The Comparative Effects of Dyad Mathematics Interventions on Improving Multiplication Proficiency

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Abstract

Proficiency in mathematics across the United States has become an area of concern as performance on state accountability tests continues to demonstrate that less than 50% of students in 4th and 8th grade are proficient in mathematics (NCES, 2013). With such large numbers of students lacking proficiency in mathematics, systematic interventions that target many students need to be utilized to intervene when large numbers of students are demonstrating limited proficiency.

The current study implemented a classwide intervention with 133 third- and fourth-grade students to increase proficiency in multiplication. Students were randomly assigned to the cognitive strategy (CSI), timed drill (TD), or a control condition. The intervention occurred for 15 school day sessions for 30 minutes. Results indicated that students in the timed drill (TD) condition improved on measures of basic multiplication and near transfer measures of division fluency, and students in the CSI condition improved on a measure of multiplication word problem solving. Findings from this study indicate that classwide dyad interventions can be used to help classrooms with large numbers of students struggling with basic multiplication. Future research is needed to determine what interventions should continue for those students not making adequate progress in tier 1 interventions, but given the promising data found here and the pressing need for increased student mathematics proficiency, additional research seems warranted. Implications for research and practice, limitations, and future directions are discussed.

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CHAPTER 1

INTRODUCTION

The number of students who are not meeting proficiency on state accountability tests for mathematics has become a growing concern to educators. According to the Nation's Report Card (National Center for Educational Statistics, NCES), in 2013 only 34% of 4th grade students were considered proficient in mathematics and 8% were considered advanced. In 8th grade, only 27% of students were proficient in mathematics, while 9% were considered advanced. These alarming statistics call for an urgent need to intervene in early mathematics instruction to increase the number of students proficient in mathematics.

Statement of the Problem

With the focus of increasing proficiency through intervening in early mathematics education, there needs to be a clear understanding of what to teach and how to teach. Teaching students to become proficient in foundational arithmetic skills is important to help students succeed in higher-level mathematics concepts (Hasselbring, Goin, & Bransford, 1987). Silbert, Carnine, and Stein (1981) stated:

> The mastery of basic facts is critical if students are to develop fluency in working mathematical problems. The negative attitudes many children have about math can be traced to not having mastered the basic facts. (p. 44)

Therefore, targeting basic multiplication facts while simultaneously focusing on increasing students' attitude towards mathematics is key to increase proficiency. This recommendation aligns with the National Research Council's (2001) five strands of mathematical proficiency. The five strands include: conceptual understanding, procedural knowledge, strategic competence, adaptive reasoning, and productive disposition, and it is recommended that these five components are included when teaching mathematics (NRC, 2001).

When a large number of students in a class have not mastered a given skill, a classwide intervention can be implemented supplemental to core instruction to teach that skill (Poncy, Skinner, & O'Mara, 2006; VanDerHeyden, McLaughlin, Algina, & Snyder, 2012), and can be a cost-saving approach when there are limited resources (Case, Speece, & Molloy, 2003; Fuchs, 2003). However, identifying the appropriate classwide interventions to specifically target multiplication skills is a continued area of future research. While there have been increases in the number of classwide interventions in mathematics being implemented, previous studies have not examined the effect on number of students who become proficient with the implementation of different classwide interventions.

Study Purpose

The purpose of this study is to implement classwide mathematics interventions to specifically target multiplication computation fluency. I identified a school with low mathematics proficiency as evidenced by performance on the Measures of Academic Progress (MAP-M; Northwest Evaluation Association, 2004), which predicts student performance on state accountability tests. The goal of the study was to implement a classwide intervention for third- and fourth-grade students for 30 minutes per day for 3 weeks (15 school days) in order to increase the total number of students identified as proficient in multiplication computation fluency. Dyads were randomly assigned to one of three conditions: 1) cognitive strategy instruction (CSI), targeting metacognitive strategies to solve multiplication word problems; 2) timed drill (TD) condition, targeting multiplication computation fluency; or 3) control condition, engaging in other mathematical areas not related to multiplication such as data analysis or telling time. In addition to targeting the increase in multiplication proficiency, this study investigated the near transfer of learned multiplication facts to students' ability to solve basic division facts and division word problems.

Significance of the Study

This study was designed to provide guidance to future research and to provide direct implications for schools. I used a randomized design to examine the effectiveness of increasing multiplication proficiency and the ability for students to transfer these skills to basic division problems and division word problems. Findings from this study will result in increased knowledge about effective multiplication interventions related to improving multiplication proficiency, and strategies to implement them in a school naturalistic setting.

Research Questions

The following research questions guided the study:

1. What is the relative effect of a fact fluency intervention and an application intervention on a multiplication fact fluency measure?
2. What is the relative effect of a fact fluency intervention and an application intervention on a multiplication application measure?
3. What is the relative effect of a fact fluency intervention and an application intervention on a division fact fluency measure?
4. What is the relative effect of a fact fluency intervention and an application

intervention on a division application measure?

5. To what extent does intervention affect levels of student proficiency in multiplication fact fluency?

Definitions

Adaptive Reasoning: The capacity to think logically about the relationships among concepts and situations (NRC, 2001). Students with adaptive reasoning are able to understand all components of a problem, apply different concepts and procedures to identify solutions and understand that they have arrived at the correct solution based on deductive reasoning.

Application Problem Solving: Applying previously learned skills to solve word problems that contain real world or situations outside of mathematics (i.e., everyday life; Blum & Niss, 2001).

Cognitive Strategy Instruction: Involves teaching techniques of modeling, corrective feedback, verbal rehearsal, self-questioning, and cueing with direct instruction in the verbal, rehearsal, self-questioning. It also involves cueing with direct instruction in the verbal math problem solving techniques of paraphrasing, visualizing, detecting relevant information, locating the question, hypothesizing, estimating, labeling, and checking (Montague & Bos, 1986).

Conceptual Understanding: An integrated and functional grasp of mathematical ideas (NRC, 2001) demonstrated by an ability to comprehend the meaning of a problem and to organize information in a manner that allows for easy recall and the ability to make connections with preexisting knowledge (Geary, 1995; Hiebert & Lefevre, 1986). Conceptual understanding may sometimes be referred to as conceptual knowledge.

Curriculum-based Measurement – Mathematics (M-CBM): A brief assessment approach that allows for, repeated administration of skill-specific domains, to assess the level and trend of academic performance and growth on many subjects including reading, spelling, writing, and mathematics (Deno, 1985). M-CBM applies curriculum-based measurement principles to mathematics by assessing specific mathematic skills such as computation, concepts, and application.

Procedural Fluency: Knowledge of when and how to use specific mathematics computation rules and in performing them flexibly, accurately, and efficiently (NRC, 2001). Students with procedural fluency will understand the rote steps used when solving specific problems, while potentially not understanding the meaning of the problem, and can solve basic computation problems without needing to rely on tables, finger counting, or other resources and strategies (Hiebert & Lefevre, 1986; Star, 2005). Procedural fluency is sometimes referred to as procedural knowledge, or computation fluency.

Productive Disposition: "The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that stay effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131).

Strategic Competence: The ability to formulate mathematical problems, represent them, and solve them (NRC 2001). Strategic competence involves open-ended or unstructured discovery learning, and encourages students to invent, share, and streamline their own concrete models and written procedures (Baroody, 2003). Strategic competence is sometimes to referred to as problem solving.

Delimitations

The following delimitations define the scope and boundaries of this current study.

1. The study occurred in an elementary school with third and fourth grade students. I chose to focus on those grades because students in the following years (e.g., fifth and sixth) require foundational skills in mathematics to solve more complex skills such as pre-algebra and geometry. Targeting students early can assist students in developing these foundational skills at an earlier age.
2. The current study only implemented two interventions targeting multiplication skills. The cognitive strategy instruction (CSI) condition contained 4 recommended strands of proficiency from the National Research Council (NRC, 2001), and the timed drill (TD) only contained 1 strand of proficiency. However, a motivational component was added to both intervention conditions, which allowed for an additional strand (productive disposition) to be included in the intervention conditions.
3. The current study occurred in the spring over 15 school days for 30 minutes a session, over the last few months of the year, and results could have varied had the study occurred during a different time of the year for a longer duration. I chose this time of year because I was able to provide mathematics interventions to students prior to state annual testing.
4. The dependent variable of this study was student performance on multiplication and division computation fluency and application measures. I selected these areas because both are relevant factors to reaching proficiency in multiplication.

Organization of the Dissertation

This dissertation contains four additional chapters that outline the study, describe key features, and discuss findings related to research and practice. Chapter 2 contains a literature review, which defines theoretical components related mathematical proficiency, describes features of teaching multiplication, and using intervention frameworks in schools with large populations of students demonstrating low levels of mathematical proficiency. Chapter 3 contains the Methods of the study, and describes the participants and school setting, pre- and posttest measures, conditions, procedures, and how data were analyzed. Chapter 4 contains the Results of the 5 questions explored in this study. Chapter 5 contains the Discussion, and reviews the findings of the study, provides implications for research and practice, discusses limitations, and provides future directions for research.

CHAPTER 2

LITERATURE REVIEW

With the implementation of No Child Left Behind (NCLB) in 2001, there has been a growing focus to ensure all students are proficient in reading and mathematics. Unlike reading instruction, which is primarily divided into five large areas (phonemic awareness, phonics, fluency, vocabulary, and comprehension (National Reading Panel, 2000), mathematics is comprised of a multitude of skills including number sense, calculation skills, numerical estimation skills, and measurement skills (McCloskey, 2007). Mathematics skills are applied in everyday situations including understanding spatial relations when reading maps, calculating budgetary expenses and estimating and calculating expenses related to purchases at the store or when buying a home or car (McCloskey, 2007). The foundational skills related to these mathematical concepts must be developed in the elementary years to foster proficiency in mathematics for all students (National Research Council, NRC, 2001).

Historically the development of mathematics proficiency has focused on conceptual and procedural understanding (Resnick & Ford, 1981). In the 1930s, the terms drill theory, meaning theory, and incidental-learning theory emerged as key components to mathematics development (Brownell, 1935). However, as a national crisis emerged in the 1950s in the area of mathematics achievement, reform emerged urging educators to enhance technical and mathematical skills to advance the competing nation (Herrera & Owens, 2001). In the 1970s, the teaching of mathematics reverted to the “back to basics” era where the focus shifted to being able to compute accurately and quickly (Herrera & Owens, 2001; National Research Council [NRC], 2001). Finally, in the 1980s and 1990s mathematics instruction shifted once again, but focused on reasoning, solving problems, connecting mathematical ideas, and communicating mathematics to others (NRC, 2001).

Within the United States, lack of proficiency in mathematics continues to be a relevant topic. While there have been gains in the number of students from the United States meeting mathematics proficiency as compared to other countries, 4^{th} and 8^{th} grade students in the United States continue to fall behind other countries in mathematics performance (National Center for Educational Statistics; NCES, 2012). According to the Nation's Report Card, in 2013 (NCES), only 34% of 4^{th} grade students were proficient in mathematics and 8% were considered advanced. In 8^{th} grade, only 27% of students were proficient in mathematics, while 9% were considered advanced. Combined in both 4^{th} and 8^{th} grades, less than 50% of students in the United States were proficient in mathematics. However, that number decreases to only 33% of 4^{th} graders and 22% of 8^{th} graders when considering students who receive a reduced-price lunch, and only 22% of 4^{th} graders and 16% of 8^{th} graders eligible for free lunch were proficient in mathematics (NCES, 2013).

The inadequate progress in mathematics achievement of students in this country has led to an increased interest in mathematics interventions using Response to Intervention (RTI) as a framework to intervene with students demonstrating inadequate progress in mathematics. Mathematics interventions are "instructional practices and activities designed to enhance the mathematics achievement of students" (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009, p. 1,205). Interventions are not meant to replace core instruction; rather they are designed to supplement core instruction and can be administered by general education teachers, special education teachers, or trained specialists and can be used to differentiate instruction within the classroom (Kovaleski, 2007). Before discussing the implementation of mathematics interventions, it is important to understand the definitions of mathematics proficiency. Below I will describe the evolving definitions and components of mathematical proficiency.

Definitions and Components of Mathematics Proficiency

Defining mathematics proficiency has proved to be difficult due to the multitude of skills and abilities related to mathematics. Unlike reading which is generally comprised of five main areas (phonemic awareness, phonics, fluency, comprehension, and vocabulary; NRP, 2000), mathematics is comprised of skills in multiple areas including a.) number properties and operations, b.) measurement, c.) geometry, d.) data analysis, statistics, and probability, and e.) algebra (National Assessment for Education Progress; NAEP, 2013) and can be applied to a variety of everyday skills ranging from basic skills such as telling time and using money, to more advanced skills such as calculating interest rates or measuring materials to build a house (McCloskey, 2007). The National Center for Education Statistics defines mathematics proficiency as students being able to "apply integrated procedural knowledge and conceptual understanding to problem solve in the five NAEP areas," while those meeting advanced proficiency are able to "apply integrated procedural knowledge and conceptual understanding to complex and nonroutine real-world problem solving in the five NAEP content areas." (NAEP, 2013). The terms conceptual and procedural knowledge still remain unclear (Baroody, 2003; Star, 2005; Zamarian, López-Rolón, Delazer, 2007) and have continued to evolve over the century.

Brownell's Theory of Arithmetic. Brownell (1935) was among the first to define mathematics proficiency. At the time, being proficient in mathematics essentially meant being able to recall mathematics facts (Drill Theory) and teaching involved (a) students imitating the skills and knowledge demonstrated by adults, (b) associating new knowledge with otherwise unrelated stimuli, (c) teaching students that they do not necessarily need to understand the material to make connections between learned stimuli, and (d) strengthening the bonds between new and previously learned stimuli through repeated drill and practice (Brownell, 1935). However, Brownell (1935) expanded the concept of mathematics proficiency by suggesting that for students who are unable to succeed with traditional instruction (drill instruction), should receive instruction that focuses on decomposing the meaning of mathematics problems, which he called Meaning Theory. Brownell (1935) suggested three connected factors that teachers must utilize when instructing the meaning behind problems, or using Meaning Theory.

First, teachers must focus on the complexity of arithmetic learning. The traditional drill theory instructional approach focused on teaching students specific strategies that would allow for them to develop skills to memorize procedural steps to follow when problem solving. However, Brownell (1935) argued that this approach would result in teachers neglecting to teach conceptual principles related to the problem. Instead, teachers should focus on more informal methods of teaching strategies such as student-invented strategies that promote conceptual understanding. Research has indicated that the use of student invented strategies can result in many positive benefits including students making fewer errors, developing number sense, creating more efficient methods to problem solve rather than relying on algorithms, and generally having skills on par to assist them with solving standard algorithms in the area of multi-digit computation (Fuson, 2003; Gravemeijer & van Galen, 2003).

Second, teachers must become mindful of the pace of instruction when teaching. Drill instruction focuses on teaching the student to quickly memorize a problem while instructing on the meaning of the problem focuses on ensuring that the student has a deeper understanding before moving onto the next concept. Grasping the meaning of the problem does not have a specific time restraint, as it is viewed as a much slower process. Teaching until the student demonstrates mastery of that concept can result in a more secure knowledge of the concept and knowledge that is more easily transferred (Baroody, 2003).

Third, when teaching the meaning behind problems, teachers should emphasize relations or connections between similar types of problems. When teaching using the drill approach, instruction should not include teaching how 7 + 2 and 6 + 3 both equal 9. Instead, the drill approach focused on teaching students to quickly produce the answer. When teaching the meaning behind problems, instruction should focus on why the two problems are similar, and should be taught as generalizations (Brownell, 1935). When teaching facts that have similar answers, teachers should identify different facts that produce similar answers, so students can develop generalizations of different problems, which would assist students in identifying relationships between different problems.

