

Essays on Labor Market Frictions and Institutions

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Abstract

I explore different aspects of the unemployment insurance (UI) in the U.S., including its design, optimality, and interactions with the labor market and business cycle. Chapter 2 comes from my job market paper, for which I used the name “Zoe Xie.” Chapters 1 and 2 are coauthored works with Yun Pei.

The U.S. government makes UI more generous during recessions. Chapter 1 investigates whether government commitment plays a role in shaping such a cyclical policy. In a stochastic general equilibrium search-and-matching framework, we model a government that optimally chooses the benefit level given to all unemployed workers. The government with commitment (Ramsey) implements the optimal policy, whereas the policy without commitment is characterized by a Markov-perfect equilibrium. Because the Ramsey government can use an ex-ante committed benefit to stimulate job search and vacancy posting (reduce moral hazard), the steady-state optimal benefit is lower than in the Markov economy, and the unemployment rate is significantly lower under the Ramsey government. In response to a fall in productivity, the optimal benefit level rises initially, while the Markov benefit level falls slightly below its steady-state level. The different initial responses arise because the fall in productivity reduces the ex-ante moral hazard cost of high benefits for the Ramsey government, while lower productivity and wages drive down the value of insurance for the Markov government. As the unemployment rate picks up, both the ex-ante moral hazard cost and the value of insurance rise, and as a result, the optimal benefit level falls significantly, while the Markov benefits increase monotonically.

Chapter 2 seeks to understand the incentives driving these increases, and how duration affects unemployment and welfare. Because of the trade-off between insurance and job search incentives, the classic time-inconsistency problem arises. This paper endogenizes a time-consistent UI policy in a stochastic general equilibrium search model, where a government without commitment to future policies chooses the UI benefit level and expected duration each period. A longer duration increases unemployed workers’ consumption but reduces job search, leading to higher future unemployment. Quantitatively, the

model is able to rationalize 80% of the variation in benefit duration from 2009 to 2013. We find that these benefit extensions increased the unemployment rate by 3 percentage points compared to the counterfactual of leaving UI policies at their 2007 settings. At the same time, the extensions also improved welfare: insurance is provided to more unemployed workers during recessions when the cost of reducing job search incentives is relatively low.

Chapter 3 argues that unemployed workers do not always lose uncollected UI benefits when they start a new job, contrary to assumptions in the UI literature. Instead, they may postpone the collection of leftover benefits to future unemployment spells. Further, using cross-time and cross-state variations in UI policies, the paper finds empirical evidence that allowing unemployed workers to delay the collection of benefits increases their incentives to find a job during recessions when wages are low, job separation rates are high, and UI benefits are extended. I quantify the effects of the policy of allowing delayed collection of benefits on aggregate unemployment by introducing endogenous search effort, benefit eligibility, and wage-indexed benefits into a standard search-and-matching framework. The model demonstrates how the policy increases the future value of employment even though more generous UI benefits in general reduce the net value of employment. Using a calibrated model, I find that allowing delayed benefit collection raises the proportion of unemployed workers receiving benefits and reduces the unemployment rate during 2009–2012.

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Chapter 1

Government Commitment and Unemployment Insurance over the Business Cycle

1.1 INTRODUCTION

A recent literature finds that the optimal (Ramsey) unemployment insurance (UI) policy is more generous at the start of a recession and less generous as the recession gets more severe to induce a recovery.¹ This pattern differs from the movement of benefits in the U.S. over the business cycle. In particular, benefits are more generous the deeper the recession gets as measured by unemployment.² In addition, Congress voted frequently on extending UI benefits from 2008 to 2013 during the Great Recession, which reflects the government's lack of commitment over UI policies. In this paper, we argue that when the government cannot commit to a prescribed path of benefit policies, it cannot use UI policy to induce recovery in recessions. Such a policy more closely resembles the U.S. policy. We characterize and compare the benefit policies with and without commitment and study its impact on the labor market.

The model integrates risk-averse workers and endogenous search intensity by unemployed workers into the Diamond-Mortensen-Pissarides framework, with business cycle drive by shocks to aggregate labor productivity. Three types of entities inhabit the model economy: workers, firms and government. Employed workers work for firms and get paid

¹ See, for example, [Philip Jung and Keith Kuester \(2015\)](#) and [Kurt Mitman and Stanislav Rabinovich \(2015\)](#).

² By generosity we refer loosely to either an increase in the benefit level received by unemployed workers or the potential duration that a benefit-eligible unemployed worker can stay on benefits.

wages. Unemployed workers receive UI benefits and choose how much to search. Job search incurs utility cost, but also increases the probability that an unemployed worker finds a job. Firms matched with workers produce and pay workers wage, while unmatched firms post job vacancies at a fixed cost. Wages are determined through a Nash bargaining process.

Because the focus of the present paper is on the comparison between UI policies with and without government commitment, we abstract from the distinction between benefit level and the expected duration that an unemployed worker can receive benefits (“benefit duration”) and instead use the benefit level alone to capture the generosity of UI policies. The government chooses UI benefits financed by a non-distortionary tax. More generous *future* UI benefits reduce unemployed worker’s current search intensity. Through general equilibrium effects, higher benefits reduce firm’s vacancy posting by increasing worker’s outside option in the wage bargaining process.

We compare two economies. In the first economy, the optimal state-contingent UI policy is the solution to a Ramsey problem of the government, which takes competitive equilibrium conditions as constraints. Because the government can commit to future policies, it optimally chooses the level of UI benefits for all periods of time and for all possible realizations of productivity shocks. The Ramsey policy, however, is not time consistent without commitment. In particular, the current government would like to reduce promised future benefits *before* unemployed workers choose how much to search (“ex-ante policy”), and increase benefits *after* they have chosen job-search level (“ex-post policy”).³ Because the government wants to implement different policies ex-ante and ex-post, such a policy is time inconsistent and cannot be implemented without government commitment.

In the second economy, we characterize and solve for the time-consistent UI policy. We use the concept of Markov-perfect equilibrium, à la [Klein, Krusell and Ríos-Rull \(2008\)](#). Each period the government chooses current policies to maximize current and future welfare, taking as given future government’s policy rules. Because the Markov government only chooses policies for the current period, its policies differ from the Ramsey policies. First, it does not consider how its policies affect private-sector choices in the previous period, whereas the Ramsey government internalizes this effect. Second, the Markov government can only indirectly influence future policies through the states of the economy, while the Ramsey government predetermines a full sequence of state-contingent policies at time 0.

³ The ex-ante government incentive is to stimulate search by promising a low consumption level of unemployment. The ex-post incentive is to insure the unemployed workers who did not find a job against job-finding risk.

We calibrate the Markov-perfect equilibrium to match the U.S. economy. This choice of target is motivated by the view that the Markov economy is a description of the reality. Our baseline calibration delivers low volatility in both economies. As an alternate calibration, we introduce wage rigidity to amplify cyclical fluctuations in the Markov economy. Overall, the Markov economy has more generous UI benefits, lower search intensity, lower job postings and higher unemployment rate than the Ramsey economy. This is not surprising, given that the Markov government has no commitment and chooses more generous benefits. This highlights the importance of commitment.

An important result concerns the dynamic responses of different governments. The Ramsey UI benefits decrease when current labor productivity is higher or unemployment rate is higher, whereas the Markov UI benefits increase when current labor productivity or unemployment rate is higher. The intuition is that the optimal (Ramsey) benefits are lower in states of the economy where the marginal social benefit of job creation is higher, because lower benefits encourage job search and vacancy posting. The marginal social benefit of job creation is higher when productivity is higher, because each worker-firm pair produces higher output; it is also higher when unemployment is higher, because the probability of filling a job is higher. The Markov government considers the previous period *bygone*, and so it does not internalize these social gains. Instead, the Markov government increases UI benefits when productivity is high, because higher wages (increase in productivity shared between workers and firm) increase the gain from redistribution. The Markov UI benefits are also higher when unemployment is high, because more unemployed workers imply a larger gain from insurance.

Because of different policy responses, the Ramsey and Markov economies have very different dynamics. In response to a one-time negative productivity shock, the Ramsey government initially increases and then slowly reduces UI benefits to the pre-shock level. The initial rise is to help workers smooth consumption, while the subsequent fall creates incentive for search and job posting. The adverse impact of higher benefit on job creation in the initial periods is mitigated by the government's commitment to lower benefits in the future. In response to the same shock, the Markov government lowers benefits immediately, because the costs of financing benefits increase when productivity is low. As the economy recovers, the Markov government gradually increases benefits to the pre-shock level. The richer dynamics of the optimal policy reflects the benefit of commitment—because the Ramsey government has commitment over future policies, it can use temporary changes in the UI policy to smooth consumption over the business cycle. As a result, the Ramsey economy experiences relatively fast recovery in unemployment, while the Markov economy undergoes a much slower recovery.

Several simplifying assumptions are made for tractability. First, neither workers nor government can save or borrow. Allowing workers to save will reduce the cyclical responses of both Ramsey and Markov governments as savings provide self-insurance. If government can save or borrow, there will be larger cyclical responses as the government is not constrained by budget constraints every period. Both assumptions, however, will not affect the comparison of the Ramsey and Markov policies. Second, the Markov government in our setup makes decision every period. Given the weekly frequency, it means that the government makes UI policy decisions every week in our setup, which is much more frequent than in reality. Changing model frequency to quarterly, however, will be a departure from the standard frequencies used in analysis of the labor market.

A review of the related literature and our contributions to the literature is given next.

1.1.1 Related literature

This paper is closely related to two strands of literature: the literature on UI and the literature on time-consistent public policy.

The literature on UI dates back to [Mortensen \(1977\)](#), who argues that unemployment insurance reduces job search effort by the unemployed. Since Mortensen, the majority of this literature takes one of two approaches: either studying the effects of actual UI policy or looking for an optimal policy. The present paper aims to bridge the two approaches by endogenizing government choice of UI policy. Here we take the stance that the government, when making UI policies, is unable to commit to future policies.

One of the classic empirical results in public finance is that social insurance programs such as UI reduce labor supply. Earlier works include [Moffitt \(1985\)](#) and [Meyer \(1990\)](#), who show that a 10% increase in unemployment benefits raises average unemployment durations by 4–8% in the U.S. [Krueger and Meyer \(2002\)](#) and [Gruber \(2007\)](#), for example, interpret this finding as evidence that UI has significant moral hazard costs. Our framework relies on a similar mechanism. When the *expected* payoff from unemployment is high relative to future wages, unemployed workers search less actively. [Chodorow-Reich and Karabarbounis \(2015\)](#) construct a time series of the opportunity cost of employment and find that the cost is procyclical and volatile over the business cycle.

More recently, [Chetty \(2008\)](#) explores an alternate explanation for the link between unemployment benefits and duration. He argues that unemployment benefits increase cash on hand for the unemployed, and thus reduces search intensity. This effect is stronger for unemployed workers with tighter liquidity constraints. Because this “liquidity effect”

has the socially beneficial effect of correcting credit market failure, the truly optimal benefit level, as [Chetty \(2008\)](#) argues, should be higher than if such an effect were ignored. For tractability, the present paper abstracts from the credit market and therefore cannot directly control for the liquidity effect.

Since the Great Recession of 2007-2009, there has been renewed interest in evaluating the quantitative effect of UI on unemployment rate. The reason behind this renewed interest is the extension of expected maximum unemployment benefit duration from 26 to 99 weeks at the peak of the recession and afterward. On the one hand, [Rothstein \(2011\)](#) estimates a small effect of duration extensions on unemployment, contributing to only 0.1-0.5 percentage-point increase in unemployment rate. On the other hand, [Hagedorn et al. \(2015\)](#), using the natural experiment of neighboring counties in two states having similar labor market conditions but different unemployment benefit durations, find a large effect of duration extensions on employment (0.0161 log-point increase in employment as a result of a 1% drop in benefit duration). In between the two extremes, [Nakajima \(2012a\)](#) estimates that duration extensions led to 1.4 percentage-point increase in unemployment rate during 2007–2012 (or approximately 30% of the overall increase). [Fujita \(2010\)](#), using data from the Current Population Survey, obtains an estimate between 0.8 and 1.8 percentage points. Our paper answers a related but more relevant question—what are the welfare implications of duration extensions? And the answer is that despite raising unemployment rate, duration extensions are actually welfare-improving.

The literature on optimal UI has traditionally adopted a principal-agent framework (e.g., [Hopenhayn and Nicolini 1997](#), [Wang and Williamson 2002](#), [Shimer and Werning 2007, 2008](#), and [Golosov, Maziero and Menzio 2013](#)). This framework allows moral hazard frictions to be characterized in a steady state, but it becomes intractable when extended to a business cycle environment. The literature typically shows that the optimal benefit should decline with the unemployment duration of an individual worker. For tractability, our paper abstracts from duration-dependent benefits. More recently, [Mitman and Rabinovich \(2015\)](#) study optimal benefits over the business cycle in a search-matching framework with endogenous unobservable search intensity. [Jung and Kuester \(2015\)](#) take a more general approach by studying the optimal mix of unemployment benefits, hiring subsidies, and layoff taxes in a recession. These papers assume that the government is able to commit to future policies. Although this assumption is innocuous and standard for normative analysis, such policies are time inconsistent and thus hardly implementable without government commitment. The empirical section of the present paper documents frequent voting sessions on UI-related issues during recession, which indicates that the government does not follow a prescribed policy rule when making UI policies. Our paper complements

the literature by characterizing a time-consistent UI policy. Intuitively, the Ramsey government wants people to search hard in normal times when unemployment is low and the marginal return to search is high. To this end, the Ramsey government promises less generous benefits during recessions.

The current paper is also related to the literature on time-consistent public policy (see, e.g., [Alesina and Tabellini 1990](#), [Chari and Kehoe 2007](#), [Battaglini and Coate 2008](#), and [Yared 2010](#)).⁴ Methodologically, our paper follows [Klein, Krusell and Ríos-Rull \(2008\)](#) to characterize the Markov-perfect equilibrium of a dynamic game in terms of a generalized Euler equation (GEE). Whereas [Klein, Krusell and Ríos-Rull \(2008\)](#) focus on a deterministic economy, we are interested in how government policy responds to business cycle fluctuations. Recent applications of Markov-perfect equilibrium include [Song, Storesletten and Zilibotti \(2012\)](#), who study intergenerational conflict over debt in a politico-economic environment.

The rest of the paper proceeds as follows. Section 2 describes the model environment and defines the private-sector competitive equilibrium. Section 3 presents the government's problem first as an optimal policy problem, and then as part of a Markov-perfect equilibrium. We characterize the solutions and solve both government's problems in this section. Section 4 describes the calibration strategy. Section 5 presents the quantitative results and discusses alternative scenarios. Section 6 concludes. We relegate all derivations, sensitivity analysis and additional figures to the Appendix.

1.2 MODEL

In this section, we describe the model environment and characterize the competitive equilibrium. The model is based on a Diamond-Mortensen-Pissarides model with aggregate productivity shocks.

1.2.1 Model environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Workers are risk-averse and maximize expected

⁴ See [Klein and Ríos-Rull \(2003\)](#) for a detailed review of the earlier literature.

lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where \mathbb{E}_0 is the period-0 expectation factor, β is the time discount factor. Period utility $U(c, s)$ takes consumption of goods c and search intensity s as inputs. Utility is increasing in c and decreasing in s . Only unemployed workers supply positive search intensity, i.e. there is no on-the-job search. Each period, an employed worker gets paid wage from production. Wages are determined through a canonical bargaining process to be specified later in the section. An unemployed worker receives unemployment benefits b . In addition, an unemployed worker also produces h , which we take as the combined value of leisure, home production and welfare. There is no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β . A firm can be either matched to a worker (and producing) or vacant. A vacant firm posting a vacancy incurs a flow cost κ .

Unemployed workers and vacancies form new matches. Let u and v denote the measure of unemployed worker, and the measure of vacancies posted, respectively. Then the number of new matches formed in a period is given by the matching function $M(su, v)$, where the quantity su is the measure of efficiency units of search by the unemployed in the economy. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of potential matches : $M(su, v) \leq \min\{su, v\}$. The job-finding probabilities per efficiency unit of search intensity, f , and the job-filling probability per vacancy, q , are functions of labor market tightness, $\theta = v/(su)$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(su, v)}{su} = M(1, \theta) \\ q(\theta) &= \frac{M(su, v)}{v} = M\left(\frac{1}{\theta}, 1\right) \end{aligned}$$

Following the assumptions made on M , $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . The job finding probability for an unemployed searching with intensity s is $sf(\theta)$. Existing matches are destroyed exogenously with constant job separation probability δ .

Only a matched pair of a worker and a firm can produce. Each matched pair produces z , where z is the aggregate labor productivity. z is constant \bar{z} in the steady state, and time-varying z_t in the economy off steady-state.

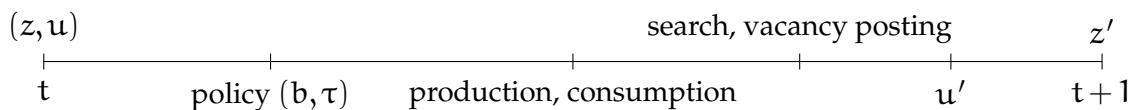


Figure 1.1: Timing of events.

1.2.2 Government

The government cannot borrow or lend; instead it balances budget each period. The government finances unemployment benefits b through a lump sum tax, τ , on all workers, both employed and unemployed.⁵ The government budget constraint is

$$\tau = ub \tag{1.1}$$

The government decides on the generosity of the unemployment insurance program by varying benefit level, $b \geq 0$. Once a benefit level is determined, all unemployed workers receive the same benefit in that period. This way of modeling the unemployment insurance system is a simplification of the reality where not all unemployed workers receive benefits. This assumption is common in the literature, for example, [Landais, Michailat and Saez \(2010\)](#) and [Jung and Kuester \(2015\)](#) both assume all unemployed receive benefits. The advantage of this setup is reduced computational complexity while still allowing the generosity of the unemployment insurance program to change. The unemployment benefits here can be thought of as compounding the potential duration and level of unemployment benefits.⁶

1.2.3 Timing

The timing of events within a period is illustrated in Figure 1.1 and is as follows. The economy enters a period t with a level of unemployment u . The aggregate shock z then realizes. (z, u) are the aggregate states of the economy. Government policies (b, τ) for the period is known to workers and firms.

Employed workers produce z and receive a bargained wage w . Unemployed workers produce h and receive benefits b . All workers pay a lump sum tax τ out of wage or benefit.

⁵ We experiment with alternative tax structure where either only employed workers pay tax, or only firms pay tax (in the form of a lump sum tax on profits). Results are not presented in the paper but are available upon request.

⁶ Equivalently, it can be thought of as compounding benefit level and proportion of unemployed workers on benefit at any time.

Given aggregate states and government policies for the period, unemployed workers choose search intensity s . At the same time, firms decide how many vacancies to post, at cost κ per vacancy. The aggregate search is then su , and the market tightness is equal to $\theta = v/(su)$. The fraction of unemployed workers who find jobs is $f(\theta)s$. At the same time, a fraction δ of the existing $1 - u$ matches are exogenously destroyed. The law of motion of unemployed workers is

$$u' = \delta(1 - u) + (1 - f(\theta)s)u \quad (1.2)$$

1.2.4 Workers

Denote by g the government policy (b, τ) . A worker entering a period unemployed consumes $h + b$ and chooses search intensity s . With probability $f(\theta(z, u; g))s$, he finds a job and starts working the following period. Let $V^e(z, u; g)$ and $V^u(z, u; g)$ be the values of an employed and an unemployed worker, respectively, with the beginning-of-period unemployment u and realized aggregate shock z , given government policy for that period $g = (b, \tau)$. An unemployed worker's optimization problem is

$$\begin{aligned} V^u(z, u; g) = \max_s & U(c, s) + \beta f(\theta(z, u; g))s \mathbb{E}V^e(z', u'; g') \\ & + \beta(1 - f(\theta(z, u; g))s) \mathbb{E}V^u(z', u'; g') \end{aligned} \quad (1.3)$$

A worker entering a period employed produces and consumes his wage w . With probability δ , he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period. The Bellman equation of an employed worker is then

$$V^e(z, u; g) = U(w(z, u; g), 0) + \beta(1 - \delta) \mathbb{E}V^e(z', u'; g') + \beta\delta \mathbb{E}V^u(z', u'; g') \quad (1.4)$$

Notice that market tightness θ and wage w are functions of the economy's states, (z, u) . This is because they are objects determined in an equilibrium. As mentioned before, job separation rate δ is taken to be constant through time.

1.2.5 Firms

In order to be matched with a worker and produce, a firm posts a vacancy.⁷ A firm that posts a vacancy incurs a flow cost κ . With probability $q(\theta(z, u; g))$, a vacancy is filled and ready for production the following period. Let $J^u(z, u; g)$ be the value of an unmatched

⁷ The firms can be viewed as a representative firm with a collection of jobs and posts several vacancies.

firm posting a vacancy. The Bellman equation of an unmatched firm is

$$J^u(z, u; g) = -\kappa + \beta q(\theta(z, u; g)) \mathbb{E} J^e(z', u'; g') + \beta(1 - q(\theta(z, u; g))) \mathbb{E} J^u(z', u'; g') \quad (1.5)$$

where $J^e(z, u; g)$ is the value of a matched firm. In equilibrium, under free-entry condition, the firm will post vacancies $v(z, u; g)$ until $J^u(z, u; g) = 0$.

A matched firm receives output net of wages $z - w(z, u; g)$. With constant probability δ , a match is destroyed at the end of period. The Bellman equation of a matched firm is

$$J^e(z, u; g) = z - w(z, u; g) + \beta(1 - \delta) \mathbb{E} J^e(z', u'; g') + \beta \delta \mathbb{E} J^u(z', u'; g') \quad (1.6)$$

1.2.6 Wage determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(su, v)$. A realized match produces some economic rent that is shared between the firm and the worker through Nash bargaining. The assumption of Nash bargaining allows comparison with the literature. We assume that wages are set period by period, so equilibrium wages respond to the state of the economy.

Worker's surplus is the difference between the values of working at wage w and being unemployed and receiving benefit b . As a result, higher benefits increase worker's surplus, and tends to drive up bargained wage. Firm's surplus is the difference between the value of a match and that of running a vacancy. As explained before, vacant firm posts vacancies until its value is zero. Thus, the firm's outside option is zero in equilibrium.

In particular, wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus when the state of the economy is (z, u) and government policy is $g = (b, \tau)$. The worker-firm pair thus solves

$$\max_w \left(V^e(z, u; g) - V^u(z, u; g) \right)^\zeta \left(J^e(z, u; g) - J^u(z, u; g) \right)^{1-\zeta} \quad (1.7)$$

where $\zeta \in (0, 1)$ is the bargaining power of the worker. $V^e(z, u; g) - V^u(z, u; g)$ is the worker's surplus, and $J^e(z, u; g) - J^u(z, u; g)$ is the firm's surplus from the match. The solution to this bargaining problem, denoted $w(z, u; g)$, is a function of the economy's states.

1.2.7 Competitive equilibrium

DEFINITION 1.1. (competitive equilibrium) Given a policy $g = (b, \tau)$ and an initial condition (z^-, u^-) , a competitive equilibrium consists of (z, u) -measurable functions for wages $w(z, u; g)$, worker's search intensity $s(z, u; g)$, market tightness $\theta(z, u; g)$, unemployment rate $u'(z, u; g)$, and value functions $V^e(z, u; g)$, $V^u(z, u; g)$, $J^e(z, u; g)$, $J^u(z, u; g)$ such that for all $(z, u; g)$

- the value functions satisfy the worker and firm Bellman equations (1.3)-(1.6)
- the search intensity s solves the unemployed worker's maximization problem of (1.3)
- the market tightness θ is consistent with the free-entry condition, $V^u(z, u; g) = 0$
- the wage w solves the maximization problem of (1.7)
- unemployment satisfies the law of motion equation (1.2)

1.2.8 Characterization

The competitive equilibrium can be characterized by three optimality conditions.⁸ Appendix A.1 contains derivation of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

The optimal choice of search intensity s for the unemployed worker is characterized by

$$\begin{aligned} \frac{-U_s(h + b - \tau, s)}{f(\theta)} &= \beta \mathbb{E} [U(w' - \tau', 0) - U(h + b' - \tau', s')] \\ &+ \beta \mathbb{E} \left[(1 - f(\theta')s' - \delta) \frac{-U_s(h + b' - \tau', s')}{f(\theta')} \right] \end{aligned} \quad (1.8)$$

The worker's optimality condition states that the marginal cost (left-hand side) of increasing the job finding probability equals the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search of the unemployed worker weighted by the aggregate job finding rate per efficiency unit of search. The marginal benefit is the sum of utility gain from being employed next period and the benefit of economizing on future search cost. A higher future benefit b' reduces the utility gain from being employed the next period, and thus lowers the marginal benefit of search today.

⁸ To economize on notation, we suppress the dependence on $(z, u; g)$. It should be understood throughout that the optimal decisions are functions with arguments $(z, u; g)$.

From firm's free-entry condition

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} \left[z' - w' + (1 - \delta) \frac{\kappa}{q(\theta')} \right] \quad (1.9)$$

where the marginal cost (left-hand side) equals the marginal benefit (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal benefit is the profits from employing a worker. Because a newly formed match does not become operational until the next period, the benefit from production only has components from the next period.

Finally, Nash bargaining implies a relationship between the worker's surplus from being employed and the firm's surplus from hiring a worker.

$$\frac{\left[\mathbb{U}(w - \tau, 0) - \mathbb{U}(h + b - \tau, s) + (1 - f(\theta)s - \delta) \frac{-\mathbb{U}_s(h + b - \tau, s)}{f(\theta)} \right] / \mathbb{U}_c(w - \tau, 0)}{z - w + (1 - \delta) \frac{\kappa}{q(\theta)}} = \frac{\zeta}{1 - \zeta}$$

The left-hand side of the equation is the ratio of worker's to firm's surplus weighted by marginal utility of higher wage. The worker's surplus (top part) comes from utility gain of being employed and reduced search cost (employed worker searches zero). Because workers are risk-averse, changes in wages have non-linear effect on his utility, as represented by $\mathbb{U}_c(w - \tau, 0)$. The firm's surplus (bottom part) derives from profit and reduced vacancy posting cost (producing firm posts zero vacancy). The right-hand side of the equation is the ratio of the worker's to firm's bargaining power. Equilibrium wage then equates the weighted ratio of worker/firm surplus to the ratio of their respective bargaining power.

Rearranging terms into a more compact condition for the equilibrium wage

$$\begin{aligned} & \zeta \mathbb{U}_c(w - \tau, 0) \left[z - w + (1 - \delta) \frac{\kappa}{q(\theta)} \right] \\ = & (1 - \zeta) \left[\mathbb{U}(w - \tau, 0) - \mathbb{U}(h + b - \tau, s) + (1 - f(\theta)s - \delta) \frac{-\mathbb{U}_s(h + b - \tau, s)}{f(\theta)} \right] \end{aligned} \quad (1.10)$$

A higher future benefit b' lowers worker's surplus. Given the future Nash bargaining condition, next period's workers demand higher wage w' , and thus firm's surplus is lower in equilibrium. The free-entry condition of (1.9) then implies a lower θ , and thus a lower job-finding rate per efficiency search unit $f(\theta)$.

A contemporaneous decrease in the aggregate labor productivity z reduces firm's surplus in (1.10) and as a result wages fall. Because current wage and productivity do not enter worker's and firm's optimality conditions (1.8)-(1.9), the contemporaneous fall in z does not directly affect search or job-finding rate. Now consider a fall in the expected fu-

ture productivity z' . In this latter case, firm reduces vacancy posting since expected return to future production is lower. Worker reduces search intensity, both because fewer vacancies lead to lower per-search-unit job-finding probability, and because lower expected aggregate productivity implies lower expected future wages.

Because different z' lead to different equilibrium search and job-finding rate, the government will optimally tailor its unemployment benefit policy to current and future economic conditions. In the next section, we analyze how a benevolent government, under assumptions of commitment and non-commitment, designs benefit policy.

1.3 GOVERNMENT POLICIES

In this section, we describe how a Ramsey government and a Markov government choose their policies respectively. To highlight the key difference between these two types of governments, we first illustrate the time inconsistency of the Ramsey policy using a simple example. We then describe the full Ramsey and Markov problems. We assume both governments are utilitarian planners, who maximize the expected value of the worker's utility. Both governments have the same policy instruments, which are unemployment benefit b and tax τ . However, the Ramsey government can commit to future policies, while the Markov government does not have the ability to commit. In the last subsection, we compare the Ramsey and Markov outcomes.

1.3.1 A simple example to illustrate time inconsistency

Before describing the full Ramsey problem, we consider a simple example to illustrate the presence of time inconsistency in the Ramsey problem. There are two periods and a unit measure of workers. Workers search in the first period and consume in the second period. Assume no time discounting and firms. In the first period, $1 - \bar{u}$ of workers are guaranteed a job in the second period. The remaining $\bar{u} = 0.05$ workers choose how much to search, $s \in (0, 1)$, for a job starting in the second period. Search incurs utility cost governed by the convex function $v(s)$. Worker's utility of consumption is given by $U(c)$.

With probability s , the worker finds a job and receives wage $\bar{w} = 1$ in the second period; otherwise he receives unemployment benefit $b \in (0, 1)$. Optimal choice of search is thus characterized by $v_s(s) = U(\bar{w}) - U(b)$. The number of unemployed workers in the second period is $u = (1 - s)\bar{u}$.

Government in this economy chooses b at the beginning of period 1 to maximize

average utility

$$\begin{aligned} W &= (1-u)U(\bar{w}) + u[U(b) - v(s)] \\ \text{subject to } u &= (1-s)\bar{u} \\ v_s(s) &= U(\bar{w}) - U(b) \end{aligned}$$

Essentially, the government is solving

$$\max_{s \in (0,1)} [1 - (1-s)\bar{u}]U(\bar{w}) + (1-s)\bar{u} [U(\bar{w}) - v_s(s) - v(s)]$$

with first-order condition given by

$$\bar{u} [v_s(s) + v(s)] - (1-s)\bar{u} [v_{ss}(s) + v_s(s)] = 0$$

Let $U(c) = \log(c)$ and $v(s) = \frac{s^2}{2}$. The government optimally chooses $s^* = 0.549$, $b^* = 0.578$ and $u^* = 0.0226$, with average utility $W^* = -0.0158$.

Now suppose the government can revise benefit after workers have chosen s . Then the *ex-post* optimal policy is $\hat{b} = 1$, with *ex-post* average utility given by $\hat{W} = (1-u^*) \log \bar{w} + u^* [\log \hat{b} - (s^*)^2/2] = -0.0034 > W^*$. In fact, any $\hat{b} > b^*$ will result in higher *ex-post* average utility. The fact that there exists a better policy *ex-post* illustrates the time inconsistency in this setup; time inconsistency, in turn, means lack of commitment leads to different policy outcomes than an economy with government commitment.

1.3.2 Ramsey government

In this section, we set up the full Ramsey problem. The modeling of the Ramsey government is very similar to that in [Mitman and Rabinovich \(2015\)](#). Since the Ramsey government has commitment to all its future policies at the beginning of time, the government's decision problem is therefore to choose a sequence of unemployment benefits and taxes $\{b_t, \tau_t\}_{t=0}^{\infty}$ in order to maximize the worker's utility, taking into account how the private sector will respond to these policies. At time 0, the government decides on its policies for all future periods and for all possible realizations of shocks. The private sector takes government policies as given and follows the timing described in Section 2.

To reduce the number of policy instruments in the government's problem, we use the following function derived from the government's budget constraint to express tax

$$\mathcal{T}(u, b) := ub.$$

Then the government's problem can be equivalently written as one of choosing policies $\{b_t\}_{t=0}^\infty$, and allocation and prices $\{w_t, s_t, \theta_t, u_{t+1}\}_{t=0}^\infty$ to maximize utility subject to the government budget constraint and competitive equilibrium conditions.⁹ Formally, the *government period return function* at time t is given by

$$R(u_t, b_t, w_t, s_t) = (1 - u_t)U(w_t - \mathcal{F}(u_t, b_t), 0) + u_t U(h + b_t - \mathcal{F}(u_t, b_t), s_t).$$

DEFINITION 1.2. (Ramsey policy) Given an initial unemployment rate u_0 and aggregate labor productivity z_0 , the optimal government policy with commitment consists of a sequence of benefits and taxes $\{b_t\}_{t=0}^\infty$ that solves

$$\max_{\{b_t, w_t, s_t, \theta_t, u_{t+1}\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(u_t, b_t, w_t, s_t)$$

over the set of all policies that satisfy the worker's law of motion equation (1.2) and the competitive equilibrium conditions (1.8)-(1.10), for all time t and aggregate shock $\{z_t\}_{t=0}^\infty$.

For easy exposition, we rewrite the competitive equilibrium conditions sequentially and use auxiliary functions $\tilde{\eta}_0$, $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$ to denote the flow equation and the three private-sector optimality conditions (1.8)-(1.10) respectively,¹⁰

$$\tilde{\eta}_0(u_t, s_t, \theta_t, u_{t+1}) = 0 \quad (1.11)$$

$$\tilde{\eta}_1(u_t, b_t, s_t, \theta_t, u_{t+1}, b_{t+1}, w_{t+1}, s_{t+1}, \theta_{t+1}) = 0 \quad (1.12)$$

$$\tilde{\eta}_2(\theta_t, z_{t+1}, w_{t+1}, \theta_{t+1}) = 0 \quad (1.13)$$

$$\tilde{\eta}_3(z_t, u_t, b_t, w_t, s_t, \theta_t) = 0 \quad (1.14)$$

where the three private-sector optimality conditions play the role of incentive constraints in the optimal policy problem, similar to the incentive constraints in a principal-agent setup, e.g. [Hopenhayn and Nicolini \(1997\)](#).

To derive a set of conditions that characterize the Ramsey policy, we let $\beta^t \pi^t \lambda_t$, $\beta^t \pi^t \mu_t$, $\beta^t \pi^t \gamma_t$ and $\beta^t \pi^t \nu_t$ be the Lagrange multipliers on (1.11)-(1.14), where π^t is the probability of a history realization $\{z_0, z_1, \dots, z_t\}$ given an initial condition z_0 . The optimal government policy can be characterized by the following government's first-order conditions

⁹ This is the primal approach to Ramsey problem.

¹⁰ See Appendix A.1 for more details.

with respect to b_t, w_t, s_t, θ_t and u_{t+1} for all time $t > 0$

$$\begin{aligned}
\mu_{t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \mu_t \tilde{\eta}_{1b,t} + \nu_t \tilde{\eta}_{3b,t} &= R_{b,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1w',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{2w',t-1}}{\beta} + \nu_t \tilde{\eta}_{3w,t} &= R_{w,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1s',t-1}}{\beta} + \lambda_t \tilde{\eta}_{0s,t} + \mu_t \tilde{\eta}_{1s,t} + \nu_t \tilde{\eta}_{3s,t} &= R_{s,t} \\
\mu_{t-1} \frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \dots \\
\cdots + \lambda_t \tilde{\eta}_{0\theta,t} + \mu_t \tilde{\eta}_{1\theta,t} + \gamma_t \tilde{\eta}_{2\theta,t} + \nu_t \tilde{\eta}_{3\theta,t} &= 0 \\
\lambda_t \tilde{\eta}_{0u',t} + \mu_t \mathbb{E}_t \tilde{\eta}_{1u',t} &= \beta \mathbb{E}_t \{ R_{u,t+1} - \lambda_{t+1} \tilde{\eta}_{0u,t+1} \\
&\quad - \mu_{t+1} \tilde{\eta}_{1u,t+1} - \nu_{t+1} \tilde{\eta}_{3u,t+1} \} \quad (\text{RAM})
\end{aligned}$$

where primes denote next period, and subscripts are derivatives.

The period- t solution is state dependent. It depends on the current productivity z_t and the beginning-of-period unemployment level u_t , as well as multipliers $(\mu_{t-1}, \gamma_{t-1})$. μ is the marginal value of relaxing the optimal search condition for the unemployed worker (1.8), and γ is the marginal value of relaxing the firm's equilibrium free-entry condition (1.9). The presence of μ_{t-1} and γ_{t-1} as states in the optimal policy captures commitment—the Ramsey government in period t has to deliver these marginal values, which it promised for the worker and firm in period $t-1$.

Note that commitment is assumed in the Ramsey case. If given the choice to break promise, the government will deviate from the sequence of policies prescribed by the government at time 0. The government of period t has an incentive to promise low future unemployment benefits to encourage search and vacancy posting, because, as explained in Section 2, current search and job-finding probability are higher when the future benefits are expected to be lower. But after employment outcome of period t has realized, the government has an incentive to smooth workers' consumption by providing high benefits. This incentive to deviate from original plan is what constitutes time inconsistency in the Ramsey problem.

1.3.3 Markov government

In this section, we consider government policies that are time consistent. We use the concept of Markov-perfect equilibrium, similar to that in [Klein, Krusell and Ríos-Rull \(2008\)](#). By construction, the government policy in such an equilibrium is time consistent. As a result, lack of commitment does not make a difference in policy outcome.

Intuitively, one can think of the economy as having a sequence of governments, each lasting only one period. Each successive government only chooses current policy, taking future governments' policies as given. It neither considers how its policy affects previous periods, nor can it directly choose policies for future periods. Like [Klein, Krusell and Ríos-Rull \(2008\)](#), we focus on equilibria where government policy depends differentially on the state of the economy.

The timing of events is illustrated in [Figure 1.1](#). At the beginning of each period, the government chooses its benefit and tax policy for the current period. The private-sector agents (firms and workers) then move to choose its current period search, vacancy posting and wage, as described in [Section 2](#). Because the economy consists of a mass of workers and firms, each of measure zero, the private-sector agents take future government policies as given.

The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the state of the economy. The *government period return function* in this case is given by

$$R(u, b, w, s) = (1 - u)U(w - \mathcal{T}(u, b), 0) + uU(b - \mathcal{T}(u, b), s).$$

DEFINITION 1.3. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function G , government policy function Ψ , and private decision rules $\{W, S, \Theta, \Pi\}$ such that for all beginning-of-period unemployment u and aggregate productivity z , $b = \Psi(z, u)$, $w = W(z, u)$, $s = S(z, u)$, $\theta = \Theta(z, u)$ and $u' = \Pi(z, u)$ solve

$$\max_{b, w, s, \theta, u'} R(u, b, w, s) + \beta \mathbb{E}G(z', u')$$

subject to

- the worker's law of motion

$$\eta_0(u, s, \theta, u') = u' - \delta(1 - u) - [1 - f(\theta)s]u \tag{1.15}$$

- the private-sector optimality conditions below

$$\begin{aligned}
0 &= \eta_1(u, b, s, \theta, z', u'; \Psi, W, S, \Theta) \\
&:= \frac{-U_s(b - \mathcal{F}(u, b), s)}{f(\theta)} \\
&\quad - \beta E [U(W(z', u') - \mathcal{F}(u', \Psi(z', u')), 0) - U(\Psi(z', u') - \mathcal{F}(u', \Psi(z', u')), S(z', u'))] \\
&\quad - \beta E \left[(1 - f(\Theta(z', \Psi(z', u')))) S(z', u') - \delta \frac{-U_s(\Psi(z', u') - \mathcal{F}(u', \Psi(z', u')), S(z', u'))}{f(\Theta(z', u'))} \right] \quad (1.16)
\end{aligned}$$

$$\begin{aligned}
0 &= \eta_2(\theta, z', u'; W, \Theta) \\
&:= \frac{\kappa}{q(\theta)} - \beta E \left[z' - W(z', u') + (1 - \delta) \frac{\kappa}{q(\Theta(z', u'))} \right] \quad (1.17)
\end{aligned}$$

$$\begin{aligned}
0 &= \eta_3(z, u, b, w, s, \theta) \\
&:= \zeta U_c(w - \mathcal{F}(u, b), 0) \left[z - w + (1 - \delta) \frac{\kappa}{q(\theta)} \right] \\
&\quad - (1 - \zeta) \left[U(w - \mathcal{F}(u, b), 0) - U(b - \mathcal{F}(u, b), s) + (1 - f(\theta)s - \delta) \frac{-U_s(b - \mathcal{F}(u, b), s)}{f(\theta)} \right] \quad (1.18)
\end{aligned}$$

and

- the government value function satisfies the functional equation

$$G(z, u) \equiv R(u, \Psi(z, u), W(z, u), S(z, u)) + \beta \mathbb{E} G(z', \Pi(z, u)) \quad (1.19)$$

Note that equation (1.19) reflects that future planners follow the policy rules $\{\Psi, W, S, \Theta, \Pi\}$. The Markovian assumption is reflected in the policy functions being time independent and only a function of the aggregate state. For ease of exposition, we have used the auxiliary functions $\eta_0, \eta_1, \eta_2, \eta_3$. Because policies are functions of aggregate productivity and unemployment, the auxiliary functions η_1 and η_2 are functions of z' and u' . In comparison, the Ramsey auxiliary functions $\tilde{\eta}_1$ and $\tilde{\eta}_2$ as defined in (1.12) and (1.13) are direct functions of next period decisions such as b', s' .

