Essays on Flows of Capital and Labor Across Countries

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Dedication

To my parents: Edna and Roberto, and to my bothers: Andres, Diego, Fatima and Roberto.

Abstract

My dissertation consists of three chapters. In the first chapter, I summarize the literature on international labor flows and on international capital flows, investment, and demographic transitions. I start with a historical recount of seminal papers on international labor and capital flows. I then focus on high-income, high-skilled labor flows of free mobility areas and point to where traditional migration models fail to explain these flows, and on the investment and the demographic transition of Latin America which has not been systematically studied. In the next chapters of this dissertation I address these two issues.

The second chapter of my dissertation, joint work with Daniela Viana Costa, studies labor flows of workers with similar skill-level and across countries with similar income. This chapter revisits empirical evidence on migration within the European Union-15, disaggregated by occupation. We find that most workers are highly educated and that they move to countries where their type (occupation and education level) is relatively abundant among natives. This is at odds with traditional models of migration. We develop a model with external economies of scale that generates an agglomeration force in high-educated occupations. Our main result is that a country that is relatively abundant in highly educated labor force will attract foreign labor of the same type. We argue this model is more suitable to analyzing migration flows between countries of similar income level.

The third chapter of my dissertation studies the behavior of investment during demographic transitions. In particular, I focus on the period of time where the working age to population ratio reaches its maximum, namely the demographic window. I document that in Europe, Asia, and Oceania investment rates are higher 15 years before and during the window than in other periods of time, whereas in Latin America they are lower. To understand the relation between investment and a demographic window, I build an overlapping generations model with demographic change and variable degree of financial openness. Within this framework, I conduct several exercises and counterfactuals involving potential drivers of the investment behavior. I find that the demographic behavior in conjunction with the region-specific financial openness, can

explain the main pattern of investment for the demographic window in Latin America vis-a-vis Europe, Asia and Oceania.

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Chapter 1

Critical Literature Review

1.1 Introduction

In this chapter I review the literature on labor flows, and capital flows in the context of capital investment and demographic transitions. The chapter is divided into two sections.

In the first section, I summarize the literature on labor flows, in particular migration for work purposes. I begin with a historical recount of some seminal papers on labor flows in general and then I focus on migration by country of origin distinguishing between two different approaches. The *south-north* approach, which accounts for the largest portion of the literature concerning migration, explains labor flows from low to high income countries. The *north-north* approach, which has recently gained attention with selective migration policies and the consolidation of free labor mobility areas of similar income and skill level, studies labor flows from and to high income countries. This part of my literature review is written in the present context of the high-income, high-skilled labor flows of the European Union 15 (EU15). Therefore, the review of the *south-north* works points out to where these papers fail to explain and predict these flows, and the review of the *north-north* works emphasizes features of the models that are relevant for these analysis. In the second chapter of this dissertation, my coauthor Daniela Costa and I, develop a model that is suitable for analyzing intra-EU15 labor flows.

In the second section of this chapter, I summarize the literature on capital flows and their implications on demographic transitions. I start by defining the concept and categories of international capital flows and by reviewing some of of the most important works pertaining this subject. I distinguish between capital flows generated by trade of goods, intertemporal capital flows, and capital flows for diversification. I then discuss empirical findings that have challenged the theory of capital flows, so called “empirical puzzles”, and some important works which have tried to addressed these puzzles. Then, I focus on intertemporal capital flows, investment and demographic transitions. This part of the review is divided into two sections: the first introduces the concept of a demographic transition (DT) and discusses some of the most prominent writings on the causal relation between economic development and DTs; while the second part discusses the impact of capital flows on the effects that DTs have on economic outcomes. This section is pertinent in its relation to the third chapter of this thesis where I study the relationship of investment rates before and during demographic windows and the degree of openness to trade in capital of the economies.

1.2 International Labor Flows

1.2.1 The theory of migration

The study of labor flows, dates back to the classical economist. In the “Wealth of Nations”, Smith (1776) discusses “Of the Wages of Labour”. Smith points out how because wage variations across the United Kingdom were higher than variations of the prices of goods,[1] one would expect labor flows to be higher than than the trade of goods, contrary to what he observed. Based on these observations, he concluded that man is difficult to be transported. This digression, has been interpreted by some, like Bodvarsson et al. (2015), as an early reference to migration costs.

Note that Smith specifies “man” as a unit of analysis, not inputs or factors of production. This specification reveals that although not explicit, his interpretation of labor flows pertained to migration, where workers decide to reallocate, and not of labor flows as movements of a factor of production which might result automatically from production patterns. Some Economists write of international labor flows as only the result of trade of goods, for instance, the Ricardian-Viner model of trade and the Heckscher-Ohlin theory. These models do not consider the decision of migrating which is

[1] The variation of the prices of goods are also across the United Kingdom.

the ultimate causal mechanism for international labor flows in the theoretical framework for this dissertation. For this reason, these are excluded from the literature review. For the purpose of this section I use the terms "labor flows" and "migration" interchangeably to refer to labor flows as migration for working purposes.

Still among the early works on migration, Ravenstein (1885) wrote seven laws of migrations based on his analysis of census data and migration statistics of the United Kingdom. These are: (1) Most migrants move a short distance and mostly to centers of commerce or industry; (2) Rapidly growing towns attract immigrants from immediate surroundings first, and then from more remote districts; (3) Dispersion of immigrants is inverse to absorption and has similar characteristics; (4) Migrations generate a "compensating counter-current"(p. 199);[2] (5) Distant migrants prefer to move to great centers of commerce or industry; (6) People from urban towns migrate less than rural ones; and (7) Women migrate more than men.

Four years later in a new paper with the same title, Ravenstein (1889) extended his analysis to what he called the principal countries of continental Europe and North America (Canada and the United States).[3] The first law was modified to something along the following lines: most migrants travel short distance and they do to "better themselves in material respects" (p. 284) because of overpopulation, commercial and industrial expansion, or to procure productive unoccupied land. This very last observation applied to the United States and Canada with a lot of arable unoccupied land as opposed to Europe.

More formal theories of migration would follow. For instance Stouffer (1940) focused on migration and distance and formulated a theory that established a relation between these two elements. Stouffer's theory was that "the number of persons going a given distance is directly proportional to the percentage increase in opportunities in that distance" (p. 846) and he provided some equations for establishing this relationship. His idea was similar to the refined first law of Ravenstein. Zipf (1946) went a step further by including both migration costs and population sizes in a gravity model to explain the

[2]Later in his second paper Ravenstein would refer to this law as "Every inflow of foreign elements is largely compensated by an outflow of natives" (p. 278), but of less proportion in America.

[3]The European group included: the German Empire, Luxembourg, Switzerland, Austria, Hungary, the Balkan Peninsula, Bulgaria, Romania, Denmark, Norway, Sweden, Finland, Russia, France, Italy, Spain, and Portugal.

magnitude of the labor flows between pairs of communities of the United States.[4] Zipf's theory predicted that the lowest the distance (and therefore the transportation cost) and the bigger the population of the source or the host region, the bigger the labor flow will be. Note that this idea was introduced before, in Ravenstein's laws (1), (2), and (5), however Zipf made a direct connection between migratory flows, costs and population sizes. Furthermore, this paper is among the first to provide a mathematical model for the study of migration and his model successfully accounted for the magnitude of the labor flows of his data. The model however did not try to explain the decision making process of migration, instead taking a decision to migrate as a given.

In the 1960s the literature focused on explaining the migration decision, i.e., how and why do workers decide to migrate. Sjaastad (1962) for example, analyzed the choice of migration as an outcome of comparing monetary and non-pecuniary costs and returns of migrating to those of not migrating. In his analysis Sjaastad considered the differences in earnings and discussed migration by occupation and age. Lee (1966) proceeded in a similar way and identified four factors behind migration decisions: those associated with area of origin, those associated with area of destination, intervening obstacles, and personal factors. Lee emphasized that the balance in favor of migrating must overcome what he calls the natural inertia, of staying in my interpretation. He also mentioned that some migration decisions were made at the family level and that migration involved self selection and life cycle.

Current traditional theory of migration, though more refined and precise, is still centered on modeling migration as a decision that results of a weighing of options, a theory originated in the 1960s as described above. Simply put, only when the perceived benefits outweigh the perceived costs (including opportunity costs) do people choose to migrate and cross the border. International labor flows generate a reallocation of labor that is jointly determined by the demand and the supply of both native and foreign workers. Once net wages equalize in both countries, this theory predicts that migration flows will cease.

Further on, the literature evolved to include several extensions such as changing the static framework for a dynamic one, focusing on life cycle behavior, introducing

[4] The author referred to the this movement across communities as "intercity" in the title of the paper.

uncertainty and imperfect information, modeling of more complex migration costs, incorporating the government, making migration an activity for human capital formation or one with human capital destruction, modeling of remittances, brain drain, return migration, business cycles, and choices at a family level, and introducing income inequality, self selection, and more.

In the next section I focus on self selection —of occupations— and on how and why workers move from either poorer to richer regions, or between rich regions. I start by reviewing the literature on international labor flows that emphasizes the differences in income between the source and host countries. I introduce the traditional model and then focus on self selection of occupations and skills. I also discuss concentration patterns that are observed in some labor flows and then review some papers related to these concentration patterns.

1.2.2 South-North Labor Flows

A large proportion of the literature on international labor flows that emphasizes the characteristics of the source and home country, focuses on migration of workers from low-income countries (the south) to high-income countries (the north). An emblematic case is the Mexico-US migration. Henceforth I refer to these type of labor flows as *south-north*. In the south-north literature, workers move from poorer regions where a type of labor, usually a low-skilled one, is abundant, to richer ones where it is scarce. This labor flow is the result of higher expected expected earnings abroad, because of differences in countries' income levels and because of the relative scarcity of the migrant's type of labor in the host country.

In recent years selective policies of migration aimed to attract high-skilled foreign labor force, and the creation of free labor mobility areas of similar income-level countries —like the Schengen Area— have driven attention to labor flows of high-skilled workers and within high-income countries.[5] One example of this labor flow is the intra-EU15 work-related migration. This type of flows cannot be explained by the south-north literature because its basic mechanisms are contradicted from the beginning. This section reviews some of the migration literature that falls in within the south-north traditional approach and points to where the works in this literature fail to explain and

[5] In the next chapter, my coauthor Daniela Costa and I refer to this type of labor flow as *north-north*.

predict the *high-income, high-skilled* labor flows.

Beyond the basic south-north model described above, authors began to modify the standard framework by including additional features to allow further self-selection of immigrants and to rationalize the observed heterogeneity (in the performance, earnings, and country of origin of immigrants) of some of these labor flows. Some these features include heterogeneity of schooling and skills, mobility costs, loss of ability of newcomer-foreign workers, differences in the size of the source and host countries, among others.

Regarding self-selection of occupations, the seminal paper Roy (1951) "Thoughts on the distribution of earnings" looked at how workers select occupations based on their skills (and on the correlation of their skills with those of others), and at the effect of this selection on the distribution of income and productivity across occupations. Roy added a detailed explanation of self-selection across two simple occupations —hunter and fisherman— from which he generalizes further on. He even allowed for geographical costs of changing occupations which are taken into account in the occupational choice. However, his paper studied labor choices in a closed economy framework and therefore its focus is not the flows of labor across countries. In addition, he did not include a mathematical model to accompany his explanations. Some scholars, refer to Borjas (1995) as the first paper to formalize a Roy Model.

Borjas (1995) provided a model where self-selection is driven by the correlation between skills across countries and the dispersion of the distribution of their relative earnings. With this model, Borjas aimed to explain migration flows towards the United States and the differences in the earnings of immigrants by country of origin. Moreover, he allowed for immigrant cohorts from the same source country to differ according to changes in the relative rate of return to skill. Despite his claim that his model can explain migration flows for many host countries, this is only true when the host country is relatively richer than the source country.

Urrutia (2001) went a step further in this line of thought and modeled migration flows from Mexico and India to the United States in a south-north approach. He extended the analysis by allowing for differences in migration costs due to distance and language barriers, which he models as a fixed cost and as a temporary loss of ability respectively. By considering these two aspects, Urrutia generates a self-selection pattern that can account for heterogeneity in the performance of immigrants from different source countries, which is observed in the data. His main result was that immigrants from distant countries are more likely to belong to the top abilities distribution. However, this result runs contrary to what is observed in the context of migration from and to free mobility areas, where migrants move across proximal countries and yet belong to the top abilities distribution for some occupations.

Lopez-Real (2011) incorporated a new source of heterogeneity of workers. In this model, workers are heterogeneous in years of schooling and ability. Lopez-Real finds that self-selection in ability is always positive and that differences in total factor productivity (TFP) determine whether self-selection in schooling is positive or negative. Nonetheless, his model cannot explain the concentration patterns that some occupations with high educational content display in high income countries of free mobility areas.[6] Moreover Lopez-Real assumed that the host country is a large open economy while the source country is a small open economy. This is not always the case for countries of free labor mobility areas that are similar in terms of size and openness. Additionally, some of the results of Lopez-Real depend on exogenous TFP differentials, which might no be present in integrated and synchronized areas.

In a more descriptive work, Dustmann and Frattini (2012) provided an overview of immigration to Europe between the Second World War and the early 2010s. The authors documented the disparities between immigrants born in the European Union (EU) and those born outside of the EU, with special focus on labor markets. Overall, they found that EU-born immigrants are more similar to the native population than immigrants coming from from elsewhere, for example with respect to occupational and educational attainment distributions. Non-EU immigrants were found to be more concentrated than EU nationals in less skilled occupations. Hence their observations indicate that the intra-EU migration flow (a case of high-income, high-skilled labor flows) is different from the aggregate migration flow to the EU15 in that they mimic the characteristics of the national population.

Dustmann and Frattini calculated the employment probability of EU-born and non-EU-born immigrants working in the EU, controlling for gender, education, region and age. They foun that non-EU immigrants are in disadvantage in all countries studied, lagging behind by at most 20 percentage points in employment probability compared to

[6] In the next chapter, Daniela Costa and I document these concentration patterns in the EU15.

natives with the same characteristics. Meanwhile EU immigrants are at most 8 percentage points behind. The differences across immigrants documented in their work pose relevant questions that are left unanswered by the traditional south-north literature. Why are migration flows across EU15 countries more intense than what is predicted by the south-north models provided that those countries are considerably more homogeneous? Why do the EU15 migrants are so different from those coming from other parts of the world?

1.2.3 North-North labor flows

I now review a second strand of the literature of international labor flows. The papers in this section are related to the high-income, high-skilled work-related migration. This type of flows are those where workers with similar characteristics, usually high-skill labor, move across high-income countries (the north). In parallel to the term south-north, I refer to this as the *north-north* approach.

As mentioned in the previous section, one of the characteristics of the high-income, high-skilled labor flows is the concentration of foreign workers in regions where the native workers of their same type are abundant.[7] One important paper related to these concentration patterns is Chipman (1970). Chipman developed a model with perfect competition and increasing returns to scale. This paper focused on proving that increasing returns to scale are possible in a competitive environment, and did not look specifically at migration. However, his framework (the economies of scale) can be adapted to generate agglomeration of workers in line with the aforementioned concentration of of immigrants of high-skilled occupations.

Another important paper that is related to these concentration patterns is Krugman (1991). This paper studied the geographical location of labor. Krugman's model differres from Chipman's in that the former used a monopolistic framework, in that he considered economies of scale that were not external to the firm, and in his consideration of transportation costs. His model, under certain conditions, was able to generate a regional concentration in the manufacturing industry. Although he framed his paper in the context of the clustering of workers in few regions of the United States, some of the

[7] This is a general feature of the high-skilled or high-educated labor force, and also of the labor force which has any characteristic that generates spillovers or increasing returns.

mechanisms of his paper are suitable for the analysis of the international concentration patterns of workers with similar characteristics.

Haupt and Uebelmesser (2010) analyzed highly-educated labor flows and provided a partial equilibrium model with increasing returns to scale and migration choice where workers with high human capital concentrate in a single high-skill industry. The authors focused on the negative result of that the geographical concentration of trade in high-skilled labor generates distortions in the host country for less able individuals to become highly educated. However their paper provides a basis for explaining both the high-skilled and high-income characteristics of the north-north approach.

In the next chapter of this thesis, my coauthor Daniela Costa and I document the concentration patterns in the north-north intra-EU15 migration. These concentration patterns that are present in occupations with high-educated labor and across-high income countries. We propose a model with external economies of scales in the highly-educated sector which generates the concentration patterns documented in the area.

1.3 Capital Flows, Demographic Transitions and Investment

1.3.1 The theory of international capital flows

Although international capital flows, date as far back as initial contact between early societies with borrowing and lending across them, the study of these flows has only recently become a major area of study in Economics.[8] In this section I define the concept of international capital flows and review some of the seminal papers that study the causes of these flows.

International capital flows consist of outflows and inflows. Ruffin (1984) defines a capital outflow as "the accumulation of ownership claims against capital located in a foreign country" (p. 246), similarly a capital inflow is the accumulation of a liability

[8]For instance, Obstfeld and Rogoff (1995) mentioned that "trade credits" (borrowing and lending across countries) "were already common in biblical times" (p. 1734).

that increases the domestic capital.[9] In the neoclassical literature, one possible classification of the international capital flows is: capital flows implied by trade of goods, intertemporal capital flows, and capital flows for international diversification.

In the first models of international trade, capital flows were a result of production patterns that emerged when exploiting comparative advantages in technologies and factor endowments.[10] An example these models is the basic Heckscher-Ohlin. In an influential paper, Mundell (1957) showed that in a Heckscher-Ohlin two-country, two-factors, two-goods model with identical homogeneous-of-degree-one production functions,[11] where one good always requires a greater share of one factor than the other good, and where endowments are such that there is no specialization, trade of commodities is perfectly substitutable with trade of factors of production. I.e., in the Heckscher-Ohlin framework trade of goods is equivalent to trade of factors of production.[12] These models assume that trade balances period by period and therefore the current account is always equal to zero.

By the late 1960s and early 1970s some economists like Bruno (1970) emphasized the lack of treatment of prolonged trade imbalances in the international literature. Bruno refered to Bardhan (1967) and Hamada (1969) as some of the few models, developed before his, which allowed for foreign borrowing. However, these models did not allow for trade of goods, hence he went a step further by developing a model with both trade of goods and financial assets (borrowing via international capital flows). Prolonged trade imbalances could then be explained as intertemporal borrowing and lending across countries.[13]

The aforementioned literature started to gain attention in the 1980s and early 1990s.

[9] The definition of Ruffin emphasizes the exchange of capital inputs against claims for saving or borrowing. Other definitions directly treat capital as an asset, for instance Schularick (2016) defines international capital flows as “changes in the foreign ownership of domestic assets and the domestic ownership of foreign assets” (p. 1). Schularick’s definition, for instance, is more related to capital flows for international diversification.

[10] Note that this does not apply to the Ricardian-Viner model of trade, discussed in section 1.2.1, where labor is mobile across countries but capital is not.

[11] The presence of economies of scale break the result as Mundell showed.

[12] Ruffin (1984) made reference to other works that suggested the equivalence between trade of goods and factors. This works are: Heckscher (1919), Meade (1951), Lerner (1952), and Samuelson (1968).

[13] In the basic model of intertemporal trade, there could be trade imbalances period by period, however the present value of the current account must be equal to zero, provided that there are no ponzi-schemes.

One of the most cited papers in this literature which explains capital flows as net intertemporal savings, is Obstfeld and Rogoff (1995). In their paper "The intertemporal approach of the current account" net capital flows are a result of households' dynamic savings choices, to smooth consumption across time, and firms' investment choices.[14] The mechanism behind the flow of capital is the chase for high returns: capital flows from countries with low autarkic rates of return to countries with higher rates of return. Intertemporal capital flows serve for international trade of present consumption for future consumption and can generate a separation between domestic savings and domestic investment choices.[15] This last feature is of special importance for the literature that analyzes the effects of capital flows on economic outcomes. In the next section I elaborate on this topic.

The view of capital flows for international diversification emphasizes environments under uncertainty with country-specific risk that can be diversified away by exchanging assets (capital) of one country for assets in another country. Note that this trade does not necessarily require intertemporal flows; assets can be exchanged period by period and net positions canceled out without imbalances.[16] The theoretical literature on this subject dates back to the late 1960s. Among the first papers Grubel (1968) considered a framework where international capital flows were determined not only by interest rate differentials, but also by country specific variables: the growth rate of total assets of the country, as well as changes in monetary or fiscal policy.[17] In this framework, there are three assets in each country: bonds, money and risky assets. The risk of the assets is country-specific and it is diversified away via trade. Grubel's model is reminiscent of the pricing literature and did not provide much detail about the agents and the environment. However, he was is among the firsts to emphasize that gains of trade extended beyond comparative advantages to diversification.

[14]The authors emphasized that their model provided insights beyond those models that ignore intertemporal household behavior like the Mundell-Fleming framework. Note that this criticism is different from Bruno's which emphasized the lack of international models of borrowing and savings.

[15]There are situations in which this separation breaks down like when having incomplete assets-markets where there are no contingent claims.

[16]Obstfeld and Rogoff (1995) studied an environment with uncertainty and complete markets where countries trade to eliminate local shocks which then become global. The authors claim that if countries have identical and isoelastic preferences, there would be no current account imbalances.

[17]Grubel assumed that the government of each country regulates the quantity and interest rate of bonds to guarantee full employment.

Other models of diversification would follow, for instance John C. Cox (1985) developed a general equilibrium model with intertemporal trade, uncertain production, complete markets, identical households, and no transaction costs. The paper focused on the study of asset prices and one of its main results is that trade reduces risk by decreasing the drift of the determinants of the fundamentals by a risk premium factor. Other papers like Obstfeld (1994) focused on the growth effects generated by the gains of diversification. The innovation of the model of Obstfeld with respect to the model of Cox is that the latter was more general in allowing for more flexibility in the assumptions of the agents, in particular in the consumers. In addition, the model of Obstfeld allowed for "aggregate shifts in the global portfolio of risky assets" (p. 1311) which improved upon previous works like Devereux and Smith (1994). Another seminal paper in this literature is Bencivenga and Smith (1991) which took a different route in the source of uncertainty by abstracting from production risk and focusing on stochastic preferences on liquidity, and in considering a life cycle model. Although an ever-growing field, most of the literature on this subject is still centered on the same principle: capital flows internationally to diversify country-specific risk.

1.3.2 Empirical puzzles

After the seminal papers, discussed above, which established the drivers and results of international capital flows, each of these theories would be questioned by empirical findings that contradicted their implications. These contradictions are known as the "puzzles in international economics".[18] In response, some of the literature would evolve to solve some puzzles or at least to reduce them and restrict them to more specific environments.

Regarding capital flows generated by trade of goods, there is the criticism that in the data capital flows less than goods (recall that this criticism is similar to the observation by Smith regarding international labor). To overcome this criticism, or rather to explain why capital flows less than commodities, costs of capital flows, tariffs

[18]Strictly speaking, the six major "puzzles in international economics", as described by Obstfeld and Rogoff (2001) also include puzzles about trade of goods and about prices. Among these puzzles, the *home bias in trade* (McCallum (1995)) has been addressed by the introduction of trade costs and preference for national goods in the consumers' preferences. Regarding prices, the puzzles mentioned by Obstfeld and Rogoff are the *purchasing power parity puzzle* and the *exchange rate disconnect puzzle*.

and taxes and adjustment costs have been considered different the models.

Theories on intertemporal capital flows have also been challenged by puzzles. The *allocation puzzle* introduced by Lucas (1990) states that capital does not flow from North to South, or even within the regions it does not flow to high productivity countries,[19] in accordance to what the chase for higher returns would predict, and the *Feldstein and Horioka (1980) puzzle* that indicates a high correlation between investment and savings. The first of these puzzles has been addressed by considering risk-premium factors, valuation effects, and restricting the variables that originate the puzzle like in Gourinchas and Jeanne (2013) to savings only, while the second one by considering flows of capital of different length, revising datasets and methodologies like in Curcuru et al. (2008), and introducing financial frictions like limited enforcing and limited spanning as in Bai and Zhang (2010).

Lastly, as to capital flows for diversification there is the *equity home bias French and Poterba (1991) puzzle* where countries hold few international risky assets, and the *international consumption correlations puzzle* where consumption and domestic output are highly correlated. The first puzzle or bias has been addressed by introducing factors like human capital hedging as in Julliard (2002), risk of diversion like in Mendoza et al. (2007), among others. The second puzzle has been addressed by trading frictions and transportation costs, among other avenues. For instance, Backus et al. (1992) showed that transportation costs can reduce the difference between cross-country correlations of output and consumption.[20]

Having discussed the concept and categories of capital flows and some of the corresponding seminal papers, I now turn to the literature that analyzes the economic impact of intertemporal capital flows.[21] I focus on the implications of openness to capital flows

[19] Gourinchas and Jeanne (2013) distinguished between the original Lucas critique and this puzzle. They referred to the allocation puzzle as only the observation of capital not flowing to high productivity countries across developing countries. Gourinchas and Rey (2014) documented and emphasized this two findings and explained that a standard model of intertemporal trade can not explain the documented pattern of net flows between the south and the north or within developing countries.

[20] Backus et al. found that transportation costs can reduce the volatility of investment and net exports and the difference between consumption and output correlation. However, they were not able to generate cross-country correlations of consumption in line with the data. In their model, under several specifications, these correlations exceeded that of the data and because of this the authors called this the consumption/output anomaly.

[21] Some authors like Koepke (2015) and Mejía (1999) divide the literature in international capital flows into three categories: one that studies the causes of these flows, one that studies the effects of

for demographic transitions.

1.3.3 Openness to capital flows and demographic transitions

In this section I review some of the works that study the economic effects of long-run demographic changes and their relationship with international capital flows. I begin by establishing a connection between economic development and demographic transitions, and then I review the literature on the relationship between international capital flows and the effects of the demographic transitions on capital investment. Finally, I emphasize demographic transitions asynchronicity and I point to a void in the literature: capital flows, investment and the demographic transition of Latin America which I study in the final chapter of this dissertation.

Economic development and demographic transitions

There is an extensive literature that studies economics and demographics changes. Part of the literature focuses on the effects of economic development on demographic transitions, and another studies just the opposite: the economic effects of demographic changes. Regarding the first strand, the general consensus is that as economic development generates demographic transitions.

The concept of a *demographic transition* (DT) was introduced in the early 1900s. In his paper "Population", Thompson (1929) analyzed the composition and evolution underwent by the population of several countries over the 1800s and early 1900s. He classified countries into three groups according to their changes in birth and death rates: the more developed Europe where birth and death rates were sharply declining, Southern Europe and the Slavic Central Europe where death rates were declining much more than birth rates, and Russia, Asia, Africa and South America where birth rates were less under controll. Although Warren did not use the term demographic transition, he described the process of *the demographic transition model* where countries transition from higher birth and death rates to lower ones. Among the first authors to formalize the DT model, Notestein (1945) explained that a DT occurs in four stages: pre-transition with high birth rates and low population growth, early transition where death rates start

these flows in economic variables (outcomes), and one that analyzes on policy implication of these flows. This literature review focuses on the first and second categories.

to decrease, late transition where birth rates start to decrease, and post-transition with low birth and death rates. These first works on demographic change already described some of the aspects of the economic development and industrialization of the countries under consideration and their demographic stages.

Coale (1972) studied the determinants and the process of the changes in the age structure of a country's population. His description of a demographic transition was in line with previous findings.[22] He also wrote about the fact that in the 1960s the oldest populations were those of Europe, while the youngest were those in the less developed and non-industrialized countries. He made a lot of emphasis on economic development: "the highly industrialized countries have older populations than the underdeveloped ones" (p. 48). He also noted that in the previous century countries with older populations had a tendency to become older faster than younger countries whose age structure was not changed significantly.[23] Lastly, he explained that the decrease in mortality rates were an important factor for the increasingly young populations, as it was the case of the United States.

These observations would later become stylized facts of the relationship between economic development and demographic transitions which start during a country's development phase when mortality drops and the young population increases. Further on, as the economy keeps growing and developing, fertility rates drop and most of the population becomes middle-aged, and finally they begin the aging process. At the beginning of this transition the growth rate of the population increases and then it becomes decreasing as countries approach the middle-aged stage and from there on.[24]

Capital flows, demographic transitions and their effect on investment

Regarding the literature that analyzes the economic effects of demographic changes, the study of how demographic changes affects economic growth dates back to the 1700s. Malthus (1798) studied the bilateral relationship between demographic growth and economic growth. Malthus theorized that since population increased at geometric rates

[22] The causes of the transition were also in line with previous theory. For Coale, the main determinants of the age distribution were for young populations high birth-rates and for old populations low fertility rates. He also acknowledged the increase in life expectancy, but in view these factors affected the age distribution less than the changes in fertility.

[23] Thompson also mentioned that death rates were falling faster in the 1900s than before.

[24] Section 3 of this thesis provides a detailed explanation of this process and more references.

while goods for subsistence increased at arithmetical rates, population growth and economic growth self-regulate each other. In the modern growth literature, the economic effects of demographic transitions can be studied through the effect that a DT has on the growth rate of a population as well as by using lifecycle frameworks and directly modeling the change in the age-profile of the economy. In the rest of this section I restrict myself to the literature on the effects of the demographic transition on capital investment.

Modern analyses of the effects of a DT on investment rates make use of the standard neoclassical growth models with population growth like the models of Solow (1956) and Ramsey (1928)-Cass (1965)-Koopmans (1965). And also, incorporate theories like the life cycle hypothesis first posed in Modigliani (1966). In the standard Solow model, the decrease in the growth rate of the population that is observed as countries transition from young to middle age, increases total investment but leaves savings and investment rates unaffected since they are assumed to be exogenous. In a Ramsey-Cass-Koopmans framework where investment and savings are endogenous, a decreasing sequence of growth rates of the population generates an increase in savings and investment rates.[25] The second result is what the life cycle hypothesis would predict for agents transitioning from young to middle ages. Analogously for countries transitioning from middle-aged to old, the life cycle hypothesis predicts reductions in savings and investment generated by the disaccumulation of assets during retirement.

Note that the effect the DT on capital investment rates crucially depends on the relationship between savings and investment. In the aforementioned analysis there is the implicit assumption that the economy is closed to intertemporal capital flows and therefore national investments and savings are jointly determined. The first authors to study the changes on investment generated by the DT assumed closed economy frameworks,and it was not until the 1970s that the connection between the DT and capital flows was made.

Some seminal papers on the study demographic transitions and their effect on investment rates are Coale and Hoover (1958) and Leff (1969).[26] As cited by Mason

[25] For an explanation of why does this happen see section 3.5.2 of this thesis and footnote 64.

[26] The aim of this papers was to study demographic transitions and their effect on savings, however they also serve as foundation of the study of effects of capital flows on investment and demographic transitions.

(1988), in the Coale-Hoover theory "the share of national output devoted to savings and investment can be adversely affected by rapid population growth". Note that this analysis assumed a closed framework.[27] Leff (1969) had similar ideas and findings about the relationship between the DT and savings but he also introduced intertemporal, international capital flows. In his theoretical analysis he indicates that some of the savings can come from the current account. However, Leff did not consider appropriate for his study to model forward-looking decisions of agents, and instead he borrowed from the life cycle hypothesis of Modigliani and focused on the empirical estimation of savings as a function of the DT. When discussing the policy implications of his findings, Leff talked about the same effects of the DT on investments and savings, which confirms the fact that there was no separation between savings and investment choices that otherwise could have been possible by the inclusion of intertemporal capital flows in the current account.

Among the first to focus on the age-structure changes of the demographic transition, and not only on the changes of the growth rate of the population, is Fry and Mason (1982).[28] This paper studied the demographic transitions of some East Asian countries that were moving towards middle-aged. The authors considered an environment open to international capital flows. In particular, in the econometric analysis they distinguished between national and foreign savings. Their model endogenized the effect of the DT on the growth rate of Gross National Product percapita and documented positive effects of a DT on this growth rate. The authors also discussed how an increase in foreign savings, or an international capital inflow, could decrease national savings rates while increasing investment rates and the growth rate of the economy. Later on, this paper together with Kelley and Schmidt (1996), Toh (1997), Bloom and Williamson (1998), Deaton and Paxson (1998), Mason (2001), among others, would be cited in studies of the literature *East Asian economic miracle*, for being among the ones that documented positive demographic effects on investment and growth in East Asia.

[27] This observation extended to more recent papers, some authors like Henriksen (2002) emphasized that most of the work about the economic effects of demographic changes focused on closed economies. Among these papers, Ríos-Rull (2001) developed a general equilibrium, overlapping generations, closed economy model to study the the changes on savings for retirement generated by the aging of the postwar generation of Spain. The analysis focused on the transition between demographic steady states.

[28] Fry and Mason (1982) was based on the model developed in Mason (1981), however the latter talked about investment and considered an open economy framework while the former studied only national savings.

In the 1990s the focus of the effect of intertemporal capital flows and demographic transitions on investment was made explicit. Among the seminal papers on this matter Taylor and Williamson (1994), "Capital Flows to the New World as an Intergenerational Transfer", explained how capital flowed from older Europe into the younger "New World" of the late 1800s as intertemporal capital flows driven by differences in the age structure of this regions. Other influential papers in this line of research are Lane and Milesi-Ferretti (2002) and Domeij and Floden (2006). The former took the empirical route and documented, for a big set of countries, decreases in the net foreign capitals in younger populations and increases in older ones. And, the latter had the innovation of studying international capital flows, investment and demographic transitions in a theoretical framework of a neoclassical growth model with lifecycle behavior. They also analyzed the capital flows of eighteen OECD countries vis-a-vis their model which explained a "small but significant fraction of international capital flows" (p. 1).

As other regions of the world became more developed and the asynchronicity of demographic transitions grew larger, the study of demographic transitions has benefited from additional case studies. Among these studies: Henriksen (2002) analyzed capital flows between the younger United States and the older Japan. The model of Henriksen can rationalize the net-foreign asset position of the United States vis-a-vis the position of Japan by the age differences of these countries. Krueger and Ludwig (2007) analyzed the effects on international capital flows and on welfare of the European Union and the developed OECD aging faster than the United States. The main result of this paper is that the aging asynchronicity between these regions together with the current financial openness to capital flows will generate, bigger price effects in the United States in detriment of the old cohorts and to the gain of the youngest. The openness to international capital flows, seems to be somehow a negative result, as otherwise price effects which have a negative effect of the largest cohorts would be smaller, according to authors "the US is importing the more severe aging problem of other regions".[29]

[29]Its worth noting that the demographic transition of the United States has been characterized by long stages because of migration that keeps the country mostly comprised of working age population. The paper of Krueger and Ludwig studies future effects of aging. Regarding current trends, there is a big demographic cohort that is going though its own "demographic transition" and this is the baby boomer generation. The effects of population aging of this generation have been studied by some like Karahan et al.. Karahan et al. (2015) focuses on analyzing the effects of the aging of the baby boomers on firm dynamics. There is another literature that focuses on aging in the United States and financing

In sharp contrast to the aforementioned result of financial openness, Attanasio and Violante (2000) studied the capital flows between the relatively older Europe and Northern America, and the younger Latin America. Attanasio and Violante found that financial openness would benefit the relatively younger region, in this paper it is Latin America while in Krueger and Ludwig (2007) it is the United States. The reason behind this results is simply that in Latin America the biggest cohort is not expected to age soon but instead it is moving to its peak earning years making them benefit the most from general equilibrium price effects (increase in wages). In a similar spirit, Attanasio et al. (2006) analyzed capital flows form developed to less developed countries.[30] In both of these papers the authors use a general equilibrium, overlapping generations framework. They compared a fully closed economy model to a fully open economy model and analyzed the transition between demographic steady states. Their models are closely related to Ríos-Rull (2001), that analyzed the effect of the aging of the baby boomers of Spain on savings for retirement, with the addition of an open economy analysis.

The aforementioned papers focused on younger regions, about his phenomenon, the latest region to undergo a demographic transition has been Latin America and there are only few studies about it.[31] Most of this studies are descriptive of the demographic phenomenon, and only briefly discuss the effects of the DT on empirical estimations of percapita GDP growth rates.[32] Some this studies are: de Carvalho (1997), Bloom et al. (2001),Van Der Ven and Smits (2011), Saad (2011), Guzmán et al. (2006), among others.

To my knowledge, there are no systematic empirical estimations of differences in the relationship between investment and the current demographic transition of Latin America and the equivalent of other regions of the world. In the last chapter of this dissertation I document that in Latin America investment rates behave differently during DTs as compared to other regions of the world, most of which have been discussed in this review. I provide a theoretical framework for analyzing this observation and find

savings for retirement, however its discussion is beyond the scope of this review.

[30]One of the shortcomings of this paper is that the less developed countries are not in the same stage of their demographic transition, hence the flow of capital is not as clear as one that would result from analyzing interactions across different groups of comprised of countries with similar transitions.

[31]In particular the current stage of the DT of Latin America is the period of the demographic window (for a definition and discussion on this topic see chapter 3).

[32]For the reader that is familiar with the demographic window, this effects are known as the demographic dividend.

that the main determinant behind it is that Latin America is nowadays more open to capital flows that what other regions were at the same stage of their demographic transition.

