state of continual fluctuation, and only words to be constrained into a degree of stability. I can quite well believe that such confusions were present in the historical situation; my complaint against you is that you seem to see no need to explicate the issues, or to resolve the confusions.

10. I here speak somewhat loosely, without care for the distinction between the Newtonian and Minkowski space-time structures.

11. Similarly, when I said, in the passage you quote on p. 329, that "I see no way to confront the former question"—namely, whether a certain relation is involved in the structure of "the space-time manifold itself, considered apart from all other entities"—"independently of the latter" (how to explicate the notion of "the space-time manifold itself"—to draw a line, so to speak, between it and "all other entities"); and that yet the converse may also seem to hold, etc.: I was not claiming to offer a *proof* of vicious circularity in the enterprise under discussion. So your reply, in the Uncle-Jack-and-President-Giscard passage, aimed at refuting in general terms a charge of "necessary circularity," is in my opinion not to the purpose: *I still see* no way—you have certainly not shown me one—to confront the first question independently of the second, or to answer the second without begging the first.

12. *De gravitatione et aequipondio fluidorum*, in Hall and Hall, ed., *Unpublished Scientific Papers of Isaac Newton* (Cambridge University Press, 1962), pp. 131–132, 136; Latin on pp. 99–100, 103. Hall and Hall most irritatingly render *affectio* throughout as "disposition"—one among many seriously misleading mistranslations in their English version of Newton's Latin text.

13. Again the Halls give a really terrible mistranslation: they have *"it is not among the proper dispositions that denote substance."* But it should be plain to anyone with a rudiment of philosophic discrimination that when Newton writes *"Non est* substantia *tum quia non absolute per se . . .* subsistit; *tum quia non* substat *ejusmodi propriis affectionibus quae substantiam denominant,"* the two verbs in the two dependent clauses—*subsistit* and *substat*—are deliberately chosen for their association with *substantia*: substance is what is self-subsistent, and is also the substrate or supporter of properties. The meaning of *substat* has to be, not "stands *among*," but "stands *under*": extension is not a substance because it does not support—underlie—stand under—the "characteristic denominations" (what Frege would call the "*Merkmale*") of a substance.

14. The article may, of course, be disputed, since Latin possesses no articles. Newton's text reads: *Spatium est entis quatenus ens affectio*; and the Halls render: "Space is a disposition of being *qua* being." The expression "being *qua* being" has indisputable standing in the metaphysical tradition—but not, I think, as denoting an individual subject of attributes, something that has "affections" (or "dispositions"). The translation I have given seems to me consonant with the sentences that follow.

15. Not, as the Halls translate, "of the first existence of being"!—The "first existing thing," of course, according to Newton, is God (of whom he has previously characterized space as "an emanative effect (as it were)"; but it is noteworthy that the *reason* he gives for his statement that space is "an emanative effect of the first existing thing" is *quite independent of what thing that may be*.

16. In what you call Riemann's "DH," it is this *Wirkliche*, rather than the spatial manifold itself, whose "binding forces" are said to give rise to the "Massverhältnisse" in the manifold: that is, Riemann does not speak of binding forces as *acting upon the spatial manifold* (a notion it is hard to make any sense of), but as *acting upon "the real."*

MICHAEL FRIEDMAN

Simultaneity in Newtonian Mechanics and Special Relativity

1. Introduction

Everyone will agree, I think, that the transition from Newtonian mechanics to special relativity taught us something of fundamental importance about time and simultaneity. Many philosophers have urged that there is a significant *semantic* lesson to be learned from this transition. For example, the following kinds of views have been expounded: Einstein was aided in his discovery of special relativity by an analysis of the "concepts" of time and simultaneity; the transition from Newtonian mechanics to special relativity resulted in a profound "change of meaning" of 'time' and 'simultaneous'—a change that was so extensive as to make any comparison of the two theories problematic; in a special-relativistic world the notion of simultaneity is in an important sense conventional—statements about distant simultaneity lack truth-value, they are mere "definitions."

Defenders of such views rarely provide explicit semantic theories within which their claims can be evaluated. There are, however, philosophical theories of meaning lurking in the background. Thus claims that Einstein analyzed the "concept" of simultaneity fit naturally into an operationalist account of meaning, since what Einstein did was to discuss ways of measuring time and distant simultaneity. Similarly, claims about the conventionality of distant simultaneity are supported (at least in Reichenbach's case) by a verificationist theory of meaning. Statements about distant simultaneity in a special-relativistic world lack truth-value, it is argued, because they are unverifiable in principle. On the other hand, views that make much of the noncomparability or "incommensurability" of the meanings of 'time' and 'simultaneous' in Newtonian mechanics and special relativity seem to involve some kind of "contextual" theory of meaning—meaning is to be identified with "role in theory" or the like.

In this paper I would like to see what light can be shed on these semantic issues using an approach to the theory of meaning that has been the subject of much contemporary discussion. The approach I have in mind takes reference rather than meaning as the central notion of semantics. According to this approach, the semantical properties of a sentence—its truth-value or lack of it, its inferential connections with other sentences, etc.—are determined by (a) the referential properties of its component words—the objects denoted by its singular terms and the sets (properties) determined by its predicates; and (b) the "logical form" of the sentence—how it is built up from its component words by means of grammatical constructions like truth-functions, quantifiers, etc. Let us call this kind of approach *referential semantics*. It is plausible to suppose that referential semantics can be an illuminating framework for discussing traditional semantic issues about the transition from Newtonian mechanics to special relativity, because many of these issues involve claims about truth. Thus the conventionalists hold that statements about distant simultaneity lack truth-value in a special relativistic universe. The "meaning change" and "incommensurability" theorists hold that we cannot apportion truth and falsity to the statements involving 'time' and 'simultaneous' made by a Newtonian physicist according to the truth and falsity of corresponding statements of special relativity, because such statements have different meanings in their different theoretical contexts. Similarly, according to the "meaning change" theorists, we cannot say that such statements made by a Newtonian physicist are approximately true, that some statements of Newtonian mechanics are logical consequences of special relativity, etc. From the point of view of referential semantics, all claims of this kind must depend on peculiarities in the referential properties of 'time' and 'simultaneous' (assuming that there is nothing problematic about the grammatical structure of the sentences in question).

A second feature of my approach is that I shall treat both Newtonian mechanics and special relativity as *space-time* theories. I view both theories as theories about a four-dimensional manifold, space-time, and the geometrical structures that characterize it. Where the two theories differ is with respect to the geometrical structures that space-time actually possesses. In particular, differences between the two theories as to time and simultaneity are to be understood as differences in the geometrical properties predicated of space-time. I adopt this view of the two theories

because it seems to me to make their similarities and differences—their comparison—especially clear. However, I shall not argue directly for this view here (see, e.g., Earman, 1970, and Earman and Friedman, 1973). Nor shall I argue directly for referential semantics (see, e.g., Field, 1972 and 1973). Instead, I hope to show that the conjunction of these views provides a fruitful framework for the discussion of traditional philosophical issues relating to Newtonian mechanics and special relativity. Of course, if I am successful, this paper will constitute an indirect argument for the referential approach to semantics and the space-time approach to our two physical theories.

My argument will proceed as follows. In section 2 I shall briefly sketch four-dimensional formulations of Newtonian mechanics and special relativity, as such formulations will probably be unfamiliar to most readers. In section 3 I shall discuss the question of whether 'time' and 'simultaneous' underwent a "meaning change" in the transition from the former theory to the latter. I shall argue that with respect to the kind of meaning that is most relevant to questions about the truth of statements in the two theories—i.e., with respect to reference—it is plausible to suppose that there has been no change. In section 4 I shall discuss the issue of the conventionality of simultaneity in special relativity. I shall argue that conventionalists have not given us a good reason to regard statements of distant simultaneity as truth-valueless in the context of special relativity.

