

## *Popper's Propensity Interpretation of Probability and Quantum Mechanics*

This paper is a critique of Popper's interpretation of quantum mechanics and the claim that the propensity interpretation of probability resolves the foundational problems of the theory. The first two sections are purely expository. In section 1, I sketch Popper's conception of a formal theory of probability and outline the propensity interpretation. Section 2 is a brief description of the gist of Popper's critique of the Copenhagen interpretation of quantum mechanics and his proposed solution to the measurement problem (the problem of the "reduction of the wave packet"). In section 3, I show that the propensity interpretation of probability cannot resolve the foundational problems of quantum mechanics, and I argue for a thesis concerning the significance of the transition from classical to quantum mechanics, which I propose as a complete solution to these problems. Finally, section 4 contains some general critical remarks on Popper's approach to the axiomatization and interpretation of formal theories of probability.

### 1

In Appendix \*iv of *The Logic of Scientific Discovery*,<sup>1</sup> Popper criticizes Kolmogorov<sup>2</sup> for failing to carry out his program of constructing a purely formal theory of probability: Kolmogorov assumes that in an equation like

$$p(a,b) = r$$

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<sup>1</sup> K. R. Popper, *The Logic of Scientific Discovery* (New York: Harper Torchbooks, 1965).

<sup>2</sup> A. N. Kolmogorov, *Foundations of the Theory of Probability* (New York: Chelsea, 1950).

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where  $p(a,b)$  is the conditional probability of  $a$  given  $b$ , the elements  $a$  and  $b$  are sets. Popper's proposal is that nothing should be assumed concerning the nature of the elements  $a,b,c, \dots$  in the set  $S$  of admissible elements on which the probability functions  $p(a,b)$  are defined, beyond the closure of the set  $S$  under the two operations: *product* (or *meet*, or *conjunction*) and *complement*. For the axioms of the formal theory of probability determine an algebraic structure of a certain kind for the set  $S$ , if we define equivalence in  $S$  by

$$a = b \text{ if and only if } p(a,c) = p(b,c) \text{ for every } c \text{ in } S.$$

With Popper's axioms,  $S$  is a *Boolean algebra*. (This is demonstrated at the end of Appendix \*v.) Thus, the elements of  $S$  may be interpreted as sets, or predicates, or sentences, etc.

Popper distinguishes the task of finding one or more suitable *axiomatizations* of the formal calculus of probability from the task of finding one or more suitable *interpretations*. These he classifies as subjective or objective. Among the objective interpretations are the *classical* interpretation of de Moivre and Laplace (in which  $p(a,b)$  is interpreted as the proportion of equally possible cases compatible with the event  $b$  which are also favorable to the event  $a$ ), the *frequency* interpretation of Venn and von Mises (where  $p(a,b)$  is the relative frequency of the events  $a$  among the events  $b$ ), and Popper's own *propensity* interpretation.

The propensity interpretation may be understood as a generalization of the classical interpretation. Popper drops the restriction to "equally possible cases," assigning "weights" to the possibilities as "*measures of the propensity, or tendency, of a possibility to realize itself upon repetition.*"<sup>3</sup> He distinguishes *probability statements* from *statistical statements*. Probability statements refer to frequencies in *virtual* (infinite) sequences of well-defined experiments, and statistical statements refer to frequencies in *actual* (finite) sequences of experiments. Thus, the weights assigned to the possibilities are measures of conjectural virtual frequencies to be tested by actual statistical frequencies: "In proposing the propensity interpretation I propose to look upon *probability statements* as statements about some measure of a property (a physical property, comparable to symmetry or asymmetry) of the *whole experimental arrangement*; a measure, more precisely, of a *virtual frequency*; and I

<sup>3</sup> K. R. Popper, "Quantum Mechanics without 'The Observer,'" in M. Bunge, ed., *Quantum Theory and Reality* (New York: Springer-Verlag, 1967), p. 32.