Chapter 2

North-North Migration and Agglomeration in the European Union 15

2.1 Introduction

In light of current selective migration reforms, this paper revisits empirical evidence on migration within the European Union 15[1] (EU15), disaggregated by occupation. We document that foreign-born workers within this area live in countries where their type is relatively more abundant among natives. This is at odds with traditional models of migration. We build a model with a sector exhibiting external economies of scale that allows for international labor flows between countries that are similar both in terms of income and individual characteristics of workers. The main result is that, if a country has a relatively larger fraction of native population working in a high educated intensive sector, this country will attract foreign labor of the same type. This is consistent with migration patterns observed in high-educated occupations in our sample of analysis.

The share of total immigrants relative to the population in Europe is now similar to that of the United States (US), number wich was much smaller around 1960 (Dustmann and Frattini, 2012). Regardless of the migratory inflows generated by the decolonization

[1]The EU15 comprised the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom.

process and the incorporation of Eastern European countries to the European Union (EU), 20% of the immigrants in the EU15 are native from other EU15 countries. Recent policy changes are likely to be behind these numbers. Two examples of these changes are the creation of a free mobility area, established by the consolidation of the Schengen Area in 1995, and the changes in national policies that formalize agreements reached under the framework of the European Higher Education Area (EHEA).

The EHEA is the result of a series of agreements signed between 1999 and 2009, involving changes in national educational policy by the member states[2]. These changes include the transferability of academic credits and the mutual recognition of degrees across the EHAE. While the Schengen Area is just one of many labor free mobility areas in the OECD (OECD, 2012), the EHEA represents the first *human capital free mobility area*. For our analysis, this means that, nowadays, skills are more transferable and workers are more mobile within this area, which enhances the importance of the intra EU migration phenomenon. These two types of free mobility policies, of workers and skills, are likely to reinforce each other.

Selective migration policies have gained weight, among industrialized countries, in detriment of traditional quotas and family reunification. This type of policies favor inflows of highly skilled labor. Within the EU, for instance, the United Kingdom is considering adopting a point-based immigration scheme, where potential immigrants earn points on the basis of their qualifications and skills, among other factors. More recently, the Great Recession has brought up concerns that were not present at the time the free mobility agreements were signed.

Most models that examine migration analyze labor flows from poorer regions, where a type of labor is abundant, to richer regions, where it is scarce. We refer to this approach as the *south-north* approach[3]. In this setup, individuals migrate because of the relative scarcity of their own type of labor in the host country and the difference in countries' in income levels, which rewards those that cross the border with higher labor earnings. This is not the case between the majority of the workers of the EU15.

Most of the workers of the EU15 are highly educated, and the EU15 countries have similar and high income levels. In addition, in this paper we document that if a EU15

[2]All European countries except for Belarus are part of the EHEA.

[3]For a description and summary of these models see section 1.2: with a literature review on labor flows.

country has a relatively larger fraction of native population in a high-educated occupation, this country will attract foreign EU15 labor of the same type. We also document that high-educated occupations display concentration patterns in that workers in those occupations cluster in specific countries. In light of this findings, the intra-EU15 migration is one where workers migrate to countries similar to their source country and where their type is relatively abundant. This labor flow is classified as what we call *north-north* migration.

We develop a model with external economies of scale where wages are strictly increasing in the amount of high-educated (HE) labor, both foreign and native, employed in a country. Hence, at the individual level, it is worthy for the most able households to become HE and to move to the country where there are more HE native workers. This is consistent with the migration pattern observed in high-educated occupations in our sample of analysis, a north-north migration pattern. Therefore, by incorporating the previously described agglomeration mechanism, the model we propose in this paper successfully generates the EU15 migration flows and concentration patterns for our sample data.

2.1.1 Related Literature

The model of this paper is based on Chipman (1970). For our model, the most important feature of his theoretical framework is the presence of increasing returns to scale in the production of the skilled intensive good, that are external at the firm level. This is the main force that induces agglomeration of workers with similar characteristics after migration. In this paper, we use the aforementioned feature and generalize the model of Haupt and Uebelmesser (2010), which focus on high-educated flows, by creating a second sector with a constant returns to scale (CRS) technology, and by having a general equilibrium environment. We incorporate features of their model by allowing households to simultaneously choose education and migration in the skilled intensive sector. In the future our aim is to open the migration option for workers in the CRS sector too, in order to generate heterogeneity of migration flows.

The rest of the paper is organized as follows. Section 2 describes the data and documents patterns of intra-EU15 migration by occupation. Section 3 describes the model and the equilibrium and section 4 reports and discusses the results. Section 5 concludes.

2.2 Data and Empirical Evidence

2.2.1 Data and Classifications

One of the main limitations of the analysis of migration patterns across different countries is the lack of comparable data. It is often the case that each country uses a different definition of immigrant based on either nationality or country of origin. Harmonized data on migration status and occupation for European countries are available from two sources: the European Labor Force Survey (EU-LFS), which consists of repeated cross-sections of individuals from 1983-2013; and the Database on Immigrants in OECD Countries (DIOC), which reports aggregate numbers of workers by different demographic and labor market cateogries based on Census data, with a comprehensive list of variables and countries only for the year 2001[4]. In this analysis, we use the EU-LFS and we consider a worker to be immigrant if she was born in a country different from the one in which she works.

The European Union Labor Force Survey Database

The European Union Labor Force Survey (EU-LFS hereafter) is a harmonized household sample survey that contains quarterly detailed information on individuals per country for 28 European countries. The data covers the years from 1983 onwards, due to availability of our variables of interest we keep the years of 1996 to 2010, for a total of 31,663,252 observations.

The core variables in the analysis are country of residence, country of birth, educational attainment, employment status, hours worked, and occupation (identified by the 1988 International Standard Classification of Occupations, ISCO-88 hereafter). Out of the 28 countries available in the EU-LFS, we keep EU15 countries.

[4] A version for 2005 is available, but the information disaggregated by occupation is incomplete for a large part of the countries.

The ISCO-88: Description and Relation to Educational Classification

The ISCO-88 is one of the occupational classifications published by the International Labour Office (ILO, 1990). It uses information on national coding for over 80 countries and organizes them into a standard classification of occupations.

Even though each occupation presents a different skill specialization content in terms of tasks, we find convenient that its ordering coincides with its corresponding educational level. In particular, eight of the nine major ISCO-88[5] groups are ordered with reference to education levels[6] defined for ISCO-88 (See Table A.2 for a detailed description of each group). Five out of nine major ISCO-88 groups (4, 5, 6, 7 and 8) have the same average education level (lower or upper secondary education). These five groups, together with Elementary Services (group 9),[7] will be considered *Low-Educated* in the analysis below. The remaining groups (1-3) include occupations that require tertiary education and therefore will be classified as *High-Educated*.

2.2.2 Empirical Analysis

This section is organized in three parts. In the first part, we provide empirical support to our north-north approach showing that EU15 immigrants are different from non EU15. Henceforth we limit the study to foreign-born workers whose country of origin is a EU15 member. Then we focus on the distribution of foreign-born workers across occupations, compared to the distribution of native-born workers. In the second part, we compute the correlation between the occupational distributions of foreign-born workers and that of native-born workers, for each country. We use this correlation to explain the relation between natives and foreigners. Finally in the third part, we compute a proxy for concentration of total workers depending on their occupation. For this measure, we use the educational component of the ISCO-88.

[5] We exclude Armed Forces (group 0).

[6] The ISCO levels of education are based on the first (1976) version of the International Standard Classification of Education (ISCED). ISCO defines 4 levels of education: 1 for primary education, 2 for lower and upper secondary education, 3 for tertiary education not leading to a university degree and 4 for tertiary education leading to a university degree.

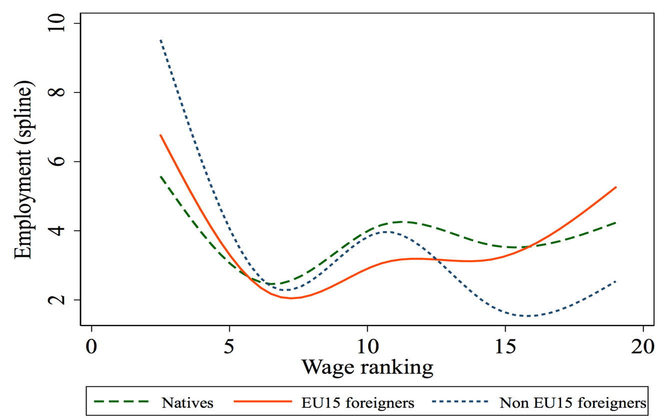
[7] Elementary Services is the only major group in which the average education level is primary school.

North-North Empirical Analysis

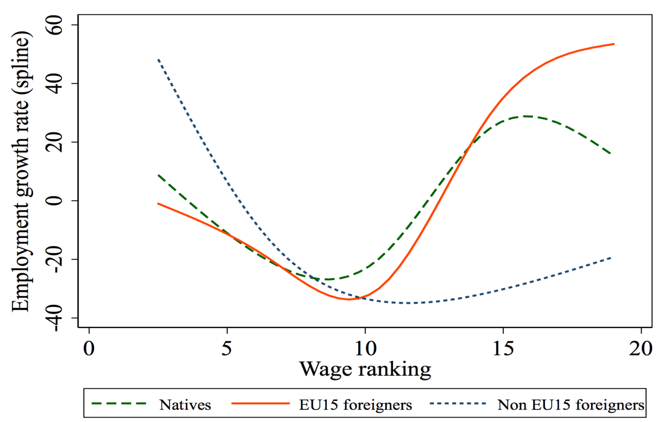
Many authors[8] have emphasized the differences between EU15 immigrants and other foreigners. In particular, EU15 immigrants have similar characteristics as natives, for example both groups have high levels of education and are more concentrated in skilled occupations. Figure 2.1 depicts the distribution of employment across occupations, ordered by wage, for natives and foreigners.

Figure 2.1: **Employment distribution**

(a) Percentage Employment (2010)



(b) Growth rate of employment share (1996-2010)



Notes: Wage ranking orders occupations by their mean wage across 10 European countries across all years, following wage information in Goos et al. (2014). We restrict the number of countries to match their wage ranking. Panel (a) plots the median spline of employment shares by migrant status, pooled across countries. Panel (b) plots the median spline of the growth rate of these employment shares from 1996-2010.For detailed information on occupations refer to Table A.1 in appendix A.

[8] Dustmann and Frattini (2012) among others, see section **??**.

In line with previous findings, we observe that EU15 foreigners behave in a similar fashion as natives and very differently from non EU15 immigrants. Additionally, in panel (a) we observe that the share of EU15 foreigners exceeds that of natives in the top paid occupations. Panel (b) in Figure 2.1 shows the evolution of this distributions. Notice that this graph is reminiscent of the polarization literature: employment growth is larger in both tails of the wage distribution and negative in the middle, for natives and EU15 immigrants. We emphasize that both the share and growth rate of EU15 workers in the top paid occupations exceed that of any other group. In this paper we will concentrate on this particular group of occupations.

Occupational Migration Patterns

"Do people migrate to countries where there are more native workers of their type or where there are less?" This subsection addresses this question from an empirical perspective.

First, we define the occupational distribution of foreign and native workers in country i as:

$$S_N^i = \left(s_{N1}^i, s_{N2}^i, ..., s_{NJ}^i\right), \text{ where } \quad s_{Nj}^i \equiv \frac{\text{\# native workers in occupation } j \text{ and country } i}{\text{\# native workers in country } i}$$

$$S_I^i = \left(s_{I1}^i, s_{I2}^i, ..., s_{IJ}^i\right), \text{ where } \quad s_{Ij}^i \equiv \frac{\text{\# EU15-immigrant workers in occupation } j \text{ and country } i}{\text{\# EU15-immigrant workers in country } i},$$

respectively, where J is the number of subgroups considered, $J = 26$ for ISCO 2-digit.

Then, for each occupation j , we extract the shares by country for both native and immigrant workers: $(s_{Nj}^1, ..., s_{Nj}^I)$ and $(s_{Ij}^1, ..., s_{Ij}^I)$. Next, we compute the correlation between the occupational distribution of native workers and that of EU15 foreign-born workers. We interpret this as an empirical measure of how EU15 foreign-born workers allocate themselves across countries based on the given distribution of natives. More specifically, we regress S_N^i on S_I^i and a set of year dummies. Table 2.1 shows an example of how this shares look like for for subgroup 12 (Corporate Managers).

We obtain a correlation of 0.66, significant at the 1% level. As a comparison, this number is only 0.40 for immigrants from outside the EU15. The result of this empirical analysis is that if a country has a relatively larger fraction of native population working in a high-educated occupation, Managers in the example shown in Table 2.1, this country will attract foreign labor of the same type.To answer the question that we posed at the beginning of the subsection, people do migrate to countries where there are more native workers of their type. This is an example of what we previously referred to as north-north migration patterns.

Table 2.1: **Share of Natives and Immigrants by Country of Residence 12: Corporate Managers, 2010**

Country	Share Natives	Share Foreign EU-15
Austria	3.56	6.02
Belgium	7.40	14.12
Denmark	2.81	4.40
Spain	2.52	3.64
Finland	6.51	4.70
France	5.88	5.95
Greece	1.69	2.00
Ireland	9.08	14.60
Italy	2.09	2.75
Luxembourg	1.51	3.36
Netherlands	5.38	4.72
Norway	5.48	5.75
Portugal	2.07	3.98
Sweden	4.57	3.74
United Kingdom	12.07	11.53

Column 2 reports the ratio between the number of native-born workers in occupation 12 and the total number of native-born workers. Column 3 reports the ratio between the number of foreign-born workers in occupation 12 and the total number of foreign-born workers.

Concentration Patterns

Do countries keep a balanced distribution of workers across occupations or are some groups of workers more concentrated in one country? To answer this question, we present a measure of concentration of workers.

First, we define the occupational distribution of the total working population in country i as:

$$S^i = \left(s^i_1, s^i_2, ..., s^i_J\right), \text{ where } \quad s^i_j \equiv \frac{\text{\# all workers in occupation } j \text{ and country } i}{\text{\# all workers in country } i}.$$

Recall that ISCO-88 occupations are ordered according to educational level. In the data description we have defined *high* and *low-educated* occupations. Notice that we can express S^i as $S^i_{HE} \bigcup S^i_{LE}$, where S^i_{HE} contains the shares corresponding to subgroups of groups 1-3, and S^i_{LE} those to subgroups of groups 4-9. Then, for each pair of countries in our sample, i and h, we compute $Corr(S^i_{HE}, S^h_{HE})$ and $Corr(S^i_{LE}, S^h_{LE})$, for each year of the sample. Then for each country i and year we compute the average (across education levels) of each pairwise correlation with the other countries in the sample. Finally we calculate the average across years.

We want to emphasize the difference with respect to the previous analysis. In this case, the population of analysis is total working population in each occupation, regardless of their country of birth. Second, this correlation is computed over the occupational distribution, by education group.

We interpret a positive correlation as evidence that the country keeps a balanced structure in that education group. A negative correlation means that a country has a lower share of its working population in occupations where other countries have a high share. We take this as evidence of concentration. Table 2.2 shows the average correlations.

Table 2.2: **Concentration Patterns (1996-2010)**

Country	Average Correlation High Skill	Average Correlation Low Skill
Austria	.31	.75
Belgium	.34	.65
Denmark	.49	.76
Spain	.58	.69
Finland	.58	.68
France	.47	.77
Greece	.15	.39
Ireland	-.15	.79
Italy	.25	.66
Luxembourg	.57	.53
Netherlands	.58	.76
Norway	.38	.64
Portugal	.45	.44
Sweden	.57	.67
United Kingdom	-.43	.74

We find two results: First, for low-educated occupations, countries keep a more balanced structure: average correlations are positive and high in general. Second, for high-educated occupations, there are concentration patterns. In this occupation group, the results are more heterogeneous, yet correlations are generally lower and even negative for some cases.

We will use these two main findings of education and concentration as input in our model. We will have two sectors: One will exhibit constant returns to scale and employ only low-educated labor. The other one will exhibit increasing returns to scale and employ only high-educated workers.

2.3 The Model

Framework

We consider a static, one-period model of education and migration choice. An economy consists of firms, households and governments. There are two countries: 1 and 2. Both countries have identical production technologies and initial size. We normalize initial population size to 1 in each country.

There are two productive sectors in each country: one displays constant returns to scale (CRS) and the other one increasing returns to scale (IRS) at the industry level, i.e. IRS are external to the firm. They produce using High-Educated (HE) and Low-Educated (LE) labor, respectively.

Households are heterogeneous in ability and mobility. They make consumption, education and migration decisions and supply labor inelastically. Their education decision determines the type of labor they will supply (HE or LE) and their migration choice determines their country of residence. Finally, there is a government that collects education payments of high-educated workers and transfers them to households in a lump sum way.

We begin with a closed economy, in which there is no migration choice. Next, we analyze a two-country open economy model, where we allow for free mobility of goods, labor and degrees (skills).[9]

[9]In order to capture the free transferability of academic credits and the mutual recognition of degrees

2.3.1 Closed Economy

Production

There are two goods in the economy: Y and Z. Sector Y is composed of a continuum of symmetric firms in the interval $[0,1]$ that use HE labor as their only input. Output of firm $k \in [0,1]$ is:

$$y_k = A(H) \cdot h_k, \quad \text{where} \quad H = \int_0^1 h_k dk, \tag{2.1}$$

where $A' > 0$, $A'' < 0$ and h_k is the amount of HE labor used by firm k.

Production of good Y exhibits IRS at the country-industry level, but these are external to individual firms. The more HE workers in the economy, the higher the output of each producer. However, each firm $k \in [0,1]$ is atomless and does not internalize its effect on aggregate demand of HE labor in their country. Therefore, each individual firm considers the productivity term in the production function as a constant and behaves competitively. Inverse demand of HE workers is given by:

$$w_H = A(H) \cdot P_Y. \tag{2.2}$$

For simplicity, and since firms are identical, we characterize the equilibrium using a representative firm that demands h and produces Y.

Good Z is produced by a representative firm. The only input for production is LE labor and it has the following CRS technology:

$$Z = B \cdot L, \tag{2.3}$$

where $B \geq 1$ and L denotes the amount of LE labor used.

The inverse demand for LE workers is given by:

$$w_L = P_Z \cdot B. \tag{2.4}$$

Households and Government

Households are heterogeneous in ability. They are born low educated and can decide to remain uneducated and earn w_L working in the CRS sector. Alternatively, they can choose to acquire high education by paying an individual specific cost[10] and then earn

across the EHAE.

[10] In general, the education cost can be interpreted as effort or ability.

w_H working in the IRS sector. Regardless of their choice, they supply labor inelastically since there is no disutility from working.

At the beginning of the period, each household $j \in [0,1]$ makes an ability draw that determines her education cost θ_j, which is negatively related to ability. For the most able individual, education will be free. For the least able individual, the cost will be the highest possible cost, $\bar{\theta}$. Education costs are uniformly distributed in the interval $\left[0, \bar{\theta}\right]$.

Given ability, prices, wages and transfers $\left(\theta^j, P_Y, P_Z, w_H, w_L, T\right)$, each household j chooses an education level and consumption bundle $\left\{e^j \in \{HE, LE\}\,,\ \ c_Y^j, c_Z^j\right\}$ to solve:

$$\begin{aligned} \max_{\{e^j, c_Y^j, c_Z^j\}} \quad & \lambda \log c_Y^j + (1-\lambda) \log c_Z^j \\ \text{s.t.} \quad & P_Y c_Y^j + P_Z c_Z^j \leq W^j \\ & W^j = w_H - \theta_j + T \quad \text{if } e^j = HE \\ & W^j = w_L + T \qquad\quad\;\; \text{if } e^j = LE \\ & c_Y^j \geq 0, \quad c_Z^j \geq 0. \end{aligned}$$

The last agent of our economy is the government (G). This agent collects education payments and transfers them equally to all households in a lump-sum fashion . The revenue of the government is given by:

$$G^R = \int_{\mathcal{H}} \theta_j dj = \int_0^{\theta^*} \theta_j dF(\theta) = \int_0^{\theta^*} dF(\theta), \tag{2.5}$$

where F is the *cdf* of θ_j, $\mathcal{H} \equiv \left\{j \in [0,1] \mid e^j = H\right\}$ and θ^* is a threshold that characterizes the set of educated workers. We will elaborate further on this cutoff value in the next subsection. Government expenditures are equal to:

$$G^E = \int_0^1 T dj, \tag{2.6}$$

where T is the percapita transfer.

Definition 2.3.1. ***Autarky Equilibrium:*** *Given the ability distribution $U[0, \bar{\theta}]$, a competitive equilibrium for this economy is: (i) education and consumption choices from households $\left\{\left(e^j, c_Y^j, c_Z^j\right)\right\}_{j\in[0,1]}$; (ii) production plans (Z, L, Y, h); (iii) lump-sum transfers T; (iv) prices $\{P_Z, P_Y, w_H, w_L\}$; and (v) an endogenous threshold θ^* such that:*

1. *Given prices, transfers the ability draw* θ_j, (c_Y^j, c_Z^j, e^j) *solves j's problem,* $\forall j$.

2. *Given prices, the production plan of the IRS sector,* (Y, h), *satisfies* $w_H = A(H)P_Y$.

3. *Given prices, the production plan of the CRS sector* (Z, L) *satisfies* $w_L = BP_Z$.

4. *The government budget balances*

$$\int_{\mathcal{H}} \theta_j dj = \int_0^1 T dj.$$

5. *The labor markets clear*

$$H^s \equiv \int_{\mathcal{H}} j dj$$
$$H \equiv h = H^s$$
$$H + L = 1.$$

6. *The goods markets clear*

$$Y = \int_{\mathcal{H}} c_Y^j dj + \int_{\mathcal{H}^c} c_Y^j dj$$
$$Z = \int_{\mathcal{H}} c_Z^j dj + \int_{\mathcal{H}^c} c_Z^j dj.$$

Characterization of the Equilibrium

Households consumption demand functions are:

$$c_Y = \frac{(\lambda)}{P_Y} \cdot W^j \tag{2.7}$$

$$c_Z = \frac{(1-\lambda)}{P_Z} \cdot W^j. \tag{2.8}$$

Given the ability draw θ_j, households maximize net labor income in order to maximize utility. Their education decision boils down to:

$$e^j = \begin{cases} HE & \text{if } w_H - \theta_j \geq w_L \\ LE & \text{otherwise.} \end{cases} \tag{2.9}$$

This inequality determines individual supply of HE labor. Using the distribution of education costs $\theta_j \sim U[0,\bar{\theta}]$, we get an expression for aggregate supply of HE labor:

$$H = F(w_H - w_L) = \frac{w_H - w_L}{\bar{\theta}}. \tag{2.10}$$

Notice that equation 2.10 implies that, all else equal, the higher the maximum cost of acquiring education, the smaller the share of HE people working in the IRS sector. We emphasize this result, since it will be the main difference between countries in the integrated economy. Note that the education decision is determined by a cutoff rule: every household j with $\theta_j \leq \theta^*$ will become HE, where $\theta^* = w_H^* - w_L^*$.

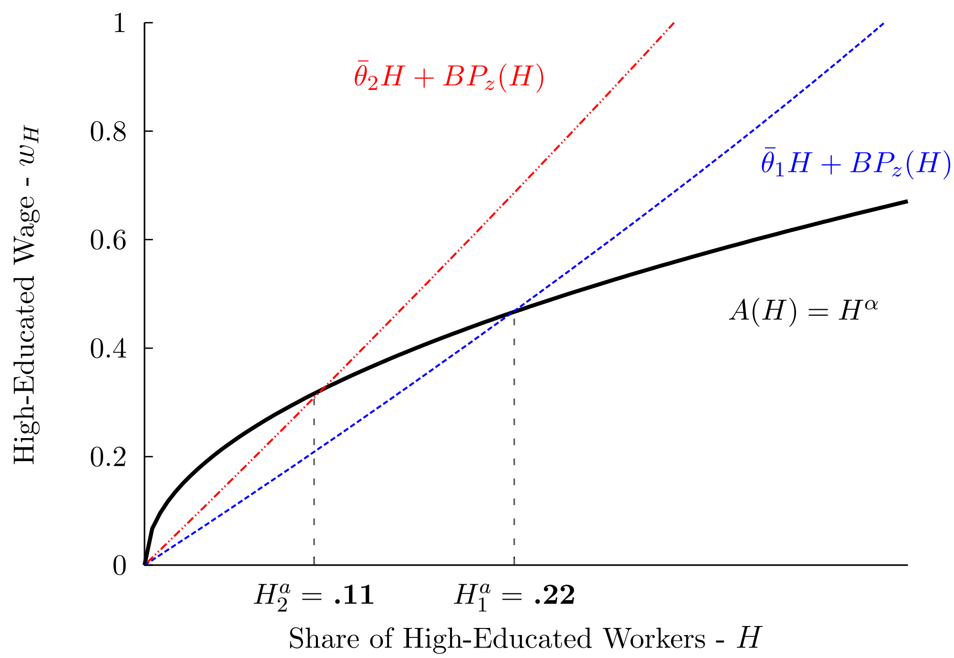
To complete the characterization of the equilibrium, we normalize $P_Y \equiv 1$. Combining the firm's inverse demand for labor and using the market clearing conditions:

$$H^* = \frac{A(H) - BP_Z^*(H)}{\bar{\theta}} \tag{2.11}$$

$$P_Z^*(H) = \frac{(1-\lambda)HA(H)}{\lambda B(1-H)}. \tag{2.12}$$

In Figure 2.2, we illustrate the equilibrium under different ability supports.

Figure 2.2: **Autarky Equilibrium**



In the previous exercise, we used the following specification: $A(H) = H^{\alpha}$, $\alpha = 0.5$, $\lambda = 0.8$, $B = 1$, $\bar{\theta}_1 = 2$ and $\bar{\theta}_2 = 3$.

2.3.2 Integrated Economy

We now consider an integrated world economy consisting of two countries indexed by i. Countries are identical in production technologies, preferences and population sizes, but differ in the distribution of ability. The world population is normalized to 2 (1 for each country). Both goods are tradable and both countries are big.

Households are heterogeneous in mobility and ability. In each country, there is an exogenous fraction $\gamma \in (0, 1)$ that is perfectly mobile, and a fraction $(1 - \gamma)$ that is perfectly immobile. Both types of labor, HE and LE, are perfect substitutes across countries. That is, natives and immigrants are assumed to be equally productive. The ability distribution is $\mathcal{U}\left[0, \bar{\theta}^i\right]$, where $\bar{\theta}^i$, $i \in \{1, 2\}$, is now country specific. The distribution of ability is the same across mobile and immobile groups. People pay for education in their country of origin and can freely transfer their degree across countries.

Firms' problems in each country remain unchanged. Since both goods are perfectly tradable, prices will equalize across countries. This, together with the fact that the CRS sectors are identical, imply that wages of the low educated sector are also equalized across countries ($w_L^1 = w_L^2$).

In the open economy we have two types of households: mobile and immobile. Immobile ones face the same problem as in the closed economy. Mobile households, however, now have the additional choice of migration. More formally, a mobile worker j in country i chooses:

(i) Her education level: $e_j^i \in \{HE, LE\}$, which determines the sector where she will work.

(ii) Her migration status: $m_j^i \in \{N, M\}$, where N stands for *Native* and M for *Migrant*. This decision, given (i), determines her country of residence and, therefore, her wage.

Given mobility, ability, prices, wages and transfers $\left(\gamma, \theta_j^i, P_Y, P_Z, w_H^i, w_L^i, T^i\right)$, each household j from country $i \in \{1, 2\}$ chooses an education level, a migration status and a consumption bundle: $\left\{e_j^i \in \{HE, LE\}, m_j^i \in \{N, M\},\ \ cy_j^i, cz_j^i\right\}$ to solve:

$$\begin{aligned}
&\max_{\{e_j, m_j, cy_j, cz_j\}} \lambda \log cy_j^i + (1-\lambda)\log cz_j^i \\
&s.t. \quad P_Y cy_j^i + P_Z cz_j^i \leq W_j^i \\
&where \quad W_j^i = w_H^i - \theta_j^i + T^i \qquad \text{if } e_j^i = HE \quad m_j^i = N \\
&\qquad\qquad W_j^i = w_H^{-i} - \theta_j^i + T^i \qquad \text{if } e_j^i = HE \quad m_j^i = M \\
&\qquad\qquad W_j^i = w_L^i + T^i \qquad\qquad \text{if } e_j^i = LE \quad \forall m_j^i \\
&s.t. \quad cy^j \geq 0 \quad cz^j \geq 0.
\end{aligned}$$

Without loss of generality, we assume that, in the case of indifference, a worker remains in her country of origin. This implies that low-educated individuals will always stay in their home country, i.e. $m_j^i = N \ \text{ if } e_j^i = LE.$

Since households pay for education in their country of origin, regardless of their migration status, government revenue in country i revenue becomes:

$$G^{Ri} = \int_{\mathcal{H}_N^i} \theta_j^i dj + \int_{\mathcal{H}_M^i} \theta_j^i dj = \int_{\mathcal{H}_N^i \bigcup \mathcal{H}_M^i} \theta_j^i dj,$$

where $\mathcal{H}_N^i$ denotes the set of HE workers born in country i that choose to stay home, and $\mathcal{H}_M^i$ is the set of HE workers that choose to work abroad. Formally:

$$\begin{aligned}
\mathcal{H}_N^i &= \left\{j \in [0,1] \mid e_j^i = HE \text{ and } m_j^i = N\right\} \\
\mathcal{H}_M^i &= \left\{j \in [0,1] \mid e_j^i = HE \text{ and } m_j^i = M\right\}.
\end{aligned}$$

Thresholds and HE Labor Supply

In the integrated economy, education level and migration status are jointly determined. Because of mobility heterogeneity, there are two cutoffs for each country: one for immobile and one for mobile workers. The first one is determined by the education premium at home, and the second one can be decomposed in a migration premium and an education premium.

For immobile workers, the education decision follows the same rule as in the closed economy. Therefore an immobile worker j from country i becomes HE if her net wage of working in the HE intensive sector of i is higher than the one she would earn in i if she remains LE, i.e., $w_H^i - \theta_j^i \geq w_L$. The threshold for immobile workers in country i is thus given by:

$$\theta_{immobile}^{i*} = w_H^i - w_L \tag{2.13}$$

and every immobile individual j with $\theta_j^i \leq \theta_{immobile}^{i*}$ will choose $e_j^i = HE$.

We define $w_H^i - w_L$ as the education premium at home, this is the spread between the wage of a high educated worker and a low educated one. Notice the threshold for immobile workers is given by this premium and every worker that needs to pay a a cost below this value will choose to be high educated and benefit from the spread.

For mobile workers, the education decision must now incorporate the possibility of higher earnings abroad. Hence a mobile worker j in country i decides to become HE if her net wage of working in the HE intensive sector of one of the two countries is higher than the one she would earn in i if she remains LE, i.e., $\max\left\{w_H^i, w_H^{-i}\right\} - \theta_j^i \geq w_L$. The education cost threshold for mobile workers is thus given by:

$$\theta_{mobile}^{i*} = \max\left\{w_H^i, w_H^{-i}\right\} - w_L \tag{2.14}$$

and every mobile individual j with $\theta_j^i \leq \theta_{mobile}^{i*}$ will choose $e_j^i = HE$.

Regarding the migration decision, a HE worker will go wherever she gets a higher wage. This is, $\forall j$ with $e_j^i = HE$:

$$\begin{aligned} m_j^i &= N \quad \text{if } w_H^i \geq w_H^{-i} \\ m_j^i &= M \quad \text{otherwise.} \end{aligned}$$

Further manipulation of the threshold for mobile workers (**??**) show that it can be decomposed in a migration and an education premium as follows:

$$\theta_{\text{mobile}}^{i*} = \max\left\{w_H^i, w_H^{-i}\right\} - w_L = \underbrace{\max\left\{w_H^i, w_H^{-i}\right\} - w_H^i}_{\text{Migration Premium}} + \underbrace{w_H^i - w_L}_{\text{Education Premium}}.$$

If the migration premium is positive, mobile HE workers benefit from both premia. They earn higher wages because they are high educated and on top of that they get access to even higher wages abroad because they migrate.

Definition 2.3.2. ***Integrated Equilibrium:*** *Given mobility γ and ability distributions $\mathcal{U}[0, \bar{\theta^i}]$, $i \in \{1, 2\}$, a competitive equilibrium for the two-country economy is:*

- *households' education, migration and consumption* $\left\{e_j^i, m_j^i, cy_j^i, cz_j^i\right\}_{j \in [0,1], i \in \{1,2\}}$,
- *production plans* $\left\{Y^i, h^i, Z^i, L^i\right\}_{i \in \{1,2\}}$,
- *lump-sum transfers from the government* $\left\{T^i\right\}_{i \in \{1,2\}}$,
- *prices* $\left\{w_H^i, w_L^i\right\}_{i \in \{1,2\}}$, P_Z^w, P_Y^w, *and*
- *cutoff education costs* $\left\{\theta^{i*}_{immobile}, \theta^{i*}_{mobile}\right\}_{i \in \{1,2\}}$ *such that:*

1. *Given prices, transfers, mobility, and ability* θ_j^i $\left\{e_j^i, m_j^i, cy_j^i, cz_j^i\right\}_{i \in \{1,2\}}$ *solve j's problem* $\forall j$.
2. *Given prices, production plans of IRS sectors* $\left\{Y^i, h^i\right\}_{i \in \{1,2\}}$ *satisfy* $w_H^i = A(H^i)P_Y^w$.
3. *Given prices, production plans of CRS sectors* $\left\{Z^i, L^i\right\}_{i \in \{1,2\}}$ *satisfy* $w_L = B \cdot P_Z^w$.
4. *The government budget balances in each country* $(i = 1, 2)$*:*

$$\int_{\mathcal{H}_N^i \bigcup \mathcal{H}_M^i} \theta_j^i dj = \int_0^1 T^i dj.$$

5. *The labor markets clear (in each* $i = 1, 2$*):*

$$\begin{aligned} H^{si} &\equiv \int_{\mathcal{H}^i} j dj \\ h^i &= H^i \qquad\qquad H^i = H_N^i + H_M^{-i} \\ L^i &= L_N^i + L_M^{-i} \\ 1 &= H_N^i + H_M^i + L_N^i \qquad wlog \quad L_M^{-i} = 0. \end{aligned}$$

6. *The good markets clear:*

$$\begin{aligned} Y^1 + Y^2 &= \int_{\mathcal{H}^1} c_Y^H dj + \int_{\mathcal{H}^2} c_Y^H dj + \int_{\mathcal{L}^1} c_Y^L dj + \int_{\mathcal{L}^2} c_Y^L dj \\ Z^1 + Z^2 &= \int_{\mathcal{H}^1} c_Z^H dj + \int_{\mathcal{H}^2} c_Z^H dj + \int_{\mathcal{L}^1} c_Z^L dj + \int_{\mathcal{L}^2} c_Z^L dj, \end{aligned}$$

where $\mathcal{H}^i = \mathcal{H}_N^i \bigcup \mathcal{H}_M^i$ *and* $\mathcal{L}^i = \left(\mathcal{H}_N^i \bigcup \mathcal{H}_M^i\right)^c$.

Characterization of the Integrated Equilibrium

For households, optimal consumption choices are as in the closed economy. However, the introduction of migration choice generates two equilibrium objects in the HE sector: (i) Aggregate supply of native HE labor H^i_N and (ii) Aggregate supply HE emigrants H^i_M. Each one is determined in equilibrium according to individual-specific mobility and ability draw, following the cutoff rules described above. These thresholds are determined by the relative price of final goods, which, at the same time, depend on the aggregate stock of HE workers.

Individual supply of HE immobile labor is given by

$$e^i_j = HE \text{ if } w^i_H - \theta^i_j \geq w_L.$$

Using the distribution of θ^i_j and the share of immobile workers $1-\gamma$, we get an expression for the aggregate supply of immobile HE native workers:

$$H^i_{N,immobile} = \left(\frac{w^i_H - w^i_L}{\bar{\theta}^i}\right) \cdot (1-\gamma). \tag{2.15}$$

The individual supply of HE mobile labor is given by:

$$e^i_j = HE \text{ if } \max\left\{w^i_H, w^{-i}_H\right\} - \theta^i_j \geq w_L.$$

Using the distribution of θ^i_j and the share of mobile workers γ, we get an expression for the aggregate supply of mobile HE natives and emigrants:

$$H^i_{N,mobile} = \left(\tfrac{w^i_H - w^i_L}{\bar{\theta}^i}\right) \cdot \gamma \qquad H^i_M = 0 \qquad \text{if } w^i_H \geq w^{-i}_H \tag{2.16}$$

$$H^i_{N,mobile} = 0 \qquad H^i_M = \left(\frac{w^i_H - w^i_L}{\bar{\theta}^i}\right) \cdot \gamma \qquad \text{otherwise,} \tag{2.17}$$

where

$$\begin{aligned} H^i &= H^i_N - H^i_M + H^{-i}_M, \quad H^i_N = H^i_{N,mobile} + H^i_{N,immobile} \\ w^i_H &= A(H^i) \\ w^i_L &= BPz \\ Pz &= \frac{(1-\lambda)\left(A(H^i)\cdot H^i + A(H^{-i})\cdot H^{-i}\right)}{\lambda(2 - H^i - H^{-i})}. \end{aligned}$$

This completes the characterization of equilibria for the general case. In the next section, we provide a discussion of specific cases.

