Implementing Standards-Based Mathematics: Toward Improving Conceptual Understanding of Key Ideas of Linear Functions for Middle School Immigrant Students

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Abstract

Mathematics education research on immigrant students indicates that schools are often not prepared to support students with cultural and language barriers (Campbell, Adams, & Davis, 2007; Civil 2014). This descriptive case study documents the ways in which middle school immigrant students demonstrated understanding of key ideas of linear functions prior to and after participation in an eight-week Constructivist Teaching Experiment (CTE) (Cobb & Steffe, 2011; Cohen, Manion & Morrison, 2011). The purpose of this study is to show that all students-- regardless of their English language or mathematical proficiency ratings-- can learn mathematics with conceptual understanding (Hansen-Thomas, 2009; Silver & Stein, 1996; Whitney, 2010).

Results prior to the CTE indicated the following. First, middle school immigrant students answered a majority of questions correctly without justifying when it was not required on the pre-assessment. And second, pre-assessment questions influenced the ways that middle school immigrant students communicated and used representations. Results after the CTE revealed three major findings. One, all middle school immigrant student participants increased their conceptual understanding on the post-assessment. Two, post-assessment questions with a real-life situation that asked for multiple representations influenced the ways that middle school immigrant students communicated and used representations. Three, all students filled in procedural and conceptual gaps after listening to peer presentations, individually and as a team. In summary, positive outcomes were associated with student-centered and problem-centered mathematics instruction that aligned with equity goals (National Council of Teachers of Mathematics, 2000).

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Chapter 1

Introduction

This study examines the ways that seven middle school immigrant students enrolled in an intervention mathematics course demonstrated conceptual understanding of key ideas of linear functions prior to and after participation in an eight-week constructivist teaching experiment. The content of the intervention included: identifying missing y-values, describing slope, graphing, writing and solving equations, interpreting solutions using a real-life situation from a table. The pedagogical model implemented reflected characteristics of a Standards-based mathematics classroom (NCTM, 2000).

To build a case for this study a discussion of deficit views of immigrant students in mathematics education and the underlying reasons for this view is presented. Then the potential for Standards-based mathematics teaching approaches to overturn existing disparities is discussed. The chapter closes with the research questions and an outline for the rest of the dissertation.

Deficit Views of Immigrant Students in Mathematics Education

Today, one in five children in the United States is the child of immigrants, and by 2040 it is projected that one in three children will fit this description (Suarez-Orezco, 2009). Immigrant students in this study are defined as, “a case when students (or their parents) were born in a country other than the one they are currently living in and attending school,” and include students from low socio-economic backgrounds that consistently performed in the bottom third on national standardized mathematics assessments (hereafter *immigrant students*) (Civil, 2014, p. 277; National Center for

Education Statistics [NCES], 2007). The literature surrounding immigrant students in U.S. schools shows that immigrant students face barriers that their native born peers do not, but still are expected to achieve at the same rate (Civil, 2014; Freeman & Crawford, 2008). Additionally, immigrant students are victims of deficit discourses around policies which make their way into schools (Civil, 2014). One such example is the unintended consequence of the No Child Left Behind Act of 2001, where immigrant students are too often placed in low-track mathematics courses in segregated settings that aligned with teaching practices reflecting a *pedagogy of poverty* (Freeman & Crawford, 2009; Haberman, 1991; Secada & Carey, 1990; U.S. Department of Education, 2001). Haberman (1991) coined the term *pedagogy of poverty,* to describe how schools contribute to the achievement gap by accepting a basic approach to teaching, where students are expected to repeat rote problems to demonstrate understanding and are punished for non-compliance.

Wang and Goldschmidt (1999) and Paul (2005) found that a relationship exists between immigrant status and English proficiency, and initial algebra one course placement in that students assigned to low-initial algebra one course negatively impacted students' mathematics and higher education trajectory. Results from two studies, Wang and Goldschmidt (1999) and Paul (2005), asserted that unless middle school immigrant students' achievement was raised by the end of 8^{th} grade, little can be done in high school to change their mathematics trajectory associated with preparation for higher education. In order to ensure that middle school immigrant students are mathematically proficient, there is a need to disrupt this deficit view and failed policies surrounding immigrant students that prevent immigrant students from having access to rigorous initial algebra courses prior to 8^{th} grade. Therefore, research should contribute to an understanding of the factors leading to deficit views and failed policies that negatively impact immigrant students as well as study alternative instructional models that reflect the belief that all children can learn mathematics in a meaningful way.

Little empirical evidence exists to support deficit views that immigrant students who speak more than one language cannot be successful in mathematics (Barwell, Barton & Setati, 2007). The literature shows that immigrant students can learn mathematics with conceptual understanding if provided structured instructional supports in heterogeneous learning environments coupled with high teacher expectations (Silver & Stein, 1996). While deficit views of immigrant students have led to long-term underachievement, there is promise in implementing structured learning environments that pay attention to differences in language and prior experiences that immigrant students bring to school. For this reason, if immigrant students are to increase their mathematics achievement, then a focus on structuring instructional strategies that move away from deficit views and toward a view of mathematics teaching and learning as envisioned by the NCTM Standards document has potential for narrowing the achievement gap between immigrant students and majority students.

Standards-Based Instruction

Implementing Standards-based mathematics teaching approaches aligns with equity goals espoused in *Principles and Standards of School Mathematics* (PSSM) documents (Senk & Thompson, 2003; NCTM, 2000). Standards-based mathematics teaching approaches challenge traditional views about what it means to do mathematics. Standards-based teaching is both inquiry-based and student-centered, where the roles of the teacher include choosing rich, open-ended problems that emphasize mathematical reasoning, predicting student reasoning, and organizing tasks that help students to engage in mathematical discussions. In Standards-based mathematics classrooms, students are often presented with more realistic and complex mathematical problems where they are encouraged to work in pairs or in teams as resources for working through problems, and where strategies and solutions are shared via whole-class discussions.

Cited in Stein, Smith, and Hughes (2009), Ball (1993) described the role of the reform teacher as, "no longer 'dispenser of knowledge' and arbiter of mathematical 'correctness' but an engineer of learning environments in which students actively grapple with mathematical problems and construct their own understanding" (p. 315). Smith (1996) argued that teachers should serve as a coach for student thinking, pose math questions, and motivate learning by not being the direct knowledge provider because, "good telling cannot guarantee that students will learn" (p. 392). Standards-based mathematics teaching approaches are aligned with the five processes of effective mathematics teaching as highlighted by NCTM (2000): *problem solving, reasoning and proof, communication, connections,* and *representations.* In particular, this study aims to address communication and representations for middle school immigrant student participants.

Several studies documented positive outcomes for immigrant students that received structured supports that align with Standards-based mathematics teaching approaches as evidenced by increase in mathematics achievement on standardized achievement and higher order problem solving on conceptual assessments (Garcia, 1991; Hansen-Thomas, 2009; Senk & Thompson, 2003; Silver & Stein, 1996; Whitney, 2010). Further, research on successful implementation of Standards-based mathematics teaching approaches for immigrant students included de-tracking in mathematics classrooms, and teaching approaches that encouraged team work on rich open-ended problems, communication of mathematical process, whole class discussions, and use of multiple representations (Hansen-Thomas, 2009; Silver & Stein, 1996; Whitney, 2010). Silver and Stein (1996) asserted that when schools move away from blaming low performance of mathematics for poor immigrant students as due to lack of student ability or potential, rather than as a result of a set of instructional practices that fail to provide students with equitable environments, schools are more likely to increase achievement outcomes. For example, results from the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), a reform project aimed at poor urban and immigrant middle school students, showed significant gains for middle school immigrant students' conceptual understanding of mathematics, and increase in higher order thinking, problem solving, and reasoning (Silver & Stein, 1996). Over a decade later, similar results were documented by Hansen-Thomas (2009) with middle school English Language Learners (ELLs) that learned from Standards-based mathematics curricula that emphasized mathematical discussions and structured support with attention to students' unique language needs. Therefore, Standards-based mathematics teaching approaches that align with literature on how people learn and emphasize structured teams in heterogeneous learning environments, communication of mathematical processes, and multiple representations lead to meaningful learning (Bransford, Brown & Cocking, 2000).

Several studies that documented the benefits of Standards-based mathematics teaching approaches for middle and high school immigrant students promoted productive mathematical discussions that paid special attention to unique language needs of immigrant students, and problem solving that encouraged multiple representations. For example, students that were encouraged to use informal language and semi-formal language in mono-lingual team settings were associated with positive outcomes on standardized assessments (Cramer, 2001; Garcia, 1991; Hansen-Thomas, 2009). Additionally, students that were encouraged to use multiple strategies and multiple representations demonstrated increased positive outcomes on both standardized assessments and open-ended assessments that examined conceptual understanding (Silver, Lane & Smith, 1991; Silver & Stein, 1996). Therefore, two frameworks that were associated with positive mathematics achievement for immigrant students, and that highlighted the salient features of Standards-based mathematics teaching approaches are used in this study, namely the Lesh Translation Model (LTM) and Orchestrating Productive Mathematical Discussions (Lesh, Post & Behr, 1987; Cramer, 2003; Senk & Thompson, 2003; Stein et al, 2009). Instruction based on the LTM supports students to move among representations to express the same mathematical ideas in multiple ways and is associated with characteristics of higher order thinking and reasoning (Lesh et al, 1987; NCTM, 2000; Silver & Stein, 1996). NCTM (2000) states that, “When students gain access to multiple representations and the ideas they represent, they have a set of tools that significantly expands their capacity to think mathematically" (p. 67). A focus on productive discourse is the second framework used in the study as it has been shown that effective classroom discourse engages immigrant students in sense-making in authentic ways individually and in small team settings (Hansen-Thomas, 2009; Stein et al, 2009).

In summary, it has been argued thus far that immigrant students benefit from structured team settings in heterogeneous learning environments that pay attention to language differences, productive mathematical discussions, and multiple representations. This shift from traditional teaching approaches to teacher-centered to student- and problem-centered mathematics teaching approaches align with research from cognitive science on how people learn and benefit all students.

Purpose of the Study

The purpose of this study is to investigate the characteristics of seven middle school immigrant students' understandings prior to and after participation in a constructivist teaching experiment that emphasized Standards-based pedagogy. In particular, emphasis was given to the role of mathematical discussions and use of multiple representations in building understanding of key ideas of function. This study enables the researcher to understand how structured learning environments that align with Standards-based mathematics teaching approaches support immigrant students' conceptual understanding of key ideas of linear functions.

A major assumption underlying this study is that by enacting mathematics pedagogical approaches that are aligned with both constructivist teaching approaches as outlined by *PSSM* documents and equity goals, immigrant students that were once victims of deficit teaching approaches will demonstrate conceptual understanding of more challenging key ideas of linear functions. There is an implicit assumption that there will be a change in middle school immigrant students' understandings who participate in the constructivist teaching experiment as studies have previously documented. It has been observed that middle and high school immigrant students in former studies that participated in classrooms that structured mathematics teaching in ways that aligned with Standards-based mathematics teaching approaches, experienced improvements in their mathematics scores and their mathematics reasoning and problem solving. For instance, some studies documented that middle and high school immigrant students performed as well or higher than their native-born peers with English as a first language on standardized assessments, as well as on open-ended assessments aimed to measure conceptual understanding (Paul, 2005; Silver & Stein, 1996; Wang & Goldschmidt, 1999). Building on the result of prior research that documented middle and high school immigrant students' increased performances on conceptual and quantitative measurements within Standards-based classrooms, it is likely that middle school immigrant students, at the end of the eight-week constructivist teaching experiment, may demonstrate improvement on their conceptual understanding of key ideas of linear functions. In effect, there is a potential for Standards-based mathematics teaching approaches to be identified as a model for teaching middle school immigrant students that are often victims of low-expectations and low-track mathematics courses (Freeman & Crawford, 2008; Hansen-Thomas, 2009; Silver & Stein, 1996).

Statement of the Problem

The key issues to be addressed in this study is the impact of Standards-based mathematics teaching approaches that emphasize productive mathematical discussions and multiple representations have on middle school immigrant students' conceptual understanding. If there are any changes in middle school immigrant students' conceptual understanding, what are the characteristics of the changes in middle school immigrant students' mathematical processes? That is, how does the constructivist teaching experiment that emphasized mathematical discussions and multiple representations support conceptual understanding as evidenced by student response scores that closely examines both students' solutions and justifications of key ideas of linear functions? Such a study and its findings will enable mathematics teachers of immigrant students and mathematics educators of pre-service teachers to glean a better understanding of how implementing structured heterogeneous learning environments increase middle school immigrant students' achievement on standardized assessments as well as on conceptual understanding assessments.

Research Questions

The following questions will guide this study:

1. In what ways do immigrant students demonstrate their understanding of key ideas of linear functions by expressing translations between and within modes of representations prior to participating in mathematical activities that emphasize Lesh Translation Model?

2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions after participating in mathematical activities that emphasize multiple representations from Lesh Translation Model?
3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Organization of the Dissertation

Chapter 1 presented the case for investigating the three research questions. Chapter 2 of this dissertation presents a review of the literature that is relevant to this study. Chapter 3 explains the research design and research methodology used in this qualitative study. Chapter 4 presents the descriptive results of the study from the analyses of the study that address three research questions. Chapter 5 summarizes the findings, discussions, and implications, and recommendations for future studies.

Chapter 2

Review of Literature

Introduction

The purpose of this study is to respond to the goals espoused in the *Principles and Standards of School Mathematics* (PSSM) that outlined “equity” as their first principle for school mathematics, “Excellence in mathematics requires—high expectations and strong support for all students” (National Council of Teachers of Mathematics [NCTM], 2000, p. 11). Additionally, NCTM (2000) noted that, “Equity requires accommodating differences to help everyone learn mathematics” (p. 13). This study addresses the mathematics achievement and opportunity gaps from the literature that showed immigrant students were victims of deficit views that made way into schools and that often placed immigrant students in lower-track mathematics courses in segregated settings absent of rich problem solving (Secada & Carey, 1996; Freeman & Crawford, 2008). Further, this study builds a case for the benefits of Standards-Based mathematics teaching approaches that paid attention to language differences, emphasized mathematical discussions, and encouraged multiple representations, to disrupt deficit views (Civil, 2014; Hansen-Thomas, 2009; Silver & Stein, 1996). A constructivist teaching experiment approach was utilized to support middle school immigrant students in well structured heterogeneous learning environments that included students with varying English language and mathematics proficiency (hereafter *heterogeneous learning environments*) (Senk & Thompson, 2003; Silver & Stein, 1996). Therefore, the purpose of this study is to counter deficit views of mathematics teaching approaches for middle school immigrant students, and increase middle school immigrant students' opportunities to learn mathematics with conceptual understanding as highlighted by NCTM (2000) equity goals (Campbell, Adams, & Davis, 2007).

In this chapter, an overview of the literature related to this study will be presented. First, the literature review uncovered overwhelming deficit views of mathematics teaching approaches surrounding immigrant students. Second, empirical literature that countered deficit views of immigrant students in mathematics settings and that documented positive mathematics achievement outcomes are presented. Third, salient features of Standards-based mathematics curriculum and instructional strategies that supported immigrant students' conceptual understanding are presented. Fourth, a brief history of the failure of a "back to basics" movement that mirror current deficit teaching approaches for immigrant students are presented to further argue for more Standards-based mathematics teaching approaches. Finally, two tools used in the study are introduced: The Lesh Translation Model and Orchestrating Productive Mathematical Discussions. The chapter concludes with a description and benefits of a constructivist teaching experiment, the role of the teacher, addresses methodological rigor, and research goals. Therefore, the literature review will identify existing problems and justify the intervention tools. Following the literature review, a theoretical framework will be presented to guide the intervention further, and then the research questions are presented.

Deficit Views of Immigrant Students in Mathematics Education

The literature surrounding immigrant students in U.S. schools revealed that immigrant students face mathematics language, English language, and cultural barriers that their native born peers do not, yet they were expected to achieve at the same levels as their English-speaking peers (Freeman & Crawford, 2008). Wang & Goldschmidt (1999) reported that a relationship existed between students' immigrant status and English proficiency, and placement in low-track mathematics courses. Furthermore, Paul (2005) found that initial algebra one grouping in middle school mattered, and that race and ethnicity were significantly associated with differences in enrollment in algebra two or higher mathematics courses. The reports asserted that immigrant students in low-track mathematics courses in middle school had very different opportunities to prepare for college and direct entry in the workforce. Recent reports documented that immigrant students were victims of negative discourses around immigration and education policies that made way into schools (Freeman & Crawford, 2008). Such an example is the No Child Left Behind (NCLB) Act that pressured schools to meet strict goals for all students no matter their cultural or language barriers (United States Department of Education [USDE], 2001). Immigrant students are victims of deficit discourses around policies which make their way into schools (Civil, 2014). One such example is the unintended result of the NCLB Act of 2001, where immigrant students were often placed in low-track mathematics courses in segregated classrooms that aligned with teaching practices reflecting a *pedagogy of poverty* (Freeman & Crawford, 2008; Haberman, 1991; Secada & Carey, 1990; USDE, 2001). Haberman (1991) coined the term *pedagogy of poverty,* to describe how schools contribute to the achievement gap by accepting a basic approach to teaching, where students are expected to repeat rote problems to demonstrate understanding and are punished for non-compliance. Needless to say, immigrant students have had to face several challenges that their native born peers did not.

Deficit views of immigrant students have implications for mathematics teaching and learning. Gates (2006) reported that, "In many parts of the world, teachers—mathematics teachers—are facing the challenges of teaching in multiethnic and multilingual classrooms containing immigrant, indigenous, migrant and refugee children, and if research is to be useful it has to address and help us understand such challenges" (p. 391, cited in Civil, 2014, p. 277). Previously, discussion surrounding immigrant students underscored a general public discourse that framed immigrant students as a problem. These deficit views of immigrant students are problematic and, "such will affect teachers who may view diversity approaches to doing mathematics that immigrant students often bring to mathematics classrooms as problematic rather than as an opportunity to learn" (Civil, 2014, p. 278). To further exacerbate the problem, Campbell et al. (2007) documented that approximately one in two teachers reported they taught students with limited English proficiency, and were not prepared to teach students with cultural and language differences. Therefore, mathematics teachers of immigrant students faced challenges of teaching English language, mathematics language and mathematics content in their classrooms; and more studies need to address these (Campbell et al, 2007).

The 2008 International Congress of Mathematical Education report called for improvements in mathematics education that were more inclusive for immigrant students that challenged the overwhelming deficit views (Civil, 2014). To begin, the report suggested that mathematics teachers of immigrant students ought to highlight immigrant students' experiences as funds of knowledge. Planas and Civil (2009) supported the view that immigrant students brought valuable mathematics knowledge to school, and teachers and schools ought to view diversity as a resource and gain deeper understanding of immigrant students' experiences rather than as obstacles. Furthermore, research called for mathematics teachers to incorporate different forms of doing mathematics as resources, including asking students to write and discuss in their native language and use informal language, to consider that diversity of language backgrounds play a prominent role in mathematics education of immigrant students (Cramer, 2001; Campbell et al, 2007; Garcia, 1991; Hansen-Thomas, 2009). These recommendations are starting points for mathematics teachers of immigrant students to create more inclusive learning environments and align with the Equity Principle (NCTM, 2000).

NCTM (2000) emphasized that mathematics teachers need to teach content with conceptual understanding to *all* students. However, in order to achieve the goals espoused in *PSSM* documents, a better understanding of ways to disrupt current deficit views of mathematics teaching approaches to immigrant students is needed. The empirical literature presented thus far report that different forms of mathematics exist, and that immigrant students bring rich mathematics experiences with them to school that were not always valued in the curriculum (Civil, 2014). One approach, as advocated by Garcia (1991), suggested that teachers encouraged students to use their native language, or to withhold using formal language in mathematics classrooms. This view that students ought to be able to use informal or semi-formal to learn challenging mathematics concepts was also supported by Cramer (2001) when teaching students linear algebra with conceptual understanding. Addressing attention to differences in languages that immigrant students bring to mathematics classrooms merely served as starting points to counter deficit views, and provide only a glimpse of the possible solutions. The remaining sections of the literature review will focus on features of successful mathematics programs and teaching approaches that have been shown to overturn the disparities in mathematics education for immigrant students. Therefore, the literature will also support the intervention tools selected for this study.

Immigrant Students Can Learn Mathematics with Conceptual Understanding

Immigrant students are capable of achieving success in mathematics as their native born peers (Paul, 2005; Wang & Goldschmidt, 1999). Empirical studies have documented middle immigrant students' success in mathematics, and highlighted how properly structured instructional strategies were associated with increasing students' opportunities to learn with conceptual understanding. Research by Wang and Goldschmidt (1999) and Paul (2005) asserted that middle school immigrant students taking challenging algebra courses are capable of achieving as their native born peers, and argued that immigrant students are not inferior in learning challenging mathematics concepts because they do not speak English as a first language. In particular, Wang and Goldschmidt (1999) examined the performances of middle school ELL students from various mathematics tracks and found that mathematics courses that were structured and provided support that were aimed at unique needs of ELL students, were as beneficial for ELL students as standard mathematics are for English proficient students. Similarly, Paul

(2005) examined middle school algebra students' mathematics placement, and found that students' initial algebra one grouping for immigrant students mattered. Results from these two studies illustrated that immigrant and ELL students had positive mathematics achievement outcomes that were comparable, or higher than their native born peers when they were provided structured support that emphasized communication of mathematical reasoning and use of multiple representations tailored to their unique experiences.

Findings from empirical studies so far support the argument that middle school immigrant students who had opportunities to learn mathematics in ways that aligned with NCTM (2000) learned mathematics with conceptual understanding. Namely, the strategies included structuring heterogeneous learning environments that encouraged team work, rich problem solving, communication of mathematical processes, whole class mathematical discussions, and multiple representations. To further support previous findings, Silver and Stein (1996) reported statistically significant gains in middle school immigrant students' conceptual understanding of mathematics and higher order thinking as a result of careful attention to language support, multiple representations, and mathematical communication. Similar results were reported by Hansen-Thomas (2009) with middle school English Language Learners (ELLs) that learned from Standards-based mathematics curricula that emphasized mathematical discussions and encouraged students' use of their native and informal language. The results showed that immigrant students that received consistent support through modeling and eliciting student participation in mathematical discussions, passed standardized mathematics exams at a greater rate than students who did not receive consistent support. These empirical studies, therefore, support that immigrant students can learn mathematics with conceptual understanding.

The literature review so far presented long term deficit views of teaching mathematics to immigrant students, and documented possible ways to counter deficit views of immigrant students in middle school mathematics classrooms. Taken together, the purpose of the literature review thus far was to argue for the need for research grounded in empirically tested intervention to improve mathematics education for middle school immigrant students (Hansen-Thomas, 2009; Silver & Stein, 1996). In the following sections, empirically grounded Standards-based mathematics teaching strategies are introduced that support conceptual understanding are presented. First, to further support the argument for more Standards-based mathematics teaching approaches, the failure of "Back to Basics" movement are presented. Then, salient features of Standards-based mathematics teaching approaches are elaborated, including structuring productive mathematical discussions and using multiple representations to support conceptual understanding. After that, the importance of algebra as a gatekeeper are discussed to further support the rationale for improving algebra instruction in the middle grades. Finally, a description and the benefits of a constructivist teaching experiment are presented.

The Failure of the "Back to Basics" Movement

In the late 1950s, there was a "new math" movement that shifted the national focus from computational skills to teaching mathematics with understanding. Two decades later, there was a push for a "back to basics" movement, which emphasized mathematical computations over meaningful applications. As a result of the shift to "back to basics," mathematics education was no longer challenging for U.S. students and the negative impact was later published in a document entitled, "A Nation at Risk" in the early 1980s (Senk & Thompson, 2003). The document highlighted that U.S. students were scoring below the international average, and proved that the "back to basics" movement was not effective. In 1987, the negative effect of the "back to basics" movement was further highlighted after the Second International Mathematics Study (SIMS) was published, which placed the U.S. below the international average out of twenty countries (Senk & Thompson, 2003). As a response to the international test results, mathematics reform efforts began in the late 1980s to the early 1990s, and are still going on today almost three decades after the SIMS report.

In 1989, NCTM intensified their efforts to reform mathematics education and introduced the *Curriculum and Evaluation Standards for School Mathematics* (CESSM), which was aimed to speak to the profession about a new vision for school mathematics and to politicians. The CESSM document was created for K-16, and included a list of standards to guide mathematics instruction that included: (a) problem solving, (b) communication, (c) reasoning, and (d) mathematical connections (NCTM, 1989). CESSM document challenged a traditional teacher-centered mathematics setting, where the teacher was responsible for student learning through direct instruction, where students often worked individually to reproduce what the teacher has shown them. In contrast, CESSM documents urged teachers to pose problems for students to work in small groups and that might be solved by using various strategies with no ability grouping. The vision of CESSM is revised in NCTM's *Principles and Standards for School Mathematics* (PSSM) document (2000).

For over twenty-five years, there have been heated debates about the merits of traditional versus Standards-based mathematics teaching approaches (Kieran, 2007). Standards-based mathematics teaching approaches challenged strongly held beliefs about what mathematics was most important and how it was taught most effectively (Senk & Thompson, 2003). Traditional approaches to teaching mathematics included the view that the teacher was the leader of discussions, and therefore, was responsible for imparting knowledge to students (Senk & Thompson, 2003). Whereas, advocates for Standards-based mathematics teaching approaches, argued that good telling did not ensure that students have learned; and instead, advocated for *judicious telling* (Smith, 1996). Although debates about how to effectively teach mathematics to students continue, the benefits of Standards-based mathematics for all students cannot be denied.

Standards-Based Mathematics Teaching to Overturn Existing Disparities

Standards-based mathematics teaching approaches are a possible solution for overturning existing disparities for immigrant students (Hansen-Thomas, 2009; Silver & Stein, 1996; Whitney, 2010). In Standards-based mathematics classrooms, teachers often began by posing problems for students to work on individually at first. Next, students worked in small, mixed ability groups that encouraged problem solving using multiple representations, and last, students summarized and presented their mathematical processes to the whole class (Cramer, 2003; Stein et al, 2009). Standards-based mathematics teaching challenged traditional views about what it meant to do mathematics from largely focusing on procedures and notations where algebra is seen as a tool for symbolic manipulation and solving problems, to an emphasis on understanding the intricacies within a single representation, as well as how representations are connected and related (Kieran, 2007; Senk & Thompson, 2003). Therefore, we need more studies that document successful implementation of Standards-based mathematics teaching approaches to middle school immigrant students.

Research by Billstein and Williamson (2003), Ridgway, Zawojewski, Hoover and Lambdin (2003), and Romberg and Shafer (2003) provide three studies that documented positive outcomes of Standards-based mathematics curriculum on middle school students from diverse socio-economic settings, to engage students in higher order thinking and mathematical discourse. Silver and Stein (1996) documented positive outcomes of Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), an educational reform project aimed at urban middle school students in low socio-economic status communities. The QUASAR Project drew from cognitive research that, “learning does not proceed lock-step fashion,” but required a more “meaning-based approaches” that encouraged development of conceptual understanding in mathematics (Silver & Stein, 1996, p. 480). The QUASAR project was implemented based on the premise that low performance of mathematics for poor urban and immigrant students were not due to primarily to a lack of student ability or potential, but rather to a set of educational practices that fail to provide them with high-quality mathematics learning opportunities. Evidence of the effectiveness of the reform program included growth in students’ understanding, problem solving, reasoning and communication on written exams, higher student performance on the National Assessment of Educational Progress (NAEP) than demographically similar students, and “increased number of students qualifying to take algebra in grade 9 at QUASAR schools, with QUASAR middle schools having the second highest passing rate” (Silver & Lane, 1993, cited in Silver & Stein, 1996, p. 506). Therefore, Silver and Stein (1996) argued that middle school urban and immigrant students needed appropriately enhanced and more structured forms of instruction to support learning mathematics with conceptual understanding.

Despite the documentation of success of Standards-based mathematics teaching approaches for *all* students, recent literature showed that there continues to be a prevailing “Back to Basics” approach in schools with high immigrant student populations. Recall that Haberman (1991) coined the term, *pedagogy of poverty*, to identify teaching approaches that aligned with a “Back to Basics” approach, where the teacher demonstrated problems to students, and students repeated back to demonstrate understanding, and were punished for non-compliance. Therefore, more research is needed that document the positive outcomes of Standards-based mathematics teaching approaches, and that document ways to support immigrant students in mathematics that align with the goals of NCTM (2000). The following sections will further elaborate on the characteristics of Standards-based mathematics teaching approaches that are aligned with the needs of immigrant students, as well as the intervention tools that were utilized in this study.

Productive Mathematical Discussions to Support Immigrant Students

Standards-based mathematics teaching approaches called for productive mathematical discussions in heterogeneous learning environments (Moschkovich, 1999; Senk & Thompson, 2003; Stein et al, 2009). Further, Standards-based mathematics teaching approaches emphasized problem solving, critical thinking through discovery learning, communication and discussion in group situations (Hansen-Thomas, 2009). With updated NCTM (2000) standards that set high expectations for all students, there were increased challenges on the part of all students to be able to not only to compute and calculate, but also to be able to talk "the mathematical talk," in order to be successful in academic endeavors (NCTM, 2000, p. 88-89). According to Hansen-Thomas (2009), successful teachers at engaging immigrant and ELL students in mathematical discussions included, "students' focused and engaged participation in group activities, their ability to respond appropriately to teacher elicitations, students' success on oral and written classroom assignments and tasks, and students' success on classroom and standardized math tests and assessments" (p. 93). In sum, productive mathematical discussions have been shown to support middle school immigrant and ELL students.

Despite documentation of success of structured mathematical discussion to engage immigrant students, it was not without critique. According to Ramptom, Roberts, Leung, and Harris (2002), one major critique of a collaborative approach in Standards-based mathematics teaching approaches were, "the lack of appropriate mechanisms to fully characterize language learning by not taking into account the complexities that culturally and linguistically diverse students experience in schools" (cited in Hansen-

Thomas, 2009, p. 91). Ramptom et al. (2002) argued for mathematical discussions that gave attention to students' diverse backgrounds further to support immigrant students in mathematics classrooms. Therefore, it was not enough to incorporate mathematical discussions, teachers needed to prepare in advanced ways to include students with language differences, and cultural experience differences (Campbell et al, 2007; Stein et al, 2009). This view aligned with Stein et al. (2009) that challenged the role of the teacher to include *anticipating, monitoring, selecting, sequencing* and *connecting*. Therefore, despite critiques of mathematical discussions as productive for immigrant and ELL students, evidence from the literature support that Standards-based mathematics teaching approaches engaged immigrant students when teachers took into consideration the needs of immigrant students' diverse experiences (Hansen-Thomas, 2009).

To continue with suggestions from Ramptom et al. (2002), reports supported that immigrant students need access to mathematics curriculum and instructional strategies that included attention to unique language needs of immigrant students required for achievement in mathematics (Cuevas, 1984; Hansen-Thomas, 2009). Mathematics teachers cannot assume that immigrant students will be familiar with linguistic structures encountered in the mathematics classrooms. Therefore, mathematics teachers need to ensure that:

> "…mathematics must not be taught by the teacher writing symbols on the blackboard, rearranging them, getting 'answers,' asking the class to copy the process and to learn it by heart. Instead the teacher must be trained to involve the

> children in carefully structured activities, investigations and discussions which will ensure understanding. (Morris, 1975, p. 52, cited in Cuevas, 1984, p. 139)

Standards-based mathematics teaching approaches take into consideration the above suggestions, and advocate for all students to learn mathematics with conceptual understanding. Further, the passage suggested that immigrant students need instruction that supports the use of students' native language including allowing informal language, before learning formal language of mathematics are supported by other studies (Cramer, 2001; Garcia, 1991; Campbell et al, 2007; Hansen-Thomas, 2009; Silver & Stein, 1996; Whitney, 2010). For example, Silver and Stein (1996) suggested that tasks, "intended to provoke students to engage in conceptual understanding, reasoning, or problem solving" have been linked with long term mathematics success for immigrant students (p. 483). Furthermore, Silver, Edward, and Lane (1991) suggested that effective strategies ought to include collaborations with researchers co-creating with teachers of immigrant students, where mathematical tasks emphasized communication and multiple representations and moved away from the "assembly line" approach (p. 3). Therefore, the goals of this study incorporated suggestions from Silver et al. (1991) to support middle school immigrant students with structured support for productive mathematical discussions that include opportunities to use their native language, or informal language.

The following sections of the literature review will shift to focus on the importance of algebra in middle school since algebra has been identified as a gatekeeper for students' mathematics and higher education trajectory (Kieran, 2007; Moses & Cobb, 2001).

The Importance of Algebra

Engaging immigrant students in algebra early on was seen as critically important as algebra education was deemed a gatekeeper for higher education (Kieran, 2007; Moses & Cobb, 2001; Paul, 2005). Moses and Cobb (2001) contended that, "Today, the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy" (p. 5). In response to the empirical literature that insist there is relationship between success in mathematics and economic access, since 2011 several states including Minnesota required all 8th graders to master the linear aspects of algebra as defined by MDE (2007). Although it was only recently that several states, including Minnesota, recognized the importance of algebra in the middle grades, success in algebra and higher education trajectory has been documented (Paul, 2005; Wang & Goldschmidt, 1999). Therefore, the following sections will be dedicated to arguing for intervention in middle school settings, and support the goals of this study further.

Paul (2005) asserted that, "The capacity to think and problem solve with mathematical training associated with completion of algebra should be the benchmark of defining mathematics literacy" (p. 263). Unfortunately, problem solving in algebra as a benchmark for mathematics literacy was becoming less of a reality for immigrant students as evidenced by the rate students were placed in lower-track algebra courses (Freeman & Crawford, 2007). Findings from empirical studies on middle school immigrants taking algebra suggested that unless student achievement in mathematics was raised by the end of eighth grade, little can be done in high school to change the percentage of students prepared for regular and competitive mathematics courses associated with preparing students for college (Wang & Goldschmidt, 1999; Paul, 2005). Needless to say, the quality of algebra education must be improved and expectations be raised for immigrant students. Therefore, these differences in algebra experiences of lower-tracks meant a potential for widening the achievement gap in high school among students in different ethnic groups.

Current initiatives have identified algebra as a central concern in mathematics education in the United States and have recast it as a longitudinal strand for students in Kindergarten through Grade 12 (NCTM, 2000). Middle school students need to be proficient in linear function ideas that include the following topics:

> "In Grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can understand concepts of variable, expression, and equation; represent situations and number patterns with tables and graphs… and develop confidence in solving linear equations using concrete, informal, and formal methods" (NCTM, 1989, cited in Moschkovich, Schoenfeld & Arcavi, 1993, p. 69).

These core ideas align with more recent NCTM (2000) documents that set high expectations for all students. For example, *PSSM* documents described linear algebra goals that include emphasizing mathematical connections and to solve novel problems rather than teaching disconnected facts and procedures to commit to memory (NCTM, 2000). As stated earlier, success in linear algebra leads to greater access to higher level mathematics (Paul, 2005; Wang & Goldschmidt, 1999). Unfortunately, school mathematics do not always emphasize the conceptual understanding of key ideas of linear functions, where communication and multiple representations between tables, graphs, and equations are emphasized (Moschkovich et al, 1993). The following section will detail effective intervention tools utilized in this study that were grounded in empirical literature.

The Lesh Translation Model

Lesh, Post, and Behr (1987) asserted that students with mathematical proficiency had the capacity to move freely among representational forms of the same mathematical ideas as a characteristic of competent performance. This view was supported by Silver and Stein (1996), that added that, "the ability to generate or use multiple representations was associated with characteristics of higher order thinking and reasoning" (p. 484). Moschkovich (2002) advocated for multiple representations to help increase comprehension of mathematics for immigrant and ELL students. All of these views aligned with NCTM (2000) that, "When students gain access to multiple representations and the ideas they represent, they have a set of tools that significantly expands their capacity to think mathematically" (p. 67). Additionally, NCTM (2000) added that supplying context as a tool further supported student communication. Huntley, Marcjus Kahan, and Miller (2007) found that as students became more mathematically sophisticated, they developed, "an increasingly large repertoire of mathematical representations as well as knowledge of how to use them productively" (p. 116). An example of mathematically sophisticated understanding included translating between and within representations. Therefore, one can argue that multiple representations are necessary to implement Standards-based mathematics teaching approaches to support student communication of mathematical ideas.

Figure 2.1 displays the *Lesh Translation Model* (LTM), a tool that mathematics teachers have used to plan and implement lessons that supported teaching for conceptual understanding (Cramer, 2003). The creation of the LTM was heavily influenced by cognitive scientists such as Bruner (1961), Dienes (1960) and Piaget (1970) and emphasized that meaningful mathematics lessons required instruction that emphasized connections between and within representations: manipulatives, pictures, real-life context, verbal symbols, and written symbols (Cramer, 2003). Further, the arrows in in the model required that teachers guided students to represent mathematical ideas in multiple ways, and to make connections among different embodiments. Each time students translated from one representation to another representation, they must reinterpret their understanding and make mathematical connections. That is, translations between various modes of representations helped to make mathematical ideas more meaningful for immigrant students.

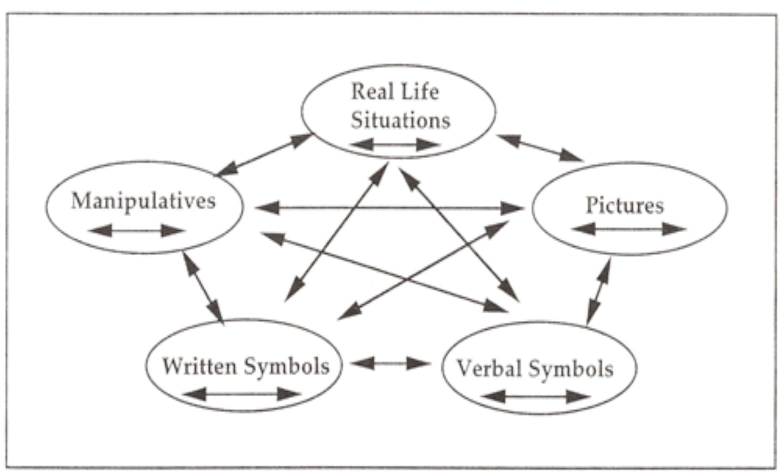


Figure 2.1. The *Lesh Translation Model* (Cramer, 2003).

Mathematics teachers have used LTM with students of all levels, from elementary to pre-service teachers, including with immigrant students to guide their instructional decisions and their views of assessment with gains in students' mathematical achievement (Cramer, 2003; Hofacker, 2006; Whitney, 2010). Further, it has been documented that learning algebra with multiple representations was effective for *all* students, and not just immigrant students (Garcia, 1991; Hansen-Thomas, 2009; Senk & Thompson, 2003; Silver & Stein, 1996; Whitney, 2010).

In alignment with *Standards*-based mathematics teaching approaches, LTM supported learning mathematics with understanding through multiple modes of representation, such as promoting immigrant students' thinking, reasoning and problem solving, and communication to a much greater extent as aligned with goals of NCTM (2000). Unlike in traditional curricula, where mathematics was characterized with written symbols and where immigrant students were often taught manipulation of symbols too prematurely, LTM aligned with NCTM (2000) goals to teach with emphasis on process skills such as mathematical reasoning and justification (Cramer, 2003; Secada & Carey, 1990). Therefore, LTM challenged traditional mathematics curricula that did not give students full opportunities to understand or make sense of topics and often emphasized memory rather than understanding, and will support planning and teaching in this study. The next section introduces supports for orchestrating productive mathematical discussions.

A Framework for Orchestrating Productive Mathematical Discussions

Mathematical discussions can engage immigrant students in sense-making in authentic ways and help move the class toward the development of important and worthwhile ideas in the discipline. In a widely cited article, "Keeping an Eye on the Mathematical Horizon," Ball (1993) argued that the mathematics field needs to take responsibility for helping teachers to learn how to continually "size up" whether important mathematical ideas were being developed in these discussions and be ready to step in and redirect the conversation when needed (cited in Stein et al, 2009, p. 319). Because Standards-based mathematics was grounded in theory for productive mathematics engagement, teachers must learn to orchestrate discussions in ways that engage students' sense-making in authentic ways and to move the class as a whole toward the development of important mathematical ideas.

Table 2.1

Five Practices for Orchestrating Productive Mathematical Discussions (Stein et al, 2009)

Role of the Teacher	**Description**
Anticipating	*Anticipate* likely student responses to cognitively demanding tasks
Monitoring	*Monitor* students' responses to the tasks during the explore phase (recall: launch, explore and discuss" phases of reform)
Selecting	*Select* particular students to present their mathematical responses during the discuss/summarize phase
Sequencing	*Sequence* or purposefully sequencing the student responses that will be displayed
Connecting	*Connect* or help the class make mathematical connections between different students' responses and between students' responses and the key ideas

Table 2.1 above summarized five strategies that teachers used to support mathematical communication in their classrooms: *anticipating, monitoring, selecting, sequencing* and *connecting* (Stein et al, 2009). These were strategies that were shown to be useful for all students, but that could be elaborated in order to address needs of immigrant students. Grounded in theory for productive disciplinary engagement, the five practices were designed to support Standards-based mathematics instructional practices that were inquiry-based and student-centered. The model, therefore, supported students' accountability through a set of teaching practices that took students' ideas as a launching point, built on resources provided, where the teacher selected responses in order that supported instructional goals through careful selection and steering (Stein et al, 2009). Additionally, attention to mathematical discussions encouraged students to be authorities of their mathematical learning through problem solving, where their ideas are publicly credited and students were held accountable for their ideas, and not only their solutions in Standards-based mathematics classroom settings.

Earlier, there was discussion around the criticisms despite well documented successes with Standards-based approaches to teach middle school immigrant students mathematics with conceptual understanding. Stein et al. (2009) described some of the challenges of Standards-based teaching approaches in three ways. First, there was lack of support for reform classroom teachers, who were often not trained in selecting and planning cognitively demanding tasks. Second, there was criticism when the "near-complete control of the mathematical agenda is relinquished to students with respect to what teachers *could* do to encourage rigorous mathematical thinking and reasoning"

(Stein et al, 2009, p. 319). Third, a common challenge for teachers in Standards-based settings was, "efforts to increase student accountability may lead to a message to students that strategies need to be validated by the teacher, thus students tend to no longer report what they actually think about a problem but what they believe their teachers will respond to favorably" (Stein et al, 2009, p. 333). These criticisms are important and guided the implementation of the framework in the study that utilized a constructivist teaching experiment. The following section describes constructivist teaching experiments as an appropriate methodology for the goals of this study, and present the research questions.

Constructivist Teaching Experiments

Teaching mathematics to immigrant students requires designing equitable environments that accommodate differences to help everyone learn mathematics with high expectations and strong support (NCTM, 2000). Cobb and Steffe (2011) described constructivist teaching experiments (CTEs) as researchers acting as teachers and interacting with a single student, or groups of students to attempt to guide constructive activities (Cobb, Yackel & Wood, 2011). This study sought to understand how utilizing Standards-based mathematics teaching approaches combined with LTM and a framework for orchestrating productive mathematical discussions supported immigrant students' conceptual understanding of key ideas linear functions. Because this study includes using Standards-based mathematical activities to help make transparent immigrant students' mathematical processes orally and in writing, CTE methods are appropriate for answering the research questions.

A CTE approach to answering the research questions articulated for this study helps to answer three research questions in more rich ways, but the approach also helped address three criticisms presented earlier about Standards-based mathematics that: (a) teachers are often not trained in selecting and planning cognitively demanding tasks, (b) teachers should encourage more rigorous mathematical thinking and reasoning, and (c) teachers need to increase student accountability of their mathematical explanations (Stein et al., 2009). Further, CTEs challenged traditional mathematics teaching that emphasized rote skills that teach facts in isolation, and aligned with Standards-based mathematics teaching that were associated with teaching higher-order thinking skills (Cobb, Yackel & Wood, 2011). CTEs are a form of design experiments that involve both developing instructional designs to support particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them. The central tenets for CTEs are (a) be experientially real for students, (b) guide students to reinvent mathematics using their commonsense experience, and (c) provide opportunities for students to construct their own understanding based on their experiences. The purpose of CTEs as a methodology is, therefore, to enable researchers to investigate the *process* by which individual students reorganize their mathematical ways of knowing and thus, inform teachers in their decision making (Cobb, 2003). Further, Cobb (2003) pointed that the emphasis of CTEs are on the interpretation of students' mathematical reasoning rather than on the development of the intervention (instructional design). Given the pragmatic focus on individual students' reasoning and conceptual understanding, CTEs require an analytic approach that takes into account diverse ways in which students participate in smaller teams and whole class settings. More important, CTEs addressed deficit mathematics teaching approaches (Civil, 2014). Further, Cobb (2000) reported that teachers who participated in teaching experiments were more likely to be effective in supporting students' mathematics development.

Theoretical Framework

The literature review presented here has been used to expand the understanding of the problem, and the theoretical framework will depict the literature base that was used to shape the investigation. In the following sections, the theoretical framework summarizes the "lens" through which data was collected and analyzed (McKinney & Reeves, 2012).

Lesh and Sriraman (2005) claimed that mathematics education literature showed little research on advancing teaching and learning, and therefore needs to respond to both practical needs of classroom teachers. Further, Cobb (2007) argued for more mathematics education research as a design science involving developing, testing, and revising designs for supporting envisioned learning processes. Therefore, the CTE aligned with the need for more research to solve practical problems grounded in theory in a real world classroom context. Guided by a review of the literature on how people learn, this study drew upon constructivist theories of learning. Campbell et al. (2007) described constructivist teaching theories as having, "as many answers as there are theoreticians" (p.135). To further clarify constructivist teaching theory, this section presents an empirical collection of Paul Cobb's "greatest" contributions to the field of mathematics education research according to Yackel, Gravemeijer and Sfard (2011). In, *A Journey in Mathematics Education Research,* CTEs were richly documented. Cobb and Steffe (2011) described constructivist teaching as an “emphasis placed on activity of modeling children’s realities” (p.21), where teachers’ mathematical knowledge played a crucial role in decision on what could be constructed by students, and that teachers should make a *conscious* attempt to “see” both their own point of view and children’s point of view. Therefore, for the purpose of this study, the theoretical framework that guided the study design was a learning theory called *Constructivism*.

Constructivism was born from cognitive science, that the belief that learning mathematics with understanding was process-oriented with emphasis on mathematics reasoning with written and oral explanations, not merely symbolic manipulation and rote skills (NCTM, 2000). Constructivism posited that student learning was an active process and that students were not passive learners, and did not enter mathematics classrooms as blank slates. Constructivist mathematics teachers viewed students’ unique experiences and prior knowledge as important to their learning such as how they received and constructed new knowledge. Therefore, constructivism goals emphasized the role of the teacher to create meaningful mathematical experiences such as using multiple representations to support immigrant students in algebra as evidenced by the literature. Cobb and Steffe (2011) said, “Children’s construction of mathematical knowledge is greatly influenced by the experience they gain through interaction with their teacher (p. 19). For the purpose of this study, constructivism guided the design and focus on students’ mathematical learning *process* through interventions that emphasized the use of multiple representations and mathematical discussions. Contributions from cognitive psychology to learning theories advanced mathematics teaching and learning theory, and further justified the study goals. Therefore, constructivism guided the vision to support learning environments that were developmentally appropriate as well as attended to immigrant students' prior knowledge. The following section presents three research questions that guided this study.

Research Questions

This study builds on the literature on the effectiveness of Standards-based mathematics teaching approaches, the use of multiple representations as described in the Lesh Translation Model, and the importance of mathematical discussions for conceptual understanding. However, few studies documented the effectiveness of these strategies working together for middle school immigrant students. Due to the gaps in the literature, this study will address the following research questions. The questions are restated here:

1. In what ways do immigrant students demonstrate their understanding of key ideas of linear functions by expressing translations between and within modes of representations <u>prior</u> to participating in mathematical activities that emphasize LTM?
2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions <u>after</u> participating in mathematical activities that emphasize multiple representations from LTM?
3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Overview of Subsequent Chapters

Chapter 2 provides a review of the literature that guided the rationale for the research questions and a review of the research upon which this study builds. Chapter 3 presents the research methodology, research design and research instruments. It ends with how the data is analyzed and aligns the data sources with the related research questions. Chapter 4 presents the findings from numerical and descriptive coding from this study. Chapter 5 presents a summary of the study, conclusion, implications and recommendations for future studies.

Chapter 3

Research Methodology

Chapters 1 and 2 argued that immigrant students' performance in middle school algebra impacted their access to competitive mathematics courses in high school and preparation for higher education. It was also argued that immigrant students were capable of learning mathematics with conceptual understanding as their native born peers, if provided with structured heterogeneous learning environments that emphasized communication, attention to language differences, and multiple representations in team settings. This study responds to the literature that middle school immigrant students that consistently perform in the bottom third of standardized mathematics assessments, need access and opportunities to learn rigorous mathematics with conceptual understanding (National Council of Teachers of Mathematics [NCTM], 2000; National Center for Educational Statistics [NCES], 2007). One factor identified as being a major cause of underachievement for immigrant students included lack of teacher preparation and overemphasis on a "back to basics" teaching approach in segregated settings. Rather than blaming underachievement to differences in immigrant students' language and cultural barriers, teachers ought to view differences as resources for teaching. Recommendations from the empirical literature support inclusive strategies that paid attention to language differences (i.e. encourage use of native/informal language, and delay formal symbolic manipulation), productive mathematical discussions, and multiple representations in heterogeneous team settings (Campbell, Adams, & Davis, 2007). Therefore, the purpose of this study was to document the ways that middle school immigrant students demonstrate conceptual understanding of key ideas of linear function before and after participation in a constructivist teaching experiment that emphasized Standards-based mathematics teaching approaches.

This research study was conducted to provide evidence of whether implementing Standards-based mathematics teaching approaches that emphasized multiple representations and mathematical discussions supported 7th grade immigrant students' conceptual understanding of key ideas of linear functions. Studying students' written and oral communication through peer-problem solving in teams, whole class presentations of group work, and partner interviews working on tasks that emphasized the Lesh Translation Model [LTM] and communication of mathematical processes, allowed the researcher to examine the following research questions (Cramer, 2003):

1. In what ways do immigrant students demonstrate their understanding of key ideas of linear functions by expressing translations between and within modes of representations prior to participating in mathematical activities that emphasize the LTM?
2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions after participating in mathematical activities that emphasize multiple representations from the LTM?
3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Further, this study utilized a constructivist teaching experiment (CTE) with a group of 7th grade immigrant students enrolled in an intervention mathematics course (Cobb, 2011). The CTE involved implementing modified problems from a Standards-based curriculum, *College Preparatory Mathematics* (CPM), that offered more accessible problem solving opportunities for immigrant students in heterogeneous team settings, and whole class discussion opportunities in the following format: *launch, explore* and *summarize/discuss* (Dietiker & Baldinger, 2006; Stein, Smith, & Hughes, 2009).

Current initiatives in mathematics education in the United States focus on algebra standards that span from kindergarten through Grade 12 (NCTM, 2000; U.S. Department of Education, 2008). Policy changes focused on algebra in younger grades (hereafter, *early algebra*), tying success in early algebra as a critical path to students' long term mathematical success (Katz, 2007, cited in Blanton, Brizuela, Gardiner, Sawrey & Newman-Owens, 2015, p. 512; Paul, 2005). The importance of success in early algebra was also seen through policy supported from several states including Minnesota that now require algebra for all 8th graders. However, recent standardized test scores for 8th grade students in Minnesota indicated that immigrant and ELL students were at risk compared to their Caucasian counterparts (Minnesota Department of Education [MDE], 2014). Results for Minnesota 8th graders reported that ELL students were one third as likely to be prepared for higher level mathematics that were required for college entrance (MDE, 2014). Standardized test reports supported recommendations for more research on teaching and learning early algebra as a high-priority area for further research (RAND Mathematics Study Panel, 2003, cited in Huntley, Marcjus, Kahan & Miller, 2007, p.

115). These large mathematics achievement gaps pointed to the premise that unequal access to academic curricula produced inequities in student learning of mathematics, and the slogan, *algebra for everyone* has been popular in reform as a response to inequalities (Gamoran & Hannigan, 2000). This research study aimed to create early algebra access for 7th grade immigrant students, and to place early algebra learning with conceptual understanding at the forefront to prepare *all* students for competitive majors in STEM fields. In this chapter, the theoretical framework, context of the study and participants, and the research design are discussed.

Research Design

This research is a descriptive case study of a constructivist teaching experiment [CTE] in order to contribute to the literature on how Standards-based mathematical teaching approaches that emphasized multiple representations and mathematical discussions supported 7th grade immigrant students' conceptual understanding of key ideas of linear functions (Cobb, 2011; Hansen-Thomas, 2009; Silver & Stein, 1996; Yin, 2014). A CTE involves a researcher acting as the teacher and interacting with students to attempt to guide constructive activities (Cobb & Steffe, 2011; Cobb, Yackel & Wood, 2011). The descriptive case study method allows the researcher to focus on a particular setting in smaller groups, and to manage ongoing data collection, while closely examining the data in order to make revisions to the intervention (McKinney & Reeves, 2012). A descriptive case study is appropriate for this study because the purpose of a case study is to provide a more detailed account of the event or phenomenon under study with, "real people in real situations, enabling readers to understand ideas more clearly," and for more explanatory power of the effectiveness of the intervention (Cohen, Manion & Morrison, 2011, p. 289). The data sources in this study were artifacts from seven middle school immigrant students' participation in a CTE that emphasized key ideas of linear functions.

Patton (2003) described purposeful criterion sampling as a method in which the researcher chooses participants that meet a certain criterion that allow for rich data collection. This study utilized purposeful criterion sampling to ensure that information-rich cases supported the goals of the study to examine the effectiveness of implementing Standards-based mathematics teaching approaches, multiple representations, and mathematical discussions. The criteria for this study required that the school had high immigrant population, students had above average class attendance, students were willing to be a part of the study that required video and audio recorded interviews of them working on mathematical tasks, and students were not currently using Standards-based curriculum. Also, the teacher must not already be using Standards-based mathematics curricula.

Settings and Participants

Participants in the study were eight, 7^{th} grade immigrant students, six students that identified as "does not meet," or "partially meets" grade level standardized reading and mathematics tests, enrolled in a semester-long intervention mathematics classroom; and two students that identified as "meets" or "exceeds" grade level standardized reading and mathematics tests who were added to the case study. Heterogeneous learning environments align with Standards-based mathematics teaching practices and the goals of the CTE, and therefore, support more fruitful discussions (Cobb, 2011). The researcher and the 7th grade mathematics teacher made up the "research team." The roles of the research team included: select and modify rich mathematics problems from CPM; conduct anticipatory thought experiments and write open-ended questions; implement the intervention, reflect as a team, and modify subsequent lessons (Cobb, 2011; Dietiker & Baldinger, 2006; Stein et al, 2009). The researcher is an experienced teacher of the course content with a valid 5-12 Minnesota mathematics teaching license.

The intervention course met twice a week for 80 minutes. During the course of two months, a total of ten mathematical tasks were implemented, and a total of three interview assessments of students working in pairs took place: a pre-assessment took place prior to the study, a mid-assessment took place after five tasks were implemented, and post-assessment took place after all ten tasks were implemented.

The School Context

Harmony Charter School (pseudonym) was located in an inner-city middle class neighborhood, and was part of the largest school district in a Midwestern state. Harmony served students from grades five to eight: approximately 90% of students were Hispanic and approximately 10% of students were either Black, American Indian/Alaskan Native, White, or Asian/Pacific Islander. At Harmony, 61 % of students were classified as ELLs, and over 95% of students received free or reduced lunch (MDE, 2015). According to MDE (2015) a majority of Harmony teachers were considered "highly-qualified."

Selection and Description of the Participants: The 7th Grade Teacher, Mr. Champion

At Harmony, there was one mathematics teacher for each grade level from grades five to eight. The 7th grade mathematics teacher, Mr. Champion (pseudonym) was selected for this study. Mr. Champion was identified by the principal as one of the top mathematics teachers at Harmony. He entered teaching via an alternative pathway through Teach for America, and this was his third year teaching at Harmony. Mr. Champion is a middle-aged Caucasian male with English as his first language, holds a bachelor's degree in Economics, and only recently received a Minnesota 5-12 mathematics teaching license midway through the study. At Harmony, Mr. Champion held several roles: he wrote the 7th grade curriculum that had been used for the past three years, taught mathematics and led professional development. In a semi-structured interview prior to the study, Mr. Champion described his teaching style as traditional, and emphasized test taking skills and rote procedures over conceptual understanding. The pre-interview data revealed that Mr. Champion met criterion sampling (Patton, 2003).

Prior to the study, the researcher served as a volunteer in Mr. Champion's mathematics and intervention classrooms to ensure that the school, teacher and students all met purposeful criterion sampling (Patton, 2003). During a typical period in Mr. Champion's class, he stood at the front and presented examples that students were expected to repeat to demonstrate competency. For example, students were asked to identify slope and write equations for proportional relationships without connecting the table, graph, or real-life context to make sense of the equation. An electronic timer was displayed to "motivate" students to stay on task. Students were often asked to enter their solutions via technology that was translated into a frequency chart that displayed a summary statistic, emphasizing solutions over process. This teaching style is reflective of the *pedagogy of poverty*, where students were given busy work absent of real-world problem solving, and were punished for non compliance (Haberman, 1991). Additionally, Mr. Champion spoke Spanish as a second language, the primary home language of the majority of his students. However, this skill was applied as a form classroom management to communicate with parents rather than as part of his pedagogy to engage a majority of students.