Brownell (1935) also summarized an approach to mathematics instruction referred to as Incidental-Learning Theory, in which students explored the world, identified regularities (patterns and relations), and self-invented their own understanding of concepts and procedures. Instruction based on incidental-learning should promote mathematical learning through student exploration and curiosity. However, Brownell

(1935) also identified three constraints towards this method of learning. First, this strategy is slow and time-consuming. Second, this strategy can result in fragmented learning due to its lack of structure. Lastly, teachers often lack the expertise to use this instructional strategy. Brownell's three theories to learning arithmetic have assisted in developing more current views of mathematical development, which are discussed below.

National Assessment for Education Progress. The National Assessment for Educational Progress (NAEP, 2013) established a more recent view of mathematics proficiency, which they defined as "applying integrated procedural knowledge and conceptual understanding to problem solve in the five NAEP areas" including: a.) number properties and operations, b.) measurement, c.) geometry, d.) data analysis, statistics, and probability, and e.) algebra. However, to become proficient in mathematics, NAEP suggested developing mathematical abilities (NAEP, 2003). They describe three mathematical abilities as part of the framework used when teaching mathematics. The three abilities include: conceptual understanding, procedural knowledge, and problem solving. Students who have conceptual knowledge can recognize, label and generate examples of concepts, identify and apply principles, compare and contrast problems and apply learned definitions to problems. Students with procedural knowledge can correctly select and apply appropriate procedures, correctly employ an algorithm and extend or modify a procedure when solving mathematical problems. Lastly, students who demonstrate problem-solving skills will use multiple strategies when solving a problem, apply reason to justify their answer, and combine concepts, procedures and reason to problem solve. The NAEP (2003) mathematical abilities align well with Brownell's (1935) Drill, Meaning, and Incidental Learning Theories. The National Research Council

(NRC, 2001) attempted to expand on these areas by developing a comprehensive definition that took into account other student variables beyond what the NAEP (2003) considered to be mathematical achievement.

National Research Council. Mathematics proficiency, according to the NRC (2001), is comprised of five major strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These five strands are considered to be interdependent and collectively contribute to the development of mathematical proficiency. Teaching only one or two of these strands in isolation cannot develop mathematical proficiency; rather the strands of mathematical proficiency should be viewed as multi-dimensional and can be used to help students be successful (NRC, 2001). Cooperatively, these five strands reflect a body of research representing effective components to math achievement.

Conceptual understanding. Conceptual understanding refers to “an integrated and functional grasp of mathematical ideas” (NRC, 2001, pp. 118). Students who have conceptual understanding demonstrate an ability to comprehend the meaning of a problem and are able to organize information in a manner that allows them to easily recall it and make connections with preexisting knowledge. Students with conceptual understanding can verbally explain the concept, but it is possible that they could develop a conceptual understanding before they have the ability to verbalize it (Geary, 1995).

Students with conceptual understanding can demonstrate this ability using multiple strategies (NRC, 2001). For example, a student with conceptual understanding could utilize several strategies when solving the problem 3 x 6. A student solving this problem could use manipulatives (e.g., blocks, cubes, etc.) or pictures to represent 3 groups with 6 objects in each group or they could use repeated addition strategies by adding the number 3, 6 times. They could also use number sentences (e.g., 3 x 6 =?) to solve the problem. This flexible thinking allows for the student to make connections, as the student would be able to understand that multiple strategies lead to the same answer and thus are connected through a network of related strategies. In addition to understanding how to apply different types of strategies, students with conceptual understanding will also demonstrate an understanding of math principles that can be applied to different problems.

A strong conceptual understanding of mathematics can be beneficial to students' mathematical achievement. Hiebert and colleagues (1986) suggest that understanding mathematical symbols can enhance conceptual principles. That is, symbols can represent a complex concept and students who understand these symbols can solve more complex problems. For example, a student with strong conceptual understanding who understands the division (÷) symbol in the problem 6 ÷ 2, not only understands that the problem is a division problem, but should also understand that this problem represents 6 objects that will be divided into halves. Moreover, students with conceptual understanding may reduce the time of having to learn new facts because they can reduce the number of problems they have to learn using other strategies such as commutativity and associativity (NRC, 2001; Baroody, Wilkins, & Tiilikainen, 2003).

Overall, students who spend more time understanding the conceptual underpinnings of a mathematical concept are likely to make fewer errors when problem solving because they have a deeper understanding of the components of the problem (Zamarian et al., 2007; Rittle-Johnson, Siegler, & Alibali, 2001). However, some conceptual ideas appear much harder for children with mathematical learning disabilities to grasp, such as counting (Geary, Bow-Thomas, & Yao, 1992), estimation skills, and the understanding of place value (Hanich, Jordan, Kaplan, & Dick, 2001). Students with a learning disability in mathematics may also demonstrate difficulties in more basic principles such as finger counting or regrouping (Mazzocco, 2007).

Procedural fluency. Procedural fluency refers to "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (NRC, 2001, p. 121). Students with procedural fluency will understand the rote steps used when solving specific problems, while potentially not understanding the meaning of the problem. Additionally, they will be able to solve basic computation problems without needing to rely on tables, finger counting, or other resources and strategies. Students' accuracy and efficiency in solving mathematical facts can be improved through repeated practice, thus improving their overall procedural fluency (Burns, 2005; Poncy, Skinner, & Jaspers, 2007; Tournaki, 2003). Unlike conceptual understanding, procedural fluency does not directly connect to previously learned material; rather the student learns the rote algorithmic facts or rules of how to solve a specific kind of problem.

Procedural knowledge is comprised of two key components (Hiebert & Lefevre, 1986). The first is the formal language of mathematics often represented by symbols. This formal representation does not refer to the contextual meaning of the symbols. For example, if a student with only procedural knowledge were solving the problem $6 \div 2$, he or she may only understand the necessary steps of division, without understanding the meaning of the problem's stem or answer. In other words, students with this type of skill only have an understanding of the surface features of the problem and lack a deeper understanding of concepts related to the problem (Star, 2005).

The second feature of procedural knowledge refers to the understanding of the steps necessary to solve the problem. This involves the rote memorization of the problems. Students with procedural fluency will be able to solve problems simply because they know the steps necessary for that particular kind of problem. However, their answer may result in errors if they misread or misinterpret the problem due to a lack of conceptual understanding and use of the wrong procedures.

Like conceptual understanding, students' comprehension in procedural fluency has some benefits (Hiebert & Lefevre, 1986). Mostly, the understanding of symbolic representations results in students' ability to understand mathematical visual patterns. Students with procedural fluency will have an understanding of what rules need to be applied to certain problem stems based on the presented symbols. Also, students with procedural fluency tend to have an overall understanding of the rote procedures needed for solving certain types of problems (Fuchs, Fuchs, Schumacher, & Seethaler, 2013). Moreover, students with better developed fluency in basic problems are statistically more likely to perform better on algorithmic computation problems, such as double-digit addition and subtraction problems and problems that involve regrouping (e.g., 57 + 28; 62 – 47; Fuchs, Fuchs, Compton, et al., 2006). Students who learn to solve more advanced algorithms will develop an understanding of the general structures of mathematics (e.g., mathematics is highly organized, filled with patterns, and can be predictable), which can be used to complete routine tasks (NRC, 2001).

Conceptual understanding and procedural fluency appear to be distinct forms of mathematical knowledge, yet they can influence one another. Rittle-Johnson and Alibali (1999) argue that improved conceptual understanding leads to improved procedural fluency. For example, students who understand the meaning of place value are more likely to correctly apply steps for solving place value problems (Cauley, 1988). Rittle-Johnson et al. (1999) also suggest that procedural knowledge, especially in cases where mathematical problems require more complex skills such as multiplying or dividing fractions, may enhance conceptual understanding. Linking the two types of knowledge could result in improved acquisition and application of each type of knowledge (Hiebert & Lefevre, 1986); however, the influence of procedural knowledge on conceptual understanding might only occur under specific circumstances, for example, after repeated exposure of the procedure or if the relationship between the concept and procedure is relatively clear (Rittle-Johnson et al., 1999). Computational proficiency requires both fluency and a solid understanding of core concepts (e.g., commutative, distributive, and associative properties; National Mathematics Advisory Panel, NMAP; 2008)

Strategic competence. Strategic competence refers to "the ability to formulate mathematical problems, represent them, and solve them" (NRC 2001, pp. 124). Strategic competence is similar to problem solving in that it involves open-ended or unstructured discovery learning, and encourages students to invent, share, and streamline their own concrete models and written procedures (Baroody, 2003). Students with strategic competence are able to understand all components of a problem by building a mental representation of the problem, eliminating irrelevant information from it, and using a specific and organized approach to find the solution (NRC, 2001). Novice problem solvers will tend to focus on surface features of the problem, such as the variables in the problem, while more expert problem solvers will focus on the structural relationship within the problem, such as patterns or clues that could be used to get the answer (Bransford, Brown, & Cocking, 2001).

Students with strategic competence demonstrate flexibility in their thinking (NRC, 2001), which is developed through the exposure of nonroutine problems. An example of a routine problem is a typical algorithmic problem like 332 + 589, because common strategies are applied to other similar types of triple-digit addition problems. Routine problems involve two or more steps to solve and require a deeper analysis of the problem (van Garderen & Montague, 2003) and generally require a higher cognitive demand (Smith & Stein, 1998). A nonroutine problem is one that when students first read the problem, they do not know how they are going to solve the problem (NRC, 2001). An example of a nonroutine problem is as follows (Mathematical Processing Instrument, Hegarty & Kozhevnikov, 1999):

> There are 8 animals on a farm. Some of them are hens and some are rabbits. Between them they have 22 legs. How many hens and how many rabbits are on the farm?

A student solving this problem could use multiple strategies to solve it including using visual representations, algebraic equations (e.g., $h + r = 8$ and $2h + 4r = 22$), and guess-and-check strategies. However, a student with strategic competence would demonstrate flexible thinking skills by applying multiple strategies when problem solving (NRC, 2001). Additionally, students develop conceptual understanding when they are engaged in productive struggle and make connections to other related math concepts (Hiebert & Grouws, 2007).

Strategic competence is related to both conceptual understanding and procedural knowledge. One aspect of strategic competence is the ability to organize and recognize patterns within the problem, which relates to conceptual understanding in that students must be able to understand the organizational aspects of the problem (e.g., distance-rate-time problems, interest problems, discount problems; Jitendra, DiPipi, & Perron-Jones, 2002). Additionally, knowledge related to the mathematical structure of the problem can assist in activating relevant patterns or dimensions of the problem (Jitendra et al., 2002). Students will also develop procedural fluency through the repeated exposure to solving problems, and they will learn to adapt new and more efficient procedures (e.g., moving from using representations to using algorithms, NRC, 2001), thus increasing their flexibility skills.

Adaptive reasoning. Adaptive reasoning refers to "the capacity to think logically about the relationships among concepts and situations" (NRC, 2001, pp. 129). Students with adaptive reasoning are able to understand all components of a problem and understand how they derived the answer. They can apply different concepts and procedures to identify solutions and understand that they have arrived at the correct solution based on deductive reasoning. When a student without adaptive reasoning is unsure if they have come to the correct conclusion, they may defer to another individual or an answer key to check their answer, while a student with adaptive reasoning is be able to evaluate their own work by checking to see if their reasoning is correct. This skill extends beyond explaining how the answer was derived, but also demonstrates inductive reasoning related different patterns, analogies, and metaphors (NRC, 2001).

Students are thought to develop reasoning skills when 3 conditions are present

(Alexander, White, & Daugherty, 1997). First, students must have sufficient knowledge base related to the concepts of the problem they are solving. In other words, they must have a strong conceptual understanding related to the types of problems they are attempting to solve. Second, they must be familiar and comfortable with the material, which occurs through repeated exposure. Third, the task must be understandable and motivating. Students must learn to find the material to be relevant and applicable to everyday problems. The ability to make connections between mathematical problems for the classroom and real-world problems, and developing a motivation to continue learning mathematical problems, strongly relates to the importance of productive disposition.

Productive disposition. Productive disposition refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that stay effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131). As instructors teach mathematics to students, they must also teach the importance and relevance of mathematics and make connections to everyday life. Even more recently, there has been a bigger focus on fostering a positive attitude towards mathematics as new initiatives have emerged in the science, technology, engineering and mathematics (STEM) fields. Many of these initiatives are targeting middle school and high school students (DeJarnette, 2012), and therefore, it is critical that educators assist in developing this productive disposition in elementary school.

Students who have developed a productive disposition will demonstrate greater confidence with mathematics and will view mathematics as useful and relevant to everyday life. The concept of productive disposition is related to how students view intelligence. Students who believe that intelligence is a fixed trait tend to develop performance goals and are motived by demonstrating their knowledge to others, but students who believe intelligence is a malleable trait, tend to set learning goals and are motivated to increase their mastery of a skill (Dweck, 1986). Research indicates that students who are not confident in mathematics will avoid taking more rigorous mathematics classes in high school (NRC, 2001; Boaler, 2008). The NMAP (2008) reported that children display greater perseverance with mathematics when they believe that their effort is what makes better at mathematics. Students who develop a productive disposition will see their mathematical ability as malleable and will understand that they are capable of developing skills to become successful at mathematics. Thus, having a productive disposition could lead to students being more willing to attempt a rigorous mathematics course.

The benefits of the interdependent strands. Simultaneously teaching the five strands can result in numerous benefits for students (NRC, 2001). First, conceptual knowledge can foster students' ability to understand different procedures (i.e., procedural knowledge) and thus develop the understanding to identify relationships and patterns within problems and learn to apply concepts differently to get the same answers (i.e., strategic competence). Furthermore, children who demonstrate an understanding of the problem structure are more likely to effectively acquire and use procedures (Anderson, 1983; Hiebert & Wearne, 1996; Rittle-Johnson & Alibali (1999). When students have a developed conceptual understanding of an idea, they can use reason to determine which procedures are appropriate depending on the varying type of problem (Gelman & Meck, 1983). In other words, students who only received procedural instruction will make gains on measures of conceptual understandings, but are not able to adapt procedures to similar, but slightly modified problems. Second, it becomes easier to explain how the answer was derived when a student has learned to understand the concepts and procedures related to the problem. When students learn the concepts behind problems, the transferability is significantly enhanced, thus overall enhancing a students' ability to problem solve (Schoenfeld, 1992). Lastly, when students can clearly justify their answer, they develop confidence in their mathematical ability, and may wish to further pursue more difficult mathematics courses (Dweck, 1986; NRC, 2001). Students who understand mathematical concepts and procedures can see how it becomes relevant to everyday situations such as estimating at the store, planning an event, budgeting, or measuring objects (NRC, 2001). Furthermore, the NMAP (2008) reported that the developmental of conceptual understanding, computational fluency, and problem solving skills must all be included when teaching mathematics to prepare students for more complex skills such as algebra.

Synthesis of Multi-Component Interventions

Combining intervention strategies with multiple proficiency strands has been supported through meta-analytic research. For example, using modeling and drill or a combination of three or more treatments was beneficial for students learning multiplication facts (Codding, Burns, & Lukito, 2011), and using two instructional components (e.g., teaching students with heuristics while using explicit instruction) resulted in a unique contribution than using the techniques separately (Gersten, et al., 2009) in mathematics. Swanson and Hoskyn (1998) found in a meta-analysis that using a combination model of cognitive strategy and direct instruction produced better outcomes for students. However, previous meta-analytic research did not examine mathematical interventions through the lens of mathematics proficiency and there did not seem to be a conceptual rationale for why particular interventions were combined.