2.4 Results and Discussion

We are interested on the comparison of countries with different distribution of native HE population. In particular we want model two countries, one with a higher fraction of native HE workers to show it will attract HE immigrants, in line with our empirical analysis. In our model we generate a heterogeneity on HE native shares by considering different abilities distribution, which in turns translates into different education costs.

Assumption 2.4.1. *The maximum education cost in country 1 is lower than in country 2, i.e.* $\bar{\theta}_1 < \bar{\theta}_2$.

Assumption 2.4 implies that, in autarky, both the share of HE workers and their wage are higher in country 1 than in country 2 [11].

Since the sector that uses HE labor as input exhibits external economies of scale, we expect that, by opening the borders and allowing for labor mobility, foreign HE workers will flow to country 1. This will be analyzed in *Case 1*.

We acknowledge that in our model, as in other models with external economies of scale, there is a multiplicity of equilibria. For our analysis, this means that, even if country 1 has a higher labor and wage in the HE intensive sector in autarky, it is possible for HE labor to cluster in country 2. For this equilibrium to arise, people would need to expect higher wages in country 2 even though they are lower in autarky. For completeness, we will briefly discuss this equilibrium in *Case 2*[12]

For *Case 1* we conjecture and impose[13] that $w_H^{1w} > w_H^{2w}$.

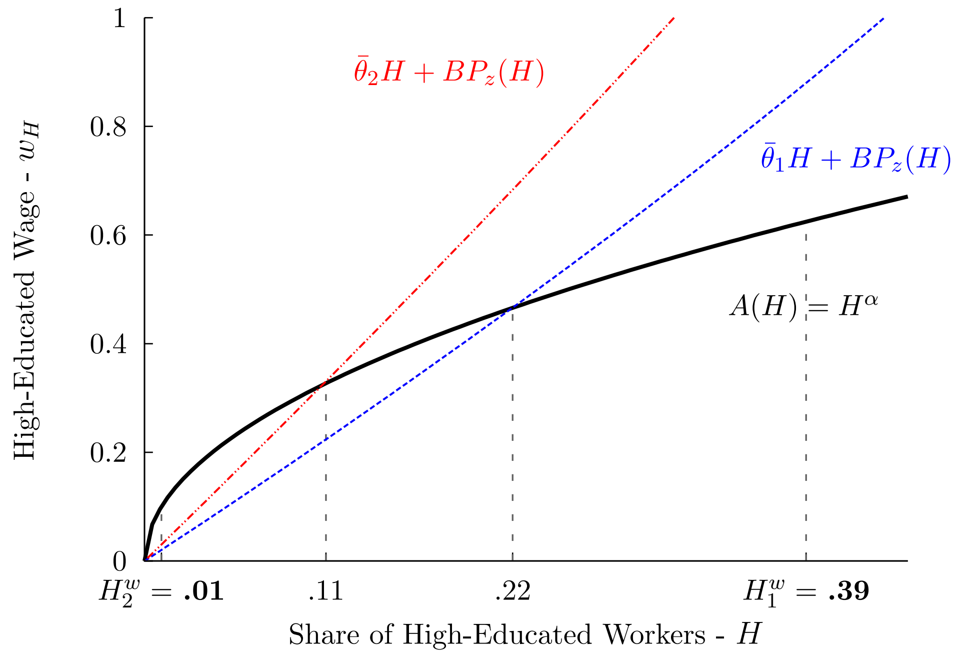
As a result, every mobile household from 2 will migrate to country 1, whereas every mobile household from country 1 will stay there. Figure 2.3 illustrates this equilibrium and contrasts it to the equilibrium in autarky.

[11]See Figure 2.2.

[12]We plan to extend the analysis of this Case by introducing beliefs in the model. However at this stage of the paper, we have not modeled them modeled beliefs explicitly, this is an additional reason for why we will only comment briefly on *Case 2.*

[13]After this conjecture we verify with the numerical results, following a guess and verify approach.

Figure 2.3: **Specialization Patterns: Case 1**



For the open case, we set the ratio $\bar{\theta}^2/\bar{\theta}^1$ to 1.5 and the mobile fraction γ to 0.5 [14]. In particular we use $\bar{\theta}_1 = 2$ and $\bar{\theta}_2 = 3$ to account for the differences in ability. Table 2.3 contains the numerical results, and Figure 2.3 provides an illustration. In autarky, 22% of the total population in country 1 is HE, in contrast to 11% in country 2. When workers are allowed to move freely, these numbers change to 39% and 1%, respectively. This is, 39% of the residents of country 1 are employed by the IRS sector, while only 1% of the residents of country 2 are HE.

[14] The rest of the parameters are the same as in Figure 1 of the closed economy

Table 2.3: **Numerical Results (Case 1)**

	Autarky		**Integrated**	
	Country 1	Country 2	Country 1	Country 2
Residents	1	1	$1 + H_M^2$	$1 - H_M^2$
HE	21.66	10.47	39.14	1.13
LE	78.34	89.53	70.64	89.08

The relation between HE immigrants and HE natives in country 1 is given by:

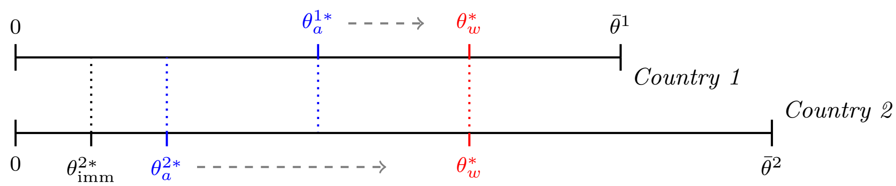
$$\frac{H_M^2}{H_N^1} = \frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma. \tag{2.18}$$

Equation 2.18 establishes a linear relation between HE natives and immigrants. Given a stock of H_N^1, the share of workers who migrate from country 2 to 1 is determined by two factors, depending only on parameters of the model. The first one is trivial: immigration increases with mobility (higher γ). The second factor is the relative difference in education costs: the higher the education costs in country 2 with respect to country 1, the lower the share of immigrants for a given number of HE natives. For a constant level of $\bar{\theta}_1$, increases in $\bar{\theta}_2$ discourage workers from country 2 from becoming HE since the cost they have to pay at home does not compensate for the higher wages abroad.

As *Case 1* and Figure 2.3 illustrate, the main result of our model is that if a country has a relatively larger fraction of native population working in the high educated (IRS) sector, this country will attract foreign labor of the same type. This is consistent with "north-north" migration patterns observed in HE occupations our sample of analysis. The model we propose in this paper allows for the possibility of workers migrating to a place where there are more of their type, as opposed to the standard south-north approach where migration flows are due to scarcity: workers migrating to places where there are less of their type.

Additionally we find that in a human capital free mobility are with free mobility of labor and transferability of education, the interaction between migration and education decisions increase the total HE labor stock. Other workers decide to become HE to take advantage of migration premium and spillovers from the IRS sector. This translates into agents willing to pay higher education costs (see Figure 2.4).

Figure 2.4: **Thresholds**



In our numerical exercise, the threshold for becoming HE increases for both countries under the integrated economy. In the host country HE wages increase with the inflow of HE immigrants due to the production externality and therefore more people become HE. In the source country mobile workers are willing to pay more for their education since they can benefit from higher wages abroad.

In *Case 2*, even though we begin with $w_H^{1a} > w_H^{2a}$, we conjecture and impose $w_H^{1w} < w_H^{2w}$. As a result, there is agglomeration of HE labor in country 2 [15]. Despite the fact that this case is possible, it is not desirable because it induces a lower level of world GDP, consumption and welfares (see Table 2.4).

Table 2.4: **Sectoral Output, World GDP and Welfare**

	Output			**Welfare**		
	$Y_1 + Y_2$	$Z_1 + Z_2$	Total GDP	Cou.1	Cou.2	Total
Case 1	0.25	1.60	0.31	-1.60	-2.39	-3.99
Case 2	0.18	1.64	0.23	-2.25	-2.14	-4.40

[15] See the system of equations that determines the equilibria and the figure in appendix A.

2.5 Conclusions

In this paper, we have first provided evidence that, if a EU15 country has a relatively larger fraction of native population working in a high-educated occupation, this country will attract foreign EU15 labor of the same type. This result is in line with what we have refer to as north-north migration pattern, where workers migrate to countries similar to their source country and where their type is relatively more abundant. We have also documented that high-educated occupations display concentration patterns in the sense that workers in those occupations tend to cluster in specific countries.

As we have emphasized, the intra EU15 migration phenomenon cannot be studied under the traditional south-north approach. The reason is that this framework assumes differences between the source and the host countries in terms of both income and characteristics of their labor force that are not observed in the EU15. To fill this gap, in this paper we propose a model that allows for workers flows between similar countries. Moreover, our model successfully generates the EU15 migration patterns we have documented and it also rationalizes agglomeration consistent with the concentration findings we have documented.

The mechanism of our model is the following: wages for HE labor are strictly increasing in the amount of HE labor, both foreign and native, employed in a country. This is achieved via external economies of scale in the sector intensive in HE labor. Hence, at the individual level, it becomes worthy for the most able households to become HE and to move to the country where there are more HE native workers.

To properly analyze selective policies of migration, it is imperative to propose better mechanisms of analysis that allow for bilateral flows of workers from economically similar countries. The theoretical framework developed in this paper provides useful insight about how to model a mechanism capable of driving migration and generating agglomeration.

Our model can be extended along several dimensions. For instance, we used the relation between the ISCO-88 occupation classification and the education level to relate our model to the data. However, we acknowledge that reducing the types of workers to be based only on education limits the applications of our model. In this sense, allowing for occupational differences might be insightful. We think important differences arise in terms of transferability of skills and mobility between occupations, regardless of their education level. Additionally we could allow for migration in the CRS sector.

Another direction is to include more structure in the non-IRS sectors. In particular we could include an additional sector with constant or even decreasing returns to scale that is attached to the size or structure of the population in each country. This could be interesting since there are differences within the low-educated (LE) group in the data. Service Elementary occupations workers behave very differently from, for example, Machine Operators[16]. This could happen because the former group faces a considerably inelastic demand and is directly attached to the population size. In contrast, the latter group is more exposed to country-specific sectoral shocks. For instance, we think of the 2000's construction boom in Spain as an exogenous increase in Spain's construction labor productivity, B^i, that can drive a positive correlation between the share of native and immigrants.

[16]Groups 91 and 81 at the 2-digit ISCO-88 level, respectively.

Chapter 3

Investment for the Demographic Window in Latin America

3.1 Introduction

Regions of the world undergo a common demographic transition that occurs in three stages: first the child-dependent cohort is large, then the working-age cohort is at its maximum size, and finally, an aging-dependent population increases. A *demographic window* is defined by the United Nations (UN) as a period of time when "the fraction of the population under age 15 falls below 30% and the fraction of the population 65 years and older is still below 15%."[1] Thus, during a demographic window there is a temporary increase in the working-age population, as a percent of the total population, moving to its peak earning years; while the dependent population is still low.

For policy-makers, like the multilateral organizations, this window is seen a period of opportunity because of the *demographic dividend:* the potential increase in resources per capita that might result from having a smaller dependent population and a larger working-age population. During a demographic window, dependency ratios fall to their minimum level,[2] freeing up resources for alternative uses other than consumption. In this sense the demographic window is seen as a temporary moment where dependency

[1] In the *World population to 2300* report (United-Nations, 2004).

[2] For example, in Latin America, during the middle of the demographic window, the total dependency ratio (the number of old and young dependents per worker) is projected to be at its minimum. See figure B.2 in appendix B.1.

is the lower and there are many workers to invest in. After this window, the needs of the elderly will increase and there will no longer be a big labor force.

The aforementioned argument of decrease in dependency and increase in working-age population has been used to emphasize the importance of investment in preparation and during the demographic window. Case studies of East Asian countries, which had their demographic windows from the 1960s to the 1990s, have documented positive demographic effects on investment rates and GDP per capita growth.[3] Motivated by these findings, multilateral organizations like the United Nations,[4] the World Bank (WB),[5] and the International Monetary Fund (IMF)[6] have issued as a general policy recommendation to increase investment in capital, both human and physical, before and during a demographic window to take full advantage of the demographic dividend.

Despite the fact that the stages of a demographic transition are generally common across countries, their speed and timing are not. For instance, Europe and some Asian countries have already gone through their demographic windows and are now aging, while in contrast Latin America is just entering its window, and Africa has yet to do so. In this paper I focus on Latin America to analyze a relevant current economic phenomenon: the demographic change that this region is undergoing and its relationship with the observed patterns of investment rates.

In this paper I analyze the macroeconomic impact of a country's population structure passing through its demographic window. In particular, I focus on the behavior of investment leading up to and during the demographic window in Latin America compared to other regions of the world. I document that in Europe, Asia, and Oceania, investment rates are higher for the 15 years before leading up to and during the demographic window than they are in other periods of time. However, this is not the case for Latin America, where investment rates decrease during this period.

I build an overlapping generations (henceforth OLG) model with variable degree of

[3]For example, Mason (2001) reports positive contributions of investment induced by demographics to GDP per capita growth rates in Japan, Taiwan, South Korea, and Thailand from 1965 to 1990.

[4]*State of World Population Report* (United-Nations, 2014).*Remarks of Chuma (2015)*, International Labor Organization (ILO), in the framework of the *2030 Agenda for sustainable development* of the UN.

[5]*Realizing the Demographic Dividend: Challenges and Opportunities, Ministry of Finance and Development 2011*

[6]*What is the demographic dividend?* (Lee and Mason, 2006).

financial openness and demographic change that can rationalize the documented pattern of investment. The main result is that in financially open economies, investment rates decrease in preparation for and during the demographic window, while in closed economies they increase. The main mechanism behind this result is that in closed economies the decrease in child dependency generates an increase in savings rates and therefore in investment rates. On the other hand, in financially open economies, since savings and investment need not to be equal, investment decreases because of the decrease in the growth rate of the labor force that comes with a demographic window.

In terms of the observed patterns, the fact that European and Asian countries are less open before and during their demographic windows, can help to explain why they have higher investment rates before and during their windows. Similarly, greater financial openness before and during the window in Latin America can explain why investment rates are lower during this period. While it might seem surprising that Europe and Asia are considered as relatively closed financially and Latin America as relatively open, the data support this claim.[7] In addition, is important to keep in mind that I am comparing Europe and East Asia in the 1970s to Latin America in the present days.[8]

The contribution of this paper is to document the relationship between investment and the demographic window for different regions of the world and to try to explain the differences behind this relationship, with a special focus on Latin America. Additionally, this paper addresses and provides empirical context to a concern of both the governments of Latin American countries and international organizations: lack of investment for the demographic window.

The model proposed is consistent with the documented facts for both Latin America and Europe, Asia, and Oceania, and it can not rationalize the general recommendation to increase investment for Latin America. Per the model, in an open economy the privately optimal response to a demographic window is to decrease investment rates since the economy is slowing down.

The rest of the paper is organized as follows. Section 2 describes common trends

[7]The average Chinn-Ito index of financial openness is higher in the demographic windows of the countries of Europe, Asia, and Oceania (0.325), than the average index in the windows of Latin American countries (0.362).

[8]The 1970s is the average of the European and East-Asian demographic windows. In my sample these windows happened from the 1960s to the early 1990s, and in the data some European windows date back to the early 1900s.

that are observed before and during a demographic window. Section 3 documents the empirical findings regarding investment rates for the demographic window. Section 4 describes the model and the equilibrium. Section 5 reports and discusses the results of the model. Section 6 concludes.

3.2 The Demographic Window

In this section I describe the worldwide long run demographic transition, I define the concept of a demographic window, and describe the common demographic trends that are observed in both the demographic transition and the demographic window.

3.2.1 The Demographic Transition

Countries undergo a common long run demographic transition that occurs in three stages: first the child-dependent cohort is large, then the working-age cohort is at its largest and finally an aging-dependent population increases.

The first stage, characterized by a large child cohort, is usually reached during a country's development phase when it is able to meet basic needs in terms of food and health, mortality rates drop,[9] and population growth accelerates. In terms of indicators, during this stage the population pyramid is *expansive*: triangular and horizontally expanding at its base. The growth rate of newborns is high,[10] the child to population ratio is increasing, and the working age- to total-population ratio is decreasing.

In general, what triggers the movement from the first to the second stage is a reduction in fertility rates. The growth rate of newborns starts a sustained, sometimes sharp, decreasing trend and population growth slows down. As the growth of the youngest cohort decreases and the last (and large) child cohorts from stage one scale up the population pyramid, the working-age population becomes the widest group of the pyramid. In terms of indicators, the population pyramid is growing towards its center.[11] The

[9]For example, according to Lee (2003) the first stages of mortality reduction are caused by a decrease in diseases and better nutrition. Also, according to the United-Nations (2014) "mortality rates among children fall because of interventions like safe water and sanitation."

[10]The growth rate of the child population (population aged 0-14) is high and not drastically decreasing.

[11]Here there are two options: the population pyramid will be *stationary* if the growth rate of newborns stays positive, or *contracting*— shrinking at the base—if the growth rate of newborns becomes negative. In Mexico and Latin America, for instance, the latter is the case, and in the US the former applies.

child to population ratio is decreasing and the working age- to total population-ratio increasing—a reversal of the previous trend. It is important to mention that during this stage, and from there on, even though the working age-to total population-ratio might be increasing, the growth rate of the working-age population is not. This growth rate is decreasing since each year there are fewer children who will be part of the future working-age population.

Finally, the aging process begins, and countries transition from the second to the third stage. During this transition, the large working-age population cohort of the second stage joins the elderly population. In the data there is an increasing elderly population to total population ratio, and the working age- to total population-ratio shifts from increasing to decreasing. As countries enter the third stage, the increasing elderly population makes it to the top of the demographic pyramid as a result of increases in life expectancy. The pyramid of this stage is thus widening at the top.[12]

3.2.2 The Demographic Window

The aforementioned transition is part of what the concept of the demographic window captures. Officially, a *demographic window* (DW) is defined by the United Nations as a period of time when "the fraction of the population under age 15 falls below 30% and the fraction of the population 65 years and older is still below 15%." In terms of the stages described above we can think of a demographic window as a phenomenon that occurs as countries transition into the second stage and before they reach the third one— a phenomenon where the child cohort is shrinking, the working-age population cohort is expanding, and the elderly population has not increased substantially.

It is true that other phenomena could generate temporary demographic windows, like the immigration of young people and baby booms.[13,14] However, in this paper I focus on demographic windows that are part of this long run worldwide demographic transition because they are more commonly occurring around and have more permanent consequences. These windows are triggered by a sustained reduction in the number of newborns and characterized by a temporary increase in the working age- to total

[12] Japan is an example of a country that has this type of population pyramid.

[13] These phenomena can also contribute to an increase in the duration of a demographic window.

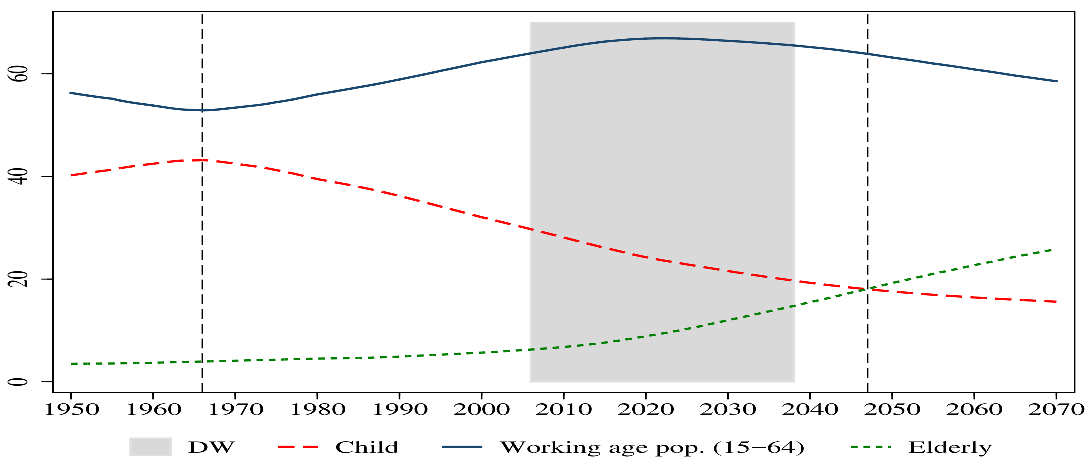
[14] And extreme cases, like an always-steady population, the sudden death of all the elderly or children, could also generate or extend demographic windows. This kind of cases are however rarely seen.

population-ratio, which reaches its maximum during the window. I use these two features to generate and characterize a demographic window in the model of this paper.

Finally, it is important to mention that while the stages of a demographic transition are generally common across countries, their speed and timing are not. For instance, Europe and some Asian countries have already gone through their demographic windows and are now aging, while Latin America is just entering its window and Africa will do so in the future (see figure B.4). In this paper I focus on Latin America, the region currently undergoing this process. Figure 3.1 shows the demographic indicators and trends described above for this region.

Figure 3.1: **Demographic transition in Latin America**

(a) Population by age group (%)



(b) Population pyramids

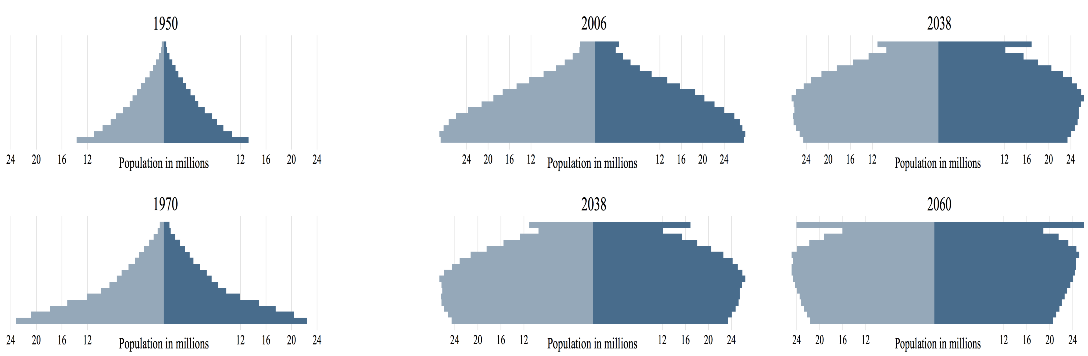
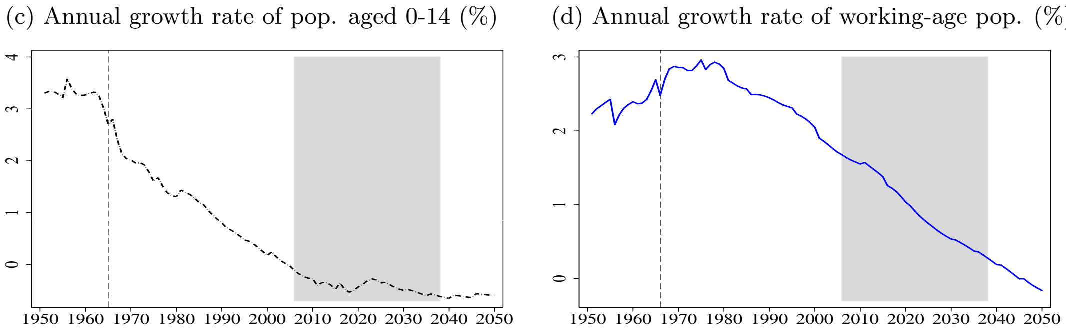


Figure 3.1 (Cont.): **Demographic transition in Latin America**



Notes: The gray areas indicate the years of the demographic window. The dotted vertical lines in panel (a) indicate the approximate years where stages 1 and 2 end. Panels (a) and (b) are aligned horizontally to show the change in population structure in terms of both age groups and population pyramids.

In the next section, I document the different relationship between the demographic window and investment in Latin America and the rest of the world. Then in the model I use Mexico as a benchmark case for Latin America since their demographic transitions are very similar (see figure B.1 in appendix B.1), and since I have the data to perform the growth accounting exercises required for the model for Mexico.

3.3 Data and Empirical Motivation

In this section, I first document that there are different patterns of investment for the demographic window across regions. Specifically, in Latin America the investment rates before and during their demographic windows are lower than outside of these periods, whereas elsewhere the opposite is true. I then provide suggestive evidence of the relationship between financial openness and investment for the demographic window and its importance for the different patterns across regions.

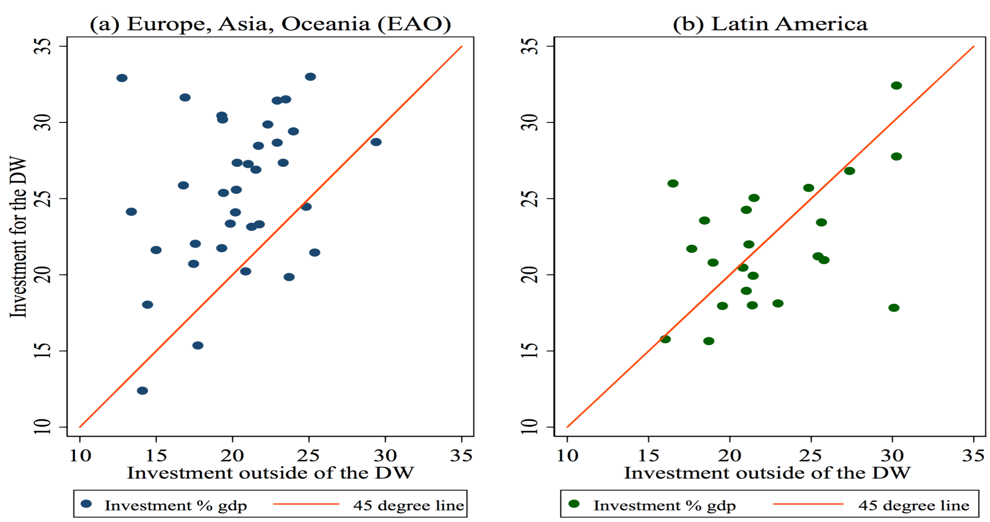
3.3.1 Data Description

I use six databases for the empirical analysis: World Development Indicators (WB), International Financial Statistics (IMF), the 2012 World Population Projects Database (United Nations), Penn World Tables (7.1, 8.0, and 8.1), the Chinn and Ito (2008) index, the Total Economy Database (2016), and the LABORSTA database (ILO). The core variables in the analysis are gross capital formation, gross domestic product, the number of people by age, labor force, and financial openness. Due to availability of the variables of interest, for the main sample I kept 59 countries with demographic transitions in Europe, Asia, Oceania, and Latin America, and the years 1960 to 2013.[15]

3.3.2 Investment and the Demographic Window

The main empirical result is that in Europe, Asia, and Oceania, *investment for the demographic window* (i.e., investment rates observed during the window and in the 15 years before it) is higher than investment (rates) outside of this period, while in Latin America, the opposite is observed. To derive this result, I grouped countries in two regions (1) Europe, Asia, and Oceania; and (2) Latin America.[16] As a first approach to the question *Are investment rates for the demographic window in Latin America are different from other regions?*, I compared average investment for the demographic window against average investment rates outside of this period.[17] Figure 3.2 gives an illustration.

Figure 3.2: **Investment rates for the demographic window (DW)**



The figure plots the average across years of investment rates in DW and the 15 years before it on the horizontal axis, and the average across years of investment rates outside of this period on the vertical axis. Each dot is a country.[18]

[15] African countries and countries with economic systems designated as transitional by the IMF and WB are not included in the main sample.

[16] Latin America is *Latin America and the Caribbean* using UN geographic sub-regional classification.

[17] Average across years. For this analysis *investment rate* is the gross capital formation to GDP ratio.

[18] The countries below the 45 degree line in panel (a) are Fiji, Israel, Latvia, Myanmar, Oman, Pakistan, and Spain. See figure B.3 in appendix B.1.

The conclusion from this initial analysis is that investment rates for the demographic window in Europe, Asia, and Oceania are on average and in general higher than investment rates outside of this period. For Latin America, the pattern is different.

Then, to formally obtain my main finding I regressed investment rates on dummies for region and demographic window, controlling for country and year fixed effects:

$$Inv_{c,t} = \beta_0 + \beta_1 dw_{c,t} + \beta_2 dw_{c,t} \cdot LA_c + f_c + f_t + \epsilon_{c,t}, \tag{3.1}$$

where c denotes the country and t the year. $dw_{c,t} = 1$ if the country is in its demographic window or in the 15 years before it and 0 otherwise, and $LA_c = 1$ if the country belongs to Latin America. f_c and f_t denote country and time fixed effects. Table 3.1 reports the results of this specification.[19]

The results are that in Europe, Asia, and Oceania, investment rates for the demographic window are associated with an increase of 4.23 percentage points. In contrast, in Latin America, investment rates for the demographic window are associated with a decrease of 0.79 percentage points.

Table 3.1:

Regression Estimates:

β_1 (dw)	4.23***	(0.26)
β_2 $(dw \times LA)$	-5.02***	(0.47)
Observations	2770	

Robust s.e. in parentheses, *** $p < .01$.

Results:

Hypothesis	value	P-val alternative
$\beta_1 > 0$	4.23	0.00
$\beta_1 + \beta_2 < 0$	-0.79	0.01

The estimates and hypothesis testing in table 3.1 illustrate these findings. The coefficient estimate of the variable dw_{ct} is positive and significant, indicating that the period for the demographic window is associated with an increase of investment rates in Europe, Asia, and Oceania. This hypothesis can not be rejected, as the P-value of the alternative hypothesis is zero. For Latin America, the effect of being in the period for the demographic window is given the coefficient of dw_{ct} plus its interaction with the region $dw_{ct} \times LA_c$. The sum of these coefficients is negative, indicating that for

[19]For the econometric analysis, investment rates are defined as the trend component of gross capital formation (GCF) over gross domestic product (GDP). The ratio of GCF to GDP was hp-filtered and the cyclic component removed. The idea behind this procedure is to consider long term variations of investment.

Latin America the period for the demographic window is associated with a decrease in investment rates; and this hypothesis can not be rejected at the 1% level.

These results are robust and they hold under different specifications. In particular, they hold when using different definitions of investment for the demographic window, and when dropping postwar years to consider possible effects of World War II on investment rates (see tables B.1 and B.4 in appendix B.1 for these and other specifications, including different set of controls and samples).

3.3.3 Empirical Relationship Between Investment for the Demographic Window and Openness

In this section I document the empirical relationship between investment for the demographic window and the degree of financial openness measured by the Chinn and Ito index of capital openness[20]. To do so, I estimate a variant of the baseline econometric model including as explanatory variables the Chinn and Ito index and its interaction with the demographic window, shown as

$$Inv_{c,t} = \beta_0 + \beta_1 dw_{c,t} + \beta_2 dw_{c,t} \cdot LA_c + \beta_3 kaopen_{c,t} + \beta_4 kopen_{c,t} \cdot dw_{c,t} + f_c + f_t + \epsilon_{c,t}, \tag{3.2}$$

where *kaopen* is the normalized Chinn-Ito index of capital account openness ranging from 0 to 1. The normalization assigns a value of 1 to the highest degree of financial openness and zero to the lowest; the lesser the restrictions on cross-border financial transactions, the higher the index.

The main result is that before and during a demographic window, more financially open economies have lower investment rates ($\beta_3 + \beta_4 < 0$). This evidence is suggestive of the importance of openness in the relationship between investment rates and the window. Empirically, there is a negative relationship between investment for the demographic window and the degree of financial openness of an economy.

[20] The main reason for using the Chinn and Ito index is that it is one of the few indices that cover many countries and many years. There are other index of capital openness like Quinn (1997) and Lane and Milesi-Ferretti (2007) which also includes many countries but the former it is only available until 2004, while the latter starts only in 1970. Other indices with a wide coverage of countries are much more restrictive in terms of the years, for instance Fernandez et al. (2015) covers 100 countries and the period of 1995-2013.

Table 3.2: **Demographic window and openness**

Regression Estimates:

β_1 (*dw*)	6.48***	(0.49)
β_2 (*dw* × LA)	-3.92***	(0.52)
β_3 (*kaopen*)	3.39***	(0.60)
β_4 (*kaopen* × dw)	-5.20***	(0.69)
Observations	2189	

Robust s.e. in parentheses,*** $p < .01$.

Results:

Hypothesis	value	P-val alternative
$\beta_1 > 0$	6.48	0.000
$\beta_1 + \beta_2 > 0$	2.56	0.000
$\beta_3 > 0$	3.39	0.000
$\beta_3 + \beta_4 < 0$	-1.81	0.001

Regarding the level of openness, in Latin America the average (across countries and years) openness using the normalized Chinn-Ito index in the 15 years before and during the window is 0.52, whereas outside of thar period the level is 0.31.[21] Using the previous regression this translates into an effect of -0.94 percentage points on investment rates in the 15 years before and during the window, and an effect of 1.08 percentage points outside of this period of time.[22]

The fact that countries in Latin America are more open before and during their windows than Europe, Asia, and Oceania could account for the differences in investment behavior across regions.

In this section I have documented that investment rates in Latin America display a different pattern in the presence of demographic changes, and that there is a negative relationship between investment for the demographic window and the degree of financial openness of the economies, when openness is measured by the Chinn-Ito index. I find this evidence suggestive of the importance of openness in both (1) the relationship between the demographic window and investment rates, and (2) the potential of openness in accounting for the documented different behavior of investment for the demographic window across regions. Motivated by this evidence, in the next section I develop an OLG model with demographic change and variable degree of financial openness.

[21]The average is computed as simple average across years and countries.

[22]On average, the normalized Chinn-Ito index in the 15 years before and during the windows in Latin America is 0.52 and in Europe, Asia, and Oceania it is 0.50. Furthermore, the level of openness in Latin America increased from 0.32 before this period to 0.52. In Europe, Asia, and Oceania the corresponding levels are 0.50 and 0.67.

3.4 The Model

In order to study the relationship between investment and the demographic window, I build an overlapping generations model with demographic change and a variable degree of financial openness. In this section, I first present the general model. Then, to provide some intuition, I discuss the mechanisms of the two extreme cases: the fully-open and the fully-closed economies.

3.4.1 The General Model

The Environment

An economy consists of households, firms, banks, and a government. In every period, four generations of households coexist. Generation t has N_t identical households who live for four periods and make economic choices during three of them. They feed their children, consume, save, and supply labor (inelastically). When the economy is financially open, they save in domestic capital and in foreign bonds; and when it is fully is closed, all savings are directed to domestic capital.[23] Banks take the savings, transform them into capital, and rent that capital to firms that produce goods using capital and labor services. Banks are also in charge of international savings or borrowing, and for that they pay taxes. The degree of financial openness of an economy is captured by the level of these taxes. There are two taxes, one on interest payments for foreign debt (τ_1) and another on total returns for foreign asset holdings (τ_2).[24] These taxes are paid at home, and they are collected by the government, and redistributed to the workers in a lump sum fashion. The economy is small relative to the rest of the world in the sense that it does not influence the price of foreign bonds.

The evolution of the population is determined by a simple law of motion for newborns. There is no early death in the model; however, in the numerical exercises, the growth rates of newborns are adjusted to the growth rates of the working-age population and therefore include mortality between these age groups.

[23]Both instruments, domestic capital and foreign bonds, have the same net return in equilibrium.

[24]I allow for taxes on capital inflows (τ_1) and capital outflows (τ_2) to be different since a country might have a different degree of financial openness for capital inflows and outflows.

Demographic Structure

In each period there are four generations alive: children, young workers, middle-aged workers, and retirees. Because of no early death, the size of the generations (N) is indexed by date of birth subscript.[25]

At any given period t, total population is:

$$Pop_t = \underbrace{N_t}_{\text{children}} + \underbrace{N_{t-1}}_{\text{Young workers}} + \underbrace{N_{t-2}}_{\text{Middle-aged workers}} + \underbrace{N_{t-3}}_{\text{retireees}}.$$

The evolution of newborns is given by the law of motion:

$$N_t = (1 + g_{nt})\, N_{t-1}.$$

The ratio of working-age to total population, or WAP ratio is:

$$WAP_t : \qquad \frac{N_{t-1} + N_{t+1}}{Pop_t} = \left[\frac{(1 + g_{nt})\,(1 + g_{nt-1})}{2 + g_{nt-1}} + 1 + \frac{1}{(1 + g_{nt-2})\,(2 + g_{nt-1})} \right]^{-1},$$

where N_t is the number of newborns in t, and g_{nt} is the growth rate of N_t.

Remark 3.4.1. *A decrease in the growth rate of newborns leads to a temporary increase in the WAP ratio.*

This is an important remark, since it allows me to generate a demographic window (characterized by a temporary increase in the WAP ratio) by imposing a decreasing growth rate on newborns, as observed in the data.

The growth rate of the working-age population is:

$$g_{WAPt} = \frac{(1 + g_{nt-2})\,(2 + g_{nt-1})}{(2 + g_{nt-2})} - 1.$$

Remark 3.4.2. *If the growth rate of newborns is decreasing, then the growth rate of the working-age population is also decreasing.*

[25] The number of households of generation t alive in period s N_s^t is equal to the number of households of generation t born in t $\equiv N_t$. Note that choice variables are indexed by time subscript and generation superscript.