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propose to look upon the corresponding *statistical statements* as statements about the corresponding *actual frequency*.”<sup>4</sup>

### 2

In his article “Quantum Mechanics without ‘The Observer,’ ”<sup>5</sup> Popper characterizes the Copenhagen interpretation as involving what he calls “the great quantum muddle.”

Now what I call the great quantum muddle consists in taking a distribution function, i.e. a statistical measure function characterizing some *sample space* (or perhaps some “population” of events), and treating it as a *physical property of the elements of the population*. . . . Now my thesis is that this muddle is widely prevalent in quantum theory, as is shown by those who speak of a “duality of particle and wave” or of “wavicles.” For the so-called “wave” — the  $\psi$ -function — may be identified with the mathematical form of a function,  $f(P, dP/dt)$ , which is a *function of a probabilistic distribution function*  $P$ , where  $f = \psi = \psi(q,t)$ , and  $P = |\psi|^2$  is a density distribution function. . . . On the other hand, the *element* in question has the properties of a particle. The wave shape (in configuration space) of the  $\psi$ -function is a kind of accident which poses a problem for probability theory, but which has nothing to do with the physical properties of the particles.<sup>6</sup>

Thus, it is a confusion to interpret Heisenberg’s uncertainty relations as setting limits to the precision of our measurements. Rather, these relations are statistical scatter relations: they relate the dispersions of conjugate magnitudes such as position and momentum in quantum ensembles.

With this clarification, Popper proposes that the measurement problem of quantum mechanics be resolved in the following way: He considers the example of a symmetrical pin board, constructed so that a number of balls rolling down the board will ideally form a normal distribution curve at the bottom, representing the probability distribution for each single experiment with each single ball of reaching a certain final position. The probability distribution of reaching the various final positions for those balls which actually hit a certain pin will be different from the original distribution, and this conditional probability can be calculated from the probability calculus. Now Popper’s thesis is this:

<sup>4</sup> *Ibid.*, pp. 32, 33.

<sup>5</sup> *Ibid.*

<sup>6</sup> *Ibid.*, pp. 19, 20.

“In the case of the pin board, the transition from the original distribution to one which assumes a ‘position measurement’ . . . is not merely analogous, but identical with the famous ‘reduction of the wave packet’. Accordingly, this is not an effect characteristic of quantum theory but of probability theory in general.”<sup>7</sup> Again:

Assume that we have tossed a penny. The probability of each of its possible states equals  $\frac{1}{2}$ . As long as we don't look at the result of our toss, we can still say that the probability will be  $\frac{1}{2}$ . If we bend down and look, it suddenly “changes”: one probability becomes 1, the other 0. Was there a quantum jump, owing to our looking? Was the penny influenced by our observation? Obviously not. (The penny is a ‘classical’ particle.) Not even the probability (or propensity) was influenced. There is no more involved here, or in any reduction of the wave packet, than the trivial principle: if our information contains the result of an experiment, then the probability of this result, relative to this information (regarded as part of the experiment's specification), will always trivially be  $p(a,a) = 1$ .<sup>8</sup>

3

For the purposes of this analysis, I propose to characterize a statistical theory as involving a set of physical magnitudes forming an algebraic structure of a certain kind, together with an algorithm for assigning probabilities to ranges of possible values of these magnitudes. The idempotent magnitudes, whose possible values are 0 or 1, represent the propositions of the theory, i.e., the idempotent magnitudes are associated with the properties of the systems described by the theory. I shall refer to the given algebraic structure of the idempotent magnitudes as the *logical space*  $L_1$  of a statistical theory. Roughly,  $L_1$  represents all possible ways in which the properties of the systems described by the theory can hang together, or the possible events open to the systems. It is not required that  $L_1$  be Boolean.

I shall say that two physical magnitudes,  $A$  and  $B$ , are statistically equivalent in a statistical theory just in case

$$p_W(\text{val}(A) \in S) = p_W(\text{val}(B) \in S)$$

for every statistical state  $W$  (i.e., for every probability assignment generated by the statistical algorithm of the theory) and every Borel set  $S$  of

<sup>7</sup> *Ibid.*, p. 36.