Let $\lambda, \mu, \gamma, \nu$ be the Lagrange multipliers on (1.15)-(1.18), respectively. The benefit policy in a Markov equilibrium can be characterized by the following government's first-order conditions with respect to b, w, s, θ , and u' :

$$\begin{aligned}
\mu \eta_{1b} + \nu \eta_{3b} &= R_b \\
\nu \eta_{3w} &= R_w \\
\lambda \eta_{0s} + \mu \eta_{1s} + \nu \eta_{3s} &= R_s \\
\lambda \eta_{0\theta} + \mu \eta_{1\theta} + \gamma \eta_{2\theta} + \nu \eta_{3\theta} &= 0 \\
\lambda \eta_{0u'} + \mu \mathbb{E} \eta_{1u'} + \gamma \mathbb{E} \eta_{2u'} &= \beta \mathbb{E} G'_u = \beta \mathbb{E} \{ R'_u - \lambda' \eta'_{0u} - \mu' \eta'_{1u} - \nu' \eta'_{3u} \} \quad (\text{MAR})
\end{aligned}$$

where primes denote next period, and subscripts are derivatives. Note that because η_1 and η_2 contain functions of next period unemployment u' in the form of next period policy functions, derivatives of $\eta_{1u'}$ and $\eta_{2u'}$ contain policy function derivatives.

The Markov-perfect equilibrium is then characterized by a system of *functional* equations (1.1), (1.2), (1.16)-(1.18) and (MAR). An analytical characterization of the Markov-perfect equilibrium is not available. We solve for the equilibrium numerically using a standard cubic spline projection method to approximate the policy functions. As a robustness check, we use Chebyshev collocation and obtain identical results.

1.3.3.1 The Generalized Euler Equation

To build some intuition for how b is determined, we combine the government first-order conditions into a single equation that characterizes the Markov benefit policy¹¹

$$\begin{aligned}
& \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \\
& + \mathbb{E} \left(-\frac{\eta_{1b}}{\eta_{1u'}} \right) \left\{ \begin{array}{l} \left(-\frac{\eta_{0u'}}{\eta_{0s}} \right) \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right] \\ + \left(-\frac{\eta_{2u'}}{\eta_{2\theta}} \right) \left[-\frac{\eta_{1\theta}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \right] \\ + \left(-\frac{\eta_{2u'}}{\eta_{2\theta}} \right) \left[-\frac{\eta_{0\theta}}{\eta_{0s}} \left(R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right) - \frac{\eta_{3\theta}}{\eta_{3w}} R_w \right] \end{array} \right\} \\
& + \beta \mathbb{E} \left(-\frac{\eta_{1b}}{\eta_{1u'}} \right) \left\{ R'_u - \frac{\eta'_{0u}}{\eta'_{0s}} \left[R'_s - \frac{\eta'_{1s}}{\eta'_{1b}} \left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w \right) - \frac{\eta'_{3s}}{\eta'_{3w}} R'_w \right] - \frac{\eta'_{3u}}{\eta'_{3w}} R'_w \right\} \\
& + \beta \mathbb{E} \underbrace{\left(-\frac{\eta_{1b}}{\eta_{1u'}} \right) \left(-\frac{\eta'_{1u}}{\eta'_{1b}} \right)}_{db'/db \text{ holding } \eta'_i=0, u'' \text{ unchanged}} \left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w \right) = 0 \quad (\text{GEE})
\end{aligned}$$

Because of the presence of policy function derivatives in auxiliary function derivatives, the above equation is also known as the Generalized Euler Equation or GEE. From the GEE, it is obvious any change in b has three trade-offs. First, the trade-off between consumption of unemployed and employed (first line). Second, the trade-off of current welfare through search and job posting (second and third lines) against future welfare through unemployment (fourth line). In particular, a higher b increases u' , leading to lower s since b' is higher in expectation. Lower s increases today's welfare. At the same time, a higher u' reduces next period's welfare R both directly and indirectly through effects on search and wages. Lastly, the next period's trade-off between consumption of unemployed and em-

¹¹ Appendix A.1 contains two derivations of the GEE, one with policy functions defined as before, e.g. $S(z, u)$, and the other with policy functions such as $\hat{S}(z, b, u)$. It can be shown that the equilibria based on the two definitions are in fact equivalent.

ployed (last line). The weight on the last term can be thought of as db'/db holding $\eta'_1 = 0$ and u'' unchanged. The government determines current benefit by setting the net marginal value of b to zero.

Note that the GEE does not contain explicitly the derivative of Ψ ; it appears indirectly in private-sector auxiliary function derivative $\eta_{1u'}$. This reflects an important point made earlier—the successive governments agree on a policy rule Ψ . The Markov government does not try to manipulate its successor through changing current b , hence the absence of derivatives of Ψ directly from the GEE. The fact that Ψ affects private-sector auxiliary function derivative captures the fact that how much a lower b increases private-sector search (and other decisions) depends on how the extra search will reduce next period unemployment. This makes the Markov government differ significantly from a government in a dynamic game setting. In that case, each successive government manipulates the next government to set a lower b than it chooses. Such a strategy leads to high consumption (high current b) and low future unemployment (low future b and hence high search).

1.3.4 The role of commitment

By comparing first-order conditions of the Ramsey government (RAM) and the Markov government (MAR), two key differences emerge.

First, the Markov optimality conditions do not contain promised marginal values from the previous period (Lagrange multipliers μ_{t-1} and γ_{t-1}) as the Ramsey conditions do. μ_{t-1} and γ_{t-1} are the marginal values to the unemployed workers and firms in period $t-1$, respectively. These marginal values are affected by expected policy and allocations in period t . For example, higher benefits in period t reduce expected gain from search and vacancy posting thus decreasing search and vacancy posting in period $t-1$. The presence of these marginal values as state variables in the Ramsey optimality conditions means that the Ramsey government needs to choose policies that can deliver these promises. In contrast, because the Markov government lacks commitment, it does not internalize how current policy affects incentives in the previous period, and thus does not deliver these promises.

The second difference is the presence of policy derivatives in the Markov auxiliary

functions. In particular,

$$\begin{aligned}\eta_{1u'} &\equiv \frac{\partial \eta_1}{\partial u'} + \underbrace{\frac{\partial \eta_1}{\partial b'} \psi'_u + \frac{\partial \eta_1}{\partial w'} W'_u + \frac{\partial \eta_1}{\partial s'} S'_u + \frac{\partial \eta_1}{\partial \theta'} \Theta'_u}_{\text{disciplining effect}} \\ \eta_{2u'} &\equiv \underbrace{\frac{\partial \eta_2}{\partial w'} W'_u + \frac{\partial \eta_2}{\partial \theta'} \Theta'_u}_{\text{disciplining effect}}\end{aligned}$$

These policy derivatives illustrate how the current Markov government can induce the future governments to choose certain policy by changing the states of the economy. More specifically, the Markov government, by choosing current benefit level, affects search intensity in the current period¹², thus affecting next period's unemployment (and next period's state) holding other things constant. Next period's unemployment in turn influences how the government in the next period chooses its policy. This *disciplining effect* is captured by the presence of policy derivatives in the auxiliary functions. In contrast, because the Ramsey government can commit to future policies, its policy function is a sequence of states-contingent plans. In particular, the Ramsey government chooses in period 0 a sequence of state-contingent policies and allocations for all future periods.

1.4 CALIBRATION

We describe our calibration strategy in this section. The model period is taken to be one week. We calibrate the Markov equilibrium to match important features of the U.S. labor market.

The utility function is

$$u(c, s) = \frac{1}{1-\sigma} \left(\left[\frac{c}{v(s)} \right]^{1-\sigma} - 1 \right)$$

where $v(s)$ is the cost of search. For $\sigma \neq 1$, this utility function represents a preference non-separable in c and s . We assume $v(\cdot)$ is a non-negative, strictly increasing and convex function, with the property that $v(0)$ is bounded and $v(0) > 0$. We choose the search cost function to be

$$v(s) = \exp \left(\frac{A}{1+\phi} [(1-s)^{-(1+\phi)} - 1] - (A-1)s \right)$$

This functional form is chosen to guarantee that s is strictly less than 1. In particular,

¹² The effect of benefit level on current period search works through non-separability of preference. Under our parametrization, lower benefit level increases current search intensity.

for any $A > 0$, v exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, $v(1) = v_s(1) = \infty$, and $v(0) = v_s(0) = 1 > 0$. With this functional form for search cost, when $\sigma = 1$, the utility function can be shown (using L'Hospital's Rule) to reduce to $\log c - \log v(s)$, which is utility function often used in the literature.¹³ When $\sigma \neq 1$, the utility function features non-separability between consumption and search. This allows the Markov government to have disciplining power over its successor.

We adopt the matching function from [den Haan, Ramey and Watson \(2000\)](#), which is also used in [Hagedorn and Manovskii \(2008\)](#) and [Mitman and Rabinovich \(2015\)](#)

$$M(su, v) = \frac{(su)v}{[(su)^x + v^x]^{1/x}}$$

Together with the search cost function, this matching function guarantees that both the job-finding rate, $f(\theta)$ s and the job-filling rate $q(\theta)$ are always strictly less than 1.

As in [Shimer \(2005\)](#), labor productivity z_t is taken to be average real output per person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume the shock to z follows an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \sigma_\epsilon \epsilon_t$$

where $\rho \in [0, 1)$, $\sigma_\epsilon > 0$, and ϵ_t are i.i.d. standard normal random variables. The parameters are estimated to be $\rho = 0.9895$ and $\sigma_\epsilon = 0.0034$ at the weekly frequency.

We set the discount factor $\beta = 0.99^{1/12}$, giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion is $\sigma = 0.75$. Although this number is small compared to those used in the macro literature, we believe it is within reasonable range given the weekly frequency.¹⁴ [Hagedorn and Manovskii \(2008\)](#) estimate weekly job separation rate to be 0.0081. They also estimate the costs of vacancy creation to be 58% of weekly labor productivity. Therefore, we set the job separation parameter $\delta = 0.0081$ and cost of vacancy posting $\kappa = 0.58$.

The value of non-market activity is taken to be $h = 0.4$, a common value used in the search literature.¹⁵ The worker's share in wage bargaining is set at 0.5, so that worker and firm have equal share of bargaining power.¹⁶ We set the search cost curvature parameter ϕ

¹³ Appendix A.2 contains analysis on separable preference.

¹⁴ [Hopenhayn and Nicolini \(1997\)](#) also set the relative risk aversion coefficient in the range of 0.5 – 0.75 for a weekly frequency.

¹⁵ [Hall \(2006\)](#) estimates a value of leisure relative to productivity at about 43%.

¹⁶ The literature uses a wide range of bargaining power parameter. [Shimer \(2005\)](#) uses a higher bargaining power parameter of 0.72. [Hagedorn and Manovskii \(2008\)](#) use elasticity of wage to productivity to estimate bargaining power share at 0.052. [Nakajima \(2012b\)](#) estimates a bargaining share of 0.07 in a setup with elastic

Table 1.1: Summary of Calibration

Parameter	Description	Value
β	Discount factor	$0.99^{1/12}$
σ	Coefficient of relative risk aversion	0.75
h	Value of leisure	0.4
A	Disutility of search	0.024
ϕ	Search cost curvature	1
ζ	Bargaining power of worker	0.5
δ	Separation rate	0.008
χ	Matching parameter	0.644
κ	Vacancy posting cost	0.58
ρ	Persistence of productivity	0.9895
σ_ϵ	Std of innovation to productivity	0.0034

to 1 in the benchmark model and investigate the sensitivity of main results under different values of ϕ .

We jointly calibrate (1) the matching function parameter χ , and (2) the level parameter of search cost A , to match the mean job-finding rate and mean job-filling rate. These data targets are directly measured in the U.S. data from 1951-2004. [Shimer \(2005\)](#) reports a monthly job-finding rate of 0.45, and job-filling rate of 0.71. We convert them to weekly job-finding rate of 0.139, and job-filling rate of 0.266.¹⁷ The calibration results are summarized in [Table 1.1](#). [Table 1.2](#) compares some labor market statistics in the U.S. economy and the calibrated Markov economy. The calibrated model does a good job generating the relevant correlations. In particular, the model delivers negative correlation between unemployment and vacancy, thus preserving the Beveridge-curve relationship. Because we target only first moments, the calibrated model generate much lower volatility than the U.S. economy. We address this issue by introducing wage rigidity in later section.

1.5 QUANTITATIVE ANALYSIS

In this section, we describe the quantitative results of the model. In order to investigate how the economy behaves under the Ramsey policy and Markov policy, we bring the labor supply.

¹⁷ We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See [Zhang, Conn and Scheinberg \(2010\)](#) for details.

Table 1.2: Summary Statistics

Statistic	z	u'	v	v/u
<i>Quarterly U.S. data 1951-2004</i>				
standard deviation	0.013	0.125	0.139	0.259
Correlation matrix	z	1	-0.302	0.460
	u'	-	1	-0.919
	v	-	-	1
	v/u	-	-	1
<i>Calibrated Markov economy</i>				
standard deviation	0.013	0.005	0.009	0.013
Correlation matrix	z	1	-0.954	0.974
	u'	-	1	-0.861
	v	-	-	1
	v/u	-	-	1

Note: Standard deviations and correlations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

calibrated parameters into the model under each policy.

1.5.1 Steady-state comparison

Table 1.3 compares steady states under the Ramsey and Markov policies. Not surprisingly, the Ramsey economy performs better than the Markov economy. The Markov government gives a much higher unemployment benefit than does the Ramsey government. The replacement ratio is 58% in the Markov economy, as opposed to 33% in the Ramsey economy. Because we use the non-commitment economy as calibration target, the replacement ratio in the Markov economy is much closer to the 60% replacement ratio as found in the U.S. economy. With higher benefit level, unemployed workers have less incentives to search. In fact, the Markov economy has much lower search intensity than the Ramsey economy.

Higher benefits give workers higher outside options, so wages are slightly higher in the Markov economy. Higher wages indicate lower profits for firms, and hence lower vacancy posting under the Markov policy. Lower search by workers and lower vacancy post-

Table 1.3: Steady-state Statistics

Statistic	Ramsey	Markov
benefit, b	0.318	0.564
wages, w	0.976	0.981
search, s	0.690	0.329
vacancy, v	0.036	0.028
unemployment, u	0.023	0.054
replacement ratio(%)	32.6	57.5
average consumption	0.963	0.950
consumption equivalent welfare change(%)	–	1.52

Note: Steady states are computed using parameters calibrated to Markov equilibrium.

ing by firms lead to much higher unemployment level in the Markov economy. Therefore, output is lower and agents consume less in the Markov economy. In terms of welfare, the average consumption in the Markov economy has to increase by 1.52% to be equivalent to the Ramsey economy in steady state.

Table 1.3 highlights the importance of commitment. The government has two opposing incentives. One is insurance, through providing higher unemployment benefit to help workers smooth consumption. The other incentive is job creation, by giving lower benefits to encourage search and vacancy posting, thereby lowering unemployment and increasing output. Ideally, the government would like higher benefit in the current period and lower benefits in the following periods because, as discussed before, current search and job posting are mainly affected by expectations of future benefits. But when the government lacks the ability to commit to future policies—as in the case of the Markov economy and almost all governments in reality—it cannot make any credible promise of lower benefit in the future. As a result, such government consistently provides higher than optimal benefits and leads the economy into a state of high unemployment, low output and low welfare.

1.5.2 Policy functions

In this section, we present and compare Ramsey and Markov equilibrium policy functions solved using cubic spline projection method. In Appendix A.3 we include Markov equilibrium solved using Chebyshev collocation method.

Figure 1.2 plots the Ramsey policy (left) and the Markov equilibrium policy (right) functions for benefit (top panels) and next period unemployment (bottom panels), hold-

ing productivity at the steady state level.¹⁸ In each plot, the solid line represent policy function, and the dashed line indicates steady state unemployment rate.¹⁹

First, consider unemployment benefit (the top panels). The optimal unemployment benefit in the Ramsey case is decreasing in unemployment level, whereas the Markov benefit is increasing in unemployment. One key difference between these two governments is that the Ramsey government internalizes the impact of its current policy on the actions of private sector in previous periods. When unemployment level is high, the marginal social benefit of job creation is higher, because the expected output gain of increasing vacancy posting is proportional to the number of unemployed workers. Thus the Ramsey government reduces unemployment benefit when unemployment is high, in order to induce more search and vacancy posting in the previous period.

In contrast, the Markov government considers the previous period *foregone* and hence does not internalize how previous period's expectation of current policy impacts the economy in the past. At the same time, as more workers are unemployed, the Markov government, with a utilitarian objective function, has a stronger motive to provide insurance and help smooth consumption. So the Markov government increases unemployment benefits by redistributing more from the employed to the unemployed. Although the government also has an incentive to encourage search, its inability to commit means the government keeps postponing the action to the next period.

The bottom panels of Figure 1.2 plot the next period unemployment policy functions u' associated with the Ramsey policy (left) and the Markov policy (right). In both cases, the policy function is increasing in current unemployment and coincides with the 45-degree line once at the steady state. Notice that the slope of the Ramsey unemployment is flatter than that of the Markov unemployment. This is because the Ramsey government, by planning a sequence of policies at time 0, has more control over the economy, and thus can move the next period unemployment further away from current unemployment. The Markov government, in contrast, can only influence the next period economy through the *disciplining effect* on the next government, and thus has smaller power over the state of the economy.

Figure 1.3 plots the Ramsey (left) and the Markov equilibrium (right) benefit policy functions, holding unemployment at the steady state level. The Ramsey and Markov un-

¹⁸ The Ramsey policy function plots also hold promised marginal utilities μ_- and γ_- at their respective steady state level. Note that even though we solve Ramsey policies as functions, the solution to a Ramsey problem really should be understood as *sequences of variables* from $t = 0$ to $t = \infty$, given some initial state, (u_0, z_0) in this case.

¹⁹ Appendix A.4 contains other policy function plots, holding either unemployment or productivity at steady state.

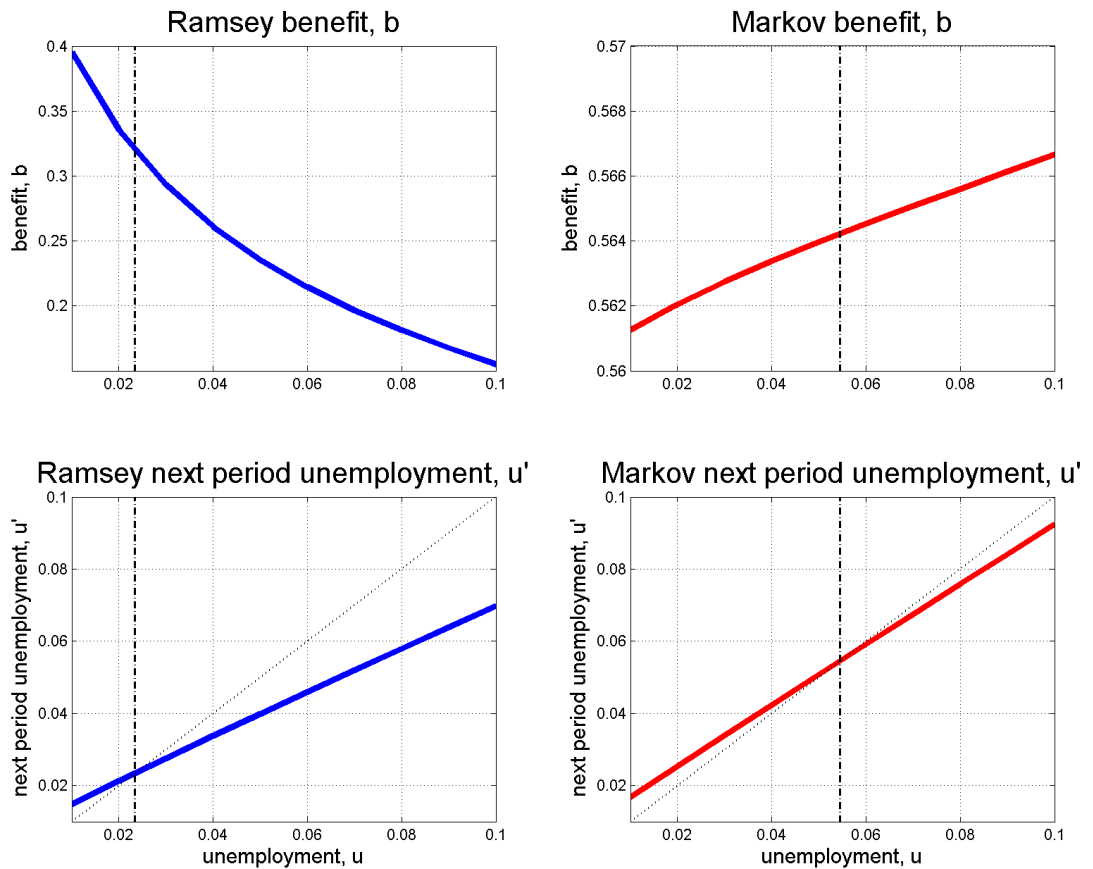


Figure 1.2: Ramsey (left) and Markov (right) benefit (top panels) and unemployment (bottom panels) policy functions holding productivity at steady state.

employment benefits are decreasing and increasing, respectively, in productivity. In other words, when productivity is low, optimal benefit is high whereas the Markov government provides low benefit. The difference comes again from the lack of commitment by the Markov government. From the perspective of the Ramsey government, the marginal social benefit of job creation is lower when productivity is low, since the output of each firm-worker pair is low. As a result, the marginal social cost (in the form of lower search and fewer vacancy postings) of unemployment benefit is low. So the Ramsey government provides high benefit.

In contrast, the Markov government does not internalize the changing social marginal cost of benefits in the form of job creation. The Markov government weighs the welfare gain from redistribution against the financing cost of benefits. When productivity is low,

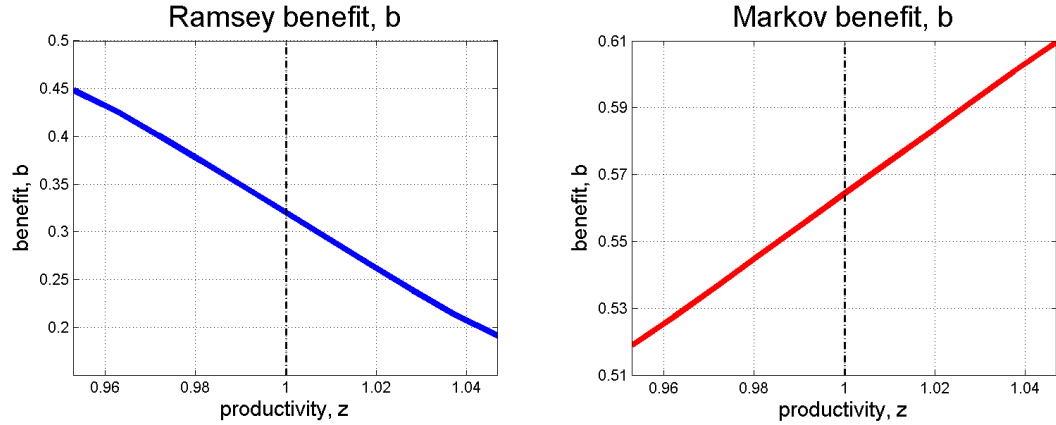


Figure 1.3: Ramsey (left) and Markov (right) benefit policy functions holding unemployment at steady state.

output and the aggregate resource in the economy are low. As a result, the marginal cost of financing benefits is high, and so the Markov government provides low benefits. In addition, with persistent shocks, low productivity implies that future productivities are also likely to be low. With expectations of low future productivity, firms reduce vacancy posting. The Markov government reduces unemployment benefit to encourage current period search and vacancy posting,²⁰ thus increasing future aggregate resources.

1.5.3 Dynamics

To understand how the Ramsey and Markov economy behave over time, we simulate each economy. Figure 1.4 plots and compares the dynamic responses of key variables in the Ramsey and Markov economy to a 1% drop in productivity.²¹ The optimal benefit level initially jumps up, then falls for about 30 weeks following the shock, and slowly reverts to its pre-shock level. Unemployment rises in response to the drop in productivity and continues rising for about 10 weeks before falling back to its pre-shock level. The Markov government, however, reduces benefits in response to lower productivity, and slowly raises benefits back to its pre-shock level. Because of the different initial responses in benefit policy, unemployment in the Markov economy also responds markedly differently compared to the Ramsey economy. Unemployment jumps up only slightly when the shock hits, then

²⁰ This effect works because preferences are non-separable in consumption and search intensity. Under our parameterization, the cross derivative of utility in benefit and search is negative. So when benefits are low, the marginal utility (cost) of search is high (low), and thus search is high. Higher search intensity increases the per-vacancy job-filling rate, so firms have more incentive to post vacancy.

²¹ Appendix A.4 contains figures of dynamic responses for other labor market variables.

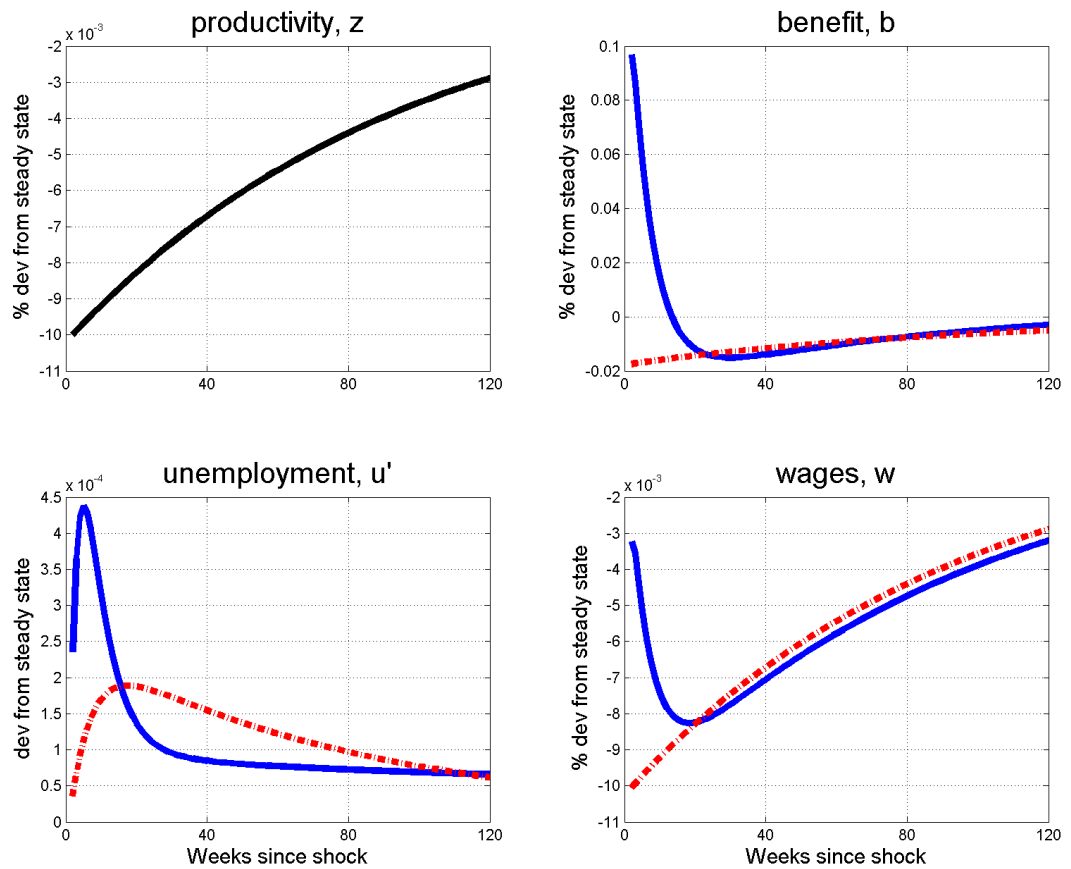


Figure 1.4: Ramsey (solid blue line) and Markov (dashed red line) responses to a 1% drop in productivity.

rises for 15 weeks, and slowly falls back to pre-shock level.

Such responses to a negative shock are consistent with the properties of the Ramsey and Markov policy functions. Immediately after the negative shock, productivity is low, and so is the social value of employment. As a result, the Ramsey government tolerates the rise in unemployment. The Markov government, not internalizing the changing social value of job creation, lowers benefits in response to lower aggregate output after the shock. As unemployment rises, the social benefit of creating more vacancies increases relative to the benefit of providing insurance, and the Ramsey government therefore cuts unemployment benefits to reduce unemployment. The Markov government, however, does not internalize the effect of benefits on job creation in the previous period; instead, the Markov benefit rises as unemployment rate increases (more incentive to redistribute) or as pro-

Table 1.4: Simulated Statistics under Markov and Ramsey Policies

Statistic	Productivity	Benefit	Unemployment	Wages	Search	Vacancy
<i>Markov policy</i>						
mean	1	0.564	0.054	0.980	0.329	0.028
standard deviation	0.013	0.023	0.005	0.013	0.000	0.013
<i>Ramsey policy</i>						
mean	1	0.319	0.023	0.976	0.689	0.036
standard deviation	0.013	0.055	0.013	0.012	0.007	0.013

Note: Means are reported in levels. Standard deviations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

ductivity rises (more resources to redistribute). Because the Markov government does not increase benefits immediately after the shock, unemployment does not peak until 15 weeks after the shock, and peaks at a lower level, in deviation terms, than unemployment in the Ramsey economy. But because the Markov government keeps raising benefit following the shock, the Markov economy features a much *slower* recovery in unemployment.

After about 20 weeks, the responses of benefits in the Ramsey and Markov economies are very similar, but the unemployment dynamics still differ markedly. In particular, unemployment in the Markov economy recovers much more slowly. The reason is as follows. We can rewrite equation (1.2), the law of motion of unemployment, as

$$\hat{u}' \approx [1 - f(\theta)s - \delta]\hat{u}$$

where $\hat{u} = u - \bar{u}$ is the deviation from the steady state. Due to higher search intensity and higher job posting, the job finding rate, $f(\theta)s$, is higher in the Ramsey economy than that in the Markov economy. It means that the unemployment deviation decays at a faster rate in the Ramsey economy. Or equivalently, unemployment in the Markov economy recovers at a slower pace.

Turning to wages. Wages fall less, in percent deviation terms, in the Ramsey economy than they do in the Markov economy. This is because the initial rise in Ramsey benefits smooths the fall in wages through an increase in the worker's outside option. Wages also fall for a longer period—for about 15 weeks before picking up—under the optimal policy, whereas wages in the Markov economy dip upon impact, and rise monotonically back to their pre-shock level.

Table 1.4 reports the moments of key variables when the model is simulated under the optimal policy and the Markov policy. Compared to the optimal policy, search intensity under the Markov policy has minimal volatility (rounded to zero). This is because the

Table 1.5: Summary Statistics with Wage Rigidity

Statistic	z	u'	v	v/u
<i>Quarterly U.S. data 1951-2004</i>				
standard deviation	0.013	0.125	0.139	0.259
Correlation matrix	z	1	-0.302	0.460
	u'	-	1	-0.977
	v	-	-	1
	v/u	-	-	1
<i>Calibrated Markov economy</i>				
standard deviation	0.013	0.125	0.254	0.360
Correlation matrix	z	1	-0.938	0.976
	u'	-	1	-0.847
	v	-	-	1
	v/u	-	-	1

Note: Standard deviations and correlations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Markov policy, unlike the Ramsey policy, is not designed to increase search incentive when the economy is in recovery. The low search volatility contributes to *lower* volatility of unemployment in the Markov economy. These results show that although the optimal policy creates faster recovery, the Markov economy has less cyclical fluctuations in unemployment. The higher unemployment volatility under the optimal policy reflects the fact that the Ramsey government aims to reduce volatility of average consumption (and welfare), at the cost of higher unemployment fluctuation. In fact, consumption (welfare) volatility in the Markov economy is 13.68% (13.58%), as opposed to 12.36% (12.34%) under the Ramsey government.

Vacancy posting, on the other hand, has the same volatility under both policies, because vacancy posting is mainly driven by expected future aggregate productivity. Since the two economies follow the same productivity process and experience the same shocks, the volatility of vacancy posting (and wages) do not differ much between the two economies.

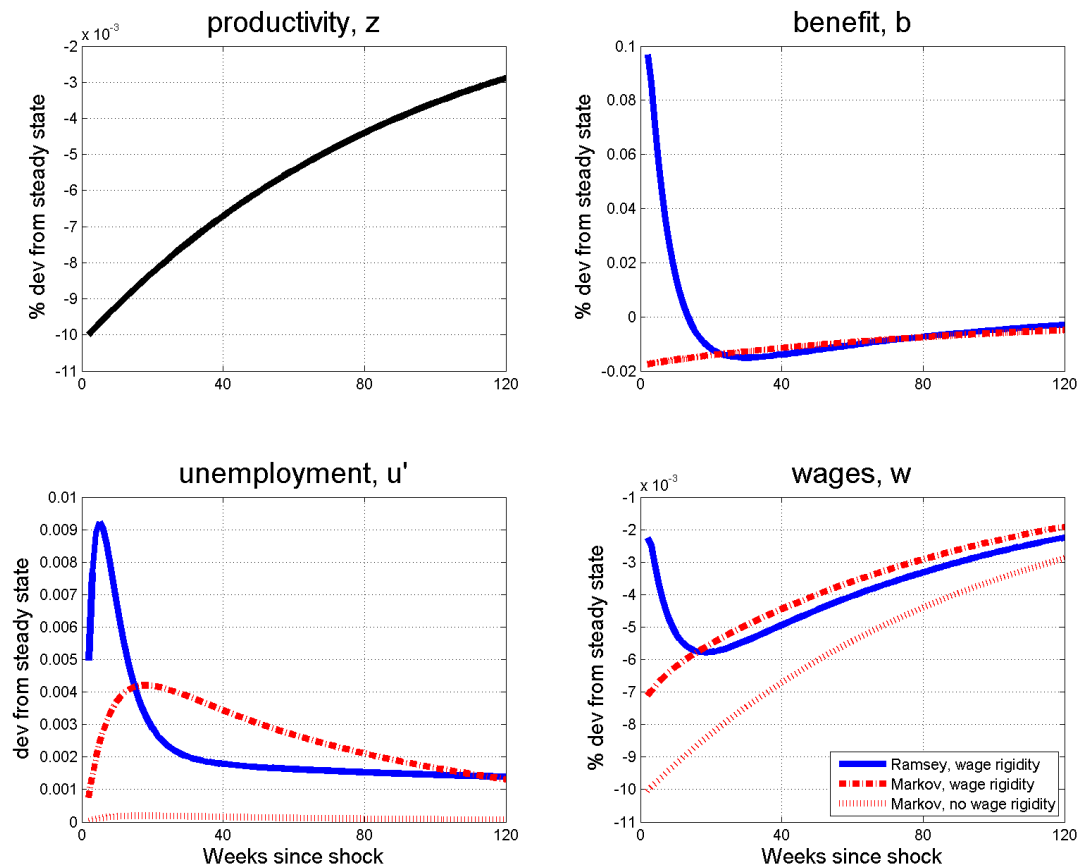


Figure 1.5: Responses to 1% drop in productivity: Ramsey (solid blue line) and Markov (dashed red line) with wage rigidity, Markov without wage rigidity (dotted red line)

1.5.3.1 Wage rigidity

It is well-known that without wage rigidity the search and matching model cannot easily generate the scale of cyclical fluctuations observed in reality.²² This is why the second-moments in Table 1.4 are much lower than empirical moments. To see whether the behavior of the Ramsey and Markov economies change when the economy has more realistic volatility, we introduce wage rigidity in the form of countercyclical bargaining power of

²² See, for example, Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008) for more details on wage rigidity.

worker. Specifically, we assume that the worker’s bargaining power follows²³

$$\zeta_t = \exp(\log \bar{\zeta} + \epsilon_\zeta \log z_t)$$

where $\epsilon_\zeta < 0$ and is calibrated in the Markov economy to match the volatility of unemployment observed in the data. [Hagedorn and Manovskii \(2008\)](#) document quarterly standard deviation of unemployment between 1951 to 2004 to be 0.125. This implies $\epsilon_\zeta = -6.6$, so for a 1% drop in productivity, the bargaining power of workers increases by 6.6%. [Table 1.5](#) compares labor market statistics of the U.S. economy with the Markov economy calibrated with wage rigidity. With wage rigidity, the Markov economy does a good job capturing the cyclical properties of the U.S. labor market.

[Figure 1.5](#) plots responses of Ramsey and Markov economies to an unexpected 1% drop in productivity with wage rigidity. Both economies behave similar as the economies without wage rigidity. For easier comparison, [Figure 1.5](#) also plots Markov policy responses without wage rigidity. As expected, unemployment exhibits much large cyclical fluctuation with wage rigidity, as do wages.

1.6 CONCLUSION

This paper studies how a welfare-maximizing government chooses UI benefit policy when the government can and cannot commit to future policies. We use the Ramsey policy to describe government policy with commitment, and the Markov-perfect equilibrium for a government without commitment. The Markov equilibrium has higher benefits and higher unemployment rate than the Ramsey economy. Over the business cycle, the Ramsey policy—which is optimal—raises benefits at the beginning of a recession, and gradually decreases benefits over time. In contrast, the Markov policy—which is time consistent—reduces benefits at the onset of a recession, and slowly increases benefits as the economy recovers. Our findings thus highlight the importance of commitment when designing the optimal unemployment insurance policy over the business cycle.

An important direction for future research is to study the duration of UI benefits. The U.S. government extends UI durations in recessions to provide insurance to more unemployed workers, which is why UI duration is a more relevant margin for policy discussions²⁴.

²³ Similar assumptions are common in the literature. [Landais, Michaillat and Saez \(2010\)](#) and [Nakajima \(2012a\)](#) both directly specify $w_t = \exp(\log \bar{w} + \epsilon_w z_t)$. [Jung and Kuester \(2015\)](#) use a cyclical bargaining power structure similar to ours in their benchmark calibration.

²⁴ Chapter 2 looks at time-consistent UI policy allowing for changes in both the benefit level and benefit duration.

Another interesting direction is to allow the government and the private sector to change their behavior at a different frequency. In the paper, both the private sector and government make decisions at a weekly frequency. It will be more realistic if the government makes policy decisions on a quarterly basis while the labor market react to policy changes and productivity shocks at a weekly frequency.

Chapter 2

A Quantitative Theory of Time-Consistent Unemployment Insurance

2.1 INTRODUCTION

Unemployment insurance (UI) programs are important components of social safety nets in many countries. These programs help jobless workers smooth consumption while they search for jobs, but the UI programs also reduce incentives to search. The U.S. government has extended UI benefit duration during every recession since the 1970s. In particular, during and following the Great Recession, UI duration reached an unprecedented level, and the whole extension program lasted for more than four years.¹ At the same time, the unemployment rate reached 10% for the first time in 30 years, leading many to argue that the benefit extensions have contributed to high unemployment rates and are therefore bad for the economy. Motivated by this debate, this paper asks three questions. First, why does the government extend UI benefits during recessions; second, to what extent does longer duration raise unemployment; and third, would fixing durations improve welfare?

