Measurement Error Effects on Estimates from Linear and Nonlinear Regression Whole Stand Yield Models in Minnesota

by

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Table of Contents

Page

List of Tables

List of Figures

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Abstract

Forest stand models provide practitioners and managers with tools for projecting future yields given specified starting conditions. In Minnesota, Walters and Ek (1993) developed a system of linked yield equations for several forest attributes that only require stand age (years) and site index (ft) as input variables. However, these variables are difficult to measure with accuracy, and use of erred input values may produce biased results. Therefore, this study conducted a sensitivity analysis to examine the effect of measurement error on outputs from each model. Measurement error came from two sources: 1) systematic error (from 0 to ±50% of the original input value) and 2) stochastic error (error normally distributed with mean zero and standard deviation 0 to 30% of the input value). Before running the models, error was added to stand age, site index, or both, and the results were compared to the *true* model estimates. In addition, two hypothetical management scenarios were provided to exemplify the implications of using affected model output.

Results showed the models had mixed sensitivity to the various sources of measurement error. Large systematic measurement error in one input variable tended to produce moderate to large percent changes in all models. In particular, errors were the most pronounced in the height and volume equations (often >50% change in projections). With systematic error in both input variables, the effect was magnified, especially for young, less productive stands. Extreme measurement errors produced the largest impacts (though positive error rates in site index tended to offset equivalent negative error rates in stand age) (due to original coefficient values). The results for stochastic error showed that although most models had a nonlinear form, the observed bias was negligible due to canceling errors. However, the variability in model estimates was extreme (particularly in the height and volume models at young ages and low site indices), with several relative standard errors surpassing 50% of the model estimate. Finally, the management case study revealed the potential of large differences in projected revenue in the presence of low to moderate measurement error. Collectively, these results suggest that moderate to high systematic measurement error will considerably alter model estimates, and although random measurement error will not bias results (on average), the resulting inflation of variability from even low additional error will substantially increase uncertainty in model forecasts. Further, the study highlights the necessity of careful crew training before acquiring stand ages and site indices.

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1. Introduction

The Lake States region has a long history of forest growth and yield model development (e.g., Brown and Gevorkiantz 1934; Gevorkiantz and Duerr 1938; Ek and Brodie 1975; Dixon and Keyser 2008; Zobel et al. [In press]). In particular, the Walters and Ek (1993) model has seen use by the Minnesota Department of Natural Resources (MNDNR) and Minnesota Forest Industries (MFI), among others. The model represents a system of linked yield equations that describe several forest stand attributes (basal area, quadratic mean diameter, trees per acre, stand height, gross volume, merchantable volume, and biomass). Walters and Ek (1993) fit these models using linear and nonlinear regression forms and constructed the system so that the only required inputs were stand age and site index. Calibration and validation data came from the 1977-1990 USDA Forest Service, Forest Inventory and Analysis (FIA) database for Minnesota (USDA 2012).

Both linear and nonlinear regression techniques assume the explanatory variables are exact and have no error in the measured values. However, this may not be a tenable assumption for the Walters and Ek (1993) FIA calibration data, particularly for stand age. When describing this variable, O'Connell et al. (2014) states, "Age is difficult to measure and therefore [stand age] may have large measurement errors" (p. 57). The similar (though independent) procedures for determining site index may also lead to considerable measurement error. If this error does exist, the regression slope (rate) coefficients are smaller than those fitted to data without measurement error (due to the increased variability in the independent variables). This results in flatter model curves and estimates that suffer from attenuation (biased towards zero). When regression models include independent variables with measurement error, interpretation of model coefficients may lead to faulty conclusions, but model estimates will remain unbiased (if the same affected variables are used during projections) (Carroll et al. 2006). Since the Walters and Ek (1993) models are predictive rather than descriptive, measurement errors similar to those found in FIA will not affect projections (on average).

Further, assuming the measurement error is random and normally distributed, the affected explanatory variables may be defined as $\mathbf{X}^* = \mathbf{X} + \mathbf{e}$, where $e_i \sim N(0, \sigma_i^2)$ and i = stand age or site index. Thus, the forestry models all have the form $y = f(\mathbf{X}^*, \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ represents the vector of parameters. Note that for stand age and site index, forest inventory efforts may never obtain $\mathbf{X}$, but rather $\mathbf{X}^*$, due to the difficulty in measuring these attributes. Current (and most likely future) techniques involve some level of subjectivity. A standardized process does not exist for measuring each attribute, and creation of such a methodology would be inefficient in many situations. Even well-trained, experienced crews may select different trees to represent the stand, leading to similar yet different values for age and productivity. Thus, most if not all measurements of stand age and observations of site index will come from the erred distributions. For practical purposes, $\mathbf{X}^*$ is treated as $\mathbf{X}$ in Walters and Ek (1993) and many other forestry models.

When applying the Walters and Ek (1993) models, input values must come from $\mathbf{X}^*$ (rather than $\mathbf{X}$) to produce credible predictions. However, what if the measured values of $\mathbf{X}^*$ have additional error (e.g., abnormal measurement error occurs during data collection)? The observed variables are now $\widetilde{\mathbf{X}} = \mathbf{X}^* + \mathbf{e}^*$, where $\mathbf{e}^*$ may follow any pattern. For example, FIA crews likely have the most intense training and preparation of any field crews in the United States. Measured values are subject to strict tolerance levels, and the USDA Forest Service maintains high levels of quality assurance/quality control. Even for these crews, measurement error in stand age and site index still occurs due to the subjectivity of representative tree selection (data follows $\mathbf{X}^*$). However, for less experienced crews, or those with improper training, measurement of stand ages and observation of site indices may suffer from abnormal measurement error (data follows $\tilde{\mathbf{X}}$). Also, if crews use measurement techniques that differ from those used in FIA, then observed stand ages and site indices come from the distribution of $\tilde{\mathbf{X}}$. In this case, however, $\mathbf{e}^*$ arises from a combination of "technique error" and measurement error. Whatever the source, this abnormal error (hereafter referred to as measurement error) may be random or systematic. The interest lies with how $\mathbf{e}^*$ (with various patterns) affects Walters and Ek (1993) model projections. From an applied perspective, how do stand projections differ when using $\tilde{\mathbf{X}}$ rather than $\mathbf{X}^*$? The answer may have significant management implications.

This study seeks to examine the effect of measurement error (in both stand age and the variables used to compute site index) on Walters and Ek (1993) model estimates through a sensitivity analysis. The initial effort will focus on the aspen forest type (due to its commercial importance in Minnesota) and will follow two steps: (1) an analytic solution will be sought for directly computing measurement error and its impact, with various levels of measurement error propagated through the system; if no solution is readily available, (2) simulation techniques will approximate the effects of different levels of measurement error. Simulation techniques will closely follow the work of Gertner and Dzialowy (1984), who conducted a similar study for testing the Lakes States variant of the USDA Forest Service, Forest Vegetation Simulator (LS-FVS). They considered both systematic and stochastic measurement error propagated through the component models and across the entire projection system. Results from this study will help guide current and potential users of the Walters and Ek (1993) models (and similar mathematical equations) as to the credibility and utility of projection estimates when inputting variables with abnormal measurement error.

Note that consideration of the interaction between model residual standard error and that error introduced from poor measurements was beyond the scope of this study. Walters and Ek (1993) provide a ratio adjustment that essentially shifts the average yield curve to pass through the stand values of interest at time zero. This ratio between estimated and observed values is then used to adjust future projections, with the intention of providing more realistic results. This study assumed (1) users will employ the ratio adjustment in their projections and (2) after adjustment, stands at least roughly follow the average development pattern (as modified by the ratio). Under these assumptions, the models produce fairly accurate estimates (all residuals negligibly different from zero) and any variability in estimates is due to the introduced measurement error.

2. Methods

2.1 Models

The objective of Walters and Ek (1993) was to provide a set of forecasting models that allowed for easy projection of forest attributes using readily available inventory data. Equations were fit using both linear and nonlinear regression techniques for several stand level variables. The models of most interest are given below.

$$B = a_1 S^{a_2} A^{a_3} \quad (1)$$

$$D = b_0 + b_1 A + b_2 S \quad (2)$$

$$N = \exp(c_0 + c_1 \ln(D)) \quad (3)$$

$$H = g_1 S^{g_2} (1 - e^{g_3 A})^{g_4 S^{g_5}} \quad (4)$$

$$V = d_1 B^{d_2} H^{d_3} \quad (5)$$

where B = cohort basal area (ft^2/ac) for trees ≥ 0.95-in diameter breast height (dbh), S = site index (ft), A = stand age (years), D = stand quadratic mean diameter (in), N = number of trees per acre, H = average total height (ft) of dominant/codominant trees in the stand, V = gross volume (ft^3/ac) of trees ≥ 5-in dbh above a 1-ft stump, and a_1-a_3, b_0-b_2, c_0-c_1, g_1-g_5, and d_1-d_3 are forest type specific parameters (see Walters and Ek (1993) for coefficient estimates). Ek (1971) provided the height model as parameterized by Hahn (1984).

This system of linked equations requires only stand age and site index as input variables, though others may be used if available. Note that site index is a function of age and height, thus suggesting site index and stand age are interdependent (i.e., measurement error in stand age should also lead to error in site index). However, according to the FIA field guide, determining site index and stand age are independent processes (USDA 2012). For stand age, the measurement error pertains less to tree ring counts and more to the selection of cored trees and estimation of the percentage of the stand they represent. Deriving site index also involves selection of representative trees, but these 'site trees' form permanent records for comparisons to future remeasurements. Although similar processes, the two groups of cored trees are chosen independently (except in unique circumstances).

2.2 Measurement Error

This research will focus on two types of error: (1) systematic measurement error and (2) stochastic measurement error. The former relates to situations where the measurement device has not been calibrated (e.g., for tree heights) and provides erroneous readings consistently off in the same direction and by the same magnitude. Another example includes poor field crew training and data collection such that representative trees and site trees are selected that provide consistently low (or high) age and site index values, respectively. In addition, use of different data collection techniques may lead to systematic differences in observed stand values.

Systematic measurement errors were added to observations of age only, site index only, and both age and site index. The latter allows for comparison of the interaction between measurement error in multiple, independent input variables. Following the protocol within Gertner and Dzialowy (1984), model runs included error rates of ±10%, 20%, 30%, 40%, and 50% (of the true value of each observation). Results were then compared with runs having no measurement error (see Figure 1 for a schematic demonstrating the general effect of systematic measurement error on model estimates).