Implementing Mathematics Interventions Using Response to Intervention

Improving mathematics proficiency remains a high priority due to low levels of mathematics proficiency across the country as previously discussed. Historically, school settings with high populations of poverty and students of color have students who are at risk for academic failure at disproportionate levels (Gordon & Yowell, 1994; Natriello, McDill, & Pallas, 1990), and students who identify as both students of color and poor, have lower academic self-efficacy (Borman & Overman, 2004). School settings that have a high population of students of color, free and/or reduced price lunch, and/or are located in urban settings, may be challenged with having multiple students within a given classroom demonstrating academic difficulties (Sullivan & Bal, 2013), but are limited with resources to address those needs (Burns, Pulles, Helman, & McCommas, 2016). Many schools are addressing a high proportion of students with academic difficulty by implementing an intervention framework such as response to intervention (RTI), which utilizes a multi-tiered service delivery model designed to provide support to all students within a given setting (Batsche et al., 2005). An RTI framework allows for students to receive interventions in a class-wide, small group, or an individualized setting, while using data-based decisions and implementing evidence-based interventions (Kovaleski, 2007).

Data-based decisions. An RTI model begins with universal screening. Universal screening is designed to identify those who are struggling, or those who may not be struggling, but are at risk to struggle in the near future (Albers & Kettler, 2014). The process involves screening all students (e.g., school wide or class-wide) and using this data to determine if students are demonstrating need in a specific area such as mathematics (Kettler, Glover, Albers, & Freney-Kettler, 2014). Specifically within mathematics interventions, universal screening can be used to identify and evaluate the mathematical curriculum and ensure that students are receiving high quality instruction (Clarke, Doabler, & Nelson, 2014). Effective screening is important because students identified with mathematics difficulties (MD) early on in the school year (e.g., fall) tend to have higher growth trajectories than students identified with MD later in the school year (e.g., spring; Morgan, Farkas, & Wu, 2009).

Gersten and colleagues (2012) identified several screening measures for mathematics including singular aspects of whole numbers such as magnitude comparison, word problem solving involving simple arithmetic and retrieval of basic arithmetic facts. Measures of fact fluency can also be used as a universal screener and can have important relevance to future mathematics performance. Research by Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) found concurrent validity coefficients of fact fluency measures with the Stanford Achievement Test-Tenth Edition (SAT-10; Pearson, 2003) of .55 for first-grade students and .59 for second-grade students. Additional research by Clarke, Gersten, Dimino, & Rolfhus (2012) examined the predictive validity with a fact retrieval measure and the Terra Nova (CTB/McGraw-Hill, 2008) achievement test, and found a correlation coefficient of .50. Fact fluency measures can be used to predict student responsiveness to an intervention (Codding, Shiyko, Russo, Birch, Fanning, & Jaspen, 2007).

Measures of specific components from the scope of the school's mathematics curriculum or from state standards (e.g., Common Core State Standards) can also serve as screeners (Gersten et al., 2012). Examples of these measures are ones developed by Fuchs, Fuchs, & Zumeta (2008) and Clarke and colleagues (Clarke et al., 2012; Clarke et al., 2011). Timed computation measures that are aligned with local learning standards have been used to determine mathematical risk or predicting mathematics failure (Foegen, Jiban, & Deno, 2007).

The monitoring of student progress during intervention is a key component within the RTI model. One frequent form of measuring student progress is through curriculum-based measurement (CBM; Deno, 1985), which is often used as a general outcome measure (GOM) to track overall student improvement. CBM results in data that are psychometrically sound (Christ, Scullin, Tolbize, & Jiban, 2008; Marston, 1989; Tindal & Marston, 1996; Good & Shinn, 1990) and when used to make instructional decisions, can result in improved student learning (Fuchs, Fuchs, Hamlett, & Stecker, 1991; Fuchs, Fuchs, Hamlett, & Whinnery, 1991; Fuchs, Fuchs, & Maxwell, 1988).

Previous studies have utilized a classwide mathematics intervention using universal screening data to determine fluency need (Ardoin, Witt, Connell, & Koenig; 2005; Bentz, Fuchs, Fuchs, Hamlett, & Phillips; 1994). VanDerHeyden, McLaughlin, Algina, and Snyder (2012) screened 50,000 students in a school district located in southeastern United States. After screening, classrooms performing below benchmark received a fact fluency intervention, which resulted in significantly more growth than students in the control group. Fourth grade students demonstrated significant improvements on a multiplication CBM with an effect size of .20 in August, .56 in December, and .75 in May (VanDerHeyden et al., 2012).

Synthesis of Implementing Mathematics Interventions Using Response to Intervention

In 2009, Gersten and colleagues compiled an Institute of Education Science Practice Guide discussing using RTI to assist elementary and middle school students in mathematics. In it they describe tier 1 as the core mathematics instruction that all students in a classroom receive as well as evaluating core instruction. It also includes universal screening procedures and methods to determine eligibility for tier 2 and tier 3 interventions. While many agree that a part of tier 1 includes a strong core (Clarke, Doabler, and Nelson, 2014) and is important as it can reduce learning difficulties when correctly implemented (Doabler, Fien, Nelson-Walker, & Baker, 2012), the use of incorporating interventions at tier 1 are discussed less frequently.

Delivering Effective Mathematics Interventions

RTI models can include classwide, small-group, and individual interventions. An important component of a classwide (tier 1) intervention is the inclusion of high-quality core instruction (Clarke, et al., 2014; Gersten, Chard, et al., 2009; National Mathematics Advisory Panel, 2008). High quality core instruction should include several key features including prioritizing instruction around critical content, preteaching requisite skills, carefully selecting instructional examples, scaffolding instruction, modeling and demonstrating instructional tasks, providing frequent and meaningful practice and review opportunities, using visual representations, and delivering timely feedback (Doabler, et al., 2012). Gersten and colleagues (2009) created several recommended strategies when using RTI with mathematics interventions including screening all students, teaching whole numbers to elementary students, using explicit and systematic instruction,

instruction on solving word problems, and using visual representations. Additionally, Gersten et al. (2009) recommended including 10 minutes in each session to building fluent retrieval of basic arithmetic facts, monitoring progress of students receiving supplemental interventions, and incorporating motivational strategies with tier 2 and tier 3 interventions. Other strategies include student think-alouds and peer-assisted learning (Ketterlin-Geller, Chard & Fien, 2008).

Incorporating the strands of proficiency in intervention instruction. Effective mathematics instruction and intervention should integrate the five strands of proficiency outlined by the NRC (2001) and described above because doing so transfers procedural and conceptual skills to problem solving. When students develop a connected network of ideas and concepts, they can transfer knowledge to novel tasks (Schoenfeld, 1992). Moreover, students who demonstrate automaticity of basic facts are also more efficient with word problem solving tasks (Fuchs et al., 2010) and automaticity with basic number combinations reduces the cognitive load when solving problems as less time is spent on retrieving basic facts (Goldman & Pellegrino, 1987). While certain single-digit multiplication fluency facts are easier to master, facts with digits 6-8 (e.g., 6 x 2 – 8 x 9;) require the most practice before mastery (Nelson, Burns, Kanive, & Ysseldyke, 2013).

The use of visual representations has been identified as an evidenced-based practice for students with mathematical difficulties (Jitendra, Nelson, Pulles, Kiss, & Houseworth, in press) including the use of schema diagrams (Jitendra et al., 1998, 2013; Xin et al., 2005, 2011) and number lines (e.g., Woodward, 2006, Woodward & Brown, 2006). Using visual representations can help develop conceptual understanding as students learn to represent a problem using other methods (NRC, 2011).

Explicit instruction is characterized by several important components including teacher demonstration with guided practice, presentation of examples, student verbalization, and corrective feedback (National Mathematics Advisory Panel, 2008; Zannou, Ketterlin-Geller, & Shivraj, 2014). When teaching mathematics in a class-wide setting, the teacher would begin by modeling a problem using a think aloud technique, where the teacher would verbally model what he or she was thinking while completing each step needed to solve the problem. Then the student would attempt a problem, would receive immediate corrective feedback along each step of the problem, and finally would attempt to solve the problem independently (Burns et al., 2014). Explicit instruction has been identified as an effective strategy with an effect size of *d* = 1.22 (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2011).

Cognitive strategy instruction. Instructional strategies known to help students improve mathematics achievement can include the use of visual representations (Haas 2005; Gersten, Beckmann et al., 2009) and cooperative learning models where students work together towards a common goal and can include peer tutoring (Gersten, Chard et al., 2009). Cognitive strategy instruction (CSI), developed by Montague (1992) incorporates many of these strategies, as it focuses on teaching cognitive and metacognitive process, strategies and mental activities to enhance learning (Montague, Enders, & Dietz, 2011). Metacognition refers to:

> One's knowledge concerning one's own cognitive processes or anything related to them, e.g., the learning-relevant properties of information or data … among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear"

(Flavell, Friedrichs, & Hoyt, 1970, p. 232).

This definition encompasses other related cognitive areas including individuals' declarative knowledge about their cognitive processes and self-regulatory procedures (Schoenfeld, 1992) which is why cognitive strategy instruction incorporates metacognitive and self-regulation strategies as they are intertwined. Metacognitive skills have been correlated with mathematical achievement, in particular with problem solving (Lucangeli & Cornoldi, 1997). Students with academic deficits often do not possess metacognitive skills, and it is therefore imperative that they are taught these skills to become more effective problem solvers (Montague, 2003).

CSI utilizes a seven-step checklist to foster cognitive and meta-cognitive skills. The checklist includes the following steps: 1.) read the problem for understanding; 2.) paraphrase the problem; 3.) visually represent the problem; 4.) estimate the answer; 5.) hypothesize; 6.) compute; and 7.) check their work. Within each of these stages, students use a "say, ask, check" model where they talk through the problem, ask question and check their work, all of which helps enhance their meta-cognitive skills (Montague, 2003).

Word problems add an additional complexity than rote problems in that they require reading. Reading difficulties (RD) refers to students demonstrating a weakness in decoding or recognizing written words, and mathematical difficulties (MD) refers to students performing below the 35^{th} percentile (Jordan, 2007). Students with RD and MD have lower performance than students with only MD even when controlling for factors such as intelligence, SES, ethnicity and gender (Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, Hanich, 2002). While students with MD and RD show difficulties with word problems, students with only MD tend to perform better on mathematics tasks related to language such as story problems and verbal counting but demonstrate difficulties with content related to numerical understanding like estimation and rapid fact retrieval (Hanich, Jordan, Kaplin, & Dick, 2001). Due to these differences, it is thought that MD and RD + MD present different types of MD resulting in different developmental trajectories (Fletcher, 2005). Due to the additional complexities associated with mathematical word problems, using multiply strategies to teach word problems can be an effective strategy. This intervention incorporates multiple strategies recommended by NRC (2001) including conceptual understanding, procedural knowledge, strategic competence, and adaptive reasoning. However, this intervention primarily focuses on developing conceptual understanding, strategic competence, and adaptive reasoning. Productive disposition is not a core component of this intervention, but an additional reward or motivational system can be added to enhance productive disposition. CSI has been shown to be an effective intervention strategy (Swanson, 1999; Swanson & Sachse-Lee, 2000), with a large effect size of $d = 0.74$ by Xin and Jitendra (1999) and $d = .86$ (Lee, 2000).

Timed drill. Timed drill (TD) is an intervention aimed at increasing computational fluency or automaticity and can be a helpful strategy for those students identified as having mathematical difficulties or learning disability (Ashcraft & Christy, 1995; Codding, Burns, & Lukito, 2011; Geary, 1996). A typical TD intervention involves the teacher or interventionist introducing a set number of facts with the corresponding answers (Woodward, 2006). When new facts are introduced it is critical that the teacher or interventionist use a direct instruction approach teaching each fact and having each student say the fact and the respective answer (Stein, Silbert, & Carnine, 1997). After all new facts have been introduced, students are administered a 2-min timed drill worksheet containing a variety of facts including the newly learned targeted facts. Students try to complete as many facts as possible within the 2 min time period. At the conclusion of the drill, facts are corrected either by the teacher or interventionist, or could be read orally. Students make changes to any incorrect answers. To enhance motivation, students can graph the total number of problems read correctly (Miller & Heward, 1992). Many studies have implemented TD in combination with other interventions such as strategy instruction (Tournaki, 2003; Woodward, 2006) and found that students tend to perform better when using other strategies in combination with an effect size of $d = .68$ (Woodward, 2006).

Synthesis of Mathematics Interventions

When teaching mathematics instruction, it is important to include a variety of components including explicit instruction, visual representations, and feedback (Doabler, et al., 2012; Gersten et al., 2009). Many interventions assess growth through the use of M-CBM as a way to determine students' response to a given intervention; however, limited research exists on the number of students reaching proficiency on a given skill, and future research studies could benefit from calculating number of students proficient (Burns, VanDerHeyden, & Jiban, 2006).

Summary and Research Questions

With the focused attention on increasing the number of students reaching mathematical proficiency, it is critical to identify efficient intervention strategies to reduce the number of students making inadequate progress. Since the implementation of

No Child Left Behind in 2001, schools are under more pressure to ensure every child is proficient in mathematics and reading. Implementing a classwide intervention, specifically targeting multiplication facts can be useful for students in 4th and 5th grades, as it is a foundational step in their mathematical instruction.

This study aimed to answer the following research questions:

1. What is the relative effect of a fact fluency intervention and an application intervention on a multiplication fact fluency measure?
2. What is the relative effect of a fact fluency intervention and an application intervention on a multiplication application measure?
3. What is the relative effect of a fact fluency intervention and an application intervention on a division fact fluency measure?
4. What is the relative effect of a fact fluency intervention and an application intervention on a division application measure?
5. To what extent does intervention affect levels of student proficiency in multiplication fact fluency?

CHAPTER 3

METHODS

Participants and Setting

Data collection for this study occurred in the spring of 2015 with approval from the University of Minnesota's Institutional Review Board and approval from the local education agency at which the study took place. This study was conducted in a suburban elementary school located in the upper Midwest with third- and fourth-grade students. Students were working in dyads, and dyads were randomly assigned to one of three intervention conditions for 15 sessions. The principal investigator and graduate student research assistants administered pre- and posttest measures and delivered instruction for the intervention sessions.

Setting. The intervention occurred in an elementary school located in the upper Midwest. The school had approximately 423 students enrolled in kindergarten through 5th grade. Across the entire school, 40.4% students were White (not of Hispanic origin), 37.8% were African American (not of Hispanic origin), 12.8% were Hispanic, 7.3% were Asian, and 1.7% were American Indian/Alaskan Native. Approximately 61.7% of students were eligible for free/reduced price lunch, 10.2% received special education services and 10.6% of students were identified as having limited English proficiency.

In 2014, just one year prior to when the current study took place, 36.5% of students were proficient on the state accountability exam in mathematics, compared to the entire state in which the study occurred, where 61.4% of students were proficient in mathematics. The low number of student proficiency in mathematics at this school was concerning and called into question whether basic multiplication skills could be targeted with students in 3rd and 4th grade who could benefit to improve their mathematics proficiency.

Participants. This study employed an experimental design with 133 students, with 64 students (48.1%) from three third-grade classrooms and 69 (52.9%) from three fourth-grade classrooms. There were 64 students from third grade and 69 students from fourth grade who participated. Students participated in the intervention within their own grade. Both grades reflected diversity across multiple demographic characteristics (see Table 1). A total of 70 (52.6%) students were male. Student ethnic and racial identity was diverse with 91 (68.4%) students identifying as non-White. Forty-two percent of students were identified as African American, 31% as White, 14% Hispanic, 11% Asian, and 2% American Indian. In regards to special education services, 15 (11.3%) students received services. Data on the specific special education category was unavailable; therefore, it is unclear if any students were receiving services under a specific learning disability related to mathematics. The number of students identified as English language learners was relatively equal across conditions.

Table 1

Demographic Data for 3rd and 4th Grade Participants.

