for the whole development will be the concept of *unconditional alternative metrizability*, which I shall proceed to define. To what extent this concept can be usefully applied to spaces that are not differentiable manifolds is not clear to me at present.

We begin by regarding a differentiable manifold X_n of finite dimension n (≥ 1) as a space of dimension n that can be covered by a finite number of coordinate neighborhoods, each of which can be provided with a coordinate system. A coordinate system for a coordinate neighborhood is an assignment of n-tuples of real numbers to the points of that neighborhood; specifically, it is a bicontinuous one-to-one mapping of a region of the n-dimensional space R_n of real numbers onto the points of the coordinate neighborhood of X_n. Since R_n is isomorphic to the n-dimensional Euclidean space E_n, we can equally well conceive the coordinatization of a neighborhood of X_n as a one-to-one bicontinuous mapping of that neighborhood onto the points of a region of E_n which has already been endowed with Cartesian coordinates. The coordinates of the points of our neighborhood of X_n are simply the coordinates of their image points in E_n under that mapping. Since the supply of one-to-one bicontinuous mappings is very large, a given coordinate neighborhood of X_n may be outfitted with a wide variety of distinct systems of coordinates. It should be explicitly stated, moreover, that the region of E_n involved in the coordinatization of a given neighborhood of X_n is by no means uniquely determined by the choice of the neighborhood of X_n.

With the wide latitude of choice of coordinate systems available, it is useful to consider tranformations from one system of coordinates to another. If we begin with a set of coordinates x^i $(i = 1, \ldots, n)$, we can transform to a new set of coordinates $\bar{x}^j$ $(j = 1, \ldots, n)$, where

$$\bar{x}^j = f^j(x^1, \ldots, x^n), \tag{11}$$

and the functions f^j are continuous, possess continuous partial derivatives to some specified order, and possess inverses. For present purposes, let us assume that the functions f^j are of a class C^∞. Any system of coordinates that results from an admissible coordinate system by such a transformation is considered an admissible coordinate system.

Let us now endow our coordinate neighborhood of X_n with a metric g_{hk}, which is a field of type (0, 2) symmetric tensors. At each point P of our neighborhood we have n^2 quantities that are given as functions of the coordinates of the point. To keep the functional dependency of the components of g_{hk} on the coordinates x^i clearly in mind, let us write explicitly

$$g_{hk} = g_{hk}(x^i) = g_{hk}(x^1, \ldots, x^n). \tag{12}$$

Each member of our tensor field is attached to a point of our coordinate neighborhood, and the point is identified by its coordinates x^i.

Let us now recoordinatize our neighborhood by performing a coordinate transformation to a new set of coordinates $\bar{x}^j$. Let us further define a *new* tensor field on the same coordinate neighborhood by the rule,

$$g_{hk}(x^i) = \tilde{g}_{hk}(\bar{x}^i). \tag{13}$$

This simply has the effect of carrying a tensor attached to a point p with coordinates x^i in the old coordinate system to a *different* point $\bar{p}$ whose coordinates $\bar{x}^j$ relative to the new coordinate system are equal to those of p relative to the old system, i.e.,

$$x^i(p) = \bar{x}^i(\bar{p}). \tag{14}$$

This *new* tensor field $\tilde{g}_{hk}$ is a *different* metric for our coordinate neighborhood.

We may now ask whether the two metrics g_{hk} and $\tilde{g}_{hk}$ are *equally legitimate* metrizations of our coordinate neighborhood. This is a difficult

question, to which such authors as Poincaré, Reichenbach, and Grünbaum have devoted considerable attention.[35] It seems to me that they have effectively established a general affirmative answer to the question of equal legitimacy; for example, this would seem to be the import of Reichenbach's theory of equivalent descriptions of physical space based upon the admissibility of alternative coordinating definitions of congruence. It is not necessary to try to argue the case again here; for purposes of the present discussion, I shall simply assume that we know how to answer the question at issue. I now propose the following definition:

Definition 1. A coordinate neighborhood of X_n is *unconditionally alternatively metrizable* if and only if every remetrization of the type defined in (13) with respect to *every admissible coordinate* system for that neighborhood is an equally legitimate metrization of that neighborhood.