2. Four-Dimensional Formulations of Newtonian Mechanics and Special Relativity

According to the space-time point of view, the basic object of both our theories is a four-dimensional manifold. I shall use R^4, the set of quadruples of real numbers, to represent the space-time manifold. Both theories agree that there is a natural system of straight lines defined on this manifold. If (a_0, a_1, a_2, a_3), (b_0, b_1, b_2, b_3) are two fixed points in R^4, then a *straight line* is a subset of R^4 consisting of elements (x_0, x_1, x_2, x_3) of the form

(1) $$\begin{aligned} x_0 &= a_0\mathrm{r} + b_0 \\ x_1 &= a_1\mathrm{r} + b_1 \\ x_2 &= a_2\mathrm{r} + b_2 \\ x_3 &= a_3\mathrm{r} + b_3 \end{aligned}$$

where r ranges through the real numbers. A *curve* on R^4 is a (suitably continuous and differentiable) map $\sigma: R \to R^4$. Such a curve $\sigma(u)$ is a *geodesic* if and only if it satisfies

$$\begin{aligned} x_0 &= a_0 u + b_0 \\ x_1 &= a_1 u + b_1 \\ x_2 &= a_2 u + b_2 \\ x_3 &= a_3 u + b_3 \end{aligned} \tag{2}$$

where $(x_0, x_1, x_2, x_3) = \sigma(u)$ and the a_i and b_i are constants. So if a curve is a *geodesic* its range is a straight line. Note that the geodesics are just the curves that satisfy

$$d^2x_i/du^2 = 0 \qquad i = 0, 1, 2, 3. \tag{3}$$

The importance of straight lines and geodesics is due to the fact that both theories agree that the trajectories of free particles are straight lines in space-time. So we can represent such trajectories as geodesics in R^4.

A *coordinate system* is a one-one (suitably continuous and differentiable) map $\phi: R^4 \to R^4$. A coordinate system is *affine* if and only if it is a linear transformation of R^4, i.e., it satisfies

$$y_i = \sum_{j=0}^{3} a_{ij}x_j + b_i \qquad i = 0, 1, 2, 3 \tag{4}$$

where the a_{ij} and b_i are constants and $(y_0, y_1, y_2, y_3) = \phi(x_0, x_1, x_2, x_3)$. Affine coordinate systems are precisely those that preserve the condition

$$d^2y_i/du^2 = 0 \qquad i = 0, 1, 2, 3 \tag{5}$$

for geodesics. As we shall see, such coordinate systems are a natural representation of the physicist's frames of reference.

So far, Newtonian mechanics and special relativity agree on the structure of space-time. But the two theories differ over what further structures exist on the space-time manifold, and, in particular, over the individual natures of space and time. In what follows I shall deal only with the kinematical aspects of our two theories, since these aspects are most relevant to the role of time and simultaneity. However, it should be noted that dynamics—i.e., gravitational interaction in the case of Newtonian mechanics, and electromagnetic interaction in the case of special relativity—can be easily dealt with within this framework as well (see Anderson, 1967, Earman and Friedman, 1973, Havas, 1964, and Trautman, 1966).

(a) Newtonian Mechanics

The central object that Newtonian kinematics postulates on the space-time manifold is an *absolute time*: a real-valued function $t: R^4 \to R$ defined by $t(x_0, x_1, x_2, x_3) = x_0$. Think of t as assigning a time to each point (event) in space-time. The hypersurfaces t = constant are called *planes of absolute simultaneity*. Two points in R^4 are *simultaneous* if and only if they lie on the same t = constant hypersurface. Furthermore, on each plane of absolute simultaneity Newtonian kinematics postulates a Euclidean metric, h, defined by

$$h((t, x_1, x_2, x_3), (t, x_1', x_2', x_3'))^2 = (x_1 - x_1')^2 + (x_2 - x_2')^2 + (x_3 - x_3')^2. \tag{6}$$

Now any geodesic curve $\sigma(u)$ satisfies $x_0 = a_0u + b_0$, so if we use $x_0 = t$ as a parameter for σ it remains a geodesic: i.e., $\sigma(t)$ satisfies

$$d^2x_i/dt^2 = 0 \qquad i = 1, 2, 3. \tag{7}$$

This is just Newton's law of inertia.

An *inertial* coordinate system is an affine coordinate system which is generated by a *Galilean* transformation; i.e., y_0, y_1, y_2, y_3 is inertial if and only if

$$\begin{aligned} y_0 &= x_0 = t \\ y_i &= \sum_{j=0}^{3} a_{ij}x_j + b_i \qquad i = 1, 2, 3 \end{aligned} \tag{8}$$

where the a_{ij}, $i, j = 1, 2, 3$ form an orthogonal matrix: $\Sigma\, a_{ij}a_{kj} = \delta_{ik} = 1$ if $i = k$, 0 if $i \neq k$. Inertial coordinate systems are just those that preserve the above form of the law of inertia and the above form of the spatial metric h. I shall say that an inertial coordinate system y_0, y_1, y_2, y_3 is *adapted* to a trajectory $\sigma(t)$ if and only if $\sigma(t)$ satisfies the equations $y_0 = t$, $y_i = 0$, $i = 1, 2, 3$. Thus one can think of σ as representing a particle at rest at the origin of y_0, y_1, y_2, y_3. There exists an inertial coordinate system adapted to σ if and only if σ is a geodesic. So if σ is a geodesic and ϕ is an inertial coordinate system adapted to σ, I shall call the pair $\langle\sigma, \phi\rangle$ an *inertial frame*. In inertial frames free particles satisfy Newton's first law.

(b) Special Relativity

In Newtonian kinematics time is represented by the function t, while space is represented by a t = constant hypersurface, endowed with a three-dimensional Euclidean metric h. In special relativity we capture the roles of both time and space by a single object: a four-dimensional pseudo metric g defined by

(9) $$g((x_0, x_1, x_2, x_3), (x'_0, x'_1, x'_2, x'_3))^2 = (x_0 - x'_0)^2 - (x_1 - x'_1)^2 - (x_2 - x'_2)^2 - (x_3 - x'_3)^2.$$

g is called the *Minkowski* metric. Two points $p, q \in R^4$ have *timelike separation* if $g(p, q)^2 > 0$, *spacelike separation* if $g(p, q)^2 < 0$, *null separation* if $g(p, q)^2 = 0$. A curve is *timelike* if every point on it has timelike separation from every other point, and similarly for spacelike and null. Equivalently, a curve $\sigma(u)$ is timelike if and only if

(10) $$\sum_{ij} \eta_{ij} \frac{dx_i}{du}\frac{dx_j}{du} > 0$$

everywhere, and similarly for spacelike and null—where $\eta_{ij} = 1$ if $i = j = 0$, -1 if $i = j = 1, 2, 3$, and 0 if $i \neq j$. We require that the trajectories of free particles be timelike geodesics.

For any timelike curve $\sigma(u)$, we can define its *length* τ by the formula

(11) $$\tau(u) = \int^u \sqrt{\sum_{ij} \eta_{ij} \frac{dx_i}{du}\frac{dx_j}{du}}\, du.$$

τ is called the *proper time* of σ. On timelike curves we can use τ as a parameter, and if $\sigma(\tau)$ is a timelike geodesic it satisfies the law of motion

(12) $$d^2x_i/d\tau^2 = 0.$$

An *inertial* coordinate system is an affine coordinate system which is generated by a *Lorentz* transformation; i.e., y_0, y_1, y_2, y_3 is inertial if and only if

(13) $$y_i = \sum_j a_{ij}x_j + b_i$$

where

(14) $$\sum_{ik} a_{ij}a_{kl}\eta_{ik} = \eta_{jl}.$$

Inertial coordinate systems are just those that preserve the above form of the law of motion and the above form of the space-time pseudo metric g. Since in an inertial coordinate system a timelike geodesic $\sigma(\tau)$ satisfies $y_0 = a_0\tau + b_0$, we can use y_0 as a parameter on curves as well without disturbing the condition for timelike geodesics. The y_0 coordinate of an

inertial system is called the *coordinate time* of the system. From now on I shall denote such a coordinate time by 't.' Thus in an inertial system the law of motion can be written in the form

(15) $$d^2y_i/dt^2 = 0 \qquad i = 1, 2, 3.$$

I shall say that an inertial coordinate system is *adapted* to a trajectory $\sigma(\tau)$ if and only if $\sigma(\tau)$ satisfies the equations $y_0 = t = \tau$, $y_i = 0$, $i = 1, 2, 3$—where τ is the proper time of σ. There exists an inertial coordinate system adapted to σ if and only if σ is a timelike geodesic. If σ is a timelike geodesic and ϕ is an inertial coordinate system adapted to σ, I shall call the pair $\langle \sigma, \phi \rangle$ an *inertial frame*. Relative to a given inertial frame we have hypersurfaces $t =$ constant, where t is the coordinate time of the frame. These hypersurfaces are spacelike (every point in one has spacelike separation from every other point) and are endowed with a Euclidean metric by g (if p, q have spacelike separation, define $h(p, q)^2 = -g(p, q)^2$). Two points $p, q \in R^4$ are *simultaneous with respect to the given inertial frame* if and only if they lie on the same $t =$ constant hypersurface.