The Classroom Context: Grade 7 Intervention Class

Mr. Champion taught three traditional 50-minute pre algebra courses in the morning, and one 40-minute intervention course with high immigrant student population in the afternoon. The afternoon intervention class was built into the school schedule and was reserved for students who were identified as "does not meet," or "partially meet" mathematical proficiency as determined by state standardized mathematics test results from the previous year (MDE, 2014). The intervention class met twice a week on Tuesdays and Thursdays for a total of 80 minutes per week. Students who were in the intervention class comprised of 50% female and 50% male. The class had 14 students ranging from low to medium English proficiencies as determined by their state standardized reading test scores (MDE, 2014). The 40-minute intervention class was set up with the expectation that students worked individually to complete worksheets that contained up to forty drill problems that were absent of real-life applications. The worksheets were often hand written, or cut and pasted from traditional online materials the hour prior to the start of the intervention class. The afternoon class was unlike his traditional morning mathematics class since there was little to no formal teaching, or teacher interaction, and no discussion or real-world problem solving. Instead, students sat quietly and waited to be helped by the teacher as needed. If students had questions the common practice was to ask their teacher and not their peers. The afternoon intervention class was selected for the constructivist teaching experiment since the profile of the teacher and the students matched closely with purposeful criterion sampling goals of the study (Patton, 2003). The teacher taught a high group of immigrant population, and students were not currently using Standards-based mathematics curricula.

The Student Participants: N = 8 Students

Fourteen students ranging from Low English Proficiency (LEP) to Medium English Proficiency (MEP) made up the afternoon intervention class. For the duration of the study (ten class periods), two students, one Medium English Proficient (MEP) and one High English Proficient (HEP) student, that were willing to participate in the study were added to the afternoon class for a total of sixteen students to create a more heterogeneous learning environment (Senk & Thompson, 2003). Of the sixteen students, eight students with various reading levels and mathematics levels were selected to take part in the study: four males and four females with no single gender groups, to ensure maximum variation and a representative sample of the student population (Cohen, Manion, & Morrison, 2011). Studying eight students closely allowed the researcher to more richly document students' interactions and responses to mathematical activities, enabling the researcher to conduct deeper analysis of three research questions (Patton, 2003).

A total of seven out of eight students remained part of the study from start to finish. One LEP student was dropped from the study because she failed to complete the post-assessment and post-interview partner assessment. Three students, Anna (27th percentile), Joe (27th percentile), and Olvin (14th percentile) were identified as *Low-English proficient (LEP)* from their Measures of Academic Progress (MAP) reading percentile with all scoring less than the 33rd percentile. Three students, Lucas (59th percentile), Sandy (56th percentile) and Brooke (53rd percentile) were identified as *Medium-English proficient (MEP)* from their MAP reading scores as they scored above the 33rd percentile, but less than the 66th percentile. One student, Leon (87th percentile), was identified as *High-English proficient (HEP)* from his MAP reading score above the 66th percentile. Table 3.1 displays the percentile ranking from the MAP scores of eight middle school immigrant students that were selected to participate in the study (MDE, 2014). Students' MAP mathematics percentile rankings were included in the table to present a holistic picture of the heterogeneous learning environment. It is important to point out that although some students were identified as LEP based on their reading scores, their mathematics scores were not necessarily low as well. For example, two students who identified as LEP, Olvin and Anna, identified as *Medium Proficient in Mathematics* (MPM) with both scoring above the 33rd percentile. Whereas three MEP students, Jessica, Sandy, and Lucas, identified as *Low Proficient in Mathematics* (LPM) based on their MAP mathematics scores with all scoring below the 33rd percentile. One

HEP student, Leon, had reading and mathematics scores that aligned, and identified as *High Proficient in Mathematics* (HPM) based on his reading and mathematics proficiency levels being above the 66^{th} percentile.

Table 3.1

MAP Reading and Mathematics Percentile Rank for Eight 7^{th} Grade Participants

Student Names*	**MAP Reading Percentile**	**English Language Proficiency**	**MAP Mathematics Percentile**	**Mathematics Proficiency**
Olvin	14^{th}	Low (LEP)	37^{th}	Medium (MPM)
Anna	21^{st}	Low (LEP)	40^{th}	Medium (MPM)
Joe	27^{th}	Low (LEP)	4^{th}	Low (LPM)
Jessica**	35^{th}	Medium (MEP)	15^{th}	Low (LPM)
Brooke	53^{rd}	Medium (MEP)	63^{rd}	Medium (MPM)
Sandy	56^{th}	Medium (MEP)	21^{st}	Low (LPM)
Lucas	59^{th}	Medium (MEP)	32^{nd}	Low (LPM)
Leon	87^{th}	High (HEP)	89^{th}	High (HPM)

**pseudonyms*
***dropped out of the study*

Timeline

Table 3.2 shows the timeline of the study.

Table 3.2

Timeline of the Study

Description	Timeline
Pre Interview & Pre Assessment 1	November 2015
Mathematical Tasks (1-5)	November 2015
Mid Interview Assessment 2	November 2015
Mathematical Tasks (5-10)	December 2015
Post Assessment 3	December 2015
Post Interview 3	January 2016

The Intervention

Constructivist teaching experiments (CTEs) are considered a type of design research. In design research, Lesh and Kelly (2000) advocated going beyond changing one aspect of student learning, namely the curriculum, and creating environments that produced changes in subjects whose knowledge or abilities were being investigated. The intervention instruments for the study are discussed in this section. To begin, the research team selected a National Science Foundation (NSF) funded algebra curriculum, *College Preparatory Mathematics* (CPM), with an overarching goal to provide students with skills and knowledge necessary to reason and communicate mathematically (Dietiker & Baldinger, 2006). The curriculum was selected to serve as a guide since it contained rich linear function problems that aligned with the overarching goals of the study. Next, the research team narrowed the focus on key ideas of linear functions to graphing and writing linear equations based on middle school algebra mathematics goals (MDE, 2007).

Prior to the study, middle school immigrant students mostly worked with linear functions that followed a proportional linear pattern in the form, $y = kx$, where k is the *unit rate*, or *slope*. During the constructivist teaching experiment, students worked with non-proportional linear patterns in the form, $y = mx + b$, where m is the *pattern* (slope), and b is the *starting value* (y-intercept). In the constructivist teaching experiment, students used their current understanding of key characteristics of proportional linear relationships to learn about non-proportional linear relationships. As this study was interested with the ways in which 7th grade immigrant students communicate and use representations to demonstrate their conceptual understanding of key ideas of linear functions, data related to the students' understandings of the concept were included on the pre-assessment interview questions. The key ideas of linear function in the pre-assessment included identifying slope, making predictions, graphing, writing equations, and writing original real-life situations about proportional linear patterns from a table of values. Table 3.3 displays the pre-assessment questions as they aligned with MDE (2007) middle school algebra benchmarks and lists the embedded translations. The pre-assessment questions aligned with the following algebra benchmarks: (1) determine the unit rate; (2) understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate; (3) understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form $y = kx$; (4) represent real-world relationships using an equation; (5) represent proportional relationships with verbal descriptions, and translate from one representation to another.

Table 3.3

Pre-Assessment Questions Aligned with Algebra Benchmarks and Embedded Translations

Pre-Assessment Questions	**Algebra Benchmark**	**Embedded Translations**
4a. Given a table, if you know x, can you find y? How do you know that is correct?	Determine the unit rate (*constant of proportionality* or *slope*).	*Picture (Table) to Verbal & Written Models (Equation)*
4b. Graph data from a table. What would the graph look like? How do you know? (Discuss steepness).	Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (*constant of proportionality*).	*Picture (Table) to Picture (Graph), Written & Verbal Models (Equation)*
4c. Write an equation from a table in y = *k*x format. What is k? Explain how you know the answer is correct.	Understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form y=*k*/x, or y = *k*x.	*Picture (Table) to Written Model (Equation)*
4d. Given a real-life situation, what does k stand for? 5. Write a real-life situation from a table.	Represent real-world or mathematical situations using equations involving variables and positive and negative rational numbers. Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.	*Written Model (Equation) to Real Life Situation* *Picture (Table) to Real Life Situation, & Verbal Model*

Table 3.4 displays the post-assessment questions as they aligned with MDE (2007) middle school algebra benchmarks and lists the embedded translations. The post-assessment questions aligned with topics from the constructivist teaching experiment, and focused on non-proportional linear function patterns that aligned with the following algebra benchmarks: (1) identify graphical properties of linear functions including slope and y-intercept, (2) know that slope equals the rate of change, and that the y-intercept is

zero when the function represents a proportional relationship; (3) understand that a function is linear if it can be expressed in the form y = *m*x + b; (4) use linear equations to represent situations involving constant rate of change, including proportional and non-proportional relationships; (5) represent linear functions with verbal descriptions; and translate from one representation to another.

Table 3.4

Post-Assessment Questions Aligned with Algebra Benchmarks and Embedded Translations

Post-Assessment Questions	**Algebra Benchmark**	**Embedded Translations**
1a. Given a table with a real-life situation, describe the pattern for x and y. 2a. Give a picture, describe the pattern using words or a table. Then draw figures 0 & 4. 3a. Given a graph with a real-life situation, describe the pattern using words or a table 1d & 2c. Is the pattern proportional? Give two reasons.	Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship. Understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form y = *k*x.	*Picture (Table) to Verbal Model* *Picture (Table) to Written Model (Equation)*
1b. Write an equation using the pattern and the starting value. 2b. Write a rule for the pattern. How many tiles will the 50^{th} figure have? How do you know? 3c. Write an equation using the pattern and the starting value. 4c. How do you write an equation for this graph?	Understand that a function is linear if it can be expressed in the form f(x) = *m*(x) + b, or if its graph is a straight line. Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships.	*Picture (Table) to Verbal Model* *Picture (Table) to Written Model (Equation)*
1c. How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check? 3b. What is the meaning of the starting value and pattern? 4a-b. How many tiles are in	Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.	*Picture (Table) to Verbal Model* *Picture (Table) to Written Model (Equation)* *Picture to Real-Life*

figure 100? List three possible strategies. Pick one of the above strategies and use it to find the number of tiles in figure 100.		

Table 3.5 displays the lesson objectives, big questions, and detailed lesson objectives from CPM that were selected for the study (Dietiker & Baldinger, 2006).

Table 3.5

Select Topics on Graphing and Writing Linear Functions from CPM (Dietiker & Baldinger, 2006)

Lesson Objectives	**Big Questions**	**Detailed Lesson Objectives**
Extending Patterns and Finding Rules	*What is the rule?*	Identify rule for a pattern and state rule in words
Using Tables, Graphs, and Rules to Make Predictions	*How can I make a prediction?*	Find rule for patterns and write rule algebraically using symbolic notation; evaluate algebraic expressions to make predictions about a pattern

From Table 3.5, the research team redesigned ten mathematical tasks that reflected Standards-based mathematical practices to better fit the needs of current 7th grade student participants. The modifications included the following criteria: (a) select rich problems that allowed for multiple perspectives and multiple entry points, (b) pay attention to language differences and encourage students to restate the question using informal and native language in team settings, (c) ask students to use two or more representations to justify their solution in writing and orally, (d) ask students to present their work and reflect on peer presentations (see Figures 3.1 and 3.2 below) . Recall that Standards-based mathematics teaching approaches were defined as student- and problem-centered learning environments that encouraged use of multiple representations and promoted rich mathematical discussions in heterogeneous team settings (Senk & Thompson, 2003). Table 3.6 lists a description of ten mathematical tasks along with the type of translation being assessed (Cramer, 2003).

Table 3.6

Ten Mathematical Tasks Completed by 7th Grade Intervention Student Participants

Task	**Translations Embedded in Ten Tasks**
1-2	Given tile patterns, *identify* a rule for a pattern, *draw* missing figures, *explain* how you know using 2 representations: Table of values, Graph, Equation, Diagram, Numeric or written explanation (*Picture-tiles to Verbal/Written Models, & Picture-tiles/graphs*)
3	Given a table of values with a real-life context, *identify* missing values, *estimate* large x values, and *represent* situation using an equation (*Picture-table to Verbal/Written Models*)
4	Given a table of values with a real-life context, *identify* missing values, *plot* the points on a graph, *identify* y-values from a graph given x = 0 and x = a decimal value, *write* an equation using the table or graph, *identify* y-value given large x value using the context (*Picture-table to Verbal/Written Models, Picture-graph*)
5	Given a graph with real-life context, *interpret* a single point using real-life context, *discuss* "pattern" (slope) and "starting value" (y-intercept) in an equation using a table and a graph. Given a table, and a graph, *write* an equation, and a story to represent each relationship (*Picture-graph to Verbal/Written Models, & Real-Life Situation*)
6	Given algebra pattern tiles, *draw* missing figures, *represent* the pattern using a table and a graph, *explain* how you wrote your equation, *identify* future values and *justify* using two representations (*Picture-tiles to Verbal/Written Models, & Real-Life Situation*)
7	Given tables with real-life contexts, *describe* the pattern in words, *identify* the pattern and starting value, *test* the equation and graph the relationship; *describe* similarity and differences (*Picture-table to Verbal/Written Models*)
8-9	Given graphs with no context, *describe* the pattern using words or a table, *identify* y-intercept informally, *write* an equation and test the equation, *describe* how to write an equation given a graph (*Picture-graph to Verbal/Written Models*)
10	Given a table with an incorrect graph: *graph* correctly, *explain* what the student did wrong (use a graph, table or words to explain). Given a graph with context, *list* three strategies to find x = 100, *select* the best strategy, and *writ*e an equation (*Picture-table/graph to Picture-graph, Verbal/Written Models*)

The research team met weekly to plan prior to the implementation date. The purpose of planning weekly, and not planning all ten detailed lessons in advance was because in constructivist teaching experiments and design research each design was determined by the previous outcome (Cobb, 201; McKinney & Reeves, 2012). Participating students' written work and whole class presentations, as well as the research team's reflections informed the direction of the next task, often leading to deeper discussions. The following section will first describe the framework that guided the design of the mathematical task using the Lesh Translation Model (Cramer, 2003). Following a description of the model, a framework that guided the implementation of the teaching experiment, Orchestrating Productive Mathematical Discussions, will be discussed (Stein, et al, 2009).

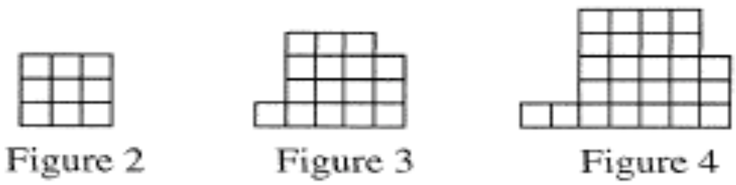
The Lesh Translation Model

The Lesh Translation Model [LTM] is a planning and teaching tool for teachers that supports teaching for conceptual understanding in diverse settings (Cramer, 2003). For this research study, the LTM guided the development of process-oriented questions that supported 7th grade immigrant students' communication and use of representations individually and in heterogeneous team settings for ten mathematical tasks. For example, given algebra tile patterns for Figure 2, Figure 3, and Figure 4, middle school immigrant students were asked to use two or more representations to find the total algebra tiles for Figure 6 and Figure 10 (i.e. table of values, graph, equation, diagram, numeric or written explanation). The teaching experiment contained mathematical tasks that encouraged translations *between* and *within* representations to support 7th grade immigrant students' conceptual understanding of key ideas of linear functions. An example of a translation *between* representations included, given a real-life situation of a linear pattern in a table, students were asked to describe the patterns using words to graph the pattern, write an equation, and/or interpret the meaning of the pattern and starting value that represented the linear pattern. At times, students were asked to justify their solutions using representations, for example, use a table of values to verify their equation. An example of a translation *within* representations included, given a picture of tile patterns that represented a linear pattern, students were asked to produce a graph of a line or a table (different kinds of pictures) to show the same pattern. The five representations from the LTM are: pictures, written model, verbal model, and real-life situations (Cramer, 2003).

Figure 3.1 displays an example of Mathematical Task #1 that the research team designed and the individual workspace emphasizing LTM for one mathematical task about, *Extending Patterns and Finding Rules* from CPM (Dietiker & Baldinger, 2006). The first two tasks in the CTE introduced students to tile pattern pictures with non-linear and linear patterns, and were selected to provide immigrant students with rich opportunities to communicate about different kinds of patterns using words, tables, numeric and algebraic models. The mathematical tasks all followed an open ended format and provided immigrant students with multiple entry points that encouraged students to first try the problem using their unique approaches and prior experiences individually during the first phase, *launch* (Stein et al, 2009).

Task One: Extending Patterns and Finding Rules Activity

Individual Workspace:
Try to solve the problem below. What do figures 6 and 10 look like? How many total squares are in each figure?



Explain how you know with at least 2 representations (Table of values, Graph, Equation, Diagram, Numeric or written explanation)

Figure 3.1. Sample Standards-based mathematical task page 1 of 2.

Figure 3.2 below displays the Team Checklist for the second phase, *explore* (Stein et al, 2009).

Task One: Extending Patterns and Finding Rules Activity

Team Checklist:

1. **Discuss** with your teammate (in English or Spanish)
   - What is the problem asking?
   - Have you seen a problem like this before?
   - Share your ideas about how to solve the problem with your team
2. **Solve** the problem on your poster with your team you must find a way to represent your solution using at least *two* different representations
   - Table of values
   - Graph
   - Equation
   - Diagram
   - Numeric explanation
   - Written explanation
3. Make sure your group knows how to **explain** each step clearly and how the representations connect to each other.
4. **Present** your solution to the class.
5. **Reflect.** As you watch each group think about the following questions. What similarities do you notice? What is different? Would you change your process/solution?

Figure 3.2. Sample Standards-based mathematical task page 2 of 2.

The Team Checklist served two purposes, to encourage use of first language, and to attempt to introduce Standards-based mathematics teaching approaches that include introducing new socio-mathematical norms for students (to be discussed further in the next section). The purpose of the Team Checklist included providing opportunities for immigrant students to communicate using their first language, Spanish or English to restate the problem in their own words, and to communicate their individual work in a heterogeneous team setting (Campbell et al, 2007; Anhalt, 2014). Heterogeneous learning environments with structured team settings supported preparation for the third phase in a Standards-based mathematics setting from Stein et al. (2009), *summarize/discuss.* During the third phase, middle school immigrant students were asked to present their mathematical processes as teams to the whole class using at least *two* different representations (Cramer, 2003). Additionally, the individual and team work checklist acted as a support for the shift in the roles of the research team as advocated by Stein et al. (2009) to support students' mathematical connections among peer presentations. Therefore, the last part of the Team Checklist required each team to think about similarities and differences among the peer presentations. It is important to point out that Mathematical Task #1 was heavily scaffold using a checklist approach because Standards-based mathematics teaching approaches and new socio-mathematical norms were being introduced formally for the first time (Cobb, Gravemeijer & Yackel, 2011). The scaffolding became less of a list as we moved from task one to task ten. All ten tasks are available as appendixes (Appendix C to Appendix L). Plans for implementation, including a framework for orchestrating productive mathematical discussions are discussed in the following section.

Introducing Standards-Based Socio-Mathematical Norms

Lesh and Kelly (2000) advocate the creation of learning environments that support the goals of CTEs. To further enhance the CTE that utilized Standards-based mathematics teaching approaches and emphasized LTM, the research team attempted to introduce new socio-mathematical norms. We acknowledge that Standards-based socio-mathematical norms included emphasis on justifying processes and discussing both *calculational* explanations that emphasized procedures and *conceptual* explanations that emphasized mathematical reasoning. Cobb, Gravemeijer, and Yackel (2011) described *calculational* explanations as less desirable, as they focus only on procedures without justifying methods. *Conceptual* explanations involve an account of both the solution and justifying methods is preferred. Figure 3.1 and Figure 3.2 above demonstrated how the research team explicitly asked for and modeled new socio-mathematical norms that aligned with goals of Standards-based mathematics pedagogy. It was the intent of the research team to introduce the new socio-mathematical norms through verbal and written communication of expectations at the beginning of each class, during interactions with students in small group settings, and during whole-class discussions. Introducing Standards-based socio-mathematical norms complimented the CTE goals to improve conceptual understanding, as evidenced by 7th grade immigrant students' communication and use of representations. Additionally, Boaler (2008) found that students that had a clear sense of the characteristics of high quality work were more likely to produce desirable reform outcomes. Therefore, structuring mathematical tasks with Standards-based socio-mathematical norms served several purposes: to enhance and assess the effectiveness of the intervention, to help with design revisions, and to aid students to produce more rich data sources to analyze.

A framework for orchestrating productive mathematical discussion techniques was used to support planning for the implementation. The structure of the implementation had students first working on rich Standards-based mathematical tasks individually (Figure 3.1 above), and then in teams (Figure 3.2 above) before presenting their mathematical processes to the whole class (Senk & Thompson, 2003, Stein et al, 2009). The role of the research team prior to the implementation was to *anticipate* student responses and to create questions that support the type of communication and representations aligned with the goals of the study. During implementation, the roles of the research team was to *monitor* students' responses, *sequence* the group work presentations that led up to big ideas, and help students make mathematical *connections* between representations (Stein et al, 2009).

Table 3.7 summarizes the five practices: *anticipating, monitoring, selecting, sequencing,* and *connecting* (Stein et al, 2009). The following section provides a more detailed plan of the collaborative process from planning to implementation.

Table 3.7

Five Practices for Orchestrating Productive Mathematical Discussion (Stein et. al, 2009)

Strategy	Role of the Teacher
Anticipating	*Anticipate* likely student responses to cognitively demanding tasks
Monitoring	*Monitor* students' responses to the tasks during the explore phase (recall: launch, explore and discuss" phases of reform)
Selecting	*Select* particular students to present their mathematical responses during the discuss/summarize phase
Sequencing	*Sequence* or purposefully sequencing the student responses that will be displayed
Connecting	*Connect* or help the class make mathematical connections between different students' responses and the key ideas

Planning the Constructivist Teaching Experiments: A Collaborative Process

Constructivist teaching experiments (CTEs) are collaborative in nature (Cobb, 2011). Cobb (2011) suggested three processes to support the research team when planning: (a) identify goals, (b) formulate conjectures, and (c) focus on means to support the goals. In the first step of the collaborative process, the research team identified possible learning goals and created a possible sequence of lesson objectives for middle school immigrant students' development of linear functions that focused on conceptual understanding. An outline of mathematical activities that aligned with state standards, was approved by the cooperating teacher prior to the start of the study (Table 3.3 and Table 3.4 above). While planning each of the ten mathematical tasks, the research team conducted *anticipatory thought experiments* (Table 3.6 above). Anticipatory thought experiments involved envisioning how each of the activities might be realized by students in the classroom. The process included envisioning the student- teacher and peer-to-peer interactions, as well as envisioning how students' interpretations and solutions might evolve as the students participate (Cobb, 2011).

Second, the research team formulated a *conjecture* about student learning and the learning environment, and then developed means of supporting and organizing the intervention (Cobb, 2011). Conjectures represented the researchers' lens, and were "inferences based on inconclusive or incomplete evidence" (Confrey & LaChance, 2000, p. 234-235). The research team wrote the following conjecture to guide their planning and implementation:

> "Immigrant students need rich problems that allow multiple entry points and encourage the use of multiple representations in heterogeneous group settings to learn key ideas of linear function with conceptual understanding" (Research Team, November 2015).

Additionally, one important aspect of the planning included selecting rich problems from Standards-based mathematics curricula, CPM, that paid attention to experientially real activities given immigrant students' prior and out of school experiences, were selected.

Third, the research team focused on the *means* of supporting the intervention through identifying specific mathematical practices and activities. The Team Checklist was an example of the research team using various questions to guide students working in heterogeneous teams to: *discuss, solve, explain, present* and *reflect* on their mathematical processes (Figure 3.2 above). Therefore, discussion and team work strategies were examples of means to support middle school immigrant students in developing and elaborating models of their informal mathematics activity, shifting from teaching formal mathematics first, to starting with informal mathematical reasoning using a variety of tools (i.e. LTM) to influence learning with understanding (Cramer, 2001; Cramer, 2003). Table 3.8 summarizes the CTE collaboration planning process and elaborates the roles of the researcher and teacher supported by theoretical grounding (Cobb, 2011). The research team utilized the table as a flexible guide and not as a strict rule.

Table 3. 8

Constructivist Teaching Experiment Collaboration Process (Cobb, 2011)

Researcher and Teacher Role	**Theoretical Grounding**
Identify possible goals for students' mathematical development and outline provisional sequence of instructional activities	Designer carries out *anticipatory thought experiment* (Simon, 1996).
Formulate conjectures about student learning and learning environment; and develop means of supporting and organizing the intervention	*Choose* realistic mathematics problems: provide experientially real activities given their prior and out of school experiences; and justifiable activities in terms of learning goals.
Focus on means of supporting evolution of classroom mathematical practices and construct activities	*Support* students by encouraging use of informal and native language; support students shifting from informal to formal mathematical reasoning. *Use* tools (i.e. LTM). The tools with which people act profoundly influence the understanding they develop (Vygotsky, 1987)

Table 3.9 lists an outline of algebra topics from CPM 7th grade content goals and embedded translations.

Table 3.9

An Outline of Algebra Topics from CPM 7th Grade Content Goals and Embedded Translations

Topics from CPM	**Algebra Content Benchmarks and Embedded Translations**
Extending Patterns and Finding Rules	Find the rate and describe the slope or rate of change from a picture, table, or graph (*Picture-tables/graphs to Written/Verbal Models, Real-Life Situations, & Pictures-graphs*)
Using Tables, Graphs, and Rules to Make Predictions	Write a linear equation to represent the relationship from a table or words Represent the relationship in words, tables, and as an equation (*Picture-tables to Written/Verbal Models, Real-Life, & Situations*)

Data Sources and Collection Procedures

Data used in this study were collected from several sources, including (a) student written work and interview transcripts in pairs on pre-and post-assessment; (b) student written work on team posters; (c) video transcripts of team presentations of posters; and (d) individual student written reflections at the beginning (Mathematical Task #1), middle (Mathematical Task #4), and end (Mathematical Task #8) of the constructivist teaching experiment. These instruments are described in the following sections. Table 3.10 shows how the data sources related to the three research questions.

Table 3.10

Data Sources Used in the Study as They Corresponded to the Research Questions

Research Questions	**Data Sources**
1. In what ways do immigrant students demonstrate their understanding of key ideas of linear functions by expressing translations between and within modes of representations prior to participating in mathematical activities that emphasize LTM?	*Student Written Work and Interview Transcripts in Pairs on the Pre-Assessment:* Given a table absent of a real life situation (proportional): Questions 4a: find a missing y-value and justify solution; Question 4b: graph; Question 4c: write an equation in y = kx format; Question 4d: interpret the meaning of slope, k, given a real-life situation; Question 5: write a real-life situation and graph given a table
2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions after participating in mathematical activities that emphasize multiple representations from LTMl?	*Student Written Work and Interview Transcripts in Pairs on the Post-Assessment:* Given a table with a real-life situation (non-proportional): Question 1a: finding two missing y-values and describe x- and y-patterns; Question 1b: write an equation in y = mx + b format; Question 1c: solve equation and interpret solution using a real-life situation; Question 1d: explain if the pattern is proportional using two or more representations
3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?	*Student Written Work on Team Posters and Video Transcripts of Team Poster Presentations:* Students' written work on posters from various points in the CTE from the beginning, middle and end* *Individual Student Written Reflections:* Individual students' written reflections from various points in the CTE from the beginning, middle and end* (*Mathematical Task #1, Mathematical Task #4, and Mathematical Task #8)

Student Pre- Interview and Pre-Assessment

A pre-interview was conducted with heterogeneous student pairs prior to the constructivist teaching experiment to learn more about students' past and current mathematics learning experiences, as well as to assess immigrant students' current understanding of key ideas of linear functions. Three non-math questions included questions about a typical day in their current mathematics class to learn about classroom socio-mathematical norms, differences and similarities about mathematics classes students took in elementary and middle school to learn about students' experiences with different representations, and how much of the content they are currently learning reflects their real-life situations, an important aspect of conceptual understanding (Cobb, Gravemeijer, & Yackel, 2011). Two mathematics questions with proportional linear patterns and multiple subparts that aligned with what students were currently studying in class followed. Question 4 presented a table of values without a context and four subparts, 4a to 4d, to gain insight on middle school immigrant students' conceptual understanding of key ideas of linear function. Question 4a asked students to find a missing y-value and to explain their solution. Question 4b asked students to graph the data and describe the direction and "steepness" of the line as they did in class since slope was not formally introduced yet. Question 4c asked students to write an equation for the table. Question 4d asked students to interpret the meaning of *k,* given a real-life situation. Question 5 presented a new table with positive non-integer y-values, and asked students to write a real-life context and graph. In sum, the pre-interview questions supported answering research question one about student's current experiences with translating between and within representations. Figure 3.3 below displays questions as well as the embedded translations from the mathematics portion of the pre-assessment interview. A complete copy of the pre interview questions is included in the appendix (Appendix A).

<table>
<tr><th>Questions</th><th>Embedded Translation</th></tr>
<tr><td>Question 4a. If you know x, can you find y? How do you know that is correct?

Table 1. Linear Data with positive slope, k
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>
</td><td><u>Translate between</u>: Picture (Table) to Verbal Model</td></tr>
<tr><td>Question 4b. If you were to graph the data from Table 1, what would the graph look like? How do you know?</td><td><u>Translate within:</u> Picture (Table) to Picture (Graph)</td></tr>
<tr><td>Question 4c. Using Table 1, can you write an equation in y = kx format? What is k? Explain to me how you know your answer is correct.</td><td><u>Translate between:</u> Picture (Table) to Written Model (Equation)</td></tr>
<tr><td>Question 4d. What if I told you a real-life situation to describe the table? The x column stands for the number of hours you work as a Spanish tutor and y stands for the amount of money in dollars. Now that you have a real-life situation, what does the k stand for in y = kx?

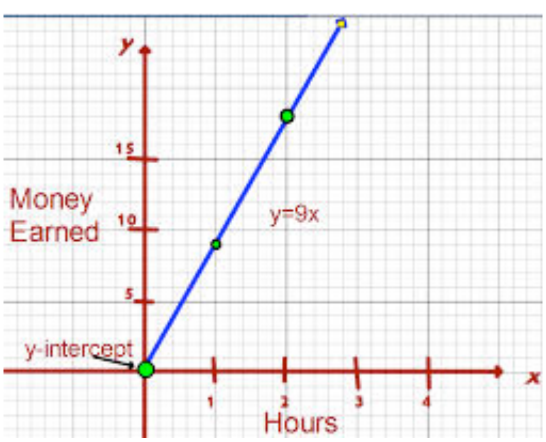

</td><td><u>Translate between:</u> Picture (Graph) to Verbal Model, & Real-Life Situation</td></tr>
</table>

Figure 3.3. Mathematics pre-assessment sample questions.

Post-Assessment Interviews

The post-assessment included a non-proportional linear function problem with four subparts that were designed to collect qualitative data from participating students in the constructivist teaching experiment, to reveal students' mathematical processes, and to document students' ability to translate between and within representations (Cramer, 2003). Recall that prior to the study, students only worked with proportional linear function problems in $y = mx$ format, and during the constructivist teaching experiment, students worked with non-proportional linear function problems in $y = mx + b$ format. The post-assessment problems were rich open-ended problems designed to match as closely as possible with the types of problems students have been working on during the constructivist teaching experiment. Figure 3.4 displays the post-assessment mathematics questions as well as the embedded translations. The question presents student with a table given a real-life situation, and the subparts asked students to: (1a) find two missing y-values and describe x- and y-patterns using words, (1b) write an equation using the *pattern* and *starting value*, (1c) use the equation to solve for a y-value given x-value of 50 hours, interpret the y-value solution using a real-life situation, and verify the solution, and (1d) decide if the pattern in the table follows a proportional linear pattern and justify solution using two or more representations. Students were required to complete multiple translations from LTM on the post-assessment (as well as on the pre-assessment) (Cramer, 2003). The post-assessment data supported answering research question two.

<table>
<tr><th>Question</th><th>Embedded Translation</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

<table><tr><th>Hours worked</th><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><th>Money in Piggy Bank ($)</th><td></td><td></td><td>41</td><td>48</td><td>55</td></tr></table>
a) Complete the table. Describe the pattern for x and y.
b) Write an equation using the pattern and the starting value.
c) How much money will Jane have in the bank after 50 hours of babysitting? How can you check?
d) Is the pattern proportional? Give <u>two</u> reasons. (Use the table, pictures, words or a graph)</td><td>Picture (Table) to Verbal Model
Picture (Table) to Written Model (Equation)
Picture (Table) to Real-Life Situation</td></tr>
</table>

Figure 3.4. Post-assessment questions and embedded translations.

The post-assessment was given at the end of the teaching experiment after ten tasks. The post-assessment served as the final exam for topics around key ideas of linear functions, namely, finding missing y-values using pattern, or slope; writing an equation, solving the equation, and interpreting the solution given a real-life situation from a table. The post-assessment was not timed, but was designed to be completed in about one hour. Students were told in writing and verbally that they could have as much time as they needed to try every problem. One student did not complete the post-assessment given two class periods, and was dropped from the study. The interview pairs also matched the intervention classroom teams of four students, but with two students instead of four students like in the CTE so that each student would have increased opportunity to share their mathematical processes in a smaller setting. The pairs remained the same for both interviews on the pre- and post assessments. This is so that students were comfortable and contributed to more rich discussions. The researcher proctored all exams.

Students took the post-assessment the same way as the pre-assessment, first individually using a pencil and later students had the opportunity to discuss and make changes using a pen during their interview with a partner. The teacher suggested that students first tried the problems on their own so the discussions were more fruitful. Reflections from the constructivist teaching experiment implementations revealed that increasing the individual work time for students led to more contributions from different members of the team during whole class discussion (i.e. *summarize/discuss* phase). The post-assessment interview questions supported answering research question two. Post-assessment interviews were done outside of the normal forty-minute class time since interviews lasted between thirty minutes to one hour. A complete copy of the post-assessment is available in the appendix (Appendix B). The following section discusses the data sources that supported answering research question three.

Student Written Work on Team Posters, Video Transcripts of Team Presentations, and Individual Student Written Reflections

To answer research question three about the characteristics of middle school immigrant students' understandings after participating in the constructivist teaching experiment (CTE) that emphasized whole class mathematical discussion and listening to peer presentations, several data sources were collected and analyzed. Data from various time frames in the CTE included students' written work on team posters, video transcripts of team presentations of posters and individual student written reflections at the beginning (Mathematical Task #1), middle (Mathematical Task #4), and end (Mathematical Task #8) of the CTE. Further, the data analysis was divided into three parts: *before, during,* and *after* whole class discussion for each of the three selected mathematical tasks. Student written work on team posters took place *before* whole class discussion; video transcripts of team presentations of posters took place *during* whole class discussion; and individual student written reflections took place *after* whole class discussion were presented by teams. Documenting student participation on mathematical tasks at various points in the teaching experiment at the beginning, middle, and end, as well as a closer examination of each mathematical task (*before, during,* and *after)* each of the tasks supports answering research question three in rich ways. Table 3.11 summarizes the data presentation plans for three mathematical tasks from the CTE to answer research question three.

Table 3.11

Data Presentation from Three Mathematical Tasks from the Constructivist Teaching Experiment

Mathematical Task #	**Data Presentation Plans**	**Embedded Translations**
Beginning: *Mathematical Task #1*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models, & Picture (tiles to equation/graph)*
Midway: *Mathematical Task #4*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models, & Picture (table to equation/graph)*
End: *Mathematical Task #8*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models) (graph to equation)*

The last two paragraphs in this section will briefly explain other data sources collected from this study, but were not used directly to answer the research questions. Multiple data sources support triangulation of findings and therefore, were collected to help the researcher understand the phenomenon taking place in rich ways (Patton, 2003).

Video Transcripts of Lessons and Audio Transcripts of Research Team Reflections

Video transcripts were taken of the *launch, explore,* and *summarize/discuss* phases (Stein et al, 2009). This included the teacher introducing the mathematical tasks, students working in team settings on mathematical tasks, and students presenting their solutions and justifications at the end of each class. The videos captured the teacher communicating the expectations of students' written and oral work (Standards-based socio-mathematical norms), student conversations while solving problems, and whole class presentations of their solutions with explanations for all ten mathematical tasks. Recall that the whole class presentations were selected and sequenced by the research team from least conceptual to conceptual justifications. Students were asked to reflect on the whole class presentations immediately after the presentations individually in writing and orally as a whole class. Video recordings helped the researcher to answer all three research questions in rich ways. Audio recordings of research team reflections immediately after each implementation, and during every planning period of the *anticipatory thought experiments,* were used to enhance modifications for subsequent tasks. Few times when the research team was not able to reflect together, the teacher would audio record his reflections and electronically send them to the researcher to be transcribed.

Field Notes

The role of the researcher was also the co-teacher in the teaching experiment and therefore, was actively interacting with students throughout the class period, field notes were taken in the form of post-implementation reflections immediately after the lesson via “jottings” about observations about student learning, including challenges and strengths of the intervention implementation. More thoughtful written reflections were completed after each lesson with the teacher when possible. This is common in research where the researcher is a participant-observer (Patton, 2003). Field notes supported the researcher during data analysis to triangulate findings. The following section discusses the research design and data analysis in detail.

Research Design and Data Analysis

A case study where the case of interest is a single mathematics intervention class participating in a constructivist teaching experiment [CTE] was designed to support immigrant students’ conceptual understanding of key ideas of linear functions (Cobb, 2011). A descriptive case study design was used in this study in order to gain more rich insight into 7th grade immigrant students’ conceptual understanding of key ideas of linear functions through their ability to translate between and within representations (Cramer, 2003). As described in previous sections, the research team designed and implemented ten mathematical tasks guided by NSF-funded curriculum, CPM (Dietiker & Baldinger, 2006). The researcher designed the pre-assessment to closely mirror students’ current understanding of linear function ideas, and the post-assessments to reflect the topics from the CTE with feedback from the teacher. The researcher also designed and conducted all interviews. The researcher and teacher acted as co-designers and co-teachers for the CTE. The analysis for this research was conducted using middle school immigrant students’ written work and interview transcript from pre- and post-assessments in pairs, video transcripts of students’ presentation of team posters, and students’ individual written reflections from various points of the CTE, taken by the researcher during the the eight-week CTE.

To answer each of the three research questions in this study regarding middle school immigrant students' conceptual understanding of key ideas of linear functions, the researcher engaged in numeric and descriptive data analysis focused around two NCTM (2000) process skills, *communication* of mathematical processes, and use of *representations* to make connections. For example, given table with a real-life situation, are students able to justify their solutions using two or more representations (i.e. example of *solution* coding), and are students able to highlight relationships between the two strategies, or are the strategies treated as isolated facts/rules (i.e. example of *justification* coding) (Cramer, 2003; Hiebert & Lafevre, 1987)? Therefore, the researcher focused on two types of numerical coding: solution and justification, and a final score was given. Following numerical coding, descriptive summaries of student responses using interview transcripts and students' written work were presented. The following sections discuss the details of the multiple cycle coding process.

To begin the data analysis, the researcher first transcribed all interview data starting the same day after the interviews or implementation, and no later than one week after the implementation. The researcher also wrote analytic memos while transcribing to assist in the description and eventual creation of themes around communication and representations (NCTM, 2000). The memos included comments about student reasoning or strategies. For example, the researcher wrote an analytic memo on the transcription to document a conversation with Brooke (MEP) from the post-assessment about verifying an equation to note new strategies that previously were not seen on the pre-assessment, "*This was a checking technique they learned from intervention lesson, this was not done prior to being shown in class*."

Second, after the researcher transcribed all data, the researcher examined students' solutions and justifications through multiple cycles. Individual pre-and post-assessment questions were coded in two ways: solutions were coded as: *correct (2), partially correct (2),* or *incorrect (0);* and justifications were coded as: *conceptual (2), procedural (1), none/incorrect (0).* Hiebert and Lefevre (1986) discussed procedural knowledge as being useful and necessary for conceptual understanding, but that students also needed to be able to assess the reasonableness of procedures and explain why and how to move from procedural to conceptual. Characteristics of justifications coded as *procedural* included student reasoning using, "algorithms, or rules, for completing mathematical tasks," or a step-by-step procedure that was prescriptive when completing a task in a linear sequence (Hiebert & Lefevre, 1986, p. 4). Conceptual knowledge was considered rich in relationships and as a network in which ideas are linked, and suggested a higher level of reflection, where students took a step back and reflected on the information being connected. Students with conceptual understanding were able to understand how and why procedures work in more than one way and pointed out similarities in relationships. Characteristics of justifications coded as *conceptual* included student reasoning moving away from presenting "isolated pieces of information," and toward recognizing the, "relationship to other pieces of information" (Hiebert & Lefevre, 1986, p. 4). Additionally, conceptual knowledge involved the student being able to monitor and evaluate reasonableness and anticipate the consequence of possible actions. It is important to point out that justifications coded as *conceptual* were dependent upon each individual question on the pre- and post-assessment questions because each question covered different key ideas of linear function topics and solutions/justifications looked different depending on topic.

The researcher utilized multiple cycle coding of student responses for both solutions and justifications, to ensure reliability and consistency in coding (Miles, Huberman & Saldana, 2014). It is important to point out that the researcher modified what counted as conceptual justification after reviewing all student work to take into account the unique entry points that immigrant students bring to school with them (Civil, 2014; Planas & Civil, 2009). For example, the researcher initially had in mind a strategy that students *ought to* justify finding a missing y-value using the concept of slope, but several times students justified in ways that were unique, but were still coded as conceptual using multiplication, division and addition, and concept of slope informally. Because the ways students made connections between and within representations was of interest in this study, and the data could not be easily captured viewing students written work on assessments alone, the researcher used additional strategies to analyze data from video interview transcripts of student interview in pairs. The strategies were as followed: a) view attentively video data, b) describe the video data, c) identify critical events, d) transcribe, e) code, f) construct storyline, and g) compose narrative (Powell, Francisco, & Maher, 2003, cited in Pettis, 2015, p. 92). These strategies were used when analyzing transcriptions of student interviews working on pre- and post-assessments as well as for student presentations of team posters.

The stages of data analysis were guided by Miles, Huberman, and Saldana (2014), who strongly advised analysis concurrent with data collection, interweaving data collection and data analysis from the very start. As stated earlier, the researcher wrote analytic memos, comments, and reflections after each implementation as well as reflected with the teacher on the outcomes immediately after the intervention, paying close attention to students' contributions in small group work and whole class discussion to inform future designs. Miles et al. (2014) suggested two cycles of coding, First Cycle and Second Cycle Coding. Prior to the cycles of codes and coding, Miles et al. (2014) suggested the researcher first consider two things: *description* and *application* of data sources. *Description* of data serves as a "critical link" between the data collection and their explanation of the meaning, and requires deep reflection about the meaning of data, and sets the stage for further data analysis and drawing conclusions. Analytic memos served as preliminary round of reflection, followed by Two Cycles of Coding (Miles et al, 2014). *Application* of data considers the multiplicity of data sources and forms, and suggests a selective process of what data will best answer the research questions, using the description process to trigger deeper reflection. In sum, considering the description and application of the multiple data sources as it applied to each research question helped the researcher to narrow the data sources for each research question (see Table 3.10 above). Additionally, the researcher used two cycles of coding as a heuristic, or a method of discovery of themes, for careful reading and reflection (Miles et al, 2014). The following details the two cycles of coding to answer three research questions.

Coding and analysis of the data proceeded in three phases: the first phase was more discussed as informal writing of analytic memos while transcribing; and these sections will discuss two rounds of formal coding of student solutions and justifications, The First Cycle Coding and Second Cycle Coding (Miles et al, 2014). In the First Cycle Coding, the researcher tallied the total possible points for a correct solution as two points: *correct (2), partially correct (1),* or *incorrect (0);* and the total possible points for justification as two points: *conceptual (2), procedural (1), or none/incorrect (0).* Therefore, a maximum of 4 points was awarded for individual student response to pre- and post-assessment questions. The purpose for the First Cycle Coding (Miles et al, 2014) was to provide a numeric account of the data from the pre-and post-assessments.

Simultaneously, during the First Cycle Coding, the researcher wrote comments to provide descriptive account of the data and to summarize student responses on individual pre- and post-assessment questions to clarify solution and justification codes assigned. For example, comments from pre-assessment codes were focused around *communication*, including: "Students used more than one representation when asked to do so," or "a real-life situation supported conceptual justifications." The purpose of the first cycle of codes comments were to identify patterns in student approaches, challenges and strengths in their processes translating between and within representations. A similar process followed when commenting on the post-assessments. However, for the post-assessment, new sets of codes were added since new ideas around the LTM appeared more frequently. Comments from the post-assessment included, "Students describe the slope, or pattern, as two changing variables in writing," and "students used two or more representations when asked to do so." In sum, all data were coded to focus on NCTM (2000) process skills, *communication* and *representations* as a way to support coding for themes about conceptual understanding. This study was designed to investigate students' conceptual understanding of key ideas of linear functions and the majority of data segments were simultaneously coded in more than one category. The overlapping codes were used to identify areas of connections or disconnections in student's conceptual understanding during the First Cycle Coding.

The Second Cycle Coding was a way to make sense of themes and codes further, or to group data or codes from the First Cycle Coding (Miles et al, 2014). Pattern coding during this stage helped the researcher to reassemble the data into dominant themes and related subthemes. For example, after several cycles of review of multiple data sources for research question three, the following preliminary themes emerged: (1) students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$); (2) students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings; and (3) students' English language proficiency was not correlated with conceptual explanations. Themes that resulted form multiple cycle coding for all three research questions are presented in detail in Chapter 4.

Role of the Researcher

For this constructivist teaching experiment, the researcher served as co-designer of ten mathematical tasks with the teacher, the co-teacher during the study, and the designer of the pre-, mid- and post-assessments with feedback from the teacher. The researcher as co-teacher roles allowed the researcher to actively contribute to richness of mathematical discussions, as well as monitor and suggest sequencing of whole class discussions, from least conceptual to conceptual. Additionally, the design and implementation of all mathematical tasks were influenced by the researcher's knowledge of Standards-based mathematical practices from the literature, and with feedback from the classroom teacher's knowledge of his students. Therefore, it was expected that student interactions were influenced by the researcher. For example, the researcher interacted with students prior to the start of the study in an informal setting. The researcher was the sole grader and coded multiple data sources collected for this study.

Trustworthiness

The researcher utilized several methods to enhance the credibility of the study. First, the multiplicity of data sources along with the systematic documentation of the processes by which the data was collected and analyzed, supported the researcher in answering the research questions with high-quality evidence and transparency (McKinney & Reeves, 2012). Transparency included describing in detail the process, making it possible to retrace the teaching experiment and check the data used to justify the claims and analysis codes (Patton, 2003). Second, the researcher as the co-teacher enhances the credibility of the study as teaching and interviewing over extended periods along with intensive interactions with students are common in CTEs (Cobb & Steffe, 2011). Third, the multiplicity of data included student interviews, video documentation of implementation, research team reflections, and teacher interviews, supported triangulation of data, to ground the findings from the experiment (Patton, 2003).

The results of the study are not generalizable to a larger population due to a small sample size and purposeful criterion sampling strategy used (Patton, 2003). However, this study seeks to support the literature on development of teaching culturally and linguistically diverse students' development of key ideas of linear functions in ways that emphasize NCTM (2000) process goals, namely *communication* and *representations*. No one single model currently exists that supports teaching for conceptual understanding, therefore the results of the teaching experiment serves as one transferrable guide where the frameworks and findings from the study may be used to test future teaching experiments. The rich descriptions gleaned from this study can be used to enhance and/or inform future studies, and the frameworks used in the study can serve as models to be built upon when supporting diverse students.

Conclusion

This chapter summarized the research design, participant selection process, collaborative design process, the intervention frameworks, data collection and analysis. The trustworthiness and transferability of the study were also discussed. Chapter 4 presents the descriptive results of the study from the analyses of the study that address three research questions. Chapter 5 summarizes the findings, discussions, and implications, and recommendations for future studies.

Chapter 4

Results

This chapter contains a description of the data collected during an eight-week constructivist teaching experiment that focused on these key ideas of linear functions: extending patterns and finding rules and, using tables, graphs, and rules to make predictions. The study was set in a Midwestern inner-city mathematics intervention classroom designed for students that “did not meet” or “partially meet” mathematics state standards (Minnesota Department of Education [MDE], 2014). A purposeful criterion selection strategy was used to select the school, teacher and students (Patton, 2003). A school with high immigrant population, high English Language Learner (ELL) population, and high number of students from low socio-economic background that consistently underperformed in standardized national assessments in mathematics was selected (National Center for Educational Statistics, 2007). The teacher was not currently using Standards-based mathematics curricula and therefore, met criterion sampling.

Data were collected from multiple sources including students’ pre- and post-assessments interview transcripts designed by the researcher with input from the teacher, students’ written work from team posters, video transcripts of student presentations, and individual student written reflections completed after whole class discussion that emphasized *connections* among the peer presentations. Individual student written reflections after the whole class discussion contained prompts to compare and contrast mathematical processes among peer team presentations and asked students to answer similar function questions from the team poster. For example, *which strategy do you* *prefer and why? Using any strategy find the number of squares in the 15th figure.* Seven students from various reading and mathematics levels that had above average class attendance, and who agreed to be video taped working on mathematical tasks and assessments, were selected for the study. Three students, Anna (27th percentile), Joe (27th percentile), and Olvin (14th percentile) were identified as *Low-English proficient (LEP)* from their Measures of Academic Progress (MAP) reading percentile with all scoring less than the 33rd percentile. Three students, Lucas (59th percentile), Sandy (56th percentile) and Brooke (53rd percentile) were identified as *Medium-English proficient (MEP)* from their MAP reading scores as they scored above the 33rd percentile, but less than the 66th percentile. One student, Leon (87th percentile), was identified as *High-English proficient (HEP)* from his MAP reading score above the 66th percentile.

This study was guided by three research questions:

1. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions by expressing translations between and within modes of representations <u>prior</u> to participating in mathematical activities that emphasize LTM?
2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions <u>after</u> participating in mathematical activities that emphasize multiple representations from LTM?

3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Data Collected

In order to answer these research questions, video transcripts of student semi-structured interviews on the pre- and post-assessments, data from video transcripts of student team presentations of mathematical task posters, and individual student written reflections after whole class discussions that emphasize connections of key ideas of functions among peer-presentations are presented. The purpose of this study was to document the ways in which immigrant students demonstrated their conceptual understanding before and after participating in the constructivist teaching experiment designed to support students' ability to communicate and use representations to extend patterns and find rules, and use tables, graphs, and rules to make predictions. Recall that ten mathematical tasks and pre- and post-assessment questions were discussed in detail in Chapter 3. Questions related to immigrant students' conceptual understanding of their current ideas of linear functions were included in the pre-assessment, whereas new key ideas of linear function questions related to immigrant students' conceptual understanding from the constructivist teaching experiment were included in the post-assessment.

Prior to the study, students mostly worked with linear functions that followed a proportional linear pattern in the form, $y = kx$, where k is the *unit rate*, or *slope*. During the constructivist teaching experiment, students worked with non-proportional linear patterns in the form, $y = mx + b$, where *m* is the *pattern* (slope), and b is the *starting value* (y-intercept). In the constructivist teaching experiment, students used their current understanding of key characteristics of proportional linear relationships to learn about non-proportional linear relationships. Students had the opportunity to compare and contrast proportional and non-proportional relationships. For example, in the post-assessment students were given a non-proportional relationship table with a real-life situation and asked to explain if the pattern was proportional or not using two representations to justify their reasoning. Students could use a picture, table, graph, equation or words to explain. Questions related to students' understandings of key ideas of linear functions as it related to their ability to communicate and use representations were included on the pre- and post-assessments are presented herein.

As this study was interested with the ways in which 7th grade immigrant students communicate and use representations to demonstrate their conceptual understanding of key ideas of linear functions, data related to the students' understandings of the concept were included on the pre-assessment interview questions. The key ideas of linear function in the pre-assessment included finding a missing y-value using pattern (or slope) and justifying the solution, making predictions, graphing, writing and solving equations, and writing original real-life situations about proportional linear patterns from a table of values. The pre-assessment questions reflected algebra topics covered prior to the eight-week constructivist teaching experiment that aligned with the following algebra benchmarks: (1) determine the unit rate; (2) understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate; (3) understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form y = *k*x; (4) represent real-world relationships using an equation; (5) represent proportional relationships with verbal descriptions, and translate from one representation to another (MDE, 2007). The post-assessment questions reflected topics from the constructivist teaching experiment, and focused on non-proportional linear function patterns that aligned with the following algebra benchmarks: (1) identify graphical properties of linear functions including slope and y-intercept, (2) know that slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship; (3) understand that a function is linear if it can be expressed in the form y = *m*x + b; (4) use linear equations to represent situations involving constant rate of change, including proportional and non-proportional relationships; (5) represent linear functions with verbal descriptions; and translate from one representation to another (MDE, 2007).

The data in this chapter are organized according to individual participant responses and then according to student-pair responses to the pre- and post-assessment interview questions for the first two research questions. Student written work on team posters, video transcripts of student team presentations, and individual student written reflections after whole class discussions that emphasize *connections* among peer-presentations are presented for the third research question. In all cases, interviewees' response scores are presented first, followed by a summary of all students' responses. Data are organized in tables by student and his or her corresponding numeric score and descriptive summaries of their solutions in pairs. The researcher designed the pre-assessment to closely reflect students' current understanding of linear function ideas, and the post-assessment to reflect the topics from the constructivist teaching experiment with feedback from the teacher. Due to the fact that the pre-assessment was designed to measure students' current understanding of proportional linear function content objectives, there is a slight asymmetry in the pre- and post-assessment topics. The ways in which students communicated and translated between representations were of interest in this study, therefore, similar translations were highlighted in both pre- and post-assessments. For example, both the pre- and post-assessment presented students with a table and asked students to find missing y-values using the pattern, or slope, predict future values, and interpret data using a real-life situation. The post-assessment did not ask students to graph data from a table. Instead, students were asked to explain how they know if the pattern was proportional using two or more representations. Additionally, the constructivist teaching experiment offered students opportunities to discuss the similarities and differences between proportional and non-proportional relationships. At the end of each section, the themes that emerged from student responses in the pre- and post-assessments are summarized as it relates to communication and representations. The ways that these themes relate to the research questions are also discussed.

Understandings of Linear Function Ideas Prior to Intervention

The following sections present the data from students' written work as well as students' oral explanations in pairs from the pre-assessment interview transcripts from Question 4 with multiple subparts and Question 5, as the data from these questions relate to the first research question:

1. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions by expressing translations between and within modes of representations prior to participating in mathematical activities that emphasize LTM?

The first three questions were non-mathematics interview questions designed to gain a snapshot of students' typical mathematics class period, students' experiences using multiple representations, and how much of students' current mathematical experience had applications to real-life situations. The first three pre-interview questions were used to present the study context in Chapter 3. Figure 4.1 displays the pre-assessment questions as well as the embedded translations. To answer the first research question, throughout this chapter, the pre-assessment questions are presented first, provided by a brief description of the problem including expectations from students, and followed by a table of individual student scores coded numerically. Students received a solution score, justification score, and final score for each individual pre-assessment question. A summary of student responses in pairs is then presented. Recall from Chapter 3 the total possible points for a correct solution were two points: *correct (2), partially correct (1),* or *incorrect (0);* and the total possible points for justification were two points: *conceptual (2), procedural (1), or none/incorrect (0).*

<table>
<tr><th>Pre-Assessment Questions</th><th>Embedded Translations</th></tr>
<tr><td>4. Table 1. Linear data with positive slope, *k*.
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>?</td></tr>
</table></td><td></td></tr>
<tr><td>a) If you know x, can you find y? How do you know that is correct?</td><td>*Picture (Table) to Verbal Model*</td></tr>
<tr><td>b) If you were to graph the data from Table 1, what would the graph look like? How do you know?</td><td>*Picture (Table) to Picture (Graph)*</td></tr>
<tr><td>c) Using Table 1, can you write an equation in y = *k*x format? What is *k*? Explain to me how you know your answer is correct.</td><td>*Picture (Table) to Written Model (Equation)*</td></tr>
<tr><td>d) The x column stands for the number of hours you work as a Spanish tutor and y column stands for the amount of money in dollars. Now that you have a real-life situation, what does the slope, *k,* stand for in y = *k*x?</td><td>*Picture (Table) to Real-Life Situation*</td></tr>
<tr><td>5. Given Table 2, explain how to graph the values and write or tell a real-life situation to match the table.
Table 2. Linear data with positive slope, k
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>3.5</td></tr>
<tr><td>2</td><td>7.0</td></tr>
<tr><td>3</td><td>10.5</td></tr>
</table></td><td>*Picture (Table) to Verbal/Written Models (Equation), and Real-Life Situation*</td></tr>
</table>

Figure 4.1. Pre-assessment questions with embedded translations.