<sup>8</sup> *Ibid.*, p. 37.

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real numbers. (For “ $\text{val}(A) \in S$ ”, read: The value of the magnitude  $A$  lies in the set  $S$  of real numbers.) So if

$$p_W(\text{val}(X) \in S) = p_W(\text{val}(A) \in g^{-1}(S))$$

for every statistical state  $W$  and every Borel set  $S$ , where  $g:R \rightarrow R$  is a real-valued Borel function on the real line, then  $X$  and  $g(A)$  are statistically equivalent. (The magnitude  $g(A)$  will satisfy the condition

$$p_W(\text{val}(g(A)) \in S) = p_W(\text{val}(A) \in g^{-1}(S))$$

for every possible probability assignment, not only those generated by the statistical states of the theory.) With respect to this equivalence relation, I shall say that two magnitudes  $A$  and  $A'$  are compatible if and only if there exists a magnitude,  $B$ , and Borel functions  $g:R \rightarrow R$  and  $g':R \rightarrow R$  such that<sup>9</sup>

$$\begin{aligned} A &= g(B) \\ A' &= g'(B). \end{aligned}$$

This definition of the compatibility of two magnitudes is due to Kochen and Specker.<sup>10</sup> A linear combination of two compatible magnitudes may be defined as the linear combination of associated functions:

$$aA + a'A' = (ag + a'g')(B)$$

where  $a, a'$  are real numbers, and similarly the product may be defined as

$$AA' = (gg')(B).$$

With linear combinations and products of compatible magnitudes defined in this way, the set of magnitudes of a statistical theory forms a *partial algebra*,<sup>11</sup> and the set of idempotent magnitudes forms a *partial Boolean algebra*. I shall refer to the partial Boolean algebra of idempotents defined in this way as the *logical space*  $L_2$  of a statistical theory.

Evidently, the two algebraic structures,  $L_1$  and  $L_2$ , are different. I

<sup>9</sup> I.e., such that

$$\begin{aligned} p_W(\text{val}(A) \in S) &= p_W(\text{val}(B) \in g^{-1}(S)) \\ p_W(\text{val}(A') \in S) &= p_W(\text{val}(B) \in g'^{-1}(S)) \end{aligned}$$

for every statistical state  $W$  and every Borel set  $S$ .

<sup>10</sup> S. Kochen and E. P. Specker, *Journal of Mathematics and Mechanics*, 17 (1967), 59. Kochen and Specker use the term “commensurability” instead of “compatibility.” I avoid their term because it suggests a particular and, I think, misleading interpretation of the relation.

<sup>11</sup> The term is due to Kochen and Specker, who have investigated the properties of these algebraic systems.

shall say that a statistical theory is complete if and only if  $L_1$  and  $L_2$  are isomorphic. For this is the case if and only if the statistical states of the theory generate all possible probability measures on the logical space  $L_1$ .

A partial Boolean algebra is essentially a partially ordered set with a reflexive and symmetric (but not necessarily transitive) relation — compatibility — such that each maximal compatible subset is a Boolean algebra. Thus, a partial Boolean algebra may be pictured as “pasted together” in a certain well-defined way from its maximal Boolean subalgebras. By a probability measure on a partial Boolean algebra,  $L$ , I mean any assignment of values between 0 and 1 to the elements of  $L$ , which satisfies the usual conditions for a probability measure (as defined, say, by Kolmogorov's axioms) on each maximal compatible subset of  $L$ .