This paper makes two main contributions. Theoretically, it is the first to study time-consistent UI policy in a general equilibrium search and matching framework. The literature has characterized the optimal cyclical UI policy as the solution to a Ramsey problem

¹ The potential maximum duration an unemployed worker can receive UI benefits was gradually increased from 26 weeks during normal times to 99 weeks from late 2008 to 2010 and stayed long until the end of 2013. During this period, Congress voted frequently on whether to keep or change UI duration, which suggests that the government does not follow a pre-committed policy rule when making UI duration policies. Appendix B.1 provides a detailed timeline of UI benefit extensions from 2008 to 2013.

of the government (see, for example, [Jung and Kuester \(2015\)](#) and [Mitman and Rabinovich \(2015\)](#)). However, the cyclical variations of the optimal policy does not resemble what is actually observed.² One reason is that the optimal policy is not time consistent and thus not implementable by a government without commitment to future policies.³ This paper characterizes the time-consistent UI policy using the concept of Markov-perfect equilibrium. In particular, a welfare-maximizing government chooses UI policy each period depending on the level of unemployment and aggregate productivity.⁴ In equilibrium, the government follows the same policy rule each period. The time-consistent UI policy has characteristics that are consistent with U.S. policy. In particular, the UI benefit level stays roughly constant, whereas the benefit duration increases during recessions.

The paper's second contribution is to quantify the impact of benefit extensions on unemployment and welfare. When calibrated to match the U.S. economy in steady state, the model produces cyclical volatilities of unemployment and vacancies that are close to the data. Furthermore, after calibrating to the observed path of unemployment, the model matches 80% of the variations in benefit duration from 2008 to the end of 2013. Compared to the scenario where the government keeps UI duration at its 2007 level, the unemployment rate associated with the time-consistent UI policy is 2 to 3 percentage point higher from 2009 to 2013. Nevertheless, the benefit extensions improved welfare by 0.16% of consumption relative to a no-extensions policy. This welfare assessment relies on disentangling the insurance and moral hazard effects of UI policy, which is made possible through our theoretical framework.

In addition to the two main contributions, this paper also makes empirical and methodological contributions. Empirically, we document that UI duration increased during all recessions since the 1970s. We find consistent patterns across these recessions. First, UI durations reach their highest levels around the time that unemployment rates peak. Second, recessions with higher peak unemployment rates are also associated with larger UI duration increases. Methodologically, this paper is the first to solve for a Markov-perfect equilibrium with two endogenous states by approximating policy rules. The challenge

² The Ramsey UI policy is less generous when the unemployment rate is high, because the committed policy maker can use policy to incentivize people to search hard when unemployment is high. [Appendix B.5](#) explores the Ramsey policy in steady state.

³ Intuitively, more generous benefits increase the current consumption of unemployed workers at the cost of higher future unemployment due to reduced job search incentives. As a result, the government always has incentives to deviate from ex-ante policies by giving more generous benefits ex-post, which constitutes a classic case of the time-inconsistency problem. [Appendix B.5](#) provides a numerical example to illustrate this problem.

⁴ Because we want to use the model to study observed UI variations, we keep the policy parameters as close to reality as possible. For example, we abstract from the type of employment history-dependent UI policy analyzed in [Hopenhayn and Nicolini \(1997\)](#).

here is computing policy derivatives in two dimensions.

Underlying these results is a general equilibrium business cycle model with a government and private sector consisting of workers and firms. A central government chooses the UI benefit level and the probability that benefit expires (“benefit exhaustion probability”), financing the UI program with a lump-sum tax on all workers. Modeling the benefit exhaustion probability rather than a fixed length of benefits keeps the government’s decision tractable.⁵ At the same time, the inverse of benefit exhaustion probability gives the expected duration, which allows for comparison with empirical evidence on benefit extensions. A key assumption here is that once benefits expire, the unemployed worker does not regain eligibility until he finds a job. Such a UI system is consistent with the actual UI policy in the U.S. Under this assumption, the unemployed workers with benefits search less than those without benefits. As a result, benefit duration policy today through, changing the proportion of unemployed workers with and without benefits, directly affects the states of the economy (unemployment and the measure of benefit-eligible unemployed workers) inherited by the future government. As such, the policy choice today influences future policy choices.

We model the private sector using the Diamond-Mortensen-Pissarides model with risk-averse workers, endogenous search intensity by the unemployed, and business cycles driven by shocks to aggregate labor productivity. Employed workers are matched with firms and receive a wage. With an exogenous probability, jobs are destroyed and the worker becomes unemployed. Unemployed workers enjoy home production (or leisure), and choose how much to search for a job. Job search comes at a utility cost, and higher job search intensity leads to higher job-finding probability. Because we want to study the UI duration dimension of the government policy, we distinguish between unemployed workers who are eligible and ineligible for UI benefits. If eligible for benefits, the unemployed worker receives UI benefits in addition to private home production. Eligibility is gained through working on a job and expires with the exhaustion probability chosen by the government. On the other side of the labor market, firms matched with a worker today produce and pay wages tomorrow, while unmatched firms post job vacancies at a cost.

The government’s choice of the UI benefit level is static in our model. In particular, the benefit level does not affect future unemployment, and so there is no intertemporal dimension to the choice of current benefit level. The government sets benefits so that the net marginal gain from insuring unemployed workers is zero. In contrast, the choice of expected duration is more sophisticated. When the government decides the duration of

⁵ Using a probability to model the duration policy allows us to use one variable—the measure of benefit-eligible unemployed workers—to summarize the relevant states instead of keeping track of discrete durations.

benefits, it faces the trade-off insurance and incentives. On the one hand, higher duration increases the UI coverage today, thus raising insurance for unemployed workers. On the other hand, a longer duration policy, through reducing today's average search, raises future unemployment. In addition, because today's duration policy affects future unemployment, it also affects the expectations of future policies, which the government takes into account.

The cyclical properties of the time-consistent UI policy are consistent with observed U.S. policy. Over the business cycle, the benefit level is slightly procyclical, increasing in labor productivity and falling in unemployment, but the magnitude of fluctuations is tiny. In contrast, the expected UI duration is strongly countercyclical. In particular, in response to a drop in productivity, expected future productivity is low, which implies a low marginal return to production tomorrow and a low marginal gain from job creation today. As a result, the cost of a higher expected duration is low, and the government raises UI duration. As the unemployment rate rises,⁶ the marginal gain from increasing UI duration is higher as more unemployed workers receive benefits, and as a result, the expected duration increases further.

Because our theoretical framework generates cyclical movements of UI policy that are consistent with empirical observations, the model can potentially be used to answer quantitative questions about UI benefit extensions. Before that, we validate our model along three dimensions. First, we check if a calibrated version of the model correctly predicts the generosity of UI policy in steady state. Calibrated to the steady-state U.S. labor market and average replacement ratio, the model generates a steady-state UI duration of 26.3 weeks, compared to 26 weeks in the U.S. Second, without directly targeting the business cycle properties in calibration, the calibrated model generates labor market volatilities that are very close to the data.⁷ Lastly, we test to see how well our model can account for variations in UI duration in recessions. We apply the model to the U.S. economy during the Great Recession by feeding in the exogenous job separation rates from the data and calibrating labor productivity to match the observed path of unemployment rates from 2008 to the

⁶ By fixing the proportion of unemployed workers with benefits, a higher unemployment rate implies a larger measure of benefit-eligible unemployed workers.

⁷ Economists in the labor search literature have suggested many ways to address the famous “Shimer puzzle”—named after [Shimer \(2005\)](#)—that the standard calibration of the search and matching model fails to account for the cyclical properties of unemployment and vacancies (see, for example, [Hagedorn and Manovskii \(2008\)](#) and [Hall and Milgrom \(2008\)](#)). The reason that our calibrated model generates large volatilities in unemployment and vacancies is two-fold. First, following [Hagedorn and Manovskii \(2008\)](#) we calibrate the wage elasticity with respect to productivity to data, and obtain a high home production value from targeting. These two values help generate relatively large cyclical variations in job search and vacancy posting. Second, because UI duration responds countercyclically to business cycle, it reduces workers' search in a recession, thus contributing to large cyclical movements in unemployment and vacancies.

end of 2013. Overall, our model matches the variations in duration very well, generating the correct timing of duration changes as well as 80% of the overall increase in UI duration.

An implication of our theory is that the Markov policy, by increasing UI duration in recessions, contributes to higher unemployment. Using the calibrated model, we ask how much of the increase in the unemployment rate can be accounted for by UI benefit extensions as opposed to general economic conditions. In particular, we compare the Markov-perfect equilibrium to an economy where the government does not change UI policy in recessions. The same paths of job separation rates and productivity shocks that generate a peak of 10% unemployment rate in the Markov equilibrium, lead to a peak of only 6.7% under the no-extensions policy. In other words, about 55% of the total increase in the unemployment rate during this period can be accounted for by rising UI benefit extensions. An important mechanism of our model works through expectations: when unemployment is high, the unemployed workers expect longer UI durations in the future, and in response reduces job search. To investigate how much workers' expectation contributes to the 3-percentage point unemployment gap, we turn off the expectations effect and obtain an unemployment gap of less than 1%, which implies that more than two-thirds of the total unemployment gap is driven by expectations.

Even though longer UI durations during recessions contribute to higher unemployment rate, the policy is in fact welfare improving. Compared to the economy where government policy does not respond to changes in the economic conditions, the Markov policy results in higher average welfare during the Great Recession, equivalent to 0.16% of average consumption. This is because the welfare loss from low job search due to longer durations is low in a recession, and as a result, the welfare gain from higher consumption of unemployed workers outweighs the loss from higher unemployment.

While a long tradition of literature has studied optimal UI policy (see, for example, [Hopenhayn and Nicolini \(1997\)](#), [Wang and Williamson \(2002\)](#), [Shimer and Werning \(2008\)](#), [Chetty \(2008\)](#), and [Golosov, Maziero and Menzio \(2013\)](#)), the cyclical response of optimal UI is a relatively new topic. Even less is known about time-consistent UI policy. The present paper fills the gap by characterizing time-consistent UI policy, focusing on its cyclical responses to changes in the underlying economic conditions. In addition to generating realistic cyclical responses, our theory is able to deliver quantitatively relevant variations in UI duration, thus giving us a framework to address quantitative questions.

The present paper contributes to the policy evaluation of UI benefit extensions (see, for example, [Fujita \(2010\)](#), [Rothstein \(2011\)](#), [Nakajima \(2012a\)](#), and [Hagedorn, Manovskii and Mitman \(2015\)](#)). Our contribution is that we use the model with endogenous government policies to evaluate the extensions policy. Our approach addresses the potential

policy endogeneity of the empirical works in this literature. Compared with the structural studies, which usually impose a deterministic and exogenous policy path, our endogenous policies take into account the policy effects on the households' expectations and how these expectations affect government's policy choices. In addition, little attention is given to the welfare implications of benefit extensions. Using the theoretical framework of time-consistent UI policy, our paper asks whether these extensions reduce welfare by raising unemployment. The answer is, surprisingly, that benefit extensions are welfare improving, despite contributing to up to 55% of the overall increase in the unemployment rate during 2008-2013.

Our paper is related to the literature on time-consistent public policy (see, for instance, [Alesina and Tabellini \(1990\)](#), [Klein and Ríos-Rull \(2003\)](#), [Chari and Kehoe \(2007\)](#), [Battaglini and Coate \(2008\)](#), and [Yared \(2010\)](#)). Methodologically, this paper follows [Klein, Krusell and Ríos-Rull \(2008\)](#) to characterize the Markov-perfect equilibrium government policy using the generalized Euler equation (GEE). Whereas [Klein, Krusell and Ríos-Rull \(2008\)](#) focus on the properties of a deterministic economy in steady state, we are interested in how government policy responds to business cycle fluctuations. Recent applications of Markov-perfect equilibrium include [Song, Storesletten and Zilibotti \(2012\)](#), who study intergenerational conflict over debt in a politico-economic environment.

The idea that the welfare gains and costs of UI vary over the business cycle is not new. For example, [Krueger and Meyer \(2002\)](#) argue that the efficiency loss from reduced search effort may be smaller during a recession than during a boom. More recently, [Kroft and Notowidiglo \(2015\)](#) use empirical results to estimate the moral hazard cost and consumption smoothing benefit of UI benefits. They find that the marginal welfare cost from generous benefits is procyclical, whereas the marginal welfare gain is modest and varies positively with unemployment rate. While they focus on the changing moral hazard effect of UI benefits on individual workers, we investigate the variations in the efficiency lost from the government's perspective.

Another channel through which UI may affect economic activities is through changes in aggregate demand. A transfer of income in the form of unemployment insurance may raise aggregate demand and, hence, employment, because unemployed workers may have a higher marginal propensity to consume; see, for example, [Albertini and Poirier \(2015\)](#) and [Kekre \(2015\)](#). However, the empirical evidence provided by [Hagedorn, Manovskii and Mitman \(2015\)](#) suggests that the benefit extensions lowered employment during 2008-2013, implying that the aggregate demand effect is not as powerful as the labor market channel. We therefore choose to focus on the effect of UI through labor supply. More importantly, we rationalize and evaluate endogenous UI benefit extensions in a stochastic

environment without government commitment. We thus offer a positive account of UI insurance and its impact. In contrast, [Kekre \(2015\)](#) derives a (Ramsey) optimal UI benefit formula with government commitment in the tradition of [Baily \(1978\)](#) and [Chetty \(2006\)](#), and evaluates exogenous benefit extensions in a deterministic setting.

The rest of the paper proceeds as follows. Section 1 describes the model environment and defines the private-sector competitive equilibrium. Section 2 defines the Markov-perfect equilibrium. We characterize the solution to the government’s problem using the GEE and solve the equilibrium. Section 3 describes the parametrization strategy. We conduct equilibrium analysis in Section 4 by presenting the Markov government’s policy rules and discussing their implications for the labor market. Section 5 provides quantitative analysis of UI benefit extensions during recessions. Section 6 concludes. We relegate all derivations, sensitivity analysis, additional figures, and a detailed review of the related literature to the Appendix.

2.2 MODEL

In this section, we describe the model environment and characterize the competitive equilibrium. The model is based on a Diamond-Mortensen-Pissarides model with aggregate productivity shocks.

2.2.1 Model environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Some unemployed workers receive UI benefits. Workers are risk averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\mathbb{U}(c_t) - v(s_t)]$$

where \mathbb{E}_0 is the period 0 expectation factor, and β is the time discount factor. Period utility comprises of utility from consumption of goods $\mathbb{U}(c)$ and disutility from job search activity $v(s)$. Utility is increasing in c and decreasing in s . To study the insurance incentive of the government we assume that $\mathbb{U}(\cdot)$ is a concave function. Only unemployed workers choose positive search intensity; that is, there is no on-the-job search. Each period, an employed worker gets paid wages from production. Wage determination technology is specified later in this section. An unemployed worker, if on unemployment benefits, receives b from the

government. In addition, an unemployed worker also produces h , which we take as the combined value of leisure, home production, and welfare. There are no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β . A firm can be either matched to a worker (and producing) or vacant. A vacant firm posting a vacancy incurs a flow cost κ .

Unemployed workers and vacancies form new matches. Let I and V denote the aggregate search by unemployed worker and the aggregate vacancy posting by firms, respectively. Then the number of new matches formed in a period is given by the matching function $M(I, V)$. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of expected matches: $M(I, V) \leq \min\{I, V\}$. The job-finding probability per efficiency unit of search intensity, f , and the job-filling probability per vacancy, q , are functions of labor market tightness, $\theta = V/I$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(I, V)}{I} = M(1, \theta) \\ q(\theta) &= \frac{M(I, V)}{V} = M\left(\frac{1}{\theta}, 1\right). \end{aligned}$$

Following the assumptions made on M , $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . The job-finding probability for an unemployed searching with intensity s is $sf(\theta)$. Existing matches are destroyed exogenously with constant job separation probability δ .

Each period, a matched pair of a worker and a firm produces z , where z is the aggregate labor productivity. z is equal to \bar{z} in steady state.

2.2.2 Government policy

The government cannot borrow or lend; instead, it balances the budget each period. The government finances unemployment benefits b through a lump sum tax, τ , on all workers, both employed and unemployed. The government budget constraint is

$$\tau = u_{\text{benefit}} b. \tag{2.1}$$

The government decides the generosity of the UI program by varying (1) benefit level, $b \geq 0$, and (2) the benefit exhaustion probability d (so $1/d$ is the expected duration). Once a benefit level and exhaustion probability are determined, previously benefit-eligible unemployed workers receive benefits b with probability d .

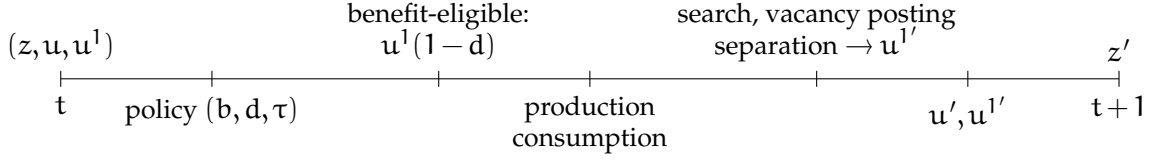


Figure 2.1: Timing of events.

2.2.3 Timing

The timing of events within a period is illustrated in Figure 2.1 and is as follows. The economy enters period t with a measure of the total unemployed workers u and a measure of the benefit-eligible unemployed workers u^1 . The aggregate shock z is then realized. (z, u, u^1) are the aggregate states of the economy.

Once government policies (b, d, τ) for the period are announced, $u_{\text{benefit}} = u^1(1-d)$ workers receive benefit. In other words, with probability d , unemployed workers previously with benefits lose benefit status in this period.

Employed workers produce z and receive wages w . Unemployed workers produce h and, if receiving benefits, receive b . All workers pay a lump sum tax τ .

Given aggregate states and government policies for the period, unemployed workers with and without benefits choose search intensity s^1 and s^0 , respectively. At the same time, firms decide how many vacancies to post, at cost κ per vacancy. The aggregate search is then $I = u^1(1-d)s^1 + (u - u^1(1-d))s^0$, aggregate vacancy posting is V , and market tightness is equal to $\theta = V/I$. The fraction of unemployed workers with and without benefits who find jobs is $f(\theta)s^1$ and $f(\theta)s^0$, respectively. At the same time, a fraction δ of the existing $1-u$ matches are exogenously destroyed. Newly unemployed workers and unemployed workers with benefits constitute next period's state u^1 .⁸

⁸ Effectively, newly unemployed workers qualify for benefits with probability $1-d$. In reality, newly unemployed workers qualify for benefits with at least two quarters of earnings and must pass an "earnings test" that depends on individual state policies. We model it as a probability for simplicity here.

The laws of motion of unemployed workers are

total unemployment:

$$u' = \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^0)(u-u^1(1-d)) + (1-f(\theta)s^1)u^1(1-d)}_{\text{previously unemployed who didn't find job}} \quad (2.2)$$

unemployed with benefit:

$$u^{1'} = \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^1)u^1(1-d)}_{\text{benefit-eligible unemployed who didn't find job}}. \quad (2.3)$$

2.2.4 Workers

Denote by g the government policy (b, d, τ) . In what follows we suppress the functional arguments in θ , which is an object determined in equilibrium. Wage w also depends on states of the economy, and may be an equilibrium object. The wage determination process is specified later. A worker unemployed and with benefits consumes $h + b - \tau$ and chooses search intensity s^1 ; an unemployed worker without benefits consumes $h - \tau$ and chooses search intensity s^0 . With probability $f(\theta)s$, $s = \{s^0, s^1\}$, he finds a job and starts working the following period. Let $V^e(z, u, u^1; g)$ and $V^u(z, u, u^1; g)$ be the values of an employed and an unemployed worker, respectively, with the beginning-of-period measures of unemployed workers with and without benefits (u, u^1) and realized aggregate shock z , given government policy for that period $g = (b, d, \tau)$.

The optimization problem of an unemployed worker *without* benefits (superscript 0 denotes no benefits) is

$$V^0(z, u, u^1; g) = \max_{s^0} U(h - \tau) - v(s^0) + f(\theta)s^0\beta\mathbb{E}V^e(z', u', u^{1'}; g') \\ + (1 - f(\theta)s^0)\beta\mathbb{E}V^0(z', u', u^{1'}; g'), \quad (2.4)$$

and the problem of an unemployed worker *with* benefit is

$$V^1(z, u, u^1; g) = \max_{s^1} U(h + b - \tau) - v(s^1) + f(\theta)s^1\beta\mathbb{E}V^e(z', u', u^{1'}; g') \\ + (1 - f(\theta)s^1)\beta\mathbb{E} \left[d'V^0(z', u', u^{1'}; g') + (1 - d')V^1(z', u', u^{1'}; g') \right]. \quad (2.5)$$

A worker entering a period employed produces and consumes his wage w minus tax τ . With probability δ , he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least

one period. The Bellman equation of an employed worker is then

$$V^e(z, u, u^1; g) = U(w - \tau) + (1 - \delta)\beta \mathbb{E}V^e(z', u', u^1; g') + \delta\beta \mathbb{E} \left[d'V^0(z', u', u^1; g') + (1 - d)'V^1(z', u', u^1; g') \right]. \quad (2.6)$$

2.2.5 Firms

To be matched with a worker and start production, a firm posts a vacancy.⁹ A firm that posts a vacancy incurs a flow cost κ . With probability $q(\theta)$, a vacancy is filled and ready for production the following period. Let $J^u(z, u, u^1; g)$ be the value of an unmatched firm posting a vacancy. The Bellman equation of an unmatched firm is

$$J^u(z, u, u^1; g) = -\kappa + q(\theta)\beta \mathbb{E}J^e(z', u', u^1; g') + (1 - q(\theta))\beta \mathbb{E}J^u(z', u', u^1; g'), \quad (2.7)$$

where $J^e(z, u, u^1; g)$ is the value of a matched firm. In equilibrium, under free-entry condition, the firm will post vacancies $v(z, u, u^1; g)$ until $J^u(z, u, u^1; g) = 0$.

A matched firm receives output net of wages $z - w$. With constant probability δ , a match is destroyed at the end of period. The Bellman equation of a matched firm is

$$J^e(z, u, u^1; g) = z - w + (1 - \delta)\beta \mathbb{E}J^e(z', u', u^1; g') + \delta\beta \mathbb{E}J^u(z', u', u^1; g'). \quad (2.8)$$

2.2.6 Wage determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(I, V)$. A realized match produces some economic rent that is shared between the firm and the worker. To introduce wage rigidity, we set wages to be a function of aggregate productivity. In particular, wages increase in labor productivity z , but less than one to one, where the elasticity is estimated from data. As such, workers and firms share risk arising from fluctuating labor productivity.

As a robustness check, we solve the equilibrium where wages are set each period through Nash bargaining, and the outside value to a worker is unemployed without benefits.¹⁰ In this latter case, equilibrium wages respond to the unemployment states of the

⁹ The firms can be viewed as a representative firm with a collection of jobs and several vacancies.

¹⁰ The specification reflects the fact that in the U.S., unemployment benefits qualification is conditional on

economy. We obtain results qualitatively similar to the specification with wage rigidity.

2.2.7 Competitive equilibrium

DEFINITION 2.1. (Competitive equilibrium) Given a policy $g = (b, d, \tau)$ and initial conditions (z^-, u^-, u^{1-}) , a competitive equilibrium consists of (z, u, u^1) -measurable functions for worker's search intensity $s^0(z, u, u^1; g)$ and $s^1(z, u, u^1; g)$, market tightness $\theta(z, u, u^1; g)$, total unemployment $u'(z, u, u^1; g)$, and the measure of unemployed workers with benefit $u^1(z, u, u^1; g)$, and value functions $V^e(z, u, u^1; g)$, $V^0(z, u, u^1; g)$, $V^1(z, u, u^1; g)$, $J^e(z, u, u^1; g)$, and $J^u(z, u, u^1; g)$ such that for all $(z, u, u^1; g)$:

- the value functions satisfy the worker and firm Bellman equations (2.4)-(2.8);
- the search intensities s^0 and s^1 solve the unemployed worker's maximization problem of (2.4) and (2.5), respectively;
- the market tightness θ is consistent with the free-entry condition,

$$V^u(z, u, u^1; g) = 0;$$

- measures of unemployment satisfy the laws of motion (2.2)-(2.3).

2.2.8 Characterization of private-sector optimality

The competitive equilibrium can be characterized by three optimality conditions.¹¹ Appendix B.3 contains derivations of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

The optimal choice of search intensity s^0 and s^1 for the unemployed worker is characterized by

no benefit:

$$\frac{v_s(s^0)}{f(\theta)} = \beta \mathbb{E} \left[\mathbb{U}(w' - \tau') - \mathbb{U}(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \quad (2.9)$$

reason for job separation. Workers who quit do not qualify for benefits. We view separation due to wage bargaining break-down a form of quitting. In addition, this specification also makes wages less volatile over the business cycle, and hence allows our model to generate greater cyclical variations in unemployment.

¹¹ To economize on notation, we suppress the dependence on $(z, u, u^1; g)$. It should be understood throughout that the optimal decisions are functions with arguments $(z, u, u^1; g)$.

with benefit:

$$\begin{aligned} \frac{v_s(s^1)}{f(\theta)} = & \beta \mathbb{E} d' \left[\mathbb{U}(w' - \tau') - \mathbb{U}(h - \tau') + v(s^{0'}) + (1 - f(\theta') s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \\ & + \beta \mathbb{E} (1 - d') \left[\mathbb{U}(w' - \tau') - \mathbb{U}(h + b' - \tau') + v(s^{1'}) + (1 - f(\theta') s^{1'} - \delta) \frac{v_s(s^{1'})}{f(\theta')} \right]. \end{aligned} \quad (2.10)$$

The worker's optimality conditions state that the marginal cost (left-hand side) of increasing the job-finding probability equals the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search of the unemployed worker weighted by the aggregate job-finding rate per efficiency unit of search. The marginal benefit is the sum of utility gain from being employed in the next period and the benefit of economizing on future search cost. A higher future benefit b' or a higher future duration $1/d'$ reduces the utility gain from being employed in the next period and thus lowers the marginal benefit of search today.

From firm's free-entry condition

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} \left[z' - w' + (1 - \delta) \frac{\kappa}{q(\theta')} \right], \quad (2.11)$$

where the marginal cost (left-hand side) equals the marginal benefit (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal benefit is the profits from employing a worker. Because a newly formed match does not become operational until the next period, the benefit from production has components only from the next period.

2.3 MARKOV EQUILIBRIUM

In this section, we define the Markov-perfect equilibrium in our economy. We assume the government is a utilitarian planner, who maximizes the expected value of a worker's utility. The government policy instruments include benefit level b , expected duration $1/d$, and tax τ . We consider government policies that are time consistent using the Markov-perfect equilibrium, à la [Klein, Krusell and Ríos-Rull \(2008\)](#).

Intuitively, one can think of the economy as having a different government each period. Each successive government chooses only current policy, taking future governments' policies as given. In other words, today's government cannot directly choose future policies. Instead, both today's government and private sector take form an expectation about future government policy rules when making decisions. Like [Klein, Krusell and Ríos-Rull](#)

(2008), we focus on equilibria where government policy depends *differentiably* on the aggregate states of the economy.¹²

The timing of events is illustrated in Figure 2.1. Because the economy consists of a mass of workers and firms, private-sector agents take future government policies as given. The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the state of the economy. To reduce the number of policy instruments in the government's problem, we use the following function derived from the government's budget constraint to express tax

$$\mathcal{T}(u^1, b, d) := u^1(1-d)b.$$

The *government period return function* is equal to the average welfare of all workers, and is given by

$$\begin{aligned} R(z, u, u^1, b, d, s^0, s^1) = & \text{worker} \\ & (1-u)U(w(z) - \mathcal{T}(u^1, b, d)) \\ & \text{unemployed without benefit} \\ & + (u - u^1(1-d)) [U(h - \mathcal{T}(u^1, b, d)) - v(s^0)] \\ & \text{unemployed with benefit} \\ & + u^1(1-d) [U(h + b - \mathcal{T}(u^1, b, d)) - v(s^1)] \end{aligned}$$

DEFINITION 2.2. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function G , government policy rules Ψ^b and Ψ^d , and private decision rules $\{S^0, S^1, \Theta, \Gamma, u^1\}$ such that for all aggregate productivity z and unemployment states (u, u^1) , $b = \Psi^b(z, u, u^1)$, $d = \Psi^d(z, u, u^1)$, $s^0 = S^0(z, u, u^1)$, $s^1 = S^1(z, u, u^1)$, $\theta = \Theta(z, u, u^1)$, $u' = \Gamma(z, u, u^1)$, and $u^{1'} = \Gamma^1(z, u, u^1)$ solve

$$\max_{b, d, s^0, s^1, \theta, u', u^{1'}} R(z, u, u^1, b, d, s^0, s^1) + \beta \mathbb{E}G(z', u', u^{1'})$$

subject to

- The worker's laws of motion

$$\begin{aligned} 0 &= f_1(u, u^1, d, s^0, s^1, \theta, u^{0'}) \\ &:= u' - \delta(1-u) - f(\theta)(s^0 - s^1)u^1(1-d) - (1-f(\theta)s^0)u \end{aligned} \quad (2.12)$$

$$\begin{aligned} 0 &= f_2(u, u^1, d, s^1, \theta, u^{1'}) \\ &:= u^{1'} - \delta(1-u) - (1-f(\theta)s^1)u^1(1-d) = 0; \end{aligned} \quad (2.13)$$

¹² While there is not a proof for the existence and uniqueness of Markov-perfect equilibrium, [Chatterjee and Eyigungor \(2014\)](#) provide argument for the existence of Markov-perfect equilibrium with continuous decision rules.

- The private-sector optimality conditions below, writing $\theta = (z, u, u^1)$ to economize on notation

$$\begin{aligned}
0 &= \eta_1(s^0, \theta, \theta'; \Psi^b, \Psi^d, S^0, S^1, \Theta) \\
&:= \frac{v_s(s^0)}{f(\theta)} \\
&\quad -\beta \mathbb{E} \left[\mathbb{U}(w(z') - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) - \mathbb{U}(h - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) + v(S^0(\theta')) \right] \\
&\quad -\beta \mathbb{E} \left[(1 - f(\Theta(\theta'))) S^0(\theta') \frac{v_s(S^0(\theta'))}{f(\Theta(\theta'))} - \delta \frac{v_s(S^1(\theta'))}{f(\Theta(\theta'))} \right]
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
0 &= \eta_2(s^1, \theta, \theta'; \Psi^b, \Psi^d, S^1, \Theta) \\
&:= \frac{v_s(s^1)}{f(\theta)} \\
&\quad -\beta \mathbb{E} \Psi^d(\theta') \left[\mathbb{U}(w(z') - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) - \mathbb{U}(h - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) + v(S^0(\theta')) \right] \\
&\quad -\beta \mathbb{E} \Psi^d(\theta') \left[(1 - f(\Theta(\theta'))) S^0(\theta') \frac{v_s(S^0(\theta'))}{f(\Theta(\theta'))} - \delta \frac{v_s(S^1(\theta'))}{f(\Theta(\theta'))} \right] \\
&\quad -\beta \mathbb{E} (1 - \Psi^d(\theta')) \left[\mathbb{U}(w(z') - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) \dots \right. \\
&\quad \quad \quad \left. \dots - \mathbb{U}(h + \Psi^b(\theta') - \mathcal{T}(u^1, \Psi^b(\theta'), \Psi^d(\theta'))) + v(S^1(\theta')) \right] \\
&\quad -\beta \mathbb{E} (1 - \Psi^d(\theta')) [1 - f(\Theta(\theta')) S^1(\theta') - \delta] \frac{v_s(S^1(\theta'))}{f(\Theta(\theta'))}
\end{aligned} \tag{2.15}$$

$$0 = \eta_3(\theta, \theta'; \Theta) := \frac{\kappa}{q(\theta)} - \beta \mathbb{E} \left[z' - w(z') + (1 - \delta) \frac{\kappa}{q(\Theta(\theta'))} \right]; \tag{2.16}$$

- The government value function satisfies the functional equation

$$G(\theta) \equiv R(z, u, u^1, \Psi^b(\theta), \Psi^d(\theta), S^0(\theta), S^1(\theta)) + \beta \mathbb{E} G(z', \Gamma(\theta), \Gamma^1(\theta)).$$

The Markov government chooses current policy, knowing how the private sector will behave given the policy.¹³ More specifically, the current Markov government weighs the trade-off between current and future welfare. By choosing a longer expected duration $1/d$, the current government increases the share of unemployed workers receiving benefits today, thus raising the current welfare. At the same time, because of moral hazard problem, unemployed workers on benefits choose lower search intensity, and as a result higher duration reduces the average search intensity, leading to higher future unemployment and lower future welfare.¹⁴

¹³ Equivalently, in the setup here, the government chooses both policy and private-sector allocations, taking into consideration private-sector optimality conditions.

¹⁴ A secondary effect exists through taxation. With longer duration, more unemployed workers receive

In addition, because all successive governments follow the same set of policy rules, the current government, by choosing current policy, affects the policies of future governments through changing future states of the economy. This *disciplining effect*, through private-sector expectations of future policies, affects job search of unemployed workers on benefits today, and through general equilibrium effects, affects job search of benefit-ineligible unemployed and firms' vacancy posting. The current government correctly anticipates this effect when choosing today's policy. Proposition 1 provides the conditions that characterize government decisions. The proof involves deriving the GEE and is included in Appendix B.3.

PROPOSITION 2.1. Given the aggregate states of the economy and the private-sector optimality conditions, the unemployment benefit policy b in the Markov-perfect equilibrium is characterized by

$$R_b = 0, \quad (2.17)$$

and policy d associated with the expected benefit duration can be characterized by the GEE

$$\begin{aligned} 0 = & R_d - f_{1d}\lambda \\ & + \frac{f_{2d}}{f_{2u}'} \left\{ \frac{\eta_{1u}'}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u}'}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \dots \right. \\ & \quad \dots + \frac{\eta_{3u}'}{\eta_{3\theta}} \left[-\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) \dots \right. \\ & \quad \quad \left. \left. \dots - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left(R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \right\} \\ & + \beta \mathbb{E} \left(-\frac{f_{2d}}{f_{2u}'} \right) [R_{u^1} - \lambda' f'_{1u^1}] \\ & + \beta \mathbb{E} \left(-\frac{f_{2d}}{f_{2u}'} \right) \left(-\frac{f'_{2u^1}}{f'_{2d}} \right) [R_d' - \lambda' f'_{1d}], \end{aligned} \quad (2.18)$$

benefits and the lump-sum tax is higher. The scale of this marginal cost of longer duration is small relative to the other two effects.

where λ is the shadow price of unemployment characterized by

$$\begin{aligned}
0 = \lambda & \\
& + \left\{ \frac{\eta_{1u'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u'}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \dots \right. \\
& \quad \dots + \frac{\eta_{3u'}}{\eta_{3\theta}} \left[-\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) \dots \right. \\
& \quad \quad \quad \left. \left. \dots - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left(R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \right\} \\
& - \beta \mathbb{E} [R'_e - \lambda' f'_{1e}] \\
& - \beta \mathbb{E} \left(-\frac{f'_{2u}}{f'_{2d}} \right) [R'_d - \lambda' f'_{1d}]. \tag{2.19}
\end{aligned}$$

The benefit level b affects only current welfare and does not have an effect on the states of future economy.¹⁵ As a result, b is set at a level that equates the marginal benefit (in the form of higher consumption for unemployed workers with benefits) and marginal cost (from a higher lump-sum tax) on current welfare. The equation $R_b = 0$ reflects such incentives.

In contrast, the choice of d is more complex. The GEE (Equation 2.18) provides a way to interpret the incentives of the government when choosing d . From (2.18), any change in d has four effects. First, it directly affects the trade-off between current consumption and future unemployment (first line). In particular, a lower d (higher expected duration) increases current welfare by increasing the share of unemployed workers receiving benefits. This is the *insurance* effect. At the same time, a lower d also reduces average search, thus increasing future unemployment.¹⁶ Second, through changing the expectation of future duration it affects current job search of benefit-eligible unemployed workers. This is the *moral hazard* effect. Changes in search of benefit-eligible unemployed workers in turn affect average job search and vacancy posting through general equilibrium effects (second and third lines). Third, any change in d affects future consumption through changing future unemployment (fourth line). This and the second effect together represent the “search/leisure” trade-off—lower d increases future unemployment, which in equilib-

¹⁵ One criticism of this assumption is that current benefit level does not affect search behavior of unemployed workers. In other words, there is no moral hazard cost in setting a high current benefit level. Some empirical evidence exists that shows that more generous UI benefit is associated with longer spells of unemployment, with an elasticity of about 1.0; see [Krueger and Meyer \(2002\)](#) for a survey of the earlier literature. More recently, [Krueger and Mueller \(2010\)](#) use time use data to find that for a subgroup of benefit-eligible unemployed workers, more generous benefits reduce their job search time. At the same time, they find that search activity of the benefit-eligible unemployed spikes as benefit exhaustion (26 weeks) approaches, which offers more support for moral hazard associated with expected duration.

¹⁶ $f_{1d} = f(\theta)(s^0 - s^1)u^1$ is the marginal change in unemployment when d changes.

rium reduces search today through a higher expected d' , thus increasing current welfare. Lastly, through changing d' , any change in d changes the future trade-off between consumption and unemployment (last line). The weight on the last line can be thought of as dd'/dd holding the two flow equations at zero and unemployment after the next period unchanged. The government determines current d by setting the net marginal value of d to zero.

Note that the GEE does not contain explicitly the derivative of Ψ^d ; it appears indirectly in the private-sector auxiliary function derivatives. In particular, the derivatives with respect to u' are

$$\begin{aligned}\eta_{1u'} &\equiv \frac{\partial \eta_1}{\partial u'} + \underbrace{\frac{\partial \eta_1}{\partial b'} \Psi_u^{b'} + \frac{\partial \eta_1}{\partial d'} \Psi_u^{d'} + \frac{\partial \eta_1}{\partial s^{0'}} S_u^{0'} + \frac{\partial \eta_1}{\partial s^{1'}} S_u^{1'} + \frac{\partial \eta_1}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}} \\ \eta_{2u'} &\equiv \frac{\partial \eta_2}{\partial u'} + \underbrace{\frac{\partial \eta_2}{\partial b'} \Psi_u^{b'} + \frac{\partial \eta_2}{\partial d'} \Psi_u^{d'} + \frac{\partial \eta_2}{\partial s^{1'}} S_u^{1'} + \frac{\partial \eta_2}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}} \\ \eta_{3u'} &\equiv \frac{\partial \eta_3}{\partial u'} + \underbrace{\frac{\partial \eta_3}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}},\end{aligned}$$

and the derivatives with respect to $u^{1'}$ are

$$\begin{aligned}\eta_{1u^{1'}} &\equiv \frac{\partial \eta_1}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_1}{\partial b'} \Psi_{u^1}^{b'} + \frac{\partial \eta_1}{\partial d'} \Psi_{u^1}^{d'} + \frac{\partial \eta_1}{\partial s^{0'}} S_{u^1}^{0'} + \frac{\partial \eta_1}{\partial s^{1'}} S_{u^1}^{1'} + \frac{\partial \eta_1}{\partial \theta'} \Theta_{u^1}'}_{\text{disciplining effect}} \\ \eta_{2u^{1'}} &\equiv \frac{\partial \eta_2}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_2}{\partial b'} \Psi_{u^1}^{b'} + \frac{\partial \eta_2}{\partial d'} \Psi_{u^1}^{d'} + \frac{\partial \eta_2}{\partial s^{1'}} S_{u^1}^{1'} + \frac{\partial \eta_2}{\partial \theta'} \Theta_{u^1}'}_{\text{disciplining effect}} \\ \eta_{3u^{1'}} &\equiv \frac{\partial \eta_3}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_3}{\partial \theta'} \Theta_{u^1}'}_{\text{disciplining effect}}.\end{aligned}$$

This reflects an important point made earlier—that successive governments agree on a policy rule Ψ^d . The Markov government does not try to manipulate its successor through changing current d , hence the absence of directives of Ψ^d directly from the GEE. The fact that Ψ^d affects private-sector auxiliary functions captures the fact that how much a higher d (lower expected duration) increases private-sector job search and vacancy posting depends on how the extra search will reduce next-period unemployment.

The Markov-perfect equilibrium is then characterized by a system of *functional* equations (2.1), (2.12)–(2.16), and (2.17)–(2.19). An analytical characterization of the Markov-

perfect equilibrium is not possible; instead, we solve for the equilibrium numerically by approximating the government policy rules and private-sector decision rules using the Chebyshev collocation method.

2.4 PARAMETRIZATION

We describe our calibration strategy in this section. The model period is one month. We calibrate the parameters of the Markov equilibrium to match important features of the U.S. labor market between 2003.I and 2007.IV.