Hiebert and Lefevre (1986) discussed procedural knowledge as being useful and necessary for conceptual understanding, but that students also needed to be able to assess the reasonableness of procedures and explain why and how to move from procedural to conceptual. Characteristics of justifications coded as *procedural* included student reasoning using, “algorithms, or rules, for completing mathematical tasks,” or a step-by-step procedure that was prescriptive when completing a task in a linear sequence (Hiebert & Lefevre, 1986, p. 4). For example, Question 4a on the pre-assessment, asked students to find a missing y-value when x-value was four given a table absent of a real-life situation. An example of a solution coded as *correct,* and a justification coded as *procedural* was from Anna (LEP), “I know 36 is correct because you can just multiply by nine in the table.” When asked to explain, Anna said, “It came from like, I think it came from like the table because you have to multiply across x, you need to find the y.” Anna did not connect the solution to other patterns embedded in the task, for example, nine is the slope and represents two changing quantities involving x- and y-values, or the ratio, y/x, relationship embedded in the task.

Conceptual knowledge was considered rich in relationships and as a network in which ideas are linked, and suggested a higher level of reflection, where students took a step back and reflected on the information being connected. Students with conceptual understanding were able to understand how and why procedures work in more than one way and pointed out similarities in relationships. Characteristics of justifications coded as *conceptual* included student reasoning moving away from presenting “isolated pieces of information,” and toward recognizing the, “relationship to other pieces of information” (Hiebert & Lefevre, 1986, p. 4). Additionally, conceptual knowledge involved the student being able to monitor and evaluate reasonableness and anticipate the consequence of possible actions. For example, Question 4b on the pre-assessment asked students to graph given a table absent of a real-life situation. An example of a solution coded as *correct,* and a justification coded as *conceptual* was from Olvin (LEP): he correctly graphed and described the line as increasing and linear because of the "same rate" of change. He mentioned that a y-axis interval selection of 9 versus 3 changed the appearance of the graph. His justification was coded as *conceptual* because he answers the question correctly and was able to evaluate the consequence of possible actions, evidenced by his reasoning that connects the characteristics of the graph (line) to the concept of slope as constant rate of change (Hiebert & Lefvre, 1986)

It is important to point out that justifications coded as *conceptual* were dependent upon each individual question on the pre-assessment. For example, Question 4a on the pre-assessment asked students to find a missing y-value given a table absent of a real-life situation, and justify their solution. An explanation coded as *conceptual* for Question 4a included students going beyond calculating the missing y-value, but linking the two changing quantities involving x- and y-values in more than one way. For example, students could point out that y = *9*x is related to *9* = y/x, or use a verbal model to explain the concept of slope, that as x-values increased by one unit, y-values increased by 9 units. Whereas, Question 4b on the pre-assessment asked students to write an equation from a table in y = *k*x format, explain the meaning of slope *k*, and explain how to check the solution. A possible explanation coded as *conceptual* included students writing the equation in the format, y = *9*x, where *9* is the slope, and verify the equation by plugging in additional coordinate points from the table (linking table to equation). A maximum of 4 points was awarded for individual student response to pre- and post-assessment questions. The following sections present data from the pre-assessment semi-structured interview transcripts.

Question 4a. Find a Missing Y-Value (*Picture to Verbal Model*)

Figure 4.2 shows Question 4a that asks students to find a missing y-value from a table that is absent of a real-life situation. Students are also asked to justify the solution. The table presents the translation embedded in the task. Question 4a was designed by the researcher with input from the teacher to reflect students' current mathematical tasks in their intervention classroom, and presented students with a table with a proportional linear relationship absent of a real-life situation. Students were expected to find a missing y-value of 36 by identifying a pattern, or slope, of nine from the table. Explanations coded as *conceptual* required that students went beyond calculating the missing y-value, but seeing a relationship between at least two ideas embedded in the task. For example, linking the two changing quantities involving x- and y-values in more than one way (i.e. *as the x-values increased by one, the y-values increased by 9*; *this is the same value that you multiply x by to get y*); or linking the x- and y-value relationships with an additional representation (i.e. insert a real-life situation to relate two quantities, $9 *earned per hour, so $36 earned for 4 hours to make sense of the rate of change in this task*). Two examples of justifications coded as *procedural* are, "It is going up by nine because one times nine is nine, two times nine is 18;" and "It came from like, I think it came from like the table because you have to multiply across x, you need to find the y." The reasoning presents isolated pieces of information that follow rules with no relationship to other pieces of information.

Pre-Assessment Question	Embedded Translation
4a. If you know x, can you find y? How do you know that is correct? Table 1. Linear Data with positive slope, k <table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>9</td></tr><tr><td>2</td><td>18</td></tr><tr><td>3</td><td>27</td></tr><tr><td>4</td><td>?</td></tr></table>	*Picture (Table) to Verbal Model*

Figure 4.2. Pre-assessment question 4a on finding a missing y-value.

Students first answered pre-assessment questions individually using pencil before answering pre-assessment questions in pairs using pen to make changes, if any. Table 4.1 presents individual student scores on student-pair solutions and explanations from the pre-assessment interview transcripts, followed by a description of codes. All students answered the task correctly but used solely procedural strategies in their justification.

Table 4.1

Question 4a Scores

Student	Conceptual Understanding Score
Anna (LEP)	3 (2, correct 1, procedural)
Leon (HEP)	3 (2, correct 1, procedural)
Lucas (MEP)	3 (2, correct 1, procedural)

Olvin (LEP)	3 (2, correct 1, procedural)
Sandy (MEP)	3 (2, correct 1, procedural)
Joe (LEP)	3 (2 correct 1, procedural)
Brooke (MEP)	3 (2, correct 1, procedural)

Anna and Leon (Q4a). When asked to find a missing y-value from a table absent of a real-life situation, Anna (LEP) used a procedure involving multiplication to describe the proportional linear relationship between x-values and y-values, and correctly calculated a missing y-value of 36 and identified pattern of 9. She explained, "I know 36 is correct because you can just multiply by nine in the table." When asked to explain, Anna said, "It came from like, I think it came from like the table because you have to multiply across x, you need to find the y." The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because she answered the question correctly from a table and calculated the missing y-value using a rule in isolation. She did not link her reasoning to other possible relationships between x- and y-values from the table (i.e. y = 9x is related to 9 = y/x). Her partner, Leon (HEP), shared two strategies. Leon agreed with his partner to use multiplication, and added a second procedure to calculate the missing y-value that involved addition, "I have two ways. Since the pattern by y is increasing by nine, I added 27 to nine and that equals 36. And I also did Anna's way by multiplying nine by four to get 36." The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he answered the question correctly from a table and calculated the missing y-value using two rules in isolation. He did not link the two strategies to show the relationship between x- and y-values from the table (i.e. adding consecutive y-values by 9 worked because x-values increased by *one* unit each time; +9 is linked to "multiply by 9" from $y = 9x$). In sum, Anna and Leon came up with two procedures to calculate the missing y-value of 36. Neither partner saw the relationships between the strategies.

Olvin and Lucas (Q4a). When asked to find a missing y-value from a table absent of a real-life situation, Olvin (LEP) used a procedure involving multiplication to describe the proportional linear relationship between x-values and y-values, and correctly calculated the missing y-value of 36. He described the strategy using multiplication, "It is going up by nine because one times nine is nine, two times nine is 18." Olvin also shared that he noticed the relationship could be addition, but then dismissed it, "Well I know it's going up so its multiplication or addition. But for me it's multiplication, it is times nine." He was correct that both procedures worked, but could not elaborate on addition. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he answered the question correctly from a table and calculated the missing y-value using a rule in isolation. He did not link his reasoning to other possible relationships between x- and y-values from the table (i.e. $y = 9x$ is related to $9 = y/x$, or adding by 9 to consecutive y-values worked because x-values increased by one unit; the value 9 is also interpreted as slope or rate of change). His partner, Lucas (MEP) agreed with multiplication as one possible explanation for calculating slope, and offered a second procedure to calculate and verify the solution involving division, “I agree because when you put y and x on the bottom and… you put nine and one, you divide that and it equals nine” (*y is the numerator and x is the denominator, or 9/1 = 9*). His written work showed, 36 divided by four was nine, and he wrote 36 in the table for the missing y-value. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he answered the question correctly from a table and calculated the missing y-value using two rules in isolation. He did not link his reasoning that y = 9x is related to 9 = y/x or, for example, that finding this quotient (y/x) is a strategy for finding the slope in proportional situations. In sum, Olvin and Lucas came up with two procedures to calculate the missing y-value of 36. Neither partner connected the relationship between the strategies.

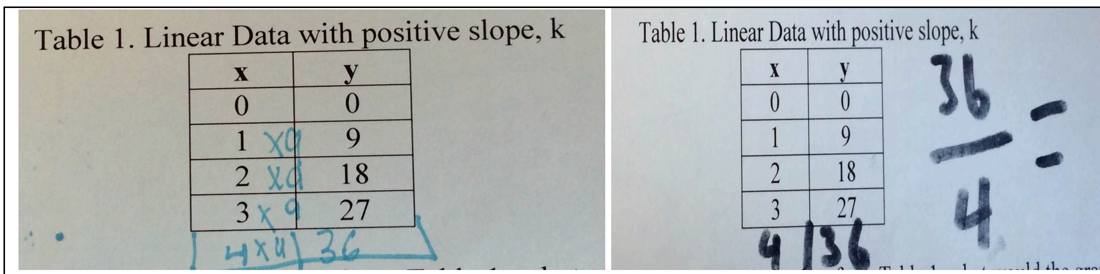
Sandy and Joe (Q4a). When asked to find a missing y-value from a table absent of a real-life situation, Sandy (MEP) explained two ways to identify the missing y-value of 36 verbally using multiplication and division, “There’s two ways, you could just multiply it or you could put y over x.” In her written work, she wrote about multiplication and division, “Nine divided by one is nine, and four times nine is 36.” The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because she answered the question correctly from a table and calculated the missing y-value using two rules in isolation. She did not link her strategies explaining how finding the rule, y = 9x is related to 9 =

y/x. Her partner, Joe (LEP), used two different strategies to calculate slope in his written and verbal explanations. In his written work, Joe used multiplication and division, "Four times nine is 36," but in his verbal explanation he used division, "I think you could divide it too… nine divided by one. Nine is the slope." The solution was coded as *correct (2).* The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he answered the question correctly from a table and calculated the missing y-value using two rules in isolation. Although Joe mentioned the slope was nine, his calculations did not demonstrate understanding of the *meaning* of slope as a relationship that, as x-values increased by one unit, y-values increased by 9 units. Hiebert and Lefevre (1986) described a characteristic of procedural understanding as rules or "step by step" (p. 4). In sum, Sandy and Joe used two procedures involving multiplication and division to calculate the missing y-value of 36. Neither partner saw relationships between the strategies beyond procedural calculations as rules.

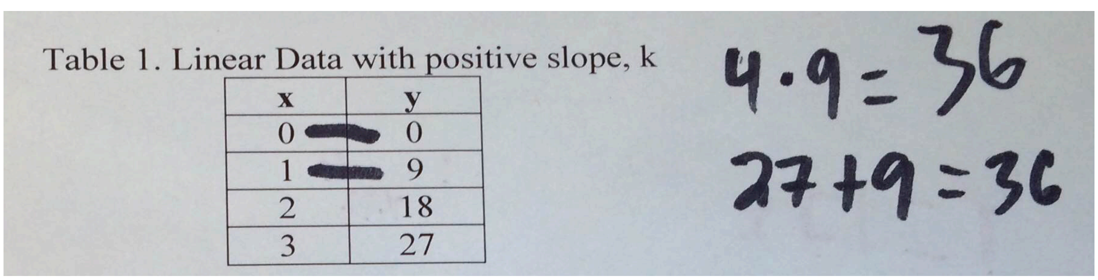
Brooke (Q4a). When asked to find a missing y-value from a table absent of a real-life situation, Brooke (MEP) did not show her written work on the paper test to describe the proportional linear relationship between x-and y-values, but correctly calculated the missing y-value of 36. In her verbal explanation, Brooke used every coordinate pair from the table to verify the slope was nine using multiplication, "x times nine equals y… because I saw that one and nine and that will be one times nine, and then two times nine would equal 18, and three times nine would equal 27, so four times nine would equal 36." When probed for another strategy to verify the pattern, or slope, was nine and the missing y-value was 36, Brooke shared a second procedure using addition, "So the y is just going up by nine" (*to explain that y-value of 27 plus nine is 36*). Brooke came up with two procedures to calculate the slope of nine and missing y-value of 36. The solution was coded as *correct (2).* The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because she answered the question correctly from a table and calculated the missing y-value using rules in isolation. She did not link the strategies that y = 9x is related to adding by 9 to the y-values worked because the x-values increased by *one* unit, and that adding by 9 would *not* work if x-values increased by more than one unit. In sum, Brook answered the question correctly, but did not link relationships between the strategies.

Summary of Picture (Table) to Verbal Model Translation. When students were asked to find a missing y-value from a table absent of a real-life situation, all students correctly identified that the pattern was nine and calculated the missing y-value of 36 using up to two operations. All students correctly calculated the missing y-value procedurally in isolation, but none of the students linked the pattern of 9 in their strategies with other relationships between the x- and y-values. Students with conceptual justifications needed to point out other relationship between at least two ideas embedded in the task. For example, linking the two quantities involving x- and y-values in more than one way: *as x-values increased by one, the y-values increased by nine*, or linking the x- and y-value relationships with an additional representation (i.e. insert a real-life situation to relate two quantities, *$9 earned per hour, so $36 earned for 4 hours*). Students could also point that y = 9x is related to 9 = y/x because the slope is nine, and that adding y-values by 9 worked because x-values increased by *one* unit, but would not work if x-values increased by more than one unit. Three types of calculations emerged from the data: multiplication, addition and division (*picture-table to verbal/written models*). First, all students used multiplication to describe the relationship between x- and y-values from the left column to the right column in the table, where x-values multiplied by 9 obtained the corresponding y-values. Second, two students, Leon (HEP), and Brooke (MEP) used addition to describe the changes in y-values and neglected the changes in x-values by one unit, where adding the slope to y-values was used to determine consecutive y-values. This change of x-values by one unit is important for the addition strategy, any increase above or below one unit would not work. Olvin (LEP), Brooke (MEP), and Leon (HEP), used the addition approach. Third, three students, Joe (LEP), Lucas (MEP), and Sandy (MEP, used division to describe the relationship between x- and y-values as *y divided by x*, where the quotient was the pattern, 9. Division was the most common strategy and aligned with the primary classroom teaching strategy.

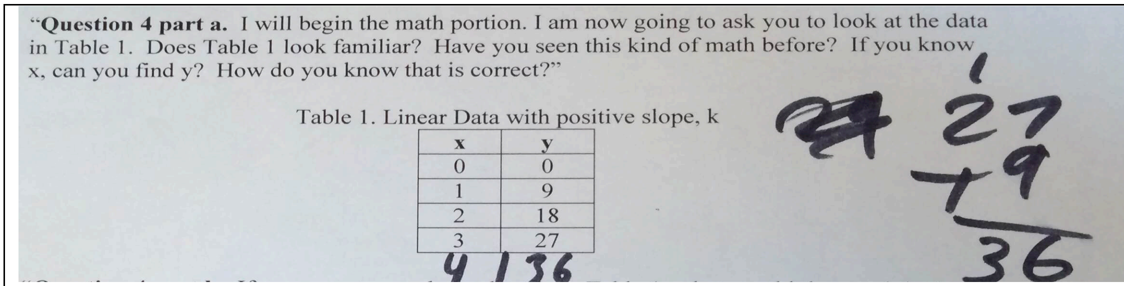
Further, the data showed that when students were probed to explain their reasoning, student explanations showed that students treated calculations as separate strategies and did not highlight the relationship between multiplication, division and addition by 9. Interestingly, none of the students acknowledged the origin when calculating the slope of nine. For example, when using multiplication to calculate slope, all students calculated using coordinate pair, (1, 9), and not at the origin (0, 0), since any number times zero resulted with zero. Second, the addition strategy was the least common approach. Using the addition approach emphasized attention to how y-values were changing and neglected changes in the x-values. Third, less than half of the students described the pattern, or slope as, *y divided by x,* starting with coordinate pair (1, 9) and not the origin, similar to the multiplication procedure. Using division closely aligned with students' current classroom experiences when taught how to find missing y-values. In sum, all students correctly calculated the missing y-value, but none of the students linked their calculations, or connected the relationships involving x- and y-values explicitly. Figure 4.3 shows students' individual written work on Question 4a from the pre-assessment.



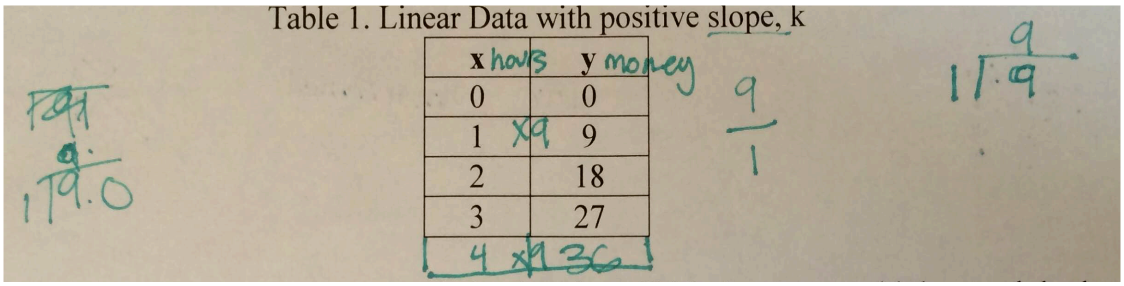
Joe (top left) uses multiplication and Lucas uses division (top right) to identify slope and missing y-value when x is 4.



Leon uses multiplication and addition to identify slope and missing y-value when x is 4.



Olvin uses addition to identify slope and missing y-value when x is 4.



Sandy uses multiplication and division to identify slope and missing y-value when x is 4.

Figure 4.3. Student individual written work on Question 4a about finding a missing y-value from a table.

The following sections present data for pre-assessment Question 4b about graphing from a table.

Question 4b. Graph from Table (*Picture to Picture*)

Figure 4.4 shows Question 4b on graphing from a table on the pre-assessment as well as the embedded translation. Question 4b was designed by the researcher with input from the teacher to reflect students' current mathematical tasks in their intervention classroom, and was designed to highlight students' process when translating data from a table as coordinate pairs to the Cartesian plane in the first quadrant, where x-values and y-values are all positive. Students were expected to choose intervals that fit the domain and range from the table, discuss steepness, and explain how the intervals affect the appearance of the graph. Hiebert and Lefevre (1986) discussed characteristics of conceptual knowledge that included linking relationships between ideas; for example, students discussed that the *appearance* of the steepness of the line changed with varying interval selections for the y-axis. For example, the x-axis interval of one and y-axis interval of nine looks different from x-axis interval of one and y-axis interval of three, but the line is still increasing. An example of a justification coded as *procedural* is if students graph but do not explain the direction and the steepness of the line, and do not discuss how the interval selection changes the appearance of the graph. An example of a justification coded as *conceptual* is from Olvin (LEP), “It’s going to look a little weird if we count by nine, if we count by three the graph would be bigger… zooming in;” and the graph is “growing up, going up fast.” Olvin discussed the direction of the line and how the appearance of the graph changes with different intervals, and correctly described the direction of the line.

<table>
<tr><th>Pre-Assessment Question</th><th>Embedded Translation</th></tr>
<tr><td>Table 1. Linear Data with positive slope, k
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>
4b. If you were to graph the data from Table 1, what would the graph look like? How do you know?</td><td>Picture (Table) to Picture (Graph)</td></tr>
</table>

Figure 4.4. Pre-assessment question 4b on graphing from a table.

Table 4.2 presents individual student scores on student-pair solutions and explanations from the pre-assessment interview transcripts, followed by a description of codes. Three students answered the task partially correctly and four students answered the task correctly, and three students used procedural strategies and four students used conceptual strategies in their justification.

Table 4.2

Question 4b Scores

Student	Conceptual Understanding Score
Anna (LEP)	4 (2, correct 2, conceptual)
Leon (HEP)	2 (2, correct 2, conceptual)
Lucas (MEP)	3 (2, correct 1, procedural)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q4b). When asked to graph data from a table, Leon (HEP) labeled the x-axis interval counting by one from zero to twelve, and labeled the y-axis interval counting by three from zero to 39. His partner, Anna (LEP) contributed verbally to the graphing process. She agreed with her partner on the interval selections, and added an appropriate reasoning of her own, that the y-axis interval selection could also be another number and does not need to be three, “It doesn’t need to be three, it can be some number else. I agree cuz you can get either way the answer of 36” (*she agreed with y-axis interval of 3*). Anna’s explanation showed that she thought through the problem and that a factor of 36, including three, works. She did not just agree with her partner conveniently. Anna also correctly agreed with her partner that the line was increasing, and said that steepness of the line depended on interval selection, “it would depend on the number” (*it would depend on the y-axis interval selection*). Anna described that steepness of the graph depended on the interval selection. Her response showed that she understood there was relationship between the interval selection and the *appearance* of the graph. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly, and discussed the relationship between the interval with the appearance of line. She correctly described the direction of the line as increasing. This is an example of what Hiebert and Lefevre (1986) described as displaying reflection and monitoring and evaluating reasonableness to anticipate the consequence of possible actions.

Her partner, Leon, graphed all the points from the table, starting with (1, 9) to (4, 36), and skipped the origin (0, 0). He went back to graph the origin when probed by the researcher. When asked to describe the graph, Leon described the direction of the line correctly as increasing and that the line was, “slanted… and then the line goes up.” Leon described the relationship between the interval selection and the appearance of the line, or steepness, “it would *depend* on what the scale went up by.” He is correct that the appearance of the steepness of the graph would change if the y-axis interval changes from counting by three to nine. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly, and discussed the relationship between the interval and the appearance of the line. He correctly described the direction of the line as increasing. This is an example of what Hiebert and Lefevre (1986) described as displaying reflection and monitoring and evaluating reasonableness to anticipate the consequence of possible actions. In sum, Anna and Leon correctly graphed the data, described the direction of the line and described the relationship between the interval selection and the appearance of the line.

Olvin and Lucas (Q4b). When asked to graph data from a table, Olvin (LEP) and his partner, Lucas (MEP) graphed the data from the table and labeled the x-axis interval counting by one from zero to seven, and labeled the y-axis interval counting by nine and not by three from zero to 36. While graphing, Olvin thought out loud noticing that counting by nine for the y-axis interval affected the graph appearance, “It’s going to look a little weird if we count by nine, if we count by three the graph would be bigger.” When asked to clarify further what he meant, Olvin said that the points would remain the same no matter the interval, but the graph with a smaller interval showed an example of “zooming in.” When asked to describe the shape of the line, he used formal language to describe the shape of the line as, “straight and never a curve because, it is going up by the same rate.” Olvin and his partner graphed all points from the table in order from the origin, (0, 0) to (4, 36). The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a graph and a verbal model to explain what the graph looked like with different interval selections. He connected the type of graph (line) with concept of equal rate. He realized that the scale does not affect the rate or steepness. This is an example of what Hiebert and Lefevre (1986) described as displaying reflection and monitoring and evaluating reasonableness to anticipate the consequence of possible actions.

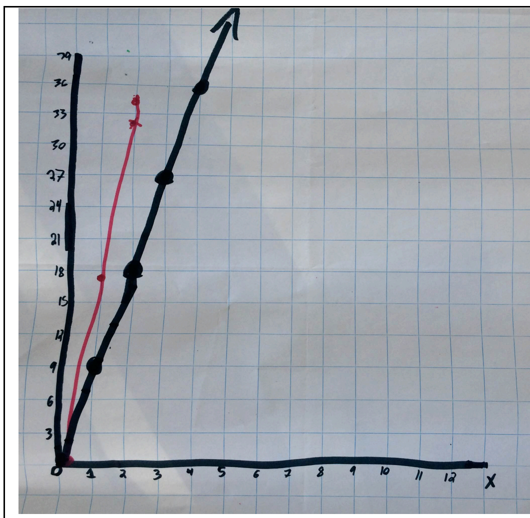
His partner, Lucas, explained that counting by nine for the y-axis was appropriate, but that counting by three also worked well. He said the location of the points would not change even if the interval changed, but was less clear about how the changes in the y-axis interval from counting by nine versus counting by three affected the graph. Lucas thought that choosing a y-axis interval of three instead of nine is an example of "zooming out." He did not explicitly describe the direction or steepness of the graph, just that the location of the points would not change with different intervals. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he answered the question correctly using a graph, and he was not clear when he described the relationship between the different intervals of three or nine, or did not describe the direction of the line explicitly. In sum, Olvin and Lucas correctly graphed and explained their interval selection. Only Olvin connected the line graph with the concept of equal rate and that the scale does not affect the rate or steepness. This is an example of what Hiebert and Lefevre (1986) described as displaying reflection and monitoring, and evaluating reasonableness to anticipate the consequence of possible actions

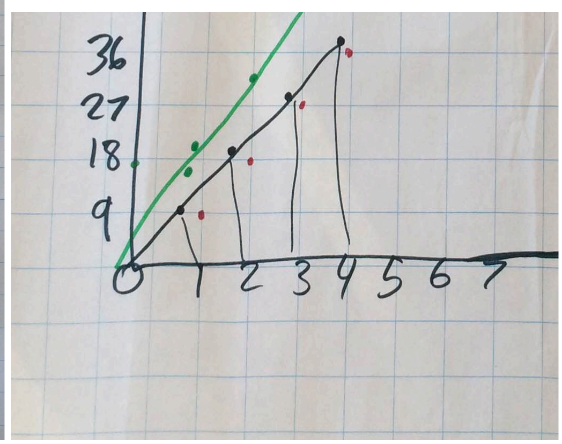
Sandy and Joe (Q4b). When asked to graph data from a table, Sandy (MEP) and Joe (LEP) approached graphing data from a table as a team, and split the work evenly. Sandy labeled the y-axis interval counting by nine from zero to 36, and Joe labeled the x-axis interval counting by one from zero to fifteen. They took turns graphing all the points from the table, from the origin, (0, 0) to (4, 36), connected the points and constructed a straight line. Sandy described the steepness of the line graph as steep and increasing, "growing up, going up fast." The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a graph (picture), and she used a verbal model to describe the increasing line as steep or, "growing fast," connecting the patterns from the table to the graph. Hiebert and Lefevre (1986) described this as connecting the pattern embedded in the table with the picture of a steep, "fast," line rather than just step by step graphing the line with no explanation.

Her partner, Joe, also correctly described the graph as an increasing and steep line, "a straight, steep line that keeps on going… going up every time by 9." Joe agreed with his partner that the graph was increasing and "growing fast," and that the line showed an "increase." The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a graph and linked the type of graph with an idea of constant rate, or slope. Hiebert and Lefevre (1986) described this as connecting the pattern embedded in the table with the picture of a steep, "growing fast," line rather than just step by step graphing the line with no explanation. In sum, Sandy and Joe correctly graphed the data and explained their interval selection, and both students correctly described the direction of the line as increasing, and Joe linked the line with the idea of slope.

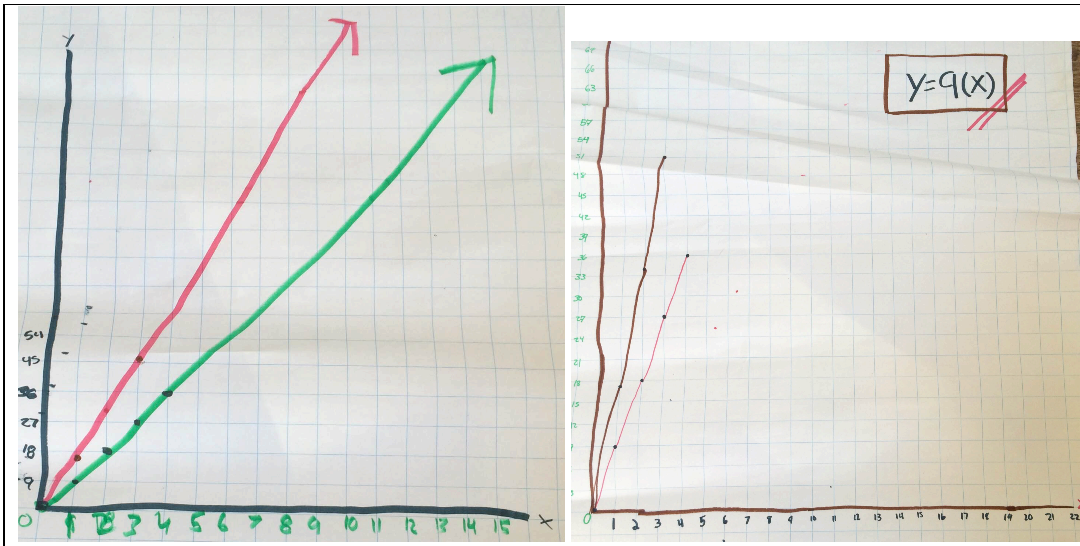
Brooke (Q4b). When asked to graph data from a table, Brooke (MEP) labeled the x-axis interval counting by ones from zero to 22, and the y-axis interval counting by three from zero to 75. In her verbal explanation, she clarified that counting by three or nine for the y-axis both made sense, "As long as you can divide by 3." Recall that the maximum y-value in the table was 36. It was not clear why she choose three, but counting by three matched the values in the table since three was a factor of the maximum y-value of 36. Brooke graphed all points from the table except for the origin. When probed by the researcher, she went back to graph the origin and connected the points to make a steep and increasing straight line. When asked to describe the shape of the line graph, Brooke was the only student to use hand gestures to motion a vertical motion, signaling a steep line. She described the line correctly as increasing, "Like straight up… very steep." The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. This response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a graph to discuss the direction and steepness of the line, and she described the steepness of the line using gestures to point straight to represent a steep line. Hiebert and Lefevre (1986) described this as connecting the pattern embedded in the table with the picture of a steep, "like straight up," line rather than just step by step graphing the line with no explanation. In sum, Brooke correctly correctly graphed and described the direction of the line as increasing and steep.

Summary of Picture (Table) to Picture (Graph) Translation. When students were asked to translate from a table with no real-life situation to the Cartesian plane, two themes emerged. First, all students labeled the x-axis interval as counting by ones, as shown by the table of values (*picture to verbal model*). However, when selecting an interval for the y-axis, students differed slightly. All students agreed on choosing a factor of 36, the maximum y-value, including three or nine for the y-axis interval. Four students, Olvin, Lucas, Sandy and Joe, used the coordinates from the table exactly as shown and counted by nine for the y-axis. Three students, Anna, Leon, and Brooke, chose to count by three for the y-axis. Second, all students used correctly described the shape and steepness of the graph correctly as an increasing straight and steep line. Olvin was the only student to use formal language and mentioned that the line had a *constant rate of change*. Olvin and his partner, Lucas, explained that changing the interval to three instead of nine had a *zooming in* (zooming out) effect, but that the interval did not change the location of the points or the steepness. Three students, Anna, Leon, and Lucas, pointed that the steepness of the line was dependent on the interval choice. Additionally, when asked to describe the steepness of the graph, six out of seven students, Leon, Lucas, Olvin, Sandy, Joe and Brooke, correctly described the direction of the line as increasing when explaining the shape of the graph in different ways, “straight,” “going up by the same rate,” “a slanted line,” “straight up,” “growing fast,” and “going up.” One student, Joe, used formal language to describe the direction of the line as an, “increase.” One student, Brooke used hand gestures and semi-formal language to signal a vertical line that was going, “straight up.” In sum, all students correctly graphed the data from the table, and about half of the students pointed that the y-axis interval affected the (appearance) steepness of the line. Hiebert and Lefevre (1986) described these explanations that pointed to interval selection changing the steepness of the line as a form of reflecting, rather than just step by step graphing the line with no explanation. Figure 4.5 shows student written work on Question 4b from the pre-assessment.





Anna and Leon graph y = 9x (black line).   Olvin and Lucas graph y = 9x (black line)



Sandy and Joe graph y = 9x (green line). Brooke graphs y = 9x (pink line).

Figure 4.5. Student work in pairs for question 4b on graphing data from table.

The following section presents data from pre-assessment Question 4c about writing equation and interpreting slope.

Question 4c. Write an Equation from a Table (*Picture to Written Model*)

Figure 4.6 shows Question 4c about writing an equation and interpreting the meaning of slope from the pre-assessment, as well as the embedded translation. Question 4c was designed by the researcher with input from the teacher to reflect students' current mathematical tasks in their intervention classroom, and presented students with a table with a proportional linear relationship absent of a real-life situation. Question 4c asked students to write an equation for the proportional relationship from a table. Students were expected to use the slope of 9 from Question 4a to write the equation, y = *9*x. One way to justify why the rule works would be to verify the equation by plugging in more than one coordinate pair (x, y) from the table. For example: 9 = 9(1), 18 = 9(2), 27 = 9(3), or 36 = 9(4). This was the strategy taught in the class. Another possible example of a justification coded as *conceptual* is, "*k* is 9. 9 is the unit rate or the *constant of proportionality*," and verifies using multiplication of the slope of nine with each x-values form the table produces the correct corresponding y-values. An example of a justification coded as *procedural* is, "y divided by x," but does not plug in coordinate pairs to verify or connect this strategy to another pattern embedded in the table or key idea related to linear functions.

<table>
<tr><th>Pre-Assessment Questions</th><th>Embedded Translation</th></tr>
<tr><td>Table 1. Linear Data with positive slope, k
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>

4c. Using Table 1, can you write an equation in y = kx format? What is k? Explain to me how you know your answer is correct.</td><td>Picture (Table) to Written Model (Equation)</td></tr>
</table>

Figure 4.6. Pre-assessment question 4c on writing equation and interpreting slope.

Table 4.3 presents individual student scores on student-pair solutions and explanations from the pre-assessment interview transcripts, followed by a description of codes. All students answered the task correctly, but three students used procedural strategies and four students used conceptual strategies in their justification.

Table 4.3

Question 4c Scores

Student	Conceptual Understanding Score
Anna (LEP)	3 (2, correct 1, procedural)
Leon (HEP)	4 (2, correct 2, conceptual)
Lucas (MEP)	4 (2, correct 2, conceptual)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	3 (2, correct 1, procedural)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	3 (2, correct 1, procedural)

Anna and Leon (Q4c). When asked to write an equation given a proportional linear relationship from a table, Anna (LEP) and Leon (HEP) used the correct pattern, or slope, to write the equation as, $y = 9x$. In their explanation, Anna and Leon used multiplication and division similar to their explanation from Question 4a, to verify how they calculated the missing y-value of 39 using the pattern of nine. Leon verified the equation using multiplication of x-values with the pattern of 9 to produce the corresponding y-values. Further, when asked to describe the meaning of k = 9, Leon used formal language to explain the definition of slope as the *unit rate,* or *constant of proportionality*, "k is 9. 9 is the unit rate or the constant of proportionality," and he explained that using multiplication by pattern of 9 with each x-values from the table produced the correct corresponding y-values. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and connected the idea of k from the equation y = kx to the concept of unit rate and constant of proportionality. He further verified the equation using coordinate pairs from the table, linking coordinate pairs to the equation. Hiebert and Lefevre (1986) described this as connecting relationships from different pieces of information and pointing out similarities. His partner, Anna, added a second procedure for calculating missing y-value using a pattern of nine with division as another way to check the pattern was nine, "y divided by x gives 9." Unlike Leon, she did not plug in coordinate pairs from the table to verify or connect this rate to the slope or constant of proportionality idea. The solution was coded as *correct (2).* The explanation was coded as *procedural (1).* The response was coded as *some understanding (Score: 3)* because she answered the question correctly using a written model, but she did not verify the equation using coordinate pairs from the table or connect the pattern to other embedded relationships from the table. Hiebert and Lefevre (1986) described this as procedural since there were not connections between pieces of information or pointing out of similarities. In sum, both partners explained how to calculate slope of nine. Only Leon linked relationships with equation using coordinate pairs from the table.

Olvin and Lucas (Q4c). When asked to write an equation given a proportional linear relationship from a table, Lucas (MEP) and Olvin (LEP) used the correct pattern, or slope, to write the equation as, y = 9x. In his explanation, Lucas used formal language to describe the meaning of a slope as *unit rate* and *slope.* Further, he explained how to calculate the slope using similar procedures from Question 4a using division as, "9/1, 18/2, 27/3 36/4." He verified the equation using coordinate pairs from the table. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified the equation using coordinate pairs from the table using a verbal model and a written model. He made connections between the value 9 and the idea of slope and unit rate. His partner, Olvin, approached the problem first by writing the expected format, "y = *k*x," followed by the plugging in the slope of nine into the equation for *k*, so, "y = 9x." In his explanation, Olvin agreed with his partner that the slope was nine using multiplication. However, when probed further, Olvin confused the coordinate (1, 9) as the slope and was not able to verify the equation correctly at first. For example, he was expected to show that, 9 = 9(1). When asked where to plug in (1, 9) from the table into the equation, Olvin pointed to the slope, 9 because y = 9, but did not know where to plug in x = 1. With partner discussion and questioning from the research team, Olvin, correctly verified the equation using a coordinate pair, (2, 18), from the table:

SC: Now do you know how to check to see if your answer is correct?
O: Nope.
SC: Lucas do you know how to check? Your answer is correct by the way gentlemen. But do you know how to check to make sure it is correct?
L: Yeah we could (inaudible).
SC: Ok what do you mean? Tell me.
L: Divide 9 over 1, 18 over 2 and 27 over 3 and 36 over 4.
SC: That's excellent. Did you know there is another way to check?
O: Nope.
SC: Okay, so pick points that are nice and easy for ya, from the table pick anyone one of those pairs.
O: 1 and 9.
SC: 1 and 9. Ok where does 1 go in your y = 9x and where does 9 go?
O: Right here.
SC: So looking at this equation right here, where does 1 go and where does 9 go?
L: 9.
SC: So if you look here…
L: Oh 1 is right here.
SC: And where does 9 go?
L and O: Right there.
SC: Right there, where?
O: Where 9 is.
SC: So if x is 1 and y is 9 where do you plug in x?
O: Right next to it.
SC: Uh huh, and then y is 9 where do you put 9? Y is 9, where is y?
L: Ohhhhh.
SC: Ah, cuz you kept pointing at the 9. So if you put 1 in here you have 9 times 1, what is 9 times 1? (*pointing to x from the equation, y = 9x*)
L: 9.
SC: 9, and what did we say y was?
L: Unit rate. (*Lucas confuses slope of 9 with output when y = 9*)
SC: What did we say 9 was?
O: 9. (*Students mentioned 9 was the slope earlier*)
SC: Here, what is 9?" (*pointing to y = 9 on the table*)
L and O: y.
SC: y, so you are getting (1, 9) from the table a little mixed up with the unit rate. So here if you put 1 and here you put 9 you have 9 times 1 equals 9. Now Olvin, can we pick a different pair or Lucas pick a different pair to check?
L: 18.
SC: 18, but that's not a pair. That's one number I need a pair.
L: And 2.
SC: (2, 18), where does the 2 go?
O: In the x.
SC: Uh ha, and where does the 18 go?

L and O: In the y. (*both students answered in unison*)
RT: Ahhhh, so 9 times 2 is?
O: 18.
SC: Ahh, that's another way to check! Did you know that?
O: No.
SC: Now you know.
L: How about this way? (*pointing to their written work, y/x*)
SC: You can do it that way, too. So let's put 18 in for y and 2 in for x, what do we get?
L: 9.
SC: Yeah that's another way to check. Excellent.

The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified the equation using coordinate pairs from the table making connections between rule and data in the table. Hiebert and Lefevre (1986) described this as connecting relationships from different pieces of information and pointing out similarities. In sum, both partners explained how to calculate slope correctly and verified the equation using coordinate pairs from the table.

Sandy and Joe (Q4c). When asked to write an equation given a proportional linear relationship from a table, Sandy (MEP) and Joe (LEP) used the correct pattern, or slope, to write the equation as, y = 9x. Both students first wrote a division problem, "y/x = 9," to show their work for slope before writing the correct equation, "y = 9x." In their explanation, Joe and Sandy discussed how to calculate slope using multiplication similar to Question 4a, and explained that a slope of nine was a steep line. Sandy did not verify the equation by plugging in points or connecting the rule they found to other possible relationships embedded in the task. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some* *understanding (Score: 3)* because she answered the question correctly using a written model, but did not justify the equation by plugging in coordinates pairs from the table using a verbal model or connecting the rule to other possible linear function ideas. Hiebert and Lefevre (1986) described this as procedural since there was no connecting of relationships from different pieces of information or pointing out of similarities. Her partner, Joe, verified the equation by plugging in three coordinate points, "*k* is the slope because we multiply by 9… we get the answer because 1 x 9 = 9, 2 x 9 =18, 3 x 9 = 27…" The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. This response is coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified the equation by plugging in coordinate points from the table to link the pattern, or slope, from the equation, y = 9x, to the table of values where y-values increasing by 9. Hiebert and Lefevre (1986) described this as connecting relationships from different pieces of information and pointing out similarities. In sum, both partners explained how to calculate slope correctly and wrote the correct equation. Only Joe connected the table to the equation and justified by plugging in coordinate pairs from the table.

Brooke (Q4c). When asked to write an equation given a proportional linear relationship form a table, Brooke (MEP) used the correct pattern, or slope, from Question 4a to write the correct equation as, "y = 9(x)." She was the only student to use parenthesis in her equation around x. Her verbal explanation involved two procedural calculations to verify her slope, multiplication and division. However, Brooke did not verify the equation by plugging in coordinate pairs from the table as taught in class or making connection to other linear function ideas that could justify why the rule is correct. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because she answered the question correctly using an equation showing the pattern, or slope of 9, but she did not verify the equation by plugging in coordinate pairs from the table or connecting the pattern with other relationships embedded in the task that could justify the rule is correct for all values in the table. Hiebert and Lefevre (1986) described this as procedural since she did not connect relationships from different pieces of information or pointing out of similarities. In sum, Brook wrote the correct equation but did not explain or verify her solution.

Summary of Picture (Table) to Written Model (Equation) Translation. When students were asked to write an equation for the proportional relationship from a table, all students correctly wrote the equation as y = 9x (*picture to written model*). However, when probed further, three out of seven students, Anna, Sandy, Brooke, repeated how to calculate the slope from Question 4a, including multiplication and division. Although the solution was coded as *correct,* the explanation was coded as *procedural* because students did not verify the equation by plugging in one or more coordinate pairs from the table into the equation or connect to other linear function ideas that could justify why the rule works for all coordinate pairs in the table. Four out of seven students, Leon, Lucas, Olvin, and Joe, verified the equation using coordinate pairs from the table (*picture to written model*). The explanation was coded as *conceptual* because students verified the equation by plugging in one or more coordinate pairs from the table into the equation and showed the relationship between the x- and y-values in the table as multiplication. Hiebert and Lefevre (1986) described this as connecting relationships from different pieces of information and pointing out similarities. As shown by the pre-assessment interview transcripts, one student, Olvin, confused the coordinate (1, 9) with the slope of nine at first. When asked where to plug in (1, 9) from the table into the equation, Olvin pointed to the slope, 9 for y = 9, but did not know where to plug in 1 for x = 1. With questioning from the researcher, Olvin and his partner, Lucas, correctly verified their equation using coordinate pair (2, 18) from the table as 18 = 9(2). The data from Question 4c on the pre-assessment showed students' ability to translate from a table to equation, about less than half answered procedurally and more than half answered conceptually. The following sections present data from the pre-assessment Question 4d about interpreting slope using a real-life situation.

Question 4d. Interpret Slope Using a Real-Life Situation (*Picture to Real-Life Situation*)

Figure 4.7 shows Question 4d about interpreting slope given a real-life situation as well as the embedded translation. Question 4d was designed by the researcher with input from the teacher to reflect students' current mathematical tasks in their intervention classroom, and presented students with a table with a proportional linear relationship, but this time with a real-life situation. Question 4d asked students to interpret the meaning of slope, $k = 9$, given a real-life situation where x-values represented the number of hours worked, and y-values represented the amount of money in dollars earned for working as a Spanish tutor. Students were expected to explain that a slope of nine represented two related real-life quantities, where nine is the amount of money in dollars earned *per* hour working as a Spanish tutor, or \$9 per hour. Examples of justifications coded as *procedural* are if students switched the x- and y-value real-life meaning, “9 hours for a dollar,” or neglected to relate the two real-life quantities “9 dollars.” There is an example of a solution coded as *partially correct* with no explanation, “Nine represents the, yeah the dollars.” Two examples of justifications coded as *conceptual* are, “Dollars earned like every hour. You work one hour to earn \$9;” and, “18 is the money earned for two hours, and 27 is the money earned in three hours.” Student justifications coded as *conceptual* included interpreting more than one coordinate pairs from the table that the more hours worked, the more money earned and connecting these symbolic values to a meaningful rate based on context. For example, (0, 0) represented zero dollars earned for working zero hours, (1, 9) represented nine dollars earned for working one hour, (2, 18) represented 18 dollars earned for working two hours, and so on.

<table>
<tr><th>Pre-Assessment Questions</th><th>Embedded Translation</th></tr>
<tr><td>Table 1. Linear Data with positive slope, k
<table>
<tr><th>x
(hours)</th><th>y
(dollars)</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>2</td><td>18</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>
4d. The x column stands for the number of hours you work as a Spanish tutor and y column stands for the amount of money in dollars. Now that you have a real-life situation, what does the slope, *k,* stand for in y = *k*x?</td><td>*Picture (Table) to Real-Life Situation, & Verbal Model*</td></tr>
</table>

Figure 4.7. Pre-assessment question 4d on interpreting slope using a real-life situation.

Table 4.4 presents individual student scores on student-pair solutions and explanations from the pre-assessment interview transcripts, followed by a description of codes. Two students answered the task partially correctly and five students answered the task correctly, and two students did not not explain their reasoning and five students used conceptual strategies in their justification.

Table 4.4

Question 4d Scores

Student	**Conceptual Understanding Score**
Anna (LEP)	1 (1, partially correct 0, no explanation)
Leon (HEP)	4 (2, correct 2, conceptual)
Lucas (MEP)	4 (2, correct 2, conceptual)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	1 (1, partially correct 0, no explanation)
Joe (LEP)	4 (2 correct, 2, conceptual)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q4d). When asked to interpret the meaning of the slope given a real-life situation, Leon (HEP) correctly answered the question as two related, real-life quantities, the amount of dollars earned per hour worked, "Dollars earned like every hour. You work one hour to earn $9." Leon does not include other values from the table to interpret that the more hours worked, the more money is earned. However, he did relate two real-life quantities, dollars and hours. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a real-life situation to connect the constant rate of change of nine with two related quantities, amount of money earned per hour worked. He linked the meaning of slope as a constant rate within context. His partner, Anna (LEP), did not describe the slope as two related, real-life quantities. Anna recognized that nine represented the amount of money in dollars earned, but did include the relationship to the second quantity, time in hours. She explained that, "Nine represents the, yeah the dollars." The solution was coded as *partially correct (1).* The explanation was coded as *incorrect/none (0)*. This response was coded as *limited understanding (Score: 1)* because she answered the question partially correctly and was unable to interpret the symbolic meaning of nine as a rate using a real-life situation. She did not have a complete explanation of the real-life situation and therefore, the justification is coded as incorrect. The response did not show how or why, but repeating of facts without connecting the relationship between x- and y-values. In sum, Leon related two real-life quantities, dollars and hours, correctly, and Anna only discussed the amount of money earned in dollars, coded as partially correct.

Neither partner used other values from the table to interpret that the more hours worked, the more money in dollars earned, or connected the idea of constant rate of change to the context to show other relationships in the table.

Olvin and Lucas (Q4d). When asked to interpret using the real-life context, Olvin (LEP) and Lucas (MEP) interpreted the context correctly. Lucas thought a slope of nine represented the number of hours, and later corrected himself that nine represented the amount of dollars earned. After some discussion, Lucas correctly added an additional coordinate pair to interpret the real-life context that the more hours worked, the more money in dollars earned, "18 was dollars earned for two hours of work." He related two real-life quantities, dollars and hours. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified his interpretation with other coordinate pairs from the table using a real-life situation. He understood the relationship between slope of 9 in a relationship to a rate within context and that the more hours worked the more dollars earned. With support from his partner, Olvin correctly explained his answer that the slope of nine represented the amount of money earned *per* hour, "Because you work one hour you earn nine dollars. In one hour you earn nine dollars." He does not include other coordinate pairs from the table to demonstrate his understanding that the more hours worked the more money earned. However, he related two real-life quantities, dollars and hours. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified his interpretation with other coordinate pairs from the table using a real-life situation showing an understanding of slope as 9 and as a rate within context. In sum, Lucas and Olvin correctly answered the question using two-related quantities, dollars and hours. Unlike Lucas, Olvin did not use other values from the table to verify the interpretation of slope that the more hours worked, the more money earned.

Sandy and Joe (Q4d). When asked to interpret the slope given a real-life situation, Sandy (MEP) and Joe (LEP) at first did not interpret the slope of nine correctly using the real-life situation as two related real-life quantities, dollars per hour. Sandy and Joe at first confused the slope of nine as the average hours worked, and then correctly explained that nine represented the amount of money and not the average hours worked. Sandy did not explain two related quantities, \$9 *per* hour. The explanation was coded as *partially correct (1)*. She also did not use other points from the table to clarify that the more hours worked the more money earned. The explanation was coded as *incorrect/none (0)*. The response was coded as *limited understanding (Score: 1)* because she answered the question partially correctly using a real-life situation and did not show an understanding of the pattern, or slope, of nine and as a rate within context. She did not link the concept with other relationships embedded in the table. Her partner, Joe, explained the meaning of the pattern, or slope of 9, using more than one coordinate pairs from the table to support his interpretation that nine was the amount of money earned per hour worked, and that the more hours worked the more money earned, "18 is the money earned for two hours, and 27 is the money earned in three hours." He related two real-life quantities, dollars and hours. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and verified his interpretation using other coordinate pairs from the table using a real-life situation. His explanation showed that he linked the meaning of constant rate of change with the context that as x-values, or hours, increased by one hour, y-values increased by $9. In sum, Sandy answered the question *partially correct* without an explanation and she did not use other values from the table to verify her interpretation or connect other embedded relationships from the table to the real-life situation. Her partner, Joe, answered the question correctly and used more than one coordinates from the table to interpret that the more hours worked, the more money earned.

Brooke (Q4d). When asked to interpret the slope using a real-life context, Brooke (MEP) was the only student to answer the question using more than two representations combined with more than one example. At first, Brooke used the graph to explain that the x-axis represented the hours worked and y-axis represented the amount of dollars earned. She explained that it was possible to verify the points from the graph with the table. Next, Brooke connected the table to the graph using the real-life situation, "Nine is dollars per hour. 18 is dollars for two hours." She explained that the more hours worked the more money earned, and related two real-life quantities, dollars and hours. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly and verified her interpretation using other coordinate pairs from the table using a graph, a real-life situation and connected the understanding of slope as 9 and as a rate within context. In sum, Brooke answered the question linking three representations, table, real-life situation, and graph, correctly.

Summary of Picture (Table) to Real-Life Situation Translation. When asked to interpret the meaning of the slope given a real-life situation, where x-values represented the hours worked and y-values represented the amount of money earned (*picture to real-life situation, & verbal model*). Five out of seven students, Leon, Lucas, Olvin, Joe, and Brooke, correctly interpreted the meaning of the slope as two related real-life quantities, dollars earned per hour worked. Three students, Anna, and Sandy answered the question *partially correct* using dollars, but did not include the second related quantity, hour. However, when probed for further explanations, three students, Lucas, Joe and Brooke shared more than one coordinate pairs to interpret the meaning of the slope, that the more hours worked, the more money earned. When students explained the real-life situation and the relationship to the x- and y-values in the table, this showed that they linked the patterns embedded in the table with the context. The following sections present data from pre-assessment Question 5 about graphing and writing a real-life situation from a table with positive non-integer y-values.

Question 5. Graph, Write Equation and a Real-Life Situation for a Table (*Picture to Real-Life Situation and Verbal Model*)

Figure 4.8 shows Question 5 about graphing and writing a real-life situation from a table with positive non-integer y-values. Question 5 presented students with a table with positive integer x-values and positive non-integer y-values, where x-values ranged from zero to three with an interval of one, and y-values ranged from zero to 10.5 with an interval of three point five. Students were expected to choose x- and y-axis intervals that included the domain and range values from the table and provide an original real-life situation. An example of a justification coded as *procedural* would be students assigning x- or y-values a real-life context without relating the two quantities in their interpretation: "*Let x-values represent the number of students attending a field trip and let y-values represent the total bus fare*. So, 3.5 is $3.50." This is a procedural justification because students did not interpret the meaning of 3.5 as two related quantities as, $3.50 per student, and treated y-value as an isolated quantity. Two examples of justifications coded as *conceptual* are, "I think it can stand for days, and hours worked. So, in two days, seven hours worked, 10.5 hours in three days" (*x-values represented number of days worked and y-values represented number of hours worked*); and "How about fruit? One person eats-- zero people eats zero oranges, one person eats three oranges and a half." These examples are coded as conceptual because the x- and y-values are interpreted as being related, that an increase in one quantity is an increase in the other by a constant rate, and because students used more than one coordinate pairs from the table to interpret a real-life situation from that as x-values increased by one unit, y-values also increased by a constant of 3.5 units. Students linked the constant rate of change to multiple coordinate pairs from the table.

Pre-Assessment Question	Embedded Translation
5. Given Table 2, explain how to graph the values and write or tell a real-life situation to match the table. Table 2. Linear Data with positive slope, k. <table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>3.5</td></tr><tr><td>2</td><td>7.0</td></tr><tr><td>3</td><td>10.5</td></tr></table>	*Picture (Table) to Real-Life Situation, & Verbal Model*

Figure 4.8. Pre-assessment question 5 on graphing and writing a real-life situation from table.

Table 4.5 presents individual student scores on student-pair solutions and explanations from the pre-assessment interview transcripts, followed by a description of codes. All students answered the task correctly and all used used conceptual strategies in their justification.

Table 4.5

Question 5 Scores

Student	Conceptual Understanding Score
Anna (LEP)	4 (2, correct 2, conceptual)
Leon (HEP)	4 (2, correct 2, conceptual)
Lucas (MEP)	4 (2, correct 2, procedural)
Olvin (LEP)	4 (2, correct 2, conceptual)

Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q5). When asked to write a real-life situation and explain how to graph the positive non-integer data, Anna (LEP) said x-values represented the number of days worked, and y-values represented the amount of hours worked. In her explanation, Anna used more than one coordinate pair from the table to interpret the real-life situation, where x-values represented the number of days worked and y-values represented the number of hours worked. She said, "I think it can stand for days, and hours worked. So, in two days, seven hours worked, 10.5 hours in three days" (*x-values represent number of days worked and y-values represent number of hours worked*). She used more than one coordinate pairs to explain an original real-life situation and selected appropriate intervals. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a real-life situation to describe the coordinate pairs in the table to describe the meaning of a constant rate of change of 3.5 as 3.5 hours worked in one day. She used more than one coordinate pairs from the table to interpret that the more days worked, the more total hours added by a constant of 3.5, linking the concept of slope, or constant rate of change to the context.

Her partner, Leon (HEP), used a more original real-life example to represent the new data, where the x-values represented the number of bags of Takis snack that were sold at the school store, and the y-values represented the weight of the snack in ounces. Leon wrote his real-life situation as a word problem, “I have some bags of Takis. Each bag weighs 3.5 oz. How much will 5 bags weigh?” He used an original real-life situation to write a word problem and selected appropriate intervals. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly to write the answer as a word problem using a real-life situation and he connected his word problem with the slope as constant rate of 3.5 ounces per bag, relating two quantities, weight in ounces and number of bags. In sum, when asked to describe how to graph data from a table with positive non-integer values, Anna and Leon chose appropriate intervals that matched the table values exactly, counting by one for x-axis and counting by three point five for the y-axis. Anna graphed the origin as the first point (0, 0) and her partner graphed two points from the table, (1, 3.5) and (2, 7). Only Anna used more than one coordinate pairs to interpret the real-life situation. Both students answered the question correctly using appropriate intervals and linked a real-life situation by relating two quantities (days and hours, and bags and total weight in ounces) to the constant rate of change, 3.5.

Olvin and Lucas (Q5). When asked to write a real-life situation and explain how to graph the positive non-integer data, both Olvin (LEP) and Lucas (MEP) used similar real-life situations, where x-values represented the number of people and y-values represented the amount of food consumed. Olvin described x-values as the number of people and y-values as the number of orange fruit consumed. Olvin first answered the question with a question, “How about fruit? One person eats-- zero people eats zero oranges, one person eats three oranges and a half.” He used more than one coordinate to explain an original real-life situation and selected appropriate intervals. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a real-life situation to relate two quantities, number of people and number of orange consumed; and he showed that as the number of people increased the number of orange consumed increased at a constant rate of 3.5, linking slope and context.

