An Investigation into Three Core Practices in a Standards-Based Elementary Mathematics Methods Class: The Case of Six Preservice Teachers

A Dissertation
SUBMITTED TO THE FACULTY OF
UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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June, 2016

Acknowledgements

First, I would like to thank the Almighty God for His abundant grace, which has sustained me throughout my educational endeavors, my wife Mrs. Nellie Tackie, our children Theophilus Nii Ardey Tackie and Laurenda Ellen Tackie, my mother, Madam Agnes Annan, my siblings, and the entire Kwakyi family for their prayers, support and motivation throughout my graduate program. I cannot forget my dear sister and friend, Linda Boatemaa, whose words of encouragement and advice inspired me to pursue graduate school in the United States.

Secondly, I would like to thank my advisors, Dr. Kathleen Cramer and Dr. Lesa M. Covington Clarkson for their guidance and support throughout my studies, especially during the research and dissertation writing process. In addition, I would like to thank you for the funding opportunities you offered me.

Thirdly, I would like to express my sincere appreciation to my doctoral committee members, Prof. David O'Brien, Dr. Julie Brown, and Dr. Erin Baldinger. I really appreciate the feedback, support, and guidance each of you provided me during the proposal meeting to my dissertation defense.

My next thank you goes to Dr. Tamara Moore, Prof. Gillian Roehrig, faculty, staff, and colleagues at the STEM Education Center for the support you offered me during my studies at the University of Minnesota. To Dr. Tamara Moore and Prof. Gillian Roehrig, thank you for the EngrTEAMS funding opportunities you provided me. A special thank you also goes to 3M for their Fellowship Award, which has enabled me to fund my doctoral degree education. I cannot forget Dr. Sue Staats for her immense support and love demonstrated to me since I met her.

To my District Pastor, Pastor John Kudolf Ansah and wife Mrs. Florence Ansah, my former District Pastor, Pastor Mbanyane Socrates Mhango and wife Mrs. Phyllis Mhango, and the entire members of the Minnesota District of the Church of Pentecost, I would like to thank you for your prayers, encouragement and support. To the church members in Maplewood Assembly, I want to express my profound gratitude for all that you have done for me.

I would also like to thank my brothers, Forster Ntow, Alex K. Tsikudo Adovor, Hayford Manu, David Kimori and all my colleagues who have supported me in diverse ways since I came to the University of Minnesota. To everyone, I say God richly bless you and your families.

Dedication

This dissertation is dedicated to my wife, Mrs. Nellie Tackie and our children, Theophilus Nii Ardey Tackie and Laurenda Ellen Tackie for their love, support, and prayers. I have been away from my family for nearly six years, but Nellie and our children have stood tall by my side to see me graduate successfully. I say, God richly bless you.

Abstract

Over the past several years, the field of teacher education has faced the perennial problem of identifying productive ways to prepare teachers to meet the realistic work of teaching. As a result, the field is still undergoing several transitions in teacher preparation programs. There has been a shift from an era where teacher educators and researchers focused on detailing the knowledge base teachers needed for teaching toward teaching practices that entail knowledge and enactment (McDonald, Kazemi, & Kavanagh, 2013). Preparing preservice teachers to teach using the concept of "core practices" (Ball & Forzani, 2009) is an attempt to centralize preservice teachers' learning to teach directly the practical work of teaching. Although the concept of core practices for teacher preparation seems promising, not much is known about how preservice teachers learn to enact these practices as they engage in lesson planning and teaching.

The purpose of the study was to describe the learning goals that are set by preservice teachers, the learning tasks they design to help students meet the learning goals, and their implementation of Stein et al's., (2008) five practices of orchestrating whole-class discussion around the learning tasks. The researcher also explored the factors that influence preservice teachers as they learn to enact these practices. A multiple-case design (Yin, 2014) with six preservice teachers was used in this study. The preservice teachers were enrolled in a standards-based elementary mathematics methods class. Data were analyzed using an inductive-deductive method.

The results revealed that in designing learning tasks, preservice teachers used their understandings from the mathematics methods class to design low- and high-level

cognitive demand tasks that engaged students in using multiple modes of representations in inquiry-based learning settings to support students' learning. Although preservice teachers setting of learning goals were influenced by their understandings from the mathematics methods class, they were highly influenced by their practicing schools' mathematics curricula and their cooperating teachers. The preservice teachers had varying experiences and levels of success in implementing the five practices for orchestrating whole-class discussion discussed in Stein et al. (2008).

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Chapter 1

Introduction

Over the past several years, the field of teacher education has faced the perennial problem of identifying productive ways to prepare teachers to meet the realistic work of teaching. As a result, the field is still undergoing several transitions in teacher preparation programs. There has been a shift from an era where teacher educators and researchers focused on detailing the knowledge base teachers needed for teaching toward teaching practices that entail knowledge and enactment (McDonald et al., 2013). The underlying assumption for this shift is that teacher educators need to seek ways to better prepare pre-service teachers to learn how to apply their knowledge to the practical work of teaching (Ball & Forzani, 2009). Preparing preservice teachers to teach using the concepts of "core practices" or "high-leverage practices" (Ball & Forzani, 2009) is an attempt to centralize preservice teachers' learning to relate directly on the practical work of teaching rather than focusing on theoretical foundations that may not help pre-service teachers acquire the skills and knowledge necessary for the practical work of classroom teaching (Forzani, 2014). By their very nature, core practices seek to detail the routine teaching activities that compel teachers to make on-the-fly decisions during teaching.

This transition to teaching practices has led some researchers in this area to organize the work of teacher education in terms of core practices of K-12 teaching with the view that the adoption of this core practices approach by teacher education would improve educational success for all students (McDonald et al., 2013). As a result of seeking to improve students' learning, efforts to design teacher education programs around core practices have focused on three important ideas about teaching and teacher education: (a) that instruction should be aimed at learning goals that are underpinned by the expectation to help all students learn high-level thinking, reasoning and problem-solving skills; (b) that, teaching will help students learn content for these goals is contingent on the kind of classroom interaction that occurs during teaching; and (c) that, this kind of teaching requires making the subject-matter of instruction an important component of the goals and activities that have been outlined in the curriculum (Forzani, 2014).

In the context of mathematics education, these ideas imply that teacher preparation programs designed around core practices would seek to prepare teachers who will be able to set mathematical learning goals to identify the mathematical knowledge students should be able to demonstrate at the end of an instructional period. In addition to this, preservice teachers should be prepared to set learning tasks that promote students' reasoning and problem solving to address the learning goals. In order to set learning tasks that will address learning goals, teachers would have to identify the mathematical knowledge that is espoused by the learning goals in order to design specific tasks to demonstrate the required mathematical knowledge.

This means that setting learning goals and designing learning task to meet those goals are two core practices. Finally, preservice teachers should also be prepared to orchestrate effective discussions around learning tasks to help all students meet the intended learning goals. The learning goals therefore suggest the tasks to use which in turn suggest the kind of classroom discussions teachers should orchestrate to structure teaching and learning to achieve a desired learning goal. From this perspective, the three core practices, namely, setting learning goals, designing tasks, and orchestrating classroom discussions seem to be interrelated.

While the fundamental ideas for designing teacher education around core practice can afford teacher educators the opportunity to focus on the pertinent issues preservice teachers may encounter (Grossman, Hammerness, & McDonald, 2009a), if the 'natural' synergies that exist among core practices are not explored, it is likely that preservice teachers will learn several core practices, and yet they will not be able to effectively connect those practices in order for them to teach effectively. Because teaching requires that teachers make several pedagogical, curricula, and classroom management decisions concurrently, it is important that researchers in the core practice movement delve deeper into finding the connections that exist among core practices.

In this study, three core practices are investigated concurrently: setting mathematical learning goals; designing learning tasks that align with the stated goals; and orchestrating discussions around the learning tasks to achieve those goals. The rationale for this study is to describe the learning goals that are set by preservice teachers, the learning tasks they design to help students meet the learning goals, and their practices during whole class discussions, as well as to uncover factors that influence preservice teachers as they learn to enact these practices. The choice of these three core practices was informed partly by the existing literature which found that these core practices are crucial in teaching, and partly by the fact that these practices are the focus of the methods class that is the context of the study. In the following section, a brief review of relevant literature on standards-based learning environment and the mathematics proficiency required for students, and preparing preservice teachers for teaching using core practices are examined.

Standards-Based Learning Environment and Mathematical Proficiency Needed

Since the release of *An Agenda for Action* by the National Council of Teachers of Mathematics (NCTM) in 1980, there have been several efforts to reform mathematics education. These efforts have contributed to the development of mathematical standards initiated by the NCTM which suggest how mathematics should be taught in our schools (Dossey, Halvorssen, & McCrone, 2012). In the advent of mathematical standards from the NCTM, provisions were made to guide the kinds of learning environments that are expected in mathematics classrooms. Consequently, the NCTM (2000) provided six guidelines as a framework for standards-based teaching and learning environment which require the following: (a) teachers need to have high expectations for all students; (b) there is coherence and structure in mathematics curriculum to be learned; (c) teachers need to know and build on their students' prior knowledge; (d) students should be actively engaged in the teaching and learning process to construct new knowledge themselves; (e) teachers must use meaningful assessment techniques to support important mathematics content and at the same time elicit feedback for teachers and students; and (f) technology should be integrated meaningfully to enhance students' learning of the mathematics. This framework is expected to guide teachers in making effective instructional decisions to create learning environments where all students can learn mathematics meaningfully. Ensuring that students learn the required mathematics and attain the desired proficiency led to the National Research Council report, *Adding it Up: Helping Children Learn Mathematics,* which outlines the strands of mathematical knowledge students should acquire for mathematics proficiency.

In this report, Kilpatrick, Swafford, and Findell, (2001) proposed that to be mathematically proficient, students must have: (a) a conceptual understanding which entails the comprehension of mathematical concepts, operations, and relations; (b) procedural fluency which demands that students should be able to carry out mathematical procedures flexibly, accurately, efficiently, and appropriately; (c) strategic competence which entails students' ability to formulate, represent, and solve mathematics problems; (d) adaptive reasoning—the capacity for logical thought, reflection, explanation, and justification; and (e) a productive disposition which stipulates that students develop "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficiency" (p.116). These strands of mathematical proficiency are interrelated. Therefore, in the process of teaching, teachers are expected to keep track of their learning goals, design appropriate tasks, and orchestrate class discussions in a way that will assist all students in attaining the desired goals set by the teacher.

With this view of mathematics instruction where students are expected to actively engage in mathematics lessons, the role of the teacher has shifted from lecturing mathematics and being an "arbiter of mathematical correctness" (Stein, Engle, Smith, & Hughes, 2008, p.315) to an orchestrator of instructional environments where students actively grapple with engaging tasks to construct mathematical knowledge (NCTM, 2000). As a result of this, teachers should be able to structure instructional environments that will enable students to co-construct mathematical knowledge with teachers (Bransford, Brown, & Cocking, 2000). In order to achieve this goal, it is important that future are prepared to enact core practices involved in the work of teaching so that they will be able to make good decisions to support students' learning.

Preparing Preservice Teachers for Teaching Using the Core Practice Approach

In the era of mathematics standards where teachers are expected to engage students meaningfully to acquire mathematics proficiency, we cannot leave teacher preparation to chance because teachers' performance affect what students learn in the classroom, which in turn, affect any attempt to improve teaching and learning. According to Darling-Hammond (1995), the success of educational reforms partly depends on teachers. She argues that standards place learning at the "center of the teaching enterprise, articulate a strong knowledge base, and acknowledge that good teaching is contingent upon considerations of students, goals, and contexts, and must be conducted in a reciprocal exchange between teachers and students" (p. 20).

Along this line of thinking, and considering that there exist some relationships among teachers' sense of efficacy, the patterns of teacher-student interactions, and students' achievements (Kilpatick, et al., 2001), it is imperative that teacher education builds a strong capacity of teaching force that will operationalize reform-based teaching practices to improve mathematics teaching and learning. This will mean that teachers who are prepared to teach should be equipped with the necessary skills and knowledge to enable them carry out their practical work in the classroom. If preservice teachers are trained to understand and enact the core practices in teaching, which is the central task of their work in the classroom, it is likely that they will be able to help students learn meaningfully. In this regard, teacher preparation should be carefully organized to effectively prepare future teachers.

Organizing teacher preparation program around core practices would mean that teacher educators will identify specific core practices and help preservice teachers learn to enact those practices before they begin their independent teaching. What one considers a core practice, therefore, may depend on how one perceives teaching, and what one may consider as a central teaching activity.

Focusing teacher preparation programs on core practices affords teacher educators not only the opportunity to address the complexity of teaching, but also, it enables teacher educators to respond to pertinent issues preservice teachers may encounter when they begin their independent teaching (Grossman, Hammerness, & McDonald, 2009a). From this perspective, the core practice approach to teacher education may help bridge the gap between research and practice. This is because as preservice teachers learn the theoretical underpinnings of these core practices and learn to enact them through “approximations” of practice (Grossman, Compton, Igra, & Williamson, 2009), preservice teachers will be able to transfer this practical knowledge to the field when they are asked to teach. Currently, there is a call by the NCTM to bridge the gap between research and practice by ensuring that teachers enact core practices that are central to teaching in order to help all students learn meaningfully (NCTM, 2014).

The NCTM, through *Principles to Action: Ensuring Mathematics Success for All,* refers to core practices as “Mathematics Teaching Practices” (NCTM, 2014), and argues that these teaching practices “represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9). The Mathematics Teaching Practices (MTPs) proposed by NCTM include: (a) establishing mathematical goals to focus learning, (b) implementing tasks that promote reasoning and problem solving, (c) using and connecting mathematical representation, (d) facilitating meaningful mathematical discourse, (e) posing purposeful questions, (f) building procedural fluency from conceptual understanding, (g) supporting productive struggle in learning mathematics, (h) and eliciting and using evidence of student thinking. In the elementary mathematics method class which is the context of this study, the class was designed around these eight teaching practices with a special focus on establishing mathematical goals, implementing tasks that promote reasoning, and facilitating meaningful mathematics discourse. In the following section, these three core practices are examined.

Using mathematical goals to focus learning. Effective teaching of mathematics requires that teachers establish learning goals that will articulate the mathematics students will learn and how students will demonstrate their understanding in relation to given state standards. These learning goals should not be merely restating the standards. Rather, preservice teachers should be able to clearly identify specific mathematical concepts, skills, facts, and methods students are expected to understand at the end of a given lesson or series of lessons (William, 2011). Setting explicit learning goals and explaining them to students can possibly guide both teachers and students in the teaching and learning process. While teachers may use the learning goals to guide their instructional decisions, students may use the learning goals as a way of assessing their learning.

These learning goals may be determined for broader units, or for daily lessons. When broader goals are derived from given state standards, teachers may have to obtain sub-goals or specific learning objectives for individual lessons, which when achieved, could lead to achieving the broader goals. For instance, in teaching and learning of fractions, if the overarching goal at the end of a unit is to ensure that students will be able to add two unlike fractions (fractions with different denominators), then specific learning goals that will lead to this bigger goal need to be considered. For example, while the long-term learning goal may be for students to add fractions symbolically, a sub-goal may initially be set for students to add fractions concretely using a model like fraction circles to give students experience with direct modeling before making connections to symbols. Appropriate sub-goals may include representations students need to use to ensure conceptual understanding of the long-term goal.

The challenge is that these goals and sub-goals may not be explicitly stated for teachers. In some cases, teachers may rely on textbooks and other curricular materials for learning goals for selected state standards. The question one may ask is, if teachers resort to textbooks for learning goals, how can they ensure that these learning goals will be suitable to the needs of all the students in their class? Inasmuch as curricula materials may be good resource materials for preservice teachers to obtain learning goals, it may be important for preservice teachers to be able to set their learning goals that will meet the needs of their students, and at the same time satisfy the needs of the state standards being addressed.

Although establishing clear and explicit learning goals can possibly set a good foundation for effective mathematics teaching and learning, how teachers formulate these learning goals is not clearly known. The assumption has been that when teachers are given textbooks and standards, together with other teaching logistics, then teaching may follow naturally. However, this assumption may be far from the reality of teaching. Developing explicit learning goals is a complex task which demands teachers to consider several factors (including the teaching context, differing backgrounds of students, the scope of mathematical knowledge to be learned, different representations that can be used to scaffold learning, and how the knowledge learned could be applied to other settings) before setting their learning goals. For preservice teachers who are learning to teach mathematics, not much is known about the kinds of resources the preservice teachers may draw from to set mathematical learning goals that will meet the requirements of state standards. In thinking about helping preservice teachers learn to enact core practices, it is important for us to know the resources preservice teachers may draw from to set mathematical learning goals. In addition to this, it is also important to know how preservice teachers use these resources within their teaching contexts to set learning goals for specific lessons

Designing tasks that promote reasoning and problem solving. In addition to setting explicit mathematics learning goals, the NCTM (2014) also believes that "effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies." (p.10). The implication is that teachers should use appropriate learning tasks that will align with the intended goals in order to serve the desired purposes (Stein et al., 2008). Because learning tasks serve as the route that leads students to desired learning goals (Clements & Sarama, 2012), if learning tasks do not align with the intended learning goals, it is possible that students will not be challenged to work towards achieving the learning goals.

According to Stein, Grover, and Henningsen (1996), a mathematical task is "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea." (p. 460). From this definition, one could consider all activities aimed at developing a mathematical concept, skill, fact, procedure, etc. as mathematical tasks. Consequently, based on the intended learning goals of a lesson, or series of lesson, tasks could vary in complexity, demands, or extensiveness (Stein et al., 1996). These tasks should engage students actively in the learning process and also challenge them to be thinkers and problem solvers. The tasks should also enable students with differing backgrounds to use varied strategies in solving them. In order to achieve this, teachers would have to develop realistic tasks that will be meaningful to students (Cross et al., 2012).

Stein, Grover, and Henningsen (1996) identify two main levels of tasks: tasks with low-level cognitive demands and tasks with high-level cognitive demands. Low-level cognitive demand tasks call for memorization of mathematical facts, or the use of mathematical formulas, algorithms, or procedures without making connections to concepts, understandings, or meaning. High-level cognitive tasks, on the other hand, require students to use mathematical formulas, algorithms, or procedures with making connections to concepts, understanding, or meaning. When solving high-level cognitive demand task, may use their prior knowledge as a basis for making necessary mathematical connections to solve the given task. Another dimension of the high-level cognitive demand task (doing mathematics) requires students to engage in cognitive activity "including complex mathematical thinking and reasoning activities such as making and testing conjectures, framing problems, looking for patterns, and so on." (Stein et al., 1996, p. 466).

Teachers can use different tasks to serve different purposes and so it is important that teachers consider their students' backgrounds and their learning goals in order to design tasks that will help students achieve the learning goal. Deciding on which learning tasks to use in a lesson is very important because the task students engage in account for what they learn (Doyle, 1988). Research studies (e.g., Boaler & Staples, 2008) indicate that when tasks demand high-level students' thinking and reasoning, students do better than when low-level tasks are used. Stein et al., (1996) however, caution that, because of the way teachers may set up tasks in the classroom, or the way students will interpret the tasks, may change the cognitive demands of tasks. In view of this, teachers will have to ensure that tasks are set up and implemented to maintain their cognitive demands.

Being able to use tasks to engage all students would require teachers to design engaging tasks to meet the needs of all learners. Where possible, teachers can also adopt and adapt tasks from textbooks and other resources to support students' learning. In the context of preservice teachers enrolled in mathematics methods course, not much is known about the kinds of resources and understandings they draw from to design mathematical learning tasks to meet specific learning goals.

Facilitating mathematical discussion. Discussions are important component of mathematics instruction (Boerst, Sleep, & Ball, 2011). In the era of reform-based teaching in which students are actively engaged in the teaching and learning process, the role of the mathematics teachers is shifting from being mathematical knowledge dispenser to someone who create conducive learning environment for students to grapple with mathematics and co-construct mathematical knowledge with the teacher. Students are encouraged to solve tasks using diverse approaches and then asked to share their solutions. In some cases, the class engages in whole-class discussions to assess students' solution strategies. As students and teachers engage in whole-class discussions, teachers are expected to facilitate the discussions meaningfully by using students' generated ideas, examine multiple solutions, compare different representations, and terms as a platform for mathematical sense making to support students' mathematical learning (Boerst et al., 2011).

Holding effective class discussions require teachers to have mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). Teachers should also be able to "size up mathematical ideas flexibly, frame strategic questions, and keep an eye on core mathematical points" (Boerst et al., 2011, p. 2845). It is therefore expected that teachers should design learning tasks that will elicit the kind of discussions necessary to achieve the intended goal. In this case, when teachers fail to use *strategic* tasks, it is likely that the class may not be able to engage in meaningful discussions to support students' learning. When students are not actively engaged in whole-class discussions to construct mathematical knowledge, teachers may resort to "show and tell" (Ball, 2001), without engaging students in the learning process. Using *strategic* tasks means that teachers will carefully use tasks that will enable them drive the class into the expected discussions. When *strategic* tasks are used, there is the likelihood that teachers can elicit students' responses and engage the class in successful discussion to achieve the desired goal. Along this line of thinking, it is important that the learning tasks are carefully designed not only to align with the learning goals, but also facilitate class discussions.

Facilitating class discussion may not be an easy task for all teachers. Most teachers find it difficult to orchestrate meaningful mathematical discourse (Stein et al., 2008). While experienced teachers may be able to hold class discussion effectively, novice teachers may not be able to do so. Consequently, Stein et al., (2008) argue that novices and inexperienced teachers need a set of practices that can prepare them facilitate mathematics discussions effectively and help them learn to become expects discussion facilitators with time. The authors proposed five practices for orchestrating meaningful mathematics discourse:

(1) Anticipating likely students' responses to cognitively demanding mathematics tasks.

(2) Monitoring students' responses to the tasks.

(3) Selecting particular students to present their mathematical responses for discussion.

(4) Purposefully sequencing the student responses that will be displayed.

(5) Helping the class make mathematical connections between different students' responses and between students' responses and key ideas.

These practices, according to Stein et al., (2008), can serve as a starting point for novices and inexperienced teachers to learn how to facilitate effective whole-class discussions.

Purpose of the Study

From the preceding sections, what counts as a core practice and the rationale for organizing teacher education around core practices have been outlined. In its very nature, teaching is a complex activity that naturally entails interactions among various core practices. For example, when leading a class discussion, teachers need to elicit students' responses and use them to facilitate learning. Teachers are expected to better understand their students, have a clear idea of learning goals, and figure out the productive ways to orchestrate class discussions so that students can be assisted to reach desired learning goals.

Currently, studies on core practices have looked at core practices in isolation. However, the complex nature of teacher requires teachers to apply several practices concurrently. Therefore, instead of looking at core practices in isolation, it may be helpful to study different core practices simultaneously to see how teachers enact them during lesson planning and teaching. In addition, not much is known about the resources preservice teachers use from their learning communities in the process learning to enact core practices. As the field of teacher education lends more toward core practices, we need to develop a firm foundation to guide teacher education programs. As noted by Forzani, (2014), "if the current work is to gain momentum and significantly influence teacher education practice, some more permanent infrastructure for teaching and learning core practice-based methods will need to be built, and the common view that teaching cannot be specified or taught repudiated" (p. 366). Building on this notion, and to address the gap in the literature on core practices, which is lack of knowledge about the resources preservice teachers use in learning to enact core practices, this study seeks to describe how preservice teachers construct learning goals, learning tasks, and implement discussion practices, as well as to unpack the factors that influence preservice teachers as they engage in these practices. The following research questions guided the study:

1. In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from given state standards to support elementary school students' mathematics learning? What are the characteristics of the learning goals?
2. In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks?
3. In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?

Better understanding of the resource preservice teachers use in a learning community and how those resources are used to plan and enact core practices in the context of an elementary mathematics methods course may provide new knowledge to guide teacher preparation programs which are centered around core practices. These three core practices are very important because teaching and learning processes is mainly centered on these core practices. This is because while learning goals set the stage for effective teaching and learning, designing and implementing learning tasks is imperative for teachers to achieve their learning goals. Carefully planned learning tasks can be used to connect mathematical representations to support students' learning. Also, good tasks can be used to help build procedural fluency from conceptual understanding. Finally, at the heart of teaching and learning mathematics is orchestrating meaningful class discussions. During class discussions, teachers can elicit and use evidence of students' thinking, support productive struggle in learning mathematics, pose purposeful questions, and make mathematical connections. Therefore, in studying these three teaching practices (setting learning goals, designing learning tasks, and orchestrating discussions), it is expected that the findings will contribute meaningfully to teaching and learning of mathematics.

Organization of the rest of the Dissertation

This study is organized into five chapters. Chapter Two is a review of related literature. Research on the theoretical underpinnings of core practices teaching concepts, learning to teach in a community, setting mathematical goals, designing and implementing mathematical tasks, and facilitating class discussions, will be reviewed. A conceptual framework will be generated from the related literature to show how the study was approached.

The research design, data collection strategies, and analysis methods for this study are discussed in Chapter Three. This section explains the decisions made about the data collection, design of the study, and the general direction of the study. Furthermore, this chapter has detailed explanation of how data are analyzed for single and cross-cases.

In Chapter Four, a qualitative description of all the single cases and cross-case analysis is presented. These descriptions are presented with respect to the research questions. Chapter Five provides the overall perspective of the study. An in-depth discussion on the findings on the three core teaching practices with the related literature is presented. In addition, conclusions and implications for teacher education are presented. Finally, directions for future research are discussed.

Chapter 2

Literature Review

The purpose of this study was to find out how preservice teachers enrolled in a standards-based mathematics methods class engaged in three core practices: setting mathematical learning goals, designing learning tasks, and facilitating whole-class discussions around the learning tasks. Also, the study focused on how the learning communities in which preservice teachers engaged in these practices influenced their decision-making processes as they planned and enacted these teaching practices. As this study builds on research in these areas (standards-based learning environment, teachers' content knowledge for teaching, core practices, and learning communities), the literature review is organized around each area.

First, the standards-based teaching and learning environment and the use of multiple representations is described. Second, a brief review of the pedagogical content knowledge from Shulman (1986, 1987) and the mathematical knowledge for teaching (Ball et al., 2008) are presented to provide a background for the mathematical knowledge teachers use in enacting their practices. Third, the chapter briefly reviews literature on learning communities and how preservice teachers learn to teach in a community. Fourth, literature on using core practices in teacher education is presented by describing core practices, how teacher education can focus on core practices, and a description of the three core practices (setting learning goals, designing learning tasks, and orchestrating class discussions) which is the focus of this study. Finally, the chapter ends with the conceptual framework of the study.

Standards-Based Teaching and Learning Environment

The importance of mathematics at the academic and nonacademic sectors places it in the spotlight of national affairs, to the extent that poor performance in mathematics raises public concern. For instance, in the 1980s there was a perception that the quality of mathematics instruction was declining amidst national need to train citizens not only for mathematics proficiency, but also, for economic empowerment. To that end, there were calls for reforms in mathematics education in terms of curriculum developments and evaluation studies to shape the teaching and learning of mathematics (Dossey et al., 2012). Not only have the national calls led to the development of mathematics standards, but also to identify the kind of mathematical instruction that should be exhibited in the classrooms.

These reform efforts and standards documents (e.g., NCTM,1991; NCTM, 2000) have advocated that mathematics teaching and learning should focus on problem solving, reasoning and proof, communication, connections, and representations in order to prepare students to have the mathematical skills they need. Although some mathematics teachers may be designing their lessons to meet these goals, others may engage in using a lecturing approach to teaching mathematics, in which the teachers present mathematical knowledge with the hope that students would be able to apply the knowledge to solve mathematical tasks (Phillip et al., 2007). Using this teaching approach may not engage students to actively co-construct mathematical knowledge with teachers. Rather than using a teaching approach that may not engage students in the teaching and learning process, teachers should endeavor to make their mathematics lessons engaging by enacting teaching practices that will enable students to participate in class discussions.

One way teachers can do this is to set up mathematical tasks which lend themselves to inquiry-based learning in which students will be able to work collaboratively, using multiple modes of representation, to construct mathematical knowledge.

Students in inquiry-based classrooms are expected to demonstrate understanding of mathematical concepts and ideas to enable them apply their mathematical knowledge in new settings. To this end, the emphasis of teaching and learning mathematics in inquiry-based classroom is on problem solving, communicating mathematical ideas, reasoning, and making mathematical connections (NCTM, 1998; 2000). Reformers expect that students are actively engaged in the teaching and learning process in order for them to develop mathematical understanding. With this view of mathematics teaching and learning, the role of the teacher has shifted from lecturing mathematics and being an "arbiter of mathematical correctness" (Stein et al., 2008, p. 315) to an orchestrator of instructional environments where students actively grapple with mathematics tasks to construct mathematical knowledge (NCTM, 2000).

At the heart of engaging students meaningfully in a inquiry-based mathematics class is the use of multiple representations, including concrete materials, pictures, written and verbal symbols, and real objects in teaching and learning mathematics (NCTM, 2000). Students are expected to "create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems" (NCTM, 2000, p.67).

Using multiple representations to teach for conceptual understanding. Using a variety of representations in teaching and learning mathematics affords teachers the opportunity to present mathematical concepts in diverse ways such that students will be able to make abstractions that leads to conceptual understanding (Dienes, 1960). Bruner (1966) also expressed the idea of multiple representation using three stages of representations, namely, enactive stage, iconic stage, and symbolic stage. At the enactive stage, students use concrete materials to model mathematical ideas, whereas at the iconic stage pictures and drawings are used in teaching-learning situation. The last stage is for symbolic representations of mathematical ideas. According to Bruner, teachers should ensure that learners develop in one stage before proceeding to the next stage. One way of helping students develop conceptual understanding through multiple modes of representations is using the Lesh Translation Model (LTM) (Figure 1), (Lesh & Doerr, 2003).

The model consists of multiple modes of representation, namely, manipulative (concrete models), symbolic, language, pictorial, and realistic (real-world contexts) representations. The use of the LTM is to enable students to represent mathematical concepts using multiple modes of representation and being able to translate between and within multiple modes of representation (Cramer, 2003).

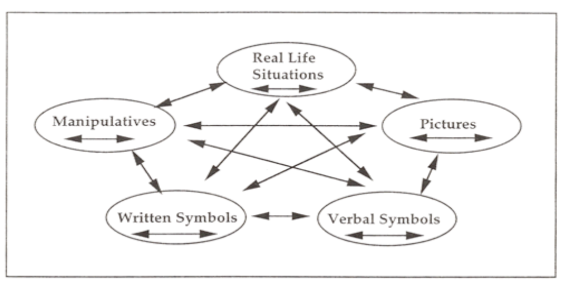

Figure 1. The Lesh Translational Model.

The Lesh translation model has been used as a framework to develop classroom instruction and curricular materials in the Rational Number Project (RNP) research. According to Behr, Lesh, Post, and Silver (1983), the RNP has identified that when students are able to represent fractions using multiple ways, or when fraction ideas are linked to students' lived experiences, students are able to develop conceptual understanding. In view of this, it is important for mathematics teachers to have experience with the Lesh translation model so that they will be able to help students learn mathematics effectively (Cramer, 2003).

A translation is the ability to model a mathematical concept within the same representation or between different representations (Cramer, Monson, Wyberg, Leavitt, & Whitney, 2009). For example, when students are able to model a fraction using an area as a unit, students are translating from the symbolic form to the pictorial representation—that is, translating between models. On the other hand, when students model a fraction from the area model using a number line, they are translating within representations. Researchers (e.g., Cramer, Post, & delMas, 2002) have argued that enforcing the translation within and among representations in teaching mathematics supports students' conceptual understanding and problem solving skills in learning fractions.

Teacher Knowledge for Teaching

Teaching is a complex activity (Grossman et al., 2009a) that requires that teachers take into consideration a number of factors before making instructional decisions. These decisions are subject to teachers' value-judgment occasioned by the context within which they operate. As such, teachers have the prerogative to enact productive teaching practices which promotes deep learning for students. The core ideas and philosophies underpinning these national standards in mathematics education and the attempts to empower citizens to be mathematically proficient largely depend on the classroom teachers—the agents of change in educational circles (Darling-Hammond, 1995).

Shulman's professional knowledge domains. Shulman (1986) began a new era of research that focused not only on finding teachers' content knowledge, but also, finding other domains of teacher knowledge that contributed to effective teaching and learning. Shulman, in his paper, *Knowledge Growth in Teaching,* asserted that issues that deal with teachers' subject matter knowledge and how teachers transform their knowledge into teaching were not addressed. Consequently, Shulman proposed that teachers' content knowledge could be distinguished into three main categories: subject matter knowledge, pedagogical content knowledge, and curricular knowledge (Shulman, 1986). In another study, Shulman proposed that there is a knowledge base for teachers. Shulman categorized this knowledge base as: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. (Shulman, 1987). Among these categories of teacher knowledge, Shulman indicated that *pedagogical content knowledge* (PCK) is of special interest because it portrays the unique knowledge teachers use in teaching by blending content knowledge with pedagogy for effective teaching and learning. Shulman's work began a new era of research by mathematics educators to clarify the PCK construct further. For example, Ball, Thames, and Phelps (2008) introduced the construct of mathematical knowledge for teaching and its accompanying conceptual framework.

Mathematical knowledge for teaching (MKT). After Shulman's categorization of teacher knowledge and the idea of pedagogical content knowledge, more research continued to investigate teachers' knowledge. Using a longitudinal study, Ball, Thames, and Phelps (2008) analyzed video recordings of teaching episodes to investigate teacher practice to find out the kind of mathematics knowledge that is used in teaching. From their analysis, Ball et al., (2008) proposed a theory-based construct, "mathematical knowledge for teaching (MKT)." According to the Ball et al., (2008), mathematical knowledge for teaching is "the mathematical knowledge needed to carry out the work of teaching mathematics." (p. 395). The MKT deals with the tasks pertained with teaching and the mathematics demands that emerge from teaching.

Central to the definition of mathematical knowledge of teaching is how Ball and her colleagues conceptualize teaching. They define teaching as,

> everything the teacher must do to support the learning of their students...the interactive work of teaching lessons in classrooms and all the tasks that arise in the course of the work...planning for those lessons, evaluating students' work, attending to concerns for equity, and dealing with the building principal who has strong views about the math curriculum (p.395)

Inherent in this definition is the notion that mathematics teachers engage in several practices during the teaching and learning process. Teachers will have the responsibility to plan lessons, teach them in class, and evaluate the teaching. Performing these teaching practices require teachers to draw from from their MKT to set learning goals, select or design learning tasks, facilitate class discussions, and enact other teaching practices to achieve desired learning goals. Ball et al. (2008) identified six domains of MKT, namely, *common content knowledge* (CCK), *specialized content knowledge* (SCK), *knowledge of content and students* (KCS), *knowledge for content and teaching* (KCT), *knowledge at the mathematical horizon,* and *knowledge of the curriculum*.

While CCK refers to the mathematical knowledge and skills that can be used in several settings, SCK on the other hand, is the unique form of mathematical knowledge that is used for teaching. Common content knowledge can be conceived as the mathematical content people know. For example, people who have some mathematics background can identify that one-half is greater than one-fourth. Specialized content knowledge, however, may not be easily accessible to people who do not have any mathematical pedagogical skills. Specialized content knowledge is the mathematical knowledge unique to teaching. This is the mathematical knowledge that teachers have, which general users of mathematics do not.

Knowledge of content and student (KCS) entails a combination of knowing about mathematics and knowing about your students. Mathematics teachers with KCS will likely be able to anticipate students' possible thought processes as far as mathematics is concerned and be able to envision students' possible misconception and difficulties in learning specific mathematical concepts. The implication is that if teachers do not have KCS for teaching specific mathematical content, there is the likelihood that teachers will not be able to draw from students' possible misconceptions as a guide to teach effectively. If teachers are aware of students' misconceptions, they may be able to help them learn effectively and to help students undo their misconceptions (Ball, et al., 2008).

Knowledge of content and teaching (KCT) is the knowledge that combines teachers' mathematical knowledge and knowledge about teaching. When teachers have KCT they will be able to use appropriate teaching and learning materials for teaching a specific mathematics content and at the same time, be able to structure lessons effectively. For example, in order to decide whether to use concrete material or pictorial representation for a given teaching and learning situation, teachers have to make decisions in order to enhance students' learning. While some teachers are able to make good instructional decisions to enhance mathematics learning, others may have difficulties using appropriate learning materials to teach the same content under similar conditions. For example, Suppose a student argues that $\frac{1}{6}$ is greater than $\frac{1}{3}$ because 6 is greater than 3. In this case, the teacher's goal will be to help students understand that their reasoning is incorrect. How can teachers enact effective teaching practices, like using some mathematical activities and tasks and leading an effective discussion for students, for students to be convinced that $\frac{1}{3}$ is rather greater than $\frac{1}{6}$? Teachers with KCT will be able to design appropriate learning activities to help students see that $\frac{1}{3}$ is greater than $\frac{1}{6}$.

Although these knowledge domains are useful for effective teaching, teachers, will have to use their knowledge base in practice (Grossman et al., 2009a). As noted already from Ball et al., (2008) conception of teaching, it is evident that mathematics teachers will have to make several decisions when planning and teaching mathematics lessons. During teaching, these decisions will be exhibited in the form of teacher practices. Because of this, some researchers (e.g., Grossman, Hammerness, & McDonald, 2009; McDonald, Kazemi, & Kavanagh, 2013; Forzani, 2014) argue that instead of detailing teacher's mathematical knowledge as a way of measuring teacher effectiveness, we should rather concentrate on preparing teachings to engage in the core practices of teaching. This does not mean that in thinking about using core practices for teacher education one would should not consider the mathematical knowledge needed to teach. Rather, researchers who argue for core practice concept of teacher education are of the view that if preservice teachers are prepared to enact the core practices of teaching, they may be effective teachers.

Learning Community and Community of Practice

As noted in chapter 1, in a standard-based learning environment, teachers do not engage in the didactic teaching, but rather, they facilitate learning by organizing students and all other resources in a learning community. Cross (1988) defines learning community as groups of people engaged in intellectual interaction for the purpose of learning. Defined this way, the mathematics methods class where preservice teachers and their instructor engaged in intellectual interactions with the aim of learning to teach can be considered a learning community. The intellectual interaction will require that members of the community establish relations as a platform for successful interaction to thrive. When this happens, the learning community can be conceptualized as what Lave and Wenger (1991) call "Community of Practice."

According to Lave and Wenger (1991), "community of practice is a set of relations among persons, activity, and the world over time and in relation with other tangential and overlapping communities of practice." (p. 98). Using "community of practice" twice in the definition suggests that when an individual belongs to a community of practice X and also belong to a community of practice Y, in such a way that there exist a "tangential" or "overlapping" relation between the two communities of practice, then the individual can be said to belong to a larger community of practice which is a combination of the two communities of practice. This is because using the terms "tangential" or "overlapping" relationship implies that any form of connection that exists between two or more communities of practice warrants those communities of practices to be considered as a bigger community of practice.

