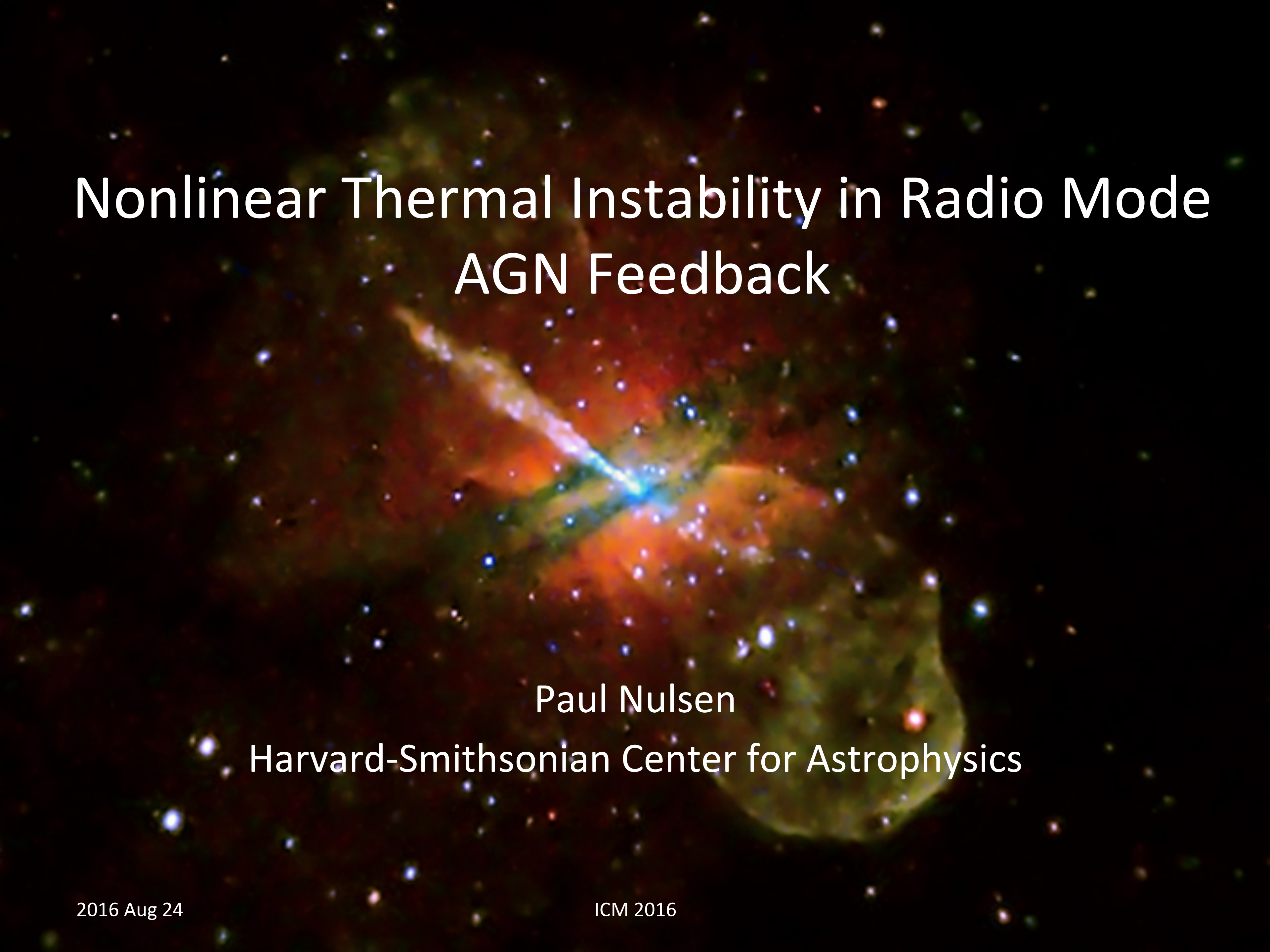
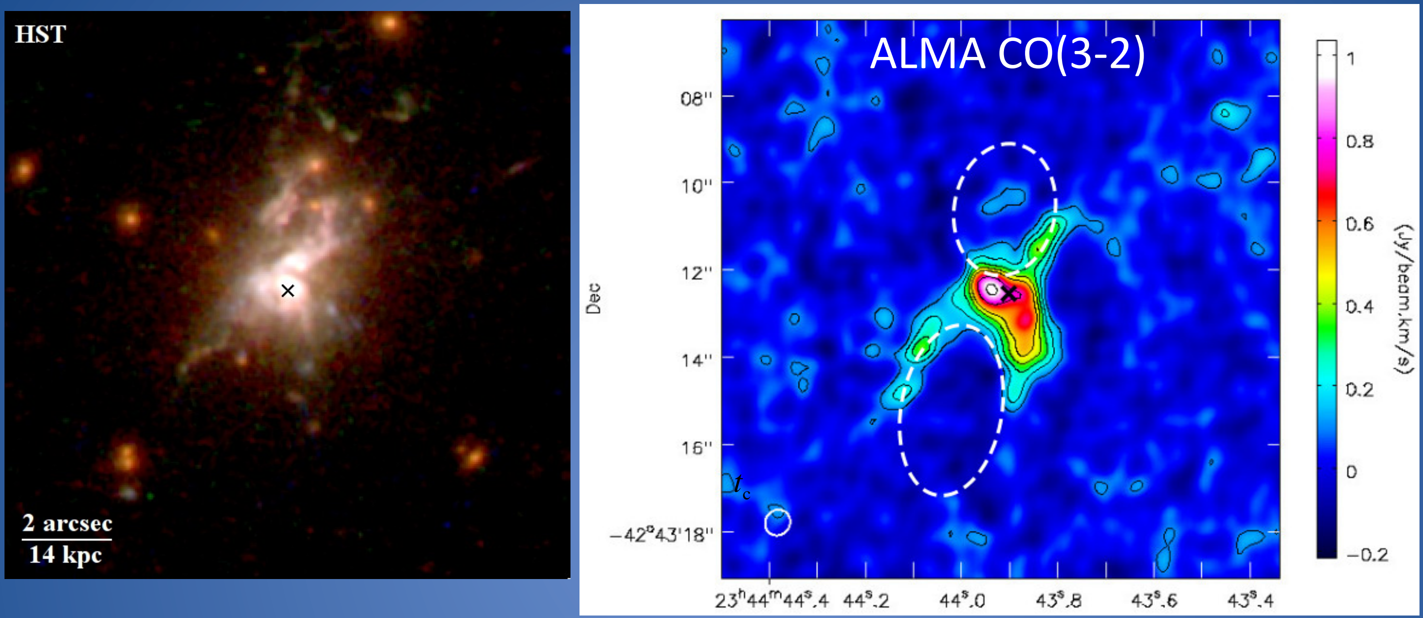