	CSI (*n* = 42)		TD (*n* = 45)		Control (*n* = 46)		Total (*n* = 133)	
	n	%	*n*	%	*n*	%	*n*	%
Grade								
3rd	21	50.0	21	46.7	22	47.8	64	48.1
4th	21	50.0	24	53.3	24	52.2	69	52.9
Gender								
Male	26	61.9	20	44.4	24	52.2	70	52.6
Female	16	38.1	25	55.6	22	47.8	63	47.4
Race/Ethnicity								
African Am.	20	47.6	16	35.6	19	41.3	55	41.4
American In.	0	0	1	2.0	1	2.1	2	1.5
Asian	4	9.5	6	13.3	5	10.9	15	11.3
Hispanic	4	9.5	7	15.6	8	17.4	19	14.3
White	14	33.3	15	33.3	13	28.3	42	31.6
Special Ed.								
Yes	3	7.1	5	11.1	7	12.2	15	11.28
No	39	92.9	40	88.9	39	84.8	118	88.72
English learner								
Yes	6	14.3	7	15.6	6	13.0	19	14.3
No	36	85.7	38	84.4	40	87.0	114	85.7

Note. African Am. = African American; American Ind. = American Indian; CSI = Cognitive Strategy Instruction; Special Ed. = Special Education; TD = Timed Drill

Power analysis. Statistical power is typically calculated priori to the study using a power analysis to increase the likelihood that statistically significant findings will result with sufficient power (Hedges & Pigott, 2001). Power analyses are conducted to determine an appropriate sample size to allow for enough power for the smallest effect size to be present in the given sample size given (Hedges & Pigott, 2001). Conducting a power analysis requires several parameters including the estimated effect size, an alpha level, power, degrees of freedom (*df*), number of groups, and number of co-variates. For the current study, an effect size of 0.56 was selected based on a previous classwide mathematics intervention study aimed at increasing computational fluency (VanDerHeyden, McLaughlin, Algina, & Snyder, 2012). The alpha level was set at 0.05 with a power of 0.80, 2 *df*, 3 groups, and 3 co-variates. The power analysis resulted in a sample size of 35 participants needed per condition to detect significance.

Research assistants. The principal investigator (PI) recruited 22 research assistants to assist with pre- and posttest data collection and deliver instructions for the intervention. The research assistants were graduate students in Educational Psychology with minimum of 1 year of collecting data and/or delivering interventions and/or classroom instruction in school settings. Ninety-one percent of the research assistants self-identified as White and 9% percent identified as Asian/Pacific Islander. Additionally, 91% were females and 9% were males. The number of times individual research assistants assisted varied from 1 session to 12 sessions, depending upon their availability.

The PI trained all research assistants prior to assisting with the study by modeling each condition to the interventionists, and having the interventionists practice the intervention with one another. The PI used a checklist that contained all of the important components related to the intervention. All research assistants needed to demonstrate mastery by modeling the intervention strategy to the PI prior to assisting with the study. At the start of the training session, the principal investigator reviewed general instructions that the research assistants would give to the dyads. General procedures included reminding students to quietly transition between rooms, instructing students to find their folder upon entering the room, and to sit quietly with his or her partner.

During the intervention, a minimum of one research assistant was present in each condition room and was responsible for modeling the activity to the entire class prior to the students splitting in pairs. Additionally, the research assistant helped students get started on work with their partner and helped with managing behavior. An additional research assistant was present in each room on minimum of 25% of sessions to assist with fidelity of implementation. Research assistants also administered pre-and posttest measures and were blind to which conditions students were assigned during administrations.

Training Cognitive Strategy Instruction.

Research assistants trained in the Cognitive Strategy Instruction condition first watched the PI lead a practice lesson and then each research assistant was required to lead a practice lesson with the principal investigator. The PI read a multiplication word problem, and then walked through each of the seven-steps on the checklist (e.g., read, paraphrase, visualize, hypothesize, estimate, compute, and check the answer) with the research assistants acting as dyads. Then the PI asked a student (i.e., research assistant) to read the second problem. This time the PI asked leading questions (e.g., what is our first step), to allow students to participate. Lastly, the PI allowed the students (i.e., research assistants) to practice solving the problems on their own. When students finished solving the remaining problems, the PI gathered the students and reviewed the answers, by calling on a student to read the problem, describe how they solved it, and review the answer allowing students to practice using think alouds. All research assistants participating in this would practice leading this lesson prior to leading the intervention. See Appendix J for specific wording of directions.

Training Timed Drill. Research assistants trained in the Timed Drill condition first watched the PI lead a practice lesson and then each research assistant was required to lead a practice lesson with the PI. The PI began by introducing the 5 target facts and saying the specific answers to each target fact (e.g., "5 x 7 = 35. What does 5 x 7 = ? Yes, 5 x 7 = 35.") After each of the 5 facts were introduced, the PI would then read the directions for the 2-minute timed drill (see Appendix I) for specific wording of directions. After the drill, the PI would instruct students (i.e., research assistants) to correct answers with his or her partner using the answer key. The PI would model counting the total problems correct, and then model how to graph the number of the total problems correct. The research assistants then had to demonstrate these same steps to the PI.

Measures

A total of five measures were used in the study. One was used as a screener and four measures were administered to students as pre- and posttest measures. The measures are described below.

Screener. The Measures of Academic Progress (MAP-M; Northwest Evaluation

Association, 2004) winter data were used to partner students into dyads. Data reported by the test publishers indicated acceptable reliability for the MAP-M with students in 3rd and 4th grade. Estimates of internal consistency were .94 for both grades and test-retest reliability from fall to spring was .84 and .88 for 3rd and 4th grade respectively (Northwest Evaluation Association, 2004). School personnel administered the tests in January as part of their regular universal screening process.

Multiplication fact fluency assessment. To assess multiplication fact fluency students were administered a Multiplication Fluency Measure. Each probe contained 70 problems total, with 35 facts randomly order (using a random list generator) on each side of a page. On each probe, the facts one each page were organized by 7 rows with 5 problems in each row. The facts included in this probe were 35 unique facts ranging from 6 x 2 – 6 x 9 and 6 x 12 through 12 x 2 – 12 x 9 and 12 x 12. Facts with the digits of 10 and 11 were excluded, allowing students to focus on more difficult facts. Commutative problems were only included once. For example, 6 x 7 was a target fact, but 7 x 6 was not a target fact. The pre- and posttest measures were 2 unique forms. The inverse fact that came first in numerical order (5 x 8) was included while the inverse (8 x 5) was excluded.

This measure was group administered. Students were given two minutes to complete as many problems as possible. Answers were converted to digits correct per minute (DCPM) metric. Data acquired from single-skill mathematics assessments have established high reliability (Burns, VanDerHeyden, & Jiban, 2006) and are reliable for criterion-referenced decisions regarding that skill (Hintze, Christ, & Keller, 2002). Two raters scored this measure for pre- and posttest on over 25% of all measures in order to assess inter-observer agreement (IOA). The number of items that were rated as correct or incorrect by both raters was divided by the total number of items and multiplied by 100, which resulted in 94% IOA for both multiplication fact fluency pre- and posttest measures. Disagreements were discussed to reach 100% agreement and the data were recoded.

Multiplication word-problem assessment. To measure the application of basic multiplication to word problems, students completed an untimed multiplication word problem-solving measure. Items for the assessment were selected from several standardized assessment subtests categorized as measures of application skills, including (a) Math Concepts and Applications subtests of the Kaufman Test of Educational Achievement, Second Edition (KTEA-II; Kaufman & Kaufman, 2004), (b) Application subtest of the Key Math Revised (Connolly, 2000), (c) Math Reasoning subtest of the Wechsler Individual Achievement Test, Second Edition (WIAT-II; Psychological Corporation, 2001), (d) Practical Applications subtest of the Comprehensive Math Abilities Test (CMAT; Hresko, Schlieve, Herron, & Sherbenau, 2003), (e) Diagnostic Achievement Battery, Third Edition (DAB-III; Newcomer, 2001), and (f) Applied Problems subtest of the Woodcock Johnson Achievement Test, Third Edition (WJ-Ach III; Woodcock, McGrew, & Mather, 2001).

Students were required to apply simple multiplication computation skills to solve the problems, which include multiplication of numbers 2 through 9 and 12. This measure consisted of 19 items. Each item was worth two points. Students received 1 point for using the correct number sentence, which needed to include the correct number operation. For example, if the problem required the student to multiple 4 x 6, the student needed to write "4 x 6= ". Students received an additional half point for the correct answer. They received this half point if they had the correct answer regardless if their work was shown. Lastly, students received an additional half point if the student wrote a label with the numerical answer. They could receive this half point regardless if they had the correct or incorrect answer. If there was not a mathematics sentence (i.e. operation; 4 x 6=), but the answer was correct then 1.50 points were awarded for the correct answer. If the correct answer and label were used, then a total of 2.00 points were awarded.

Internal consistency with the present sample was calculated and resulted in $r = .95$ for the pretest measure and $r = .97$ for the posttest measure. Two raters scored the measure for pre- and posttest on over 25% of all measures, and inter-observer agreement was calculated as described above. The IOA for the word-problem assessment was 97% and 94% respectively on the multiplication application pre- and posttest measures. Disagreements were discussed to reach 100% agreement and the data were recoded.

Division fact fluency assessment. To assess students near transfer of basic multiplication fact fluency, a Division Fact Fluency Assessment was administered to students. Similar to the multiplication fact fluency measure, this timed measure contained 70 facts, with 35 unique facts on each side of the paper, arranged in 7 rows of 5 problems. The facts used were the inverse of the basic multiplication problem. The facts used were the reciprocal of the basic multiplication problem. For example, because 6 x 7 was used in the multiplication probe, 42 ÷ 7 was included in the Division Fact Fluency Assessment. The pre- and posttest measures were two unique forms. This measure was group administered. Students were given 2 minutes to complete as many problems as possible. Answers were converted to a DCPM metric.

Two raters scored this measure for pre- and posttest on over 25% of all measures, and IOA was computed as described above. The IOA for this measure was 94% and 97% respectively for Division Fact Fluency pre- and posttest measures. Disagreements were discussed to reach 100% agreement and the data were recoded.

Division word-problem assessment. To measure the near transfer skill of basic multiplication, an untimed, division word problem assessment was administered to students. This measure was developed by the researcher and intended to mirror the multiplication application measure. Similar to the multiplication application measure, this measure contained 19 problems all of which were basic division facts. The 19 facts were selected at random. The facts were the came from the same pool of multiplication problems used in the multiplication application measure. Similar to the multiplication application measure, each item was worth two points. Students received 1 point for using the correct number sentence, which needed to include the correct number operation. For example, if the problem required the student to divide 60 ÷ 5, the student needed to write "60 ÷ 5 =". Students received an additional half point for the correct answer. They received this half point if they had the correct answer regardless if their work was shown. Lastly, students received an additional half point if the student wrote a label with the numerical answer. They could receive this half point regardless if they had the correct or incorrect answer. If there was not a mathematics sentence (i.e. operation; 60 ÷ 5=), but the answer was correct then 1.50 points were awarded for the correct answer. If the correct answer and label were used, then a total of 2.00 points were awarded.

Internal consistency with the present sample was calculated and resulted in $r = .98$ for the pretest measure and $r = .99$ for the posttest measure. Two raters scored this measure for pre- and posttest on over 25% of all measures in order to assess IOA. The

IOA for the measure was 97% and 94% respectively for division application pre- and posttest measures. Disagreements were discussed to reach 100% agreement, and the data were recoded.

Curriculum-Based Measure of Reading. Curriculum-based measures of reading (CBM-R) were used as a co-variate. Because state assessments of mathematics proficiency often involve reading (e.g., reading word problems or reading multiple choice word answers), reading should be taken in to account when implementing word problem solving interventions. Further, higher levels of reading proficiency have been correlated with higher performance on outcomes on mathematics state proficiency exams (Crawford, Tindal, & Stieber, 2001; Helwig, Rozek-Tedesco, Heath, & Tindal, 1999). Data from the March CBM-R universal screening were used for the study. CBM-R data were collected independently from the principal investigator and the research team. Each classroom teacher administered a 1-minute, grade level reading probe to each student using the Formative Assessment System for Teachers (FAST, Christ et al., 2014). The data consisted of the number of words read correctly in 1 minute. The median internal consistency for 4^{th} and 5^{th} reading passages is .90, ranging from .89 to .91 (Christ et al., 2014).

Procedures

Within the schools, the teachers referred to this study as the mathematics rotation. Pre-tests were administered within 1 week of the start of the intervention for both grades. Graduate research assistants were trained on administering directions for all measures. The posttests were administered the day after the last session for fourth grade, and two school days for third grade. However, the posttest measure was administered 6 school days after the last session for 2 classrooms due to delayed state testing that took precedence.

Data from the winter administration of the MAP-M were used to partner students into dyads. Within each classroom, students were rank ordered from highest to lowest on the basis of students' winter MAP-M RIT score. The class list was then split in half to create dyads. The highest performing student was paired with the student ranked at the top of the lower half of the list, or at the 50^{th} percentile in the class. This method was used to prevent unbalanced mathematics skills within the dyads, such as having two high skilled or two low-skilled partners working with the next student below the 50^{th} percentile. This method was used to prevent unbalanced mathematics skills within the dyads, such as having two high-skilled or two low-skilled partners working together, and has been used in other classwide partner interventions (McMaster, Fuchs, Fuchs, & Compton, 2005). After all partners were made, each pair was randomly assigned to one of the following conditions: Cognitive Strategy Instruction (CSI), Timed Drill (TD), or control. For example, the first pair was assigned to the cognitive strategy instruction condition, the second pair was assigned to the timed drill condition, and the third pair was assigned to the control condition. The condition assignment repeated in the same order until all partners were placed in a condition. Each classroom followed this same partnering activity and condition assignment. Within each grade, one-third of each classroom stayed within their homeroom, while the other two-thirds split into one of the other two classrooms where the other two intervention conditions were occurring. A total of 21 (31.8%) dyads were assigned to the CSI condition, 23 (34.8%) were assigned to the TD condition, and 22 (33.3%) were assigned to the control condition.

The intervention occurred for 15 sessions. The sessions were on consecutive school days for third grade. Fourth grade had 13 consecutive school days and then had a 6 school day recess for spring break. Students resumed the remaining two sessions after break. At the beginning of the mathematics rotation period, three research assistants each entered one classroom. Classroom teachers did not have a direct a role in the intervention. They remained present in the classroom during the intervention and assisted with behavior and transition.

The timed drill condition and the cognitive strategy instruction condition each had five new target problems daily. A total of 35 unique facts were selected as the target facts to teach over the 15 sessions. A random list generator was used to randomly select which of the 35 facts would be taught on the session. After the seventh session, all 35 facts had been taught at least once. The 35 facts were randomly ordered again for sessions 8 through 14. For the 15^{th} session, five more facts were randomly selected from a random list generator. At the end of the 15 sessions, all facts were taught at least twice, with five facts being taught three times.

To assist with a smooth classroom transition and to increase motivation among students, they were instructed that they could earn up to two stickers every day for their positive behavior. They were told they could receive the first sticker if they were able to quietly transition between rooms. They were instructed that they could earn a second sticker if they were working hard with their partner. The research assistant would walk around the room and give one sticker to each student if the student transitioned quietly to the appropriate room prior to the start of the intervention, and another if the student was on-task and working hard during the intervention session. On every fifth session, students received small prizes (i.e., pencils, erasers) if they 8 or more stickers for that given week. On the last sessions, students were allowed to take home their sticker chart. The majority of students would receive 2 stickers each day, but specific data on the exact number of stickers students received each day was not recorded.

Pre- and post-measures. Of the four pre-and post-measures, two measures were timed, and two measures were untimed. All pre- and post-measures were administered over 2 half-hour sessions. The pre-test measures were administered one school day prior to the start of the intervention and posttest measures were administered the day after the last session for fourth grade, and two school days for third grade. However, the posttest measure was administered 6 school days after the last session for 2 third grade classrooms due to delayed state testing that took precedence.