The utility function is

$$U(c, s) = \frac{c^{1-\sigma}}{1-\sigma} - v(s),$$

where $v(\cdot)$ is the search cost function. We assume $v(\cdot)$ is a non-negative, strictly increasing, and convex function, with the property that $v(0)$ is bounded and $v(0) \geq 0$. We specify the search cost function to be consistent with the literature:

$$v(s) = \gamma \frac{s^{1+\phi}}{1+\phi}.$$

For any $\gamma > 0$, v exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $v(0) = v_s(0) = 0$.

We adopt the matching function from [den Haan, Ramey and Watson \(2000\)](#), which is also used in [Hagedorn and Manovskii \(2008\)](#) and [Krusell, Mukoyama and Sahin \(2010\)](#) among others,

$$M(I, V) = \frac{V}{[1 + (V/I)x]^{1/x}},$$

where I is the aggregate job search and V is the aggregate vacancy posting in the economy. This matching function guarantees that both the job-finding rate,

$$f(\theta) = \frac{\theta}{[1 + \theta x]^{1/x}},$$

and the job-filling rate,

$$q(\theta) = \frac{1}{[1 + \theta x]^{1/x}},$$

are always strictly less than 1.

We pick three parameters related to preferences. The discount factor β is $0.99^{1/3}$, giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion σ is set to 1 (log utility). Finally, the search cost curvature parameter ϕ is set to 1 following the

Table 2.1: Externally Calibrated Parameters

Parameter	Description	Value
δ	U.S. job separation rate	0.02
κ	Vacancy posting cost	0.58
ρ	Persistence of productivity	0.968
σ_ϵ	Standard deviation of innovation to productivity	0.0060
ϵ_w	Elasticity of wage with respect to productivity	0.446

Note: Calibration targets are monthly statistics of the U.S. economy.

average estimate in the literature.¹⁷

The externally calibrated parameters are summarized in Table 2.1. Following the methodology outlined in Shimer (2005), we calculate the average monthly job separation rate from aggregate-level CPS data.¹⁸ This gives an average job-finding rate during 2003.I-2007.IV of 0.40, and an average separation rate $\delta = 0.02$.¹⁹ We set the costs of vacancy creation κ to be 58% of monthly labor productivity following Hagedorn and Manovskii (2008).

As in Shimer (2005), labor productivity z is taken to be the average real output per employed person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume an AR(1) process for the shock to z :

$$\log z' = \rho \log z + \sigma_\epsilon \epsilon,$$

where $\rho \in [0, 1)$, $\sigma_\epsilon > 0$, and ϵ are i.i.d. standard normal random variables. We target a quarterly autocorrelation of 0.762 and an unconditional standard deviation of 0.013 for the HP-filtered productivity process. At a monthly frequency this means setting $\rho = 0.9680$ and $\sigma_\epsilon = 0.006$.

Wages are function of productivity with the following functional form,

$$w(z) = \exp(\log \bar{w} + \epsilon_w \log z),$$

¹⁷ Imposing ϕ equal to 1 gives a quadratic search cost function. This restriction is consistent with estimates by Yashiv (2000), Christensen et al. (2005), and Lise (2013), and calibration work of Nakajima (2012a).

¹⁸ To be consistent with our model, we do not adjust for time aggregation error when computing the job separation rate. Therefore, the job separation rate from the data is $\delta_t = u_{t+1}^s / e_t$, where u^s is short-term (one to four weeks) unemployment, and e is total employment.

¹⁹ Although some may argue that the U.S. economy during 2003.I-2007.IV is *above* the long-run trend, we believe it is an appropriate period to target for the labor market, especially because of the secular downward trend in job separation rate documented by, for example, Fujita (2012). Appendix B.2 also documents a declining trend in job-finding rate since 1951. Given these trends, using the long-run average job-finding and separation rates would overestimate the recent steady-state numbers.

Table 2.2: Internally Calibrated Parameters

Parameter	Description	Value
h	Value of home production	0.595
γ	Disutility of search	1.706
χ	Matching parameter	3.462
\bar{w}	Steady-state wage	0.979
Target	Data	Model
Average replacement ratio	40%	38.1%
Average job-finding rate	0.40	0.416
% unemployed with benefits	45	45.8
Average job-filling rate	0.66	0.661

Note: Calibration targets are monthly statistics of the U.S. economy 2005.I-2007.IV.

where \bar{w} represents the steady-state share of output for the worker, and ϵ_w is the elasticity of the average wage with respect to aggregate productivity. We use data on labor productivity and real wages (constructed using labor shares data) between 1951.I and 2014.IV to estimate $\epsilon_w = 0.446$. This means a 1 percentage point increase in labor productivity is associated with a 0.446 percentage point increase in real wages. Our estimate is close to the estimate of 0.449 for 1951.I-2004.IV obtained by [Hagedorn and Manovskii \(2008\)](#).

We jointly calibrate four parameters using steady-state moments. The four parameters are (1) the value of home production (and leisure) h , (2) the matching function parameter χ , (3) the level parameter of search cost γ , and (4) the steady-state wage level \bar{w} . We use four steady-state moments as targets: (1) the expected UI replacement ratio, (2) the average job-finding rate, (3) the average job-filling rate, and (4) the proportion of unemployed workers with benefits.²⁰ We follow [Shimer \(2005\)](#) and set the replacement ratio at 40%. The average job-finding rate is the monthly rate at which unemployed workers become employed, and it is 0.40 for 2003.I-2007.IV. Over the same period, the job-filling rate is 0.66.²¹ Table 2.2 reports these internally calibrated parameter and the matching of calibration targets. The calibrated model delivers a benefit duration of 26.3 weeks, very close to the benefit duration of 26 weeks in the U.S. during normal times, thus delivering the first

²⁰ We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See [Zhang, Conn and Scheinberg \(2010\)](#) for details.

²¹ The job-filling rate is calculated as the job-finding rate divided by the vacancy-unemployment ratio, where the latter is computed using the national unemployment rate reported by the BLS and the nonfarm job openings from the Job Openings and Labor Turnover Survey. The estimate for the 2003.I-2007.IV period is close to [den Haan, Ramey and Watson \(2000\)](#) who use plant-level data during 1972.II-1988.IV and get a job-filling rate of 0.71.

Table 2.3: Summary Statistics: Cyclicality

Statistic	Productivity z	Unemployment u	Vacancy v	v-u ratio v/u
<i>Quarterly U.S. data 1951.I-2014.IV</i>				
Standard deviation	0.013	0.126	0.141	0.261
Correlation matrix	z	1	-0.230	0.331
	u	-	1	-0.969
	v	-	-	1
	v/u	-	-	1
<i>Calibrated Markov economy</i>				
Standard deviation	0.013	0.147	0.167	0.273
Correlation matrix	z	1	-0.908	0.982
	u	-	1	-0.909
	v	-	-	1
	v/u	-	-	1

Note: Seasonally adjusted unemployment, u , is constructed by the BLS from the CPS. Vacancy-posting, v , is [Barnichon \(2010\)](#)'s spliced series of seasonally adjusted help-wanted advertising index constructed by the Conference Board and the job-posting data from the JOLTS. Both u and v are quarterly averages of monthly series. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1,600.

model validation.

Table 2.3 compares labor market statistics in the U.S. economy and the calibrated Markov economy. The calibrated model does a good job of generating the relevant cyclical properties, which provides the second model validation for our theory. In particular, the model produces negative correlation between unemployment and vacancy, thus preserving the shape of the Beveridge-curve (inverse relation between unemployment and vacancy). Two parameters allow our model to match cyclical properties well. First, we calibrate the elasticity of wage with respect to productivity to match data counterparts. The relatively low wage elasticity means firm's profit and hence vacancy posting are volatile over the business cycle. Second, by targeting the average replacement ratio, we obtain a high value of home production, which contributes to high volatility in job search. Unlike [Shimer \(2005\)](#) and [Hagedorn and Manovskii \(2008\)](#), the benefit level in our model is endogenously chosen by the government and is a function of home production in steady state. As such, we can use the home production value to target replacement ratio.

2.5 EQUILIBRIUM ANALYSIS

In this section, we present the Markov government policy rules and discuss their effects on the equilibrium labor market.

2.5.1 Markov equilibrium policy rules

Figure 2.2 plots the Markov equilibrium policy rules for UI policies holding productivity at the steady state level.²² In each plot, the solid line represents policy rule, and the dashed line represents steady-state unemployment.

The expected UI duration $1/d$ increases in total unemployment. The decision on UI duration involves a trade-off between insurance (for higher current consumption) and job-creation (for higher future welfare). When unemployment is high, both the insurance and job-creation incentives are high—the former because more people are unemployed, and the latter because shorter duration would increase job-search incentives for more people through their expectations of future UI policies. In equilibrium, the increase in insurance incentive outweighs the higher job creation incentive, and expected duration is longer at higher unemployment.

In contrast, the UI benefit level b is lower at higher unemployment, but the scale of change is minuscule, falling by less than 1% from steady state to 10% unemployment. Intuitively, when unemployment is high, the cost of taxation rises slightly more than the gain from insurance. The almost flat policy rule of b reflects that when unemployment increases the rise in cost of taxation is almost entirely offset by the higher gain from insurance.

Figure 2.3 plots the Markov equilibrium government policy rules, holding unemployment (both total unemployment and benefit-eligible unemployment) at the steady-state levels. The expected duration increases dramatically with lower labor productivity, especially when productivity is below its steady state. This is because when productivity is low, the expected productivity next period is also low, assuming a persistent productivity process. As such, the marginal return from production tomorrow (for both workers and firm) is low, as is the cost of low job creation today (high unemployment tomorrow). As a result, the marginal cost of longer duration in the form of lower job creation is low, and the government chooses long duration. The unemployment benefit level b , in contrast, increases with higher labor productivity, but the slope is fairly small, indicating very small changes with respect to labor productivity.

²² We also hold the proportion of unemployed workers with benefits at the steady-state level.

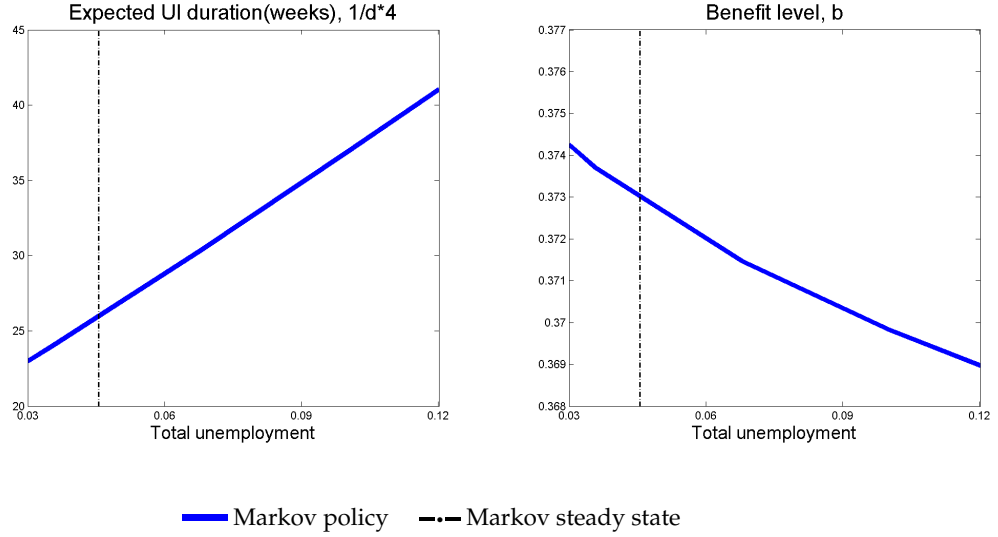


Figure 2.2: Markov equilibrium government policy rules holding productivity and proportion of benefit-eligible unemployed workers at steady state.

2.5.2 Comparative static analysis of government incentives

Because the government’s choice of d (which directly translates into expected duration) involves the trade-off between insurance and moral hazard, we conduct comparative static analysis to understand changes in these two incentives that drive the movements in expected duration. Figure 2.4 shows responses of these two incentives to changes in unemployment (left panel) and productivity (right panel).²³

As total unemployment rises, both insurance gains and moral hazard cost are higher. More specifically, when unemployment is higher, longer UI duration has a larger insurance effect because it extends benefit coverage to more unemployed workers. In terms of the GEE (Equation 2.18), this effect affects the marginal effect of government choice d (and hence expected duration) on current welfare (part of R_d):

$$- u^1 \times \underbrace{[U(h + b - \tau) - U(h - \tau)]}_{\text{welfare gain from giving benefits to an additional worker}} \quad (2.20)$$

With higher unemployment (u) and fixing the proportion of benefit-eligible unemployed workers (u^1/u), u^1 in expression (2.20) is larger, which increases the marginal welfare gain from a smaller d (longer expected duration). At the same time, higher unemployment makes future unemployment more sensitive to changes in search induced by changes in

²³ The responses over unemployment hold constant the proportion of benefit-eligible unemployed workers and productivity at steady state. The responses over productivity hold total and benefit-eligible unemployment at steady state. Both subplots hold UI policies at steady state.

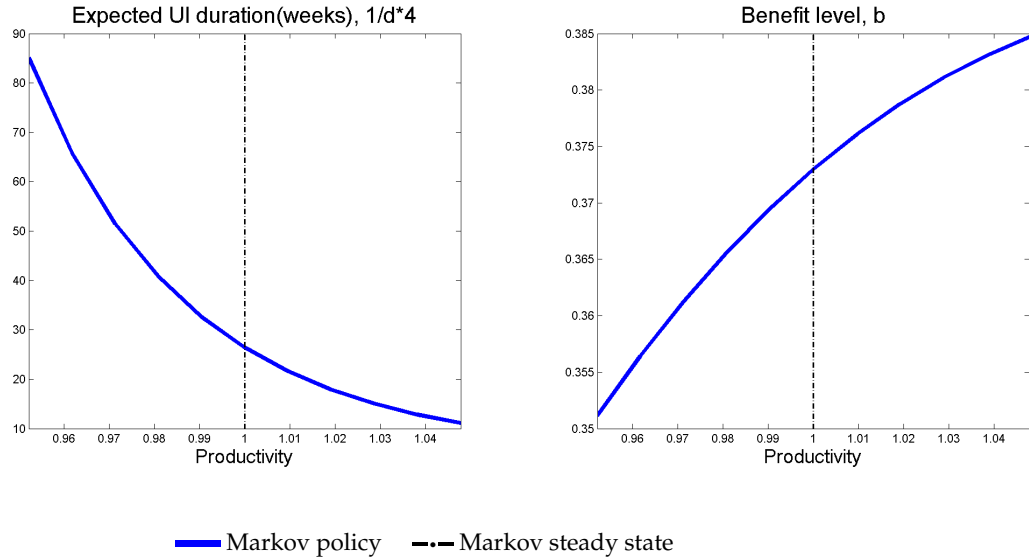


Figure 2.3: Markov equilibrium government policy rules holding unemployment at steady state.

the current UI policy, and as a result, longer UI duration has a larger moral hazard cost. The left panel of Figure 2.4 shows that a 1% increase in total unemployment raises the insurance incentive by 16% and moral hazard cost by 9%. The higher increment in insurance incentives means that at a higher unemployment level, the government has a stronger incentive to increase expected UI duration.

In response to a drop in productivity, both insurance gains and moral hazard cost are lower. In particular, lower productivity leads to lower wages, which increases the employed workers’ marginal utility of consumption and reduces the marginal gain from insurance. This effect is small, and disappears if workers are risk neutral. In contrast, the drop in moral hazard cost is larger. In response to a lower productivity, the expected future productivity and wages are also lower, which means that future unemployment leads to a smaller reduction in average consumption. In other words, there is lower moral hazard cost as a result of lower average search induced, and the government can “afford” to choose longer duration. The drop in moral hazard cost is amplified by a drop in job posting—result of lower productivity and hence lower expected future profit—which lowers the response of future unemployment to changes in duration.

The variations of marginal welfare gain and cost here are consistent with recent empirical findings by Kroft and Notowidiglo (2015). First, they find that the moral hazard cost is procyclical. The marginal cost of moral hazard here varies positively in both unemploy-

ment and productivity and is overall procyclical.²⁴ Second, they find that the marginal welfare gain from consumption smoothing varies positive with the unemployment but variations are small. The marginal gain from insurance in our mechanism also varies positively with unemployment, but the scale of variation is large. This is because the gain from consumption smoothing that they document is only part of the gain from insurance in our mechanism. Most of the variations in the gain from insurance in the left panel of Figure 2.4 come from an *extensive* margin: when unemployment is high, the gain from extending benefits to more unemployed workers is high because it increases the average consumption of all unemployed workers.

2.5.3 Impulse response in policy and labor market

We now consider the economy's response to a one-time, unanticipated drop in productivity. Figure 2.5 shows the response of the economy to a 1% drop in productivity z at time 0. We first focus on the responses with Markov policy (solid blue lines). We then compare the responses of the economy with and without (dotted red lines) government policy changes to understand the driving forces behind movements in the labor market.²⁵ Because the transition dynamics are relatively slow, it takes a long time for the economy to return to a steady state. In Figure 2.5, the time horizon is 90 months or approximately seven and half years.

Upon shock, expected duration rises immediately from 26.3 weeks to 33 weeks and then falls slowly as productivity recovers. By month 30, expected duration has fallen to 29 weeks. Since unemployment is a slow-moving process, it peaks at around month 7, when productivity has already recovered one-fifth of the 1% drop. Because expected duration rises with higher unemployment, the drop in expected duration after the initial rise is slowed by the rising unemployment. Benefit level, in contrast, falls to below steady state on impact, with less than 1% total change, and slowly recovers to the pre-shock steady state as both productivity and unemployment recover.

Search by both benefit-eligible and benefit-ineligible unemployed workers fall upon impact, which drives average search down by about 10%. While benefit-eligible unemployed workers search less in response to much longer expected UI duration, the benefit-ineligible unemployed workers respond mainly to lower expected future output and wages. Vacancy posting also falls initially but the recovery is much quicker than the job search re-

²⁴ While we distinguish between a drop in productivity and an increase in unemployment, the empirical work of Kroft and Notowidiglo (2015) does not. So their result that moral hazard cost is higher when the unemployment rate is lower should not be directly compared to the left panel of Figure 2.4.

²⁵ Appendix B.4 provides impulse responses of other labor market statistics.

covery. By month 6, vacancy posting is more than half-way back to the pre-shock steady-state level. This is because vacancy posting depends on expected future productivity and aggregate search. As search by individual unemployed workers recovers, and with high unemployment during the first few months after shock, *aggregate* search is high.²⁶ Because higher aggregate search increases the marginal return from vacancy posting, vacancy posting responds to aggregate (and not average) search. Total unemployment increases rapidly to peak in month 7, before gradually falling back to its steady-state level.

To understand to what extent the rise in unemployment is driven by changes in policy versus productivity, we shut down changes in government policy and only allow the labor market to respond to changes in productivity. Compared to unemployment increases with policy changes (solid blue lines), the increases without policy change (dotted red lines) are much more muted (1% versus 5%). Behind the difference in unemployment are smaller drops in both search and vacancy posting without policy change. In particular, the average search drops by less than 1%, compared to a 9% decrease with policy changes. The drop in vacancy posting without policy changes is about half of the drop with policy change. The larger difference in search reflects that job search incentives are distorted by policy changes whereas vacancy posting incentives respond mostly to productivity changes.

2.6 UI DURATION EXTENSIONS IN RECESSION

Because the cyclical properties of Markov equilibrium policy rules are consistent with those of the U.S. policy, in this section we use the theoretical framework to study recessions. We first validate the model by using the model to account for expected duration variations during and after the Great Recession (December 2007 to December 2013). We then compare the Markov policy to alternative UI policies to study the impact on unemployment and welfare.

2.6.1 Empirical evidence of UI benefit extensions in recessions

We first document variations in UI duration during each recession since the 1970s. Figure 2.6 plots variations in unemployment and UI duration during all five recession episodes.²⁷ The shaded areas represent National Bureau of Economic Research (NBER) official recessions.

²⁶ Additional impulse response in Appendix B.4 shows above steady-state aggregate search during the first few months after shock.

²⁷ The recession from January to July 1980 was both shorter and milder than the other recessions. In addition, it was followed immediately by the much longer recession from July 1981 to November 1982. We therefore left out the former recession period.

sion dates. For each recession episode, the dotted red line (right axis) plots the unemployment rate, and the solid blue line (left axis) plots the maximum expected UI duration in weeks. The timing and extent of changes in UI duration follow the specifics of the federal unemployment compensation laws, which are available from the U.S. Department of Labor Employment and Training Administration (DOLETA) website. Two things are worth noting. First, during all recession episodes, UI duration reached its highest level around the time unemployment peaked. Second, comparing across recessions, recession with higher unemployment is associated with in general higher expected UI duration. This, however, was not true for the 1980s recession. The fact that Markov benefit duration rises with total unemployment is consistent with the above historical evidence.

Because more detailed data are available for the Great Recession, Figure 2.7 documents the frequency of legislation on UI policy during and following the recession. The vertical dotted lines indicate the timings of legislation. The frequency of legislation increased substantially from the mid-2008, especially from late 2009 to 2011. This provides suggestive evidence that the federal government does not follow a prescribed policy rule and instead makes policy choices depending on the contemporary states of the economy.²⁸ It is therefore a natural choice to use Markov policy, which is time consistent and without commitment, to describe policy changes during the recession.

Because state-level implementations of UI benefit extension are conditional on state economic conditions, especially on the state's insured unemployment rate (IUR) and total unemployment rate (TUR), we use the two statistics to compute whether the state was eligible for longer durations in the month a UI-related legislation was passed during the Great Recession. We then create a weighted measure of expected UI duration across states using the number of total insured unemployed workers in each state as the weight. Appendix B.1 contains more details. Figure 2.7 plots weighted expected UI duration with a dashed blue line. For the quantitative analyses, we use this weighted average series as the empirical counterpart because it is a more accurate description of the UI duration policy implemented.

2.6.2 The Great Recession

To further our theory, we put the model in an environment similar to the U.S. economy during and following the Great Recession from December 2007 to December 2013. Be-

²⁸ The fact that UI legislation involved debates and voting in Congress suggests that the government did not follow a pre-committed series of state-contingent policies as a government with commitment would optimally do.

cause our theory focuses on UI, we specify a path for productivity to match the observed unemployment path during the period.²⁹ We use a piecewise linear productivity process consisting of the decline, the trough and, the recovery. It turns out that this simplified way of calibrating the productivity path generates good fit for unemployment. The job separation rate is exogenous and calculated from aggregate-level CPS data. Figure 2.8 plots the shock processes.³⁰

Model Fit Given the shock processes, Figure 2.9 plots the variations in UI duration, labor market variables, and average welfare generated by the Markov equilibrium (solid blue line). Compared to the U.S. economy (dashed black line), the Markov equilibrium matches well the variations in UI duration and the vacancy-unemployment ratio. In particular, the Markov expected duration policy rises from 26 weeks to slightly below 80 weeks, compared to the maximum of 90 weeks in the U.S. economy. The Markov policy also generates a decline in expected duration, but the decline starts earlier than in the U.S. economy. The Markov policy captures the sudden drop and slow rise in the vacancy-unemployment ratio as observed in the U.S. economy, but underestimates the scale of the drop. One reason for the smaller drop in the vacancy-unemployment ratio is that the model does not have job-to-job transition. During recessions, job-to-job transition, in addition to unemployment-to-job transition, declines, and as a result, vacancy posting should decline more when job-to-job transition is taken into account.

2.6.2.1 Policy Evaluation

One interesting question we study using the model is whether and how much do benefit extensions contribute to higher the unemployment rates. We use the counterfactual where the government does not do benefit extensions but instead keeps benefit duration at 26 weeks throughout the recession (and the private sector fully understands it). Fig-

²⁹ Appendix B.4 includes two alternative calibrations where we use productivity z to either match the path of UI duration or get a best fit for both unemployment and duration. The benchmark calibration of productivity process here is the preferred one because while unemployment is a smooth process (and reported on a weekly basis), UI duration is not because Congress meets on a relatively fixed schedule that does not respond to changes in economic conditions. Therefore, matching to UI duration is subject to underlying assumptions about meeting schedules.

³⁰ The process we specify for labor productivity is in fact not far-fetched. Labor productivity as measured by average production per person in the nonfarm business sector by 3% from the end of 2007 to the beginning of 2009, which is more than 4% in detrended terms. The difference between our process and the U.S. process is the recovery path. U.S. labor productivity recovered swiftly to pre-2008 levels by the end of 2010, whereas our process follows a slower recovery process. For theories on slow and/or jobless recovery following the Great Recession, see, for example, [Stock and Watson \(2012\)](#), [Shimer \(2012\)](#), and [Heathcote and Perri \(2015\)](#). In addition, [McGrattan and Prescott \(2010, 2014\)](#) suggest that labor productivity calculated from the data—especially during the 1990s and the Great Recession—are mismeasured. This paper does not take a stand on what the true labor productivity is and instead use the observed unemployment path to discipline productivity.

ure 2.9 shows that, in contrast to the no-extension policy (dotted red line), the Markov benefit extensions policy leads to higher unemployment rates. At the peak of unemployment, unemployment is lower by more than 3 percentage points in the economy without extensions. At the same time, average welfare is with Markov extensions. The difference in welfare over the transition path is roughly translated into 0.16% of average monthly consumption.³¹

A key prediction of our theory is that search effort is procyclical, that it falls during recessions. This feature is present in the standard search model with endogenous search effort. The empirical findings on the cyclical nature of search effort are mixed.³² More recently, [Gomme and Lkhagvasuren \(2015\)](#), after controlling for heterogeneity in the unemployed worker's past wages and hours, find evidence that search is procyclical, consistent with the prediction of structural search literature.

2.6.2.2 The effect of expectation

To isolate the effect of expectation in our result, we shut down the channel of private sector's expectations. In particular, in this exercise we assume that the private sector expects future UI duration (and benefit level) to stay at the steady-state level. At the same time, we assume that the government implements the same policy as before ex-post. In other words, all benefit extensions above the regular 26 weeks are "unanticipated." Figure 2.10 illustrates the experiment by comparing the economy under (1) Markov benefit extensions policy (solid blue line), (2) no benefit extensions policy (dotted red line), and (3) unexpected extensions policy (dashed green line).

The unemployment gap between Markov extensions and unexpected extensions policies is large, accounting for about 70% of the unemployment gap between the Markov extensions and the no-extensions economics. This reflects the importance of expectations. When benefit-eligible unemployed workers rationally expect the government to follow Markov policy, they reduce their job search activities when productivity is low or when unemployment is high, because the expected long UI duration next period distorts their search incentives. In contrast, when unemployed workers expect the government to maintain a no-extensions policy, they do not reduce search as much.

³¹ Even though shocks to both productivity and job separation rate contribute to cyclical fluctuations in the model, the shock to productivity actually drives most of the movement. Appendix B.4 provides an analysis where productivity is kept at its steady-state level to isolate the effect of shock to job separation rate.

³² [Shimer \(2004\)](#) and [Mukoyama, Patterson and Şahin \(2006\)](#), for example, find countercyclical search effort, while [DeLoach and Kurt \(2013\)](#) find evidence of procyclical search effort. See [Gomme and Lkhagvasuren \(2015\)](#) for a brief review of the empirical literature on search effort.

Because the distortion on search incentives works only through unemployed worker's expectation of future UI policies, this exercise is also a decomposition of the unemployment gap into the "search" wedge and the "composition" wedge. More specifically, the unemployment difference between Markov extensions and the unexpected extensions represents the effect of policy distortion on search behavior; the unemployment difference between the unexpected extensions and no-extensions economics represents the effect of policy in changing the composition of the unemployed population—longer duration increases the proportion of unemployed workers with benefits, thus reducing average search.

Interestingly, the average welfare of under unexpected extensions is higher than in the Markov equilibrium. This is not surprising, because the former economy has both high current consumption—from the ex-post Markov policy—and low future unemployment—due to the expectation of no-extensions policy. But such an economy is in a sense "unsustainable" as it requires that the government always be able to "fool" the private sector.

2.6.2.3 Drivers of the welfare gap

One thing worth noting is the welfare evaluation is essentially an *ex-post* calculation, meaning that it is done knowing the realization of shocks during the transition path. This comparison is instructive as a policy evaluation, as are similar policy evaluation experiments performed in the literature. Maybe equally interesting is the *ex-ante* welfare comparison. The difference here is the ex-ante evaluation is done without the knowledge of how the recession would pan out, and agents in the economy expect future shocks to follow an AR(1) process. We assume that an unexpected productivity shock occurs in January 2008, and perform the welfare comparison after the realization of this shock. The question addressed by this exercise is, "should the government follow the Markov rule or the no-extensions policy given this shock?" Interestingly, the no-extensions policy gives slightly higher (less than 0.1% in consumption equivalent) average welfare than the Markov policy rule ex ante. The reason for this welfare reversal is that the recession during this period turns out to be both long and severe, which gives more justification for the Markov policy.

The welfare gap between the Markov and no-extensions policies in Figure 2.9 are driven by two opposing forces: the higher unemployment under the Markov policy reduces the average welfare in the Markov economy relative to the no-extensions policy, whereas the higher proportion of unemployed workers on benefits increases the average welfare of unemployed workers in the Markov economy. Figure 2.11 illustrates these two forces.

The proportion of unemployed workers on benefits (middle panel) is calculated as

$u^1(1 - d)/u$. Under the no-extensions policy, the proportion falls early in the recession. While both the measure of benefit-eligible unemployed worker, u^1 , and the total unemployment, u , increase in response to rising job separation rates and falling productivity, the rise in total unemployment is larger because the job-finding rates of both benefit-eligible and benefit-ineligible unemployed workers fall. In contrast, under the Markov policy, the initial rise in the proportion is mainly driven by the longer duration policy (lower d).

The gap in benefit coverage between the two policies increases over time. At the time when unemployment peaks, about 60% of unemployed workers are covered by UI benefits under the Markov policy, whereas only 40% have benefits under the no-extensions policy. The higher benefit coverage under the Markov policy leads to higher average welfare among unemployed workers relative to the no-extensions policy. The welfare gap (right panel) between the Markov and no-extensions policies is then the result of the welfare cost of higher unemployment being outweighed by the welfare gain from higher benefit coverage ratio. An important reason for this result is that wages are low during the recession, which lower the marginal cost of unemployment.³³

2.6.2.4 Relation to [Hagedorn, Manovskii and Mitman \(2015\)](#)

The comparison between Markov equilibrium and the economy under no-extensions policy is in line with [Hagedorn, Manovskii and Mitman \(2015\)](#), who exploit discontinuity at state borders to identify the effect of unemployment benefit extensions. In particular, one way to interpret their empirical result is follows. In states where firms and workers expect good exogenous shocks to the economy, e.g. an oil boom, they also expect lower or no benefit extensions. This increases the expected value of employment to a worker and in turn increases job creation compared to states with bad economic outlooks. This interpretation is very similar to our theory. With the Markov policy, firms and workers *expect* longer benefit durations in recessions—analogue to states with bad economic outlooks—whereas under the no-extensions policy, the private sector *expects* no-extensions policy—an extreme case of states with good exogenous shocks.

One difference between our theory and [Hagedorn, Manovskii and Mitman \(2015\)](#) lies in the mechanism underlying our results. In their model, longer benefit duration increases unemployed worker's outside option, thus increasing their reservation wage. Higher reservation wage then reduces firm's vacancy posting by reducing profit margin.

³³ Another reason is that the long duration policy discourages search by the benefit-eligible unemployed workers, which means even if benefit coverage were the same under the two policies, the average welfare of unemployed workers would still be slightly higher under the Markov policy.

In our model, longer expected duration makes unemployment less painful and thus reduces job search activity. Reduced search activity then reduces the marginal return to vacancy posting, lowering overall vacancy posting. To see how much of the unemployment gap in Figure 2.9 comes from differences in vacancy posting as opposed to job search, Figure 2.12 compares the Markov equilibrium and the economy under no-extensions policy along these two dimensions. Both vacancy posting and average search are higher in the economy with no-extensions UI policy, but the gap is much larger for average search. This comparison reflects that in our model both vacancy posting and search contribute to the unemployment gap, but average search contributes more.

2.7 CONCLUSION

This paper develops a quantitative theory of how a welfare-maximizing government uses UI policy to balance the incentives of insurance against the cost of moral hazard arising from distortion in unemployed workers' job search incentives. We use the concept of Markov-perfect equilibrium to study a time-consistent UI policy, where the government makes decisions each period contingent on payoff-relevant aggregate states of the economy. In the steady state, our theory delivers an expected UI duration close to the U.S. policy. Over the business cycle, the UI benefit level stays roughly constant and the expected duration rises during recessions. Both the steady state and cyclical properties of our theory are consistent with policies in the U.S. since the 1970s.

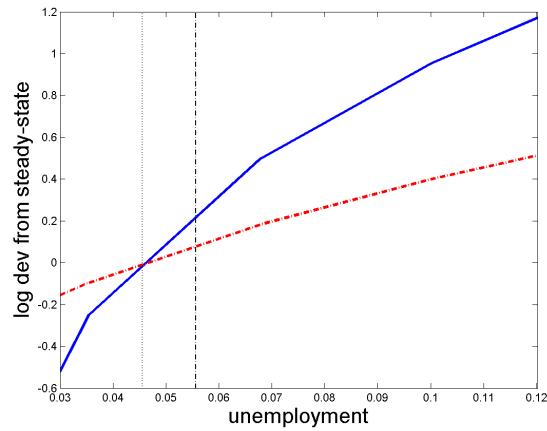
We then use the theoretical framework to study benefit extensions in the U.S. from 2008 to 2013. We find that compared to a UI policy fixed at the 2007 level, the Markov policy, which matches most of the variations in benefit duration observed in the data, leads to an increase of 3 percentage points in the unemployment at its peak. Of this unemployment gap, we find that more than two-thirds is driven by private-sector expectations: unemployed workers expect longer future UI durations in recessions and as a result reduce job search. More importantly, we find that a longer UI duration during recessions is welfare improving. Compared to the scenario where the government does not change UI policies, the benefit extensions lead to higher welfare equivalent to 0.16% of average consumption. This finding provides a new perspective to the debate over whether UI benefit extensions are good policy.

Several simplifying assumptions are made for tractability. First, neither workers nor government can save or borrow. Because savings provide self-insurance to workers, allowing workers to save will reduce the need for government-provided insurance policy. At the same time, credit access may reduce search by the unemployed (see, for example,

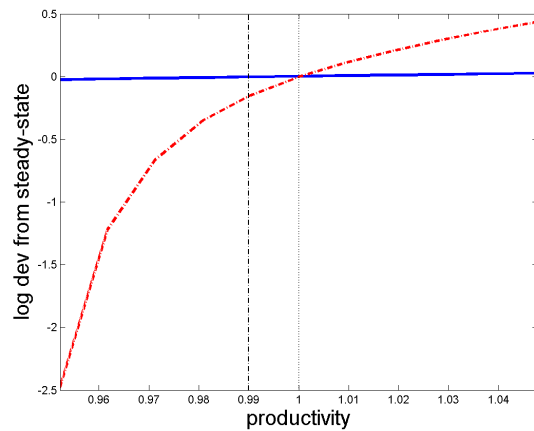
[Herkenhoff \(2015\)](#)), thus exacerbating moral hazard associated with search. The reduced need for insurance and increased moral hazard problem will likely reduce the cyclical response of benefit duration. Allowing government access to the credit market will increase the cyclical responses of the benefit duration and likely make the benefit level less procyclical (or more acyclical). This is because the government can borrow to finance a generous benefit policy in bad times and pay back the debt with tax revenue in good times.

The second assumption is that government policy takes effect right away. In reality, there often is a time lag between legislation and implementation. Allowing the government to announce policy changes before implementation gives workers and firms time to react to the potential changes, which may mitigate the effect of policy changes (“announcement effect”). However, by looking at UI legislations during the Great Recession, we find that most extensions came into effect shortly after announcement. Furthermore, the announcement effect, if any, is likely small quantitatively. [Nakajima \(2012a\)](#), for example, incorporates announcement effect in his evaluation of benefit extensions and finds minor quantitative effect associated with announcement.

Responses to a 1% increase in unemployment



Responses to a 1% drop in productivity



— Marginal gain from insurance
 - - - Marginal cost of moral hazard
 ····· Steady state
 - - - 1% increase in unemployment (or drop in productivity)

Figure 2.4: Responses of marginal gain from insurance and marginal moral hazard cost to a 1% change in unemployment or productivity, holding government policies at the steady state.

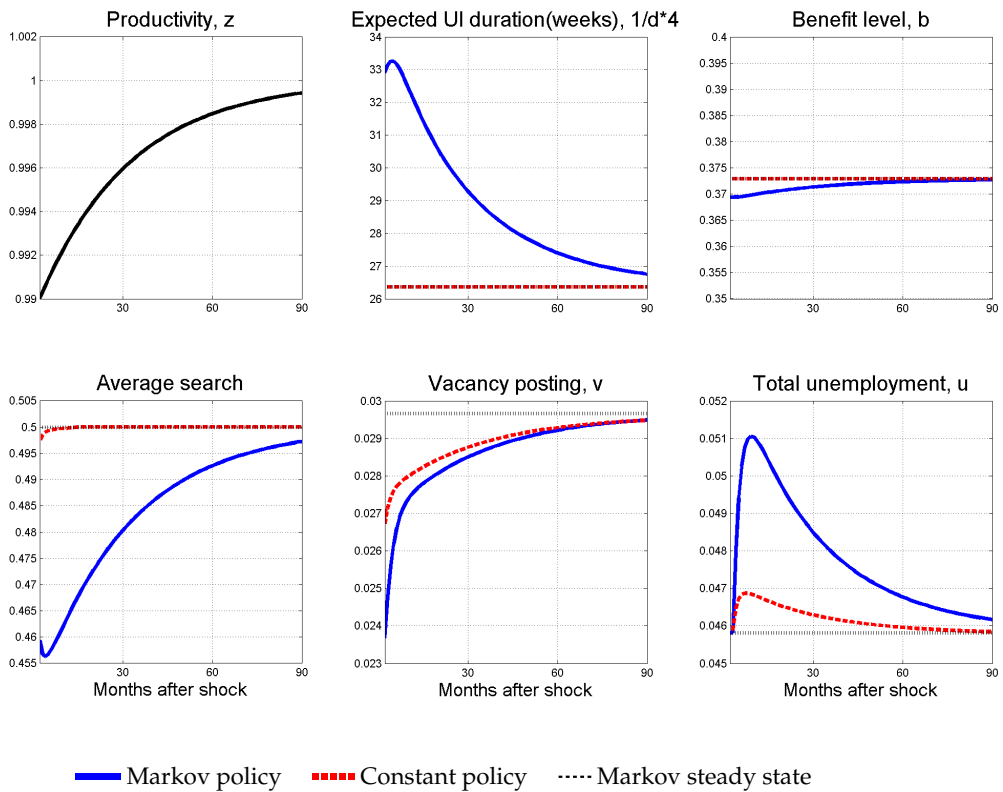
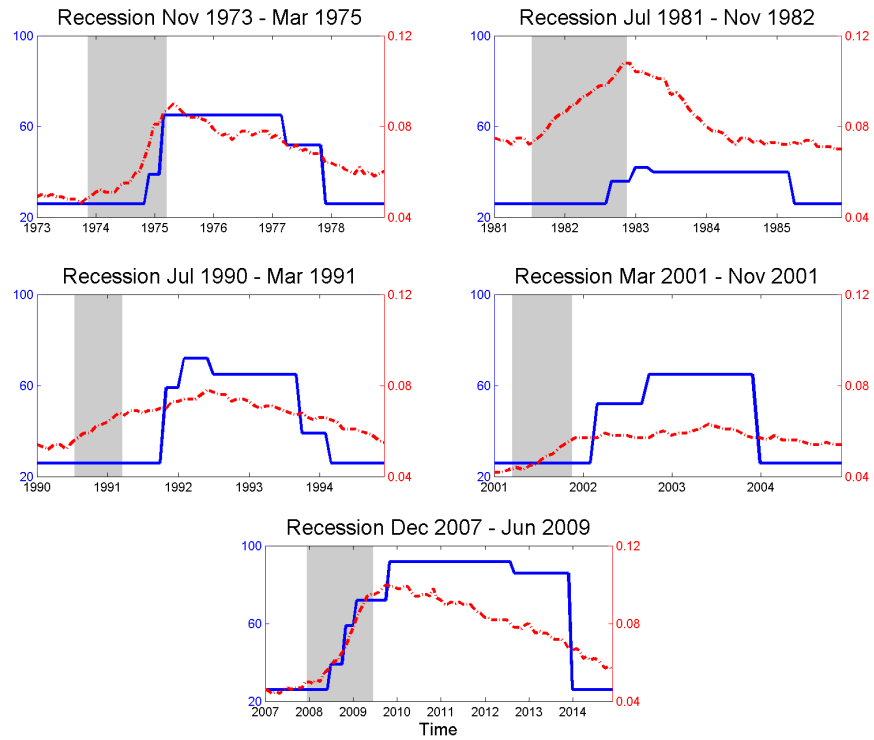
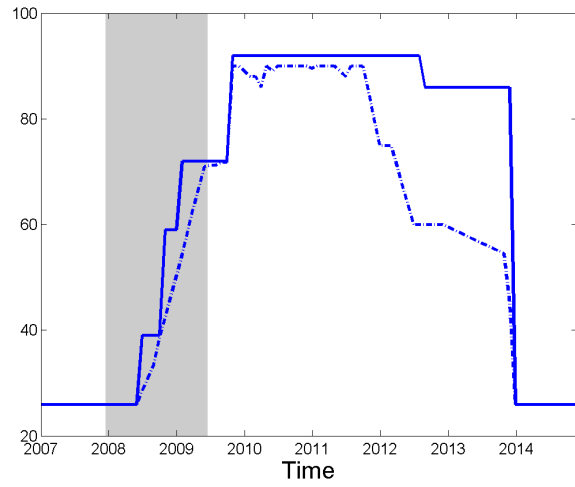


Figure 2.5: Impulse response to a 1% drop in productivity.