Nonlinear Thermal Instability in Radio Mode AGN Feedback

Paul Nulsen

Harvard-Smithsonian Center for Astrophysics

Thermally Unstable Cooling



Phoenix cluster (Russell+ in prep)

Thermally unstable cooling in hot atmospheres is a critical element of the “radio mode” feedback cycle

Linear growth of thermal instability is weak, unless the cooling time, $t_c < t_{ff}$, free-fall time

Linear Thermal Instability

Linear perturbations of a plane parallel atmosphere, in which heating balances cooling, are discussed in McCourt+ (2012)

When the cooling (heating) time is long compare to the dynamical time, the perturbations are primarily sound waves or internal gravity waves

Thermal instability develops from the latter (slower) modes, which are characterized by the Brunt-Väisälä (angular) frequency,

$$\omega_{\rm BV}^2 = \frac{g}{\gamma}\frac{d\ln\Sigma}{dR} = \frac{1}{\gamma}\frac{v_{\rm K}^2}{R^2}\frac{d\ln\Sigma}{d\ln R}$$

In the Boussinesq approximation (no sound), with a net cooling rate of Θ = D – Q per unit volume, the dispersion relation is

$$\omega^2 + i\omega\frac{(\gamma-1)\mu m_{\rm H}}{\gamma k\rho_{\rm e}}\left(\frac{\partial\Theta_{\rm e}}{\partial T}\right)_p - \omega_{\rm BV}^2\sin^2\theta = 0$$

=> Linear perturbations are always unstable for $\left(\partial\Theta_{\rm e}/\partial T\right)_p < 0$

Saturation of Linear Thermal Instability

McCourt+ (2012) found perturbations saturate at finite amplitude, unless the cooling time is shorter than about the free-fall time, $t_c < t_{ff}$

Why? Nonlinear effects tend to damp oscillations, causing perturbations to saturate

For a cooling function scaling as T^α and a heating rate per unit volume that is independent of the density, $\left[(\gamma-1)\mu m_{\mathrm{H}}/(\gamma k\rho)\right]\left(\partial\Theta_{\mathrm{e}}/\partial T\right)_p = -(2-\alpha)/(\gamma t_{\mathrm{c}})$