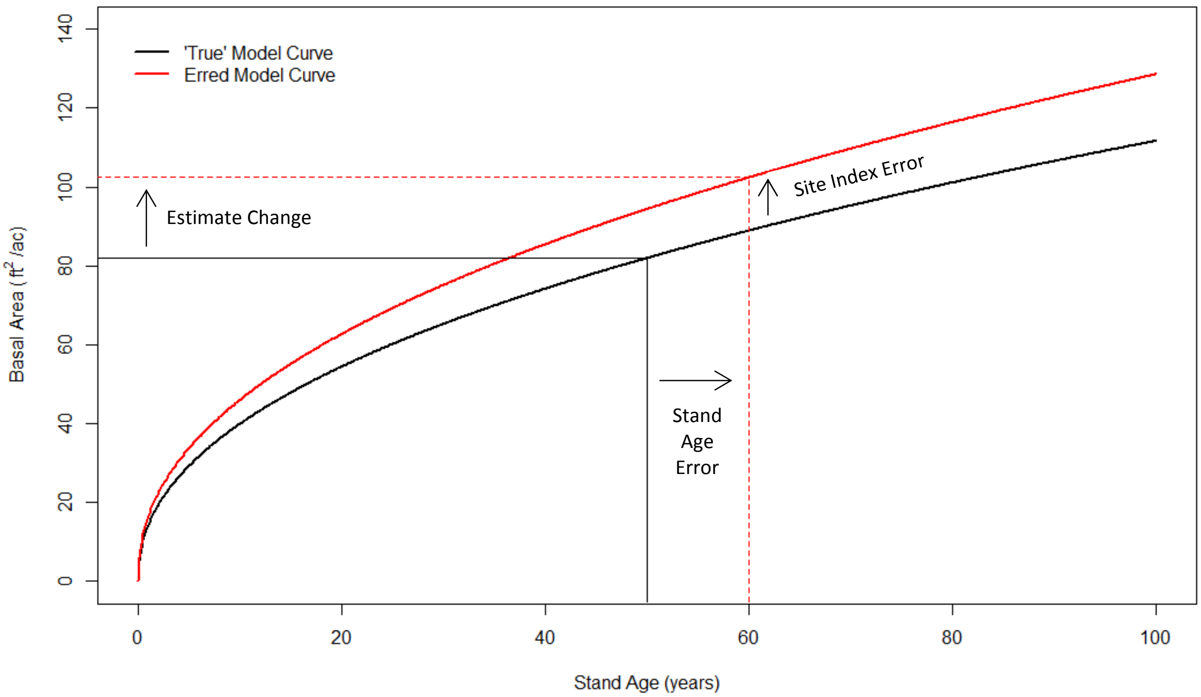

Figure 1. Schematic of the measurement error effect on Walters and Ek (1993) model projections. The example shown is for a 50-year old, 60-foot site index aspen stand with (red curve) and without (black curve) +20% systematic measurement error in both variables.

Stochastic measurement error occurs in an unspecified, often unrepeatable pattern that most often arises from crew misuse of equipment or inconsistent judgment during data collection. This error occurs randomly with no set direction or magnitude. For example, as the day and/or field season continues, increasing crew sloppiness or fatigue may negatively impact the quality and precision of measurements. Another example includes the subjective selection of representative trees and site trees by inexperienced or poorly trained crews under widely varying stand conditions. These situations make the choice of trees difficult (if not impossible) to standardize and obtain accurate data. Finally, use of different data collection techniques may lead to distributions of stand values with the same center but different spreads.

Stochastic errors were added to observations of age only, site index only, and both age and site index. Unlike systematic error, these errors represent random deviations (in either direction) from the true values, and thus the errors may cancel given an appropriate model form and error structure. However, regardless if errors cancel, the introduced standard error of model estimates will increase proportionally to the measurement error. To investigate both outcomes, random errors were added to the *true* data before running the models. These errors were normally distributed with mean zero and standard deviation 0%, 5%, 10%, 15%, 20%, 25%, or 30% of the true observed value (Gertner and Dzialowy 1984). Results were then compared to those without measurement error to observe the extent of noncanceling errors (bias) and the inflated standard errors of the estimates.

2.3 Analytic Solution

Quantifying the relationship between measurement error and model estimates may have a straightforward mathematical solution (particularly for the simple equations in Walters and Ek (1993)). If so, those terms provide a direct method for calculating the effects of any level of measurement error without the need for simulated results. Therefore, attempts were made to solve the Walters and Ek (1993) equations to provide simple expressions for describing the effects of measurement error. Initial investigation showed that analytic solutions were available for all models under systematic measurement error. However, for stochastic error, all attempts at finding an analytic solution proved unsuccessful, and thus the effect of random measurement error was estimated via simulation.

2.4 Simulation Solution

Monte Carlo simulation techniques were used to observe the extent of noncanceling errors and to quantify the inflation of variability in estimates due to the addition of random measurement error. Running simulations for every combination of possible stand age and site index was prohibitive, but four hypothetical stands were created to provide coverage of the general relationship between input values and measurement error. The four combinations included (A = 20 years, S = 40 ft), (A = 20 years, S = 60 ft), (A = 50 years, S = 40 ft), and (A = 50 years, S = 60 ft). The last combination represents the traditional rotation age for aspen in Minnesota and average site index class. Since most of the yield curves maintain a rough linear form after the young ages, the upper extremes (e.g., A = 80, S = 80) were not investigated, as the effect of measurement error will resemble that for moderate stand ages and site indices. Also, aspen is a short-lived species in Minnesota, with many stands deteriorating soon after rotation age.

During each iteration, random measurement error was added to the stand age and site index for each stand. The Walters and Ek (1993) models were then run using the modified data, and deviations were computed between model estimates with and without measurement error. The general simulation model and its components are provided below.

$$\tilde{y}_{i,j,k} = WE|_{\tilde{A}_{i,j},\tilde{S}_{i,k}} \quad (6)$$

$$\tilde{A}_{i,j} = A_i + \alpha_{i,j} \quad (7)$$

$$\tilde{S}_{i,k} = S_i + \alpha_{i,k} \quad (8)$$

$$d_{i,j,k} = \tilde{y}_{i,j,k} - \hat{y}_i \quad (9)$$

where $\tilde{y}_{i,j,k}$ = model estimate of the attribute of interest using observations from the ith stand and from the jth trial (stand age) and kth trial (site index); WE = Walters and Ek (1993) model of B, D, N, H, or V; $\tilde{A}_{i,j}$ = modified value of stand age for the ith stand from the jth trial; $\tilde{S}_{i,k}$ = modified value of site index for the ith stand from the kth trial; $\alpha_{i,j}$ = random measurement error for the ith stand from the jth trial and $\alpha_{i,j} \sim N(0, \varphi_j A_i)$; $\alpha_{i,k}$ = random measurement error for the ith stand from the kth trial and $\alpha_{i,k} \sim N(0, \varphi_k S_i)$; φ_j = specified coefficient of variation for the jth trial; φ_k = specified coefficient of variation for the kth trial; $\hat{y}_i$ = model estimate for the ith stand without measurement error; $d_{i,j,k}$ = deviation between the erred model estimate of interest and the 'true' model estimate for the ith stand from the jth and kth trial; i = 1, 2, 3, and 4; j = 1, 2, …, 7 (0%, 5%, 10%, 15%, 20%, 25%, 30%); k = 1, 2, …, 7 (0%, 5%, 10%, 15%, 20%, 25%, 30%).

Note that measurement error was added independently such that each level of error in stand age (jth trial) was combined with each level of error in site index (kth trial) (totaling 49 combinations). Each simulation used 10,000 iterations, totaling 49 * 10,000 * 4 = 1.96 x 10^6 records. The behavior of the deviations and the standard errors of the estimates (standard deviation of iterated values) were used to describe the effect of stochastic measurement error. All simulations were run in R using custom functions (Appendix 3) (R Development Core Team 2011).

2.5 Management Scenario

Two hypothetical management scenarios were constructed to demonstrate the impacts of measurement error on revenue from timber management. Table 1 provides the particulars of interest for a 6,000 acre tract in Minnesota. Data postulation included various levels of stand size (acres), age, and site index. Using the results from the analytic and simulated solutions, systematic measurement errors of 10% (in stand age and site index) and random measurement errors with 10% variability were introduced into each stand. Per acre and total volumes were estimated, along with their associated gross revenues using the most current price sheet (MNDNR 2015).

Table 1. Tract descriptions for a hypothetical forest management situation in Minnesota. Total tract size is 6,000 acres.

Stand #	Size (acres)	Age (years)	Site Index (ft)
1	3,000	40	65
2	500	80	55
3	1000	20	60
4	300	30	65
5	200	65	70
6	1000	5	65

3. Results and Discussion

3.1 Analytic Solutions

Analytic solutions were readily available for isolating the systematic measurement error impact within each model (Table 2). Appendix 1 contains the derivations of these mathematical solutions.

Table 2. Systematic measurement error analytic solutions to the basal area (B), quadratic mean diameter (D), trees per acre (N), height (H), and gross volume per acre (V) equations. Note that α_1 and α_2 represent the percent measurement error (expressed as decimals) in stand age and site index, respectively, and that the solutions give percent changes in model estimates due to measurement error.

Model	Age Only	Site Index Only	Both Age & Site Index
B	$(1+\alpha_1)^{a_3}-1$	$(1+\alpha_2)^{a_2}-1$	$(1+\alpha_1)^{a_2}(1+\alpha_2)^{a_3}-1$
D	$\alpha_1 b_1 A/D$	$\alpha_2 b_2 S/D$	$(\alpha_1 b_1 A+\alpha_2 b_2 S)/D$
N[a]	$\left(1+\delta_{D_A}\right)^{c_1}-1$	$\left(1+\delta_{D_S}\right)^{c_1}-1$	$\left(1+\delta_{D_{AS}}\right)^{c_1}-1$
H[b]	$\left(\frac{1-e^{f_1(1+\alpha_1)}}{1-e^{f_1}}\right)^{f_2}-1$	$\frac{(1+\alpha_2)^{f_3}}{(1-e^{f_1})^{f_4}}-1$	$\frac{(1+\alpha_2)^{f_3}\left(1-e^{f_1(1+\alpha_1)}\right)^{f_5}}{(1-e^{f_1})^{f_2}}-1$
V[a]	$\frac{\left(1+\delta_{B_A}\right)^{d_2}}{\left(1+\delta_{H_A}\right)^{-d_3}}-1$	$\frac{\left(1+\delta_{B_S}\right)^{d_2}}{\left(1+\delta_{H_S}\right)^{-d_3}}-1$	$\frac{\left(1+\delta_{B_{AS}}\right)^{d_2}}{\left(1+\delta_{H_{AS}}\right)^{-d_3}}-1$

[a] δ_{i_j} = percent change in the 'true' ith model estimate due to measurement error in the jth variable, where i = B, D, or H and j = A, S, or both (AS).
[b] $f_1 = g_3 A$, $f_2 = g_4 S^{g_5}$, $f_3 = g_2$, $f_4 = f_2(1-(1+\alpha_2)^{g_5})$, $f_5 = f_2(1+\alpha_2)^{g_5}$

Initial examination of Table 2 reveals several noteworthy results. Only the basal area (B) model has percent changes independent of the input variable magnitude. Regardless of the value of stand age and site index, the percent change remains constant for given levels of measurement error. For the other models, the effect of measurement errors depends on the values of the input variables. Increasing age or site index in the trees per acre (N) and quadratic mean diameter (D) models (at a given level of measurement error) leads to larger deviations between the true estimates and those affected by measurement error. In the height (H) and gross volume (V) models, increasing the magnitude of the input variables leads to reductions in the percent change between the original and erred estimates. To help demonstrate this behavior, the percent change from *true* model estimates was computed at each level of introduced systematic measurement error in stand age or site index (Figure 2 and Appendix 2, Tables A2.1 and A2.2).