Similarly, Lucas described x-values as the number of people and y-values as the number of biscuits consumed. He first answered the question in the form of a question, “Is pounds ok? Zero people eat zero pounds of biscuits. One person eats three and a half pounds of biscuits, two people eat seven pounds of biscuits, three people eat ten and a half pounds of biscuits.” He used more than one coordinate to explain an original real-life situation and selected appropriate intervals. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a real-life situation to relate two quantities, number of people and number of biscuits consumed; and he showed that as the number of people increased the number of biscuits consumed increased at a constant rate of 3.5. In sum, when asked to describe how to graph data from a table with positive non-integer values, Olvin and Lucas discussed that the choice for intervals depended on the values from the table and selected intervals of one for x-axis and three point five for y-axis to match the data exactly. Both partners used more than one coordinate pairs to explain their real-life situation, and answered the question correctly using appropriate intervals and real-life situations.

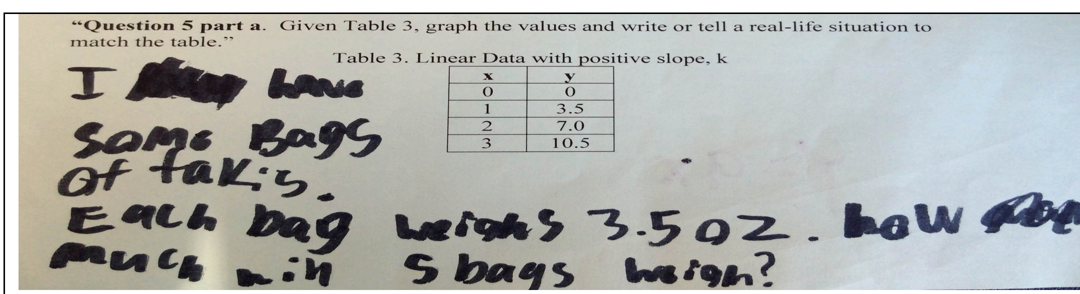
Sandy and Joe (Q5). When asked to write a real-life situation for a table with positive non-integer values and describe how to graph, Sandy (MEP) and Joe (LEP) both used extra curricular activities including swimming and soccer to help make connections. Sandy described x-values as miles swam and y-values as hours swam, "For every hour a swimmer swims 3.5 miles. The x column stands for how many miles. They y stands for how many hours." To graph, she explained that the x-axis interval was one and y-axis was 3, and 3.5 was "just a little past 3." She used an original real-life situation and selected appropriate intervals with her partner. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a real-life situation that related two quantities, hours and miles; and connected the concept of constant rate of 3.5 from the table to explain that as, "for every hour."

Her partner, Joe, made up a new rule for soccer, and described x-values as the number of goals stopped by the goalie and y-values as the number of points earned as a result of goals stopped. His written work did not answer the question correctly, "3.5 x 2, 3.5 x 3." In his verbal explanation, Joe answered the question correctly. He clarified his answer, "This is kind of tricky. The x for this one will be the number the goalie stops, and for this one (y) how many they scored. The x equals how many times the goalie stopped it and the y means how, how much goals it is worth, they score." Joe verified his interpretation of the real-life situation using two coordinate pairs from the table, "One stop means three point five points. Three stops mean they earn 10.5 points." He used more than one coordinate pairs to interpret the real-life situation and chose appropriate intervals. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a real-life situation that related two quantities, number of goals stopped and number of points scored; and he interpreted all coordinate pairs from the table to show that as the number of goals stopped increased by one, the number of goals scored increased by a constant rate. He expressed a generalized real-life context, and then he used examples from coordinate pairs from the table. In sum, when asked to describe how to graph data from a table with positive non-integer values, Sandy and Joe chose x-axis interval as one, but chose not to match the y-values from the table exactly counting by three. They explained the location of the first point, "3.5 is between 3 and 6, just a little past 3." Both Sandy and Joe used more than one coordinate pairs to interpret their real-life situations correctly. In sum, both students answered the question correctly using appropriate intervals and real-life situations to link constant rate of change, and context.

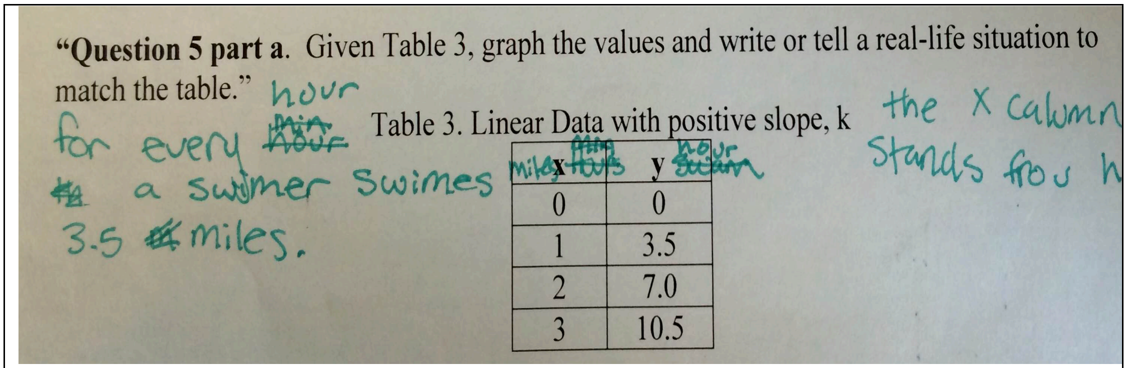
Brooke (Q5). When asked to write a real-life situation and explain how to graph the positive non-integer data, Brooke (MEP) labeled the x-values as hours and the y-values as miles. She verbally answered the question with a question and attempted an answer, "Can you use miles and hours? Tatsumi skateboarded 3.5 miles every hour, x is hours and y is miles. (2, 7) means 7 miles in 2 hours." Her written work showed an interpretation of coordinate pair (1, 3.5), "Tatsumi skateboarded 3.5 miles for 1 hour." When asked to describe how to graph data from a table with positive non-integer values, Brook chose one for the x-axis interval, and explained that one half and one for the y-axis interval would work, "I would count by one or .5 because the biggest number is 10.5." When probed further to choose the best interval, she decided on 3.5 as the best interval for the y-axis. She used more than one coordinate pairs to interpret the real-life situation and chose appropriate intervals. The solution was coded as *correct (2).* The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a real-life situation to relate two quantities, hours and total miles, and she interpreted several coordinate pairs from the table to show that as the number of hours increased by one, the total miles increased by a constant rate of 3.5 miles. In sum, she answered the question correctly using appropriate intervals and interpreted the real-life situation that connected constant rate with the the context, emphasizing two real-life quantities.

Summary of Picture (Table) to Picture (Graph) and Real-Life Situation Translations. When asked to write a real-life situation to represent the positive non-integer data, all students completed the task and correctly interpreted the meaning of the data (*picture to real-life situation, and verbal model*). Five out of seven students, Anna, Olvin, Lucas, Joe and Brooke, used more than one coordinate pairs pairs from the table to link the constant rate of change with the context. Two students, Leon and Sandy, used exactly one pair to connect the constant rate of change to the real-life situation.

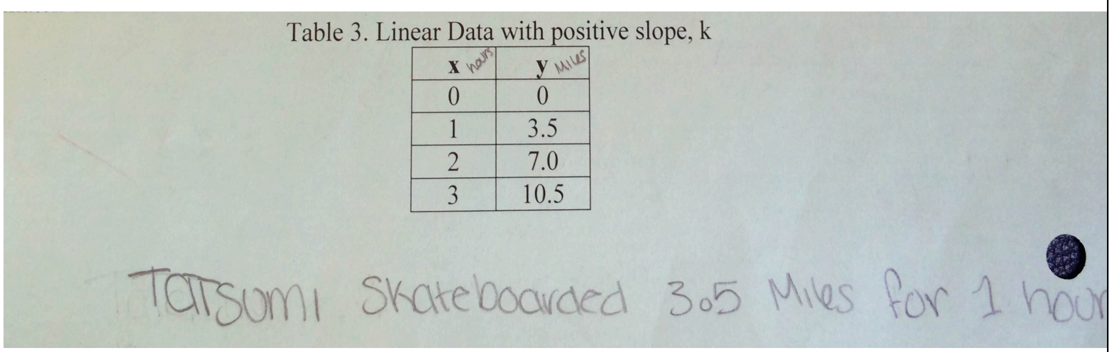
Proportional relationships, where input of zero resulted in an output of zero, appeared to be more intuitive for all students as evidenced by their ability to write original real-life situations that made sense of the data with positive non-integer values. All students correctly wrote an original real-life situation to match the new data with decimal y-values and connected the constant rate of change to the context. One student, Anna, used a slightly similar context as seen in a previous problem. All students were able to correctly interpret that an input of zero resulted in an output of zero. A closer examination showed that four out of seven students, Anna, Leon, Olvin and Lucas, copied the y-axis intervals directly from the table to the x-axis and y-axis, and three out of seven students, Brooke, Sandy and Joe, used unique y-axis intervals. Additionally, Leon, Sandy, and Brooke, wrote word problems to answer the question. In sum, all students answered the question correctly using appropriate intervals and real-life situations. All student explanations were coded as *conceptual.* All student responses were coded as *conceptual understanding* because students related two real-life quantities and the concept of constant rate of change from the table to show that as x-values increased by one, the y-values increased by 3.5 units using the real-life situation. Figure 4.9 shows student written work on Question 5 from the pre-assessment.



Leon writes a word problem using a real-life situation.



Sandy labels the column x as “miles” and column y as “hour,” and writes a sentence using the real-life context.



Brooke labels the x column as “hours” and the y-column as “miles,” and writes a sentence using a real-life context to interpret one coordinate pair.

Figure 4.9. Student work for question 5 on writing a real-life situation from a table.

The following sections present a summary of understandings of middle school immigrant students before participating in the eight-week constructivist teaching experiment.

Summary of Understandings Prior to Intervention

The following section will present a summary of seven middle school immigrant student responses to five pre-assessment questions numerically and descriptively as they relate to research question one:

1. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions by expressing translations

between and within modes of representations prior to participating in mathematical activities that emphasize LTM?

Recall from Chapter 3 the total possible points for a correct solution was two: *correct (2), partially correct (1),* or *incorrect (0);* and the total possible points for justification was two: *conceptual (2), procedural (1), or none/incorrect (0).* A maximum of 4 points was awarded for each correct solution with conceptual explanation. Table 4.6 displays a summary of students' numerical coding individually by question and translation type on the pre-assessment using three numeric scores: solution, justification, and a final score in bold. Following that, a descriptive summary of findings from student performance on the pre-assessment is presented and then how the data answers research question one is presented. The last three *rows* provided three different summaries of student performance: (1) students' average scores by question out of five possible questions and embedded translations out of 4 possible points (in ascending order: 3.0, 3.1, 3.6, 3.85, and 4 on five pre-assessment questions); (2) solution summary: total correct, partially correct, and incorrect questions out of 7 students; (3) justification summary: total conceptual, procedural, and incorrect justifications out of 7 students. The last *column* provided four different summaries: (1) overall average scores by individual students out of 4 possible points (in ascending order: 3.0, 3.0, 3.6, 3.6, 3.8, 3.8 and 3.8 out of 4 possible points); (2) overall average scores on five pre-assessment questions as a group out of four possible points (3.51 out of 4 possible points); (3) overall solution summary of total correct (33/35), partially correct (2/35), and incorrect solutions (0/35) out of 35 possible correct solutions; and (4) overall justification summary of total conceptual (20/35), procedural

(13), and incorrect (2/35) justifications out of 35 possible justifications. The following sections provide a closer examination of student responses on the pre-assessment by interview pairs.

Table 4.6

Detailed Summary of Student Scores from Pre-Assessment by Question and Translations

	Question 4a ***(Picture to Verbal Model)***	**Question 4b** ***(Picture to Picture)***	**Question 4c** ***(Picture to Written Model)***	**Question 4d** ***(Picture to Real-Life Situation)***	**Question 5** ***(Picture to Real-Life)***	**Overall Average**
Anna (LEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	3 (2, correct, 1, procedural)	1 (1, partially correct, 0, incorrect)	4 (2, correct, 2, conceptual)	3.0
Leon (HEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	3.8
Lucas (MEP)	3 (2, correct, 1, procedural)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	3.6
Olvin (LEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	3.8
Sandy (MEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	3 (2, correct, 1, procedural)	1 (1, partially correct, 0, incorrect)	4 (2, correct, 2, conceptual)	3.0
Joe (LEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	3.8
Brook (MEP)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	3 (2, correct, 1, procedural)	4 (2, correct, 2, conceptual)	4 (2, correct, 2, conceptual)	3.6
Average by Question	3	3.85	3.6	3.1	4	3.51
Solution Summary	7 correct 0 partially correct 0 incorrect	7 correct 0 partially correct 0 incorrect	7 correct 0 partially correct 0 incorrect	5 correct 2 partially correct 0 incorrect	7 correct 0 partially correct 0 incorrect	33/35 2/35 0/35
Justification Summary	0 conceptual 7 procedural 0 incorrect	6 conceptual 1 procedural 0 incorrect	4 conceptual 3 procedural 0 incorrect	5 conceptual 0 procedural 2 incorrect	7 conceptual 0 procedural 0 incorrect	20/35 13/35 2/35

Anna and Leon. Anna (LEP) was one of two students with the lowest overall average score of 3.0 out of 4 points on the pre-assessment, and coded as having *some understanding* of key ideas of linear functions about identifying slope, graphing, writing equation, interpreting a real-life situation and writing a real-life situation from a table. Anna responded to four out of five questions correctly about identifying slope, graphing, writing equation, and writing a real-life situation from a table with proportional linear relationship. She answered one out of five questions partially correct about interpreting data using a real-life situation. A closer examination showed that Anna conceptually explained two questions about writing a real-life situation to match a table of values with positive non-integer values and writing an equation from a table. She answered two questions procedurally about finding slope and graphing from a table. Anna left one question explanation blank about interpreting data from a table using a real-life situation.

Her partner, Leon (HEP), was one of three students that scored the highest overall average score of 3.8 out of 4 points on the pre-assessment, and coded as having *some understanding* of key ideas of linear functions. Leon answered all five questions correctly about identifying slope, graphing, writing an equation, interpreting data from a table using a real-life situation and writing a real-life situation from a table with positive non-integer values. A closer examination showed that Leon answered four questions conceptually about finding slope, graphing, interpreting data using a real-life situation writing a real-life situation from a table with positive non-integer values. He explained one problem procedurally about writing an equation from a table. In sum, Anna had one of the lowest overall average scores and Leon had one of the highest overall average scores on the pre-assessment out of seven students.

Lucas and Olvin. Lucas (MEP) scored the second highest overall average score of 3.6 out of 4 points on the pre-assessment, and coded as having *some understanding* of key ideas of linear functions about identifying slope, graphing, writing equation, interpreting a real-life situation and writing a real-life situation from a table. Lucas answered all five questions correctly about identifying slope, graphing, writing equation, interpreting data using a real-life situation, and writing a real-life situation from a table with proportional linear relationship. A closer examination showed that Lucas answered three questions conceptually about finding slope, interpreting data using a real-life situation writing a real-life situation from a table with positive non-integer values. He explained two problems procedurally about graphing, and writing an equation from a table.

His partner, one of the lowest English proficient students, Olvin (LEP) scored the *highest* overall average score of 3.8 out of 4 points on the pre-assessment, and coded as having between *some understanding* and *conceptual understanding* of key ideas of linear functions. Olvin answered all five questions correctly about identifying slope, graphing data, writing an equation, interpreting data from a table using a real-life situation and writing a real-life situation from a table with positive non-integer values. A closer examination showed Olvin explained four questions conceptually about graphing, writing equations, interpreting a real-life situation from a table, and writing a real-life situation to match a table of values with positive non-integer values. He explained one question procedurally about identifying slope. In sum, Lucas had the second highest overall average score, and Olvin had the highest overall average score on the pre-assessment out of seven students.

Sandy and Joe. Sandy (MEP) was the other student that scored the lowest overall average score of 3 out of 4 points on the pre-assessment, and coded as having *some understanding* of key ideas of linear functions about identifying slope, graphing, writing equation, interpreting a real-life situation and writing a real-life situation from a table. Sandy answered four out of five questions correctly about identifying slope, graphing, writing equations, and writing a real-life situation from a table with positive non-integer values. Sandy answered one out of five question partially correct about interpreting a real-life situation from a table. A closer examination showed she answered two questions conceptually about graphing and writing a real-life situation from a table with positive non-integer values. Sandy answered two questions procedurally about identifying slope and writing equations. She left one explanation to a question blank about interpreting a real-life situation from a table.

Her partner, one of the lowest English proficient students, Joe (LEP) scored the *highest* overall average score of 3.8 out of 4 points on the pre-assessment, and coded as having between *some understanding* and *conceptual understanding* of key ideas of linear functions. Joe answered all five questions correctly about identifying slope, graphing, writing an equation, interpreting data from a table using a real-life situation and writing a real-life situation from a table with positive non-integer values. A closer examination showed Joe explained four questions conceptually about graphing, writing an equation from a table, and writing a real-life situation from a table with positive non-integer values. He explained one question procedurally about identifying slope and graphing from a table. In sum, Sandy had the second lowest average overall average score and Joe had one of the highest overall average score on the pre-assessment out of seven students.

Brooke. Brook (MEP) had the second highest overall average score of 3.6 out of 4 points on the pre-assessment, and coded as having *some understanding* of key ideas of linear functions about identifying slope, graphing, writing equation, interpreting a real-life situation and writing a real-life situation from a table. Brooke answered all five questions correctly about identifying slope, graphing, writing an equation, interpreting data from a table using a real-life situation and writing a real-life situation from a table with positive non-integer values. A closer examination showed Brooke explained three questions conceptually about graphing, interpreting a real-life situation from a table and writing a real-life situation from a table with positive non-integer values. She answered two questions procedurally about identifying slope and writing equation from a table. In sum, Brooke had the second highest overall average score on the pre-assessment out of seven students. The following paragraph examined how middle school immigrant students performed on the pre-assessment by question and by translation types.

A closer examination by questions and by translation types on the pre-assessment showed that middle school immigrant students had the lowest overall average score of 3 out of 4 points when asked to identify slope and missing y-value (*picture to verbal model*), and the second lowest overall average score of 3.1 out of 4 points when asked to interpret data from the table given a real-life situation (*picture to real-life situation*) as well as when asked to graph from a table (*picture to picture*). One average, middle school immigrant students scored the second highest overall average score of 3.6 out of 4 points when asked to write an equation from a table (*picture to written model*), and on average, students scored the highest overall average score of 4 out of 4 points when asked to write an original real-life situation and select appropriate intervals to match a data table (*picture to a real-life situation*). The following sections provide a summary of understanding prior to participation in the constructivist teaching experiment by themes after multiple cycle coding.

Summary of Understanding Prior to Intervention by Themes

In the previous sections, data from the pre-assessment interview transcripts and students' written work were presented that documented the nature of seven middle school immigrant students' conceptual understanding of linear function ideas as they related to identifying slope, graphing, writing equations, interpreting real-life situations, making predictions, and writing a real-life situation from a table. The following sections first present a descriptive summary of characteristics of the pre-assessment questions that supported conceptual understanding of linear function ideas that focus on two mathematics processes, communication and representations, as they relate to research question one (NCTM, 2000). Figure 4.10 displays a summary of findings from the pre-assessment data that showed the approaches that supported middle school immigrant students conceptual understanding as evidenced by justification coding (i.e. procedural or conceptual). The data suggested that: (1) students gave procedural justifications when asked solution-oriented questions (versus process-oriented questions); (2) students gave conceptual justifications when asked open-ended questions that encouraged multiple perspectives; and (3) a real-life situation supported more conceptual justifications. The following sections summarize findings by individual pre-assessment questions as well as the embedded translations that support themes displayed in Figure 4.10.

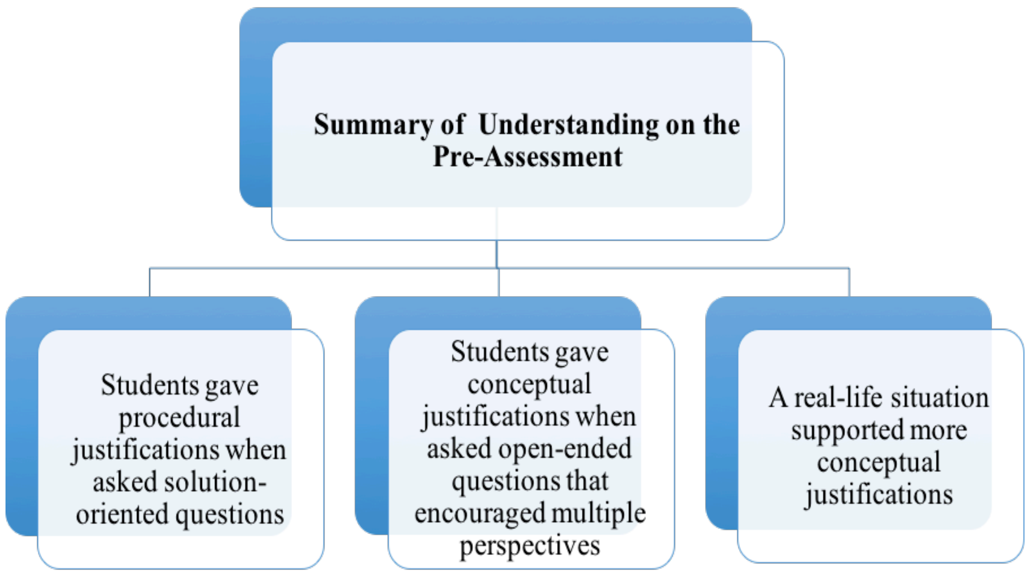


Figure 4.10. Summary of understanding on the pre-assessment.

Find a Missing Y-Value and Justify Solution (*picture-table to verbal model*)

When students were asked to find a missing y-value using a pattern, or slope, and justify the solution given a table absent of a real-life situation, all students were able to correctly calculate the missing y-value using the pattern, or slope of nine. Students used procedures in ways that aligned with the intervention classroom prior to the constructivist teaching experiment. For example, in the pre-assessment, some students were adding given y-values by the slope of 9 to calculate the missing y-value of 36 in the table, and neglected changes in x-values; or students used multiplication or division to calculate slope. However, none of the students connected the strategies to the concept of slope and explained slope as *two* simultaneously changing quantities, where change in y-values were constant for each unit of x-value increase. Students neglected that changes in x-values since they were working with unit rates. A closer examination of the question further pointed to a solution-oriented response, *if you know x, can you find y?* Therefore, the way the question was posed encouraged student responses coded as *procedural*. This finding suggests that the role of the teacher needs to include selecting rich problems and writing open-ended questions to support more *conceptual* justifications.

Graph and Describe Steepness of the Line (*picture-table to picture-graph*)

When students were asked to graph given a table absent of a real-life situation, two themes emerged. Students either chose x- and y-axis that matched the values in the table exactly, or students chose an x-axis interval to match the values in the table, but used factors of 36 for y-axis intervals. Recall that the number 36 was the maximum y-value in the table. The finding from the data also suggested that students confused steepness of the slope as dependent on the interval selection. Although it is true that the *appearance* of the steepness changed with y-axis intervals of 3 versus a y-axis interval of 9. However, students were not clear that a slope of 9 was considered steep no matter the interval selection and despite the appearance of the graph. This finding suggested a gap in student understanding about the concept of steepness, and therefore, suggested opportunities to teach about interval selection and steepness further.

Write and Verify Equation (*picture-table to written model*)

When students were asked to write and verify an equation given a table absent of a real-life situation, all students wrote a correct equation in $y = kx$ format, where k was the slope. All students plugged in the slope from Question 4a, $y = 9x$. Upon closer examination, students did not all verify their equation using coordinate pairs from the table. Four of seven students used more than one coordinate to verify the equation worked for two or more coordinate pairs. This finding aligned with students' current mathematics teaching as confirmed by the teacher. Students were not asked to verify equations in their traditional mathematics class either. Therefore, this finding suggests that the role of the teacher needs to ask for verification, and provide probing questions to encourage student explanations. For example, probing in the pre-assessment interview revealed that there was confusion about the coordinate (1, 9) and a slope of 9. The first represented a coordinate point on the line, and the second described the "direction" from one point to another on a given line (rise over run). In sum, finding from the data suggested that asking students to plug in slope, k, into an equation, y = kx, was not sufficient to support conceptual understanding or encourage a more conceptual response.

Interpret Slope Given a Real-Life Situation (*picture-table to real-life situation, & verbal model*)

When students were given a real-life situation for a table and asked to interpret the meaning of slope, a majority of students (5 out of 7 students) correctly interpreted a slope relating two real-life quantities, $9 *per* hours worked. This was an improvement from Question 4a where none of the students described slope as two changing or related quantities involving x- and y-values. Further, about half of students (3 out of 7 students) interpreted two or more coordinate pairs from the table to show that the more hours worked, the more money earned. This finding suggested that the role of the teacher that included providing a real-life situation increased students' conceptual responses about the concept of slope, and supported students' meaningful understanding of the concept of slope as a relationship between x- and y-values, or two real-life quantities, dollars earned *per* hour worked.

Write a Real-Life Situation for a Table (*picture-table to real-life situation, & verbal model*)

When asked to write a real-life situation for a table, all students correctly wrote an original real-life situation to match data from the table. Further, a majority of students (5 out of 7 students) used more than one coordinate pairs to interpret the real-life situation to show that as x-values increase, so does the y-value by the slope of 3.5. This finding suggested that open-ended problems that allowed for multiple perspectives increased students' responses coded as *conceptual,* and that the role of the teacher that included writing open-ended questions and/or selecting rich problems to motivate students increased justifications coded as *conceptual*.

In summary, students answered questions with procedural justifications when asked questions that pointed to solution-oriented responses. Students increased their conceptual justifications when given a real-life situation, or when asked to answer open-ended question that allowed for multiple perspectives. Students also gave more conceptual responses when the researcher-teacher encouraged students to explain further/verify, and did not accept a correct answer. This finding suggested that the role of the teacher is to select rich problems, or give students opportunities to explain further their solutions (i.e. verify) in ways that encouraged *why* and *how* (process), and not just *what* (solution).

In the previous sections, data from the pre-assessment interview transcripts and students' written work were presented that documented the nature of seven middle school immigrant students' conceptual understanding of linear function ideas as they related to identifying slope as two changing quantities, graphing, writing an equation, interpreting a real-life situation and writing a real-life situation from a table. The following sections present the findings for research question two.

Understandings of Linear Function Ideas Post Intervention

The following sections will present the data from students' written work as well as students' oral explanations in pairs on the post-assessment questions with subparts as the data from these questions relate to the second research question:

2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions by expressing translations between and within modes of representations after participating in mathematical activities that emphasize LTM?

The post-assessment questions were designed by the researcher with input from the teacher to reflect linear function ideas from the constructivist teaching experiment that emphasized communication and multiple representations. Figure 4.11 displays the post-assessments as well as the embedded translations. To answer the second research question, the post-assessment questions are presented first, followed by first a brief description of the problem including expectations from students, and then by a table of individual responses coded numerically. Students received a solution score, a justification score, and a final score for each individual post-assessment question. A summary of student responses in pairs is then presented. The total possible points for a correct solution was two points: *correct (2), partially correct (1),* or *incorrect (0);* and the total possible points for justification was two points: *conceptual (2), procedural (1), or none/incorrect (0).* Hiebert and Lefevre (1986) discussed procedural knowledge as being useful and necessary for conceptual understanding, but that students also needed to be able to assess the reasonableness of procedures and explain why and how to move from procedural to conceptual. Characteristics of justifications coded as *procedural* included student reasoning using, "algorithms, or rules, for completing mathematical tasks," or a step-by-step procedure that was prescriptive when completing a task in a linear sequence (Hiebert & Lefevre, 1986, p. 4).

<table>
<tr><th>Post-Assessment Question 1</th><th>Embedded Translations</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.
<table>
<tr><th>Hours worked</th><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><th>Money in Piggy Bank ($)</th><td></td><td></td><td>41</td><td>48</td><td>55</td></tr>
</table>
</td><td></td></tr>
<tr><td>e) Complete the table. Describe the pattern for x & y.</td><td>Picture (Table) to Verbal Model</td></tr>
<tr><td>f) Write an equation using the pattern & the starting value.</td><td>Picture (Table) to Written Model (Equation)</td></tr>
<tr><td>g) How much money will Jane have in the bank after 50 hours of babysitting? How can you check?</td><td>Picture (Table) to Real-Life Situation</td></tr>
<tr><td>h) Is the pattern proportional? Give <u>two</u> reasons. (Use the table, pictures, words or a graph).</td><td>Picture (Table) to Verbal Model</td></tr>
</table>

Figure 4.11. Post-assessment questions with embedded translations.

For example, Question 1a, asked students to complete the table for two missing y-values and describe slope as two patterns involving x- and y-values given a table with a real-life situation. An example of a solution coded as *correct,* and a justification coded as *procedural* was from Brooke (MEP), “y is going up by 7, x is going up by 1,” and she completed the table correctly with (0, 27) and (1, 34). Her verbal explanation included two isolated procedures, addition and subtraction by the slope of seven, “I said for 0 it was 27 because it started, it was fill out out only at 2 and before it wasn’t. I just saw what it was going up by for 2 and 3 and 4 so I just subtracted it from 41 each time. And then from the next number I got, I just subtracted 7 and got 27 for 0. I said y is going up by 7 and x is going up by 1,” (*The first 2 refers to x = 2; the 2, 3, and 4 refers to x-values, and “subtracted it” refers to subtracting a slope of 7*). Hiebert and Lefevre (1986) described this explanation as procedural, because the strategies were treated as two isolated pieces of information, emphasizing algorithms and rules to complete the task.

Conceptual knowledge was considered rich in relationships and as a network in which ideas are linked, and suggested a higher level of reflection, where students took a step back and reflected on the information being connected. Students with conceptual understanding were able to understand how and why procedures work in more than one way and pointed out similarities in relationships. Characteristics of justifications coded as *conceptual* included student reasoning moving away from presenting “isolated pieces of information,” and toward recognizing the, “relationship to other pieces of information” (Hiebert & Lefevre, 1986, p. 4). Additionally, conceptual knowledge involved the student being able to monitor and evaluate reasonableness and anticipate the consequence of possible actions. For example, Question 1d asked students to use two or more representations to explain if the given table represented a proportional pattern and Leon (HEP) said, "No, it is not because the y when x is 0 is not 0, and you *add* to the pattern, 27" (*he points to the table and the + 27 in the equation, y = 7x + 27*). Leon used more than one representation to support his reasoning, the table and the equation showed a y-intercept of (0 27). Hiebert and Lefevre (1986) described this explanation as pointing out similarities and relationships to other pieces of information where the student connected the table to the equation point to the y-intercept as above the origin at (0, 27).

It is important to point out that justifications coded as *conceptual* were dependent upon each individual question on the post-assessment. For example, Question 1a asked students to find two missing y-values and describe slope as two patterns, x- and y-values. An explanation coded as *conceptual* included students going beyond calculating the missing y-values, but linking two changing quantities involving x- and y-values in more than one way. For example, students could show that subtracting y-values from right to left by 7 in the table gives a starting value (y-intercept) of 27, and therefore, the equation, $y = 7x + 27$. From the equation, students can relate the 7 from 7x with adding by 7 in the y-values. Students could also use a verbal model explain the concept of slope as linking the x- and y-value relationships with an additional representation (i.e. insert a real-life situation to relate two quantities, *dollars earned per hour, with 27 dollars as a balance*). Whereas, Question 1b asked students to write an equation from a table in $y = m\text{x} + \text{b}$, where m is the slope and b is the y-intercept, and verify the equation by plugging in additional coordinate points from the table, or connecting the constant rate of change of 7 in the y-values to the 7x in the equation. A maximum of 4 points was awarded for individual student response to pre- and post-assessment questions. The following sections present the data from the post-assessment semi-structured interview transcripts from Question 1a on identifying slope and missing y-values.

Question 1a. Identify Slope and Two Missing Y-Values (*Picture to Verbal Model*)

Figure 4.12 shows Question 1a that asks students to identify slope and two missing y-values from a table given a real-life situation. The table presents the translation embedded in the task. Question 1a was designed by the researcher with input from the teacher to reflect mathematical tasks from the constructivist teaching experiment, and presented students with a table with a non-proportional linear relationship given a real-life situation. Students were expected to calculate a slope of seven and two missing y-values of 27 and 34 for x = 0 hour and x = 1 hour, respectively. Explanations coded as *conceptual* required that students went beyond calculating missing y-values using slope of 9, but seeing relationship between two ideas embedded in the task. For example, linking the *two* patterns involving changes in x- and y-values in more than one way (i.e. *as the x-values increased by one hour, the y-values increased by $7*); or linking x- and y-value relationships with an additional representation (i.e. insert a real-life situation to relate two quantities, *$7 earned per hour, with $27 as a starting balance*). An example of a justification coded as *procedural* is: “y is going up by 7, x is going up by 1,” with the correct missing y-values of 27 and 34 in the table and no justification or connection with other patterns embedded in the table, or no interpretation using a real-life situation. An example of a justification coded as *conceptual* is, "x increases by 1, y increases by 7… I put for *hours* I have it increases by 1, and for the *piggy bank* it increases by 7. I put 27 for 0 and 34 for 1" (*for each hour increase, the amount of money in the piggy bank increases by $7*). The student connected the concept of constant rate, or slope, of 7 with two related real-life quantities, $7 per hour, given a real-life situation.

<table>
<tr><th>Post-Assessment Question</th><th>Embedded Translation</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

<table>
<tr><td>Hours worked</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Money in Piggy Bank ($)</td><td></td><td></td><td>41</td><td>48</td><td>55</td></tr>
</table>

a) Complete the table. Describe the pattern for x and y.</td><td>Picture (Table) to Verbal Model</td></tr>
</table>

Figure 4.12. Post-assessment question 1a on identifying slope and two missing y-values.

Table 4.7 presents individual student scores on student-pair solutions and explanations from the post-assessment interview transcripts, followed by a description of codes. Two students answered the task partially correctly and five students answered the task correctly, and two students used procedural strategies and five students used conceptual strategies in their justification.

Table 4.7

Question 1a Scores

Student	Conceptual Understanding Score
Anna (LEP)	3 (1, partially correct 2, conceptual)
Leon (HEP)	4 (2, correct

	2, conceptual)
Lucas (MEP)	2 (1, partially correct 1, procedural)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	3 (2, correct 1, procedural)

Anna and Leon (Q1a). When asked to find the missing y-values in the table given a real-life situation, and to identify the pattern (slope) as two changing quantities involving x- and y-values, Anna (LEP) had a *partially correct* solution. She described slope as two changing quantities, where x-values increased by one and y-values increased by seven. However, in her written description using words, she accidentally switched the numbers one and seven, "The pattern for x is going up by seven. The pattern for y is one." With partner discussion she rearranged the values so that the pattern for x-values increased by one and y-values increased by seven. Anna incorrectly completed the table since she thought the table followed a *proportional* linear pattern, and wrote (0, 0) and (1, 7), instead of (0, 27) and (1, 34) in the table. When probed by the researcher for an explanation, Anna used a correct interpretation of the real-life situation to explain that zero hours worked equaled zero dollars earned, "I got zero and seven because the reason it has to start at zero is because zero is the hours worked so I put zero there because that would match the hour worked." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> A: I wrote the pattern is for x, going up by 7 and the pattern for y is 1.
> SC: You said x is going up by 7 is that true? (*Researcher pointed to table of values, row by row.*)
> A: Yeah.
> SC: Leo is x going up by 7?
> L: No.
> SC: What is it going up by?
> L: It is going up by 1.
> SC: And what is y going up by?
> L: It is going up by 7
> SC: Anna, would you like to agree with him or would you like to keep your answer? He said x is going up by 1 and y is going up by 7. You said x is going up by 7 and y is going up by 1. One of you is correct.
> A: I would keep my answer.
> SC: Leon, would you like to convince her why you think your answer is correct?
> L: Yes, right here it says that this is x and this is y. (*He correctly labels the rows in the table with "x," and "y," next to "hours worked," and "money in piggy bank," respectively.*)
> SC: Anna, do you agree with him?
> A: Yes.
> SC: Would you like to change your answer?
> A: Yes.
> SC: So go ahead and use the pen to change your answer. Moving on, so you described the pattern, now you have to complete the table. I see that you agree on the written description of the pattern, but your table disagrees. So, Anna, can you tell me where you got 0 and 7? (*She has (0,0) and (1, 7), and not (0, 27) and (1, 34*) *on the table)*
> A: I got 0 and 7 because the reason it has to start at 0 is because 0 is the hours worked so I put 0 there. And I added 7…
> SC: Okay go back here. You said it had to start at 0, why?
> A: Because that would match the hour worked.
> SC: That's a really good explanation. However, could she have money in the piggy bank before she babysat, possibly?
> L/A: Yeah.
> SC: Does it have to be 0?
> L/A: No.

The solution was coded as *partially correct (1)*. The explanation was coded as *conceptual (2)*. The response was coded as *some understanding (Score: 3)* because she answered the question partially correctly but she correctly interpreted the real-life situation to describe slope as $7 per hour, but neglected the balance of $27 in the piggy bank before adding earnings from babysitting. She connected the context to the slope and showed some higher level reflection that working 0 hours means you earn $0. Hiebert and Lefevre (1986) described this as a form of monitoring and evaluating reasonableness.

Her partner, Leon (HEP) had the *correct* solution. He described slope as two changing quantities, where x- values increased by one and y-values increased by seven, "x is going up by one, y is going up by 7," and he wrote the correct y-values, (0, 27) and (1, 34) in the table. After partner discussion, his verbal explanation included a real-life interpretation of slope of 7 as, "how much money she earns," and with questions from the researcher, he added, "*per* hour." The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because Leon answered the question correctly using a real-life situation and connected the context with the constant rate of change from the table. Hiebert and Lefevre (1986) described this as pointing out relationships and similarities from the pattern in the table and interpreting the relationship using context. In sum, Anna and Leon correctly identified the slope as two changing quantities where x-values increased by one and y-values increased by seven. Leon completed the table correctly on his own, and Anna answered the question *partially correct* on her own. Both students showed conceptual understanding through use of multiple representations including a real-life situation, verbal and written models.

Olvin and Lucas (Q1a). When asked to find the missing y-values in the table given a real-life situation, and to identify the pattern (slope) as two changing quantities involving x- and y-values, Olvin (LEP) had a *correct* solution. He described slope as two changing quantities, where y-values increased by seven and x-values increased by one. He wrote, "+7" above the table in between each y-values. His verbal explanation described changes in both x- and y-values, "y, I think is the number that is adding by 7 and x is adding by 1." To complete the table with missing y-values, he wrote three subtraction problems using given y-values and the slope, "55 - 48 = 7; 41 - 7 = 34; 34 - 7 = 27." He correctly wrote (0, 27) and (1, 34) in the table. Additionally, Olvin included a real-life situation to interpret the meaning of slope of seven as the amount of dollars earned *per* hour worked. He said slope of seven was, "The money in the piggy bank and x is hours worked." With questions from the researcher, he added, "*each* hour she works." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> O: We have the same answers
> L: We added by 7 and that's how
> SC: So are you telling me your answer or are you reading me your answer?
> L: I'm telling you.
> SC: Both work. Ok, Olvin do you have something different from what he said?
> O: What did he say?
> SC: It says describe the pattern for x and y. It sounds to me like Lucas only did one of them. Did you do x or did you do y?
> L: Both.

SC: So do you want to answer the question again? The first question is, “Complete the table.”
L: Oh. The first one I got (0, 27) and I added 7 and I got 34. No, (1, 34)
SC: I really like the way you answered that, Lucas. You answered that as a coordinate pair. You said, “The first one I got (0, 27), and then I added 7 and I got (1, 34).” Beautiful. You didn’t just say the answers in the box. It says describe the pattern for x and y. You said a number. Is that x or y?
L: I don’t know. Oh wait, y.
SC: So Olvin, you laughed but you didn’t help answer the question. So you should probably answer that in two sentences.
O: y, I think is the number that is adding by 7 and x is adding by 1.
SC: Ok, Lucas, what do you think about that answer? Now could you use these words right here to answer the question, instead of using x and y? You answered it completely correct. But this time instead of using x, this time using the story. (*Researcher pointed to the context in the table*)
O: y is money is piggy bank and x is hours worked.
SC: If y is money in piggy bank, and you said it was changing by 7, can you tell me what that means? So in a sentence can you use the number 7 and money in piggy bank in a sentence? Put that in a sentence to explain to me what the 7 means? Lucas, you can help, too.
O: The 7 means that she is adding 7 dollars each, she is getting 7 dollars each
SC: Each what?
O: Each hour she works
SC: Thank you so much for doing that. You did more than I asked on the paper.

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because answered the question correctly using a real-life situation and connected the y-value patterns in the table with a context. Hiebert and Lefevre (1986) described this as recognizing relationships between table and context.

His partner, Lucas (MEP) had a *partially correct* solution. He did not describe slope as two changing quantities involving x- and y-values. Lucas described *one* change, the y-values. He said the change in y-values was seven and created a vertical table to subtract y-values, 55 and 48 to show the slope of seven, “55 – 7 = 48.” His verbal explanation included a second procedure, addition. He said you could *add* seven to 48 for a sum of 55, “We added by 7 and that’s how.” To complete the table with missing y-values, he answered using coordinate pairs, (0, 27) and (1, 34), “The first one I got (0, 27) and I added 7 and I got 34. No, (1, 34).” Lucas did not use a real-life situation to interpret slope. The solution was coded as a *partially correct (1).* The explanation was coded as *procedural (1).* The response was coded as *some understanding (Score: 3)* because unlike Olvin, he answered the question partially correctly and and he did connect the pattern to the real-life situation, *dollars and hours*. Hiebert and Lefevre (1986) described this explanation as presenting rules in isolation. Instead, Lucas relied on addition and subtraction procedures to explain how to calculate without pointing out similarities. In sum, Olvin and Lucas correctly identified slope as two changing quantities where x-values increased by one and y-values increased by seven, and completed the table. Olvin showed conceptual understanding through use of multiple representations including a real-life situation, verbal and written models. Lucas used only procedural calculations.

Sandy and Joe (Q1a). When asked to find the missing y-values in the table given a real-life situation, and to identify the pattern (slope) as two changing quantities involving x- and y-values, Sandy (MEP) had a *correct* solution. She described slope as two changing quantities involving x- and y-values, where x-values increased by one and y-values increased by seven. Sandy labeled the first row in the table with “x,” next to “Hours Worked,” and the second row in the table with “y,” next to “Money in Piggy Bank.” She also wrote a description for slope as two changing quantities as, “x increases by 1, y increases by 7,” and completed the table with (0, 27) and (1, 34). Sandy interpreted the meaning of a slope using a real-life situation as a relationship between amount of money in piggy bank and hours worked, "I put for hours I have it increases by 1, and for the piggy bank it increases by 7. I put 27 for 0 and 34 for 1" (*for each hour increase, the amount of money in the piggy bank increases by $7*). The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher:

> S: I put for hours I have it increases by 1, and for the piggy bank it increases by 7 (*Sandy used a real-life situation without pushing or probing from the researcher*)
> J: I have the same thing.
> SC: You do? Okay.
> S: I put 27 for 0 and 34 for 1.
> J: x is increasing by 1 and the y is increasing by 7.

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because Sandy answered the question correctly using a real-life situation to connect the pattern with the context. Hiebert and Lefevre (1986) described this explanation as recognizing relationships and pointing out similarities.

Her partner, Joe (LEP) also had a *correct* solution. He described slope as two changing quantities, where x-values increased by one and y-values increased by seven as, "x is going up by1's y is adding up by 7's." With partner discussion, Joe agreed with Sandy on the real-life interpretation of slope, "I have the same thing, x is increasing by 1 and the y is increasing by 7." He completed the table correctly with (0, 27) and (1, 34). The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a real-life situation and connected the pattern from the table with the context. Hiebert and Lefevre (1986) described this explanation as recognizing relationships and pointing out similarities. In sum, Sandy and Joe correctly identified slope as two changing quantities where x-values increased by one and y-values increased by seven, and completed the table correctly. Both students showed conceptual understanding through use of multiple representations including a real-life situation, verbal and written models.

Brooke (Q1a). When asked to find the missing y-values in the table given a real-life situation, and to identify the pattern (slope) as two changing quantities involving x- and y-values, Brooke (MEP) had a *correct* solution. She described slope as two changing quantities where x-values increased by one and y-values increased by seven, "y is going up by 7, x is going up by 1," and completed the table correctly with (0, 27) and (1, 34). Her verbal explanation included two procedures, addition and subtraction by the slope of seven, "I said for 0 it was 27 because it started, it was fill out out only at 2 and before it wasn't. I just saw what it was going up by for 2 and 3 and 4 so I just subtracted it from 41 each time. And then from the next number I got, I just subtracted 7 and got 27 for 0. I said y is going up by 7 and x is going up by 1," (*The first 2 refers to x = 2; the 2, 3, and 4 refers to x-values, and "subtracted it" refers to subtracting a slope of 7*).
She did not use a real-life situation to interpret slope. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because she answered the question correctly, but she does connect the pattern with a real-life situation as, *dollars and hours*. Hiebert and Lefevre (1986) described this explanation as implementing rules or steps in isolation. In sum,

Brooke correctly identified slope as two changing quantities where x-values increased by one and y-values increased by seven, and correctly completed the table. Her explanation was limited to procedural calculations that were not connected or similarities were not pointed out. She did not connect her explanation with a real-life interpretation of slope.

Summary of Picture (Table) to Verbal Model Translation. When asked to find the missing y-values in the table given a real-life situation, and to identify the pattern (slope) as two changing quantities involving x- and y-values, six out of seven students had a correct solution. Leon, Olvin, Lucas, Sandy, Joe and Brook described slope as two changing quantities where x-value increased by one and y-value increased by seven. One student, Anna answered the question *partially correct* at first. She switched the description in her written work, where she described x-values as increasing by seven and y-values increasing by one. Although it was a simple error, Anna demonstrated a *conceptual* justification verbally. She interpreted a slope of seven using a real-life situation to explain that working zero hours resulted in zero dollars earned. This showed that she reflected and assessed reasonableness of the given context (Hiebert & Lefevre, 1986). Five out of seven students, Anna, Leon, Olvin, Sandy and Joe, interpreted the slope using a real-life situation in their explanations and connected the context to the constant rate of change, or pattern of 7. Two students, Lucas and Brooke, did not interpret slope using a real-life situation. Brooke used two calculations, addition and subtraction, to verify that the slope was seven and to complete the table. Hiebert and Lefevre (1987) described this as procedural and implementing algorithms and rules in isolation. In all, six out of seven students completed the table correctly except for Anna, who thought the table followed as a proportional linear pattern. The following sections present data for the post-assessment Question 1b about writing an equation from a table.

Question 1b. Write an Equation from a Table (*Picture to Written Model*)

Figure 4.13 shows Question 1b about writing an equation from a table given a real-life situation as well as the embedded translation. Question 1a was designed by the researcher with input from the teacher to reflect mathematical tasks from the constructivist teaching experiment, and presented students with a table with a non-proportional linear relationship given a real-life situation. Question 1b asked students to write an equation using the *pattern* (slope) and *starting value* (y-intercept) from a table given a real-life situation. Students were expected to use the slope of \$7 and y-intercept of \$27 from Question 1a to write an equation in the form, $y = mx + b$, where m is the *pattern* and b is the *starting value*. An explanation coded as *conceptual* required students to correctly write the equation, $y = 7x + 27$, and connect the equation with the table by plugging in one or more coordinate pairs (x, y), or by linking the context with the slope and starting-value. For example, $27 = 7(0) + 27$; $34 = 7(1) + 27$; $41 = 7(2) + 27$; $48 = 7(3) + 27$; and $55 = 7(4) + 27$. An example of a justification coded as *procedural* is, "y = 7x + 27… Because 7 is the pattern and not the starting value" (*when asked why 7 is multiplied by x*). An example of a justification coded as *conceptual* is, "y = 7x + 27," and the student connected the equation to the table of values by plugging in (0, 27) and (1, 34) from the table to verify the equation: "27 = 7(0) + 27; 34 = 7(1) + 27." Hiebert and Lefevre (1986) described this explanation as linking ideas and higher level connections showing how and why an equation works for table of values.

<table>
<tr><th>Post-Assessment Question</th><th>Embedded Translation</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.
<table><tr><td>Hours worked</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Money in Piggy Bank ($)</td><td></td><td></td><td>41</td><td>48</td><td>55</td></tr></table>
b) Write an equation using the pattern and the starting value.</td><td>Picture (Table) to Written Model (Equation)</td></tr>
</table>

Figure 4.13. Post-assessment question 1b on writing equation from a table.

Table 4.13 presents individual student scores on student-pair solutions and explanations from the post-assessment interview transcripts. All students answered the task correctly; but three students used procedural strategies and four students used conceptual strategies in their justification.

Table 4.8

Question 1b Scores

Student	Conceptual Understanding Score
Anna (LEP)	3 (2, correct 1, procedural)
Leon (HEP)	3 (2, correct 1, procedural)
Lucas (MEP)	4 (2, correct 2, conceptual)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	3 (2, correct 1, procedural)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q1b). When asked to write an equation using the pattern (slope) and starting value (y-intercept) from the table, Anna had a *correct* solution. With partner discussion, Anna (LEP) modified her incorrect equation at first from her individual attempt as, y = 7x + 0, to the correct equation, y = 7x + 27. In her written work, Anna wrote a slope-intercept template from the teaching experiment, "y = __x + __," and filled the blanks with a slope of seven and a y-intercept of zero, "y = 7x + 0."

Her verbal explanation included semi-formal language from the teaching experiment, she used the words *pattern* for slope and *starting value* for y-intercept, "7 is the pattern and 27 is starting value." She said a slope of seven was to be multiplied by x, and a starting value of 27 was to be added to the product, 7x. She did not use coordinate pairs to verify. The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> SC: Because you made some changes to yours could you update your answer so it matches the marker in purple? Your pattern is what? (*Recall that Anna edited Question 1a from y = 7x + 0 to y = 7x + 27 after partner discussion.)*
> A: y.
> SC: Your pattern is? (*The researcher pointed to Anna's written work, at her equation*)?
> A: 7.
> SC: Your starting value is?
> A: 0.
> SC: Is it 0? What is it?
> A: 27.
> SC: Yeah. Update it so it matches and then compare it with Leon's. Leon, what did you get?
> L: I have 7x + 27.
> SC: And then Anna, what did you get?
> A: 7x + 27. (*For her original work, she had y = 7x + 0*)
> SC: Good so you both agree.
> A: Yes.
> SC: Okay, so how do we know that the 7 goes here and not here? (*Researcher pointed to slope and y-intercept in the equation, y = 7x + 27*)
> L: Because the 7 is the pattern and not the starting value.
> SC: Yes, Anna do you have a different explanation?
> A: The 7 is the pattern.

The solution was coded as *correct (2).* The explanation was coded as *procedural (1).* The response was coded as *some understanding (Score: 3)* because she answered the question correctly, but she did not connect the meaning of the slope or y-intercept with the context, or link the equation with the table by plugging in points to verify the equation. Hiebert and Lefevre (1986) described this explanation as showing little understanding of why or how, and does not point out relationships or assess reasonableness.

Her partner, Leon (HEP), also had a *correct* solution. He wrote the equation as, y = 7x + 27, where slope was seven and y-intercept was 27. His verbal explanation also included semi-formal language from the teaching experiment, he used *pattern* for slope and *starting value* for y-intercept, "Because 7 is the pattern and not the starting value," (*when asked why 7 is multiplied by x*). He did not use other coordinate pairs from the table to verify the equation. The solution was coded as *correct (2)*. The explanation was coded as *procedural (1)*. The response was coded as *some understanding (Score: 3)* because he did not connect the meaning of the slope or y-intercept with the context, or link the equation with the table by plugging in points to verify the equation. Hiebert and Lefevre (1986) described this explanation as showing little understanding of why or how, and does not point out relationships or assess reasonableness. In sum, Anna and Leon wrote the correct equation to represent the data in the table and used informal language to explain their equation, where the pattern (slope) was seven and starting value (y-intercept) was 27. Neither partner used one or more coordinate pairs from the table to verify their equation.

Olvin and Lucas (Q1b). When asked to write an equation using the pattern (slope) and starting value (y-intercept) from the table, Olvin (LEP) had a *correct* solution. In his written work, he wrote a slope-intercept template that summarized how to write an equation for non-proportional linear functions from the teaching experiment, "y = __x +

__,” and filled the blanks with a slope of seven and a y-intercept of 27 for the correct equation as, “y = 7x + 27.” Olvin also verified the equation using two coordinate pairs, (1, 34) and (2, 41), “34 = 7 x 1 + 27 = 34; 41 = 7 x 2 + 27.” His verbal explanation included partner discussion to verify that the equation worked for both coordinate points. His verbal explanation also included supporting his partner, Lucas (MEP), who had two equations, one incorrect equation, y = 27x + 7, and one correct equation, y = 7x + 27. The researcher asked Olvin to explain why the equation, y = 27x + 7, was incorrect and Olvin at first shared a procedural explanation, or a memorized fact. He said slope was the value to be multiplied by x, and y-intercept was the value to be added in an equation. Rather than explaining that the equation was a rule for all coordinate pairs from the table and plugging in values into the equation to verify the rule. With probing from the researcher, Olvin and his partner made connections between the equation and table of values (*picture to written model*). They verified the equation, y = 7x + 27, using two coordinate points from the table, (1, 34) and (2, 41).   The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> SC: How do we check? Did we check it?
> O: Yeah we used the equation right here.
> SC: What is another way to check?
> L: The table?
> SC: Yeah. Pick a point on the table.
> (L points to a coordinate on the table)
> SC: Ou, big number. 55 goes where in the equation, x or y?
> O: y.
> L: x.
> SC: Ok so Lucas said 55 goes x or y, I heard x and I heard y.
> L: I got confused.
> SC: Where does it go?

O: y.
L: y.
SC: y. Another way to check is if we put 55 here, right? So 55 equals 7 times, what is x?
O: 4.
SC: 4, what is 7 times 4?
O: 28.
SC: What is 28 plus 27?
O: 55.

The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and connected the equation to the coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as recognizing relationships and pointing out similarities, as well as evaluating reasonableness (why and how).

His partner, Lucas, had a *correct* solution. His written work shows two equations, one incorrect equation, y = 7 + 27x and one correct equation, y = 7x + 27. His verbal explanation included only the correct equation but he was not confident, "I got y = 7x + 27. No wait, that is wrong." After his partner, Olvin, shared that he had the same equation, Lucas became more certain and then confirmed the correct equation, "The final answer is, y = 7x + 27." His verbal explanation did not include a conceptual understanding as to why the incorrect equation did not represent the table, "I wrote 7 + 27x. I wrote my first one as a starting value. I put 7 as the starting value and the 27 as the pattern." With questioning from the researcher, Lucas and his partner verified their equation using two coordinate points (1, 27) and (2, 41), as demonstrated in the interview transcript above. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and connected the equation to the coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as recognizing relationships and pointing out similarities, as well as evaluating reasonableness (why and how). In sum, Olvin and Lucas wrote the correct equation to represent the data in the table explain their equation, where the pattern (slope) was seven and starting value (y-intercept) was 27. Both partners used one or more coordinate pairs from the table to connect the rule to the table and verify their equation.

Sandy and Joe (Q1b). When asked to write an equation using the pattern (slope) and starting value (y-intercept) from the table, Sandy (MEP) had a *correct* solution. In the upper right corner of her paper test, she wrote a slope-intercept template that summarized how to write an equation for non-proportional linear functions from the teaching experiment, "y = __x + __," and filled the blanks with a slope of seven and a y-intercept of 27 for the correct equation, "y = <u>7</u>x + <u>27</u>." In the upper right hand corner of her test, she also drew two arrows, each pointing to the blank lines; the first arrow was labeled *pattern* (slope), and the second arrowed was labeled *starting value* (y-intercept), semi-formal language that reflected mathematical tasks from the teaching experiment. Her verbal explanation included a procedural description of the equation, "The 7 came from this because you add 7 every—it increases by 7. And 27 is the starting value" (*pointing to table of values, in particular, the y-values*). She verified the equation using three coordinate pairs from the table, (1, 34), (2, 41), and (3, 48) as, "34 = 7(1) + 27; 41 = 7(2) + 27; and 48 = 7(3) + 27." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> S: I got y = 7x + 27.
> J: I got the same thing, too.
> S: y = 7x + 27.
> SC: Where did the 7 come from and where did the 27 come from?
> S: The 7 came from this because you add 7 every—it increases by 7. And 27 is the starting value. (*Sandy pointed to the y-values from the table*)

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because she correctly answered the question and connected the equation to the coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as recognizing relationships and pointing out similarities, as well as evaluating reasonableness (why and how). she verified the equation using more than one coordinate pairs from the table.

Her partner, Joe (LEP) also had a *correct* solution. His written work showed three equations, two incorrect equations, "y = x + 7; y = 27x + 0," and one correct equation, "y = 7x + 27." His verbal explanation included a short partner discussion in which he agreed with Sandy on the equation, y = 7x + 27. The solution was coded as *correct (2).* The explanation was coded as *procedural (1).* The response was coded as *some understanding (Score: 3)* because he answered the question correctly, but he did connect the patterns from the table to the equation and did not verify the equation. Hiebert and Lefevre (1986) described this explanation as procedural step-by-step (guessing using a rule/template). There is no assessment of reasonableness or why/how. In sum, Sandy and Joe wrote the correct equation to represent the data in the table to explain their equation, where the pattern (slope) was seven and starting value (y-intercept) was 27. Sandy verified the equation using three coordinate pairs from the table. Joe did not verify his equation using coordinate pairs from the table.