In other words, suppose we begin with a legitimate metrization of our neighborhood, then perform a coordinate transformation as given in (11), and finally introduce a new metric according to (13). If this *always* results in an equally legitimate metrization of the neighborhood, the neighborhood is unconditionally alternatively metrizable.

The idea behind this definition is straightforward. If the legitimacy of a metrization of a space (or a coordinate neighborhood thereof) is totally insensitive to coordinate changes, then, in view of the evident arbitrariness of the assignment of coordinates, the metric cannot represent any intrinsic property of that space. Consider a simple example. Suppose we begin with a two-dimensional space which we recognize intuitively as a Euclidean plane, and suppose further that it is provided with a coordinate system which we recognize intuitively as Cartesian. The metric is given by

$$ds^2 = (dx^1)^2 + (dx^2)^2; \; g_{hk} = \delta^h_k \tag{15}$$

We now transform to a coordinate system we recognize intuitively as polar (with the pole falling outside our coordinate neighborhood).[36] Nevertheless, we do not change the *form* of the metric, writing

$$d\tilde{s}^2 = (d\bar{x}^1)^2 + (d\bar{x}^2)^2; \; \tilde{g}_{hk} = \delta^h_k \tag{16}$$

This constitutes a drastic change of metric, in the sense that the distance between two points p and q will in general be changed, and intervals that were congruent under the old metric will in general not be congruent under the new metric. Consider any two pairs of points (p_1, q_1) and (p_2, q_2) that represent equal intervals in the old metric, but unequal intervals in the new metric. If there is no intrinsic property of these two intervals that determines that they are either intrinsically congruent or that they are intrinsically incongruent, then there is no valid basis for maintaining that one (at most) of these metrizations is legitimate—that at least one of them must be illegitimate. If it can be argued validly that there are no intrinsic properties of this space that show that the congruences delivered by one of these metrics are correct while those produced by the other are incorrect, then that argument for the legitimacy of this kind of remetrization is an argument for the metrical amorphousness of the space.

As I said at the outset, I do not claim to have provided necessary and sufficient conditions for the applicability of such concepts as alternative metrizability and metric amorphousness in all generality.[37] I am prepared, however, to offer the following sufficient conditions:

Condition 1. A coordinate neighborhood of X_n is *alternatively metrizable* if it is unconditionally alternatively metrizable.

Condition 2. A coordinate neighborhood of X_n is *metrically amorphous* if it is unconditionally alternatively metrizable.

Condition 3. A coordinate neighborhood of X_n *has no intrinsic metric* if it is unconditionally alternatively metrizable.[38]

Unconditional alternative metrizability may be a fairly strong condition; nevertheless, it seems to exhibit a type of alternative metrizability from which metrical amorphousness can reasonably be inferred. It is strong enough to rule out those cases of alternative metrizability in which all of the alternative metrics can be considered, in any sense, trivial variants of one another. Moreover, I am convinced that the arguments of Poincaré, Reichenbach, Grünbaum, et al. are sufficient to qualify such continuous *physical* manifolds as physical space, physical time, and physical space-time for unconditional alternative metrizability. Conditions 1–3 thus seem to apply to the physical manifolds of interest.

Notes

1. J. A. Wheeler, *Geometrodynamics* (New York: Academic Press, 1962), p. 225.
2. J. A. Wheeler, "Curved Empty Space-Time as the Building Material of the Physical World," in E. Nagel, P. Suppes, and A. Tarski, eds., *Logic, Methodology and Philosophy of Science* (Stanford, Calif.: Stanford University Press, 1962), p. 361.