As people interact in the community of practice, through active participation in the community, they internalize information and consequently learning occurs (Lave, & Wenger, 1991). In a standards-based learning environment, learning communities and communities of practices are used to enable students to learn. Similarly, in teacher education programs, the concept of learning communities can be used to help preservice teachers relate to colleagues and mathematics educators to learn the practices of teaching.

Learning to teach in a community. According to Hammerness et al., (2005) "new teachers learn to teach in a community that enables them to develop a *vision* for their practice; a set of *understandings* about teaching, learning, and children; *dispositions* about how to use this knowledge; *practices* that allow them to act on their intentions and beliefs; and *tools* that support their effort" (p.385). Figure 2 shows how the *vision*, *understandings*, *dispositions*, *practices* and *tools* are connected in within the learning community.

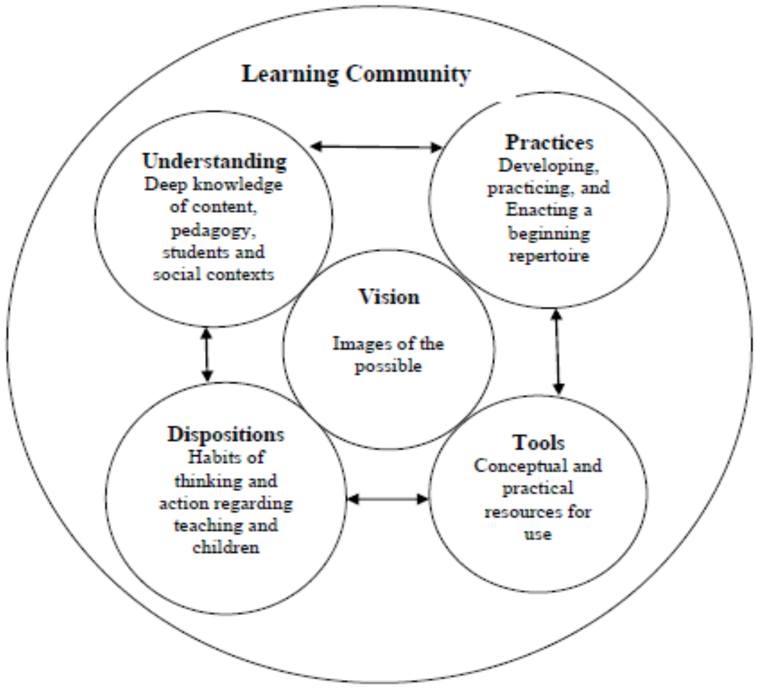

Figure 2. Learning to teach in community (adopted from Hammerness et al., 2005).

Having a clear vision of the kind of teaching to do is crucial for preservice teachers in teacher preparation because a clear vision can guide prospective teachers to direct their professional learning towards a desirable end. Without a clear vision, preservice teachers may not be able to get focused on the kind of knowledge and practices they should acquire in their teacher education program. In addition, Feiman-Nemser (2001) argues that preservice teachers must have a clear vision of good and desirable teaching practices in order to assess their teaching and students' learning. Consequently, it is important that preservice teachers develop a clear idea—a vision—of what they intend to teach, and how they can facilitate teaching in order for them to be effective teachers who can help students learn their desired learning goals.

In order for teachers to be able to carry out their vision of good teaching, they need the required knowledge to accomplish the vision. Research studies (Ball et al., 2008; Shulman, 1986, 1987) have shown that teachers must have a strong knowledge base, or understanding of the subject they teach and how they make it accessible to students. Making content knowledge accessible and comprehensible to students require that teachers have a clear understanding of students' backgrounds and their prior knowledge, as well as an understanding of how students learn. Having this knowledge base and other educational tools will enable teachers teach effectively to accomplish their visions.

According to Hammerness et al., (2005), educational tools are things teachers use for their work of teaching. These tools can be conceptual or practical. Conceptual tools include learning theories, frameworks, and ideas and concepts (such as culturally relevant pedagogy, differentiation, etc.) about teaching and learning. Practical tools, on the other hand, refer to specific instructional techniques and strategies, and resources, such as curricular materials, textbooks, assessment tools (Grossman et. al., 1999) that teachers use for teaching. Through teacher education programs, preservice teachers get exposed to several practical and conceptual tools they can use for teaching.

Teaching practices are the practical things teachers do during teaching and learning sessions with the aim of helping students to achieve a particular goal. These include instructional strategies such as giving explanations, orchestrating whole-class discussions, planning activities, developing simulations, planning debates, and organizing writing workshops (Feiman-Nemser, 2001a). Practices also include designing and teaching lesson plans, developing and implementing assessments, and providing constructive feedback. Ensuring that these practices are enacted effectively is important for the core practices approach of teacher education because in this approach teaching practices are the central focus to teacher development.

Preparing Teachers Using Core Practices

Preparing teachers to use core practices is an attempt to centralize preservice teachers' learning to teach on the practical work of teaching rather than focusing on theoretical foundations that may not help preservice teachers acquire the skills and knowledge necessary for the practical work of classroom teaching (Forzani, 2014). In order for teacher educators to effectively use core practices in teacher education, teacher educators should enact pedagogies that will help preservice teachers learn the core practices and be able to enact them effectively. In line with this, Grossman and her colleagues (2009) identified three ways teacher educators can prepare their preservice teacher using core practice.

In their study to investigate how people are prepared for professional practice, Pam Grossman and her colleagues (2009) used a comparative case study of professional education across three different professions—the clergy, clinical psychology, and teaching to propose a framework to “describe and analyze the teaching of practice” (Grossman et al., 2009, p. 2879). The study took place in eight professional education programs located in seminaries, schools of professional psychology, and universities across the United States. The participants for the study included faculty members, students, and administrators at each of these eight programs. For each of the institutions, the researchers conducted site visits that included interviews with administrators, faculty, and staff; observations of multiple classes and field-work; and focus groups with students who were either at the midpoint or at the end of their program. The researchers identified three concepts for understanding the pedagogies of practice in professional education: *representations, decomposition*, and *approximations of practice*. According to Grossman and her colleagues,

> *Representations* of practice comprise the different ways that practice is represented in professional education and what these various *representations* make visible to novices. *Decomposition* of practice involves breaking down practice into its constituent parts for the purpose of teaching and learning. *Approximations of practice* refer to opportunities to engage in practices that are more or less proximal to the practices of a profession (p. 2879).

In preparing preservice teachers to enact core practices using this framework, teacher educators are may demonstrate a variety of ways core practices can be explicitly described and represented. In addition, the framework suggests that core practices should be broken down into smaller parts to facilitate teaching and learning. In this case, when teaching preservice teachers how to learn and enact a core practice, it may be better for teacher educators to present core practices in teachable parts that can be assimilated effectively by preservice teachers. Finally, the framework also suggests that after educators have presented core practices in teachable parts, preservice teachers should be given opportunities to rehearse the practice. Allowing preservice teachers to rehearse core practices can make preservice teachers confident about enacting those core practices before their practical teaching in the classroom (Lampert et al., 2013). Because of the complexity of teaching and how teaching context may present different situations in different classrooms, it may be possible that decomposition, representations, and approximation may not always be taught sequentially.

Teacher educators have used this framework in teaching core practices in their mathematics methods class. For example, mathematics teacher educators and researchers, Boerst, Sleep, Ball, and Bass (2010), during their elementary mathematics methods course for beginning teachers, adapted Grossman et al., (2009) framework to teach their preservice teachers how to lead mathematics discussion. Boerst and his colleagues conceptualized decomposition of teaching as nested practices of varying components that connect techniques and domains. Conceptualizing decomposition in this way, the researchers were able to articulate their course content, their choice of representations of practice and the design and sequencing of approximations. This study suggests that teacher educators can adapt or adopt the concepts of decomposition, representation, and approximations of practice in a way that will enable them to teach preservice teachers core practices that are central to the work of teaching.

Unlike the practiced-based approach to teacher preparation where preservice teachers spend significant amount of time observing and engaging in practices which are not specifically directed at acquiring specific skills (Forzani, 2014), the core practice concept of teacher preparation focuses the work of preservice teachers on teaching. Similarly, the core practice approach to teacher education differs from the realistic teacher education concept. In the realistic teacher education, teachers' concerns and needs are the central focus of teacher preparation whereas the core practice approach to teacher development centers on a set of practices that are core to teaching.

Focusing teacher preparation on teaching practices will require that teacher educators "organize the curriculum around a set of core practices and help novices develop professional knowledge, and skills, as well as an emerging professional identity around these practices." (Grossman et al., 2009 a, p.277). Also, according to Grossman et al., (2009 a), the core practices of teaching would provide the "warp threads" for the curriculum, while the knowledge and skills needed to enact the core practices constitute the "weft."

Defining core practices. Organizing teacher preparation program around core practices requires teacher educators to identify specific core practices and help preservice teachers learn to enact those core practices before they begin their independent teaching. What one considers a core practice, therefore, depends on how one perceives teaching, and what one considers as a central teaching activity. Because of possible conceptions of core practices, researchers in this field have not yet agreed on a common definition of core practice (Ball & Forzani, 2009). Grossman et al. (2009 a) have identified criteria that can be used to identify core practices. According to Grossman et al., (2009 a), core practices share the following characteristics:

- Practices that occur with high frequency in teaching.
- Practices that novices can enact in classrooms across different curricula or instructional approaches.
- Practices that novices can actually begin to master.
- Practices that allow novices to learn more about students and about teaching.
- Practices that preserve the integrity and complexity of teaching.

- Practices that are research-based and have the potential to improve students' achievement.

(Grossman et al., 2009 a, p. 277).

Based on the criteria, core practices include learning about student understanding (Grossman et al., 2009 a), eliciting and responding to students' ideas (Lampert et al., 2013), setting and maintain high expectations (Ball & Forzani, 2009) using learning tasks to promote student learning (Doyle, 1988; Stein et al., 1996), and leading class discussion (Boerst et al., 2011).

In focusing teacher preparation on core practices, Grossman et al., (2009 a) argue that, teacher educators "must attend to both the conceptual and practical aspects associated with any given practice." (p. 278). For example, consider the core practice of eliciting students' thinking in teaching. When teaching practices related to this core practice, teacher educators should explain the underlying theoretical principles of why it is necessary to elicit students' thinking when teaching so that preservice teachers can learn when and how to elicit students thinking. In addition, teacher educators should provide preservice teachers the opportunity to learn and enact instructional routines associated with eliciting students thinking. While preservice teachers learn to enact core practices, it enables them to develop a sense of professional identity built around their role as teachers. (Grossman et al., 2009 a).

Focusing teacher preparation programs on core practices affords teacher educators not only the opportunity to address the complexity of teaching, but also to be able to respond to pertinent issues preservice teachers may encounter when they begin their independent teaching (Grossman et al., 2009 a). From this perspective, the core practice approach to teacher education can help bridge the gap between research and practice because as preservice teachers learn the theoretical underpinnings of core practices and learn to enact them through "approximations" of practice (Grossman et al., 2009) preservice teachers will be able to transfer their practical knowledge to the field when they are asked to teach.

Currently, there is a call by NCTM to bridge the gap between research and practice by ensuring that teachers enact core practices that are central to their work in the classroom so that they can help students learn mathematics meaningfully. NCTM, through *Principles to Action: Ensuring Mathematics Success for All,* refers to core practices as "Mathematics Teaching Practices" (NCTM, 2014), and argues that these teaching practices "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9). In this study, three core practices were investigated. These include: setting mathematical learning goals, designing learning tasks to address the goals, and orchestrating whole-class discussions around the learning tasks to achieve the learning goals. Because these three core practices were the focus of this study, they are examined in the following sections.

Using Mathematical Goals to Focus Learning

Setting learning goals that will articulate the mathematics students will learn and how they will demonstrate their understanding is important in mathematics education because formulating "clear, explicit learning goals sets the stage for everything else" to follow in the classroom (Hiebert, Morris, Berk, & Jansen, 2007, p.51). Mathematical learning goals detail the important knowledge, skills, facts that mathematics teachers use to determine what they want their learners to acquire at the end of an instructional period or over several instructional periods (Jansen, Bartell, & Berk, 2009). In line of this thinking, learning goals shape the content knowledge that is established and built upon in the course of learning period. This means that by setting learning goals, teachers will determine the kind of knowledge to build and how to build that knowledge. These learning goals should not be merely restating the standards. In preparing future mathematics teachers, therefore, it is important to prepare Rather, preservice teachers should be able to clearly identify specific mathematical concepts, skills, facts, and methods students are expected to understand at the end of a given lesson or series of lessons (Wiliam, 2007).

Because teachers usually have the responsibility to set learning goals, the goals they set depends on their value judgment about what student should learn and be able to do at the end of an instructional period. Setting these learning goals may also be informed by educational standards (e.g., state standards), previous experience, research findings, and vision. Because of this, learning goals construction is influenced by several factors, including teachers' personal judgment, educational policies, students' learning, and the learning environment (Baroody, Cilbuskis, Lai, & Xia, 2004).

Teachers' past learning experience and content knowledge may influence the type of learning goals they articulate for their students. For example, teachers who believe that learning goals should scaffold students' learning are likely to set goals that will scaffold one another to a desired overall goal. On the contrary, teachers who have high expectations for all students may not consider scaffolding learning goals, but rather, these teachers may set high-level learning goals which may not be accessible to all students. In order to minimize possible discrepancies in setting learning goals, teachers' pedagogical and mathematical content knowledge may be crucial in unpacking mathematical content in order to set reasonable learning goals for students. Reasonable learning goals are learning goals that are within the reach of all learners in the classroom.

Characteristics of learning goals. In order for learning goals to achieve the intended purpose, they are to be carefully stated to meet certain characteristic. For example, Jansen and her colleagues (2009) argue that for learning goals to facilitate knowledge building, the goals should be "targeted" and "shared".

Targeted learning goals are learning goals that "must be sufficiently well specified to suggest interventions for supporting learners in achieving them and to indicate the types of evidence needed to determine if the goals have been achieved" (Jansen, et al., 2009, p.525). Making learning goals sufficiently well specified implies that learning goals should be stated in clear and explicit terms to identify what students are expected to do and how they will demonstrate their learning. In this regard, teachers should be clear about the learning goals they want students to achieve in a given instructional session. Clear and unambiguous language should be used in stating learning goals in measureable terms. In order to clearly state what students are to learn and how evidence of students' learning will be ascertained, it may be important to let students demonstrate the type of representation(s) (verbal, pictorial, symbolic) they are expected to use to demonstrate their understanding of the learning goal.

Setting clear and explicit learning goals also requires that teachers unpack bigger learning goals (that come from state standards) into “digestible” bits for students using their knowledge expertise (Hiebert, Morris, Berk, Jansen, 2007). Teachers may be required to break broader learning goals down to obtain sub-goals or specific learning objectives for individual lessons. For instance, in the teaching and learning of fractions if the overarching goal at the end of a unit is to ensure that students will be able to subtract two unlike fractions with denominators, then specific learning goals that will lead to this bigger goal need to be considered. For example, while the long-term learning goal may be for the students to subtract fractions symbolically, a sub-goal may initially be set for students to subtract fractions concretely using a model like fraction circles to connect steps with the model to symbols.

In a study to find out how clarity and coherent learning goals scaffold students’ learning in Physics class, Seidal, Rimmele, and Prenzel (2005) rated video recordings of introductory Physics lessons with respect to clarity and coherence of the lesson structure. The results indicated that there was a positive effect of classes with high goal clarity and coherence on students' supportive learning conditions, self-determined motivation, and organizing learning activities. Also, the researchers found out that over the course of the school year, high goal clarity and coherence resulted in positive competence development. These findings support the view that when teachers articulate explicit, measureable, and coherent learning goals, they can facilitate students’ learning.

According to Jansen et al., (2009), another characteristic of learning goals that make them functional is that they must be shared. That is, learning goals “must be mutually understood and committed to by all participants in the knowledge-building process" (p.525). This means that in a given learning community, teachers must communicate learning goals to learners in a way that learners will fully understand those learning goals. Understanding the learning goals will mean that learners will know what they are required to do during, or after the instructional period and how they are supposed to demonstrate their knowledge. In this case, the targeted learning goals should be shared with leaners at the beginning of the lesson in order that learners can know the road map ahead of them during the instructional period.

Sharing the targeted learning goals in the beginning stages of the lesson and making sure that students and teachers have clear understanding of the objective can facilitate teaching and learning. On the part of teachers, when targeted learning goals are shared explicitly, it can guide their instructional decisions and assessment strategies. This is because teachers can use formative assessment techniques to elicit students' thinking to see the extent to which the learning goals are being achieved. On the part of the students, knowing the learning goals at the beginning of the lesson can motivate them to focus their learning in the sense that they can also assess their learning by measuring their knowledge acquired relative to the objectives of the class.

Although establishing targeted learning goals and sharing these learning goals with learners can set a good foundation for effective mathematics teaching and learning, not much is known about how teachers formulate these learning goals from given broader educational goals like what is stated in state standards. The assumption of educational stakeholders has mainly been that when teachers are given textbooks and standards together with other teaching logistics, then it is likely that teaching will follow naturally. However, this assumption may be far from reality.

Establishing targeted learning goals is a complex task which demands that teachers consider several factors (including the teaching context, differing backgrounds of students, the scope of mathematical knowledge to be learned, multiple representations that can be used to scaffold learning, and how the knowledge learned can be applied to other settings) before developing learning goals. This calls for careful planning and deeper reflection on the part of teachers and their ability to draw from their mathematical knowledge and pedagogical knowledge for teaching (Ball, Thames, & Phelps, 2008; Shulman, 1986; 1987). When learning goals are clearly defined, that can set a good foundation for teachers to plan the learning tasks to use in order to achieve the desired learning goals.

Use of Mathematical Tasks in Teaching

Over the past several years, efforts have been made to better understand the affective, cognitive, and metacognitive aspects of problem solving in mathematics and other subject areas (e.g., Schoenfeld, 1992; Silver, 1985; Cai, 2003). In contrast, problem-posing research is relatively new in mathematics education (Brown & Walter, 1993) even though mathematics educators have viewed problem posing not only as the means to understand students' mathematical thinking but also as a means to teach mathematics with understanding (Cai et al., 2015). If problem posing is viewed as a tool for understanding students' mathematical thinking and to teach for understanding, then it is imperative that mathematics teachers take a serious look at how they will select and pose mathematics problems. Selecting and posing problems will require that teachers have a clear understanding of the concept of problem posing.

Problem posing is conceptualized differently by researchers. For example, while Kilpatrick (1987) perceives problem posing as reformulation of existing problem, Pelczer and Osana (2015) conceptualize problem posing in broader terms. According to Pelczer and Osana, problem posing "involves asking questions during classroom interactions to assess students understanding, modifying existing problems to adjust the difficulty level of a task, and creating problems to meet instructional objectives." (p. 469). These definitions suggest that during teaching and learning sessions, mathematics teachers can engage in problems posing for a variety of reasons. One such reason can be that teachers will pose problems that will help them achieve their instructional goals. While some of these problems posed by teachers may come on the fly during teaching, others are prepared during the lesson planning stage. These planned tasks may be teacher-generated, selected and/or adapted tasks from curriculum materials, and/or from other sources during lesson planning.

While questions that are posed during the class sessions may be important, these questions may arise as a result of reactions that are generated from other tasks teachers pose to students. During lesson planning, as teachers plan their learning goals, they also think about the learning tasks they will be using during the teaching sessions. These learning tasks are conceptualized as "*lesson-driven tasks*" in the sense that they drive the direction of the lesson, and as such, they may form a central part of the teaching and learning process for a given lesson. These "*lesson-driven tasks*" are activities teachers will require students to respond to during the teaching and learning process in order to elicit students' understanding of what is being taught in the class. These tasks are central tasks as compared to other "*peripheral tasks*" that arise in the course of teaching and learning because these are the tasks teachers intentionally select, adapt, or design to use in order to meet instructional goals.

The mathematics tasks teachers pose in their classrooms deserve a lot of importance because they convey inherent messages about the nature of mathematics, what mathematics is and is not, and students begin to identify themselves as mathematical persons or not (Crespo, 2003; Doyle, 1988). Tasks that are used during teaching and learning also operate as the "proximal cause" of students learning (Shavelson, Webb, & Burstein, 1986).

Doyle (1988) conceptualizes academic task in terms of four general components:

- A product, such as numbers in the blanks on a worksheet, answers to a set of test questions, oral responses in class, or a solution to a word problem.
- Operations to produce the product, for example copying numbers off a list. Remembering answers from previous lessons, applying a rule (e.g., "Invert and multiply") to select appropriate answers, or formulating an original algorithm to solve a problem.
- Resources, such as notes from lectures, textbook information, conversations with other students, or models of solutions supplied by the teacher.

- The significance or “weight” of a task in the accountability systems of a class; for example, a warm-up exercise in math might count as a daily grade, whereas a unit test might equal 30% of the grade for a term. (Doyle, 1988, p. 169)

The concept of task, from Doyle’s (1988)’s perspective, requires that in a teaching and learning session, students should be working towards a desired product. This product basically means providing a certain kind of response to a given task. The response can be demonstrated in multiple forms of representation (verbal, symbolic, written word, pictorial, or a combination of these forms). The cumulative effect of students’ ability to generate the required products should lead to a general learning goal teachers want students to achieve. In mathematics classrooms, these learning goals may be teachers’ intended learning objectives they want to achieve. In view of this, teachers will have to set clear and explicit learning goals that will guide the mathematical tasks they will use in their teaching.

The second point explains the needed resources and conditions that will enable students accomplish a given goal. In the context of mathematics teaching and learning, these resources and setting problem space are conditions mathematical teachers have to create in the learning environment for students. These conditions can include teachers’ explanation of mathematical concepts, leading class discussions, eliciting students’ ideas and organizing classroom resources to assist student grapple with mathematical tasks to work towards desired learning goals. Inextricably linked to creating resources is helping students to use appropriate operations to harness the available resources provided in order to solve tasks.

Finally, the last component of task, according to Doyle (1988), is the relative importance of tasks towards achieving the overall learning goals in the class. This means that teachers may consider some tasks carrying more weight than others. In this case, teachers' decisions around learning tasks can impact what students learn in the class (Crespo, 2003). Mathematics teachers, therefore, are required to use mathematical tasks because they engage students' intellect, make them apply their knowledge in order to form coherent framework for mathematical ideas (NCTM, 1991).

Cognitive demands of mathematical tasks. According to Doyle (1988), the cognitive level of an academic task "refers to the cognitive processes students are required to use in accomplishing it" (p. 170). Based on this idea of categorizing a learning task, Doyle categorizes learning tasks as memory work, using formulas and algorithms to solve tasks, search and match, and higher cognitive tasks. Memory tasks are tasks that require students to recall or use information they have already seen. For example, asking students to memorize their multiplication facts to find the product of two single-digit whole numbers will be considered as a memory task.

Other forms of tasks require students to apply mathematical formulas or algorithms to solve the task. For example, in a mathematics class, when students are asked to calculate the area of a rectangle given the dimensions of the lengths and width of the rectangle, that is considered as a task. With this task, students will invoke a formula and mathematical operations to solve the task. Unlike the first two types of tasks, students sometimes use the search-and-match strategy in which they use key words or phrases in the task to find the algorithm to apply. These three types of tasks are frequently used in the classroom (Doyle, 1988). On the other hand, high cognitive demand tasks require students to use their mathematical concepts, skills, facts, formulas, and other approaches to tackle mathematical tasks in order to solve them. Usually, high cognitive demand tasks, require students to transfer their knowledge to other settings in order to make sense of tasks in order to solve them.

Building on Doyle's work on tasks, Stein, Grover, and Henningsen (1996) identified two main categories of cognitive level demand for tasks. Table 1 shows the cognitive demand levels of mathematical tasks identified by Stein, Grover, and Henningsen (1996).

Table 1

Cognitive Demands of Learning Tasks Framework

Lower-Level Demands	Higher-Level Demands
Memorization Memorization of mathematical facts	Procedures with connections Use mathematical procedures by making appropriate connections.
Procedures without connection Using mathematical procedures and standard algorithms without making connections to any context	Doing mathematics Transfer mathematical knowledge to novel situations to design solution strategies to solve tasks

Cognitive demands of learning tasks (adapted from Stein, Grover, and Henningsen,1996).

While all the tasks levels may be necessary for students of varying abilities, it is important that teachers endeavor to set high-level cognitive demand tasks to help students learn. Studies in Cognitively Guided Instruction (CGI) suggest that teachers who are able to design good tasks for students are able to assist students' learning (Fennema, Carpenter, & Peterson, 1989). Apart from categorizing tasks based on their cognitive demand levels, tasks can appear as familiar or novel to students. Familiar tasks include the tasks students have seen before, or actually solved in class. On the other hand, novel tasks include tasks students come across for the first time when learning. Generally, familiar tasks do not pull any surprises to students because they may have some knowledge about how to respond to those tasks. Unlike familiar tasks, novel tasks may pull some surprises for students because they may not be able to readily know how to respond to those task (Doyle, 1988). Therefore, teachers should know their students in order to use appropriate tasks in teaching. This requires teachers to use their mathematical and pedagogical knowledge for teaching.

Sequencing of task is another important feature in using mathematical tasks. Teachers need to sequence mathematical tasks in a coherent manner that will enable students make necessary connections and apply relevant previous knowledge in solving new tasks. This situation does not occur in most mathematical classrooms because tasks are sometimes sequenced based on management considerations, personal preferences, or perceived motivational requirement (Doyle, 1988). Another important thing is that teachers sometimes focus on checking correct answers at the detriment of checking students' work for conceptual understanding.

Getting mathematical work "correct" may not necessarily mean that students understand the mathematical concept underlying the task (Schoenfeld, 1988). When teachers focus on checking correct answers, they may tend not to see students' misconceptions and errors. Instead of focusing on seeking correct answers, usually at the detriment of conceptual understanding, teachers should consider the bigger picture of the mathematical goals students need to achieve so that they can structure their tasks to connect with other tasks. Because of the nature of learning tasks and the role they play in teaching and learning it is important that teachers learn to select, adapt, or design good mathematical tasks that will help them achieve their learning goals. In the case of helping preservice teachers learn how to set tasks, teacher educators can design their programs to integrate the learning of tasks construction and/or selection so that preservice teachers will be able to enact this core practice effectively.

Crespo (2003) examined the changes in the problem posing strategies of a group of elementary preservice teachers as they posed mathematics problems to their fourth grade students through exchange of letters. The context of the letter writing between the preservice teachers and their students was to provide authentic teaching experience "in that it paralleled and simulated three important aspects of mathematics practice: posing tasks, analyzing pupils' work, and responding to pupils' ideas"(Crespo, 2003, p. 246). Preservice teachers wrote and received six mathematics letters. The letter writers determined the content and focus of the letters; the only requirement was to include a mathematics problem for their students to solve. Although preservice teacher posed their own problems, during letters 3, 4, and 6 they were required to at least pose one problem from a common set that had been explored in their methods class. The exchange of letters was initiated by the fourth graders. In addition to the letters, preservice teachers' weekly mathematics journals and final reports were analyzed.

The results indicated that preservice teachers' later problem posing practices significantly differed from their earlier ones. Instead of posing traditional single-step and computational problems, these preservice teachers posed problems that had multiple approaches and solutions, were open-ended and exploratory, and were cognitively more complex. In addition, preservice teachers' problem posing styles also changed. "Rather than making adaptations that made students' work easier or narrowed the mathematical scope of the problem, their adaptations became less leading and less focused on avoiding pupils' errors" (p. 243). According to Crespo (2003), the reported changes indicate a significant shift in preservice teachers' views and beliefs about worthwhile mathematics problems and about their students' mathematical errors. Crespo further reported that preservice teachers' newly adopted practices suggest that they viewed mathematical problems as a means to challenge and probe students' mathematical thinking. Further, it was reported that rather than working to avoid or bypass students' mathematical errors at the expense of trivializing students' mathematical work, the preservice teachers appeared not to be pushed back by students' errors, but continued to challenge students' mathematical thinking.

The changes that occurred for the preservice teachers are encouraging and can help these preservice teachers in their teaching endeavor. What contributed to those changes can serve as guidelines for teacher educators in preparing preservice teachers to learn how to design mathematical tasks. Crespo however, cautioned that the changes "did not happen overnight and were not self-generated" (p.264). Rather, Crespo believes that while it was difficult to pinpoint factors that contributed to these changes, having an *authentic audience* that interacted with the problems supported the reported changes. This suggests that as preservice teachers analyzed the solutions from their fourth graders, they were able to draw from those analyses to improve upon their tasks construction.

Nicol and Crespo (2006) investigated how four preservice teachers enrolled in a post-baccalaureate elementary teacher education program at a large Canadian University interpret and use textbooks while learning to teach mathematics during university coursework and practicum teaching. As part of the teacher education program, preservice teachers were required to complete two assignments related to curriculum analysis. One assignment required preservice teachers to develop a collection of 10 mathematical problems (from a range of resources including textbooks as well as problem solving and mathematical puzzle book), solve them, and provide written analysis of the problems. The researchers found out that the preservice teachers were able to analyze curriculum materials during the course work and were able to select, adapt, and/or develop mathematical problems for their students. It was also reported that there were varying differences in how preservice teachers adapted or created their own tasks to suit their students' context.

Findings from the above studies suggest that teacher education programs can be structured to help preservice teachers learn how to design, select, or adapt learning tasks to meet the needs of their students. Although in these studies preservice teachers learned to pose mathematical problems to students, the studies did not require preservice teachers to pose "intentional tasks" aimed specifically at helping students achieve desired learning goals. In real context, teaching is aimed at achieving a particular goal. Therefore, it is important that as preservice teachers are trained to design, select, and adapt tasks, they should do so with a purpose, which is to use the tasks to meet learning goals. Also, when tasks are carefully planned and aimed at reaching a desired goal, preservice teachers will also have to be able to orchestrate effective classroom discussions around these learning tasks in order to achieve the goals.

Orchestrating Mathematics Discussions

Because of the importance of discussions in teaching and learning mathematics, there have been calls for teachers to engage students in quality discussions in mathematics classrooms (e.g., Smith, Hughes, & Stein, 2009; National Council of Teachers of Mathematics [NCTM],1991; Lampert & Cob, 2003). Although much is being done to clearly define what form mathematics discussion should take, there is currently not a consensus about what is an ideal way to conduct discussions. (Boerst et al., 2011). Pimm (1987) argued that although the term "discussion" affords the opportunity to outline a "category of verbal exchange," it is still not well-defined and not bounded.

To some extent, the unbounded nature of leading class discussion may be a laudable idea considering the fact that mathematics discussions can take several forms and for different purposes depending on teachers' desired goals for their students. For example, mathematical discussions can be used to elicit students' ideas, share students' solution strategies with the whole group, guide students to make discoveries, engage students to work in groups etc. From these perspectives, it follows that mathematics discussions are not "monolithic, but comes in a variety of forms as a function of both design and necessity" (Boerst et al., p. 2846). Because of the non-monolithic nature of mathematics discussions, teachers need to consciously plan mathematics discussions in order to facilitate meaningful mathematics learning. This requires teachers to be grounded in mathematical content and pedagogical knowledge for holding mathematics discussions.

Holding effective mathematics discussions require teachers to have mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008) and the skill to elicit and use students' ideas effectively. With these knowledge domains, mathematics teachers will be able to facilitate mathematics discussions effectively because they will be able to "size up mathematical ideas flexibly, frame strategic questions, and keep an eye on core mathematical points" (Boerst, et al., 2011, p. 2845). It is therefore expected that teachers should design learning tasks that will elicit the kinds of discussions necessary to achieve the intended learning goal. In this case, when teachers fail to use 'strategic' tasks, it is likely that there will be no discussion and when that happens, it is possible that teachers will resort to "show and tell" (Ball, 2001), without engaging students in the learning process.

In an inquiry-based mathematics lesson, which incorporates whole class discussions, teachers design their lesson in three phases (Sherin, 2002). It begins with "launching" the mathematical goals by the teacher. During the launch phase, teachers share their target goals with the students by introducing the overarching mathematical goal students will be working to achieve, the resources that are available and the evidence that would be required of them. The launch stage is followed by the "explore phase," in which students work on mathematical tasks to produce the targeted goal. At this stage, teachers adopt a variety of pedagogical approaches to help students learn the needed mathematics content. Some of these approaches include asking students to work in pairs or in small groups depending on the classroom dynamics and the nature of the learning task. In keeping with the vision of the reform pedagogy, teachers sometimes encourage students to use diverse solution strategies. The lesson concludes with whole-class discussions and summary of the students' solution strategies in order to share students' ideas with the whole class.

Considering the structure of teaching mathematics described earlier, preservice teachers using this structure are expected to facilitate class discussions meaningfully by using students' generated ideas, examining multiple solutions strategies, comparing different representations, and terms as a platform for mathematical sense making to support students' mathematical learning (Stein et al., 2008). In view of this, discussions play a key role in inquiry-based mathematics instruction (Kazemi & Stipek, 2001; Nathan & Knuth, 2003). Consequently, it is important that mathematics teachers are able to enact effective mathematics discussions in order to help students learn meaningfully.

Although leading mathematics discussion is seen as a tool for facilitating effective teaching activities envisioned in the reformed-based teaching, not all teachers can effectively orchestrate mathematics discussions. While some teachers may be good in holding discussions, others, especially novices, may not be able to hold effective discussions (Schoenfeld, 1998). Stein and Smith lament that,

> novices are regularly surprised by what students say or do, and therefore often do not know how to respond to students in the midst of a discussion. They feel out of control and unprepared, which then reduces their efficacy as teachers, making discussion based pedagogy a lot less attractive" (Stein et al., 2008, p. 321).

In order to assist novices to be able to learn the practice of holding mathematics discussions, Stein et al., (2008) argue that "novices need a set of practices they can do both prepare them to facilitate discussions and help them gradually and reliably learn how to become better discussion facilitators over time." (p.321

Five practices for orchestrating mathematics discussion. Facilitating class discussion may not be an easy task for all teachers; most teachers find it difficult to orchestrate meaningful mathematical discussions (Stein et al., 2008). While experienced teachers may hold class discussion effectively, novice teachers may not be able to do so. Consequently, Stein et al., (2008) argue that novices and teachers who may not be experienced with leading class discussion need a set of practices that can prepare them facilitate mathematics discussions effectively and help them learn to become expects discussion facilitators with time. The authors proposed five practices for orchestrating meaningful mathematics discussion, namely, anticipating, monitoring, selecting, sequencing, and making connections. These practices, according to the authors, can serve as a starting point for novices to learn how to facilitate effective class discussion.

Anticipating. The first practice for leading effective whole-class discussion is being able to anticipate students' mathematical responses to instructional tasks(s). This goes beyond teachers' ability to assess difficulty level or appropriateness of mathematical tasks to conceptualizing "expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices" that teachers would like students to learn. (Stein et al., p. 323).

Furthermore, anticipation requires that teachers will do the mathematical tasks they intend to give to their students. In doing so, teachers can assess the difficulty level of the tasks, identify different solution strategies in order to be equipped with some background knowledge about what to anticipate from students when they are working on such tasks. When teachers fail to work on tasks they plan to give to their students, it is possible that there could be some surprises in the class on the day students are asked to work on the tasks. Some of the surprises can be that teachers may assign tasks that are either too easy or too difficult for students, or even tasks that may pose some challenges to the teacher.

Posing tasks without first solving them is what Crespo (2003) refers to as posing task "blindly." Instead of posing problems blindly, teachers should solve tasks first so that they can anticipate possible solution paths students might use. This can also help teachers anticipate possible misconceptions that might arise in the class when students are working on the tasks. Teachers can also anticipate students' solution strategies from research literature. For example, studies in the Cognitively Guided Instruction (CGI) (Fennema et al., 1996) have provided some insight about students' thinking on operations on whole numbers. When teachers explore all available recourse in search of students' possible strategies in order to be able to anticipate what student are likely to do in tackling a mathematical task, teachers will add to their knowledge base and be better prepared to lead mathematical discussions.

Monitoring. Monitoring students' mathematical responses involves "paying close attention to the mathematical thinking in which students engage as they work on a problem during the explore phase." (Stein et al., 2008, p. 326). To do this, teachers usually circulate around the classroom while students work on tasks. By monitoring students work, teachers identify a variety of mathematical ideas, skills, theories, representations, procedures, facts, etc. students use in solving tasks. This offers teachers the opportunity to hone in on which students' strategies to use and how to use them during whole-class discussions. Another way teachers can assess students' mathematical ideas is by listening to group conversations and taking notes of which part of the conversations can be used during whole-group discussions.

Selecting. Purposefully selecting students' responses for whole-class discussion require that teachers select specific students to share their work with the whole class. As teachers have made their anticipations, and monitored students' work, they can be guided by their learning goals to select the type of students' responses, representations, etc. that are considered worthwhile relative to the overarching learning goals. Where teachers encounter unfamiliar solution strategies, that can serve as a learning opportunity for them. Teachers should allow students to explain the rationale behind their mathematical ideas and not shut them off completely when they see unfamiliar strategies.

In some cases, however, teachers can decide which strategies should not be shared with the class. Usually, teachers can make such decisions based on the intended learning goal and the extent to which particular students' work may, or may not be worthwhile to share. If sharing some work may potentially lead the class totally from its goal, and at the same time not be helpful for the students, teachers should be able to make good decisions about whether to use those tasks or not.

Sequencing. After selecting students' work to be shared with the whole class, teachers are required to purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goals. By sequencing students' work for whole-class discussions, teachers can make necessary connections of the ideas to help students build conceptual understanding of what is being learned and ultimately, help students achieve their learning goals. In addition, by sequencing solution strategies, teachers can make necessary comparisons to find similarities and differences of different approaches and the affordance each approach offers.

Purposefully sequencing solutions strategies can also enable teacher deal with students' misconceptions because as teachers can identify possible errors in students' solutions strategies and guide them to correct those errors during the whole-class discussions. The overarching force behind teachers' decision making process about how to sequence student-generated mathematical ideas for further discussions should be the prerogative of mathematics teachers who have positioned themselves to helping students achieve a desired goal.

Connections. The final stage of the five practices is for teachers to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully. By making connections, teachers can help students build a holistic view of mathematical knowledge that will enable them to answer problems. This can be achieved by teachers helping students see the relationships that exist between mathematical operations, concepts, facts, etc. This is the stage where teachers bring everything together to make sense for students.

In applying these practices, teachers will have to be ‘quick thinkers’ in order to make informed decisions. Stein et al., (2008) argue that these five practices can be explicitly taught to novices who are learning to lead effective mathematics discussions. In that sense, it is not out of place to argue that preservice teachers who have been enrolled in standards-based mathematics methods course, in which these five practices are taught and modeled will be able to enact these practices during teaching.