Conditions. This study implemented 3 conditions simultaneously: a control group and 2 types of interventions. Students were randomly assigned to 1 of the 3 conditions. Interventions for 4th grade occurred in a 30-minute block outside of core instruction, while the 3rd grade intervention occurred in a 30-minute block during the 90-minute core mathematics instruction.

Cognitive strategy instruction condition. A cognitive strategy instruction (CSI) intervention was used to teach word problem solving (WPS). CSI combines cognitive processes (e.g., visualization) and meta-cognitive or self-regulation strategies (e.g., self-questioning) to help students to become proficient problem solvers and strategic learners through a specific sequence of steps to analyze problems (Montague & Dietz, 2009). CSI is comprised of highly organized and structured lessons that include the following components: appropriate cues and prompts, guided and distributed practice, cognitive modeling, interaction between teachers and students, immediate and correct feedback on performance, positive reinforcement, overlearning, and mastery (Montague, 2003).

Montague's (1992) 7-step model to solve mathematical word problems is an example of the CSI intervention. This intervention consists of the following steps: (1) reading the problem for understanding; (2) paraphrasing by putting the problem into one's own words; (3) visualizing by drawing a schematic representation; (4) hypothesizing or setting up a plan; (5) estimating or predicting the answer; (6) computing; and (7) checking that the plan and answer are correct. Additionally, the 7-step model includes a self-regulation component where student *say* the instructions, *ask* themselves questions, and *check* their work as they respond.

During this condition, students were given 5 different word problems solve with a partner using this 7-step model. These five problems were the same target problems taught in the timed drill condition. Each student had a checklist in his or her folder with each of the 7-steps, and each of the 7-steps were also listed on the worksheet with the problems. At the beginning of each intervention session, the interventionist reviewed the 7-steps by modeling the first question. After the first question was modeled, the interventionist instructed the students to work with their partners to solve the remaining problems. The interventionist would walk around the room and answer any questions the students might have had. After students finished answering all 5 questions, the interventionist reviewed the answers together with the class.

Timed drill condition. A timed procedural intervention was used to enhance multiplication computation (Burns, Riley-Tillman, & VanDerHeyden, 2012). The aim of this intervention condition was to have students increase their computational fluency of basic multiplication facts. Every day, students would pick up their individual folder. They would begin by completing a practice sheet with five target problems. This quarter sheet paper had a total of 10 problems with 2 rows of problems and 5 problems in each row. The first row of problems contained the 5 target problems. The second row of problems contained the commutative of the targeted problems. For example, if 7 x 9 was a target problem in the top row, 9 x 7 was listed in the second row in random order. If the target problem was multiplying the same number twice (e.g., 8 x 8), the same problem was listed again in the second row (i.e., 8 x 8 would have been in the top and in the bottom row). Students were instructed to practice answering these 10 untimed problems. After answering the target problems, students switched papers with their partners to correct the answers. The answers were located on the back of the quarter sheet. If students had any incorrect answers, they would mark the answer as incorrect and return the sheet back to their partner to correct.

Once all partners were done practicing the target problems, they were given a multiplication probe, this probe was similar to the one administered at pre- and posttest. A new probe was created for each session. The interventionist set a timer for 2 minutes and followed standardized instructions (Shinn, 1989). The interventionist told the students, “When I say begin, do your best to answer these math problems. Start here [interventionist points to first problem on top left corner] and go across the page trying each problem. If you come to a problem you don’t know, put an X on it and move onto the next problem. Ready? Begin.”

After the students finished answering the problems on the timed drill, an answer key was passed out. Students switched papers with their partner. Students then counted the total number of correct problems out of 70 possible problems. Students were then instructed to graph the total number of problems correct on a graph located in their folder. The purpose of graphing was to motivate students to improve their score from the previous session. The use of timed drills has been supported in increasing computational fluency (Miller, & Heward, 1992; Woodward, 2006; VanDerHeyden & Burns, 2005).

Control condition. Students in the control condition did not receive any fact fluency intervention instruction. Students worked in pairs throughout this condition completing worksheets in the content areas of telling time, data analysis, measurement, and money. Worksheets were retrieved from a website called Math Aids (www.math-aids.com). The content of the worksheets would change every three to five sessions. At the start of the intervention, the research assistant would begin by modeling the first problem on the worksheet. Then the assistant would model a problem, but would call on students to help solve the problem together. Lastly, students would work with their partner on the remaining problems and the research assistant would walk around and help if students were struggling. After 20 minutes, the research assistant would get the students' attention and review the answer together as a large group.

Core mathematics instruction. The Principal Investigator interviewed the lead teacher in 3^{rd} and 4^{th} grade for a description of the core curriculum. The third grade lead teacher reported using *Math in Focus* (Cavendish, 2013). *Math in Focus* uses Singapore Math, which emphasizes teaching mathematical concepts using concrete, pictorial, and abstract representations and bar modeling (Cavendish, 2013). The mathematics core block began with a 15-minute anchor lesson. Students then worked in small groups on content from the opening lesson for 20 minutes. Simultaneously, the teacher worked with a small group of students who need more individualized support. The teacher then pulled the students together for another whole group lesson. Following students worked independently on the skill, and completed an exit slip with multiple problems of the content they learned. Students also had the opportunity to come up with their own problems and share their work with their peers.

In addition to core instruction, the third-grade lead teacher also reported doing other mathematics activities during a 30 minute intervention block, 4 days a week. While the content varied across subjects, when mathematics was the content area covered they included reviewed multi-digit addition/subtraction and place value. The teacher also incorporated mathematics in other daily activities including during the morning meeting where she would provide students with the answer and a piece of the problem and have students solve the missing number. The third grade teacher reported that this practice was similar for the other 2 third grade teachers.

The fourth-grade teacher lead reported using the same curriculum and similar strategies for core instruction as the 3rd grade team. She also reported having computer centers for students to play mathematics games related to the concept of the week. Similar to third grade, fourth grade also reported utilizing a 30-minute intervention block 4 days a week to focus on specific skills identified by as areas needing improvement based on strand data from the mathematics section of the MAP-M.

Fidelity of implementation. Fidelity of implementation was observed across multiple settings. First, fidelity of pre-and posttest administration was observed. An additional research assistant was present when another research assistant was providing instructions on administrating the measures. This additional research assistant used a checklist with the directions for each measure, and checked to make sure each step was done correctly. The total number of items on the checklist that was observed was divided by the total number of items and multiplied by 100. The resulting fidelity was 100% for both pre- and posttests administration.

Second, fidelity of implementation was recorded for the interventionists (research assistants) on the first day across all conditions and additional sessions for a total of 25% of sessions to ensure that the intervention instruction in each condition was being delivered with fidelity. An additional research assistant was present on the first session for each condition. This additional research assistant used a checklist to ensure that the interventionist was completing each step when delivering instructions of the intervention. The number of steps correctly implemented was divided by the total number of steps and multiplied by 100 to get a percentage. Results for each condition in 3rd and 4th grade resulted 100% fidelity.

Lastly, because dyads had times where they would work together, independent from the interventionist, fidelity was also recorded for 25% of all sessions across all dyads all conditions in both grades with the same checklist. The number of steps correctly implemented was divided by the total number of steps and multiplied by 100 to get a percentage. In third and fourth grade, fidelity of implementation within the control condition was 100% among dyads. In the timed drill condition, fidelity of implementation was also 100% in both grades. In the CSI condition, fidelity was 95% for third grade and 90% for fourth grade.

Research Design

This study implemented a between subjects design study. Students were partnered based on MAP-M winter data within their homeroom. After dyads were formed, the dyads were randomly assigned to one of three conditions. Prior to the start of the intervention, students were administered all four measures as described above. The following research questions guided the study:

1. What is the relative effect of a fact fluency intervention and an application intervention on a Multiplication Fact Fluency Measure?
2. What is the relative effect of a act fluency intervention and an application intervention on a Multiplication Application Measure?
3. What is the relative effect of a fact fluency intervention and an application intervention on a Division Fact Fluency Measure?
4. What is the relative effect of a fact fluency intervention and an application intervention on a Division Application Measure?
5. To what extent does intervention affect levels of student proficiency in multiplication fact fluency?

Data analysis. The first four research questions compared the effects of the three conditions on all four measures. The posttest score of the four measures were compared with four different analyses of covariance (ANCOVA; 3x1 factorial design) using the posttest score as the dependent variable, the pre-test score, grade and spring CBM reading score as the covariates. There was an increased likelihood for family-wise error given the number of analyses. A corrected alpha level given the four analyses would be .0125 (.05 / 4). Therefore, a conservative alpha of .01 was used to be more certain that family-wise error did not occur. Eta squared was also calculated and was interpreted as .01 as a small effect, .06 as a moderate effect, and .14 as a large effect (Cohen, 1988).

Planned post-hoc analyses using Tukey's procedure were also conducted to determine differences between conditions and a Cohen's (1988) *d* was computed to examine size of the effect. When interpreting the effect size using Cohen's *d, 0*.20 is considered small, 0.50 is considered medium, and 0.80 is considered large (Cohen, 1988).

The final research question was analyzed using a non-parametric, Kruskal-Wallis *H* to compare pre- and posttest proficiency scores between conditions. The Kruskal-Wallis *H* test was selected because it can be used to analyze non-parametric data with more than two independent groups (Kruskal, 1952). To answer this question pre- and posttest scores using digits correct per minute were converted to dichotomous variables of either proficient or non-proficient. Dyads received a 1 if their DCPM score on the Multiplication Fluency Measure was above the cut-off score of 17 or higher for 3^{rd} grade or 29 or higher for 4^{th} grade (VanDerHeyden & Burns, 2008), and students received a 0 if their DCPM score was below this cut-off, respectively. The number of dyads identified as either proficient or non-proficient were compared between pre- and posttest performances.

CHAPTER 4

RESULTS

Prior to answering the research questions, the mean number of sessions attended by dyads within each condition was calculated. Dyads had a mean of 13 sessions across all conditions with no statistical differences between conditions $F = (65, 2)\ 0.60$, $p = .55$). Descriptive statistics for all measures were also reported including skewness and kurtosis for each distribution (see Table 2). The majority of the distributions were normal; however, two measures presented an abnormal distribution. The multiplication fact fluency pretest measure had a positive skew of 2.19 and a kurtosis value of 5.52 and the division fact fluency pretest had a positive skew of 2.34 and a kurtosis value of 7.72. These values suggest that students had a similar low performance on these measures. The pretest scores were used as covariates for the analyses with the posttest scores as the primary dependent variable. Student performance on all posttest measures was normally distributed. Because the pretest data were covariates, two non-normal distributions were acceptable to conduct parametric analyses.

Table 2

Descriptive Statistics for Fact Fluency and Application Measures.

	Pretest				Posttest			
	M	*SD*	Skew	Kurtosis	*M*	*SD*	Skew	Kurtosis
Multiplication Fact Fluency	14.46	9.47	2.19	5.52	19.32	12.69	1.31	1.37
Multiplication Application	21.28	8.01	-0.53	-0.01	24.23	8.60	-0.64	-0.56
Division Fact Fluency	4.80	4.41	2.34	7.72	7.83	5.80	1.14	1.56
Division Application	15.45	8.09	0.06	-0.40	18.49	9.66	-0.11	-0.63

Other assumptions were tested including homogeneity of regression slopes and homogeneity of variance using the Levene's Test. All assumptions were met except for the homogeneity of regression of slopes for the Multiplication Fact Fluency Measure, which resulted in a significance value of $p = .017$ and the homogeneity of variance resulted in a significance for the multiplication fact fluency measure with a significance value of $p = .002$.

A one-way ANOVA was conducted to examine differences in the MAP-M winter administration across conditions, which resulted in a nonsignificant finding F (2, 62) = 1.19, $p = .31$. There were no significant differences between groups on any of the pre-test measures (Multiplication Fact Fluency F = (2, 66) 0.36 $p = .70$; Multiplication Application F = (2, 66) 1.25 $p = .30$; Division Fact Fluency F = (2, 66) 0.69 $p = .51$; and Division Application F = (2, 66) 0.50 $p = .61$).

A Chi-Square was performed to test for significant differences of the demographics across the three conditions. Results indicated that there were no significant differences between the three conditions and the demographic descriptors for gender X^2 (2, N = 133) = 2.66, $p = .64$, race/ethnicity X^2 (2, N = 133) = 3.25, $p = .92$, limited English proficiency status X^2 (2, N = 133) = 0.12, $p = .94$, and special education status X^2 (2, N = 133) = 1.42, $p = .49$.

Attrition for the study was low, with only 4 (3%) of the students not completing the study. In third grade, 1 student assigned to the CSI condition moved on the day of the last intervention session. In fourth grade, 1 student assigned to the CSI condition was administered the measures after receiving parental consent, but then was absent for 13 of 15 sessions and therefore was withdrawn from the study. Additionally, 1 third-grade student assigned to the control condition received special education services during the time of the delivered intervention and therefore did not receive any measures or interventions, and 1 fourth-grade student assigned to the control condition, had a parent declined consent and therefore did not receive any measures or interventions. None of these 4 students' demographic data are represented in the demographic data above. What Works Clearinghouse (2010) states that attrition needs to be equivalent across all conditions to keep attrition from minimizing the size of the effects. In this case, 2 students assigned to the word problem solving condition and 2 students assigned to the control condition were not able to participate or complete the study, thus resulting in relatively equal attrition across conditions.

Relative Effect of Interventions on Multiplication Fact Fluency

The first research question inquired about the relative effect of the two interventions on multiplication fact fluency. Table 3 presents the results of the relative effect of a fact fluency intervention and an application intervention on a Multiplication Fact Fluency Measure. An ANCOVA was used to evaluate intervention effects on a multiplication fact fluency measure with multiplication fact fluency pretest score, grade, and spring CBM-R scores serving as covariates. Results showed that the mean DCPM significantly differed across conditions, F (2, 66) = 32.11, $p < .01$, with observed power of 0.99. Results also indicated a large effect size of $\eta^2 = .52$.

Table 3

Descriptive and Inferential Statistics for the Relative Effect of a Multiplication Intervention on Multiplication Fact Fluency.

	CSI (n = 21)		TD (n = 22)		Control (n = 23)		F	η^2
	M	SD	M	SD	M	SD		
Pre-Int.	13.04	4.75	16.04	11.82	14.18	10.26		
Post-Int.	15.11	6.37	29.58	14.79	12.60	7.11	32.11*	0.52

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; *F* is an analysis of covariance using the pretest, grade, and spring curriculum based measure of reading.

*$p < .01$

Planned post-hoc analyses were conducted using Tukey's procedure to analyze the difference between conditions (see Table 4). Results indicated that students in the TD condition performed significantly better than students in the CSI condition with a mean difference of 14.47 DCPM (p < .01, d = 1.27) and students in the control condition with a mean difference of 16.98 DCPM (p < .01, d = 1.46). There were no significant differences between the CSI and control condition with a respective mean difference of 2.51 (p = .71, d = 0.37).

Table 4

Tukey's Post Hoc Analysis for Differences Between Conditions on Multiplication Fact Fluency.

	CSI			TD			Control		
	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*
CSI	--	--	--	14.47	<.01	1.27	2.51	. 71	-0.37
TD	14.47	<.01	-1.27	--	--	---	16.98	<.01	-1.46
Control	2.51	.71	0.37	16.98	<.01	1.46	--	--	--

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; Mean Difference = Difference in Digits Correct Per Minute

Relative Effect of Interventions on Multiplication Application

Research question 2 sought to answer what is the relative effect of a fact fluency intervention and an application intervention on a Multiplication Application Measure. Table 5 presents the results of the relative effect of a fact fluency intervention and an application intervention on a Multiplication Application Measure. An ANCOVA was used to evaluate intervention effects on a Multiplication Application Measure with Multiplication Application pretest score, grade, and spring CBM reading scores serving as covariates. Results showed that the mean number of problems correct significantly differed across conditions, F (2, 66) = 15.42, $p < .01$, with observed power of 0.99. Results also indicated a large effect size of $\eta^2 = 0.34$.