Figure 2 and Tables A2.1 and A2.2 also show the effect of model form on the severity of measurement error direction. For D, the linear form led to equality of positive and negative errors at each level of measurement error. However, in the remaining models, the nonlinear forms led to negative measurement errors (those estimates smaller than the truth) generally having a larger effect on model estimates than positive errors. The increased effect ranged from 4.7% to 38.4% higher in absolute percent change when compared to the extremes in stand age (±50% error). For site index, the added projection error ranged from 4.7% to 17.8% for the extremes. The only exception was the volume model with slightly higher percent changes at the positive extreme (up to 2.8% higher). Also, the model for N showed the only opposite effect, where positive measurement errors led to negative percent changes in model estimates. All other models had percent changes in the same direction as the measurement error.

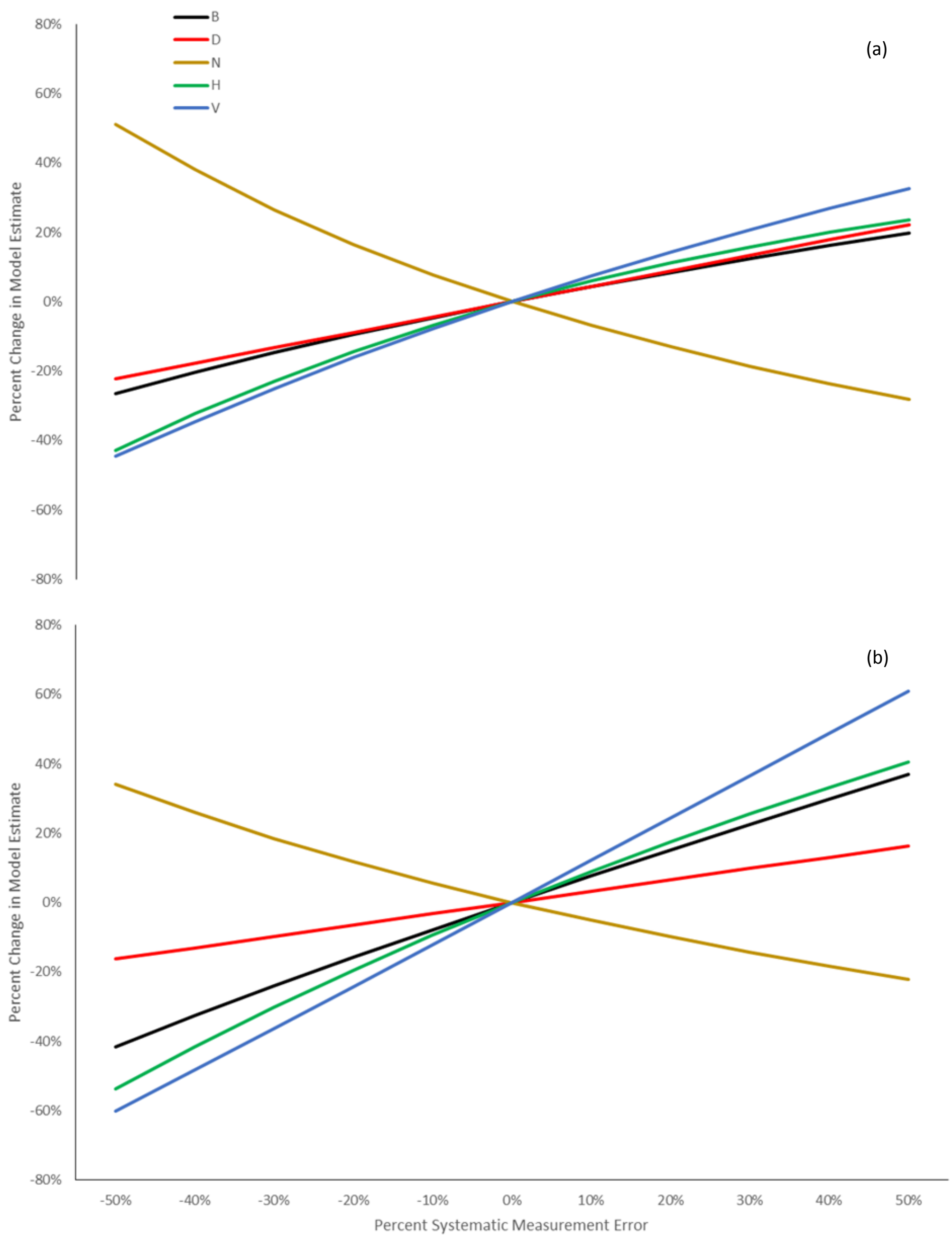

Figure 2. Percent changes in model estimates due to the introduction of various levels of systematic measurement error. Results are given for errors in (a) stand age only and (b) site index only as used in the basal area (B), quadratic mean diameter (D), trees per acre (N), height (H), and volume (V) models.

Table 2 also provides expressions for the joint effect of measurement error in both stand age and site index. These expressions show that the measurement error effects in each variable are multiplicative, except for D (where they are additive). Table 3 shows summary results for the extremes using the four hypothetical stands described in section 2.4. Appendix 2 contains additional tables for every combination of measurement error in the two input variables (Tables A2.3 to A2.7). (Due to the plethora of possible tables, only those for rotation age [50 years] and average site index class [60 ft] were reported in Appendix 2.)

Table 3. Percent changes between erred model estimates and the 'true' estimates for measurement error in both ***stand age and site index***. Results are presented for the most extreme combinations of measurement error and for four sets of stand age (A – years) and site index (S – ft).

Model	Stand Age Error	**Stand Age/Site Index Combination (**Site Index Error**)**							
		A = 20, S = 40		**A = 20, S = 60**		**A = 50, S = 40**		**A = 50, S = 60**	
		-50%	*50%*	*-50%*	*50%*	*-50%*	*50%*	*-50%*	*50%*
B	*-50%*	-0.571	0.005	-0.571	0.005	-0.571	0.005	-0.571	0.005
	50%	-0.299	0.640	-0.299	0.640	-0.299	0.640	-0.299	0.640
D	*-50%*	-0.316	0.032	-0.343	0.102	-0.371	-0.127	-0.385	-0.059
	50%	-0.032	0.316	-0.102	0.343	0.127	0.371	0.059	0.385
N	*-50%*	0.870	-0.051	0.999	-0.147	1.146	0.250	1.227	0.104
	50%	0.055	-0.364	0.193	-0.385	-0.178	-0.405	-0.089	-0.415
H	*-50%*	-0.992	0.053	-0.961	0.111	-0.938	-0.134	-0.856	-0.022
	50%	-0.650	2.395	-0.553	1.380	-0.277	0.867	-0.283	0.613
V	*-50%*	-0.966	0.033	-0.922	0.063	-0.900	-0.072	-0.840	-0.008
	50%	-0.594	2.074	-0.536	1.533	-0.397	1.219	-0.399	1.049

Again for B, the measurement error effect is independent of stand age or site index at a given level of error, as seen in the table. However, since site index has a larger coefficient, the measurement error in this variable (both magnitude and direction) had a larger effect on model estimates than measurement error in stand age. Given its linear form, the model for D showed symmetric patterns between opposed combinations of measurement error (e.g., (+20%, +20%) versus (-20%, -20%)), though the results had opposite signs. In addition, since stand age has a larger coefficient, the effect of error in stand age exceeded that of error in site index. This relationship was naturally carried through into the N model, but the nonlinear form produced percent changes over twice as large at the negative (rather than the positive) extremes for all ages and site indices. At a fixed level of both positive or both negative measurement error, the D and N models had the largest percent changes at the upper extremes of each input variable (i.e., error effect grew with increasing stand age and site index). However, this disparity was low to moderate (absolute differences between the two extreme stands in Table 3 was 6.9% for D and ranged from 5.1% to 35.7% for N).

Unlike D, the measurement error in site index had a much larger impact on H model estimates than errors in stand age (due to the specific coefficient values and the model form). This characteristic of the H model (and B model) carried through into the V model, with error in site index having a larger effect than error in stand age. In addition, both B and V demonstrated opposite behavior to that of D and N, with higher percent changes at the positive extremes of measurement error. This behavior continued regardless of input values for V, but reversed for H in older stands (negative extremes produced higher percent changes). In addition, the effect of measurement error increased with decreasing stand age and site index (at a fixed level of both positive or both negative measurement error). This disparity was often substantial, with absolute differences between the extreme stands ranging from 13.6% to 178.2% for H and 12.6% to 102.5% for V.

Interestingly, the effect of positive measurement error in site index tended to offset the effect of negative measurement error (of the same magnitude) in stand age for all models and input values. Closer examination, however, suggested this cancellation was due to the specific coefficient estimates for aspen. In other words, this offsetting will only occur for additional forest types with similar relationships between coefficient estimates.

3.2 Simulation Solutions

All attempts at finding analytic solutions for stochastic measurement error proved unsuccessful, and thus the effect of random measurement error was estimated via simulation. Similar to the systematic results, simulations of random measurement error produced a large volume of results. Table 4 provides summaries (maximums) for each model. Individual model results are provided in Appendix 2 (Tables A2.8 to A2.16).

Table 4. Mean deviations (estimate with measurement error—estimate without measurement error) and relative standard error of the model estimates at given levels of stand age and site index. Relative standard errors were used to facilitate comparison across models. The reported values are maximums across the 49 combinations of measurement error in stand age and site index.