Now, the statistical algorithm of quantum mechanics involves the representation of the statistical states of a mechanical system by a certain class of operators in a Hilbert space (the statistical operators), and the physical magnitudes of the system by hypermaximal Hermitian operators in the Hilbert space. Each statistical operator  $W$  assigns a probability to the range  $S$  of the magnitude  $A$  according to the rule

$$p_W(\text{val}(A) \in S) = \text{Tr}(WP_A(S))^{12}$$

where  $P_A(S)$  is the projection operator associated with the Borel set  $S$  by the spectral measure of  $A$ . The peculiarity of quantum mechanics as a statistical theory lies in the fact that the logical space  $L_1$  of a quantum mechanical system is not Boolean. The idempotent magnitudes are represented by the projection operators, and the structure of the logical space  $L_1$  is given by the algebra of projection operators in Hilbert space. There is a 1-1 correspondence between projection operators and subspaces in Hilbert space, each projection operator corresponding to the subspace which is its range.

The completeness problem for quantum mechanics was first solved by Gleason. Gleason's theorem<sup>13</sup> states that in a Hilbert space of three or more dimensions, all possible probability measures on the partial Boolean

<sup>12</sup> The right-hand side of this equation is the trace of the product of two Hilbert space operators: the statistical operator  $W$  and the projection operator  $P_A(S)$ , corresponding to the range  $S$  of the magnitude  $A$ . The trace of an operator is an invariant. In the case of an  $n$ -dimensional Hilbert space, the trace is the sum of the elements along the diagonal of the matrix of the operator.

<sup>13</sup> A. M. Gleason, *Journal of Mathematics and Mechanics*, 6 (1957), 885.

algebra of subspaces may be generated by the statistical operators  $W$ , according to the algorithm

$$p_W(K) = \text{Tr}(WP)$$

where  $P$  is the projection operator onto the subspace  $K$ . It follows immediately that the logical spaces  $L_1$  and  $L_2$  are isomorphic.

Kochen and Specker pointed out a further corollary to Gleason's theorem: Because there are no dispersion-free probability measures (two-valued measures) on  $L_1$ , except in the case of a two-dimensional Hilbert space, there are no two-valued homomorphisms on  $L_1$ . The nonexistence of two-valued homomorphisms on  $L_1$  means that it is impossible to imbed  $L_1$  in a Boolean algebra. It follows that it is impossible to represent the statistical states of quantum mechanics by measures on a classical probability space in such a way that the algebraic structure of the magnitudes of the theory is preserved.<sup>14</sup> In other words, it is impossible to introduce a "phase space,"  $X$ , and represent each physical magnitude by a real-valued Borel function on  $X$ , in such a way that each maximal compatible set of magnitudes is represented by a set of phase space functions which preserve the functional relationships between the magnitudes. What cannot be achieved for magnitudes  $A, A', \dots$  which are all functions of the magnitude  $B$ , i.e.,  $A = g(B), A' = g'(B), \dots$ , is that if  $B$  is represented by the phase space function  $f_B$ , then  $A$  is represented by the phase space function  $f_{g(B)} = g(f_B)$ ,  $A'$  is represented by the phase space function  $f_{g'(B)} = g'(f_B)$ , etc. Thus, there is no phase space<sup>15</sup> reconstruction of the quantum statistics which preserves the functional relations between compatible sets of physical magnitudes.

<sup>14</sup> A classical probability space is a triple  $(X, F, \mu)$ , where  $X$  is a set,  $F$  is a  $\sigma$ -field of subsets of  $X$  (the measurable sets), and  $\mu$  is a probability measure on  $X$ . Classical statistical mechanics is explicitly formulated as a statistical theory on a classical probability space, with  $X$  the phase space of classical mechanics. The axiomatization of the classical theory of probability along these lines is due to Kolmogorov, *Foundations of the Theory of Probability*. What is fundamental is the notion of probability as a measure function on an event structure or propositional structure represented as the algebra generated by the subsets of a set under the operations of union, intersection, and complement, i.e., a Boolean algebra. It is always possible (trivially) to represent the statistical states of an arbitrary theory by measures on a classical probability space, if the algebraic structure of the magnitudes of the theory (or the logical space  $L_1$ ) is not preserved. See Kochen and Specker, *Journal of Mathematics and Mechanics*.