Let us call a triple $\langle R^4, t, h \rangle$, where t is an absolute time and h is a Euclidean metric on the hypersurfaces $t =$ constant, *Newtonian space-time*; a pair $\langle R^4, g \rangle$, where g is the Minkowski metric, *Minkowski space-time*.[1] The basic claim of Newtonian kinematics is that our universe is a Newtonian space-time; the basic claim of special relativity is that our universe is a Minkowski space-time. Differences between the two theories over the roles of time and simultaneity turn on structural differences between Newtonian and Minkowski space-times. Thus in Newtonian space-time there is a unique global time determined by t, and a unique relation of simultaneity $\mathcal{S}$ such that $p\mathcal{S}q$ if and only if p and q lie on the same $t =$ constant hypersurface. Both time and simultaneity are independent of coordinate system or reference frame.

In Minkowski space-time, on the other hand, there is no such unique global time. Time is in the first instance a local property; the proper time of a particular timelike curve. Being local, proper time cannot be used to define a relation of simultaneity at all; it can be used only to compare the times of points lying on the same trajectory. However, relative to a particular inertial frame F there is a global time t_F—the coordinate time of the inertial coordinate system determined by F. Thus in Minkowski space-time there is a multitude of simultaneity relations. For each inertial frame F there is a simultaneity relation $\mathcal{S}^F$ such that $p\mathcal{S}^F q$ if and only if p and q lie on the same t_F = constant hypersurface. So in special relativity (global) time and simultaneity are coordinate or frame dependent. It makes no sense to say that two events are simultaneous *simpliciter*, but only *relative to* this or that inertial frame or coordinate system.

3. The "Meaning" of 'Simultaneous' in Newtonian Mechanics and Special Relativity

If special relativity is true, Newtonian mechanics as a whole is false. Our world is a Minkowski space-time, not a Newtonian space-time; and neither a frame-independent global time nor a frame-independent simultaneity relation exists. Nevertheless, although the whole system of beliefs about time and simultaneity held by Newtonian physicists was false, we might plausibly (and perhaps naïvely) suppose that some of these beliefs were true. For example, we might suppose that when a Newtonian physicist uttered a sentence such as

(16) Events e_1 and e_2 are simultaneous in frame F,

he said something true. On the other hand, when he uttered a sentence like

(17) If e_1 and e_2 are simultaneous in frame F, then e_1 and e_2 are simultaneous in frame F',

he said something false. Our reasoning here is that (16) is true and (17) false because a relativistic physicist would accept (16) and reject (17), and we believe that special relativity is true.[2] Furthermore, although (17) is strictly false, we might plausibly (and perhaps naïvely) suppose that it is approximately true—as long as e_1 and e_2 are not widely separated in space and the relative velocities of F and F' are small. This is because of the following derivation in special relativity: Let events e_1 and e_2 have coordinates (x_0, x_1, x_2, x_3) and (x_0, x'_1, x_2, x_3) respectively in frame F. If y_0 is the coordinate time of event e_1, and y'_0 is the coordinate time of e_2 in frame F', it follows that their difference is given by

(18) $$|y'_0 - y_0| = v\,|x'_1 - x_1| / \sqrt{1 - v^2}$$

where v is the velocity of frame F' relative to frame F (I assume that F' is moving along the x_1-axis of F and that $c = 1$). This difference is small if v is small and $|x'_1 - x_1|$ is small. Thus, if e_1 and e_2 are simultaneous in F, they will be approximately simultaneous in F' whenever they are spatially close and the velocity of F' relative to F is small.

However, a "meaning change" theorist would not be happy with this way of looking at the matter (see, e.g., Feyerabend, 1962 and Kuhn, 1962). He would deny that the fact that a relativistic physicist would accept (16) and reject (17) gives us a reason to think that a Newtonian physicist said something true when he uttered (16) and said something false when he uttered (17). For, according to the advocate of "meaning change," (16) and (17) do not express the same things when uttered by a Newtonian physicist and by a relativistic physicist; (16) and (17) have different meanings in their different theoretical contexts. Similarly, a "meaning change" theorist would deny that the fact that (18) is derivable in special relativity gives us a reason to think that (17) is approximately true in the context of Newtonian mechanics. The sentence that is derivable in special relativity is not an approximation to (17) as a principle of Newtonian mechanics, for the two have radically different meanings.

Now the first thing to notice is that the relevant issue here is not whether 'simultaneous' has different *meanings* in the two different theoretical contexts, but whether it has different *referents*. For, if there is anything right about the referential approach to semantics, truth-value is a function of the referents of the component words of the sentences in question. Thus, as long as 'simultaneous' has the same referent in our two theoretical contexts, (16) and (17) will have the same truth-values in the two contexts, whether or not they have the same meanings. As long as the reference of 'simultaneous' is preserved, our argument that (16) is true and (17) false in the context of Newtonian physics because (16) is true and (17) false in special relativity is correct. Similarly, if reference is preserved, we can regard (18) as an approximation to (17), and we can therefore regard (17) as approximately true. Thus, if the problem of "incommensurability" relates to the comparison of the truth-values of sentences in our two theories—e.g., if the problem is whether sentences in the two theories can *contradict* each other, whether sentences in one theory can be *derived* from sentences in the other, whether sentences in one theory can be *approximations* to sentences in the other, etc.—then the crucial issue is over the referents of words like 'time' and 'simultaneous,' not their meanings. The "meaning change" theorist must argue that 'time' and 'simultaneous' have different referents in their different theoretical contexts, not merely that they have different meanings.

How does the "meaning change" theorist argue for his view? Characteristically, he appeals to the radical differences in the theoretical principles involving time and simultaneity in the two theories. For example, in Newtonian mechanics time is an "absolute," nonrelational, frame-independent quantity; while in special relativity it is a relational, frame-dependent quantity. How could terms embedded in such radically different theoretical principles have the same meaning? This line of thought can support claims about the referents of 'time' and 'simultaneous' if we adopt the view that the reference of a theoretical term is determined by the theoretical principles containing the term. That is, we can use theoretical differences as an argument that 'time' and 'simultaneous' have different referents in Newtonian mechanics and special relativity if we adopt the view that a theoretical term refers to whatever satisfies the theoretical principles containing the term, or whatever satisfies a sufficient number of such principles, or the like. Thus if 'time' refers to anything at all in the context of Newtonian mechanics, it refers to an "absolute," frame-independent quantity; if 'simultaneous' refers to anything at all, it refers to an "absolute," frame-independent relation. On the other hand, in special relativity 'time' refers to a relational, frame-dependent quantity; 'simultaneous' refers to a frame-dependent relation. Therefore these terms cannot possibly have the same referents in the two different theories.

This view of how the reference of theoretical terms is determined—that a theoretical term refers to whatever satisfies (a sufficient number of) the theoretical principles containing the term—is closely analogous to the Russell-Searle account of how the reference of proper names is determined. According to the Russell-Searle account, the referent of a proper name is whoever satisfies (a sufficient number of) the descriptions we "associate" with the name. This account of the reference of proper names has been the subject of much recent critical discussion (see Kripke, 1972). It seems to me that the parallel account of theoretical terms has very similar flaws. In particular, if the account of theoretical terms in question is correct, it is hard to see how a theory can ever turn out to be false (or at least hard to see how the "central" principles of a theory can turn out to be false). If this account is correct, there either is an entity satisfying (a sufficient number of) the theoretical principles involving a given term or there is not. If there is, the principles are true; if there is not, the given term lacks a referent and the principles are truth-valueless. So, for example, if this account is correct, we cannot say that Newtonian mechanics represents a false view *of* time and simultaneity. Newtonian mechanics is

not a theory *about* anything. The terms 'time' and 'simultaneous' have no referents, and, consequently, the theoretical principles involving these terms are not false but truth-valueless.

An obvious way out of this difficulty is to view the theoretical principles involving 'time' and 'simultaneous' of Newtonian mechanics as *existential* assertions; or, what amounts to the same thing, to view theoretical terms as analogous to definite descriptions, and to adopt Russell's rather than Frege's view of the truth-values of sentences containing nonsatisfied definite descriptions. That is, we construe Newtonian mechanics as containing assertions of the form

(19) There exist a quantity t and a relation $\mathcal{S}$ such that ________,

where the conjunction of the various theoretical principles involving 'time' and 'simultaneous' is put in the blank. This construal allows us to say that Newtonian mechanics as a whole is false, since there exists no such quantity t and no such relation $\mathcal{S}$. However, it does not allow us to say anything about the truth-values of individual sentences of Newtonian physics. We cannot say, for example, that (16) is true and (17) false. Note, that it will not do to construe individual theoretical sentences again as existential assertions. We cannot, e.g., construe (16) as

(20) There exists a relation $\mathcal{S}$ such that e_1 bears to e_2 in Frame F, and (17) as

(21) There exists a relations $\mathcal{S}$ such that if e_1 bears to e_2 in frame F then e_1 bears $\mathcal{S}$ to e_2 in frame F'.