Brooke (Q1b). When asked to write an equation using the pattern (slope) and starting value (y-intercept) from the table, Brooke (MEP) had a *correct* solution. She wrote the equation, "y = 7x + 27," where slope was seven and y-intercept was 27. In her verbal explanation, she plugged in more than one coordinate pairs from the table to verify the equation, (0, 27) and (1, 34). The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> B: I thought the equation was 7x + 27 because at the beginning, if you multiply anything by 0 it's just 0 and since the starting value is 27 I thought that is what you had to add on each time.
> SC: I like how you explained your equation, you said if I were to put in a 0 in… go ahead and say it again. You put points in to show that your equation is correct. (*This was a checking technique they learned from intervention lesson, this was not done prior to being shown in class*)
> B: If I were to put a zero where x is, anything multiplied by 0 is just 0, you have to add on 27 to get to 27.
> SC: Thank you so much, Brooke, for that. Does it work for other points or just 0?
> B: For every point.
> SC: Can you pick just one more point just to make sure?
> B: 7 times 1 would equal 7, and then plus 27 would equal 34
> SC: So it does work.

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly and she connected the coordinates pairs from the table with the equation and verified the rule works. Hiebert and Lefevre (1986) described this as assessing reasonableness, and the why and how as well as connecting relationships between table and equation using the pattern. In sum, Brooke wrote the correct equation to represent the data in the table and used informal language to explain their equation, where the pattern (slope) was seven and starting value (y-intercept) was 27. She verified the equation using more than one coordinate pairs from the table.

Summary of Picture (Table) to Verbal Model Translation. When asked to write an equation using the pattern (slope) and starting value (y-intercept) and verify the equation, $y = 7x + 27$, using coordinate pairs from the table, six out of seven students, Leon, Lucas, Olvin, Sandy, Joe and Brooke, correctly wrote the equation (*picture to verbal/written models*). One student, Anna, at first wrote an incorrect equation following a proportional linear relationship pattern. With partner discussion, she corrected her equation to match the non-proportional linear pattern data in the table. Three out of seven students, Anna, Sandy, and Olvin, used the template that summarized how to write an equation for non-proportional linear functions from the teaching experiment, "y = __x + __," where the first blank represented the *pattern* (slope) and the second black represented the *starting value* (y-intercept). This strategy aligned more with procedural strategies applying a rule or steps with little explanations. In their verbal explanations, all students used semi-formal language, *pattern* and *starting value,* for slope and y-intercept, respectively. Students used semi-formal language that reflected strategies from the teaching experiment to write and explain that the equation was the pattern (slope), \$7, multiplied by x, and added to starting value (y-intercept), \$27. Students connected the patterns from the table with the real-life situation, this is characteristic of conceptual explanations. In their written work, four out of seven students, Lucas, Olvin, Sandy and Brooke used two or more coordinate pairs from the table to verify their equation. Hiebert and Lefevre (1986) described this explanation as characteristic of conceptual understanding, showing students connecting table to equation and assessing reasonableness by checking the why and how. The following section presents data from post-assessment Question 1c about solving the equation, y = 7x + 27, for unknown y-value given x = 50 hours, and interpreting y-value using a real-life situation.

Question 1c. Solve Equation and Interpret Solution (*Picture to Real-Life Situation*)

Figure 4.14 shows Question 1c on determining the amount of money in the piggy bank after 50 hours of babysitting, and interpreting y-value given a real-life situation, as well as the embedded translations. Question 1c was designed by the researcher with input from the teacher to reflect mathematical tasks from the constructivist teaching experiment, and presented students with a table with a non-proportional linear relationship and a real-life situation. Students were expected to use the equation from Question 1b, y = 7x + 27, to determine the total amount of money in the piggy bank after adding 50 hours of pay. A justification coded as *conceptual* required students to plug in x = 50 into the equation to solve, and connect the solution with a real-life situation as, $377 after 50 hours of pay has been added to the piggy bank, or linking the equation with the pattern to point out where the pattern shows up in the equation: y = 7(50) + 27 = $377. A procedural explanation includes solving the problem without explanation of the meaning, or plugging 50 into x-value for an incorrect equation like Anna (LEP) before discussing with her partner. Two examples of justifications coded as *conceptual* are: "y = 7 x 50 + 27… = 350, 350 + 27 = $377; 377 represents the money. Jane will have $377 after 50 hours of babysitting;" and, "She would have $377 after babysitting 50 hr;" and the student verified that the equation worked for three other coordinate points from the table including (1, 27), (2, 34) and (3, 41): “y = 7x + 27; y = 7 x 1 + 27 = 34; y = 7 x 2 + 27 = 41; y = 7 x 3 + 27 = 48.”

<table>
<tr><th>Post-Assessment Question</th><th>Embedded Translation</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.
<table>
<tr><th>Hours worked</th><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><th>Money in Piggy Bank ($)</th><td></td><td></td><td>41</td><td>48</td><td>55</td></tr>
</table>
c) How much money will Jane have in the bank after 50 hours of babysitting? How can you check?</td><td>Picture to Real-Life Situation</td></tr>
</table>

Figure 4.14. Post-assessment question 1c on solving the equation for y-value when x-value is 50, and interpreting the meaning of the y-value given a real-life situation.

Table 4.9 presents individual student scores on student-pair solutions and explanations from the post-assessment interview transcripts, followed by a description of codes. All students answered the task correctly and solely used conceptual strategies in their justification.

Table 4.9

Question 1c Scores

Student	**Conceptual Understanding Score**
Anna (LEP)	4 (2, correct 2, conceptual)
Leon (HEP)	4 (2, correct 2, conceptual)
Lucas (MEP)	4 (2, correct 2, conceptual)
Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q1c). When asked to determine the amount of money earned after 50 hours of work, where x-value was 50, and y-value was unknown, Anna (LEP) had a *correct* solution. With support from her partner, she edited her incorrect proportional linear equation from a proportional linear equation, y = 7x + 0, to the correct non-proportional linear equation to, y = 7x + 27. She thought the table followed a proportional linear pattern at first. In her verbal explanation, Anna said, “Jennifer will have 62 after baby sitting. I can check this by using the pattern and the equation." It is not clear where the 62 came from when probed by the researcher. With partner discussion, Anna adjusted her equation to y = 7x + 27, and solved the problem correctly. She plugged in 50 for x-value to solve for the unknown y-value: "y = 7 x 50 + 27." Anna used the lattice method to multiply seven by 50. Her correct answer showed two steps, multiplication and addition, "7 x 50 = 350, 350 + 27 = $377." Additionally, Anna interpreted the solution of 377 using a real-life situation, "377 represents the money. Jane will have $377 after 50 hours of babysitting." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> SC: So this is the part that helps answer my question earlier that how do we know the 7 goes here and not here? Anna, where do we put 50? If it is 50 hours, does it go here or here? (*The researcher pointed to the equation, emphasizing the location of the slope and y-intercept when speaking the terms, pattern and y-intercept*)
> A: It goes here. (*Pointed correctly to the y-value)*
> SC: Okay so can you do it for me? Write it here.
> A: Do I write the equation?
> SC: Write whatever the answer will be, so show your work. (*Anna wrote the correction work to evaluate the y-value when x = 50 hours: y = 7 x 50 + 27*)
> SC: Yep. And then Leon, what did you get?
> L: For my work, I got 7 x 50 + 27.
> SC: And what does that answer equal?
> L: 377.
> SC: Ok and Anna?
> A: 377.
> SC: Ok so what does 377 represent? Look at the question, how much…
> A: Oh it represents the money.
> SC: Yep. Don't forget the dollar sign. $377. In a sentence, can you answer the question for me? How much money will Jane have in the bank after 50 hours of babysitting? Using a sentence can you answer the question?
> A: Jane will have $377 after 50 hours of babysitting.
> SC: Beautiful. Thank you, Anna.

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because she correctly answered the question connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain.

Her partner, Leon (HEP) had a *correct* solution. In his written work, he plugged 50 in for x-value in the equation and solved for unknown y-value, "You can use the equation. 7 x 50 + 27 = 377." With partner discussion, Leon edited his solution of 377 to include a real-life situation to $377. In his verbal explanation, his calculations matched that of his partner, "For my work I got 7 x 50 + 27 is $377." The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he correctly answered the question and connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain. In sum, Leon and Anna correctly plugged in 50 for x-value in the equation and solved for unknown y-value of 377. Both students used a real-life situation to answer the question that 377 is the amount of money in dollars earned after 50 hours of work. Multiple representations were used.

Olvin and Lucas (Q1c). When asked to determine the amount of money earned after 50 hours of work, where x-value was 50, and y-value was unknown, Olvin (LEP) had a *correct* solution. At first he set up a proportion to attempt to solve for y-value, the amount of money after 50 hours. With partner discussion, Olvin decided to plug in 50 for the x-value in the equation, instead of setting up a proportion, to find the unknown y-value, "7 x 50 + 27 = 377." In his verbal explanation, Olvin said that multiplication by 50 made sense, "In the problem it said 50 so I thought in my mind we had to multiply by 50." The researcher asked Olvin to clarify the meaning of 50 using the table and Olvin said he was not sure, "I don't know." With help from his partner, Lucas (MEP), who interpreted the problem using a real-life situation, that 50 represented *hours*, or the x-value. Olvin agreed that 50 represented the number of hours, and that 50 was to be plugged in for the x-value in the equation, "Oh, 50 is hours." With questions from the researcher about how to verify that y = 7(5) + 27 was correct, Olvin and his partner chose multiple coordinates from the table to check their equation correctly, including (4, 55), (1, 34), and (2, 41). Olvin said, "You put 34 in y and then 7 x 1 + 27 will give you 34; I got 41 = 7 x 2 + 27." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> L: I got 7 times 50 + 27.
> SC: Olvin, what did you do? Share what you did, even if it's wrong you can change your answer.
> O: I'm not sure if I even did it.
> SC: That's okay, can you share what you wrote because you wrote something.
> O: I wrote 5/62 x 50, but it won't give me the answer (*He set up a proportion, a similar strategy from class used for proportional linear function patterns*)

SC: Okay, can you tell me what you were thinking when you were doing that and then can you tell me what you are going to change about it?
O: In the problem it said 50, so I thought in my mind we had to multiply by 50.
SC: Ok, so if you look here, what does the 50 represent? Which one of those does it represent, the first row or the second row? (*Researcher pointed to table*)
O: The first row.
SC: Okay, so it looks to me like you did a proportional relationship, right? Like one fraction equals another, right? Is this a proportional relationship? We are kind of jumping to part d. Can we do it this way?
O: I don't know.
SC: It's okay not to know. Talk to Lucas, I think Lucas might be able to help. Lucas, what did you do? Tell Olvin what you did to convince him that that is the right answer. So you just read it, I don't think Olvin was listening.
L: I used the pattern. Yes, the pattern, then I multiplied it with 50 hours then I added the pattern again.
SC: The pattern again? There are two patterns? Where is the pattern? Is this the pattern or is this the pattern? (*Researcher pointed to slope of 7 and y-intercept of 27 to clarify*)
L: Oh.
SC: These are your two vocabulary words, pattern and starting value.
L: Oh wait, no no no, I forgot.
S: Okay, so tell Olvin what you did.
L: I multiplied 7 times 50 plus 27.
SC: Ok so maybe you should write in 27 because you wrote 7. Does that make sense?
L: Yeah.
SC: So now try to convince Olvin what you are doing because Olvin isn't sure. Alright explain it again.
L: Okay, so you use your pattern and then where the x is, between the plus, you put 50, and then you add 27 and I got 377.
SC: And how do we know that that 50 goes in here and not here? You're right, but how do you know that? Olvin you helped earlier, you said 50 was what? (*Researcher pointed to x- and y- in the equation, y = mx + b, to ask for clarification*)
L: Hours.
O: Oh. 50 is hours.
SC: Hours. So that's why it goes where?
L: In the x.
SC: In the x, right. This is x and this is y. Very good explanation. Now you have to make sure Olvin agrees with you. Lucas, you did a good job explaining it. Olvin, do you believe him? (*Researcher pointed to row one and row two, to emphasize x and y*)
O: Yes

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because answered the question correctly and connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain.

His partner, Lucas, had a *correct* solution. He plugged 50 in for x-value in the equation, and explained that he multiplied 50 with the slope of 7 and added y-intercept of 27 to get the solution of 377, "7 x 50 + 27 = 377." His verbal explanation matched his written work, "I got 7 x 50 + 27." His verbal explanation included an interpretation of the solution using a combination of semi-formal language and a real-life situation, "I used the pattern, then I multiplied it with 50 *hours* then I added the pattern again… I multiplied 7 times 50 plus 27." With questioning from the researcher about how to verify the equation, Lucas and his partner, Olvin, offered multiple coordinates from the table, (4, 55), (1, 34) and (2, 41). Lucas plugged in coordinate pair, (1, 34) into the equation y = 7x + 27, "34 is not x, it is y. 34 equals 7 times 1 plus twenty-*siete* (27 in English-*Spanish*)." The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours, and included other coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain. In sum, Olvin and Lucas plugged in 50 for x-value in the equation and solved for unknown y-value. Olvin answered the question *correctly.* Both students verified the equation using more than one coordinate pair from the table, and used a real-life situation to answer the question that 377 was the amount of money in dollars earned after 50 hours of work, and included other coordinate pairs from the table. Multiple representations were used.

Sandy and Joe (Q1c). When asked to determine the amount of money earned after 50 hours of work, where x-value was 50, and y-value was unknown, Sandy (MEP) had a *correct* solution. She plugged in 50 for x-value into the equation, y = 7x + 27, and solved for unknown y-value as 377, "y = 7 x 50 + 27 = 377." Additionally, Sandy interpreted her solution using a real-life situation and wrote, "She would have $377 after babysitting 50 hr." In her verbal explanation, Sandy verified that the equation worked for three other coordinate points from the table including (1, 27), (2, 34) and (3, 41): "y = 7x + 27; y = 7 x 1 + 27 = 34; y = 7 x 2 + 27 = 41; y = 7 x 3 + 27 = 48." The following excerpt from the semi-structured post-assessment interview transcript highlighted the role of the researcher to push students to elaborate on their explanations:

> SC: Compare with each other and then tell me the answer
> S: I got 377
> J: I didn't write nothing. I just wrote the equation.
> SC: You have a pen so you can discuss and change your answer.
> J: Yeah she is correct. It equals 377.
> SC: How can you check?
> S: 377.
> J: Well what I did was multiply 7 times 50 and then plus 27.
> SC: And how do you know that it works? It works for 50.
> S: Because of the formula.
> SC: Because of the formula, very good. Is there another way to check to see if the formula is correct, besides putting 50 in for x?

S: You can add other numbers that are already on the chart. (*She referred to the table*)
SC: Exactly. So could you pick another pair for me just to make sure your equation is right? (*This verification strategy was taught in the teaching experiment to support connections, and conceptual understanding*)
S: 1 times 7 plus 27 equals 34 (*She used (1, 34) to verify*)
SC: Excellent. Thank you so much, Sandy. Joe, can you pick another point and test it out? She picked (1, 34), can you pick another point?
J: 7 times 41 (*41 is a y-value, not an x-value*)
SC: Sandy can you help him out? He picked… (*Sandy pointed to x-values from the table, to get Joe started*)
SC: So you are right, Joe, to pick (2, 41). Where does 2 go and where does 41 go in this equation you have?
J: 2 goes first and then 41.
SC: 2 goes where, x or y?
J: x.
SC: So put 2 in for the x, so 7 times what?
J: 2, which is 14.
SC: Plus 27.
J: Which equals 41.
SC: Cool. So you know your equation works.

The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because she correctly answered the question and connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours, and included other coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain.

Her partner, Joe (LEP) had a *correct* solution. He wrote the equation, "y = 7 x 50 + 27," and with the help of his partner, Sandy, he added the solution of 377, "7 x 50 + 27 = 377." His verbal explanation showed he was unsure on his individual work. His partner, Sandy, verified the equation, and he contributed by verifying that the equation works for another coordinate pair from the table, (2, 41), “y = 7 x 2 + 27 = 41.” Joe agreed with his partner on the interpretation using a real-life situation. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly and connected the solution using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours, and included other coordinate pairs from the table. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain. In sum, Sandy and Joe plugged in 50 for the x-value to solve for y-value of 377. Both students verified the equation using more than one coordinate pair from the table, and used a real-life situation to answer the question that 377 is the amount of money in dollars earned after 50 hours of work, and included other coordinate pairs from the table.

Brooke (Q1c). When asked to determine the amount of money earned after 50 hours of work, where x-value was 50, and y-value was unknown, Brooke (MEP) had a *correct* solution. She plugged in 50 for the x-value in the equation and solved for unknown y-value as, “7(50) + 27 = 377.” Her verbal explanation included a real-life situation interpretation that 377 was the amount of money earned after 50 hours of work, “I thought she was going to have $377 because I used my equation to find out what she would get after 50 hours. I did 7 x 50 + 27 = 377.” When asked to verify that 50 was the x-value, Brooke included a real-life situation to interpret, “Because it is part of the x, it says hours,” (*she pointed to x-values in the table showing hours*). The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a real-life situation to explain the meaning of 377 as dollars earned for working 50 hours. Hiebert and Lefevre (1986) described this explanation as connecting relationships between table of values, patterns, and equation with real-life situation. Multiple representations were used to explain. In sum, Brook correctly plugged in 50 for x-value in the equation and solved for unknown y-value, and she used a real-life situation to answer the question that 377 is the amount of money in dollars earned after 50 hours of work. She did not verify using more than one coordinate points from the table.

Summary of Picture (Table) to Real-Life Situation. When asked to determine the amount of money earned after 50 hours of work, where x-value was 50, and y-value was unknown, all students answered correctly and solely used conceptual strategies in their justification (*picture to real-life situation*). At first, Anna had an incorrect equation when working individually because she thought the table followed a proportional linear pattern from her work in Questions 1a. With partner discussion, she corrected her answer and explained using a real-life situation to show y = 7 (50 hours) + \$27 = \$377, or \$377 dollars in the piggy bank after 50 hours of babysitting. Three students, Anna, Olvin, and Joe, arrived at the correct solution with partner support. Two students, Sandy and Anna, used the real-life context to answer the question in writing. Five students, Lucas, Olvin, Sandy, Joe, and Brook students, verified their equation using more than one coordinate pairs. Lucas and Olvin verified with researcher questioning. All students interpreted the solution using a real-life situation that after 50 hours, the amount of money earned will be

$377 verbally. The following section presents data from the post-assessment Question 1d about identifying characteristics of proportional patterns using two representations or more: the table, pictures, words, or a graph.

Question 1d. Describe Characteristics of Proportional Relationships Using Two Representations (*Picture to Verbal Model*)

Figure 4.15 shows Question 1d on identifying characteristics of proportional patterns using two representations or more: the table, pictures, words, or a graph as well as the embedded translations. Question 1d was designed by the researcher with input from the teacher to reflect mathematical tasks from the constructivist teaching experiment, and presented students with a table with a non-proportional linear relationship and a real-life situation. Students were expected to explain that the pattern does *not* follow a proportional linear relationship and use two representations to explain their reasoning. An explanation coded as *conceptual* required students to use two representations to explain their reasoning: a table, graph, and/or words in writing or verbally. An example of a justification coded as *procedural* is answering the question correctly and providing an incomplete explanation or calculation without explaining, for example: it is not proportional because y/x is not a constant pattern. Two examples of justifications coded as *conceptual* are, "No, it is not because the y when x is 0 is not 0, and you add to the pattern, 27" (*he points to the table and equation*); and, "No because proportional means it starts at the origin… I said not because proportional usually means that it starts at the origin. Then also if you put it on the table, 0 would have to be 0 and then you wouldn't have to be adding anything each time you multiply. If it is proportional you usually don't have to add on anything you just have to multiply" (*she refers to the table and equation*).

<table>
<tr><th>Post-Assessment Question</th><th>Embedded Translation</th></tr>
<tr><td>1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

<table>
<tr><td>Hours worked</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Money in Piggy Bank ($)</td><td></td><td></td><td>41</td><td>48</td><td>55</td></tr>
</table>
d) Is the pattern proportional? Give <u>two</u> reasons. (Use the table, pictures, words or a graph)</td><td>Picture (Table) to Verbal Model</td></tr>
</table>

Figure 4.15. Post-assessment question 1d on describing a non-proportional relationship using two or more representations, including a table, pictures, words, or a graph.

Table 4.10 presents individual student scores on student-pair solutions and explanations from the post-assessment interview transcripts, followed by a description of codes. One student did not answer the task correctly and six students answered the task correctly, but all students used conceptual strategies in their justification.

Table 4.10

Question 1d Scores

Student	Conceptual Understanding Score
Anna (LEP)	2 (0, incorrect 2, conceptual)
Leon (HEP)	4 (2, correct 2, conceptual)
Lucas (MEP)	4 (2, correct 2, conceptual)

Olvin (LEP)	4 (2, correct 2, conceptual)
Sandy (MEP)	4 (2, correct 2, conceptual)
Joe (LEP)	4 (2, correct 2, conceptual)
Brooke (MEP)	4 (2, correct 2, conceptual)

Anna and Leon (Q1d). When asked to examine the linear pattern from the table with a real-life context and decide if the pattern was proportional, and give two reasons using the table, words or a graph, Anna (LEP) had an *incorrect* solution. Her written work showed an explanation that matched her original incorrect proportional linear equation from Question 1a, where the table values are (0, 0) and (1, 7) instead of (0, 27) and (1, 34); and Question 1b, y = 7x + 0, "This is the pattern proportional because it does start at the origin and matches the pattern." Anna used more than one representation to explain her reasoning (*She points to the equation and the table*). Although Anna answered the question incorrectly, her explanation showed translation between multiple representations. The solution was coded as *incorrect (0).* The explanation was coded as *conceptual (2).* The response was coded as *rote understanding (Score: 2)* because she answered the question incorrectly that the table followed a proportional pattern, but the equation that she wrote initially was y = 7x + 0, so using the incorrect equation, she displayed conceptual explanations to match using a table and an equation. Hiebert and

Lefevre (1986) described this explanation as showing some reflecting to assess reasonableness and connecting to the context.

Her partner, Leon (HEP), had a *correct* solution. He said that the pattern was not proportional since it did not start at at the origin. He used the table and equation to explain his reasoning, “No, it is not because the y when x is 0 is not 0, and you add to the pattern, 27” (*he points to the table and the + 27 in the equation,* $y = 7x + 27$). Leon used more than one representation to support his reasoning. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly that the table followed a proportional pattern, and using a table and an equation to explain his reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a table and equation. In sum, Anna and Leon used more than one representation to explain their reasoning. Anna did not answer the question correctly. Her partner Leon answered the question correctly.

Olvin and Lucas (Q1d). When asked to examine the linear pattern from the table with a real-life context and decide if the pattern was proportional, and give two reasons using the table, words or a graph, Olvin (LEP) had a *correct* solution. His written work described the pattern as not proportional since the pattern did not include the origin from the table. He also drew a graph with an arrow pointing to the y-intercept at (0, 27), and not at the origin, (0, 0). His verbal explanation matched his written work, “No because it doesn’t start in the origin. Also if you put it on a graph it would be a little bit above the 0.” Olvin used more than one representation to explain his reasoning. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly that the table followed a proportional pattern using a table and a graph to explain his reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a table and graph.

His partner, Lucas (MEP) had a *correct* solution. He said that the pattern was not proportional since the data in the table did not include the origin, (0, 0). In his verbal explanation, he included the table and an equation from question 1b. He pointed that the equation started at (0, 27), and not at the origin (0, 0), "This, yeah, the plus 27," (*he points to the equation*). Lucas used two representations to support his answer. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he correctly answered the question that the table followed a proportional pattern using a table and an equation to explain his reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a table and equation. In sum, Olvin and Lucas used more than one representation to explain their reasoning. Both students answered the question correctly.

Sandy and Joe (Q1d). When asked to examine the linear pattern from the table with a real-life context and decide if the pattern was proportional, and give two reasons using the table, words or a graph, Sandy (MEP) had a *correct* solution. She said the pattern was not proportional since the pattern did not go through the origin on a graph. She also used an equation to explain her reasoning. She said the equation, $y = 7x + 27$, required adding a nonzero value of 27, and the data in the table did not follow a proportional linear pattern. Her verbal explanation matched her written work, “I said not because if it was on a graph, it didn’t go through the origin. You have to add another value to get the next value,” (*she pointed to the nonzero y-intercept value*). Sandy used more than one representation to explain her reasoning. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a table, a graph, and an equation to explain her reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a table and equation.

Her partner, Joe (LEP), had a *correct* solution. He wrote, “No it is not proportional cause it doesn’t start from the origin, (0, 0), it starts at (0, 27). Also cause if you do a table it starts with a 27,” (*points to the graph from his partner work, Sandy*). His verbal explanation matched his written work, “And because it starts at 0 point 27, yeah, (0, 27). It doesn’t start at (0, 0), it starts at (0, 27).” He also agreed with his partner with a third representation, equation. Joe used more than one representation to explain her reasoning. The solution was coded as *correct (2)*. The explanation was coded as *conceptual (2)*. The response was coded as *conceptual understanding (Score: 4)* because he answered the question correctly using a table and an equation to explain his reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a table and graph. In sum, Sandy and Joe used more than one representation to explain their reasoning. Both students answered the question correctly.

Brooke (Q1d). When asked to examine the linear pattern from the table with a real-life context and decide if the pattern was proportional, and give two reasons using the table, words or a graph, Brooke (MEP) had a correct solution. She wrote, "No because proportional means it starts at the origin." Her verbal explanation elaborated the table and equation, "I said not because proportional usually means that it starts at the origin. Then also if you put it on the table, 0 would have to be 0 and then you wouldn't have to be adding anything each time you multiply. If it is proportional you usually don't have to add on anything you just have to multiply" (*referring to the equation*). The solution was coded as *correct (2).* The explanation was coded as *conceptual (2).* The response was coded as *conceptual understanding (Score: 4)* because she answered the question correctly using a table, and an equation to explain her reasoning. Hiebert and Lefevre (1986) described this explanation as pointing out similarities between pieces of information, in this case a graph, table, and equation. In sum, Brooke answered the question correctly using more than one representation.

Summary of Picture (Table) to Verbal Model Translation. When asked to examine the linear pattern from the table with a real-life context and decide if the pattern was proportional, and give two reasons using the table, words or a graph, six out of seven students, Leon, Olvin, Lucas, Sandy, Joe and Brooke, correctly described the relationship as not proportional. One student, Anna, incorrectly thought the pattern was proportional. A closer examination showed that all students, including Anna, who got an incorrect answer, used more than one representation to explain their reasoning, including graphing, table and equation. Anna and Leon used a table and equation to explain their reasoning. Olvin and Lucas used a table and a graph to explain their reasoning. Sandy and Joe used a graph and an equation to explain their reasoning. Brooke used all three representations including a graph, table and equation to explain her reasoning. Six out of seven solutions were coded as *correct* with *conceptual understanding*. One solution was coded as *incorrect* with *conceptual understanding*. The following sections present a summary of understandings of middle school immigrant students after participating in the eight-week constructivist teaching experiment.

Summary of Understandings Post Intervention

This section will present a summary of seven middle school immigrant student responses to four post-assessment questions numerically and descriptively as they relate to research question two:

2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions by expressing translations between and within modes of representations <u>after</u> participating in mathematical activities that emphasize LTM?

Recall that the total possible points for a correct solution was two: *correct (2), partially correct (1),* or *incorrect (0);* and the total possible points for justification was two: *conceptual (2), procedural (1), or none/incorrect (0).* A maximum of 4 points was awarded for each correct solution with conceptual explanation. Table 4.11 displays a summary of students' numerical coding individually by question and translation type on the pre-assessment using three numeric scores: solution, justification, and a final score in bold. Following that, a descriptive summary of findings from student performance on the pre-assessment is presented and then how the data answers research question one is presented.

Table 4.11

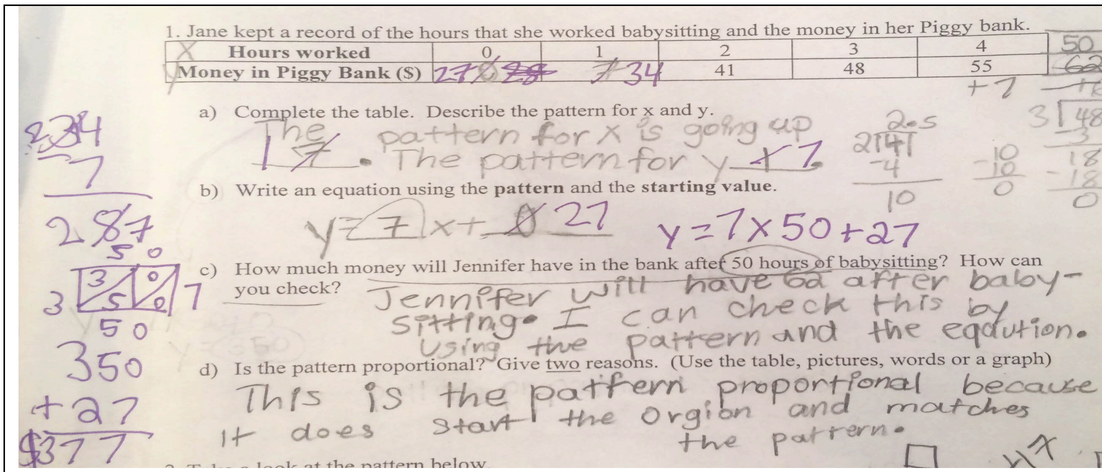
Detailed Summary of Student Scores from Post-Assessment by Question and Translations

	Question 1a: ***(Picture to Verbal Model)***	**Question 1b:** ***(Picture to Written Model)***	**Question 1c** ***(Picture to Real-Life Situation)***	**Question 1d:** ***(Picture to Verbal Model)***	**Overall Average**
Anna (LEP)	3 (1, partially correct; 2 conceptual)	3 (2, correct, 1, procedural)	4 (2, correct; 2, conceptual)	2 (0, incorrect; 2 conceptual)	3.00
Leon (HEP)	4 (2, correct; 2, conceptual)	3 (2, correct; 1, procedural)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	3.75
Lucas (MEP)	2 (1, partially correct; 1, procedural)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	3.50
Olvin (LEP)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4.0
Sandy (MEP)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4.00
Joe (LEP)	4 (2, correct; 2, conceptual)	3 (2, correct; 1, procedural)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	3.75
Brook (MEP)	3 (2, correct; 1, procedural)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	4 (2, correct; 2, conceptual)	3.75
Average by Question	3.40	3.57	4.0	3.71	3.67/4
Solution Summary	5, Correct 2, Partially Correct 0, Incorrect	7, Correct 0, Partially Correct 0, Incorrect	7, Correct 0, Partially Correct 0, Incorrect	6, Correct 0, Partially Correct 1, Incorrect	25/28 2/28 1/28
Justification Summary	5, Conceptual 2, Procedural 0, Incorrect	4, Conceptual 3, Procedural 0, Incorrect	7, Conceptual 0, Procedural 0, Incorrect	7, Conceptual 0, Procedural 0, Incorrect	23/28 5/28 0/28

The last three *rows* provided three different summaries of student performance: (1) students' average scores by question out of four possible questions and embedded translations out of 4 possible points (in ascending order: 3.40, 3.57, 3.71, and 4.0 on four post-assessment questions); (2) solution summary: total correct, partially correct, and incorrect questions out of 7 students; (3) justification summary: total conceptual, procedural, and incorrect justifications out of 7 students. The last *column* provided four summaries: (1) overall average scores by individual students out of 4 possible points (in ascending order: 3.0, 3.5, 3.75, 3.75, 3.75, 4.0, and 4.0 out of 4 possible points); (2) overall average scores on five post-assessment questions as a group out of 4 possible points (3.67 out of 4 possible points); (3) overall solution summary of total correct (25/28), partially correct (2/28), and incorrect solutions (1/28) out of 28 possible correct solutions; and (4) overall justification summary of total conceptual (23/28), procedural (5/28), and incorrect (0/28) justifications out of 28 possible justifications. The following sections provide a closer examination of student responses on the post-assessment by interview pairs.

Anna and Leon. Anna (LEP) scored the lowest overall average score of 3 out of 4 points on the post-assessment, and coded as having *some understanding* of key ideas of linear functions about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. Anna responded to two out of four questions correctly about writing an equation, and solving linear equations and interpreting solution using a real-life situation from a table of values given a real-life situation. She answered one question partially correct about completing a table and describing slope as two-changing quantities from a table of values given a real-life situation; and one question incorrect about discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Anna conceptually explained three questions about completing a table and describing slope as two-changing quantities, writing an equation, and solving linear equations and interpreting solution using a real-life situation from a table of values given a real-life situation. She incorrectly explained one question about discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation.

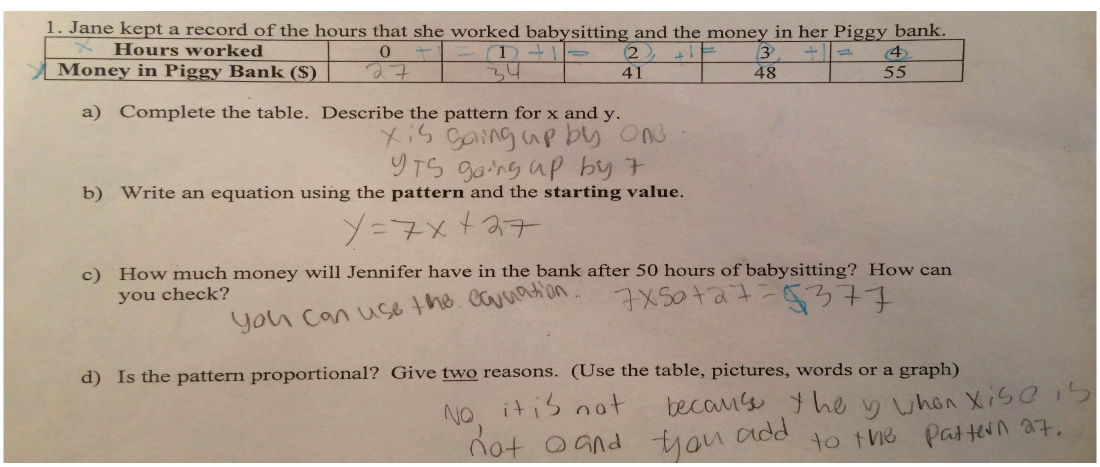
Her partner, Leon (HEP) scored the second highest overall average score of 3.75 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. He answered all four questions correctly about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Leon answered three questions conceptually about completing a table and describing slope as two-changing quantities, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. He answered one question procedurally about writing an equation from a table of values given a real-life situation. In sum, Anna had the lowest overall average score of 3.0 out of 4 points on the post-assessment. She improved 0 point from the pre- to post-assessment. Her partner, Leon, had the second highest overall average score of 3.75 out of 4 points on the post-assessment. He decreased slightly by 0.05 point from the pre- to post-assessment. Figure 4.16 shows students' written work on Question 1a to Question 1d from the post-assessment for Anna and Leon.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	~~27~~ ~~28~~	~~7~~ 34	41	48	55

a) Complete the table. Describe the pattern for x and y.
The pattern for x is going up 1. The pattern for y +7
b) Write an equation using the pattern and the starting value.
y = 7x + 27   y=7x50+27
c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?
Jennifer will have 62 after baby-sitting. I can check this by using the pattern and the equation.
d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)
This is the pattern proportional because it does start the orgion and matches the pattern.

Anna shows her individual work in pencil and partner support in pen.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	27	34	41	48	55

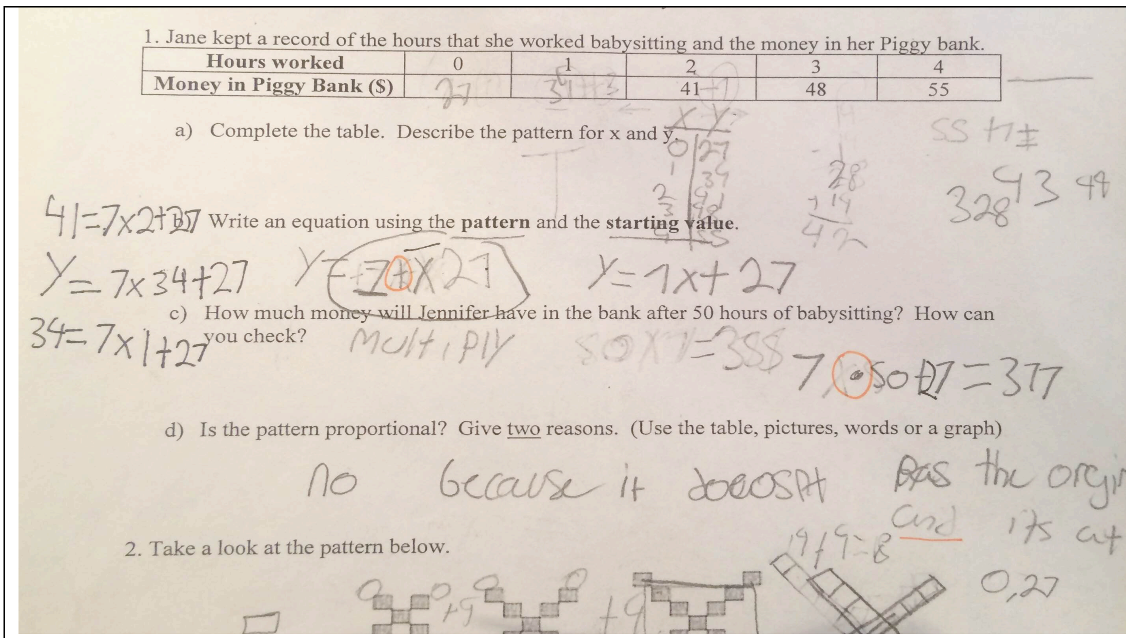
a) Complete the table. Describe the pattern for x and y.
x is going up by one
y is going up by 7
b) Write an equation using the pattern and the starting value.
y=7x+27
c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?
you can use the equation. 7x50+27=$377
d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)
No, it is not because the y when x is 0 is not 0 and you add to the pattern 27.

Leon shows his individual work in pencil and partner support in pen.

Figure 4.16. Student individual (pencil) and partner work (pen) on post-assessment including translations from a picture (table) with a real-life context to verbal and written models, and interpreting solutions using a real-life situation for Anna and Leon.

Olvin and Lucas. Olvin (LEP) was one of two students that scored the highest overall average score with a perfect 4 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. Olvin responded to all questions correct about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Olvin conceptually explained all questions about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation.

His partner, Lucas (MEP) scored the second lowest overall average score of 3.5 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. Lucas responded to two questions correctly about writing an equation and solving linear equations, and interpreting solution using a real-life situation from a table of values given a real-life situation. He answered two questions partially correct about completing a table and describing slope as two-changing quantities, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Lucas conceptually explained three questions about writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. In sum, Lucas had the second lowest average overall score of 3.5 out of 4 points. He decreased slightly by 0.10 point from the pre- to post-assessment. His partner, Olvin, had the one of the highest average overall scores of 4 out of 4 points. He improved 0.20 point from the pre- to post-assessment. Figure 4.17 shows students' written work on Question 1a to Question 1d from the post-assessment for Lucas and Olvin.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	27	34	41	48	55

a) Complete the table. Describe the pattern for x and y.

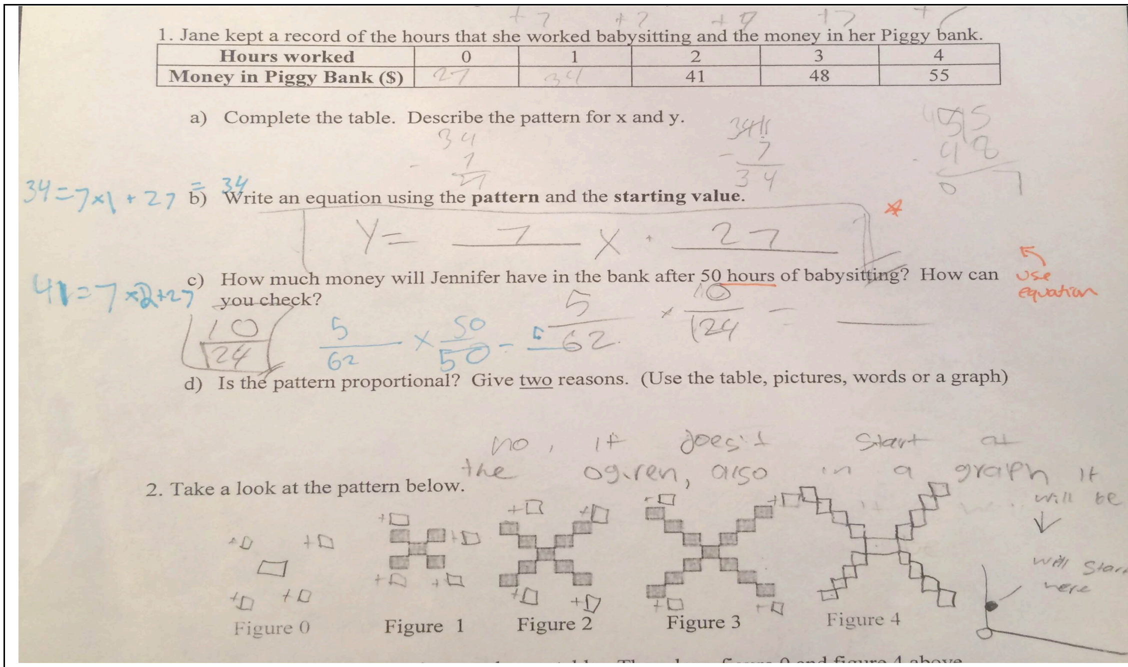
b) Write an equation using the **pattern** and the **starting value**.

c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?

d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)

2. Take a look at the pattern below.

Lucas shows his individual work in pencil and partner support in pen.

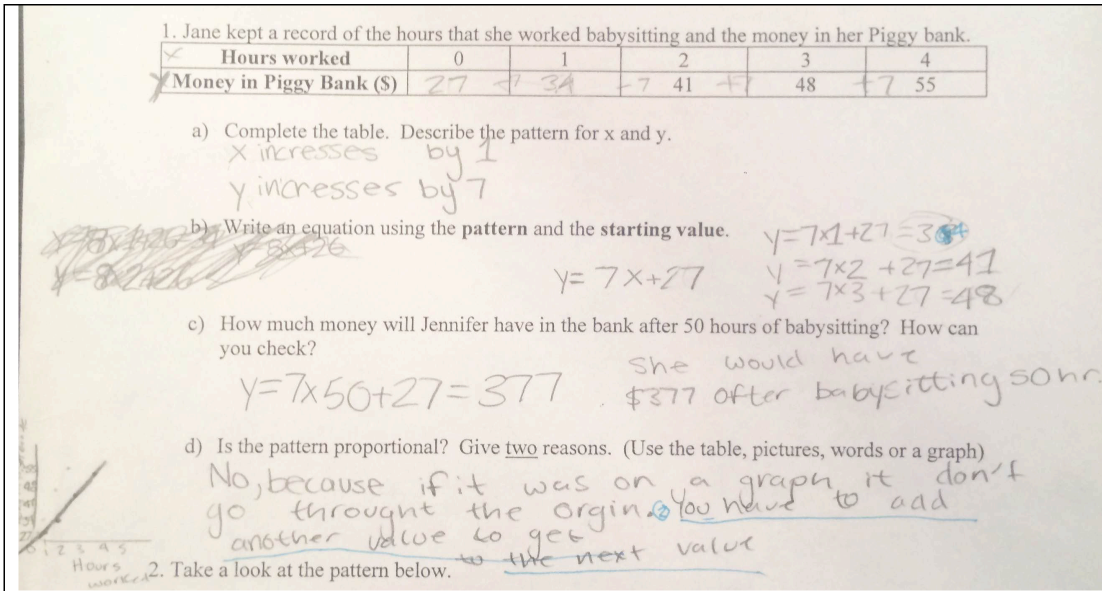


Olvin shows his individual work in pencil and partner support in pen.

Figure 4.17. Student individual (pencil) and partner work (pen) on post-assessment including translations from a picture (table) with a real-life context to verbal and written models, and interpreting solutions using a real-life situation for Lucas and Olvin.

Sandy and Joe. Sandy (MEP) was one of two students that scored the highest overall average score with a perfect 4 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. Sandy responded to all questions correctly about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Sandy conceptually explained all questions about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. She was the only student with a perfect score of 4 out of 4 possible points on the post-assessment.

Her partner, Joe (LEP) scored the second highest overall average score of 3.75 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. Joe responded to all questions correctly about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Joe conceptually explained three questions conceptually about completing a table and describing slope as two-changing quantities, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. Joe answered one question procedurally about writing equations from a table of values given a real-life situation. In sum, Sandy (MEP) had the highest average overall score of 4 out of 4 points on the post-assessment. She improved 1 point from the pre- to post-assessment, with the highest increase out of all students. Her partner, Joe (LEP), had the second highest average overall score of 3.75 out of 4 points on the post-assessment. He *decreased* 0.05 point from the pre- to post-assessment. Figure 4.18 shows students' written work on Question 1a to Question 1d from the post-assessment for Sandy and Joe.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	27 +7	34 +7	41 +7	48 +7	55

a) Complete the table. Describe the pattern for x and y.
x increases by 1
y increases by 7

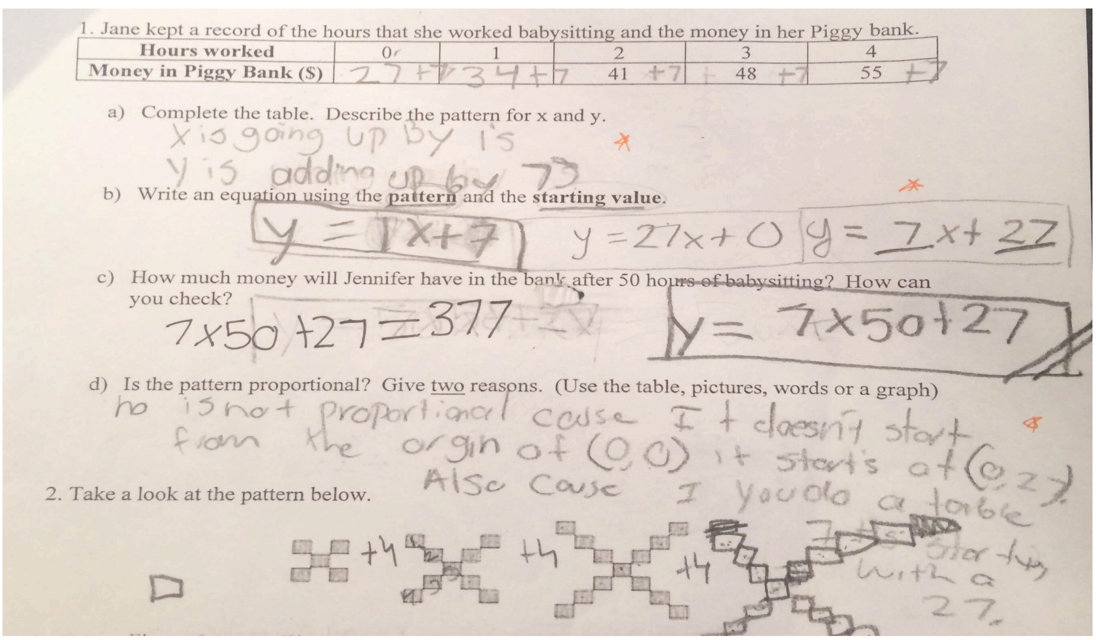
b) Write an equation using the **pattern** and the **starting value**.
y = 7x+27
y=7x1+27=34
y=7x2+27=41
y=7x3+27=48

c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?
y=7x50+27=377
She would have $377 after babysitting 50hr.

d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)
No, because if it was on a graph it don't go throught the orgin. You have to add another value to get to the next value

2. Take a look at the pattern below.

Sandy shows her individual work in pencil and partner support in pen.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	27 +7	34 +7	41 +7	48 +7	55 +7

a) Complete the table. Describe the pattern for x and y.
x is going up by 1's
y is adding up by 7's

b) Write an equation using the **pattern** and the **starting value**.
y = 7x+7 y = 27x + 0 y = 7x+ 27

c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?
7x50 +27 = 377 y = 7x50+27

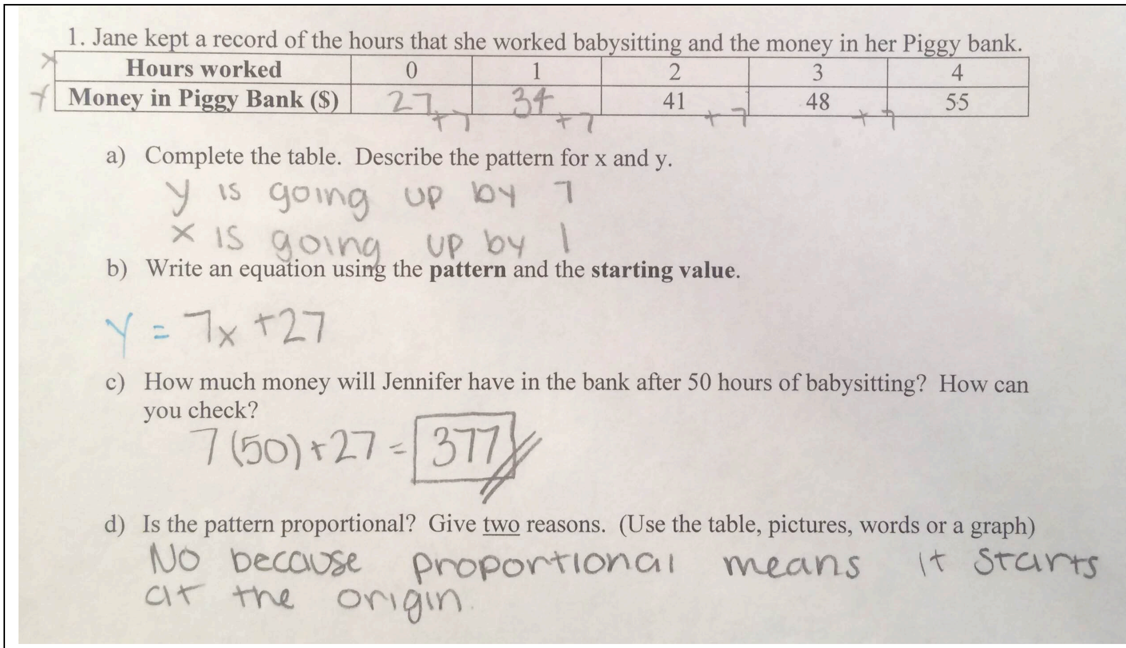
d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)
no is not proportional cause It doesn't start from the orgin of (0,0) it start's at (0,27) Also cause I you do a table start with a 27.

2. Take a look at the pattern below.
+4 +4 +4

Joe shows his individual work in pencil and partner support in pen.

Figure 4.18. Student individual (pencil) and partner work (pen) on post-assessment including translations from a picture (table) with a real-life context to verbal and written models, and interpreting solutions using a real-life situation for Sandy and Joe.

Brooke. Brooke (MEP) scored the second highest overall average score of 3.75 out of 4 points on the post-assessment, and coded as having *conceptual understanding* of key ideas of linear functions. Brooke responded to all questions correctly about completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. A closer examination showed that Brooke conceptually explained three questions about writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. She answered one question procedurally about completing a table and describing slope as two-changing quantities from a table of values given a real-life situation. In sum, Brooke had one of the second highest scores of 3.75 out of 4 points on the post-assessment. She improved 0.15 point from the pre- to post-assessment. Figure 4.19 shows students' written work on Question 1a to Question 1d from the post-assessment for Brooke.


1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)	27	34	41	48	55

+7 +7 +7 +7

a) Complete the table. Describe the pattern for x and y.

y is going up by 7
x is going up by 1

b) Write an equation using the **pattern** and the **starting value**.

$y = 7x + 27$

c) How much money will Jennifer have in the bank after 50 hours of babysitting? How can you check?

$7(50) + 27 = \boxed{377}$

d) Is the pattern proportional? Give <u>two</u> reasons. (Use the table, pictures, words or a graph)

No becouse proportional means it starts at the origin.

Brooke shows her individual work in pencil and her changes during the interview in pen.

Figure 4.19. Student individual (pencil) and partner work (pen) on post-assessment including translations from a picture (table) with a real-life context to verbal and written models, and interpreting solutions using a real-life situation for Brooke.

Summary of Understanding Post Intervention by Themes

In the previous sections, data from the post-assessment interview transcripts and students' written work in pairs were presented that documented the nature of seven middle school immigrant students' conceptual understanding of linear function ideas as they related to completing a table and describing slope as two-changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. The following sections first present a descriptive of characteristics of the post-assessment questions that supported conceptual understanding of linear function ideas that focus on two mathematics processes, communication and representations, as they relate to research question two (NCTM, 2000). Figure 4.21 displays a summary of findings from the post-assessment data that showed the approaches that supported middle school immigrant students' understanding as evidenced by justification coding (i.e. procedural or conceptual). The data suggest that: (1) students gave conceptual justifications when asked process-oriented questions; (2) students used more than one representation to justify when asked to do so; and (3) students gave conceptual justifications using a real-life situation when given a meaningful real-life situation at the beginning. The following sections summarize findings by individual post-assessment questions as well as the embedded translations that support themes displayed in Figure 4.20.

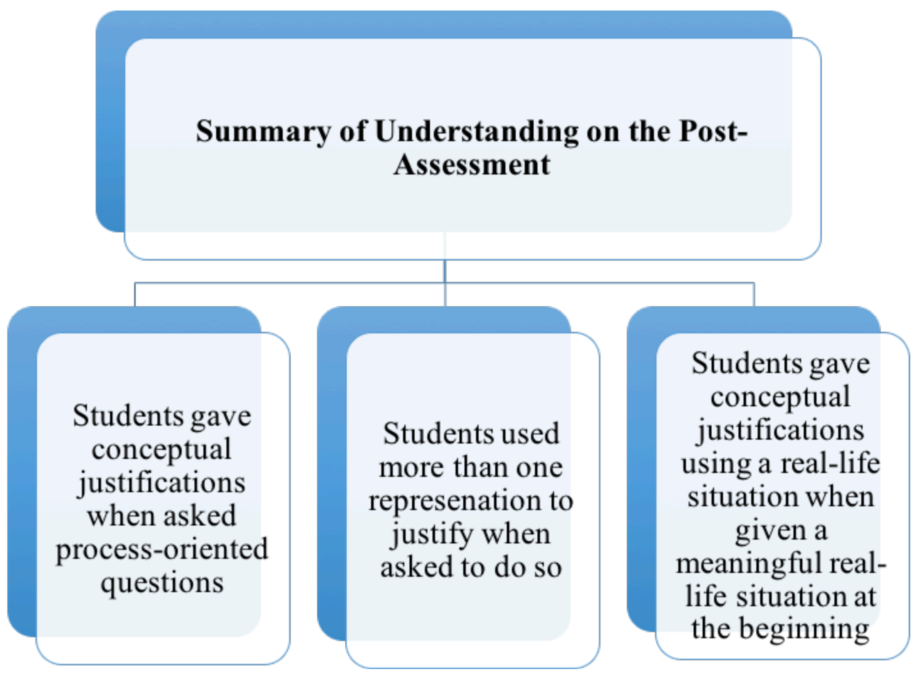


Figure 4.20. Summary of understanding on the post-assessment.

Find Two Missing Y-Values and Describe Slope Given a Real-Life Situation (*picture-table to verbal model*)

When students were asked to complete the table with two missing y-values and describe slope as two patterns given a table with a real-life situation, six out of seven students correctly calculated missing y-values using the pattern, or slope. One student misunderstood the table as following a proportional linear pattern and found the correct slope of 7, but did not correctly calculate the y-intercept of 27. However, all students correctly described slope as *two* patterns involving x- and y-values. A closer examination of the question, *complete the table and describe the pattern for x and y,* revealed that when the question explicitly asked students to notice *two* patterns in the table, all students responded accordingly. Whereas, in the pre-assessment, all students neglected the pattern for the x-values in the table. This approach that includes asking students to describe two patterns from a table is aligned with strategies from the constructivist teaching experiment. In addition, giving students a real-life situation early on was associated with an increase in the number of students interpreting their solution using a real-life situation. Therefore, the findings suggest that the way questions are posed encourage student responses coded as *conceptual*, and suggests that the role of the teacher that includes selecting rich problems and writing open-ended questions supports more *conceptual* justifications.

Write and Verify Equation Given a Real-Life Situation (*picture-table to written model*)

When students were asked to write and verify an equation given a table with a real-life situation, six out of seven students wrote a correct solution in y =*m*x + b format, where *m* was the slope and b was the y-intercept. Upon closer examination of the questions, a majority of students verified the equation using two or more coordinate pairs. This finding aligned with the expectations and strategies from the constructivist teaching experiment. The classroom teacher confirmed that verifying equations using coordinate pairs was previously not expected of students. Therefore, findings show that when the roles of the teacher include encouraging students to verify their equation by plugging coordinate pairs from the table, support more *conceptual* justifications. For example, one student used all coordinate pairs from the table to verify that the equation. In sum, the data suggests that when students are asked to go beyond writing an equation and are asked to verify, students are more likely to make the connection between the table and the equation, thus supporting more conceptual justifications.

