Moment of Inertia Estimation Using a Bifilar Pendulum

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Abstract

The objective of this project was to investigate experimental methods for estimating rotational moments of inertia. The moments of inertia of an aircraft are important in understanding its aerodynamic properties and thus its translational and rotational motion during flight. A current method used in the Unmanned Aerial Vehicle (UAV) Laboratory to estimate moments of inertia includes a bifilar pendulum, which will be described in this report. An investigation of the bifilar pendulum includes determining the accuracy of the experiment and understanding its experimental process. It was found that the variance for ten experiments was small, allowing confidence to be had when estimating moments of inertia of a given aircraft. However, it should be noted that uncertainty in aircraft properties could affect the comparative analysis between analytical values and experimental results. Additionally, this investigation provides insight into the experimental process of moment of inertia estimation and motivates future research in the area.

Overview

A bifilar pendulum consists of suspending an aircraft from two parallel wires, or filars, that allow it to rotate freely about a given axis. The experiment is to measure the moment of inertia for the axis of rotation parallel to the filars. A small moment is then applied to the aircraft to measure its period of oscillation, which allows further calculation of its angular frequency, as denoted by omega (ω). The moment of inertia can then be calculated using the following equation.

$$I = \frac{mgd^2}{4L\omega^2} \tag{1}$$

Where m is the mass of the aircraft, g is the acceleration due to gravity, d is the distance between the filars, and L is the length of the filars. Equation 1 was obtained from the nonlinear mathematical model of a bifilar pendulum, which was developed from first principles using Lagrange equations.[1]

It is important to verify that the center of gravity of the aircraft is aligned with the points of suspension (i.e. the plane of the filars); a misalignment can lead to significant errors. The error associated with a misaligned center of gravity can be bound by the parallel-axis theorem.

Procedure

The filars were attached parallel to and on opposite sides of the center of gravity via hooks (Figure 1). The hooks were approximated as point masses and their moments of inertia

Figure 1: Filar suspension points

were subtracted from the total inertia of the model. Kevlar strings were used for the filars, and were attached above to pivot points consisting of metal screws atop a metal plate to minimize frictional forces, which cause damping, during oscillations (Figure 2). The model

Figure 2: Pivot points

used for the experiment was constructed using PVC pipe and configured as a cross (Figure 3). To verify the location of the center of gravity, a level tool was used to measure the tilt of the model (e.g. a center of gravity ahead of the points of suspension would result in a forward tilt). Since the tilt was found to be zero, it was known that the center of gravity was aligned with the points of suspension. Per experiment, the period for 10 oscillations was measured using a stopwatch for 6 trials. The average value was then used to calculate its frequency. Observations shown that the period for 3, 5, and 10 oscillations never exceeded a difference of more than one hundredth of a second; the minimal amount of damping in the system resulted in a less than 0.01% error in the moment of inertia and consequently was neglected. To reduce random error, the filar lengths were adjusted to different heights per experiment. A total of 10 experiments were conducted. Table 1 lists the mass and length measurements for the setup.

Table 1: Setup mass and length measurements

Property	Value
Mass (m)	2.150 kg
Distance between filars (d)	.13452 meters

Figure 3: Bifilar pendulum setup

Additionally, a drawing of the model seen in Figure 1 was created in the CAD program, SOLIDWORKS, for analytical purposes (Figure 4). The drawing was able to verify that the center of gravity lies symmetrically, in the plane of the filars.