This makes (16) come out true, all right, but it also makes (17) true. For (21) is certainly true; there exist plenty of frame-independent relations, e.g., the relation of having spacelike separation! This last move makes it far too easy for an individual theoretical sentence to be true.

These considerations suggest that it is a mistake to view the reference of theoretical terms as determined by the theoretical principles within which they occur. If we say that a theoretical term either refers to an entity that satisfies (a sufficient number of) the theoretical principles containing the term or to nothing at all, we make it too difficult for such theoretical principles to turn out false. On the other hand, if we construe theoretical terms as analogous to Russellian descriptions, and thereby construe theoretical principles as basically existential assertions, we make it too difficult for such principles to turn out true—for in this latter case, only the theory as a whole can be true or false. And note that this holds even if the theory as a whole is completely and exactly true—we still have no general method for apportioning truth to the individual sentences of the theory. However, if the reference of a theoretical term is not determined by the theoretical principles within which it occurs, how is it determined? In my opinion, so-called causal theories of reference are on the right track. That is, it seems to me that what a theoretical term refers to is not a matter of which entity (if any) satisfies the theoretical principles involving the term, but rather, a matter of which actual entities have the right sort of "historical" connection with the use of the term (see Kripke, 1972 and Putnam, 1973).

Now I grant that this way of talking is extremely vague, and I do not know how to give a precise account of what the right sort of "historical" connection is. Nevertheless, in my view, this way of looking at the reference of theoretical terms does not leave us at a total loss either. On the contrary, I think we have enough intuitive ideas about what the "right sort of connection" is to at least get plausible candidates for the referents of most theoretical terms. For example, such questions as: 'What actual quantities are being measured by the measuring procedures used to determine values for the quantities postulated by the theory?' and 'What entities are actually responsible for the phenomena explained by the theory?' seem highly relevant for determining which quantities and relations the theoretical terms of our theory actually refer to. Furthermore, although the "historical" connection view of reference does not have anything very precise to say about just what the reference relation is, it says enough to free us from the implausibilities of the satisfaction-of-theoretical-principles account. That is, it shows us how even the central principles of a theory can turn out to be false, and it allows us to attribute truth and falsity to the individual sentences of a theory in a plausible way.

The case of 'time' and 'simultaneous' in Newtonian mechanics provides a good illustration of these points. In determining the referents of these terms, we should not look for entities that satisfy the theoretical principles of Newtonian physics—there are no such entities! Rather, we should proceed as follows: given the entities—quantities, relations, etc.—that our best current theory postulates, we look for some among these which (a) give a plausible distribution of truth-values for the sentences involving 'time' and 'simultaneous' used by Newtonian physicists; (b) are actually responsible for the phenomena explained by Newtonian mechanics; (c)

are actually measured by the measuring procedures used to test Newtonian mechanics. Supposing for a moment that special relativity is our best current theory, and using these (admittedly rough and incomplete) guides, I suggest we obtain the following results about the referents of 'time' and 'simultaneous' in Newtonian mechanics:

(i) In a context like 'time . . . in frame F,' 'time' refers to t_F—the coordinate time of frame F. In a context like 'simultaneous . . . in frame F,' 'simultaneous' refers to $\mathcal{S}^F$—the relation of lying on the same hypersurface t_F = constant.

(ii) Where 'time' or 'simultaneous' occurs without explicit qualification as to reference frame, but other features of the context "attach" the sentence to a particular reference frame—e.g., the sentence is uttered within a particular laboratory frame on the surface of the earth—'time' refers to the coordinate time t_F of that frame and 'simultaneous' refers to $\mathcal{S}^F$.

(iii) Where the context neither explicitly nor implicitly "attaches" the sentence to a particular inertial frame, 'time' and 'simultaneous' have no referents.

These suggestions accord with (a)–(c) above. We have the intuitively plausible consequence, for example, that (16) is true and (17) false; we are able to attribute truth and falsity to the individual sentences used by Newtonian physicists; and we make it neither too hard nor too easy for such sentences to come out true. The quantity assigned to 'time'—i.e., the coordinate time of a particular frame in a particular context—is the quantity actually responsible for the phenomena explained by Newtonian kinematics. The central explanatory principle of Newtonian kinematics is the law of inertia (7); and, according to special relativity, the correct form of this law is (15)—which determines the trajectory of a free particle as a function of *coordinate time*. Finally, the quantity assigned to 'time' is the quantity actually measured by (ideal) clocks. According to special relativity, (ideal) clocks measure the proper time along their trajectories. So a clock at rest at the origin of a particular inertial frame F measures the coordinate time of F.

If (i)–(iii) are correct, 'time' and 'simultaneous' have referential properties analogous to *indexical* words like 'I,' 'you,' 'here,' and 'now.' Just as indexical words refer to different things relative to different contexts—relative to different speakers, hearers, places, and times—'time' and 'simultaneous' refer to different things relative to different inertial reference frames. (And, as in the case of indexical words, the relevant context may be either explicit or implicit.) Just as the truth-values of sentences containing indexical words can vary with context, the truth-values of sentences containing 'time' and 'simultaneous' vary with inertial frame. Neither kind of sentence possesses a truth-value *absolutely*, but only relative to this or that context (reference frame). Thus, when the sentence in question is not "attached" to any context (reference frame) of the appropriate kind, it lacks a truth-value and its component words lack referents.

If I am right, the transition from Newtonian mechanics to special relativity *has* taught us a semantic lesson. In a special relativistic world the referents of 'time' and 'simultaneous' have to be taken as dependent on reference frame; 'time' and 'simultaneous' must be seen as possessing referential properties analogous to those of indexical words. If the world were Newtonian, this would not be necessary; 'time' and 'simultaneous' would have unique, frame-independent referents. However, it is not necessary to suppose that 'time' and 'simultaneous' have *changed* their referential properties in this transition. Since our world is and always was (so we believe—modulo note 2) a special-relativistic world, not a Newtonian world, the words 'time' and 'simultaneous' have and always had referential properties appropriate to a special relativistic world. Thus, when used by a relativistic physicist, 'time' and 'simultaneous' have the same referential properties as they did when used by a Newtonian physicist: i.e., (i)–(iii) still hold. (Of course, if a relativistic physicist is careful, case (iii) will never occur!) One is able to argue for a significant semantic change in the transition from Newtonian mechanics to special relativity only by employing wildly implausible theories about the reference of theoretical terms.

4. The Conventionality of Simultaneity in Special Relativity

The problem of the conventionality of simultaneity is typically introduced in the following way: we are asked to imagine two points, p_0 and p_1, in a given reference system. Situated at each of the points is a clock. A light signal is sent from p_0 to p_1, where it is reflected back to p_0. The light signal leaves p_0 at t_1—as determined by the clock at p_0—and returns to p_0 at t_2. Our problem is to synchronize the clock at p_1 with the clock at p_0; to say when, according to p_0-time, the light signal arrives at p_1. We must determine which event between t_1 and t_2 at p_0 is simultaneous with

the event E at p_1. According to the conventionality thesis it is a matter of definition which event between t_1 and t_2 is simultaneous with E; no choice is any "truer" than any other. Of course, if we assume that the velocity of light is the same from p_0 to p_1 as it is on the return trip, the p_0 time of E would be unambiguously determined as

(22) $\quad t = t_1 + \frac{1}{2}(t_2 - t_1)$

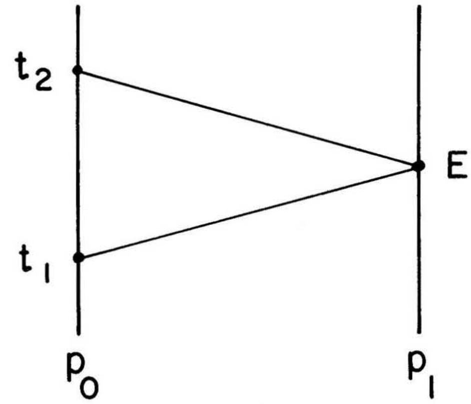

Figure 1

However, conventionalists argue that any claim about the one-way velocity of light—as distinct from its round-trip velocity—is just as conventional. They argue that

(23) $\quad t = t_1 + \epsilon(t_2 - t_1)$

is just as good as (22) for determining the p_0-time of E, where ϵ is any real number such that $0 < \epsilon < 1$. Only computational simplicity can favor the choice $\epsilon = \frac{1}{2}$ over any other admissible value of ϵ. There are no facts that make (22) true and (23) false.