— Expected UI duration (weeks) - - - Unemployment rate

Figure 2.6: Empirical changes in unemployment (right axis) and UI duration (left axis) during recessions since the 1970s.



— Expected UI duration (weeks) - - - Weighted UI duration (weeks) ····· UI legislation

Figure 2.7: Empirical changes in UI duration and timing of UI-related legislation during the Great Recession.

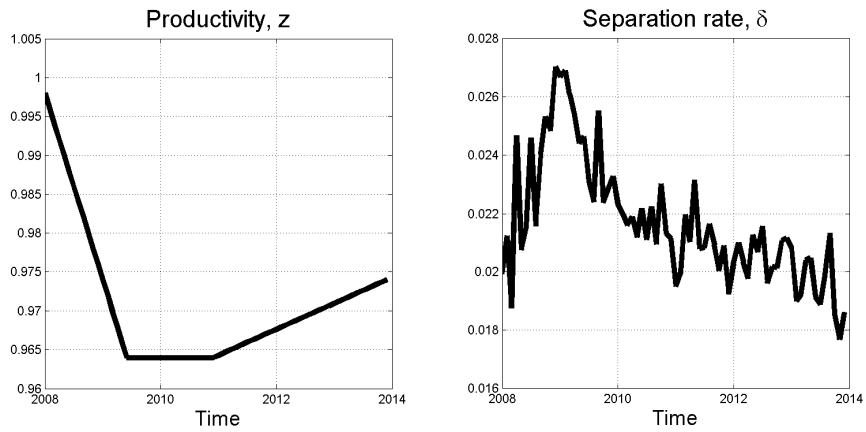


Figure 2.8: Exogenous shock processes during the Great Recession.

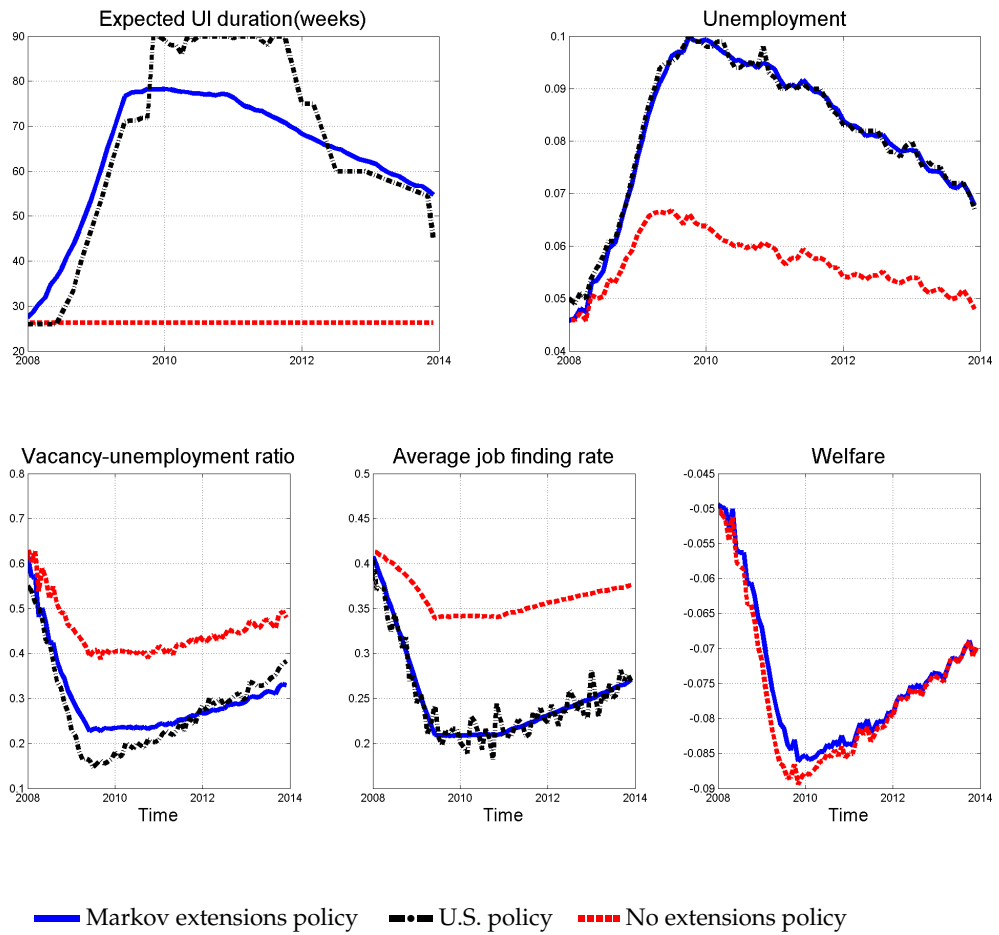
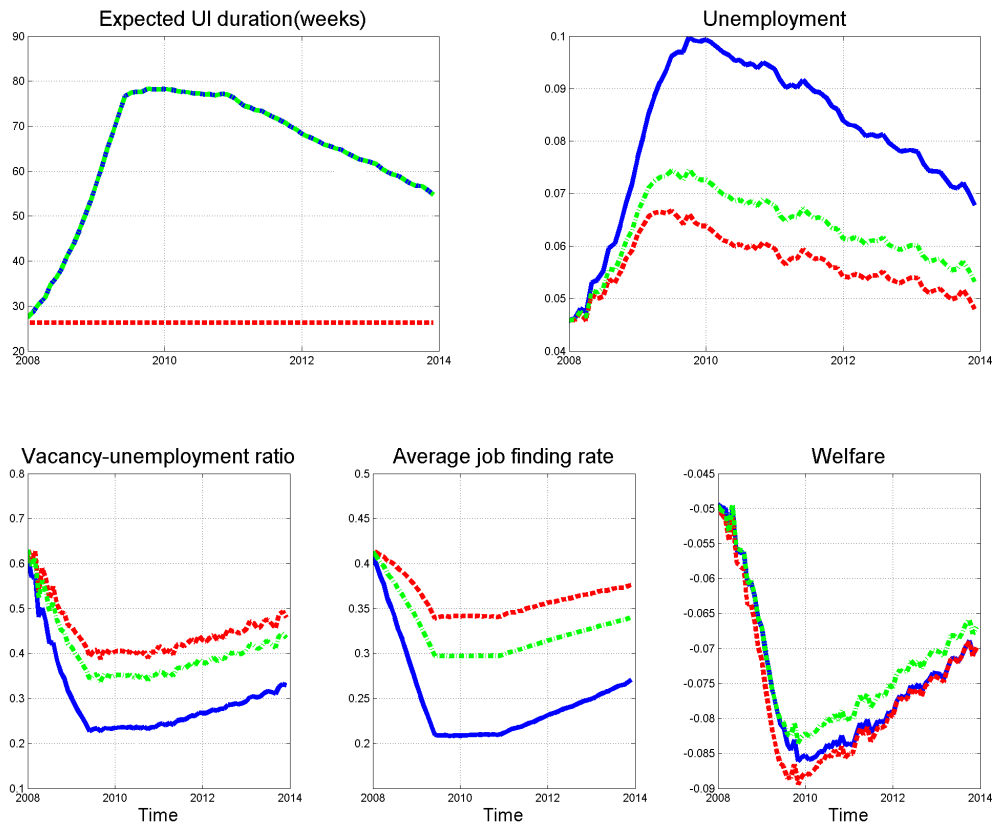


Figure 2.9: UI duration, unemployment, and welfare during the Great Recession: Model, data, and counterfactual policies.



— Markov extensions policy - - - No extensions policy ··· Unexpected extensions policy

Figure 2.10: UI duration, unemployment, and welfare during the Great Recession: Effect of expectation.

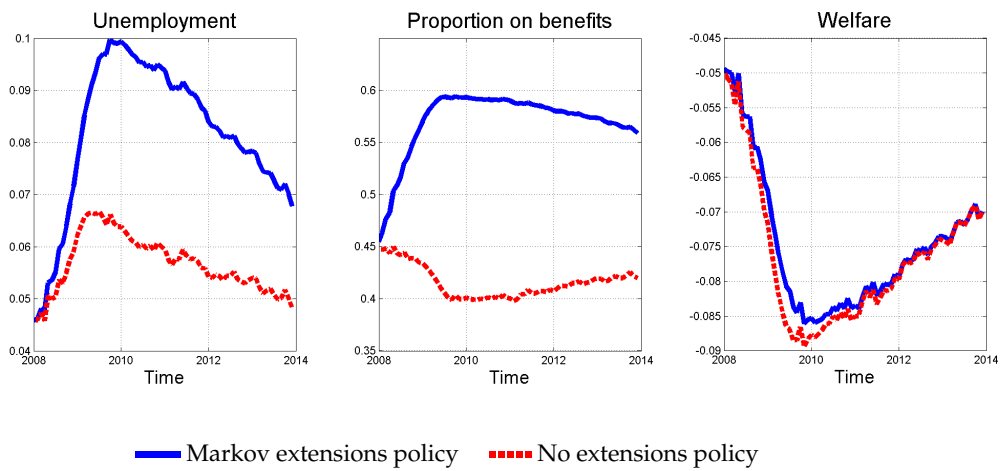


Figure 2.11: Drivers of welfare gap between Markov extensions and no extensions policies: Unemployment and proportion unemployed on benefits.

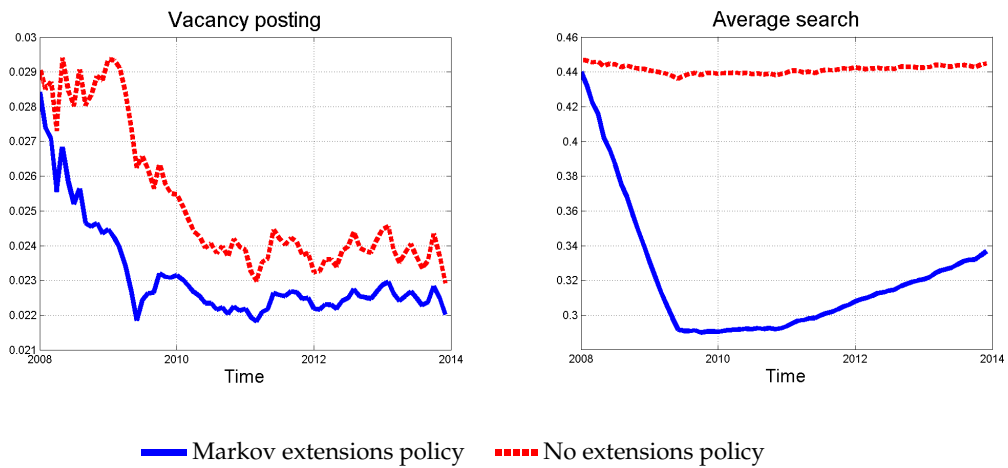


Figure 2.12: Markov extensions versus no extensions policy: Vacancy posting and average search.

Chapter 3

Delayed Collection of Unemployment Insurance during Recessions

3.1 INTRODUCTION

Two assumptions are common in the literature on unemployment insurance (UI) policy. First, only unemployed workers receive UI benefits. Second, once an unemployed worker finds a job, she loses any uncollected benefits. The first assumption is true to the extent that unemployed workers also include underemployed workers such as part-time workers who are actively seeking full-time employment.¹ The present paper examines the second assumption both empirically and quantitatively.

Empirically, this paper documents that during recessions workers in the U.S. can and do delay the collection of UI benefits to future unemployment spells (“retention policy”). In addition, there are both cross-state and cross-time differences in how easy an unemployed worker can delay collection of leftover benefits as a result of policy variations. Exploiting these differences, I find empirical evidence that making delayed benefit collection easier encourages unemployed workers to search for a job in recessions. Intuitively, because UI benefits are proportional to wage income, low-paying jobs qualify for lower benefits. As a result, an unemployed worker has less incentive to take a low-paying job because of low future benefits when she becomes unemployed again. With the retention policy, if the worker has leftover benefits when she finds a job, she can delay collecting the leftover benefits to future unemployment spells.

¹See, for example, [McCall \(1996\)](#) and [Le Barbanchon \(2014\)](#) on extending UI benefits to part-time workers.

Motivated by the empirical evidence, this paper extends the standard search and matching model to incorporate features of the retention policy. In particular, unemployed workers actively search for jobs. Search incurs a utility cost, and job-finding probability is proportional to individual's search level. Employed workers may qualify for UI benefits through working. If qualified for benefits, an unemployed worker receives a fixed amount of benefits each period for a finite number of periods. The level of benefits received each period are proportional to the wages prior to unemployment. Without the retention policy, once an unemployed worker finds a job, any leftover benefits will be forfeited. With the retention policy, any uncollected benefits stay on the worker's record, and when she becomes unemployed again she can choose between any newly qualified UI benefits or the leftover benefits.² By giving the worker a choice in future benefits, the retention policy increases the expected value of employment by (weakly) increasing the value of future unemployment. As a result, the unemployed workers increase their search effort, and aggregate job-finding probability becomes higher.³

The effect of retention policy on workers' search depends on three factors of the economic condition. First, the effect is larger when the probability of losing a job is larger. A larger job separation probability reduces the discount on the value of future unemployment, thus raising the effect of future unemployment on current search incentives. Second, the effect of retention policy is larger when the difference in wages between previous and future jobs is (positive) larger. A positive wage difference implies that the current UI benefits are higher than the expected future benefits. The larger the wage difference, the larger the difference between current and future benefits, and the more retention policy raises the value of employment for unemployed workers. Third, the effect is larger when the benefit duration is longer such as during a recession. Longer durations give unemployed workers more benefits to carry over to future unemployment spells, thus increasing the effect of retention policy. I use a simple model to illustrate the effect of the retention policy and how it changes with the underlying economic conditions, across states and across time.

Because the three factors change with the underlying economic conditions during a recession, and because the empirical evidence is snapshots at one or more points along a recession, we need a quantitative model to evaluate the effect of retention policy on job-

² As an example, suppose a worker qualifies for 26 weeks of benefits of \$200 per month. He finds a job after collecting benefits for 6 weeks. Without retention policy, he loses the 20 weeks of benefits, and whether he qualifies for benefits when he becomes unemployed again depends on how much he works between the two unemployment spells. With the retention policy, if he does not qualify new benefits when unemployed again, he can continue collecting the 20 weeks of benefits.

³ A key assumption here is that workers do not quit into unemployment. This assumption is supported by the fact that workers who quit do not qualify UI benefits, although quits are sometimes hard to distinguish from other reasons of unemployment; see, for example, [Zhang and Faig \(2012\)](#).

finding probability during a recession. For example, an unemployed worker who loses her job at the beginning of a recession is affected most by the retention policy, because (1) the gap between her past (pre-recession) and future (during recession) wages is likely high, (2) the job separation probability during her future employment is likely high, and (3) the benefit duration she qualifies is likely long because of benefit extensions during recession. To incorporate changing economic conditions, I extend the simple model to a quantitative model and analyze a transition from a pre-recession economy without UI extension to the economy between 2011 and 2012. I calibrate the model steady state to match the pre-recession economy, and feed in changing job separation probability, aggregate productivity and UI duration between 2008 and 2012. I find that while UI extensions discourage search and raise unemployment, the retention policy mitigates the adverse incentive effect of UI extensions and lowers the unemployment rate by about 0.8 during 2008–2012.

This paper contributes to the understanding of how UI policy affects unemployed workers' search behavior. Previous literature has studied how benefit level and benefit duration affect search. Empirical evidence in the literature suggests that more generous UI benefits are associated with longer spells of unemployment, with an elasticity of about 1.0; see [Krueger and Meyer \(2002\)](#) for a survey of the earlier literature. More recently, [Krueger and Mueller \(2010\)](#) use time use data to find that for a subgroup of benefit-eligible unemployed workers, more generous benefits reduce their job search time. At the same time, they find that search activity of the benefit-eligible unemployed spikes as benefit exhaustion (26 weeks) approaches, which suggests that longer UI duration is associated with less search by the unemployed workers. The present paper argues that because of the retention policy in recessions, higher benefit levels do not always mean lower search. In fact, higher expected benefits during future unemployment spells may increase search by current unemployed workers.⁴

Another strand of the literature on UI policy looks at specific policy details and studies how they affect search behavior. For example, [Zhang and Faig \(2012\)](#) examine how endogenous UI eligibility affects search, and find that when UI benefits must be earned with employment, generous UI becomes an additional benefit to working. The current paper complements their work by incorporating both endogenous eligibility and wage-indexed wages, so that working at a high-paying job has the additional benefit of qualifying for higher benefits in unemployment. And with the retention policy, this benefit may extend into future unemployment spells. [McCall \(1996\)](#) and [Le Barbanchon \(2014\)](#) look at the effect of

⁴ Because the effect of the retention policy requires that workers form expectations about future benefits, government commitment to these expected future benefits is implicitly assumed. Chapter 2 examines the effects of government commitment to UI policies on the optimal cyclical UI benefits and on the labor market.

partial benefits on unemployed workers' incentives to take part-time job. The present paper complements their works by looking at how delayed benefit collection affects another "alternative" form of employment, short-term employment.

Because the effect of retention policy is quantitatively relevant only during recessions when the maximum potential UI duration is extended, the study of the retention policy has ramifications on the optimal cyclical UI policy; see, for example, [Jung and Kuester \(2015\)](#), and [Mitman and Rabinovich \(2015\)](#). In addition, because the effect of the retention policy also changes with the scale of UI extensions, incorporating the retention policy in models evaluating the UI extensions will change the quantitative results. In fact, quantitatively I find that the retention policy mitigates the effect of UI extensions. This paper thus contributes to the literature on the impact of UI extensions; see, for example, [Fujita \(2010\)](#), [Rothstein \(2011\)](#), [Nakajima \(2012a\)](#), [Hagedorn et al. \(2015\)](#).

The rest of the paper proceeds as follows. Section 2 describes the policy background and empirical evidence pertaining to policy variations. In Section 3 I introduce a simple version of the model to illustrate the effect of policy variation. Section 4 presents a quantitative model and analysis based on calibrated parameters. Section 5 concludes.

3.2 POLICY BACKGROUND AND EMPIRICAL EVIDENCE

The retention policy allows unemployed workers to delay collection of UI benefits to future unemployment spells.⁵ Two important ingredients here are that workers *qualify* benefits through work—so not all unemployed workers receive benefits—and the monthly benefit payout amount ("benefit level") is proportional to wages received during the most recent employment. The retention policy changes over the business cycle. In addition, while the majority of states have the retention policy, states differ in how easy it is for unemployed workers to take advantage of the retention policy.

3.2.1 Cross-time policy variations

During normal times, when the maximum potential UI duration is 26 weeks, an unemployed worker has up to one year ("benefit collection window") to use all 26 weeks. As an example, an unemployed worker who qualifies 26 weeks of benefits and starts collecting

⁵ This section focuses on explaining the cross-time and cross-state variations of the retention policy and abstracts from the variations in UI extensions. While UI extensions also vary over time in recessions, it does not affect the cross-sectional empirical analysis. The variations across states are controlled for in the empirical analysis by taking data from a time when states in the sample implemented roughly the same length of UI extensions.

on January 1, 2005 may collect during anytime before January 1, 2006. During this time, if she finds a job after collecting benefits for 13 weeks, then she may collect the other 13 weeks if she loses her job again before January 1, 2006. A complication arises if the employment during this one-year period qualifies her with a new UI segment. When this happens, the worker may choose whether to continue collecting the 13 weeks left over from before, or start the new UI segment. However, she may not keep both UI segments.

The impact of the retention policy is small during normal times for two reasons. First, because the benefit collection window is one year for a typical 26 weeks of benefits, any job taken has to last less than six months for the workers to keep the leftover UI segment. During normal times, when long-term employment opportunities are relatively abundant, the probability of having a job lasting for less than six months is low. Second, the opportunity to choose between leftover benefits and new UI segment makes a difference if the benefit level is high enough on the leftover benefits that the workers will choose the leftover benefits. During normal times, because wage scarring is relatively low, it is unlikely that unemployment will create a big wage gap that will lead to large differences in benefit levels.

The retention policy plays a more important role in recessions for two reasons. First, the benefit collection window is extended with UI benefit extensions such as during the Great Recession when the UI benefits were extended from 26 to over 90 weeks. With these extensions, the benefit collection window is also extended from one year to as long as extensions are in effect. For example, during the Great Recession, extensions were in effect for four years (from 2008 to the end of 2013). In addition, long-term jobs are harder to find in a recession. With short-term employment, it is more likely that at the end of a job the worker is still within the collection window of the benefit that she started before taking the job.

Second, the benefits that a worker qualifies from jobs before the start of the recession are likely higher than the benefits from working during the recessions. This is true with wage scarring or if during the recession she is forced to take a job that pays much less than the job before the recession. With wage scarring, the wages she gets after being unemployed are lower than wages before unemployment. As an example of different types of job, suppose a worker laid off from a regular, long-term job at General Electric cannot find similar type job during the recession and is forced to work a temporary job at McDonald's. The two jobs potentially differ in two dimensions—wages and job security. Wages are likely higher at General Electric even without wage scarring. Job security is likely worse at McDonald's especially if the job is temporary. Because of lower wages, if the worker qualifies new UI benefits at McDonald's, the new benefit level is lower than her benefits

qualified from working at General Electric. At the same time, because of lower job security, the job at McDonald's may not qualify for new UI benefits, in which case being able to continue collecting any leftover benefits will be even more valuable.

It is worth noting that unlike during normal times when the worker can *choose* between leftover benefits and new qualified benefits, in recessions, the retention policy is more restrictive. In particular, once the worker qualifies new benefits, she may not continue collecting any leftover benefits from before.⁶ This restriction was in place until July 2011, when a federal legislation awarded the choice of UI segments to the unemployed worker.

Because the effect of the retention policy differs substantially over the business cycle, I will treat the policy as time dependent. For tractability, I assume that the retention policy has negligible effect during normal times, and only plays a role in recessions.

3.2.2 Cross-state policy variations

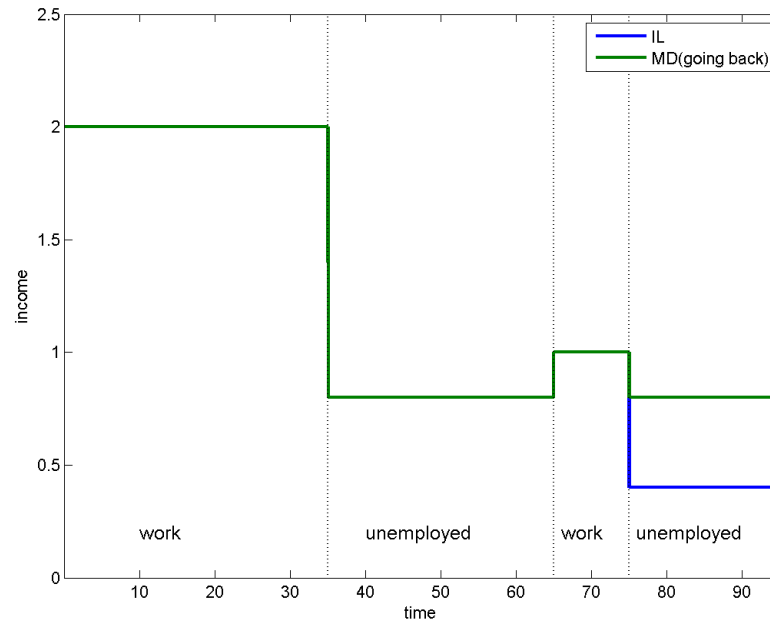
In addition to variations across time, the retention policy also differs across states. Because the policy effect is small during normal times, in what follows I focus on the policy differences in recessions.

States differ in how hard it is to take advantage of the retention policy. More specifically, states differ in how much work is needed to qualify for a new UI segment. As noted in the previous subsection, the restriction on retention policy before July 2011 means that when an unemployed worker qualifies a new UI segment, she has to start collecting the new segment. Given this restriction, it is *easier* for an unemployed worker to collect leftover benefits in states where it is *harder* to qualify new UI benefits.

Figure 3.1 illustrates the cross-state difference for a worker who becomes unemployed twice within a 90-week period. Because benefit level is dependent on wages, the income she gets during unemployment is proportional to the income from work. For illustration purpose, suppose Illinois does not allow delayed collection of benefits (or that the worker qualifies a new UI segment in Illinois from the short period of work). In Illinois (blue line), the worker has to start collecting new benefits which are lower than the benefits before. In contrast, in Maryland (green line) where delayed collection of benefits is possible (or that the wages from the short period of work do not qualify new UI in Maryland), she can

⁶ The reason for this feature is that extended benefits in recessions come from federal funding, whereas the first 26 weeks of benefits are funded through states. Under the UI regulation prior to July 2011, benefits funded by states must be collected first before collecting federal benefits, and in cases where more than one benefit segments are open, state-funded benefits take precedence.

Figure 3.1: Illustration of cross-state difference in retention policy.



continue collecting the leftover benefits from the first unemployment spell.

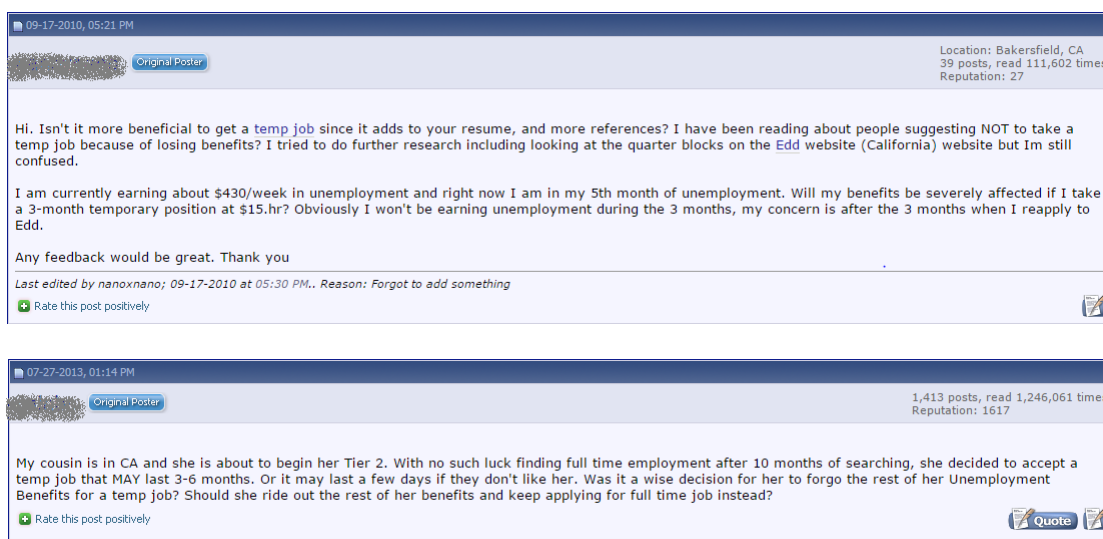
Because of the difference in benefit levels, when searching for jobs during the first unemployment spell, the worker will anticipate the difference in consumption level during the second unemployment spell. In Maryland, where the expected consumption during the second unemployment spell is relatively high, the worker has more incentives to find any job (especially a job with lower wages and/or shorter expected duration) during the first unemployment spell, knowing that life won't be too hard if she gets unemployed again soon. Following this logic, the probability that an average worker finds a job within a set period of time will be higher in Maryland than in Illinois.⁷

⁷ In addition to benefit level, the duration of benefits is also a factor in worker's choice. Because newly qualified benefits start from week 1 of benefits, the duration on the new UI segment is most likely longer than any leftover benefits. But from the empirical patterns, duration of benefits does not seem to be as important as benefit level. One possible reason is that in recessions unemployed workers are credit constrained and value liquidity more than total expected income. See [Chetty \(2008\)](#) for a discussion of how liquidity effect affects workers' search incentives.

3.2.3 Evidence of policy effects on individual choice

An important question is whether the unemployed workers are knowledgeable and rational enough to take into account such complicated policy structures and anticipate consumption changes during future unemployment spells? On the Unemployment forum of the web site City-Data,⁸ a popular forum site for U.S. city information with alleged 1.5 million members, many unemployed workers discussed the how taking a temp job or part-time job would impact their UI benefit receipts. Figure 3.2 presents two examples of questions asked on the forum that are related to the retention policy. These two posts illustrate that the unemployed workers do indeed consider how taking a job, especially a temp or part-time job impact the benefits they will get when the job ends.

Figure 3.2: Example discussions on the retention policy and short-term job on City-Data.



3.2.4 Empirical evidence on aggregate effect of retention policy

To empirically document the effect of the retention policy on job search, the ideal dataset is microdata with administrative details on UI recipients including benefit receipts and employment history. In the absence of such data, I exploit policy variations across states before July 2011 to capture the aggregate effect of retention policy.

As explained in the previous subsection, the state policy on how much a worker needs to earn to qualify a second UI segment determines the probability that an unemployed

⁸ <http://www.city-data.com/forum/unemployment>.

worker can delay collection of benefits. Because most states require a worker to earn over a one-year period a multiple (from 3 to 10) of the benefit amount to qualify a new UI segment, I place states according to the multiple into three groups. A higher multiple represents a state where it is harder to qualify new benefits and thus easier to delay collection of benefits.⁹ One issue is that the effect of the retention policy potentially depends on the expected duration of benefits and states implement different UI extensions in recession. I exclude states that did not implement the second tier of extension before November 2009 in the sample created from survey data of 2010. This restriction drops North Dakota, Nebraska, South Dakota, and Utah out of the sample. Table 3.1 presents summary statistics of sample of workers from the the Current Population Survey (CPS) Displaced Worker Supplement by state group.

Table 3.1: Summary Statistics of State Groups.

	Group I	Group II	Group III
Number of States	8	16	22
Workers in sample	258	613	558
In-sample UI recipients	76	217	204

Note: CPS Displaced Worker Supplement 2010, restricted to workers who lost job in the year prior to survey and are in labor force at time of survey. Sample restricted to states implementing similar benefit extension tiers: states with no EUC 2 before Nov 2009 ND, NE, SD, UT) are excluded; state with no retention policy (VA) is excluded.

I use data from the CPS Displaced Worker Supplement to compute a state-level average job-finding probability among unemployed workers who lost job in the previous year. The CPS is a representative sample of U.S. households, but does not contain information about UI eligibility or receipt. The Displaced Worker Supplement of the CPS contains this information but is limited by its biennial frequency. I use the 2010 survey because it contains information on workers who lost their jobs at the starts of the recession (2008 and 2009). I also use the 2012 survey because the surveys happened after the July 2011 federal law took effect.

I use the individual responses to questions about UI benefit receipt, labor force status at the time of survey, and the timing of unemployment to create a sample of workers who lost job in the year prior to the survey year and are still in the labor force at the time of survey. I then separate the sample into UI recipients and non-recipients. I then use responses to questions about job finding to construct the measure of aggregate job-

⁹ I group states as follows. Group I: multiple equal to 3 or 4, Group II: 5 or 6, and Group II: 8 or 10. No states have a multiple 7 or 9. While this way of grouping is somewhat arbitrary, it balances the number of states in each group.

finding probability for the two groups of workers. Table 3.2 shows the aggregate job-finding probability measure for each state group and by UI recipient status.

Table 3.2: Cross-State Difference in Aggregate Job-Finding Probability Measure.

	Group I (8 states)	Group II (16 states)	Group III (22 states)
	Easier to collect previous benefits →		
UI recipients	0.29	0.34	0.37
Non-recipients	0.59	0.59	0.61
	After July 2011: No cross-state difference in retention policy		
UI recipients	0.41	0.40	0.42

Note: CPS Displaced Worker Supplement 2010 and 2012, restricted to workers who lost job in the year prior to survey and are in labor force at time of survey. Sample restricted to states implementing similar benefit extension tiers: states with no EUC 2 before Nov 2009 ND, NE, SD, UT) are excluded; state with no retention policy (VA) is excluded.

Consistent with the intuition for how cross-state policy differences affect workers' job finding incentives, the first panel of Figure 3.2 shows that among UI recipients, the aggregate job-finding probability is *higher* in states where it is *easier* to collect previous benefits. Among the control group of non-recipients, the job-finding probability does not differ consistently across state groups. The pattern in the non-recipient group controls for conditions that may differ consistent across state groups (for example, firm-side conditions, aggregate shocks). As a second control group, I look at the aggregate job-finding probability among UI recipients after July 2011, when a federal law removed the effect of cross-state policy difference on benefit collection. The second panel of Figure 3.2 shows that in this group, again, there is no consistent pattern across state groups.

To the extent that the two control groups capture any economic conditions that vary consistently across state groups, the pattern among UI recipients prior to 2011 shows that the retention policy, by allowing unemployed workers to collect previous benefits (or equivalently, delay collection of benefits), increases workers' job search incentives in a recession. Several assumptions are made here. First, I have assumed that most of the cross-state difference in aggregate job-finding probability is driven by unemployed workers' decisions to take short-term jobs. Second, I have assumed that for these workers, the benefit level of their first UI segment is higher than subsequent benefit levels. While these assumptions are hard to verify without information on the type of jobs taken and detailed wage or benefit history, the evidence provided in the previous subsection should demonstrate that these assumptions are not far from reality.

Because the empirical evidence is cross-sectional, the numbers are hard to interpret especially due to the presence of business cycle variations and endogeneity. Thus, in the two two sections I develop a structural model that incorporates the necessary elements of the retention policy.

3.3 A SIMPLE MODEL

This section uses a simple model to illustrate the effect of the retention policy on the job-finding incentives of unemployed workers. The model extends the standard Mortensen-Pissarides framework to include endogenous search, UI eligibility, and wage-indexed benefits. For simplicity, I only model two types of jobs, which is enough for the simple illustration. Because the effect of the retention policy should be small without duration extensions such as during normal times, to best illustrate the effect of the policy I assume the economy is in a state of recession with UI extensions.¹⁰

3.3.1 Model setup

Environment Time is discrete and infinite. The economy consists of a mass of infinitely lived workers. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Some unemployed workers receive UI benefits. For simplicity, I assume risk-neutral workers here. Workers maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [c_t - v(s_t)]$$

where \mathbb{E}_0 is the period 0 expectation factor, and β is the time discount factor. Period utility comprises of utility from consumption of goods c and disutility from job search activity $v(s)$, which is an increasing, convex function with $v(0) = 0$. Utility is increasing in c and decreasing in s . Only unemployed workers choose positive search intensity; that is, there is no on-the-job search. Each period, an employed worker gets paid wages w , which depend on type of job. With an exogenous job separation probability δ each period, the worker becomes unemployed. An unemployed worker get h from non-monetary benefits such as leisure and home production. If on unemployment benefits, she also receives b which is a function of previous wages w^- . In addition, all unemployed workers search at the utility cost $v(s)$, and finds a job with probability $\alpha(s)$, where $\alpha(\cdot)$ is an increasing and concave

¹⁰ For the analysis in this section, I assume the recession is long enough to reach a near steady state.

function. There are no private insurance markets and workers cannot save or borrow.

UI policy structure Not all unemployed workers receive benefits. With probability λ a newly unemployed worker qualifies new benefits. UI benefits are indexed on wages of previous employment through $\gamma\omega$, where γ has the interpretation of the monetary replacement ratio, and ω denotes wages at the previous job. Wages depend on type of jobs, which differ in two dimensions: wages and job separation probability. The good jobs pay higher wages and have better job security than bad jobs, or $w_g > w_b$ and $\delta_g < \delta_b$. The proportion ρ of all job openings are bad jobs. Benefits expire with an exogenous probability e each period. This is a simple way to capture the fact that benefits do not last forever.

Worker's problem An unemployed worker has individual state ω , which equals the wages at her previous employment $\{w_g, w_b\}$ if she has benefits, or 0 if no benefits. The unemployed worker chooses how much to search s . Her problem can be written recursively as follows for $\omega = \{w_g, w_b, 0\}$,

$$\begin{aligned} U(\omega) = \max_s \quad & \gamma\omega + h - v(s) + \underbrace{\beta(1 - \alpha(s)) \left[eU(0) + (1 - e)U(\omega) \right]}_{\text{doesn't find a job}} \\ & + \underbrace{\beta\alpha(s) \left[\rho W_b(\omega) + (1 - \rho)W_g(\omega) \right]}_{\text{finds a job}}, \end{aligned} \quad (3.1)$$

where U is the unemployed worker's value function, and W_k is the value function of an employed worker at job type $k = \{g, b\}$ and is given by

$$W_k(\omega) = w_k + \underbrace{\beta(1 - \delta_k)W_k(\omega)}_{\text{keeps job}} + \underbrace{\beta\delta_k \left[\lambda U(w_k) + (1 - \lambda)U(\omega) \right]}_{\text{loses job}}. \quad (3.2)$$

In the above unemployed worker's problem, her current consumption consists of base consumption h and benefits $\gamma\omega$ (if no benefits then $\omega = 0$). Search incurs a utility cost $v(s)$, and the probability of finding a job in the current period is given by $\alpha(s)$. If she doesn't find a job, then with probability e she loses benefits next period. If she finds a job, with probability ρ it is a bad job (b), otherwise it is a good job (g). A workers at job type k gets paid wages w_k . With the type-specific job separation probability δ_k she loses her job. With probability λ the newly unemployed worker qualifies new benefits with benefit level γw_k , otherwise she can collect any leftover benefits from when she was last unemployed at the benefit level $\gamma\omega$.

It is important that an employed worker inherits the individual state ω from when

she was last unemployed. This is to keep track of her previous benefit status the difference between past and current benefits is important to the retention policy. In particular, $\omega = 0$ means she did not have benefits (or benefits ran out) during her previous unemployment spell, and so when she becomes unemployed again she either gets new benefits if her current job qualifies new benefits or no benefits.

3.3.2 Retention policy and search incentives

From the unemployed worker's problem we can derive a condition for the optimal level of search s of an unemployed worker with benefits $\gamma\omega$

$$\frac{v_s(s)}{\beta\alpha_s(s)} = \underbrace{[\rho W_b(\omega) + (1-\rho)W_g(\omega)]}_{\text{future value of finding a job}} - \underbrace{[eU(0) + (1-e)U(\omega)]}_{\text{future value of not finding a job}}, \quad (3.3)$$

where the left-hand side is the marginal cost of search weighted by the marginal probability of job finding and expressed in future terms (divided by time discount β), and the right-hand side expresses the marginal gain from additional search as the net value of finding a job versus not finding a job.