Conceptual Framework

The purpose of this study was to find out how preservice teachers set mathematical learning goals that articulate the mathematics students will learn; how they design learning tasks and how they orchestrate effective mathematics discussions. The study also investigated the influence of the learning communities on preservice teachers as they learn to enact these core practices. The literature review provides a guideline into how this study was conceptualized in order to address the research questions.

From the Learning to Teach Framework by Hammerness et al., (2005), the learning community include the mathematics methods class, preservice teachers’ practicing school, families and friends as well as social media. The communities were categorized as ‘formal’ and ‘informal.’ The formal communities include the university where preservice teachers meet with their colleagues and the course professor to learn how to teach mathematics, and the practicing schools. These two communities are termed ‘formal’ because there were structured activities in these communities that were aimed at helping preservice teachers gain knowledge for learning and about teaching. At the university, there were organized teaching curricula that were followed. Also, preservice teachers engaged in structured activities in the course of the study. Similarly, at the practicing schools, preservice teachers are required to engage in apprenticeship of observation. On the other hand, the 'informal' community comprise families, friends, and other social media that impacts preservice teachers' way of learning to teach. These settings were considered 'informal' because there were no structured learning activities organized for preservice teachers.

Preservice teachers' understandings were considered to be the knowledge gained by participating in all the activities in these 'formal' and 'informal' communities in the process of learning to teach. These understandings provided preservice teachers with conceptual and practical tools in order for them to teach according to their vision. In the context of this study, preservice teachers' practices include the three core practices: setting learning goals, designing learning tasks, and orchestrating class discussions.

Figure 3 shows that preservice teachers acquire understanding and develop some dispositions as a result of their interaction with the learning communities. These understandings and dispositions are exhibited in their practices as they set learning goals, design tasks, and orchestrate classroom discussions. The three practices, setting learning goals, designing learning tasks, and orchestrating classroom discussions, are conceptualized to be connected because of the possible interactions that may exist among them. For example, in thinking about the learning tasks to use in a mathematics lesson and the kind of discussions to facilitate, preservice teachers may consider the overall learning goals they want to achieve at the end of the teaching-leaning session. Along this line of thinking, the learning goals will influence the tasks to use and how discussions should be held. This relationship is shown by the forward arrows from the learning goals to the learning tasks and the discussions. Similarly, the learning tasks may also suggest how discussions should be facilitated around the tasks in order to reach the learning goals. This is also shown by the forward arrow from the learning task to the discussions.

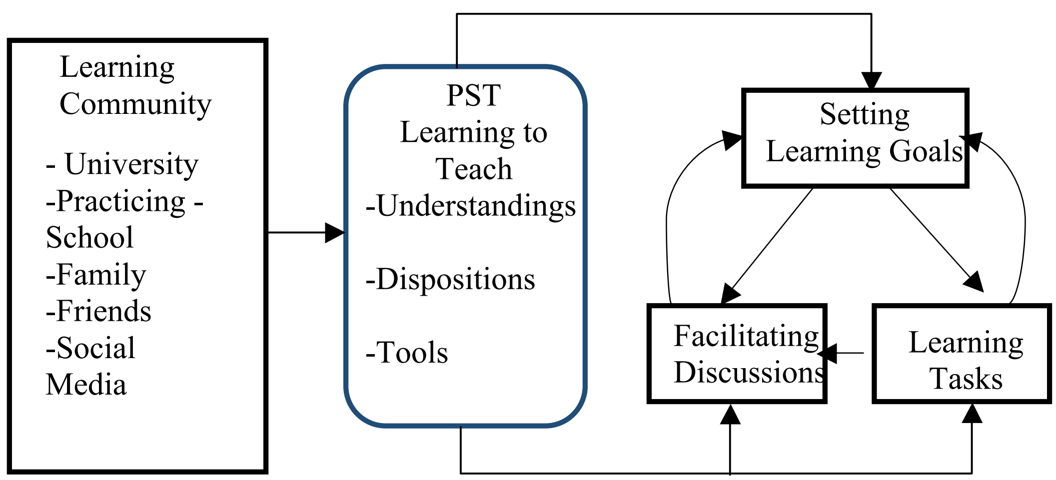

Figure 3: Conceptual framework for learning to teach.

As preservice teachers plan their learning tasks and the kinds of discussions they may facilitate for their lessons, they may be reflecting on the learning goals in order to make sure that the tasks and the discussions will be directed toward the learning goals. This is shown with the curved arrows from the learning tasks and the discussions to the learning goals.

Summary

National calls for reforms in mathematics education and standards documents have led to the kind of mathematical knowledge to teach and how to teach it. Instead of teaching mathematics in a way that does not engage students in the teaching and learning process, it is recommended that mathematics teachers use a teaching approach in which teachers create learning environments that support students to grapple with mathematical tasks in order to co-construct mathematical knowledge. In a mathematics class where teachers engage students in knowledge construction, teachers may use multiple modes of representation to enhance conceptual understanding. Also, teaching mathematics in a reform class requires teachers to be able to enact the core practices of teaching. Core practices involve the practical work of teaching, that is, the practices mathematics teachers enact during teaching and learning sessions. These core practices include setting learning goals, designing learning tasks to meet the goals, and orchestrating mathematics discussion around the tasks to meet the goals. Learning goals should be "targeted" and "shared" in order to serve their purpose.

Learning tasks comprise all activities in the learning segments students will engage in with the aim of learning a mathematical idea. Depending on the cognitive demands, learning tasks can serve several purposes. Leading mathematical discussions effectively is an important component in teaching mathematics. In order to lead mathematics discussions effectively, five practices of discussion facilitation are considered, namely, anticipating, monitoring, selecting, sequencing, and connecting. Although there are studies on the core practices for this study, such studies focus on these core practices in isolation. However, in teaching mathematics, these core practices are inextricably connected. Consequently, it is imperative to find out how these three core practices are related in the context of mathematics teacher education program. It is also important to know what kinds of resources preservice teachers use from their learning communities as they learn to enact the three core practices.

The literature and associated conceptual framework guided the research methodology for the study. Chapter 3 provides a detailed description of the research methodology. This is followed by the results presentation in chapter 4. Chapter 5 provides a discussion of the results of the study.

Chapter 3

Research Methodology

In the preceding chapters, it was argued that teacher development has undergone series of shifts with the aim of seeking productive ways of preparing teachers for teaching mathematics effectively. While some researchers hold the view that we need to detail teachers' mathematical knowledge needed to teach effectively, others believe that mathematics educators should concentrate on the practical work teachers do in the classroom. Researchers who take the latter stance on teacher preparation seek to place more emphasis preparing teachers to enact core practices that can make them effective mathematics teachers. The underlying philosophy for the core practices concept to teacher education has been that if teachers can enact teaching practices, for example, setting mathematical goals, designing tasks, leading effective discussions, they will be better prepared to teach effectively.

Inasmuch as one would support the view that teacher education should focus preservice teacher training on core practices, it is prudent to state that teacher educators who adopt the core practices concept of teacher preparation should be guided by empirical findings about these core practices and how preservice teachers enact them during the process of learning to teach. Knowing this will enable teacher educators to make informed decisions about teacher preparation and core practices. This study therefore, investigated three core practices: setting mathematics learning goals, designing learning tasks, and facilitating class discussions, in the context of an elementary mathematics methods class.

Overview of Research Questions and Rationale

The following research questions were formulated to guide the study:

1. In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from given state standards to support elementary school students' mathematics learning? What are the characteristics of the learning goals?
2. In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks?
3. In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?

The rationale for the first research question was to understand the practical road map preservice teachers undertake to translate mathematical standards into "digestible" bits of daily learning goals for students. The State Standards for mathematics (and now the Common Core State Standards) are often broad mathematical learning goals that students at specified grade levels are required to achieve at the end of a given learning unit. Usually, it takes several teaching days for teachers to help students learn a given mathematical standard. The implication is that teachers have to break down the mathematical standards into daily learning goals so that the cumulative effect of the daily learning goals will be helping students achieve the benchmark in the state standards.

Similarly, in the case of preservice teachers, when preparing lesson plans for teaching, they outline specific learning goals they want students to achieve at the end of a learning session. These learning goals are carved out of the mathematical standards with the assumption that if students achieve the specific daily learning goals, then they will also achieve the mathematical standard from which the learning goals were carved. Along this line of thinking, the learning goals that are set from the mathematical standards need to be targeted and purposeful in order to help students achieve the desired mathematical standard. Setting targeted and purposeful learning goals implies that each learning goal maps onto the mathematical standard. In this regard, care needs to be taken when setting mathematical learning goals from a given mathematical standard. Therefore, a clear knowledge about how these learning goals are articulated and the resources used in articulating these learning goals may serve as a guide in training future teachers on how to set learning goals from standards.

The second research question seeks to understand the characteristics of the mathematical learning tasks preservice teachers design and use to ensure that their students achieve the desired learning goals set by the teacher. In addition, how preservice teachers draw on resources from their learning communities to design learning tasks was also explored. The tasks include how preservice teachers adapted or adopted learning tasks from textbooks or other sources to help them achieve desired learning goals. As explained in chapter 2, learning tasks are the segments in the teaching and learning process, in which students are asked to perform certain activities with the aim of attaining specific skills, concepts, facts, theorems, etc. This includes the tasks that are adapted or adopted from textbooks or other sources. From this point of view, the learning tasks should be directly linked to the desired learning goals so that by performing those tasks students will be helped to achieve the learning goals. Consequently, knowing how preservice teachers design learning tasks to meet specific learning goals will provide guidelines as to how future teachers can be trained to design learning tasks.

Findings from the third research question was used to shed more lights on how the participants of this study planned and enacted the five practices for facilitating whole-class discussions, discussed in Stein et al., (2008), around learning tasks to achieve desired goals. By knowing the effective practices for facilitating classroom discussions, teachers will be aware of productive practices for orchestrating mathematical discussions so that they can focus their classroom discussions meaningfully towards desired learning goals.

The study also sought to find out how the formal and informal learning communities as well as preservice teachers' personal experiences influenced their decision making processes around these three core practices. Having a better understanding of the impact of the learning community will help teacher educators shape teacher preparation to ensure that preservice teachers obtain the productive information from the learning communities. In addition, having a better understanding of how preservice teachers' personal experiences influence their decision-making around the core practices, may help teacher educators shape preservice teachers' perceptions about teaching and learning so that preservice teachers will be better positioned to draw from their past experience to make informed decisions around core the practices.

Although there have been several studies on core practices, there is still much to know about how preservice teacher translate mathematical standards into daily learning goals and how discussions are facilitated around specific learning tasks towards the learning goals. Therefore, findings from this study will shed more light on the kinds of resources preservice teachers draw on to set mathematical learning goals and design mathematical tasks that will address the learning goals. In addition, findings from the study will provide new knowledge on how the five practices for orchestrating whole-class discussion described by Stein et al., (2008) are planned and enacted by preservice elementary mathematics teachers.

The purpose of this chapter is to delineate the procedures used to answer the research questions in order to contribute to the literature on how teacher education can be structured to help preservice teachers learn to enact these three core practices. The study context, access to the mathematics methods class, participant selection procedure, and research design are described first. The data sources and instruments used and the analysis procedure are also outlined. Finally, measures that were taken to ensure trustworthiness of this study are presented.

Context for the Study

Description of the methods class. The context for the study is a three-credit mathematics methods class, *Teaching Mathematics in the Elementary School*, and its associated student teaching components designed for preservice elementary mathematics teachers in a large Midwestern University. Most of the students in this program completed their undergraduate degree in elementary education foundations and are completing their licensure program and Master of Education (M.Ed.) as part of a fifth year program. The class was designed to support teaching mathematics as envisioned by the National Council of Teachers of Mathematics' *Principles and Standards* document (NCTM, 2000). It was taught by Dr. Shelly, an associate professor of mathematics education who has developed and taught several mathematics content and methods courses for elementary preservice teachers at the university.

The methods class was designed for preservice teachers to meet at the university for class sessions, and also work with a cooperating teacher in a local school. At the university setting, preservice teachers were expected to participate in all class meetings and complete all class assignments, activities, and projects. The other part of the course required preservice teachers to engage in a practicum in an elementary school under the mentorship of a cooperating classroom teacher. Stemming from a master-apprenticeship relationship, the preservice teachers were expected to observe the cooperating teacher to learn the work of teaching. In addition, the preservice teachers were encouraged to co-plan and co-teach some lessons with the cooperating teacher until preservice teachers did their full student teaching.

Goals of the methods class. The goals of the methods class include: helping preservice teachers develop a philosophy for how to teach young children mathematics, which reflect what is currently known about how students learn; the NCTM standards and effective mathematics teaching practices (same as Core Practices); helping preservice teachers appreciate the importance of teaching mathematics for conceptual understanding by using multiple representations and emphasizing connections within and between representations; and be conversant with the NCTM eight practices for teaching mathematics with particular emphasis on establishing mathematical goals, designing and implementing tasks to promote reasoning and problem solving, and facilitating meaningful mathematical discussions.

Materials used for the methods class. The *Principles to Action: Ensuring Mathematics Success for All* (NCTM, 2014), *Making Number Talks Matter* (Humphreys & Parker, 2015) *and Cognitive Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson, 2015) were three required books for the class. All the preservice teachers had copies of these books and were required to read assigned sections before class. In addition, preservice teachers were assigned several readings which they had to complete before class started.

The *Principles to Action* (2015) contains eight Mathematics Teaching Practices (the same as core practices) which are considered important mathematics practices for effective teaching. These eight teaching practices, which are all researched-based, are considered to be effective teaching practices teachers are required to enact for effective mathematics teaching and learning. Among these eight Mathematics Teaching Practices, three of them were the focus of the methods class. These include setting mathematical learning goals, designing learning tasks, and orchestrating effective mathematics discussions. These core practices were the focus of this study.

The second resource material was the *Cognitively Guided Instruction* (CGI) book from Carpenter et al., (2015). The overarching idea of the CGI book is that children invent their own strategies to solve mathematical tasks and, as such, teachers should elicit students' thinking by using appropriate tasks to help them learn mathematics meaningfully. The book also shares ideas on students' thinking related to early number acquisitions. When asked to solve tasks, children use direct modeling with manipulative, representations, and verbal explanations to express their thinking processes. During the methods class, there were series of discussions about how to elicit students' thinking and how to guide them when they are engaged in performing mathematical tasks. In some cases, the class watched videos of children solving mathematical tasks as a platform for class discussions. These videos are part of the CGI book that shows how elementary students draw from their repertoire to operate on whole numbers. While the CGI book helped the preservice teachers to understand that children come to the class with a wealth of knowledge and that such knowledge should be explored, the third book explained how preservice teachers can engage children in mental mathematics.

Another required text for the methods class was *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grade 4-10.* According to Humphreys and Parker (2015), *Making Number Talks Matter "*is about helping students take back the authority of their own reasoning through a short, fifteen-minute daily routine called Number Talks, in which they reason mentally with numbers" (p.1). Number Talks, the authors explain, is "a brief daily practice where students mentally solve computation problems and talk about their strategies" (p. 5). In this sense, using Number Talks as a daily routine requires that preservice teachers understand students' abilities and the type of computational tasks students can be asked to solve during Number Talks.

During the methods class, preservice teachers were introduced and engaged in Number Talks. They were also required to write a project on Number Talks, in which they selected computational tasks and engaged students in Number Talks. This project was submitted to Dr. Shelly. Unlike the CGI which requires teachers to allow students to demonstrate their invented strategies through the use of direct modeling, pictures, and symbols, Number Talks requires that student use only verbal explanation to communicate their thought processes. In addition to these books, preservice teachers were assigned to read some peer-reviewed articles that were connected to the topics discussed in class.

The peer reviewed journal articles were used to study the theoretical groundings of the mathematics teaching practices and other important practices about teaching and learning. Among the peer reviewed journal articles is *Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell.* In this paper, Stein, Smith, and Hughes (2008) argue that in order to effectively lead mathematics discussion, without simply telling students answers, teachers should use five practices to guide class discussions. As already explained, these five practices require that teachers anticipate the mathematical thoughts processes students are likely to bring to the learning environment, monitor students as they engage in activities, select exemplary works from students that could be shared with the whole class, sequence the selected exemplars, and connecting all the selected mathematical ideas and thoughts meaningfully to achieve a desired goal. These five practices were discussed and modeled by Dr. Shelley during the methods class sessions.

Pedagogy and content of the methods class. As a standard-based methods class, preservice teachers were exposed to group work, activity-based learning, problem solving tasks, and other reform-based teaching and learning practices that have been set forth by the NCTM. These teaching strategies were facilitated by modeling the five practices (anticipating, monitoring, selecting, sequencing, and connecting). As Dr. Shelly modeled the five practices, preservice teachers' attention was drawn to those practices for them to see how each of the practice was modeled. Mediating the explanations and enacting of the practices was the use of multiple representations to facilitate the teaching and learning process.

The Lesh Translation Model (LTM) (Figure 1), (Lesh & Doerr, 2003) was used to introduce preservice teachers to multiple representations. The model consists of multiple modes of representation which enables students represent mathematical concepts using multiple modes of representation and being able to translate between and within multiple modes of representation (Cramer, 2003). A translation is the ability

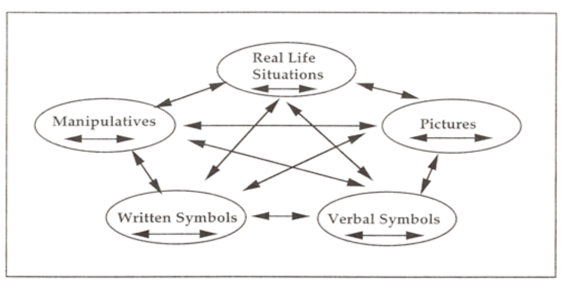

Figure 1. The Lesh Translational Model.

to model a mathematical concept within the same representation or between different representations (Cramer, et al., 2009). By using the LTM, preservice teachers were taught how they could use multiple representations to facilitate students' learning. These pedagogical skills were demonstrated across several mathematical content.

The methods class prepared preservice teachers on several mathematics content and pedagogical skills. Table 2 shows the topics that were covered in the methods class and the rationale for those topics.

Table 2

Content of the Math Methods Course and their Objective

Topics treated	Objectives for the the pre-service teachers
NCTM and Common Core Standards, Learning Theories, State Frameworks, Modeling a Standards-based lesson	To be conversant with the various mathematical standard, some learning theories.
Learning theories, Lesh Translational Model, Instructional examples with Steins five practices	To understand the use of multiple representations and acquire skills for leading effective mathematics discussions.
Lesson Planning and classroom discourse; Addition and Subtraction problems from Cognitive Guided Instruction	To understand that students have invented strategies for operating on whole numbers. Lesson planning
Multiplication and Division problems from Cognitive Guided Instruction; Developing student interviews; Number Talks	Using Number Talks to engage students in mental mathematics.
Place Value and Whole Number Arithmetic; Number Talks	To understand place value
Fraction concepts, order and equivalence	To understand the concept of Fractions and the materials for teaching Fractions.
Fraction operations	To gain conceptual understanding in Fraction operation.
Number line model; Decimal and Academic Language	To use different models for teaching Fractions and Decimals.
Algebra for K-6,	Introduce pre-service teachers to Algebra.
Geometry—Van Hiele model	Introduce pre-service teachers to Geometry
Number Talks; Assessing student work	Making sense of students’ work

Participants

Access to the methods class. During the Summer of 2015, permission was sought from Dr. Shelly about using her mathematics methods class and some of the preservice teachers for this study. Following the initial agreement with Dr. Shelly, an e-mail was sent to the class to inform them that a graduate student will be visiting the class to explain a research study and ask for volunteers to participant in the study. On the first day of the methods class, in Fall 2015, information regarding the study was shared with the preservice teachers. A class survey was administered to all the preservice teachers (See Appendix A) to collect data on their demographic information, mathematics content and pedagogy classes taken since high school, challenges and areas of strengths in foundational mathematics classes taken, their beliefs about the nature of mathematics and how it should be taught, rationale to be mathematics teachers, and their professional experiences.

The course instructor explained to them that their responses would serve two purposes. First, the survey responses were to enable Dr. Shelley better understand preservice teachers' backgrounds and needs so that she could make informed decisions about how to teach and also make necessary modifications to the course. The other reason was that the responses would help the researcher select potential participants for the study. All the 33 preservice teachers in the class authorized the instructor to share their survey responses with the researcher in order to select participants for the study. The surveys were completed electronically and were anonymous. Thus, no student was asked to write the name on the survey. This ensured that students' identities were not revealed at the time of analyzing the surveys for participant selection.

Participant selection. Preservice teachers' survey responses were read to find out those who responded to all the questions. Students who did not provide detailed information, or did not respond to some of the survey items were not considered to be participants of the study because it was assumed that those preservice teachers may not provide detailed information when they are selected for the study. The survey responses for potential participants were read again to identify if there were common themes about the characteristics of the preservice teachers enrolled in the methods class. After analyzing the remaining survey responses, three categories of preservice teachers were identified: the "*enthusiasts*," the "*optimists*," and the "*uncomfortable*." Six preservice (two from each category) teachers were purposefully selected and contacted for the study. According to Patton (2002), purposeful sampling enables researchers to select "information-rich cases for study in depth" (p. 230). Along this line of thinking, the purposive sampling method was appropriate for this study because it helped the researcher to obtain key informants for the study.

The *Enthusiasts* were the preservice teachers who indicated that they like mathematics and are very enthusiastic about being mathematics teachers. Those in the second category, the *Optimists*, were the preservice teachers who indicated that although they have not been very good in mathematics and that they have had negative experiences as mathematics students, they would do "whatever it takes" to be good mathematics teachers. Unlike the preservice teachers in these two categories who were positive about teaching mathematics, the third group of future teachers, the *Uncomfortable*, were the preservice teachers who indicated that they were uncomfortable about teaching mathematics, or that teaching mathematics was “scaring” for them.

The preservice teachers in each category were ranked in order of preference as participants based on how they responded to the survey. Students who provided in-depth responses to the survey items were ranked higher than those who did not provide detailed responses. This was done with the assumption that preservice teachers who provided more information from their survey responses will be more likely to participate fully in the study than those who did not provide more information.

The first two candidates from each category were given to Dr. Shelley who contacted them through e-mail to seek their consent about participating in the study. Out of the six students who were contacted, five accepted to participate in the study immediately. Because the sixth preservice teacher was delaying in responding to the email, after being prompted, another student from that category was contacted and confirmed as a participant for the study. Table 2 shows the three categories of participants for this study.

Two students from different categories were selected to present cases that will either predict similar results or those that will predict contrasting results (Yin, 2013). Selecting participants with varied backgrounds (expectations from the course, rationale for being a mathematics teacher, and confidence in teaching mathematics) helped to see how participants’ background influences their decision making process around the three core practices.

Table 3

Description of Participants' Categories and Sample Quotes

Category	Description	Selected Quote
Enthusiasts (Jocelyn and Phoebe)	Participants who are very enthusiastic about teaching mathematics	"I love kids and I love math! Teaching has always been something that I've been interested in"
Optimists Anna and Rachel	Participants who believe that the bitter experiences as math students will energize them to do whatever it takes to be good mathematics teachers	"I know what it feels like to be a struggling math student and I would do whatever it takes so my students do not have to feel this way"
Uncomfortable Samantha and Morgan	Participants who are not comfortable teaching mathematics, but have found themselves in the math teachers web	"Teaching math is something I am uncomfortable with; it is essentially, "scary" for me. I want to overcome the fear of teaching it"

Participants' mathematics background. The preservice teachers in the methods class were elementary preservice teachers who have taken the *Mathematics and Pedagogy for Elementary Teachers I and II* foundational courses as undergraduates in the elementary education foundations major at the University. In the first of these two courses, the preservice teachers learned content related to functions, probability, number and numeration in an environment that modeled K-6 mathematics pedagogy that they are expected to implement in their graduate program leading to licensure. The course was an integrated model of content and methods designed around problem solving, connections, communication, reasoning, and representations. The second part of the course, *Mathematics and Pedagogy for Elementary Teachers II*, differs from the first part by the mathematics content (Geometry, Measurement, Probability, and Statistics) that was taught. All the preservice teachers had taken courses in College Algebra. Apart from these courses, the students had varying backgrounds in terms of mathematics content courses they had taken from high school, or at the college.

Contexts of the Practicing Schools

The six preservice teachers taught in four schools, namely, Woods, Adams, Birch, and Atomic Elementary from four school districts: Oxford, Ashtown, Brandon, and Maranatha. Table 3 shows preservice teachers' practicing schools, their school districts, and the mathematics curricula that are used by the districts. Two school districts (Ashtown and Maranatha) used the same mathematics curricula, *Everyday Math* (University of Chicago Center for Elementary Math and Science Education, 2004) whereas the other two school districts (Oxford and Brandon) used the *Investigations* (Technical Education Research Centers, 2004) and *Math Expressions* (Houghton Mifflin Math Expressions, 2004) curriculum respectively. In addition to the mathematics curricula that were used, Oxford School District uses *Focused Instruction* as a teaching guide for instruction. Focused Instruction aligns content to be taught with how it should be taught and what to assess. It is focused on ensuring that there is high learning expectation for all students in the district. One way to support Focused Instruction is to provide curriculum guide and unit assessment to guide teachers in pacing their lessons (Oxford School District, 2015). The following table summarizes the mathematics curriculum used in the various schools.

Table 4

Participants' Schools, School Districts, and the Math Curriculum used

Participant	School Taught (Elementary)	Math Curriculum used	School District
Anna	Woods	Investigations	Oxford
Jocelyn	Adams	Everyday Math	Ashtown
Morgan	Birch	Math Expressions	Brandon
Phoebe	Atomic	Everyday Math	Maranatha
Rachel	Adams	Everyday Math	Ashtown
Samantha	Atomic	Everyday Math	Maranatha

Another important context for the practicing schools was the demography of the students as well as the type of student population in each of the schools. The type of student population refers to students who are categorized as (a) English Learners, (b) needing special education, (c) beneficiaries of free/reduced priced lunch, and (d) homeless. Table 5 illustrates student enrollment by demography and population type.

Table 5

Students Enrollment by Demography and type of Population

	Practicing Schools			
Ethnicity	Woods Elementary	Adams Elementary	Birch Elementary	Atomic Elementary
American Indian/Alaskan Native	17 (3.4)	7 (1.3)	13 (1.2)	1 (0.6)
Asian/Pacific Islander	15 (3.0)	31(5.7)	197 (17.8)	24 (15.4)
Hispanic	135 (26.7)	221(40.6)	255 (23.1)	19 (12.2)
Black, not of Hispanic Origin	253 (50.0)	176 (32.4)	490 (44.4)	95 (60.9)
White, not of Hispanic Origin	86 (17.0)	109 (20.0)	149 (13.5)	17 (10.9)
All Students	506 (100)	544 (100)	1,104 (100)	156 (100)
Special Population				
English Learner	253 (50.0)	215 (39.5)	259 (23.5)	45 (28.8)
Special Education	79 (15.6)	44 (8.1)	145 (13.1)	15 (9.6)
Free/Reduced priced lunch	418 (82.6)	472 (86.8)	901 (81.6)	138 (88.5)
Homeless	23 (4.5)	18 (3.3)	21 (1.9)	13 (8.3)

Note. First number is the total count and the second number is the percentage of the count.

Research Design

In order to understand how preservice teachers develop effective mathematics teaching practices within different contexts, it was important to take a deeper inquiry into the real life experiences—that is, preservice teachers' interaction within their formal and informal communities—in the process of learning to teach. As such, a case study design was used for this study. Case study enables the researcher to gain deeper insight of a real life "case" to be studied (Yin, 2013). In this study, a "case" constitutes preservice teachers and the learning communities that influence their decision-making as they enact the three core practices, namely, setting mathematical learning goals, designing learning tasks that align with the goals, and orchestrating effective classroom discussions around those tasks with a focus on the learning goals.

A multiple-case design (Yin, 2013) with six preservice teachers was used in carrying out this study. Each of the preservice teachers served as a case for analysis to gain better understanding of how individual's background and context influenced their decision-making around these three teaching practices. After analyzing each case, a cross-case analysis was performed to see if there were overlapping themes among the cases. Several instruments for data collection were used to collect data to answer the four research questions.

Data Sources and Instruments

The data collected for this study were categorized into three forms, namely, observational data, interactive data, and artifacts. Observational data consist of the data collected during observation of the methods class and students teaching sessions. These data were taken in the form of field notes. Interactive data, on the other hand, is the data collected from all audio recordings. These include audio recordings of interviews, methods class interactions, student teaching interactions, and debriefing from student teaching sessions. Artifacts include written materials (articles, course syllabus, class work) from the mathematics methods class, preservice teachers' written assignments (lesson plans, Number Talk project, Student Interview project), and sample of students' work collected during preservice teachers' teaching practice.

Observational data. Two types of observational data were collected, namely, observation of the mathematics methods class and student teaching observations. During the mathematics methods class and students teaching observations, field notes were taken carefully to document processes and actions closely as they unfolded in the research sites. In view of this, close attention was given to the specific things that were central to this study. In this case, an observational protocol (see Appendix D) was used to observe all the methods class sessions and the preservice teachers' teaching practice. The protocol served as a guide to document specific classroom activities around the three core practices (setting learning goals, designing tasks, and facilitating classroom discussions).

Methods class observation. During the methods class sessions, an audio recorder was placed on top of Dr. Shelley's desk in front of the class while filed notes were taken at the back left corner of the classroom. Occasionally, there was the need to circulate in the class to observe the activities preservice teachers engaged in. After each class session, these observational notes (often in scratched form) were typed and a reflection from the observation written. In addition to observing issues around the three core practices closely, the general class activities were also observed and documented.

All the whole-class interactions between Dr. Shelley and the preservice teachers were recorded. Also, students' interactions and how they were engaged were monitored and recorded. A special attention was also given to the six participants in the study.

Participants' contributions in the class, as well as their participation in class discussions and activities were noted. All the interactions that occurred in the methods class were also recorded for triangulation purposes during the analysis stage of the study. The observations helped build deeper understanding of the context of the study in the sense that it afforded the opportunity to understand the activities of the preservice teachers. Further, observing the methods class also helped gain better understanding of the study in general.

Student teaching observations. In order to better understand preservice teachers' discussion facilitation and how they enact the core practices, the observational protocol (See Appendix D) was used to keep track of preservice teachers' discussion facilitation around the tasks and the learning goals. The five practices for facilitating effective mathematics discussions (anticipating, monitoring, selecting, sequencing, and connecting) were specifically looked out for during the teaching. Similar to the methods class, all activities that occurred during student teaching were observed and recorded. This afforded the chance of gaining deeper understanding of how preservice teachers enacted the core practices. The field notes were used to correspond to preservice teachers' lesson plans to find out if preservice teachers enacted all the things they had written in their lesson plans.

As part of the requirement of the methods class, the preservice teachers were required to prepare and teach a three-day lesson plan in their practicing schools. The lesson plans were to be prepared and submitted to Dr. Shelley, the course professor, for reviews and feedback before they were taught. Sample lesson plan (See Appendix F) was given to the class as a guide for writing their lesson plans. The unit/topics taught were identified by the preservice teachers in consultation with their cooperating teachers. These units followed from the class schedules that were being used. In the case of Anna, the school district pacing guide, *Focus Instruction* was used to determine the mathematics content to teach. As a result of the mathematics content selection process, the preservice teachers did not have a choice of what to teach. Table 6 shows the mathematical standards and grade levels preservice teachers taught during the three-day teaching periods. Students' teaching sessions were observed.

In order to record the class interactions, audio recorders were sometimes given to the preservice teachers to place it at their point of convenience, or it was put somewhere that did not distract the learners as field notes were taken. These decisions about where to place the audio recorder were done in consultation with the preservice teacher and the classroom teacher. The purpose was to ensure that the students were not distracted by the audio recorder. Field notes were taken to record all the class activities with a special focus on the three core practices. The audio recordings were played back to fill in the field notes.

Interactive data. Interactive data include data obtained from audio recordings from the mathematics methods class conversations, student teaching conversations, debriefing sessions with preservice teachers, and interviews. As stated earlier, all the methods class sessions (with exception of the first day), the student teaching interactions, and debriefing sessions with the preservice teachers were recorded during the course of the study. In addition to this, two interview sessions were administered to the preservice teachers.

Table 6

Student Teachers Grade Levels Taught and the Standards Addressed

Name of Participant	Grade Taught	State Standard
Anna	First	1.3.2.3 Identify pennies, nickels, and dimes; find the value of a group of these coins, up to one dollar.
Jocelyn	Fourth	4.1.1.1 Demonstrate fluency with multiplication and division facts
Morgan	Fourth	3.1.1.4 Round numbers to the nearest 10,000, 1000, 100 and 10. Round up and round down t estimate sums and differences. 3.1.2.1 Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place values, including standard algorithms. 3.1.2.2 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of the results.
Phoebe	KG	K.1.1.1 Recognize that a number can be used to represent how many objects are in a set or to represent the position of an object in a sequence. K.3.2.1Usewords to compare objects according to length, size, weight and position. K.3.2.2 Order 2 or 3 objects using measurable attributes, such as length and weight.
Rachel	Third	3.1.1.1 Read, write and represent whole numbers up to 100,000. 3.1.1.2 Use place value to describe whole numbers between 1000 and 100,000 in terms of ten thousands, thousands, hundreds, tens and ones.
Samantha	Second	2.2.2.2 Identify, create, and describe simple number patterns involving repeated addition or subtraction, skip counting and arrays of objects such as counters or tiles. Use patterns to solve problems in various contexts.

The interactive data helped gain better understand about how preservice teachers set their learning goals, designed learning tasks, and orchestrated classroom discussions around these tasks as well as the factors that influence their decision-making around these teaching practices. By interviewing the preservice teachers, it gave them a voice in the data collected (Talmy, 2010). Giving prospective teachers the chance to express themselves through interviews, not only enhanced better understanding of the data collected, but also provided the opportunity to elicit more information from the preservice teachers.

Preservice teachers' interviews. The first interview was to follow up with the preservice teachers to gain better understanding of their survey responses. These interviews were conducted about two weeks into the methods class. It occurred after the preservice teachers' survey responses had been analyzed and the six participants selected. The survey items were used as a guide for the interviews. Before the interviews, preservice teachers were asked to verify whether the survey that was to be used for the interview was the correct survey response. This was to ensure that the participants were asked the right questions. The interviews lasted between 15-30 minutes depending on the probing and follow-up questions that emerged from the interview.

The second interviews were conducted after preservice teachers had written their lesson plans. An interview protocol (See Appendix E) consisted of open questions around how preservice teachers set learning goals, design tasks, how they intend to facilitate classroom discussions, as well as the factors they considered in making their decisions around the three core practices was used to guide the conversation. In addition, the resources that were used to set learning goals and designing learning tasks were also explored. However, there were a number of follow-ups during the interview, allowing the researcher to explore for details. These interviews were conducted in a conversational manner to enable preservice teachers speak deeply about the issues. With the exception of Megan, all the preservice teachers were interviewed before they started teaching with their lesson plans. Megan was interviewed after her first day teaching. This happened because she needed more time to complete her lesson plan.

The data generated from the interviews served two purposes. In one sense, they served as preservice teachers' perceptions about setting learning goals that meet state standards, designing learning tasks, mathematics discussion facilitation, and how that helps teachers in the teaching and learning process, as well as the influence of the learning community on their decision-making process. In the other sense, the data enabled triangulation of findings thereby enhancing the validity of this study.

Debriefing sessions. Three debriefing sessions were conducted after the participants had taught each of the lessons. These were conducted immediately after preservice teachers had taught their lessons, or some hours after their teaching, based on preservice teachers' schedule. With the exception of one debriefing session which was rescheduled because the preservice teacher was busy, all the others were done on the same day of teaching. These debriefing sessions served two purposes.

The first purpose was for my research study. For this purpose, the debriefing sessions (average length of 15 minutes) were used to seek clarification of the preservice teachers' teaching moments and actions that were not clear to me, and also, to offer opportunity for the preservice teachers to share their thoughts about their teaching. The session started by asking preservice teachers to give a general overview of their lesson, state what went well and what may be done differently. Some of the questions include, “How did the lesson go?” “What are your plans for the next lesson?” Specific questions about preservice teachers’ learning goals, the tasks they used in the class, as well as their discussion facilitation were asked. This was done to better understand preservice teachers’ perception about how they worked towards achieving their learning goals using the tasks. Asking preservice teachers about their next plans for the next lesson was intended to see their dispositions and reflections for their teaching. The second reason for the debriefing sessions was to give preservice teachers some feedback about their teaching style. This was done in such a way that the data collected for the study were not compromised. Questions asked to give feedback included, “How did you see your teaching?” “What went well in the lesson?”

Although the debriefing was in the form of an interview, it was more conversational and in a relaxed mood. This enabled the preservice teachers to share their deeper thoughts about their teaching. Table 7 summarizes the times of observation for the participants.

Table 7

Student Teaching Observation and Data Collected

Participants	Days of Teaching	Time of Teaching	Lesson Observed	Audio-Recorded Lesson	Debriefed
	1				Yes
Anna	2	9:30am -10:20am	Yes	Yes	Yes
	3		Yes	Yes	Yes
	1		Yes	Yes	Yes
Jocelyn	2	10:00am - 11: am		Yes	Yes
	3		Yes	Yes	Yes
	1		Yes	Yes	Yes
Morgan	2	1:30pm – 2:30pm	Yes	Yes	Yes
	3		Yes	Yes	Yes
	1		Yes	Yes	Yes
Phoebe	2	2:10pm – 3:00pm	Yes	Yes	Yes
	3			Yes	
	1		Yes	Yes	Yes
Rachel	2	8:15am – 9:15am	Yes	Yes	Yes
	3		Yes	Yes	Yes
	1		No teaching		
Samantha	2	9:50am –10:55am	Yes	Yes	Yes
	3		Yes	Yes	Yes

Notes. Samantha taught for two days because her cooperating teacher used the first day to complete a work she had started with the students. The teaching times for Anna and Jocelyn were overlapping so that did not allow me to fully observe all of their three lessons. Researcher was not available to observe Phoebe's third lesson.

Preservice teachers audio-recorded their lessons during the times they were not observed. All the audio-recorded lessons were played several times to compare with the self-report from the preservice teachers. The self-report, the audio recordings, and the debriefing sessions provided information about how preservice teachers facilitated classroom discussions around the learning tasks to achieve the learning goals. All the interactive data were replayed to annotate the field notes. In addition to the interactive and observational data, written data were also collected.

Artifacts. Artifacts consist of written documents from the methods class, preservice teachers' written assignments, and students work collected during the student teaching observations. The methods class documents include the course textbooks, syllabus, lecture notes, and research articles that were assigned for reading. These materials served as reference materials that were read to gain better understanding of what was taught in the methods class. Artifacts from preservice teachers include their written assignments (students interview project, Number Talk project, and Lesson Plan) and in-class activity sheets were used to address the specific research questions. Preservice teachers' lesson plans were used in the analysis to answer the research questions. During the student teaching observations, samples of students' work were also collected. These work samples were used to compare with the enacted tasks students engaged in during the teaching and learning process.