I think this problem can be greatly clarified by looking at special relativity from the space-time point of view of section 2. Our discussion will be facilitated if we consider Minkowski space-time as a two-dimensional manifold—i.e., as R^2 instead of R^4. This device simplifies the algebra without essentially changing the conceptual situation. Our theory remains the same, except that the Minkowski pseudo-metric takes the simpler form

$$g((x_0, x_1), (x'_0, x'_1))^2 = (x_0 - x'_0)^2 - (x_1 - x'_1)^2$$

in R^2. Thus, inertial coordinate systems y_0, y_1 are characterized by the condition

$$g(p, q)^2 = (y_0 - y'_0)^2 - (y_1 - y'_1)^2$$

where p has coordinates (y_0, y_1) and q has coordinates (y'_0, y'_1). To set up the problem in this framework, consider a given inertial frame associated with the time-like geodesic $\sigma(\tau)$. Let there be given two null geodesics (light rays) which intersect $\sigma(\tau)$ at $y_0 = \tau_1$, and $y_0 = \tau_2$ respectively, and intersect each other at E. Since null geodesics have constant unit velocity in inertial systems (I have set $c = 1$), it is clear that if we fix the time at point E according to the synchronization role (22)—i.e., if we let the time of E be

$$t = \tau_1 + \tfrac{1}{2}(\tau_2 - \tau_1)$$

we are merely adopting the coordinate time $t = y_0$ of our given inertial frame as our global time. That is, the rule (22) amounts to fixing the time

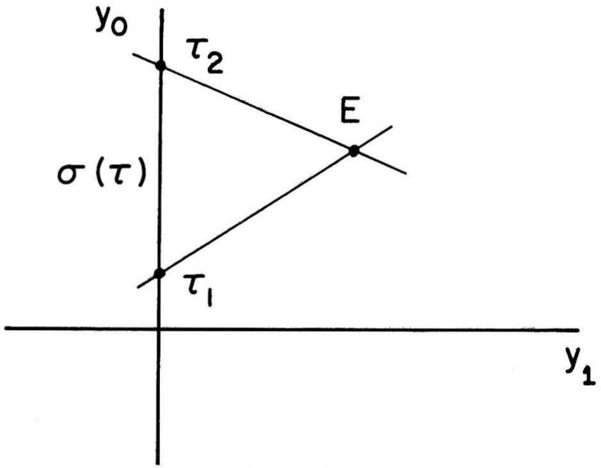

Figure 2

of events not on the trajectory $\sigma(\tau)$ by means of the coordinate time of an inertial coordinate system adapted to $\sigma(\tau)$.

What are we doing if we use (23) instead of (22) to fix the time of E—i.e., if we let the time of E be

(23) $\quad \bar{t} = \tau_1 + \epsilon(\tau_2 - \tau_1)$

with $\epsilon \neq$ ½? This latter procedure can be viewed as using the coordinate $\bar{t} = z_0$ of a *noninertial* coordinate system $z_0\, z_1$ as our global time. It amounts to fixing the time of events not on the trajectory $\sigma(\tau)$ by means of the $z_0 = \bar{t}$ coordinate of a noninertial coordinate system adapted to $\sigma(\tau)$ (in the sense that $\sigma(\tau)$ satisfies $z_0 = \tau$, $z_1 = 0$ in z_0, z_1).

The relation between the noninertial system z_0, z_1 and our original inertial system y_0, y_1 is easily seen to be

(24) $$\bar{t} = z_0 = t + 2\delta y_1$$
$$z_1 = y_1$$

where $\delta = \epsilon -$ ½. The inverse relation is of course

(25) $$t = y_0 = \bar{t} - 2\delta z_1$$
$$y_1 = z_1$$

Thus, using $\epsilon \neq$ ½ in (22) amounts to performing the coordinate transformation (24) and using the "coordinate time" $\bar{t}$ of the new system to define simultaneity. Using (25) we find that the Minkowski metric takes the form

(26) $$g(p, q)^2 = (z_0 - z'_0)^2 - 4\delta(z_0 - z'_0)(z_1 - z'_1) + (4\delta^2 - 1)(z_1 - z'_1)^2$$

in our new system z_0, z_1. Now a minimal condition for a z_0-coordinate to be a temporal coordinate is that the curves z_0 = constant be spacelike. It follows from (26) that this is the case if and only if

(27) $\quad 4\delta^2 - 1 < 0.$

If we substitute $\epsilon -$ ½ for δ in (27), this minimal condition becomes

(28) $\quad \epsilon(\epsilon - 1) < 0.$

(28) implies that $0 < \epsilon < 1$. So the "coordinate time" of an ϵ-*system*—a system in which (26) holds—is a suitable temporal coordinate only if $0 < \epsilon < 1$.

Some useful facts about ϵ-systems are the following. First, if a trajectory has velocity $v = dy_1/dt$ in an inertial coordinate system and "velocity" $\bar{v} =$

$dz_1/d\bar{t}$ in an ϵ-system, it follows from (24) and (25) that the two are related by

(29) $\quad v = \bar{v}/(1 - 2\delta\bar{v}).$

Second, the relation between the coordinate time t and the proper time τ of a trajectory in an inertial system is given by

(30) $\quad d\tau = \sqrt{1 - \bar{v}^2}\, d\bar{t}.$

In ϵ-systems (30) becomes

(31) $\quad d\tau = \sqrt{1 - 4\delta\bar{v} + (4\delta^2 - 1)\bar{v}^2}\, d\bar{t}$

or (32) $\quad d\tau = \sqrt{(1 - \bar{v}(2\epsilon - 1))^2 - \bar{v}^2}\, d\bar{t}.$

Finally, we know that two inertial systems, t, y_1 and t^*, y_1^* are related by a Lorentz transformation

(33) $\quad t^* = (t - vy_1)/\sqrt{1 - v^2}$

$$y_1^* = (y_1 - vt)/\sqrt{1 - v^2}$$

where v is the relative velocity of the two systems. How are two different ϵ-systems related?

Let there be given two ϵ-systems, I and II, with coordinates $\bar{t}$, z_1 and $\bar{t}^*$, z_1^* respectively. Let the respective values of ϵ in the two frames be ϵ_1 and ϵ_2, and let frame II move with "velocity" v with respect to frame I. We can use the following procedure to find the transformation connecting the two frames: (1) use (25) to transform I into an inertial frame t, y_1; (2) use (28) and (32) to transform t, y_1 into a second inertial frame t^*, y_1^* moving with velocity v with respect to the first; (3) use (24) to obtain the frame II. This procedure results in some tedious algebra and

$$\bar{t}^* = z_0^* = \frac{(2\bar{v}(1 - \epsilon_1 - \epsilon_2) + 1)\bar{t} - (2(\epsilon_1 - \epsilon_2) + 4\bar{v}\epsilon_1(1 - \epsilon_1))z_1}{\sqrt{(1 - v(2\epsilon_1 - 1))^2 - v^2}}$$

(34)

$$z_1^* = \frac{z_1 - \bar{v}\bar{t}}{\sqrt{(1 - \bar{v}(2\epsilon_1 - 1))^2 - \bar{v}^2}}$$

Note that when $\epsilon_1 = \epsilon_2 = \frac{1}{2}$, $\bar{v} = v$ and (34) reduces to a Lorentz transformation (33).

John Winnie (1970) derives the above transformations from a completely different point of view.[3] He calls the relations (34) the *ϵ-Lorentz transformations*. The purpose of Winnie's paper is to argue that special relativity as formulated using the standard synchronization rule (22) is "kinematically equivalent" to a formulation using the nonstandard rule (23) with $\epsilon \neq \frac{1}{2}$—thus vindicating, according to Winnie, the thesis of the conventionality of simultaneity. In the present framework, Winnie's claim is that special relativity as formulated in ϵ-systems is equivalent to special relativity as formulated in inertial systems. It seems to me that there is one sense in which this claim is obviously true, but completely trivial; and there is a second sense in which it is not at all obvious, and completely unsupported by Winnie's arguments.