Table 5

Descriptive and Inferential Statistics for the Relative Effect of a Multiplication Intervention on a Multiplication Application Measure.

	CSI (n = 21)		TD (n = 22)		Control (n = 23)		F	η^2
	M	SD	M	SD	M	SD		
Pre-Int.	20.81	6.97	22.85	7.72	20.07	9.26		
Post-Int.	28.12	5.74	24.31	8.00	24.23	8.60	15.42*	0.34

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; F is an analysis of covariance using the pre-test, grade, and spring curriculum based measure of reading.

*$p < .01$

Planned post-hoc analyses were conducted using Tukey's procedure to analyze the difference between conditions (see Table 6). Results indicated that students in the CSI condition performed significantly better than students in the control condition with a mean difference of 7.69 DCPM ($p < .01$, $d = 0.94$). There were no significant differences between the CSI and TD condition with a respective mean difference of 3.80, ($p = .28$, $d = 0.55$) or between the TD and the control condition with a respective mean difference of 3.89 ($p = .25$, $d = 0.43$). However, the effects were moderate and favored the CSI condition.

Table 6

Tukey's Post Hoc Analysis for Differences Between Conditions on Multiplication Application Measure.

	CSI			TD			Control		
	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*
CSI	--	--	--	3.80	0.28	-0.55	7.69	<.01	-0.94
TD	3.80	0.28	0.55	--	--	---	3.89	0.25	-0.43
Control	7.69	<.01	0.94	3.89	0.25	0.43	--	--	--

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; Mean Difference = Difference in Number of Problems Correct

Relative Effect of Interventions on Division Fact Fluency

Research question 3 sought to answer what is the relative effect of a fact fluency intervention and an application intervention on a Division Fact Fluency measure. Table 7 presents the results of the relative effect of a fact fluency intervention and an application intervention on a Division Fact Fluency measure. An ANCOVA was used to evaluate intervention effects on a Division Fact Fluency measure with division application pretest score, grade, and spring CBM reading scores serving as covariates. Results showed that the mean DCPM significantly differed across conditions, F (2, 66) = 10.61, $p < .01$, with observed power of 0.99. Results also indicated a large effect size of $\eta^2 = 0.26$.

Table 7

Descriptive and Inferential Statistics for the Relative Effect of a Multiplication Intervention on a Division Fact Fluency Measure

	CSI (n = 21)		TD (n = 22)		Control (n = 23)			
	M	SD	M	SD	M	SD	F	η^2
Pre-Int.	4.14	3.72	5.74	5.31	4.45	4.00		
Post-Int.	6.69	4.74	11.01	6.95	5.59	3.81	10.61*	0.26

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; F is an analysis of covariance using the pre-test, grade, and spring curriculum based measure of reading.

*$p < .01$

Planned post-hoc analyses were conducted using Tukey's procedure to analyze the difference between conditions (see Table 8). Results indicated that students in the TD condition performed significantly better than students in the CSI condition with a mean difference of 4.32 DCPM ($p < .05$, $d = 0.73$) and students in the control condition with a mean difference of 5.42 DCPM ($p < .01$, $d = 0.97$). There were no significant differences between students in the CSI and control condition with a mean difference of 1.10 ($p = .78$, $d = .26$).

Table 8

Tukey's Post Hoc Analysis for Differences Between Conditions on Division Fact Fluency.

	CSI			TD			Control		
	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*
CSI	--	--	--	4.32	<.05	0.73	1.10	0.78	-0.26
TD	4.32	<.05	-0.73	--	--	---	5.42	<.01	-0.97
Control	1.10	0.78	0.26	5.42	<.01	0.97	--	--	--

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; Mean Difference = Difference in Digits Correct Per Minute

Relative Effect of the Interventions on Division Application

Research question 4 examined the relative effect of a fact fluency intervention and an application intervention on a Division Application Measure. Table 9 presents the results of the relative effect of a fact fluency intervention and an application intervention on a Division Application Measure. An ANCOVA was used to evaluate intervention effects on a Division Application Measure with Division Application pretest score, grade, and spring CBM reading scores serving as covariates. Results showed that the mean number of problems correct did not significantly differed across conditions, F (2, 66) = 1.81, p = .17, with observed power of 0.36. Results also indicated a moderate effect size of $\eta^2 = 0.06$.

Table 9

Descriptive and Inferential Statistics for the Relative Effect of a Multiplication Intervention on a Division Application Measure.

	CSI (n = 21)		TD (n = 22)		Control (n = 23)			
	M	SD	M	SD	M	SD	F	η^2
Pre- intervention	24.36	5.93	16.93	9.52	14.97	8.38		
Post-intervention	17.94	7.77	20.72	9.88	16.69	10.95	1.81	0.06

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; F is an analysis of covariance using the pre-test, grade, and spring curriculum based measure of reading.

*p < .01

Planned post-hoc analyses were conducted using Tukey's procedure to analyze the difference between conditions (see Table 10). Results indicated no statistically significant differences between conditions. The CSI and TD conditions resulted in a mean difference of -2.79 (p = .61, d = -.31). The CSI and control conditions resulted in a mean difference of 1.24 (p = .91, d = .13). The control and TD conditions resulted in a mean difference of 4.03 (p = .35, d = -.39).

Table 10

Tukey's Post Hoc Analysis for Differences Between Conditions on Division Application Measure.

	CSI			TD			Control		
	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*	Mean Difference	*p*	*d*
CSI	--	--	--	-2.79	0.61	0.31	1.24	0.91	-0.13
TD	-2.79	0.61	-0.31	--	--	---	4.03	0.35	-0.39
Control	1.24	0.91	0.13	4.03	0.35	0.39	--	--	--

Note: CSI = Cognitive Strategy Instruction; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; Mean Difference = Difference in Number of Problems Correct

Effects on Levels of Student Proficiency in Multiplication Fact Fluency

Research question 5 examined to what extent does intervention affect levels of student proficiency in multiplication fact fluency (M-CBM multiplication). Table 7 describes the results of the Kruskal-Wallis *H* analysis for the pre- and posttest comparison of the number of dyads proficient in basic multiplication fluency. At pre-test the mean rank was 33.71, 33.30, and 33.50 for the CSI, TD, and control condition respectively. Results indicated that the number of dyads identified as proficient in basic multiplication fluency were not significantly different at the pre-test X^2 (66, 2) = 0.01, p = .99, but were significantly different at posttest X^2 (66, 2) = 24.76, $p < .01$. There were 3 (14.29%) dyads proficient in the CSI condition at pretest and 4 dyads (19.05%) proficient at posttest. The number of proficient dyads in the TD condition increased from 3 (13.04%) dyads proficient at pretest to (15) dyads 65.23% dyads proficient at posttest. The number of dyads in the control condition decreased from 3 dyads (15.79%) proficient at pretest to 0 dyads (0%) proficient at posttest. The mean rank changed to 45.52 for the TD condition, 30.29 for the CSI condition, and 24.00 for the control condition.

Table 11

The Number of Dyads who Became Proficient in Multiplication Computation Compared to Pre- and Posttest Intervention.

	CSI			TD			Control			X^2
	n	%	MR	*n*	%	MR	*n*	%	MR	
Pre-Int.										
Prof.	3	14.3	33.71	3	13.0	33.30	3	13.6	33.50	0.01
N.P.	18	85.7	N/A	20	87.0	N/A	19	86.4	N/A	
Post-Int.										
Prof.	4	19.0	30.29	15	65.2	45.52	0	0	24.00	24.76*
N.P.	17	81.0	N/A	8	34.8	N/A	22	100	N/A	

* $p < .01$

Note: CSI = Cognitive Strategy Instruction; MR = Mean Rank; TD = Timed Drill; Pre-Int. = Pre-Intervention; Post-Int. = Post-Intervention; Prof. = Proficient; N.P. = Non-Proficient.

CHAPTER 5

DISCUSSION

The purpose of this chapter is to synthesize the findings of this study. The chapter begins by reviewing the purpose of the study, and then reviews the findings of the research questions. The chapter concludes by discussing limitations of the study, implications related to this and previous studies, and future directions of research.

Review of Study Purpose

Students who struggle with basic mathematics skills may demonstrate difficulties as they continue to progress with more advanced mathematical concepts (Hasselbring, et al., 1987). Often teachers are faced with multiple students struggling with a given topic, but have limited resources to intervene (Gersten et al., 2009); however, implementing a classwide intervention can target specific skill deficits across a large number of students (Burns et al., 2016; Poncy, Skinner, & O'Mara, 2006; VanDerHeyden, McLaughlin, Algina, & Snyder, 2012). Classwide interventions often incorporate peer-tutoring in which students work in dyads (Cook, Scruggs, Mastropieri, & Casto, 1985; Fuchs et al., 1997).

The purpose of the current study was to better understand how to increase the number of students identified as being proficient in basic multiplication computation fluency through classwide intervention. Specifically, the aim of the study was to implement two distinct mathematics intervention conditions (timed drill, TD and cognitive strategy instruction, CSI) and one control condition to determine which would result in a larger number of students being identified as proficient in multiplication computation fluency. Additionally, this study investigated the near transfer of learned multiplication facts to students' ability to solve basic division facts and division word problems. The study was conducted with third- and fourth-grade students in the spring of 2014 over 3 weeks for each grade.

Research Question 1: What is the Relative Effect of a Fact Fluency Intervention and an Application Intervention on a Multiplication Fact Fluency Measure?

Results indicated that students in the timed drill (TD) condition performed significantly better than students in the cognitive strategy instruction (CSI) condition with a mean difference of 14.47 digits correct per minute (DCPM; $p < .01$, $d = 1.27$) and students in the control condition with a mean difference of 16.98 DCPM ($p < .01$, $d = 1.46$). There were no significant differences between the CSI and control condition with a respective mean difference of 2.51 ($p = .71$, $d = 0.37$). There was a large effect size between the TD and CSI condition and the TD and control condition indicating that students in the CSI and control conditions performed worse on the measure assessing multiplication computation fluency than students in the TD condition.

These findings support previous research on timed drill interventions and their effectiveness on increasing computational fluency (Ashcraft & Christy, 1995; Codding, Burns, & Lukito, 2011; Geary, 1996) such as a multiplication fact fluency study by Woodward (2006) found that fourth grade students who received a timed drill intervention did better at learning facts than students in the control group, and students who received timed drill in addition to other cognitive strategies (e.g., doubling numbers, using a number line) did better on learning common facts performed even better, with a moderate Cohen's *d* effect size of 0.68. This study suggests that TD can be used as a foundational intervention with other additive components to enhance the intervention.

It is possible that students from the current study in the TD condition performed better because they were exposed to more problems at each session than students in the other conditions. Students in the TD condition had the opportunity to practice each of the five target problems twice because during the practice time, students would answer the target problem twice before starting the drill. Once the drill started, students then had the opportunity to answer all 36 problems up to two times if they were able to get to the back side of the worksheet within 2 minutes as the drill worksheets had each of the 36 problems listed twice (one set of the 36 target problems in random order on the front side of the drill worksheet and the same set of 36 target problems in random order on the back side of the drill worksheet). Therefore, students in the TD sessions had many more opportunities to practice answering the facts than students in the other conditions. Having increased opportunities to practice responding is a recommended strategy when delivering interventions (Burns, VanDerHeyden, & Zaslofsky, 2014).

Research Question 2: What is the Relative Effect of a Fact Fluency Intervention and an Application Intervention on a Multiplication Application Measure?

Results indicated that students in the CSI condition performed significantly better than students in the control condition with a mean difference of 7.69 DCPM ($p < .01$, d = 0.94). There were no significant differences between the CSI and TD condition with a respective mean difference of 3.80, ($p = .28$, d = 0.55) or between the TD and the control condition with a respective mean difference of 3.89 ($p= .25$, d = 0.43). However, both effects were moderate and favored the CSI condition. While the difference in performance between the TD and control condition was not significant, performance by the TD condition was still slightly higher indicating that students who received intervention support on increasing fact fluency, performed better on multiplication word problems than students who were in the control condition.

The current findings support previous research that CSI can increase students' ability to problem solve (Swanson, 1999; Swanson & Sachse-Lee, 2000; Xin & Jitendra, 1999). A study by Montague, Krawec, Enders, and Dietz (2014) used CSI to improve problem solving skills for 1,059 7th grade students using a randomized control trial, and results indicated that over a school year, students receiving the intervention performed better than the comparison group with a moderate Cohen's *d* effect size of .61. Research also synthesized this study (Montague, Krawec, Enders, and Dietz, 2014) and a study by Montague, Enders, & Dietz (2011) on the intervention program called *Solve It!,* which found a large intervention effect size of 0.88.

Overall, results from this research question indicated that students in the CSI condition performed better than students in the TD or control condition, suggesting that students who received an intervention that specifically taught strategies to apply when solving word problems performed better when solving word problems requiring application of multiplication. The lack of significant differences between the TD and control conditions and the TD and CSI conditions could be a result of power. The power analysis suggested that 35 dyads should be present; however, there were only 21, 22, and 23 dyads in the CSI, TD and control conditions, respectively.

Research Question 3: What is the Relative Effect of a Fact Fluency Intervention and an Application Intervention on a Measure?

Research question 3 addressed what is the relative effect of a fact fluency intervention and an application intervention on a division fact fluency measure, which examined the effects of learning multiplication on a near transfer measure of basic division. Results indicated that students in the TD condition performed significantly better than students in the CSI condition with a mean difference of 4.32 DCPM ($p < .05$, $d = 0.73$) and students in the control condition with a mean difference of 5.42 DCPM ($p < .01$, $d = 0.97$). There was no significant difference between students in the CSI and control condition with a mean difference of 1.10 ($p = .78$, $d = 26$). These results indicated students in the TD condition performed significantly better on a near transfer measure of basic division facts than students in the CSI and control condition with a large effect size of .74.

Previous research found a relationship between multiplication and division (Baroody, 2006; Greer, 1992; Jitendra, DiPippi, & Perron-Jones, 2002; Montague & Jitendra, 2006). A study by Cook and Reichard (1996) found students who learned division facts, were able to generalize them on multiplication measures. It is possible that students from the current study in the TD condition improved because they had more opportunities to practice the 36 multiplication target facts that were also of similar fact families the pre- and posttest division measure (e.g., they practice 4 x 9 = 36, so it might have been easier for them to solve 36 ÷ 4).

Research Question 4: What is the Relative Effect of a Fact Fluency Intervention and an Application Intervention on a Division Application Measure?

Research question 4 sought to identify the relative effect of a fact fluency intervention and an application intervention on a division application measure, which examined the effects of learning multiplication on a near transfer measure of division word problem solving. Results indicated no statistically significant differences between conditions. The CSI and TD conditions resulted in a mean difference of -2.79 (p = .61, *d* = -.31), suggesting the CSI condition performed slightly worse on the division application measure than the TD condition. The CSI and control conditions resulted in a mean difference of 1.24 (p = .91, *d* = .13), suggesting no differences between performance on the division application measure between the CSI and control condition. Lastly, the control and TD conditions resulted in a mean difference of 4.03 (p = .35, *d* = .39), suggesting that there were no significant differences between the control and TD condition; however, there was a moderate effect size favoring the TD condition. There were no significant differences found on this near transfer measure and all of the effects were small to moderate. Again this lack of significance could be due to the limited power from the sample size. Furthermore, the observed power for this measure was only .36.