Data analysis procedures. In seeking to find out how preservice teachers set learning goals that meet state standards, design learning tasks which address to the learning goals, and facilitate classroom discussions effectively, as well as exploring the effect of the learning communities on preservice teachers, multiple strategies and analytic frameworks were used to analyze the data in order to answer the research questions.

For each of the research questions, data were analyzed on case-by-case basis to answer the research questions as pertained to individual cases before cross-case analysis.

To begin with the analysis, interviews were transcribed and multiple copies were made for separate analyses for each research question. Also, the NVivo software was used for coding. Deductive and inductive coding were used as deemed appropriate for each research questions. While deductive codes enabled me to fit data into existing frameworks, inductive coding was used to develop new codes that did not fit into the prescribed codes. For all the inductive coding, an open coding approach was used to assign new codes. After the first round of coding, multiple rounds of coding were done to check for consistency in the codes (constant comparative analysis) (Cohen, Manion, & Morrison, 2011). Constantly comparing the codes allowed me to confirm the codes, or to find discrepancies in the codes.

After careful and thorough coding, patterns were observed. These patterns were used to form categories and possible themes that could emerge from the entire analysis. After analyzing the data of a case in relation to the research questions, similar analyses strategies were used to analyze all the other cases for the same research question to ensure objectivity in the analysis. Ensuring objectivity in the analysis helped to validate the findings from the study. Finally, cross-case analyses were conducted to check for possible overlapping themes among the cases. In order to make the final findings valid, different data sources and methods were used for triangulation to ensure that the final findings were well grounded and warranted by data.

The triangulation was done using matrices and data displays (Miles & Huberman, 1994) for possible findings to check how different data sources (lesson plans, field notes, interviews, reflections, and project work) or methods (observation, interviews) confirmed or disconfirmed the findings. Looking for confirming or disconfirming cases enabled me to warrant findings and assertions (Erickson, 1986). In the following section an outline of how each research question was analyzed is given.

Research questions and analysis strategies. *Research Question 1: In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from given state standards to support elementary school students' mathematics learning?* Three data sources were used to answer research question 1. These include, preservice teachers' lesson plans, interviews, and field notes. In their lesson plans, preservice teachers were required to state content and academic language goals for each lesson. The content goals specified the mathematics content students were going to learn and the academic language goals illustrated the mathematical vocabulary students were required to acquire to demonstrate their understanding of the mathematics knowledge learned. All the learning goals that were stated in the lesson plans were identified. The goals were analyzed using a rubric (see Appendix B) to assess how the learning goals addressed the Minnesota State standards, and how the goals identified the mathematical knowledge students were going to learn at the end of an instructional session

The survey follow-up interviews provided information about what preservice teachers perceived about learning goals as they came into the program. The lesson plan interviews were used to explore the resources preservice teachers used to set learning goals as well as how those resources were used to set the learning goals. The transcripts from the interviews were coded using an inductive-deductive coding approach (Miles &

Huberman, 1994). The deductive component of the coding resulted from the fact that the researcher wanted to categorize the resources as university-based resources, practicing-school-based resources, personal decisions, or resources from other sources. Using these categories, an open coding approach was used to code the interview transcripts inductively. These codes were refined as other cases were analyzed to ensure consistency in the codes. Table 8 shows sample codes and their categories. These were codes from analyzing the learning goals.

Table 8

Sample Codes and Categories Generated from Analyzing the Learning Goals

Codes	Category
Consulting district curriculum	
Cooperating Teacher help	School-based resources
Using state standard	
Breaking down benchmark	
Focus on end-goal	Personal decisions
Importance of math	
Knowledge about students	

Research Question 2: In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks? The second research question was used to explore the resources and factors that were used to set the learning tasks, the characteristics of the learning tasks and how the learning tasks addressed the learning goals. Interview transcripts related to the learning tasks were coded for university-based resources, practicing school-based resources, personal decisions, and other resources. In order to categorize the learning tasks, all the tasks that were enacted during the student teaching practice were identified and categorized.

According to Stein, Grover, and Henningsen (1996), a mathematical task is "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea." (p. 460). Using this as a working definition, all the learning tasks preservice teachers used during their teaching were identified. In line with the working definition, tasks comprised all the mathematical related activities preservice teachers asked students to do during the teaching and learning process in order for students to learn a particular mathematical idea. From this perspective, all activities related to a given mathematical idea was considered as part of a give task. For example, asking students to make up 50 cents using a combination of pennies, nickels, and dimes was considered as a task. Also, asking students to write their division word problems was considered as a mathematical task. Tasks that were stated in preservice teachers' lesson plans but were not enacted in the class were not used for the analyses. This was because the enacted tasks were used to help students achieve the learning goals.

In analyzing the learning tasks, an inductive coding approach was used to code all the learning tasks based on what the tasks demanded students to do. In order to characterize the learning tasks, Stein, Grover, and Henningsen (1996)'s frame work for categorizing learning (See Appendix C) tasks was used to develop deductive codes to categorize the learning tasks. The framework categorizes tasks based on cognitive demands. There are two major cognitive demands, namely, low-level cognitive demand tasks and high-level cognitive demand tasks. Low-level cognitive demand tasks were further categorized as memorization tasks and tasks that require students to use mathematical procedures without making connections to concepts. High-level cognitive demand tasks were also categorized as tasks that require students to use mathematical procedures with making connections to concepts and tasks that required students to "do mathematics" that is, engage in high-cognitive processes like using mathematical knowledge in novel situations to solve problems.

In addition to categorizing the learning task based on their cognitive demands, the learning tasks were also mapped to the learning goals in order to see how the learning tasks address the learning goals. In doing this mapping, the mathematical knowledge provided by the learning tasks were assessed to see whether the mathematical knowledge addressed the essential ideas required by the state standards.

Research Question 3: *In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?* In seeking to answer this research question, preservice teachers' lesson plans and the field notes were used for analyses. The field notes were used to confirm whether preservice teachers enacted all the discussion practices stated in their lesson plans. Only enacted practices were coded from the lesson plans. Coding was done using a deductive-inductive approach.

The deductive codes consist of Stein's five practices for orchestrating effective mathematics discussions. These include anticipation, monitoring, selecting, sequencing, and connecting. Based on the definitions from the authors and the literature, the five practices were used as deductive codes (See Appendix D) for specific classroom instances that fitted into each of the five codes. The inductive coding was used to code all other classroom discussions that did not fit into any of the five prescribed codes. While the deductive coding helped me to categorize specific discussions practices, the inductive coding enabled me to find possible rationale and linkages among the practices that were enacted around the learning tasks to with the aim of helping students achieve the learning goals.

The above research questions were used to explore the effects of the learning communities on preservice teachers' decision-making processes around the three core practices. In order to achieve this goal, the conceptual and practical tools and understandings from the formal and informal communities as well as preservice teachers' personal dispositions were explored to find out how those factors influenced preservice teachers in the processes of engaging in these core practices. In order to do that, all the data from the interview sessions were used for analysis. From the preservice teachers' lesson plans all the resources that were used were noted. During the interview, preservice teachers were asked to explain how they used those resources in setting learning goals, designing the learning tasks, and facilitating effective classroom discussions. Inductive coding was used to code how the resources or preservice teachers' personal experiences shaped their decision making processes around the three core practices.

Table 9 shows the research questions and the associated data sources that were used to for the analysis in order to answer respective questions.

Validity of instruments. In qualitative study, validity and reliability issues are important because they indicate the degree of trustworthiness of the study. Guba and Lincoln (1981) proposed that the concept of trustworthiness in qualitative study consist of four components, namely, credibility, transferability, dependability, and conformability. There are a number of methodological strategies that can be used to demonstrate the trustworthiness of a study. Among the methods that can be used is triangulation which is accomplished by the consideration of multiple sources of data from a diverse group of individuals and settings, using several different methods (Miles, Huberman, & Saldana, 2014). In relation to this study, multiple data were collected from the six preservice teachers and the methods course. By considering multiple participants and the data that related to them in terms of interviews, class documents, reflections, and field notes it was ensured that the results are not bound to particular cases.

Considering categories of events and themes across participants made it possible to confirm the emerging assertions from the data, beyond particular instances and cases In addition, the use of multiple data sources in the form of interviews, class observations, and document enhanced the validity of the study. For example, interviews were used to confirm some interpretations that were made during the observations, and also served as a complementary data source for the other data that were collected.

Table 9

Research Questions, Data Source, and Analysis Strategies

Research Question	Data Source	Data Analysis
In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from a given state standard to support elementary school students' mathematics learning?	Lesson plans, Interviews Field notes Students Reflections	Code field notes, interviews transcripts and reflections. Identify patterns and emerging themes. Check that the learning goals align with the State Standards
In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks?	Students lesson plans, Interview Data Students' Reflections Field Notes	Code field notes, interviews transcripts and students' reflections. Identify patterns and emerging themes. Check how learning tasks align with learning goals. Categorize learning tasks.
In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?	Lesson plans, Field notes Interview Data Interviews Lesson plans	Use Stein (2008) five practices as deductive codes to code field notes and interviews transcripts. Identify patterns and emerging themes. Identify new codes from inductive coding and compare codes.

Summary

In summary, this study used multiple analytic frameworks to provide a deeper understanding about how pre-service teachers set their mathematics learning goals, design learning tasks to meet those goals, and facilitate effective classroom discussions. Further, the influence of the formal and informal learning communities on pre-service teachers' decision making process were also explored. The use of artifacts, interactive, and observational data enhanced triangulation of the findings within and across all the cases. In chapter 4 detail analysis and findings of the study are presented. Results are presented for each case and then for cross-cases.

Chapter 4

Findings and Discussions

This study was conducted to investigate three core practices, namely, setting mathematics learning goals, designing learning tasks to address the learning goals, and orchestrating classroom discussions, in the context of elementary mathematics methods class. In order to understand preservice teachers' vision to become mathematics teachers and to uncover how the preservice teachers learn to enact these three core practices within their respective context, a multiple case study design involving six preservice elementary mathematics teachers was used for this study. Data used for the study include preservice teachers' survey responses, interviews, lesson plans, and filed notes from student teaching observations. Qualitative data analysis techniques were used to analyze the data to answer the three research questions that guided the study.

This chapter reports results of the single and cross-case analyses. For each case, an overview of the preservice teacher's mathematics background and teaching context are presented, followed by results to answer the three research questions. For research question 1 description and evaluation of the learning goals are presented. In addition, the resources and processes used by the preservice teachers to set the learning goals are presented. For research question 2, a presentation of the learning tasks and how the tasks address the learning goals are described. Also, the resources used from the learning community in designing the learning tasks and how they were used are described. For the third research question, the results are presented to describe how preservice teachers enacted the five practices in orchestrating whole-class discussions. Finally, results for cross-case analysis are presented around each of the three research questions in order to see the similarities and differences about how the preservice teachers enacted each of the three core practices and used resources from the learning community.

The Case of Anna

Anna's mathematics background. Prior to the post-secondary education, Anna had mixed experiences about learning mathematics at the high school both positive and negative. Algebra and trigonometry were Anna's favorite subjects while geometry did not come easy for her. In her freshman year, Anna took algebra which she liked very well because of the nature of the course. She indicated that, "I always liked algebra because of the way it was kind of clear cut. My teacher will say $x + xy$ equals this and like there is one way to solve it." (Anna, Interview 1). To Anna, the nature of algebra made her like the subject because she knew that always there was one clear rout to solve algebraic tasks. Similarly, Anna also liked advance algebra and trigonometry during her junior year in high school because she "had a really good teacher for that" (Anna, Interview 1). While Anna had a good handle on algebra and trigonometry, she struggled with geometry during her sophomore year in high school. The nature of the geometry content and how it was presented by the teacher made it difficult for Anna to grasp the concepts. She lamented that,

> I really struggled, I mean it was like, I just every day I was so frustrating it was oh it was so hard. I never got it, I mean we didn't have the best teacher. We will just have proofs and learn how to solve the proofs and I just remember like, I go home and when I am doing my homework I will cry and my parents didn't know how to help me because they didn't know how to do it either. And our teacher was kind of like not available for extra help. So I just never really learned it and I think I

> got C or C+ and I have never got, I have always got a really good grades. I was pretty much an A student all the way through until that point. Just so like upset that I got a bad grade and I was doing bad in the test and there was no one in my school that would, that could tutor me, I don't know why they just wasn't available.

Although Anna did very well in algebra and trigonometry, she struggled in geometry. Being an A student, Anna was poised to do well in geometry. This encouraged Anna to seek assistance in order to improve upon her geometrical skills. Unfortunately, there was no assistance for her. The lack of support from the teacher and colleagues deepened her frustrations with learning geometry. Anna's negative experiences with mathematics made her less confident and thus, made her shy away from taking higher mathematics courses like pre-calculus and calculus. Although Anna expressed regrets for not taking those courses, she also believes that she made the right decision at the time.

> I never took pre-calculus or calculus because I never was like confident in myself to do that. Looking back, I wish I would have but at the time I was like, I didn't like math, I haven't really had good math teachers why would I take pre-calculus so I didn't. (Anna, Interview 1).

During Anna's senior year, she took a statistic class and did well. Generally, although her geometry class was an adverse learning experience for Anna, she was still optimistic that she could do well in mathematics and become a great mathematics teacher. She carried her optimism to the college level where she had a new type of mathematics learning experience.

At the college level, Anna took three mathematics courses, including, *College Algebra, Mathematics and Pedagogy for Elementary Teachers* I, as well as *Mathematics and Pedagogy for Elementary Teachers II*. Anna's mathematic learning experiences were different from that of her high school years. At the college level, the mathematic pedagogy was generally inquiry-based and engaging. Unlike the high school where the content was "cut and dry," (Anna, interview 1) that is, presented without engaging activities, at the college level, the mathematics content was presented with contexts in which learners were actively involved in the teaching and learning process. The *Mathematics and Pedagogy for Elementary Teachers I and II* were foundational courses for all preservice elementary mathematics teachers. Not only did Anna learned mathematics content in an engaging manner from these two courses, she also learned how to teach mathematics using an activity-based approach in which students' thinking was elicited and valued for knowledge construction. Anna' dream of becoming mathematics teacher was coming back to life as she expressed how these two courses changed her perception about the nature of mathematics.

> Those classes were just like really eye opening because I never learned math the way they were teaching us to teach math. So it just like a totally new math for me and it time I remember like why am I learning this and being frustrated but now like…looking back like it all relate like am like I know exactly why she did this because that is how kids learn now and I don't even know kids were learning, like these derived facts in counting methods because I so am like that make total sense like I just wish I would have had more trust than now was the way l…doing it that make sense (Anna, Interview 1).

As expressed by Anna, the pedagogical approaches used by the course instructors for the two foundational courses were an "eye-opener" for her. Everything seemed like "new math" because she had not experienced mathematics taught in this manner. The content was not "cut and dry" and also, she realized that, unlike her high school algebra where there appeared to be only one way to do mathematics, with these foundational courses, Anna learned that there could be multiple ways of arriving at a mathematical solution. Although these two courses were generally good for Anna, she encountered some challenges with some the assignments. However, because of her determination to be a good mathematics teacher, she persevered through all the frustrations. Anna, indicated that the struggles she endured through these courses were good learning moments.

Anna's vision as a mathematics teacher. Because Anna had wanted to be a teacher her entire life and truly believes that she can make a difference in students' lives (Anna, Survey), she was hopeful that she would be a good mathematics teacher. This vision underpinned her motivation to have positive attitude towards the mathematics methods course (Teaching Mathematics in Elementary School) she was enrolled at the university. Anna had four general goals for the class, namely, to learn the content she will be teaching, to overcome her personal "stigma" that math is challenging, to learn various teaching techniques, and to fall in love with math. Guided by these goals, Anna accepted the challenge to work hard and overcome all challenges in order for her to be a good mathematics teacher.

> This course is very important to me because I struggled through math as a child and all the way through college, it was never something that just "clicked" for me. I know that I am going to have to work harder and smarter in order to become an expert math teacher and I am willing to do this. I want to learn the content completely before I teach a lesson, but also learn different methods of teaching the content (Anna, Survey).

The above quote summarizes Anna's mathematics experience since elementary school to college. She has experienced the difficulties in learning mathematics and has also experienced good teaching from good mathematics teachers and so she believes that her rich experiences will set her to be a good mathematics teacher.

Anna's teaching context. As part of the requirements of the mathematics methods course, Anna prepared a three-day mathematics lesson that was taught at a practicing school as part of her field experience. Before this teaching experience, Anna had observed the cooperating teacher and had assisted her in classroom duties including co-teaching some mathematics lessons. Anna taught a first grade class at the Woods Elementary school in the Oxford School District. The school used the *Investigations* (Technical Education Research Centers, 2004) curriculum and *Focused Instruction* (Oxford School District, 2016) pacing guide. The pacing guide stipulates the mathematics content to be taught at each grade level. Table 10 shows the demographic and population type for Woods Elementary.

Table 10

Woods Elementary Enrollment by Demography and Population Type

Ethnicity	Woods Elementary
American Indian/Alaskan Native	17 (3.4)
Asian/Pacific Islander	15 (3.0)
Hispanic	135 (26.7)
Black, not of Hispanic Origin	253 (50.0)
White, not of Hispanic Origin	86 (17.0)
All Students	506 (100)
Special Population	
English Learner	253 (50.0)
Special Education	79 (15.6)
Free/Reduced priced lunch	418 (82.6)
Homeless	23 (4.5)

Note. First number is the total count and the second number is the percentage of the count.

Results from Mathematics Learning Goals

Research Question 1: *In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from a given state standard to support elementary school students' mathematics learning? What are the characteristics of these goals?* Anna taught a first grade lesson on money. Table 11 shows the state standards and corresponding benchmark Anna addressed during her student teaching.

Table 11

Mathematical Strand and Benchmark for Anna's Class

Strand	Standard	No.	Benchmark
Geometry and Measurement	Use basic concepts of measurement in real-world and mathematical situations involving length, time, and money	1.3.2.3	Identify pennies, nickels and dimes; find the value of a group of these coins, up to one dollar.

Note: first grade, geometry and measurement, second standard and second benchmark.

Expected big ideas and reflection on the mathematics benchmark. In order to address the mathematical benchmark for the first grade class, the state mathematical framework identifies what students should be able to do in relation to the benchmark. For this lesson, students are required to be able to identify pennies, nickels, and dimes; know the value of a penny, a nickel and a dime; count groups of pennies; count groups of nickels; count groups of dimes; count a group of pennies, nickels and dimes up to a dollar; and use the cents symbol-¢.

Identifying pennies, nickels, and dimes and knowing their values require that students would be able to visually recognize each of the coins and be able to associate monetary values to each of them. One way to ensure that students have really had a good grasp of the three coins will be their ability to differentiate between the three coins. When students are able to identify the coins and their values they can draw from their previous knowledge in addition of whole numbers to find the value of group of coins. This means that identifying coins is foundational knowledge on which the other goals associated with this benchmark can be achieved. For example, if students cannot correctly identify the coins and their respective values, it may not be likely that they will be able to find the value of a group of these coins. Finding the value of group of coins up to one dollar requires students to be able to tell the value of a group of these coins. Because the benchmark requires that students should be able to count group of money up to a dollar, it becomes the responsibility of the teacher to structure the lesson so that students would be assisted to acquire the desired learning goal.

Anna's mathematics learning goals. In order to meet the given benchmark in the standard for the first grade lesson on money, Anna formulated four content goals. Table 12 shows Anna's content goals for the three-day lessons.

Table 12

Anna's Learning Goals for the Three-day Lessons

Code	Learning Goals
AC1	I can name pennies, nickels, and dimes and tell how much each coin is worth
AC2	I can count groups of pennies, nickels and dimes
AC3	I can identify the value of combinations of coins.
AC4	I can name pennies, nickels and dimes and count a combination of coins to $1.00.

I can name pennies, nickels, and dimes and tell how much each coin is worth. This content goal was used to help student name all the three coins and tell the value of each of the coins. The learning goal was clearly stated because it identified the mathematics knowledge students were required to learn. In addition, the goal addressed the benchmark in the state standards.

Students were able to associate the correct names to each of the three coins and identified their respective values. Anna used sentence frames to help students attain this goal. At the end of the lesson students were able to state that a penny is worth 1 cent; a nickel is worth 5 cents; and a dime is worth 10 cents. In addition to this, students were also taught to be able to find the relationship between the values of the all the coins. For example, students were able to tell that 5 cents equal 1 nickel; 10 cents equal 1 dime; and 2 pennies equal 1 dime. In helping students to identify the coins and tell their values, Anna used manipulatives, pictures, written words, and spoken language to help students translate from one mode of representation to another. In translating between modes of representation, students were able to draw pictures, use symbols and written words as well as spoken word to state the value of a group of coins. While the use of the multiple representation enabled students to demonstrate their understanding by translating between different representations, it also afforded students different ways to learn to count coins. Although the benchmark does not explicitly state that students should be able to tell the value of each coin, this knowledge is implicitly implied by the benchmark because if students cannot tell the values of each coins, then it is likely that they may not be able to find the value of a group of coins. Finding the values of a group of coins is ultimately the goal of the benchmark so in order to achieve that goal, it is imperative that students build a strong foundational knowledge for that goal. For example, if student do not know that a nickel is worth 5 cents or 5 pennies and a dime, 10 cents, how would they be able to find value of group of nickels and dimes? Consequently, helping students to be able to find the value of each of the three coins will help them find the value of a group of coins.

I can count groups of pennies, nickels and dimes. This goal was used to help students count a group of pennies, group of nickels, and group of dimes. This learning goal addressed the benchmark because students were able to count group of coins and state the value of the coins as required by the benchmark. The goal was clearly stated to show what students should be able to do.

For this goal, Anna used several activities to help students counts the coins. Initially, students were not given any restrictions about how many coins they were required to count. Students were encouraged to count as many coins as possible and state the value of the group of coins they had counted. For example, some students were able to count 20 pennies and stated the value as 20 cents; counted 7 dimes and assigned the value of 70 cents; and counted 10 nickels as 50 cents. In finding out the values of the group of coins, students used their own strategies. While some used skip counting by 5s and 10s, others combined skip counting strategies and counting on strategies. Because Anna encouraged students to use their invented strategies to find the value of collection of coins, students were able to learn at the own pace thereby enhancing their confidence in learning to count money.

I can identify the value of combinations of coins. The third learning goal was used to help students find the value of a group of coins. The purpose of this learning goal was to prepare students for counting a combination of coins up to a dollar. Anna, indicated that this goal was to scaffold students' learning so that those who could not count up to a dollar would be able to count to other values before counting to a dollar. Because of the mathematical knowledge inherently related to this learning goal, it helped students to work toward achieving the overall goal of the benchmark. Therefore, the goal addressed the benchmark of the state standards. The goal was however, not clear in terms of the kinds of combination of coins that were to be used to make up the respective monetary values.

Anna used different mathematical tasks to help students work in pairs to find the value of a given group of coins. The group of coins comprised any possible group of pennies, nickels, and dimes. For example, in one activity, students worked in pairs. In their pairs, each student had a cup full of combination of pennies, nickels, and dimes. The two students were required to count and determine the value of each other's coins and then determine who had the larger amount of money. In helping students achieve this goal, Anna encouraged them to count up to one dollar where possible and in cases where students were not able to count up to a dollar, they were encouraged to count up to any value they could.

I can name pennies, nickels and dimes and count a combination of coins to $1.00. Finally, the last content goal, combines all the first three content goals. It is the re-stating of the benchmark that is being addressed with a change of words and phrases. The word “identify” in the benchmark was substituted with “name” and the phrase “find the value” substituted with “count.” This goal was stated clearly to show the mathematical knowledge students should be able to demonstrate at the end of the lesson. The goal addressed the benchmark in the states standards. For this goal, students were given the opportunities to be able to count any combination of coins up to a value of one dollar. All the four learning goals built on one another, that is, each learning goal served as a relevant knowledge to achieve other learning goals. Table 13 shows an evaluation of the learning goals in relation to the essential mathematical knowledge needed to address the benchmark.

Table 13

Assessment of Anna’s Learning Goals for Clarity and Alignment with State Benchmark

Student’s Learning goal	Clarity of student’s goals	Connection to state benchmark
I can name pennies, nickels, and dimes and tell how much each coin is worth	The goal explicitly shows what students are expected to be able to do. They should be able to name the coins and be able to tell the value of each coin.	The goal is linked to the benchmark.
I can count groups of pennies, nickels and dimes	The goal is clear in terms of what students should be able to do—counting group of coins.	This will scaffold students learning to find the values of group of coins. It is linked indirectly to the benchmark.
I can identify the value of combinations of coins.	This goal is not clear how students will “identify” the values of the combination of the coins.	This is linked to the benchmark.
I can name pennies, nickels and dimes and count a combination of coins to $1.00	This is clear because it shows what students should be able to do.	This goal connects to the benchmark.

Setting learning goals. In formulating the four learning goals to address the given mathematical benchmark for the state standards, Anna used several resources from the learning community and also considered multiplicity of factors.

School-based resources. The resources consulted from the practicing school include the cooperating teacher, the school district pacing guide (Focused Instruction), and the state standard. Firstly, the mathematical content to be taught were identified by Anna and the cooperating teacher by following the school district pacing guide. The pacing guide stipulates the mathematics content that should be taught in the various grade levels. Because of the pacing guide, Anna could not select her choice of topic to teach; she was compelled to follow the pacing guide. Secondly, Anna consulted the state standard to find out the actual mathematical benchmark that had to be addressed. Finding out the benchmark to be addressed from the state standard enabled Anna to know the scope of mathematical content she was required to teach. Thirdly, while *Focused Instruction* provided examples of learning goals for Anna to frame her learning goals, the benchmark in the state standard served as a guide in the process of breaking down the benchmark to obtain the learning goals.

These steps were repeated when Anna was asked to explain how she set the learning goals. She indicated that, "focus Instruction puts *I ca*n statement up, so I looked at them, like what the idea was, then I just wrote my own, but there are very similar and they can, like based off the standards." The Focused Instruction and the state standards served as the major resources Anna used in identifying the mathematical content to address in the learning goals and how the learning goals should be framed. Anna however, used her personal experiences and judgments to set the final learning goals. She summarized the process this way,

> So the Minnesota state standard is: Identify pennies, nickels, and dimes, find the value of a group of these coins up to one dollar. So I took that as like the main goal at the end of my three days, and then I made '*I can*' statements that all will help with that goal…I just, I looked at the state standards and what I knew and wanted my students should be able to do and then I kind like broke it down to like smaller parts. Before they can just count to a dollar I could teach other things, before they can just know how to count up to a dollar so I broke it down to just being able to identify pennies nickels and dimes and then counting them, and then identifying the combination for counting them so it's almost like a step process so I will say my first three content goals kind like all lead up to my final content objective. (Anna, interview 2).

While the curricula materials and the cooperating teacher directed Anna about the mathematical content knowledge to teach and how to frame the learning goals, the way the learning goals were broken down was based on Anna's personal experiences and dispositions.

Personal Decisions. Anna's experiences from learning mathematics and her desire to "do whatever it takes" to make her students comfortable with mathematics influenced how she set her learning goals from the mathematical benchmark. From Anna's perspective, before students can achieve the big mathematical goal stated in the benchmark, there is the need to scaffold students' learning by breaking the standard into smaller parts such that the cumulative effect of all the learning goals will help students achieve the mathematical knowledge needed to address the benchmark in the state standard.

Breaking the mathematical benchmark from the state standards depends on individual judgment. What one considers to be important will influence their decision making processes. To Anna, her knowledge about her students and what she considered important were the factors that influenced her decision making concerning how the state mathematical benchmark was broken down. She indicated that “just knowing my students and their previous abilities, and then what I think is important in the practice of learning how to count up to a combination of a dollar and I was able to look at the previous assessment” (Anna, Interview 2). From the students’ assessment result, Anna identified the ability levels of the students which served as a basis for her to break down the mathematical benchmark. During the interview on the learning goals, Anna indicated that not all students would be able to count up to a dollar initially so she wanted to scaffold students’ learning to enable all the students to attain the learning goal of counting up to a dollar eventually. Anna’s knowledge about students’ learning influenced her decision-making process in setting her learning goals.

Summary for Research Question 1 for Anna

In setting mathematical learning goals to address the benchmark in the state standards, Anna decomposed the benchmark into four learning goals in which each learning goal served as a foundation for the subsequent learning goals. That is, knowledge from the first learning goal served as a prerequisite for the second learning goal and the first two learning goals served as prerequisite for the third learning goal before the final learning goal. In this case, the learning goals were used to scaffold students’ learning towards achieving the bigger idea of the benchmark. In order to set the learning goals, Anna used resources from her practicing school and the university mathematics methods class. The resources from the practicing school included the school district pacing guide, the state standards, and the cooperating teacher. These resources assisted Anna to identify the mathematics content that was to be taught. On the other hand, Anna's understandings from the mathematics methods course enabled her in breaking down the learning goals. Based on Anna's knowledge about her students' performance, and her knowledge about students' thinking, she decomposed the learning goals to meet the needs of all the students.

Results from Learning Tasks

Research Question 2: In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks? In seeking to address the first grade state benchmark on counting money, Anna designed and adapted several learning tasks to support students' learning. Table 14 shows the learning tasks that were used during Anna's three-day teaching practice. These tasks were identified using Stein eta l., working definition of mathematical task.

Asking students to make up 15 cents, 20 cents, and 50 cents with different coin combinations were considered as mathematical tasks because because students were required to recall their knowledge about pennies, nickels, and dimes to make specific values.

Table 14

Learning Tasks used in Anna's First Grade Lessons

Code	Task	Student responsibility
AT1	Counting pennies	Students color given number of pennies on a work sheet up to a total of 100 pennies.
AT2	Talking about dimes and nickels.	Students identify nickels and dimes and state how much each is worth.
AT3	Reviewing previous knowledge on pennies, nickels, and dimes and finding the relationship among the coin.	Students are asked to recall their previous knowledge on pennies, nickels, and dimes by identifying them and stating the value of each coin. Students are also asked to state the relationship between the coins. For example, how many nickels make a dime.
AT4	Make up the value of 15, 20, and 50 cents.	Students are required to make up 15, 20, and 50 cents in multiple ways. They can use one type of coin or any combination of coins.
AT5	Clean up the money (Math game).	Math game used for students to learn finding the value of group of coins.
AT6	Finding the value of coins	Given a group of coins, students are required to find the total value of the coins.
AT7	Make one dollar with only one type of coin.	Student use one type of coin to make one dollar.
AT8	Make one dollar with different coin combinations.	Student use combination of coins to make one dollar.

Categorizing the learning tasks. Using Stein et al., (1996) framework of cognitive demands, the learning tasks were categorized to identify the cognitive demand they posed to students as they learned to count money. Table 15 shows the categorization of the learning tasks used in Anna's class. There were a mixture of low-and high cognitive demand tasks. None of the task was categorize as "doing mathematics."

Table 15

Cognitive Demands of Anna's Learning Tasks

Lower-Level Demands	Higher-Level Demands
Memorization Counting pennies; Finding the value of pennies, nickels, dimes.	Procedures with connections Find the value of a combination of coins. Making up a given value using combinations of coins Making one dollar with combination of coins.
Procedures without connection Make value of money with only one type of coin. For example, make 20 cents using only nickels. Finding the relationship among the coins.	Doing mathematics

Cognitive demands of learning tasks (adapted from Stein et al., 1996).

Counting pennies was considered as a low-level demand task that used memorization because students had to use their previous knowledge on counting whole numbers. Similarly, when students were discussing the pennies, nickels, and dimes, they used their previous knowledge to state the values of each of the coins. Students already knew that 1 penny is worth 1 cent, 1 nickel is worth 5 cents, and 1 dime is worth 10 cents. Another task categorized as low level but required students to use mathematical procedures without necessarily making any connection was asking student to make up a given amount of money using only one type of coin. For example, when students were asked to make up 20 cents using only nickels, they knew that 1 nickel worth 5 cents so they added four-5cents to make up 20 cents.

Making up a given value up to one dollar using a combination of coins was considered as a high-level cognitive demand task which required students to make connections between their previous knowledge on the values of the the three coins and their addition strategies to find combinations of coins to make up the given value. For example, when students were asked to make 50 cents using a combination of coins, their thinking about the possible combinations of 1 cent, 5 cents, and 10 cents to get to 50 cents. While some used only two types of coins to make up 50 cents others used all three coins to make up the 50 cents. In all these activities, students were given manipulatives to work with. In addition to the coins given to the students they were also required to write out the number of each coin in their combination that made up the given value. Further students were asked to say their coin combination aloud.

Mapping the learning tasks to the learning goals. In seeking to find the connection between the core practices of setting learning goals and using learning tasks to address the goals, the learning demands of the learning tasks were assessed to see how they supported students' mathematical knowledge building in order to meet the stated learning goals. Table 16 shows the learning goals and the related learning tasks that map onto the learning goals. All the learning tasks enacted provided learning opportunities for students to have mathematical knowledge that helped them achieve the learning goals.

Table 16

Anna's Learning Goals and Associated Learning Tasks

Learning Goals	Codes for Learning tasks
I can name pennies, nickels, and dimes and tell how much each coin is worth.	AT1, AT2
I can count groups of pennies, nickels and dimes.	AT1, AT2, AT3
I can identify the value of combinations of coins.	AT1, AT2, ,AT3, AT4, AT5, AT6
I can name pennies, nickels and dimes and count a combination of coins to $1.00.	AT1, AT2, AT3, AT4, AT5, AT6, AT7, AT8

The low-level cognitive demand tasks, AT1 and AT2 (counting pennies and finding the value of pennies, nickels, and dimes) provided fundamental knowledge for all the learning goals. That is, the mathematical knowledge provided from the initial tasks served as a prerequisite knowledge for the high-level tasks. The learning goals and learning tasks were in hierarchical. A preceding learning goal served as a prerequisite knowledge for a new learning goal. Similarly, the learning tasks provided a scaffold to support students' learning of high-level cognitive demand tasks. In order for Anna to help her students achieve the benchmark, she used several learning tasks to scaffold students' learning to meet the overarching goal which is addressing the benchmark.

Setting learning tasks. In order to use appropriate learning tasks that will help students achieve the learning goals, several resources from the learning community were used. In addition to this, Anna's personal dispositions also influenced her decision-making process around the learning tasks. All the interview transcript related to the learning tasks were coded for school-based resources, university-based resources and personal decisions.

Resources from practicing school. Anna used curricula materials and ideas from the cooperating teacher to find out the mathematical tasks that were to be used to meet the needs of the district and the cooperating teacher. The district pacing guide, *Focus Instruction* and the mathematics curriculum, *Investigations* were used to find out the mathematics games that were given to Anna to use. During the second day of the lessons, Anna used a mathematical game, Clean Up the Money, to help student count combinations of coins. When asked how the game was going to help students achieve the stated goals, Anna explained that the cooperating teacher wanted her to use the game for the second day because in the past, the cooperating teacher had always used that game to help students count money. According to Anna, the cooperating teacher believed that using the game was "the best way for them to learn how to count money." (Anna, Interview 2). The district mathematics curriculum and the cooperating teacher were the two main factors that influenced Anna's learning goals for class.

Personal Decisions. In addition to using resources from the practicing school, the preservice teacher used her personal decisions to design and adapt learning tasks to suit the needs of the students. In adapting the tasks, Anna's personal mathematical learning experiences, her knowledge about students' learning abilities and the understandings gleaned from the mathematics methods class influenced the decisions she made.

> This game, they have like one kind like in a book where it collects a dollar but then I made collect a 50 cents for students that might not be able to get to a dollar right away. I just made it a little bit easier, and I picked this game because it also uses addition, and that kind of like, serves as like review and I thought it just like a good way to incorporate more than just money into our lessons (Anna, Interview 2).

The above quote shows how Anna adapted the mathematical game to meet the needs of the students. For this mathematical game, Anna considered students' prior knowledge in adding whole numbers in selecting the game. In order to adapt it, Anna indicated that she considered the ability levels of her students to scaffold the tasks in order to help students who might not be able to grasp the bigger concepts easily.

These personal decisions were based on Anna's personal judgments about learning mathematics and the needs of the students. In order to make sure that students achieve the final big goal of the lesson, that is, being able to identify pennies, nickels, and dimes and finding the values of a combination of the coins up to one dollar, Anna made several decisions about the learning tasks to use. Although the cooperating teacher wanted the students to use the mathematics game, Clean Up the Money, to learn how to count money, Anna wanted the student to "dig deep" into the lesson on the third day. She stated that,

> The third day, we have to really kind of dig into sorting money, and then making different combinations up to a dollar because that's the overall goal. So am still kind of working on what to do with that so I am thinking myself and I wanna do stations so I know that am gonna model each station and I know that I am gonna play the game collecting 50 cents and then that will be one station and that will be for most of the kids and I will also have another station but collect a dollar and that will be a little more challenging but it will be like the same game (Anna, Interview 2).

Because Anna wanted the students to "dig deep" into counting money up to one dollar, she planned mathematical activities that could be used at different stations. The stations were to create avenue for students with differing abilities to progress gradually from making values of money less than one dollar till they reached the one-dollar goal.

University-based Resources. The tool that was taken from the university methods course in planning the learning tasks was the direct modeling concept from the *Cognitively Guided Instruction* and the use of multiple representations from the Lesh Translation Model. The preservice teacher indicated that "the best way to really learn about pennies nickels and dime and how to use them is just being hands-on" (Anna, Interview 2). Engaging students in hands-on activities requires that teachers designed learning tasks that will sill support students' learning in an engaging manner. In addition to using hands-on activities, Anna provided the opportunities for students to demonstrate their understanding of the mathematical concepts they learned using multiple modes of representations including concrete objects, pictures, verbal language and written words. By structuring the learning tasks around hands-on activities, students were engaged in the lessons and were able to grapple with mathematical tasks to construct their knowledge. The activities also afforded students the opportunity to work collaboratively.

Although Anna used several resources from the methods class, she was not able to apply all the tools she learned from the course. For example, Anna did not use the idea of Number Talks even though she had initially decided to use Number Talk in her lessons. According to Anna, because Number Talks was used for operating on whole numbers, she thought it was not applicable in learning to count money. During my interview with Anna, she indicated that Number Talks and counting money are “two separate things” (Anna, interview 2). Anna could have used Number Talks to engage students about adding money. This could have facilitated students’ ability to find values of combination of coins. Anna responses during about why she did not use Number Talks revealed that she did not think about using Number Talks when she was planning her lessons because she thought Number Talks was only for operating on whole numbers.