This is also an important remark, since in the model, in equilibrium, the working-age population will be equal to the labor force. Therefore a demographic window, generated by a decrease in the growth rate of newborns, will generate a decrease in the growth rate of labor force.[26]

Regarding the choice of the number of generations: One must consider at least three generations (one being children) in the model to have decreasing growth rates of newborns before and during the demographic window and increasing WAP ratios. Furthermore, one must have at least four generations to have WAP ratios higher than 50% in the demographic window; this being the case, four generations provides a better demographic fit.[27]

Households

Households live for four periods and make economic choices during three of them. Because they care about their children, they feed their offspring. In this way, children consume what their parents give them. The discounted utility of a representative agent of generation $t-1$ is:

$$U^{t-1} = u\left(c_t^{t-1}\right) + \gamma\phi_{t-1}u\left(c_t^t\right) + \beta u\left(c_{t+1}^{t-1}\right) + \beta^2 u\left(c_{t+2}^{t-1}\right).$$

There is no disutility of working, but there is exogenous retirement in the last period of life. The one-period utility over consumption is of the constant relative risk aversion (CRRA) form, i.e., $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.[28] Altruism towards children is captured by the parameter γ, which is the weight of each child's utility in their parents' utility.[29] In the benchmark model, γ is set to 0.5 using the "OECD equivalence scale."[30] ϕ_{t-1} is the number of children per worker of generation t-1, and since households are representative, $\phi_{t-1} = \frac{N_t}{N_{t-1}} = (1 + g_{nt})$.[31]

[26] For more details on this see section B.2.2 in the Appendix.

[27] Given that the model excludes early death, adjusting the growth rate of newborns to the growth rate of age cohorts gives a better demographic fit with four generations than with three.

[28] The assumption of no disutility of working is sensible since in the model it implies that the growth rate of the labor force is equal to the growth rate of the working-age population. This has as empirical counterpart that both these rates are similar in levels and trends. See section B.2.2 in the Appendix.

[29] Parents care equally for each of their children.

[30] For an explanation of this choice see appendix B.2.1. For a sensitivity analysis on γ, see Section 3.5.3.

[31] See appendix B.2.1 for a proof of this result and for a more detailed expression of these preferences.

A representative agent of generation t-1 chooses (once she becomes active) in t: her labor supply, her consumption during working years and retirement, her savings, and the consumption by her children (c_t^t). In other words, in t, generation t-1 chooses $\{\ell_t^{t-1}, \ell_{t+1}^{t-1},\ c_t^{t-1}, c_{t+1}^{t-1}, c_{t+2}^{t-1},\ a_{t+1}^{t-1}, a_{t+2}^{t-1},\ c_t^t\}$ to solve:[32]

$$\begin{aligned} \max \quad & u\left(c_t^{t-1}\right) + \gamma\phi_{t-1}u\left(c_t^t\right) + \beta u\left(c_{t+1}^{t-1}\right) + \beta^2 u\left(c_{t+2}^{t-1}\right) \\ s.t. \quad & c_t^{t-1} + \phi_{t-1}c_t^t + a_{t+1}^{t-1} \leq w_t\ell_t^{t-1} + TR_t^{t-1} \\ & c_{t+1}^{t-1} + a_{t+2}^{t-1} \leq w_{t+1}\ell_{t+1}^{t-1} + \left(1 + r_{t+1}^b\right)a_{t+1}^{t-1} + TR_{t+1}^{t-1} \\ & c_{t+2}^{t-1} \leq \left(1 + r_{t+2}^b\right)a_{t+2}^{t-1} \\ & \ell_t^{t-1} \leq 1, \quad \ell_{t+1}^{t-1} \leq 1, \end{aligned}$$

where TR_t^{t-1} and TR_{t+1}^{t-1} are transfers from the government in t and t+1 respectively.

Banks

A representative bank transforms domestic and international savings into capital, rents this capital to the firms, buys international bonds, and it is competitive. This bank can finance itself from national and international markets, paying taxes domestically on external interest payments or earnings. I assume that foreign investors have access to a similar bank in their home countries and therefore the international returns on assets and capital are equal, i.e., $r_{t+1}^{b*} = r_{t+1}^{k*} - \delta \equiv r_t^w$, and exogenous to this economy. Because of the tax structure, I distinguish between external borrowing D_{t+1} and lending B_{t+1}. There are two cases to consider. If the economy is an:

(i) External borrower ($D_{t+1} > 0$): Then the bank will issue national and international debt promising to repay the principal and interests in the next period. It will then transform this debt into capital K_{t+1} and rent this capital to the firm in t+1. For each unit of capital, the bank will earn a revenue of r_{t+1}^k and a liquidation value of $(1-\delta)$. Therefore raising $\left(1 + r_{t+1}^k - \delta\right)$ and repaying $\left(1 + r_{t+1}^b\right)$ at home, and $\left(1 + r_{t+1}^w\right)$ abroad. In addition it has to pay a tax of $\tau_1 r_{t+1}^w$ at home. In this way the total cost of external financing is $r_{t+1}^w\left(1 + \tau_1\right)$.

(ii) External lender ($B_{t+1} > 0$): Then the bank will issue national debt, collecting

[32] Choices are indexed by time subscript and date of birth superscript.

savings (a^{FII}_{t+1}) from domestic households. It will then allocate some of this savings to domestic capital K_{t+1} and some to foreign bonds B_{t+1}, raising $\left(1+r^k_{t+1}-\delta\right)$ and $\left(1+r^w_{t+1}\right)$ respectively, and paying a tax of $\tau_2 r^w_{t+1}$. Thus the net external return on international bonds is $r^w_{t+1}\left(1-\tau_2\right)$.

The bank chooses national $K^{F_{II}}_{t+1}$, a^{FII}_{t+1} and international assets D_{t+1}, B_{t+1} to maximize:

$$(1+r^k_{t+1}-\delta)K^{FII}_{t+1}+\left(1+r^w_{t+1}\left(1-\tau_2\right)\right)B_{t+1}-(1+r^b_{t+1})a^{F_{II}}_{t+1}-\left(1+r^w_{t+1}\left(1+\tau_1\right)\right)D_{t+1}$$

$$s.t. \qquad K^{FII}_{t+1}\leq a^{FII}_{t+1}-B_{t+1}+D_{t+1} \qquad \text{tech. constraint}$$

$$D_{t+1}>0 \text{ and } B_{t+1}=0 \quad \text{or} \quad B_{t+1}>0 \text{ and } D_{t+1}=0.$$

Firms

A representative firm is competitive and produces goods using capital and labor with a constant returns to scale technology with labor augmenting technological change.[33] It rents capital and labor services from the bank and the households respectively to solve:

$$\min_{\{K_t\ ,L_t\}} \quad w_t L_t + r^k_t K_t$$

$$s.t. \quad Y_t \leq K^\alpha_t \left(A_t L_t\right)^{1-\alpha}.$$

The Government

Taxes are paid domestically and the government collects its revenues and redistributes them back to the each worker in a lump sum fashion. Therefore, every period the government budget balances

$$D_t r^w_t \tau_1 + B_t r^w_t \tau_2 = N_{t-1} TR^{t-1}_t + N_{t-2} TR^{t-2}_t, \tag{3.3}$$

where TR^{t-1}_t and TR^{t-2}_t are transfers in t to each young and middle-aged worker.

Definition 3.4.3. ***Taxed Economy Equilibrium:*** *Given sequences of productivity A_t, the interest rate of the rest of the world r^w_t, population dynamics N_t, ϕ_t, taxes τ_1, τ_2, initial domestic bonds a^0_1, a^{-1}_1, and initial capital stock K^0_1, a competitive equilibrium for this economy is: (i) consumption plans, savings (asset holdings), and labor supply*

[33]Technology evolves according to $A_t = (1+g_{at})\,A_{t-1}$, where g_{at} is the growth rate of A_t.

choices from households of generation t-1 $\{c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}\}_{t-1=2}^{\infty}$, $\{a_{t+1}^{t-1}, a_{t+2}^{t-1}\}_{t-1=2}^{\infty}$, $\{\ell_t^{t-1}, \ell_{t+1}^{t-1}\}_{t-1=2}^{\infty}$; *(ii) production plans of the firm* $\left\{L_t^{FI},\ K_t^{F_I}\right\}_{t=1}^{\infty}$;
(iii) capital allocations, national bonds, purchases and sales of international bonds of the bank $\left\{K_{t+1}^{FII},\ a_{t+1}^{F_{II}}, D_{t+1}, B_{t+1}\right\}_{t=0}^{\infty}$; *(iv) transfers to young and middle-aged workers* $\{TR_t^{t-1},\ TR_t^{t-2}\}_{t=1}^{\infty}$; *and (iv) prices* $\{w_t,\ r_t^b,\ r_t^k\}_{t=1}^{\infty}$, *such that:*

1. *Given prices and transfers, generation's t-1 choices* $\{c_t^{t-1} c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}, a_{t+1}^{t-1}, a_{t+2}^{t-1}, \ell_t^{t-1}, \ell_{t+1}^{t-1}\}$ *solve the household problem.*

2. *Given prices, the banks' choices* $\left\{K_t^{F_{II}}\ , a_t^{F_{II}}, D_{t+1}, B_{t+1}\right\}$ *solve the bank's problem.*

3. *Given prices, the firms' choices* $\left\{K_t^{F_I}\ , L_t^{F_I}\right\}$ *solve the goods producing firm problem.*

4. *Given prices, the government chooses transfers* $\{TR_t^{t-1},\ TR_t^{t-2}\}_{t=1}^{\infty}$ *to satisfy its budget constraint.*

5. *The domestic bonds market clears:*
$$\sum_{i=1}^{N_{j-1}} a_{t+1}^{j-1} + \sum_{i=1}^{N_{j-2}} a_{t+1}^{j-2} = a_{t+1}^{F_{II}}.$$

6. *The capital and labor inputs markets clear:*
$$\begin{aligned} K_t^{FI} &= K_t^{FII} \\ L_t^{FI} &= N_{t-1} \cdot \ell_t^{t-1} + N_{t-2} \cdot \ell_t^{t-2}. \end{aligned}$$

7. *The goods market clears:*
$$N_{t-1}c_t^{t-1} + N_t c_t^t + N_{t-2}c_t^{t-2} + N_{t-3}c_t^{t-3} + K_{t+1} - (1-\delta)K_t ...$$
$$... + B_{t+1} - (1+r^w)B_t - D_{t+1} + (1+r^w)D_t = Y_t.$$

8. *The assets market clears:*
$$\sum_{i=1}^{N_{j-1}} a_{t+1}^{j-1} + \sum_{i=1}^{N_{j-2}} a_{t+1}^{j-2} = K_{t+1} + B_{t+1} - D_{t+1}.$$

Characterization of the Equilibrium

The solution to the household problem can be summarized by her assets holdings:

$$a_{t+1}^{t-1} = \frac{\left[\beta^{\frac{1}{\sigma}}\left(1+r_{t+1}\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1+r_{t+1}^{b}\right)\left(1+r_{t+2}^{b}\right)\right]^{\frac{1}{\sigma}-1}\right]\left(w_t \ell_t^{t-1} + TR_t^{t-1}\right)}{1+\gamma^{\frac{1}{\sigma}}\phi_t + \beta^{\frac{1}{\sigma}}\left(1+r_{t+1}^{b}\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1+r_{t+1}^{b}\right)\left(1+r_{t+2}^{b}\right)\right]^{\frac{1}{\sigma}-1}} \ldots$$

$$\ldots - \frac{-\left(1+\gamma^{\frac{1}{\sigma}}\phi_t\right)\frac{w_{t+1}\ell_{t+1}^{t-1}+TR_{t+1}^{t-1}}{1+r_{t+1}^{b}}}{1+\gamma^{\frac{1}{\sigma}}\phi_t + \beta^{\frac{1}{\sigma}}\left(1+r_{t+1}^{b}\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1+r_{t+1}^{b}\right)\left(1+r_{t+2}^{b}\right)\right]^{\frac{1}{\sigma}-1}} \tag{3.4}$$

$$a_{t+2}^{t-1} = \frac{\beta^{\frac{2}{\sigma}}\left[\left(1+r_{t+1}^{b}\right)\left(1+r_{t+2}^{b}\right)\right]^{\frac{1}{\sigma}-1}\left[\left(1+r_{t+1}^{b}\right)\left(w_t \ell_t^{t-1} + TR_t^{t-1}\right) + w_{t+1}\ell_{t+1}^{t-1} + TR_{t+1}^{t-1}\right]}{1+\gamma^{\frac{1}{\sigma}}\phi_t + \beta^{\frac{1}{\sigma}}\left(1+r_{t+1}^{b}\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1+r_{t+1}^{b}\right)\left(1+r_{t+2}^{b}\right)\right]^{\frac{1}{\sigma}-1}} \tag{3.5}$$

her optimal labor supply choice $\ell_t^{t-1} = 1$, and her consumption demand functions.

Capital and labor demands from the goods-producing firm are given by:

$$(1-\alpha) A_t k_t^{\alpha} = w_t \tag{3.6}$$

$$\alpha k_t^{\alpha-1} = r_t^k, \tag{3.7}$$

where $k_t \equiv \frac{K_t}{A_t L_t}$ is defined as capital per effective units of labor (p.e.u.l).

From the bank's allocations, we have the no-arbitrage conditions of domestic and international bonds, where the net return on domestic capital is equalized to the after-tax return abroad:

$$\left(1 + r_{t+1}^{k} - \delta\right) = (1 + r_{t+1}^{b}) \tag{3.8}$$

$$\left(1 + r_{t+1}^{k} - \delta\right) = \left[1 + r_{t+1}^{w}(1+\tau_1)\right] \qquad \text{if } D_{t+1} > 0 \tag{3.9}$$

$$\left(1 + r_{t+1}^{k} - \delta\right) = \left[1 + r_{t+1}^{w}(1-\tau_2)\right] \qquad \text{if } B_{t+1} > 0. \tag{3.10}$$

The conditions that determine whether an economy will be borrowing, lending, or in autarky are as follows:

$$D_{t+1} > 0 \quad \text{if} \quad F_1'\left(k_{t+1}^{aut}, 1\right) - \delta > r_{t+1}^{w}(1+\tau_1) \quad \text{and} \quad K_{t+1} = a_{t+1}^{FII} + D_{t+1} \tag{3.11}$$

$$B_{t+1} > 0 \quad \text{if} \quad F_1'\left(k_{t+1}^{aut}, 1\right) - \delta > r_{t+1}^{w}(1-\tau_2) \quad \text{and} \quad K_{t+1} + B_{t+1} = a_{t+1}^{FII} \tag{3.12}$$

$$\text{Autarky if } (1-\tau_2) r_{t+1}^{w} \leq F_1'\left(k_{t+1}^{aut}, 1\right) - \delta \leq r_{t+1}^{w}(1+\tau_1) \quad \text{or} \quad K_{t+1} < a_{t+1}^{FII} + D_{t+1} - B_{t+1}, \tag{3.13}$$

where k_{t+1}^{aut} is the capital (p.e.u.l) of being in financial autarky in t+1, given the history of capital and transfers.[34]

Let $s_t^{t-1} \equiv \frac{a_{t+1}^{t-1}}{A_t}$ and $s_{t+1}^{t-1} \equiv \frac{a_{t+2}^{t-1}}{A_{t+1}}$ be savings per effective units. Using the assets market clearing condition and the optimal savings, the amount of borrowing/lending (p.e.u.l) is:

$$\frac{1}{(1+g_{at+1})(1+g_{Lt+1})}\left[\frac{1+g_{nt-1}}{2+g_{nt-1}}s_t^{t-1}+\frac{s_t^{t-2}}{2+g_{nt-1}}\right]-k_{t+1}=\begin{cases}-d_{t+1} & \text{if borrower}\\ b_{t+1} & \text{if lender}\\ 0 & \text{if autarky.}\end{cases} \tag{3.14}$$

Capital per effective unit of labor is pinned down by the after-tax interest rate on international assets if the economy is open, and by a law of motion derived from optimal savings, prices, and equation (3.14) if it is closed:

$$k_t = \left(\frac{r_t^w(1+\tau_1)+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \quad \text{if} \quad d_{t+1}>0 \tag{3.15}$$

$$k_t = \left(\frac{r_t^w(1-\tau_2)+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \quad \text{if} \quad b_{t+1}>0 \tag{3.16}$$

$$k_t = k_{t+1}^{aut}\left(\{k_t\}_t, \{tr_t^j(d_t, b_t, \tau)\}_t\right) \quad \text{if} \quad \text{in autarky,} \tag{3.17}$$

where the terms in brackets are included to emphasize the dependence of the history of capital and transfers on savings. tr_t^j are transfers per effective worker, which are the tax revenues divided equally between young and middle-aged workers ($\theta_t^y = \theta_t^m = 0.5$):

$$tr_t^{t-1} = \theta_t^y \frac{d_t L_t r_t^w \tau_1}{N_{t-1}} \qquad tr_t^{t-2} = \theta_t^m \frac{d_t L_t r_t^w \tau_1}{N_{t-2}} \qquad \text{if} \quad d_t>0 \tag{3.18}$$

$$tr_t^{t-1} = \theta_t^y \frac{b_t L_t r_t^w \tau_2}{N_{t-1}} \qquad tr_t^{t-2} = \theta_t^m \frac{b_t L_t r_t^w \tau_2}{N_{t-2}} \qquad \text{if} \quad b_t>0. \tag{3.19}$$

This completes the characterization of the equilibrium.[35] I now turn to an important remark regarding the determination of openness of the economies.

Remark 3.4.4. *From conditions (3.11), (3.12), and (3.13), given the demographics of*

[34] See equation 3.20 in the next section.

[35] See section B.2.4 in for a description of the algorithm of the computation of the equilibrium.

an economy, there are thresholds on the taxes τ_1 and τ_2 such that for a tax level higher than the threshold, the economy will be closed and capital (p.e.u.l) will be determined endogenously. For lower tax levels, capital (p.e.u.l) will be given by the after-tax return on international assets.

This is important because for the demographic window exercises, the driver behind the different results in the fully-open and the fully-closed economies is also whether capital (p.e.u.l) is determined endogenously or exogenously. Therefore, it suffices to discuss the extreme cases which will extend to the various degrees of openness, depending on which side of the threshold the taxes of the economy fall.

3.4.2 Fully-Closed and Fully-Open Economies

Since the mechanisms in the model with variable degree of openness are those of the extreme cases,[36] In this section I lay down the main equations of the fully-closed and fully-open economies to provide intuition. In the next section I fully explore and discuss the effect of the demographic window in these cases.

In the fully-closed economy, capital is equal to the savings of young and middle-aged workers, and both are determined simultaneously. In contrast, in the fully-open economy, capital (p.e.u.l) is determined by the interest rate of the rest of the world, independently of savings, and foreign assets are the difference between national savings and this capital.

Fully-Closed Economy

When the economy is fully closed, the total capital stock is equal to the supply of domestic savings of the workers. The law of motion of capital per effective units of labor reads:

$$(1 + g_{at+1})(1 + g_{Lt+1})\, k_{t+1} = \underbrace{\frac{N_{t-1}}{L_t}}_{\text{\% of young savers}} s_t^{t-1} + \underbrace{\frac{N_{t-2}}{L_t}}_{\text{\% of middle-age savers}} s_t^{t-2}, \tag{3.20}$$

[36] See remark 3.4.4, figure 3.4, and numerical exercise in Section 3.5.1.

where s_t^{t-1} and s_{t+1}^{t-1} are functions of capital (p.e.u.l), demographic variables and parameters. The equilibrium can be summarized by a sequence of capital per effective units of labor.[37] The rest of the equilibrium allocations are expressed in terms of this variable.

In the closed economy, the demographic window will operate via savings and the drivers of the savings will be the drivers of investment for the demographic window.

Fully-Open Economy

For the extreme case of a fully-open economy, assume there are no taxes on capital flows (i.e., τ_1=τ_2=0). The no-arbitrage conditions for international assets are reduced to:

$$\left(1 + r_{t+1}^k - \delta\right) = (1 + r_{t+1}^w). \tag{3.21}$$

Since the interest rate of the rest of the world is given, the rental price of domestic capital is also determined by the rest of the world. Therefore, the firm demands capital up to the point where its net[38] marginal product equalizes the return abroad and the expression of capital (p.e.u.l) is:

$$k_t = \left(\frac{r_t^w + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{3.22}$$

Capital per effective units of labor is pinned down by the return abroad, savings are determined independently of capital (p.e.u.l), and foreign asset holdings are the difference between effective savings and domestic capital (p.e.u.l):[39]

$$(1 + g_{at+1})\,(1 + g_{Lt+1})\left[k_{t+1}(r_{t+1}^w) + b_{t+1} - d_{t+1}\right] = \frac{N_{t-1}}{L_t}\;\; s_t^{t-1}(r^w) + \frac{N_{t-2}}{L_t}\;\; s_t^{t-2}(r^w). \tag{3.23}$$

The main divers of the results of the demographic window exercises will be the facts that savings are independent of capital p.e.u.l and that in open economies investment and savings need not to be equal.

[37]This sequence can be reduced to a third-order difference equation for a numerical solution, see section B.2.4.

[38]Net of depreciation.

[39]Expression 3.23 (and expression 3.20) is derived from the assets market clearing condition.

3.5 The Demographic Window: Main Results of the Models

Recall that in the data the child to population ratio is decreasing both before and during the demographic window, and that during the window there is a temporary increase in the working age- to total population-ratio.[40] In the model, I generate a demographic window by imposing an decrease in the growth rate of newborns, which generates a temporary increase of the WAP ratio, and a decreasing (and low) children to population ratio.[41]

This paper studies the behavior of investment leading up to and during the demographic window. I do so through the lens of the models by analyzing the response of the investment rate to this decrease in the growth rate of newborns. The exercise consists of calculating the transition between two steady states and analyzing the change in investment rates before and during the demographic window, which is part of this transition. The first steady state has a high newborns' growth rate and the second one, a low growth rate. The shock consists of imposing a decreasing growth rate of newborns g_{nt} through which the large cohort of children becomes part of the working-age population, generating a demographic window. As the population stabilizes, the economy moves out of the demographic window and into the second steady state. The change in the growth rate of newborns is perfectly foreseen by everyone alive during the transition.

In this section I explore the effects that different variables have on investment for the demographic window. I begin by analyzing openness. Then I analyze changes in the return on foreign assets and in productivity, as well as the effects of capital depletion after World War II (WWII). The conclusion is that openness is the determining factor behind the documented differences in investment behavior. Next, I conduct a sensitivity analysis on some parameters of the model. Finally, I compare the results of the benchmark model to the investment rates observed in Brazil.

[40]These is true in the data in general, and in particular for Latin America.

[41]Recall that in this paper I focus on the demographic windows that are part of the long-run demographic transition. These windows are triggered by a reduction in fertility rates and characterized by a temporary increase in the WAP ratio, which reaches its maximum level during the window. There are other shocks besides a decreasing growth rate for newborns that could generate a demographic window in the framework of this model, however I do not study those cases since they are not the focus of this paper.

3.5.1 Investment and openness

The main result of the model is that in open economies, investment rates decrease before and during the demographic window, whereas in closed economies they temporarily increase. The investment rates, as a function capital (p.e.u.l), can be expressed as:

$$ir_t = (1+g_{Lt+1})(1+g_{at+1})\frac{k_{t+1}}{k_t^{\alpha}} - (1-\delta)k_t^{1-\alpha}, \tag{3.24}$$

In the open economy, capital (p.e.u.l) is determined by the exogenous return on foreign assets, and therefore investment rates move together with the growth rate of the effective labor force. The growth rate of the labor force is in turn decreasing during the demographic transition since in each period there are fewer children that will join the labor force (recall remark 3.4.1).[42] Even considering improvements in labor productivity, the growth rate of the effective labor force does not increase since, in the data, demographic transitions are stronger than technological changes.[43]

In the closed economy, investment is determined by national savings, and the rates of both are equal. The first term in the above investment equation is the savings rate of workers, and the second is the dissavings rate:[44]

$$ir_t = \underbrace{\frac{N_{t-1}}{L_t}}_{\text{\% young savers}} sr_t^{t-1} + \underbrace{\frac{N_{t-2}}{L_t}}_{\text{\% middle savers}} sr_t^{t-2} - \underbrace{(1-\delta)k_t^{1-\alpha}}_{\text{dissavings rate}}. \tag{3.25}$$

Workers' savings rates[45] increase for three reasons. First, there are fewer mouths to be fed. Second, the bulk of the labor force moves towards middle-aged workers, who are the ones saving the most[46]. Third, labor earnings increase because of price effects.[47] The dissavings rate increases more slowly, and therefore the overall hence investment rate increases. This last result follows from capital (p.e.u.l) being increasing, as it is the

[42] For a more detailed discussion on this see section B.2.2.

[43] See section 3.5.2 for specific exercises that allow for changes in productivity.

[44] The second therm is the dissavings rate since in OLG each period agents sell their undepreciated capital.

[45] For the expressions of sr_t^{t-1} and sr_t^{t-2}, take equations B.3 and B.4 of appendix B.2.3 and divide by k_t^{α}.

[46] See figure B.7 in appendix B.2.3.

[47] Price effects do not seem to be the determining factor behind the savings behavior, as discussed below.

only vehicle of savings.[48] As the economy moves to the long run, savings stabilize and investment rate decreases to its long-run value. Overall, there is a temporary increase in the investment rates before and during the demographic window".

The cause for the aforementioned result in investment, per the model, is that in small open economies there is a complete separation between savings and investment decisions. In a demographic window, the former are modified by changes in child dependency and in savers' cohort sizes and the latter by the growth rate of effective labor; they move in opposite directions.[49] In closed economies, however, savings and investment move in the same direction, so it is possible to have increasing investment rates.

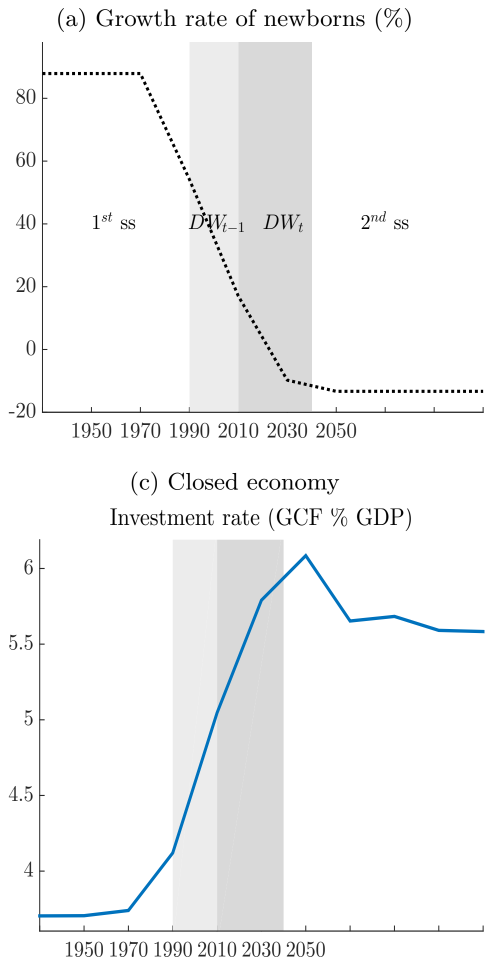
Figure 3.3 shows the benchmark numerical exercise of Mexico. The first steady state is characterized by the average growth rate of newborns from 1950–1970, and the second by the projection for 2050. I plot this growth rate in panel (a).[50] Panel (b) shows the change in the WAP ratio that this generates. Notice that the ratio peaks in the demographic window. Panels (c) and (b) show he investment and savings rates of the closed and the open (frictionless) economies respectively.

[48] For a visual illustration see figure B.8 in appendix B.2.3.

[49] Child dependency is measured by children per worker in this model.

[50] 20-year cumulative growth rate: average for 1950-1970, observed for 1990 and 2010, projection for 2050.

Figure 3.3: **Benchmark exercise, Mexico**



Notes: This figure plots the Mexican demographic transition (i.e., the reduction in the growth rates of newborns in panel) (a) and the demographic window that this generates in panel (b). The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before the window. In panels (c) and (d) I plot the investment and savings rates throughout this transition.

Note that the evolution of savings rates in the closed and the open economies is similar —they increase before and during the demographic window and decrease as the economy converges to the long run— this suggests that price changes are not the dominant effects.

In the open economy, the savings of young workers increase since they bring less money from the future for their children's consumption (decrease of dissavings) and as a result of more available income for themselves. The savings of the middle-aged workers also increase, because of the latter.

In the closed economy total worker's savings also increase. However, young workers' savings decrease the period before and during the demographic window, while middle-age savings increase. Savings of young workers, increase in the first period of the transition because of the reduction in child dependency.[51] After that, once price effects kick in, savings of the young decrease through the rest of the transition. Borrowing increases since the labor income profile is increasing through the transition, and the cost of financing decreases, making it cheaper to bring the increased future labor income to the present and smooth consumption. However, the magnitude of this change is small and the young savers cohort shrinks at the expense of the middle-aged workers' cohort, which makes their savings the determinant ones. Savings of middle-aged workers increase through the entire transition. In the first period, this occurs because the decrease in child dependency increases the available income. Further on, this higher available income compounds with higher labor earnings to generate an increase in savings. Even though returns for savings decrease, savings increase since that is the only way to consume during retirement.

Recall that the results of the extreme cases extend to the model with variable degrees of openness. Figure 3.4 illustrates this point, with the previous exercise of Mexico using the model with variable degree of openness for high and low taxes.[52] When taxes are high, investment for the demographic window increases as in the fully-closed economy, and when taxes are low it decreases as in the fully-open economy.[53]

In these numerical exercises, the growth rate of technology and the rate of return on the rest of the world are set to the average between 1950 and 2014. Both variables were calculated through growth accounting exercises on Mexico and on the United States—which represents the rest of the world.[54] In the next section I relax these assumptions.

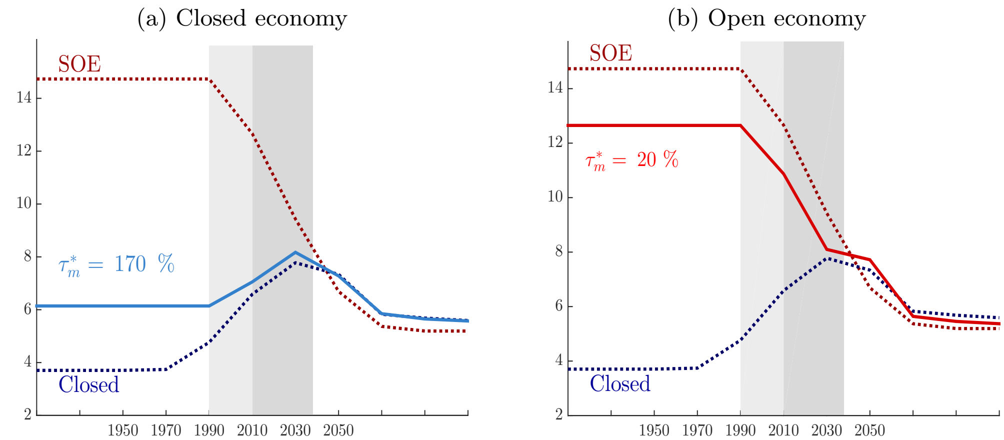
[51]The first period of the demographic transition is 1970-1990, the period before the window is 1990-2010.

[52]Figure B.9 in appendix B.2.3 plots the capital (p.e.u.l) of this exercise.

[53]For more details of the open taxed case see the open economy section in appendix B.2.4.

[54]I compute the growth rate of total factor productivity (TFP) of Mexico using the model specification and retrieving TFP as a residual. The rate of return of the rest of the world is the marginal productivity of capital (p.e.u.l) net of depreciation of the US. In the growth accounting exercises, a la Kehoe, I use data on investment, CFC, GDP, and WAP, with a depreciation of 5% and a capital share of income of 0.3.

Figure 3.4: **Benchmark exercise of Mexico: varying degree of openness**



Notes: This figure plots the Mexican demographic transition in a framework of variable openness. The dark gray bar indicates the years of the demographic window and the light gray bar, the 15 years before it. In panels (a) and (b), in solid lines, I plot the investment rate throughout this transition in the taxed-closed and the taxed-open economies. The dotted line are the the investment rates in the frictionless model. In the closed economy $\tau_1 = 20\%$ and $\tau_2 = 5\%$ and in the open economy $\tau_1 = 170\%$,and $\tau_2 = 7\%$.

3.5.2 Robustness to alternative confounding factors

In this section, I allow for changes in productivity, international returns, and negative shocks on the capital stock. I explore whether these variables can generate, on their own, different responses of investment for the demographic window and whether they can change the main result of openness.

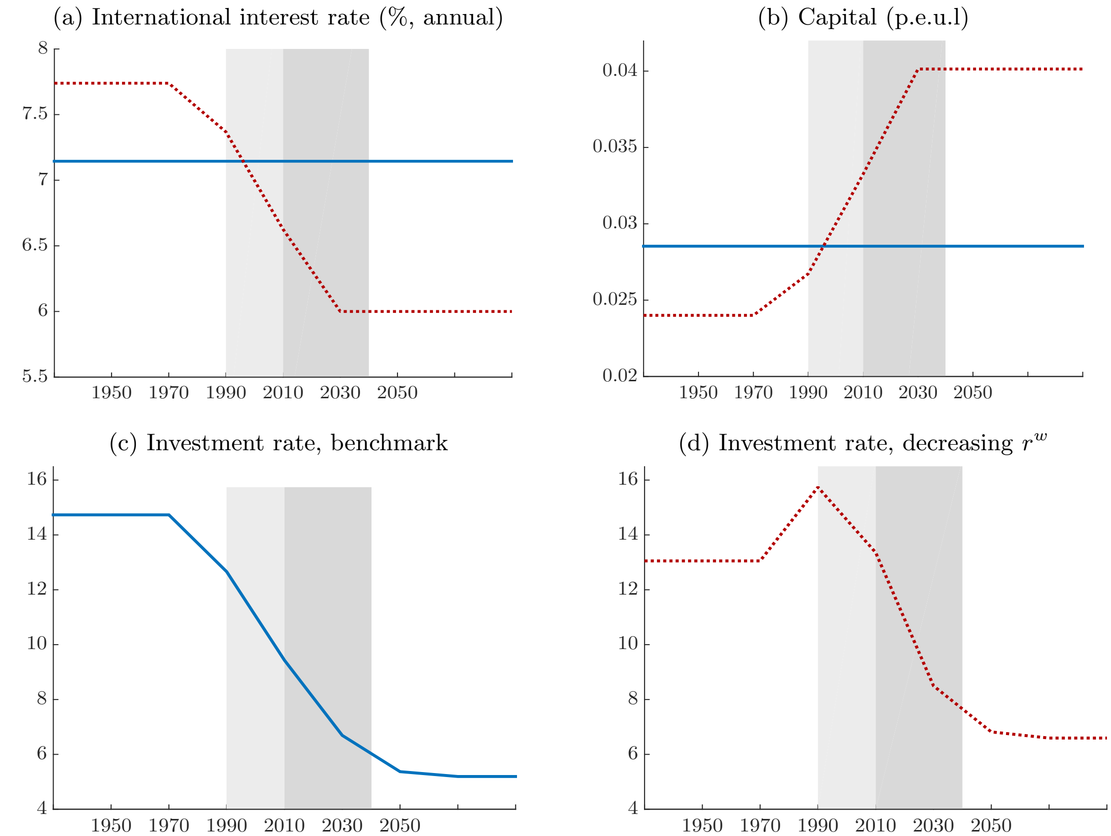
Return on international assets

In this section, I relax the assumption of a constant rate of return on international assets and examine the effects that the observed decrease in this rate could have on investment for the demographic window. The exercise consists of introducing the observed decreasing path of the rate of return of the rest of the world, measured as the net marginal productivity of capital of the United States, and computing the equilibrium in the open (frictionless) economy through the Mexican demographic transition.[55] Figure

[55]The annualized international rates of return are 7.7% (1950-1970), 7.4% (1970-1990), 6.6% (1990-2010), and 6% (2030 on). I chose 6% for 2030 on to abstract from the drop in rates of the Great Recession (the value of 2007 was 6.14% and afterwards rates dropped to 5.5%). A second criterium is

B.10 shows the results of this exercise and compares it to the benchmark.[56]

Figure 3.5: **Decreasing international rate of return**



Notes: This figure shows capital (p.e.u.l) and investment rates (GCF % GDP) through the Mexican demographic transition under two scenarios. The first scenario, shown by the dashed lines, is characterized by the observed decreasing international return until 2010 with a projection of 6% for 2030 on. The second scenario, shown by the solid lines, is the benchmark exercise with an annual constant return of 7.15% (the average between 1950 and 2014).

The decrease in the return of international assets generates an increase in domestic capital (p.e.u.l) to lower its marginal productivity and equalizes the return of domestic and international assets (see panels (a) and (b) of figure B.10). Investment rates increase in the first period of the demographic transition because of the increasing capital (p.e.u.l). However, once the effects of the demographic window kick in, investment rates decrease. In particular, during the period before and in the first period of the

to use the average of the most recent decade, 2004–2014, which is also around 6% (5.8).

[56]To see the results on savings rate, refer to figure B.10 in appendix B.2.3.

demographic window, investment rates go down since the decrease in future labor force associated with the demographic window is large enough to offset the increase in domestic capital (p.e.u.l) generated by the changes in the interest rate.[57] The end result, as depicted in panel (d) of figure B.10, is a decrease of investment for the demographic window.