To understand how the retention policy affects search incentives we can write (3.3) more explicitly:

$$\begin{aligned} \frac{v_s(s)}{\beta\alpha_s(s)} &= \rho w_b + (1-\rho)w_g - e[h - v(s'_0)] - (1-e)[\gamma\omega + h - v(s')] \\ &+ [1 - \delta_g - \alpha(s')] \frac{v_s(s')}{\alpha_s(s')} + e \left[\alpha(s') \frac{v_s(s')}{\alpha_s(s')} - \alpha(s'_0) \frac{v_s(s'_0)}{\alpha_s(s'_0)} \right] \\ &+ \underbrace{\beta\rho(\delta_g - \delta_b) \left\{ W_b(\omega) - U(\omega) \right\}}_{\text{job security effect}} + \underbrace{\beta e(1 + \delta_g - e) \left\{ U(\omega) - U(0) \right\}}_{\text{benefit eligibility effect}} \\ &+ \beta\lambda \underbrace{\left\{ \rho\delta_b [U(w_b) - U(\omega)] + (1-\rho)\delta_g [U(w_g) - U(\omega)] \right\}}_{\text{retention effect}}. \end{aligned} \quad (3.4)$$

The first two lines on the right-hand side are standard conditions for optimal search: the first line represents the marginal gain of employment from higher utility next period, and the second line represents the marginal gain of employment from not having to search next period. The last part on the second line represents the marginal gain (loss) from future search if benefits expire in the next period. Here, s_0 is the search level by an unemployed

worker without benefits.

There are three other effects in addition to the standard gains from employment. All three effects take place in the periods after the next period (thus discounted by β). First, the *job security effect* represents the marginal gain (loss) of having a bad job relative to a good job. This effect disappears if two job types have the same job security ($\delta_g = \delta_b$). Second, the *benefit eligibility effect* represents the marginal gain from having benefits, which disappears when all unemployed workers get benefits ($e = 0$) or if the unemployed worker does not have benefits today ($\omega = 0$). Third, the *retention effect* comes from the difference between the current ($\gamma\omega$) and future (γw_b or γw_g) benefit levels and is affected by the retention policy.

For easy interpretation, the retention effect can be equivalently written as

$$\beta\lambda \left\{ 1_{\omega=w_g}(-\rho)\delta_b + 1_{\omega=w_b}(1-\rho)\delta_g \right\} \left[U(w_g) - U(w_b) \right]. \quad (3.5)$$

The effect is negative if the current benefit effect is high ($\omega = w_g$), and it is positive if the current benefit level is low ($\omega = w_b$). In a recession when good jobs are scarce (ρ is large)¹¹ or bad jobs are even less secure (δ_b is large), the negative effect on unemployed workers receiving high benefits is amplified. The size of the effect is larger when it is easier to qualify new benefits (λ is larger).

The retention effect here reflects the effect of allowing delayed collection of benefits without giving workers a choice between old and new benefits. More specifically, the unemployed worker receiving high benefits today is discouraged from searching for jobs because of the prospect of lower benefits in future unemployment spells, whereas someone receiving low benefits now is encouraged to search today because of potentially higher benefits when she is unemployed again. The variable λ here captures the cross-state difference in policy explained in the previous section. A larger λ corresponds to states where it is easier to qualify new benefits. Because of the restriction that once qualified new benefits the unemployed worker must collect new benefits, the negative incentives on search by high-benefit unemployed workers are also larger in these states.

The retention effect is larger for higher job separation rates δ_b and δ_g . Higher separation rates reduce the discount on the difference in future value of unemployment, and making the policy effects on search more pronounced. The effect is also larger for larger wage gap, $w_g - w_b$, and hence larger utility difference $U(w_g) - U(w_b)$. The larger the wage difference, the more the retention policy matters for the expected value of future

¹¹ In the quantitative section, I endogenous ρ as an outcome of firm-side decisions, and find rising ρ in recessions.

unemployment. The effect of the retention policy should also depend on how long the unemployed worker can collect benefits, because the longer she can collect, the more benefits she can potentially carry over to future unemployment spells. But because all unemployed workers face the same benefit exhaustion probability e , this effect is not present in the simple model. For the quantitative model in the next section, unemployed workers have maximum UI entitlements, and so workers at different point in their unemployment spell face different benefit exhaustion probabilities.

After July 2011 Consider the policy after July 2011, when a federal law removed the cross-state differences in retention policy by giving workers a choice between new and old benefits. Now the retention effect becomes

$$\begin{aligned} & \beta\lambda \left\{ \rho\delta_b \left[\max\{U(w_b) - U(\omega), 0\} \right] + (1-\rho)\delta_g \left[U(w_g) - U(\omega) \right] \right\} \quad (3.6) \\ \equiv & \begin{cases} 0 & \omega = w_g \\ \beta\lambda(1-\rho)\delta_g \left[U(w_g) - U(w_b) \right] & \omega = w_b \end{cases} \end{aligned}$$

In this case, because the unemployed worker can choose between the leftover benefits ($\gamma\omega$) and new benefits, the retention effect is nonnegative. The retention policy does not have any additional incentive effects on the worker with higher benefits ($\omega = w_g$), and it increases the search incentives of workers with low benefits now ($\omega = w_b$).

3.4 QUANTITATIVE MODEL

This section extends the simple model in the previous section to include discrete UI duration, human capital of workers and firm-side decision. Allowing discrete UI duration means an unemployed worker collecting her first week of benefits will make different decision from someone at her last week of benefits. This difference is both consistent with empirical findings in the literature and relevant for the choice between new and old benefits. Human capital depreciation from unemployment reduces wage income from future work, and as a result, unemployed workers have an additional incentive to find a job sooner rather than later. Firm-side decision allows differences in unemployed worker's job-finding decision to affect the overall job creation through firms' job posting.

The purpose of this section is to quantify the effect of the retention policy during a recession. The empirical evidence presented in Section 2 motivates provides directions for modeling choice, but because the evidence is cross-section, it does not reflect the effect of changing economic conditions. A structural model allows for analyses over a transitional

path.

3.4.1 Model

Model environment Instead of risk-neutral workers assumed in the simple model, the workers here are risk averse with utility from consumption at time t given by $u(c_t)$. Unemployed workers with benefits qualify for entitlement of benefits, which is the length that an unemployed worker can receive benefits while unemployed. As with the simple model, benefit levels are proportional to wages to most recent employment.

In addition to employment and UI status, workers also differ by the level of skills. In particular, there are finite levels of skill $h \in \{h_1, h_2, \dots, h_N\}$ with $h_1 < h_2 < \dots < h_N$. A worker accumulates skill from work with probability h_{up} , and loses skill from unemployment with probability h_{down} .

As with the simple model, there are two types of jobs, regular job (indexed by g) and temp job (indexed by b),¹² operated by regular and temp firms, respectively. That is, the labor market on the firm-side is segregated by type, and a firm may only post one type of jobs.¹³ Regular firms are more productive, pay higher wages and lower exogenous job separation rates. In particular, the productivity of regular firms varies over the cycle and depends on the worker's skill level, whereas the productivity of temp firms is fixed and skill-independent. Given the time-varying labor productivity z_t and a lower-bound productivity level \underline{z} , the output of a match between a worker with skill h and a position is $p_g(z_t, h)$ for a regular job and p_b for a temp job, where

$$p_g(z_t, h) = hz_t > p_b = \underline{z}.$$

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β . A firm can be either matched to a worker and producing or vacant. A vacant firm posting a vacancy incurs a flow cost κ which is assumed independent of job type.

Unemployed workers and vacancies are randomly matched according to the matching function $M(S, V)$, where S is the aggregate search effort of all unemployed workers, and V is the aggregate vacancy posting by all firms, both regular and temp. The matching function is assumed strictly increasing and strictly concave in both arguments, and is

¹² "b" stands for bad jobs and "g" for good jobs. These are to be consistent with the analyses in the previous section.

¹³ Although many establishments in reality hire both long-term and temp workers, certain type of jobs are only filled by temp workers, for example, seasonal jobs, lower-ranking sales jobs. As such, an establishment does not choose whether these positions are filled by long-term or temp workers. In the setup here, a firm can be viewed as a collection of positions, some filled and others vacant.

bounded above the number of expected matches: $M(S, V) \leq \min\{S, V\}$. The job-finding probability per efficiency unit of search, f , and the job-filling probability per vacancy, q , are functions of labor market tightness $\theta = V/S$. More specifically,

$$f(\theta) = \frac{M(S, V)}{S} = M(1, \theta)$$

$$q(\theta) = \frac{M(S, V)}{V} = M\left(\frac{1}{\theta}, 1\right).$$

Following the assumptions on M , $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . The job-finding probability for an unemployed worker searching with intensity s is then $sf(\theta)$. The job-filling probability of a temp vacancy is $\rho q(\theta)$, where ρ is the proportion of all vacancies posted by temp firms. As with the simple model, existing jobs are destroyed exogenously with constant and job type-dependent probability δ_k for job type k .

The wages of regular jobs are determined through a bargaining process to be specified later, and so they depend on the workers' skill levels and outside options. In contrast, wages of temp jobs are fixed and independent of worker characteristics. The different wage determination processes mirror the reality: temp jobs often have fixed wages around minimum wage levels, whereas regular jobs involve wage bargains that take into account the worker's skills and outside options.

In this model, changes in productivity z drives the business cycle. Job separation rate δ , output of positions p , and the maximum UI entitlement J potentially vary with productivity over the cycle.

Worker's problem The worker's problem is similar to before. An unemployed worker has individual state (ω, j, h) , where ω is her previous wages, j is her UI entitlement for the period, and h is her current skills level. While ω does not change during the same unemployment spell, j changes with probability 1 each period, and h follows a Markov process. UI entitlement $j \in \{1, 2, \dots, J\}$, where J is the maximum potential duration of benefits that an unemployed worker can get. J is larger in recessions than during normal times because of benefit extensions. Each period, the unemployed worker chooses how much to search for a job. Her problem can be written as

$$U_t(\omega, j, h) = \max_s u(\gamma\omega + h) - v(s)$$

$$+ \beta(1 - f(\theta_t)s)\mathbb{E}_{h'|h}^- \left[\mathbf{1}_{j=1} U_{t+1}(0, 0, h') + \mathbf{1}_{j>1} U_{t+1}(\omega, j-1, h') \right]$$

$$+ \beta f(\theta_t)s\mathbb{E}_{h'|h}^- [\rho_t W_{b,t+1}(\omega, j-1, h') + (1 - \rho_t)W_{g,t+1}(\omega, j-1, h')], \quad (3.7)$$

where $\mathbb{E}_{h'/h}^-$ is the expectation of future skill level conditional on today's skill level in unemployment ($-$ denotes loss of skill from unemployment). As long as the worker still collects benefits ($j \geq 1$), her entitlement decreases by 1 each period in her unemployment. When she is at her last period of UI benefits ($j = 1$), her benefits expire next period ($\omega = 0$ and $j = 0$).

The Bellman equation of an employed worker at job $k = \{g, b\}$ is given by

$$W_{k,t}(\omega, j, h) = u(w_{k,t}) + \beta(1 - \delta_{k,t})\mathbb{E}_{h'/h}^+ W_{k,t+1}(\omega, j, h') \quad (3.8)$$

$$+ \begin{cases} \text{(recession, before July 2011)} \\ \beta\delta_{k,t}\mathbb{E}_{h'/h}^+ \left[\lambda_{k,s,t}(\omega)U_{t+1}(\gamma w_k, J_t, h') + (1 - \lambda_{k,s,t}(\omega))U_{t+1}(\omega, j-1, h') \right] \\ \text{(recession, after July 2011)} \\ \beta\delta_{k,t}\mathbb{E}_{h'/h}^+ \left[\lambda_{k,s,t}(\omega) \max \left\{ U_{t+1}(\gamma w_k, J_t, h'), U_{t+1}(\omega, j-1, h') \right\} \dots \right. \\ \quad \left. \dots + (1 - \lambda_{k,s,t}(\omega))U_{t+1}(\omega, j-1, h') \right] \\ \text{(non-recession)} \\ \beta\delta_{k,t}\mathbb{E}_{h'/h}^+ \left[\lambda_{k,s,t}(\omega)U_{t+1}(\gamma w_k, J_t, h') + (1 - \lambda_{k,s,t}(\omega))U_{t+1}(0, 0, h') \right] \end{cases}$$

where $\mathbb{E}_{h'/h}^+$ is the expectation of future skill level conditional on today's skill level at work ($+$ denotes accumulation of skill from work). During work, any leftover benefit entitlement j stays the same. Once the job is exogenously destroyed, the probability of qualifying new benefits is given by a function of previous benefit level $\lambda_{k,s}(\omega)$ and depends on type of job (k), and state (s). More specifically,

$$\lambda_{k,s,t}(\omega) = (1 - \delta_{k,t})^{X_s \gamma \omega / w_k},$$

where X_s is the multiple that differs across U.S. states and used to group states in the empirical section. In a recession, when the retention policy has the most impact, a newly unemployed worker who does not qualify new benefits can use any leftover benefits from previous unemployment spells. Before July 2011, if she qualifies new benefits, she has to start the new UI segment; after July 2011, she can choose whether to start new UI segment or to continue collecting leftover benefits. During normal times, the retention policy does not apply, and thus if a newly unemployed worker does not qualify new benefits, she becomes unemployed without benefits.

Newly qualified UI benefits have the maximum potential entitlement J_t which varies over time according to the legislation. To reduce computational complexity, I assume that the government can commit to previous entitlements. In other words, if a worker qualified for an entitlement longer than the current maximum entitlement, $J_{t-1} > J_t$, she can poten-

tially collect all J_{t-1} periods of benefits even though the current maximum benefit period lower than her entitlement.

Firm's problem To be matched with a worker, a firm posts a vacancy at a flow cost κ . A temp job and a regular job is filled with probability q_b and q_g , respectively, where

$$q_b = \rho q(\theta), \quad q_g = (1 - \rho)q(\theta).$$

Production takes place the following period. The Bellman equation of a vacant firm of type $k = \{b, g\}$ is

$$V_{k,t} = -\kappa + \underbrace{\beta(1 - q_{k,t})V_{k,t+1}}_{\text{vacancy not filled}} + \underbrace{\beta q_{k,t} \frac{1}{\mu_t^u} \sum_{(\omega,j,h)} \left[\mu_t^u(\omega,j,h) \mathbb{E}_{h'|h}^- F_{k,t+1}(\omega,j,h') \right]}_{\text{vacancy filled}}, \quad (3.9)$$

where given the distribution of workers μ_t , μ_t^u is the total measure of unemployed workers and $\mu_t^u(\omega, j, h)$ is the measure of type- (ω, j, h) unemployed workers. If the vacancy is filled, then the firm's future value is the expected value of a position filled by a worker with skill h , weighted by the unemployed worker's type distribution. The ratio $\mu_t^u(\omega, j, h) / \sum \mu_t^u$ represents the proportion of type- (ω, j, h) unemployed workers among all unemployed workers. It therefore gives the probability that a vacancy is filled by an unemployed worker of type (ω, j, h) . Under free-entry conditions for each type of firms, the value of a vacancy is zero,

$$V_{k,t} = 0. \quad (3.10)$$

Given the free-entry condition, the Bellman equation of a type- k position filled by a worker of type (ω, j, h) is

$$F_{k,t}(\omega, j, h) = p_k(z_t, h) - w_{k,t} + \beta(1 - \delta_{k,t}) \mathbb{E}_{h'|h}^+ F_{k,t+1}(\omega, j, h'), \quad (3.11)$$

where $p_b = \underline{z}$ and $p_g(z_t, h) = z_t h$ are the firm's productivity. Wages of a temp job are fixed, $w_{b,t} = \bar{w}$, and wages of a regular position, $w_{g,t}$, is the outcome of wage bargaining.

Wage determination Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(S, V)$. A realized match produces some economic rent shared between the firm and worker in a way that is job type-dependent. Temp jobs have fixed wages, $w_b = \bar{w}$, while wages of regular jobs come from a Nash bargaining process. I assume that wages of regular jobs are set every period, so the

equilibrium wages respond to the aggregate productivity and worker's skill level.

More specifically, worker's surplus from working on a regular job is the difference between the values of working at wage w and being unemployed without benefits. This specification of the outside option follows the rules of UI that workers who quit do not qualify benefits. Firm's surplus is the difference between the values of a filled position and running a vacancy. In particular, wages are chosen to maximize the weighted product of the worker's surplus and the firm's surplus when the aggregate productivity of the economy is z and the worker's individual state (ω, j, h)

$$\left(W_{g,t}(\omega, j, h) - U_t(0, 0, h)\right)^\zeta \left(F_{g,t}(\omega, j, h) - V_{g,t}\right)^{1-\zeta}, \quad (3.12)$$

where $\zeta \in (0, 1)$ is the bargaining power of the worker, $W_{g,t}(\omega, j, h) - U_t(0, 0, h)$ is the worker's surplus, and $F_{g,t}(\omega, j, h) - V_{g,t}$ is the firm's surplus from the match. The solution to this bargaining problem, denoted w_t , varies over times and by worker's individual states.

Competitive equilibrium Given an initial condition for worker type distribution μ_0 and a sequence of time-varying parameters $\{z_t, \delta_{k,t}, J_t\}_{t,k}$ a competitive equilibrium consists of a sequence of regular job wages $w_{g,t}$, labor market tightness θ_t , job type distribution ρ_t , worker's optimal decision rules of search $s_t(\omega, j, h)$, worker type distribution μ_t , and value functions $U_t(\omega, j, h)$, $W_{k,t}(\omega, j, h)$, $F_{k,t}(\omega, j, h)$ such that:

- The value functions satisfy the worker's and firm's Bellman equations (3.7), (3.9), (3.9), and (3.11).
- The search intensity $s_t(\omega, j, h)$ solves the type- (ω, j, h) unemployed worker's maximization problem in (3.7).
- The market tightness θ_t and job type distribution ρ_t are consistent with the free-entry conditions of both types of firms in (3.10).
- The wage of regular jobs $w_{g,t}$ maximizes the Nash product of (3.12).
- The law of motion associated with worker distribution μ_t is consistent with worker's optimal choices of search and stochastic processes.

3.4.2 Parametrization

This section provides a description of the parametrization strategy. Table 3.3 summarizes the values of parameters. The model period is one week. I calibrate the steady-state com-

petitive equilibrium to match key statistics of the U.S. labor market.

The utility of consumption takes the following functional form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

I assume the search cost function is non-negative, strictly increasing, and convex, with the property that $v(0)$ is bounded and $v(0) \geq 0$. I specify the search cost function to be consistent with the literature:

$$v(s) = \xi \frac{s^{1+\phi}}{1+\phi}.$$

For any $\xi, > 0$, $v(\cdot)$ exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $v(0) = v_s(0) = 0$. The matching function takes the Cobb-Couglas form of

$$M(S, V) = \eta S^\alpha V^{1-\alpha},$$

where S is the aggregate job search and V is the aggregate vacancy posting in the economy.

I pick two parameters related to preferences. The discount factor β is set to give a quarterly discount factor of 0.99. The coefficient of relative risk aversion σ is set to 1 (log utility). Following [Nakajima \(2012a\)](#) I calibrate the level parameter of the search cost function (ξ) to match the average time an unemployed person spends on job search. The curvature parameter of search (ϕ) is calibrated to match a 50% of unemployed workers receiving benefits in the steady state.

I normalize the steady-state aggregate productivity (\bar{z}) to 1. The temp job productivity (\underline{z}) is set relative to the steady-state aggregate productivity at 0.6. The level parameter of matching function (η) is calibrated to match a steady-state unemployment rate of 4.5%, and this gives $\eta = 0.207$. I follow [Shimer \(2005\)](#)'s estimate of the elasticity of matching function to set $\alpha = 0.72$. Following the Hosios efficiency condition, the worker's bargaining power is set to equal the elasticity of matching function, $\zeta = \alpha = 0.72$. Temp job wages are set to 0.6, according the estimate of average temp job wages in a 2004 BLS study. Because the job separation rates are potentially time varying, I calibrate the steady-state job separation rate from regular jobs to match the transition probability from employment to unemployment between 2005 to 2007, which gives $\bar{\delta}_g = 0.003$. The steady-state job separation rate from temp jobs ($\bar{\delta}_b$) is calibrated to the average length of temp jobs of about one quarter.

The UI replacement ratio (γ), the ratio of benefits and wages, is set at 0.4 following [Shimer \(2005\)](#). The value of non-monetary benefits (h) is set at 0.3 following [Nakajima \(2012b\)](#)'s estimate. I choose three skill levels ($N = 3$) to capture the skill losses from average unemployment and from long-term unemployment. The step size in skill (Δh) is set at 0.15

Table 3.3: Summary of Parametrization.

Parameter	Description	Value
<i>Preferences</i>		
β	Time discount factor	$0.99^{1/13}$
σ	Coefficient of relative risk aversion	1
ξ	Level parameter of search cost	2.134
ϕ	Curvature parameter of search cost	0.98
<i>Labor Market</i>		
\bar{z}	Steady-state aggregate productivity	1
\underline{z}	Temp job productivity	0.6
η	Level parameter of matching function	0.207
α	Elasticity of matching function	0.72
ζ	Worker's bargaining power	0.72
w_b	Wages on temp job	0.6
$\bar{\delta}_g$	Steady-state regular job separation rate	0.003
$\bar{\delta}_b$	Steady-state temp job separation rate	1/13
<i>UI-Related</i>		
γ	UI replacement ratio	0.4
h	Value of non-monetary benefits	0.3
<i>Workers' Skills</i>		
N	Number of skill levels	3
Δh	Step size in skill levels	0.15
$\pi_{h' h}^-$	Probability of skill loss during unemployment	1/12
$\pi_{h' h}^+$	Probability of skill accumulation during employment	1/250

to reflect estimates by [Farber \(2010\)](#) that job losers experience about 15% of real weekly earnings loss on average. The probability of skill loss during unemployment ($\pi_{h'|h}^-$) is set at 1/12 based on the average unemployment duration of around 3 months. The probability of skill accumulation during employment ($\pi_{h'|h}^+$) is set at 1/250, according [Kambourov and Manovskii \(2009\)](#)'s estimates of a 12-20% increase in wages during the first 5 years of occupational tenure.

3.4.3 Policy experiment over transition path

I compute a transition path between two steady states. The initial steady state resembles the pre-recession economy of 2005-2007 without UI extensions, and the end steady state

is the economy of 2011-2012 when both the job separation rates and UI extensions stayed roughly unchanged and thus can be approximated as a steady state.

Exogenous processes Over the transition path between the two steady states, the aggregate productivity, z_t , the separation rate, $\delta_{k,t}$, and the maximum potential UI entitlement, J_t , change over times. When computing the transition path, I assume that these paths are revealed at the beginning of the transition path. In other words, it is a perfect foresight equilibrium with respect to the exogenous shocks. The assumption of perfect foresight makes the solving the equilibrium computationally manageable.

Figure 3.3: Paths of exogenous processes 2008-2012: Data and smoothed.

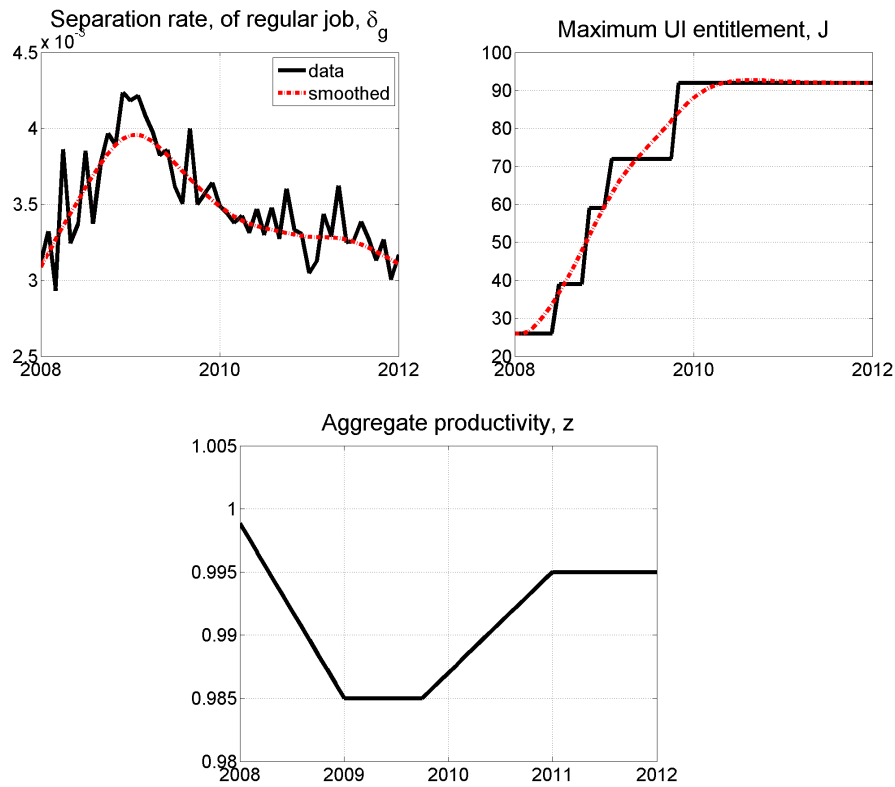


Figure 3.3 shows the paths of the exogenous processes calculated from the data and smoothed as model inputs from 2008 to 2012. I compute the job separation rates following the methodology outlined in Shimer (2005) and using the aggregate-level monthly CPS data, and then convert it to weekly values. The path for the maximum UI entitlement is take directly from the U.S. Department of Labor Employment and Training Administration (DOLETA) website. I smooth the job separation and UI entitlement series before

feeding them into the model to compute the transition path. The rationale for using the smoothed series is to make the assumption of a transition path with perfect foresight more reasonable.¹⁴

The path of aggregate productivity is calibrated so that the unemployment rate reaches around 10% in the second half of 2009. This requires a drop of 1.5% from the beginning of 2008 to 2009 and stays low until late 2009 before recovering to 0.5% below the pre-recession level. Note that the three exogenous paths correspond to the three economic conditions that affect the effect of the retention policy. Higher separation rates and longer UI entitlement both amplify the effect of the retention policy in reducing search disincentive. Lower aggregate productivity, in contrast, leads to lower regular wages and hence smaller wage gap between regular and temp jobs. Smaller wage gap in turn reduces the effect of the retention policy.

Policy experiments The purpose of computing the transition path is to find out the aggregate effect of having the retention policy. To this end, I assume the restriction on delaying the collection of benefits has been removed. In other words, the federal law of July 2011 is implemented from the start of the recession. In addition, I take the median probability of the qualifying new benefits across all states ($X_s = 5$).

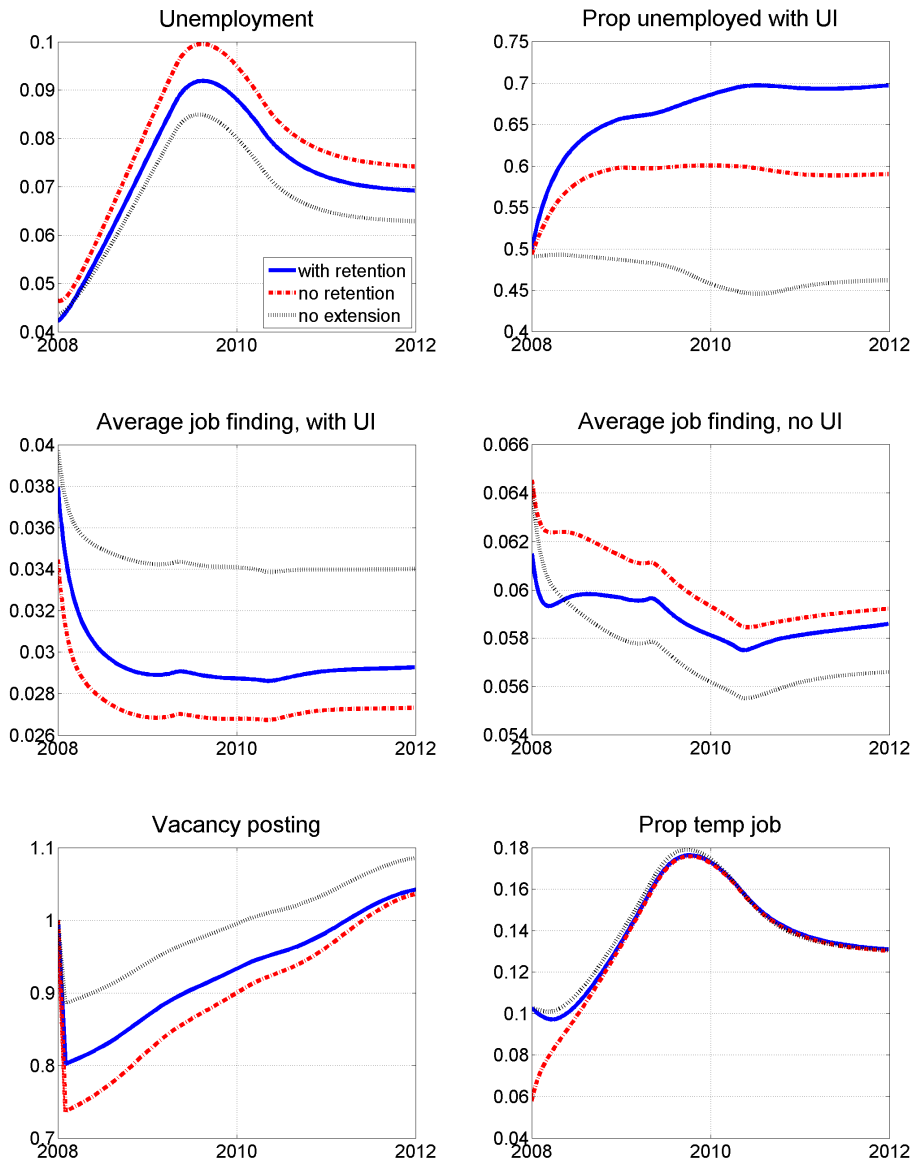
Figure 3.4 compares the transition paths of the economy with retention to two alternative economies.¹⁵ In the first alternative economy, there is no retention policy and any newly unemployed worker who does not qualify new benefits are unemployed without benefits. Because in reality the use of the retention policy is restricted before July 2011, the U.S. economy is between the baseline and first alternative economies. In the second alternative economy, I assume that there is no UI extension, and so the maximum UI entitlement stays at 26 weeks. All three economies are otherwise identical with the same shock processes. The comparison between the economies with and without retention tells us the effect of the retention policy over the transition, while the comparison between the no-retention economy and the no-extension economy highlights the effect of UI extensions.

The Unemployment rate peaks in the second half of 2009. Unemployment is lower by about 0.8% in the economy with the retention policy than without. The economy without UI extensions has the lowest unemployment, with an unemployment gap of about 1.5% with respect to the economy without retention policy but with UI extensions. With UI

¹⁴ It would be hard to imagine that the workers and firms perfectly foresee the timing of changes in UI entitlement which resulted in the zigzag pattern in Figure 3.3 or the short-term fluctuations in the job separation rates.

¹⁵ Vacancy posting is normalized to 1 at the end of 2007.

Figure 3.4: Simulated paths of labor market 2008-2012: Retention policy and UI extension.



extensions, the proportion of unemployed workers receiving benefits increases over the transition path, partly because of the gradually increasing maximum UI entitlement as shown in Figure 3.3. The proportion is highest on the transition in the economy with retention policy, because some unemployed workers who don't work enough to qualify

new UI can use leftover benefits from before.

Average search and hence the average job-finding rate among unemployed workers with benefits falls in all economies mainly because of falling aggregate productivity which reduces the return from search activity. The fall is more substantial in the two economies with UI extensions because of rising maximum entitlements further reduce the incentives to search. Because workers on the transition paths have perfect foresight about the maximum entitlements in the future, they reduce search effort very early on the transition and both search and job-finding rate stay low over the entire transition path. Consistent with intuition, the unemployed workers with benefits reduce search less in the economy with retention policy, because being able to delay collection of leftover benefits increases their expected future value of employment. In contrast, search by unemployed workers without benefits fall the most in the economy without UI extensions. This is because while falling aggregate productivity reduces the incentives to search in all three economies, the larger fall in benefit-search as a result of UI extensions increases the return to search through a general equilibrium effect, and thus unemployed workers without benefits have slightly higher search and job-finding rates with UI extensions.¹⁶

Turning to the firm side, vacancy posting is normalized to 1 in the initial steady-state economy. Because firms have perfect foresight about the shock processes, vacancy postings fall when the shock paths are revealed at the beginning of the transition paths. Because of lower expected search in the economies with UI extensions, vacancy postings fall more in these economies. Because the retention policy creates incentives to search, it mitigates the negative incentive effects of UI extensions. The expected higher search with retention policy raises vacancy posting relative to the economy without retention policy. The proportion of vacancies posted by temp firms rises initially in response to falling aggregate productivity. While the marginal product of regular firms is linked to the aggregate productivity, the marginal product of temp firms stay unchanged over the business cycle. As a result, the drop in vacancy posting by regular firms is larger than by temp firms, leading to rising proportion of temp job postings.¹⁷ Finally, the proportions differ very little across the economies, because worker's search is random and not directed.

¹⁶ But because of rising benefit proportion and the small changes in no-benefit search relative to benefit search, average search across all unemployed workers falls more in the economy without retention, followed by with retention, and the least in the economy without UI extensions.

¹⁷ This pattern is broadly consistent with empirical studies that document a rise in temp agency hirings during the Great Recession.

3.5 CONCLUSION

This paper examines an assumption used in the modeling of UI in the literature that an unemployed worker forfeits any uncollected benefits at the time she finds a job. The paper first documents that during recessions workers in the U.S. can and do delay the collection of benefits to future unemployment spells. Exploiting cross-state and cross-time policy differences, this paper then provides empirically support that the allowing workers to delay benefit collection can have quantitatively relevant aggregate effects. I then extend a standard search and matching model to incorporate elements necessary to study the impact of such policies, and use a simple framework to illustrate how the policy can create incentives for unemployed workers to search for jobs. Using a quantitative version of the model, I then study a transition path that resembles the U.S. economy from 2008 to 2012. The policy to allow delayed benefit collection lowers the unemployment rate during the period by mitigating the negative incentive effects of UI extensions.

An interesting future direction for research is to allow workers to save. Because savings provide self-insurance to workers, introducing savings decision will reduce the response of unemployed workers to changes in UI policy. [Nakajima \(2012a\)](#) incorporates borrowing and savings choices in the evaluation of UI extensions, and finds that the job-finding rate of borrowing-constrained unemployed workers are higher and more responsive to changes in UI generosity. Because wages in my model are determined through a bargaining process, incorporating borrowing and savings choices considerably complicates computation.¹⁸

¹⁸ A version of the model with savings choice and endogenous wage determination can be solved using method outlined in [Krusell, Mukoyama and Sahin \(2010\)](#).

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Appendix

A APPENDIX FOR CHAPTER 1

A.1 Derivations

A.1.1 Derivation of private sector optimality conditions

Solving unemployed person's problem by taking derivative with respect to s

$$\frac{-U_s(h+b-\tau, s)}{f(\theta)} = \beta \mathbb{E}[V^e(z', u') - V^u(z', u')] \quad (\text{A.13})$$

Using worker's bellman equations

$$V^e(z, u) - V^u(z, u) = U(w-\tau, 0) - U(h+b-\tau, s) + \beta(1-f(\theta)s-\delta)\mathbb{E}[V^e(z', u') - V^u(z', u')]$$

Combining the two equations

$$V^e(z, u) - V^u(z, u) = U(w-\tau, 0) - U(h+b-\tau, s) + (1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau, s)}{f(\theta)} \quad (\text{A.14})$$

Update one period, take expectations and substitute into (A.13)

$$\frac{-U_s(h+b-\tau, s)}{f(\theta)} = \beta \mathbb{E} \left[U(w'-\tau', 0) - U(h+b'-\tau', s') + (1-f(\theta')s'-\delta)\frac{-U_s(h+b'-\tau', s')}{f(\theta')} \right]$$

From unmatched firm's value function, assuming free entry, i.e. $J^u(z, u) = 0$

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} J^e(z', u') \quad (\text{A.15})$$

Then firm's value function can be rewritten as

$$J^e(z, u) = z - w + (1-\delta)\frac{\kappa}{q(\theta)} \quad (\text{A.16})$$

Update one period, take expectations and substitute into (A.15)

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} \left[z' - w' + (1 - \delta) \frac{\kappa}{q(\theta')} \right]$$

Take first-order condition of the Nash bargaining problem (1.7) with respect to w

$$\zeta \mathcal{U}_c(w - \tau, 0) \left[J^e(z, u) - J^u(z, u) \right] = (1 - \zeta) \left[V^e(z, u) - V^u(z, u) \right]$$

Substitute in (A.14) and (A.16)

$$\begin{aligned} & \zeta \mathcal{U}_c(w - \tau, 0) \left[z - w + (1 - \delta) \frac{\kappa}{q(\theta)} \right] \\ = & (1 - \zeta) \left[\mathcal{U}(w - \tau, 0) - \mathcal{U}(h + b - \tau, s) + (1 - f(\theta)s - \delta) \frac{-\mathcal{U}_s(h + b - \tau, s)}{f(\theta)} \right] \end{aligned}$$

A.1.2 Definition of auxiliary functions in the Ramsey problem

$$\begin{aligned} & \tilde{\eta}_1(u_t, b_t, s_t, \theta_t, u_{t+1}, b_{t+1}, \tau_{t+1}, w_{t+1}, s_{t+1}, \theta_{t+1}) \\ = & \frac{-\mathcal{U}_s(h + b_t - \mathcal{S}(u_t, b_t), s_t)}{f(\theta_t)} \\ & - \beta \mathbb{E}_t \left[\mathcal{U}(w_{t+1} - \tau_{t+1}, 0) - \mathcal{U}(h + b_{t+1} - \mathcal{S}(u_{t+1}, b_{t+1}), s_{t+1}) \dots \right. \\ & \left. \dots + (1 - f(\theta_{t+1})s_{t+1} - \delta) \frac{-\mathcal{U}_s(h + b_{t+1} - \mathcal{S}(u_{t+1}, b_{t+1}), s_{t+1})}{f(\theta_{t+1})} \right] \end{aligned}$$

$$\tilde{\eta}_2(\theta_t, z_{t+1}, w_{t+1}, \theta_{t+1}) = \frac{\kappa}{q(\theta_t)} - \beta \mathbb{E}_t \left[z_{t+1} - w_{t+1} + (1 - \delta) \frac{\kappa}{q(\theta_{t+1})} \right]$$

$$\begin{aligned} \tilde{\eta}_3(z_t, u_t, b_t, w_t, s_t, \theta_t) & = \zeta \mathcal{U}_c(w_t - \tau_t, 0) \left[z_t - w_t + (1 - \delta) \frac{\kappa}{q(\theta_t)} \right] \\ & - (1 - \zeta) \left[\mathcal{U}(w_t - \mathcal{S}(u_t, b_t), 0) - \mathcal{U}(h + b_t - \mathcal{S}(u_t, b_t), s_t) \dots \right. \\ & \left. \dots + (1 - f(\theta_t)s_t - \delta) \frac{-\mathcal{U}_s(h + b_t - \mathcal{S}(u_t, b_t), s_t)}{f(\theta_t)} \right] \end{aligned}$$

$$\tilde{\eta}_0(u_t, s_t, \theta_t, u_{t+1}) = u_{t+1} - \delta(1 - u_t) - (1 - f(\theta)s)u_t$$

A.1.3 Derivation of Markov GEE

Throughout this section, we drop the dependence of functions on productivity shock z to economize on notation. Combine government first-order conditions,

$$\begin{aligned} & \frac{1}{\eta_{0s}} \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right] + \frac{\eta_{1u'}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \\ & - \frac{\eta_{2u'}}{\eta_{2\theta}} \left[\frac{\eta_{0\theta}}{\eta_{0s}} \left(R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right) + \frac{\eta_{1\theta}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) + \frac{\eta_{3\theta}}{\eta_{3w}} R_w \right] \\ = & \beta \Omega'_u \quad (\text{FOC}) \end{aligned}$$

Rewrite Bellman equation in shorthand

$$\Omega(u) = R(u, \Psi(u), W(u), S(u)) + \beta \Omega(\Pi(u))$$

Taking derivative of Bellman equation with respect to u

$$\Omega_u = R_u + R_b \Psi_u + R_w W_u + R_s S_u + \beta \Omega'_u \Pi_u \quad (\text{ENV})$$

Combine FOC and ENV to eliminate $\beta \Omega'_u$

$$\begin{aligned} \Omega_u = & R_u + R_b \Psi_u + R_w W_u + R_s S_u \\ & + \Pi_u \left\{ \frac{1}{\eta_{0s}} \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right] + \frac{\eta_{1u'}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \right\} \\ & - \Pi_u \left\{ \frac{\eta_{2u'}}{\eta_{2\theta}} \left[\frac{\eta_{0\theta}}{\eta_{0s}} \left(R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right) \dots \right. \right. \\ & \quad \left. \left. \dots + \frac{\eta_{1\theta}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) + \frac{\eta_{3\theta}}{\eta_{3w}} R_w \right] \right\} \end{aligned} \quad (\text{A.17})$$

Differentiate η_1 and η_2 with respect to u

$$\begin{aligned} 0 &= \eta_{1u} + \eta_{1b} \Psi_u + \eta_{1s} S_u + \eta_{1\theta} \Theta_u + \eta_{1u'} \Pi_u \\ 0 &= \eta_{2\theta} \Theta_u + \eta_{2u'} \Pi_u \end{aligned}$$

Re-arrange

$$\begin{aligned} \frac{\eta_{1u'}}{\eta_{1b}} \Pi_u &= -\frac{\eta_{1u}}{\eta_{1b}} - \Psi_u - \frac{\eta_{1s}}{\eta_{1b}} S_u - \frac{\eta_{1\theta}}{\eta_{1b}} \Theta_u \\ \frac{\eta_{2u'}}{\eta_{2\theta}} \Pi_u &= -\Theta_u \end{aligned}$$

Substitute into (A.17)

$$\begin{aligned}
\Omega_u &= R_u + R_b \Psi_u + R_w W_u + R_s S_u \\
&\quad + \Pi_u \frac{1}{\eta_{0s}} \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right] \\
&\quad - \left(\frac{\eta_{1u}}{\eta_{1b}} + \Psi_u + \frac{\eta_{1s}}{\eta_{1b}} S_u + \frac{\eta_{1\theta}}{\eta_{1b}} \Theta_u \right) \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \\
&\quad + \Theta_u \left[\frac{\eta_{0\theta}}{\eta_{0s}} \left(R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right) + \frac{\eta_{1\theta}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) + \frac{\eta_{3\theta}}{\eta_{3w}} R_w \right] \\
&= R_u + \left[\frac{1}{\eta_{0s}} \Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)} \Theta_u \right] \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \right] - \frac{\eta_{1u}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) \\
&\quad + \left[W_u + \frac{\eta_{3b}}{\eta_{3w}} \Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}} \Theta_u - \frac{\eta_{3s}}{\eta_{3w}} \left(\frac{1}{\eta_{0s}} \Pi_u + \frac{f_\theta(\theta)s}{f(\theta)} \Theta_u \right) \right] R_w \tag{A.18}
\end{aligned}$$

Given the worker flow equation

$$\Pi(u) = \delta(1 - u) + [1 - f(\Theta(u))S(u)]u$$

Differentiate with respect to u

$$\frac{1}{uf(\theta)} \Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)} \Theta_u = \frac{1 - \delta - f(\theta)s}{uf(\theta)}$$

Given $\eta_3 [u, \Psi(u), W(u), S(u), \Theta(u)] = 0$, differentiate with respect to u

$$W_u + \frac{\eta_{3b}}{\eta_{3w}} \Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}} \Theta_u = -\frac{\eta_{3u}}{\eta_{3w}} - \frac{\eta_{3s}}{\eta_{3w}} S_u$$

Substitute into (A.18)

$$\Omega_u = R_u - \frac{\eta_{3u}}{\eta_{3w}} R_w + \frac{1 - \delta - f(\theta)s}{uf(\theta)} \left[R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right] - \frac{\eta_{1u}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right)$$

Update and substitute into FOC, we get the GEE

$$\begin{aligned}
& \frac{1}{u f(\theta)} \left[\underbrace{R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w}_{\lambda \eta_{0u'}} + \underbrace{\frac{\eta_{1u'}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right)}_{\mu \eta_{1u'}} \right] \\
& - \frac{\eta_{2u'}}{\eta_{2\theta}} \left[\underbrace{\frac{f_\theta(\theta) s}{f(\theta)} \left(R_s - \frac{\eta_{1s}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) - \frac{\eta_{3s}}{\eta_{3w}} R_w \right) + \frac{\eta_{1\theta}}{\eta_{1b}} \left(R_b - \frac{\eta_{3b}}{\eta_{3w}} R_w \right) + \frac{\eta_{3\theta}}{\eta_{3w}} R_w}_{\gamma \eta_{2u'}} \right] \\
= & \beta R'_u + \beta \frac{1 - \delta - f(\theta') s'}{u' f(\theta')} \left[\underbrace{R'_s - \frac{\eta'_{1s}}{\eta'_{1b}} \left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w \right) - \frac{\eta'_{3s}}{\eta'_{3w}} R'_w}_{-\lambda' \eta'_{0u}} \right] \\
& - \beta \underbrace{\frac{\eta'_{1u}}{\eta'_{1b}} \left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}} R'_w \right)}_{\mu' \eta'_{1u}} - \beta \underbrace{\frac{\eta'_{3u}}{\eta'_{3w}} R'_w}_{\nu' \eta'_{3u}} \tag{A.19}
\end{aligned}$$

where $\tilde{\eta}_{0,t}(u_t, s_t, \theta_t, u_{t+1}) := u_{t+1} - \delta(1 - u_t) - (1 - f(\theta_t) s_t) u_t$. Re-arrange to get the GEE in the text.