Summary for Research Question 2 for Anna

In seeking to help students achieve the learning goals that were set, Anna used both high- and low-level cognitive demands tasks. These tasks provided students with the opportunity to use multiple modes of representations in order to facilitate learning. All the learning tasks provided mathematical knowledge that addressed the learning goals. The learning tasks were scaffolded to provide prerequisite knowledge for other learning tasks. While Anna designed some of the tasks, she also adapted some tasks not only to meet the learning abilities of the students, but also, to address the learning goals. In deciding on the type of tasks to use, Anna used several resources from the learning community as well as her personal dispositions to design the learning tasks. Learning community-based resources include curriculum materials, cooperating teacher's help, and the university mathematics method class.

Using the Five Practices to Orchestrate Class Discussions

Research question 3: In what ways did the mathematics methods class influence preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals? According to Anna, it was easy for her to incorporate *anticipation*, *monitoring*, and *making connections* in the lesson plan because these practices were independent of students' work. That is, she did not need to know what students were going to do in class before she would know how to enact these practices. Consequently, in her lesson plan Anna indicated specific things she was going to anticipate, how she was going to monitor students work and how she was going to make connections of students' ideas to facilitate their learning. On the other hand, Anna indicated that it was difficult for her to incorporate practices like *selecting* and *sequencing* because those practices depended on what students were going to do during the teaching and learning sessions. Although she knew what she wanted to do, it was difficult for her to incorporate those practices in the lesson plan. When Anna was asked how she was going to incorporate the five practices in her lesson she explained that,

> Well, this was a little more difficult to incorporate the five practices because, I mean, I definitely going to be monitoring the activities at all times and I think I

> can easily be able to select student like when I am walking around and monitoring like I can see them making different combinations of money, the games, and like counting in different ways, and I will stop and ask, oh, how did you use, how did you make that combination? What did you do to trade in this money? So then I am gonna write that down and have the students come back and share but …so I think just monitoring and then selecting sort of students to share and then have them have time to sit on the carpet and share is something that will be beneficial to them. Every day we will anticipate the day and I will explain what we are doing and then, just like the only connection that I will be able to make is just like a real connection (Anna, Interview 2).

During the teaching session, evidence of these practices explained above were monitored to see how they were enacted. Monitoring students' work was evident during students' work time. Anna circulated the class and monitored students' solution strategies. These solution strategies were selected and sequenced and used during the whole class sessions. Anna explained that "much of the selecting happened as we went throughout the book and activities, I was not really sure what to expect, but students were making several connections" (Anna, lesson plan reflection). Students were asked to share their work to set the stage for discussions. The learning goals determined how students' work were selected and purposefully sequenced for the whole class discussions. Anna used her discretion to call on students. Table 17 provides explanation of the five practices and how Anna enacted those practices.

Table 17

Summary of Anna's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Anna's Practices
Anticipating	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices.	I anticipate that students will identity pennies on the sheet handed out during the read aloud. I also anticipate that students will try to use 2 dimes. *20 cents*: students will most likely use 2 dimes, 20 pennies, 1 dime 10 pennies, 1 dime and 2 nickels, 5 nickels, some may try 1 dime and 4 nickels. 50 cents: this one will be more of a challenge problem and I anticipate all kinds of combinations, since I have never seen students count this high with money.
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem.	I plan on monitoring the front half of the room, while my cooperating teacher monitors that back half of the room to better reach the needs of all students
Selecting	Select specific students to share their work with the whole class.	Select students who color in the entire row of 10 pennies at one time instead of individually counting each square. Also select students that knew 1 dime and 1 nickel equaled 15 cents and colored fifteen without prompting.
Sequencing	Purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goal	Call on students as I see fit throughout the story. Go in order of 15, 20, 50 cents and have student with correct combinations share first and if students made incorrect combinations have one share second, so students can talk through the combination together as a group.
Connecting	Teacher to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully.	Connect with prior knowledge, counting by 10s and try to make connections each time penny, nickel, or dimes are mentioned in the test. Sharing these combinations as a whole group will connect with the content objectives.

Summary for Research Question 3 for Anna

In using the five practices for orchestrating whole-class discussion, Anna identified how she was going to enact each of the practices in her lesson plan. Anna had specific things she anticipated from her students as she planned her lessons. She also had specific plans about how she was going to monitor students' work, select examples of students' work to share, and how to purposefully sequence the selected work to share with the whole class. Anna's description of how she intended to make connections, however, appeared not be align with Stein et al., 2008 conception of making connections. Anna appeared to conceptualize making connections to mean her ability to connect students' ideas to their prior knowledge, rather than seeing the art of making connections as helping students to link ideas to meet desired learning goals. Although incorporating the five practices in the lesson plan was challenging, Anna implemented all the five practices effectively to support students' learning.

The Case of Samantha

Samantha's mathematics background. Samantha liked mathematics when she was young. However, as she progressed in high school, her love for mathematics was waned because she was not confident in the higher mathematics courses she took. In high school, the mathematics classes she took included algebra 1, algebra 2 and pre-calculus. Samantha did not take calculus for two reasons. First, she was "scared" to take it because she was not confident. Second, she did not need calculus to graduate so she rather opted for Spanish classes as substitute. During the first interview, Samantha was quiet and not forthcoming so not much information was gathered from her K-12 mathematics learning experiences.

At the college level, Samantha took college algebra and the foundational courses for elementary mathematics teachers: *Mathematics and Pedagogy for Elementary Teachers* I and *Mathematics and Pedagogy for Elementary Teachers II*. In these foundational classes, Samantha learned mathematics content as well as pedagogical knowledge for teaching mathematics. Comparing her mathematics learning experiences in these foundational courses to her elementary school mathematics learning, Samantha indicated that there was a sharp contrast between how mathematics was taught in these two settings. “I feel like when we were in elementary school math was taught differently and was memorizing fact and memorizing equations.” (Samantha, survey). While Samantha’s described her elementary mathematics learning as procedural and memorization, she indicated that learning mathematics at the college was inquiry-based and hands-on. Generally, Samantha’s mathematical strength included Geometry with shapes and visualizing patterns.

Samantha’s vision as a mathematics teacher. Because Samantha liked mathematics and also liked children she thought that teaching mathematics will be a perfect match. Nevertheless, she did not plan to be solely a mathematics teacher. When asked to explain why she decided to teach mathematics, Samantha explained that teaching mathematics was not an option for her because of her role as a general teacher.

> I did not choose exactly to be a “mathematics” teacher, rather a general education teacher, but obviously this will include teaching math. I loved math in elementary school because I understood and was successful in math. This is not the case for all students. I hope to be able to make math an enjoyable experience for all

> students, despite their abilities. This is how I will approach every subject I teach. I think school should be engaging and have a fun atmosphere, and I hope to be able to create this type of setting for every subject in my classroom (Samantha, survey).

Although Samantha did not originally intend to be a mathematics teacher, she believes that she can help students learn mathematics in an engaging manner. Considering her mathematics learning experiences at the elementary school, she "loved" mathematics and was determined to help all students enjoy learning mathematics.

Although her later experience with advanced classes in mathematics was not the best, Samantha was determined to learn how she can teach mathematics in an engaging manner in an atmosphere where students will be confident with math and be free to talk about math, make mistakes and correct them. With this view of mathematics teaching, Samantha wanted to learn research-based mathematics teaching and learning from the mathematics methods course.

> I am really looking forward to growing in my math understanding with the newest research. I hope to have very strong math lessons because I think math is crucial to student success. *I do not feel qualified yet to teach math in the classroom*, so I am hoping this class broadens my knowledge (Samantha, survey, emphasis added).

Because Samantha believes mathematics is crucial to students and yet not feeling qualified to teach it, she demonstrated her readiness not only to learn the mathematics content she will be teaching, but also, to learn new strategies for teaching the content.

Samantha's teaching context. As part of the field experience requirements of the mathematics methods course, Samantha prepared and taught a two-day mathematics lesson at the Atomic Elementary in the Maranatha School District. Samantha was supposed to teach for three days, but due to time constraints on the part of the cooperating teacher she taught for two days. The school used the *Everyday Math* (University of Chicago Center for Elementary Math and Science Education, 2004) curriculum. Prior to the student teaching, Samantha had observed her second grade cooperating teacher and had assisted her in classroom duties including co-teaching some mathematics lessons. Table 18 shows the demographic and population type for Atomic Elementary.

Table 18

Atomic Elementary Student Enrollment by Demography and Population Type

Ethnicity	Atomic Elementary
American Indian/Alaskan Native	1 (0.6)
Asian/Pacific Islander	24 (15.4)
Hispanic	19 (12.2)
Black, not of Hispanic Origin	95 (60.9)
White, not of Hispanic Origin	17 (10.9)
All Students	156 (100)
Special Population	
English Learner	45 (28.8)
Special Education	15 (9.6)
Free/Reduced priced lunch	138 (88.5)
Homeless	13 (8.3)

Note. First number is the total count and the second number is the percentage of the count.

Results of Mathematics Learning Goals

In order to assess how Samantha's learning goals aligned with the benchmark in the state standard, the benchmark was analyzed to identify the mathematical knowledge students are required to learn. The following table shows the mathematics benchmark Samantha addressed in the state standard.

Table 19

Mathematical Strand and Benchmark for Samantha's Class

Strand		No.	Benchmark
Algebra	Recognize, create, describe, and use patterns and rules to solve real-world and mathematical problems.	2.2.1.1	Identify, create, and describe simple number patterns involving repeated addition or subtraction, skip counting and arrays of objects such as counters or tiles. Use patterns to solve problems in various contexts. For Example: Skip count by 5s beginning at 3 to create the pattern 3,8,13,18… Another example: Collecting 7 empty milk cartons each day for 5 days will generate the pattern 7,14,21,28,35, resulting in a total of 35 milk cartons.

Note: first grade, geometry and measurement, second standard and second benchmark.

Expected big ideas and reflection on the mathematics benchmark. The mathematical ideas and understanding related to this benchmark requires that students will be able to recognize patterns in numbers and apply the pattern to predict what comes next in a number pattern; describe the rule for a given number pattern; use a given rule to extend or complete a number pattern; and use patterns to solve problems (Minnesota State Mathematics Framework, 2014). Students are required to be able to use different strategies including repeated addition or subtraction, skip counting, and arrays of objects to identify, create and describe number patterns. For Samantha's lesson, she focused on using skip counting strategy. The benchmark also requires that students will be able to use their knowledge in skip counting to solve problems. Solving problem involved students' ability to read and understand word problems and apply appropriate strategies to solve them.

Samantha's mathematics learning goals. In seeking to meet the given benchmark in the standard for the second grade lesson on skip counting, Samantha formulated 3 learning goals. Table 20 shows the goals that were formulated for the lessons.

Table 20

Samantha's Learning Goals for the Three-day Lessons

Code	Learning Goals
SC1	Students will be able to skip count by 2's, 3's, 4's, 5's, 6's, and 10's using manipulatives and written symbols.
SC2	Students will be able to explain how and why they skip counted using a certain value
SC3	Students will be able to solve word problems using skip counting strategies.

Learning goals. For the first learning goal, the preservice teacher expected children to be able to use manipulatives to skip count and and represent their results using mathematical symbols. For this goal, the preservice teacher used her personal judgment to determine how students should skip count, that is, counting in twos, threes, fours, fives, and sixes. Also, the preservice teacher requires students to be able to use manipulatives to model how they will skip count and then represent their results in symbols. By asking students to use manipulatives, that can afford students the opportunity to make mental pictures of the number patterns before they can associate the written symbols to the sequence of numbers that will be generated from the use of manipulatives. This goal addresses the benchmark in the standard and clearly stated the mathematical knowledge students should be able to do and how they should demonstrate their understanding. It addresses the benchmark in the sense that the goal seeks to have children do skip counting using different numbers. Although the benchmark did not indicate which numbers student should skip count by, Samantha wanted the students to start counting from easier numbers to bigger numbers. Finally, the goal clearly shows how students should demonstrate their thinking, which is, to be able to use manipulatives for modeling and use symbols.

The second learning goal requires that students are able to explain the rationale behind their skip counting strategies. During the lesson plan follow-up interview session, Samantha indicated that although this goal "technically doesn't fit with the standard" (Samantha, interview 2), she wanted to use it to make students work collaboratively because she felt that most of the students were not independent. When asked how collaboration would help students' learning, Samantha explained that,

> Well, first of all I feel like the kids are very a lot of them are very needy and they always want, can you help me, can you help me? come do this , come do this, and so I feel like if you put them in pairs, because they do some stuff in pairs like reading stuff, but I felt like it will help with that and give kids more confidence and accountability, they have to do it together and but I fell that having someone that you can work with the problem with and be able to talk too… (Samantha, interview 2)

From Samantha's point of view, teaching in a collaborative approach enables students to share their thoughts to help one another and also be more accountable. She thinks most of the students do not get the opportunity to work collaboratively with their friends even though they need help. Samantha also thinks that the cooperating teacher engages in a lot of teaching by telling instead of engaging the students in an inquiry-based teaching approach in which the students will grapple with mathematical tasks to construct their mathematical knowledge.

Although the benchmark does not specifically require students to explain 'how' and 'why' they skip count using a given number, Samantha believes that this goal will make students be able to explain their mathematical thinking. The inclusion of this goal is based on the Samantha's personal judgment about what counts as important for students to learn in order to meet the given mathematical benchmark.

The preservice teacher intended to use learning goal 3 to address the part of the benchmark that requires students to use patterns to solve problems in various contexts. The preservice teacher interpreted the example of the real-world problem which is *Collecting 7 empty milk cartons each day for 5 days will generate the pattern 7,14,21,28,35, resulting in a total of 35 milk cartons* as word problems and set this learning goal in that regard. From this perspective, the last learning goal is set to address the given benchmark. This goal can enable students to apply their initial knowledge on skip counting in authentic setting. With a given word problem, student will not only have to find out the number which will start a pattern, but also find out how the numbers in the pattern can be identified.

Considering the three learning goals, they build on one another. The first learning goal set the stage for students to practice with skip counting using different numbers. The goal seeks to to have students work with manipulatives to model their patterns and then use symbols as a form of representing their mathematical models. The second learning goal ensures that students can explain the rationale behind their thinking. This goal seeks to ascertain whether students have clear understanding of their skip counting activities. Finally, the last learning goal required students to solve word problems involving skip counting.

Table 21

Assessment of Samantha's Learning goals for Clarity and Alignment with State Benchmark

Learning Goals	Clarity of goals	Connection to state benchmark
Students will be able to skip count by 2's, 3's, 4's, 5's, 6's, and 10's using manipulatives and written symbols	The goal clearly shows what students should be able to do and how they should do it.	The goal is linked to the benchmark.
Students will be able to explain how and why they skip counted using a certain value.	This goal is stated in clear terms. Students are required to be able to provide rationale for why and how they skip count.	This goal is not technically linked to the benchmark. It can help students explain their reasoning for their skip counting strategies.
Students will be able to solve word problems using skip counting strategies.	This goal can be phrased to make it more specific in terms of which type of word problems students will be solving.	This is linked to the benchmark.

Setting learning goals. In setting the learning goals to address the given mathematical benchmark for the state standard, Samantha considered several factors including her perception about students' needs, how she envisioned teaching, and the cooperating teacher's teaching approach. In addition, knowledge gained from the learning community played significant role in Samantha's final decision about the learning goals.

In order to understand how Samantha considered these factors and used the resources from the learning community, the lesson plan and the interviews were coded. During the coding process all the codes related to the learning goals were identified and classified into three main categories, namely, cooperating teachers' style of teaching, university-based resources, and curricular-based resources.

Cooperating teacher's style instructional approach. Having studied the new methods of teaching mathematics from research-based teaching practices during the mathematics methods class, Samantha did not see these reform-based teaching practices being enacted in the cooperating teacher's classroom. According to Samantha, the cooperating teacher was practicing an "old-school" teaching in which she did all the teaching by telling. During mathematics lessons, students will be sitting in the class, listen to the teacher present all the mathematical knowledge with the hope that students will understand what is taught. The students did not have the chance to contribute in the knowledge constructing process.

Another aspect of the cooperating teacher's teaching strategy that does not conform to reform-based teaching was that the cooperating teacher used individualized work rather than engaging students in collaborative and hands-on activities in which students will work together to explore mathematical ideas. When asked how she set her mathematical learning goal, Samantha began by expressing her frustration with the teacher's "old school" teaching methodology.

> Well my cooperating teacher does a lot of very old school, very very old school and like today she spent 40 minutes trying to draw this adding strategy like the doubles plus 1 strategy. She spent all day trying to draw one… they didn't get it. Like she doesn't use like hands-on, she doesn't use collaboration, she doesn't use

> more than one to learn anything and so that was one of my goals because one of the big things of class is that I think it is important to be able to apply what their learning to write it to read it to be able to say it to be able to build with cubes or whatever so that is how I got that goal (Samantha, interview 2).

Because of the traditional approach (the old school way of teaching) to teaching mathematics, Samantha wanted the students to have hands-on activities and work collaboratively so that students could discuss their ideas and construct mathematical knowledge hence Samantha's second learning goal. According to Samantha, using research-based teaching practices is important to facilitate students' learning even though that learning goal might not "technically" fit into the given benchmark.

University-based resources. As discussed earlier, the mathematics methods class was designed to meet reform-based teaching practices in which authentic tasks are used as a platform for students to engage in productive discussions using multiple representations to explore several solution strategies. According to Samantha, this teaching approach was the direct opposite to what was enacted in the cooperating teacher's classroom.

> She does the one strategy, this is, the one way you gonna solve it. This is how you need to do it. So one way in that and then also she doesn't combine like written and speaking so to be able to have all the different forms and to be able to give different strategies.

In the cooperating teacher's "old-school" teaching approach, students are required to follow strict procedures to follow the one way to solve a mathematical task. Also, there is no use of multiple representation. Samantha was poised to introduce radical changes in the teaching approach that was prevalent in the classroom. She wanted to use the Lesh Translation Model in which students would be "able to combine, like the real world application, the written representation, being able to verbally explain the problem and write in symbols." (Samantha, interview 2).

Curricular-based resources. Samantha used the mathematics benchmark from the state standards as a framework for her to set some learning goals. The first learning goal about skip counting and last learning goal which required students to use skip counting in to solve real life problems were obtained from the state standards.

Summary for Research Question 1 for Samantha

In order to set mathematical learning goals that will meet the benchmark in the state standards, Samantha decomposed the benchmark to obtain two learning goals and also used the knowledge gleaned from her mathematics methods class to obtain one learning goal. In all, three learning goals were used to address the benchmark. The first and third learning goals that were decomposed from the benchmark directly addressed the benchmark, while the second learning goal was used to support students' collaborative learning. In formulating these goals, the preservice teacher wanted to offer more opportunities for the students to explore mathematical ideas to construct their mathematical knowledge. While two of the three goals clearly indicated the mathematical goals students should be able to do and how they will show their evidence of understanding, the third learning goal was not very clear. Generally, all the three learning goals were scaffold to support students' learning.

Results of Learning Tasks

Table 22 shows all the learning tasks that were used for Samantha's three-day teaching practice. These are the learning tasks that were stated in the lesson plans and enacted in the classroom.

Table 22

Learning Tasks used in Samantha's 2nd Grade Lessons

Code	Task	Student responsibility
ST1	Find multiple ways of counting number of people in the classroom.	Students are asked to find a way to count the number of people in the classroom. They are also asked to find 'faster' way of counting the people in the classroom and continue to find all possible ways they can count.
ST2	Counting cubes in multiple ways.	Working in pairs, students are required to sort cubes and count them. They are asked to count the cubes in multiple ways using the skip counting strategies they had learned at the beginning of the class.
ST3	Recording numbers from counting.	Student are asked to write down the sequence of the numbers generated from the counting.
ST4	Reviewing previous knowledge on skip counting.	Students are asked to state examples of how they used skip counting in the previous lesson.
ST5	What number is missing.	Students are required to find missing numbers in a given sequence.
ST6	Find the "odd" number in the pattern.	Give a sequence of numbers, students are asked to find a number that does not belong to the sequence.
ST7	Math game involving skip counting.	Students play this game in pairs. They roll two dice, use one number to start a sequence and the second number as the number to skip count by.
ST8	Solving word problems involving skip counting.	Students are required to interpret a given real-life task in which they they are required to use their skip counting ideas to generate a solution.

By Stein eta l., (1996)'s definition of mathematical tasks as "a segment of classroom activity that is devoted to the development of particular mathematical idea," asking students to find multiple ways of counting the total number of people in the classroom was identified as a mathematical task because this task was used to introduce the concept of skip counting to the students. According to Samantha, the activity was used to serve multiple purposes. She explained that,

> this will engage the students and get them thinking about skip counting. Students will have the opportunity to participate right away and share their thinking to the group. It will also introduce the purpose of skip counting (count groups of objects in a faster way). This also gives students a visual connection to what/ why they are skip counting (Samantha, lesson plan).

As students find "faster" ways of counting the number of people in the room, they begin to realize that when they can also count in twos, threes, etc. and that the total number of people in the classroom is independent of the method of counting. This activity, therefore, does not only introduces the concept of skip counting to the students, it also set the stage for students to see why they skip count and how they can combine their counting on strategies to skip counting in order to count a group of objects.

Similarly, counting cubes in multiple ways was considered as a mathematical task because it was an extension of the first activity but in this case, students used manipulatives. This activity reinforced the skip counting strategies students discovered during the introductory stages of the lesson. Using the manipulatives and working in groups enabled the students to share their ideas meaningfully. During this lesson, students were actively engaged in the skip counting process. They used several skip counting strategies and counting on strategies to count collections of cubes. Where students faced difficulties, the preservice teacher assisted them appropriately. counting the people in the classroom afforded students the opportunity to have experience with concrete materials. The context used for this activity (the classroom) provided concreate experience for the students.

Categorizing the learning tasks. Using Stein et al., (1996) framework of cognitive demands, the learning tasks were categorized to identify the cognitive demand they posed to students as they engaged in skip counting. Table 23 shows the categorization of the learning tasks used in Samantha's class.

Table 23

Cognitive Demands of Samantha's Learning Tasks

Lower-Level Demands	Higher-Level Demands
Memorization	Procedures with connections
Counting by ones. Recording numbers from modeling skip counting. Review previous knowledge on skip counting.	Counting by 2s, 3s, 4s, 5s, etc. Finding missing numbers in a sequence. Identifying numbers which do not belong to a given sequence. Math game involving skip counting.
Procedures without connection	Doing mathematics

Cognitive demands of learning tasks (adopted from Stein et al., 1996).

Skip counting by ones was considered as a memorization task because students already knew how to count by ones and could recall their counting facts to determine the number of people in the room or collections of cubes. On the other hand, skip counting by 2s, 3s, 4s, etc. were classified as high-level demand tasks because students may make some connections to their previous knowledge to count in 2s, 3s, 4s, etc. Also, students may combine skip counting and counting-on strategies to find the total number of collection of cubes.

Mapping the learning tasks to the goals. In order to find the connection between the learning goals and the tasks, all the tasks identified in Table 24 were mapped onto the the learning goals. Table 24 shows the learning goals and the related learning tasks.

Table 24

Samantha's Learning Goals and Associated Learning Tasks

Learning Goals	Learning tasks
Students will be able to skip count by 2's, 3's, 4's, 5's, 6's, and 10's using manipulatives and written symbols	ST1, ST2, ST3, ST4
Students will be able to explain how and why they skip counted using a certain value.	ST1, ST2, ST3, ST4, ST7
Students will be able to solve word problems using skip counting strategies	ST1, ST2, ST3, ST4, ST5, ST6, ST7, ST8

Considering the cognitive demands of all the learning tasks, it is observed that the cognitive demands of the learning goals increased from the first learning goal to the third learning goal. All the learning tasks which address the first learning goal serve as foundational knowledge for the subsequent learning goals. In this sense, the learning goals build on one another. For the second learning goal, students were required to explain their rationale for using specific numbers for skip counting. In doing so, they were required to use their knowledge gained from skip counting and therefore, all the tasks that addressed the first learning goal also addressed the second learning goal.

Setting learning tasks. In choosing learning tasks that will help students achieve the learning goals, Samantha consulted different resources, including her mathematics course professor and online tools and also considered several factors in order to ensure that the learning tasks used for the lessons were “appropriate” (Samantha, interview 2). First, Samantha consulted the mathematics methods course professor, Dr. Shelley, for some ideas on the skip counting lesson she was going to teach. Based on the ideas the professor shared, Samantha checked for online resources on skip counting and adapted some tasks to meet the needs of the students and the learning goals. She lamented that,

> Well, I talked to [Dr. Shelley] very briefly and she just had a few ideas so I kind of went off from what she said and then I just looked online because my cooperating teacher doesn’t really give me much so I kind like just made, found what I could online and then adapted it to fit with the goals and what I think will be appropriate for the students (Samantha, interview 2).

Although Samantha’s cooperating teacher gave her the textbook, *Everyday Math,* she did not use it during the lesson planning stage because she did not have access to the book during her lesson planning stage. When asked why she did not use the textbook, she indicated that, “well because she just showed it to me a long time ago and then when I needed to write my lesson we were on campus, I wasn’t really getting a response and I needed to write it.” (Samantha, interview 2). Because of lack of communication between Samantha and her cooperating teacher, she could not use the mathematics textbook during the lesson planning stage. Samantha adapted the online resources to make them suitable for the intended learning goals.

In adapting the learning tasks, Samantha considered students’ grade levels, how the tasks could address the benchmark in the standards and the availability of teaching and learning materials that would be needed. Samantha explained that the learning tasks used for the lessons had to go back to the standard, that is, the tasks should address the benchmark being addressed and the goals she wanted to achieve.

> For sure it has to be something that can fit specifically under one of the criteria in the standards and also for the goals that I had decided on, and then grade level. If there is something I need to prepare, if it's like a whole big activities and I need supplies (Samantha, interview 2).

Making sure that the learning tasks fit the learning goals was Samantha's priority. She wanted to ensure that all the tasks will facilitate students' learning in order to achieve the desired learning goals and also meet the demands of the standards. Equally important to this objective was that the learning tasks should be "appropriate" for the grade level. In adapting the tasks to be appropriate for the students, Samantha considered students' academic abilities and adapted the tasks to meet the needs of the students. In the counting activity which was used to introduce the concept of skip counting for example, Samantha explained that,

> I kind like do an activity where we count the kids in the room and so that is something that like kindergarteners will have a hard time with, that fifth graders will think it was dumb getting them pairs and then getting in threes and the getting them go round counting. But I feel like that won't be too hard for them, won't be too easy, I guess, and then just knowing that the level of some most of them is pretty low and so adding is difficult for a lot of them so it's nothing like skip counting by the numbers, at least for the first day, unless I'm totally wrong but fairly simple for them (Samantha, interview 2).

Clearly, Samantha adapted the learning tasks to ensure that students will be able to work through them successfully. In addition to adapting the tasks to meet students' academic capabilities and at the same time address the desired learning goals, Samantha also adapted some tasks to include multiple representation. For example, in the dice game, she indicated that “I wanted to have all the different components like the speaking, the writing, the manipulatives so I just built on what I found and made sure that I added in time for all the different components” (Samantha, interview 2). Not only did Samantha adapt learning tasks, she also designed some of the tasks. For example, Samantha made up some word problems related to skip counting. Because Samantha taught in two days she could not use the word problems on the third day as was planned.

Summary for Research Question 2 for Samantha

During the skip counting lesson for her second graders, Samantha adapted and designed tasks that addressed the benchmark being addressed in the state standards and also meet the learning goals. In obtaining these learning tasks, Samantha used available resources form the learning community including the mathematics methods course professor, the mathematics textbook, online resources and the state standards. While these resources provided Samantha with the mathematical content that could be used, Samantha used her personal discretion to adapt learning tasks appropriately to meet the needs of the students. According to Samantha, making learning tasks appropriate required that the learning tasks were neither too easy nor too hard for the students. Also, Samantha ensured that the learning tasks required students to use multiple modes of representation, namely, verbal, symbols, pictures, manipulatives, to demonstrate their understanding of their mathematical knowledge. Samantha’s decision to set learning tasks that encouraged students to use multiple modes of representations stemmed from her understandings about students thinking and how multiple representations support students’ learning. In addition, Samantha indicated that she wanted her students to experience inquiry-based mathematics lesson as opposed to the lecture approach of teaching mathematics, which was mainly used by the cooperating teacher. A combination of low-and high level cognitive demand tasks which addressed the benchmark were used to support students learning.

Using the Five Practices to Orchestrate Class Discussions

Research question 3: In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?

In order to find out how Preservice teacher's whole-class discussion aligned with Stein and Smith (2008)'s five practices (anticipating, monitoring, selecting, sequencing, and connecting) for orchestrating effective class discussions, Samantha's whole-class discussions were explored to find how she enacted these five practices. In addition to the class discussions, the lesson plan was also used to find out Samantha's plan for orchestrating those practices.

Although Samantha saw her course professor implement the five practices several times in the mathematics methods class, she did not know how she was going to implement them in her lessons because she indicated that she never saw her cooperating teacher use the five practices. To Samantha, she wanted to see how the five practices were implemented directly with her elementary students before she could see how she could also implement them. When asked how she was going to implement the five practices, she explained that, "that is the hardest part because there is never a discussion. I don't know exactly how I am going to do it yet" (Samantha, interview 2). Samantha knew all the components of the five practices because she had seen her course professor implement them several times in the methods class.

> Kind of what we are learning in [Dr. Shelly's] class. She will just give us a question or something to work through or talk about and then as we work together, she comes around helps us and then at the end it's kind of debriefed and with the teacher and the students give their input on how to solve or whatever the problem would be and then through by the end then all the students know or kind of have like consensus of how to solve the problem (Samantha, interview 2).

Samantha's difficulty with implementing the five practices was not because she did not see the five practices being implemented before, but because she did not actually see the practices being implemented with elementary students. Because Samantha did not know how to incorporate the five practices in her lesson plan she did not think about the practices when she was planning the lessons. Also, she did not think incorporating the five practices in the lesson plan was the main priority.

> I wasn't thinking about it, we kind like did but it was a long time ago, but I think we are learning it more indirectly, because [Dr. Shelly] does this a lot and we talked about these steps maybe once, explicitly talked about this different steps but not very recently. I also designed it not exactly thinking this is the main priority (Samantha, interview 2).

The quote above indicates that Samantha wanted to see the five practices being explicitly stated and demonstrated during the mathematics methods class. She also wanted the course professor to explain the importance of implementing the five practices during their student teaching. This means that it is important for teacher educators to make practices explicit to students and encourage them to use them as they plan their lessons.

Samantha, however, identified that she was going to have some anticipation during the lesson. In her lesson plan, the things she planned to anticipate included the kind of support students would need and the counting strategies that would be used.

Knowing the level of her students, Samantha anticipated that some students would have difficulty with the skip counting. In view of this, she was ready to look out for possible counting errors that were going to be made by students so that she could help them accordingly. She indicated that,

> I am anticipating that there will be some incorrect counting on the recording sheet. It may be difficult to both record and say the skip counting, so I will be expecting to see some mistakes, which we can address together (Samantha, lesson plan).

Samantha's anticipation positioned her during the teaching and learning sessions. She circulated in the class and monitored students' work. During the monitoring stage she took notes of students work that was shared during the closure stage of the lesson. At this stage, Samantha called students to share their mathematical ideas and guided the class in meaningful discussion to ensure that students understood what was learned. Although Samantha enacted the five practices during the teaching and learning stage, she could not write all the practices in her lesson plan. While it was easy for her to write what she was anticipating and the kind of work she would select, she could not identify specific examples of students work she was going to select and how to sequence them. Similarly, making mathematical connections to build mathematical ideas could not be explicitly stated in the lesson plan because Samantha did not know what students would do during the teaching and learning stage. She lamented that,

> See, it's going to be hard because and I need to figure out how to do it but with skip counting and since that is the main focus there is not that many ways to count 2, 4, 6, 8. I'm I guess am having hard time coming up with what I will select and sequence if I guess I don't know. I need to work on this…. What's hard is my teacher never does this closer so when I was planning this lesson I didn't think about this (Samantha, interview 2).

Samantha indicated that incorporating the five practice in the lesson plan was very difficult because she never saw the cooperating teacher implementing them in her class.

Table 25
Summary of Samantha's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Samantha's Practices
Anticipating	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices.	I am anticipating that there will be some incorrect counting on the recording sheet. It may be difficult to both record and say the skip counting, so I will be expecting to see some mistakes, which we can address together.
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem.	As I walk around the room, I will be assisting students but also taking pictures of the work I want to show to the class (different recording strategies, different values students can skip count by).
Selecting	Select specific students to share their work with the whole class.	I will select student work that builds upon our discussion from Day 1. I will also select students who can explain their skip counting by writing out their numbers and explaining their strategy to the class.
Sequencing	Purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goal	Identified order in which students would be called to share their work.
Connecting	Teacher to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully.	Students will be able to skip count by numbers 2-6, and I will make the connection to how this is repeated adding.

Another reason why Samantha found it difficult to implement the five practices was that she did not practically learn how to enact the five practices in an elementary classroom. Table 25 summarizes how Samantha planned to enact the five practices for orchestrating whole-class discussion.

The Case of Jocelyn

Jocelyn's mathematics background. During her high school years, Jocelyn took several mathematics classes including geometry, college algebra, pre-calculus and one semester of calculus. Jocelyn described her high school mathematics learning experiences as procedural, with an emphasis on memorization of mathematical facts. Rather than helping students to learn conceptually in which they will be able to apply their knowledge to solve mathematical tasks, students were taught to master mathematical formulas and techniques for solving specific mathematical tasks. Jocelyn explained that,

> They would say, so if you see a problem that looks like this, you use this strategy. And they would say, okay practice that, and then they give us another example, a different problem with its corresponding strategy. But then if you saw something that was maybe even like slightly different than what you've seen in class, like say we're taking like a standardized test or something and it has like a slight variation, it was like, I was stuck, because even though it wasn't that much different than what I have learned, it was different enough where I didn't have like a precedent to work with (Jocelyn, interview 1).

The teaching strategy described by Jocelyn indicates that mathematics was taught in a using a traditional teaching approach, in which students were required to apply specific strategies whenever they encountered mathematical tasks. In this teaching approach, it is likely that some students may find it difficult to cope with mastering mathematical procedures and formulas. Students who are taught in this way may find it difficult to develop conceptual understanding of the mathematical content that is presented to them. For example, in Jocelyn's case, she explained that "it was all memorize this, memorize this, and I think that's why I struggled. Because it was like if I encountered a problem that I didn't have like a formula for, I couldn't do it" (Jocelyn, interview 1). Although Jocelyn struggled in mathematics during high school, she described her mathematics performance as "slightly, if it's above average at all, it's very slightly above average." (Jocelyn, interview 1).

At the college level, Jocelyn took college algebra*, Mathematics and Pedagogy for Elementary Teachers I* and *Mathematics and Pedagogy for Elementary Teachers II*. The mathematic pedagogy classes were inquiry-based and provided avenues for students to collaborate with one another to work on mathematical tasks. This approach to teaching mathematics engaged students to learn mathematics conceptually. Unlike Jocelyn's high school mathematics experience where she encountered several challenges, she "felt comfortable with nearly every topic" (Jocelyn, interview 2) in these foundational courses.

Irrespective of Jocelyn's mathematical struggles at the high school, she learned to persevere with mathematics so she did not easily give up on mathematical tasks that were challenging. Instead, she worked hard to overcome all struggles that came her way when learning mathematics. This positive attitude made Jocelyn confident and comfortable with mathematics.

> I'm comfortable with math, because I have gotten over the apprehension when you don't get something right away. I've learned to work through things. But as a child I would get frustrated with something and I would stop, and that was it didn't come easier. I didn't work harder at it, so over time I overcame that. but I will still have moments where if something just isn't working out I would have to like distance myself from it and then come back (Jocelyn, interview 1).

Unlike her childhood years where she got frustrated, she is now committed to persevere with mathematics learning in order to be good in mathematics. Even if she did not "get something right away" (Jocelyn, interview 1), she works hard to ensure that she finally understands it. Whenever necessary, Jocelyn reflects on difficult tasks in order to develop new solution strategies to solve the task.

Jocelyn's vision as a mathematics teacher. Jocelyn's decision to be an elementary mathematics teacher was underpinned by the fact that she believed she would excel in mathematics and be able to help students learn mathematics. When asked why she decided to be a mathematics teacher, she explained that, "I decided to be an elementary mathematics teacher because I felt that I would excel in the subject, and because I enjoy math I would be better prepared to help my students excel as well. (Jocelyn, survey).

Looking back to her high school mathematics experience, Jocelyn sees that there is a new direction for mathematics education in which students are not taught to memorize mathematical formulas and strategies, but they are "thought to reason through problems" (Jocelyn, interview 1). Jocelyn seem to be excited about the focus of mathematics education to engage students in the teaching and learning process in which students actively think through mathematical tasks to generate solutions. She explained that, in the mathematics methods class, which was the context of this study, she learned how to engage students to collaborate in working on mathematical tasks in order to construct solutions. Helping students to work collaboratively, Jocelyn indicated, would enable her to help students learn mathematics.

> I think it will help me a lot, in that the students will be more engaged. that's a big thing. The models that [Dr. Shelley] is showing us, they really promote involvement. Like, you are not relying on a student to volunteer to answer a question, you are giving them the opportunity to work with other people and develop solutions together, so then when it comes time to like come back to the group, everyone should have something to contribute. whereas what I remember is, you would either get what the teacher was trying to teach you in the 50 minutes and then if you didn't, too bad, because we were moving on to something else the next day (Jocelyn, interview 1).

In contrast to Jocelyn's initial experiences with learning mathematics in which students worked individually on mathematical tasks, she saw that her mathematics methods class was preparing her to learn to teach differently. In this new teaching approach, students are encouraged to work in groups to construct mathematical knowledge. Because of Jocelyn's commitment to teaching mathematics, she was anticipating to learning effective mathematics teaching strategies that would enable her to teach all students meaningfully. She indicated that the mathematics methods class was important to her in her teaching endeavor.

> This course is important to me because I anticipate that it will prepare me to teach mathematics using the methods that are currently expected to be the most effective. I thoroughly enjoy teaching math, so I am looking forward to learning about how to encourage meaningful discourse in my classroom (Jocelyn, survey)

As a result of Jocelyn's enthusiasm for teaching mathematics, her personal goals for taking the mathematics methods class was to learn about productive ways for teaching mathematics across all grade levels.

> One goal I have for myself is to help my students enjoy math and in part help erode the stigma that only "math people" can excel in math. Relating to this goal is the hope that I will help my students grow over the course of the year, regardless of their ability (Jocelyn, survey).

The above statement indicates how Jocelyn is committed to help all students learn mathematics. Because of her past experiences in learning mathematics, Jocelyn believes that she will understand students better in order to help them learn mathematics meaningfully. Her decision to be a mathematics teacher was predicated on her belief that she will be a good mathematics teacher.

Jocelyn's teaching context. During her filed experience, Jocelyn taught a 4th grade class at Adams Elementary school in the Ashtown District. The school used the *Everyday Math* curriculum (University of Chicago Center for Elementary Math and Science Education, 2004). Table 26 shows the demographic and population type for Adams Elementary.