Results

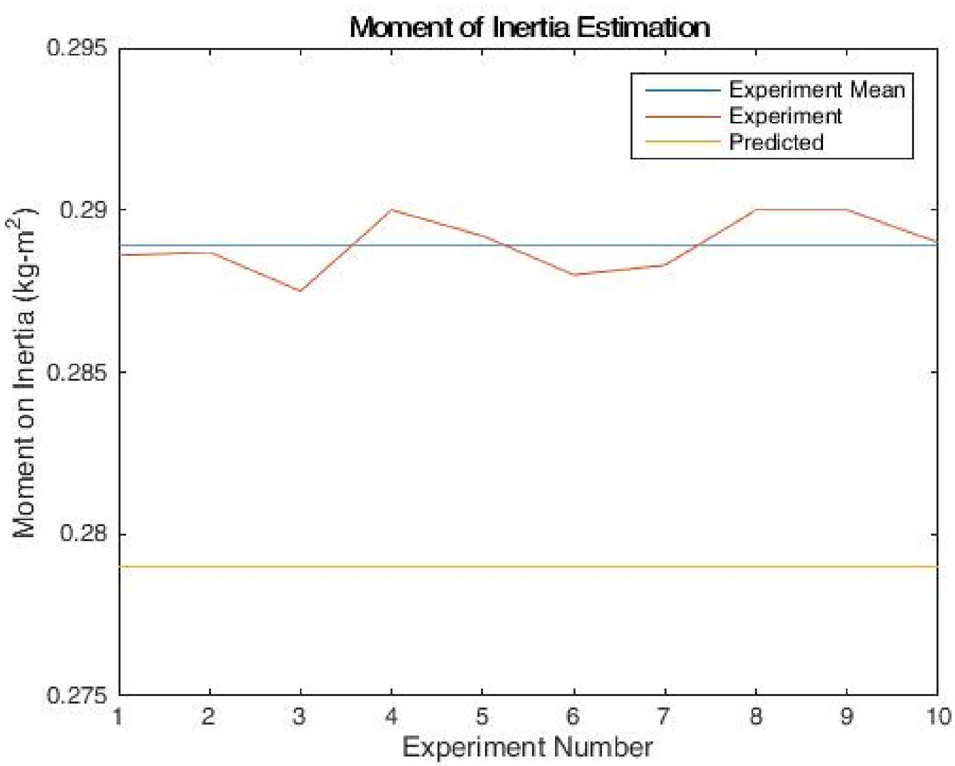
Table 2: Bifilar testing data

Trial	Average Period (seconds)	Length of filar (L) (meters)	Moment of inertia (kg-m²)
1	132.51	1.4700	0.2886
2	133.54	1.4923	0.2887
3	133.45	1.4963	0.2875
4	134.31	1.5027	0.2900
5	134.19	1.5042	0.2892
6	134.23	1.5113	0.2880
7	134.67	1.5199	0.2883
8	135.25	1.5240	0.2900
9	134.34	1.5034	0.2900
10	130.01	1.4130	0.2890

Table 3: Moment of Inertia Results

Predicted	0.2790 kg-g²
Experimental	Mean: 0.2880 kg-g² Standard Deviation (σ): 8.81 Percent Error: +3.6%

Graph 1: Plot of Moment of Inertia Results

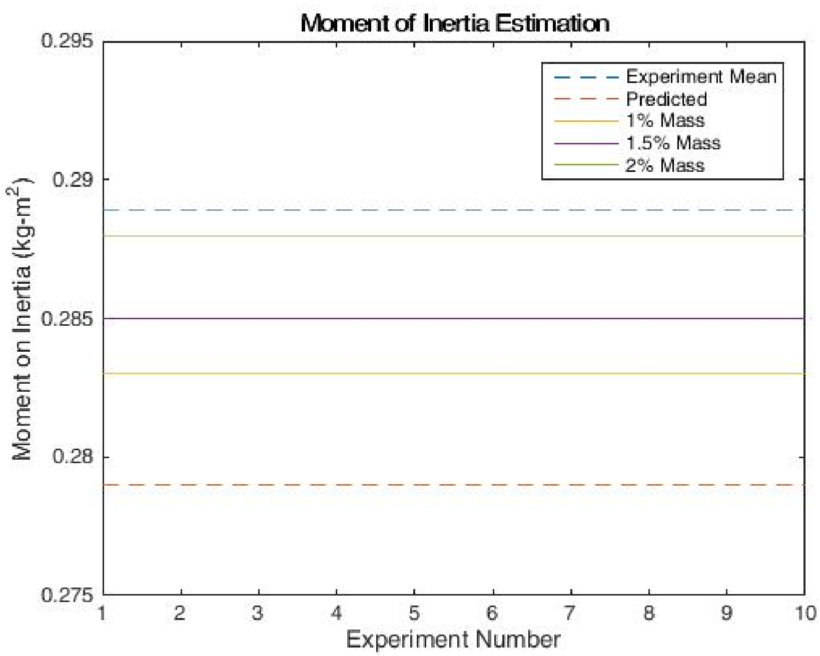


Discussion

For 10 bifilar pendulum experiments, the mean of the moment of inertia values was 0.2880 (kg-g^2) with a standard deviation of 8.81. Note that more than 20 total experiments were conducted (not shown in Graph 1). However, the experiments that yielded moment of inertia values greater than 3σ were deemed to be invalid, and were discarded. The discarded data was found to have sources of error that included unequal filar lengths, measurement error, un-parallel filars, or a center of gravity that is misaligned with the plane of the filars. It was found that a level tool is useful to verify that the model is laterally level, which ensures an equal length of the filars. Additionally, it is useful to establish reference points, for the pivots, that are equivalent in length to the distance between the filars, to ensure that the filars are parallel to each other.

The source of the +3.6% bias that occurs between the experimental mean and predicted values is assumed to be due to the non-uniform density of the PVC pipes used in the experimental model; the density of the PVC pipes is assumed to be uniform in the analytical model. In a further analysis, it was found that the moment of inertia followed a linear relationship for small variations in the mass distribution (Graph 2).

Graph 2: Mass Variance for PVC Pipes of Non-Uniform Density



The small variations in mass distribution were modeled as the following. In SOLIDWORKS, small percentages of the pipe's mass were concentrated at the end of each pipe. This concentrated mass was modeled as a small washer with the mass of 1%, 1.5%, and 2% of each pipe's mass. For each instance, the moment of inertia value was recorded.

Furthermore, it was found experimentally that flipping the orientation of the pipe-arms yielded a significant change in the moment of inertia. Both of these analyses suggest that the PVC piping is in fact non-uniform.

Conclusion

The variance for 10 given bifilar experiments proved to be small, allowing confidence to be had when estimating the moments of inertia of a given aircraft. However, it should be noted that uncertainty in the aircraft properties could affect the comparative analysis between the analytical and experimental results. Additionally, this investigation provides insight into the experiment process for estimating moments of inertia and motivates future research in the area, which includes an experimental method for estimating moment of inertia tensors.

References

1. Gupta, A. (2013). *Swing Tests for Estimation of Moments of Inertia.* Department of Aerospace Engineering & Mechanics, University of Minnesota.

Appendix

Figure 4: SOLIDWORKS drawing of PVC model