The discriminant of the dispersion relation is then $\dfrac{4}{\gamma}\dfrac{v_{\mathrm{K}}^2}{R^2}\dfrac{d\ln\Sigma}{d\ln R}\sin^2\theta - \left(\dfrac{2-\alpha}{\gamma t_{\mathrm{c}}}\right)^2$

This positive, making the perturbations oscillatory, unless the cooling time is shorter than a multiple (order unity) of R / v_K – essentially t_{ff}

For unidirectional perturbations, slowing the gas allows it more time to cool, tending to promote unstable cooling

Condition for Nonlinearity

How does linearity fail? When the Reynolds stress is comparable to the (linear) acceleration, i.e. $|\mathbf{v} \bullet \nabla \mathbf{v}| \geq |\partial \mathbf{v} / \partial t|$

For a "blob" of size r, oscillating radially, this happens for $v \geq \omega_{\mathrm{BV}} r,$ or an oscillation amplitude of $A \geq r$

Nulsen (1997): for gravity wave modes, when the shear is almost vertical, i.e. $|\sin\theta| \approx 1$, a mode is shear unstable for $A \geq r$

A mode with almost horizontal shear becomes Rayleigh-Taylor unstable for $|A \sin\theta| \geq r$

Intermediate cases can be unstable to both at different phases

If $d \ln \Sigma / \mathrm{dR} \approx 1 / R,$ these conditions can be summarized as $\dfrac{\delta\rho}{\rho_{\mathrm{e}}} > \dfrac{r}{R}$

This is more readily met than the condition $\dfrac{\delta\rho}{\rho_{\mathrm{e}}} > 1$ of Balbus & Soker (1989)

McCourt+ (2012) used 1% perturbations, whereas Sharma+ (2012) used 30%

Nonlinear Unstable Cooling

Cowie+ 1980; Nulsen 1986

An overdense gas "blob" of radius r and density ρ has terminal speed $v_{\mathrm{t}} \approx v_{\mathrm{K}} \sqrt{\frac{r\delta\rho}{R\rho_{\mathrm{e}}}}$

Nonlinearity requires $\frac{\delta\rho}{\rho_{\mathrm{e}}} > \frac{r}{R}$, while for $\frac{\delta\rho}{\rho_{\mathrm{e}}} > \frac{R}{r}$, the blob would free-fall

The energy (entropy) equation is $\rho T \frac{dS}{dt} = Q - D,$ where Q is the heating rate and D the cooling rate per unit voume

For a blob falling at its terminal speed, if it remains in local pressure equilibrium, its entropy is related to the radius by $\frac{1}{\gamma - 1} p_{\mathrm{e}} v_{\mathrm{t}} \frac{d \ln \Sigma}{dR} = D - Q$

If the cooling function ~ T^{α} and Q is independent of the density, this gives

$$\frac{d \ln\left(\Sigma / \Sigma_{\mathrm{e}}\right)}{d \ln \mathrm{R}} = \frac{t_{\mathrm{ff}}}{t_{\mathrm{c,e}}} \frac{\left(\rho / \rho_{\mathrm{e}}\right)^{2-\alpha} - 1}{\sqrt{2(\delta\rho / \rho_{\mathrm{e}})(r / R)}} - \frac{d \ln \Sigma_{\mathrm{e}}}{d \ln R}$$

The entropy of an unstably cooling blob declines inward faster than the ambient entropy, requiring the first term on the right to exceed the second

Requires small blobs (low v_{t}) or a short cooling time ($\delta\rho > \rho_{\mathrm{e}}$) – no oscillations

Uplift Creates Nonlinear Perturbations

To be lifted in the wake of a rising lobe, the terminal speed of a gas cloud must be smaller than the flow speed in the updraft, v_u

This sets a limit on the *excess* column density of the cloud, $r\delta\rho < R\rho_e \frac{v_u^2}{v_K^2}$

The density perturbation is determined determined by the entropy of the lifted gas

$$\frac{\delta\rho}{\rho_e} = \left(\frac{\Sigma_e}{\Sigma}\right)^{1/\gamma} - 1$$

This can readily be of order unity

In that case, the limit on the column density requires $\frac{\delta\rho}{\rho_e} > \frac{r}{R}$, so that the perturbation is nonlinear from the outset

Cf. Li & Bryan (2014)

Conclusions

- Linear thermal instability is ineffective in cluster cores, unless $t_c < t_{ff}$
- A perturbation is nonlinear if $\frac{\delta\rho}{\rho_e} > \frac{r}{R}$
- Low values of t_c / t_{ff} favor nonlinear thermal instability – but do not cause a sharp turn on
- Uplift in the wake of a radio lobe naturally creates nonlinear perturbations that can be thermally unstable