A.1.4 Alternative (and equivalent) definition of Markov-perfect equilibrium

This section provides an alternate and equivalent definition for the Markov-perfect equilibrium where the government chooses b only and the private sector acts optimally. The derivation of Markov GEE for this definition is given after the definition.

DEFINITION .1. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function $\Omega(u)$, government's policy function $\Psi(u)$, and private decision rules $\tilde{W}(u, b)$, $\tilde{S}(u, b)$, $\tilde{\Theta}(u, b)$ and $\tilde{\Pi}(u, b)$ solving

- for all u

$$\Psi(u) \in \arg \max_b R(u, b, \tilde{W}(u, b), \tilde{S}(u, b)) + \beta \Omega(\tilde{\Pi}(u, b))$$

- for all u and b

$$\tilde{\Pi}(u, b) = \delta(1 - u) + [1 - f(\tilde{\Theta}(u, b))] \tilde{S}(u, b) \tag{A.20}$$

$$0 = \eta_1(u, b, \tilde{S}(u, b), \tilde{\Theta}(u, b), \tilde{\Pi}(u, b)) \tag{A.21}$$

$$0 = \eta_2(\tilde{\Theta}(u, b), \tilde{\Pi}(u, b)) \tag{A.22}$$

$$0 = \eta_3(u, b, \tilde{W}(u, b), \tilde{S}(u, b), \tilde{\Theta}(u, b)) \tag{A.23}$$

- for all u

$$\Omega(u) \equiv R(u, \Psi(u), \tilde{W}(u, \Psi(u)), \tilde{S}(u, \Psi(u))) + \beta \Omega(\tilde{\Pi}(u, \Psi(u)))$$

i.e. the government moves first, choosing b and τ , then private sector moves according to (A.20)-(A.23). First-order condition of the government's problem, suppressing functional arguments, is given by

$$R_b + R_w \tilde{W}_b + R_s \tilde{S}_b + \beta \Omega'_u \tilde{\Pi}_b = 0. \quad (\text{A.24})$$

Differentiating Bellman equation with respect to u

$$\Omega_u = R_u + R_b \Psi_u + R_w [\tilde{W}_u + \tilde{W}_b \Psi_u] + R_s [\tilde{S}_u + \tilde{S}_b \Psi_u] + \beta \Omega'_u [\tilde{\Pi}_u + \tilde{\Pi}_b \Psi_u] = 0. (\text{A.25})$$

Substitute expression for $\beta \Omega'_u$ from (A.24) into (A.25)

$$\Omega_u = R_u + R_b \Psi_u + R_w [\tilde{W}_u + \tilde{W}_b \Psi_u] + R_s [\tilde{S}_u + \tilde{S}_b \Psi_u] - \frac{R_b + R_w \tilde{W}_b + R_s \tilde{S}_b}{\tilde{\Pi}_b} [\tilde{\Pi}_u + \tilde{\Pi}_b \Psi_u].$$

Update one period and substitute into (A.24)

$$0 = R_b + R_w \tilde{W}_b + R_s \tilde{S}_b + \beta \tilde{\Pi}_b \left\{ R'_u + R'_b \Psi'_u + R'_w [\tilde{W}'_u + \tilde{W}'_b \Psi'_u] + R'_s [\tilde{S}'_u + \tilde{S}'_b \Psi'_u] \dots \right. \\ \left. \dots - \frac{R'_b + R'_w \tilde{W}'_b + R'_s \tilde{S}'_b}{\tilde{\Pi}'_b} [\tilde{\Pi}'_u + \tilde{\Pi}'_b \Psi'_u] \right\}.$$

Re-arrange to get the GEE

$$0 = \underbrace{[R_b + \mathbb{E} \tilde{W}_b R_w + \mathbb{E} \tilde{S}_b R_s]}_{\text{effect of db holding } u'} + \beta \mathbb{E} \tilde{\Pi}_b \underbrace{[R'_u + \tilde{W}'_u R'_w + \tilde{S}'_u R'_s]}_{\text{effect of } du' \text{ holding } u''} \\ + \beta \mathbb{E} \tilde{\Pi}_b \left(-\frac{\tilde{\Pi}'_u}{\tilde{\Pi}'_b} \right) \underbrace{[R'_b + \tilde{W}'_b R'_w + \tilde{S}'_b R'_s]}_{\text{effect of } db' \text{ holding } u''} \quad (\text{A.26})$$

where the functions with tilde are transformations of the ones without, e.g. $S(z, u) \equiv \tilde{S}(z, b(u), u)$ and $S_u = \tilde{S}_u + \tilde{S}_b \Psi_u$. Because of the presence of policy function derivatives such as \tilde{S}_u and \tilde{S}_b , the above equation is also known as the Generalized Euler Equation or GEE. From the GEE, it is obvious any change in b has three effects. First, it affects the contemporaneous wages and search and thus both directly and indirectly changes the value of current government return function. Second, through changing u' , it changes next period's unemployment, wages and search, thus changing next period's value. Last, it also has an effect on next period's value through its effect on next period benefit b' . The government determines current benefit by setting the net marginal

value of b to zero.

A.2 Separable Preference

A.2.1 “The last emperor”: the case of separable preference

To illustrate the role of disciplining effect in the determination of benefit, we consider a special case of the Markov equilibrium. With preference separable in consumption and search, the Markov government’s optimality condition reduces to (the first line of GEE)

$$R_b + \underbrace{\left(-\frac{\eta_{3b}}{\eta_{3w}} \right)}_{=\partial w/\partial b|_{\eta_3=0}} R_w = 0 \quad (\text{A.27})$$

Notice that this condition does not contain policy derivative. With separable preference, current benefit policy does not affect current period search and vacancy posting. As a result, the Markov government has no disciplining effect over future governments, and each government chooses benefit policy to maximize current period government return function ($dR/db = 0$). Effectively, the Markov benefit policy equates the marginal utility of worker and unemployed (the first term in (A.27)), taking into account how benefit affects equilibrium wage (the second term). In a way, each successive government behaves like “the last emperor”.

Under the assumption of fixed wages (so the wage bargaining competitive equilibrium condition disappears), the Markov equilibrium with separable preference has an analytical solution.

PROPOSITION A.1. Under the assumptions of separable utility and fixed wages \bar{w} , a Markov-perfect equilibrium is given by

$$b = \bar{w} - h, \quad s = 0, \quad u = 1, \quad \theta = q^{-1} \left(\frac{1 - \beta(1 - \delta)}{\beta(\bar{z} - \bar{w})} \kappa \right).$$

Proof. The proof is straightforward. With fixed wage, (A.27) reduces to $R_b = 0$, or equivalently,

$$U_c(h + b - \tau) = U_c(\bar{w} - \tau)$$

which, given strict monotonicity of preference in consumption, entails $h + b = \bar{w}$. Since unemployment gets the same consumption as employment, it follows that $s = 0$ and $u = 1$. When wages are fixed, steady-state market tightness is also fixed. \square

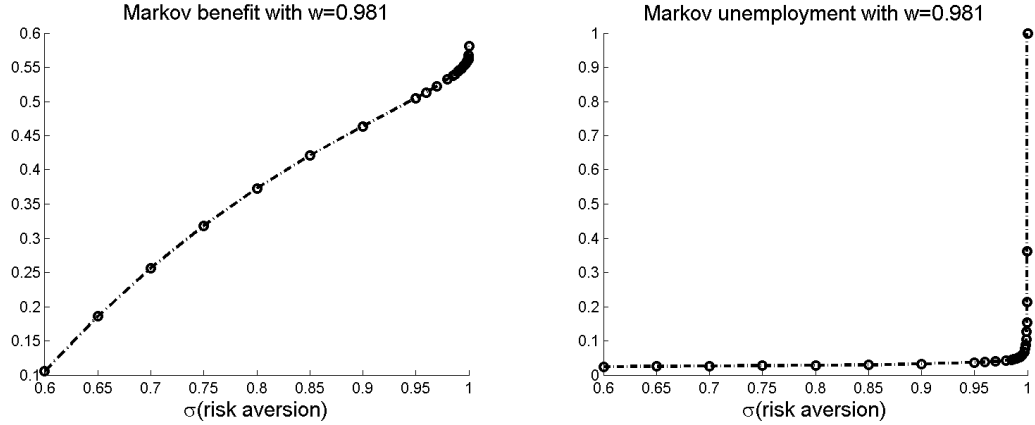


Figure A.1: Continuity of Markov-perfect equilibrium. Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1.

A.2.2 Continuity of Markov-perfect equilibrium

As in the previous literature on dynamic games, we cannot prove general existence or uniqueness results for the Markov-perfect equilibrium. But with fixed wages, we can show the continuity of Markov equilibrium policy rules. Figure A.1 shows that the Markov equilibria with non-separable preference converge smoothly to the equilibrium with separable preference as $\sigma \rightarrow 1$. The figure plots the Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1, holding all other parameters as for the case of flexible-wage and given in Table 1.1. Wages are fixed at $\bar{w} = 0.981$, the steady-state level in the baseline flexible-wage Markov equilibrium. Circles indicate the 25 values of σ for which the Markov equilibrium is computed numerically. The values for $\sigma = 1$ correspond to the equilibrium computed analytically in Proposition A.1. At $\sigma = 1$, the equilibrium features high benefit and high unemployment. As σ increases toward 1, both benefit and unemployment rise.

A.3 Sensitivity Analysis and Robustness Check

A.3.1 Sensitivity analysis on curvature parameter of search cost

For each value of ϕ , other parameters are recalibrated to match first-moment of the Markov equilibrium. The following table provides steady-state value for the Ramsey and Markov economy. The steady states are not sensitive to changes to ϕ .

Table A.1: Sensitivity Analysis for Different ϕ Values in Steady State

Statistic	$\phi = 0.5$		$\phi = 1$ (baseline)		$\phi = 2$	
	Ramsey	Markov	Ramsey	Markov	Ramsey	Markov
benefit	0.314	0.564	0.318	0.564	0.325	0.565
wages	0.976	0.981	0.976	0.981	0.976	0.981
search	0.716	0.329	0.690	0.329	0.649	0.332
vacancy	0.036	0.028	0.036	0.028	0.036	0.028
unemployment	0.022	0.054	0.023	0.054	0.025	0.054

Note: for each ϕ value, Ramsey and Markov steady states are re-computed using re-calibrated parameters.

A.3.2 Robustness check: Markov equilibrium using Chebyshev collocation

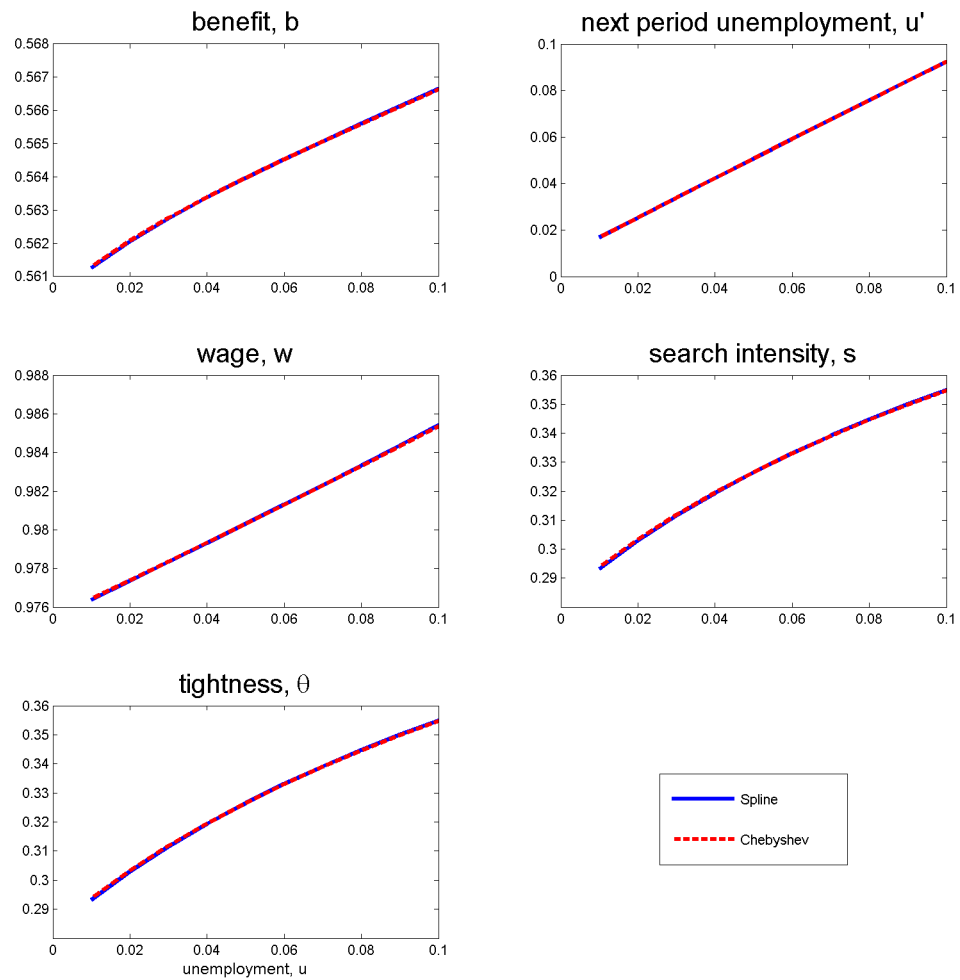


Figure A.2: Markov equilibrium policy functions approximated using cubic spline (solid blue) and Chebyshev collocation (dashed red) holding productivity at steady state. Chebyshev results are approximated using order-10 polynomials.

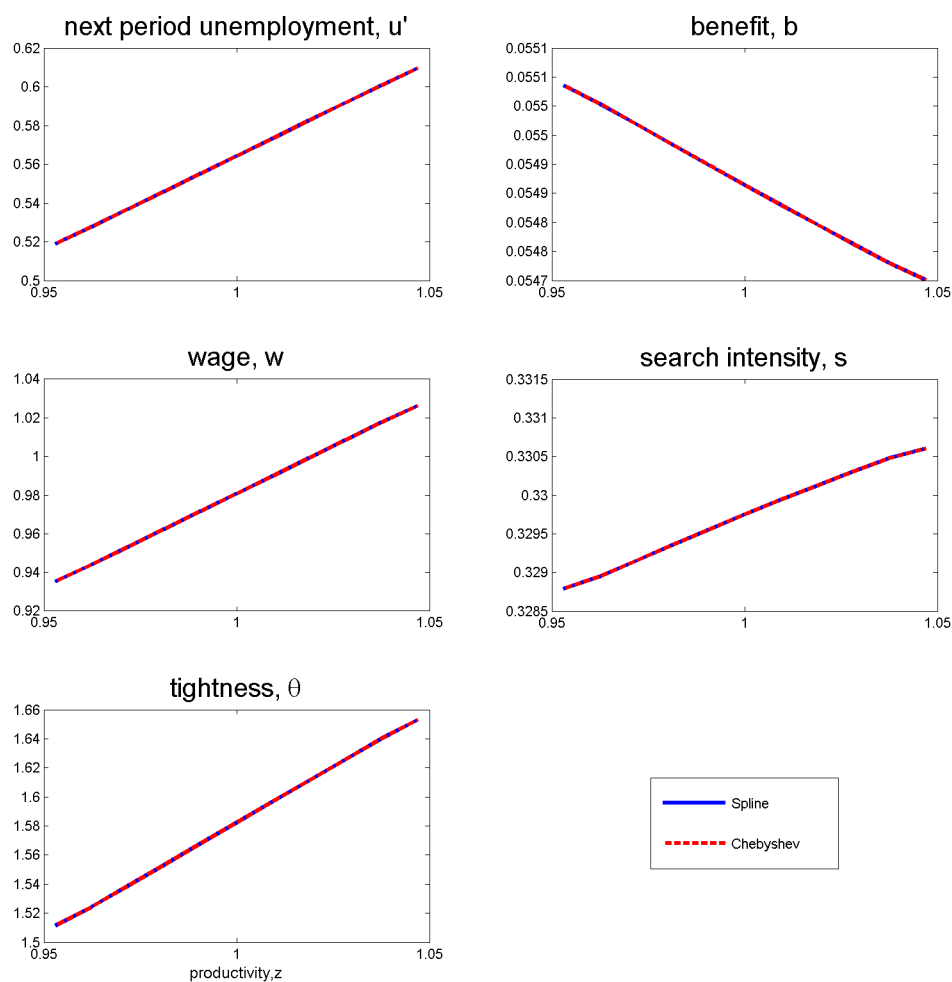


Figure A.3: Markov equilibrium policy functions approximated using cubic spline (solid blue) and Chebyshev collocation (dashed red) policy functions holding unemployment at steady state. Chebyshev results are approximated using order-10 polynomials.

A.4 Additional Figures

A.4.1 Additional policy function plots over unemployment and over productivity shock

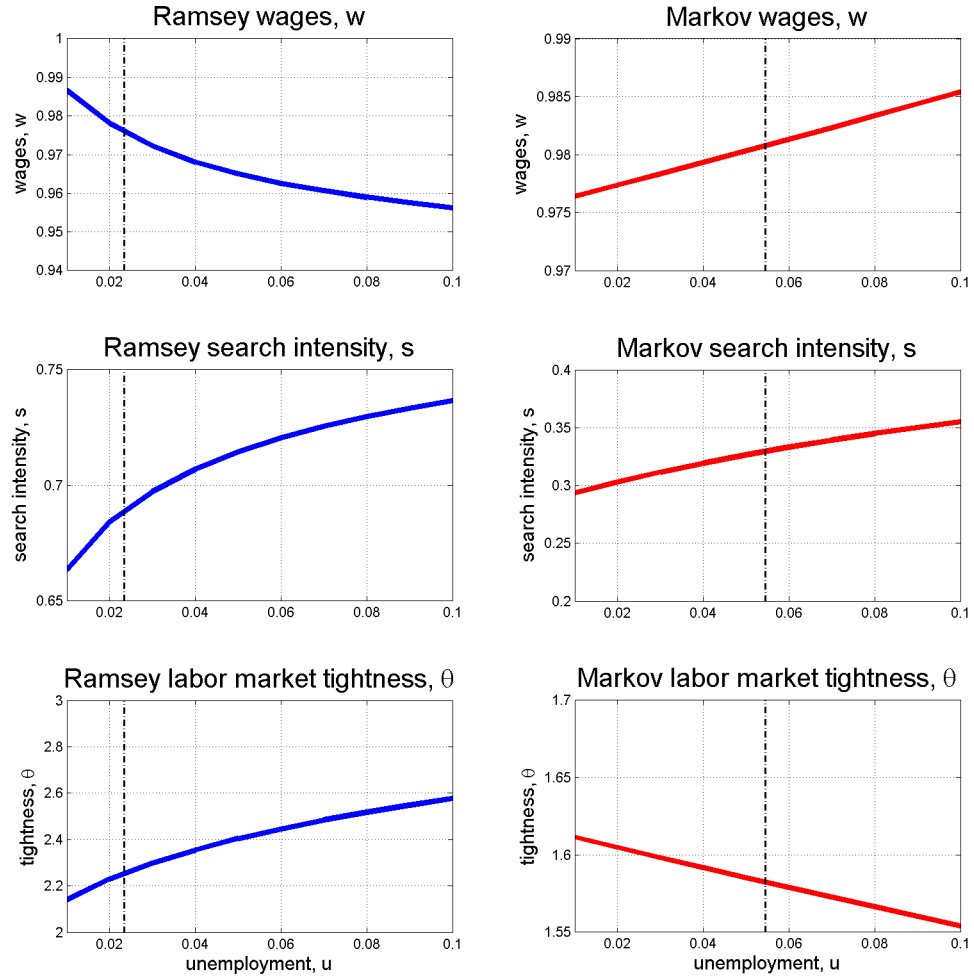


Figure A.4: Ramsey (left) and Markov (right) wage (top panel), search intensity (middle panel) and market tightness (bottom panel) policy functions holding productivity at steady state.

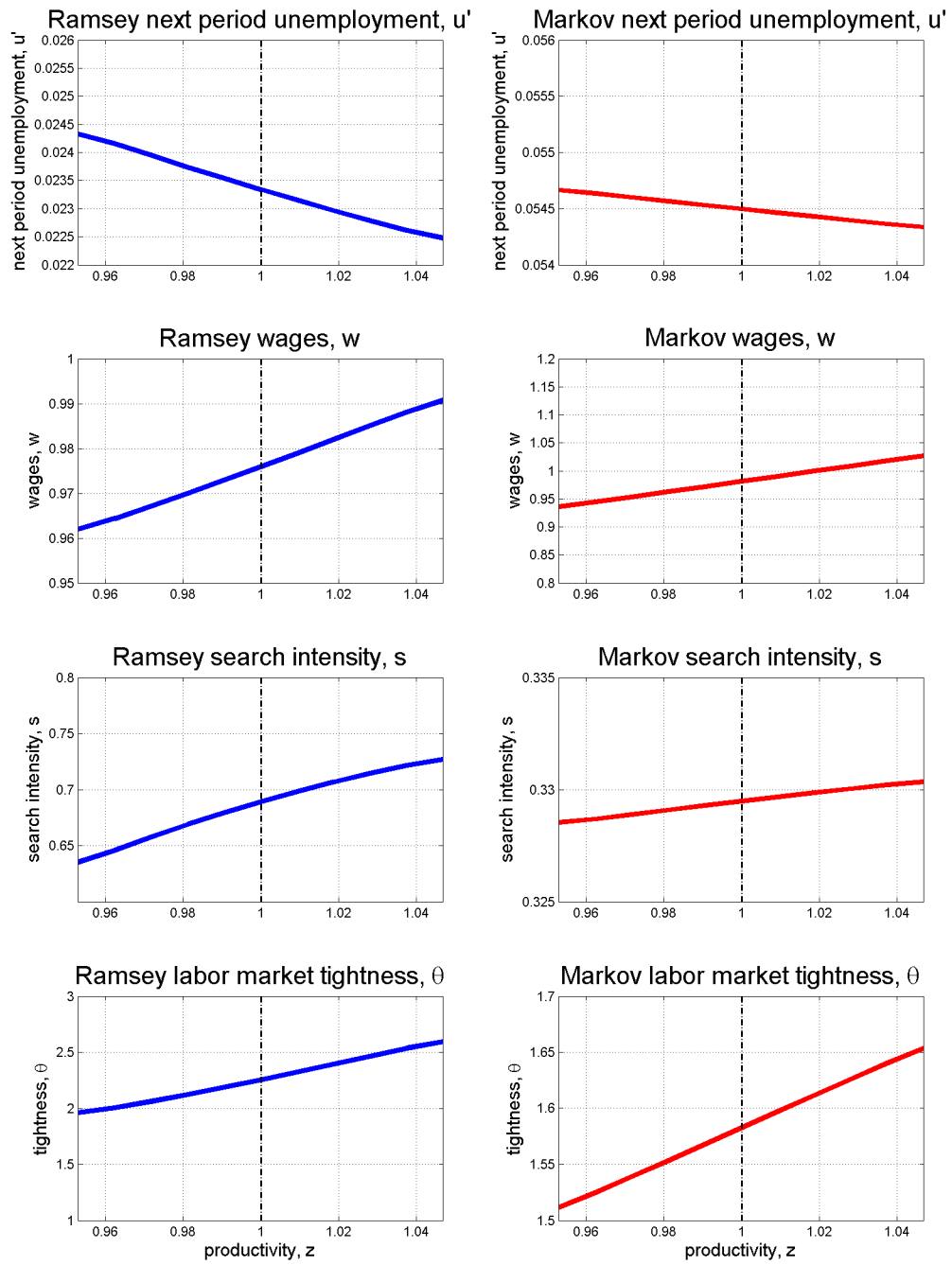


Figure A.5: Ramsey (left) and Markov (right) unemployment, wage, search intensity and market tightness policy functions holding unemployment at steady state.

A.4.2 Additional impulse response plots (baseline)

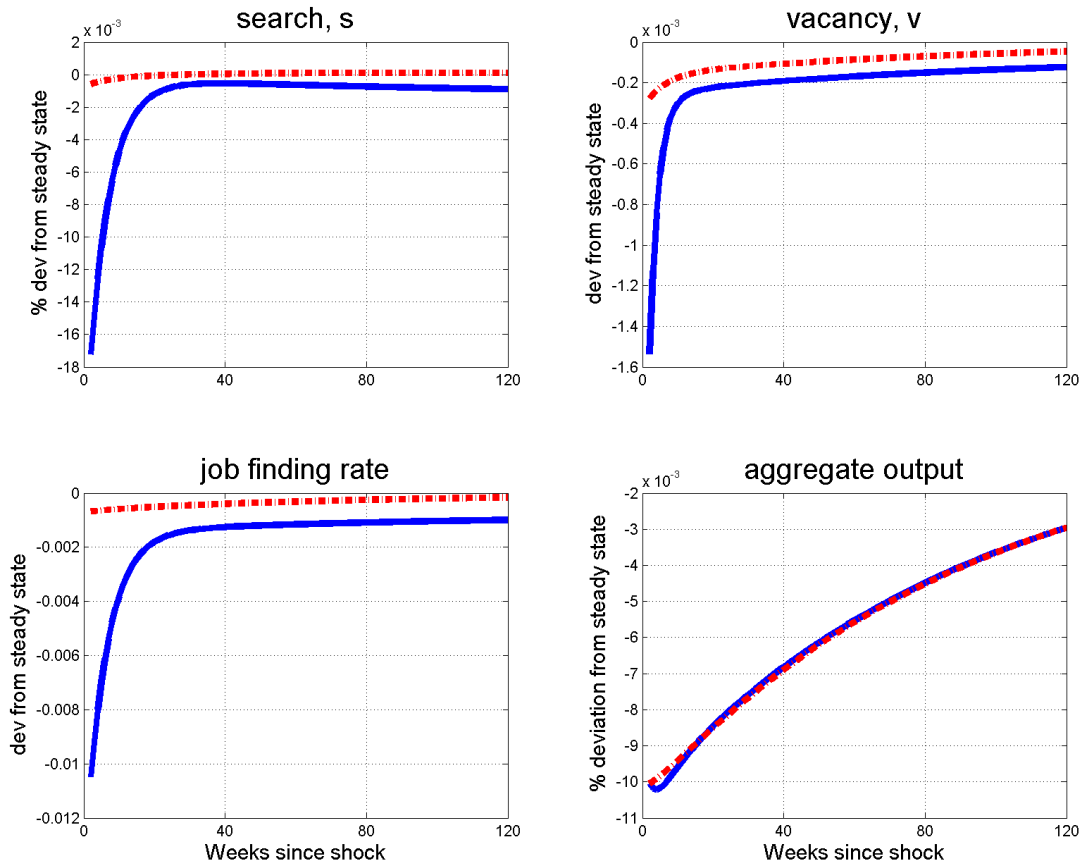


Figure A.6: Ramsey (solid blue line) and Markov (dashed red line) responses to a 1% drop in productivity.

B APPENDIX FOR CHAPTER 2

B.1 Unemployment Insurance Benefit Extensions in the Great Recession

States unemployment insurance and the federal government have adjusted unemployment benefits through the duration margin. In normal circumstances and under the regular program: Unemployment Compensation (UC), an eligible unemployed worker may receive unemployment benefits up to 26 weeks in most states. During economic downturn, automatic benefits extensions are triggered under the Extended Benefits (EB) program. The duration is 13 or 20 weeks depending on the state's insured unemployment rate (IUR) or the total unemployment rate (TUR). In addition, the Emer-

gency Unemployment Compensation (EUC08) has been launched in 2008 and has been redefined in the ARRA context in 2009. It also increases the maximum benefits duration. Four waves called “Tiers” have been implemented. The first one (Tiers I) is effective without any conditions on states’ experience with unemployment. Tiers II, III and IV require a condition on the IUR and/or TUR to be effective.

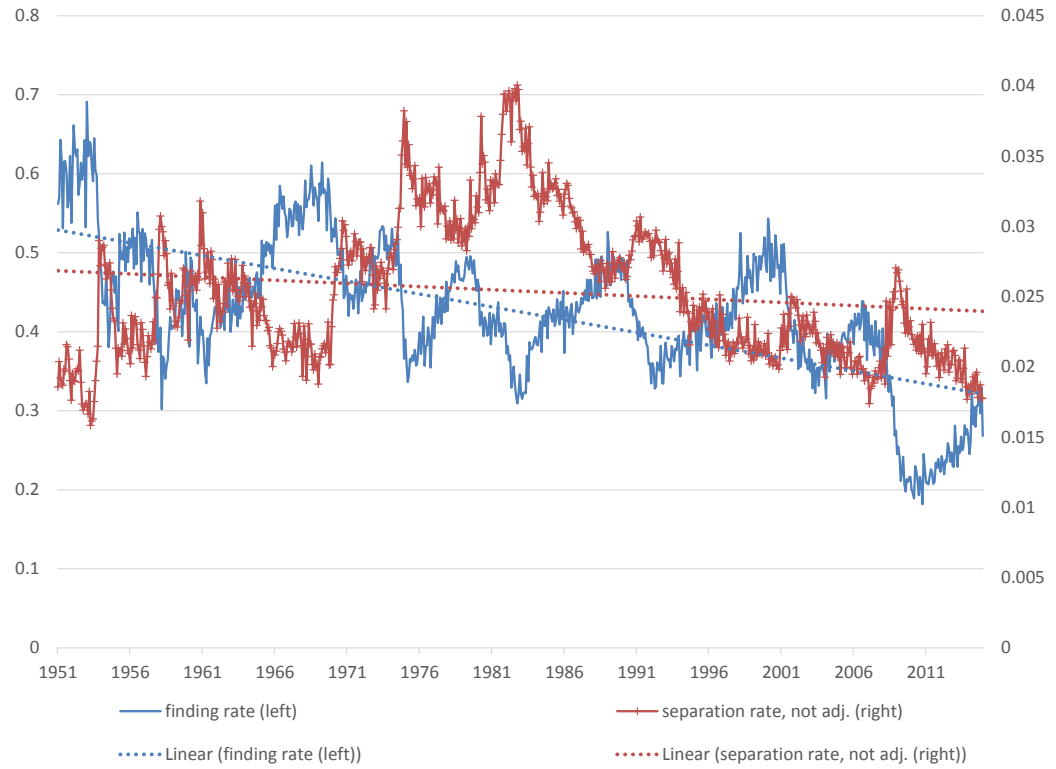
For these purposes, we extract the series of the IUR and TUR for 51 states of the US and compute if the state is eligible for the EB and the EUC08 programs. The sum of these three programs gives the maximum duration of unemployment benefits for each state. It is weighted in order to build an aggregate indicator. We assume the weights are equal to the number of total insured unemployed workers in the state divided by the total insured unemployed workers in the US. Statistics on insured unemployment population comes from the U.S. Department of Labor Employment and Training Administration. Table [B.1](#) reports a timeline for policy changes and unweighted expected maximum duration under the EUC08 and EB programs.

Table B.1: Federally-Funded Unemployment Insurance Extensions 2008-2013

Start date	Program extension of EUC08	End date	Additional Weeks
Jul 2008	13 weeks for all states	Nov 2008	13
Nov 2008	Tier I: 20 weeks for all states Tier II: 13 weeks for states with TUR $\geq 6\%$	Mar 2009	33
Mar 2009	keep existing structure	Nov 2009	33
Nov 2009	Tier I - 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states TUR $\geq 6\%$ Tier IV: 6 weeks if states TUR $\geq 8.5\%$	Dec 2009	53
Dec 2009	keep existing structure	Aug 2010	53
Mar 2010	keep existing structure	Sep 2010	53
Apr 2010	keep existing structure	Nov 2010	53
Jul 2010	keep existing structure	May 2011	53
Dec 2010	keep existing structure	Jun 2012	53
Dec 2011	keep existing structure	Aug 2012	53
Feb 2012	Tier I: 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states TUR $\geq 6\%$ Tier IV: 6 weeks if states TUR $\geq 8.5\%$ (16 weeks if no active EB and TUR $\geq 8.5\%$)	May 2012	53
Jun 2012	Tier I: 20 weeks for all states Tier II: 14 weeks if states TUR $\geq 6\%$ Tier III: 13 weeks if states TUR $\geq 7\%$ Tier IV: 6 weeks if states TUR $\geq 9\%$	Sep 2012	53
Sep 2012	Tier I: 14 weeks for all states Tier II: 14 weeks if states TUR $\geq 6\%$ Tier III: 9 weeks if states TUR $\geq 7\%$ Tier IV: 10 weeks if states TUR $\geq 9\%$	Dec 2012	47
Jan 2013	keep existing structure	Dec 2013	47
Start date	Program extension of EB	End date	Additional Weeks
Feb 2009	6.5% 13 week IUR and IUR $\geq 110\%$ of prior 3 years 8% 13 week IUR and IUR $\geq 110\%$ of prior 3 years	Dec 2013	13 26

Source: DOLETA, [Whittaker and Isaacs \(2013\)](#), [Albertini and Poirier \(2015\)](#)

B.2 Secular Decline in Job Finding and Separation Rates



B.3 Derivations and Proofs

B.3.1 Derivation of private sector optimality conditions

Throughout this section, we drop the dependence of functions on productivity shock z to economize on notation.

- Solving problem of unemployed person without benefit by taking derivative with respect to s^0

$$\frac{v_s(s^0)}{f(\theta)} = \beta[V^{e'} - V^{0'}] \quad (\text{B.28})$$

Solving problem of unemployed person with benefit by taking derivative with respect to s^1

$$\frac{v_s(s^1)}{f(\theta)} = \beta[V^{e'} - d'V^{0'} - (1-d')V^{1'}] \quad (\text{B.29})$$

Using worker's bellman equations

$$\begin{aligned} V^e - V^0 &= U(\bar{w} - \tau) - [U(h - \tau) - v(s^0)] \\ &\quad + \beta(1 - f(\theta)s^0)[V^{e'} - V^{0'}] - \beta\delta[V^{e'} - (1-d')V^{1'} - d'V^{0'}] \quad (\text{B.30}) \\ V^e - dV^0 - (1-d)V^1 &= d[U(\bar{w} - \tau) - U(h - \tau) + v(s^0) \dots \\ &\quad + \beta(1 - f(\theta)s^0)(V^{e'} - V^{0'}) - \beta\delta(V^{e'} - (1-d')V^{1'} - d'V^{0'})] \\ &\quad + (1-d)[U(\bar{w} - \tau) - U(h + b - \tau) + v(s^1) \dots \\ &\quad + \beta(1 - f(\theta)s^1 - \delta)(V^{e'} - d'V^{0'} - (1-d')V^{1'})] \quad (\text{B.31}) \end{aligned}$$

Combine (B.30) with (B.28) and (B.29)

$$V^e - V^0 = U(\bar{w} - \tau) - U(h - \tau) + v(s^0) + (1 - f(\theta)s^0) \frac{v_s(s^0)}{f(\theta)} - \delta \frac{v_s(s^1)}{f(\theta)}$$

Update one period, and substitute into (B.28)

$$\frac{v_s(s^0)}{f(\theta)} = \beta \left[U(\bar{w} - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right]$$

Combine (B.31) with (B.28) and (B.29)

$$\begin{aligned} & V^e - dV^0 - (1-d)V^1 \\ &= d \left[U(\bar{w} - \tau) - U(h - \tau) + v(s^0) + (1 - f(\theta)s^0) \frac{v_s(s^0)}{f(\theta)} - \delta \frac{v_s(s^1)}{f(\theta)} \right] \\ & \quad + (1-d) \left[U(\bar{w} - \tau) - U(h + b - \tau) + v(s^1) + (1 - f(\theta)s^1 - \delta) \frac{v_s(s^1)}{f(\theta)} \right] \end{aligned}$$

Update one period, and substitute into (B.29)

$$\begin{aligned} \frac{v_s(s^1)}{f(\theta)} &= \beta d' \left[U(\bar{w} - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \\ & \quad + \beta(1 - d') \left[U(\bar{w} - \tau') - U(h + b' - \tau') + v(s^{1'}) + (1 - f(\theta')s^{1'} - \delta) \frac{v_s(s^{1'})}{f(\theta')} \right] \end{aligned}$$

- From unmatched firm's value function, assuming free entry, i.e. $J^0(u, u^1) = 0$

$$\frac{\kappa}{q(\theta)} = \beta J^1(u', u^{1'})$$

Then firm's value function can be rewritten as

$$J^1(u, u^1) = z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta)}$$

Update one period

$$J^1(u', u^{1'}) = z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta')}$$

Substitute into the first equation

$$\frac{\kappa}{q(\theta)} = \beta \left[z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta')} \right]$$

B.3.2 Proof of proposition 2.1: derivation of Markov GEE

Throughout this section, we drop the dependence of functions on productivity shock z to economize on notation. Let $\lambda, \lambda_b, \mu, \mu_b, \gamma$ be the Lagrange multipliers on (2.12)-(2.16), respectively.