Table 26

Adams Elementary Enrollment by Demography and Population Type

Ethnicity	Adams Elementary
American Indian/Alaskan Native	7 (1.3)
Asian/Pacific Islander	31(5.7)
Hispanic	221(40.6)
Black, not of Hispanic Origin	176 (32.4)
White, not of Hispanic Origin	109 (20.0)
All Students	544 (100)
Special Population	
English Learner	215 (39.5)
Special Education	44 (8.1)
Free/Reduced priced lunch	472 (86.8)
Homeless	18 (3.3)

Note. First number is the total count and the second number is the percentage of the count.

Results from Jocelyn's Mathematics Learning Goal

Jocelyn taught a 4th grade lesson on division using the partial quotient approach. Table 27 shows the mathematics benchmark for the 4th grade division fact that was to be addressed by Jocelyn.

Table 27

Mathematical Strand and Benchmark for Jocelyn's Class

Strand	Standard	No.	Minnesota state benchmark
Number & Operation	Demonstrate mastery of multiplication and division basic facts; multiply multi-digit numbers; solve real-world and mathematical problems using arithmetic	4.1.1.1	Demonstrate fluency with multiplication and division facts.

Expected essential ideas of the benchmark. The mathematical benchmark Jocelyn addressed during her three-day teaching practice was *demonstrate fluency with multiplication and division facts.* The big ideas and essential understanding involving this benchmark requires that student will be able to demonstrate mastery of basic multiplication and division facts and use these basic facts to support their work in multiplication and division problems involving multi-digits. In addition, fourth graders are required to use their place value understanding to support multiplication by 10, 100 and 1000. Also, fourth graders are required to divide multi-digit numbers by one-and two-digit numbers in problem solving situations. Furthermore, fourth graders are required to solve these division problems using repeated addition, distributive, associative and commutative properties; place value understanding; mental strategies; and partial quotients. They are also expected to use their understanding in rounding, benchmark numbers and place values to estimate and evaluate the reasonableness of results. For the purposes of Jocelyn's students teaching, her lesson was focused on using partial quotient approach to division. Therefore, her learning goals we related to helping students develop division facts.

The big ideas and essential understanding from the state mathematical framework requires that students demonstrate mastery in using division facts and place value ideas to divide multi-digit numbers by one-and two-digits numbers using different strategies. In order for students to demonstrate mastery, they should be able to show how they can draw from their previous knowledge on division facts to solve division tasks.

In order for Jocelyn to help students achieve this learning goal, she can draw from teaching strategies from the mathematics methods class. For examples, Jocelyn could use Number Talks, the direct modeling strategies, use of multiple representations from the Lesh Translation Model, and the five practices for orchestrating effective mathematics discussions to help students learn the big ideas and the essential understandings. While Number Talks could be used to review students' knowledge on basic division facts, direct modeling will help students model division problems thereby offering them the opportunity to have concrete experience. Using these multiple modes of representations can facilitate teaching and learning by enabling students to develop mathematical concepts at their own pace (Bruner, 1966; Dienes, 1966; & Lesh, 1978).

Jocelyn's mathematics learning goals. Jocelyn formulated three content goals which are presented in Table 28. The first goal clearly shows what students should be able to do. Students should be able to divide a three-digit number by a one-digit number.

Table 28

Jocelyn's Learning Goals for Division Facts for 4th Graders

Code	Learning Goals
JC1	Students will be able to solve division problems incorporating one and three-digit numbers using the partial quotients method. For example, 246/5=x.
JC2	Students will be able to interpret the significance of remainder in a division problem within context. (For example, a remainder in a word problem involving food can be further divided as a fraction or a decimal, because this reflects a possible situation. Conversely, a remainder in a division word problem involving people cannot be further divided, because people would not be divided in a real situation.)
JC3	Students will be able to identify the dividend and divisor within a word problem, and solve for the quotient.

Goal 1 shows the approach (partial product approach) students are required to use to solve the division problem. Although the goal indicates that students should use the partial product approach, Jocelyn appeared to draw from her mathematics methods class and encouraged students to use their invented strategies to solve division problems. For example, students used base-ten blocks and the equal-sharing strategy to solve division tasks. Drawing from the Lesh Translation Model, Jocelyn asked students to draw pictures for their models, use symbolic representation, and verbally explain their solution strategies to their partners, the cooperating teacher, and herself. Using this learning goal, Jocelyn assisted the students to solve division tasks in a variety of ways including the partial quotient approach. As a result of this, the learning goal addressed the essential ideas associated with the benchmark. Because students were encouraged to use other strategies, apart from the partial quotient approach, Jocelyn could have stated the learning goal to capture that idea so that it would have been explicit that students can use either the partial quotient approach or any other approach.

The second learning goal seek to help students explain the significance of the remainder in a division problem within a given context. For this goal, when students solved division tasks that resulted in decimals, they were required to determine the significance of the fractional part. That is, they were required to state whether the decimal part should be considered as part of the solution or to be ignored. While the remainders for some division problems may be meaningful, those in other situations may not make sense. For example, when dividing 5 full pizzas between 2 people, each person will get 2.5 pizzas. In this case, the decimal component is meaningful and cannot be ignored. On the other hand, if 5 people want to share 17 chairs, each person can have 3 whole chairs and there will be left with 2 chairs. In this case, the fractional part of the solution will not make any meaning within the context of the problem. This is because the remaining two chairs cannot be broken into pieces and shared among the 5 people. This goal was clearly stated and it identifies what students should be able to do. The goal, however, does not explicitly address the essential ideas of the benchmark because students are not required to interpret remainders from division tasks. Although this learning goal did not address the benchmark, Jocelyn considered the knowledge to be important for students to know because they can apply it to their daily lives.

The final goal required students to be able to identify the dividend and divisor in a given word problem in order to solve the problem. For this goal, students were required to use either the partial quotient approach, or any method to solve a division word problem. For an extended version of this goal, students were asked to write their own division word problems even though that was not stated in the goal. According to Jocelyn, if students can write their own word problems, then they can also identify the divisor and dividend and consequently, they will be able to solve it. Although the essential ideas related to the benchmark being addressed did not indicate that students should solve word problems, it was good to help students solve word problems because word problems set the division problems in context for students. Because of that students can make meaning of the result and can also make decisions about the remainders as they were required to do in the second goal. Wring division word problems was also not stipulated in the essential ideas of the benchmark, but Jocelyn decided to let students write their own word problems because she believed that "if they can write their own problem and give it to someone to solve, then they have to really know how to solve those problems in order to be able to write their own problem" (Jocelyn, interview 2). A summary of evaluation in relation to how they meet the big ideas and essential understanding of the benchmark as well as their clarity and coherence are presented in Table 29.

Setting learning goals. In setting the three learning goals to address the given mathematical benchmark for the state standards, Jocelyn used several resources from the learning community and also considered multiplicity of factors. During the coding process all the codes related to the learning goals were identified and classified into either school-based resources/factors, or personal decisions.

Table 29

Assessment of Jocelyn's Learning goals for Clarity and Alignment with State Benchmark

Learning Goal	Clarity of goals	Connection to Minnesota state benchmark
Solving division tasks	Stated clearly because it shows what students should be able to do.	Addresses the benchmark.
Interpreting remainders	Goal was stated clearly because it shows what students should be able to do.	Does not address the benchmark.
Identifying dividend and divisors in a word problem and solving for the quotient.	Goal was clearly stated because it tells what students should be able to do.	Does not address the benchmark (Goes beyond the benchmark).

Practicing School-based resources/factors. In order to set the learning goals for her lessons, Jocelyn drew from several resources from her practicing school as well as considered some factors to enable her set her learning goals. Jocelyn obtained the learning goals from the mathematics curriculum and adapted them to make them inquiry-based. She indicated that, "a lot of [the goals] I took from the mathematics curriculum then I just adjusted how we will approach them so the goals are pretty much the same because we are held accountable to the curriculum" (Jocelyn, interview 2). Jocelyn also explained that the school system was structured in such a way that they had to keep up with the pace of the mathematics curriculum in order to teach all the content specified for each day. The implication is that Jocelyn had to ensure that she taught all the mathematics content specified for the three days. This means that when students did not understand the concepts, they will not get extra time to work on the content because they had to move on to new content.

After obtaining the mathematics content to teach, Jocelyn checked the state standards to see which benchmark corresponds with that content. In identifying the mathematics content from the curriculum, Jocelyn worked with the cooperating teacher to figure out what content to teach in order to keep up with the schedule of the class. While the curriculum materials helped Jocelyn to identify the mathematics content to teach, the classroom culture and Jocelyn's knowledge about the students' abilities enabled her to structure how the lessons would be taught. Instead of asking students to work individually, as they did with their cooperating teacher, Jocelyn wanted the students to work with partners, and in some cases students worked in groups.

> I am incorporating a lot of like partner sharing and group talk and group Work because our math lessons are, we usually start them on the carpet, they write their notes and then they go back to their desk and they do their math journal pages on their own they can work with partner but that is not necessary so am like giving them more opportunities to work with others (Jocelyn, interview 2).

Jocelyn wanted to introduce partner and group work to enable students work collaboratively instead of following the usual classroom routine. In making these decisions, Jocelyn indicated that her cooperating teacher trusted the strategies she used to engage students in the teaching and learning process.

Jocelyn's learning goals selection also depended on the availability of time. She explained that within the one hour that was allocated for mathematics lesson, part of the time had to be used for introducing the lesson. Because of this, she had to plan the learning goal she could achieve within the time for work.

Personal Decisions. Jocelyn's personal decisions for this lesson centered on instructional strategies. She wanted the students to work collaboratively and grapple with mathematical tasks in an inquiry-based format in which student will "think on their own" (Jocelyn, interview 2) before she would provide them with answers. Jocelyn seemed to drew from her understandings gleaned from the mathematics methods class about students' learning styles and the need to support all students to learn by working in groups and engaging in thoughtful discussions to construct mathematical knowledge. Jocelyn's personal comfort level about teaching mathematics and how she feels about division also influenced her general goals for the lesson. She explained that because she is very comfortable teaching mathematics, she was well prepared to set learning goals she knew she would be able to achieve.

University-based Resources. Drawing from the Cognitively Guided Instruction (CGI) ideas about teaching and learning mathematics and the Lesh Translation Model, Jocelyn indicated that she wanted to use direct modeling for the students to be able to have physical materials to work with in solving division tasks. By using these concrete materials Jocelyn believed that it will afford all students to be able to learn division in using their own strategies and also making the concepts "more approachable to them" (Jocelyn, interview 2) because some of the students needed more help with mathematics. In addition to the concrete materials, other modes of representations were also used to ensure that students understood what they learned.

Summary for Research Question 1 for Jocelyn

During her students teaching, Jocelyn set three learning goals based on her school mathematics curriculum. The learning goals were intended to address the mathematics content that were stipulated in the mathematics curriculum. The learning goals from the mathematics curriculum differed from the benchmark in the state standard. In view of this, only one of the three learning goals addressed the benchmark. All the learning goals clearly indicated what students should know and be able to do at the end of the given day's lesson. In setting the learning goals, Jocelyn used several resources from the learning community including the mathematics curriculum, state standards, help from the cooperating teacher, ideas form the mathematics methods class. Jocelyn also considered students' abilities and available time to decide the goals that should be set for each day.

Results from the Learning Tasks

During Jocelyn's three-day teaching practice, she used several learning tasks to support students' learning of partial quotient approach to division. Table 30 shows all the learning tasks used during Jocelyn's students teaching. These learning tasks were enacted during Jocelyn's three-day teaching practice.

Table 30

Learning Tasks used in's Jocelyn's 4th Grade Lessons

Code	Task	Student responsibility
JT1	Reviewing/Solving partial quotient approach to division Review in whole group and solve examples with partners	We will begin the lesson as a whole group by reviewing the partial quotient method with the problem 145/6. I will write the number model on the board as $145 \div 6 = x$ speak with a partner to determine what the first step should be to solve the problem using the partial quotient method.
JT2	Solve division tasks using the partial quotient approach (work with partner)	Students are asked to use the partial quotient approach to solve division tasks. Students work with a partner and are required to explain their work to the partner.
JT3	Solve word problem involving division	Students are required to use any appropriate strategy to solve division word problems.
JT4	Making decisions on the remainder of a division problem	Students work in pairs or groups to solve division tasks and decide whether the remainder from the division is significant or not.
JT5	Identify the operation used in a word problem and giving rationale	Students are required to identify the mathematical operation that is needed to solve a given word problem.
JT6	writing division word problem	Students are asked to write their own word problems.
JT7	Modifying subtraction problem to division problem	Students are required to change a subtraction word problem to a division word problem.

Categorizing the learning tasks. Using Stein et al., (1996) framework of cognitive demands, the learning tasks were categorized to identify the cognitive demand they posed to students as they learned to count money. Table 31 shows the categorization of the learning tasks used in Jocelyn's class.

Table 31

Cognitive Demands of Jocelyn's Learning Tasks

Lower-Level Demands	Higher-Level Demands
Memorization	Procedures with connections Making decisions about remainders from division problem; Modifying subtraction word problem to division word problem; Identifying the operation used in a word problem
Procedures without connection Reviewing partial quotient approach to division	Doing mathematics Writing division word problem

Reviewing students' prior knowledge knowledge on partial quotient's approach to solving division tasks was categorized as low-level demand task because students had already learned how to solve division tasks using the partial quotient approach. In this situation, students were required to recall their knowledge on divisions. Asking students to write their own division word problems was considered as a high-level cognitive demand task because students had to draw from their mathematical knowledge involving divisor and dividend, as well as their knowledge in sentence construction, to write word problems.

Mapping the learning tasks to the learning goals. In seeking to find out whether the learning tasks addressed the learning goals, the learning demands of the learning tasks were assessed to see how they supported students' mathematical knowledge building in order to meet the stated learning goals. Table 32 shows the learning goals and the related learning tasks that map onto the learning goals. All the learning tasks enacted provided learning opportunities for students to have mathematical knowledge that helped them achieve the learning goals.

Table 32

Jocelyn's Learning Goals and Associated Learning Tasks

Learning Goals	Learning tasks
Students will be able to solve division problems incorporating one and three-digit numbers using the partial quotients method.	JT1, JT2
Students will be able to interpret the significance of remainder in a division problem within context	JT1, JT2
Students will be able to identify the dividend and divisor within a word problem, and solve for the quotient.	JT1, JT2, JT3

While some of the learning tasks addressed the learning goals that were set for the lessons, some of the learning tasks students engaged in did not address any of the learning goals directly. Reviewing previous knowledge on partial quotient, solving division tasks and making decisions on remainders from the division tasks addressed the learning goals. Tasks that required students to identify the mathematical operations in word problems and tasks that required students to modify subtraction word problems to division word problems were used to scaffold students' ability to write their own division word problems. In those word problems, Jocelyn guided students to identify the three parts of the word problems and how each of the three parts contributed to show the mathematical operation needed to solve that word problem. For these tasks, Jocelyn provided four word problems, each one addressing addition, subtraction, multiplication, or division. By using these four word problems, students identified words and phrases that determined the mathematical operation for the word problem. These ideas helped students to see the difference between division word problem and a non-division word problem, which consequently helped students to write their own division word problems.

Setting learning tasks. In order to use appropriate learning tasks that will help students achieve the learning goals, Jocelyn used several resources from the learning community. In addition to this, Jocelyn's personal dispositions also influenced her decision-making process around the learning tasks. All the interview transcript related to the learning tasks were coded for school-based resources, and university-based resources and personal decisions.

Resources from practicing school. In designing learning tasks for the three-day lessons, Jocelyn either used tasks or adapted tasks from the school mathematics curriculum. Jocelyn indicated that she had to use the tasks in the mathematics curriculum book because all the students in 4th grade were required to answer those tasks as their homework assignments. In adapting tasks for the students, Jocelyn restructured how students were required to answer the tasks. Jocelyn explained that,

> I looked at the curriculum, there is like a book that shows like step by step what to do so I kind like looked at what they suggested. Some of them I took directly from the curriculum, like the summative work that they do that's part of the curriculum because it's homework for them and the whole grade level does that so I wanted to incorporate them in the body of the lesson I will look at what they suggested and I just change a few things (Jocelyn, interview 2).

The above quotation suggests that Jocelyn, used the school curriculum book in order to let students work on required homework tasks that were stipulated for all the 4th graders.

Personal Decisions. While Jocelyn used learning tasks from the curriculum book, she also adapted some of these tasks to enable students work collaboratively. She set the tasks in an open-ended form, asked students to use multiple paths to solve them, and at the same time, ensured that the tasks were presented with simplicity to facilitate students' learning.

> I thought of what they could do easily with partners or group so I will really want to like, incorporate the group work aspect and then I really just though about what's the most efficient way or the simplest way they can see these concepts because I didn't wanna make things so complicated and fancy that they lose site of the objective (Jocelyn, interview 2).

Jocelyn's ambition to help all students learn mathematics is shown in her decision-making process around the learning tasks. She wants to present the mathematical concepts efficiently and with simplicity so that all the students can learn the desired mathematical knowledge without distractions. Thinking of presenting the mathematical tasks efficiently will require that Jocelyn draws on students' prior knowledge and what they can do as well as to use her mathematical and pedagogical content knowledge effectively.

Also, Jocelyn encouraged students to use multiple pathways to solving mathematical tasks. Unlike the negative learning experiences (she had during her elementary school years), Jocelyn provided students with the opportunity to explore mathematical tasks to and use their invented strategies to solve those tasks. Encouraging students to use multiple solution strategies provides opportunities for students to discuss their ideas with their group members or partners. This approach helped student to actively engage in the learning process. Jocelyn believed that instead of providing solutions to students, it is important to allow students grapple with mathematical tasks to find their own solutions because "when kids find things on their way it's so much powerful than when they are just told" (Jocelyn, interview 2).

University-based Resources. Jocelyn also used ideas from the university methods course in planning the learning tasks that were used to support students' learning. These ideas were the direct modeling strategy from *Cognitively Guided Instruction* and the use of multiple representations from the Lesh Translation Model. In solving division tasks, Jocelyn asked students to use base-ten materials to model their solution strategies and also asked them to use other modes of representations to illustrate their concrete models. By asking students to use multiple modes of representation to illustrate their thinking, it is likely that students may be able to make connections from one mode of representation to another, thereby enhancing students' understanding of the concepts they learn.

> I was thinking a lot, like the cognitively guided instruction because I think those translate really well to like what my class is doing now. They think of numbers like abstract sense but along that continuum because we have so many that performing math students and very low students, so I thought about how to make this approachable to everyone because in a lot of instances that I have noticed half the class get it, quarter of the class is kind of like I need some help and then the last quarter is like they are completely lost so yes CGI was mainly what I drew from (Jocelyn, interview 2).

Summary for Research Question 2 for Jocelyn

In helping students to meet the learning goals that were set for this lesson, Jocelyn used different learning tasks to help students to the goal. Most of the learning tasks were high-level cognitive demand tasks that required students to make use of the mathematical knowledge and also draw from other knowledge base to make necessary connections in order to solve the tasks. While some of the tasks directly addressed some of the learning goals, other tasks were used to scaffold students' learning in order for them to be able to solve other tasks. In designing the tasks, Jocelyn used several resources from the learning community and also considered several factors including students' learning abilities, as well as how she wanted students to work collaboratively, using multiple modes of representations, to generate multiple solution pathways to the tasks that were posed to students. Jocelyn indicated that the understandings from the mathematics methods class enabled her to set engaging learning tasks. Ideas from the Lesh Translation Model and Cognitively Guided Instruction supported Jocelyn's learning tasks setting. The learning tasks actively engaged students to think deeply to construct mathematical knowledge. Jocelyn was the only participant who set a task which was categorized as "doing mathematics." This task required 4th graders to write their own division word problems and solve them.

Using the Five Practices to Orchestrate Class Discussions

Jocelyn's anticipation for her students included the type of solution strategies students were going to use and some specific solutions students might use in class. For the solution strategies, Jocelyn anticipated that students may use the standard algorithm, non-standard algorithm, or a direct modeling approach to solve division tasks. Because of Jocelyn's commitment to help all students learn mathematics by drawing from their previous knowledge, she was open to seeing students use their invented strategies and also use the manipulatives to generate solutions. Because of this, she did not restrict the students, but rather encouraged them to collaborate with their partners (when working in pairs), or group members in order to solve the division tasks. During the teaching sessions, Jocelyn monitored students' work by circulating in the class to look out for the strategies she had anticipated. It appeared that Jocelyn's anticipation helped her during the monitoring stage because as she circulated in the class she was able to focus on what she was looking for. In order to guide students to achieve the desired leaning goals, Jocelyn selected sample students' work and strategies, sequenced them according to how she wanted the students to share, and help students connect their ideas towards the goal for the day.

According to Jocelyn, this teaching model in which teachers set up the task and implement the five practices to help students achieve the learning goals is a powerful teaching strategy. She explained that,

> I really like the model [Dr. Shelly] showed us in class. The five steps, when the students are working look around and you will take notes of who is showing which strategies and then you will deliberately choose them to share in a particular order so that amount of control gives you the chance to show the kids like these are all different ways that we can approach a problem (Jocelyn, interview 2).

Jocelyn liked using the five practices to organize her lessons because she realized that this was productive way for her to organize her lesson in order for her to be able to see what student know and can do, share different solution strategies to students in order to help them to achieve the desired learning goal. Although Jocelyn liked the five practices, she thought that if teachers were not careful they may not engage students whose work may not be selected and shared by the teachers. Jocelyn explained that this teaching approach gave power to the teacher because the teacher controls which students get to share their work and who does not. Table 33 summarizes how Jocelyn implemented the five practices during her three-day student teaching.

Table 33

Summary of Jocelyn's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Jocelyn's Practices
Anticipation	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices	I anticipate that the use of a visual will not be represented in the student's work, since the numbers they are working with are relatively large. I anticipate that the students may bring the following strategies to the discussion: use of a standard algorithm, direct modeling, and using non-standard algorithm.
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem.	Circulating to see what students are working on. Checking for specific solution strategies..
Selecting	Select specific students to share their work with the whole class.	I will look for the following: Use of a standard algorithm to solve a problem, Use of visuals to solve a problem, Correct use of the Partial Quotient Model. I will be looking for an answer similar to: "Each child gets 5 marbles, and there is one left over.
Sequencing	Purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goal	I will ask one group that I observed using a standard algorithm to solve the problem to share their strategy with the class. The second group that I ask to share will have used a non-standard algorithm to solve the problem. The final group I will ask to share will have used direct modeling or another visual representation to solve the problem.
Connection	Teacher to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully.	We can connect these methods to approaches that students may use when solving word problems that incorporate subtraction or multiplication as well.

The Case of Morgan

Morgan's mathematics background. Morgan had a very good experience with mathematics at the elementary and high school years until she took trigonometry and calculus. According to Morgan, she could not get along well with the trigonometry teacher, a situation that caused her love trigonometry to wane. During her high school years, Morgan took advance mathematics classes including algebra 2 and advance placement calculus (AP calculus). Morgan indicated that trigonometry and calculus were the two course that made her have negative experiences with mathematics. She explained that,

> I always liked math until I got to trigonometry and calculus. Algebra 2 was fine, it was just trigonometry I had a teacher that I did not click with which I feel like it kind like destroy for a lot of people that have that one teacher, and then that really turned me off from math and then calculus I just I did it but I was never really interested in there and I think that that's kind like where I fell off and kind like lost my kind a got scared of math. Before that I was, I loved math and then my undergrad degree is architecture so like I did math for my previous job all the time, like that's it wasn't scaring in that sense (Morgan, interview 1).

Clearly, Morgan's negative experiences with her trigonometry teacher made her lose interest not only in trigonometry, but in mathematics as a whole. The negative experiences from trigonometry and calculus transcended into having some apprehension for mathematics. Although she did not like calculus she had to still take the course because she was majoring in architecture for her undergraduate degree. Because Morgan wanted to be teaching mathematics, she came back to the university to take the licensure program for teachers.

At the college level, Morgan took*, Mathematics and Pedagogy for Elementary Teachers* I, and *Mathematics and Pedagogy for Elementary Teachers II*. She indicated that although she encountered some challenges with learning to integrate technology in mathematics during the initial stages of the mathematics and pedagogy class, she became comfortable with the course because of the support she obtained from the course professors. She also explained that the mathematics and pedagogy classes were inquiry-based, in which students were introduced to the use of manipulatives to facilitate learning.

Morgan's vision as a mathematics teacher. Morgan's desire to make mathematics interesting and engaging for students encouraged her to leave architecture and business for teaching. Although Morgan loved architecture, that was not her passion. She explained that, "I want to stimulate math interest and make kids excited to learn about math. I also want to be able to create those connections to kids' lives so math becomes meaningful to them instead of worksheet after worksheet" (Morgan, survey).

Even though Morgan has the passion to be a good mathematics teacher, she did not see herself to be qualified to teach mathematics because she believed that she needed to learn how to teach mathematics in order to teach it effectively.

> I feel like I was always really good at [math] in school but teaching it is, I am really uncomfortable with it and I feel like I am not prepared to teach kids. Even now in my co-teaching I always feel like I am not confident enough in the material knowing how to present it to really do the math justice I guess. So like to be the best that I could be for the kids. I don't feel like I am at that point yet. (Morgan, interview 1).

Morgan believes that knowing mathematics content is different from being able to teach it effectively to students. Her negative experiences with mathematics teachers may possibly account for her fear in teaching mathematics. Nevertheless, Morgan was determined to acquire the mathematical and pedagogical content knowledge in order to be a good mathematics teacher. Because of Morgan's determination to be a good mathematics teacher, her personal goals for the mathematics methods class was to learn "how to make math understandable for elementary kids" (Morgan, survey) and "make it engaging and meaningful and also follow the prescribed curriculum" (Morgan, survey). When asked why the methods class was important for her, she explained that, "this class is very important to me! Teaching math is something I am uncomfortable with; it is essentially, "scary" for me. I want to overcome the fear of teaching it and be confident in my practices!" (Morgan, survey). Although Morgan desires to be a good mathematics teacher, she was scared of taking the full responsibility of being a classroom teacher. Morgan, however, believed that the methods class will equip her with teaching skills to enable her teach mathematics effectively to students.

Morgan's teaching context. As part of her field experience requirement of the mathematics methods course, Morgan prepared and taught a three-day mathematics lesson at a practicing school. Before this teaching experience, Morgan had observed the cooperating teacher and had assisted her in classroom duties including co-teaching some mathematics lessons. Morgan taught a 3rd grade class at the Birch Elementary school in the Brandon School District. The school used the *Math Expressions curriculum* (Houghton Mifflin Math Expressions, 2004). Table 34 shows the demographic and population type for Birch Elementary.

Table 34

Students Enrollment by Demography and type of Population

Ethnicity	Birch Elementary
American Indian/Alaskan Native	13 (1.2)
Asian/Pacific Islander	197 (17.8)
Hispanic	225 (23.1)
Black, not of Hispanic Origin	490 (44.4)
White, not of Hispanic Origin	149 (13.5)
All Students	1,104 (100)
Special Population	
English Learner	259 (23.5)
Special Education	145 (13.1)
Free/Reduced priced lunch	901 (81.6)
Homeless	21 (3.3)

Note. First number is the total count and the second number is the percentage of the count.

Results from Morgan's Mathematics Learning Goals

Morgan taught a 3rd grade mathematics lesson on place value, rounding and estimation. Because of the school pacing guide, Morgan was required to address more than one benchmark for the three days. Table 35 shows the mathematics benchmarks in the Minnesota state standards which were addressed.

Table 35

Mathematics Strand and Benchmark for Morgan's Class

Strand	Standard	No.	Minnesota State benchmark
Number & Operation	Compare and represent whole numbers up to 100,000 with an emphasis on place value and equality	3.1.1.4	Round numbers to the nearest 10,000, 1000, 100 and 10. Round up and round down to estimate sums and differences.
		3.1.2.1	Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.
		3.1.2.2	Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.

Note: first grade, geometry and measurement, second standard and second benchmark.

Expected essential ideas of the mathematics benchmark. Morgan wanted to address three benchmarks from the state standards: rounding whole numbers, adding and subtracting whole numbers and solving real-world problems involving addition and subtraction. Relating to rounding whole numbers, students are required to be able to round four- and five-digit whole numbers to the nearest 10,000, 1,000, 100 and 10 and use rounding to estimate sums and differences. The essential ideas related to the addition and subtraction of the second benchmark requires that students will be able to apply their knowledge of place value to add or subtract multi-digit numbers up to three- and four-digit numbers. The third benchmark requires that students will use variety of strategies including assessing reasonableness of results, use of technology, place value understanding, use of standard algorithm, and relationship between addition and subtraction to solve real-world problems related to addition and subtraction. In addition, students are required to exhibit their knowledge on decomposition and composition of numbers when finding sums or differences. For example, adding 184 and 37 could include 180 and 30, then adding 4 and 7 or 184 + 37 = (180 + 4) + (20 + 10 +7) = 180 + 20 + 10 + 4 +7.

In addressing these essential ideas, the researcher expected that Morgan may draw from all the learning theories she learned from the mathematics methods class. For example, ideas from the Cognitive Guided Instruction could be used to engage student in direct modeling of addition and subtraction using base-ten materials as well as applying invented strategies to positive and negative whole numbers. In doing so, students can be asked to work collaboratively in order that they can support one another in the process of constructing mathematical knowledge. Setting up the class in a way that will involve students' participation would enable Morgan to monitor students work, address their challenges, select exemplary work that could be shared with the entire class. When students' works are selected, Morgan can then sequence them in a way that will enhance her connect students' ideas meaningfully to help students achieve the desired learning goals.

Morgan's mathematics learning goals. Morgan formulated her learning goals for each of the three days. In table 36, the learning goals for each day are presented.

Morgan set five goals for the three days. Two of the learning goals were addressed for all the three days. For the first day, Morgan helped the students to be able to round whole numbers, estimate sums and differences using rounding ideas, and identifying the mathematics operation needed to solve a given word problem. For the first of the three goals for the first day, students were required to draw from the previous knowledge on rounding and build on that knowledge to be able to round whole numbers to the nearest 10, 100, and 1000. This knowledge was then used to build students' knowledge on estimating sums and differences. In order for students to be able to estimate a given sum, the needed to round off the given addends to the nearest 10, 100, or 1000 and use the new numbers to find the sum.

Table 36

Morgan's Learning Goals for her Three-Day Lessons

Day	Code	Learning Goals
	MC1	Round numbers to the nearest 1000, 100, or 10.
1	MC2	Estimate sums and differences using estimation and round
	MC3	Decide if a word problem will be solved using subtraction or addition
	MC3	Decide if a word problem will be solved using subtraction or addition.
2	MC2	Estimate sums and differences using estimation and rounding
	MC 4	Solve real world subtraction problems with numbers up to 10,000.
	MC 3	Decide if a word problem will be solved using subtraction or addition
3	MC 2	Estimate sums and differences using estimation and rounding
	MC 5	Solve real world subtraction problems with numbers up to 1,000,000

Rounding the numbers enabled the students to estimate the sums and differences. Building students' knowledge in rounding numbers therefore, served as a good foundation for the estimations exercises. The two learning goals were sequenced in such a way that the first learning goal provided the prerequisite knowledge for the second learning goal. The third learning goal for the first day required that students identify the mathematical operation (addition or subtraction) needed to solve a given word problem. This learning goal did not have direct connection with the first two learning goals for the day, but it introduced the students to word problems and also helped them to identify special words or phrases in the word problems and their related mathematical operation required to solve them. Once students identified addition and subtraction word problems, they also estimated the sums and differences. All the three learning goals for the first day addressed the three benchmarks. Although not all the three learning goals addressed the benchmarks, they provided initial knowledge base which served as a platform for other knowledge development.

The fourth learning goal was used to reiterate the concept of subtraction, but for larger numbers and using the context of word problems. The word problems required students to identify the numbers to subtract and how to do the subtraction. Students used their place value and estimations ideas to find the differences between two multi-digit numbers up to 10,000. Morgan decided to scaffold students learning by asking students to find sums and differences on different days. Instead of asking students to find sums and difference up to 10,000 on the first day, she decided to split this on the first two days. Splitting the learning goal provided students with the opportunity to build on their previous knowledge. Similarly, the last learning goals was used to reinforce students' knowledge on solving subtraction word problems, but to 1,000,000 on the third day.

Table 37

Assessment of the Morgan's Learning Goals for Clarity and Alignment with State Benchmark

Learning Goal	Clarity of goals	Connection to Minnesota state benchmark
Round numbers to the nearest 1000, 100, or 10.	This goal is clear because it shows what students should be able to do	This addressed the essential ideas of the benchmark
Estimate sums and differences using estimation and rounding	Learning goal shows what mathematical knowledge students should be able to demonstrate at the end of the lesson	This addressed the benchmark
Decide if a word problem will be solved using subtraction or addition	This goal is very clear because it shows what students should be able to do	This goal was used to scaffold students' learning to solve word problems involving addition and subtraction. Students' ability to identify the mathematical operation to be used to solve the word problem is the first step in solving word problems.
Solve real world subtraction problems with numbers up to 10,000.	This is clear because it identifies the mathematical knowledge students are required to learn	This goal addresses the benchmark
Solve real world subtraction problems with numbers up to 1,000,000	This goal is clear because it identifies what students should be able to do	This addresses the benchmark

Morgan's learning goals were clearly stated to show what students should be able to at the end of each day's lesson. The goals were scaffolded to ensure that students acquire the necessary prior knowledge to build on for further mathematical knowledge. For example, instead of teaching students to subtract multi-digit numbers up to 1,000,000, in one lesson, Morgan broke this down over the three days. Breaking down the lessons into smaller parts afforded students the opportunity to ask questions for clarification, and also enabled Morgan to address students' difficulties as they learned to solve word problems involving subtraction. A summary of evaluation of Morgan's learning goals in relation to how they addressed the essential ideas of the benchmark, as well as their clarity and coherence are presented in Table 37.

Setting learning goals. In formulating the five learning goals to address the given mathematical benchmarks for the state standards, Morgan used the cooperating teachers support, ideas form the mathematics curriculum and also considered her personal decisions. When asked how she formulated the learning goals, Morgan shared that, "I asked my cooperating teacher for advice and if that it will work and, referred to the curriculum for objectives and what needs to be taught" she went on to explain that "I guess a lot of [the goals] is from the curriculum and what's needed by that and then what I think my kids can do and what level I think they are" (Morgan, interview 2).

Practicing school-based resources/factors. Morgan used the mathematics curriculum and the support from the cooperating teacher to set her learning goals for the three days teaching. Because Morgan taught for the three days, she was required to adhere to the curriculum demands of the practicing school. In this case Morgan had to follow the pacing schedule for the class because she indicated that the teacher was expected to strictly follow the pacing guide even if students did not understand a particular lesson.

> They are expected to be on that day they are expected to be matching math calendar every single day. and they were told like if you don't finish the lesson for that day you move on to the next day and so it's very strict and not necessary best

> for the kids at all. But I think that the teachers are under a lot of pressure to follow that. (Morgan, interview 1).

Because of accountability on the part of the cooperating teachers, Morgan ensured that she sought advice from the cooperating teacher regarding the lesson goals she wanted to set. While the cooperating teacher assisted Morgan in deciding on what to teach, the mathematics curriculum assisted her to set the learning goals for each day, making sure that the mathematics content that had to be taught were addressed in the learning goals.

Personal experiences/decisions. Morgan's personal dispositions about what her students can do within a given time frame influenced the learning goals she set for her students. Although she wanted to address the curriculum demands, she also considered her students' abilities to set goals that could be achieved in a given time interval. Morgan's ability to factor her students learning background into setting learning goals enabled her to scaffold her learning goals meaningfully. The learning goals were set in such a way that the first learning goal provided the prerequisite knowledge for the subsequent learning goals. For example, students were required to be able to round whole numbers before they were asked to use this idea in estimating sums and differences. In addition, before students were required to solve word problems involving subtraction, Morgan introduced a learning goal that required students to be able to identify the mathematical operation that should be used to solve a given word problem. This goal although was not part of the benchmark, provided some underlying knowledge, which enabled students to differentiate between word problems that needs to be solve by addition and those that need to be solved by subtraction.

Summary for Research Question 1 for Morgan

Morgan formulated five learning goals to address the three benchmarks in the state standards. All the learning goals were clearly stated to identify the mathematics knowledge students were expected to demonstrate at the end of each of the three days. Morgan's learning goals were categorized according to the days she addressed them. The learning goals were scaffolded to provide prerequisite knowledge for one another. For example, all the learning goals for the first day provided fundamental ideas about rounding numbers, estimating sums and differences, and identifying the mathematical operation needed to solve word problems. These ideas provided prerequisite knowledge for other learning goals. The learning goals addressed the essential ideas outlined by the benchmark in the state standards. In setting the learning goals, Morgan ensured that she followed the school schedule by consulting the cooperating teacher and the mathematics curriculum to identify the mathematics content that were to be taught. While the curriculum material and the cooperating teacher assisted Morgan to obtain the mathematics content to be taught and the goals to be set, Morgan's personal decisions influenced how she categorized the learning goals for each of the three days.

Results of Learning Tasks

In order to address the learning goals for the three days, Morgan designed and used several learning tasks. For example, asking students to sort word problems into addition and subtraction categories was considered as a mathematical task because students would be required to identify word and phrases that suggest which mathematical operation is appropriate to solve the word problems. This knowledge will therefore serve as a prerequisite knowledge for solving word problems involving subtraction. Also, estimation was considered as a mathematical task because by learning to estimate, students begin to have sense of the reasonableness of solutions the will later come across. The following table shows all the tasks that were used during Morgan's teaching practice.

Table 38

Learning Tasks used in Morgan's 3rd Grade Class

Code	Task	Student responsibility
MT1	Reviewing previous knowledge on rounding numbers	Students recall their knowledge on place values to answer some questions.
MT2	Sorting word problems into addition and subtraction categories	Given bag of cards with word problems, students are required to sort them into similar categories: addition, take-away, or comparison.
MT3	Estimating difference from word problems	"I want you to each select one of your cards that is subtraction. When you have your card I want you to put it on your forehead like this (put card on forehead). Good, looks like everyone has a card. Take your card off your forehead and look at it. Now I want you to estimate the difference using what you know about rounding." Allow them to think about the answer
MT4	Addition and Subtraction of numbers up to 1,000	I will provide them with the questions. They then get the class started on these problems. The problems will ask the students to subtract and add numbers up to 1,000
MT5	Subtract multi-digit numbers up to 10,000	Students will explore in order to understand when to use subtraction. They will also explore how to approach a word problem. How do they decide what is important and what to do
MT6	Place value activity.	Using place value chart to work on addition.
MT7	Subtract multi-digit numbers up to 1,000,000	Subtract multi-digit numbers

Categorizing the learning tasks. Using Stein et al., (1996) framework of cognitive demands, the learning tasks were categorized to identify the cognitive demand they posed to students as they learned to count money. Table 39 shows the categorization of the learning tasks used in Morgan's class.