Statistic	Stand Age	Site Index	Model				
			B	D	N	H	V
Mean Deviation	20	40	-0.019	n/a	0.041	0.088	0.057
	20	60			0.052	0.015	0.028
	50	40			0.069	-0.048	-0.023
	50	60			0.067	-0.052	-0.023
Relative Standard Error*	20	40	0.272	0.133	0.252	0.796	0.690
	20	60		0.150	0.295	0.556	0.557
	50	40		0.165	0.344	0.423	0.475
	50	60		0.164	0.340	0.335	0.422

* The Center for Disease Control (CDC) uses relative standard error for reporting (Klein et al. 2002). They consider relative standard error >30% excessive uncertainty and do not report results under those circumstances.

Examination of Table 4 (and Tables A2.8 to A2.11) reveals that the B model had a low maximum mean deviation (-1.9%) across all combinations of stand age and site index. With a linear form and normally distributed errors, the D model will produce mean deviations of zero (with infinite iterations), and thus no results were reported. The largest percent change for the N model was 6.9%, and this model showed a relatively increasing effect with older and more productive stands. Even with the most extreme measurement errors, the highest percent change in H was 8.8%. The result for V was similar, with the largest change being 5.7%. For the two latter models, the effect was greatest at young ages (overestimates), but steadily decreased until having a negative effect (underestimates) at older ages and higher site indices. Even with some observable trends, the overall effect of noncanceling random measurement error appears minimal. Actual deviations during model application will likely have a much smaller range, since the Table 4 results were maximums across extreme errors in one or both input variables.

Unlike mean deviations, the standard error of the model estimates proved substantial. Table 4 (and Tables A2.12 to A2.16) reveals that for the B model, the maximum relative standard error was relatively high at 27.2% across all ages and site indices. Measurement error was less impactful in D, but was still notable (16.4%). The maximum error was large in N at 34.0%, with the effect increasing in the older ages and on the more productive sites. For the H and V models, the maximum relative standard error proved extreme (79.6% and 69.0%, respectively). As before, the effect of random error decreased with increasing stand age and site index.

Collectively, these results suggest that even with low to moderate abnormal random measurement error in one or both input variables, the introduced variability (and associated uncertainty) in model estimates may be substantial. Figure 3 and Tables A2.12 to A2.16 provide similar results when considering errors in stand age or site index only, showing particularly extreme effects on H and V estimates in young/unproductive stands.

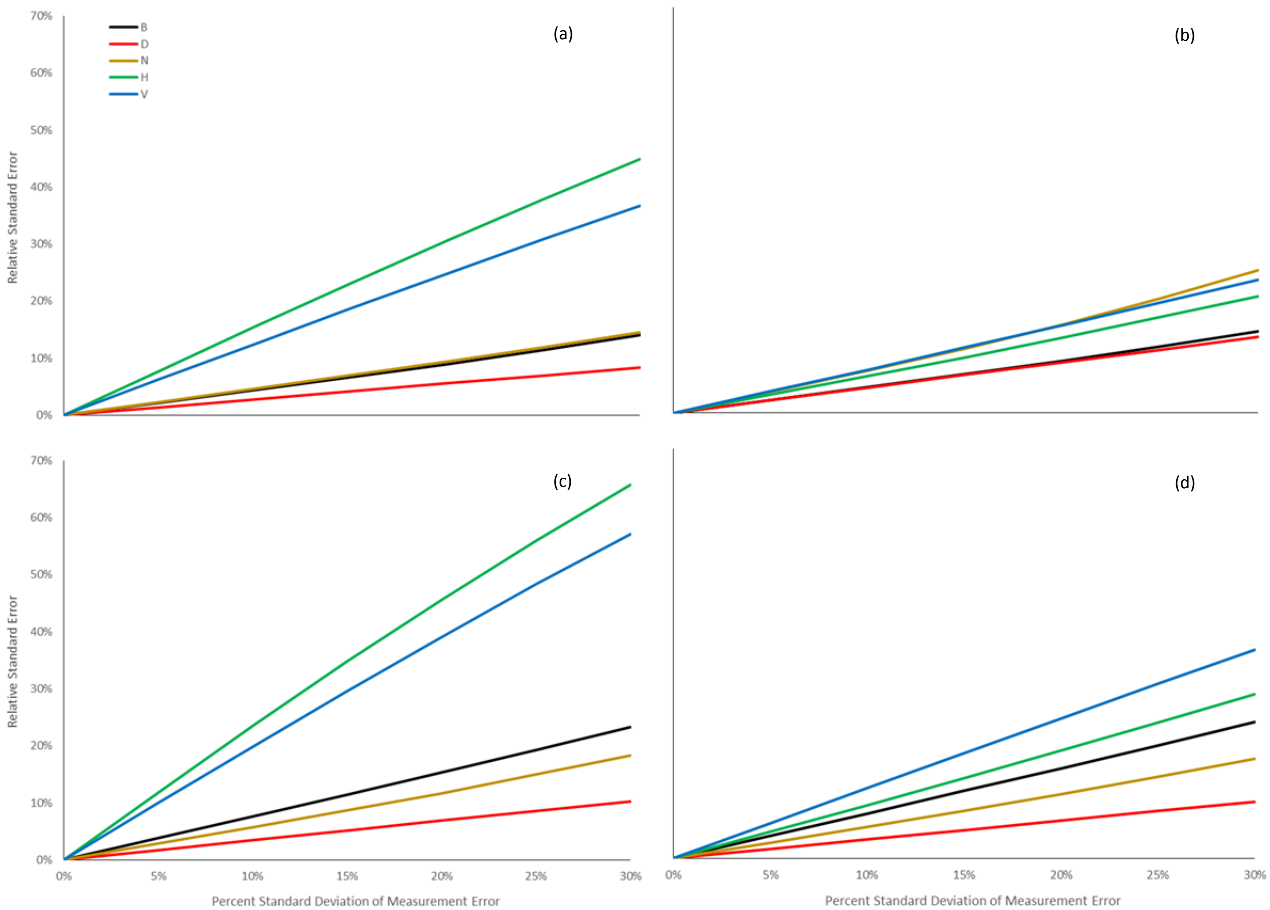

Figure 3. Relative standard error (percent) of model estimates due to the addition of various levels of stochastic measurement error in the independent variables. Results are given for errors in stand age only ([a] and [b]) and site index only ([c] and [d]) for the basal area (B), quadratic mean diameter (D), trees per acre (N), height (H), and volume (V) models. Stand age (A) and site index (S) combinations were A = 20 years and S = 40 ft ([a] and [c]) and A = 50 years and S = 60 ft ([b] and [d]).

3.3 Management Scenario

Table 5 provides the results from the management examples. For systematic errors of +10% in both stand age and site index, the effect was a $91.59/ac overestimate of gross revenue, translating to an additional $549,526 for the tract (or a 22.6% artificial increase in revenue). Introducing random measurement error (with standard deviation of 10%) produced a decrease in revenue of $89.73/ac, totaling a $538,380 decrease (-22.2% change) across the tract. Note again that this latter scenario is one example from an infinite number of possible outcomes. Overall, these results demonstrate that low to moderate abnormal measurement error may have a substantial impact on estimated yields and revenue.

Table 5. Volume per acre and total volume estimates for each stand (and across stands) when model input variables had zero, systematic, and random measurement errors. Mean (per acre) and total gross revenue estimates are provided, along with the changes between the erred estimates and the true estimates. Note that the average volume per acre across stands represents a weighted average based on stand size.

Stand	**No Measurement Error**		**Systematic Measurement Error**		**Random Measurement Error**	
	Volume/ac	*Total Volume*	*Volume/ac*	*Total Volume*	*Volume/ac*	*Total Volume*
1	16.3	48,814	19.9	59,669	10.4	31,239
2	22.7	11,375	26.9	13,462	31.0	15,507
3	7.6	7,561	9.7	9,738	5.7	5,677
4	12.7	3,800	15.8	4,727	13.6	4,086
5	25.9	5,171	30.7	6,138	15.9	3,171
6	2.1	2,078	2.9	2,906	1.6	1,640
Average/Total	13.1	78,799	16.1	96,641	10.2	61,319
Gross Revenue*	$404.50	$2,427,007	$496.09	$2,976,533	$314.77	$1,888,627
Change			$91.59	$549,526	-$89.73	-$538,380

* Price used was $30.80/cord for pulpwood and represents an average across values reported in MNDNR (2015).

4. Conclusion

The Walters and Ek (1993) models showed mixed sensitivity to the various sources of measurement error. Moderate to large systematic measurement error considerably altered projections from all models. In particular, errors were the most pronounced in the height and volume equations. With systematic error in both input variables, the effect was magnified, especially for young, less productive stands. In some cases, the changes exceeded 100%. However, positive errors in site index tended to offset equivalent negative errors in stand age (possibly unique to the aspen forest).

For stochastic error, even the most extreme error rates had negligible effect on the mean deviation between model estimates with and without measurement error. Although the errors failed to cancel in the nonlinear models, the results showed minimal change. However, the induced variability in model estimates was substantial for most models. Relative standard errors were moderate to large in the quadratic mean diameter and basal area/trees per acre models, respectively. For the height and volume models, the relative standard errors were extreme (e.g., 79.6%), with the effect much higher in young stands with lower site indices.

Finally, the management case study revealed the potential of large differences in projected revenue in the presence of low to moderate abnormal measurement error. All these results indicate the necessity of careful crew training and the inclusion of quality assurance/quality control measures (if absent). Although the presence of very small measurement errors appear to have minimal effect, increasingly large systematic or random errors will lead (or possibly lead) to highly impaired projections that may alter timings of harvest or minimum bid requirements during timber sales. Thus, practitioners using the Walters and Ek (1993) models (or similar yield models) should adequately prepare their crews for data collection (including the use of FIA procedures). Selecting representative trees with relative accuracy and consistency will help ensure unbiased model estimates without an additional source of variability.