To summarize, the observed decrease in international returns, absent of the demographic window, would generate an increase of investment rates. Since it makes it relatively less worthwhile to invest abroad, households shift part of their international assets to domestic capital which increases investment rates. However, once the demographic window effect is taken into account, namely the large decrease in growth rate of effective labor force, the aforementioned result is more than offset decreasing investment rates. The conclusion is that even considering the decrease in returns on international assets, investment for the demographic window decreases in the open economy.

Productivity Growth

Recall that in open economies investment rates grow with effective labor force. In principle, increases in the growth rate of productivity could increase investment rates. However, once we consider demographic changes, investment for the DW does not increase with technological progress since in order to offset demographic effects, changes in the growth rate of productivity need to be implausibly high.[58] To illustrate this point, in figure 3.6(a) I compare a scenario with high technological growth (the average growth of Mexico between 1950 and 1970), with the observed decreasing path.[59] Note that in both scenarios investment rate is decreasing in the transition regardless of these big technological changes.[60]

In particular, consider the case of most Western European countries and Japan.[61]

[57] In these two periods, the decreases in the future growth rate of the labor force are the biggest, since they are the decreases in current labor force that happen during the demographic window itself.

[58] The growth rate of working-age population decreases more than the increases in productivity during this period. To get a sense of the size of demographic changes, in Latin America the annual growth rate of the working-age population decreases from 3% at the beginning of the demographic transition to 0% towards the end of the demographic window. See figure B.14(a) in appendix B.2.3.

[59] Annualized TFP growth rates: 2.84% (1950-1970), -0.82% (1970-1990), -0.11% (1990-2010), 0% (2030 on).

[60] In the open economy, savings rates increase like in the closed economy, see figure B.11 in appendix B.2.3.

[61] I group Japan with Europe since the demographic window of Japan happened around the same

These countries experienced high growth from the postwar era until the mid 1970s, followed by a slowdown.[62] In all the countries in my sample that belong to this group, this period of high productivity, falls during their demographic windows.[63] One could therefore ask if we should think of these countries as open economies where investment for the DW increased because of high TFP, but as explained above, through the lens of the model we cannot.

In the same spirit, it is not possible to undo the result of the closed economy with productivity changes. In closed economies, changes in the growth rate of TFP modify the level of investment rates, but these changes do not modify the response of investment rates to demographic changes. We can see this in figure 3.6(b). Again, we might ask if we should think of Latin America as a closed economy where decreasing TFP explains decreasing investment rates. The answer is no, since not only is the behavior of investment for the demographic window unchanged, but decreasing growth rates of TFP increase investment rates, the opposite of what I document for this region.

The fact that a decrease in the growth rate of TFP increases investment rates is a standard result in closed models with technological and population growth. In a simple representative agent model of this type, a permanent decrease in the growth rate of either variable increases the steady-state level of capital (p.e.u.l).[64] At the beginning of the transition, accumulation is fast, overcompensating the decrease in the growth rate of TFP (see equation 3.24), and therefore increasing investment rates. As the economy converges to the long run, capital accumulation slows down. The same logic extends to the case of a decreasing sequence of growth rates of TFP, which implies convergence towards increasing capital (p.e.u.l) steady states and therefore increasing investment rates.[65]

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time as other European countries (1960 to 1996).

[62] High growth in terms of GDP per capita and GDP per working-age population, which in a balanced growth path is equivalent to productivity growth. For some countries this high growth lasted until 1973 and for others until 1978, the years of the oil shocks.

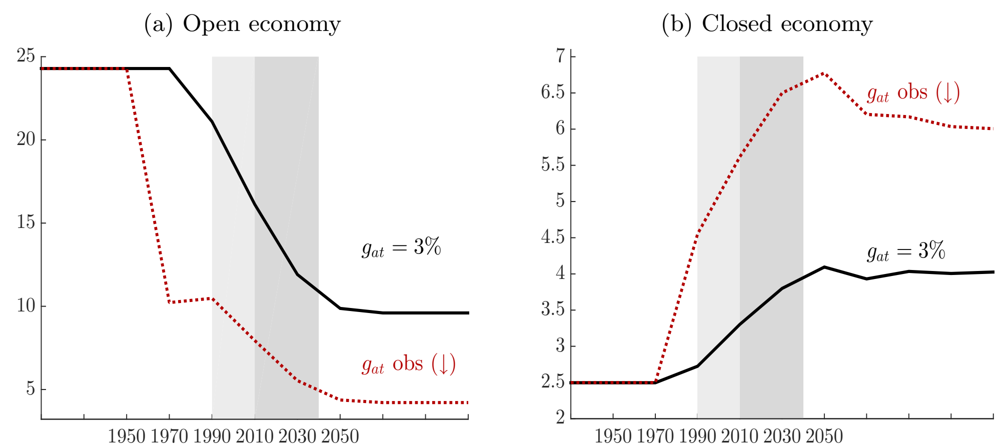
[63] "This group" being Western Europe and Japan, see figure B.12 in appendix B.2.3 for the evolution of GDP per capita, because of data availability, which is similar to the evolution of GDP per working-age population.

[64] In a balanced growth path model with CRRA utility over consumption and Cobb-Douglas labor augmenting technology, the steady state capital (p.e.u.l) is $k_{ss} = \frac{\alpha}{\frac{G^\sigma}{\beta} - (1-\delta)}^{\frac{1}{1-\alpha}}$, where G is the growth rate of technology and labor. Note that a decrease in G will increase kss for $\sigma \geq 0$.

[65] This result does not depend on the parameters of the model; in particular it holds for $\sigma \geq 0$.

In the model in this paper, there are similar mechanisms in place regarding steady state levels. A decreasing TFP growth rate increases capital (p.e.u.l) of the steady state.[66] Regarding convergence, there is an increase in savings and therefore in investment rates.[67] For young workers, since future labor income grows more slowly, less of it is brought to the present, decreasing dissavings. For middle-aged workers, since the growth of labor income was higher when they were young, they increase savings for retirement to smooth consumption. The dissavings rate increases more slowly since the economy is converging to a higher steady state. In short, in the closed economy, the response to a decreasing productivity growth is to increase savings and investment rates to smooth consumption since labor income growth slows down.

Figure 3.6: **Investment rates and technology growth**



Notes: This figure shows investment rates (GCF % GDP) through the Mexican demographic transition under two scenarios. Shown by the solid line: a "high technological growth rate," where $g_{at} = 3\%$ annually (e.g,the observed average for the first steady state 1950-1970). Shown by the dashed line: the "observed technological growth" until 2010 (see footnote 62) with a projection of 0% for 2030 on.

Finally, I analyze the possible effects of the change in the long-run growth rate of Western Europe and some Asian countries. In the 1800s, when the United Kingdom was the industrial leader, these countries generally grew at an approximate annual rate of 1%. Later, in the 1900s, when the United States took over as the lead country, they grew at an approximate rate of 2.%[68] For the countries of my sample, the change in

[66] For standard parameters, i.e., $\forall \sigma > 0$, $\forall \delta$, $\forall \gamma \geq 0$, $\forall \beta > 0$.
[67] See savings equations B.3 and B.4 in appendix B.2.3.
[68] 2% is the average growth rate of GDP per working-age population of the US over the 20th century.

growth trend happened between 1880 and 1920.[69]

In the model, the aforementioned change in long-run growth is a one-time increase in the growth rate of technology that can be incorporated as a change to a higher balanced growth path.[70] In the open economy framework, when the change in growth rate happens close to or during the demographic window, this change is offset by an even larger demographic change (see equation 3.24). To give a sense of demographic changes in this region, for instance, the annual growth rate of the working-age population decreased in Japan from 2% before its demographic window to around 0% towards its end.[71] This is a bigger change than the movement from a 2% BGP to a 1% one.

In the closed economy framework, a one-time increase in the TFP growth rate of generates on its own a decrease in the investment rate. Therefore this this change in the long run growth can not help to explain the documented increase of investment for the demographic window in these region. Once we add the demographic window on the top of the technological change, then the increasing pattern of investment emerges, as depicted in figure 3.6(b), where the only difference is that the level of investment is lower than what it would be without technological changes.[72]

Capital Depletion and Subsequent Accumulation (WWII)

In this section I address the question of whether the increase in investment for the demographic window in Europe was a result of postwar capital destruction and subsequent reconstruction. Many European countries were in their demographic windows around the World War II. In the empirical analysis, the main findings documented in this paper (i.e., the different behaviors of investment for the demographic window across regions) are robust to possible effects of the war on investment rates. As mentioned in the empirical section, the results hold even in a subsample where I dropped post-war years, e.g, the 1960s and the 1970s, for Europe, Asia, and Oceania.

[69] See figure B.13 in appendix B.2.3.

[70] Note that this is different from the discussion of the growth rate of technology during the period of 1950s to 1970s. Here, the focus is on a one-time change in long-run growth, as opposed to an increasing or decreasing path. This is could be of interest in the closed economy. See figure B.15 in appendix B.2.3.

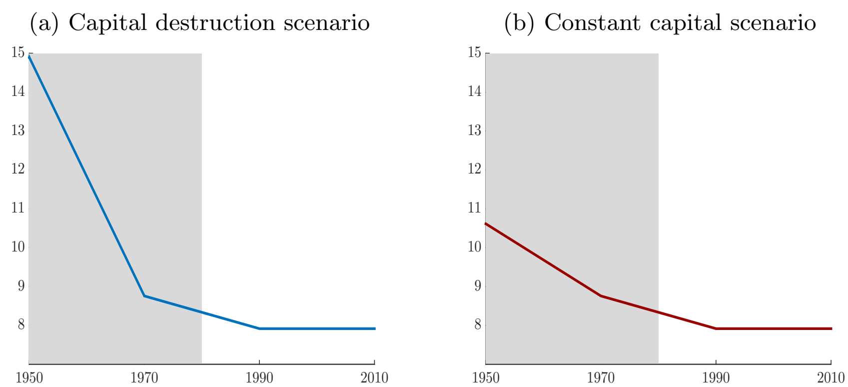
[71] I use the case of Japan since its demographic window started in 1960 and finished in 1996, and thus there is full data availability. See fig B.14(b) in appendix B.2.3

[72] For an exercise that replicates the steps followed in this explanation see figure B.15 in appendix B.2.3.

To analyze possible war effects on investment for the demographic window in the model, I consider the destruction and subsequent accumulation of capital. To do so, I assume that in the first period, the economy is below its balanced growth path. I perform this exercise in the open economy framework, in the same spirit as the productivity exercises, since the closed economy already generates increasing investment rates.

As an example of a European demographic transition I consider the demographic window of Great Britain (1920–1980).[73] This transition is introduced as a decreasing growth rate of newborns starting one period before 1950, since Great Britain was already in its window by then.[74] This growth rate stabilizes by 1990, which I take as the second steady state. I assume that in 1950 capital (p.e.u.l) was 50% below the steady state-level and that it recovered by 1970, after twenty years (one period in this model) and therefore I do not include adjustment costs.

Figure 3.7: **Investment rates, WWII exercise**



Notes: This figure plots the investment rate through this transition in a frictionless open economy under two scenarios. The first one is the WWII case, where capital (p.e.u.l) in 1950 is 50% below the steady-state level. The second one is a "constant capital scenario" where capital destruction is not taken into account.

The result, shown in figure 3.7, is that the initial investment rate is higher (compared to a scenario without capital destruction) in the first period, and afterwards it decreases towards the long-run equilibrium. Therefore, even considering capital destruction and

[73] I exclude Scottland and use census data of England and Wales because of availability data, and also because their transition exhibits the general characteristics discussed in section 3.2, see figure B.16 in appendix B.2.3. According to McDaniel and Zimmer (2013) its transition "was triggered by a rapid drop in fertility in the early 1900s".

[74] I considered the period of 1930-1990. I used the observed values of 1931, 1971, and 1991. For 1951 I used the average of 1931 and 1971 because of missing data.

reconstruction, in an open economy, investment rates decrease in the demographic window. According to the model, therefore, the increase in investment for the demographic window in Europe was not a result of the postwar capital destruction and its subsequent reconstruction in an open economy framework.

Mechanically, in the first period capital is below the steady state and it increases back to the open economy level in one period. This is the reason why the initial investment rate is high.[75] From the second period onwards, capital has recovered and is given by the after-tax foreign interest rate, independently of demographic changes, so investment rates decrease with the effective labor force. The assumption behind this result is that capital (p.e.u.l) recovers to prewar levels by 1970.

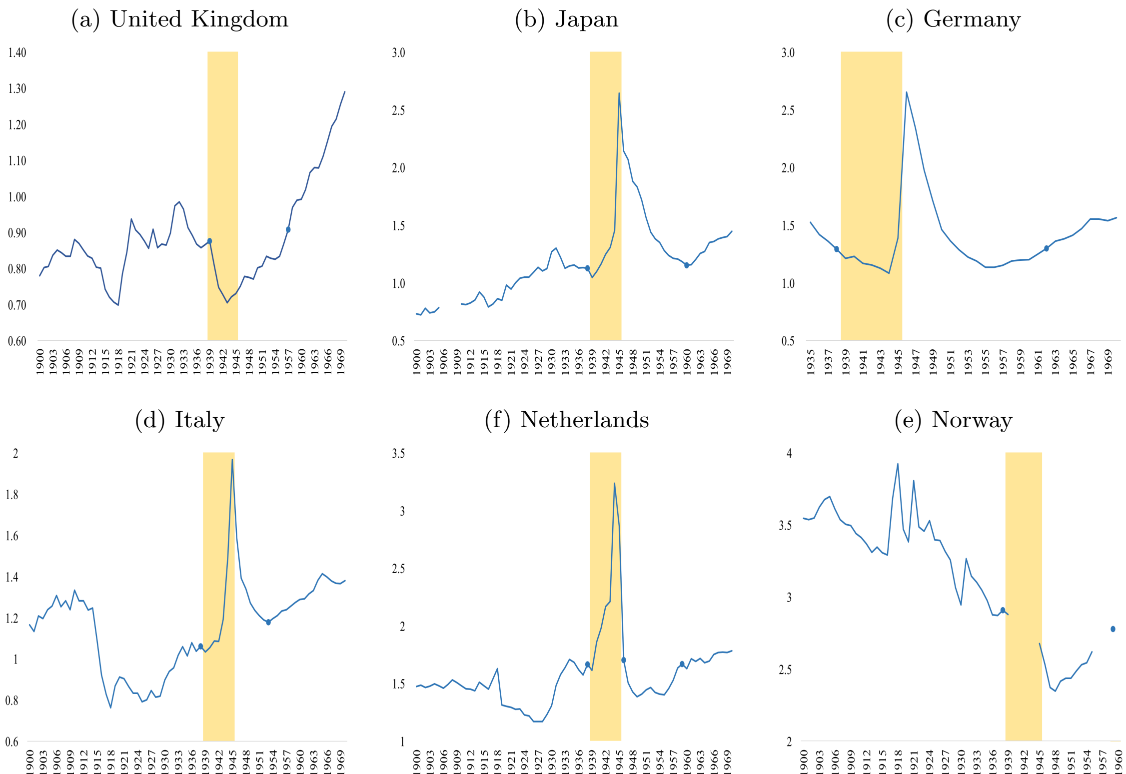
In figure 3.8, I plot estimates of the non-residential capital stock to output ratio of some European countries and Japan for the years before, during, and after WWII.[76] Notice that this ratio returns to pre-war (1938) levels by the late 1950s and early 1960s.[77] For example, in Germany, one of the most affected countries, the ratio recovers by 1962. This supports the assumption of capital (p.e.u.l) recovering at latest by 1970.

[75] We can see this directly from the investment equation (3.24) where $k_t = 0.5k_{t+1}$, $k_{t+1}(r^w, \tau)$, $k_t < k_{t+1}$.

[76] For a longer time series and a comparison with the United States, see figure B.18 in appendix B.2.3.

[77] Total non-residential capital stock recovers even faster (see figure B.17 in appendix B.2.3).

Figure 3.8: **Capital stock to output ratio and WWII**



Notes: This figure plots capital to output ratios. The shaded area indicates the years of WWII. This ratio is the estimated non residential capital stock divided by GDP. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the United Kingdom, Japan, Germany, it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Level are in 1990 international dollars, except for Italy's, which is in 2011 international dollars of 2011 because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively.

3.5.3 Sensitivity Analysis

In this section, I discuss the sensitivity of the results of the model to the selected parametrization used in the numerical exercises. Table 3.3 shows the parameter values chosen and their source or target. For the sensitivity analysis I focus on two parameters from the households' preferences, since the rest of the parameters are standard. I first analyze the benevolence of parents towards their children and then, the intertemporal elasticity of substitution.

Table 3.3: **Baseline parameter values**

Parameter	Value	Source/Target
β	$0.96^{\dagger}$ *	Annual return on risk-free bonds
σ	2	Reciprocal of IES: Conesa and Garriga (2008)
γ	0.5	OECD equivalence scale
α	0.3	Bergoeing et al. (2002)
δ	$0.05^{\dagger}$ *	Consumption of Fixed Capital/GDP
τ_1 / τ_2	20% / 5%	Open economy
τ_1 / τ_2	170% / 7%	Closed economy**

Notes: † The values of β and δ are the 20 year equivalent of the annual values of 0.96 and 0.05, respectively.

* The value of δ of Mexico was calibrated to match the average CFK/GDP ratio of the period 1970-2013.

** The minimum tax τ_1 to close the economy is 160%.

Benevolence Towards Children

In this section I analyze the effect of a decrease in the valuation of children in their parents' utility. When this valuation decreases, savings rates increase, which in the closed economy increases investment rates and in the open economy leaves them unchanged.

Households care about the utility of their children and because of that, during the first period of their working life they feed their offspring. Given the utility specification,[78] households equalize the marginal utility of consumption of the first period of their working life to the marginal utility of each of their children weighted by the parameter γ.[79]

This implies that as young workers they dissave for their own consumption, during that period, and for the consumption of their children. For each unit of consumption in the first period of their working life, this young workers set aside $\gamma^{\frac{1}{\sigma}}\phi_{t-1}$ units for their children.[80] Therefore, a decrease in the valuation of children increases savings rates for two reasons. First, it decreases dissavings of young workers. Second, it increases savings rates in general by freeing up resources for self-spending which are distributed across periods, via savings.

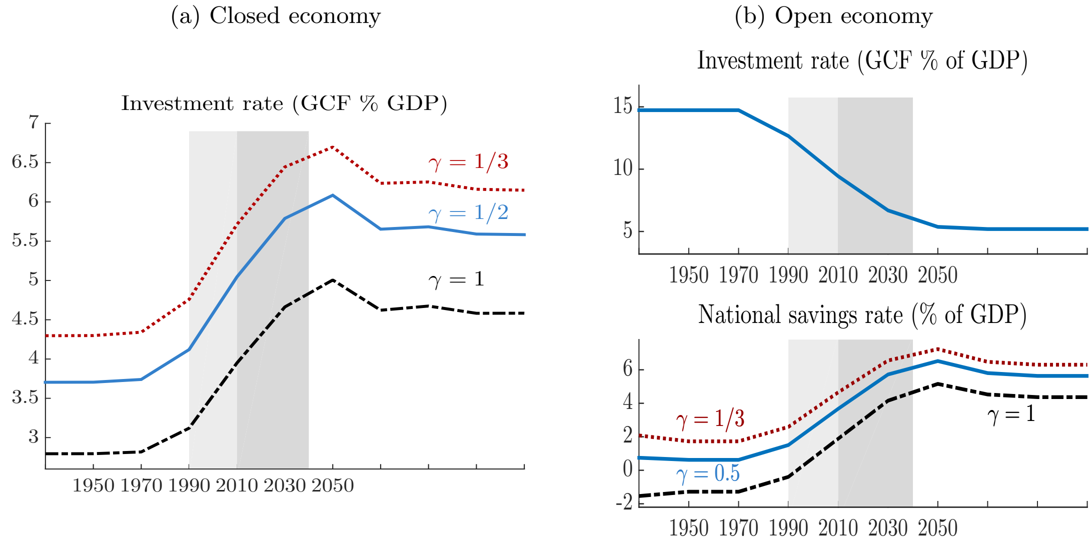
[78] $U^{t-1} = u\left(c_t^{t-1}\right) + \gamma\phi_{t-1}u\left(c_t^t\right) + \beta u\left(c_{t+1}^{t-1}\right) + \beta^2 u\left(c_{t+2}^{t-1}\right)$

[79] First order condition $u'(c_t^{t-1}) = \gamma u'(c_t^t)$

[80] See consumption equation B.2 in appendix B.2.3.

In the closed economy the increase in savings rates generates an increase in investment rates, in the open economy it does not and investment rates remain unchanged. Figure 3.9 provides an illustration using the benchmark numerical exercise.

Figure 3.9: **Sensitivity to the benevolence towards children γ**



Notes: This figure plots investment and savings rates through the Mexican demographic transition for different values of γ. The results of the closed economy are depicted in panel (a) and the results of the open economy in panel (b). The solid line shows the benchmark exercise of figure 3.3 and the dotted line shows a lower value and it illustrates the explanation provided above. The dashed line shows a higher (γ) value.

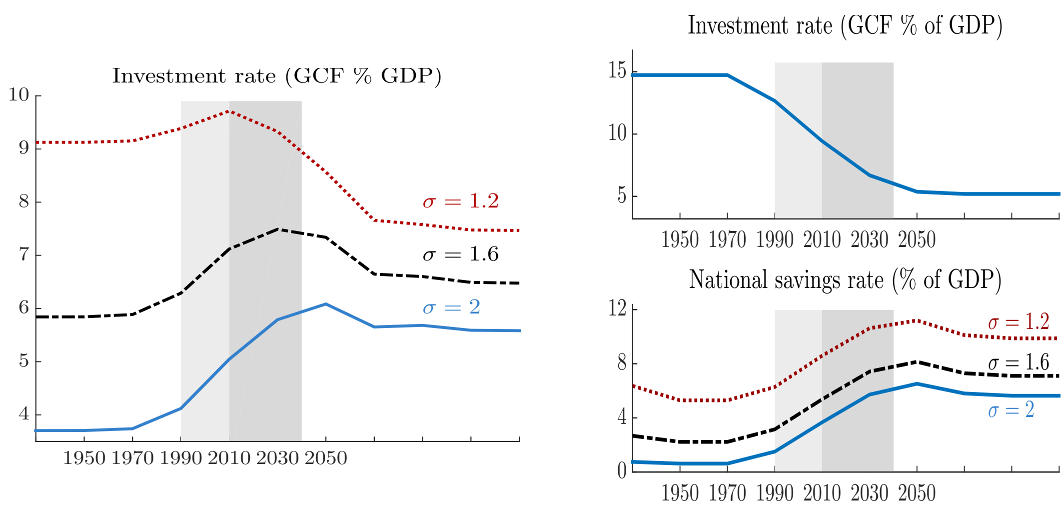
Elasticity of Intertemporal Substitution

In this section I discuss the effects of increasing the elasticity of intertemporal substitution (EIS). There are two effects: First, a higher EIS increases savings rate levels. Second, if there are general equilibrium price effects, a higher EIS reduces the effect of the demographic window: it reduces the increase in investment rates that happens before and during the window. In the open economy only the first effect is observed in savings rates, while in the closed economy both effects impact savings rates and therefore investment rates. I begin by discussing the open economy.

The specification of the utility function (CRRA) implies that an increase in the EIS is equivalent to a decrease in σ. In the open economy, a lower σ shifts up the savings rate. We can see this in equations B.3 and B.4 or in the numerical exercises of figure 3.10(b). A a lower σ increases the effective discount factor $(\beta^{\frac{t}{\sigma}}(1+r)^{\frac{t}{\sigma}-1})$, which increases savings rate levels. Note that since there are no general equilibrium price effects in the open economy, the dynamics of savings rates remain unaffected throughout the demographic transition. These changes do not affect investment rates.

Figure 3.10: **Sensitivity to the intertemporal elasticity of substitution** σ

(a) Closed economy (b) Open economy



Notes: This figure plots investment and savings rates through the Mexican demographic transition for different σ values. Panels (a) and (b) show the results of the closed and open economies. The solid line shows the benchmark exercise of figure 3.3, and the dotted and dashed lines show lower values.

In the closed economy, in addition to the increase in levels, a higher σ lowers the effect of the demographic window. The increase in savings rates that happens before and during the demographic window is bigger the lower the σ, because during the transition capital (p.e.u.l) and therefore labor income increases. The bigger the σ, the more important the response of savings to income effects as opposed to substitution effects. Thus lower values of σ decrease the response of savings to the increasing labor income profile generated by the demographic window. Investment rates decrease and the effect of the demographic window decreases as well (see figure 3.10(a)).

3.5.4 The Case of Brazil

In this section I analyze the case of Brazil, one of the biggest countries in Latin America. I compare the results of the benchmark model to the investment rates observed in Brazil, in the 15 years before and during the demographic window, and before this period.

Table 3.4 shows the results of the open frictionless benchmark model versus the data.

Table 3.4: **Investment rates (%): the model and the data**

(a) Brazil: model

period	open
1st ss	16.96
DW-15	13.58
DW	9.18
2nd ss	6.5
Decrease of inv. for DW_1	**10.42**

(c) Brazil: data

period	inv. rate
1970–1984	24.03
1985–1999	19.79
2000–2015	18.07
5.96	

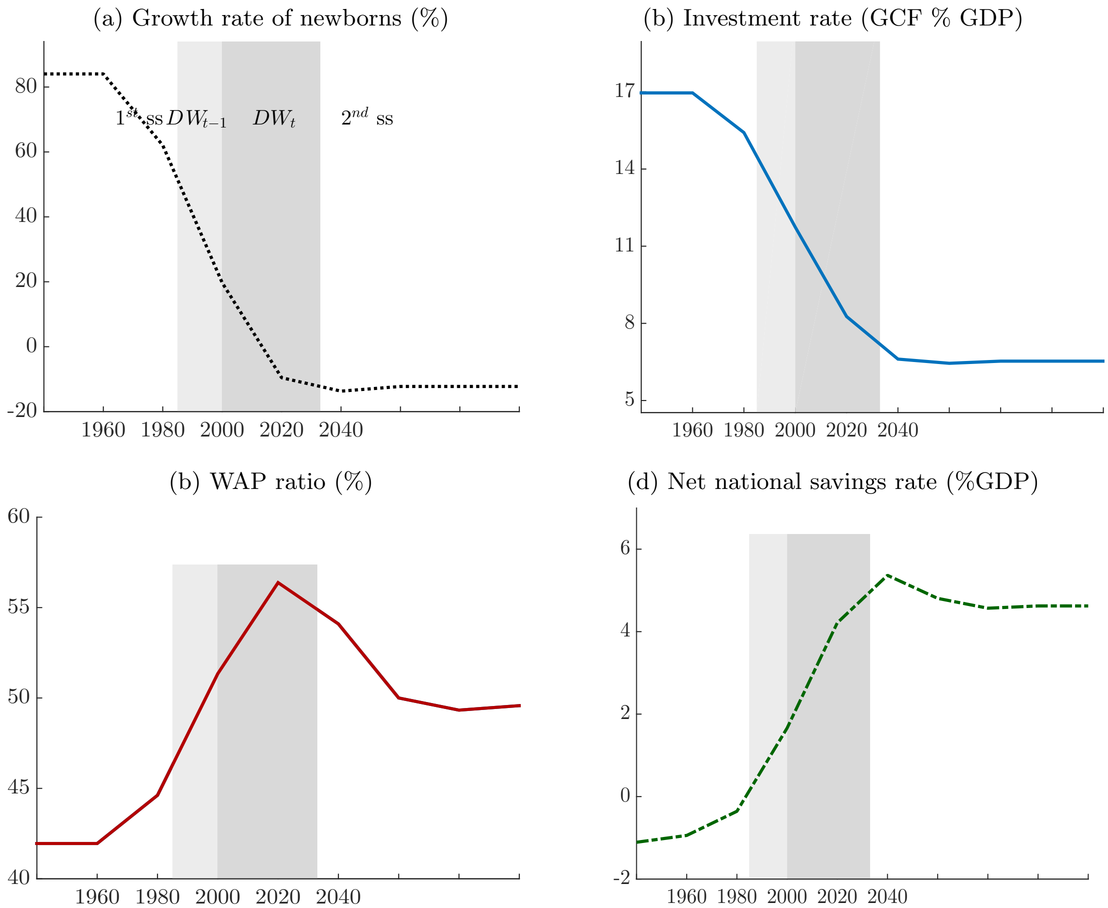
This table shows the investment rates generated in the open frictionless economy model (benchmark exercise of Brazil) in panel (a), and the observed investment rates of Brazil in panel (b). The first row is the period before the demographic transition: in the model it is the first steady state, for Brazil it is the period of 1970 to 1984.[81] The second row (DW-15) reports the investment rate of the 15 years before the demographic window: in the model it is the average of the years of 2000 and 2040 first and the last year of that period, and in the data is the average of the period indicated in the first column. The third row (DW) reports the investment rate of the demographic window up to 2014: in the model it is the average of the years of 2000 and 2040, and in the data[82] the average of the period indicated in the first column. The last row "Decrease of investment for the DW" reports the decrease of investment rates from the period before the demographic transition up the first years of a demographic window.

In Brazil, investment rates decreased from an average of 24% in the 1970s and early 1980s, to an average of 20% 15 years before the demographic window, and an average of 18% up to this point in their window (see table 3.4). This pattern of decreasing investment rates for the demographic window is consistent with the open economy result. The fact that the model implies a greater decline in investment rates than the data is suggestive of the potential of this model in explaining the documented decrease of investment for the demographic window in Latin America.

[81] The choice of the start year is to be consistent with the other periods in the table. Note that for Brazil there is no liberalization process in the 70s, which emphasizes the need for incorporating a time varying degree of financial openness in the model. I elaborate on this in the conclusions.

[82] Notes: investment rate is the ratio of GCF to GDP using data from the IMF, for Brazil because of data availability, the years of 2012 to 2015 were computed using the growth rate of GCF of the WB indicators.

Figure 3.11: **Benchmark exercise of Brazil in the open economy**



Notes: This figure plots the Brazilian demographic transition i.e., the reduction in the growth rates of newborns in panel (a) and the demographic window that this generates in panel (c). The dark gray area indicates the years of the demographic window (2000-2033) and the light gray area, the 15 years before it (1985-1999). In panels (b) and (d) I plot the investment and savings rates throughout this transition.

In the benchmark model of Brazil I consider 1960 as the first steady state and 2040 as the second one. I introduce the demographic transition as the decreasing growth rate of newborns (adjusted for mortality). Analogous to the benchmark exercise of Mexico, I use the average growth rate of TFP between 1968 and 2011. Figure 3.11 shows the result in the fully-open economy framework.

3.6 Conclusions

In this paper, I have documented that different regions display different patterns of investment before and during a demographic window. In particular, Latin America displays a different behavior in that investment rates are lower 15 years before and during their demographic window than outside of this period, whereas in Europe, Asia, and Oceania these rates are higher. Additionally, I documented a negative relationship between investment for the demographic window and the degree of financial openness of the economies, when financial openness is measured by the Chinn-Ito index. This fact indicates that financial openness is an important consideration for the relationship between investment and the demographic window.

Motivated by this finding I developed an OLG model with a variable degree of financial openness and demographic change that can rationalize the documented pattern of investment. The main result of the model is that in open economies investment rates decrease in preparation and during the demographic, window while in closed economies they increase.

The demographic windows that I study in this paper, which are part of the long-run worldwide demographic change, are triggered by a reduction in the number of newborns and characterized by a temporary increase in the working age- to total population-ratio. There are three important effects associated with these demographic window: a reduction in child dependency, an increase in the population share of middle-aged savers, and a decreasing growth rate of the labor force.

These effects generate opposite changes in investment rates depending on the level of financial openness of each economy. In closed economies, the reduction in child dependency and the increase in the share of middle-aged savers increases savings rates and therefore investment rates. The reduction in child dependency increases available income for self-spending and decreases dissavings of young workers associated with the consumption of their children. In open economies, since capital (p.e.u.l) is pinned down by the net return on international assets, and this is a growth path model, investment rates grow with the growth rate of the effective labor force, which is decreasing in a demographic window.

This emphasizes the importance of the fact that before and during a demographic window, the growth rate of the working age-population is not increasing, even though the fraction and the total working age-population might be so. The growth rate of the working age-population is decreasing since each year there are fewer children that will be part of the future working-age population, and this translates into a decreasing growth rate of the labor force. Therefore even though dependency is lower during a demographic window and there are many workers to invest in, investment as a fraction of the GDP does not necessarily need to increase. What matters for investment rates is the growth rate of the labor force, not the level, and this growth rate is decreasing.

In the model, I also explored alternative variables and hypotheses that could generate different responses of investment for the demographic window. In particular, I considered the decrease in the rate of return of international assets, both short- and long-run changes in the growth of productivity, and the effects of capital depletion and subsequent accumulation after WWII. The conclusion from these exercises is that these factors cannot undo the main result of openness nor they can generate on their own results in line with the documented patterns in investment.

The model proposed in this paper is consistent with the documented facts for Latin America, Europe, Asia, and Oceania. In particular, the fact that Europe and some Asian countries were less financially open during their demographic windows can help to explain why they have higher investment rates in those period. Similarly, being more financially open during the ongoing the demographic window in Latin America can explain why investment rates are decreasing.

The framework developed in this paper can be extended along a number of dimensions. In terms of future work, the first item in my research agenda is to perform a full calibration in order to more fully assess the model's ability to account for the changes in investment rates associated with a demographic window.[83] This direction looks promising. In the preliminary exercise I conducted to compare the performance of the model to the data, I find that the observed decrease of investment before and during the demographic window of Brazil is 6 percentage points and 10.4 percentage points in the model of the fully open benchmark economy. The observed pattern observed of decreasing investment rates for the demographic window is consistent with

[83] In the model, the parameter that was calibrated to target data moments is the depreciation rate. Taxes were chosen to simulate open and closed economies and to make the point that the results are similar to those of the fully-open and fully-closed economies.

the open economy result, and the exercise is suggestive of the potential of the model in explaining the documented decrease of investment for the demographic window in Latin America.

Related to this, an exercise where the degree of openness changes over time is also necessary to compare the predictions of the model with the data, specially for countries with demographic windows starting in the early 2000s, and in order to achieve a more realistic approach regarding the classification of an economy as open or closed. Finally, it is left to calculate the welfare to make policy recommendations.

In terms of the assumptions of the model, in this paper I take the reduction in fertility rates as exogenous, from the data, to generate a demographic window. An alternative to this approach is to model the developing process, where countries undergo through the demographic change as income increases and endogenize fertility choices. Additionally, I assume that there is no early death and adjust the growth rate of newborns to the growth rate of working-age groups. This assumption can be relaxed by including a survival probability, and it would allow for the incorporation of the decrease in mortality of the demographic transition which is a determinant of the length of a demographic window.

Another assumption is that young and middle-aged workers are equally productive. The demographic framework of the model —having two different cohorts of workers— allows for the incorporation income profiles. For countries with hump-shaped income profiles, this can be modeled as a higher efficiency of the middle-aged workers to young workers. Since in a demographic window the bulk of the labor force moves to the middle-aged workers, this will be an additional source of increase in savings. This extension could be an important determinant of patterns of investment across regions since as some authors like Lagakos et al. (2015), have documented experience-wage profiles are steeper in more developed countries than in less developed ones.

References

ATTANASIO, O. P., S. KITAO, AND G. L. VIOLANTE (2006): "Quantifying the effects of the demographic transition in developing economies," *Advances in Macroeconomics*, 6, 1–44.

ATTANASIO, O. P. AND G. L. VIOLANTE (2000): "The demographic transition in closed and open economy: A tale of two regions," Tech. rep.

BACKUS, D. K., P. J. KEHOE, AND F. E. KYDLAND (1992): "International real business cycles," *Journal of political Economy*, 745–775.

BAFFIGI, A. (2011): *Italian National Accounts 1861-2011*, Quaderni di storia economica, Banca d'Italia.

BAI, Y. AND J. ZHANG (2010): "Solving the Feldstein–Horioka puzzle with financial frictions," *Econometrica*, 78, 603–632.

BARDHAN, P. (1967): "optimum foreign borrowing," in *Essays in the theory of optimal economic growth*, ed. by K. Shell, MIT Press.

BENCIVENGA, V. R. AND B. D. SMITH (1991): "Financial intermediation and endogenous growth," *The Review of Economic Studies*, 58, 195–209.

BERGOEING, R., P. J. KEHOE, T. J. KEHOE, AND R. SOTO (2002): "A decade lost and found: Mexico and Chile in the 1980s," *Review of Economic Dynamics*, 5, 166–205.

BLOOM, D. E., D. CANNING, AND J. SEVILLA (2001): "Economic growth and the demographic transition," Tech. rep., National Bureau of Economic Research.

BLOOM, D. E. AND J. G. WILLIAMSON (1998): "Demographic transitions and economic miracles in emerging Asia," *The World Bank Economic Review*, 12, 419–455.

BODVARSSON, O. B., N. B. SIMPSON, AND C. SPARBER (2015): "Chapter 1 - Migration theory*," in *Handbook of the Economics of International Migration*, ed. by B. R. Chiswick and P. W. Miller, North-Holland, vol. 1 of *Handbook of the Economics of International Migration*, 3 – 51.

BOLT, J. AND J. L. ZANDEN (2014): "The Maddison Project: collaborative research on historical national accounts," *Economic History Review*, 67, 627–651.

BORJAS, G. J. (1995): "The economic benefits from immigration," *Journal of Economic Perspectives*, 9, 3–22.