3. Adolf Grünbaum, *Philosophical Problems of Space and Time*, 2nd ed. (Dordrecht/Boston: Reidel, 1973), chap. 22. In this chapter Grünbaum carefully documents Wheeler's *recent* renunciation of his earlier pure geometrodynamic thesis.
4. In particular, the present discussion is a supplement to section 3b—an extended footnote to p. 788, it might be said.
5. Grünbaum, *Philosophical Problems*, p. 501, carefully distinguishes between "intrinsic" (German: *inner*) in the sense applicable to Gaussian curvature, and "intrinsic" or "implicit" (German: *schon enthalten*) in the sense in which Riemann denied it with respect to the metric of continuous space. It is in this latter sense that I am using the term "intrinsic" in the present discussion. For the former concept, I am using the term "internal."
6. See chiefly Grünbaum, *Philosophical Problems*, chap. 16, "Space, Time and Falsifiability," which also appeared in *Philosophy of Science* 37 (1970): pp. 469–588.
7. Hans Reichenbach, *Philosophy of Space and Time* (New York: Dover Publications, 1958). This point had been argued with great cogency by Poincaré much earlier.
8. Reichenbach does, it is true, adopt a verifiability criterion of equivalence, but he construes this as a criterion of what can be meaningfully said about physical reality. Thus the claim that physical space actually has one of these geometries and not the other is without any possible justification, and would constitute a totally unwarranted claim *about reality* (not merely about our knowledge).
9. Some authors, including Grünbaum, prefer the term "conventionality" to "relativity" in such contexts. Reichenbach may have eschewed the former term to avoid any suggestion that he was adopting a *thoroughgoing* conventionalism of the sort be found in Poincaré. The term "relativity" emphasizes the fact that, in a given physical space, the geometry is relative to a choice of congruence standard—i.e., given a coordinating definition of congruence, the geometry of the space is a matter of empirical fact. In the presence of suitable coordinating definitions, there is nothing conventional about the geometry.
10. Grünbaum, *Philosophical Problems*, pp. 495–99, 527–32, has offered a detailed statement and elaboration of RMH.
11. Grünbaum, *Philosophical Problems*, chap. 6, especially pp. 170–72.
12. By a closed interval $[a, b]$ on a line I shall mean the set consisting of points a and b and all points lying between them. The interval is degenerate only if $a = b$. In this discussion I shall use the term "interval" to refer only to nondegenerate intervals. The term "segment" will be used as a synonym for "nondegenerate closed interval."
13. The continuity of the line, which *is* an intrinsic structural feature, is reflected in the fact that the coordinatization of the line is a continuous one-one relation between the points of the line and the continuum of real numbers. We specify that the intervals be nonoverlapping, for the inclusion of one interval entirely within another is an intrinsic relation between them, and this relationship should be reflected in the metric.
14. This point is brought out clearly in Grünbaum's comparison of the geometrical continuum of points with the arithmetical continuum of real numbers; *Philosophical Problems*, pp. 512–14, 526–31.
15. Grünbaum, *Philosophical Problems*, pp. 498–501, 529–31 explicitly acknowledges that RMH is only inductively confirmed, not a demonstrated truth. The strength of support for RMH would seem at least adequate to shift the burden of the argument to its opponents.
16. Grünbaum, *Philosophical Problems*, pp. 547–56.
17. Bas C. van Fraassen, "On Massey's Explication of Grünbaum's Conception of Metric" *Philosophy of Science* 36, no. 4, December 1969.
18. In the Appendix "Alternative Metrizability of Physical Space," I have defined *unconditional alternative metrizability* as a type of alternative metrizability that excludes cases in which the alternative metrics are simply trivial variants of one another. Unconditional alternative metrizability does, I claim, entail metrical amorphousness.
19. Clark Glymour, "Physics by Convention," *Philosophy of Science* 39, no. 3, September 1972.
20. Grünbaum's answer to Glymour's attempt to impugn Grünbaum's philosophical attack

upon the geometrodynamic program follows an entirely different tack; see Grünbaum, *Philosophical Problems*, pp. 773–88.

21. When I use the term "flat", it is to be construed in the technical sense of the vanishing of the type (1, 3) curvature tensor.

22. This is true of the metric tensor g_{hk} as well as any others—a fact which has not, to my knowledge, been mentioned explicitly in discussions of intrinsicality of metrics.

23. Because of its linearity, Euclidean space may be identified with a tangent space.

24. Note that this is precisely the sort of insensitivity to coordinate changes on the part of the affine connection as I discussed with respect to the metric in the Appendix "Alternative Metrizability of Physical Space." It is as indicative of "affine amorphousness" in this case as it was indicative of metrical amorphousness in the other case.

25. Remember that the origin is not within our coordinate neighborhood.

26. In 2-space the curvature tensor has only one *independent* component.

27. Grünbaum has argued in detail that Einstein's rods-and-clocks method, Synge's chronometric method, and the geodesic method of Weyl et al. depend equally upon extrinsic standards for the determination of the space-time metric. *Philosophical Problems*, chap. 22, section 2.