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APPENDIX 1: Derivation of Analytic Solutions

Let α_1 = the percent systematic measurement error in stand age

α_2 = the percent systematic measurement error in site index

<u>Basal Area (B)</u>

$B = a_1 S^{a_2} A^{a_3}$

Stand Age Only:

$$\Delta B = a_1 S^{a_2} \left(A(1+\alpha_1)\right)^{a_3} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} (1+\alpha_1)^{a_3} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} ((1+\alpha_1)^{a_3} - 1)$$

$$= B((1+\alpha_1)^{a_3} - 1)$$

$$\rightarrow \Delta B/B = (1+\alpha_1)^{a_3} - 1$$ □

Site Index Only:

$$\Delta B = a_1 \left(S(1+\alpha_2)\right)^{a_2} A^{a_3} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} (1+\alpha_2)^{a_2} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} ((1+\alpha_2)^{a_2} - 1)$$

$$= B((1+\alpha_2)^{a_2} - 1)$$

$$\rightarrow \Delta B/B = (1+\alpha_2)^{a_2} - 1$$ □

Stand Age and Site Index:

$$\Delta B = a_1 \left(S(1+\alpha_2)\right)^{a_2} \left(A(1+\alpha_1)\right)^{a_3} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} (1+\alpha_2)^{a_2} (1+\alpha_1)^{a_3} - a_1 S^{a_2} A^{a_3}$$

$$= a_1 S^{a_2} A^{a_3} ((1+\alpha_2)^{a_2} (1+\alpha_1)^{a_3} - 1)$$

$$= B((1+\alpha_2)^{a_2} (1+\alpha_1)^{a_3} - 1)$$

$$\rightarrow \Delta B/B = (1+\alpha_2)^{a_2} (1+\alpha_1)^{a_3} - 1$$ □

Quadratic Mean Diameter (D)

$D = b_0 + b_1 A + b_2 S$

Stand Age Only:

$$\Delta D = (b_0 + b_1 A(1+\alpha_1) + b_2 S) - (b_0 + b_1 A + b_2 S)$$

$$= b_0 + b_1 A + \alpha_1 b_1 A + b_2 S - b_0 - b_1 A - b_2 S$$

$$= \alpha_1 b_1 A$$

$$\rightarrow \Delta D/D = \alpha_1 b_1 A/D$$ □

Site Index Only:

$$\Delta D = \left(b_0 + b_1 A + b_2 S(1+\alpha_2)\right) - (b_0 + b_1 A + b_2 S)$$

$$= b_0 + b_1 A + b_2 S + \alpha_2 b_2 S - b_0 - b_1 A - b_2 S$$

$$= \alpha_2 b_2 S$$

$$\rightarrow \Delta D/D = \alpha_2 b_2 S/D$$ □

Stand Age and Site Index:

$$\Delta D = \left(b_0 + b_1 A(1+\alpha_1) + b_2 S(1+\alpha_2)\right) - (b_0 + b_1 A + b_2 S)$$

$$= b_0 + b_1 A + \alpha_1 b_1 A + b_2 S + \alpha_2 b_2 S - b_0 - b_1 A - b_2 S$$

$$= \alpha_1 b_1 A + \alpha_2 b_2 S$$

$$\rightarrow \Delta D/D = \left(\alpha_1 b_1 A + \alpha_2 b_2 S\right)/D$$ □

Trees per Acre (N)

$N = e^{c_0 + c_1 \ln(D)}$

Stand Age Only:

Let $\delta_{D_A} = \alpha_1 b_1 A/D$

$$\Delta N = e^{\left(c_0 + c_1 \ln\left(D(1+\delta_{D_A})\right)\right)} - e^{(c_0 + c_1 \ln(D))}$$

$$= e^{\left(c_0 + c_1\left[\ln(D) + \ln(1+\delta_{D_A})\right]\right)} - e^{(c_0 + c_1 \ln(D))}$$

$$= e^{\left(c_0 + c_1 \ln(D) + c_1 \ln(1+\delta_{D_A})\right)} - e^{(c_0 + c_1 \ln(D))}$$

$$= e^{(c_0 + c_1 \ln(D))} e^{\left(c_1 \ln(1+\delta_{D_A})\right)} - e^{(c_0 + c_1 \ln(D))}$$

$$= e^{(c_0 + c_1 \ln(D))} \left(e^{\left(\ln(1+\delta_{D_A})^{c_1}\right)} - 1\right)$$

$$= N\left((1 + \delta_{D_A})^{c_1} - 1\right)$$

$\rightarrow \Delta N/N = (1 + \delta_{D_A})^{c_1} - 1$ □

Site Index Only:

Let $\delta_{D_S} = \alpha_2 b_2 S/D$

Solution is the same as for stand age alone, but replace δ_{D_A} with δ_{D_S}. □

Stand Age and Site Index:

Let $\delta_{D_{AS}} = \left(\alpha_1 b_1 A + \alpha_2 b_2 S\right)/D$

Solution is the same as for stand age alone, but replace δ_{D_A} with $\delta_{D_{AS}}$. □

<u>Height (H)</u>

$$H = g_1 S^{g_2}(1 - e^{g_3 A})^{g_4 S^{g_5}}$$

Stand Age Only:

Take the natural log and solve for the natural log difference.

$$\ln(H) = \ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$\Delta\ln(H) = \ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln\left(1 - e^{g_3 A(1+\alpha_1)}\right) - \left(\ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})\right)$$

$$= g_4 S^{g_5} \ln\left(1 - e^{g_3 A(1+\alpha_1)}\right) - g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$= g_4 S^{g_5} \left(\ln\left(1 - e^{g_3 A(1+\alpha_1)}\right) - \ln(1 - e^{g_3 A})\right)$$

$$= g_4 S^{g_5} \left(\ln\left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)\right)$$

$$= \ln\left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)^{g_4 S^{g_5}}$$

$$e^{\Delta\ln(H)} = \left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)^{g_4 S^{g_5}}$$

Since the transformed natural log difference represents the multiplicative difference in the natural scale,

$$\Delta H = H\left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)^{g_4 S^{g_5}} - H$$

$$= H\left(\left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)^{g_4 S^{g_5}} - 1\right)$$

$$\rightarrow \Delta H / H = \left(\frac{1 - e^{g_3 A(1+\alpha_1)}}{1 - e^{g_3 A}}\right)^{g_4 S^{g_5}} - 1$$

□

Site Index Only:

Take the natural log and solve for the natural log difference.

$$\ln(H) = \ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$\Delta \ln(H) = \ln(g_1) + g_2 \ln\big(S(1+\alpha_2)\big) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln(1 - e^{g_3 A}) - \big(\ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})\big)$$

$$= g_2\big(\ln(S) + ln(1+\alpha_2)\big) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln(1 - e^{g_3 A}) - g_2 \ln(S) - g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$= g_2 \ln(1+\alpha_2) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln(1 - e^{g_3 A}) - g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$= \ln(1+\alpha_2)^{g_2} + \ln(1 - e^{g_3 A})^{g_4(S(1+\alpha_2))^{g_5}} - \ln(1 - e^{g_3 A})^{g_4 S^{g_5}}$$

$$= \ln\left(\frac{(1+\alpha_2)^{g_2}(1-e^{g_3 A})^{g_4(S(1+\alpha_2))^{g_5}}}{(1-e^{g_3 A})^{g_4 S^{g_5}}}\right)$$

$$= \ln\left(\frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}}(1-e^{g_3 A})^{-g_4 S^{g_5}(1+\alpha_2)^{g_5}}}\right)$$

$$= \ln\left(\frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}(1-(1+\alpha_2)^{g_5})}}\right)$$

$$e^{\Delta \ln(H)} = \frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}(1-(1+\alpha_2)^{g_5})}}$$

Since the transformed natural log difference represents the multiplicative difference in the natural scale,

$$\Delta H = H\left(\frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}(1-(1+\alpha_2)^{g_5})}}\right) - H$$

$$= H\left(\frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}(1-(1+\alpha_2)^{g_5})}} - 1\right)$$

$$\rightarrow \Delta H / H = \frac{(1+\alpha_2)^{g_2}}{(1-e^{g_3 A})^{g_4 S^{g_5}(1-(1+\alpha_2)^{g_5})}} - 1 \qquad \square$$

Stand Age and Site Index:

Take the natural log and solve for the natural log difference.

$$\ln(H) = \ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$\Delta \ln(H) = \ln(g_1) + g_2 \ln\big(S(1+\alpha_2)\big) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln\big(1 - e^{g_3 A(1+\alpha_1)}\big) - \big(\ln(g_1) + g_2 \ln(S) + g_4 S^{g_5} \ln(1 - e^{g_3 A})\big)$$

$$= g_2\big(\ln(S) + ln(1+\alpha_2)\big) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln\big(1 - e^{g_3 A(1+\alpha_1)}\big) - g_2 \ln(S) - g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$= g_2 \ln(1+\alpha_2) + g_4\big(S(1+\alpha_2)\big)^{g_5} \ln\big(1 - e^{g_3 A(1+\alpha_1)}\big) - g_4 S^{g_5} \ln(1 - e^{g_3 A})$$

$$= \ln(1+\alpha_2)^{g_2} + \ln\big(1 - e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}} - \ln(1 - e^{g_3 A})^{g_4 S^{g_5}}$$

$$= \ln\left(\frac{(1+\alpha_2)^{g_2}\big(1-e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\big(1-e^{g_3 A}\big)^{g_4 S^{g_5}}}\right)$$

$$e^{\Delta \ln(H)} = \frac{(1+\alpha_2)^{g_2}\big(1-e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\big(1-e^{g_3 A}\big)^{g_4 S^{g_5}}}$$

Since the transformed natural log difference represents the multiplicative difference in the natural scale,

$$\Delta H = H\left(\frac{(1+\alpha_2)^{g_2}\big(1-e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\big(1-e^{g_3 A}\big)^{g_4 S^{g_5}}}\right) - H$$

$$= H\left(\frac{(1+\alpha_2)^{g_2}\big(1-e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\big(1-e^{g_3 A}\big)^{g_4 S^{g_5}}} - 1\right)$$

$$\rightarrow \Delta H / H = \frac{(1+\alpha_2)^{g_2}\big(1-e^{g_3 A(1+\alpha_1)}\big)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\big(1-e^{g_3 A}\big)^{g_4 S^{g_5}}} - 1 \qquad \square$$

Volume (V)

$V = d_1 B^{d_2} H^{d_3}$

Stand Age Only:

Let $\delta_{B_A} = (1 + \alpha_1)^{a_3} - 1$

$\delta_{H_A} = \left(\frac{1-e^{g_3 A(1+\alpha_1)}}{1-e^{g_3 A}}\right)^{g_4 S^{g_5}} - 1$

$\Delta V = d_1\big(B(1 + \delta_{B_A})\big)^{d_2}\big(H(1 + \delta_{H_A})\big)^{d_3} - d_1 B^{d_2} H^{d_3}$

$= d_1 B^{d_2}(1 + \delta_{B_A})^{d_2} H^{d_3}(1 + \delta_{H_A})^{d_3} - d_1 B^{d_2} H^{d_3}$

$= d_1 B^{d_2} H^{d_3}\big((1 + \delta_{B_A})^{d_2}(1 + \delta_{H_A})^{d_3} - 1\big)$

$= V\left((1 + \delta_{B_A})^{d_2}\left(1 + \delta_{H_A}\right)^{d_3} - 1\right)$

$\rightarrow \Delta V/V = (1 + \delta_{B_A})^{d_2}(1 + \delta_{H_A})^{d_3} - 1$ □

Site Index Only:

Let $\delta_{B_S} = (1 + \alpha_2)^{a_2} - 1$

$\delta_{H_S} = \frac{(1+\alpha_2)^{g_2}}{\left(1-e^{g_3 A}\right)^{g_4 S^{g_5}\left(1-(1+\alpha_2)^{g_5}\right)}} - 1$

Solution is the same as for stand age alone, but replace δ_{B_A} and δ_{H_A} with δ_{B_S} and δ_{H_S}. □

Stand Age and Site Index:

Let $\delta_{B_S} = (1 + \alpha_2)^{a_2}(1 + \alpha_1)^{a_3} - 1$

$\delta_{H_S} = \frac{(1+\alpha_2)^{g_2}\left(1-e^{g_3 A(1+\alpha_1)}\right)^{g_4\left(S(1+\alpha_2)\right)^{g_5}}}{\left(1-e^{g_3 A}\right)^{g_4 S^{g_5}}} - 1$

Solution is the same as for stand age alone, but replace δ_{B_A} and δ_{H_A} with $\delta_{B_{AS}}$ and $\delta_{H_{AS}}$. □

APPENDIX 2: Percent Change and Relative Standard Error Tables

Systematic Error Tables

Table A2.1. Percent changes between erred model estimates and the *true* estimates for measurement error in ***stand age only***. To simplify reporting, ages were represented by three classes (young – 20 years; rotation – 50 years; and mature – 80 years) and site index was held constant at 60 ft.