BRUNO, M. (1970): "Trade, growth and capital," Working papers 65, Massachusetts Institute of Technology (MIT), Department of Economics.

CASS, D. (1965): "Optimum growth in an aggregative model of capital accumulation," *The Review of Economic Studies*, 32, 233–240.

CHINN, M. D. AND H. ITO (2008): "A new measure of financial openness," *Journal of comparative policy analysis*, 10, 309–322.

CHIPMAN, J. S. (1970): "External Economies of Scale and Competitive Equilibrium," *The Quarterly Journal of Economics, MIT Press*, 84, 347–85.

CHUMA, A. C. (2015): "Investing in youth and ensuring decent jobs to harness the demographic dividend," Remarks by Mr. Aeneas C. Chuma. ILO. Regional Director for Africa at a Side on Event Investing in Youth and Ensuring Decent Jobs to Harness the Demographic Dividend on the Occasion of the First-Ever World Youth Skills Day, during the Third International Conference on Financing for Development.

COALE, A. AND E. HOOVER (1958): *Population growth and economic development in low-income countries*, Princetion, New Jersey: Princeton University Press.

COALE, A. J. (1972): "How a population ages or grows younger," Aus: Population. The vital revolution. 1964c.

Conesa, J. C. and C. Garriga (2008): "Optimal fiscal policy in the design of social security reforms," *International Economic Review*, 49, 291–318.

Curcuru, S. E., T. Dvorak, and F. E. Warnock (2008): "Cross-border returns differentials," Tech. rep., National Bureau of Economic Research.

de Carvalho, J. A. M. (1997): "Demographic dynamics in Brazil: recent trends and perspectives," *Brazilian journal of population studies*, 5–23.

Deaton, A. (1997): *The Analysis of Household Surveys: A Microeconometric Approach to Development Policy*, A World Bank pulication, World Bank.

Deaton, A. and C. Paxson (1998): "Growth, demographic structure, and national saving in Taiwan," Papers 183, Princeton, Woodrow Wilson School - Development Studies.

Devereux, M. B. and G. W. Smith (1994): "International risk sharing and economic growth," *International Economic Review*, 35, 535–550.

Domeij, D. and M. Floden (2006): "Population aging and international capital flows," *International Economic Review*, 47, 1013–1032.

Dustmann, C. and T. Frattini (2012): "Immigration: The European Experience," Norface Discussion Paper Series 2012001, Norface Research Programme on Migration, Department of Economics, University College London.

Feldstein, M. and C. Horioka (1980): "Domestic savings and international capital flows," *Economic Journal*, 90, 314–29.

Fernandez, A., M. Klein, A. Rebucci, M. Schindler, and M. Uribe (2015): *Capital Control Measures: A New Dataset*, IMF Working Papers, INTERNATIONAL MONETARY FUND.

French, K. R. and J. M. Poterba (1991): "Investor diversification and international equity markets," *The American Economic Review*, 81, 222–226.

Fry, M. J. and A. Mason (1982): "The variable rate-of-growth effect in the life-cycle saving model ," *Economic Inquiry*, 20, 426–442.

Goos, M., A. Manning, and A. Salomons (2014): "Explaining Job Polarization: Routine-Biased Technological Change and Offshoring," *American Economic Review*, 104, 2509–26.

Gourinchas, P.-O. and O. Jeanne (2013): "Capital flows to developing countries: The allocation puzzle," *The Review of Economic Studies*, rdt004.

Gourinchas, P.-O. and H. Rey (2014): "Chapter 10 - External Adjustment, Global Imbalances, Valuation Effects," in *Handbook of International Economics*, ed. by E. H. Gita Gopinath and K. Rogoff, Elsevier, vol. 4 of *Handbook of International Economics*, 585 – 645.

Groote, P., R. Albers, and H. d. Jong (1996): "A standardised time series of the stock of fixed capital in the Netherlands, 1900-1995," GGDC Research Memorandum 199625, Groningen Growth and Development Centre, University of Groningen.

Grubel, H. G. (1968): "Internationally diversified portfolios: welfare gains and capital flows," *The American Economic Review*, 58, 1299–1314.

Guzmán, J. M., J. Rodríguez, J. Martínez, J. M. Contreras, and D. González (2006): "The Demography of Latin America and the Caribbean since 1950/La Demografia de América latina y del Caribe Desde 1950/La démographie de l'Amérique Latine et des Caraïbes dequis 1950," *Population (english edition)*, 519–620.

Hagenaars, A., K. de Vos, M. Zaidi, and S. O. of the European Communities (1994): *poverty statistics in the late 1980s: research based on micro-data*, Eurostat: Series C, Accounts, surveys and statistics, Office for Official Publications of the European Communities.

Hamada, K. (1969): "Optimal capital accumulation by an economy facing an international capital market," *Journal of Political Economy*, 77, 684–697.

Haupt, A. and S. Uebelmesser (2010): "Integration, Mobility, and Human Capital Formation," CESifo Working Paper Series 3190, CESifo Group Munich.

Heckscher, E. (1919): "The effect of foreign trade on the distribution of national income," *Ekonomisk Tidskrift*.

Henriksen, E. (2002): "A demographic explanation of US and Japanese current account behavior," *Unpublished manuscript, Carnegie Mellon University*, 1–30.

ILO (1990): "ISCO-88: International Standard Classification of Occupations," Working papers, ILO Geneve.

John C. Cox, Jonathan E. Ingersoll, S. A. R. (1985): "An intertemporal general equilibrium model of asset prices," *Econometrica*, 53, 363–384.

Julliard, C. (2002): "The international diversification puzzle is not worse than you think," .

Karahan, F., B. Pugsley, and A. Sahin (2015): "Understanding the 30-year decline in the startup rate: A general equilibrium approach," *Unpublished manuscript, May.*

Kelley, A. C. and R. M. Schmidt (1996): "Saving, dependency and development," *Journal of Population Economics*, 9, 365–386.

Koepke, R. (2015): "What drives capital flows to emerging markets? A survey of the empirical literature," MPRA Paper 62770, University Library of Munich, Germany.

Krueger, D. and A. Ludwig (2007): "On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare," *Journal of monetary Economics*, 54, 49–87.

Krugman, P. (1991): "Increasing returns and economic geography," *Journal of Political Economy, University of Chicago Press*, 99, 483–99.

Lagakos, D., B. Moll, T. Porzio, N. Qian, and T. Schoellman (2015): "Lifecycle wage growth across countries," *Unpublished Manuscript, Princeton University.*

Lane, P. R. and G. M. Milesi-Ferretti (2002): "Long-term capital movements," in *NBER Macroeconomics Annual 2001, Volume 16*, MIT Press, 73–136.

——— (2007): "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 19702004," *Journal of International Economics*, 73, 223 – 250.

Lee, E. S. (1966): "A Theory of Migration," *Demography*, 3, 47–57.

Lee, R. (2003): "The demographic transition: three centuries of fundamental change," *Journal of Economic Perspectives*, 17, 167–190.

Lee, R. and A. Mason (2006): "What is the demographic dividend?" *Finance and Development*, 43, 5.

Leff, N. H. (1969): "Dependency rates and savings rates," *The American Economic Review*, 59, 886–896.

Lerner, A. P. (1952): "Factor prices and international trade," *Economica*, 19, 1–15.

Lopez-Real, J. (2011): "A Macroeconomic Analysis on Immigrants' Self-Selection and its Implications," Working papers, Universitat Autonoma de Barcelona.

Lucas, R. E. (1990): "Why doesn't capital flow from rich to poor countries?" *The American Economic Review*, 80, 92–96.

Maddison, A. (1994): "Standardised estimates of fixed capital stock: a six country comparison," GGDC Research Memorandum 199409, Groningen Growth and Development Centre, University of Groningen.

Malthus, T. R. (1798): *An essay on the principle of population as it affects the future improvement of society [microform] : with remarks on the speculations of Mr. Godwin, M. Condorcet and other writers*, J. Johnson London.

Mason, A. (1981): "An extension of the life-cycle model and its application to population growth and aggregate saving." .

——— (1988): "Saving, economic growth, and demographic change," *Population and Development Review*, 14, 113–144.

——— (2001): *Population change and economic development in East Asia: Challenges met, opportunities seized*, Stanford University Press.

McCallum, J. (1995): "National borders matter: Canada-US regional trade patterns," *The American Economic Review*, 85, 615–623.

McDaniel, S. and Z. Zimmer (2013): *Global ageing in the twenty-first century: Challenges, opportunities and implications*, Ashgate Publishing Ltd.

Meade, J. E. (1951): "The balance of payments," *Oxford University PRess, London.*

Mejía, A. (1999): *Large capital flows: A survey of the causes, consequences, and policy responses*, IMF Working Papers, International Monetary Fund.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull (2007): "Financial integration, financial deepness and global imbalances," Tech. rep., National Bureau of Economic Research.

Modigliani, F. (1966): "The life cycle hypothesis of saving, the demand for wealth and the supply of capital," *Social Research*, 33, 160–217.

Mundell, R. A. (1957): "International trade and factor mobility," *The American Economic Review*, 47, 321–335.

Notestein, F. (1945): "Population: The long View," in *Food for the world*, ed. by T. W. Schultz, Chicago: University of Chicago Press, 36–57.

Obstfeld, M. (1994): "Risk-taking, global diversification, and growth," *The American Economic Review*, 84, 1310–1329.

Obstfeld, M. and K. Rogoff (1995): "The intertemporal approach to the current account," in *Handbook of International Economics*, ed. by G. M. Grossman and K. Rogoff, Elsevier, vol. 3 of *Handbook of International Economics*, chap. 34, 1731–1799.

——— (2001): "The six major puzzles in international macroeconomics: is there a common cause?" in *NBER Macroeconomics Annual 2000, Volume 15*, MIT press, 339–412.

OECD (1982): *The OECD list of social indicators*, Organisation for Economic Cooperation and Development Paris.

——— (2009): "What are equivalence Scales?" OECD Project on Income Distribution and Poverty, via www.oecd.org/social/inequality.htm.

——— (2012): "Free Movement of Workers and Labour Market Adjustment: Recent Experiences from OECD Countries and the European Union," Tech. rep., OECD Publishing.

Olken, B. A. (2005): "Revealed community equivalence scales," *Journal of Public Economics*, 89, 545–566.

Quinn, D. (1997): "The Correlates of Change in International Financial Regulation," *The American Political Science Review*, 91, 531–551.

Ramsey, F. P. (1928): "A mathematical theory of saving," *The Economic Journal*, 38, 543–559.

Ravenstein, E. G. (1885): "The laws of migration," *Journal of the Statistical Society of London*, 48, 167–235.

——— (1889): "The laws of migration," *Journal of the Royal Statistical Society*, 52, 241–305.

Ríos-Rull, J.-V. (2001): "Population changes and capital accumulation: The aging of the baby boom," *Advances in Macroeconomics*, 1.

Roy, A. D. (1951): "Some thoughts on the distribution of earnings," *Oxford Economic Papers*, 135–146.

Ruffin, R. J. (1984): "Chapter 5 International factor movements," Elsevier, vol. 1 of *Handbook of International Economics*, 237 – 288.

Saad, P. (2011): "Demographic trends in Latin America and the Caribbean," *Population aging: is latin America ready*, 43–77.

Samuelson, P. (1968): "International factor price equalization once again," *Economic Journal*, 58, 163–184.

Schularick, M. (2016): "International capital flows," *Oxford Handbooks Online*.

SJAASTAD, L. A. (1962): "The costs and returns of human migration," *Journal of Political Economy*, 70, 80–93.

SMITH, A. (1776): *An inquiry into the nature and causes of the wealth of nations*, no. smith1776 in History of Economic Thought Books, McMaster University Archive for the History of Economic Thought.

SOLOW, R. M. (1956): "A contribution to the theory of economic growth," *The Quarterly Journal of Economics*, 70, 65–94.

STOUFFER, S. A. (1940): "Intervening opportunities: A theory relating mobility and distance," *American Sociological Review*, 5, 845–867.

TAYLOR, A. AND J. WILLIAMSON (1994): "Capital flows to the new world as an Iitergenerational transfer," *Journal of Political Economy*, 102, 348–371.

THE-CONFERENCE-BOARD (2016): "The conference board total economy database™," May 2016, http://www.conference-board.org/data/economydatabase/.

THOMPSON, W. S. (1929): "Population," *American Journal of Sociology*, 34, 959–975.

TOH, M. H. (1997): *Savings, capital formation, and economic growth in Singapore*, East-West Center.

UNITED-NATIONS (2004): *World population to 2300*, Population Studies, United Nations. Dept. of Economic and Social Affairs. Population Division.

——— (2014): *State of world population report 2014 (F)*, United Nations Fund for Population Activities.

URRUTIA, C. (2001): "On the self-selection of immigrants," Working papers, Universidad Carlos III de Madrid.

VAN DER VEN, R. AND J. SMITS (2011): "The Demographic Window of Opportunity: age structure and sub-national economic growth in developing countries," .

ZIPF, G. K. (1946): "The P_1P_2/D hypothesis: on the intercity movement of persons," *American Sociological Review*, 11, 677–686.

Appendix A

Appendix to Chapter 1

A.1 Solving for the Equilibrium

System of equations that characterize equilibria in case 1:

$$
\begin{aligned}
H_N^1 &= \frac{w_H^1 - Bp_z\left(H_N^1, H_M^2, H_N^2\right)}{\bar{\theta}_1} \\
H_N^2 &= \frac{w_H^2 - Bp_z\left(H_N^1, H_M^2, H_N^2\right)}{\bar{\theta}_2}(1-\gamma) \\
H_M^2 &= \frac{w_H^1 - Bp_z\left(H_N^1, H_M^2, H_N^2\right)}{\bar{\theta}_2}\gamma \\
p_z\left(H_N^1, H_M^2, H_N^2\right) &= \frac{1-\lambda}{\lambda}\frac{A\left(H_N^1 + H_M^2\right)\left(H_N^1 + H_M^2\right) + A\left(H_N^2\right)\left(H_N^2\right)}{B\left(2 - H_N^1 - H_M^2 - H_N^2\right)}
\end{aligned}
$$

which can be reduced to:

$$
\begin{aligned}
\bar{\theta}_1 H_N^1 &= A\left[\left(1 + \frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma\right)H_N^1\right] - \frac{1-\lambda}{\lambda}\frac{\left[\left(1+\frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma\right)H_N^1\right]w_H^1 + H_N^2 w_H^2}{\left(2 - \left(1+\frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma\right)H_N^1 - H_N^2\right)} \\
\frac{\bar{\theta}_2}{1-\gamma}H_N^2 &= A\left(H_N^2\right) - \frac{1-\lambda}{\lambda}\frac{\left[\left(1+\frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma\right)H_N^1\right]w_H^1 + H_N^2 w_H^2}{\left(2 - \left(1+\frac{\bar{\theta}_1}{\bar{\theta}_2}\gamma\right)H_N^1 - H_N^2\right)}
\end{aligned}
$$

System of two equations that characterize equilibria in case 2:

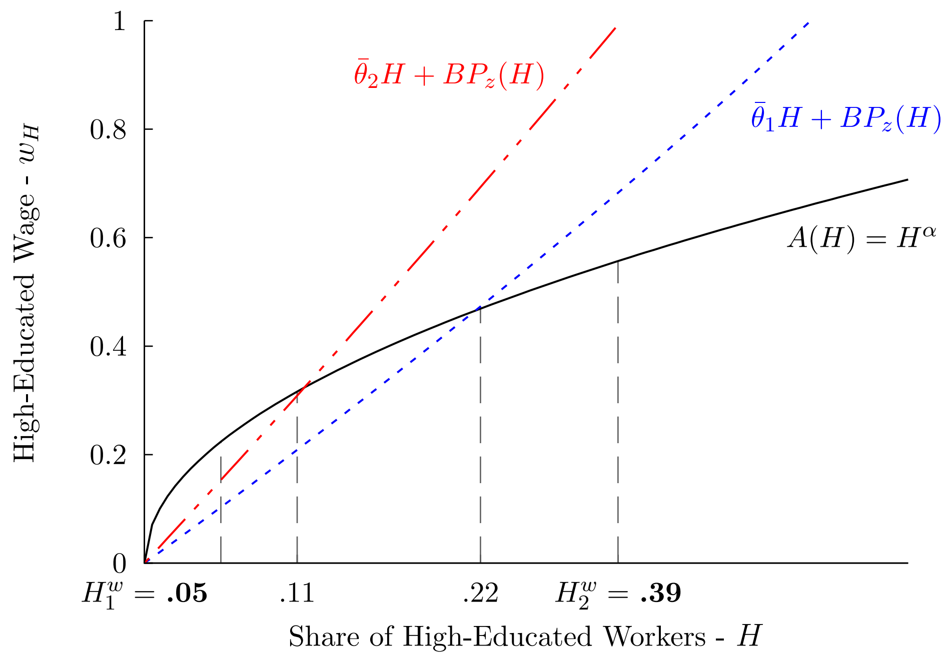
$$
\begin{aligned}
H_N^1 &= \frac{w_H^1 - Bp_z\left(H_N^1, H_M^1, H_N^2\right)}{\bar{\theta}_1}(1-\gamma) \\
H_N^2 &= \frac{w_H^2 - Bp_z\left(H_N^1, H_M^1, H_N^2\right)}{\bar{\theta}_2} \\
H_M^1 &= \frac{w_H^1 - Bp_z\left(H_N^1, H_M^1, H_N^2\right)}{\bar{\theta}_1}\gamma \\
p_z\left(H_N^1, H_M^1, H_N^2\right) &= \frac{1-\lambda}{\lambda}\frac{\left[A\left(H_N^1\right)\left(H_N^1\right) + A\left(H_N^2 + H_M^1\right)\left(H_N^2 + H_M^1\right)\right]}{B\left(2 - H_N^1 - H_M^1 - H_N^2\right)}
\end{aligned}
$$

We solved these systems using a Quasi-Newton fixed point algorithm. However, we found a solution only for a range of parameter values. This is a general feature of increasing returns to scale models that generate agglomeration and present multiplicity of equilibria. When the stock of workers of each type is exogenous, finding the set of parameter values for which there is one, multiple or no equilibria is easier. However, in the model of this paper the education choice makes the stock of HE workers endogenous which complicates the task. Further characterizing the equilibria is a priority in our research agenda.

A.2 Additional Graphs

Case 2

Figure A.1: **Specialization Patterns: Case 2**



A.3 Additional Tables

To provide a glimpse of changes in the European job and migration structure, columns 1 to 4 of Table A.1 show the employment shares of occupations, by migration status. Columns 5 to 8 show their percentage point changes between 1996 and 2010. We consider three migration categories: native-born ($Native$), born in a EU-15 country different to the one of current residency ($EU15$), and born outside the EU-15 bot working in one of our selected countries ($nonEU15$). We pool employment for each group and occupation across our 15 European countries.

Table A.1: **Summary Statistics Occupations**

	ISCO	Employment Share (2010)				$N_{2010} - N_{1996}$			
	code	Pop	Native	FB-EU15	FB-Rest	Pop	Native	FB-EU15	FB-Rest
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
High-Paying Occupations		38.16	39.74	41.64	23.50	17.58	21.82	46.47	-22.05
Corporate managers	12	4.80	4.98	6.74	2.73	8.79	12.79	36.73	-35.88
Physical, mathematical, and engineering professionals	21	4.14	4.29	4.84	2.61	51.04	59.24	52.93	-20.21
Life science and health professionals	22	2.41	2.46	2.58	1.97	9.08	12.54	52.60	-35.46
Other professionals	24	5.08	5.27	6.94	2.99	40.97	47.52	107.77	-18.27
Managers of small enterprises	13	4.70	4.73	5.71	4.20	-16.10	-16.42	16.19	-17.15
Physical and engineering associate professionals	31	4.41	4.65	4.37	2.38	15.70	20.74	13.68	-22.00
Other associate professionals	34	9.47	10.03	7.93	5.11	28.55	33.09	65.91	-1.17
Life science and health associate professionals	32	3.14	3.34	2.51	1.51	16.30	22.94	43.25	-42.30
Medium-Paying Occupations		33.81	34.20	31.19	31.20	-20.64	-20.05	-27.93	-21.24
Stationary and plant related operators	81	1.14	1.18	1.01	0.80	-19.64	-15.20	-51.45	-51.20
Metal, machinery and related trades workers	72	4.56	4.70	3.44	3.64	-26.08	-24.29	-47.88	-28.01
Drivers and mobile plant operators	83	4.59	4.60	3.20	4.88	-8.66	-9.03	-38.50	-0.18
Office clerks	41	9.71	10.30	7.64	5.19	-21.83	-18.79	-11.15	-42.21
Precision, handicraft, craft printing printing and related trade workers	73	0.38	0.38	0.44	0.36	-55.50	-56.22	-22.14	-48.90
Extraction and building trades workers	71	6.23	5.77	9.50	9.35	-10.77	-15.47	-25.21	17.12
Customer sevice clerks	42	2.42	2.50	2.25	1.82	-2.49	-1.14	58.57	-6.55
Machine operators and assemblers	82	2.99	2.99	2.47	3.12	-30.62	-29.01	-46.88	-46.14
Other craft and related trades workers	74	1.79	1.78	1.24	2.03	-39.12	-40.69	-14.95	-21.89
Low-Paying Occupations		28.03	26.06	27.17	45.30	12.37	5.92	-3.97	49.79
Laborers in mining, construction, manufacturing and transport	93	2.82	2.52	1.75	5.68	-12.84	-20.89	-35.69	39.61
Personal and protective service workers	51	11.45	11.09	10.50	14.81	18.11	14.10	15.60	56.72
Models, salespersons, and demonstrators	52	5.49	5.62	4.19	4.72	6.53	6.16	27.25	30.83
Sales and services elementary occupations	91	8.27	6.83	10.73	20.09	20.56	6.65	-18.70	53.16

Notes: Occupations are ordered by their mean wage across 10 European countries across all years, follwing wage information in Goos, Manning and Salomons (2014). Columns 1 to 4 contain employment shares by migrant status, pooled across countries. Columns 5 to 8 contain growth rates of employment shares from 1996-2010.

Table A.2: **ISCO-88 Major Groups and Skill Level**

	Major Group	ISCO Education Level
1	Legislators and Managers	4
2	Professionals	4
3	Technicians and Associate professionals	3
4	Clerks	2
5	Service and Sales	2
6	Skilled Agricultural and Fishery	2
7	Craft and Related	2
8	Plant and Machine Operators	2
9	Elementary Occupations	1

Appendix B

Appendix to Chapter 2

B.1 Data Analysis

Sample selection

The main sample has 59 countries with demographic transitions and includes data from the years of 1960 to 2013. I classify a country as having a *demographic transition* if it has data on investment and GDP during both (1) its demographic window (DW), and (2) the 15 years preceding the DW. Countries with less than 3 observations in either of these two periods were dropped from the sample. Additionally, countries with transitional economic systems according to IMF and WB classifications were excluded. African countries were also excluded since the demographic window of the continent is projected to begin in 2059. For Sections 3.2 and 3.5 I used observed data and projections of working-age population and dependency for the years of 1950-2050, and observed data and projections of labor force for the years of 1950-2020.

Investment rates

For the econometric analysis, investment rates are the trend component of the ratio of GCF to GDP. This ratio was HP-filtered using a smooth parameter of 6.25 corresponding to yearly frequency, and the cycle component was removed to consider long term variations of investment.[1]

[1]Using the non HP-filtered database, the results of the baseline specification (equation 3.1): coefficients and robust standard errors for dw and $dw \times LA$ are 4.23 (0.26)*** and -5.02 (0.47)*** respectively.

Different specifications of the model

Table B.1 reports the coefficient estimates of regressing investment rates on region and demographic window dummies for the econometric specification of equation 3.1 under different sets of controls, samples, and definitions of investment for the demographic window.

Table B.1: **Investment rates for the demographic window by region**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
DW	4.23†	5.25†	-1.17**	3.96†	2.36†	2.45†	3.37†	3.95†	3.89†	4.11†
	(0.26)	(0.31)	(0.48)	(0.26)	(0.34)	(0.34)	(0.83)	(0.26)	(0.23)	(0.27)
DW×LA	-5.02†	-6.30†			-2.65†		-3.97**	-4.34†	-5.86†	-4.86†
	(0.47)	(0.53)			(0.57)		(1.56)	(0.46)	(0.43)	(0.47)
LA		2.29†								
		(0.32)								
gr. GDP										0.14†
										(0.03)
Fixed effects										
Year	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
Sampling										
	N	N	Y	N	N	N	N	N	N	N
LatAm										
EAO	N	N	N	Y	N	Y	N	N	N	N
− 60-70s	N	N	N	N	Y	Y	N	N	N	N
+ Trans.	N	N	N	N	N	N	Y	N	N	N
For DW										
-15 years	Y	Y	Y	Y	Y	Y	Y	N	N	Y
-10 years	N	N	N	N	N	N	N	Y	N	N
-20 years	N	N	N	N	N	N	N	N	Y	N
Controls										
GDPpc	N	N	N	N	N	N	N	N	N	Y
N	2770	2770	1065	1705	1954	889	3180	2749	2824	2583

Robust s.e in parentheses. * $p < .10$, ** $p < .05$, † $p < .01$. EAO stands for Europe, Asia, and Oceania

Column 1 contains the estimates for the main sample and the baseline model. In column 2, I report the estimates of pooling the country-fixed effects by region on the dummy LA. Columns 3 and 4 contain the results of dividing the sample by region and performing a regression only on the variable DW. In columns 5 and 6, I drop the 1960s and the 1970s for Europe, Asia, and Oceania to consider possible effects of WWII on investment. In column 7, I include transitional economic systems. In columns 8 and 9, I use an alternative definition of the demographic window by defining DW investment as that which occurs during the window and the 10 years before it; in column 9 I do the same but for the DW and the 20 years before it. In column 10 I control for growth rates of constant GDP per capita.

Changes before and after the demographic window

I also analyzed changes in investment for the demographic window while distinguishing between the periods before and after the window. Regarding changes in investment before the *"lagged demographic window"* (the window and the 15 years before it), in Asia (excluding Japan) and Oceania, investment rates increased in preparation (15 years before) and during the window as compared to previous years.[2] In Latin America the opposite is true. This result is supported by dividing the sample by countries with data before and after the window and for Latin America, Asia, and Oceania estimating a regression of investment rates on a dummy for the period before the lagged demographic window $bdw_{c,t}$ and a dummy for region:

$$Inv_{c,t} = \beta_0 + \beta_1 \ bdw_{c,t} + \beta_2 \ bdw_{c,t} \cdot LA_c + f_c + f_t + \epsilon_{c,t}. \tag{B.1}$$

Table B.2:

Regression Estimates:

β_1 (*bdw*)	-3.83***	(0.53)
β_2 (*bdw* × LA)	5.36***	(0.53)
Observations	1996	

Robust s.e in parentheses *** $p < .01$

Results:

Hypothesis	value	P-val alternative
$\beta_1 < 0$	-3.83	0.00
$\beta_1 + \beta_2 > 0$	1.53	0.00

[2] I exclude Japan from this analysis since its demographic window started in 1960 and finished in 1996.

This estimation and hypothesis testing are reported in table B.2. The results are robust and hold under different specifications (see tables B.3 and B.5).

Table B.3: **Investment rates and the period before the lagged DW**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
b_dw	-3.83†	-6.04†	1.17**	-2.93†	-1.81†	-2.85†	-2.02†	-3.45†	-1.47†	-3.71†
	(0.53)	(0.57)	(0.48)	(0.63)	(0.64)	(0.77)	(0.54)	(0.53)	(0.42)	(0.56)
b_dw×LA	5.36†	7.09†			2.76†		4.06†	4.58†	5.49†	5.16†
	(0.53)	(0.62)			(0.66)		(0.56)	(0.52)	(0.47)	(0.55)
LA		-4.43***								
		(0.48)								
gr. GDP										0.14†
										(0.03)
Fixed effects										
Year	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
Sampling										
Lat Am	N	N	Y	N	N	N	N	N	N	N
AO	N	N	N	Y	N	Y	N	N	N	N
− 60-70s	N	N	N	N	Y	Y	N	N	N	N
+ Trans.	N	N	N	N	N	N	Y	N	N	N
For DW										
-20 years	Y	Y	Y	Y	Y	Y	Y	N	N	Y
-15 years	N	N	N	N	N	N	N	Y	N	N
-25 years	N	N	N	N	N	N	N	N	Y	N
Controls										
GDP pc	N	N	N	N	N	N	N	N	N	Y
N	1996	1996	1065	931	1515	450	1996	1975	2050	1816

Robust standard errors in parentheses. * $p < .10$, ** $p < .05$, † $p < .01$. AO is Asia and Oceania .

The fact that the coefficient estimate of β_1 is negative while the sum of $\beta_1 + \beta_2$ is positive indicates that in Europe and Asia investiment rates increases before and during the demographic window, while in Latin America they decrease. Neither of these hypotheses can be rejected at the 1% level. Table B.3 reports the estimates of this econometric specification (equation B.1) using different controls, samples, and definitions for the demographic window; it is analogous to table B.1.

Finally, in Europe and Japan investment rates are higher during the demographic window and in the 15 years before it than afterwards.[3] In this case, since the sample is small, the results hold at a significance levels of 8.6%. The coefficent estimate of the explanatory dummy for the period after the demographic window is 0.42.[4]

Hypothesis Testing

The main empirical result is that in Europe, Asia, and Oceania, investment rates for the demographic window are associated with an increase, whereas in Latin America they are associated with a decrease. Table B.4 reports the P-values of testing the hypothesis behind these results for different specifications of the model.[5] The specification of column (2) additionally indicates that investment rates for the demographic window are higher in Europe, Asia, and Oceania, as compared to those in Latin America, in the period for the demographic window. The results of columns (3) and (4) indicate that even taking the smallest sample, the effect of the demographic window is positive in Europe, Asia, and Oceania and negative in Latin America. Column (6) indicates that the results are robust even in the smallest sample and controlling for possible effects of the war.

Table B.5 reports the alternative hypothesis that supports the result about changes in investment before the lagged demographic window: in Latin America investment rates decrease before and during its demographic window but increase elsewhere. The table also contains the P-values of these hypothesis under the different variants of the model; it is analogous to table B.4.

---

[3] I group Japan with Europe since the demographic window of Japan happened around the same time as other European countries (1960 to 1996). In section 3.5.2 I also refer to Europe and Japan as a group.

[4] The model specification of this regression is $Inv_{c,t} = \beta_0 + \beta_1\ adw_{c,t} + \beta_c c + \beta_t t + \epsilon_{c,t}$, where $adw_{c,t} = 1$ if the country has passed its demographic window.

[5] The number at the top of each column corresponds to the model specification of the columns of table B.1.

Table B.4: **Hypothesis testing of investment for the demographic window**

	(1)	(2)	(3)	(4)	(6)
β_1	4.23	5.25	-1.17	3.96	2.45
P-val ($\beta_1 < 0$)	0.00	0.00	0.99	0.00	0.00
$\beta_1 + \beta_2$	-0.79	-1.06	.	.	.
P-va l($\beta_1 + \beta_2$) > 0	0.01	0.00	.	.	.
$\beta_2 + \beta_3$	.	-4.01	.	.	.
P-val ($\beta_2 + \beta_3$) < 0	.	0.00	.	.	

Table B.5: **Hypothesis testing of investment and the lagged demographic window**

	(1)	(2)	(3)	(4)	(6)	
β_1	-3.83	-6.04	1.17	-2.93		-6.20
Pval($\beta_1 > 0$)	0.00	0.00	0.99		0.00	0.00
$\beta_1 + \beta_2$	1.53	1.05	.	.		.
Pval($\beta_1 + \beta_2$) < 0	0.00	0.01	.		0.98	.
$\beta_2 + \beta_3$	.	2.66	.	.	.	
Pval($\beta_2 + \beta_3$) < 0	.	0.00	.	.	.	

Openness

Table B.6 reports the coefficient estimates of regression (3.2), in a sample without Europe.

Table B.6: **Demographic window and openess: 15Y without Europe**

Regression Estimates:

β_1 (dw)	6.31***	(0.58)
β_2 (dw × LA)	-4.39***	(0.59)
β_3 (kaopen)	3.45***	(0.68)
β_4 (kaopen×dw)	-4.90***	(0.81)
Observations	1561	

Robust standard errors in parentheses.

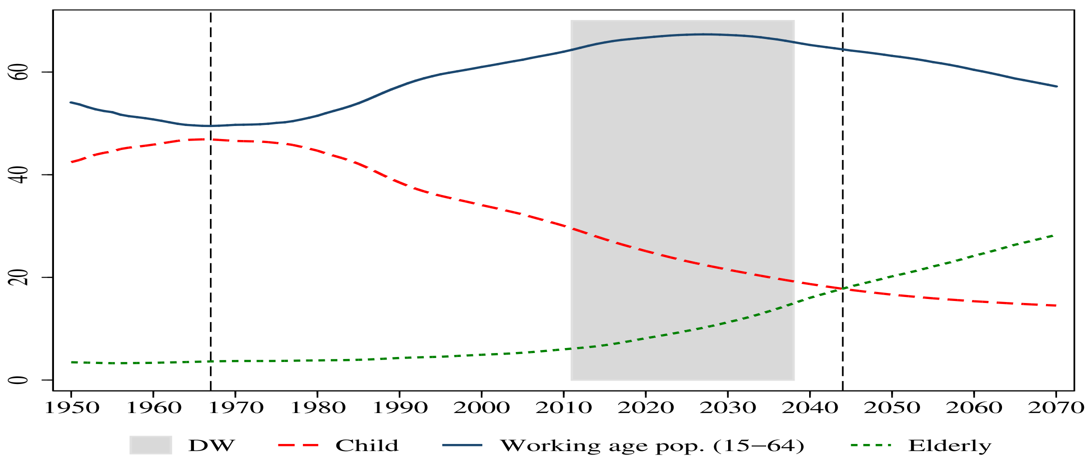
* $p < .10$, ** $p < .05$, *** $p < .01$

Results:

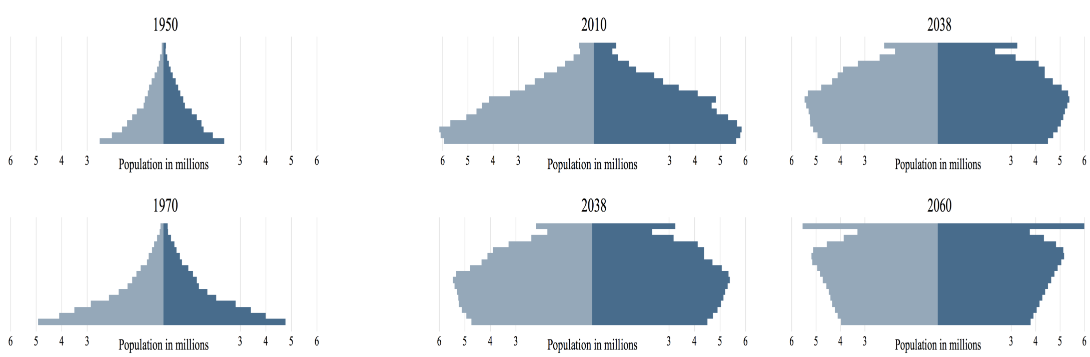
Hypothesis	value	P-val alt.
$\beta_1 > 0$	6.31	0.000
$\beta_1 + \beta_2 > 0$	1.93	0.000
$\beta_3 > 0$	3.45	0.000
$\beta_3 + \beta_4 < 0$	-1.45	0.010

Figure B.1: **Demographic transition in Mexico**

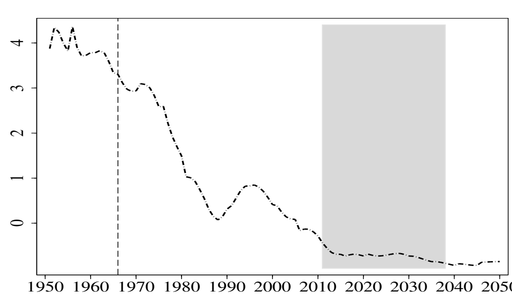
(a) Population by age group (%)



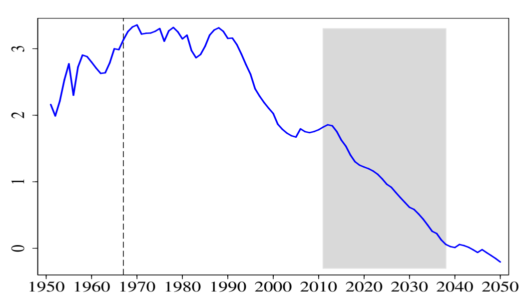
(b) Population pyramids



(c) Annual growth rate of pop. aged 0-14 (%)



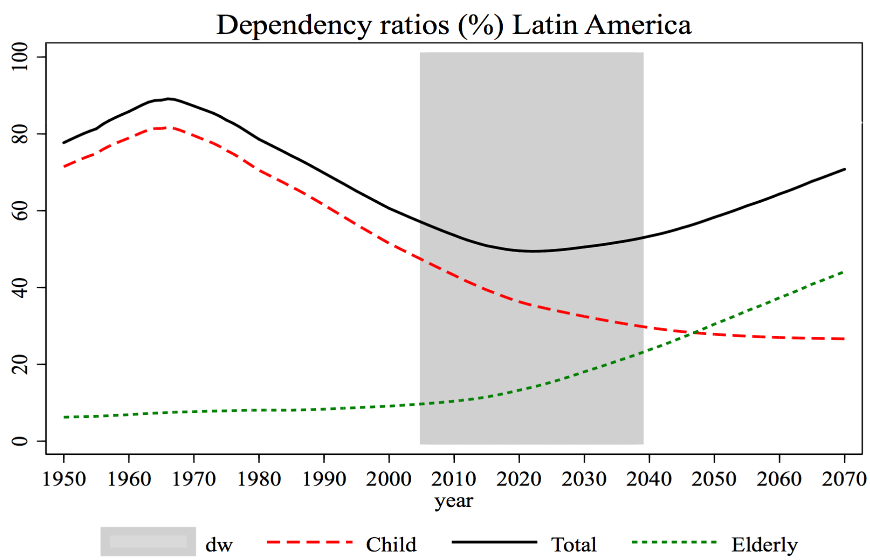
(d) Annual growth rate of working-age pop. (%)



Notes: The gray areas indicate the years of the demographic window. The dotted vertical lines in panel (a) indicate the approximate years where stages 1 and 2 end. Panels (a) and (b) are aligned horizontally to show the change in population structure in terms of both age groups and population pyramids.