28. Just as one definition of congruence might be preferable to another on grounds of *descriptive* simplicity, so also might one affine connection be preferable to another on the same grounds. Lack of descriptive simplicity does not render either a metric or an affine connection inadmissible as factually incorrect.

29. In discussion, Howard Stein made reference to a particle "sniffing out" a path through space. If we construe the term "path" as autoparallel, then the question becomes: on the basis of what "olfactory" characteristics of physical space itself can an autoparallel be detected? To "sniff out" a path requires discriminable odors, as any bird dog knows, but in a space of *homogeneous* elements, such differences in odor cannot possibly be intrinsic characteristics.

30. Whether topological structure could provide the requisite curvature is, of course, an entirely different question.

31. Especially: Gerald J. Massey, "Toward a Clarification of Grünbaum's Concept of Intrinsic Metric," *Philosophy of Science* 36, no. 4, December 1969; van Fraassen, "On Massey's Explication"; Adolf Grünbaum, "Space, Time and Falsifiability, Introduction and Part A," *Philosophy of Science* 37, no. 4, December 1970, reprinted in Adolf Grünbaum, *Philosophical Problems*, chap. 16.

32. Massey, "Toward a Clarification," p. 332. In the end Massey seems to despair of finding a reasonable explication of "alternative metrizability." See p. 345.

33. Grünbaum, *Space, Time, and Falsifiability*, part A, section 3.

34. Van Fraassen, "On Massey's Explication."

35. Henri Poincaré, *Science and Hypothesis* (New York: Dover Publications, 1952); Hans Reichenbach, *The Philosophy of Space and Time* (New York: Dover Publications, 1958); Adolf Grünbaum, *Philosophical Problems of Space and Time*, 1st ed. (New York: Alfred A. Knopf, 1963).

36. It is, of course, strictly nonsense to identify a coordinate system *as such* (without specifying a metric) as Cartesian, polar, etc. I am nevertheless using these intuitive notions simply to try to give some feeling for the definition of unconditional alternative metrizability.

37. Unlike van Fraassen's discussion, the present one applies (negatively) to various inhomogeneous spaces, such as the arithmetical spaces of real numbers. I am not sure whether it can be extended to apply to discrete spaces.

38. In this context I am, of course, referring only to nontrivial intrinsic metrics; see Grünbaum, *Space, Time, and Falsifiability* part A, section 2b, for an explication of triviality.

ADOLF GRÜNBAUM

Absolute and Relational Theories of Space and Space-Time

I wished to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality. Physical objects are not in *space*, but these objects are *spatially extended*. In this way the concept "empty space" loses its meaning.

June 9, 1952 A. Einstein

(Preface to the fifteenth edition of *Relativity: the Special and the General Theory*. New York: Crown Publishers, 1961, p. vi.)

1. Introduction

Has the issue of ontological autonomy between Newton's absolutism and Leibniz's relationalism become otiose, defunct, and perhaps even spurious in the context of present-day theories of space-time structure? Or is the dichotomy between absolutistic and relational ontologies as originally understood by Newton and Leibniz illuminatingly germane to space-time as much as it was to pre-Einsteinian space and time? Two

NOTE: This paper is based on a much shorter version delivered at a conference of the same title, held on June 3–5, 1974 in Andover, Mass., under the auspices of the Boston University Institute of Relativity Studies, directed by John Stachel.

Apart from specific other acknowledgments made within the text, I am very grateful to the various Andover Conference participants whose ideas are discussed or mentioned by name within the text, especially Howard Stein. In addition, I wish to thank John Stachel, the organizer of the conference, for the opportunity to present the first draft of this paper there and for valuable private discussions of some of its content. And I am greatly indebted to Allen Janis for having read both the first and final drafts: my treatment of the debate between the relationalist and the "semi-absolutist" had the benefit of some incisive questions from him. Both John Winnie and Alberto Coffa stimulated me to deal with some issues in section 4 (2) which I would otherwise have neglected. Jeffrey Winicour made me aware of some pitfalls in the use of the term "graviton." I also wish to thank the National Science Foundation for the support of research.