Model	Age Class	Measurement Error Rate (%)										
		-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
B	All	-0.266	-0.204	-0.147	-0.095	-0.046	0.0	0.043	0.085	0.124	0.162	0.198
D	Young	-0.121	-0.097	-0.073	-0.048	-0.024	0.0	0.024	0.048	0.073	0.097	0.121
	Rotation	-0.222	-0.177	-0.133	-0.089	-0.044	0.0	0.044	0.089	0.133	0.177	0.222
	Mature	-0.280	-0.224	-0.168	-0.112	-0.056	0.0	0.056	0.112	0.168	0.224	0.280
N	Young	0.236	0.182	0.132	0.085	0.041	0.0	-0.039	-0.075	-0.109	-0.141	-0.171
	Rotation	0.511	0.379	0.265	0.165	0.078	0.0	-0.069	-0.131	-0.186	-0.236	-0.281
	Mature	0.719	0.519	0.354	0.216	0.100	0.0	-0.086	-0.161	-0.226	-0.283	-0.334
H	Young	-0.542	-0.431	-0.321	-0.212	-0.105	0.0	0.102	0.201	0.296	0.388	0.477
	Rotation	-0.428	-0.323	-0.228	-0.143	-0.067	0.0	0.060	0.112	0.158	0.199	0.235
	Mature	-0.323	-0.230	-0.154	-0.092	-0.041	0.0	0.033	0.060	0.081	0.098	0.112
V	Young	-0.509	-0.404	-0.301	-0.199	-0.098	0.0	0.097	0.191	0.283	0.374	0.462
	Rotation	-0.446	-0.345	-0.250	-0.162	-0.078	0.0	0.073	0.142	0.207	0.268	0.326
	Mature	-0.392	-0.298	-0.212	-0.134	-0.064	0.0	0.059	0.113	0.163	0.209	0.252

Table A2.2. Percent changes between erred model estimates and the true estimates for measurement error in ***site index only***. To simplify reporting, site indices were represented by three classes (low – 40 ft; average – 60 ft; and high – 80 ft) and stand age was held constant at 50 years.

Model	Site Index Class	Measurement Error Rate (%)										
		-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
B	ALL	-0.415	-0.326	-0.241	-0.159	-0.078	0.0	0.077	0.151	0.225	0.297	0.368
D	Low	-0.122	-0.098	-0.073	-0.049	-0.024	0.0	0.024	0.049	0.073	0.098	0.122
	Average	-0.163	-0.131	-0.098	-0.065	-0.033	0.0	0.033	0.065	0.098	0.131	0.163
	High	-0.196	-0.157	-0.118	-0.079	-0.039	0.0	0.039	0.079	0.118	0.157	0.196
N	Low	0.239	0.184	0.133	0.086	0.042	0.0	-0.039	-0.076	-0.110	-0.142	-0.173
	Average	0.341	0.259	0.185	0.118	0.056	0.0	-0.052	-0.099	-0.143	-0.183	-0.220
	High	0.433	0.325	0.229	0.144	0.068	0.0	-0.061	-0.117	-0.167	-0.213	-0.256
H	Low	-0.632	-0.497	-0.366	-0.238	-0.117	0.0	0.111	0.218	0.320	0.418	0.512
	Average	-0.538	-0.416	-0.302	-0.195	-0.094	0.0	0.089	0.174	0.255	0.332	0.406
	High	-0.484	-0.371	-0.268	-0.172	-0.083	0.0	0.079	0.153	0.224	0.292	0.356
V	Low	-0.647	-0.523	-0.395	-0.265	-0.133	0.0	0.134	0.268	0.403	0.539	0.674
	Average	-0.600	-0.482	-0.363	-0.242	-0.121	0.0	0.122	0.243	0.365	0.487	0.609
	High	-0.575	-0.461	-0.346	-0.231	-0.116	0.0	0.116	0.231	0.347	0.462	0.578

Table A2.3. Percent changes between erred basal area (B) model estimates and the true estimates for measurement error in both stand age and site index. Results are presented using the historical rotation age (50 years) and the average site index class (60 ft).

Stand Age Meas. Error Rate (%)	Site Index Measurement Error Rates (%)										
	-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
-0.5	-0.571	-0.505	-0.443	-0.382	-0.323	-0.266	-0.210	-0.155	-0.101	-0.048	0.005
-0.4	-0.534	-0.464	-0.396	-0.330	-0.266	-0.204	-0.143	-0.083	-0.025	0.033	0.090
-0.3	-0.501	-0.425	-0.353	-0.282	-0.214	-0.147	-0.082	-0.018	0.045	0.107	0.167
-0.2	-0.470	-0.390	-0.313	-0.238	-0.166	-0.095	-0.025	0.042	0.109	0.174	0.239
-0.1	-0.442	-0.357	-0.276	-0.197	-0.121	-0.046	0.027	0.099	0.169	0.238	0.306
0.0	-0.415	-0.326	-0.241	-0.159	-0.078	0.000	0.077	0.151	0.225	0.297	0.368
0.1	-0.390	-0.297	-0.208	-0.122	-0.038	0.043	0.123	0.201	0.278	0.354	0.428
0.2	-0.365	-0.269	-0.177	-0.087	0.000	0.085	0.168	0.249	0.329	0.407	0.484
0.3	-0.342	-0.243	-0.147	-0.054	0.036	0.124	0.210	0.294	0.377	0.458	0.538
0.4	-0.320	-0.217	-0.118	-0.022	0.071	0.162	0.251	0.338	0.423	0.507	0.590
0.5	-0.299	-0.193	-0.091	0.008	0.104	0.198	0.290	0.380	0.468	0.554	0.640

Table A2.4. Percent changes between erred quadratic mean diameter (D) model estimates and the true estimates for measurement error in both stand age and site index. Results are presented using the historical rotation age (50 years) and the average site index class (60 ft).

Stand Age Meas. Error Rate (%)	Site Index Measurement Error Rates (%)										
	-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
-0.5	-0.385	-0.352	-0.320	-0.287	-0.255	-0.222	-0.189	-0.157	-0.124	-0.091	-0.059
-0.4	-0.341	-0.308	-0.275	-0.243	-0.210	-0.177	-0.145	-0.112	-0.080	-0.047	-0.014
-0.3	-0.296	-0.264	-0.231	-0.198	-0.166	-0.133	-0.100	-0.068	-0.035	-0.002	0.030
-0.2	-0.252	-0.219	-0.187	-0.154	-0.121	-0.089	-0.056	-0.023	0.009	0.042	0.075
-0.1	-0.208	-0.175	-0.142	-0.110	-0.077	-0.044	-0.012	0.021	0.054	0.086	0.119
0.0	-0.163	-0.131	-0.098	-0.065	-0.033	0.000	0.033	0.065	0.098	0.131	0.163
0.1	-0.119	-0.086	-0.054	-0.021	0.012	0.044	0.077	0.110	0.142	0.175	0.208
0.2	-0.075	-0.042	-0.009	0.023	0.056	0.089	0.121	0.154	0.187	0.219	0.252
0.3	-0.030	0.002	0.035	0.068	0.100	0.133	0.166	0.198	0.231	0.264	0.296
0.4	0.014	0.047	0.080	0.112	0.145	0.177	0.210	0.243	0.275	0.308	0.341
0.5	0.059	0.091	0.124	0.157	0.189	0.222	0.255	0.287	0.320	0.352	0.385

Table A2.5. Percent changes between erred trees per acre (N) model estimates and the true estimates for measurement error in both stand age and site index. Results are presented using the historical rotation age (50 years) and the average site index class (60 ft).

Stand Age Meas. Error Rate (%)	Site Index Measurement Error Rates (%)										
	-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
-0.5	1.227	1.045	0.886	0.746	0.622	0.511	0.412	0.323	0.243	0.171	0.104
-0.4	0.986	0.834	0.700	0.581	0.475	0.379	0.294	0.216	0.146	0.082	0.024
-0.3	0.784	0.655	0.541	0.439	0.348	0.265	0.190	0.123	0.061	0.004	-0.048
-0.2	0.613	0.503	0.405	0.317	0.237	0.165	0.100	0.040	-0.015	-0.065	-0.112
-0.1	0.467	0.373	0.288	0.211	0.141	0.078	0.020	-0.034	-0.082	-0.127	-0.169
0.0	0.341	0.259	0.185	0.118	0.056	0.000	-0.052	-0.099	-0.143	-0.183	-0.220
0.1	0.232	0.160	0.095	0.035	-0.019	-0.069	-0.115	-0.157	-0.197	-0.233	-0.267
0.2	0.136	0.073	0.015	-0.037	-0.086	-0.131	-0.172	-0.210	-0.246	-0.279	-0.309
0.3	0.052	-0.004	-0.055	-0.102	-0.146	-0.186	-0.223	-0.258	-0.290	-0.320	-0.348
0.4	-0.023	-0.073	-0.118	-0.161	-0.200	-0.236	-0.269	-0.301	-0.330	-0.357	-0.383
0.5	-0.089	-0.134	-0.175	-0.213	-0.248	-0.281	-0.312	-0.340	-0.367	-0.392	-0.415

Table A2.6. Percent changes between erred height (H) model estimates and the true estimates for measurement error in both stand age and site index. Results are presented using the historical rotation age (50 years) and the average site index class (60 ft).