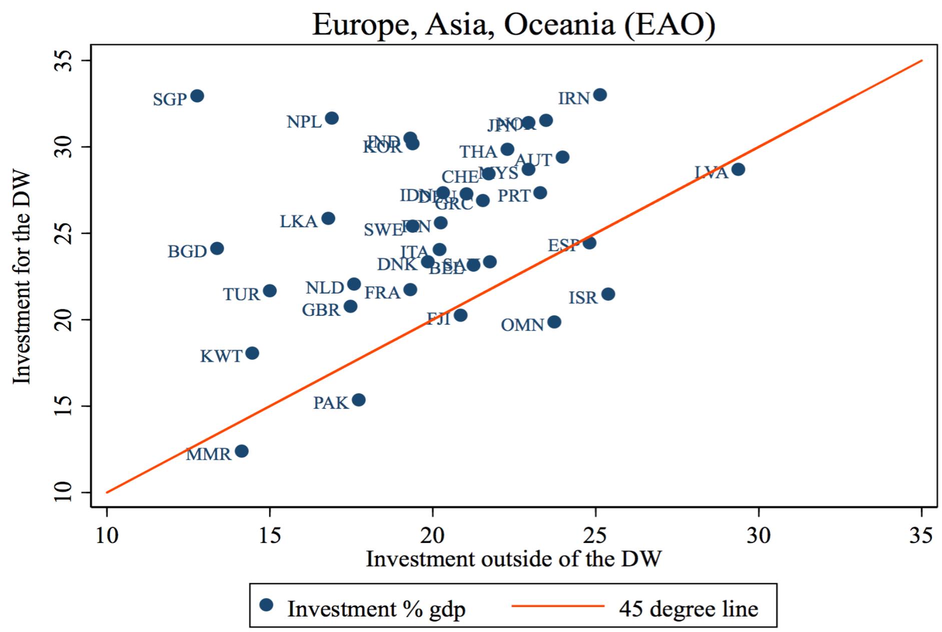
An indicator that is sometimes used to describe the demographic window and to motivate its importance is the total dependency ratio. This ratio is defined as the child plus elderly population divided by the working-age population. Dependency ratios usually reach their minimum during a demographic window, since the working-age population is increasing, the child population is decreasing, and the elderly population is low.

Figure B.2: **Demographic window and dependency rates**



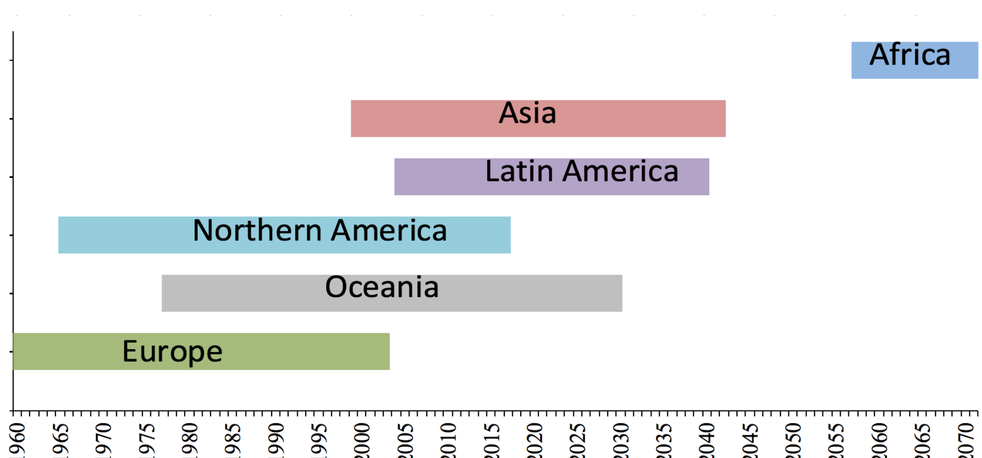
Notes: The figure plots the dependency ratios. The child dependency ratio is the ratio of child population over working-age population, and the elderly ratio is the ratio of elderly population over working-age population.

Figure B.3: **Investment rates for the demographic window**



Notes: The figure plots the average (across years) of investment rates during the demographic window and the 15 years before it on the horizontal axis, and the average (across years) of investment rates outside of this period on the vertical axis.

Figure B.4: **Timing of the Demographic Window**



Notes: The figure plots the years of the demographic windows of each continent starting in 1960 (the first year in my database). Note that the demographic window of Europe begins before the timeline shown in this graph: for example, as discussed in 3.5.2, the window for England and Wales started in 1920.

B.2 Model Appendix

B.2.1 Household Preferences over Children

The utility representation of households of generation $t-1$,

$$U^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \gamma\phi_{t-1}u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}),$$

is a simplification of the following specification:

$$U_k^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \sum_{j=1}^{Nk} \gamma_j u(c_t^{k,j}) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}),$$

where U_k^{t-1} denotes the discounted utility of agent k of generation $t-1$, and:

$c_t^{t,j}$ is the consumption of the j-th children,

γ_j is the valuation of the j-th child's utility in his parents utility.

N_k is the number of children of this household.

Since parents care equally about each of their children, $\gamma_i = \gamma_i \equiv \gamma \;\; \forall i,j \;\; \Rightarrow c_t^{t,j} = c_t^{t,i} \equiv c_t^t \;\; \forall i,j$. Hence:

$$U_k^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \gamma \cdot Nk \cdot u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}).$$

And since households are representative, the number of children in each household is the same ($N_{k1} = N_{k2} \equiv N_k \quad \forall k1, k2$) and is equal to the total child population over the population of generation t-1: $Nk = \frac{N_t}{N_{t-1}} = \phi_{t-1}$.

Choice of the value of altruism towards children γ

In the benchmark model, γ is set to 0.5 using the "OECD equivalence scales" that assign a value of 0.5 to each child in a one-household member. Since, to my knowledge, there are no official estimations of equivalence scales for Mexico, and no consistent estimations across countries, specially for Latin America, I chose an scale from an international organization: the OECD. According to the OECD (1982), their scale could be used in "countries which have not established their own equivalence scales." Their scale was used in the 1980s and the early 1990s, and since my data covers 1960 to 2013, I found it suitable for reflecting an intermediate year in this time frame.

In the late 1990s, the EUROSTAT[6] adopted the use of the "OECD-modified scales" (which were proposed by Hagenaars et al. (1994)) which assign a value of 0.3 to each child. I opted to use the original scales because they better suited the timeframe of my data. Furthermore, the value given to a child is higher and adapts better to the number of years–20–that a generation is part of the child population. In the model children are dependent on their parents until they are 19, which makes their consumption higher than what it would be if they were children until age 14.

Finally since the focus of this paper is Latin America, mostly comprised by developing countries, an additional reason for using a higher scale is that in developing countries the amount of income spent in a children as a fraction of their parents' income is around 40 to 50%[7] (Deaton, 1997)[8]. Nonetheless, in section 3.5.3, I perform a sensitivity analysis on this parameter and show that the main results are not affected by its exact choice.

B.2.2 Labor Force

The demographic transition under consideration in this paper, i.e., one triggered by a decrease in the growth rate of newborns, generates a decrease in the growth rate of labor in since each period there are fewer children that will join the future labor force.

Using the demographic specification the model, the growth rate of labor force is:

$$g_{Lt} = \frac{L_t}{L_{t-1}} - 1 = \frac{(1 + g_{nt-2})(2 + g_{nt-1})}{(2 + g_{nt-2})} - 1,$$

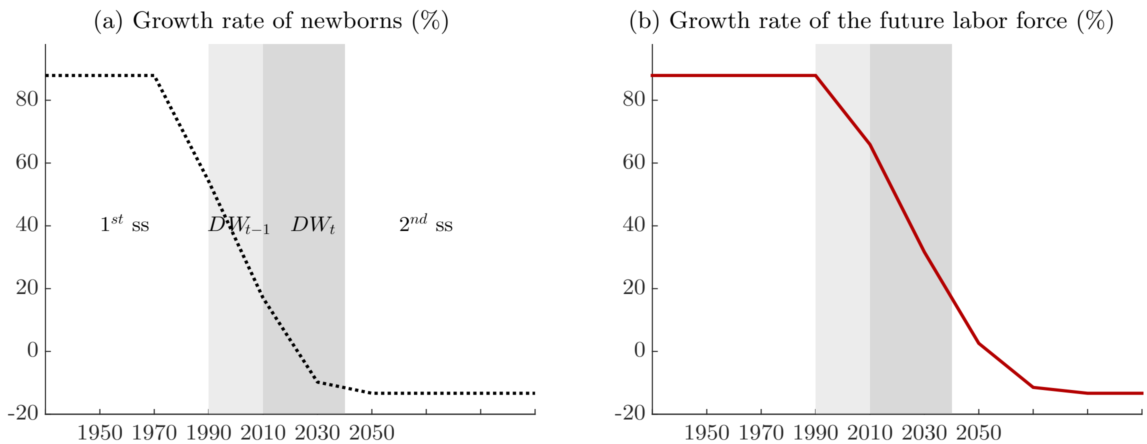
which is decreasing if the growth rate of the labor force is decreasing (see remark 3.4.1). Figure B.5 gives a graphical illustration of this for the numerical exercise of the Mexican demographic change:

[6] OECD (2009) provides an description of the evolution across time of the more commonly used scales.

[7] Per additional children and to maintain constant adult welfare.

[8] Olken (2005) finds that the value for developing countries is even higher than the originally proposed by Deaton.

Figure B.5: **Effect of the demographic window on the growth rate of labor**

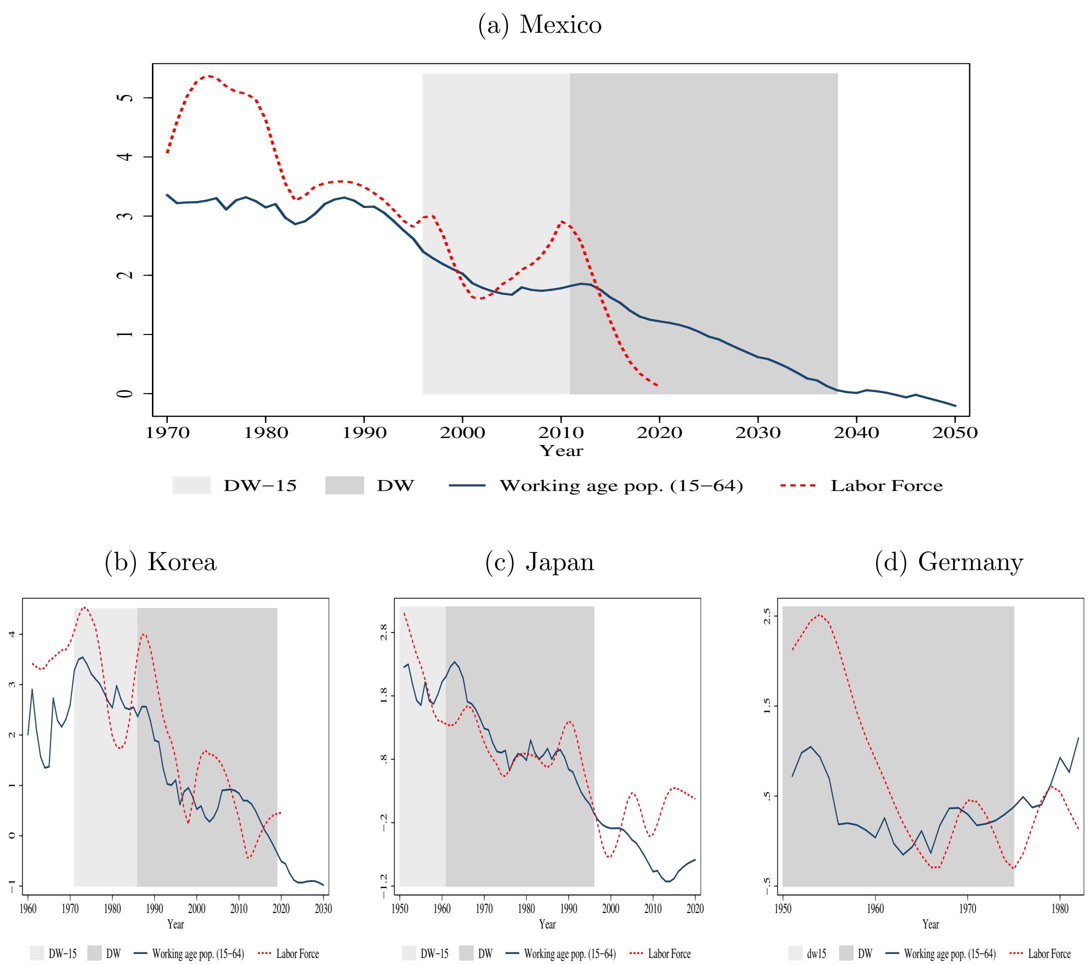


Notes: This figure plots the 20-year growth rate of newborns used in the benchmark exercise of Mexico in panel (a). In panel (b), I plot the 20-year growth rate of future the labor force, g_{Lt+1}.

Labor force and working-age population

This decrease in the growth rate of labor force is the main driver of the result in the open economy, i.e., a decrease in investment rates for the demographic window. It is important to mention that in the model, since there is no disutility of working, the labor force is equal to the working-age population. However, this is a sensible assumption since in the data the growth rates of the working-age population and of the labor force behave similarly: decreasing before and during the demographic window. Figure B.6 shows the evolution of these variables for the case of Mexico, as well as for the EAO region.

Figure B.6: **Annual growth rates of the working-age population and labor force**



Notes: for each country, this figure plots the annual growth rate of the working-age population in the solid line and the annual growth rate of the labor force in dashed line. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years preceeding it. The labor force was HP-filtered and the cyclical component removed to reflect long term trends. Labor force data was taken from the Total Economy Database for 1950–2015 and from the projections of the LABORSTA database for 2016–2020.

B.2.3 Additional Equations and Figures

The consumption demand of a young worker from generation t-1 in the closed economy is given as

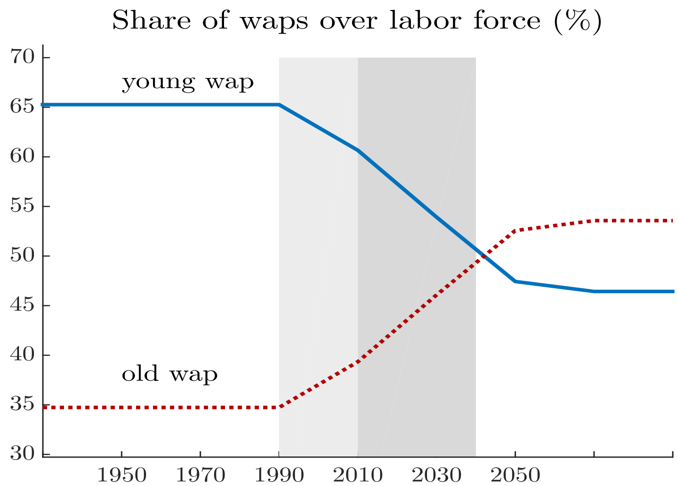
$$c_t^{t-1} = \frac{w_t + \frac{w_{t+1}}{(1+r_{t+1}^b)}}{1 + \gamma^{\frac{1}{\sigma}}\phi_{t-1} + \beta^{\frac{1}{\sigma}}\left(1 + r_{t+1}^b\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1 + r_{t+1}^b\right)\left(1 + r_{t+2}^b\right)\right]^{\frac{1}{\sigma}-1}}. \tag{B.2}$$

The expression for savings (of households of generation t-1) per effective unit of the closed economy is given as

$$s_t^{t-1} = \frac{\beta^{\frac{1}{\sigma}}\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)^{\frac{1}{\sigma}-1}\left[1 + \beta^{\frac{1}{\sigma}}\left(1 + \alpha k_{t+2}^{\alpha-1} - \delta\right)^{\frac{1}{\sigma}-1}\right](1-\alpha)k_t^{\alpha}}{1 + \gamma^{\frac{1}{\sigma}}\phi_t + \beta^{\frac{1}{\sigma}}\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)\left(1 + \alpha k_{t+2}^{\alpha-1} - \delta\right)\right]^{\frac{1}{\sigma}-1}}$$
$$... - \frac{\left(1 + \gamma^{\frac{1}{\sigma}}\phi_t\right)\frac{1 + g_{at+1}}{1 + \alpha k_{t+1}^{\alpha-1} - \delta}(1-\alpha)k_{t+1}^{\alpha}}{1 + \gamma^{\frac{1}{\sigma}}\phi_t + \beta^{\frac{1}{\sigma}}\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)\left(1 + \alpha k_{t+2}^{\alpha-1} - \delta\right)\right]^{\frac{1}{\sigma}-1}} \tag{B.3}$$

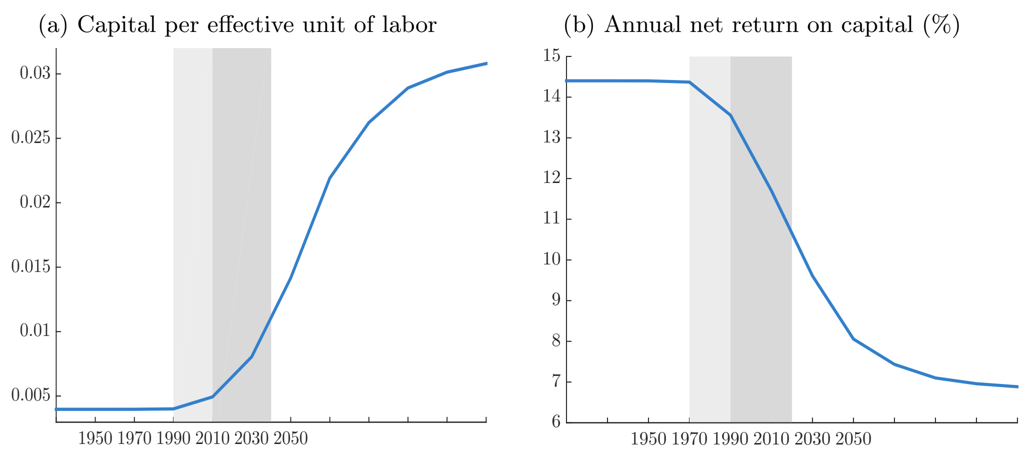
$$s_t^{t-2} = \frac{\beta^{\frac{2}{\sigma}}\left[\left(1 + \alpha k_t^{\alpha-1} - \delta\right)\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)\right]^{\frac{1}{\sigma}-1}\left[\frac{1 + \alpha k_t^{\alpha-1} - \delta}{1 + g_{at}}(1-\alpha)k_{t-1}^{\alpha} + (1-\alpha)k_t^{\alpha}\right]}{1 + \gamma^{\frac{1}{\sigma}}\phi_{t-1} + \beta^{\frac{1}{\sigma}}\left(1 + \alpha k_t^{\alpha-1} - \delta\right)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}}\left[\left(1 + \alpha k_t^{\alpha-1} - \delta\right)\left(1 + \alpha k_{t+1}^{\alpha-1} - \delta\right)\right]^{\frac{1}{\sigma}-1}} \tag{B.4}$$

Figure B.7: **Share of savers in the economy**



Notes: this figure plots the shares of young and middle age workers as a fraction of the total population. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before it. The solid line is the share of young workers and the dotted line the share of middle-age workers.

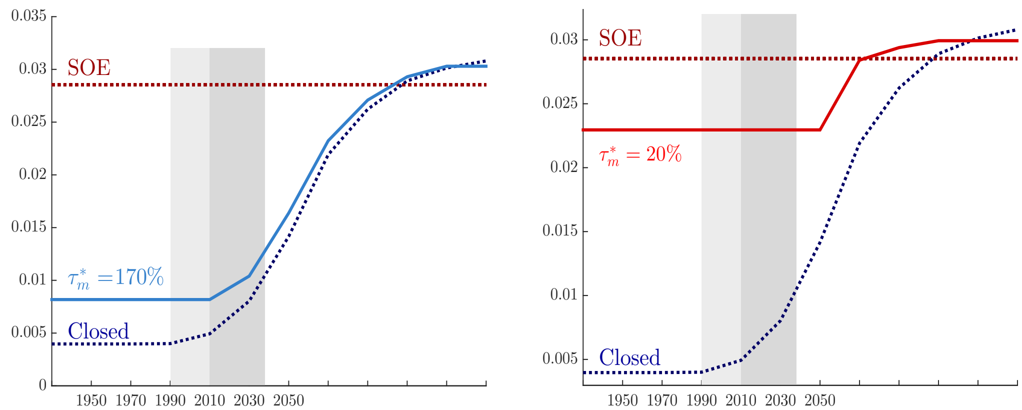
Figure B.8: **Capital and interest rate in the closed economy (benchmark ex9ercise)**



Notes: This figure plots capital (p.e.u.l) and the annualized return on capital net of depreciation of the closed economy, throughout the Mexican demographic transition of the benchmark numerical exercise (depicted in figure 3.3). Note that the net return of capital is simply the marginal productivity of the capital (p.e.u.l) net of depreciation.

Figure B.9: **Capital of the benchmark exercise with a varying degree of openness**

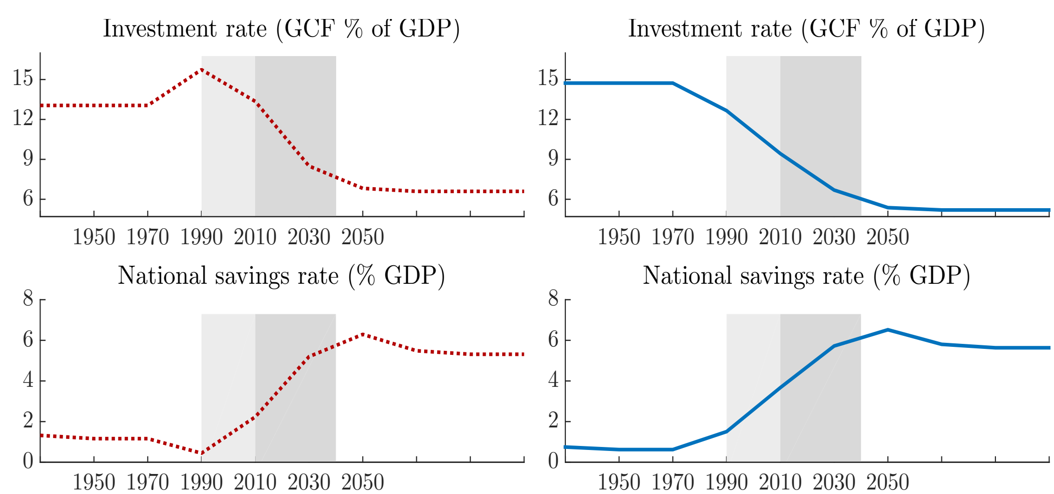
(a) Closed economy (b) Open economy



Notes: this figure plots the capital (p.e.u.l) of the benchmark exercise with a varying degree of openness (depicted in figure 3.4). The dotted line is the capital (p.e.u.l) in the frictionless model, the solid line in panel (a) is the capital (p.e.u.l) in the taxed closed economy, and the solid line in panel (b) is the capital (p.e.u.l) in the taxed open economy. In the closed economy $\tau_1 = 20\%$ and $\tau_2 = 5\%$, and in the open economy $\tau_1 = 170\%$ and $\tau_2 = 7\%$.

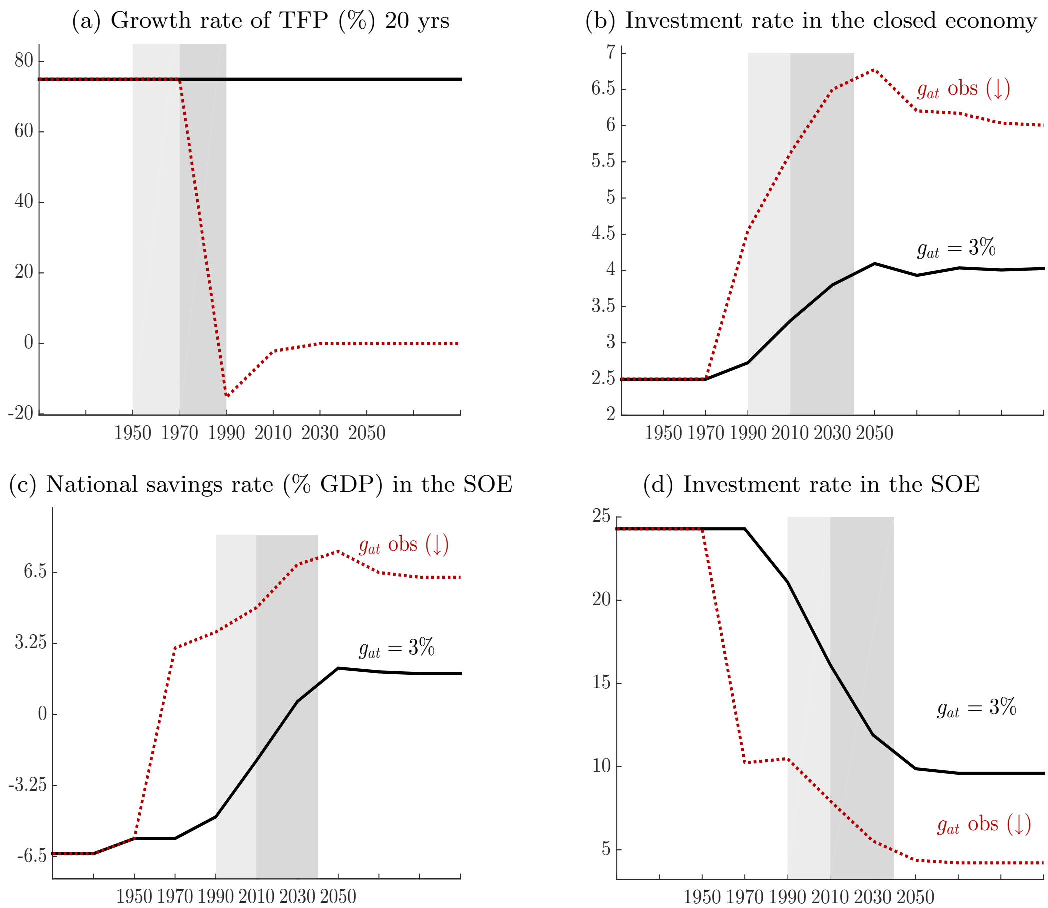
Figure B.10: **Investment and savings, decreasing international rate of return**

(a) Decreasing interest rate (b) Benchmark, constant interest rate



Notes: this figure plots investment and savings rates generated by the Mexican demographic transition under two scenarios. Panel (a) shows a scenario characterized by the observed decreasing international return until 2010, with a projection of 6% for 2030 onwards. Panel (b) shows the benchmark exercise with a constant annual return of 7.15% (average of 1950–2014).

Figure B.11: **Investment and savings rate, technology growth**



Notes: The panels in this figure show savings and investment rates generated by the Mexican demographic transition under two scenarios. The dashed line illustrate a scenario characterized by the observed decreasing growth rate of TFP until 2010, with a projection of 0% for 2030 onwards. The solid line depicts the benchmark scenario with a constant annual growth rate of 3% (average of 1950–2014). This growth rate is depicted in panel (a). In panels (b) and (d), I plot investment rates in the closed and open economies, respectively. In panel (c), I plot the savings rate in the open economy.

Figure B.12: **Growth in Western Europe and Japan (1950s–1970s)**

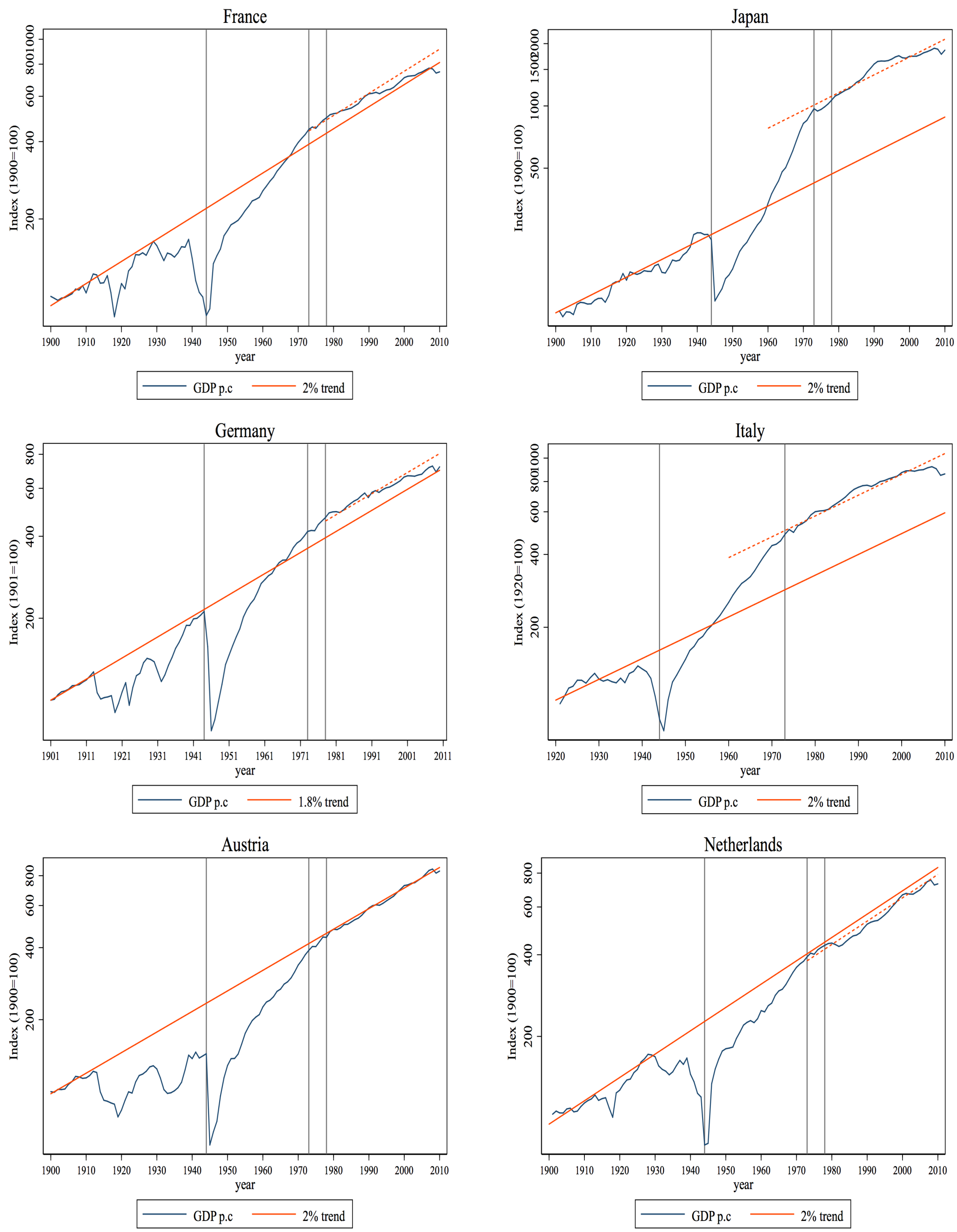


Figure B.12 (Cont.): **Growth in Western Europe and Japan (1950s–1970s)**

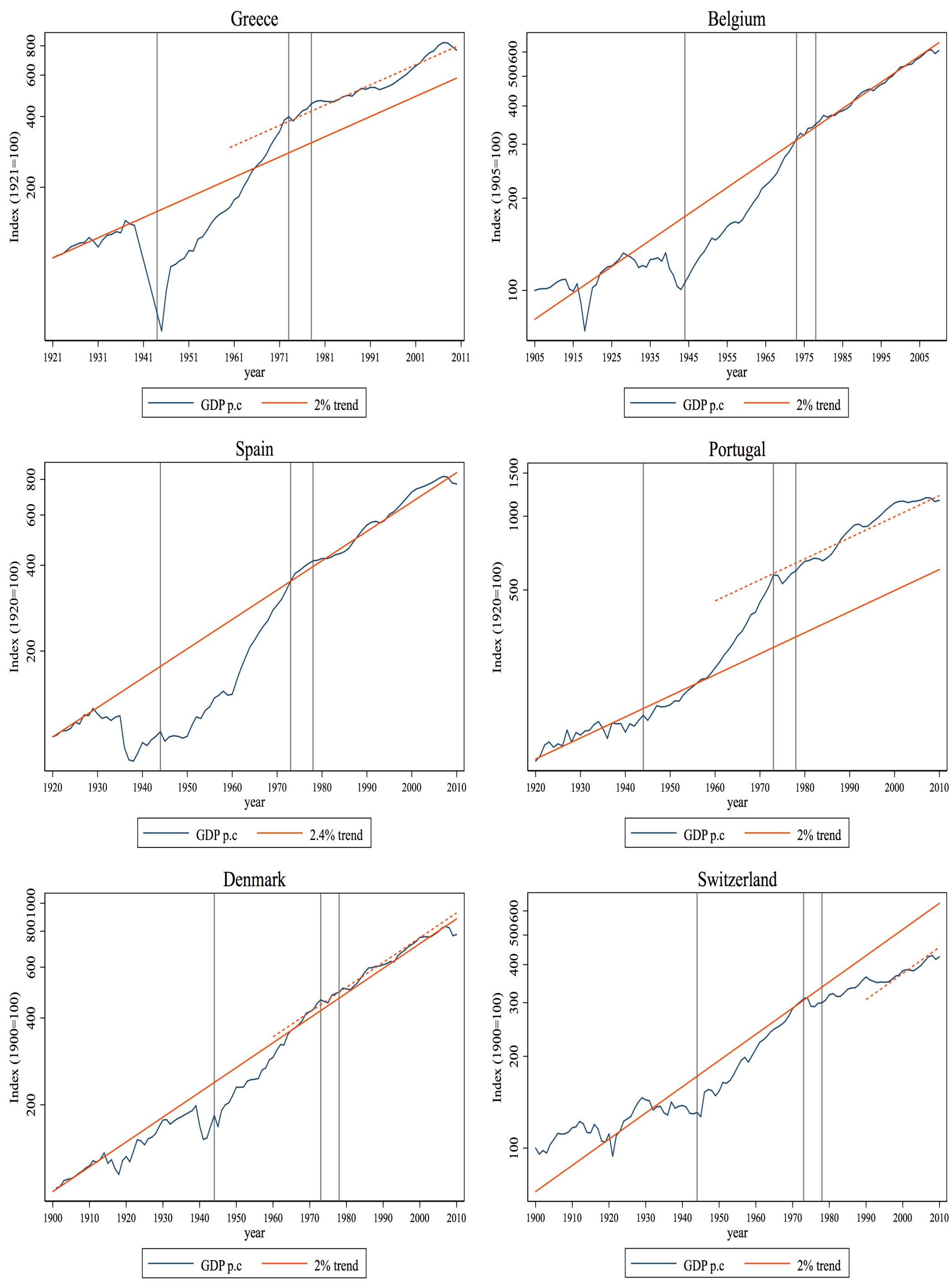
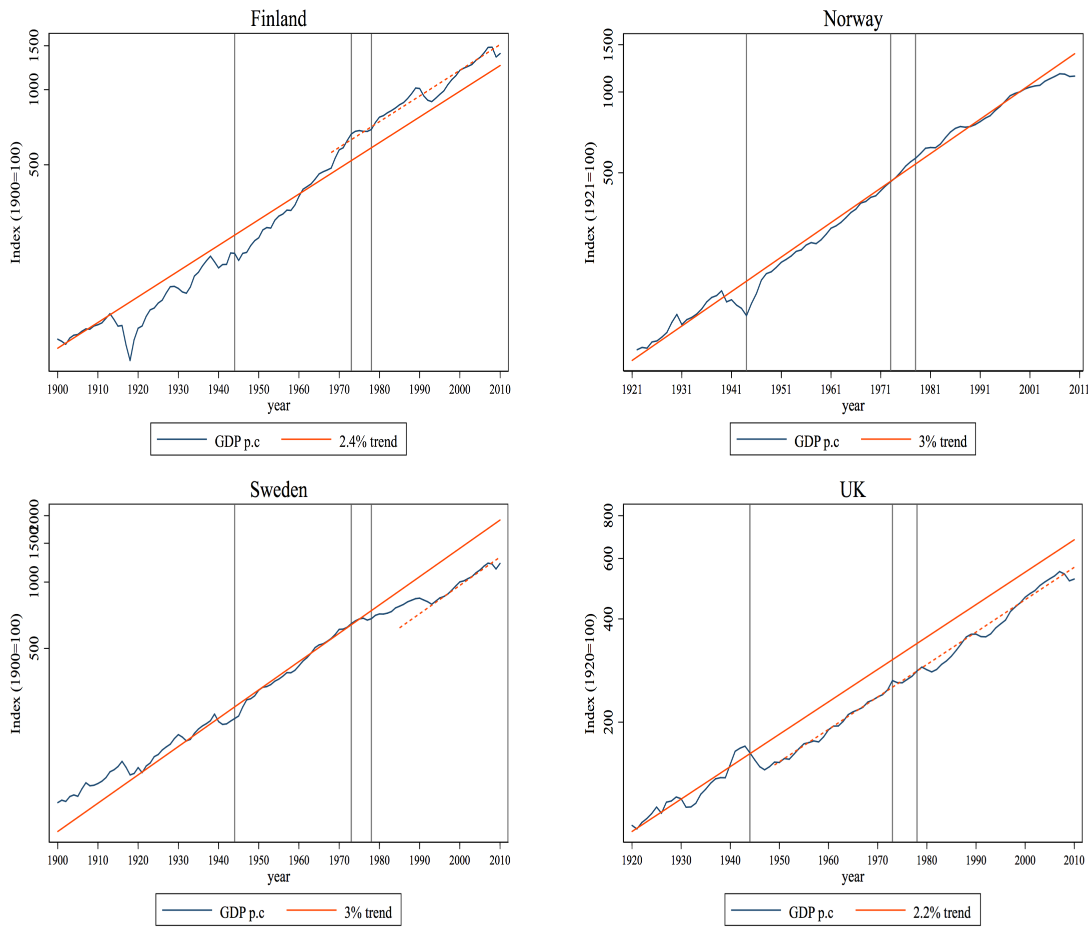


Figure B.12 (Cont.): **Growth in Western Europe and Japan (1950s–1970s)**



Notes: this figure plots an index of GDP per capita and its long-run trend for various countries. The vertical indicate the years of the end of WWII and the oil shocks in 1945, 1973, and 1978 respectively. In general, we observe that from the postwar era until the 1970s, countries experienced high growth in the sense that the slope of the GDP per capita (its growth rate) is higher than that of the long run trend. Note that in some countries, like the UK and Sweden, there was no apparent change in growth. The data of GDP per capita was taken from the Maddison Project Database (Bolt and Zanden, 2014).