Previous research found that students who learn multiplication transferred it to solve division word problems (Baroody, 2006, Greer, 1992; Jitendra, DiPippi, and Perron-Jones, 2002; Montague & Jitendra, 2006). Thus, the current data were not consistent with previous findings. It is possible that students did not receive sufficient intervention sessions to see the skill transfer to division problems, because previous studies seeing generalizability occurred year round (Montague, Appleton, & Marquard, 1993; VanDerHeyden, McLaughlin, Algina, Synder, 2012).

Research Question 5: To What Extent Does Intervention Affect Levels of Student Proficiency in Multiplication Fact Fluency?

Research question 5 sought to answer the extent intervention affected levels of student proficiency in multiplication fact fluency. At pre-test, there were 3 (14.29%) dyads proficient in the CSI condition and at posttest there were 4 dyads (19.05%)

proficient. The number of proficient dyads in the TD condition increased from 3 (13.04%) dyads proficient at pretest to 15 dyads (65.23%) proficient at posttest. The number of dyads in the control condition decreased from 3 dyads (15.79%) proficient at pretest to 0 dyads (0%) proficient at posttest. The results indicated that students in the CSI condition made minimal improvements and students in the control condition became less proficient. However, students in the TD condition made significant improvements in number of dyads being proficient at posttest, which was a significant result.

The current data support previous studies in that TD interventions increased fact fluency (Ashcraft & Christy, 1995; Codding et al., 2011; Geary, 1996), even when delivered in a classwide fashion with student dyads (VanDerHeyden & Burns, 2005; VanDerHeyden et al., 2012). Previous research also showed increases in levels of student proficiency with a randomized design (VanDerHeyden et al., 2012), but did not examine the percentage of students who were proficient. Previous reading research found that classwide interventions with student dyads led to more students being proficient (Burns et al., 2015; Burns, Pulles, Helman, & McComas, 2016), but this is the first study to examine numbers of students being proficient after classwide interventions for mathematics.

Implications

This study yielded important implications that warrant discussion. Implications for areas of practice and discussed as well as implications related to theory and research.

Potential Implications for Practice

The findings of this study provide several potential implications for practice. With the implementation of federal laws such as No Child Left Behind (2001) and the

Individuals with Disabilities Education Act (2004), teachers are required to implement evidence-based interventions with students, and are often faced with limited resources to implement interventions. Findings from the current study, and from several previous studies (VanDerHeyden & Burns, 2005; VanDerHeyden et al., 2012; VanDerHeyden, Witt, & Gilbertson, 2007) support the use classwide interventions to increase multiplication fluency. Teachers who identify a classwide need through screening for multiplication can use the classwide TD intervention with students working in dyads as a quick, effective, and not resource intensive intervention that can be implemented in less than 30 minutes each day to help students become proficient with basic multiplication facts.

Findings from the current study also support using CSI as a classwide intervention that can be implemented with students working in dyads. While this intervention did not specifically improve multiplication computation fluency, the data indicated that students did better on the word application measure with multiplication problems than students in the TD and control conditions. However, the data were less clear than for the comparison on fact fluency because the TD condition also had a moderate effect as compared to the control condition. Thus, further research with CSI as a classwide intervention is needed before it can be recommended for practice.

The current data also supported the implementation of peer tutoring as an effective practice for both interventions. While previous studies examined the effectiveness of these two interventions (Ashcraft & Christy, 1995; Codding et al., 2011; Geary, 1996; Swanson, 1999; Swanson & Sachse-Lee, 2000; Xin & Jitendra, 1999), this study was the first to compare the two as classwide interventions while incorporating peer-tutoring. Results indicated that students in both intervention conditions made significant improvements in multiplication when working with peers, in which one peer in the dyad was slightly higher according to their performance on a group measure of mathematics performance. Peer tutoring has consistently been shown to be useful for supplemental instruction (Case, Speece, & Molloy, 2003; Fuchs, 2003; Poncy, Skinner, & O'Mara, 2006; VanDerHeyden et al., 2012), but the current study used peer tutoring for both conditions. Therefore, we do not know if peer tutoring was an essential component of the intervention. Future researchers could compare the interventions with and without peer tutoring to determine the importance of working in heterogeneous dyads on the effect of the interventions.

Potential Implications for Theory

Findings from this study yield potentially important implications for theory. The current study used the five recommended strands of proficiency (National Research Council, 2001). The CSI intervention incorporated all five strands of mathematical proficiency, and the TD only incorporated two of the recommended strands of proficiency (procedural knowledge and productive disposition). Students in the TD condition made significant improvements on multiplication computation fluency, even though the intervention only addressed two of the areas of proficiency. However, students showed different improvements based on which strand the intervention more heavily focused. For example, while the CSI intervention did include components of procedural knowledge, it mainly focused on developing students' conceptual understanding, strategic competence, and adaptive reasoning. Students in this condition performed better on the application measures. Similarly, the focus for students in the TD was on procedural fluency, and students in this condition performed better on the computation measure that more closely aligned with this strand. The current data further supports the reasoning that mathematics is multifacted with many complex skills for students to master on a multitude of levels including conceptually, procedurally, and the ability to apply to solve problems (Brownell, 1935; Hiebert & Lefevre, 1986; NAEP, 2013; NRC, 2001; Resnick & Ford, 1981) and that interventions should align with the desired aspect of proficiency. Perhaps future research could combine TD and CSI approaches into one intervention and compare it to the TD and CSI conditions separately while using a measure of global mathematics proficiency, which would allow a more comprehensive evaluation of the role that aspects of proficiency had on overall mathematics skill.

Limitations

There were several limitations to consider in regards to this study. First, although students were randomly assigned to conditions after being rank-ordered, the first dyad within each class was always assigned to the CSI condition, the second dyad was always assigned to the TD condition, and the third dyad was always assigned to the control condition, with the same pattern repeating for the remainder of the students on the list. The assignment order was not randomized, and therefore, the highest performing student from each of the original classrooms (based on MAP-M data) was always in the CSI condition. However, it should be noted that analyses indicated no pre-intervention differences mathematics skills between conditions.

Second, fourth and fifth grade students each had a different cut-score to reach in order to be considered proficient. The cut-score for third grade students was 17 digits correct per minute (DCPM), while the cut-score for fourth grade students was 29 DCPM.

Growth was not calculated, and it is possible that students in both grades started at the same level of DCPM and made the same amount of gains in DCPM, making only the third grade dyad proficient and not the fourth grade dyad. For example, two dyads, one in third grade and one in fourth grade could have started at 12 DCPM, and both increased by 5 DCPM, making the third grade dyad become proficient, while the fourth grade dyad would have remained non-proficient. Related, calculating growth could have been helpful as it could have indicated the amount of growth seen within the 3 week intervention period and between the two intervention conditions. It is possible that the average DCPM score could have increased by only 1 digit for some dyads, and by multiple digits for other dyads. However, measuring growth within mathematics is an important area for future research. Due to the multiple domains within a given skill (e.g., applying multiplication to solve number and operations as well as geometry and algebra) it can be difficult to assess (Clarke, Baker, & Chard, 2008). Also assessing changing from pre-and posttest measures can be problematic because such scores are systematically related to random error, and therefore, subtracting pre-and posttest differences the true score may lead to false conclusions (Cronbach & Furby, 1970). Furthermore, future research on assessing growth and sensitivity on general outcome measures and subscale mastery probes is needed (Christ & Vining, 2006; Foegen, Jiban, & Deno, 2007; Graney, Missall, Martinez, & Bergstrom, 2009).

Third, the intervention session occurrences differed between third and fourth grade students in that the sessions for fourth grade students were delayed to begin due to scheduling difficulties, which resulted in the intervention session schedule to be interrupted by spring break. The intervention had been occurring for 13 days and was interrupted by a 6 day school break. After students returned, the remaining 2 sessions were held, followed by post-testing on the following day. Third-grade post-testing was also delayed as disruptions with the state accountability testing occurred resulting in a 6 school day delay in administering the measures.

Fourth, core mathematics instruction is a key component of an effective RTI framework as it plays an important role is supporting the development of students' mathematical proficiency (Clarke et al., 2014) and can reduce learning difficulties when correctly implemented (Doabler, Fien, Nelson-Walker, & Baker, 2012). The core instruction of the current study provided by the general education teachers was not evaluated, and therefore, it is uncertain if the lack of student proficiency was due to the quality of core mathematics instruction. Additionally, although students received this classwide intervention during the intervention time block, it is possible that some students might have been receiving additional mathematics interventions during other parts of the day.

Directions for Future Research

The findings of this study suggest several directions for future research. First future studies can calculate growth made within intervention conditions to determine if certain interventions provide better growth. Growth is important as it can tell how much improvement a student made over a select time period, and can be used to make decisions as to whether to continue or change the intervention (Deno, 1985).

Second, future studies can look at massed versus distributed practices within the interventions used in this study. Particularly, the TD intervention could result in a better effect size and better growth when implemented using distributed practices because it is a short intervention. Previous dosage studies have documented higher gains for students practicing addition facts two or four times a day versus one time per day (Schutte, Duhon, Solomon, Poncy, Moore, & Story, 2015), which suggests that dosage could be included in future extensions of this research.

Third, future researchers could use a theoretical framework for analyzing multiplication computation intervention sequences to determine the best intervention to use given a student's skill level. For example, mathematics interventions can be analyzed using the learning hierarchy (Haring & Eaton, 1978) to determine what intervention would be appropriate for students in the acquisition stage versus the adaption stage within a given skill, which could be useful in guiding instruction based specifically on students' level. For example, TD might be more appropriate for students in the proficiency phase (accurate but slow performance), but the CSI might be more effective once students are both accurate and fluent and therefore ready to learn application strategies.

Fourth, and related to the previous recommendation, future research could develop a stronger response-to-intervention (RTI) framework specifically related to multiplication and other basic foundational skills (i.e., addition, subtraction, and division). An RTI framework could be implemented across the three tiers with specific intervention strategies to use at each tier, and appropriate measures to guide the decisions made at each tier. Further, additional models could be built using this framework by expanding it to more complex skills such as multi-digit computation and algebra. Implementing these recommendations could allow for TD and CSI classwide interventions to be studied as tier-1 (whole class) or even tier-2 (small-group) interventions and could allow for core instruction and additional school-based intervention efforts to be taken into account.

Conclusion

Proficiency in mathematics across the United States has become an area of concern as performance on state accountability tests continues to demonstrate that less than 50% of students in 4^{th} and 8^{th} grade are proficient in mathematics (NCES, 2013). With such large numbers of students lacking proficiency in mathematics, systematic interventions that target many students need to be utilized to intervene when large numbers of students are demonstrating limited proficiency. Furthermore, current standards require teaches to move through mathematics content at a quick pace in order to instruct on a bevy of content areas, and many students do not have enough time to learn basic foundational concepts (Bauml, 2015; Cobb, McClain, Lamberg, & Dean, 2003; Wills & Sandholtz, 2009). Teachers could supplement core instruction with large numbers of students using a classwide intervention (Poncy et al., 2006; VanDerHeyden et al., 2012) as a way to quickly build student proficiency without extensive resources.

The current study implemented a classwide intervention with 133 third- and fourth-grade students to increase proficiency in multiplication. Results indicated that students in the Timed Drill condition improved on measures of basic multiplication and division fluency, and students in the Cognitive Strategy Instruction condition improved on a measure of multiplication word problem solving. Findings from this study indicate that classwide interventions can be used to help classrooms with large numbers of students struggling with basic multiplication. Future research is needed to determine what interventions should continue for those students not showing progress in tier 1 interventions, but given the promising data found here and the pressing need for increased student mathematics proficiency, the additional research seems warranted.

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Appendices

Appendix A. ID Number: __________

Multiplication Pre-test

12 x 5	9 x 3	7 x 3	7 x 7	12 x 12
9 x 5	6 x 2	9 x 12	12 x 2	6 x 3
7 x 2	6 x 7	7 x 12	8 x 5	7 x 9
6 x 4	8 x 12	8 x 8	8 x 4	8 x 9
12 x 4	6 x 9	7 x 5	9 x 9	9 x 4
12 x 3	6 x 8	6 x 6	6 x 5	8 x 2
7 x 8	9 x 2	6 x 12	7 x 4	8 x 3

DCPM_________ Page 1

Appendix A.

8 x 9	6 x 2	6 x 12	9 x 5	7 x 3
6 x 6	7 x 2	6 x 8	9 x 12	6 x 4
9 x 3	12 x 5	8 x 8	6 x 3	8 x 5
6 x 9	7 x 9	6 x 7	7 x 12	9 x 2
12 x 2	7 x 7	9 x 9	7 x 4	6 x 5
12 x 3	9 x 4	12 x 4	8 x 12	8 x 12
12 x 12	7 x 5	8 x 3	7 x 8	8 x 4

DCPM__________ Page 2

Appendix B. ID Number: __________

Division Pre-test

21÷7=	63÷7=	24÷12=	45÷9=	48÷6=
32÷8=	60÷12=	48÷12=	72÷6=	42÷6=
36÷6=	56÷7=	36÷12=	18÷9=	40÷8=
24÷6=	84÷7=	108÷9=	14÷7=	49÷7=
27÷9=	18÷8=	28÷7=	64÷8=	54÷6=
35÷7=	36÷9=	30÷6=	72÷8=	144÷12=
24÷8=	18÷6=	96÷8=	12÷6=	81÷9=

DCPM_________ Page 1

Appendix B.

36÷12=	18÷8=	48÷6=	48÷12=	56÷7=
24÷12=	21÷7=	36÷9=	108÷9=	18÷6=
24÷8=	64÷8=	42÷6=	63÷7=	81÷9=
30÷6=	144÷12=	35÷7=	72÷8=	14÷7=
45÷9=	72÷6=	24÷6=	28÷7=	49÷7=
36÷6=	32÷8=	96÷8=	12÷6=	60÷12=
54÷6=	27÷9=	84÷7=	40÷8=	18÷9=

DCPM__________ Page 2

Appendix C. ID Number: __________

Multiplication Posttest

6 x 5	8 x 2	6 x 9	7 x 9	7 x 3
8 x 5	6 x 8	6 x 4	6 x 7	9 x 3
9 x 9	7 x 7	12 x 4	7 x 12	6 x 2
7 x 8	8 x 8	6 x 12	9 x 2	8 x 3
12 x 5	8 x 4	9 x 4	9 x 5	12 x 12
6 x 6	7 x 4	8 x 12	12 x 2	12 x 3
8 x 9	6 x 3	7 x 2	9 x 12	7 x 5

DCPM_________ Page 1

Appendix C.

7 x 3	12 x 4	12 x 12	6 x 8	8 x 3
9 x 4	6 x 2	8 x 2	12 x 3	6 x 3
6 x 5	8 x 8	6 x 9	8 x 12	7 x 8
6 x 4	7 x 2	9 x 5	8 x 5	6 x 12
12 x 5	9 x 2	9 x 9	7 x 7	7 x 5
7 x 4	9 x 3	12 x 2	8 x 9	9 x 12
8 x 4	6 x 6	7 x 12	6 x 7	7 x 9

DCPM__________

Page 2

Appendix D. ID Number: __________

Division Posttest

42÷6=	72÷6=	14÷7=	84÷7=	49÷7=
30÷6=	45÷9=	24÷12=	63÷7=	72÷8=
35÷7=	40÷8=	108÷9=	16÷8=	60÷12=
18÷9=	27÷9=	12÷6=	21÷7=	48÷12=
56÷7=	36÷6=	36÷12=	36÷9=	24÷8=
144÷12=	18÷6=	96÷8=	64÷8=	54÷6=
32÷8=	48÷6=	81÷9=	28÷7=	24÷6=

DCPM_________ Page 1

Appendix D.