The sense in which the equivalence claim is obviously true is that Minkowski space-time can be described equally well from the point of view of ϵ-coordinate systems as from the point of view of inertial coordinate systems. Formulations of special relativity in ϵ-systems say the same thing about Minkowski space-time as formulations in inertial systems. Indeed, they are nothing but different coordinate representations of the same theory (the theory expressed in coordinate-independent form in note 4). Thus the two formulations cannot disagree about the behavior of light—light follows null geodesics independently of coordinate system; nor about the behavior of free particles—free particles follow timelike geodesics independently of coordinate system; nor about the behavior of clocks—(ideal) clocks measure the proper time along their trajectories independently of coordinate system; etc. But note that in this sense of 'equivalence' there is no need to restrict ourselves to ϵ-coordinate systems. Minkowski space-time can be equally well described from the point of view of *any* coordinate system; our theory can be represented in *arbitrary* coordinate systems. (This is especially obvious in the formulation of note 4.) Thus the equivalence of ϵ-systems and inertial systems in this sense reveals no deep facts about Minkowski space-time or special relativity. Newtonian space-time can be represented in arbitrary coordinate systems as well; Newtonian kinematics can be formulated in systems that are not inertial with no change in theory. In fact, of course, any theory expressible in *tensor form* will have this property.

Thus, if the equivalence claim is to be nontrivial, it must amount to something more than the assertion that ϵ-coordinate systems and inertial coordinate systems are equally good representations of the basic facts about Minkowski space-time hypothesized by special relativity. Let us look a little closer. According to special relativity there is no unique global time defined on space-time. However, special relativity in its usual $\epsilon = ½$ formulations associates a unique global time with every state of inertial motion. For every timelike geodesic $\sigma(\tau)$, there is a unique (up to a linear transformation) way of extending its proper time to a global coordinate time t—the y_0-coordinate of an inertial coordinate system adapted to $\sigma(\tau)$. Now a defender of the equivalence claim can be construed as asserting that there are other, equally good, ways of extending the proper time of a time-like geodesic to a global time—namely, the z_0-coordinates of ϵ-systems adapted to $\sigma(\tau)$. That is, he is claiming not merely that ϵ-systems and inertial systems are equally good coordinate representations of Minkowski space-time, but that the z_0-coordinate of an ϵ-system is an equally good candidate for the global time associated with a given state of inertial motion as the y_0-coordinate of an inertial system. The $\bar{t}$ of an ϵ-system is an equally good representation of physical time as the t of an inertial system. This explains why a defender of the equivalence thesis considers only ϵ-systems with $0 < \epsilon < 1$, and not *arbitrary* coordinate systems. For only the z_0-coordinate of an ϵ-system with $0 < \epsilon < 1$ satisfies minimal conditions for representing physical time: the hypersurfaces $z_0 =$ constant being spacelike.

If this is correct, arguments like Winnie's, which simply amount to showing how special relativity as formulated in inertial systems can be translated into a formulation in ϵ-systems, do not support a nontrivial version of the equivalence thesis. Such translation procedures merely prove that ϵ-systems and inertial systems are equally good coordinate representations of Minkowski space-time, a fact that is obvious in a tensor formulation of special relativity. In support of a stronger version of the equivalence claim, we must be given some reason to think that the z_0-coordinate of an ϵ-system is an equally good representation of physical time as the y_0-coordinate of an inertial system. Clearly the condition $0 < \epsilon < 1$ is a necessary condition for a z_0-coordinate to represent physical time—but is it sufficient? Are there any plausible additional conditions that narrow the choice of ϵ further?

The advocates of so-called *slow-transport synchrony* (see Ellis and Bowman, 1967) may be understood to propose a further such necessary condition for a z_0-coordinate to represent physical time. Consider again the problem of synchronizing two clocks in a given reference frame, one at P_0 and the other at P_1. The two are said to be in slow-transport synchrony

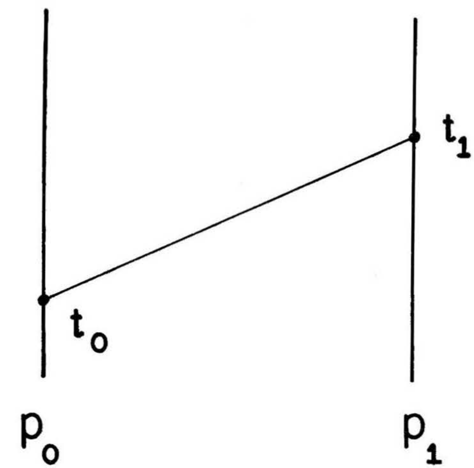

Figure 3

if a clock synchronized with P_0-time at t_0 is transported "infinitely slowly" to P_1 and is in agreement with P_1-time at t_1. (We consider only "infinitely slow" transport to avoid the velocity-dependent relativistic time-dilation effects.) More precisely, let there be given an arbitrary ϵ-system z_0, z_1 adapted to a given timelike geodesic $\sigma(\tau)$.

Consider a timelike geodesic $\rho(\tau^*)$—representing a clock transported with constant velocity—which intersects $\sigma(\tau)$ at τ_0. Consider two events E and E' on $\sigma(\tau)$ and $\rho(\tau^*)$ respectively, and let the proper time τ^* of ρ equal that of σ at their intersection: i.e., let $\tau_0 = \tau_0^*$—the two clocks are synchronized. Finally, let the "velocity" of ρ in z_0, z_1 be $\bar{v} = dz_1/dz_0$. E and E′ are *slow-transport simultaneous* if and only if $\lim_{\bar{v} \to 0} (\tau_1 - \tau_1^*) = 0$, where τ_1 and τ_1^* are the respective proper times of E and E' (Fig. 4).

Now I take the advocates of slow-transport synchrony to be imposing the further condition on a z_0-coordinate that it agree with slow-transport simultaneity; i.e., that two events are simultaneous according to z_0—they have the same z_0-coordinate—if and only if they are slow-transport simultaneous. It is not hard to show that this requirement fixes ϵ at $\frac{1}{2}$; only the y_0-coordinates of inertial systems satisfy this condition. For suppose that

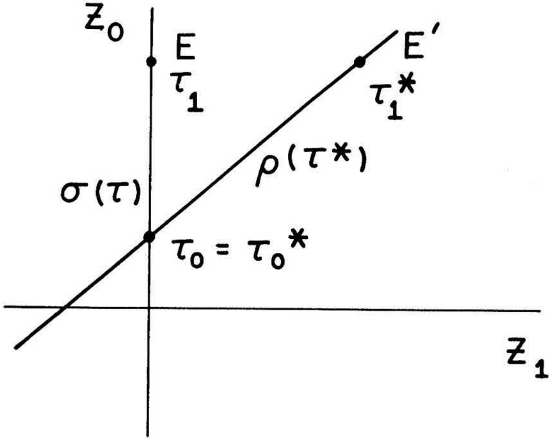

Figure 4

E and E' are z_0-simultaneous in our diagram: the z_0-coordinate of E' is just τ_1. It follows from (31) that

(35) $$(\tau_1^* - \tau_0^*) = (\tau_1 - \tau_0)\sqrt{1 - 4\delta\,\bar{v} + (4\delta^2 - 1)\bar{v}^2}$$

Expanding the "dilation term" in a binomial series we obtain

$$(\tau_1^* - \tau_0^*) = (\tau_1 - \tau_0)\,(1 - 2\delta\bar{v} - \tfrac{1}{2}(4\delta^2 - 1)\,\bar{v}^2 - \ldots$$

where the rest of the series consists of second and higher powers of $\bar{v}$. Since $\tau_0^* = \tau_0$ we have

$$(\tau_1 - \tau_1^*) = (\tau_1 - \tau_0)\,(2\delta\bar{v} - \tfrac{1}{2}(4\delta^2 - 1)\,\bar{v}^2 - \ldots$$

But $(\tau_1 - \tau_0) = z_1/\bar{v}$ where z_1 is the "spatial" coordinate of E′. So we have

$$(\tau_1 - \tau_1^*) = z_1 2\delta - \tfrac{1}{2}(4\delta^2 - 1)\,\bar{v} - \ldots$$

Letting $\bar{v} \to 0$ and substituting $\epsilon - ½$ for δ we finally get

$$\lim_{\bar{v}\to 0}\,(\tau_1 - \tau_1^*) = z_1\,(2\epsilon - 1)$$

So z_0 simultaneity agrees with slow-transport simultaneity everywhere if and only if $\epsilon = ½$, if and only if z_0 is the coordinate time of an inertial coordinate system adapted to $\sigma(\tau)$.