Interpret Slope Given a Real-Life Situation (*picture-table to real-life situation, & verbal model*)

When students were asked to calculate the y-value when x = 50 given a real-life situation, all students correctly solved the equation and interpreted the meaning of the solution of 377. Students interpreted the y-value of $377 as the total amount in the piggy bank after adding 50 hours of babysitting payment. Students calculated by plugging in 50 hours as the x-value into the equation as, y = $7(50 hours) + $27 = $377. A majority of students answered the question using a real-life situation in writing or orally, or both. The findings suggest that including a real-life situation at the beginning supports students' problem solving and interpretation of the solution, despite solving a challenging non-proportional problem. Recall that students previously mostly worked with proportional linear functions in y = *k*x format prior to the teaching experiment. Therefore, given a real-life situation and explicit questions that asked students to make connections and verify, are associated with increased conceptual justifications.

Explain Proportional Relationships Using Two or More Representations (*picture-table to real-life situation, picture-graph, & verbal/written models*)

When students were asked to describe the pattern in the table as proportional or non-proportional given a real-life situation, and to explain their reasoning using two representations, six out of seven students correctly described the linear pattern as non-proportional. A close examination of student responses, a majority of students used the table or equation to explain that the pattern had a non-zero y-intercept, thus non-proportional. Other representations that students used to justify their solution included a graph; about half of students sketched a graph from the table to justify their solution. The findings suggest that when the teacher asked students to observe, discuss and justify their solution using *two* or more representations, are all associated with increased conceptual justifications.

In summary, middle school immigrant students increased the number of responses coded as *conceptual* on the post-assessment compared to the pre-assessment. A close examination of the questions of the post-assessment questions showed more process-oriented questions that encouraged use of multiple representations. The findings suggest that increased conceptual responses were associated to the following strategies from the constructivist teaching experiment: (a) students gave conceptual justifications when asked process-oriented questions; (b) students used more than one representation to justify when asked to do so; (c) students gave conceptual justifications using a real-life situation when given a meaningful real-life situation at the beginning. The data also revealed that when students were encouraged to *discuss* their solutions *and* justifications with their partners, students were more likely to make changes toward more conceptual explanations. Therefore, the findings suggest that the role of the teacher that included selecting rich problems, anticipating student responses by writing open-ended questions, and giving students opportunities to explain in pairs, were associated with increased conceptual responses.

In the previous sections, data from the post-assessment interview transcripts and students' written work were presented that documented the nature of seven middle school immigrant students' conceptual understanding of liner function ideas as they related to completing a table and describing slope as two changing quantities, writing an equation, solving linear equations and interpreting solution using a real-life situation, and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation. The following sections present the findings for research question three.

Understandings After Participation in Whole Class Discussions Emphasizing *Connections* Among Peer Presentations

The following sections present data from team posters, video transcripts of team presentations, and individual student written reflections from various points in the constructivist teaching experiment. Data collected from the beginning, middle and end of the teaching experiment are presented in this section as the data relate directly to research question three:

3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Sixteen students made up the intervention class with four heterogeneous teams of four students. Two teams, or eight students with various English proficiency levels were selected for the study, and one student dropped out. Heterogeneous teams with mixed English and mathematics proficiency teams were grouped by the classroom teacher with input from the researcher to meet the goals of the study (see Chapter 3). *Team 1* included Leon (HEP), Lucas (MEP), Sandy (MEP), and a fourth student (LEP) that dropped out of the study. *Team 2* included Anna (LEP), Brooke (MEP), Joe (LEP), and Olvin (LEP). The research team consisted of the classroom teacher and the researcher. In this constructivist teaching experiment, mathematical tasks emphasized key ideas of linear functions were implemented each class period in three phases: *launch, explore,* and *summarize/discuss* (Stein, Smith & Hughes, 2009). During the *launch* phase, the research team introduced problems to students as a whole class, and students attempted problems individually. During the *explore* phase, students discussed their individual approaches in heterogeneous team settings, and students collaborated on solving the problem together. During the *summarize/discuss* phase, the research team sequenced student presentations from low- to high-order explanations to help students notice least complete and procedural explanations, to more complete and conceptual explanations. Low-order explanations included procedural explanations or memorized facts, while high-order explanations included multiple strategies that emphasized connections between representations based on the LTM. After all student teams presented their posters, students discussed and summarized peer presentations as a whole class. Following whole class discussions, individual students wrote reflections with prompts written by the research team. For example, given algebra tile patterns for Figure 2 to Figure 4, and after all students listened to peer team presentations about how to find the total for Figure 6 and Figure 10, students were asked to reflect on the following prompts: (1) *Which strategy do you prefer and why?* (2) *Using any strategy find the number of squares in the 15th figure.* (3) *Find the 100th figure.* In sum, the research team roles during the constructivist teaching experiment included the following: introduce the mathematics task (*launch* phase), encourage team work (*explore* phase), and ask open-ended questions while monitoring students working in groups (*explore*, and *summarize/discuss* phases).

Throughout this section, data related to research question three from the *summarize/discuss* phase taken from various points in the constructivist teaching experiment are presented. Data taken from the beginning (Mathematical Task #1), middle (Mathematical Task #4), and end (Mathematical Task #8) of the constructivist teaching experiment were selected to provide descriptive snapshots of student teamwork and individual student's understandings *before, during,* and *after* students participated in whole class discussions that emphasized *connections* among peer presentations. The following sections present three select mathematical tasks, followed by the objectives and student expectations. The data from Mathematical Task #1, Mathematical Task #4, and Mathematical Task #8, are presented in four stages and in an attempt to document characteristics of middle school immigrant students' communication of solutions and justifications, and use of multiple representations as a result of participation in whole class discussions that emphasize *connections* among peer presentations. The four stages are: (1) description of student team work from posters *before* participation in whole class mathematical discussions; (2) description of student team presentations from video transcripts *during* participation in whole class mathematical discussions; (3) description of students' individual written reflections *after* participation in whole class mathematical discussions that emphasize *connections* among peer presentations; and (4) summary of team results before and after participation in whole class mathematical discussions that emphasize *connections* among peer presentations. At the end, themes that emerged from the data are presented. Table 4.12 displays a summary of the data presentation plans for research question three as the data provides snapshots from various parts of the teaching experiment (beginning, midway, and end), and within each mathematical tasks (*before, during,* and *after*) as well as the embedded translations. Research question three seeks to document the characteristics of middle school immigrant students' understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations.

Table 4.12

Data Presentation from Three Mathematical Tasks from the Teaching Experiment

Mathematical Task #	**Data Presentation Plans**	**Embedded Translations**
<u>Beginning</u>: *Mathematical Task #1*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models, & Picture (tiles to equation/graph)*
<u>Midway</u>: *Mathematical Task #4*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models, & Picture (table to equation/graph)*
<u>End</u>: *Mathematical Task #8*	Understanding *Before* Whole Class Discussion Understanding *During* Whole Class Discussion Understanding *After* Whole Class Discussion	*Picture to Verbal/Written Models) (graph to equation)*

Understanding Before Whole Class Discussion at the Beginning of Intervention

Mathematical Task 1 (M1): Write Rules and Make Predictions for Non-linear Patterns (*picture-algebra tiles* to *verbal/written models,* and *picture-algebra tiles*). Figure 4.21 displays Mathematical Task #1 from the constructivist teaching experiment. The mathematical task was selected from a Standards-based curriculum with modifications by the research team to meet the study goals to emphasize communication and representations based on the LTM. Mathematical Task #1 asked students to examine algebra tiles that followed a non-linear pattern for Figure 2, Figure 3, and Figure 4, and asked students to examine the tile pattern in order to find the total algebra tiles for Figure 6 and Figure 10 (*picture to verbal/written models,* and *picture*). Teams that finished early were encouraged to find the total algebra tiles for Figure 100. Students with conceptual understanding were expected show their work correctly using at least two representations: table of values, graph, equation, diagram, numeric or written explanations.

Math Task #1
What do figures 6 and 10 look like? How many total squares are in each figure? Explain how you know with at least 2 representations (Table of values, Graph, Equation, Diagram, Numeric or written explanation).

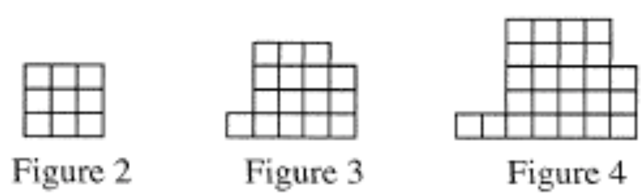


Figure 4.21. Mathematical Task #1 questions from the constructivist teaching experiment.

Team 1 student work before participation in whole class discussion (M1). Leon (HEP), Lucas (MEP), and Sandy (MEP), each had an *incorrect* solution. Figure 4.22 showed that Team 1 students used three strategies to attempt to solve the problem: pictures of algebra tiles (*picture to picture*) and numerical expressions (*picture to written model*). First, students drew pictures of algebra tiles and counted algebra tiles to find the total, and students colored the tile drawings to highlight similarities between Figure 2, Figure 3, and Figure 4 on the poster. For example, students shaded a 3 x 3 square in each of the figures, from Figure 2 to Figure 4 on the poster, and extended the shading for Figure 5 and Figure 6 on the poster. Second, students attempted to write numerical expressions that represented each figure, and then labeled the total number of tiles for each figure on the poster. For example, students correctly wrote "9" above Figure 2 on the poster, and incorrectly wrote, "9 + 2" above Figure 3 on the poster, and "9 + 3" above Figure 4 on the poster.

In sum, Team 1 students used three strategies to attempt to solve the problem, but incorrectly solved the problem for Figure 6 ad Figure 10, pictures of algebra tiles (*picture to picture*) and numerical expressions (*picture* to *written model*). Students obtained the correct totals for the given figures, but did not correctly find the totals for Figure 6 and Figure 10. Although students used three strategies to solve the word problem, numerical expressions, pictures of algebra tiles, and verbal explanations, their written work showed procedural counting strategies. The solution was coded as *incorrect (0)* and the justification was coded as *conceptual (2)*. The team poster was coded as *limited understanding (Score: 2)* because students incorrectly answered the questions, but to make connections between two or more representations to draw the figures, but did not arrive at a correct solution.

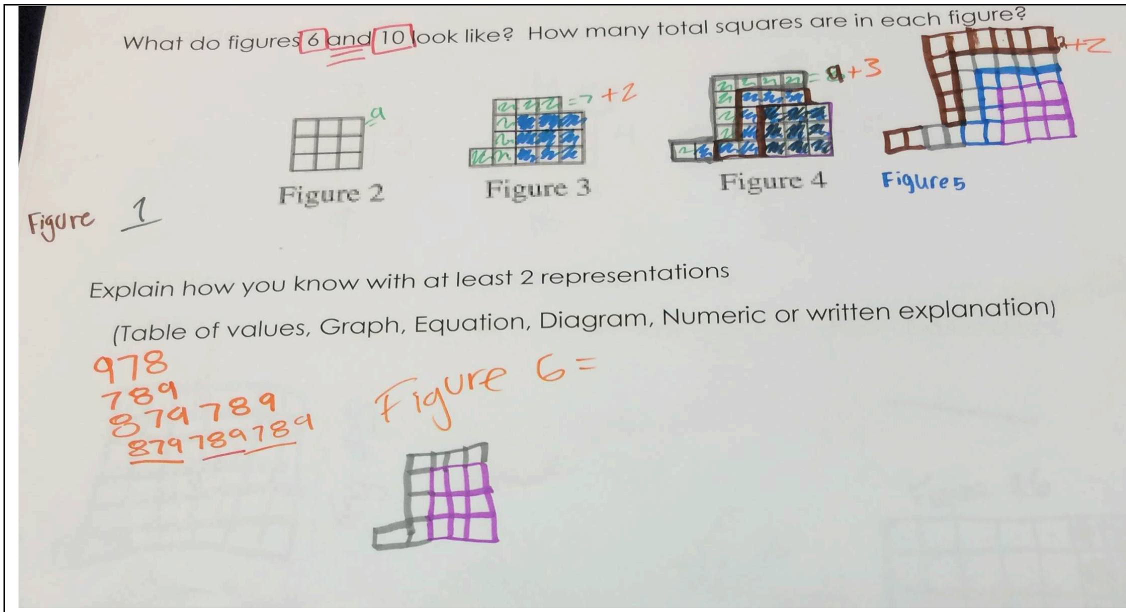


Figure 4.22. Team 1 poster of Mathematical Task #1 by Leon, Lucas, and Sandy.

Team 2 student work before participation in whole class discussion (M1). Anna (LEP), Brooke (MEP), Joe (LEP), and Olvin (LEP) had the *correct* solution. Team 2 students had a majority of LEP students, but a heterogeneous mix of mathematics proficiency levels. Figure 4.23 showed that Team 2 students used three strategies to solve the problem: pictures of algebra tiles (*picture to picture*) and numerical expressions (*picture to written model*). First, students labeled the dimensions of the figures as 3 x 3 for Figure 2, as 4 x 4 for Figure 3, and as 5 x 5 for Figure 4 on the poster. The pattern was correct for the dimensions, following the pattern, $(n + 1)^2$, where n represented the figure number. This pattern continued for Figure 5 and Figure 6 on the poster; Team 2 students also completed the pattern and found the total for Figure 100 on the poster. Students drew pictures of algebra tiles for Figure 5 as a 6 x 6 square, Figure 6 as a 7 x 7 square, and Figure 10 as an 11 x 11 square on the poster. However, Team 2 students did not show work when calculating the totals for Figure 5, Figure 6 or Figure 10 on the poster, but students wrote the correct totals: “36” (algebra tiles) squares for Figure 5, “49 squares” for Figure 6, “121 squares” for Figure 10, and “10, 201 squares” for Figure 100 on the poster.

In sum, Team 2 students used three strategies to solve the problem correctly for Figure 6, Figure 10, and for the challenge problem, Figure 100. Students used numerical expressions (*picture* to *written model*) and pictures of algebra tiles (*picture to picture*). Figure 100 was a challenge problem and students who who finished early were encouraged to attempt the problem. Team 2 student solutions correctly solved the total algebra tiles for Figure 6, Figure 10 and challenge question, Figure 100. The solution was coded as *correct (2)* and the justification was coded as *conceptual (2)*. The team poster was coded as *conceptual understanding (Score: 4)* because students correctly answered the questions and made connections between two or more representations: diagram, numerical expression, written and verbal explanations.

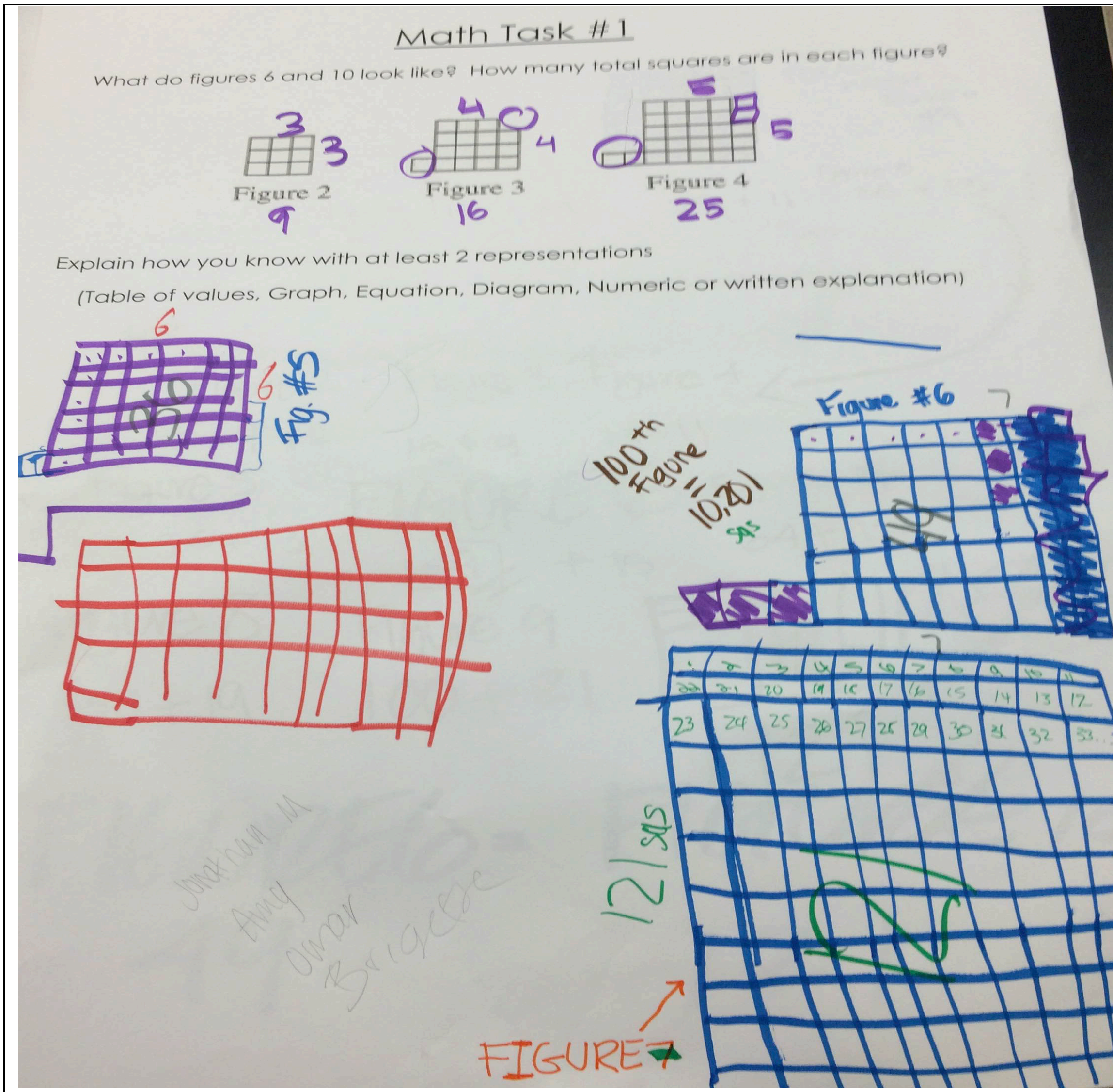


Figure 4.23. Team 2 poster of Mathematical Task #1 by Anna, Brooke, Joe, and Olvin.

The following sections present data from student teams *during* whole class discussion for Mathematical Task #1.

Understanding During Whole Class Discussion at the Beginning of Intervention

Team 1 student communication during participation in whole class discussion (M1). During whole class presentation of the team poster, Leon, Lucas, and Sandra, presented first out of four teams since their team did not answer the question correctly. Team 1 students had an *incorrect* answer for the questions and used three strategies: pictures of algebra tiles, numerical expressions, and verbal explanations. Leon presented for his team. He shared that the team first observed the figures to attempt to identify a pattern by subtracting consecutive figures:

> "So at first we tried to figure out like, like if what like there was some sort of pattern and have found something and um… After that we tried to find Figure 1, but we were, we were like, kind of stuck there. So then we tried to find a pattern, we continued trying to find a pattern and then we thought of how many there were here. (*Referring to Figure 2 to Figure 3 on the poster*)

Next, Leon discussed two strategies the team tried, one involved counting algebra tiles that the team colored blue and green, and the second involved writing numeric expressions to attempt to describe the pattern starting with, "the original 9," from Figure 2:

> "Over here, there are 7 extra from the original one, the original nine. So there is the original 9 right here in the blue, and the green ones are 1, 2, 3, 4, 5, 6, 7. And then on the second one (Figure 4) there is the original 9 there's the figure 3, 7 and there's 1, 2, 3, 4, 5, 6, 7, 8, 9 from here to here, we added nine which is two from seven." (*Leon points to Figure 3, "over here" at first, and then to Figure 2 as, "the original one," and to refer to the shaded blue 3 x 3 squares on the poster going back and forth from Figure 3 to Figure 2. Next, he counts 7 tiles that were added from Figure 3 from Figure. At the end, he counts 9 tiles from Figure 3 to Figure 4*)

The excerpt above showed Team 1 students had a difficult time identifying a pattern, and therefore, could not answer the question for Figure 6 or Figure 10. Finally, Leon shared that his team drew pictures of algebra tiles based on their counting method and wrote (incorrect) numerical expressions to describe the pattern, but the team did not have time to finish, "And over here, this is what we think Figure 5 looks like and um, we started drawing Figure 6 but we didn't finish it." In sum, Team 1 students did not answer the questions correctly and their poster presentations did not correctly connect multiple representations. Their presentation was also coded as *limited understanding (Score: 2)* because students failed to answer the questions correctly, but attempted to use multiple representations to explain their unfinished work.

Team 2 student communication during participation in whole class discussion (M1). During whole class presentation of the team poster, Anna, Brooke, Joe, and Olvin presented second out of four groups, after Team 1. Team 2 students correctly answered the questions using three strategies, pictures of algebra tiles, numerical expressions and verbal explanations. Olvin started the presentation about what his team did, but did not elaborate. The role of the research team during the *summarize/discuss* phase was to encourage team work on explanations:

> O: We found Figure 5.
> SC: Awesome how did you do that? Joe, you did a really good job starting the group when I was over there. Could you tell us what you did with figures 2, 3 and 5? See all those purple markings at the very top? Joe did that. Could you share with the class what you did?
> (*Researcher shares her observation of Joe's contribution to encourage participation from LEP student, Joe. Joe stares at his written work before responding*)
> SC: Where did 9, 16, and 25 come from?
> (*Referring to the total number of algebra tiles*)

J: I multiplied the 3 squares at the top and the 3 squares at the side to … And I got 9. And for the second one there is a small square and I just added it to the top at the corner and makes 4 and 4 and I got 16. Then the-- there are two and I just added it and I got 5 and 5 and I got 25 and that's it.
(*Joe explains Figure 2 as a 3 x 3 square dimension for a total of 9 algebra tiles, Figure 3 as a 4 x 4 square dimension for a total of 16 algebra tiles, and Figure 4 as a 5 x 5 square dimension for a total of 25 algebra tiles*)

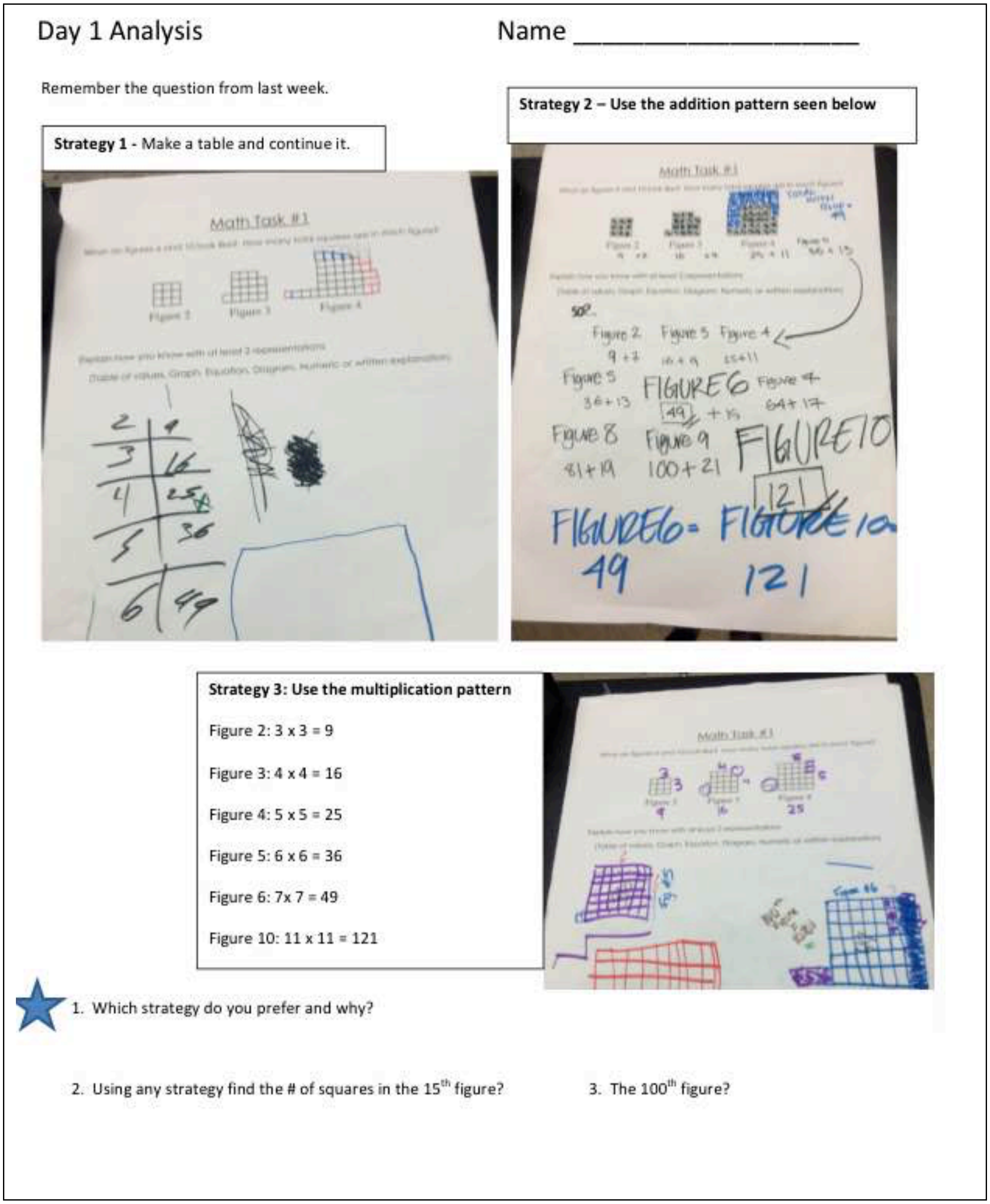
The excerpt above illustrated the role of the research team that included encouraging students using examples observed from the *monitor, select,* and *sequence* stages of the mathematical discussion framework. In the excerpt, the research team acknowledged contributions from other team members and engaged two LEP students in the presentation. Joe explained his thinking using numerical expressions to solve a higher-order, open-ended non-linear problem. First, Joe explained that labeled the dimensions of Figure 2 as a 3 x 3 square, a 4 x 4 square for Figure 3, and a 5 x 5 square for Figure 4 on the poster. The research team pushed for other team members to summarize and discuss their work for finding the total algebra tiles for Figure 6 and Figure 10:

SC: Okay, so that's Figures 2, 3 and 4. Did you answer the question though? Someone else want to answer the question? How do you get figure 6?
(*Students talk among the group*)
SC: Anna, can you help? What was Figure 6? I see the picture I just need someone to explain it. Brooke, can you help?
O: We just added a one.
SC: So point to your 6th figure… Okay someone is pointing at Figure 5 and someone is pointing at Figure 6, which one is it?
(*Olvin points to Figure 6*)
SC: Ah, that one. So tell me how many boxes are in that one? How many squares? (*Referring to Figure 6; and "boxes" refers to algebra tiles*)
O: 49.
SC: How did you get 49, briefly?
O: We multiplied 7 and 7.
SC: Uh ha. And you got 49. Thank you. We will stop at the 6th figure I don't think we will have time for the 10th."
(*The students had the correct solution for Figure 10 and Figure 100*).

In the excerpt above, Olvin explained that the team, "added a one," to the 5 x 5 square dimension to obtain a 6 x 6 square dimension, for a total of 36 algebra tiles. This pattern continued, and students added one to the 6 x 6 square dimension to obtain a 7 x 7 square for a total of 49 algebra tiles. In sum, Anna, Brooke, Joe, and Olvin, answered the questions correctly during their poster team presentation. Their presentation was also coded as *conceptual understanding (Score: 4)* because students answered the questions correctly and used multiple representations to make connections: verbal model, written words, numerical model, and diagram. The following sections present data from individual student reflections *after* participation in whole class discussions that emphasize *connections* between peer presentations for Mathematical Task #1.

Understanding After Whole Class Discussion at the Beginning of Intervention

After whole class discussion of Mathematical Task #1, students were encouraged to reflect on all peer presentations that were sequenced from low-order to high order explanations. Figure 4.24 displays a handout given to all students the following class period, *Day 1 Analysis*. The handout contained three photos with three unique peer solutions, followed by a summary of student explanations written by the research team: (1) *Strategy 1:* Make a table and continue it; (2) *Strategy 2:* Use the addition pattern seen below (*numerical* expression); and (3) *Strategy 3:* Use the multiplication pattern. Students were expected to examine peer explanations and answer similar, more challenging questions to Mathematical Task #1. The solutions were selected by the research team that represented three unique strategies from the peer presentations. For the written portion, students were asked to individually reflect on three questions written by the research team: (1) Which strategy do you prefer and why? (2) Using any strategy find the number of squares in the 15^{th} figure; and (3) Find the 100^{th} figure.



Day 1 Analysis

Name ____________________

Remember the question from last week.

Strategy 1 - Make a table and continue it.

Strategy 2 – Use the addition pattern seen below

Strategy 3: Use the multiplication pattern

Figure 2: 3 x 3 = 9

Figure 3: 4 x 4 = 16

Figure 4: 5 x 5 = 25

Figure 5: 6 x 6 = 36

Figure 6: 7x 7 = 49

Figure 10: 11 x 11 = 121

1. Which strategy do you prefer and why?

2. Using any strategy find the # of squares in the 15^{th} figure?

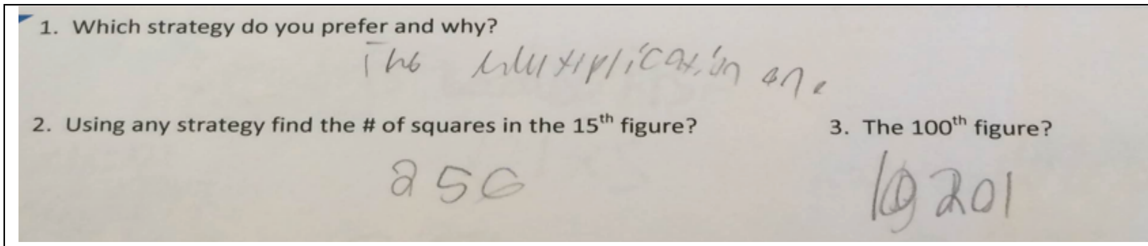
3. The 100^{th} figure?

Figure 4.24. Mathematical Task #1 reflection prompts for individual student write up.

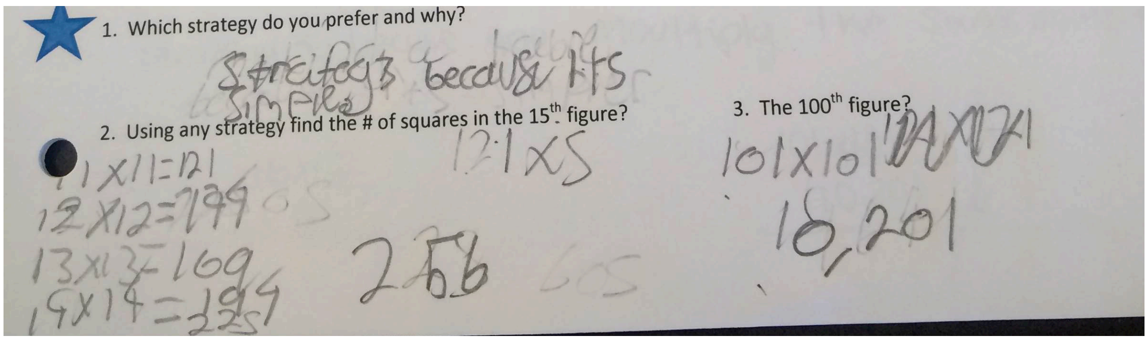
Team 1 individual student work after participation in whole class discussion (M1). Leon, Lucas, and Sandy correctly solved the new problems individually. All three Team 1 students selected *Strategy 3*, a numerical expression using multiplication to find the total algebra tiles for Figure 15 and Figure 100. Leon selected *Strategy 3* with little explanation, "The multiplication one," and correctly evaluated the expressions using mental math, 16 x 16 and 101 x 101, to find the totals. He wrote 256 algebra tiles for Figure 15, and 10, 201 algebra tiles for Figure 100, but did not show any work. Lucas explained why he chose *Strategy 3*, "Strategy 3 because it's simple." For his reflection, he wrote five multiplication sentences: "11 x 11 = 121; 12 x 12 = 144; 13 x 13 = 169; 14 x 14 = 194; 225 (he skips writing 15 x 15 = 225 and 16 x 16 = 256);" and found the correct totals for Figure 15 with, "256," and Figure 100 with, "101 x 101 = 10, 201." Sandy explained that she chose *Strategy 3* because of the multiplication pattern, and correctly found the totals for Figure 15 and for Figure 100 as 256 algebra tiles and 10, 201 algebra tiles, respectively. For her reflection, she wrote, "Strategy 3 because you multiply the same number." Her work showed two multiplication problems: "16 x 16 = 256," and "101 x 101 =10, 201."

In sum, Leon, Lucas, and Sandy did *not* answer the questions correctly about finding the total algebra tiles for Figure 6 and Figure 10 before participation in whole class mathematical discussions that encouraged *connections* between peer presentations. After listening to peer presentations of solutions with explanations connecting multiple representations (table, graph, pictures, numerical expressions), all students correctly answered similar, but more challenging questions individually for Figure 15 and Figure

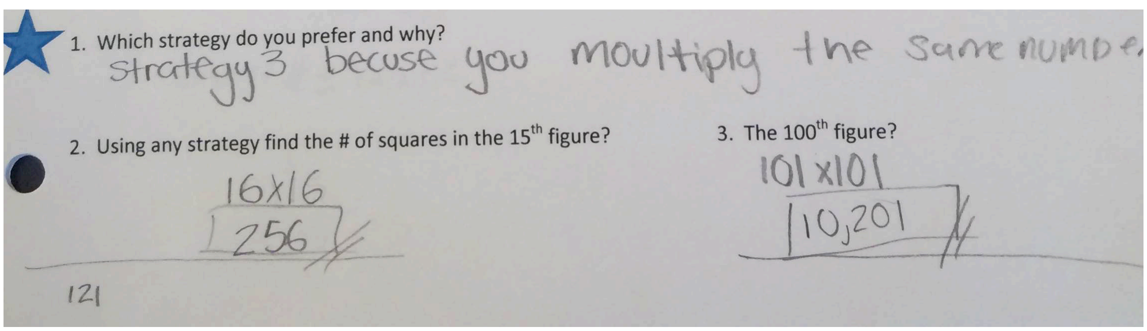
100. Their individual written work was coded as *correct.* Overall, students preferred the third strategy, *Use the multiplication pattern,* for large n-values.



Leon answered three reflection questions.



Lucas answered three reflection questions.

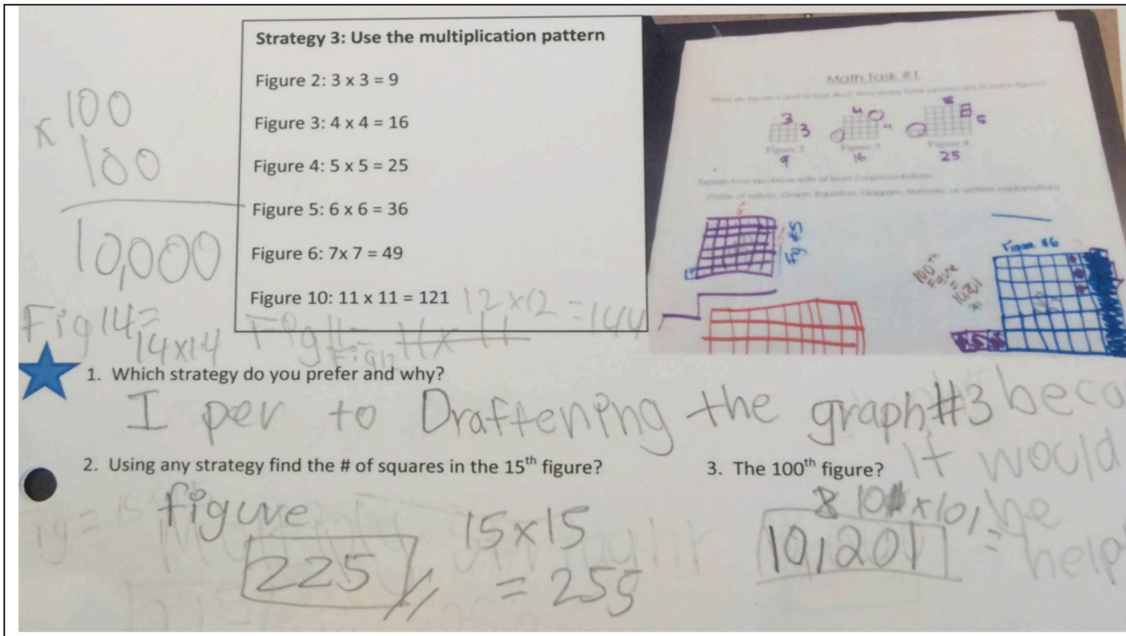


Sandy answered three reflection questions.

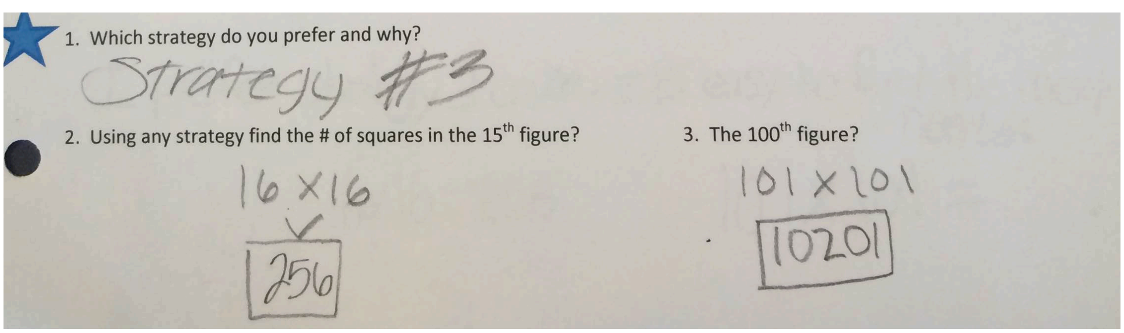
Figure 4.25. Individual Team 1 student reflections from Mathematical Task #1 by Leon, Lucas and Sandy.

Team 2 individual student work after participation in whole class discussion (M1). Anna solved the new problems *partially correctly*. In her reflection, she wrote that she preferred to graph and use numerical expressions using multiplication to solve for Figure 15 and Figure 100, "I prefer to draw the graph (and use strategy) #3 because it would be helpful." Her work was coded as *partially correct*. She found an incorrect total number of tiles for Figure 15 as 225 algebra tiles, and not 256 algebra tiles. However, she calculated the correct total algebra tiles for Figure 100 with 10, 201 algebra tiles. Her written work showed three multiplication problems: "100 x 100 = 10,000; Fig. 14 = 14 x 14; Fig. 11 = 11 x 11; 12 x 12 = 144; 15 x 15 = 255 (instead of 256);" and "101 x 101 = 10, 201." Unlike Anna, three teammates, Brooke, Joe, and Olvin solved the new problems *correctly*. Brooke chose Strategy 3 and in her reflection, she wrote that she preferred to solve with numerical expressions using multiplication, "Strategy 3," and she found the correct totals for Figure 15 with, 256 algebra tiles, and the correct total for Figure 100 with, 10, 201 algebra tiles. Her work showed two multiplication problems, "16 x 16 = 256," and "101 x 101 = 10, 201." Joe chose Strategy 3, a numerical expression using multiplication and explained, "I prefer Strategy 3 cause it's easy to find the next number." His work showed two multiplication problems, one complete, "16 x 16 = 256," and one incomplete, "101 x 101." Unlike Brook and Joe, Olvin chose a different strategy, he wrote that he preferred to solve using a table (picture), "Strategy 1 (table) because I know tables." Using a table, Olvin found the correct total algebra tiles for Figure 15 with 256 algebra tiles, and for Figure 100 with 10, 201 algebra tiles. His work showed two multiplication problems: "16 x 16 = 256;" and "101 x 101 = 10, 201."

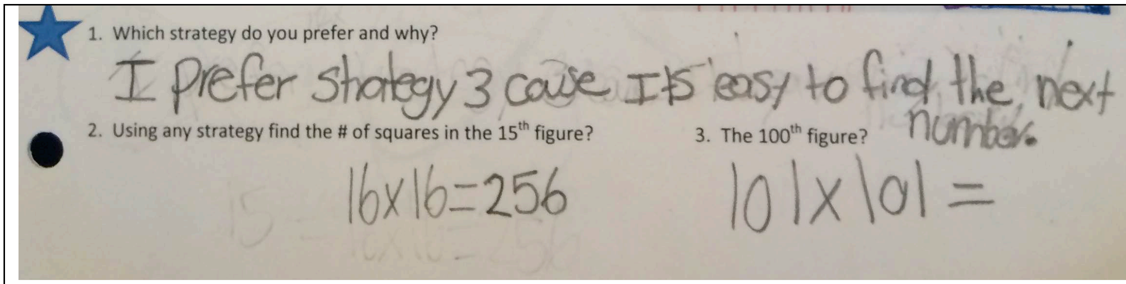
In sum, Anna, Brooke, Joe, and Olvin correctly answered the questions as a team before participation in whole class mathematical discussion that encouraged *connections* between peer presentations. After listening to peer presentations of solutions with explanations connecting multiple representations (table, graph, pictures, numerical expressions), three out of four students, Brooke, Joe, and Olvin correctly answered similar but more challenging questions individually for Figure 15 and Figure 100. Their written work was coded as *correct.* Anna answered one of two questions correctly. Her written work was coded as *partially correct.* Overall, students preferred the third strategy, *Use the multiplication pattern,* for large n-values.



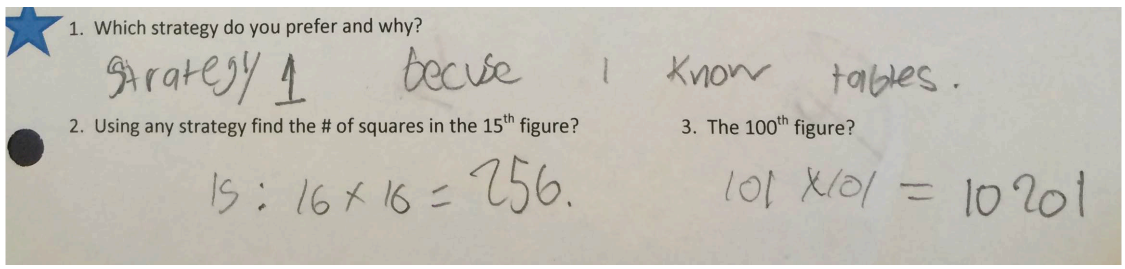
Anna answered three reflection questions.



Brooke answered three reflection questions.



Joe answered three reflection questions.



Olvin answered three reflection questions.

Figure 4.26. Individual Team 2 student reflections from Mathematical Task #1 by Anna, Brooke, Joe, and Olvin.

Understanding Before Whole Class Discussion Midway Through the Intervention

Mathematical Task 4 (M4): Graph, write an Equation and Make Predictions from a Table (*picture-table* to *verbal/written models*). Mathematical Task #4 asked students to examine an incomplete table with a real-life situation and: (a) complete the table above (when x = 0, x = 1, and x = 2); (b) plot the points that represent the height of the tree over time (label the axis); (c) find the height 1.5 years after the tree was planted and plot the point; (d) where does the line cross the y-axis? (Identify the y-intercept); (e) use the table or graph to represent this relationship with an equation (Use answer from part c to help); (f) Find the exact height of the tree after 50 years it was planted (*picture to verbal/written models*). Students were expected show their work using the slope to complete the table, label the x- and y-axis using a real-life situation, plot the coordinate pairs from the table, and solve and interpret x-values at x = 1.5 and x = 50. Students were encouraged to use the table, graph, and written models to answer the questions to demonstrate conceptual understanding. Conceptual responses are judged by how well students explained the task overall (so two or more representations to answer subparts).

Math Task #4
John found the data in the table below about his favorite tree. Use the table below to answer the following questions. Be ready to share your answers with the class.

Number of Years after Planting	0	1	2	3	4	5	6
Height of Tree (in feet)				17	21	25	

a. Complete the table above.

b. Plot the points that represent the height of tree over time (label each axis).

c. According to the graph, what was the height of the tree 1.5 years after it was planted? Plot this point.

d. Where does this line cross the y-axis.

e. Use the table or the graph to represent this relationship with an equation (use your answer in part c to help you).

f. Find the exact height of the tree 50 years after it was planted.

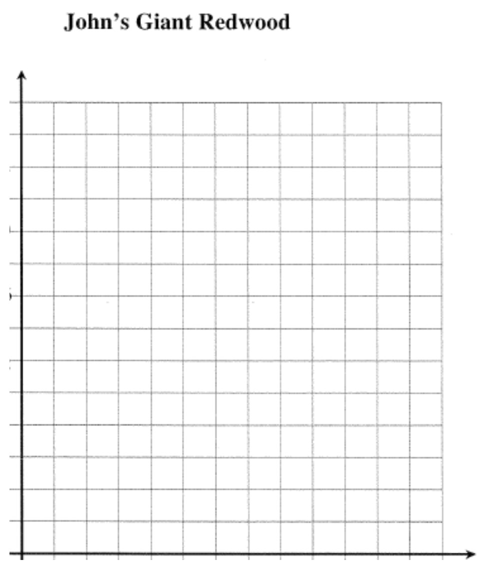


Figure 4.27. Mathematical Task #4 questions from the constructivist teaching experiment.

Team 1 student work before participation in whole class discussion (M4). Leon, Lucas, and Sandy correctly answered all five subparts. Figure 4.28 displays student work on the poster. For Question 4a, when asked to complete the table with a real-life situation, students filled in the missing values correctly using slope of 4: (0, 5), (1, 9), and (2, 13), and (6, 29). Further, students extended the table and added three more coordinate pairs, (7, 33), (8, 37), and (9, 41), and wrote numerical expressions using the slope of four to label each y-values from 0 to 9 underneath the table: “0 x 4, 5 + 0 = 5; 1 x 4 + 5 = 9; 2 x 4 + 5 = 13; 3 x 4 + 5 = 17; 4 x 4 + 5 = 21; 5 x 4 + 5 = 25; 6 x 4 + 5 = 29; 7 x 4 + 5 = 33; 8 x 4 + 5 = 37; 9 x 4 + 5 = 41.”

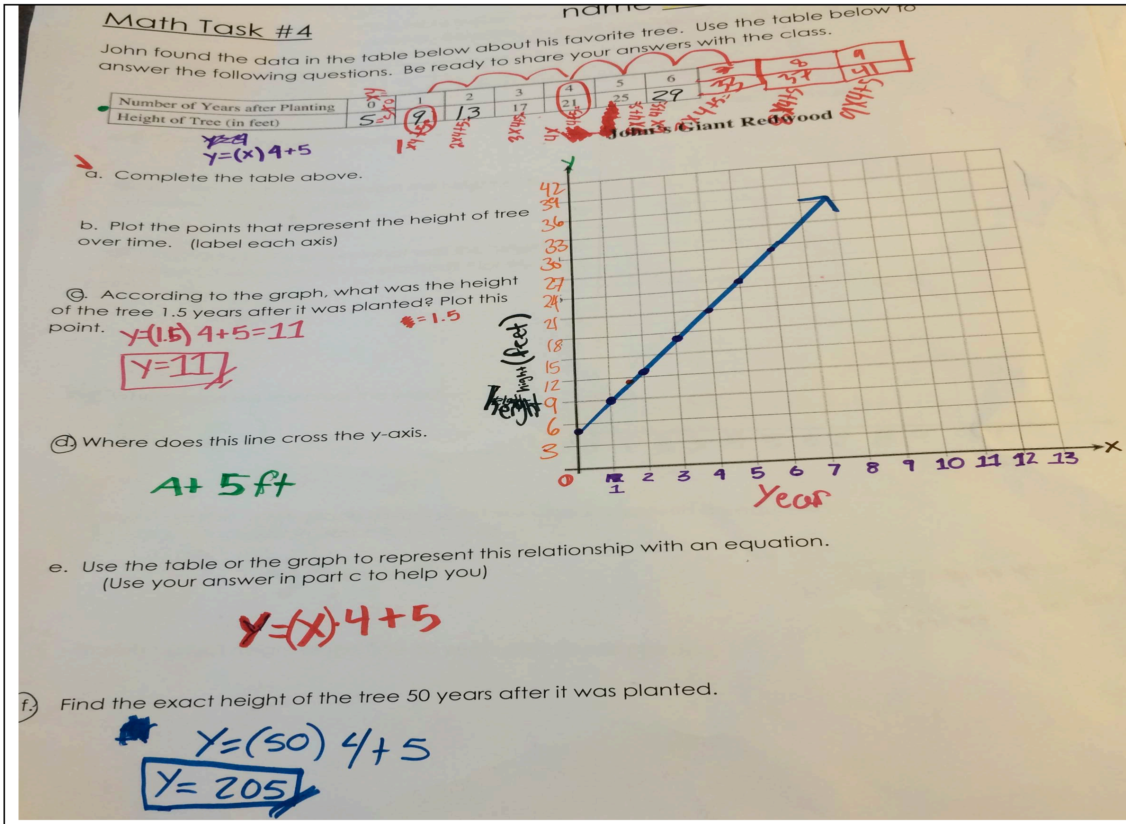


Figure 4.28. Team 1 poster of Mathematical Task #4 by Leon, Lucas, and Sandy.

Additionally, students wrote an equation to represent the table of values even when not asked to do so, “y = (x)4 + 5.” For Question 4b, when asked to plot the points that represent the height of the tree over time and label each axis, students labeled the x-axis using a real-life situation, “Year,” with intervals of one from 0 to 13; and labeled the y-axis using a real-life situation, “Height (feet),” with intervals of three from 0 to 42. For Question 4c, when asked to use the graph to find the height of the tree 1.5 years after it was planted, students used two representations, the graph and an equation, to find the height, or y-value of 11: "y = (1.5)4 + 5 = 11; x = 1.5, y = 11." For Question 4d, when asked to identify where the line crosses the y-axis (y-intercept), students wrote a correct answer of 5 and used a real-life situation, "At 5 ft." For Question 4e, when asked to use the table or the graph to represent the relationship with an equation and use part c to help, students wrote the correct equation, similar to the one from Question 4a, "y = (x)4 + 5." For Question 4f, when asked to find the exact height of the tree after it was planted, students found the correct height at 205 feet and wrote, "y = (50)4 + 5; y = 205." In sum, Team 1 students completed all five subparts correctly. The solutions were coded as *correct (2)* and the justifications were coded as *conceptual (2)*. The team poster was coded as *conceptual understanding (Score: 4)* because students correctly answered the questions and used multiple representations including table, graph, real-life situation, and written and oral words.

Team 2 student work before participation in whole class discussion (M4). Anna, Brooke, Joe and Olvin did not correctly answer all subparts. Students answered subparts 4a, 4b, 4c, and 4d correctly, but did not answer subparts 4e and 4f correctly. Figure 4.29 displays student work on the poster. For Question 4a, when asked to complete the table, students filled in the missing values correctly using slope of 4: (0, 5), (1, 9), and (2, 13), and (6, 29). For Question 4b, when asked to plot the points that represent the height of the tree over time and label each axis, students did not label the x-axis using a real-life situation, but choose appropriate intervals of one from 0 to 11; and did not label the y-axis with a real-life situation, but choose appropriate intervals of three from 0 to 42. Further, students also made a vertical table to represent the graph, with x-values from 0 to 6 and y-values from 5 to 29.

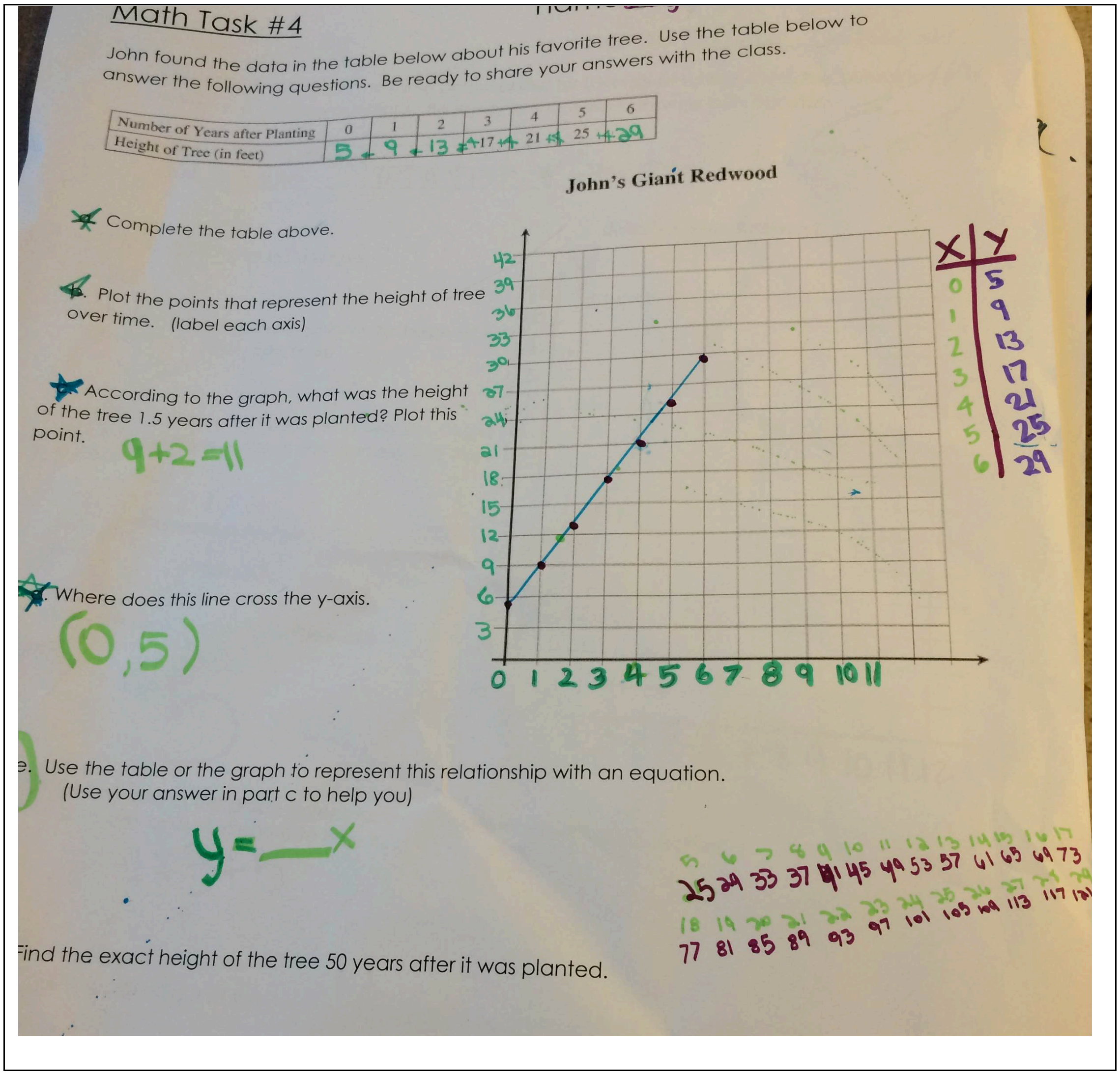


Figure 4.29. Team 2 poster of Mathematical Task #4 by Anna, Brooke, Joe, and Olvin.

For Question 4c, when asked to use the graph to find the height of the tree 1.5 years after it was planted, students used two representations, the graph and an equation, to find the height, or y-value of 11. Students used a numerical expression to correctly evaluate the height of the tree after 1.5 years it was planted, "9 + 2 = 11." For Question 4d, when asked to identify where the line crosses the y-axis (y-intercept), students wrote a correct answer using a coordinate, "(0, 5)." For Question 4e, when asked to use the table or the graph to represent the relationship with an equation and use part c to help, students wrote an incomplete equation, "y =__x." For Question 4f, when asked to find the exact height of the tree after it was planted, students did not find the correct height of 205 feet. Their work showed students listed the years from 5 to 29 consecutively as coordinate pairs from (5, 25) to (29, 121) and ran out of time. In sum, Team 2 students completed four of six subparts correctly. The solutions were coded as *partially correct (1)* and the justifications were coded as *conceptual (2).* The poster was coded as *some understanding (Score: 3)* because students answered the question partially correctly and used multiple representations including table, graph, real-life situation, and written and oral words. The following sections present data from student teams *during* whole class discussion for Mathematical Task #4.

Understanding During Whole Class Discussion Midway Through the Intervention

Team 1 student work during participation in whole class discussion (M4). During whole class presentation of the team poster, Leon, Lucas and Sandy presented after Team 2. Team 1 students correctly answered all five questions, and all students presented in this team, starting with Lucas (LU), Leon (LN), Sandy (S), and a fourth (LEP) student that dropped out of the study. The research team noticed that Team 1 students did not answer the questions from 4a to 4e in order. Instead, Team 1 students started the presentation at Question 4c, but when probed further by Mr. Champion (MC) and the researcher (SC), students shared that they started their poster work from Question 4e:

MC: Track to Leon's group. And voices off at this point. You should be tracking forward.
LU: For c we got 11 because we multiplied by times 4 … and we got 11. (*Question 4c*)
MC: Ok that's for c? Pause for a second please. You guys started at a different question. Let's start from where you started. Where did you start? (*Question 4c*)
LN: We started at e. (*Question 4e*)

The role of the research team included encouraging students to communicate their mathematical reasoning and verify their work using the real-life situation and multiple representations.