96÷8=	72÷6=	12÷6=	28÷7=	18÷6=
48÷6=	144÷12=	81÷9=	35÷7=	14÷7=
24÷12=	42÷6=	64÷8=	24÷8=	30÷6=
56÷7=	16÷8=	18÷9=	49÷7=	72÷8=
84÷7=	32÷8=	40÷8=	60÷12=	36÷6=
24÷6=	36÷12=	108÷9=	48÷12=	27÷9=
63÷7=	45÷9=	21÷7=	54÷6=	36÷9=

DCPM__________ Page 2

Appendix E. ID Number: __________

Multiplication Application Measure

1. A man is selling pizza at the fair. At $6 for a slice of pizza how much will 4 slices cost?

2. Juana counted all the boxes of pencils in the supply room. There were 7 full boxes. If each box holds 8 pencils, how many pencils were there all together?

3. A zoo has 8 times as many lions as goats. The zoo has 4 goats. How many lions does the zoo have?

4. A classroom has 12 tables. There are 6 chairs at each table. How many chairs are in the room?

5. There are 9 rows of stamps on a page. Each row has 6 stamps. In all how many stamps are there?

6. Kate has 12 bags. Each bag holds 7 marbles. How many marbles are there in all?

7. Fruit trees are growing in 8 rows. There are 9 trees in each row. How many trees are there in all?

8. There are 12 bundles of fishing poles. There are 9 fishing poles in each bundle. How many fishing poles are there in all?

9. The airplane cabin has 6 rows of 4 seats. How many seats are in the airplane cabin?

10. Diane chooses from 3 kinds of ice cream and 8 toppings. How many combinations of ice cream and toppings are there?

11. If there are 7 days in a week, how many days are in 9 weeks?

12. A parking garage charges $7.00 for each hour. How much will it cost Maria to park for 3 hours?

13. Lee is paid for each piece of furniture he builds. He is paid $8.00 for each table he assembles. This week he assembled 6 tables. What is his pay for the week?

14. A train station has 4 rows of seats. There are 12 seats in each row. How many seats are there in the station?

15. Art drove 8 miles an hour for 5 hours. How far did he drive?

16. Twelve people each have $6. How much money do they have together?

17. Highway mileage varies from car to car. Judy's car gets five miles to a gallon of gas and Bob's car gets 6 miles to a gallon of gas. How many miles can Judy drive on 6 gallons of gas?

18. If you have 9 pencils in each hand, how many pencils do you have all together?

19. A fair has 9 different contests. If each contest awards 4 ribbons, how many ribbons are awarded at the fair?

Appendix F. ID Number: __________

Division Application Measure

1. Tyler is buying bags of candy at the mall. If each bag costs $3, how many bags can Tyler buy with $27?

2. Tim was packing books in a box. He only had 9 boxes. If he had 54 books, how many books could he put equally into each box?

3. Jessica is gathering chicken eggs. She has to put them into piles of 12 eggs. How many piles will she have if she gathers 96 eggs?

4. A classroom has 42 desks with 7 desks in each row. How many rows are there?

5. Cynthia is making party bags. There are 56 toys that she wants to split equally across 7 bags. How many toys should she put in each bag?

6. Mike is planting orange trees. He needs to plant 60 trees. If he plants 5 rows of orange trees, how many orange trees would be in each rows?

7. Donyae is organizing her books on a bookshelf. She has 72 books that she can evenly put on each of 8 shelves. How many books will be on each shelf?

8. Ms. Smith has 6 boxes of pencils. There are a total of 24 pencils. How many pencils are in each box?

9. Sam is baking cookies. He has 21 cups of flour. If each batch of cookies calls for 3 cups of flour, how many batches of cookies can Sam make?

10. There are 72 combinations of ice cream and toppings at Sonny's Ice Cream Shop. If there are 12 different kinds of ice cream, how many different toppings are there?

11. It has been 63 days since Jeff saw his friend Chris. If there are 7 days in a week, how many weeks have gone by since Jeff last saw Chris?

12. Kim spent $45 on movie tickets last month. If each ticket cost $9, how many tickets did she buy?

13. Ray shovels snow for his neighbors. Last month he made $108. If he charges $12 for each sidewalk he shovels, how many sidewalks did he shovel?

14. A plane cabin has 48 seats. If there are 6 rows of seats, how many seats are in each row?

15. Reese is hosting a pizza party for his friends. If he needs 36 slices, and each pizza can be cut into 6 pieces, how many pizzas should he order?

16. Shawn has 49 grapes. He has to divide the grapes evenly among his 7 friends. How many grapes will each friend get?

17. Jon makes $7 for each car he washes. Last summer he made $84 how many cars did he wash?

18. Rebecca has 14 slices of bread. If she eats 2 slices per day, how many days will she be able to eat bread before she runs out?

19. Katie paid $28 for some candy. If the candy cost $4 per pound, how many pounds of candy did Katie buy?

Appendix G.

Pre-Test Procedures

1. **Assent (3 minutes)**
   a. Pass out assent forms to students. Read consent to the students. Allow students to ask questions. Let them know that if they do not want to participate their teacher will assign them another math activity. Students must sign their consent forms.
2. **Explain the general procedures (2 minutes)**
   a. Starting after spring break some students will be switching rooms. *Someone will call your name and tell you which room to move to. When you get to the room, you will get a folder and you will be given directions on how to do the math activity.*
   b. Explain behavior motivation. *During this math activity you have the chance to earn two stickers everyday. You can earn one if you quietly line up and walk to the room you are directed to. You can earn another sticker at the end if you have been working hard with your partner during the math activity.*
3. **Administer measures (remaining time)**
   a. Multiplication fact fluency
      i. *When I say begin, start from the top of the page. Work across the page from left to right. If you come to a problem you don't know, you may skip it. When I say stop, circle the last problem you finished. Then continue to answer the rest of the questions. Do you have any questions? Ready, begin!*
   b. Division Fact Fluency
      i. *When I say begin, start from the top of the page. Work across the page from left to right. If you come to a problem you don't know, you may skip it. When I say stop, circle the last problem you finished. Then continue to answer the rest of the questions. Do you have any questions? Ready, begin!*
   c. Multiplication Word Problems
      i. *I will read to you one problem at a time. Try your best to answer each question. Be sure to label your answers.* If students ask, you can give them an example such as, "If the problem is asking about how many dogs, your answer would include '5 dogs' as the label." Take about 30 seconds per question. It is ok if students are still working on problems along the way.
   d. Division Word Problems
      i. *I will read to you one problem at a time. Try your best to answer each question. Be sure to label your answers.* If students ask, you can give them an example such as, "If the problem is asking about how many dogs, your answer would include '5 dogs' as the label." Take about 30 seconds per question. It is ok if students are still working on problems along the way.

Appendix H.

Posttest Procedures

1. **Thank students for their hard work (2 minutes)**
    a. *Thank you so much for your hard work these last couple of weeks. Your hard will helped you get better at math. You are also helping us better understand how students learn math.*
    b. *The last thing that we will do is take a postest to see how much you learned. These are the exact same as what you did at the beginning of the year.*
2. **Administer measures (remaining time)**
    a. Multiplication Fact Fluency
        i. *When I say begin, start from the top of the page. Work across the page from left to right. If you come to a problem you don't know, you may skip it. When I say stop, circle the last problem you finished. Do you have any questions? Ready, begin!*
        ii. **After students have stopped then tell the students:** *I will give you a few more minutes to try answering as many problems as you can for two minutes. Do not go back and answer any questions from where you stopped. Do you have any questions? Ready, begin!*
    b. Division Fact Fluency
        i. *When I say begin, start from the top of the page. Work across the page from left to right. If you come to a problem you don't know, you may skip it. When I say stop, circle the last problem you finished. Do you have any questions? Ready, begin!*
        ii. **After students have stopped then tell the students:** *I will give you a few more minutes to try answering as many problems as you can for two minutes. Do not go back and answer any questions from where you stopped. Do you have any questions? Ready, begin!*
    c. Multiplication Word Problems
        i. *I will read to you one problem at a time. Try your best to answer each question. Be sure to label your answers.* If students ask, you can give them an example such as, "If the problem is asking about how many dogs, your answer would include '5 dogs' as the label." Take about 30 seconds per question. It is ok if students are still working on problems along the way.
    d. Division Word Problems
        i. *I will read to you one problem at a time. Try your best to answer each question. Be sure to label your answers.* If students ask, you can give them an example such as, "If the problem is asking about how many dogs, your answer would include '5 dogs' as the label." Take about 30 seconds per question. It is ok if students are still working on problems along the way.

Appendix I.

General Daily Procedures

1. **Greet students (~11:35 am) and get them ready to switch rooms (~11:37am)**
    a. *Today you will be breaking up into groups again to work on different math activities. When I call your name you will quietly line up at the door. I will then tell you what room to go to.*
    b. Remind students to bring a pencil. *Students moving to Ms. X's room, please quietly line up. Students moving to Ms. Y's room, please quietly line up.*
    c. Remind students that they get one sticker for **quietly lining up and quietly walking** to the room they are supposed to be in.

2. **Check that the right students came into the correct room and partner them. (~11:37 am - 11:42 am)**
    a. Stand outside room with clipboard and list of who is supposed to be in your room.
    b. Once students have entered the room remind them to sit next to their partner.

3. **Model the activity for the students (more so on first couple of days; ~11:42 am-11:47 am).**
    a. Model the activity for the student. Refer to checklist for each intervention on clipboard.
    b. Students can get their own folders.

4. **Walk around and monitor student progress. (~11:47 am - 12:05pm)**
    a. After modeling, students can start the intervention with their partner. At this point, you can walk around and give stickers to students who walked in quietly. The sticker chart is in the back right packet.

5. **Wrap-up**
    a. When students finish, have them put their folder back in the container where they got their folder. You can call students up by their homeroom, and dismiss them to their homeroom
    b. On Fridays all students will get a pencil or an eraser.

Appendix J.

Lesson Script: Cognitive Strategy Instruction

State the purpose of the lesson: (~1 min)

- ☐ *Today we are going to be working on solving word problems. You will work with your partner to solve problems. First, I will model how to solve the problems using a special checklist, and then and then you will solve the problems with your partner using the checklist*. Ask for a student to practice being your partner.

Solve problem using checklist (~5-7 min)

- ☐ Read the problem. *Now I will ask myself if I understand the problem. If I don't I will re-read it again*. Model reading the problem again.
- ☐ Paraphrase. *Next I can put the problem in my own words. I will explain to my partner what the problem was asking. When I finish solving the problem I need to add a label to my answer, so I can underline what my label for the answer will be*. Model paraphrasing problem with student partner and underlying the word that will be the label.
- ☐ Visualize. *Now that I understand the problem, I will draw a picture of the problem*. Show examples of different visualizations such as arrays or objects/groups.
- ☐ Hypothesize. *Next I will hypothesize, or make a guess about what numbers need to be multiplied*. State your hypothesis.
- ☐ Estimate. *I will estimate, or guess what the answer will be. After I solve the problem, I can look at my estimate to see if I was close. Remember an estimate is just a guess, and it does not have to be the exact answer*. Model estimating.
- ☐ Compute. *Now I can go ahead and solve. I also need to add a label*. Model how you would solve the multiplication problem, and how to identify the label. You can demonstrate other strategies such as doubling or skip counting.
- ☐ Check. *Lastly, I will check my answer. First I will see if it is close to my estimate. Then I can check my answer using division*. Model how to use division to check answer.

Independent Student Work (~15 min)

- ☐ Let students work in dyads. Walk around and assist students as needed. Help students use checklist.

Review answers (~3-5 min)

- ☐ *Now it is time for us to review the answers to the problems we solve*. Call on students to provide answers.
- ☐ Model how to solve each problem using the checklist.

Appendix K.

Cognitive Strategy Instruction Student Instructions

Work with your partner on practicing solving five word problems using the following steps.

Read ***(for understanding)***

- ☐ **Say:** Read the problem. If I don't understand, read it again.
- ☐ **Ask:** Have I read and understood the problem?
- ☐ **Check:** For understanding as I solve the problem.

Paraphrase (your own words)

- ☐ **Say:** Underline the important information. Put the problem in my own words.
- ☐ **Ask:** Have I underlined the important information? What is the question? What am I looking for?
- ☐ **Check:** That the information goes with the question.

Visualize ***(a picture or diagram)***

- ☐ **Say:** Make a drawing or a diagram. Show the relationships among the problem parts.
- ☐ **Ask:** Does the picture fit the problem? Did I show the relationships?
- ☐ **Check:** The picture against the problem information.

Hypothesize (a plan to solve the problem)

- ☐ **Say:** Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).
- ☐ **Ask:** If I…, what will I get? If I …, then what do I need to do next? How many steps are needed?
- ☐ **Check:** That the plan makes sense.

Estimate (predict the answer)

- ☐ **Say:** Round the numbers, do the problem in my head, and write the estimate.
- ☐ **Ask:** Did I round up and down? Did I write the estimate?
- ☐ **Check:** That I used the important information.

Compute (do the arithmetic)

- ☐ **Say:** Do the operations in the right order.
- ☐ **Ask:** How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
- ☐ **Check:** That all the operations were done in the right places.

Check (make sure everything is right)

- ☐ **Say:** Check the plan to make sure it is right. Check the computation.
- ☐ **Ask:** Have I checked every step? Have the checked the computation? Is my answer right?
- ☐ **Check:** That everything is right. If not, go back. Ask for help if I need it.

Appendix L.

Lesson Script: Timed Drill Intervention

State the purpose of the lesson: (~1 min)

- ☐ *Today we are going to be working on practicing multiplication facts. First you will practice a few problems. Then you will do a 2 minute multiplication activity. You will graph the number you got correct, and each day you will try to beat the number of problems you got correct.*

Directions for Practice Problems (~5-7 min)

- ☐ *First, you will begin by answering 10 practice problems. When you finish, you can switch papers with your partner. The answers are on the back of the practice problems. If any answers are wrong, cross them out for your partner, and have your partner re-do them.* Remind students not to cheat – they are not getting a grade on this.

Timed Drill (~10 min)

- ☐ Set the timer for two minutes.
- ☐ *When I say begin, do your best to answer these math problems. Start here* (point to problem in upper left hand corner) *and go across the page* (demonstrate by moving finger across the page) *trying each problem. If you come to a problem you don't know, put an X on it and move onto the next problem. Begin* (start timer).
- ☐ At the end of the 2 minutes say, *Stop*.
- ☐ Pass out the answer key. Have students check their work with partner, and have them count the total number of correct problems.

Graph Total Problems Correct (~5 min)

- ☐ *Pull out the graph in your folder. Along the bottom of the chart (left-to-right), is the X-axis, which lists the number of sessions.* **Point to x-axis.** *Along the top is the y-axis, which lists the number of problems correct.* **Point to y-axis**. *Today is session 1. Find the 1 along the X-axis. Let's say I got 10 problems correct, I would then color-in up to the number 10.* **Model on document camera.**
- ☐ *After you have graphed the total number of problems correct, you can go back and answer any problems you got incorrect.*

Extra Notes

- Make sure the students put the answer key of the practice problems in the back of their folder so they cannot see the answers during the timed drill.
- Make sure students are graphing the total number of problems they got correct. You can have them write the number of problems correct on top of each bar. This might help them try to beat their previous score.

Appendix M.

Timed Drill Student Directions

Practice

- ☐ Work on the 10 practice target problems by yourself.
- ☐ Review problems with answer key with your partner.
- ☐ If they are any incorrect answers, have your partner cross out the incorrect answer and write the correct answer.
- ☐ Cover practice problems.

Timed Drill

- ☐ Start the problems by yourself when the teacher says begin.
- ☐ Stop when the timer stops.
- ☐ Use the answer key to check answers. If there are any incorrect answers, have your partner cross out the incorrect answer and you write the correct answer.
- ☐ Count the total number of problems correct.
- ☐ Graph the total problems correct.