What does this show? It shows that *if* a necessary condition for something being a representation of physical time is that it agree with slow-transport simultaneity, *then* the z_0-coordinates of ϵ-systems with $\epsilon \neq \frac{1}{2}$ are not equally good representations of physical time as the y_0-coordinates of inertial systems. However, this refutes the conventionalist only if he concedes that this requirement—agreement with slow-transport simultaneity—is not itself conventional. And the conventionalist does not have to (nor does he in fact: see Grünbaum, 1969 and Salmon, 1969) concede this. He can maintain that just as choosing ϵ = ½ is not any "truer" or more "factual" than choosing $\epsilon \neq \frac{1}{2}$; so, requiring agreement with slow-transport simultaneity is not any "truer" or more "factual" than not requiring it. Both choices may have the advantage of simplicity over their alternatives, but not the advantage of truth. But now the debate over conventionalism begins to look hopeless. The conventionalist asserts that a certain system of description is not "factual," and produces alternative descriptions which he claims are "equally good"; the anti-conventionalist points to various asymmetries between the original system and the conventionalist's alternatives; the conventionalist replies that these differences are not "factual" either, they are merely differences in simplicity; etc. If this debate is to have any point we need some kind of *independent* characterization of the difference between "factual" and conventional statements or descriptions.

Now, if we look at the conventionality thesis from a semantic point of view, it is clear that one important difference between conventional statements and "factual" statements is that the former are supposed to have no determinate truth-value, while the latter are either determinately true or determinately false. Therefore, one possible source of an independent characterization of the difference between "factual" and conventional statements is a semantic theory that is capable of dealing with sentences that lack determinate truth-value. As I suggested earlier, I think that so-called *referential semantics* is the most promising theory of this kind. According to referential semantics, there are at least two ways in which a (grammatically well-formed) sentence can lack a determinate truth-value: (1) it can contain words that pick out *no* referents; or (2) it can contain words that have a *multiplicity* of referents. In this latter case, the sentence (like sentences containing indexical words) is neither true nor false *simpliciter*, but has different truth-values relative to different choices from among the multiplicity of referents in question. With this in mind, I would like to turn to what I think are the most important arguments for the conventionality thesis.

It seems to me that there are at bottom only two arguments for the conventionality of simultaneity in the literature: Reichenbach's and Grünbaum's. Reichenbach argues from an epistemological point of view; he argues that certain statements are conventional as opposed to "factual" because they are unverifiable in principle. Grünbaum argues from an ontological point of view; he argues that certain statements are conventional because there is a sense in which the properties and relations with which they purportedly deal do not really exist, they are not really part of the objective physical world. Thus, Reichenbach's and Grünbaum's arguments depend on two different characterizations of the difference between conventional and "factual" statements. According to Reichenbach, the "factual"/conventional distinction is just the verifiable/unverifiable distinction. According to Grünbaum, the "factual"/conventional distinction rests on a prior distinction between properties and relations that are objective constituents of the physical world and those that are not.

How does Reichenbach argue for the conventionality thesis? He considers various methods for *determining* distant simultaneity in a given reference system—various methods of *verifying* statements of the form 'Events e_1 and e_2 are simultaneous with respect to the given state of inertial motion M'—and tries to show that none of these methods furnishes an unambiguous answer in a special-relativistic world. Thus, for example, if there were no upper limit to the velocity of signals, we could determine which event at a given place P_0 is simultaneous with a given event E at P_1 by considering arbitrarily fast signals that are sent from P_0 and are reflected back from P_1 at event E. In a special relativistic world, on the other hand, there is an upper limit to the velocity of signals. Consequently, we can use signals to determine simultaneity only if we know their velocities; and knowledge of (one-way) velocity presupposes knowledge of distant simultaneity:

> Thus we are faced with a circular argument. To determine the simultaneity of distant events we need to know a velocity, and to measure a velocity we require knowledge of the simultaneity of distant events. The occurrence of this circularity proves that simultaneity is not a matter of knowledge, but of a coordinative definition, since the logical circle shows that a knowledge of simultaneity is impossible in principle (1958, 126–127).

Of course, just because one method of determining simultaneity in-

volves circularity, it does not follow that they all do; so Reichenbach considers, in addition, the possibility of determining distant simultaneity by transporting clocks from one place to another. About this method he makes two points: (1) in a special-relativistic world it does not determine a unique simultaneity relation, because the rate of clocks depends on their velocity; (2) even if the relation so determined were unique, it would still only constitute a definition, because it would depend on unverifiable assumptions to the effect that if two clocks are seen to run at the same rate when together they continue to run at the same rate when spatially separated (1958, pp. 133–135).

I think Reichenbach's treatment of the clock-transport method is not so convincing as his treatment of the signal method. First, the method of "infinitely slow" clock transport avoids problem (1). Slow-transport simultaneity is a unique simultaneity relation. Second, while it is true that slow-transport simultaneity depends on assumptions about the rates of spatially separated clocks, these appear to be *additional* assumptions. That is, we do not appear to be faced with the same kind of obvious circularity as in the signal method, in which the determination of simultaneity depends on assumptions about velocity, and assumptions about velocity depend on the determination of simultaneity. Let me try to be more precise. The uniqueness of the slow transport method—its agreement with $\epsilon = \frac{1}{2}$ simultaneity—depends on assumptions about the proper time metric. That is, we assume that the proper time metric in a particular ϵ-system is given by (31), i.e.,

$$d\tau = \sqrt{1 - 4\delta\bar{v} + (4\delta^2 - 1)\,\bar{v}^2}\,d\bar{t}$$

If we assume instead a different proper time metric, e.g.,

$$d\tau = \sqrt{1 - 4\delta\bar{v} - (4\delta^2 - 1)\bar{v}^2}\,d\bar{t} - 2\delta\bar{v}\,d\bar{t} \tag{37}$$

we can eliminate the uniqueness of slow-transport simultaneity. Thus, according to the metric (37)

$$\lim_{\bar{v}\to 0} (\tau_1 - \tau_1{}^*) = 0 \tag{38}$$

in *all* ϵ-systems. Therefore the method of slow-transport depends on assumptions about the temporal metric. However, these assumptions seem to be independent of assumptions about the value of ϵ—even if we fix the value of ϵ we are still free to choose between (31) and (37) as our proper time metric. One can argue that such assumptions about the temporal metric are themselves conventional, but this requires an *independent* argument.

In any case, the main problem with Reichenbach's argument is this: whether or not statements about distant simultaneity are in some sense unverifiable in the context of special relativity, we have been given no reason to suppose that unverifiability implies lack of determinate truth-value. It would seem that sufficient conditions for a sentence's possession of a truth-value are: (1) that it be grammatically well-formed, and (2) that its component words pick out determinate referents. If (1) and (2) are satisfied, the sentence has a determinate truth-value, regardless of its epistemic status. Thus it seems to me that Reichenbach's approach to the problem of conventionality is vitiated by his reliance on bad semantics—his reliance on the verifiability theory of meaning. Note that Reichenbach himself was perfectly explicit about his reliance on this theory. For example, in his comments on the significance of Einstein's views on simultaneity—understood as a version of the conventionality thesis, of course—Reichenbach writes:

> The physicist who wanted to understand the Michelson experiment had to commit himself to a philosophy for which the meaning of a statement is reducible to its verifiability, that is, he had to adopt the verifiability theory of meaning if he wanted to escape a maze of ambiguous questions and gratuitous complications. It is this positivist, or let me rather say, empiricist commitment which determines the philosophical position of Einstein (1949, pp. 290–291).

Grünbaum's approach to the conventionality thesis is very different. Unlike Reichenbach, he does not rely on the verifiability theory of meaning; he does not use verifiability as a criterion for possessing a truth-value. Instead, he argues that in a special-relativistic world there is no objective simultaneity relation at all, there is no genuine physical relation for 'simultaneity' to refer to. Grünbaum's argument proceeds as follows: Let us say that two events, at P_0 and P_1 respectively, are *topologically simultaneous* just in case they are connectible by no causal signal. In a Newtonian world, in which there is no upper bound to the velocity of causal propagation, there is a unique event at P_0 topologically simultaneous with a given event E at P_1. In such a world, the relation of topological simultaneity uniquely determines the relation of *metrical simultaneity*. In a special-relativistic world like our own, on the other hand, in which there is a finite upper bound to the velocity of causal propagation, there are a

multitude (in fact an infinity) of events at P_0 which are topologically simultaneous with E. In this kind of world, therefore, the relation of metrical simultaneity is not uniquely determined by the relation of topological simultaneity (see 1973, pp. 28ff; pp. 345ff.)

If this is correct,[4] in a special-relativistic world it is impossible to define a relation of metrical simultaneity solely on the basis of causal relations between events, while in a Newtonian world such a definition would be possible. But why *should* the relation of metrical simultaneity be definable solely on the basis of causal relations? Why should we take the indefinability of metrical simultaneity on the basis of topological simultaneity as a reason for concluding that there is no objective physical relation of metrical simultaneity? Why can't metrical simultaneity stand on its own feet, as it were?