MC: Why did you do that? Explain that to me, Leon.
LN: We started at e because at first we were trying to… we finished the table and then we wanted to figure out the pattern so I found out, we all agreed that e would help us solve every single other problem. (*Question 4e*)
MC: Ok so why don't you tell us the answer for e. (*Question 4e*)
LN: y = 4x + 5.
MC: What do the y and x mean?
LN: x is the number of *years* it planted. For example, 3 multiplied that by 4, add 5 to that you get 13.

The role of the research team also included encouraging all team members to participate in the whole class discussion/summary. At this point the research team explicitly pointed to a connection that was overheard during the *monitor* stage of the framework. For example, Team 1, Leon, credited a different group other than his for a correct solution. Leon said he used numerical expressions to solve the problem because he remembered from a previous task in the teaching experiment what another group did. From there, Leon, was able to successfully build off of another group to solve Mathematical Task #4:

MC: Okay so how did you know that was the right rule? How do you know? Somebody else can answer that one.
S: Because every time you went up by 4 you add ah, you would still add 5… you wouldn't get the number you'll get a decimal?

> SC: And then earlier Leon also credited a different group right? So maybe you could share what you said to me when you shared how you got part e? (*Question 4e*)
> LN: "I remember that last time we came that that group were close so I came from their prediction to my own. (*Referring to a group that was not included in the study*)

In the excerpt, Leon verified that the team equation came from observing a different team presentation.

In sum, Team 1 students answered all subparts correctly and used more than one representation to answer the questions. Their presentation was coded as *conceptual understanding (Score: 4)* because students answered the questions correctly using multiple representations including: the table, graph, real-life situation, and verbal and written words.

Team 2 student work during participation in whole class discussion M4). During whole class presentation of the team poster, Anna, Brooke, Joe, and Olvin presented first. Team 2 students answered four out of six subparts correctly. Brooke led her team in the presentation from Question 4c to Question 4f since all four teams correctly filled out the table using a slope of 4, and plotted the coordinates correctly. Brooke explained that the team used a slope of 4 to solve Question 4c about finding y-value when x-value is non-integer, at x =1.5. The problem asked students to use the graph or table:

> MC: First up to present this group is Brooke. I'll hold it up. Brooke, go ahead and stand up. We are only going to present, listen carefully, answers to c, d, e and f if we get there. Explain how you got your answer for each one. First up, c. Make sure you explain this to us make sure you project your voice. (*Questions 4c, 4d, 4e, and 4f*)
> B: For c we got 11 because we know that you always have to add a 4 to get to the next number so we just add up… to get… (*Question 4c*)

MC: Right here? (*Pointing to the y-values*)
B: Yeah.

For Question 4d, Brooke used the graph to identify the y-intercept correctly as a coordinate pair, "(0, 5)," but when asked to write an equation and use the equation to solve for large x-value at 50, students used brute force and made a list in order to work up to x-value at 50. Team 2 students did not correctly answer the question:

MC: And d? (*Question 4d*)
B: We got (0, 5) because the line touches the y-axis at (0, 5).
MC: And what were you trying to do here and here? (*Referring to Question4 e and Question 4f*)
(*Students had, "y = _x," on their poster*)
B: For e we couldn't figure out … for f we made a list of numbers to try to get…
(*Question 4e and Question 4f*)
MC: And how far did you get?
B: 29.
MC: 29, meaning what?
(*She said something that was inaudible. From the poster, 29 is x = 29*)
MC: Okay, so you just need more time. Saw some similarities we will see some similarities in just a second. Let's go to X's group right here Two claps for Brooke and her group on two, one, two. (Student *X was not included in the study*)

In sum, Team 2 students completed the first four out of six sub-questions correctly. Students used more than one representation to answer the questions, even when not asked to. This was coded as *conceptual understanding (Score: 4) because* students answered the questions correctly and used more than one representation including: table, graph, real-life situation, and written and verbal words for Question 4a to Question 4d. Students did not answer two out of four questions correctly. This was coded as *incorrect/incomplete* and *no understanding* for Question 4d and Question 4f.

Understanding After Whole Class Discussion Midway Through the Intervention'

After whole class discussion of Mathematical Task #4, students were encouraged to reflect on all peer presentations that were sequenced from low-order to high order explanations. Recall that low-order explanations included procedural explanations or memorized facts, and high-order explanations included multiple strategies that emphasized connections between representations. Figure 4.30 displays a handout given to all students, *Math Task # 4: After Group Presentations*. The handout contained two prompts written by the research team: (1) Is this relationship a proportional relationship? How do you know? (2) How did you come up with your equation? Students were expected to reflect on peer explanations before answering the questions.

Math Task #4

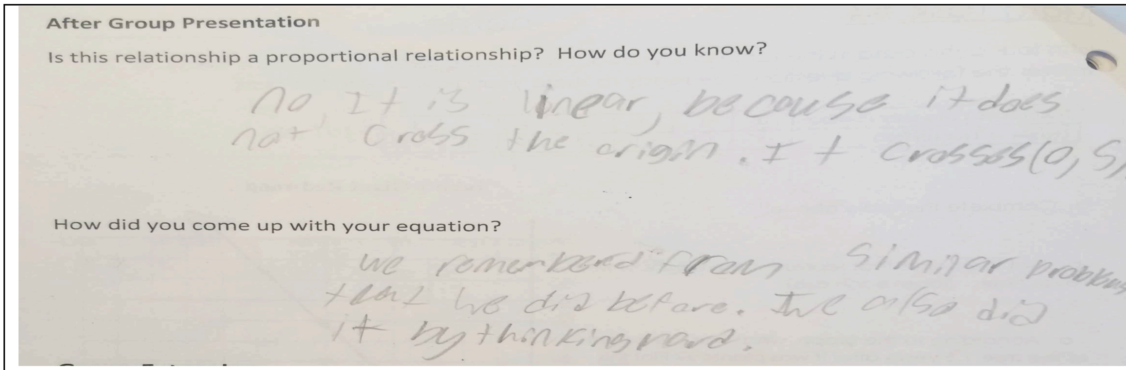
After Group Presentation
Is this relationship a proportional relationship? How do you know?

How did you come up with your equation?

Figure 4.30. Mathematical Task #4 reflection questions from the constructivist teaching experiment.

For the written portion, students were asked to individually reflect on two questions written by the research team.

Team 1 individual student work after participation in whole class discussion (M4). Leon (HEP) correctly described the relationship from the table as a non-proportional linear relationship. He used the graph to verify that the y-intercept does not cross at the origin, "No, it is linear because it does not cross the origin. It crosses (0, 5)." When asked how to write an equation, Leon made an explicit connection to a previous experience in the teaching experiment, “We remembered from similar problems that we did before, “We also did it by thinking hard” (*picture to picture, & written/verbal models*). Lucas (MEP) answered the first question incorrectly and answered the second question correctly. His response was coded as *partially correct*. He answers the second question correctly, but not the first question. He incorrectly writes that the equation is proportional, “Yes, because a proportion has y and x. It’s also has a graph and equation.” The second part is correct about writing an equation, “By multiplying by 4 and adding 5.” He described the relationship from the table as a proportional linear relationship. He first wrote no, and then wrote yes. His explanation was not clear, but included the equation and the graph, “Yes because a proportion has y and x. It also has a graph and equation.” When asked how to write an equation, Lucas described the equation, $y = 4x + 5$, “by multiplying by 4 and adding 5” (*picture to verbal/written models*). Sandy (MEP) correctly described the relationship from the table as a non-proportional linear relationship using an equation, “No because you have to add (a non zero value) and not just multiply”. When asked how to write an equation, Sandy explicitly shared that she used work presented from a different team no part of the study, to support her team, “By building off of another group” (*picture to verbal model, & equation*). In sum, two out of three students from Team 1 correctly reflected on the properties of a non-proportional linear relationship using multiple representations. One student answered the reflection questions *partially correctly*. When asked how to write an equation, all students explained using different approaches. Lucas was more explicit, using slope and y-intercept values.



After Group Presentation

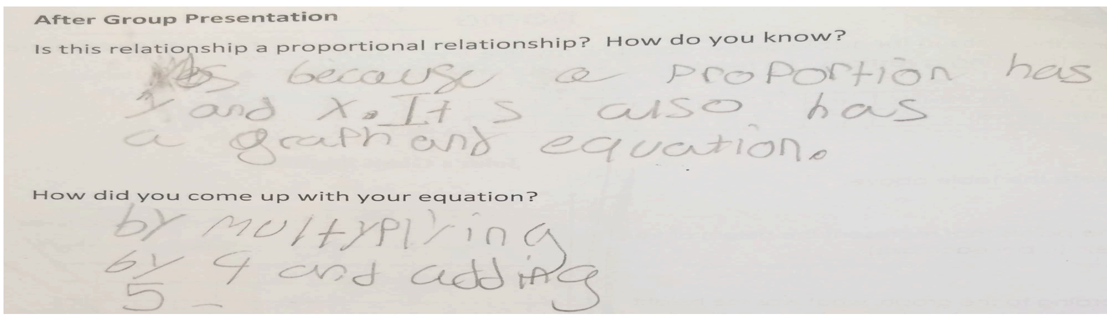
Is this relationship a proportional relationship? How do you know?

no It is linear, becouse it does not Cross the origin. It crosses (0, 5).

How did you come up with your equation?

we remenberd from simiar problems that we did before. we also did it by thinking hard.

Leon answers two reflection questions.



After Group Presentation

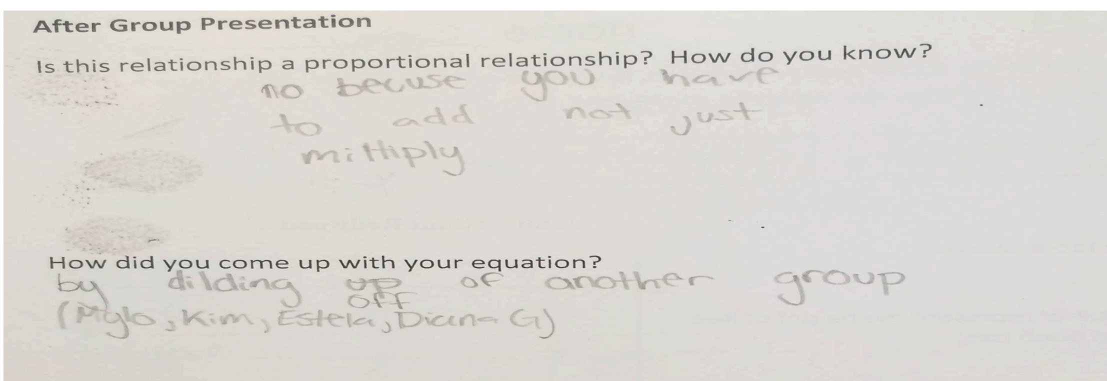
Is this relationship a proportional relationship? How do you know?

yes because a proportion has y and x. It s also has a graph and equation.

How did you come up with your equation?

by multyplying by 4 and adding 5

Lucas answers two reflection questions.



After Group Presentation

Is this relationship a proportional relationship? How do you know?

no becuse you have to add not just mitiply

How did you come up with your equation?

by dilding up off of another group (Mylo, Kim, Estela, Diana G)

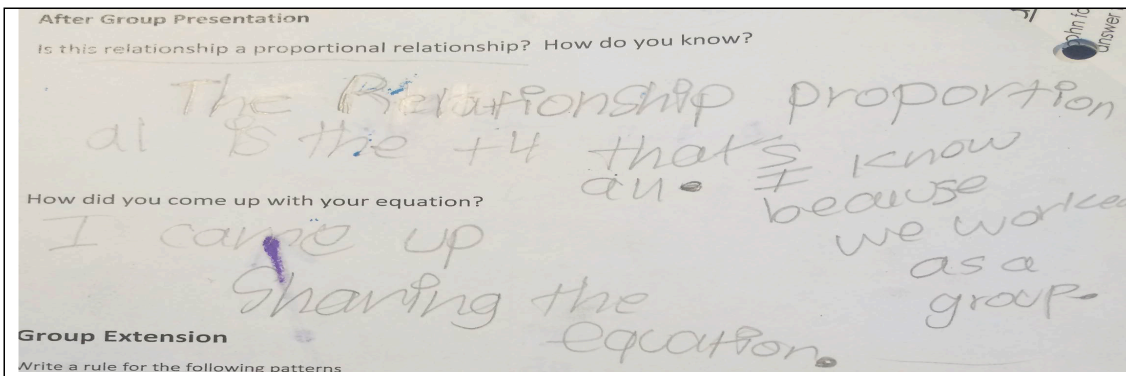
Sandy answers two reflection questions.

Figure 4.31. Individual Team 1 student reflections from Mathematical Task #4 by Leon, Lucas and Sandy.

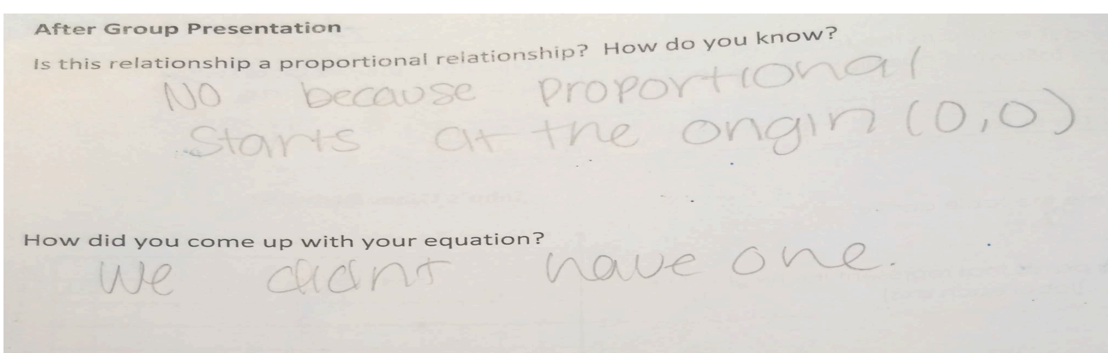
Their written reflections were coded as *correct* with *conceptual understanding* because all students used multiple representations to support their reasoning.

Team 2 individual student work after participation in whole class discussion (M4). Anna did not correctly describe the relationship from the table as a non-proportional linear relationship. Her written work showed that she tried to explain that the relationship was a non-proportional linear relationship from the team equation. She used the equation to explain her reasoning, "The relationship (is not) proportional is (because of) the +4 that's all. I know because we worked as a group." When asked how to write an equation, Anna was not clear about her work and how to write an equation, "I came up sharing the equation" (*picture to written/verbal models*). Brooke (MEP) correctly described the relationship as a non-proportional linear relation. Her explanation included the table and y-intercept, "No because proportional starts at the origin, (0, 0)." When asked how to write an equation, she did not answer the question fully, instead Brooke shared that the team did not have an equation, "We didn't have one" (*picture to verbal/written models*). Joe answered the first question correctly about describing the relationship from the table as a non-proportional linear relationship, but he did not answer the second question about how to write an equation fully, "No cause it starts with a zero (0, 0). We come up with one equation" (*It does not start at zero, so therefore, no, it is not proportional*) (*picture to verbal/written models*). Olvin (MEP) did not correctly describe the relationship from the table as a non-proportional linear relationship using an equation, "I think it's (linear) it's relationship it all went up by the same amount because it went by 4" (*picture to verbal model, & equation*). When asked how to write an equation, Olvin used the slope of 4 to show his work, "We -4 from what we had. Then we add 4 and got the same number" (*Students subtracted from from the y-values to obtain new y-values*)" (*picture to verbal/written models, & equation*).

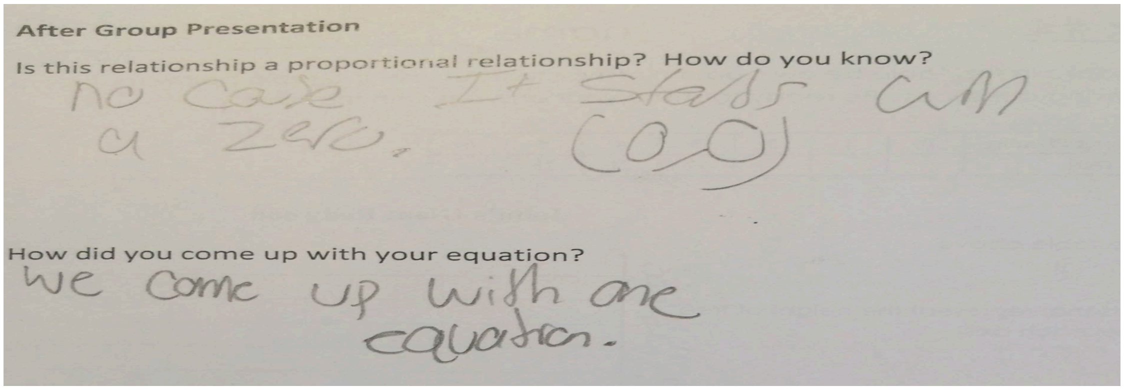
In sum, two students, Brooke and Joe, answered the first reflection question correctly, and two students, Anna and Olvin, answered the first reflection question partially correctly about the properties of a non-proportional linear relationship using multiple representations. When asked how to write an equation, three students, Anna, Brooke, and Joe, did not answer the question correctly, and one student, Olvin, answered the question partially correctly. The written reflections were coded as *conceptual understanding* because all students used multiple representations to support their reasoning.



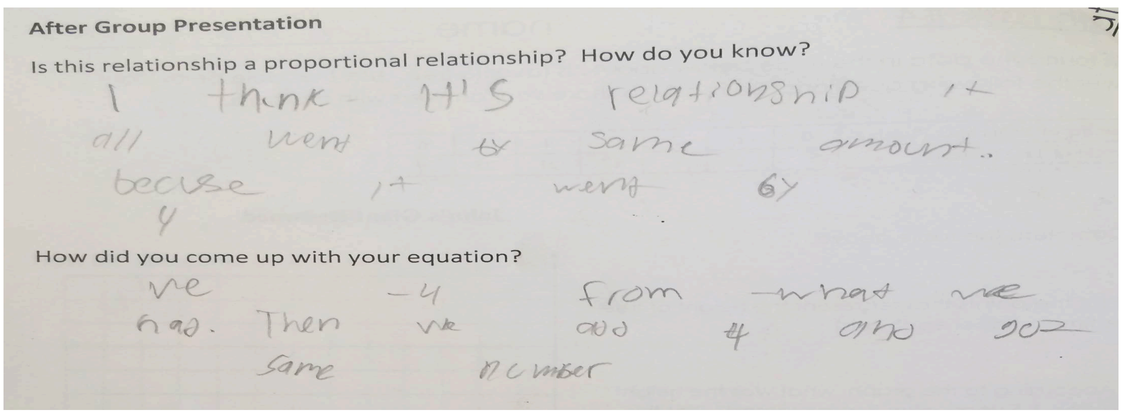
Anna answers two reflection questions.



Brooke answers two reflection questions.



Joe answers two reflection questions.



Olvin answers two reflection questions.

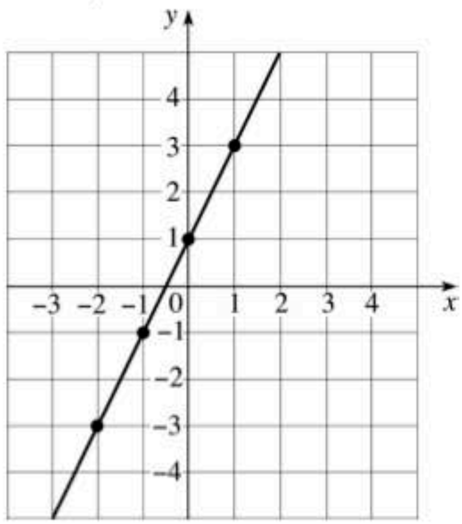
Figure 4.32. Individual Team 2 student reflections from Mathematical Task #4 by Anna, Brooke, Joe, and Olvin.

Understanding Before Whole Class Discussion at the End of the Intervention

Mathematical Task 8 (M8): Write and Verify Equation from a Graph (*picture-graph* to *verbal/written models*). Figure 4.33 displays Mathematical Task #8 from the constructivist teaching experiment. The mathematical task was selected from a Standards-based curriculum with modifications by the research team to meet the study goals emphasize communication and representations based on the LTM. Mathematical Task #8 presented students with two linear, non-proportional graphs and asked students to examine two graphs to answer two questions with multiple subparts: (a) describe the pattern using words or a table (slope), (b) when x = 0, y = ?___; (y-intercept); (c) write an equation for the relationship. Use part A and part B to help you; and (d) test your equation. Make sure it works for three coordinates. *Figure 4.33* below displays Mathematical Task #8 questions with subparts. Students with conceptual understanding were expected to show their work correctly using the graph, words, a table, and an equation.

Math Task #8

Below are two graphs that show a certain pattern. Figure out the pattern and complete the questions below.

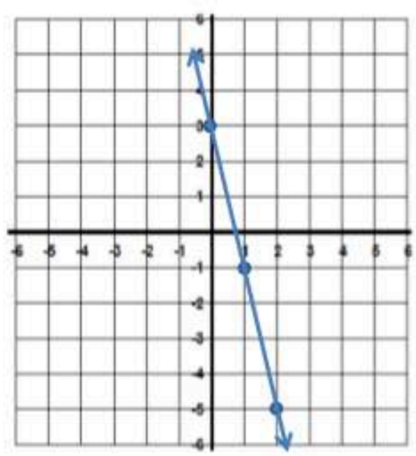


A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

D, Test your equation, make sure it works for 3 coordinates.



A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

D, Test your equation, make sure it works for 3 coordinates.

Figure 4.33. Mathematical Task #8 questions from constructivist teaching experiment.

Team 1 student work before participation in mathematical discussion (M8).

Leon (HEP), Lucas (MEP), and Sandy (MEP) answered seven out of eight question subparts *correctly*. For Question 1a, when asked to describe the pattern (slope) using words or a table, for graph one, Team 1 students created a vertical table of values with x-values from 0 to 2, and y-values from 1 to 5, “Every time the y (y-values) increases by 2” (*picture to picture, verbal/written models*). For Question 1b, when asked to find the y-intercept, students found the correct y-value of one, “When x = 0, y = 1.” For Question 1c, when asked to write an equation for the relationship and use part A and part B, students wrote the correct equation and a smiley face, “y = 2x + 1 ☺” (*picture to written model*). For Question 1d, when asked to test their equation and make sure the equation works for three coordinates, students organized their work using three columns, one for each coordinate: for (-1, 1), students wrote, “2(-1) + 1 = -2 + 1 = -1; for (1, 3), students wrote, “2(1) + 1 = 2 + 1 = 3;” and for (0, 1), students wrote, “2(0) + 1 = 0 + 1 = 1.” For Question 2a, when asked to describe the pattern using words or a table, students created a vertical table of values, x-values from 0 to 2, and y-values from 3 to -5, “Every time the y (y-values) increases (decreases) by 3” (*picture to picture, verbal/written models*). For Question 2b, when asked to find the y-intercept, students found the correct y-value of three, “When x = 0, y = 3.” For Question 2c, when asked to write an equation for the relationship and use part A and part B, students wrote the correct equation and a circled the y-intercept and labeled it, “starting value,” and wrote, “y = -4x + 3” (*picture to written model*). For Question 2d, when asked to test their equation and make sure the equation works for three coordinates, students ran out of time. In sum, Team 1 students answered seven out of eight question subparts *correctly (2)* and justified *conceptually (2)*. The team poster was coded as having *conceptual understanding (Score: 4)* because students answered correctly and conceptually, and used multiple representations to support their solutions, even when not asked to.

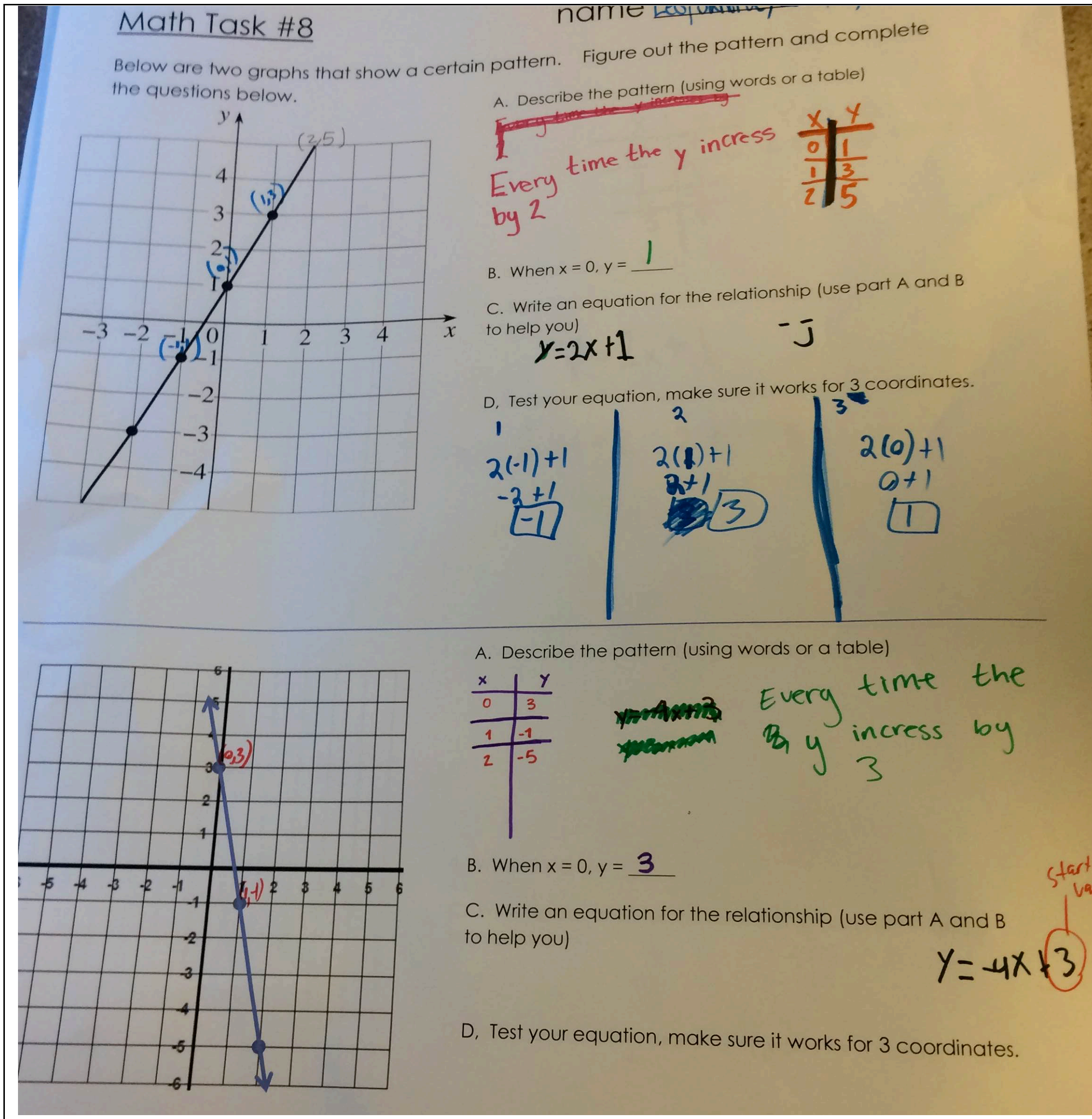


Figure 4.34. Team 1 poster of Mathematical Task #8 by Leon, Lucas, and Sandy.

Team 2 student work before participation in mathematical discussion (M8). Anna (LEP), Brooke (MEP), Joe (LEP), and Olvin (LEP) answered seven out of eight question subparts *correctly*. For Question 1a, when asked to describe the pattern (slope) using words or a table, for graph one, Team 2 students described the pattern (slope) for x- and y-values as two changing quantities, “y = it is going up by 2 and x = is going up by

1" (*picture to verbal/written models*). For Question 1b, when asked to find the y-intercept, students found the correct y-value of one, "When x = 0, y = 1." For Question 1c, when asked to write an equation for the relationship and use part A and part B, students wrote the correct equation, "y = 2x + 1" (*picture to written model*). For Question 1d, when asked to test their equation and make sure the equation works for three coordinates, students organized their work using three columns using coordinates (0, 1), (1, 3), and (2, 5): "2(0) + 1 = 1; 2(1) + 1 = 3; 2(2) + 1 = 5." For Question 2a, when asked to describe the pattern (slope) using words or a table, students correctly described x- and y-values as two changing quantities, "x is changing by 1's and y = is going down by 4's" (*picture to verbal/written models*). For Question 2b, when asked to find the y-intercept, students found the correct y-value of three, "When x = 0, y = 3." For Question 2c, when asked to write an equation for the relationship and use part A and part B, students wrote the correct equation, "y = -4x + 3" (*picture to written model*). For Question 2d, when asked to test their equation and make sure the equation works for three coordinates, students ran out of time. In sum, Team 2 students answered seven out of eight question subparts *correctly (2)* and justified *conceptually (2).* The team poster was coded as having *conceptual understanding (Score: 4)* because students answered the questions correctly and conceptually, and used multiple representations to support their solutions.

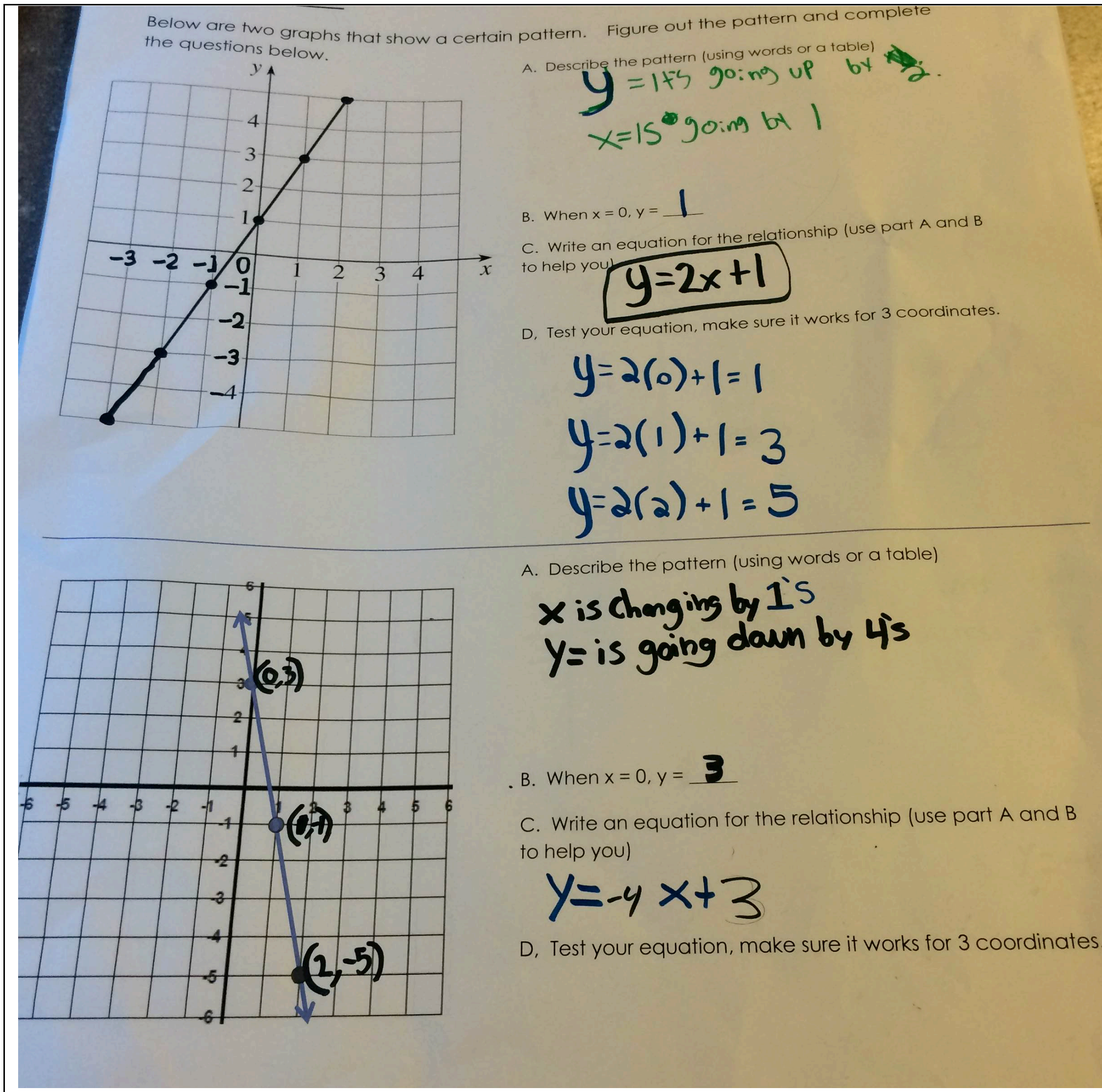


Figure 4.35. Team 2 poster of Mathematical Task #8 by Anna, Brooke, Joe, and Olvin.

The following sections present data from student teams *during* whole class discussion for Mathematical Task #8.

Understanding During Whole Class Discussion at the End of the Intervention

Team 1 student work during participation in whole class discussion (M8). During whole class presentation of the team poster, Leon, Lucas, and Sandy presented third out of three teams. The class was short on time so the research team chose three groups to present. The research team asked Team 1 students to describe how to write an equation given a graph (*picture to written model*). Leon took the lead and identified the y-intercept as a coordinate, (0, 1) and the slope as two using informal language:

> MC: Leon's group, present yours. Let's go quickly, 2 minutes. Leon I have a couple questions for you. Okay, make sure you are listening carefully. On this one, your goal Leon is to see if in your team can describe how to write the equation given the graph? How do you write the equation from the graph? Explain using the graph that is what I want you to show.
> L: When using the graph like over here.
> MC: The first one, yeah.
> L: From zero, from y is 1, you go up to 3 that is 2, and from the beginning of class since you explained that pattern goes first so and then over here the starting value is 1 so it goes right here and we have our equation. (*"From zero," refers to the y-intercept; "up to 3," refers to y-value = 3, and "2" refers to slope since Leon points to 2x from equation, y = 2x + 1. He points to the y-intercept on the graph*)

The research team used semi-formal language that matched the teaching experiment to engage in discussion with students, including "starting value" for y-intercept, and "pattern" for slope, to talk about direction of the line given a positive slope. Team 1 students used coordinates from the graph to translate their equations (*picture to verbal/written models*):

> MC: Okay, so the starting value in that case is 1 right? So where does it show up on the graph? Point to it on your graph and Leon describe where they should be pointing. (*Starting value refers to y-intercept in the teaching experiment*)
> L: Right here.
> (*Lucas points to y-intercept on the graph*)
> MC: Which is what coordinate?
> L: (0,1).
> MC: (0,1), so could you describe how would you find the starting value if you are just looking at a graph? What would you look for?
> L: You would look for from the y, from the first coordinate to the second coordinate, 0 to 1 from x, how much the y value increases by.

The role of the research team included encouraging student to explicitly translate from the graph to the equation in writing and orally (*picture to verbal/written model*). For the second question using the second graph, Leon wrote while he verbally explained how the coordinates from the graph translated to the equation, from the y-intercept to the negative slope. Once again, informal language was used including “starting value” for y-intercept and “pattern” for slope:

> MC: Okay, use that, apply that to explain how you did the equation for the second one. If you didn’t give the equation for the second one, listen carefully.
> L: Let me grab a marker really quick to write the coordinates. For the second one this one was (0, 3). (*Leon labels with the coordinate pair with a marker*)
> MC: So where does that show up in the equation write it on the poster if you don’t have it.
> L: In the equation it comes up here. (*Leon circles the 3 from the equation, y = -4x + 3*). So the starting value (*Leon labels the 4 from the equation as, “starting value”*).
> MC: So that’s the part you multiply by or you add?
> L: You add.
> MC: What number are we adding?
> L: 3.

The research team asked open-ending, pushing/probing questions about when to add a value and when to multiply, to support moving towards conceptual understanding and to identify gaps. A student from Team 2, Olvin also contributed to Team 1 presentation, but was confused the y-intercept of 3 as slope of 3. Leon from Team 1, helped to clarify that the y-intercept was 3 using informal language that mirrored the teaching experiment:

> MC: Olvin, why are we adding 3 in our equation? (*Teacher asks a student from outside of Team 1, Olvin is in Team 2*)
> O: We are going up by 3.
> MC: We are not going up by 3. He’s not clear. We are not going up by 3. Where did the 3 come from?
> L: The start. The starting value, the (0, 3).

The research team explicitly asked all students to point to the y-intercept coordinate, (0, 3) on the graph, and to connect the picture to the “+3” in the equation, y = -4x + 3. The research team also asked students to identify how they know the slope was negative from the graph:

> MC: Everybody point to that in your paper (0,3). That's where that starting value comes and you turn that into plus 3. Okay, keep going. I don't know what is going to be the multiplication number? What are you going to multiply by?
> L: It's one." (*Leon counts*)
> (*Sandra whispers*)
> S: It is right there on the white board.
> L: Well over here, it is -1.
> MC: Is it?
> L: It is (1, -1).
> MC: Oh, (1, -1).
> L: (1, -1), so from 3 to 1, you go 3, 2, 1, 0, -1, that's 4. You would decrease by 4.
> (*Earlier he mentioned -1, so it should be, "from 3 to -1," and not "from 3 to 1." Leon starts counting with his hands*)
> MC: Why is it negative? Why is it negative in this case?

In the excerpt, Leon explained that the y-values decreased by four, but he does not name the specific values as x- or y-values, so the research team encouraged Leon to explain it to a different classmate, who also was not sure. Leon described that the line was decreasing, and moved to describing specific y-values as decreasing. The research team asked Leon to state which values he was using (x- or y-values), and Leon shared correctly that he was referring to the y-values on the graph:

> L: Because it decreases by 4.
> (*deleted student input that was not part of the study*)
> MC: Say it again, Leon.
> L: Because it decreases.
> MC: Now I am going to ask you to be clear. What decreases by 4?
> L: The pattern from 3 to -1. From the coordinate over here to the coordinate over there. (*Refers to the y-intercept and then to the next coordinate*)
> MC: What coordinates the x's or the y's?
> L: The y's, not the x! No y!
> MC: Okay, what I want you to do we have 5 minutes flip your paper, small sheet of paper over. By yourself again do your best on this. Hopefully we got good explanations from our team mates so we can write these things down. You only

> have 5 minutes to do 3 questions so make sure you are working quietly and urgently. When you are done raise your hands and I will check it.

In sum, Team 1 students presented the correct solutions using multiple representations and informal language to explain their work. Their presentation was coded as *conceptual understanding (Score: 4)* because students answered the questions correctly and used multiple representations to justify.

Team 2 student work during participation in whole class discussion (M8).

During whole class presentation of the team poster, Anna, Brooke, Joe, and Olvin presented first out of three teams. The class was short on time so the research team chose three teams to present out of four teams. Olvin presented first, followed by Anna, Brooke and then Joe. For Question 1a, Olvin used the graph to describe that the y-values increased by two units, and the x-values increased by one unit:

> O: "The pattern, well what I thought the pattern was that the y each time the pattern yeah was going up by two in the y. (*y refers to the y-values*)
> (*Olvin points to y-axis on graph and motions up and down movement*)
> O: And it was going by one in the x. (*Refers to the x-values, from the poster, "x = is going up by 1" for graph 1 and, "x is changing by 1's" for graph 2 written work*)
> (*Olvin points to x axis on graph and motions left to right*).
> MC: Okay.

For Question 1b, Anna, presented the equation with the correct slope of 2 and y-intercept of 1. Brooke added that the team verified using three coordinates, (0, 1), (1, 3), and (2, 5):

> A: For the equation, the equation is y = 2x + 1.
> B: And we checked the equation by making sure that y equals 2 times 0 plus 1 equals 1. And then y would equal 2 times 1 plus 1 would equal 3. And then y equals 2 times 2 plus 1 and that equals 5. (*Walks over to look at student work*)

The role of the research team included encouraging students to verify their solutions. Joe described the changes in x- and y-values, while Brooke described by plugging in coordinate points from the graph. The research team acknowledged that their team was the only team to describe the pattern for x- and y-values. At the end of the presentation, a member from Team 1, Leon, described the pattern (slope) as changes involving x-values by one and y-values by two:

> MC: Okay, cool. How did you get your equation?
> J: We did x is changing by 1.
> MC: How did you come up with 2x + 1? Can anybody explain that? It's ok to say we just kind of guessed and that was the equation that came up that worked.
> B: Oh I know, I know. Because we at 1 if we multiplied by 2 it would equal 2 and then plus 1 it would equal 3. If we tried to multiply 1 by 3 it would equal 3 and then 2 would have to equal 6 so we have to multiply by 2 each time and add 1.
> MC: Okay, great. Describe your pattern for the second one and we are going to move on. What is the pattern for the second one?
> B: Joe wrote down that x is changing by 1s and that y is going down by 4.
> MC: Okay two things I want you guys to notice. Track to their poster. They were the only group I think that actually wrote out what the pattern is. One this else that I think is interesting is they said there are two patterns, does anybody know what the two patterns they said were? Leon? (*Leon is a member from Team 1*)
> L: That x is increasing by 1 and and y is going up by 2.
> MC: Interesting ok…Think about that.

In sum, Team 2 students presented the correct solutions using multiple representations and informal language to explain their work. Their presentation was coded as *conceptual understanding (Score: 4)* because students answered the questions correctly and used multiple representations to justify.

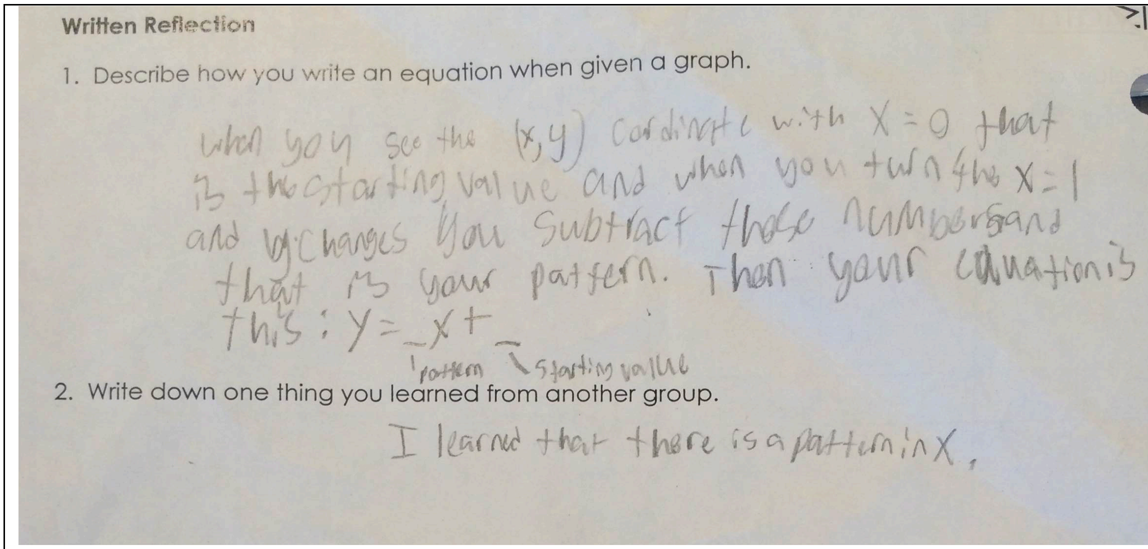
Understanding After Whole Class Discussion at the End of the Intervention

During the *connect* stage of the mathematical discussion framework (Stein et al, 2009), all students were given a handout, *Written Reflection,* with two prompts written by the research team: (1) Describe how you write an equation when given a graph. (2) Write down one thing you learned from another group.

Math Task #8
Written Reflection
1. Describe how you write an equation when given a graph.
2. Write down one thing you learned from another group.

Figure 4.36. Mathematical Task #8 reflection prompts for individual student write up. For the written portion, students were asked to individually reflect on two questions written by the research team.

Team 1 student written work after participation in mathematical discussion (M8). After peer presentations and participation in mathematical discussions that emphasized *connections*, Team 1 students answered two reflection questions individually. Leon (HEP) described how to write an equation using coordinates from a graph to identify the y-intercept when x = 0, and that the y-values patterns were the slope, "When you see the (x, y) coordinate with x = 0 that is the starting value (y-intercept) and when you turn the x = 1 and y changes you subtract these numbers and that is your pattern (slope). Then your equation is this: y = __x + ___." He drew two arrows to label the blanks as, "pattern," and "starting value," respectively. When asked to write down one thing he learned from another group, Leon said that the pattern (slope) was two changing quantities including x-values, and not only y-values, "I learned there is a pattern in x" (*picture to written/verbal models*). Lucas (MEP) described how to write an equation using a two templates, one for non-proportional linear relationships, and the other for proportional linear relationships. He wrote, "y = __x + __, or y = __x, and figure out the pattern, the graph and find the equation." When asked to write down one thing he learned from another group, he wrote that both equations were negative instead of the second graph, "That one got negative but it was both" (*picture to written/verbal models*). Sandy (MEP) described how to write an equation using the template from the teaching experiment, "y = __x + ___." She drew two arrows to label the blanks as "pattern," and "starting value," respectively and described in words work included slope and y-intercept, "First I find the starting value (y-intercept). Then I find the pattern and finally put the pattern and starting value together." When asked to write down one thing she learned from another group, she wrote about the slope as -3 and not -4, "That one group thought it was -3 instead of -4" (*picture to written/verbal models*). In sum, students reflected on writing equations in ways that aligned with the strategies in the teaching experiment. Their written work was coded as *correct* and *conceptual understanding* because students used more than one representation: written, words, equation, and graph.



Written Reflection

1. Describe how you write an equation when given a graph.

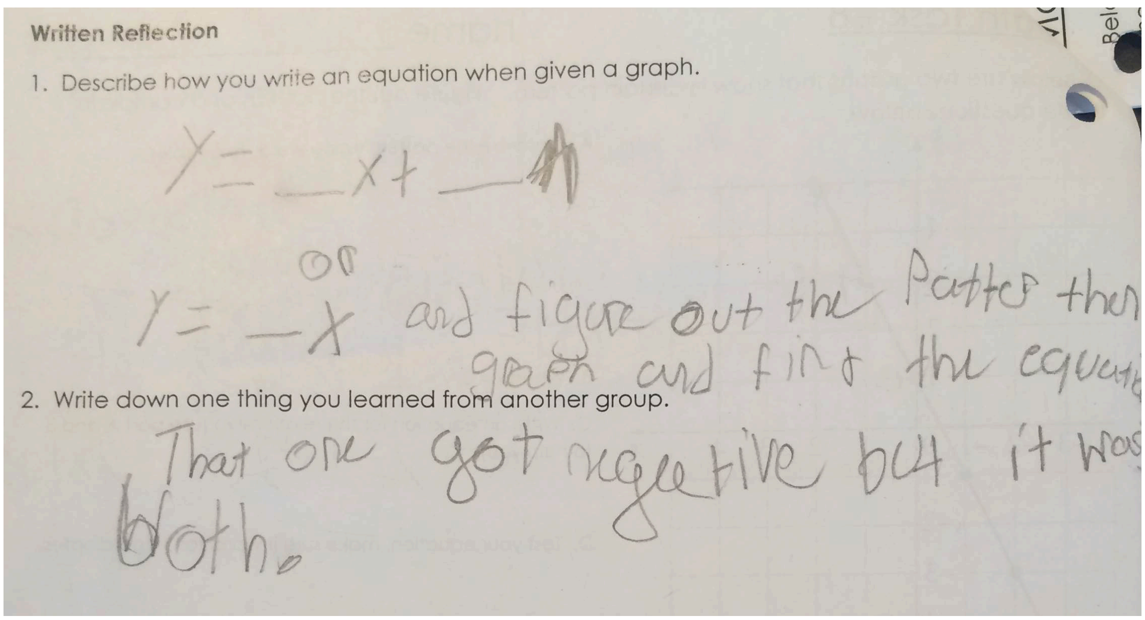
when you see the (x,y) cordinate with X = 0 that is the starting value and when you turn the X = 1 and y changes you subtract those numbers and that is your pattern. Then your equation is this: y = _x + _

pattern starting value

2. Write down one thing you learned from another group.

I learned that there is a pattern in X.

Leon answers two reflection questions.



Written Reflection

1. Describe how you write an equation when given a graph.

Y = __x + __

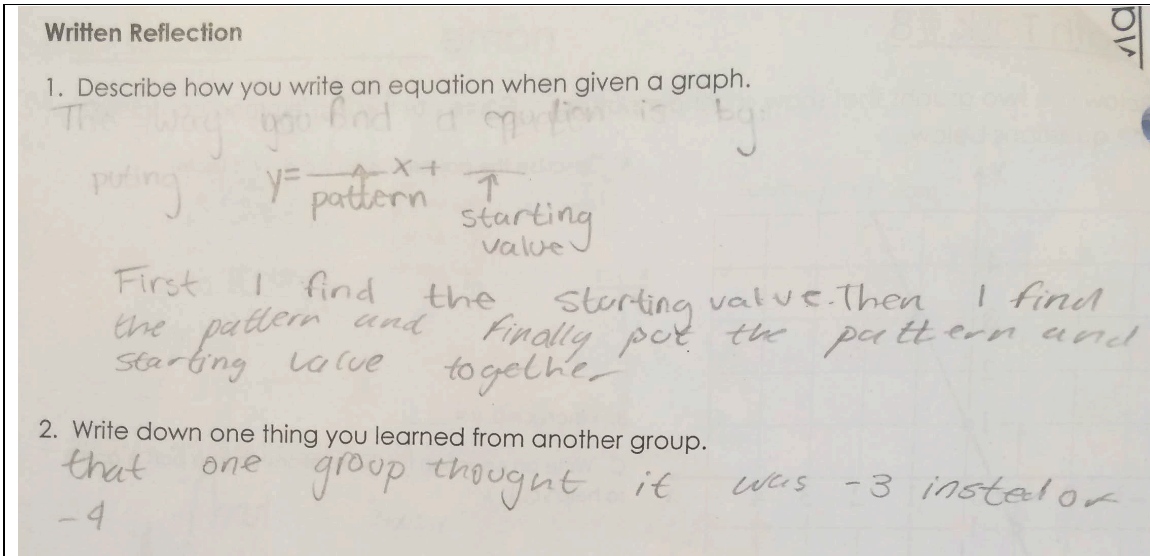
or

Y = __X and figure out the Patter then graph and find the equati

2. Write down one thing you learned from another group.

That one got negative bc4 it was both.

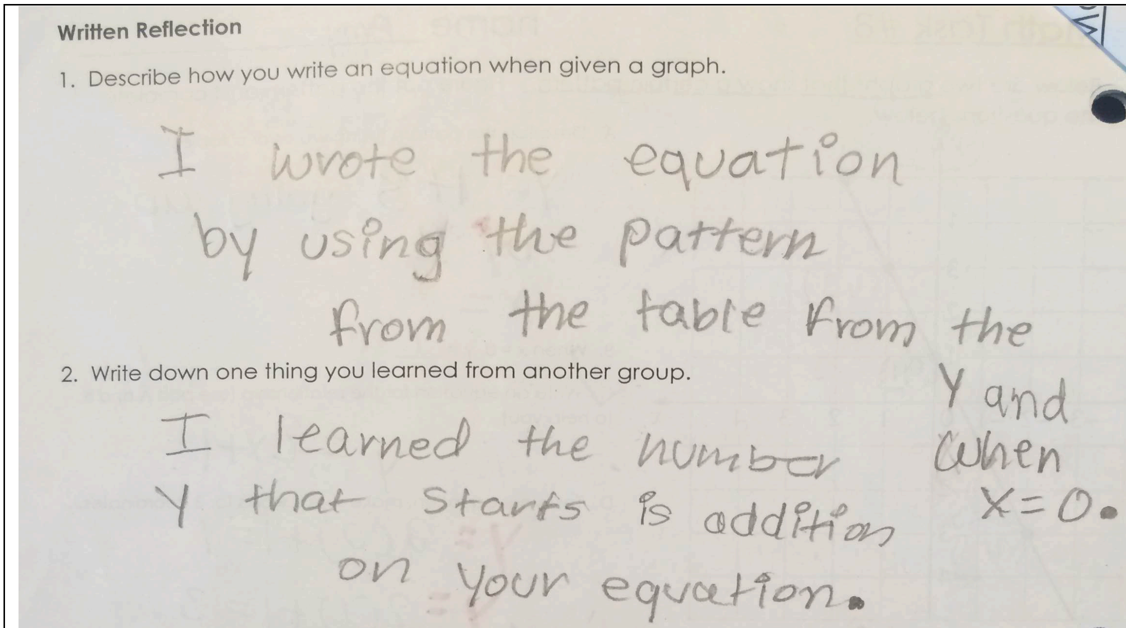
Lucas answers two reflection questions.



Sandy answers two reflection questions.

Figure 4.37. Individual Team 1 student reflections from Mathematical Task #8 by Leon, Lucas and Sandy.

Team 2 student written work after participation in mathematical discussion (M8). After peer presentations and participation in mathematical discussions that emphasized *connections*, Team 2 students answered two reflection questions individually. Anna (LEP) described how to write an equation, she explained that she used the table to identify the slope and the y-intercept, “I wrote the equation by using the pattern from the table from the y and when x = 0.” When asked to write down one thing he learned from another group, she wrote that the y-intercept is the value being added in the equation from the graph, “I learned the number y that starts is addition on your equation” (*picture to written/verbal models*). Brooke (MEP) described how to write an equation from a graph and by guessing, “I write an equation when given a graph by guessing ‘til it’s right.” When asked to write down one thing she learned from another group, she wrote that another group supported her writing an equation for the second graph, “I learned how they got their equation for #2c” (*picture to written/verbal models*). Joe (LEP) described how to write an equation as an equation with no explanation, “I wrote it like y = 2x + 1.” When asked to write down one thing he learned from another group, he wrote that the other group had different equations than his team, “One thing I learned was that other groups had kinda different equations” (*picture to written/verbal models*). Olvin (LEP) described how to write an equation as using a template from the teaching experiment using the pattern (slope) and starting value (y-intercept). He wrote, “I write y = __ x + __ in the first, the pattern and the second line the starting unit.” When asked to write down one thing he learned from another group, he wrote that his team all got the same equation, “We all got same equation” (*picture to written/verbal models*). In sum, students reflected on writing equations in ways that aligned with the strategies in the teaching experiment with more than one representation. However, they had a challenging time articulating their explanations but the data showed they were aware that slope value is to be multiplied by the x-value, *k*x, and the y-intercept is added. In sum, students reflected on writing equations in ways that aligned with the strategies in the teaching experiment. Their written work was coded as *correct* and *conceptual understanding* because students used more than one representation: written, words, equation, and graph.


Written Reflection

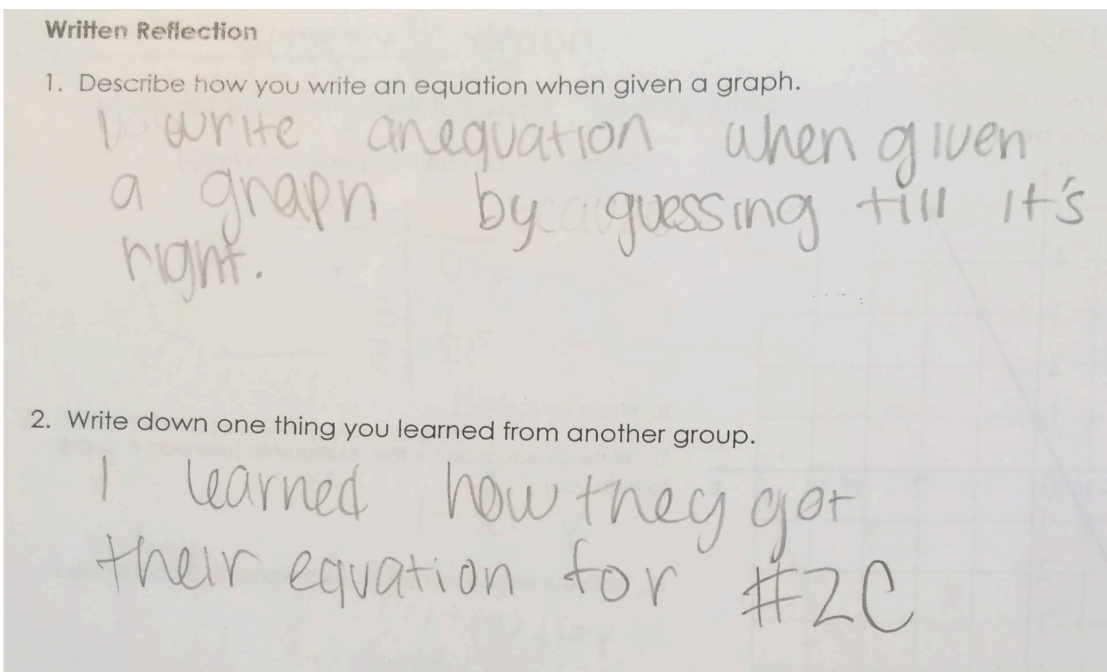
1. Describe how you write an equation when given a graph.