Stand Age Meas. Error Rate (%)	Site Index Measurement Error Rates (%)										
	-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
-0.5	-0.856	-0.776	-0.691	-0.603	-0.515	-0.428	-0.342	-0.259	-0.178	-0.099	-0.022
-0.4	-0.795	-0.701	-0.605	-0.509	-0.415	-0.323	-0.235	-0.149	-0.066	0.013	0.090
-0.3	-0.731	-0.626	-0.522	-0.420	-0.322	-0.228	-0.138	-0.052	0.031	0.111	0.188
-0.2	-0.665	-0.552	-0.443	-0.338	-0.238	-0.143	-0.053	0.033	0.116	0.195	0.271
-0.1	-0.601	-0.482	-0.369	-0.263	-0.162	-0.067	0.023	0.108	0.190	0.268	0.343
0.0	-0.538	-0.416	-0.302	-0.195	-0.094	0.000	0.089	0.174	0.255	0.332	0.406
0.1	-0.479	-0.355	-0.240	-0.133	-0.034	0.060	0.148	0.231	0.311	0.387	0.460
0.2	-0.424	-0.299	-0.184	-0.079	0.020	0.112	0.199	0.282	0.360	0.435	0.507
0.3	-0.373	-0.248	-0.134	-0.030	0.067	0.158	0.244	0.325	0.403	0.476	0.547
0.4	-0.326	-0.202	-0.090	0.013	0.109	0.199	0.283	0.364	0.440	0.512	0.582
0.5	-0.283	-0.161	-0.050	0.052	0.146	0.235	0.318	0.397	0.472	0.544	0.613

Table A2.7. Percent changes between erred gross volume (V) model estimates and the true estimates for measurement error in both stand age and site index. Results are presented using the historical rotation age (50 years) and the average site index class (60 ft).

Stand Age Meas. Error Rate (%)	**Site Index Measurement Error Rates (%)**										
	-0.5	*-0.4*	*-0.3*	*-0.2*	*-0.1*	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
-0.5	-0.840	-0.769	-0.693	-0.613	-0.530	-0.446	-0.360	-0.273	-0.185	-0.097	-0.008
-0.4	-0.792	-0.709	-0.621	-0.531	-0.439	-0.345	-0.250	-0.155	-0.059	0.038	0.135
-0.3	-0.743	-0.649	-0.552	-0.453	-0.352	-0.250	-0.148	-0.045	0.059	0.163	0.267
-0.2	-0.694	-0.591	-0.486	-0.379	-0.271	-0.162	-0.052	0.058	0.168	0.279	0.389
-0.1	-0.646	-0.536	-0.423	-0.309	-0.194	-0.078	0.038	0.154	0.270	0.386	0.503
0.0	-0.600	-0.482	-0.363	-0.242	-0.121	0.000	0.122	0.243	0.365	0.487	0.609
0.1	-0.556	-0.431	-0.306	-0.180	-0.053	0.073	0.200	0.327	0.454	0.581	0.708
0.2	-0.513	-0.383	-0.252	-0.121	0.011	0.142	0.274	0.406	0.538	0.669	0.801
0.3	-0.473	-0.338	-0.202	-0.066	0.071	0.207	0.343	0.480	0.616	0.752	0.888
0.4	-0.435	-0.295	-0.154	-0.013	0.127	0.268	0.409	0.549	0.690	0.830	0.971
0.5	-0.399	-0.255	-0.109	0.036	0.181	0.326	0.471	0.615	0.760	0.904	1.049

Stochastic Error Tables

Table A2.8. Mean deviations (expressed in percent change) between basal area (B) model estimates with and without random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		**Site Index (and Error CV)**						
		ALL						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
ALL	*0.00*	0.000	0.000	-0.001	-0.002	-0.003	-0.006	-0.008
	0.05	0.000	0.000	0.000	-0.001	-0.002	-0.003	-0.005
	0.10	-0.001	-0.001	-0.001	-0.002	-0.003	-0.004	-0.007
	0.15	-0.003	-0.003	-0.003	-0.004	-0.005	-0.006	-0.008
	0.20	-0.005	-0.006	-0.006	-0.006	-0.007	-0.009	-0.011
	0.25	-0.008	-0.009	-0.009	-0.010	-0.011	-0.012	-0.014
	0.30	-0.012	-0.014	-0.014	-0.014	-0.015	-0.017	-0.019

Table A2.9. Mean deviations (expressed in percent change) between trees per acre (N) model estimates with and without random measurement error. Results are given for four sets of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							60						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.001	0.003	0.006	0.010	0.017	0.024	0.000	0.001	0.004	0.010	0.017	0.028	0.041
	0.05	0.000	0.001	0.002	0.005	0.009	0.015	0.022	0.000	0.001	0.004	0.008	0.016	0.025	0.038
	0.10	0.002	0.002	0.004	0.007	0.011	0.017	0.024	0.001	0.002	0.005	0.010	0.017	0.027	0.039
	0.15	0.004	0.005	0.006	0.009	0.014	0.019	0.027	0.003	0.004	0.007	0.011	0.019	0.029	0.041
	0.20	0.007	0.008	0.010	0.013	0.017	0.023	0.030	0.005	0.006	0.009	0.014	0.021	0.031	0.044
	0.25	0.011	0.013	0.014	0.017	0.022	0.027	0.035	0.008	0.010	0.012	0.017	0.025	0.035	0.048
	0.30	0.016	0.018	0.020	0.023	0.027	0.033	0.041	0.011	0.013	0.016	0.021	0.029	0.039	0.052
50	*0.00*	0.000	0.000	0.001	0.003	0.005	0.008	0.012	0.000	0.001	0.002	0.005	0.009	0.015	0.021
	0.05	0.001	0.002	0.002	0.004	0.006	0.008	0.012	0.001	0.002	0.003	0.005	0.009	0.014	0.020
	0.10	0.005	0.006	0.007	0.008	0.010	0.013	0.016	0.004	0.005	0.007	0.009	0.013	0.018	0.024
	0.15	0.012	0.014	0.014	0.016	0.018	0.020	0.024	0.010	0.011	0.012	0.015	0.019	0.024	0.030
	0.20	0.022	0.024	0.025	0.026	0.029	0.031	0.035	0.017	0.020	0.021	0.023	0.027	0.033	0.039
	0.25	0.035	0.039	0.039	0.041	0.043	0.046	0.050	0.028	0.031	0.032	0.035	0.039	0.044	0.051
	0.30	0.052	0.057	0.058	0.059	0.062	0.065	0.069	0.041	0.045	0.046	0.049	0.053	0.059	0.067

Table A2.10. Mean deviations (expressed in percent change) between stand height (H) model estimates with and without random measurement error. Results are given for four sets of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							60						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.002	0.007	0.016	0.029	0.047	0.069	0.000	0.000	0.000	0.000	0.002	0.004	0.009
	0.05	0.001	0.003	0.010	0.021	0.036	0.055	0.079	0.000	0.000	0.002	0.003	0.006	0.010	0.015
	0.10	0.002	0.004	0.011	0.022	0.036	0.055	0.079	-0.001	-0.001	0.000	0.002	0.004	0.008	0.014
	0.15	0.005	0.006	0.013	0.023	0.038	0.057	0.080	-0.003	-0.003	-0.002	0.000	0.002	0.006	0.011
	0.20	0.009	0.009	0.016	0.026	0.040	0.059	0.081	-0.005	-0.006	-0.005	-0.003	-0.001	0.003	0.008
	0.25	0.015	0.014	0.021	0.030	0.044	0.062	0.084	-0.008	-0.010	-0.008	-0.007	-0.004	-0.001	0.005
	0.30	0.022	0.021	0.027	0.036	0.049	0.067	0.088	-0.011	-0.013	-0.012	-0.010	-0.008	-0.004	0.001
50	*0.00*	0.000	-0.001	-0.002	-0.006	-0.010	-0.015	-0.020	0.000	-0.001	-0.002	-0.006	-0.010	-0.016	-0.022
	0.05	-0.001	-0.001	-0.002	-0.004	-0.008	-0.012	-0.016	-0.001	-0.001	-0.002	-0.005	-0.009	-0.014	-0.020
	0.10	-0.004	-0.005	-0.006	-0.008	-0.011	-0.015	-0.019	-0.004	-0.004	-0.006	-0.008	-0.012	-0.017	-0.023
	0.15	-0.009	-0.011	-0.012	-0.014	-0.017	-0.020	-0.025	-0.008	-0.010	-0.011	-0.013	-0.017	-0.022	-0.027
	0.20	-0.016	-0.018	-0.019	-0.021	-0.024	-0.027	-0.031	-0.015	-0.017	-0.018	-0.020	-0.024	-0.028	-0.034
	0.25	-0.024	-0.027	-0.028	-0.030	-0.032	-0.036	-0.039	-0.023	-0.026	-0.027	-0.029	-0.032	-0.037	-0.042
	0.30	-0.033	-0.037	-0.037	-0.039	-0.042	-0.045	-0.048	-0.034	-0.036	-0.037	-0.040	-0.043	-0.047	-0.052

Table A2.11. Mean deviations (expressed in percent change) between gross volume (V) model estimates with and without random measurement error. Results are given for four sets of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							60						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.001	0.005	0.011	0.020	0.033	0.049	0.000	0.000	0.002	0.005	0.008	0.014	0.022
	0.05	0.000	0.002	0.007	0.015	0.026	0.040	0.057	0.000	0.001	0.004	0.008	0.013	0.019	0.028
	0.10	0.000	0.002	0.007	0.015	0.025	0.039	0.056	-0.001	0.000	0.002	0.006	0.011	0.018	0.027
	0.15	0.001	0.002	0.007	0.014	0.025	0.039	0.056	-0.002	-0.002	0.001	0.004	0.010	0.016	0.025
	0.20	0.002	0.002	0.007	0.015	0.025	0.039	0.056	-0.004	-0.004	-0.002	0.002	0.007	0.014	0.022
	0.25	0.003	0.003	0.008	0.015	0.025	0.039	0.055	-0.006	-0.007	-0.004	-0.001	0.004	0.011	0.020
	0.30	0.006	0.004	0.009	0.016	0.026	0.039	0.056	-0.008	-0.010	-0.007	-0.004	0.001	0.008	0.016
50	*0.00*	0.000	0.000	0.000	0.001	0.002	0.003	0.005	0.000	0.000	0.000	0.000	0.001	0.001	0.002
	0.05	-0.001	0.000	0.001	0.003	0.005	0.007	0.010	-0.001	0.000	0.001	0.002	0.003	0.005	0.006
	0.10	-0.002	-0.002	-0.001	0.000	0.002	0.005	0.008	-0.002	-0.002	-0.001	0.000	0.001	0.002	0.004
	0.15	-0.005	-0.006	-0.005	-0.003	-0.001	0.001	0.004	-0.005	-0.006	-0.005	-0.004	-0.002	-0.001	0.001
	0.20	-0.010	-0.011	-0.010	-0.008	-0.006	-0.004	0.000	-0.009	-0.010	-0.009	-0.008	-0.007	-0.006	-0.004
	0.25	-0.015	-0.017	-0.015	-0.014	-0.012	-0.009	-0.006	-0.015	-0.016	-0.015	-0.014	-0.013	-0.011	-0.010
	0.30	-0.020	-0.023	-0.022	-0.020	-0.018	-0.016	-0.012	-0.021	-0.023	-0.022	-0.021	-0.020	-0.018	-0.016