The answer, in Grünbaum's case, is that he holds a *causal* theory of time. He believes that all objective temporal relations are constituted by causal relations between events; the only temporal relations that objectively exist are those determined solely by causal relations:

> By maintaining that the very *existence* of *temporal* relations between non-coinciding events depends on the obtaining of some *physical* relations between them, Einstein espoused a conception of time (and space) which is *relational* by regarding them as systems of relations between physical events and things. Since time relations are first constituted by the system of physical relations obtaining among events, the character of the temporal order will be determined by the physical attributes in virtue of which events will be held to sustain relations of "simultaneous with", "earlier than", or "later than". In particular, it is a question of physical fact whether these attributes are of the kind to define temporal relations *uniquely*. . . . (1973, pp. 345–346).

So in a world in which metrical simultaneity is not definable solely on the basis of causal relations, there is no such physical relation. Note the similarity between Grünbaum's argument here and his argument for the conventionality of congruence. He argues that on a *continuous* set of spatial or temporal points there is no objective ("intrinsic") congruence relation, because on such a set congruence is not definable solely on the basis of topological properties (like cardinality) and order relations. Thus this argument depends on the claim that the only objective physical relations on a set of spatial or temporal points are those constituted by topological and ordinal relations[5]—just as the argument for the conventionality of simultaneity depends on the claim that the only objective temporal relations are those constituted by causal relations between events.

Grünbaum's argument, unlike Reichenbach's, has the advantage that *if* it were correct, we *could* draw semantic conclusions about the truth-value of sentences containing 'simultaneous' on the basis of the referential properties of their key terms. For, if Grünbaum's argument is correct, it follows that 'simultaneous' has no referent—there is no objective physical relation for it to refer to. And this would make the conventionalist contention that sentences like 'Events e_1 and e_2 are simultaneous with respect to state of inertial motion M' lack determinate truth-value highly plausible. However, it seems to me that Grünbaum's actual argument is much less persuasive than Reichenbach's. Reichenbach has given some plausibility to the claim that statements about distant simultaneity may be unverifiable within the context of special relativity. As far as I can see, Grünbaum has given us no reason to accept the view that the only objective temporal relations are constituted by causal relations. Indeed, how could one possibly support such a view? Our only grip on which properties and relations are objective constituents of the physical world is via our best theories of the physical world. The properties and relations that we hold to exist objectively are those that our best physical theories postulate. And since our best theories do not merely postulate the kind of *ordinal* (causal) temporal relations favored by Grünbaum—they postulate *metrical* relations as well—we have no reason to grant such ordinal (causal) relations the privileged ontological status that Grünbaum wants to ascribe to them.

In sum, it seems to me that we have not been given a basis for the "factual"/conventional distinction on which (a) conventional statements turn out to lack determinate truth-values, and (b) statements about distant simultaneity turn out to be conventional. Reichenbach has given a criterion for conventionality—i.e., unverifiability—which statements about distant simultaneity in a special-relativistic world can be held to fulfill with some plausibility. The verification of such statements is at least much more complicated in a special-relativistic world than it is in a Newtonian world. But there is no clear connection between Reichenbach's criterion and the lack of a determinate truth-value. Reichenbach's argument for the conventionality thesis rests on a dubious semantics. On the other hand, Grünbaum has given a criterion for conventionality—i.e., having constituent terms with no objective physical referents—which has a plausible

connection with the lack of a determinate truth-value. However, Grünbaum's argument that 'simultaneous' indeed lacks an objective referent depends on an unsupported, and seemingly unsupportable, a priori judgment as to which relations are objective. Grünbaum's argument for the conventionality thesis rests on a dubious ontology.

Notes

1. It is worth noting that both Newtonian mechanics and special relativity can be formulated within a more general point of view by starting with a four-dimensional C^∞ manifold M instead of R^4 (cf. Anderson, 1967, Earman and Friedman, 1973, Havas, 1964, and Trautman, 1966). In this framework, a *Newtonian space-time* is a quadruple $\langle M, \Gamma_{j\,k}^{\,i}, t_i, h^{ij}\rangle$, where $\Gamma_{j\,k}^{\,i}$ is a symmetric affine connection, t_i a C^∞ covector field, and h^{ij} a C^∞ symmetric tensor field of type (2, 0) and signature (0, 1, 1, 1). These objects satisfy the field equations

$$(1)\ R^i_{\ jkl} = 0$$
$$(2)\ h^{ij}_{\ \ ;k} = 0$$
$$(3)\ t_{i;j} = 0$$
$$(4)\ h^{ij}t_it_j = 0$$

where $R^i_{\ jkl}$ is the curvature tensor of $\Gamma_{j\,k}^{\,i}$.

Our law of motion is

$$(5)\ \frac{d^2x_i}{du^2} + \Gamma_{j\,k}^{\,i}\frac{dx_i}{du}\frac{dx_k}{du} = 0.$$

A Minkowski space-time is a triple $\langle M, \Gamma_{j\,k}^{\,i}, g_{ij}\rangle$, where $\Gamma_{j\,k}^{\,i}$ is a symmetric affine connection and g_{ij} is a C^∞ symmetric tensor field of type (0, 2) and signature (1, −1, −1, −1). Our field equations are just

$$(6)\ R^i_{\ jkl} = 0$$
$$(7)\ g_{ij;k} = 0$$

and our law of motion is again (5). This more general framework facilitates the comparison of these two theories with general relativity. In this context a *general relativistic space-time* is a quadruple $\langle M, \Gamma_{j\,k}^{\,i}, g_{ij}, T^{ij}\rangle$, where $\Gamma_{j\,k}^{\,i}$ and g_{ij} are as in special relativity and T^{ij} is a C^∞ tensor fields of type (2, 0) representing the mass-energy density. Our equation of motion remains the same, and we have one field equation

$$(8)\quad R^{ij} - \tfrac{1}{2}g^{ij}R = -8\pi kT^{ij}$$

where R^{ij} is the Ricci tensor of $\Gamma_{j\,k}^{\,i}$, R is the contracted Ricci tensor, and k is the gravitational constant. (The notions from differential geometry used here are explained in Hicks 1965.)

2. Of course, we really think that special relativity is only *approximately* true. However, my discussion will be much simpler if I ignore this. If I were to take account of the actual situation, I would have to change 'inertial frame' everywhere to 'approximately inertial frame,' etc.

3. Compare (33) with the relations in Winnie, 1970, p. 234, remembering that I have set $c = 1$. Note that at the end of his paper Winnie briefly alludes to the possibility of obtaining his transformations in something like the above manner—cf. pp. 236–237.

4. (Added in proof) Even this much seems actually incorrect. David Malament has recently shown that the standard $\epsilon = \frac{1}{2}$ simultaneity relation *is* (in a natural sense) uniquely definable in terms of causal relations in Minkowski space-time. See Malament, "Causal Theories of Time and the Conventionality of Simultaneity," forthcoming.

5. See Friedman, 1972 for such an interpretation of Grünbaum's argument.

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PETER A. BOWMAN

On Conventionality and Simultaneity—Another Reply

1. Introduction

In "Conventionality in Distant Simultaneity," Brian Ellis and I (1967) discussed the position Reichenbach and Grünbaum had taken on this issue. That article received considerable comment (Grünbaum and Salmon, 1969; Winnie, 1970; Feenberg, 1974), much of the critical part of which Ellis answered in "On Conventionality and Simultaneity—A Reply" (1971). Here I shall reformulate, extend, and supplement his answer to some of the critiques (Grünbaum, 1969; Salmon, 1969; van Fraassen, 1969). Elsewhere I treat the topic in a less polemical manner (Bowman, 1974 and 1976).

The conventionality of distant simultaneity, as maintained by Reichenbach and Grünbaum, is after all this commentary so widely known that it can be stated very briefly. Let us consider two points A and B which are separated from one another in an inertial frame K. For a light signal emitted from A and reflected at B back to A, we compare the time interval for the outgoing trip to that for the round trip. This ratio is called "epsilon" (ϵ). In formulating the special theory of relativity, Einstein effectively took ϵ to be $\frac{1}{2}$; thus we may use $\epsilon = \frac{1}{2}$ in defining what is now called "standard signal synchrony." Reichenbach views ϵ as restricted *only* by the causal relations involved in the signaling process. That is, the reflection of the light ray at B must take place after the ray's emission at A but before its return to A. These considerations require us to restrict ϵ between zero and one, but Reichenbach insists that within these limits values of $\epsilon = \frac{1}{2}$ "could not be called false" (1958, p. 127). He claims that there are no facts that would mediate against using these values in definitions that are now called "nonstandard signal synchrony." This allegedly

NOTE: This paper follows subsection J.1 of my dissertation (1972) with only minor expository changes except for the last page of the present subsection 2.c, which is a substantive revision.