<sup>15</sup> Of course, the phase space  $X$  need not be a phase space in the sense that it is parameterized by generalized position and momentum coordinates, as in classical mechanics. It is a phase space in the sense that the points of this space define two-

The algebra of idempotent magnitudes (propositions) of a classical mechanical system is isomorphic to the Boolean algebra of Borel subsets of the phase space of the system. The work of Gleason, and of Kochen and Specker, shows that the transition from classical to quantum mechanics involves a generalization of the Boolean propositional structures of classical mechanics to a particular class of non-Boolean structures: the propositional structure of a quantum mechanical system is a partial Boolean algebra isomorphic to the partial Boolean algebra of subspaces of a Hilbert space. This may be understood as a generalization of the classical notion of validity — the class of models over which validity is defined is extended to include partial Boolean algebras which are not imbeddable into Boolean algebras.<sup>16</sup>

Popper's position is an implicit commitment to a phase space reconstruction of the quantum statistics, i.e., the representation of the statistical states of quantum mechanics by measures on a classical probability space. His claim is that there is no more involved in the quantum mechanical "reduction of the wave packet" than the "trivial principle" according to which additional information transforms an initial probability distribution into a new conditional probability distribution in a Boolean probability calculus. But there is more involved here, because the transformation of probabilities is *non-Boolean*.

The "trivial principle" to which Popper refers is the Boolean rule for conditional probabilities. The points in a classical probability space (phase space) correspond to ultrafilters in the logical space, i.e., maximal consistent sets of propositions. The representation of the states of classical mechanics as the points of a space  $X$ , which functions in the theory like the Stone space of a Boolean logic, shows that the logical space is Boolean. The sense in which the specification of a point in  $X$  is a state-description is just this: A point in  $X$  corresponds to a maximal totality of propositions in logical space — maximal in the sense that

valued probability measures in the generalized sense, i.e., two-valued homomorphisms on  $L$ . That is to say, it is a phase space in the sense that the points of this space specify state-descriptions in an analogous sense to the points of a classical mechanical phase space: assignments of values to the magnitudes, satisfying the functional relationships given above.

<sup>16</sup> See Kochen and Specker, *Journal of Mathematics and Mechanics*, and J. Bub and W. Demopoulos, "The Interpretation of Quantum Mechanics," in R. S. Cohen and M. Wartofsky, eds., *Boston Studies in the Philosophy of Science*, vol. 13 (Dordrecht: Reidel, 1974). Also, J. Bub, *The Interpretation of Quantum Mechanics* (Dordrecht: Reidel, 1974).



these propositions are related by the logical relations of a Boolean ultra-filter. The Boolean rule for conditional probabilities is the rule

$$p(s,t) = \frac{\mu(\Phi_s \cap \Phi_t)}{\mu(\Phi_t)} = \frac{P(s \wedge t)}{p(t)}$$

where the initial probabilities are given by the measure  $\mu$ , and  $\Phi_s, \Phi_t$  are the subsets of points in  $X$  which satisfy the propositions  $s, t$  respectively. This rule derives directly from the Boolean semantics, i.e., essentially Stone's representation theorem for Boolean algebras. (See, for example, the classic paper by Los.)<sup>17</sup> Thus, the conditional probability  $p(s,t)$  is determined by a new measure  $\mu_t$ , defined by

$$\mu_t(\Phi) = \frac{\mu(\Phi \cap \Phi_t)}{\mu(\Phi_t)}$$

for every measurable set  $\Phi \subseteq X$ .

The measure  $\mu_t$  is the initial measure  $\mu$  "renormalized" to the set  $\Phi_t$ , i.e.,  $\mu_t$  satisfies the conditions:

- (i)  $\mu_t(\Phi_t) = 1$
  - (ii) if  $\Phi_u \subseteq \Phi_t$  and  $\Phi_s \subseteq \Phi_t$
- then

$$\frac{\mu_t(\Phi_u)}{\mu_t(\Phi_s)} = \frac{\mu(\Phi_u)}{\mu(\Phi_s)}$$

Condition (ii) ensures that  $\mu_t$  preserves the relative measures of subsets in  $\Phi_t$  defined by  $\mu$ .