Table 39

Cognitive Demands of Morgan's Learning Tasks

Lower-Level Demands	Higher-Level Demands
Memorization Reviewing previous knowledge on rounding numbers	Procedures with connections Sorting word problems into addition and subtraction categories. Subtract multi-digit numbers up to 10,000 Place value activity.
Procedures without connection Estimating difference from word problems Addition and Subtraction of numbers up to 1,000	Doing mathematics

Morgan used a mixture of low-and high-level cognitive demand tasks to address her learning goals. Reviewing students' previous knowledge was categorized as a low-level cognitive task because the assumption was that students would recall mathematical knowledge during this stage of the lesson. On the other hand, subtracting multi-digit numbers up to 10,000 was categorized as high-level demand task because in the process of subtracting multi-digit numbers, it is likely that students will make connections to their place value ideas.

Mapping the learning tasks to the goals. In seeking to find out the whether the learning tasks addressed the learning goals, the learning demands of the learning tasks were assessed to see how they supported students' mathematical knowledge building in order to meet the stated learning goals. Table 40 shows the learning goals and the related learning tasks that map onto the learning goals. All the learning tasks enacted provided learning opportunities for students to have mathematical knowledge that helped them achieve the learning goals.

Table 40

Morgan's Learning Goals and Associated Learning Tasks

Learning Goals	Learning tasks
Round numbers to the nearest 1000, 100, or 10	MT1
Estimate sums and differences using estimation and rounding	MT1, MT 6
Decide if a word problem will be solved using subtraction or addition	MT2
Solve real world subtraction problems with numbers up to 10,000	MT1, MT2, MT3 , MT4, MT5
Solve real world subtraction problems with numbers up to 1,000,000	MT 1, MT2, MT3, MT4, MT5, MT7

Setting learning tasks. Because learning tasks can help teachers facilitate the teaching and learning towards desired learning goals, the tasks teachers use becomes important. In addition to using tasks to meet learning goals, the kind of tasks can also determine the kind of interactions that will take place in the classroom. In the case of Morgan, when setting her learning tasks for her three-day lessons, she used research-based ideas from the mathematics methods course while at the same time considered other factors including students' performance, time constraints, and the culture of the class.

University-based Resources. Morgan's decision-making process for the learning tasks used was underpinned by what she learned from her mathematics methods course professor. When asked how she designed the learning tasks, Morgan indicated that, "I used a lot of what we learned in class with [Dr. Shelly]. Basically pretty much all of it was like what I learned with [Dr. Shelly] and then how we do things in our class" (Morgan, interview 2). Among the things Morgan learned from her course professor included engaging students in collaborative learning in which students are given the opportunity to participate in group discussions in the process of working on mathematical tasks. Also, Morgan's understandings about direct modeling and students' invented solution strategies from the Cognitive Guided Instruction studies enabled her to design tasks that made students use manipulatives and other modes of representations to construct their knowledge. Research studies from the CGI suggest that students come to the class with their invented forms of solving mathematical tasks and that, instead of teaching students to use algorithms to solve mathematical tasks, teachers should rather provide learning materials for students to explore with mathematical tasks in order to use their invented strategies to solve the tasks.

> So with [Dr. Shelly] we did an activity like this learning how to regroup the tens and then saying like when you have a 1000 like become like you are taking away this and you are becoming this kind of things so having that physical regrouping. So I had that idea and as a class we do a lot like group work so that's kind the kind of the classroom so that's what I did with that one and then the students leaders they also something that we do as a classroom but I thought that [Dr. Shelly] kind like talked about how like letting the kids talking about things so I wanted that to be a part of it (Morgan, interview 2).

Drawing from these ideas, Morgan used manipulatives and designed activities to engage students in learning to regroup in base-ten for addition and subtraction. This activity used in Morgan's class was similar to a mathematics game on place values which was played in the methods class. During the mathematics methods class at the university, when preservice teachers were learning how to teach place value concepts and operation of whole numbers, the course professor engaged the preservice teachers to play the place value game to learn the concept of regrouping in base ten and other bases. In playing the game, preservice teachers were also introduced to all the teaching and learning materials including the place value charts which were to be used for the game. Morgan was able to adapt this game for her 4th grade class to help them learn how to regroup in base-ten.

Factors from practicing school. Morgan considered several factors including students' performance, time availability and the content to be taught, and the culture of the class. Based on the students' performance, Morgan designed activities that supported students' learning of the mathematics concepts. These activities were designed to let students work collaboratively so that they were able to help one another. In addition to the students' performance, the time constraint was also considered to select the types of activities that were used. Also, the content to be taught was considered in deciding what tasks to be used for the lessons. These factors are seen in Morgan's explanation to the factors she considered in setting the learning tasks.

> The level of the kids, what we are going to be doing and how much time we have. Just depending on what I know that they know, like I have seen their test scores so that is what I can take in accounts and I have worked with them before so I know, like maybe some hurdles might be and what some road blocks are and how

> we can either break through those or like how we scaffold them so they can make for them…what time can I do in the time that I am given. I can't have them build something … I mean it has to be a realistic goal in the amount of time we have so like including like how many materials we gonna use, what's the set up like, what's the clean up like, that kind of thing…What tasks lends itself to the content. So like am not gonna have them do stuff with pizza for adding and subtracting know what I mean, like matching the content with the tasks (Morgan, interview 2).

The above quote summarizes all the factors Morgan considered before designing the learning tasks. Drawing from the mathematics methods class, as she indicated, Morgan considered multiplicity of factors to ensure that the learning tasks she was going to use were going to help students learn the desired learning goals in the given time available. Also, Morgan thought of using "appropriate" learning materials for the mathematics content that was to be taught. Using "appropriate" learning materials means that based on the mathematics content, she used the learning materials that enhanced teaching particular concept. In this case, because she was teaching addition and subtraction of whole numbers and place values, she knew that using money would be a productive teaching and learning material than using pizza. In the methods class, using pizza as a context to teach fractions was discussed to be an effective strategy, but in this case, Morgan realized that using pizza will not be the best learning material. It is important therefore, for preservice teachers to use knowledge gained from the methods classes in a way that will enhance teaching.

Summary for Research Question 2 for Morgan

In seeking to help students achieve the three benchmarks for the three-day teaching practice, Morgan designed different learning tasks. A mixture of low- and high-level cognitive demand tasks were used. Reviewing students' prior knowledge on rounding numbers was however, a low-level cognitive task which was based on memory or using procedures without connections. All the learning tasks provided mathematical knowledge that helped students to achieve the learning goals. Thus, the learning task addressed the specific learning goals. The learning tasks also provided scaffold for students to build on for other learning goals. In designing the learning tasks to use, Morgan used ideas from her course professor as the basis for setting up tasks that will engage students to work collaboratively. Ideas form the course also prompted Morgan to make informed decisions when considering the time allocation, the students' performance, and the type of manipulatives to use for the learning tasks.

Using the Five Practices to Orchestrate Class Discussions

In describing how she was going to enact the five practices, Morgan provided general ideas about how each of the five practices would be enacted. For example, her anticipation centered on students' solution strategies and the kind of misconceptions students might bring to the class. She could not state specific misconceptions or solution strategies students may bring to the class. Having a knowledge about students' specific misconceptions and solution ideas may enhance teachers' ability to monitor students' work for those specific ideas or misconceptions in order to address them effectively. During the teaching and learning sessions, Morgan circulated the the class to monitor students' work. While circulating, she took notes of students' work and shared this with the entire class during whole-class discussions. Decisions about how selected ideas were going to be sequenced were not explicitly stated in the lesson plans. Similarly, Morgan could not explain how students' ideas would be built on to make the necessary connections. The following table shows how Morgan enacted the five practices.

Table 41

Summary of Morgan's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Morgan's Practices
Anticipating	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices	How might the students answer this question? What misconceptions might this question illustrate?
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem.	Circulate while the students are working and observe work Look for students who can verbalize the steps to getting the exact answer. Look for different strategies for estimation
Selecting	Select specific students to share their work with the whole class.	Selects students who have different strategies and who you know can verbalize thinking. In the closure pick on students purposefully to bring out good estimation strategies and language for explaining subtraction algorithm with large numbers.
Sequencing	Purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goal	Decide which student work will be first. How can they build on one another?
Connecting	Teacher to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully.	How are all the responses connected?

In enacting the five practices, it appeared that because Morgan could not anticipate specific misconceptions or students' possible solution strategies, she had challenges with stating examples of specific examples she would select for sharing. Because she could not have specific examples to select, she could not also explain how sequencing would be done. Although Morgan appeared to know students' work to select and how to sequence them for whole-class sharing, it was difficult for her to state these practices in her lesson plan.

The Case of Rachel

Rachel's mathematics background. While Rachel described her elementary and middle school mathematics learning experiences as positive, she described her high school mathematics learning experiences as negative. She recalled that her high performance in mathematics in elementary school started in 4th grade when she used to complete her mathematics tasks ahead of time. Her high performance in mathematics enabled her to be placed in a special mathematics program in which she took her mathematics classes ahead of her peers. During her middles school years Rachel was always a year ahead of mathematics because she was always finishing her math "really quick" since 4th grade. She explained that,

> My math really started like elementary school in 4th grade I got to be a part of, kind like child program because I was always finishing math really quickly and so I did 4th and 5th grade math in 4th grade. In 5th grade, I did 6th grade math so it's always a year ahead of math (Rachel, interview 1).

Rachel's mathematics performance from the elementary and middle school did not however, translate to her four years of mathematics learning during her high school years.

During high school, she took algebra 2, geometry and calculus but did not do well in these classes. Rachel indicated that she began to dislike mathematics again during her high school years.

At the college level, Rachel took, college algebra, *Mathematics and Pedagogy for Elementary Teachers* I, and *Mathematics and Pedagogy for Elementary Teachers II*. Rachel indicated that the first part of the mathematics pedagogy classes was frustrating for her because she did not know what was expected of her in the class. Although Rachel saw the class as frustrating, she expressed that her perception about mathematics changed as a result of how the teacher taught by inquiry.

> I remember that our teacher constantly taught by inquiry. It was very interesting to feel like a child again, in the sense that I had no clue what was expected. Often we were sent home with little instruction on the homework and were expected to figure it out. Though this was frustrating, it definitely shaped how I feel and look at elementary mathematics. Our teacher showed us how the students would feel. They also reminded us to go back to where we had never learned any math and to try to relearn it. The first session of math pedagogy helped me to realize that math is not just a subject we have to teach, but a subject that is exciting to explore and understand (Rachel, survey).

As indicated in the above quote, Rachel's perception about mathematics changed. She had always thought that students should be taught to memorize mathematical facts and rules to apply to solve mathematical tasks. However, this perception changed as a result of the teaching approach the course professor adopted to teach the class. In addition to the fact that Rachel's perception about the nature of mathematics teaching changed, she also realized how students feel whenever they did not understand what they learned. These experiences shaped Rachel's thinking about teaching and learning mathematics. The second part of the mathematics and pedagogy classes did not pose a challenge to Rachel.

Rachel's vision as a mathematics teacher. Although mathematics was not Rachel's favorite subject to teach, when she took the mathematics and pedagogy classes she realized that mathematics could be taught in an engaging manner to support students learning. She indicated that learning mathematics is not only about numbers and facts, but to explore mathematical concepts in order to make meaning of mathematical knowledge.

> To be honest, mathematics was one of my least favorite subjects to teach when I initially started my elementary education degree in undergrad. Once I started my pedagogy classes in math, I realized how exciting it math can be. When we were allowed to just play and explore concepts, I loved exploring math. Now, I really am excited to be an elementary math teacher so I can show children how interesting and excellent math can be. It's not just facts and numbers, but also the love of exploring the world around you. I'm excited to work towards this in my math classes (Rachel, Survey).

Rachel's positive experience with the mathematics and pedagogy classes changed not only her perception about mathematics, but also her desire to be a mathematics teacher. Drawing from the exploratory nature of mathematic, Rachel hopes to engage her future students in inquiry-based learning in which students will grapple with mathematics tasks in order to explore mathematics concepts for themselves.

In view of Rachel's' desire to be a good mathematics teacher who would engage her students to explore mathematical concepts, she was determined to learn the mathematics content and the pedagogical tools that would enable her to teach mathematics effectively to students. When asked about what she wanted to learn from the mathematics methods class, Rachel expressed that she wanted to

> Learn how to successfully implement mathematics curriculum in an inquiry-based way. Too often math becomes the subject that you just work in workbooks and are

> told the facts about math. I want my students to not have to wait until college, like me, to realize that math exploration is exciting and fun. I want to learn how to use the various methods, like math talks, to build a solid structure of problem solving for my students. I also want them to realize that there really isn't a right or wrong way in problem solving. If a method works for the students, I want them to explore that and also be able to see how their peers do the same (Rachel, survey).

The above statement shows that Rachel was determined to learn the mathematics content and how to teach it in an inquiry-based approach to the students so that students will not miss the opportunity of experiencing the excitement in mathematics explorations. In addition, Rachel wanted her students to know that it is better to explore mathematical strategies than to think of getting the "right answer." To Rachel, getting the "right answer" without understanding the processes that led to the "right answer" was not meaningful. Consequently, Rachel's ambition to teach mathematics is underpinned by her desire to teach mathematics in an engaging manner.

Rachel's teaching context. Rachel prepared and taught a three-day mathematics lesson at a practicing school as part of her field experience. Before this teaching experience, Rachel had observed the cooperating teacher and had assisted her in classroom duties including co-teaching some mathematics lessons. Rachel taught a 3rd grade class at Adams Elementary school in the Ashtown School District. The school used the *Everyday Math curriculum* (University of Chicago Center for Elementary Math and Science Education, 2004). Table 42 shows the demographic and population type for Adams Elementary.

Table 42

Adams Elementary Enrollment by Demography and Population Type

Ethnicity	Adams Elementary
American Indian/Alaskan Native	7 (1.3)
Asian/Pacific Islander	31(5.7)
Hispanic	221(40.6)
Black, not of Hispanic Origin	176 (32.4)
White, not of Hispanic Origin	109 (20.0)
All Students	544 (100)
Special Population	
English Learner	215 (39.5)
Special Education	44 (8.1)
Free/Reduced priced lunch	472 (86.8)
Homeless	18 (3.3)

Note. First number is the total count and the second number is the percentage of the count.

Results from Rachel's Mathematics Learning Goal

During her teaching practice, Rachel addressed three benchmarks in the state standards. In Table 42, the mathematics benchmarks addressed for the 3rd graders are presented.

Expected big ideas and reflection on the mathematics benchmark. During the teaching practice for the three days, Rachel's lesson addressed three benchmarks of the state standard. The big ideas and essential understanding involving these benchmarks require that student will be able to read and write numbers up to 100,000; represent whole numbers up to 100,000 using words, pictures, numerals, expressions with operations, and

Table 43

Mathematics Strand and Benchmark for Rachel's Class

Strand	Standard	No.	Benchmark
Number & Operation	Compare and represent whole numbers up to 100,000 with an emphasis on place value and equality	3.1.1.1	Read, write and represent whole numbers up to 100,000. Representations may include numerals, expressions with operations, words, pictures, number lines, and manipulatives such as bundles of sticks and base 10 blocks.
		3.1.1.2	Use place value to describe whole numbers between 1000 and 100,000 in terms of ten thousand, thousands, hundreds, tens and ones.
		3.1.1.5	Compare and order whole numbers up to 100,000

Note: first grade, geometry and measurement, second standard and second benchmark.

number lines; use base-ten materials to represent numbers from 1000 to 100,000 using ten thousands, thousands, hundreds, tens, and ones; represent numbers from 1000 to 100,000 in written form; and identify value and place of digit in a given large number (Minnesota Mathematics Framework, 2014).

The essential mathematical knowledge students are required to demonstrate from Rachel's benchmarks requires students to use multiple representations to represent numbers up to 100,000. In using the multiple representations, students are required to translate between representations to demonstrate their understanding. For example, when students are given a symbolic representation of a number, they are expected to use different forms of representation, including words, pictures, base-ten materials, etc., to represent that number. In addition, students are required to identify the place value associated to a digit in a given number. Using their place value concepts and understanding, students are required to be able to write numbers in expanded form and should also be able to order numbers. In order for Rachel to be able to help students achieve these goals, her learning goals should not only address all the required essential understanding, but should also be sequenced such that knowledge gained from one learning goal would serve as prior knowledge for subsequent goals.

Rachel's mathematics learning goals. In order to address the given benchmarks in the standard, Rachel formulated 10 content goals for the three days. Table 44 shows the learning goals for each of the three days.

Table 44

Rachel's Learning Goals for the Three-Day Lessons

Day	Code	Learning Goals
1	RC1	Students will review place value through ten-thousands.
	RC2	Students will read and write multi digit whole numbers.
	RC3	Students will read and write numbers up to 100,000.
	RC4	Students will identify the places in numbers through ten-thousands and the values of the digit.
2	RC5	Students will compare and order whole numbers less than 100,000.
	RC 6	Students will read and write numbers up to 7 digits.
3	RC 7	Students will identify the places in numbers through millions and the values of the digits in those places.
	RC 8	Students will compare and order whole numbers through millions.
	RC9	Students will solve population questions and write answers in extended notation, number form, and words.

Rachel's learning goals for the first day were used to review students' previous knowledge on place values and to help them read and write numbers up to ten-thousand. Although the first learning goal indicates that students were going to review their prior knowledge on place value, it does not tell explicitly whether students will review up to hundreds, or thousand, etc. By reviewing students' prior knowledge on learning goals, students were able to apply their place value ideas to read and write numbers up to ten-thousand. The learning goal did not address the benchmarks directly, but it provided the requisite knowledge for addressing the benchmarks. Students used their place value ideas to read and write numbers up to ten-thousand. Rachel used place value chart and base-ten blocks to help students read numbers. While the place value chart provided visual representation for the students, the base-ten block provided direct modeling experiences for the students. Students explored with the base-ten blocks by forming numbers with the block, reading the numbers, and writing the numbers in symbols or in words. The second learning goal for the first day was clearly stated to demonstrate what students should be able to do. This goal provided mathematical knowledge that addressed the essential ideas related to the benchmark. Thus, the learning goal addressed the first benchmark.

The first learning goal for the second day required students to read numbers up to 100,000. This goal built on the previous day's goal in which students were required to read and write numbers up to 10,000. Rachel reviewed students' knowledge on reading and writing numbers and built on that to help students learn how to read and write numbers up to 100,000. This goal was clearly stated to show the mathematical knowledge students should be able to demonstrate. The second learning goal for the second day was used to help student identify the place value of a given digit in a given number. In achieving this goal, Rachel used the place value chart and the base-ten blocks to help students associate the correct value for a given digit in a given number. Students identified the value of a digit in multi-digit numbers up to 100,000. This learning goal demonstrated the mathematical knowledge required of students, but it did not indicate the highest numbers students were going to work with. The goal addressed the benchmark in the standard.

The last learning goal for the third day was used to help students compare and order numbers. Rachel used a card game to help students form numbers. The cards had number from 0 through 9 written on each card. The game was played in pairs. Each student was supposed to draw five cards and form a number with the cards by arranging the cards in a way that will form the largest possible number. Once each partner formed the number, students then compare the numbers and determine which is larger. As students formed and compared 5-digit numbers, they were encouraged to form larger numbers by drawing more cards. This game provided opportunity for students to think about how they could use their place value concepts to decide how they could form large numbers from given number of cards. The game also helped students to compare their numbers. In comparing numbers, students had to also draw on their previous knowledge of place values to order numbers. This goal clearly explained what students should be able to do. The goal addressed the benchmark stated in the standard.

On the third day Rachel reviewed students' previous knowledge on what was learned from the first two days and built on this knowledge to help students read, write and compare numbers up to millions. Rachel's decision to review students' previous knowledge was based on the fact that some of the students were still struggling with reading and writing numbers. Rachel therefore helped students to read and write numbers from tens, through to hundred thousand before continuing to millions. All the learning goals stated for the final day clearly identifies the mathematical knowledge students are required to demonstrate.

Table 45

Assessment of Rachel's Learning goals for Clarity and Alignment with State Benchmark

Learning Goals	Clarity of goals	Connection to benchmark
students will review place value through ten-thousands	Clearly stated	It does not address the benchmark directly but provides prerequisite knowledge for other benchmarks.
Students will read and write multi digit whole numbers	Shows math knowledge to be demonstrated	It addresses the benchmark
Students will read and write numbers up to 100,000.	It shows what students should be able to do	It addresses the benchmark
Students will identify the places in numbers through ten-thousands and the values of the	It shows the mathematical knowledge students should be able to demonstrate	This addresses the benchmark
Students will compare and order whole numbers less than 100,000.	It shows what student should be able to do.	It addressed the benchmark
Students will read and write numbers up to 7 digits.	This shows what students should be able to do	This addresses the benchmarks
Students will identify the places in numbers through millions and the values of the digits in those places.	It shows what students should be able to do	This addresses the benchmark
Students will compare and order whole numbers through millions	It shows what students should be able to do	This goes beyond addressing the benchmark.
Students will solve population questions and write answers in extended notation, number form, and words.	It is not clear the type of population questions students will solve, but it shows how students should answer the tasks	This addressed the benchmark

Also, the learning goals addressed the state standards. A summary of evaluation of how the learning goals address the essential ideas of the benchmark as well as their clarity are presented in Table 45.

Setting learning goals. In formulating the learning goals to address the given mathematical benchmarks for the state standard, Rachel used resources from the practicing school, the internet, as well as her personal decisions. These resources influenced Rachel's final learning goals.

Practicing school-based resources/factors. Resources used from the school include the cooperating teacher's support, the mathematics curriculum, and the state standards. Rachel consulted the cooperating teacher to identify the mathematics content that was to be taught. After this, the mathematics curriculum and the state standards served as resources to formulate the final learning goals. In formulating the goals from these curriculum materials, Rachel used the skills and goals stated in the mathematics curriculum, the state standards, and sample lesson plans from the internet to formulate the final learning goals for the three days. The following statement explain show Rachel set her learning goals.

> At first I had to find out what was being taught the days to teach so I didn't want to take them out of their routine and like to just throw in a random math lesson so I asked the teacher and it's place value and so then I went and looked at the standard and that addresses place value for third grade and then from the standards I and also using the kind of guide from the everyday math curriculum picked the learning goals for that day (Rachel, interview 2).

Because the cooperating teachers are held accountable for the learning targets addressed in the mathematics curriculum, Rachel ensured that she used the schedule of the mathematics curriculum to select the content to teach. The learning goals were then formulated based on the requirements of the state standards and the mathematics curriculum. Learning goals from the state standards were adapted in relation to the mathematical skills and knowledge that were to be addressed in the mathematics curriculum. Rachel explained this process as follows:

> I took most of the goals from the standards and then from what the lessons wanted them to accomplish. So everyday math lessons, they had a laid out skills that you will be able to do so I kind like took the goals from the skills that they said should be (Rachel, interview 2).

Personal experiences/decisions. In addition to using the mathematics curriculum and the state standards to set the learning goals, Rachel's personal decision about what students can do and may not be able to do as well as her views on how things work in the classroom influenced how she formulated the final learning goals. She indicated that students' first experience with place value was through the use of work sheets rather than using manipulatives and other forms of representations. In view of this, Rachel wanted to review students' prior knowledge in order to ascertain what students could do and what they might need help with. Also, because students did not use manipulatives when learning place values, Rachel used multiple modes of representation including manipulatives, pictures, words, verbal language, and symbols to engage students in collaborative work to explore how to read and write numbers.

> The first time they worked on place value it was very worksheet oriented and so I wanted to be like give them hands-on, ways to accomplish the goals but that's kind of how I picked what to do is based off of what I know they can already do and adding a little bit for the first day and next few days more and more, but just building off what they can do and what I know works in our classroom for how they learn (Rachel, interview 2).

Although Rachel did not explicitly state that she was using ideas form the mathematics methods class, it appeared that her instructional strategy to engage students in exploratory mathematics in which students use multiple modes of representations was possibly influenced by what she learned from the mathematics methods class and her initial experience with mathematics and pedagogy classes. In the mathematics methods class, the professor demonstrated the use of multiple modes of representations and encouraged preservice teachers to use these representations to support students' learning. Also, Rachel indicated in her survey response that because of her positive experience with the use of manipulatives in an inquiry-based learning environment, she would endeavor to make her students have similar learning experiences.

Other Resources. Another aspect of the learning community Rachel drew from to set her learning goals was the internet. Rachel reviewed sample lesson plans online to find out how learning goals were stated. These ideas enable her to be able to set her final learning goals. It was interesting to see that Rachel was the only participant to use online resources for setting learning goals.

Summary for Research Question 1 for Rachel

In order to help 3rd graders learn to read, write and compare numbers, during the three-day teaching practice, Rachel formulated nine learning goals. The first learning goal was used to review students' prior knowledge on place values because this was an important knowledge base for the entire lesson. Although this goal did not directly address the three benchmarks for the lessons, it reviewed students' knowledge on place values. All the other learning goals provided mathematical knowledge which addressed the essential ideas related to the three benchmarks of the state standards. The learning goals were stated to identify the mathematical knowledge students were required to demonstrate at the end of the lesson. The goals provided the necessary scaffold for students to build on their prior knowledge. In setting the learning goals, the major factor considered was to ensure that the content to be addressed in the mathematics curriculum were addressed. The state standards and the mathematics curriculum served as the two main resources for getting the content to be taught and the goals to be met. Online resources on lesson plans were also used to shape the learning goals that were formulated. Finally, Rachel, drew from her personal experiences, her students' abilities, and knowledge gleaned from the mathematics methods methods class to frame the final learning goals.

Results of Learning Tasks

Research Question 2: In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks? In order to answer this research question, all the learning tasks used during Rachel's teaching were identified and categorized using Stein et al., (1996) cognitive demand of learning tasks framework. Table 46 shows the learning tasks used by Rachel during the three-day teaching practice.

Table 46

Learning Tasks used in Rachel's Rachel's 3rd Grade Class

Code	Task	Students' responsibility
RT1	Reviewing place values	Students are required to write numbers from the word form to figures. For example Two hundred fifty-six.
RT2	Identifying place values	Students are required to recall what they already know from place values and go through each position of a given number (written in figures) to name it.
RT3	Using base-ten blocks to build numbers	Students use their own strategies to build numbers using base-ten blocks, read the numbers, and write them down in figures.
RT4	Writing numbers from pictorial representations of base-ten blocks	Students work in pairs. They discuss the pictorial representations written by the teacher in order to determine the number for that representation.
RT5	Reviewing reading numbers from the board	Students read off numbers written on the board by the teacher. Identify the digits in a given position. For example, which digit is in the tens position, thousand position, etc.
RT6	Ordering numbers	Students work in pairs to order numbers. Students are required to use their place value ideas to figure out which number is larger. Students share their strategies.
RT7	Math game with deck cards	Students are required to work in pairs. They form larger number from a given set of cards. They pull the same number of cards, say, 4 cards, and are required to form the largest possible number from the selected cards.
RT8	Applying ordering of numbers to identify cities with large population	Students are given worksheets having populations of cities. They are required to identify which two given cities have the larger population.

Categorizing the learning tasks. All the learning tasks identified from Rachel's lesson plan which were enacted in the class were categorized using Stein et al., (1996) cognitive demand framework. Table 47 shows the categorization of the learning tasks used in Rachel's class.

Table 47

Cognitive Demands of Rachel's Learning Tasks

Lower-Level Demands	Higher-Level Demands
<u>Memorization</u> Ordering numbers Identifying place values	<u>Procedures with connections</u> Math game with decker cards Ordering numbers Writing numbers from pictorial representation
<u>Procedures without connection</u> Ordering numbers Identifying place values	<u>Doing mathematics</u>

Rachel used high-cognitive demand as well as low-cognitive demand tasks to support students' learning to read, write, and compare whole numbers. Comparing and ordering numbers was categorized as high-level cognitive tasks because depending on the numbers that were compared and ordered, students are likely to use varying strategies and cognitive skills to compare and order the numbers. While some students may recall from memory to tell which number is greater than the other, others may apply some mathematical procedures like decomposing the numbers to find out the place values of the digits in order to find out which number is bigger and which number is smaller. The mathematical game was considered a high-level cognitive task that needed connections because when students picked their decker cards, they have to figure out how to place each of the digits in order to form the largest number. In doing so, students had to draw from their place value concepts to form their numbers.

Rachel's activities were structured to provide students with the opportunity to explore with mathematical tasks in order to construct knowledge. Rachel applied ideas from the cognitive guided instruction and the Lesh Translation model to engage students in inquiry-based learning. Students were encouraged to use multiple modes of representation to demonstrate how they could translate within and between modes of representations. Students' ability to translate within and between modes of representation demonstrated their level of understanding. This enabled Rachel to offer assistance to students who needed help. Students participated in all the activities. They worked collaboratively to construct knowledge. Students' ideas were elicited and shared during the summary stage of the lessons.

Mapping the learning tasks to the goals. In seeking to find out the whether the learning tasks addressed the learning goals, the learning demands of the learning tasks were assessed to see how they supported students' mathematical knowledge building in order to meet the stated learning goals. Table 48 shows the learning goals and the related learning tasks that map onto the learning goals. All the learning tasks enacted provided learning opportunities for students to have mathematical knowledge that helped them achieve the learning goals.

Table 48

Rachel's Learning Goals and Associated Learning Tasks

Learning Goals	Learning tasks
Students will review place value through ten-thousands	RT1,
Students will read and write multi-digit whole numbers	RT1, RT2,
Students will read and write numbers up to 100,000.	RT1, RT2, RT3, RT4
Students will identify the places in numbers through ten-thousands and the values of the	RT1, RT2, RT3, RT4
Students will compare and order whole numbers less than 100,000	RT1, RT2, RT3, RT4,RT5, RT6
Students will read and write numbers up to 7 digits.	RT1, RT2, RT3, RT4,RT5
Students will identify the places in numbers through millions and the values of the digits in those places.	RT1, RT2, RT3, RT4,RT5
Students will compare and order whole numbers through millions	RT1, RT2, RT3, RT4,RT5, RT6, RT7
Students will solve population questions and write answers in extended notation, number form, and words.	RT1, RT2, RT3, RT4,RT5, RT6, RT7, RT8

Setting learning tasks. In order to use appropriate learning tasks that will help students achieve the learning goals, several resources from the learning community as well as Rachel's personal resources were used when the learning tasks were designed. All the interview transcript related to the learning tasks were coded using the open coding approach. The general themes that emerged from Rachel's codes were categorized as *other resources* and *personal decisions.*

Other resources. In designing learning tasks to support students' learning, Rachel used online resources and advice from colleagues in the mathematics methods class. She indicated that she adapted learning activities that were found in sample lesson plans online. In adapting the learning tasks, Rachel considered the learning goals students needed to achieve at the end of the lessons. For example, in the card game, Rachel indicated that she adapted the game to have students form their own number and compare the size of the numbers. The following statement explains the two main resources Rachel drew from to design her learning tasks.

> Well, a handful of them were just when I have tried to research online place value activities, I read through them and then I thought about what ones I thought would reach the concepts I was trying to address for the day I was trying to address, like the card game, it technically said it was a game but I am thought that was really interesting to compare and they have to think about the size of numbers and so I realized that worked really well was my comparing contrasting, lesson 2 and so that's kind of how I picked that one and I also liked I wanted to do hands-on (Rachel, interview 2).

Personal Decisions. When asked to explain how the methods class informed her decision-making process about the learning tasks, Rachel indicated that the entire idea of setting up mathematical activities, in which students would be given the opportunity to explore mathematical concepts was derived from the mathematics methods class. Rachel wanted to engage the students using hands-on activities. This informed her decision to integrate mathematics games and other activities to engage students meaningfully. The culture of the class also informed Rachel's decision to use hands-on activities. She expressed that,

> Just the concept of having a task is from our class [methods class], I think because a lot if you are the just, go observe the schools and then especially like my classroom right now it's just kind like lecture part and then you do your work sheet and once in a while we do number blocks or something like that (Rachel, interview 2).

Rachel's passion for helping students learn mathematics meaningfully influenced her teaching strategies and the kind of mathematical tasks to use. Although in her practicing school, her cooperating teacher mainly used lecturing and work sheets, Rachel was determined to stick to her vision of designing inquiry-based learning environments for students to explore mathematical ideas.

Summary for Research Question 2 for Rachel

In seeking to help students learn to read, write, and compare multi-digit whole numbers, Rachel used high-level cognitive and low-level cognitive demand tasks. The learning tasks supported students' learning by providing the mathematical knowledge needed to achieve the learning goals. All the learning tasks addressed the learning goals that were set for the three benchmarks. The learning tasks were scaffolded such that mathematical knowledge from initial learning tasks provided a prerequisite knowledge for subsequent learning tasks. For example, the learning tasks for the first day were to review students' previous knowledge on place values which served as a foundational knowledge for other learning tasks. The nature of the learning tasks was informed by Rachel's belief that students should be supported to construct their mathematical knowledge through mathematical explorations using multiple modes of representations. In view of this, she adapted learning tasks from sample lesson plans not only to meet the content goals of the class, but also to provide students with experiences with direct modeling and the use of multiple representations. Rachel appeared to have drawn from the Cognitively Guided Instruction and the Lesh Translation Model ideas to adapt tasks to meet the needs of the students. The learning tasks provided the opportunity for students to work either in groups or with partners to explore mathematical concepts to solve tasks.

Using the Five Practices to Orchestrate Class Discussions

Rachel indicated that she did not initially plan to incorporate the five practices in her lesson plan until her colleagues in the mathematics methods class prompted her to do so because the course professor wanted to see those five practice in the lesson plan. However, Rachel indicated that after deciding to use the five practices, she realized the importance of the practices in planning the lessons. When asked how she was going to use the five practices, she indicated that,

> They are in the lesson and I think I wouldn't, honestly have taught to do it except I knew that people who had already written in their lesson said that [Dr. Shelly] really wanted them in the lesson and after I had started to write them in I realized how important it is especially in the summary in the connecting to what they will be learning next (Rachel, interview 2).

One would wonder why Rachel did not think about the five practices until her friends in the class encouraged her to use them in the lesson plan. During the methods class, these five practices were discussed for preservice teachers to realize their importance in orchestrating effective class discussions. Also, the course professor, Dr. Shelly, enacted these practices on several occasions to offer preservice teacher the opportunity to see how each of the five practices was demonstrated. Rachel later realized the importance of the five practices as she began using them in her lessons. Although she realized how the five practices would help her teach, she indicated that it was challenging for her to think about some of the practices.

Rachel's anticipation centered on general ideas about how students might solve problems and the kinds of difficulties students might have. Rachel could not provide specific solution strategies that might be used. Without being able to have some ideas about possible solution strategies students might bring to the class can possibly pose some challenges to the teacher because it is possible that the teacher might be overwhelmed with several ideas at the same time. When teachers have some knowledge about students' possible ideas, it may be possible for them to be more focused on students' work to select for sharing. Because she cannot think of the possible strategies students might use, she also indicated that sequencing was difficult for her to incorporate in the lesson plan. She lamented that,

> Even selecting I think it's hard, at least for me to think about exactly what like analyzing what mistakes they can make and what could happen during this time and then even selecting I think mine is kind of in the moment am kind of like oh I really like this person's idea and it's hard for me to think about what strategy I want them to necessarily be getting now (Rachel, interview 2).

Rachel's difficulty in finding out solutions strategies students might use in the class made it challenging for her to know how she would sequence the ideas she might want to select for making connections during the discussions phase of the lessons.

In selecting students' work, teachers can think of general or specific ideas students might bring to the learning environment if they are conversant with students' possible misconceptions about specific content areas. This would mean that it would be important for preservice teachers to learn students' possible misconceptions about specific mathematics content areas to better position them to be able to anticipate the kinds of solution strategies or ideas they might select during teaching.

Table 49

Summary of Rachel's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Rachel's Anticipation
Anticipating	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices	Students may have problems with comparing the large numbers. They may even have some struggles answering the questions using words for the numbers. Finally, some students may not completely be able to do expanded form yet. These are all things that may arise.
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem.	Circulate and ask students to show (and explain) how they came up with certain answers, taking note of the different techniques they used.
Selecting	Select specific students to share their work with the whole class.	Select students that are discussing out loud their strategies for how they made various numbers using their base-ten blocks.
Sequencing	Purposefully sequence them according to how they want to lead the mathematical discussion to the desired learning goal	At the end of the game, students I have selected and sequenced will share out some strategies they learned about comparing large numbers and how they read them.
Connecting	Teacher to help students make connections between students' mathematical ideas that have been selected and sequenced purposefully	I will use the strategies students have used to connect to our overall goal of being able to read, write, and make large numbers using place value. I will show students the connection between our base blocks and the numbers we use and discuss how these represent the same concept, only in different forms.

Knowing how to sequence your selected ideas should be informed by the goal you are trying to help students achieve. In view of that teachers would have to make on-the-fly decisions as they monitor and select students' work so that they can sequence them meaningfully. Table 49 summarizes Rachel's implementation of the five practices during the three-day teaching practice.

The Case of Phoebe

Phoebe's mathematics background. Phoebe's described her K-12 mathematics performance as positive because she indicated that she performed well in the mathematics classes she took during her elementary and high school years. Phoebe took algebra 1in middle school and several courses including algebra 2, geometry, trigonometry and analytic geometry, probability and statistics, and calculus. The calculus class was taken for college credits while she was still at the high school. Although she described the K-12 mathematics teaching as procedural, in which students were required to follow step-by-step approach to solve mathematical tasks, she performed well.

> It was a lot of use of the steps that you have to follow and sometimes the teacher will ask did anyone figure it in other ways and I was kind of good at thinking of other ways to do math problems but we won't necessarily encourage to them that way and we we had, test or homework it would give you problem and then you had to show all your work but the work you show was supposed the way that you were taught (Phoebe, interview 1).

Phoebe's ability to think in different ways about mathematics enabled her to do well in her mathematics courses. Although the mathematics teacher expected students to solve mathematical tasks in different approaches, students were not encouraged to show their strategies on tests. Learning mathematics in a way which requires students to memorize procedures and used them on test may not help teachers to find out students invented solution strategies.

At the college level, Anna took college algebra*, Mathematics and Pedagogy for Elementary Teachers* I, as well as *Mathematics and Pedagogy for Elementary Teachers II*. For the first mathematics and pedagogy class, Phoebe had some difficulties because she could not cope with the way preservice teachers in that class were required to explain their thinking behind how they solved mathematical tasks.

> In this course, I struggled with some of the hands on activities with string and with trying to explain HOW I got my answer and why it makes sense. To me, math comes easily, so I get the answer because that's the right answer, and I know how to solve it; and it's difficult for me to go back and understand why it is that answer. My strengths though were just that math is something I'm good at, so the concepts themselves were easy for me to understand (Phoebe, survey).