Figure B.13: **Growth in Western Europe and Japan (1800–1900s)**

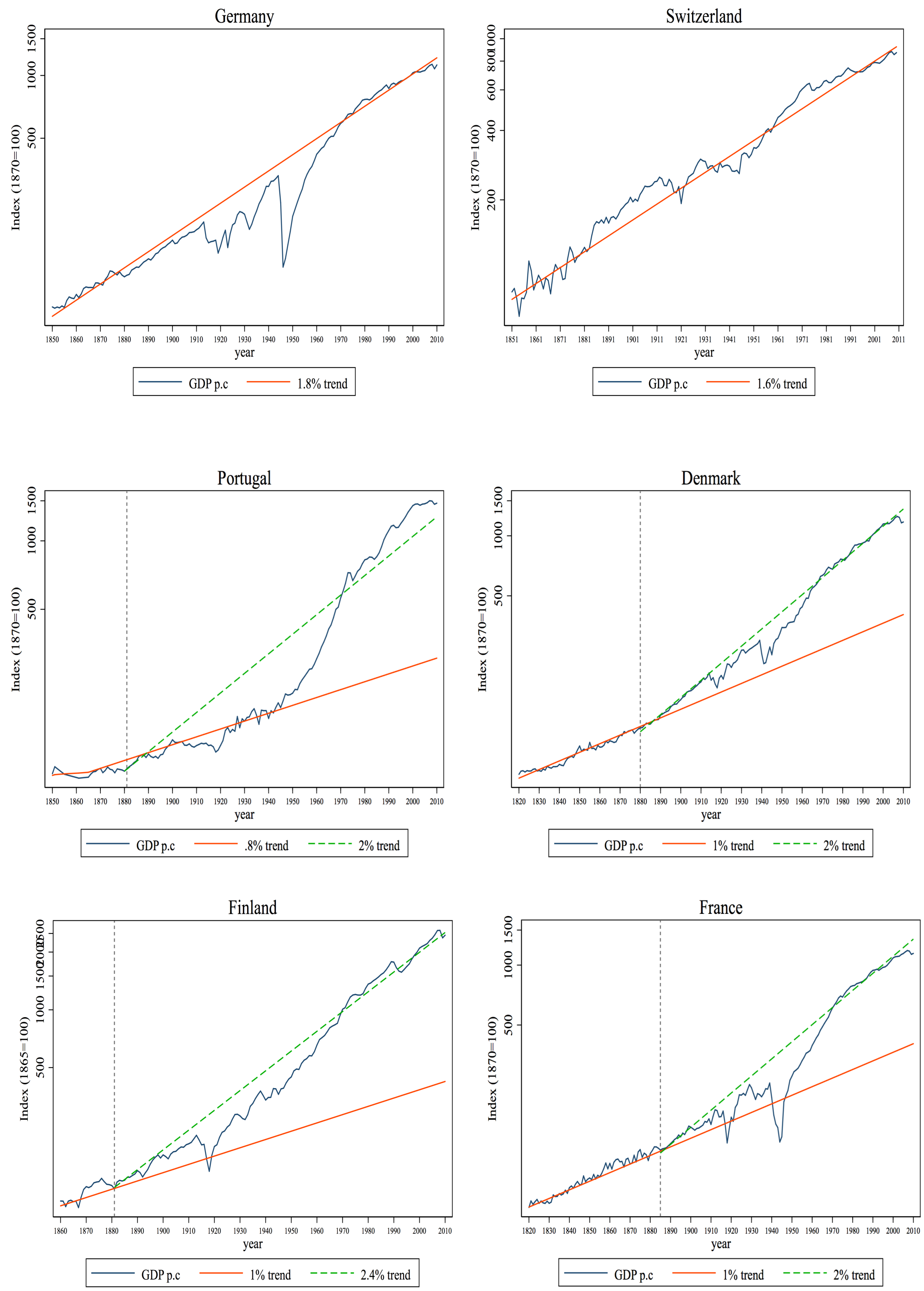


Figure B.13 (Cont.): **Growth in Western Europe and Japan (1800–1900s)**

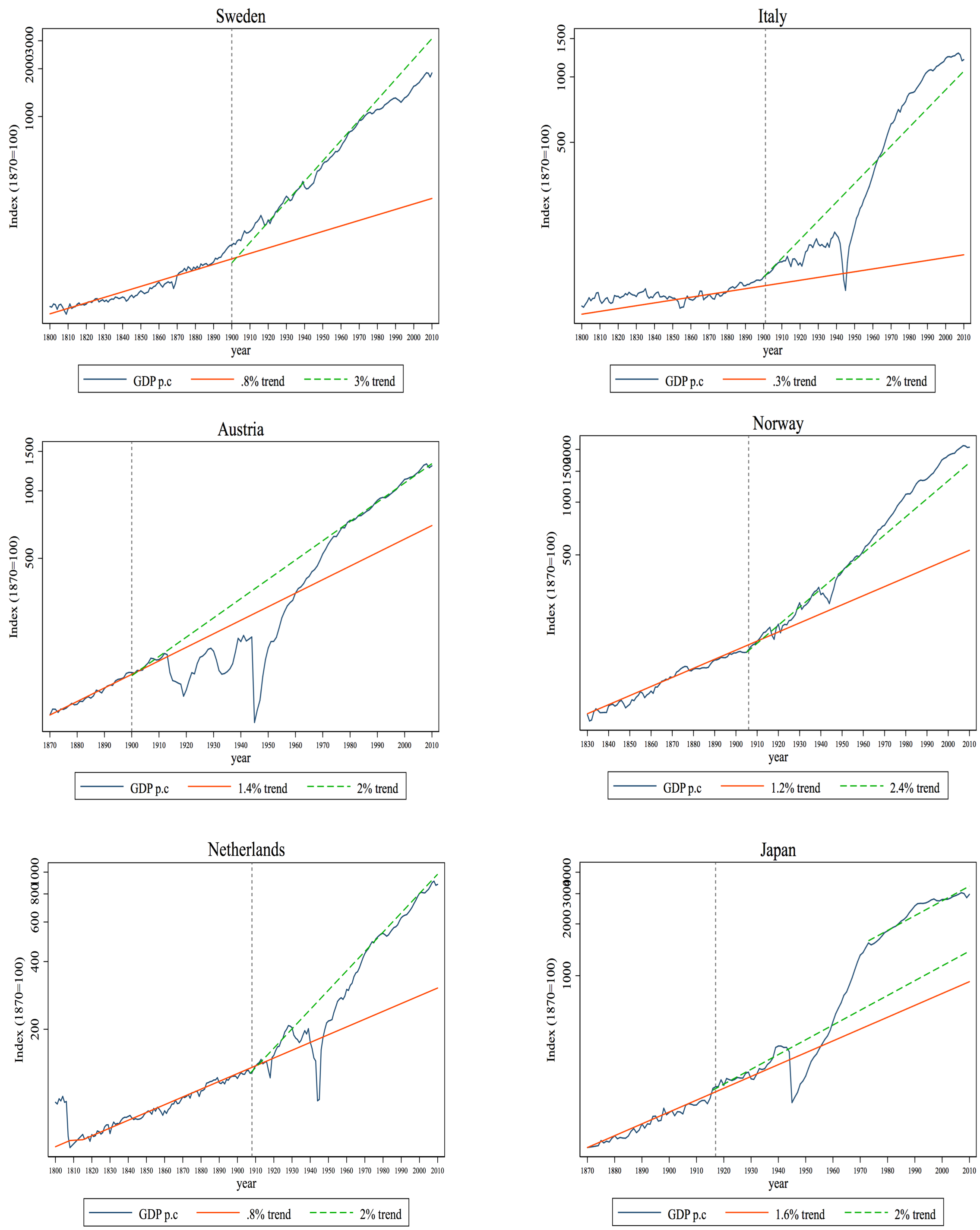
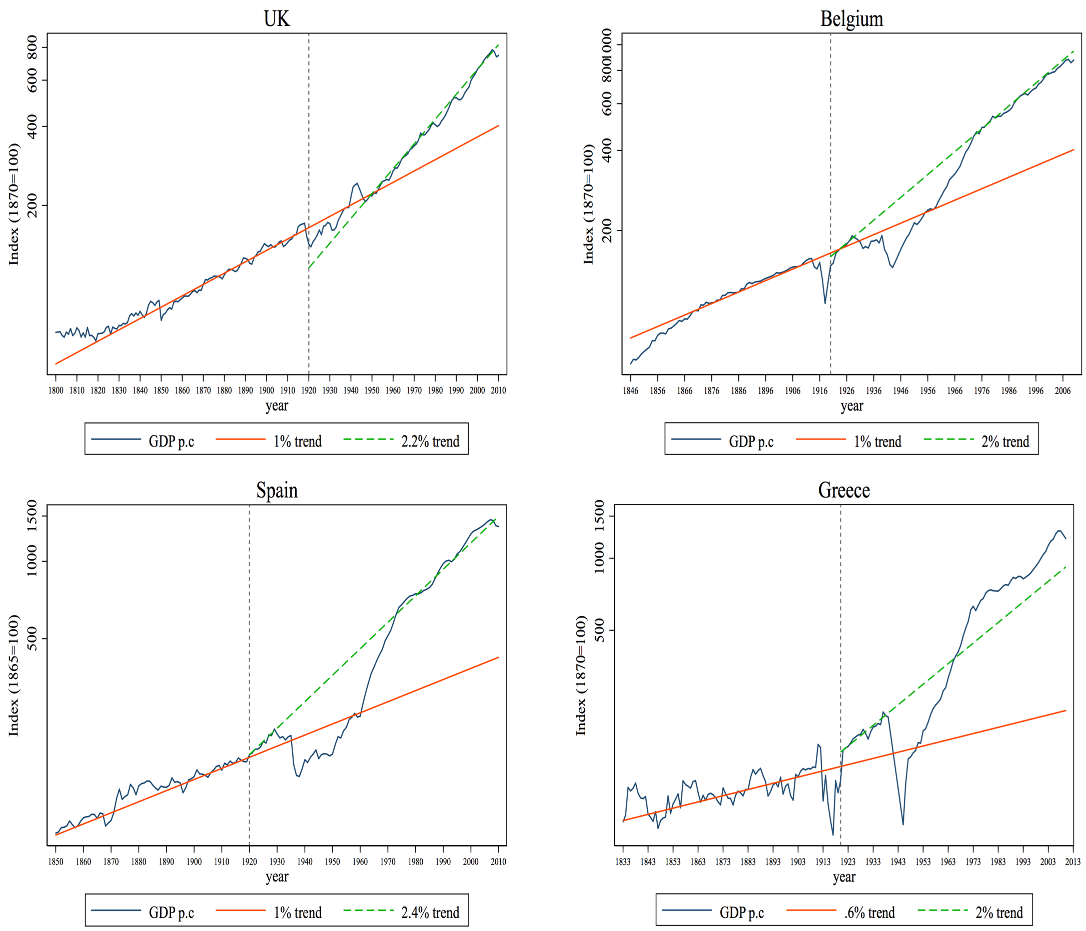


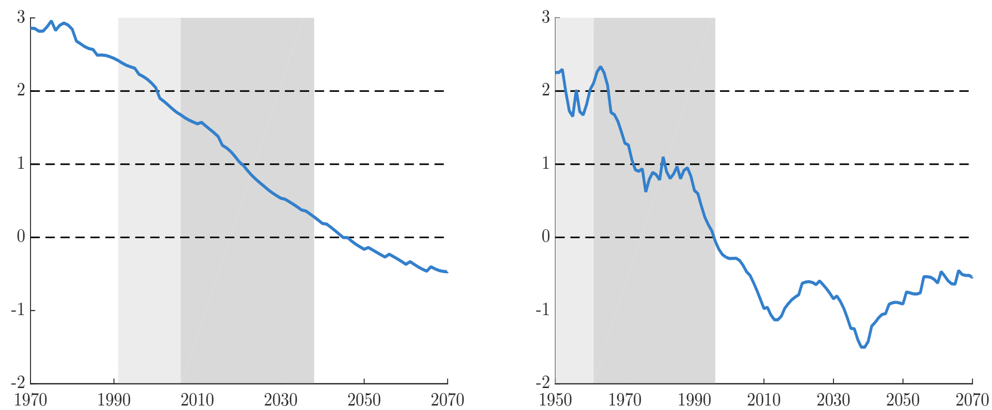
Figure B.13 (Cont.): **Growth in Western Europe and Japan (1800–1900s)**



Notes: this figure plots an index of GDP per capita and its long-run trend from the earliest available data until 2014. Vertical lines are included to indicate the approximate year where the change in long-run growth happened. The countries are ordered according to this year, starting with Finland and Norway where there was no apparent change and ending in the countries where the change happened around 1920. The data of GDP per capita was taken from the Maddison Project Database (Bolt and Zanden, 2014).

Figure B.14: **Annual growth rate of the working-age population (%)**

(a) Latin America (b) Japan



Notes: this figure plots the annual growth rate (in percentage points) of the working-age population. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before.

In the following exercise, I first introduce a change in the growth rate of technology from 1% to 2%, keeping the population growth rate constant. Second I add, on the top of the technological exercise, the Mexican demographic change to generate a demographic window. Finally, I compare the results to an exercise where the growth rate of technology is fixed at 1.5% annually throughout the demographic transition.

Figure B.15: **Investment rates, long-run technological growth**

(a) Technological change of balanced growth path (BGP)

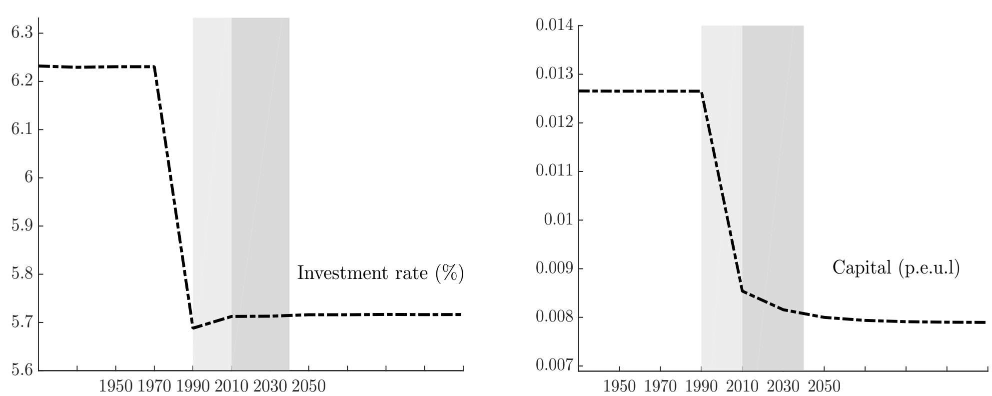
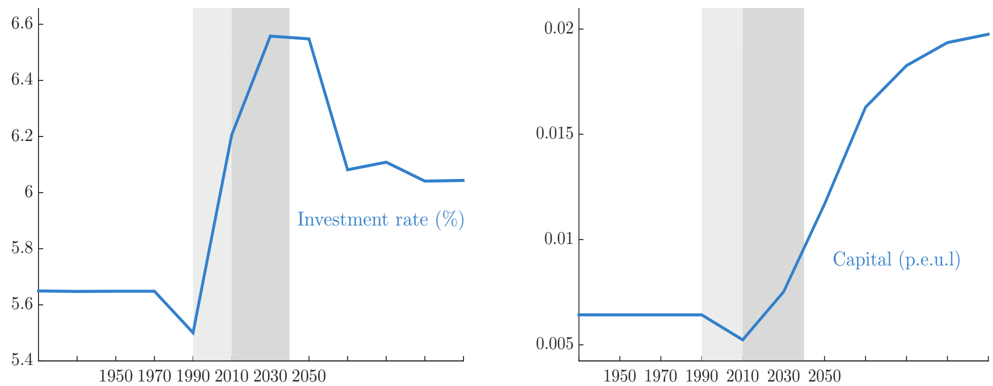
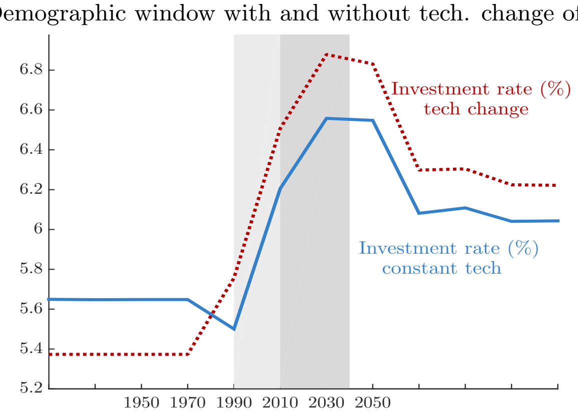


Figure B.15 (Cont.): **Investment rates, long-run technological growth**

(b) With the demographic window added



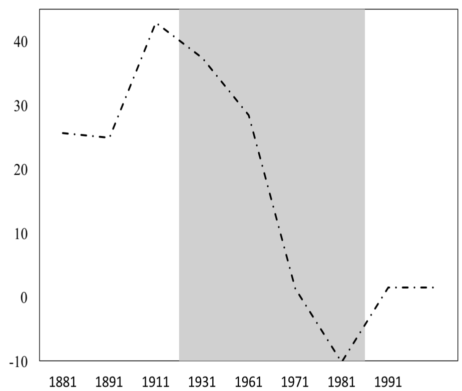
(c) Demographic window with and without tech. change of BGP



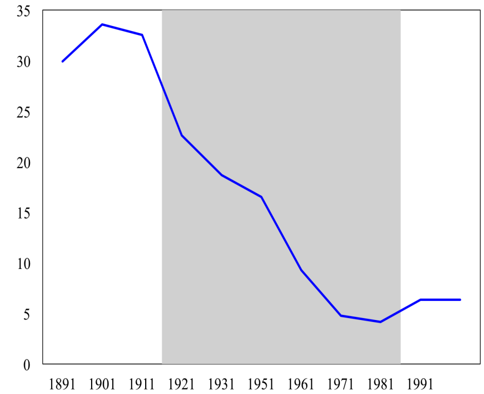
Note: in panel (a), I first introduce a change in the growth rate of technology from 1% to 2%, keeping the growth rate of population constant. Second, in panel (b) I add the Mexican demographic change. In panel (c) I compare the exercise of panel (b) to an exercise with a constant 1.5% growth rate throughout the demographic transition.

Figure B.16: **Demographic transition in England and Wales**

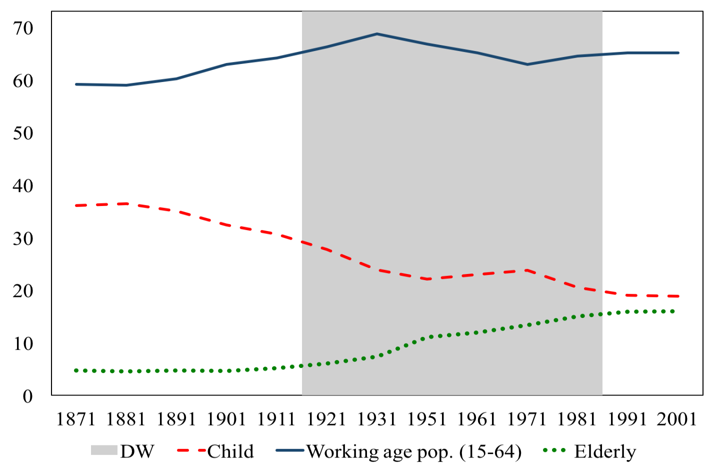
(a) 20-year growth rate of newborns



(b) 20-year growth rate of working-age pop. (%)

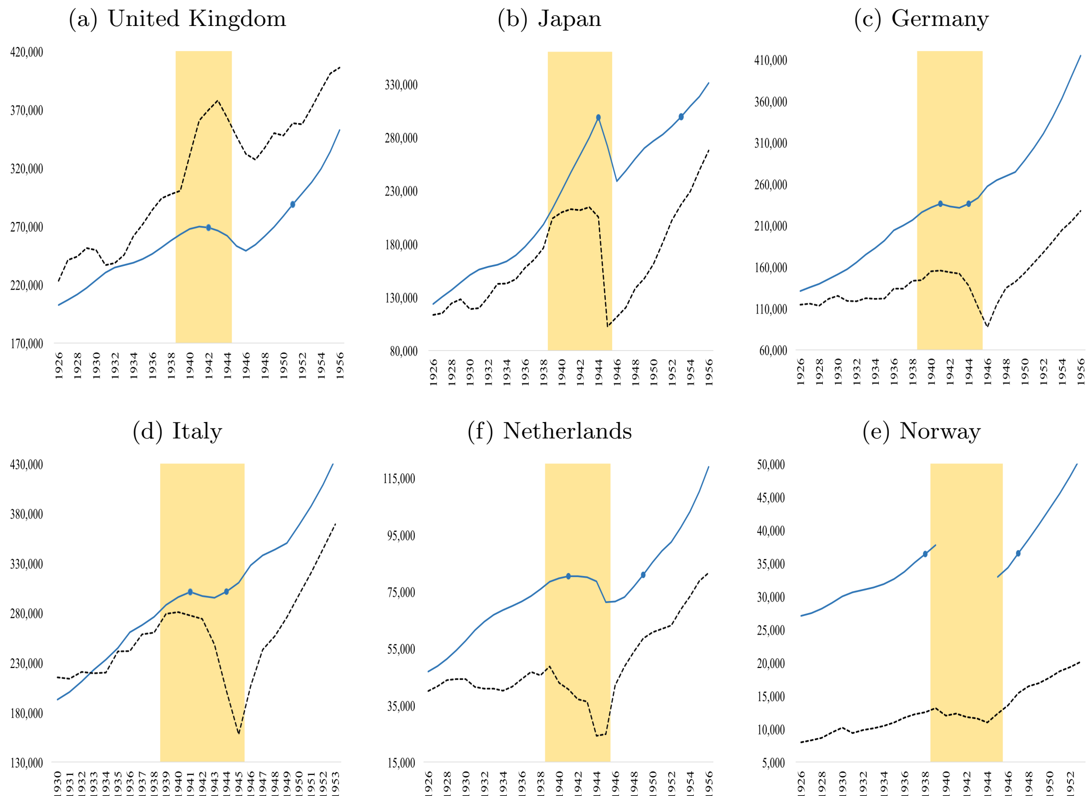


(c) Population by age group (%)



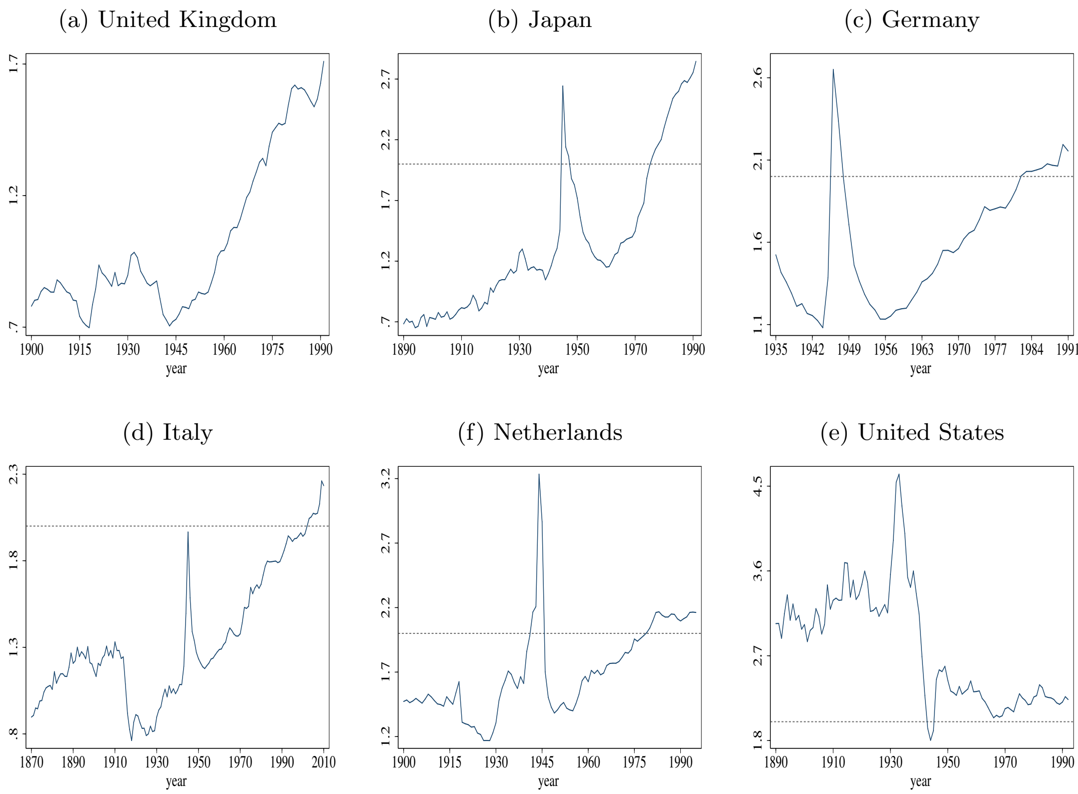
Notes: this figure plots the demographic transition of England and Wales. In panel (a), I plot the 20-year cumulative (%) growth rate of newborns used in the model. As explained in section 3.4.1, growth rates of newborns are adjusted to the growth rate of age cohorts to reflect mortality. In particular this is the forward (1 period ahead) growth rate of the cohort aged 40–59. In panel (b), I plot the 20-year cumulative (%) growth rate of the working-age population (the cohort aged 15–64). In panel (c), I plot the child, working-age population, and elderly populations as percentage of the total population. The gray area indicates the years of the demographic window.

Figure B.17: **Capital stock and GDP before and after WWII**



Note: This figure shows estimates of non-residential capital and output of some European countries and Japan. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the UK, Japan, Germany, it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Levels are in 1990 international dollars, except for Italy's, which are in 2011 international dollars because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively.

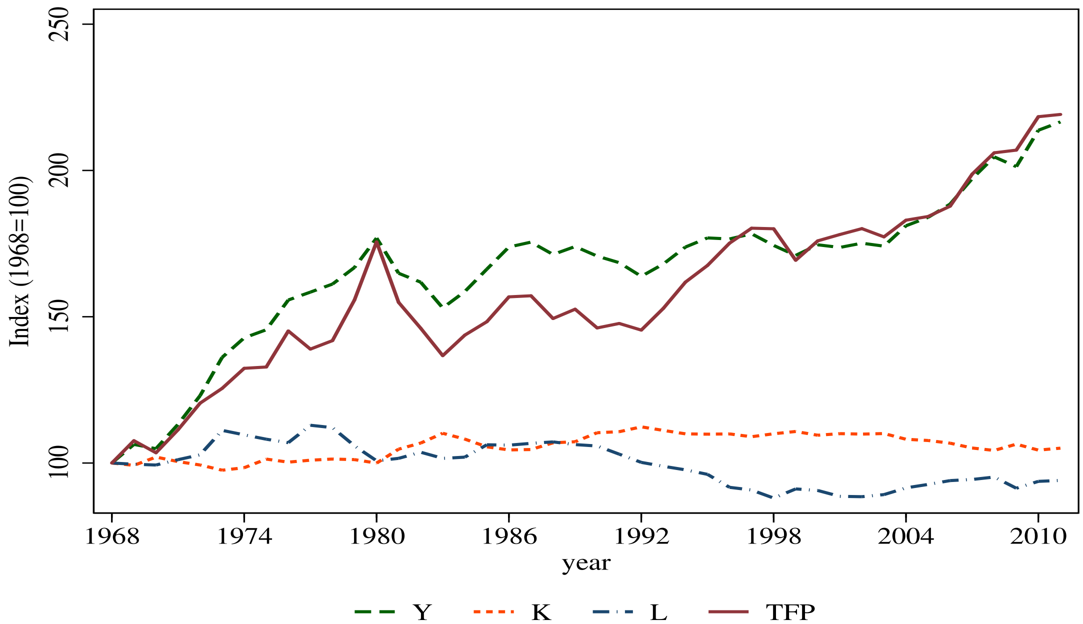
Figure B.18: **Non-residential capital to output ratio (selected countries)**



Note: This figure shows the capital to output ratio of the United States and the countries of figure 3.8 that had available data from 1990 onwards (i.e., Noway is not included). The ratio is the estimated non residential capital stock divided by GDP. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the United Kingdom, Japan, Germany, and the United States[9] it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Level series are in 1990 international dollars, except for Italy's, which is in 2011 international dollars of 2011 because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively. The horizontal line is at 2.

[9]From 1950 to 1990, the average ratio of capital stock estimated by Maddison for the United States is around 2.2. The most comparable official available series to this one is the net stock of fixed non-residential assets from the Bureau of Economic Analysis (BEA), which for the same time period is around 1.9 times GDP. Note that the latter does not include depreciation, which makes it a lower number. However, the numbers are close and I take this comparison as a check to reject the possibility of the estimates being far off the actual data. Also, notice that in later years in Japan and the rest of the European countries, the ratio of capital to GDP increases to levels that are close to the 2 of the United States. Finally, it is important to mention that the ratio of residential plus non-residential net capital stock to GDP of the BEA is around 3, a number the reader might be familiar with. However due to availability of data for prewar and interwar years, I present only non-residential capital stock.

Figure B.19: **Growth accounting exercise of Brazil**



Notes: This figure plots the growth accounting exercise for Mexico. In the vertical axis I plot indices for the corresponding elements (output, capital, labor and TFP) where the year of 1968, the first year for which there I have gathered consistent data for Brazil, is normalized to 100.

B.2.4 Computation of the Equilibrium

The closed economy

The equilibrium of the closed economy can be summarized by a sequence of capital per effective unit of labor (p.e.u.l) since the rest of the allocations can be written as a function of it, exogenous demographic variables, and the parameters of the economy. Using the law of motion of newborns and of capital (p.e.u.l) and the savings equations (B.3 and B.4) of young and middle-aged workers, the equilibrium condition can be reduced to a third-order difference equation on k_{t-1}, k_t, k_{t+1}, k_{t+2}:

$$k_{t+1} = \frac{(1+g_{nt-1})\, s_t^{t-1}(k_t, k_{t+1}, k_{t+2}, g_{at+1}, g_{nt}) + s_t^{t-2}(k_{t-1}, k_t, k_{t+1}, g_{at}, g_{nt-1})}{(1+g_{at+1})(1+g_{nt-1})(2+g_{nt})}.$$

To solve for the steady-state growth paths, given the productivity and demographic parameters of each state, I find the invariant capital (p.e.u.l) level that satisfies this nonlinear equation.[10]

To compute the transition between steady states I use a method of time path iteration. I first guess a transition path of capital (p.e.u.l) $\{k_t^g\}_{t=1}^N$ that goes from the first to the second steady state. Then, I compute the prices of this transition path and given these prices solve for the optimal savings of young and middle-aged workers (per effective unit) using their policy functions. Next, I aggregate these savings into capital $\{k_t^i\}_{t=1}^N$ using the demographic structure and the law of motion of capital. Finally, I compare the the guessed $\{k_t^g\}_{t=1}^N$ and the resulting capital paths $\{k_t^i\}_{t=1}^N$; if they do not coincide I update the guess, repeat, and iterate until convergence.

The open economy

In the open economy the variables that summarize the equilibrium are capital and foreign asset holdings per effective units of labor. Capital is either pinned down by the after-tax interest rate of international assets or by the equilibrium equation of the closed economy given the history of transfers.[11] Foreign assets (p.e.u.l) f_t satisfy the assets

[10] Since, in this model, variables grow at the product of the growth rate of newborns and technology, we define detrended variables per effective unit of labor. In the steady states the detrended variables are constant and the aggregates grow at a constant rate of $(1+g_n^{ss})(1+g_a^{ss})$.

[11] Equations 3.15, 3.16, and 3.17.

market clearing condition:

$$k_{t+1}+f_{t+1} = \frac{s_t^{t-1}\left(\{k_{t+1}\}_{i=0}^{2}, g_{at+1}, \{g_{nt+i}\}_{i=-1}^{1}, f_t, f_{t+1}\right)}{(1+g_{at+1})(2+g_{nt})} + ...$$
$$\frac{s_t^{t-2}\left(\{k_{t+1}\}_{i=-1}^{1}, g_{at}, \{g_{nt+i}\}_{i=-2}^{0}, f_{t-1}, f_t\right)}{(1+g_{at+1})(1+g_{nt-1})(2+g_{nt})},$$

where the savings equations of young and middle-aged workers s_t^{t-1} and s_t^{t-2} depend on foreign assets and additional demographic variables (since transfers themselves depend on those). We thus have a third-order difference equation on k_{t-1}, k_t, k_{t+1}, k_{t+2} and f_{t-1}, f_t, f_{t+1}, f_{t+2}.

To solve for the steady state, I first solve for capital (p.e.u.l), either equalizing the marginal productivity of capital to the after-tax international interest rate, or using the steady states of autarky. If the level of steady-state capital (p.e.u.l) of autarky is smaller (higher) than the level of capital (p.e.u.l) of the open economy and the net marginal productivity of autarkic capital is higher (smaller) than the after-tax foreign interest rate, the economy will be a borrower (lender).[12] Otherwise the capital of the steady states is the autarkic capital calculated above. Then, if the economy starts as being open, I solve for foreign assets by finding the invariant level $f*$ that satisfies this third-order difference equation.

To compute the transition in the open economy, for each period I use the conditions that determine if the economy will be borrowing, lending, or in autarky (equations 3.11, 3.12, and 3.13). Then I solve for foreign asset holdings f_{t+1} using the above difference equation given f_{t-1} and f_t.

In particular in the numerical exercise of Mexico, for low levels of taxes, i.e. "the open taxed economy," the economy begins as a borrower, then moves to autarky, and finally ends as a lender. In each period I compare the cost/return of being in autarky against the after-tax international interest rate and I check for feasibility of the assets market clearing condition. As the economy moves along the demographic transition, the autarkic return on capital increases to the point where the economy would rather

[12]If the supply of capital in autarky is smaller than the supply in open markets and the cost of financing capital inside of the economy is higher that the cost of borrowing, the economy will borrow. If instead the supply of capital in autarky is higher than in open markets and its return is higher in open markets, then the economy will lend.

be a lender. For this to happen, domestic savings need to increase beyond the capital of autarky, which takes some time. During this time the economy is in autarky until savings are high enough to become a borrower.

As soon as the autarkic return on capital rises beyond the after-tax international cost, the economy stops borrowing. Then, to determine if it will remain in international markets or move to autarky, I compute the savings of being a lender (given the history of transfers and capital) and check if savings are higher than the capital level of being a lender. If they are not, the economy cannot lend and it will move to autarky. Next, to determine how many periods it will remain in autarky, I guess the number of periods and compute savings given past and future transfers. This implies a path of capital, given by the law of motion of autarky along this transition and by the capital level of being a lender after the guessed period. Finally, I check if savings are higher than capital in the guessed period; if they are not, I update the guess and repeat.