The theorems of Gleason, and of Kochen and Specker, do not exclude the representation of the statistical states of quantum mechanics by measures on a classical probability space. They exclude a representation which preserves the algebraic structure of the magnitudes of the theory, or the structure of the logical space  $L_1$ . It follows that if  $\mu$  is a measure corresponding to some quantum mechanical statistical state in a phase space reconstruction of the quantum statistics, then the conditional probability  $\mu_t$  will not in general be a measure corresponding to a quantum mechanical state.

To see that this is so, it suffices to consider "pure" statistical states, represented by statistical operators which are projection operators onto

<sup>17</sup>J. Los, *Proceedings of the International Congress of Mathematics*, 1962, pp. 225-29.

one-dimensional subspaces or vectors in Hilbert space. Suppose  $\mu$  is the initial measure corresponding to the pure statistical state represented by the Hilbert space vector  $\psi$ . Let  $\tau$  be the eigenvector corresponding to the eigenvalue of  $t$  of  $T$ , and suppose  $\tau$  does not coincide with  $\psi$  and is not orthogonal to  $\psi$  (i.e.,  $\tau$  and  $\psi$  do not form part of any orthogonal set of basis vectors in the Hilbert space and so do not correspond to the eigenvectors of any common magnitude). I use the same symbol —  $t$  — for the value of the magnitude  $T$  and for the proposition corresponding to the property of the system represented by the value  $t$  of the magnitude  $T$ , i.e., the proposition represented by the idempotent magnitude associated with the projection operator onto the one-dimensional subspace spanned by the vector  $\tau$ . The conditional probability  $\mu_t$ , as computed by the Boolean rule, preserves the relative measures of subsets in  $\Phi_t$  defined by  $\mu$ , and so depends on  $\psi$ . But then  $\mu_t$  cannot represent a statistical ensemble of quantum mechanical systems each of which has the property  $t$ , for such an ensemble is represented by the measure  $\mu_\tau$ , corresponding to the statistical state  $\tau$ , and  $\mu_\tau$  is independent of  $\psi$  and depends only on  $\tau$ . Both measures satisfy condition (i), i.e.,

$$\mu_t(\Phi_t) = \mu_\tau(\Phi_t) = 1$$

but they cannot define the same relative measures on subsets of  $\Phi_t$ .

According to the Boolean rule for conditional probabilities, the relative probabilities of subsets in  $\Phi_t$  are unchanged by the additional information that  $t$  is true. But in a phase space reconstruction of the quantum statistics, we must assume that any initial information concerning the relative probabilities of subsets in  $\Phi_t$  is somehow invalidated by the additional information that  $t$  is true (or false), if the magnitude  $T$  is incompatible with any magnitude of which  $\psi$  is an eigenvector. The fact that an initial probability measure is reduced or "collapses" to the set  $\Phi_t$ , given the additional information that  $t$  is true, is not problematic here. What is problematic is that this reduction is apparently accompanied by a randomization process.

Popper writes:

Let us call our original specification of the pin board experiment " $e_1$ " and let us call the new specification (according to which we consider or select only those balls which have hit a certain pin,  $q_2$ , say, as repetitions of the new experiment) " $e_2$ ." Then it is obvious that the two probabilities of landing at  $a$ ,  $p(a, e_1)$  and  $p(a, e_2)$ , will not in general

be equal, because the two experiments described by  $e_1$  and  $e_2$  are not the same. But this does not mean that the new information which tells us that the conditions  $e_2$  are realized in any way changes  $p(a, e_1)$ : from the very beginning we could calculate  $p(a, e_1)$  for the various  $a$ 's, and also  $p(a, e_2)$ ; and we knew that

$$p(a, e_1) \neq p(a, e_2)$$

Nothing has changed if we are informed that the ball has actually hit the pin  $q_2$ , except that we are now free, if we so wish, to apply  $p(a, e_2)$  to this case; or in other words, we are free to look upon the case as an instance of the experiment  $e_2$  instead of the experiment  $e_1$ . But we can, of course, continue to look upon it as an instance of the experiment  $e_1$ , and thus continue to work with  $p(a, e_1)$ : the probabilities (and also the probability packets, that is, the distribution for the various  $a$ 's) are *relative probabilities*: they are relative to what we are going to regard as a repetition of our experiment; or in other words, they are relative to what experiments are, or are not, regarded as *relevant to our statistical test*.<sup>18</sup>