Table A2.12. Relative standard errors of basal area (B) model estimates with random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)						
		ALL						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
ALL	*0.00*	0.000	0.038	0.076	0.115	0.154	0.193	0.233
	0.05	0.022	0.044	0.080	0.117	0.155	0.194	0.234
	0.10	0.044	0.059	0.088	0.123	0.160	0.198	0.237
	0.15	0.067	0.078	0.102	0.133	0.168	0.204	0.242
	0.20	0.090	0.099	0.119	0.146	0.178	0.213	0.249
	0.25	0.115	0.122	0.139	0.163	0.192	0.224	0.259
	0.30	0.141	0.148	0.162	0.183	0.209	0.239	0.272

Table A2.13. Relative standard errors of quadratic mean diameter (D) model estimates with random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		**Site Index (and Error CV)**													
		40							**60**						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.017	0.034	0.051	0.069	0.086	0.103	0.000	0.022	0.044	0.066	0.088	0.110	0.131
	0.05	0.014	0.022	0.037	0.054	0.070	0.087	0.104	0.012	0.025	0.046	0.067	0.089	0.111	0.133
	0.10	0.028	0.033	0.044	0.059	0.074	0.090	0.107	0.024	0.033	0.050	0.070	0.091	0.113	0.134
	0.15	0.042	0.046	0.055	0.067	0.081	0.096	0.111	0.036	0.042	0.057	0.075	0.095	0.116	0.137
	0.20	0.056	0.059	0.066	0.076	0.089	0.103	0.117	0.048	0.053	0.065	0.081	0.100	0.120	0.140
	0.25	0.070	0.073	0.079	0.087	0.098	0.111	0.125	0.060	0.064	0.074	0.089	0.106	0.125	0.145
	0.30	0.084	0.087	0.091	0.099	0.109	0.121	0.133	0.071	0.076	0.084	0.098	0.114	0.131	0.150
50	*0.00*	0.000	0.012	0.024	0.036	0.048	0.060	0.072	0.000	0.016	0.032	0.048	0.064	0.080	0.096
	0.05	0.025	0.028	0.035	0.044	0.054	0.065	0.076	0.022	0.027	0.039	0.053	0.068	0.084	0.099
	0.10	0.049	0.051	0.055	0.061	0.069	0.078	0.088	0.044	0.047	0.055	0.065	0.078	0.092	0.106
	0.15	0.074	0.075	0.078	0.083	0.088	0.096	0.104	0.066	0.068	0.074	0.082	0.092	0.104	0.117
	0.20	0.098	0.100	0.102	0.106	0.110	0.116	0.123	0.087	0.090	0.094	0.101	0.109	0.119	0.131
	0.25	0.123	0.125	0.126	0.129	0.133	0.138	0.143	0.109	0.112	0.115	0.120	0.128	0.136	0.146
	0.30	0.147	0.149	0.151	0.153	0.156	0.160	0.165	0.131	0.134	0.136	0.141	0.147	0.155	0.164

Table A2.14. Relative standard errors of trees per acre (N) model estimates with random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							**60**						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.028	0.057	0.087	0.117	0.149	0.183	0.000	0.036	0.073	0.112	0.153	0.198	0.247
	0.05	0.023	0.037	0.062	0.090	0.120	0.151	0.185	0.020	0.041	0.076	0.114	0.154	0.199	0.248
	0.10	0.046	0.055	0.074	0.099	0.127	0.158	0.191	0.039	0.054	0.084	0.119	0.159	0.203	0.252
	0.15	0.070	0.077	0.092	0.113	0.139	0.168	0.200	0.059	0.070	0.096	0.128	0.166	0.210	0.258
	0.20	0.094	0.100	0.112	0.131	0.155	0.183	0.214	0.080	0.089	0.110	0.141	0.177	0.219	0.268
	0.25	0.120	0.125	0.135	0.152	0.174	0.200	0.231	0.101	0.109	0.127	0.155	0.190	0.232	0.280
	0.30	0.146	0.151	0.160	0.175	0.196	0.221	0.252	0.122	0.130	0.146	0.172	0.206	0.246	0.295
50	*0.00*	0.000	0.020	0.040	0.060	0.081	0.102	0.124	0.000	0.027	0.053	0.081	0.109	0.139	0.170
	0.05	0.041	0.046	0.057	0.073	0.091	0.110	0.131	0.036	0.045	0.065	0.089	0.116	0.144	0.175
	0.10	0.082	0.086	0.093	0.104	0.118	0.134	0.152	0.073	0.079	0.092	0.111	0.134	0.160	0.189
	0.15	0.126	0.130	0.135	0.143	0.154	0.168	0.184	0.112	0.116	0.126	0.142	0.162	0.186	0.213
	0.20	0.174	0.178	0.182	0.189	0.199	0.211	0.226	0.153	0.158	0.166	0.179	0.197	0.219	0.246
	0.25	0.227	0.232	0.236	0.242	0.252	0.264	0.279	0.197	0.203	0.210	0.222	0.239	0.261	0.288
	0.30	0.288	0.294	0.298	0.305	0.314	0.327	0.344	0.246	0.253	0.260	0.272	0.289	0.311	0.340

Table A2.15. Relative standard errors of stand height (H) model estimates with random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							60						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.118	0.235	0.348	0.456	0.559	0.657	0.000	0.086	0.171	0.254	0.335	0.412	0.484
	0.05	0.078	0.142	0.248	0.358	0.465	0.569	0.667	0.051	0.100	0.179	0.260	0.340	0.416	0.488
	0.10	0.156	0.196	0.282	0.382	0.484	0.583	0.679	0.102	0.134	0.199	0.273	0.349	0.424	0.494
	0.15	0.232	0.262	0.331	0.418	0.512	0.607	0.699	0.152	0.176	0.228	0.295	0.366	0.436	0.504
	0.20	0.306	0.331	0.387	0.464	0.550	0.638	0.726	0.203	0.222	0.264	0.322	0.387	0.453	0.518
	0.25	0.379	0.400	0.448	0.515	0.593	0.676	0.759	0.252	0.269	0.304	0.354	0.413	0.474	0.536
	0.30	0.448	0.468	0.509	0.568	0.640	0.717	0.796	0.301	0.316	0.345	0.389	0.442	0.499	0.556
50	*0.00*	0.000	0.056	0.113	0.169	0.226	0.282	0.336	0.000	0.045	0.091	0.137	0.184	0.232	0.280
	0.05	0.048	0.074	0.122	0.176	0.231	0.286	0.339	0.031	0.055	0.096	0.141	0.187	0.234	0.281
	0.10	0.096	0.112	0.148	0.194	0.244	0.296	0.347	0.063	0.078	0.111	0.151	0.194	0.240	0.286
	0.15	0.144	0.156	0.183	0.221	0.265	0.312	0.360	0.096	0.107	0.132	0.167	0.207	0.249	0.293
	0.20	0.192	0.202	0.223	0.254	0.292	0.334	0.378	0.130	0.139	0.159	0.188	0.223	0.263	0.304
	0.25	0.239	0.249	0.265	0.290	0.323	0.360	0.399	0.165	0.174	0.190	0.214	0.245	0.280	0.318
	0.30	0.285	0.294	0.307	0.328	0.356	0.388	0.423	0.202	0.210	0.223	0.243	0.270	0.301	0.335

Table A2.16. Relative standard errors of gross volume (V) model estimates with random measurement error. Results are given for all values of stand age (years) and site index (ft).

Stand Age (and Error CV)		Site Index (and Error CV)													
		40							60						
		0.00	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*	*0.30*
20	*0.00*	0.000	0.100	0.199	0.296	0.391	0.483	0.571	0.000	0.082	0.163	0.244	0.323	0.400	0.475
	0.05	0.063	0.118	0.209	0.304	0.398	0.490	0.579	0.048	0.095	0.171	0.250	0.328	0.405	0.480
	0.10	0.125	0.161	0.236	0.323	0.413	0.502	0.590	0.096	0.127	0.190	0.263	0.338	0.413	0.487
	0.15	0.188	0.214	0.275	0.352	0.436	0.522	0.607	0.144	0.167	0.219	0.284	0.355	0.427	0.498
	0.20	0.249	0.271	0.321	0.389	0.467	0.548	0.630	0.192	0.211	0.253	0.312	0.377	0.445	0.514
	0.25	0.309	0.328	0.371	0.432	0.503	0.579	0.658	0.240	0.256	0.292	0.344	0.404	0.468	0.534
	0.30	0.367	0.385	0.422	0.477	0.543	0.615	0.690	0.287	0.302	0.333	0.379	0.434	0.494	0.557
50	*0.00*	0.000	0.066	0.131	0.197	0.262	0.326	0.388	0.000	0.060	0.120	0.179	0.239	0.298	0.356
	0.05	0.046	0.081	0.140	0.203	0.267	0.330	0.393	0.037	0.071	0.126	0.184	0.242	0.301	0.359
	0.10	0.093	0.115	0.161	0.218	0.278	0.339	0.400	0.075	0.096	0.141	0.195	0.251	0.308	0.364
	0.15	0.140	0.156	0.192	0.242	0.297	0.354	0.412	0.113	0.129	0.165	0.212	0.264	0.319	0.374
	0.20	0.186	0.200	0.229	0.271	0.321	0.374	0.430	0.151	0.165	0.194	0.235	0.283	0.334	0.386
	0.25	0.233	0.245	0.269	0.305	0.350	0.399	0.451	0.191	0.202	0.227	0.263	0.306	0.353	0.403
	0.30	0.278	0.290	0.310	0.342	0.382	0.427	0.475	0.230	0.241	0.262	0.293	0.332	0.376	0.422