I wrote the equation by using the pattern from the table from the Y and when X=0.

2. Write down one thing you learned from another group.

I learned the number Y that starts is addition on your equation.

Anna answers two reflection questions.


Written Reflection

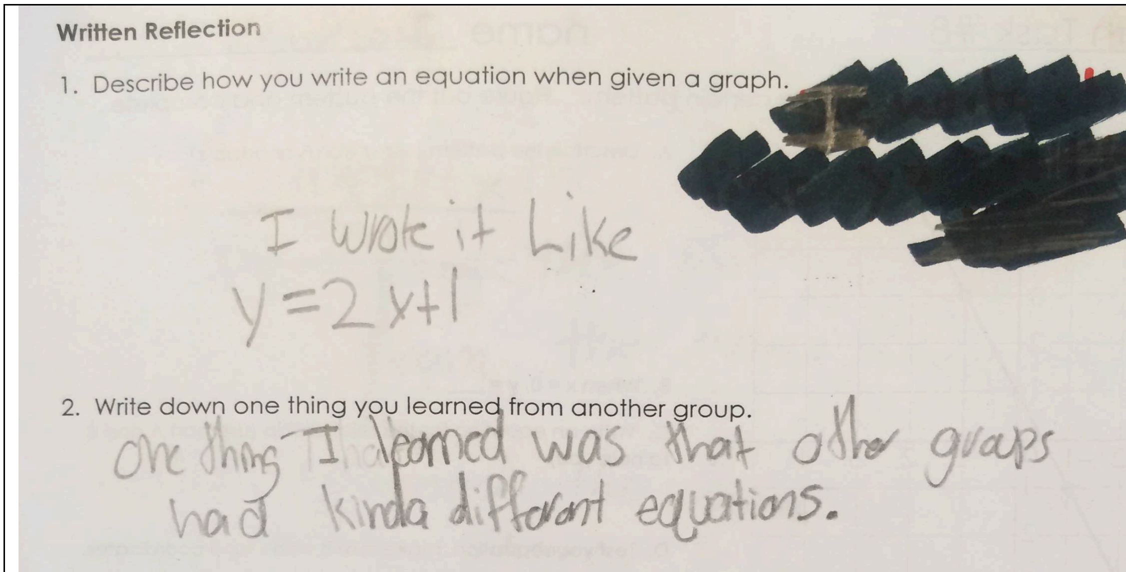
1. Describe how you write an equation when given a graph.

I write anequation when given a graph by guessing till it's right.

2. Write down one thing you learned from another group.

I learned how they got their equation for #2C

Brooke answers two reflection questions.



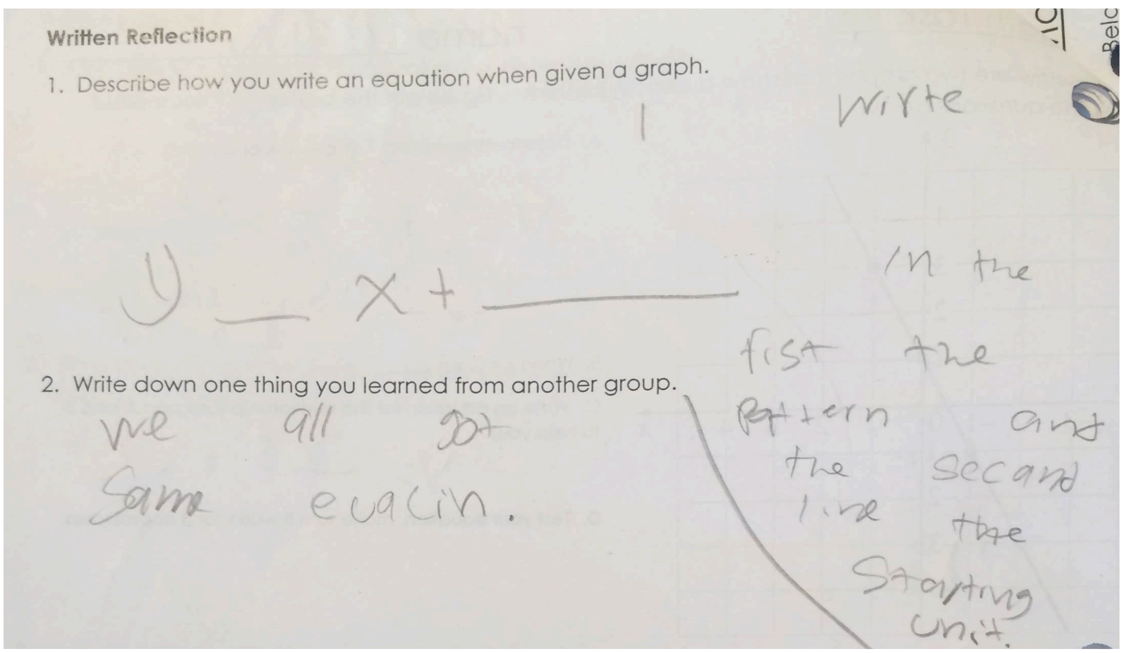
Written Reflection

1. Describe how you write an equation when given a graph. [redacted]

I wrote it Like
$y=2x+1$

2. Write down one thing you learned from another group.
One thing I learned was that other graps had kinda diffarent equations.

Joe answers two reflection questions.



Written Reflection

1. Describe how you write an equation when given a graph.

I write y ___ x + ________ in the fist the pattern and the secand line the Starting unit.

2. Write down one thing you learned from another group.
we all got Same euacin.

Olvin answers two reflection questions.

Figure 4.38. Individual Team 2 student reflections from Mathematical Task #8 by Anna, Brooke, Joe, and Olvin.

Summary of Understanding After Participation in Whole Class Discussions Emphasizing *Connections* Among Peer Presentations by Themes

In the previous sections, data from student team posters, video transcripts of student team presentations, and individual student written reflections after participation in the whole class discussions were presented that documented the nature of seven middle school immigrant students' conceptual understanding of linear function ideas as they related to identifying slope and y-intercept, graphing, writing equations, and interpreting solutions using real-life situations from graphs and tables. The following sections present a descriptive summary of students' conceptual understanding of linear function ideas that focus on two mathematics processes, communication and representations, as they relate to research question three (NCTM, 2000):

> 3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Figure 4.39 displays a summary of understandings collected from multiple data sources. The findings revealed: (1) students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$); (2) students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings; and (3) students' English language proficiency was not associated with conceptual explanations. The following sections summarize findings by individual mathematical tasks from the beginning, middle and end of the constructivist teaching experiment as well as the embedded translations to support the themes displayed in Figure 4.39.

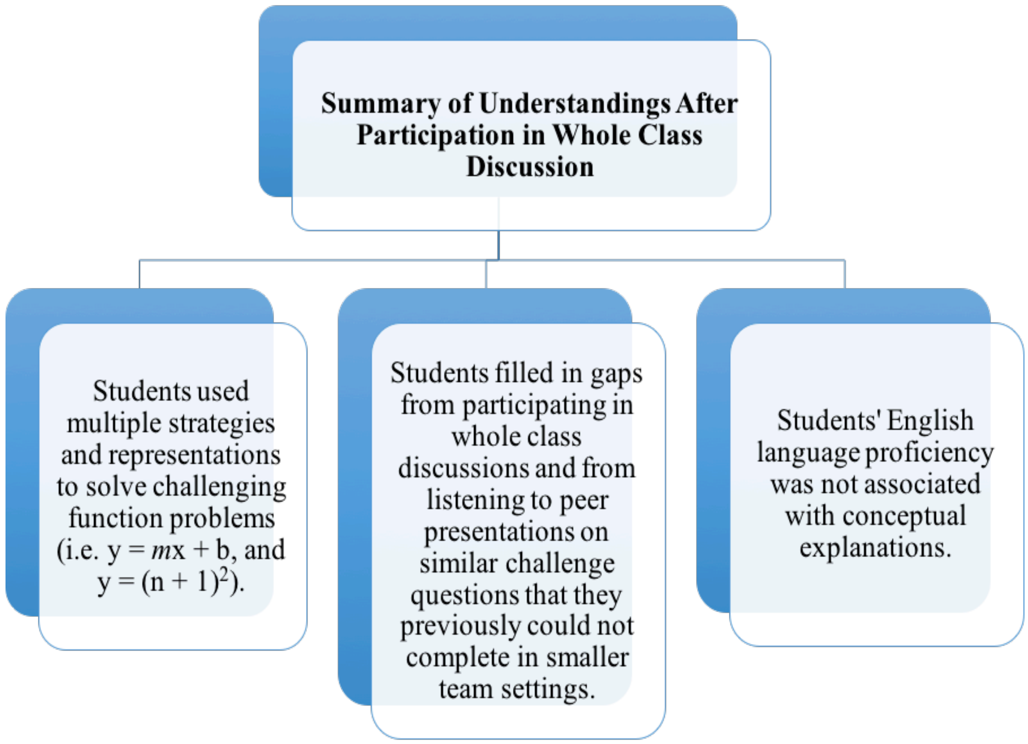


Figure 4.39. Summary of characteristics of understandings after participation in whole class discussion that emphasized connections among peer presentations.

Mathematical Task #1 (*picture-tiles to verbal/written models, and picture-tiles*)

Students were asked to find the total number of algebra tiles for Figure 6 and Figure 10 given Figure 2, Figure 3, and Figure 4. Although Team 1 students had more students with higher English proficient students with one HEP, one MEP and two LEP students (one of the LEP student dropped out of the study), the team did not answer the question correctly. Team 1 students attempted to solve the problem using multiple representations including a numeric expression, pictures of algebra tiles, and a counting strategy. Team 2 students included three LEP students and one MEP students, but with equally mixed mathematics ability as Team 1, correctly answered the question using multiple representations including numeric expressions and pictures of algebra tiles. The findings from students' written work on the team posters showed that students' language proficiency was not tied to their ability to complete tasks with conceptual understanding. Therefore, data from Mathematical Task #1 showed that, (a) *students used multiple strategies and representations to solve challenging function problems (i.e.* $y = (n + 1)^2$*);* and that (b) *students' English language proficiency was not associated with conceptual explanations.*

Further, the data from individual students' written reflections that participated in whole class discussions which sequenced least conceptual to rich conceptual explanations showed that Team 1 student responses all improved from coded as *incorrect* to coded as *correct.* The data from Mathematical Task #1 showed that prior to participation in whole class discussion, none of Team 1 students correctly found the total for Figure 6 and Figure 10. Afterwards, all students correctly found the total for Figure 15 and Figure 100 individually, using numerical expressions, from drawing algebra tiles and counting tiles. Similar findings were also true for individual Team 2 students. This is important to point out since critics of heterogeneous mathematics settings argued that LEP/MEP students are not prepared, or may rely on their MEP/HEP peers (Stein, et al, 2009). The individual student reflections revealed that *all* students completed the more challenging question on their own as a result of participation in whole class discussion and listening to peer presentations. Therefore, the findings support findings from Mathematical Task

#1 as well that, (a) *students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings;* and that (b) *students' English language proficiency was not associated with conceptual explanations.*

Mathematical Task #4 (*picture-table to picture-graph, verbal/written models, and real-life situation*)

Students were presented with a table given a real-life situation and were asked to find missing y-values, graph, identify the y-intercept, predict future values, and write and verify equation. The data showed that students correctly answered a majority or all of the questions correctly in team settings. Data showed that both teams used a written model, graph, table and a verbal model to correctly communicate their solution and justification. Team 1 students answered all questions correctly and conceptually, and Team 2 students answered all but two sub-questions correctly out of six sub-questions. Team 2 students incorrectly wrote the equation, therefore could not verify the equation correctly. This finding supports that students' language proficiency was not tied to their ability to complete tasks with conceptual understanding. Therefore, data from Mathematical Task #4 showed that, (a) *students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$);* and that (b) *students' English language proficiency was not associated with conceptual explanations.*

Further, the data from individual students' written reflections that participated in whole class discussions which sequenced least conceptual to rich conceptual explanations showed that six out of seven students improved on writing equations coded as *incorrect,* to coded as *partially correct* (1 student), and coded as *correct* (5 students). The individual student reflections showed that a majority of students improved by completing the task on their own that they previously could not. Therefore, the findings showed evidence that *students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings.*

Mathematical Task #8 (*picture-graph to verbal/written models*)

Students were presented with two non-proportional linear graphs, the first with a positive slope and the second with a negative slope, and students were asked to describe the slope as two patterns using words or a table, identify the y-intercept, write an equation, and test the equation using three coordinates. Both teams performed equally well and answered seven out of eight sub-questions correctly using multiple representations, but ran out of time. The data showed that both teams used a written model, graph, table and a verbal model to communicate their solution and justification. This data showed that language proficiency was not tied to their ability to complete tasks with conceptual understanding. Therefore, data from Mathematical Task #8 showed that, (a) *students used multiple strategies and representations to solve challenging function problems (i.e. y =mx + b);* and that (b) *students' English language proficiency was not associated with conceptual explanations.*

Further, the data from individual students' written reflections that participated in whole class discussions which sequenced least conceptual to rich conceptual explanations showed that all students correctly explained how to write an equation. Recall that midway through the teaching experiment on Mathematical Task #4, Team 2 students could not write a correct equation on their team poster. This finding suggests that *students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings.*

In summary, multiple data sources were collected to answer research question three that showed increases in students' conceptual understanding individually and in teams. This finding is evidenced by students' communication in writing and orally as well as students use of representations on mathematical tasks at the beginning, middle and end of the constructivist teaching experiment. The data showed that all students improved their solution codes from *incorrect* to *correct* throughout the teaching experiment. Furthermore, students demonstrated understanding individually as a result of participation in whole class discussions on tasks that previously could not as a team as shown by their written reflections. Therefore, findings from various points of the teaching experiment suggest several benefits of a structured heterogeneous mathematics classroom that sequenced student presentations from least conceptual to rich conceptual explanations. The results are summarized as: (1) students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$); (2) students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings; and (3) students' English language proficiency was not associated with conceptual explanations.

Summary of Responses to Three Research Questions

Research Question 1. Research question one was concerned about the ways that immigrant students demonstrated their conceptual understanding of key ideas of linear functions prior to participation in the constructivist teaching experiment, including finding a missing y-value using slope (*picture to verbal model*), graphing (*picture to picture*), writing an equation (*picture to written model*), interpreting real-life situations (*picture to real-life situation*) from a table, and writing a real-life situation for a table (*picture to real-life situation*). After several cycles of reviewing pre-assessment interview transcripts, three themes emerged around the ways in which students communicated and used representations to explain their mathematical processes. The data suggested that: (1) students gave procedural justifications when asked solution-oriented questions (versus process-oriented questions); (2) students gave conceptual justifications when asked open-ended questions that encouraged multiple perspectives; and (3) a real-life situation supported more conceptual justifications.

Research Question 2. Research question two was concerned about the ways that immigrant students demonstrated their conceptual understanding of key ideas of linear functions after participation in the constructivist teaching experiment, including completing a table with missing y-values using slope (*picture to verbal model*), writing an equation (*picture to written model*), solving linear equations and interpreting solution using a real-life situation (*picture to real-life situation*), and discussing characteristics of proportional linear functions using more than one representation from a table of values given a real-life situation (*picture to verbal/written models,* and *real-life situation*). After several cycles of reviewing post-assessment interview transcripts, three themes emerged around the ways in which students communicated their mathematical processes. The data suggest that: (1) students gave conceptual justifications when asked process-oriented questions; (2) students used more than one representation to justify when asked to do so; (3) students gave conceptual justifications using a real-life situation when given a meaningful real-life situation at the beginning.

Research Question 3. Research question three was concerned about the characteristics of immigrant students written and oral understanding of key ideas of linear functions that participated in the constructivist teaching experiment that emphasized connections among representations post whole class discussion and peer presentations. To answer research question three, data from students' written work on team posters, video transcripts of peer presentations and whole class discussion, and individual student written reflections were presented that documented the nature of seven middle school immigrant students' conceptual understanding of linear function ideas. Recall that the data presented snapshots of student work *before, during* and *after* whole class discussion from the beginning (Mathematical Task #1), middle (Mathematical Task #4) and end (Mathematical Task #8) of the constructivist teaching experiment. After several cycles of reviewing various data, three themes emerged around the ways in which students communicated their mathematical processes. The data suggested that: (1) students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$); (2) students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings; and (3) students' English language proficiency was not associated with conceptual explanations.

Conclusion

This chapter presented the results of seven middle school immigrant students' written responses and interview transcripts in pairs working on pre- and post-assessments, and on mathematical tasks that were designed by the researcher with input from the classroom teacher. All interviews were conducted by the researcher during regular school hours and lasted from 45 minutes to 1.5 hours. The pre-assessment interviews were conducted in the first week of the eight-week unit, and the post-assessment interview was conducted after eight weeks, or after ten constructivist teaching experiments. The ways that students communicate and use representations were of interest before and after to the constructivist teaching experiment. The ways that students communicated their solution and reasoning was documented, coded numerically and descriptively and summarized as well. In the next chapter, these results will be discussed, along with the contribution of the to the literature, the limitations and the implications of the study. Finally, recommendations for future research will be made.

Chapter 5

Summary, Conclusions, and Implications

There is an urgent call to disrupt deficit views of immigrant students by providing effective evidence-based strategies that support conceptual understanding for *all* students. Immigrant students are victims of lowered-expectations and often placed in low-track mathematics courses absent of rich problem-centered environments. Reports show that immigrant students consistently perform in the bottom third of national standardized assessment (Minnesota Department of Education [MDE], 2015; National Center for Educational Statistics [NCES], 2007; Senk & Thompson, 2003). In response to concerns about the quality of mathematics education for all students the professional organization, National Council of Teachers of Mathematics [NCTM] (2000), identified “equity” as its first principle. However, research shows that teachers of immigrant students are not prepared to teach students from diverse cultural and language backgrounds (Campbell, Adams, & Davis, 2007). Continued efforts are needed to improve the quality of mathematics teaching and learning and to make classrooms more inclusive for middle and high school immigrant students. These shifts from deficit views of immigrant students include access to rigorous mathematics courses and increased enrollment in more competitive mathematics and science courses associated with access to higher education (Paul, 2005; Wang & Goldschmidt, 1999). Therefore, challenging deficit views includes de-tracking mathematics classrooms and implementing more intentional pedagogical supports that align with equity goals (NCTM, 2000).

Recent studies in mathematics education surrounding immigrant students shifted to more constructivist teaching approaches to support conceptual understanding (Blanton, Brizuela, Gardiner, Sawrey & Newman-Owens, 2015; Campbell, Adams & Davis, 2007; Civil, 2014; Hansen-Thomas, 2009; Huntley, Marcjus, Kahan & Miller, 2007; Stein, Smith & Hughes, 2009; Van Eerde, 2011; Whitney, 2010). This shift was fueled by a belief that *all* students, including immigrant students, can learn mathematics with conceptual understanding if provided with structured support with attention to language differences, mathematical discussion, and multiple representations (Campbell et al 2007). Huntley et al. (2007) argued that mathematically sophisticated students were associated with fluency with multiple representations. Stein et al. (2009) encouraged structured mathematical discussions to support students' sense-making in authentic ways. Further, Van Eerde (2011) advocated for more teaching approaches that are interventionist, reflective, and involve collaboration between researchers and classroom teachers to support immigrant students' development of language required for learning in multilingual classrooms. Civil (2014) recommended that teachers recognize different forms of mathematics that immigrant students bring to school as resources for teaching. Building on this notion, and to disrupt deficit views in the literature where teachers are not prepared to teach immigrant students, the purpose of this study was to document middle school immigrant students' increased conceptual understanding from participation in a constructivist teaching experiment that paid attention to language differences, multiple representations, and de- tracking in mathematics.

Summary

This qualitative study employed a descriptive case study of a constructivist teaching experiment (CTE) to document middle school immigrant students' conceptual understanding of key ideas of linear functions prior to and after participation in Standards-based mathematics curricula as intervention (Cobb, 2011; Senk & Thompson, 2003; Yin, 2014). CTEs are when researchers act as teachers and interact with students to attempt to guide constructive activities (Cobb, Yackel & Wood, 2011). The teaching experiment utilized purposeful criterion sampling to select the school, classroom teacher and student participants: the school had a high immigrant and English language learner population, the classroom teacher was not already using Standards-based mathematics curricula, and a majority of the students "did not meet" or "partially meet" grade-level mathematics standards (MDE, 2015). Two students were added to the intervention classroom to create a heterogeneous learning environment that aligned with goals of Standards-based mathematics.

Multiple instruments were used to collect data including students' written work and oral communication from transcripts in pairs on the pre- and post-assessments, students' written team work on posters, video transcripts of poster presentations, and individual students' written reflections. The pre-assessment was to document middle school immigrant students' (the participants) understanding of key ideas of linear functions prior to participation in a constructivist teaching experiment. The post-assessment was to document middle school immigrant students' understanding of key ideas of linear functions after participating in a constructivist teaching experiment that emphasized communication and multiple representations in team settings. The post-assessment was given after all students completed ten mathematical tasks during the eight-week constructivist teaching experiment. All students completed the post-assessment in the same pairs similar to the pre-assessment interview. In the case of Brooke, however, the post-assessment interview was completed without a partner because her partner dropped out of the study. For the third research question, multiple data sources from various points in the constructivist teaching experiment at the beginning, middle, and end of the teaching experiment, included: team posters, transcripts from team presentations, and individual student written reflections.

The analysis of the data enabled the researcher to discuss the ways that students communicated and used representations, and document characteristics of understandings of key ideas of linear functions prior to and after participation in the constructivist teaching experiment that emphasized two frameworks, LTM and Orchestrating Productive Mathematical Discussions. The study was guided by three research questions. The first two research questions required an analysis of middle school immigrant students' written and oral communication to glean the ways students demonstrated conceptual understandings of key ideas of linear functions prior to and after participation in a CTE that emphasized the LTM and structured mathematical discussions. The third research question was directed at examining the characteristics of middle school immigrant students' understandings as a result of participation of structured mathematics discussions. To best address the research questions, students' written work and oral communication from transcripts in pairs on the pre- and post-assessments, students written team work on posters and video transcripts of poster presentation, and individual students' written reflections were analyzed using multiple cycle coding (Miles, Huberman & Saldana, 2014; Powell, Francisco, & Maher, 2003). The following research questions guided this study:

1. In what ways do immigrant students demonstrate their understanding of key ideas of linear functions by expressing translations between and within modes of representations <u>prior</u> to participating in mathematical activities that emphasize the LTM?
2. In what ways do immigrant students demonstrate their conceptual understanding of key ideas of linear functions <u>after</u> participating in mathematical activities that emphasize multiple representations from the LTM?
3. What are characteristics of immigrant students' written and oral understanding of key ideas of linear functions who participate in mathematical discussions that emphasize connections among representations?

Table 5.1 summarizes the results of this study from multiple data sources by themes, and lists the data sources and research questions to which each finding relates by major themes from various data sources.

Table 5.1

Summary of Results of the Study by Major Themes and Research Questions

Research Questions	**Conceptual Understanding: Communication and Representation Themes**	**Data Sources**
RQ 1	• Students gave procedural justifications when asked solution-oriented questions (versus process-oriented questions). • Students gave conceptual justifications when asked open-ended questions that encouraged multiple perspectives. • A real-life situation supported more conceptual justifications.	Pre-Assessment Written Work and Interview Transcript in Pairs *(Questions 4a to 4d; and Question 5)*
RQ 2	• Students gave conceptual justification when asked process-oriented questions. • Students used more than one representation to justify when asked to do so. • Students gave conceptual justifications using a real-life situation when given a meaningful real-life situation at the beginning.	Post-Assessment Written Work and Interview Transcript in Pairs *(Questions 1a to 1d)*
RQ 3	• Students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$). • Students filled in gaps from participating in whole class discussions and from listening to peer presentations on similar challenge questions that they previously could not complete in smaller team settings. • Students' English language proficiency was not associated with conceptual explanations.	Team Posters* Video Transcript of Team Presentations and Whole Class Discussions* Individual Student Written Reflections * *(*Mathematical Task #1, Task #4, and Task #8)*

In answering the first and second research questions, student written work and interview transcripts in pairs from the pre- and post-assessments were analyzed. Student work in pairs were coded in two ways namely, solutions and justifications. Student justifications were analyzed using conceptual and procedural definitions guided by Hiebert and Lefevre (1986). Characteristics of justifications coded as *procedural* included student reasoning using, “algorithms, or rules, for completing mathematical tasks,” or a step-by-step fashion that was prescriptive when completing a task in a linear sequence (Hiebert & Lefevre, 1986, p. 4). Characteristics of justifications coded as *conceptual* included student reasoning moving away from presenting “isolated pieces of information,” and toward recognizing the, “relationship to other pieces of information” (Hiebert & Lefevre, 1986, p. 4). Justifications coded as *conceptual* were dependent upon each individual question on the pre- and post-assessments. The third research question was asked to find out the characteristics of middle school immigrant students understandings after participation in whole class discussions and listening to peer-presentations that enacted the five practices for orchestrating productive whole class discussion (anticipating, monitoring, selecting, sequencing, and connecting). Middle school immigrant students’ team posters, video transcripts of team presentations, and individual student written reflections were analyzed to document changes in students’ responses, if any.

Overall, all middle school immigrant student participants increased the number of conceptual justifications from the pre- to post-assessment at the end of the eight-week constructivist teaching experiment. The data revealed that positive outcomes were associated with student-centered environments, where the roles of the teacher included selecting rich problems, writing open-ended questions, monitoring student work, sequencing mathematical discussions from least conceptual to conceptual justifications, and asking students to connect among peer presentations. A closer examination of the findings revealed that open-ended questions given a real-life situation at the beginning as well as asking students to use multiple representations, were associated with increased conceptual justifications. Results from the study also suggested that middle school immigrant students benefitted from participating in structured team settings that emphasized whole class discussions, and students filled in gaps from listening to peer presentations, individually and as a team. Further, findings revealed that students may benefit from a heterogeneous learning environment *bi-directionally*, where low/medium English proficient students supported medium/high English proficient partners, and medium/high English proficient students supported low/medium English proficient partners.

Results from research question one showed that students gave procedural justifications when asked solution-oriented questions versus when asked process-oriented questions. Similarly, the findings showed that students gave conceptual justifications when asked open-ended questions that encouraged multiple perspectives, and when given a real-life situation. This finding was not surprising given that students were asked questions that pointed to the solution without emphasizing reasoning with multiple representations. Middle school immigrant students answered a majority of questions correctly, but did not justify when it was not required. As a result, pre-assessment questions influenced the ways that students communicated and used representations. Additionally, students were more likely to give conceptual justifications when probed to use more representations in their justification.

Findings from the second research question revealed that middle school immigrant students benefitted from Standard-based mathematics teaching approaches given a structured heterogeneous learning environment, encouraged to communicate in writing and orally, and emphasized multiple representations in team settings. The findings showed that when the teacher presented students with rich problems given a real-life situation at the beginning, gave students time to try to solve the problem individually, required students to discuss strategies in team settings, and asked students to present to whole class, all students increased their conceptual understanding on the post-assessment. The lowest overall average score was 3 out of 4 possible points on the post-assessment. Two students earned a perfect score of 4 out of 4 possible points on the post-assessment. This finding indicated that students gave conceptual justifications when asked process-oriented questions, used more than one representation to justify when asked to do so, and gave conceptual justifications using a real-life situation, when given a meaningful real-life situation at the beginning.

Findings from the third research question showed that all students increased the number of correct solutions and the number of conceptual justifications after participation in whole class discussions and filled in gaps from listening to peer presentations, individually and in team settings. The positive outcomes were associated with selecting rich problems, preparing open-ended questions that encouraged multiple perspectives and representations, and structuring whole class mathematical discussions that emphasized connections among peer presentations, supported students conceptual understanding. All students increased not only the number of correct solutions, but in the quality of their justifications throughout the constructivist teaching experiment as evidenced by their written work on the team posters, team presentations transcripts and individual written reflections. The results further showed that middle school immigrant students used multiple strategies and representations to solve challenging function problems (i.e. $y = mx + b$, and $y = (n + 1)^2$), and that students' English language proficiency was not associated with conceptual explanations.

The data presented thus far showed that at the end of the eight-week constructivist teaching experiment, middle school immigrant student participants demonstrated higher order thinking, problem solving, and reasoning that aligned with characteristics of conceptual understanding called for by NCTM (2000). The study carefully employed tools from the empirical literature to plan and implement Standards-based mathematics teaching approaches to teach key ideas of linear functions to middle school immigrant students: the Lesh Translation Model (LTM) and a framework for Orchestrating Productive Mathematical Discussions (Cramer, 2003; Stein, Smith & Hughes, 2009). In summary, results from the study revealed evidence that Standards-based mathematics teaching approaches that encouraged multiple representations and emphasized structured mathematics discussions, were associated with increased conceptual understanding. The following sections present the limitations, implications and recommendations for future research are presented. The chapter ends with concluding remarks.

Limitations

There are several limitations that may be identified in this study conducted in an intervention mathematics class that met bi-weekly for 80 minutes for eight weeks. The role of the researcher included co-planner, and co-teacher in the intervention classroom during the eight-week that utilized a constructivist teaching experiment. The role of the researcher included interviewing students, and interacting with students in the classroom. It is expected that the researcher had influence on student outcomes from their interactions with students throughout the data collection process. Although the classroom teacher did most of the teaching and leading of whole class discussions, the role of the researcher included interviewing students on the pre- and post-assessment, asking for verification and justification. The role of the researcher included interacting with students during student team presentations, as presented in the detailed transcripts in the results section of Chapter 4. It is likely that the students may have perceived the researcher as an evaluator role, and the perception from students could have impacted the ways that students interacted with one another and the ways that students demonstrated their understandings of key ideas of linear functions. The presence of video and audio recorders also could have impacted the ways that students interacted with one another and the quality of students' written and oral responses. Therefore, the results may not be replicable in a different setting and with a different group of students, or with a different mathematics teacher using the same framework, model, and curriculum. Also, the researcher did all of the multiple cycle coding of the data so there was no inter-rater reliability. Finally, the researcher coded students' written work in two ways: solutions and justifications, as a way to verify that student written work matched their oral explanations, and vice versa. The researcher did not conduct member checking with students since data analysis occurred one week to one month after the interviews. Sharing and discussing pre- and post-assessment findings after a period of one week to one month have passed with the students may have altered the coding and therefore, was not checked. Additionally, the researcher presented various data to support findings in Chapter 4, including photos of student written work and detailed transcripts of student oral explanations in pairs on pre- and post-assessments for research questions one and two; and photos of student written work on team posters, detailed transcripts of student team presentations, as well as student written reflections for research question three, to triangulate and support assigned codes and themes to add further to the credibility of the research.

Implications and Recommendations for Future Research

This study adds to the limited empirical literature that addresses middle school immigrant students enrolled in early algebra that emphasized understandings of key ideas of linear functions (Hansen-Thomas, 2009; Silver & Stein, 1996; Whitney, 2010). This study also extends prior work on how secondary students from culturally and linguistically diverse backgrounds, learn key ideas of linear functions, and documented how Standards-based mathematics teaching approaches supported diverse students' conceptual understanding (Cramer, 2001; Silver & Stein, 1999; Whitney, 2010). This study documented the ways in which immigrant students demonstrated conceptual understanding in a structured group setting with explicit attention to needs of immigrant students using rich problems from Standards-based mathematics curriculum that focused on multiple representations and connections between them (LTM), and productive mathematical discussion framework (Stein et al, 2003). For example, the study documented the tools that the research team implemented to plan, modify and implement rich, open-ended linear function problems from Standards-based mathematics curriculum

(Cramer, 2003; Dietiker & Baldinger, 2006). The findings from this study have several pedagogical implications (teaching and learning in K-12 and higher education settings), as well as policy implications (school and district-wide levels). The following sections will discuss implications in three sections: (1) pedagogical implications in K-12 settings are discussed; (2) pedagogical implication in higher education settings in teacher preparation programs are discussed; and (3) policy implications at school and district levels are discussed. To close, a conclusion is presented.

First, pedagogical implications of findings from this study address curriculum and instruction (teaching and learning). Standards-based mathematics curriculum and teaching approaches align with NCTM (2000) goals to emphasize communication and representations, and challenge traditional roles of the mathematics teacher (Senk & Thompson, 2003; Smith, 1996; Campbell et al, 2007; Stein et al, 2009). Structuring mathematical classroom environments that emphasize communication and multiple representations support middle school immigrant students' learning with conceptual understanding. To address the first point on curricular implications, the findings from this study showed that the teacher benefitted from having an expert researcher support planning, modifying, and implementing non-traditional curriculum that aligned with professional organization goals of NCTM (2000). Further, teachers need *access* to and *opportunities to learn* about Standards-based curriculum in order to teach middle school immigrant students for conceptual understanding (Campbell et al, 2007; Stein et al, 2009). To address the second point on implications for teaching, the findings from this study suggested that the teacher benefitted from support in implementing Standards-based mathematics teaching approaches. Recall that Standards-based mathematics teaching approaches challenged traditional roles of the mathematics teacher as, "no longer 'dispenser of knowledge' and arbiter of mathematical 'correctness' but an engineer of learning environments in which students actively grapple with mathematical problems and construct their own understanding" (Ball, 1993, cited in Stein et al, 2009, p. 315). Campbell et al. (2007) discussed that teachers were often not prepared to teach students from diverse cultural and linguistic backgrounds, and therefore, need structured support or frameworks. This points to the next pedagogical implication about student learning. To address the third point on implications for learning, the findings from this study suggested that students needed structured learning environments that enhance opportunities to learn. In particular, two frameworks, the LTM and Orchestrating Productive Discussion, were used to plan, modify, and implement mathematical tasks, and engage immigrant students in a structured classroom setting. As a result, students' responses to pre- and post-assessment questions coded as *conceptual understanding* increased after participation in the eight-week constructivist teaching experiment. In sum, the findings in this study challenged what immigrant students are taught (curriculum), how immigrant students are taught linear functions (teaching), and the ways that immigrant students learn (learning), in algebra settings. A final pedagogical implication suggested that findings from this study also contribute to more fruitful and productive academic discussions in the literature. Therefore, findings from this study also contribute to the literature on the ways middle school immigrant students learn about more advanced linear functions in pre-algebra and algebra settings as their explanations illuminate strengths and challenges.

Second, pedagogical implications from this study extend to higher education, in particular, for K-12 teacher preparation programs. The findings support the literature that middle school immigrant students *can* learn mathematics with conceptual understanding if provided with structured support that paid attention to language differences, encouraged mathematical discussions and use of multiple representations. Further, professors of pre-service teachers of K-12 mathematics need to provide similar supports that were listed above, as pre-service teachers will be expected to teach mathematics for conceptual understanding that align with *PSSM* goals (NCTM, 2000). Therefore, professors of mathematics education teaching methods courses, and/or mathematics content courses need to provide pre-service teachers with *access* to Standards-based mathematics curriculum, paired with *opportunities to learn with and teach with* Standards-based mathematics curriculum. The benefits of teacher preparation programs that support Standards-based mathematics curriculum and teaching approaches are restated as: (a) Standards-based mathematics *curriculum* align with goals of the study designed around process goals of *PSSM*: communication and representations (NCTM, 2000); (b) Standards-based mathematics *teaching* approaches challenge traditional roles of the mathematics teacher (Smith, 1996; Campbell et al, 2007; Stein et al, 2009); (c) structuring mathematical classroom environments that emphasize communication and multiple representations support immigrant student learning with conceptual understanding (Hansen-Thomas, 2009; Whitney, 2010).

Third, findings from this study challenge the overwhelming tracking of immigrant students into lower-track mathematics courses as intervention. These actions have been shown in empirical studies to have long-term negative consequences on immigrant students' mathematics and higher education trajectory (Civil, 2014; Paul, 2005; Wang & Goldschmidt, 1999). Further, findings from this study that documented that middle school immigrant students' may benefit *bi-directionally* from heterogeneous team settings, where LEP/MEP students benefitted from MEP/HEP students, and MEP/HEP students benefitted from LEP/MEP students. Findings revealed increased conceptual understandings after all students participated in the eight-week constructivist teaching experiment, where students worked in mixed ability teams and had opportunities to share and discuss. In sum, immigrant students need structured heterogeneous learning environments, paired with rich, open-ended problems from Standards-based mathematics curriculum to learn challenging linear function topics with conceptual understanding. More important, the findings from this study carefully documented that English language proficiency was not indicative of immigrant students' ability to learn mathematics with conceptual understanding. Therefore, policies at the school and district levels that place students in lower-track mathematics courses ought to be eliminated.

Conclusions

Key ideas of linear functions including concepts of slope, y-intercept, and slope-intercept form are fundamental to students' success in algebra, and are required for all middle school students in Minnesota (MDE, 2015). Developing a *conceptual* understanding of key ideas of linear functions is important, and immigrant students' success in middle school algebra has been linked to long-term mathematics and higher education trajectory (Paul, 2005; Wang & Goldschmidt, 1999). The findings from students' written and oral communication and use of multiple representations from this study suggests that middle school immigrant students can learn algebra with conceptual understanding. The findings also suggest that providing structured heterogeneous learning environments with explicit attention to issues of diversity, lead to increases in students' learning opportunities and to conceptual understanding. Namely, attention to issues of diversity included a framework on productive mathematical discussions that emphasize: *anticipating*, *monitoring, selecting* and *sequencing* student responses (Stein et al, 2009). Teachers that select rich tasks, plan guided questions, and engage students by helping students build from lower- to higher-order explanations of key ideas of linear functions, support learning with conceptual understanding. Similarly, the LTM can be used as a planning and teaching tool to support teachers of immigrant students to teach in ways that align *PSSM* goals as it focuses planning around the use of multiple representations and connections between them (Cramer, 2003; NCTM, 2000). As a case study that documented the nature of seven middle school immigrant students' conceptual understanding of key ideas of linear functions using Standards-based mathematics curriculum combined with two frameworks, LTM and Orchestrating Productive Mathematical Discussion, this study supports the need for future research on a larger scale.

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Appendix A: Pre-Interview and Pre-Assessment

Student Interview

1. Describe a typical day in your current mathematics class.

2. Describe any differences and or similarities you notice between the math classes you took in elementary school and the one you are taking in middle school.

3. Discuss how much real-life situations you are studying in your math class seems to have to your real life.

"**Question 4 part a.** I will begin the math portion. I am now going to ask you to look at the data in Table 1. Does Table 1 look familiar? Have you seen this kind of math before? If you know x, can you find y? How do you know that is correct?"

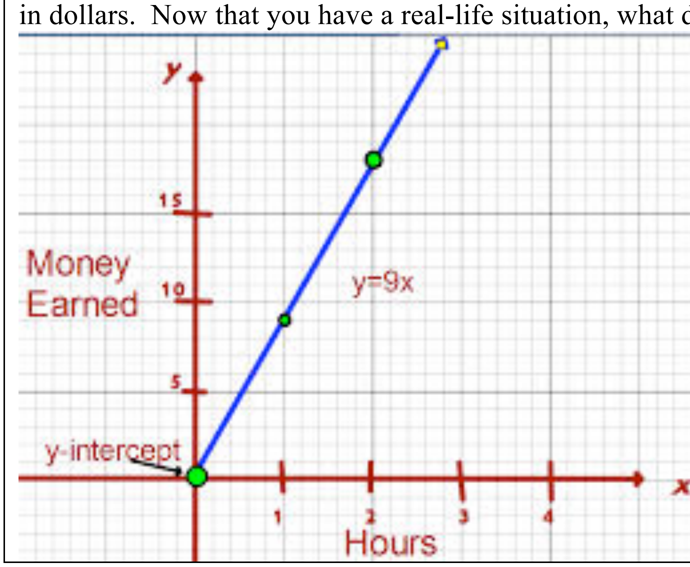
Table 1. Linear Data with positive slope, k

x	y
0	0
1	9
2	18
3	27

"**Question 4 part b.** If you were to graph the data from Table 1, what would the graph look like? How do you know?"

"**Question 4 part c.** Using Table 1, can you write an equation in y = kx format? What is k? Explain to me how you know your answer is correct."

"**Question 4 part d.** What if I told you a real-life situation to describe the table? The x column stands for the number of hours you work as a Spanish tutor and y stands for the amount of money in dollars. Now that you have a real-life situation, what does the k stand for in y = kx?"



“**Question 5 part a**. Given Table 2, graph the values and write or tell a real-life situation to match the table.”

Table 2. Linear Data with positive slope, k

x	**y**
0	0
1	3.5
2	7.0
3	10.5

Appendix B: Post-Assessment

Name: ______________________________ **Date:** ______________

Directions: *You will work individually on four math questions with sub parts similar to what you have seen in class. You will be given as much time as you need.*

1. Jane kept a record of the hours that she worked babysitting and the money in her Piggy bank.

Hours worked	0	1	2	3	4
Money in Piggy Bank ($)			41	48	55

a) Complete the table. Describe the pattern for x and y.

b) Write an equation using the **pattern** and the **starting value**.

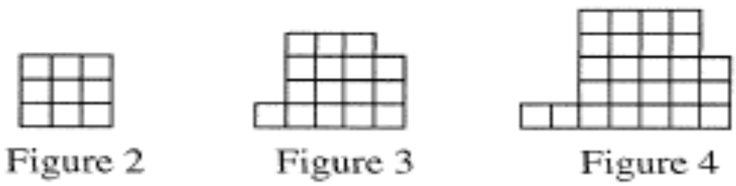
c) How much money will Jane have in the bank after 50 hours of babysitting? How can you check?

d) Is the pattern proportional? Give two reasons. (Use the table, pictures, words or a graph)

Appendix C: Mathematical Task #1

Math Task #1

Individual Work Space: What do figures 6 and 10 look like? How many total squares are in each figure?



Explain how you know with at least 2 representations (Table of values, Graph, Equation, Diagram, Numeric or written explanation)

Team Checklist:

1. **Discuss** with your teammate (in English or Spanish)
    - What is the problem asking?
    - Have you seen a problem like this before?
    - Share your ideas about how to solve the problem with your team

2. **Solve** the problem on your poster with your team you must find a way to represent your solution using at least *two* different representations
    - Table of values
    - Graph
    - Equation
    - Diagram
    - Numeric explanation
    - Written explanation

3. Make sure your group knows how to **explain** each step clearly and how the representations connect to each other.

4. **Present** your solution to the class.

4. **Reflect.** As you watch each group think about the following questions. What similarities do you notice? What is different? Would you change your process/solution?

Appendix D: Mathematical Task #2

Math Task #2 name ___________________

What do figures 6 and 10 look like? How many total squares are in each figure?

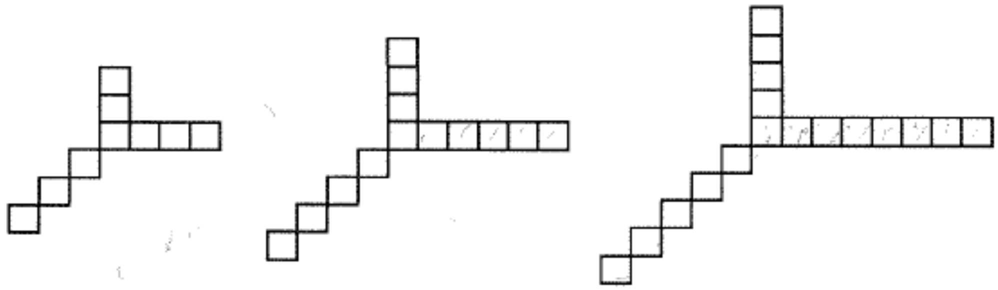


Figure 2 Figure 3 Figure 4

Explain how you know with at least 2 representations.

(Table of values, Graph, Equation, Diagram, Numeric or written explanation)

After Group Presentation

List 1 **similarity** you saw in the strategy each group used to solve this problem.

List 1 **difference** you saw in the strategy each group used to solve this problem.

Do you like your strategy or a new one you learned from a different group?

Appendix E: Mathematical Task #3

Math Task #3 name ___________________

John found the data in the table below about his favorite tree. Use the table below to answer the following questions. Be ready to share your answers with the class.

Number of Years after Planting	3	4	5
Height of Tree (in feet)	17	21	25

Explain each answer using any of the following representations
Table of values, Graph, Equation, Diagram, Numeric or Written explanation

a. How tall was the tree 2 years after it was planted? What about 7 years after it was planted? How do you know?

b. How tall was the tree the year it was planted?

c. Estimate the height of the tree 50 years after it was planted. How did you make your prediction?

d. Represent this situation with an equation

After Group Presentation

List 1 **similarity** you saw in the strategy each group used to solve this problem.

List 1 **difference** you saw in the strategy each group used to solve this problem.

Do you like your strategy or a new one you learned from a different group?

Appendix F: Mathematical Task #4

Math Task #4

name ____________________

John found the data in the table below about his favorite tree. Use the table below to answer the following questions. Be ready to share your answers with the class.

Number of Years after Planting	0	1	2	3	4	5	6
Height of Tree (in feet)				17	21	25	

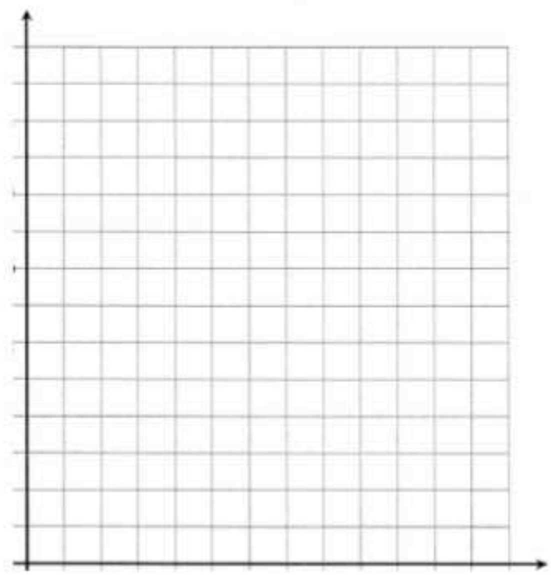
a. Complete the table above.

b. Plot the points that represent the height of tree over time. (label each axis)

c. According to the graph, what was the height of the tree 1.5 years after it was planted? Plot this point.

d. Where does this line cross the y-axis.

John's Giant Redwood



e. Use the table or the graph to represent this relationship with an equation.
(Use your answer in part c to help you)

f. Find the exact height of the tree 50 years after it was planted.

After Group Presentation

Is this relationship a proportional relationship? How do you know?

How did you come up with your equation?

Appendix G: Mathematical Task #5

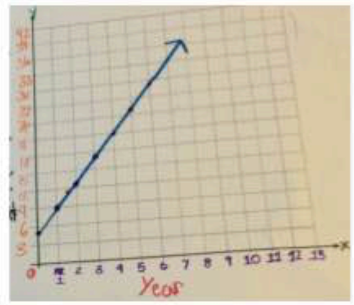
Math Task #5

name ____________________

#1 Remember from Tues. The table represents the height of a tree after planting.

Number of years after planting	0	1	2	3	4	5
Height of a Tree in ft	5	9	13	17	21	25

Also you graphed it like this →



a. What does the coordinate (4, 21) mean?

The equation two groups came up with was

$$y = 4x + 5$$

b. In this equation, what do the 4 and 5 mean? Use the **table** to explain your answer.

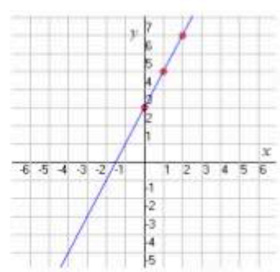
c. In this equation, what do the 4 and 5 mean? Use the **graph** to explain your answer.

#2 Using your procedures you came up with, write an equation for the following graph and table.

a. The equation is ____________________

X			2	3	4	
Y			7	10	13	

The equation is ____________________



b. Write a story to represent each relationship.

After Group Presentation

In your own words, what is an equation?

Appendix H: Mathematical Task #6

Math Task #6 name ____________________

Find the "Big C's" pattern shown below and answer the following questions in any order.

A. Draw Figure 0 and Figure 4 on the grid

Tile Pattern:

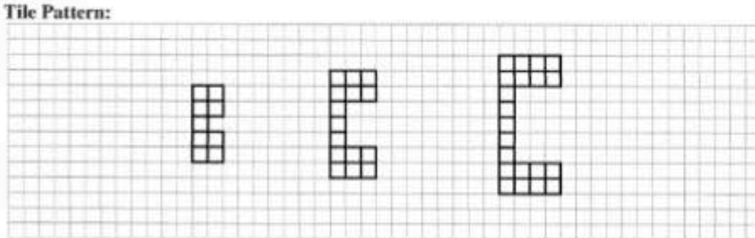


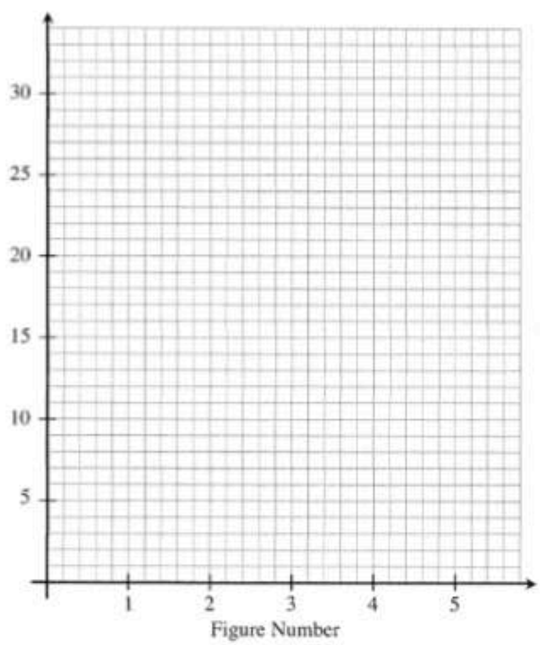
Figure 0 Figure 1 Figure 2 Figure 3 Figure 4

B. Represent the number of tiles in each figure with a table, equation, and graph.

Table:

Figure Number	0	1	2	3	4	5
Number of Tiles						

Graph



Equation

C. Explain how you wrote your equation.

D. How many tiles will be in Figure 5. Justify your answer in at least 2 different ways.

E. How many tiles will the 100th figure have? Justify your reasoning

After Group Presentation

1. What did you learn from the other groups?

2. In your own words, what is an equation?

Appendix I: Mathematical Task #7

Math Task #7 name ____________________

Below are two tables that show a certain pattern. Figure out the pattern, complete the questions below and answer the questions below.

Cost of Pandora

Months (x)	Cost (y)
1	5
2	10
3	15
4	

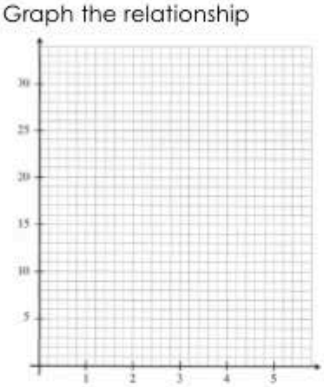
A. Describe the pattern in words.

B. The pattern starts at ______.

C Write an equation using the **pattern** and the **starting value**.

D, Test your equation, make sure it works for all values.

E. Graph the relationship



Cost of Spotify

Months (x)	Cost (y)
1	9
2	14
3	19
4	

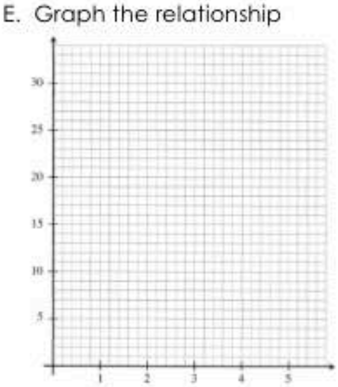
A. Describe the pattern in words.

B. The pattern starts at ______.

C Write an equation using the **pattern** and the **starting value**.

D, Test your equation, make sure it works for all values.

E. Graph the relationship



1.. How are these patterns similar? How are they different?

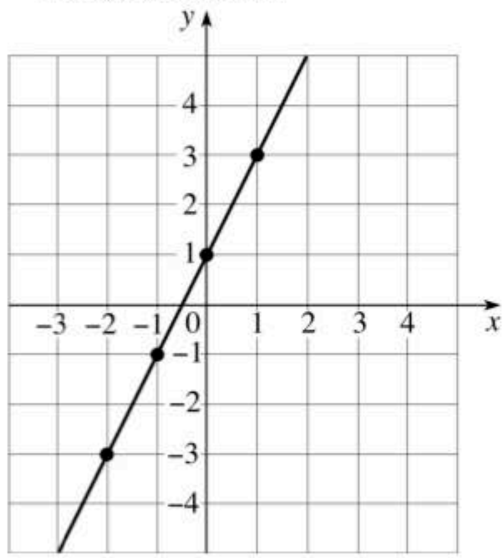
2. Are these relationships proportional? Explain your thinking.

Appendix J: Mathematical Task #8

Math Task #8

name ____________________

Below are two graphs that show a certain pattern.   Figure out the pattern and complete the questions below.

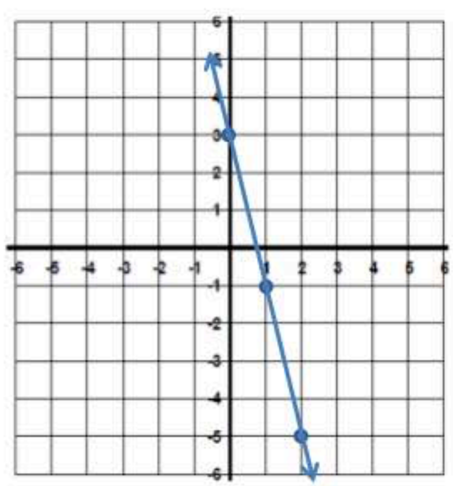


A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

D, Test your equation, make sure it works for 3 coordinates.



A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

D, Test your equation, make sure it works for 3 coordinates.

Written Reflection

1. Describe how you write an equation when given a graph.

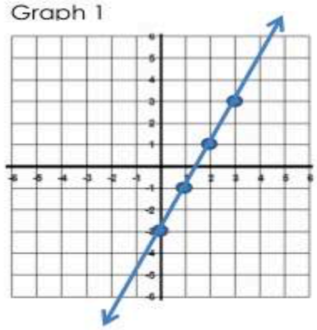
2. Write down one thing you learned from another group.

Appendix K: Mathematical Task #9

Math Task #9 name ____________________

Below are 2 graphs. Describe the pattern and write an equation for the graph.

Graph 1

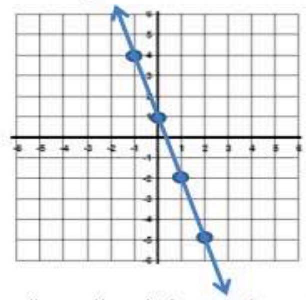


A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

Graph 2

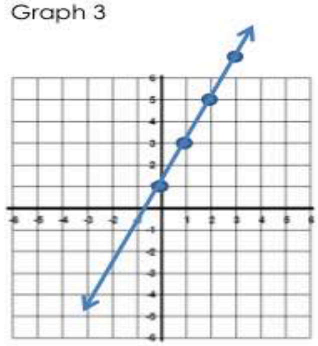


A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

Graph 3



A. Describe the pattern (using words or a table)

B. When x = 0, y = _____

C. Write an equation for the relationship (use part A and B to help you)

Present: Describe how you write an equation when given a graph.

Independent Reflection

1. Describe how you write an equation when given a graph?

2. How is graph 3 similar to graph 1.

3. How is graph 3 similar to graph 2.

Appendix L: Mathematical Task #10

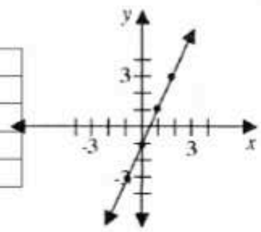
Name _______________________________________ Date ____________________

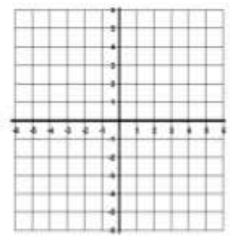
MATH TASK 10

1. A teacher had her student graph the equation $y = \frac{1}{2}x - 1$. The student made an error on the graph and table.

a. Correctly graph the equation on the graph below. Figure out the error the student made.

X	Y
0	-1
1	1
2	3
3	5



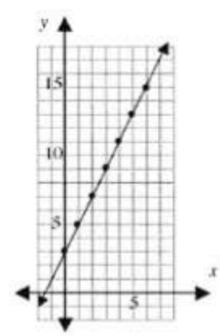


X	Y

b. Explain what the student did wrong. Use a graph, table, or words to explain.

c. Using your correct graph, explain how the 1/2 appears in the graph.

2.The graph represents a tile pattern where x is the figure # and y is the number of tiles in the figure (y is the total number of tiles).



a) How many tiles are in Figure #100? **List** three possible strategies your group could use to find the answer (do not find the answer yet).

- ________________________________
- ________________________________
- ________________________________

b) Pick one of the above strategies and use it to find the number of tiles in Figure #100.

c) How do you write an equation for this graph?

Reflection Question:

1. Describe what the graph of $y = \frac{2}{3}x + 1$ will look like?

The + 1 means that the line will ____

The $\frac{2}{3}$ means that the line will ____

2. Graph y = 2/3x on the coordiante plane below