Now this is unobjectionable, but quite irrelevant to the quantum mechanical problem. It is not the mere change from an initial probability assignment to a new probability assignment, conditional on certain information, that is problematic in quantum mechanics — i.e., it is not the fact that probabilities change if we change the reference class that is problematic. What is problematic is the *kind of change*. To refer to Popper's example: Let  $\Phi_b$  be the set of trajectories which hit the pin  $q_2$  and another pin  $q_3$ , and let  $\Phi_c$  be the set of trajectories which hit the pin  $q_2$  and  $q_4$ . Then, of course,

$$\begin{aligned} p(b, e_1) &\neq p(b, e_2) \\ p(c, e_1) &\neq p(c, e_2). \end{aligned}$$

This is not at all puzzling. But in a Boolean theory of the pin board, it would be quite incomprehensible if

$$\frac{p(b, e_1)}{p(c, e_1)} \neq \frac{p(b, e_2)}{p(c, e_2)}.$$

And this inequality is characteristic of the quantum mechanical "reduction of the wave packet."

Feyerabend has made a similar point, although his argument does not explicitly bring out the non-Boolean character of the quantum statistics: "[Popper] pleads with us not to be surprised when a change of

<sup>18</sup> K. R. Popper, "Quantum Mechanics without 'The Observer,'" pp. 35, 36.

experimental conditions also changes the probabilities. This, he says, is a 'quite trivial feature of probability theory.' . . . Quite correct — but irrelevant. For what surprises us (and what led to the Copenhagen interpretation) is not the fact that there is some *change*; what surprises us is the *kind of change* encountered."<sup>19</sup>

I have argued elsewhere<sup>20</sup> that there are two possible interpretations of the role of the Hilbert space in quantum mechanics. The "great quantum muddle" is in effect the confusion of these two interpretations. The first interpretation takes the partial Boolean algebra of subspaces of Hilbert space as the logical space  $L_1$  of a statistical theory. Vectors in Hilbert space (or, rather, one-dimensional subspaces) then represent atomic propositions or atomic events. The problem of specifying all possible probability measures on such propositional structures has been solved by Gleason: the probability calculus is generated by the set of statistical operators (in the technical sense) in Hilbert space. On this interpretation of Hilbert space — call it the *logical interpretation* — the transition from classical to quantum mechanics is understood as the generalization of the Boolean propositional structures of classical mechanics to a class of propositional structures that are essentially non-Boolean. Thus, classical and quantum mechanics are interpreted as theories of logical structure, in the sense that they introduce different constraints on the possible ways in which the properties of physical systems are structured. Just as the significance of the transition from classical to relativistic mechanics lies in the proposal that geometry can play the role of an explanatory principle in physics, that the geometry of events is not a priori, and that it makes sense to ask whether the world geometry is Euclidean or non-Euclidean, so the significance of the quantum revolution lies in the proposal that logic can play the role of an explanatory principle, that logic is similarly not a priori.

The second interpretation — call it the *statistical interpretation* — presupposes that the logical space  $L_1$  of a statistical theory is necessarily Boolean, and takes the Hilbert space as the space of statistical states of a statistical theory, with each unit vector representing a statistical state for the algorithm

<sup>19</sup> P. K. Feyerabend, "On a Recent Critique of Complementarity: Part I," *Philosophy of Science*, 35 (1968), 309. Quotation from p. 326.