1. Take derivatives of government's problem with respect to b , d , s^0 , s^1 , θ , u' and u^1

$$\begin{aligned}
b: \quad & R_b = 0 \\
d: \quad & \lambda f_{1d} + \lambda_b f_{2d} - R_d = 0 \\
s^0: \quad & \lambda f_{1s^0} + \mu \eta_{1s^0} - R_{s^0} = 0 \\
s^1: \quad & \lambda_b f_{2s^1} + \mu_b \eta_{2s^1} - R_{s^1} = 0 \\
\theta: \quad & \lambda f_{1\theta} + \lambda_b f_{2\theta} + \mu \eta_{1\theta} + \mu_b \eta_{2\theta} + \gamma \eta_{3\theta} = 0 \\
u: \quad & \lambda f_{1u'} + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} = \beta G'_u \\
u^1: \quad & \lambda_b f_{2u^1} + \mu \eta_{1u^1} + \mu_b \eta_{2u^1} + \gamma \eta_{3u^1} = \beta G'_{u^1} \quad (\text{FOC})
\end{aligned}$$

where primes denote next period, and subscripts are derivatives. The first equation above $R_b = 0$ characterize the government's decision on benefit level.

2. Take derivative of Bellman equation with respect to u and u^1 , respectively

$$G_u = R_u + R_b \Psi_u^b + R_d \Psi_u^d + R_{s^0} S_u^0 + R_{s^1} S_u^1 + \beta G'_u \Gamma_u + \beta G'_{u^1} \Gamma_u^1 \quad (\text{ENV1})$$

$$G_{u^1} = R_{u^1} + R_b \Psi_{u^1}^b + R_d \Psi_{u^1}^d + R_{s^0} S_{u^1}^0 + R_{s^1} S_{u^1}^1 + \beta G'_u \Gamma_{u^1} + \beta G'_{u^1} \Gamma_{u^1}^1 \quad (\text{ENV2})$$

substitute the last two FOCs into ENV1 and ENV2 to eliminate $\beta G'_u$ and $\beta G'_{u^1}$

$$\begin{aligned}
G_u = & R_u + R_b \Psi_u^b + R_d \Psi_u^d + R_{s^0} S_u^0 + R_{s^1} S_u^1 \\
& + \Gamma_u \{ \lambda + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} \} + \Gamma_u^1 \{ \lambda_b + \mu \eta_{1u^1} + \mu_b \eta_{2u^1} + \gamma \eta_{3u^1} \} \quad (\text{B.32})
\end{aligned}$$

$$\begin{aligned}
G_{u^1} = & R_{u^1} + R_b \Psi_{u^1}^b + R_d \Psi_{u^1}^d + R_{s^0} S_{u^1}^0 + R_{s^1} S_{u^1}^1 \\
& + \Gamma_{u^1} \{ \lambda + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} \} + \Gamma_{u^1}^1 \{ \lambda_b + \mu \eta_{1u^1} + \mu_b \eta_{2u^1} + \gamma \eta_{3u^1} \} \quad (\text{B.33})
\end{aligned}$$

3. Differentiate η_1 , η_2 and η_3 with respect to u

$$\eta_{1u'} \Gamma_u + \eta_{1u^1} \Gamma_u^1 = -\eta_{1s^0} S_u^0 - \eta_{1\theta} \Theta_u \quad (\text{B.34})$$

$$\eta_{2u'} \Gamma_u + \eta_{2u^1} \Gamma_u^1 = -\eta_{2s^1} S_u^1 - \eta_{2\theta} \Theta_u \quad (\text{B.35})$$

$$\eta_{3u'} \Gamma_u + \eta_{3u^1} \Gamma_u^1 = -\eta_{3\theta} \Theta_u \quad (\text{B.36})$$

Given the worker flow equations

$$\begin{aligned}
\Gamma(u, u^1) = & \delta(1-u) + f(\Theta(u, u^1)) \left[S^0(u, u^1) - S^1(u, u^1) \right] u^1 (1 - \Psi^d(u, u^1)) \\
& + (1 - f(\Theta(u, u^1))) S^0(u, u^1) u
\end{aligned}$$

$$\Gamma^1(u, u^1) = \delta(1-u) + (1 - f(\Theta(u, u^1))) S^1(u, u^1) u^1 (1 - \Psi^d(u, u^1))$$

differentiate with respect to u

$$\begin{aligned} \Gamma_u + f_\theta(\theta)s^0u\Theta_u + f(\theta)uS_u^0 - f_\theta(\theta)(s^0 - s^1)u^1(1-d)\Theta_u \\ - f(\theta) \left[S_u^0 - S_u^1 \right] u^1(1-d) + f(\theta)(s^0 - s^1)u^1\Psi_u^d = -\delta + (1-f(\theta)s^0) \end{aligned} \quad (\text{B.37})$$

$$\Gamma_u^1 + f_\theta(\theta)s^1u^1(1-d)\Theta_u + f(\theta)S_u^1u^1(1-d) + (1-f(\theta)s^1)u^1\Psi_u^d = -\delta \quad (\text{B.38})$$

4. Substitute (B.34)-(B.38) and the FOCs into (B.32)

$$G_u = R_u + \lambda(1-f(\theta)s^0 - \delta) - \delta\lambda_b \quad (\text{B.39})$$

Similarly, differentiate η_1, η_2, η_3 and the worker's flow equations with respect to u^1 , and substitute into (B.33)

$$G_{u^1} = R_{u^1} + \lambda f(\theta)(s^0 - s^1)(1-d) + \lambda_b(1-f(\theta)s^1)(1-d) \quad (\text{B.40})$$

5. Update (B.39)-(B.40) and substitute into the last two FOCs, respectively

$$\lambda f_{1u'} + \mu\eta_{1u'} + \mu_b\eta_{2u'} + \gamma\eta_{3u'} = \beta \left[R'_u - \lambda' f'_{1u} - \lambda'_b f'_{2u} \right] \quad (\text{B.41})$$

$$\lambda_b f_{2u^1} + \mu\eta_{1u^1} + \mu_b\eta_{2u^1} + \gamma\eta_{3u^1} = \beta \left[R'_{u^1} - \lambda' f'_{1u^1} - \lambda'_b f'_{2u^1} \right] \quad (\text{B.42})$$

6. Combine the FOCs to get rid of Lagrange multipliers (leaving only λ)

$$\lambda_b = \frac{1}{f_{2d}} [R_d - \lambda f_{1d}] \quad (\text{B.43})$$

$$\mu = \frac{1}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] \quad (\text{B.44})$$

$$\mu_b = \frac{1}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \quad (\text{B.45})$$

$$\begin{aligned} \gamma = -\frac{1}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] \dots \right. \\ \left. \dots + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \end{aligned} \quad (\text{B.46})$$

7. Rewrite (B.41)-(B.42) explicitly by substituting (B.43)-(B.46)

$$\begin{aligned}
& \lambda f_{1u'} + \frac{\eta_{1u'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u'}}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \\
& - \frac{\eta_{3u'}}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] \dots \right. \\
& \quad \left. \dots + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \\
& = \beta \left\{ R'_u - \lambda' f'_{1u} - \frac{f'_{2u}}{f'_{2d}} [R'_d - \lambda' f'_{1d}] \right\} \quad (\text{GEE1})
\end{aligned}$$

$$\begin{aligned}
& \frac{f_{2u^1'}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1u^1'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u^1'}}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \\
& - \frac{\eta_{3u^1'}}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] \dots \right. \\
& \quad \left. \dots + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \\
& = \beta \left\{ R'_{u^1} - \lambda' f'_{1u^1} - \frac{f'_{2u^1}}{f'_{2d}} [R'_d - \lambda' f'_{1d}] \right\} \quad (\text{GEE2})
\end{aligned}$$

Equation (GEE2) characterizes the government's decision on d , where λ has the interpretation of the shadow price of unemployment and is characterized by equation (GEE1). Re-arrange to get the equations in Proposition 2.1.

B.4 Additional Quantitative Analyses

B.4.1 Additional impulse responses

This section contains impulse response of some labor market statistics to a one-time 1% negative shock to productivity. This figure complements Figure 2.5.

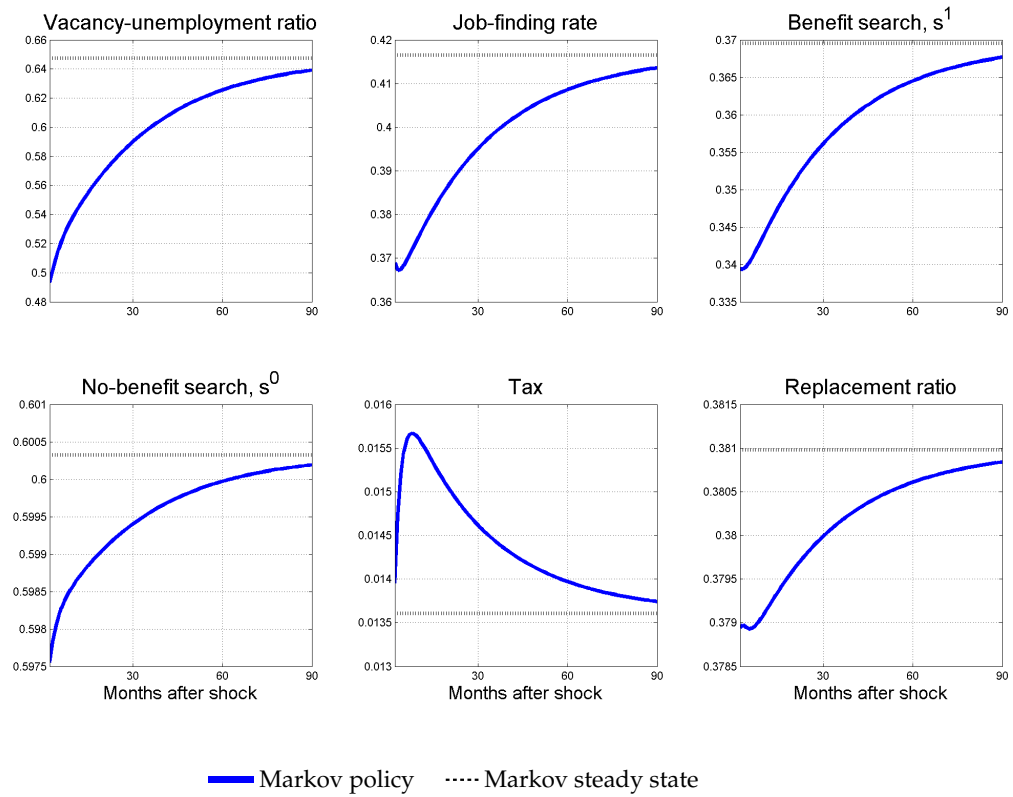


Figure B.1: Additional plots: Impulse response to 1% negative shock to productivity.

B.4.2 Alternative calibration of productivity path during the Great Recession

This section presents alternative calibration of the productivity path z_t . Compare to Figure 2.9.

Calibrating productivity path to match benefit duration

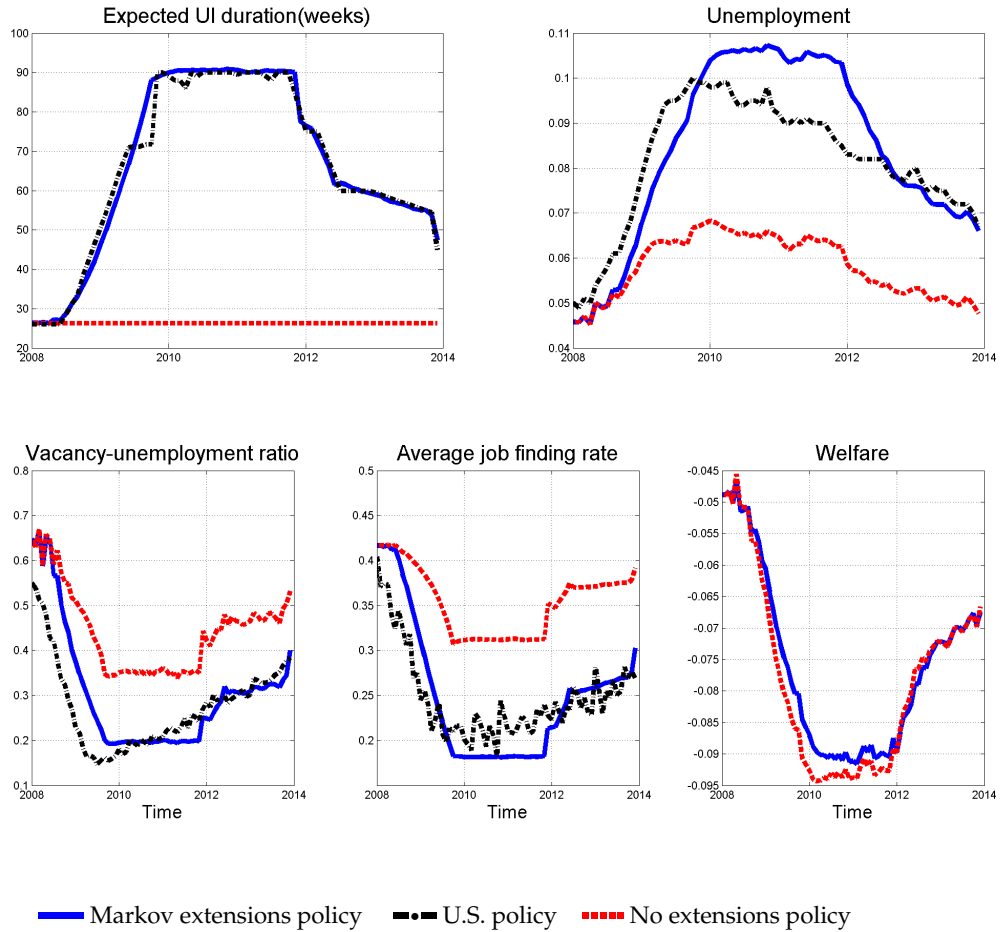


Figure B.2: Calibrating productivity to match benefit duration: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).

Calibrating productivity path for the best fit of unemployment and benefit duration

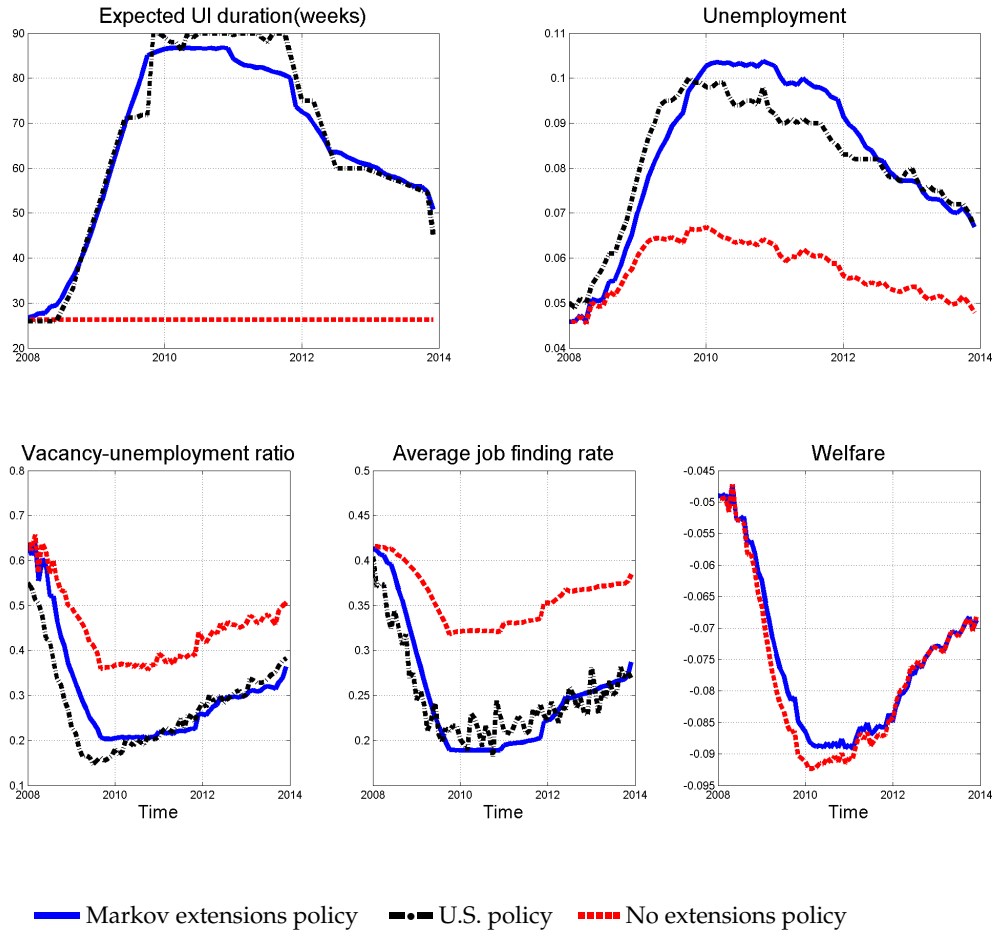


Figure B.3: Calibrating productivity path for best fit: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).

Comparison of different calibration strategies of productivity path

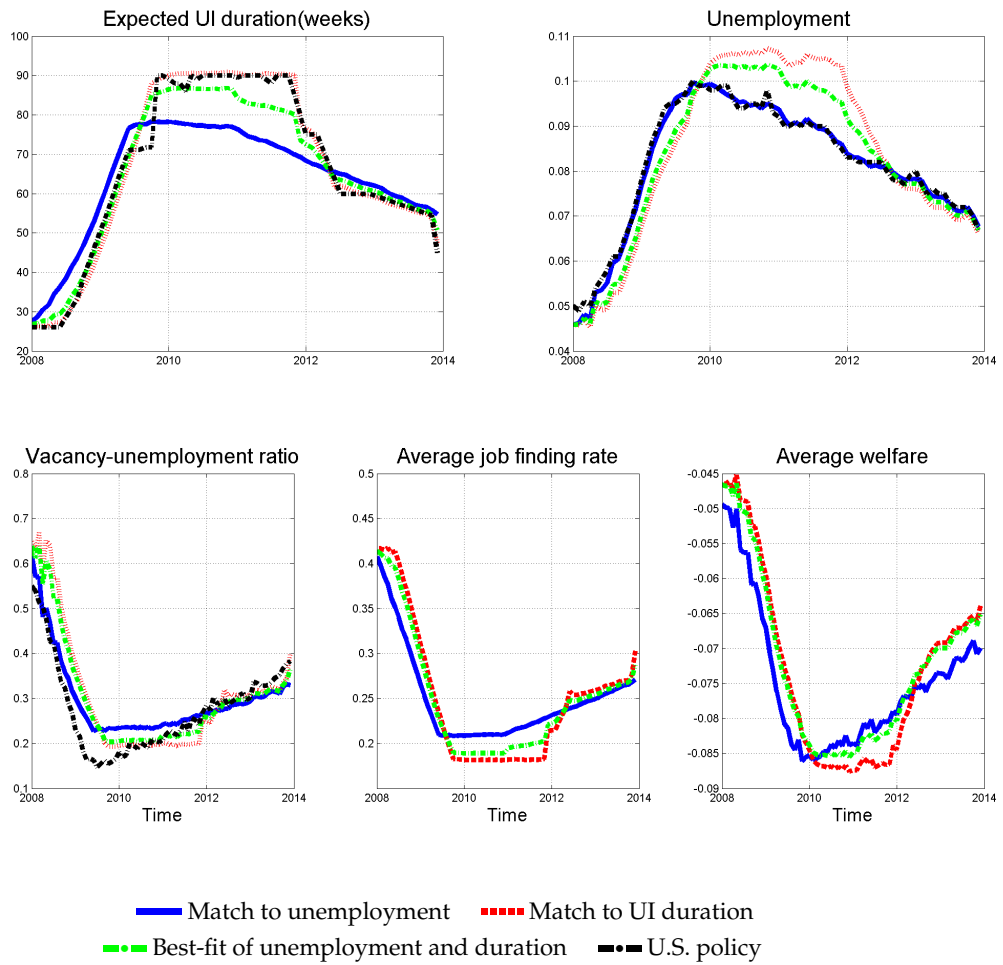


Figure B.4: Different calibrations of productivity path: Markov policy to match unemployment (solid blue line) versus Markov policy to match duration (dotted red line) versus Markov policy for best-fit (dashed green line) versus U.S. data (dashed black line).

B.4.3 Isolating quantitative effects from job separation shock

This section restricts productivity shock z to be constant at its steady-state level. The only exogenous shock here is the shock to job separation rate δ . Compared to Figure 2.9, both unemployment and expected UI duration are much lower. Thus, productivity shock (and not shock to job separation rate) drives most of the cyclical variations in the model.

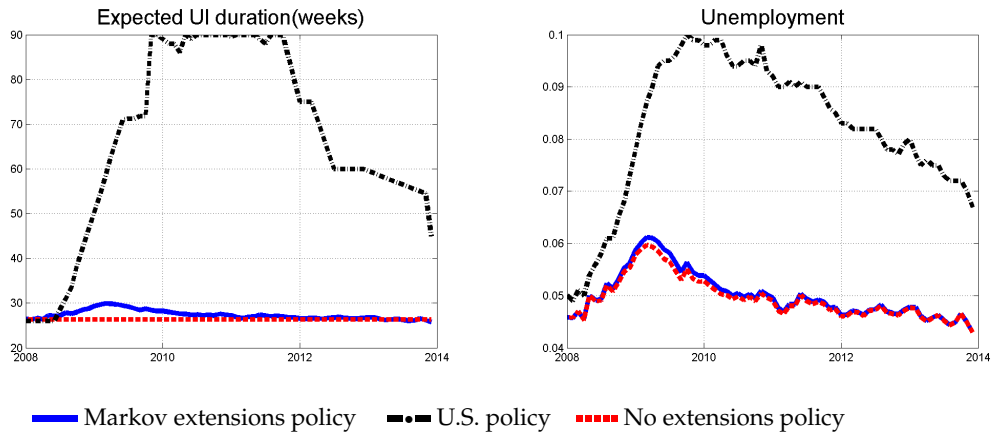


Figure B.5: Expected UI duration and unemployment: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).

B.4.4 Cyclical job separation risk

So far the cyclical job separation rate is both exogenous and unexpected. As an alternative specification, we make the job separation shock contingent on productivity in the model, so that the private sector takes into consideration the cyclical job separation rate when making decisions. We specify the job separation rate process as

$$\delta(z) = \bar{\delta} + I_{\delta}(z - \bar{z}),$$

where $\bar{\delta}$ is the steady-state job separation rate, and $I_{\delta} < 0$ is the rate of change of the separation rate with respect to aggregate productivity. This formulation has the natural interpretation that the job separation rate increases when profits are low. When labor productivity is low, wages are low as wages are also a function of productivity. Because the elasticity of wages with respect to productivity is less than 1, lower productivity means lower profit, or $z - w$ in the model. To estimate this process we use job separation and labor productivity data over 1951.I-2014.IV.

As before, productivity shock z is exogenously specified to match the unemployment process. The resulting labor productivity process requires a smaller drop than before—3.2% as opposed to 3.6% without changing separation risk. This is because the presence of countercyclical separation rate reinforces the effect of productivity shock. Figure B.6 shows the transitions for this alternate specification.

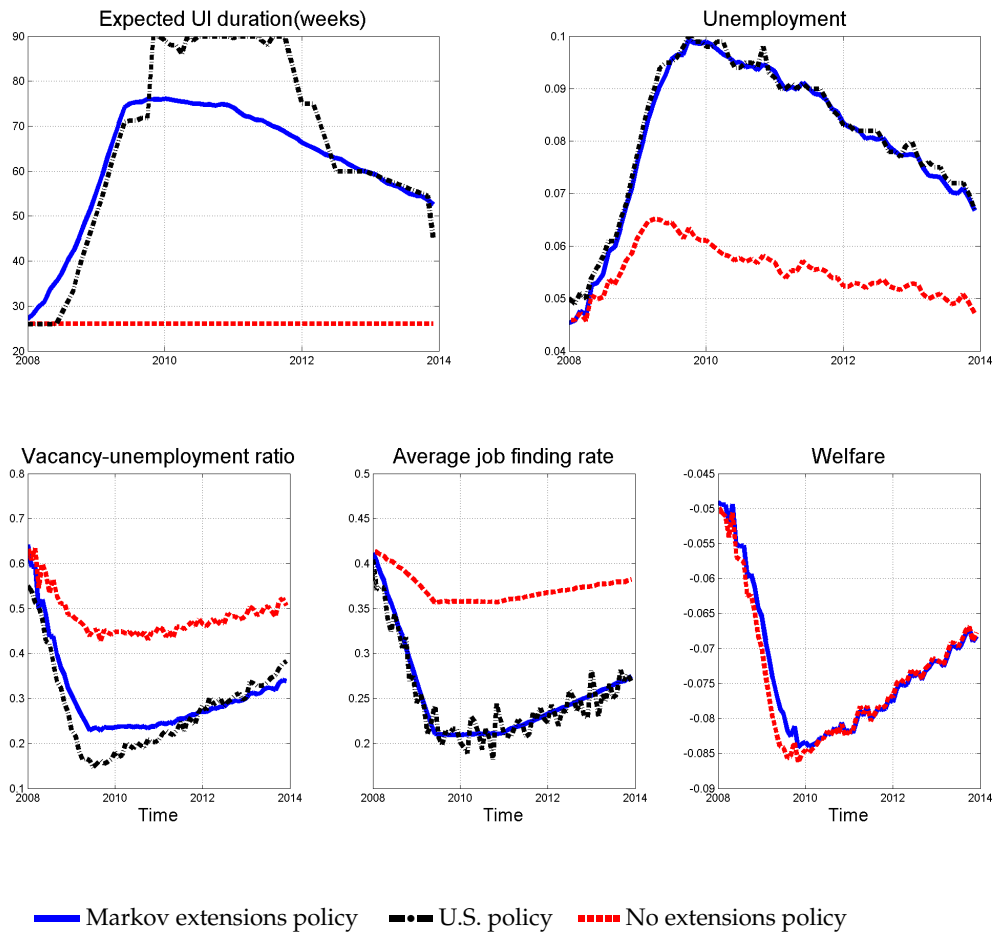


Figure B.6: With cyclical separation risk: UI duration, unemployment, and welfare during the Great Recession.

B.4.5 Other recessions

As noted in the empirical analysis, longer UI duration is not just a phenomenon during the Great Recession. Comparing across recession episodes since the 1970s, recessions with higher unemployment were associated with, in general, higher UI durations. In this section, we test whether our model delivers this characteristic. Because of the declining secular trend in job-finding and separation rates, we need to recalibrate the model parameter to the pre-recession period for each recession episode. Table B.2 summarizes the labor market statistics for the pre-recession window for each recession.

Table B.2: Calibration Targets for Other Recessions

Recession	Pre-recession period	Pre-recession labor market statistics		
		Separation	Job finding	Job filling
Nov 1973-Mar 1975	1973.I-1973.III	0.026	0.51	0.71
Jul 1981-Nov 1982	1980.II-1981.I	0.033	0.41	0.71
Jul 1990-Mar 1991	1988.I-1990.II	0.027	0.47	0.71
Mar 2001-Nov 2001	1999.I-2000.IV	0.020	0.49	0.66

Note: Job-filling rate pre-2000 are from [den Haan, Ramey and Watson \(2000\)](#).

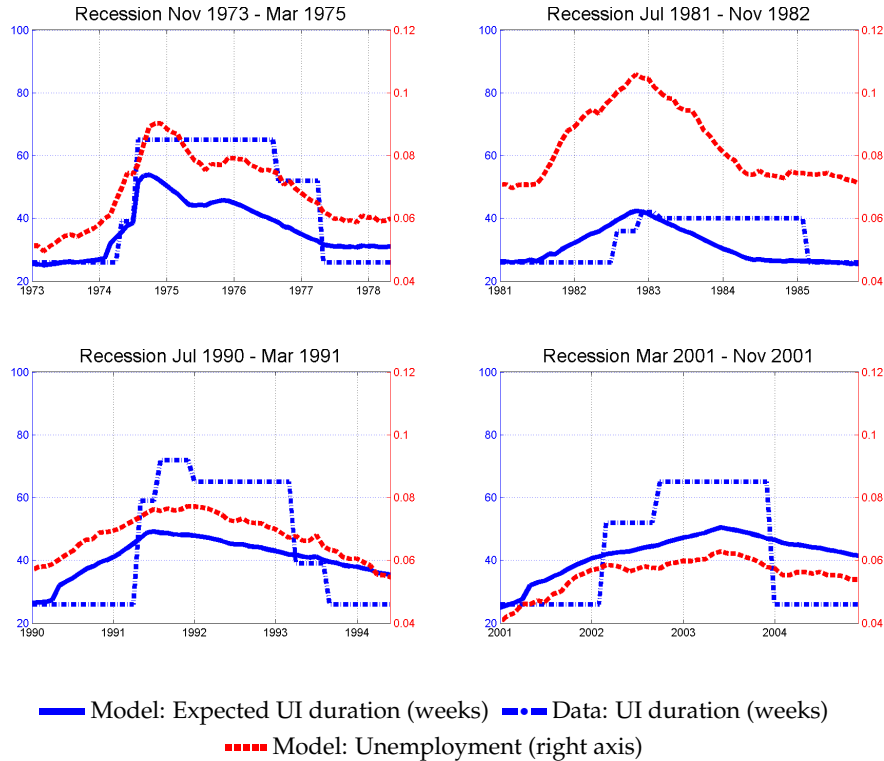


Figure B.7: UI duration and unemployment in other recessions: Model versus data

As with the Great Recession, we use the path of job separation rate from data, and target observed unemployment path to recover the path of productivity for each recession. Figure B.7 displays model-generated expected UI duration (solid blue line), unweighted UI duration from data (dashed blue line), and model-generated unemployment (broken red line, right axis) for each recession documented in the empirical analysis. Three observations are worth noting. First, the model matches increases in UI duration reasonably well, producing more than 60% of the increases (solid blue line vs. dashed blue line) in each recession. Second, consistent with patterns documented from the data, during all four recessions, model-generated UI duration reached its highest level around the time unemployment peaked. Lastly, recessions where unemployment was higher (broken red line) also had, in general, higher model-generated UI duration (solid blue line), except the 1980s recession. This evidence shows that, as an additional model validation, our theory is able to generate not only quantitatively significant UI duration increases in recessions, but also cross-time patterns consistent with the data.

B.5 The Role of Commitment: Ramsey Steady State

Our theory assumes no commitment by the government. In this section, we compare the Markov policy with the policy chosen by a Ramsey government to illustrate the role of commitment.

The Ramsey government has commitment to all its future policies at the beginning of time. The government's decision problem is therefore to choose a sequence of unemployment benefit and duration and tax policies $\{b_t, d_t\}_{t=0}^{\infty}$ to maximize the worker's utility, taking into account how the private sector will respond to these policies. At time 0, the government decides on its policies for all future periods and for all possible realizations of shocks. The private sector takes government policies as given and follows the timing described in Section 2.

Equivalently, the government's problem can be written as one of choosing policies $\{b_t, d_t, \tau_t\}_{t=0}^{\infty}$, and allocation and prices $\{s_t^0, s_t^1, \theta_t, u_{t+1}, u_{t+1}^1\}_{t=0}^{\infty}$ to maximize utility subject to the government budget constraint and competitive equilibrium conditions. Formally,

DEFINITION .2. (Ramsey policy) Given initial measures of unemployed population $(u_0, u_{b,0})$ and aggregate labor productivity z_0 , the optimal government policy with commitment consists of a sequence of benefit level and duration and taxes $\{b_t, d_t\}_{t=0}^{\infty}$ that solves

$$\max_{\{b_t, d_t, s_t^0, s_t^1, \theta_t, u_{t+1}, u_{t+1}^1\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(u_t, u_t^1, b_t, d_t, s_t^0, s_t^1)$$

over the set of all policies that satisfy the worker's flow equations (2.2)-(2.3), and the private-sector optimality conditions (2.9)-(2.11), for all time t and aggregate shock $\{z_t\}_{t=0}^{\infty}$.

For easy exposition, we use auxiliary functions \tilde{f}_1 , \tilde{f}_2 , $\tilde{\eta}_1$, $\tilde{\eta}_2$, and $\tilde{\eta}_3$ to denote the flow equations and the three private-sector optimality conditions (2.9)-(2.11), respectively. Note that the three private-sector optimality conditions play the role of incentive constraints in the optimal policy problem, similar to the incentive constraints in a principal-agent setup such as [Hopenhayn and Nicolini \(1997\)](#).

To derive a set of conditions that characterize the Ramsey policy, we let $\beta^t \pi^t \lambda_t$, $\beta^t \pi^t \lambda_{b,t}$, $\beta^t \pi^t \mu_t$, $\beta^t \pi^t \mu_{b,t}$, and $\beta^t \pi^t \gamma_t$ be the Lagrange multipliers on \tilde{f}_1 , \tilde{f}_2 , $\tilde{\eta}_1$, $\tilde{\eta}_2$, and $\tilde{\eta}_3$, where π^t is the probability of a history realization $\{z_0, z_1, \dots, z_t\}$ given an initial condition z_0 .

PROPOSITION B.2. Given initial conditions and the private-sector optimality conditions, the optimal government policy can be characterized by the following government's first-order conditions with respect to b_t , d_t , s_t^0 , s_t^1 , θ_t , u_{t+1} , and u_{t+1}^1 for all time $t > 0$ (highlights represent differences

with the Markov government optimality conditions):

$$\begin{aligned}
b: \quad & \mu_{t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2b',t-1}}{\beta} - R_{b,t} = 0 \\
d: \quad & \mu_{t-1} \frac{\tilde{\eta}_{1d',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2d',t-1}}{\beta} + \lambda_t f_{1d,t} + \lambda_{b,t} f_{2d,t} - R_{d,t} = 0 \\
s^0: \quad & \mu_{t-1} \frac{\tilde{\eta}_{1s^0',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2s^0',t-1}}{\beta} + \lambda_t f_{1d,t} + \mu_t \tilde{\eta}_{1s^0,t} - R_{s^0,t} = 0 \\
s^1: \quad & \mu_{t-1} \frac{\tilde{\eta}_{1s^1',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2s^1',t-1}}{\beta} + \lambda_{b,t} f_{2d,t} + \mu_{b,t} \tilde{\eta}_{2s^1,t} - R_{s^1,t} = 0 \\
\theta: \quad & \mu_{t-1} \frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{3\theta',t-1}}{\beta} + \lambda_t f_{1\theta,t} + \lambda_{b,t} f_{2\theta,t} + \mu_t \tilde{\eta}_{1\theta,t} \\
& + \mu_{b,t} \tilde{\eta}_{2\theta,t} + \gamma_t \tilde{\eta}_{3\theta,t} = 0 \\
u: \quad & \lambda_t f_{1u',t} - \beta \mathbb{E}_t \{ R_{u,t+1} - \lambda_{t+1} f_{1u,t+1} - \lambda_{b,t+1} f_{2u,t+1} \} = 0 \\
u^1: \quad & \lambda_{b,t} f_{1u^1',t} + \mu_t \mathbb{E}_t \tilde{\eta}_{1u^1,t} + \mu_{b,t} \mathbb{E}_t \tilde{\eta}_{2u^1',t} \\
& - \beta \mathbb{E}_t \{ R_{u^1,t+1} - \lambda_{t+1} f_{1u^1,t+1} - \lambda_{b,t+1} f_{2u^1,t+1} \} = 0 \quad (\text{RAM})
\end{aligned}$$

where primes denote next period, and subscripts are derivatives.

The period- t solution is state dependent. It depends on the current productivity z_t and the beginning-of-period unemployment level u_t , as well as multipliers $(\mu_{t-1}, \mu_{b,t-1}, \gamma_{t-1})$. μ_t and $\mu_{b,t}$ are the marginal values of relaxing the optimal search condition for the unemployed worker without and with benefit, respectively, and γ_t is the marginal value of relaxing the firm's equilibrium free-entry condition. The presence of μ_{t-1} , $\mu_{b,t-1}$, and γ_{t-1} as stated in the optimal policy captures commitment—the Ramsey government in period t has to deliver these marginal values, which it promised workers and firms in period $t - 1$.

The key difference between the conditions characterizing the Ramsey and Markov policies is that the Markov optimality conditions do not contain promised marginal values from the previous period (terms containing μ_{t-1} , $\mu_{b,t-1}$, and γ_{t-1} as highlighted in red in RAM), because the Markov government lacks commitment to future policies. μ_{t-1} , $\mu_{b,t-1}$, and γ_{t-1} represent the *marginal private values* (shadow price) of optimal job-search and vacancy-posting behaviors in period $t - 1$. These marginal values are affected by expected policy and allocations of period t . For example, more generous UI in period t reduces expected gains from search and vacancy posting, thus reducing job creation in period $t - 1$. Because the Markov government cannot commit, it does not internalize how current policy affects incentives in the previous period. As a result, its policy does not depend on the values of μ_{t-1} , $\mu_{b,t-1}$, and γ_{t-1} . In contrast, the Ramsey government chooses policies that can deliver these promises, thus their presence in the Ramsey optimality conditions.

Note that commitment is assumed in the Ramsey case. If given the choice to break a promise, the government will deviate from the sequence of policies prescribed by the government at time

Table B.3: Internally Calibrated Parameters: Ramsey

Parameter	Description	Value
h	Value of home production	0.397
γ	Disutility of search	4.432
χ	Matching parameter	2.263
\bar{w}	Steady-state wage	0.979
Target	Data	Model
Average replacement ratio	40%	37.9%
Average job-finding rate	0.40	0.400
% unemployed with benefits	45	44.6
Average job-filling rate	0.66	0.660

Note: Calibration targets are monthly statistics of the U.S. economy 2005.I-2007.IV.

0. The government at period t has an incentive to promise low future unemployment benefits to encourage search and vacancy posting, because as explained in Section 2, current search (mainly search of the benefit-eligible unemployed workers) is higher when expected future UI duration is shorter. However, after the employment outcome in period t is realized, the government has an incentive to provide insurance to more unemployed workers by choosing longer duration. This incentive to deviate from the original plan is what constitutes time inconsistency in the Ramsey problem.

B.5.1 The long-run effect of commitment

We compare the steady-state Markov and Ramsey policies. The difference here is that the Markov government lacks commitment over future policies, and hence does not consider the effect of current policy on past allocations. For a fair comparison, we re-calibrate the Ramsey economy to match the same set of steady-state statistics used to calibrate the Markov economy. Table B.3 shows these calibrated parameters and the target moments. The exogenously calibrated parameters are the same as before.

Table B.4 compares the steady-state benefit duration policy in the Markov economy and in the Ramsey economy. Because both economies are calibrated to match the same steady-state replacement ratio, benefit duration is the only source of policy difference in this comparison. Consistent with the difference highlighted before, the Ramsey policy is less generous than the Markov policy because the Ramsey government internalizes the effect of current policy on previous job creation.

Table B.4: Steady States: Markov versus Ramsey Policy

Statistic	Markov	Ramsey
Duration(weeks), $1/d$	26.3	16.8

Note: Steady states are computed using parameters calibrated to the same set of steady-state moments.