<sup>20</sup> "On the Completeness of Quantum Mechanics," in C. A. Hooker, ed., *Contemporary Research in the Foundations and Philosophy of Quantum Theory* (Dordrecht: Reidel, 1973).

$$p_{\Psi}(\text{val}(A) \in S) = \| P_A(S) \psi \|^2.$$

“Mixtures” of such “pure states” may be defined as more general statistical states, specifying probability assignments representable as weighted sums of probability assignments generated by pure  $\psi$ -states. Thus, the Hilbert space is interpreted as specifying the logical space  $L_2$  of a statistical theory, from which it follows that quantum mechanics is incomplete, since  $L_1$  is Boolean and  $L_2$  is non-Boolean. The measurement problem — the problem of the “reduction of the wave packet” or the inexplicable randomization in the transformation rule for conditional probabilities — is characteristic of the statistical interpretation. The logical interpretation avoids this problem by properly characterizing the category of algebraic structures underlying the statistical relations of the theory.

4

In the article “Quantum Mechanics without ‘The Observer,’” Popper distinguishes between *theories* and *concepts*. As scientists, we seek true theories, “true descriptions of certain structural properties of the world we live in.”<sup>21</sup> A conceptual system is merely an instrument used in formulating a theory, providing a language for the theory. Concepts, then, are “comparatively unimportant.”<sup>22</sup> In fact, “two theories,  $T_1$  and  $T_2$ , should be regarded as one if they are logically equivalent, even though they may use two totally different ‘conceptual systems’ ( $C_1$  and  $C_2$ ) or are conceived in two totally different ‘conceptual frameworks.’”<sup>23</sup>

Now it seems to me that the relation between *concepts* and *theories* (as understood by Popper) is precisely analogous to the relation between *interpretations* of probability and *formal theories* of probability (as outlined in section 1). The measurement problem which arises in quantum mechanics represents the inapplicability of Boolean probability theories to a certain domain of events. The problem cannot be avoided by using the language of propensities rather than the language of frequencies, since propensities and frequencies are both proposed as interpretations of a Boolean probability calculus. This is not to say that

<sup>21</sup> K. R. Popper, “Quantum Mechanics without ‘The Observer,’” p. 11.

<sup>22</sup> *Ibid.*, p. 14.

<sup>23</sup> *Ibid.*, p. 12.

there is in principle no difference between propensity-talk and frequency-talk, only that whatever difference there is can have no relevance for the measurement problem, just because Popper has explicitly proposed the propensity interpretation as an interpretation of a Boolean theory of probability.

Popper says, "We understand a theory if we understand the problem which it is designed to solve, and the way in which it solves it better, or worse, than its competitors."<sup>24</sup> *With respect to the measurement problem of quantum mechanics*, Popper's informal comments about propensities are quite empty. There cannot be a difference which makes a difference between propensity-talk and frequency-talk, so long as propensities and frequencies are proposed as interpretations of a Boolean probability calculus. For Popper to say something new in terms of propensities, it should be possible to formalize the difference between propensity-talk and frequency-talk in such a way that the new formal theory of probability solves those foundational problems of quantum mechanics which arise because of the inadequacy of the old theory.

It is clear, then, that Popper's propensity interpretation of probability adds nothing to the solution of the measurement problem. What of Popper's critical comments on Kolmogorov's axiomatization of the probability calculus and his own conception of a formal theory of probability? Again, there is no difference which makes a difference between Popper's axiomatization and Kolmogorov's. What the difference amounts to is this: Kolmogorov proposes axioms for a probability calculus on a Boolean logical space  $L_1$ . For completeness, the logical space  $L_2$  ought to be Boolean, and this is achieved by Kolmogorov's axiomatization. This seems to me the natural way to proceed. Popper does the same thing backward. He proposes axioms which characterize a Boolean logical space  $L_2$ . For completeness, again, the logical space  $L_1$  ought to be Boolean, and this is achieved by Popper's axiomatization. There is nothing to choose between the two approaches.

<sup>24</sup> *Ibid.*, p. 14.