Although Phoebe was good at mathematics, she struggled with justifying how she solved mathematical tasks. During her K-12 mathematics years, she was not involved in inquiry-based learning; she only learned how to apply rules and formulas to solve mathematical tasks. This method of learning mathematics posed a challenge for her in the mathematics and pedagogy class. Phoebe's experience for the second course was positive because of the support she had from the course professor. She explained that,

> In this course, I was able to have a lot more hands-on learning that really worked with my strengths. The professor did a really good job at understanding where we were at, and if we ever needed more help and didn't understand a concept, he was able to go back and give us the additional support we needed to give us a stronger understanding. That really helped me as a learner, but there wasn't a lot that I struggled with in this course (Phoebe, survey).

The course professor's support for Phoebe facilitated her mathematics learning to the extent that she did not feel any sense of struggles as she did in the mathematics and pedagogy I class.

Phoebe's vision as a mathematics teacher. Phoebe's vision of being a mathematics teacher stems from her love for mathematics and children. She believes that because of her strong mathematics content knowledge, her love for children, and her love for mathematics, she would be a good mathematics teacher who can help children learn mathematics effectively. Being a good mathematics teacher requires that the teachers have the mathematical and pedagogical content knowledge in order to make the mathematics content comprehensible to students. In addition to this, teachers should be able to relate well to their students so that they will be better prepared to understand how to present mathematical lesson to them. Phoebe believes that she has these qualities and that she would be a good mathematics teacher. The following statement explains why she believes she can be a good mathematics teacher.

> I love kids, and I love math! Teaching has always been something that I've been interested in, and I've also always loved math so the two go together pretty nicely. I think that my strong suits in math would help me to teach math to students because I understand the content well. I loved math, and a lot of students are either on one extreme or the other, where they LOVE math or HATE math, and I would like to help those kids who hate math to not hate it so much. I also have a great relationship with students, and I think that is more beneficial to them in learning versus someone who is only knowledgeable in math but doesn't have the love for children and teaching (Phoebe, survey).

Phoebe is aware that a lot of students either "LOVE" math, or "HATE" math and as a student who loves math and loves children, she is hopeful that she will be a good mathematics teacher.

Because of Phoebe's desire to be a good mathematics teacher, she wanted to learn how to teach mathematics in order to be better prepared to teach mathematics to "those students who math doesn't come easy to" (Phoebe, survey) because for Phoebe, mathematics always came natural and she did not understand why some students "didn't get it" (Phoebe, survey). In her mathematics teaching, Phoebe would like engage students in inquiry-based learning environment in which students work in pairs or groups to support one another to grapple with mathematics tasks in order to use different approaches to solve mathematical tasks.

Table 50

Atomic Elementary Enrollment by Demography and Population Type

Ethnicity	Atomic Elementary
American Indian/Alaskan Native	1 (0.6)
Asian/Pacific Islander	24 (15.4)
Hispanic	19 (12.2)
Black, not of Hispanic Origin	95 (60.9)
White, not of Hispanic Origin	17 (10.9)
All Students	156 (100)
Special Population	
English Learner	45 (28.8)
Special Education	15 (9.6)
Free/Reduced priced lunch	138 (88.5)
Homeless	13 (8.3)

Note. First number is the total count and the second number is the percentage of the count.

Phoebe's teaching context. As part of the field experience requirements of the mathematics methods course, Phoebe prepared and taught a three-day mathematics lesson at the Atomic Elementary in the Maranatha School District. The school used the

Everyday Math curriculum (University of Chicago Center for Elementary Math and Science Education, 2004). Prior to the student teaching, Phoebe had observed her kindergarten cooperating teacher and had assisted her in classroom duties including co-teaching some mathematics lessons. Table 50 shows the demographic and population type for Atomic Elementary.

Results of Mathematics Learning Goals

In order to address the benchmarks for the kindergarten mathematics lessons, Phoebe and her colleague set learning goals to meet the benchmarks they wanted to address. Table 51 shows the state standards Anna addressed during her students teaching practice.

Table 51

Mathematical Strand and Benchmark for Phoebe's Class

Strand	Standard	No.	Benchmark
Number & Operation	Understand the relationship between quantities and whole numbers up to 31	K.1.1.1	Recognize that a number can be used to represent how many objects are in a set or to represent the position of an object in a sequence.
Geometry & measurement		K.3.2.1	Use words to compare objects according to length, size, weight and position
		K.3.2.2	Order 2 or 3 objects using measurable attributes, such as length and weight.

Expected big ideas and reflection on the mathematics benchmark. Phoebe's first day lesson was used to introduce the concept of probability to the kindergarten students. During the second interview, Phoebe indicated that the lesson was to address the district mathematics curriculum and that it was not to address the state standards for kindergarteners. The goal of the lesson was to help students identify some terminologies used in probability to measure the likelihood that an event will occur or not. For example, students were to use the phrases, "more likely," or "less likely" to show that if an event is more likely to occur, then it means that event has a higher possibility of occurring than an event which is less likely to occur. An event in this case is a possible outcome one hopes to have when an activity or an experiment is conducted. For example, In the case of Phoebe's class, students were given small bags which contained blue and red bears. Students were asked to draw the bears out of the bag one at a time, with replacement, to see which color was picked most. At the end of the lesson students were able to see that the color that was picked most has its event to be more likely than the other color. Considering the mathematical knowledge that was taught, it was not used to fully address the first benchmark, K.1.1.1. Students engaged in counting activities and graphed their data. Because the first goal did not match any benchmark from the state standards, the goal was not assessed to see whether it addressed the state standard or not.

The essential understandings of the other two benchmarks require students to use the words, "heavier," "lighter" and "about the same" to compare weights of objects and to use this knowledge to order two or more objects according to their weight. In order for students to do this, they should be able to weigh objects using any appropriate means to identify which is heavier or lighter.

Phoebe's mathematics learning goals. In order to meet the given benchmark in the standards, Phoebe formulated five learning goals for the three days. Table 52 shows the goals that were formulated for the lessons.

Table 52

Phoebe's Learning Goals for the Three-Day Lessons

Day	Code	Learning Goals
1	C1	Students will be able to draw conclusions about how many of each color bear is contained in the bag.
	C2	Students will be able to graph the amount of times an object is pulled from the bag
2	C3	Students will make predictions to compare objects (which is heavier/lighter)
	C4	Students will use a pan balance to compare objects.
3	C5	Students will be able to make the pan-balance to be equal on either side using objects around the room.

The learning goals for the first day were used to help students collect and graph data in order to draw conclusions about the likelihood of an event occurring. As described earlier, this activity required that students pick objects of different colors (red and blue) from a paper bag and draw a bar graph to represent the total number of objects picked for each color. This activity provided opportunities for students to review their counting strategies and their ability to use coloring to draw simple bar graphs to illustrate the number of objects picked from the bag. Students realized that the higher the frequency of an event, the more likely is it for the event to occur. In contrast, students also realized that if the frequency of occurrence of an even is low, then that even is less likely to occur. Phoebe actively engaged the students in this activity. The whole class participated in drawing the objects from the bag to start the discussion about the concepts of more likely and less likely before students worked in pairs. By working collaboratively, students supported one another in counting, graphing their data and talking about their graph. Drawing from the mathematics methods class, Phoebe used multiple modes of representation including concrete objects, pictures, symbolic representations, and verbal explanations.

Phoebe's learning goals were not stated very clearly. The first goal, *Students will be able to draw conclusions about how many of each color bear is contained in the bag,* is not clear about the type of conclusion students are required to make. The statement could have been stated such that one can identify the mathematical knowledge students are required to demonstrate. Second, the goal of the lesson was not to let students draw conclusion about the number of each color object in a bag, rather the goal was to let students state which color object was more likely to be drawn from a bag based on the frequencies of the color objects that were drawn from the bag. The learning goal Phoebe wanted to achieve did not seem to address the goal. Stating learning goals which are not explicit can be misleading and lead to multiple interpretations. When this happens in the classroom it is possible that students may get confused about the learning goal. It is therefore important for preservice teachers to learn how learning goals can be stated in explicit terms.

Learning goal for second day. On the second day, students were required to compare objects and make predictions about which is heavier and which is lighter before using pan balance to compare the weight of the objects. Phoebe engaged students in making predications to compare the weight of objects. In comparing the weights, Phoebe used objects in the classroom which motivated students to practically engage in lifting the objects in order to make their decision about the weights. In addition to lifting objects, students also used pan balances to check the weights of objects. Students used the pan balance to explore the weights of objects. Bu using the pan balance, students were able to check the accuracy of their initial predictions. This offered students more opportunities to explore the concept of weights.

Although Phoebe's learning goals were understandable, the statements did not clearly show what students should be able to do. For example, the second goal for the second day was, *students will use a pan balance to compare objects.* This goal does not tell how many objects would be compared and also it does not tell what students are going to compare. In stating this learning goal, it is important to state that the pan balances would be used to compare the weights of two objects.

Learning goal for the third day. On the third day, Phoebe wanted the review the first two days' lessons and continue with using the pan balance to find out equal weights. This goal required that, given an object in a pan balance, students will find out how to get object(s) in the other side of the pan balance such that both pans will hold the same weights. Due to certain circumstances beyond my control, the third lesson was not observed. The lesson interactions were however recorded and played to delineate how the interactions proceeded. However, the recordings were not clear enough to provide a good sense of what transpired in the class for the third day. Notwithstanding this, the first two days' lessons provided enough information about the learning goals, the learning tasks used and how Phoebe orchestrated the classroom discussions. A summary of evaluation of how the learning goals address the essential understandings of the benchmarks as well as their clarity are presented in Table 53.

Table 53

Assessment of Phoebe's Learning Goals for Clarity and Alignment with State Benchmark

Learning Goal	Clarity of goals	Connection to Benchmark
Students will be able to draw conclusions about how many of each color bear is contained in the bag	The goal did not match what was done in the class	This goal was not addressed in the state standard. The counting component of the goal however, addressed the state standard
Students will be able to graph the amount of times an object is pulled from the bag	This goal was clear	Graphing the data did not address the state standard at that level. Phoebe used this activity to provide students with a way of visualizing the data.
Students will make predictions to compare objects (which is heavier/lighter)	This goal was clear	This goal addressed the benchmark
Students will use a pan balance to compare objects	This goal is not clear	This goal addressed the benchmark
Students will be able to make the pan-balance to be equal on either side using objects around the room.	This goal is clear	This goal addressed the benchmark

Setting learning goals. In formulating the learning goals to address the given mathematical benchmarks for the state standards, Phoebe collaborated with her colleague to plan for the learning goals. Phoebe's colleague was also a student in the mathematics methods class who also taught the same content at the kindergarten. The two preservice teachers however, taught different classes. Because of the collaboration, all the decisions made for the learning goals and the learning tasks were considered as joint decisions. The discussion section and the use of the five practices were enacted separately. In setting the learning goals, the preservice teachers used resources from the school and the university.

Practicing school-based resources/factors. In formulating the learning goals, Phoebe and her colleague consulted with their cooperating teachers to identify the mathematics content that had to be taught. Based on the content to teach, they used the mathematics curriculum to set the final learning goals. The curriculum specifies the mathematics content to be taught for each day. Based on the daily content to be taught, the preservice teachers framed their learning goals to make it inquiry-based.

> So, just kind of came up with the goals based on what the curriculum wanted them to do for that day and used that, it kind of laid out for us in kindergarten it's laid out, our curriculum is kind of like every day written for you so we kind like took what they had and kind of wrote it to match kind of content objectives and to change it a little bit so it could be more inquiry based (Phoebe, interview 2).

Because the first lesson on probability was not addressed in the state standards, the preservice teachers used their personal decisions to formulate the learning goals. For the other two learning goals which addressed the state standards, the preservice teachers combined ideas from the mathematics curriculum and the state standards together with their personal judgments to formulate their final learning goals.

Personal Decisions. While the mathematics curriculum provided the mathematical content to be taught, preservice teachers used their personal judgment about students' prior knowledge and what they thought "made sense" (Phoebe, interview 2) to teach the students informed their decisions around the learning goals. When asked to describe the process used to set the learning goals, Phoebe explained that

> First, we just looked at what the curriculum kind of idea was and used that to kind of create a lesson and then from there since there is not really a standard which goes with lesson, we kind of based it off of what we thought made sense for the students (Phoebe, interview 2).

In thinking about what made sense for the students, the preservice teachers considered students' prior knowledge about what they can do and what they would not be able to do. For example, Phoebe indicated that because students could draw objects from bags, count, and color, they decided to set learning goals that would enable students use their prior knowledge to achieve the goals. In this case, although students were working on probability lesson, Phoebe and her colleague decided not to use fractions and percentages because the students in kindergarten could not work with those numbers. Rather, the preservice teachers engaged students in counting, tallying, and graphing to teach the first lesson on probability.

Summary for Research Question 1 for Phoebe

In setting learning goals to teach the kindergarten students, Phoebe collaborated with her colleague to set learning goals for their kindergarten classes. Although the learning goals determines what mathematical knowledge students were required to demonstrate, the learning goals were not explicitly stated. Some of the learning goals were stated in such a way that could be interpreted differently. Because of the nature of the content that were taught for the the three days, the learning goals for the first day did not provide scaffold for the other learning goals. However, each of the two learning goals for the first two days provided prerequisite knowledge for the second learning goals for those days. In setting the learning goals, the preservice teachers consulted their cooperating teachers to ensure that they taught the correct mathematics content in the mathematics curriculum, used lessons specified from the mathematics curriculum to guide their learning goals, and made their personal judgments about what made sense to teach the kindergarteners for the three days. In addition, the preservice teachers drew from the mathematics methods class to make the lessons inquiry-based. Although the Phoebe and her colleague used their understandings to make personal decisions, the pacing guide dictated the mathematics content to teach for the three days.

Results of Learning Tasks

During the three-day teaching practice, Phoebe and her colleague (also a student in the methods class, but not a participant of the study) designed and used several mathematical tasks to support their students. Because the first day lesson was to introduce students to basic ideas in probability, Phoebe and her colleague designed hands-on activities to engage the students. Table 54 shows the mathematical tasks that were used in Phoebe’s class for the three days. The learning tasks encouraged students to use multiple modes of representations. Students worked with manipulatives drew graphs, explained their work in words, and also use symbolic representations. In addition, students were encouraged to work in pairs. Asking students to work in pairs offered them the opportunity to share their ideas and also compare their work to their partners.

Table 54

Learning Tasks used in Phoebe's Lessons

Code	Task	Student responsibility
PT1	Selecting colored objects from a bag and tallying	Each students gets the chance to draw a colored object from a bag. As a whole group, they tally the colors that are drawn from the bag
PT2	Counting the tally marks	Count tally marks of each color and determine which color was more.
PT3	Drawing objects from a bag and graphing results	Students pick objects from a bag and graph their results using simple bar graph.
PT4	Make predications	Students are required to make statements to show their perditions. They can use words like, 'more', 'less', 'probably', 'unlikely'
PT5	Finding out which item is heavier or lighter	Students are asked to lift some object and tell which is heavier.
PT6	Finding objects that are heavier or lighter than a given object	They are asked to state object which will be lighter than a given water bottle
PT7	Balancing a pan balance	Given a pan balance with some weights, students pairs are to figure out how to balance the pan by either adding weights or taking out weights from either sides of the pan.

Categorizing the learning tasks. Using Stein et al., (1996) framework of cognitive demands, the learning tasks were categorized to identify the cognitive demands they posed to students. Table 55 shows the categorization of the learning tasks used in Phoebe's class. The learning tasks designed by Phoebe and her colleague demanded both high-cognitive and low-cognitive skills. None of the tasks was categorized as "doing mathematics." Asking students to make predictions about which object is heavier or lighter was categorized as high-cognitive demand task because given any two objects, students would have to make some connections with the objects given to them and known weights in order to make the predictions. Students were engaged in the tasks by exploring with real objects in the classroom and also working with pan balances. Phoebe explained that the driving force of the activities was to make the lesson inquiry-based to provide students with opportunities to explore mathematical ideas.

Table 55

Cognitive Demands of Phoebe's Learning Tasks

Lower-Level Demands	Higher-Level Demands
Memorization Counting the colored bears.	Procedures with connections Predicting which objet is heavier or lighter. Finding equal weights .
Procedures without connection Making tally marks. Drawing bar graphs	Doing mathematics

Mapping the learning tasks to the goals. In seeking to find out how the learning tasks addressed the learning goals, Phoebe's learning tasks were mapped onto the earning goals. Table 56 shows the learning goals and their associated learning tasks.

Table 56

Phoebe's Learning Goals and Associated Learning Tasks

Learning Goal	Associated Learning Task
Students will be able to draw conclusions about how many of each color bear is contained in the bag	PT1, PT2
Students will be able to graph the amount of times an object is pulled from the bag	PT1, PT2, PT3
Students will make predictions to compare objects (which is heavier/lighter)	PT4
Students will use a pan balance to compare objects	PT5, PT6
Students will be able to make the pan-balance to be equal on either side using objects around the room.	PT4, PT5, PT6

Setting learning tasks. In order to use learning tasks that will help students achieve the learning goals, several resources from the learning community, as well as Phoebe's personal resources were used to set the learning tasks. All the interview transcript related to the learning tasks were coded for practicing school-based resource, university-based resources and preservice teacher's personal decisions.

University-based Resources. Phoebe explained that the big driving part of the learning tasks that were used was the idea from the mathematics methods class that students should be engaged in activities to explore mathematical knowledge for themselves. According to Phoebe, the mathematics course professor "talks a lot about inquiry and I think that was a big driving part so we were trying to make sure that what we did was inquiry and not tell the students this is it" (Phoebe, interview 2). In making the lesson inquiry-based, Phoebe and her colleague used concrete materials and pan balances to help students explore and compare weights of objects. The preservice teachers also used multiple representations during the first lesson. In this lesson, students counted objects that were drawn from bags, tallied them, used symbolic and graphical representations, and finally used verbal explanations to make conclusions about which color bear would be more likely to be picked from their bags.

School-based resource/factors. In order to ensure that the mathematical tasks students were going to be engaged in would help them learn the learning goals, the preservice teachers consulted with the cooperating teacher to find out if the tasks they had planned to use would be helpful or not. In addition to this, preservice teachers used students' prior knowledge as a basis to know what students can do and will not be able to do. Based on this idea, the preservice teachers designed tasks that used students' prior knowledge. For example, students were asked to make predictions about which object will be heavier and which will be lighter. In thinking about the weights of objects, students would draw on their knowledge about weights of objects.

Other sources. The preservice teachers also used online resources to find out how they could design activities with pan balances. These activities were adapted to meet the goals of the class. In obtaining the pan balances, the preservice teachers consulted the science lab in the school to acquire the pan balances for the lessons.

Summary for Research Question 2 for Phoebe

The mathematical tasks that were used provided mathematical knowledge that addressed the learning goals set for the lessons. While some tasks demanded low-cognitive demands, others required high-cognitive thinking. The learning tasks provided students with the opportunities to explore mathematical knowledge in order to solve the tasks that were posed to them. The tasks also helped students to work collaboratively, thereby allowing students to support one another. This atmosphere of collaboration motivated students to learn without any fear of getting wrong answers. The underlying principle that guided the development of the learning tasks may have stemmed from the learning theories that were discussed in the mathematics methods class. It is possible that Phoebe and her colleague were guided by ideas from the learning theories to consider students' prior knowledge to adapt and design activities to make their lessons inquiry-based to engage all students in the learning process. The cooperating teacher served as a resource to help the preservice teachers check if their selected tasks were within students' abilities.

Using the Five Practices to Orchestrate Class Discussions

While it may be easier to engage some students in class discussions to help them construct their mathematical knowledge, it could be challenging to engage kindergartners in effective class discussions. When Phoebe was asked how she was going to facilitate class discussions around her learning tasks to support students' learning, she indicated that because of the nature of the class, they (Phoebe, the cooperating teacher, and a supporting staff) usually use sentence frames and questions to support students' learning. The sentence frames are used to guide students to express their thoughts whereas probing and clarifying questions are used to check for students' understanding. She explained that,

> So we kind of use a lot of sentence frames because they are kindergarteners so a lot of time you ask a questions and they are like, 'I don't know.' So a lot of it is like saying things for these lessons like, 'the blank is heavier than blank' and then like as the teacher kind of guide them through like, why? How do you know? And so we do that a lot, asking them how do u know? How do you know? And helping them use vocabulary or use sentence frames to say what they want to say (Phoebe, interview 2).

Although Phoebe used sentence frames and questioning to support students' learning, she enacted the five practices. In her lesson plan as well as during the second interview, Phoebe explained how each of the five practices would be implemented in the classroom.

Phoebe's anticipation comprised general challenges and successes she expected from students. Phoebe could not state specific examples of possible solution strategies students might use to solve tasks. Concerning her monitoring practices, Phoebe identified how she was going to monitor students' work to select exemplars for sharing during the summary stage of her lessons. Because Phoebe could not anticipate specific strategies students might use, she could not state specific strategies to be selected and sequenced. Phoebe's explanation for "connecting" suggested that she probably conceptualized making connections differently from how Stein et al., (2008) conceptualize the practice of making connections. Table 57 summarizes how Phoebe enacted the five practices for orchestrating whole-class discussions.

Table 57

Summary of Phoebe's Five Practices for Orchestrating Whole-Class Discussion

Codes	Explanation	Phoebe's Implementation
Anticipating	Conceptualizing expectations about how students might mathematically interpret a problem, the array of strategies they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices	Students may have trouble with understanding how to graph the colors they choose. Students may have trouble with recording their inquiry
Monitoring	Paying close attention to the mathematical thinking in which students engage as they work on a problem	I will be walking around as students are working in their partner groups. I will be asking guiding questions, as well as making sure they are filling out the graph correctly.
Selecting/Sequencing	Select specific students to share their work with the whole class.	Selecting/Sequencing: As I am walking around, I will take note of students' responses that seem to understand the concept of weight and what it means for something to be heavier versus lighter.
Connecting	Teacher to help students make connections	As students are sharing, I will ask if anyone else had similar results.

Results and Discussion from Cross-case Analysis

In order to answer the research questions and provide general information regarding the six preservice teachers, a cross-case analysis was carried out to find out the impact of the learning community on preservice teachers' mathematical learning goals setting, the mathematical tasks they designed to teach, and how they used the five practices for orchestrating effective whole-class discussions. In the following section, the research questions are addressed with some discussion.

Research Question 1: *In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from a given state standard to support elementary school students' mathematics learning? What are the characteristics of these goals?* In setting mathematical learning goals for their respective classes during their students teaching, the preservice teachers used resources from the university mathematics methods class, their practicing school, other resources, and their personal decisions based on their dispositions and knowledge about teaching and learning. Table 58 shows all the resources used by each of the preservice teachers.

Impact of the learning community on Preservice teacher goal setting

Impact of the practicing school. When setting mathematical learning goals, the preservice teachers main focus was to meet the requirements of the school mathematics curriculum. Preservice teachers consult their cooperating teachers to ensure that they follow the school's schedule. All the preservice teachers indicated that they had no choice in selecting the mathematics content they wanted to teach because they had to keep to the class' schedule. Keeping to the class schedule was of concern to the preservice teachers because

Table 58

Resource and Personal Decisions Used in Setting Learning Goals

Participants	University-based resource/Factors	Practicing school-based resources/Factors	Personal decisions	Other resources
Anna (2nd grade topic on counting money)		School district math curriculum, Cooperating teacher, Minnesota state standards, Students' prior knowledge	Decomposing benchmark, Focusing on the end goal Importance of math	
Samantha (2nd grade topic on skip counting)	Use of multiple representation	No collaborative learning, "old-school" teaching style from cooperating teacher	Collaborative learning	
Jocelyn (4th grade topic on division facts)		Cooperating teacher, Minnesota Standards, Students' prior knowledge, School district math curriculum	Collaborative work, Inquiry-based learning, Personal feeling about division	
Morgan (3rd grade topic on place values and estimations of sums and differences)		School district math curriculum, Students' prior knowledge, Cooperating teacher Time		
Rachel (3rd grade topic on place values and ordering of whole numbers)		Cooperating teacher, School district math curriculum, Cooperating teacher's style of teaching, Students' prior knowledge, Minnesota State standards		Online lesson plans
Phoebe (KG topics: basic ideas on probability and weights		Cooperating teacher, Students' prior knowledge, School district math curriculum		Colleague from methods class

the cooperating teachers are held accountable to the mathematics content that should be taught. This finding was not surprising because in the era of high stakes testing where teachers are sometimes evaluated based on their students' performance, teachers often are under pressure to complete mathematical content. In view of this, teachers teach according to the schedule planned for the term or the academic year. Consequently, as these teachers offer their class for the preservice teachers, it was expected that the preservice teachers would be asked to keep with the schedule of the school.

The preservice teachers used the state standards to check whether the standards align with the mathematics content they were planning to teach. In some case, the mathematics content from the curriculum did not align with the state standards. For instance, when Rachel was teaching the third grade place value, the mathematics curriculum required that students are taught to read and write numbers up to 1,000,000 whereas the state standards only required students to work up to 100,000. It is possible that these conflicts between the school mathematics curriculum and the state standards could present some challenges to new teachers who might not be aware of this situation. When teachers decide to teach to satisfy the requirements of the school curriculum, care should be taken to ensure that students who might not be able to attain some mastery levels are given the necessary support to enable them. It is important also to note whether teachers should teach to the requirements of the state standards, or the school curriculum. When there are possible conflicts between the state standards and the school mathematics curriculum, which documents should be satisfied? Answer to this question may be easier for experienced teachers, but may pose some challenges to novice teachers.

The learning goals were also informed by preservice teachers' knowledge about what their students could do and not do. Effective teaching requires that teachers build their mathematical lessons on students' prior knowledge. Building on students' prior knowledge can facilitate students' understanding of the content that is being taught because students can make connections to what they know in order to learn the new content. In Phoebe's kindergarten class for example, because students can count, pick objects from a bag, and color object, one of the learning goals for the kindergarteners required students to be able to draw objects from a bag, tally the frequencies, draw a simple bar graph, and tell which event is more likely to occur. From the factors discussed above, it appears that the practicing school had a bigger impact on preservice teachers' mathematical learning goals setting than the University methods course.

Impact of the university methods class. Unlike the practicing school in which the school mathematics curriculum and the cooperating teachers were the major factors influencing how preservice teachers set their mathematics learning goals, the university resources did not directly influence the goals set by the preservice teachers. Only Samantha indicated that the use of multiple representation and collaborative learning influenced the learning goals she formulated. Samantha's vision of engaging students in collaborative learning to explore mathematical ideas using multiple representations were in sharp contrast to the teaching style of her cooperating teacher. Samantha believed that the cooperating teacher's style of teaching was an "old-school" teaching model, in which the teacher presented mathematical ideas for students to take down notes. In this teaching approach, students always worked individually and rarely used multiple representations. Using this didactic teaching approach in a class where the majority of the students are "needy," may not help students to learn mathematical meaningfully. In view of this, Samantha wanted the students to learn how to work collaboratively and also use multiple modes of representation to express their mathematical ideas.

Keeping to this goal, Samantha's learning goals required students to use multiple modes of representation to express their mathematical ideas. For example, Samantha's first learning goal was stated as follows: *Students will be able to skip count by 2's, 3's, 4's, 5's, 6's, and 10's using manipulatives and written symbols* (Samantha, lesson plan). Also, Samantha used a learning goal to help students work collaboratively with a partner: *Students will be able to explain how and why they skip counted using a certain value*. This learning goal required students to work in pairs and explain their work to each other. Samantha explained the purpose of this goal as follows: "The collaboration one wasn't technically a content one, but personally for our class that was one of my main goals that I wanted this lesson to get at more." (Samantha, interview 2). Samantha seemed to have used her knowledge from the methods class to inform her decisions about incorporating collaborative learning and the use of multiple representation in her classroom. Unlike the mathematics content that had to be dictated by the mathematics curriculum, the way students worked to achieve the learning goals were flexible for the preservice teachers. In this case, it appears that the mathematics methods shaped Samantha's teaching practice and goal setting.

Characteristics of learning goals

Clarity of learning goals. Generally, preservice teachers learning goals were clearly stated to show the mathematical knowledge they wanted students to demonstrate at the end of the instructional period. Stating clear learning goals and sharing the learning goals with students may help teachers and students to be focused on what to do during teaching and learning in order to meet the learning goals for the day. With clear learning goals, teachers can possibly direct their instruction purposefully to support students' learning. Choice of tasks and discussions strategies can also be influenced by the goals of the lesson. As noted by Jocelyn, the mathematical goal is a form of "abstraction" which can be reached through the learning tasks which serves as the "bridge" to the abstraction.

When learning goals are not stated clearly to identify the mathematical knowledge students are required to demonstrate, it is possible that when the learning goal is shared with students, they may not know what is required of them and that may pose some challenges for students to evaluate their performance in relation to the learning goals. Although in the mathematics methods class, the preservice teachers were encouraged to state learning goals that would identify mathematical knowledge students should demonstrate, not all the learning goals formulated clearly demonstrated what students were going to do. Some of the learning goals were broad rather than specific. For example, Phoebe's learning goals, *students will use a pan balance to compare objects,* does not clearly show what kind of comparison students are required to do and how students are required to state their results from the comparison they were going to do. When a new teacher picks up Phoebe's lesson plan to teach the class, that teacher may find it difficult to interpret this learning goal.

While it is important for the individual learning goals to be explicitly stated to avoid ambiguity, it is also important that when two or more learning goals are stated, the learning goals should scaffold one another. If learning goals provide the necessary scaffolding, then they can facilitate students' understanding of what they learn because the mathematical knowledge that would be provided by one learning goal may provide prerequisite knowledge for other learning goals. All the learning goals provided by the preservice teachers were scaffolded to support students' learning. The preservice teachers considered students' prior knowledge to ascertain what students would be able to do and what they will not be able to do in setting the learning goals. By knowing students' prior knowledge, Preservice teachers were able to set learning goals that provided the needed support for them.

In the case for Anna, she decomposed the benchmark in the state standards which was being addressed into her learning goals. Anna explained this process as follows: "I just, I looked at the state standards and what I knew and wanted my students should be able to do and then I kind like **broke it down** to like smaller parts." [emphasis added] (Anna, interview 2). In some cases, preservice teachers used some learning goals to provide prerequisite knowledge to support the development of other concepts. For example, Jocelyn asked her 4th graders to write their own division word problems even though this was beyond the scope of the benchmark for the unit which was being taught. When asked why she wanted the students to write word problems, Jocelyn explained that if students can write their own word problems, then they can solve those word problems. In writing their word problems, students engaged in high-level cognitive processes in order to write the tasks. Although teachers can go beyond the scope of a given benchmark, it is important that students are given the support they need to do so.

How learning goals address the benchmarks. Apart from using learning goals to guide instruction, teachers can also use learning goals as a means to find out whether the essential mathematical understandings of a given mathematical benchmark are addressed. Preservice teachers' learning goals generally addressed the benchmarks in the state standards. In the case of Anna, for example, all her learning goals were obtained from the benchmark. She decomposed the benchmark to get the learning goals. In some cases, the preservice teachers' learning goals did not directly address the benchmark; the goals were used to scaffold students' learning and to provide collaborative learning. For instance, Samantha indicated that the majority of her second graders were "needy" (because during class work, most students call for help) and as such needed to be supported through partner and group work. In view of this she deliberately set a learning goal in which she asked the students to work in pairs and explain their thinking to each other. It appeared that the preservice teacher drew from her negative learning experiences to set up supporting structures for all the students to learn. When teachers reflect on their positive and negative experiences, it is likely that their past experiences would influence the kind of support they plan for their students.

It was surprising to find out that when setting their learning goals, preservice teachers did not check the Minnesota State Mathematics Framework for the benchmarks in the state standards. This online resource provides the essential ideas that have to be addressed for every benchmark in the state standards. These essential ideas can serve as a basis for preservice teachers to set their learning goals. Although this framework was introduced and discussed in the methods class on the first day, none of the preservice teachers reported that they used the framework when setting their learning goals. Preservice teachers were more concerned about the school mathematics curriculum than other curricula materials. It appeared that because of the accountability issues, teachers tend to pay more attention to the school curriculum than other curricular materials.

Research Question 2: In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks? In order to address this research question in relation to all the preservice teachers, a summary table for the resources used are presented in Table 59.

Impact of the learning community on preservice teachers' mathematical tasks setting

Impact of the university methods class. In setting the learning tasks to address the learning goals, the preservice teachers used conceptual tools from their university methods course as well as their personal decisions. Little influence from the practicing school was noted with exception of few cases where the cooperating teachers wanted their preservice teachers to use specific tasks. The underlying principle that guided preservice teachers in selecting, adapting and designing their mathematical task was to make their lessons inquiry-based in which students would work collaboratively

Table 59

Resource and Personal Decisions Used in Setting Learning Tasks

Participants	University-based resource/Factors	Practicing school-based resources/Factors	Personal decisions	Other resources
Anna	Use of multiple representation, Direct modeling strategies from the Cognitive Guided Instruction	Adapt tasks from school district math curriculum, Cooperating teacher demands, Using the district pacing guide	Provide scaffolding for students, Wants hands-on activities, Adapting tasks	
Samantha	Use of multiple representation, Multiple teaching strategies, Collaborative learning, Methods course professor	Cooperating teacher's demands, Cooperating teacher's style of teaching—teaching by telling		Online resources for learning tasks
Jocelyn	School math curriculum, Direct modeling from CGI	School district Math curriculum, Homework requirements	Collaborative work, Inquiry-based learning	
Morgan	All the things learned from the methods class	Students prior knowledge, Time availability		
Rachel	Ideas from the methods class, Hands-on activities			Online resources, Colleague
Phoebe	Make lessons inquiry-based and engage students	Cooperating teacher, Students' prior knowledge		Colleague from methods class

with their colleagues, grapple with mathematical tasks, and use multiple representations to demonstrate their mathematical knowledge. Preservice teachers' ability to design their tasks and also adapt tasks to meet the needs of their students enabled them to engage their students in a participatory form of teaching and learning. Providing the opportunity for students to explore mathematical ideas enabled them use their invented strategies to solve mathematical tasks. The inquiry-based structure of the lessons motivated students to participate in class discussions.

In cases where preservice teachers used the mathematics curriculum, or online resources for mathematics tasks, those tasks were adapted to make them inquiry-based and to ask students to use multiple representations. For instance, Jocelyn indicated that although she used some task from the mathematics curriculum, those tasks required students to work more procedurally rather than allowing students to explore. In view of that Jocelyn adapted the tasks in such a way that the tasks required students to work collaboratively, and explore their invented strategies to solve them. Teachers' ability to adapt mathematical tasks to meet the needs of their students is important because students come to the class with varying abilities and have their invented strategies to solve mathematical tasks. In the case of the preservice teachers, their understandings from the mathematics methods class shaped their ability to set mathematical tasks that engaged students. (Carpenter et al., 2014).

In the mathematics methods class, preservice teachers discussed the use of multiple representation from the Lesh Translation Model, and how to elicit and use students' thinking from Cognitive Guided Instruction as well as how those ideas can support students' learning of mathematical concepts. It is possible that the preservice teachers drew from these ideas to design their tasks for the students. Rachel, indicated that the whole idea of setting up tasks for students to explore was as a result of the understandings from the methods class. Samantha also reiterated this fact that one of the big things in the methods class was to engage students, an idea which underpinned her decision-making process about designing learning tasks for her lessons.

Impact of the practicing school. Unlike setting learning goals where the school mathematics curriculum dictated the mathematic content to be taught, in selecting learning tasks for the students, the practicing school had little influence on the preservice teachers. While task development was greatly influenced by the University method course, the learning tasks that were selected and adapted were checked with the cooperating teachers to see whether they were suitable for the class. The preservice teachers also considered students' prior knowledge and abilities to design the learning tasks. Because students come to the class with their background knowledge, it is important that teachers know their students' abilities in order to structure their tasks to meet the needs of the students.

Characteristics of the mathematical tasks

Preservice teachers' mathematical tasks were categorized as either high-level cognitive demand tasks, or low-level cognitive demand tasks (Stein et al., 1996). The preservice teachers used the tasks based on the learning goal they wanted to achieve and what their students could do. The preservice teacher used tasks that scaffolded students' learning. For instance, Anna indicated that in using the mathematics game from her mathematics curriculum, she adapted the task to scaffold students' learning. It was not surprising to see that all the preservice teachers used tasks of varying difficulties to support students learning. This finding supports Crespo (2003)'s finding that preservice teachers can pose different tasks of varying cognitive demands. Given the necessary support preservice teachers can design their learning tasks that will enable them engage their students in teaching and learning. Preservice teachers learning tasks were structured to scaffold students' learning. Generally, the learning tasks addressed the learning goals that were set. In some cases, the tasks were used to scaffold students' learning.

Although preservice teachers used high-level cognitive demand tasks, along with low-level cognitive ones, only one task was categorized as "doing mathematics." According to Stein et al, (1996) "doing mathematics" requires that students will use their mathematical knowledge in novel situations to solve problems. Although this goal was beyond the scope of the benchmark that was being addressed, Jocelyn believed that it was a good exercise for the students. Jocelyn believed that in teaching and learning mathematics, students should be allowed to think and find things for themselves before the teachers provide answers.

In order to teach for conceptual understanding, it is important that teachers use mathematical tasks that enable students to use multiple modes of representation. All the preservice teachers used mathematical tasks that required students to use multiple modes of representations to demonstrate their thinking. By using multiple modes of representations, students were able to translate from one mode to another. For instance, Samantha asked her students to use cubes to skip count, use symbols to write the numbers in the sequence, and verbalize their final answers. Using these multiple representations afforded students the opportunity to develop conceptual understanding for skip counting. Research question *3: In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?*

Based on the requirements of the mathematics methods course lesson plans format, preservice teachers' lessons were designed to meet three components, namely, *launch, explore, and summarize*. During the launch stage, preservice teachers introduced the lesson, shared learning goals, and set up the learning activities that would be used for the lesson. At the explore stage, students are given the needed resources and support to explore mathematical ideas and concepts. These explorations were done either in pairs or in groups. For the most part, students were engaged in hands-on activities to enable them grapple with mathematical tasks in order to learn. At the summarize stage, preservice teachers gathered all the students on a carpet in front of the class to share their ideas and make necessary connections to learn the mathematical goals. At the summarize stage, teacher engaged the students in whole-class discussions where ideas were shared and students' questions were addressed. All the preservice teachers followed this lesson plan format and taught their lessons accordingly. In preparing their lesson plans and teaching, the preservice teachers enacted Stein et al, (2008)' five practices for orchestrating effective whole-class discussions. The table below summarizes how the preservice enacted the five practices.

Table 60

Summary of Preservice Teacher Five Practices

	Anticipation	Monitoring	Selecting	Sequencing	Connecting
Anna	Anticipates possible abstractions, possible difficulties Possible solution strategies—possible combinations of money	students who make the incorrect amount of change for each problem, instead of correcting, I will ask them to explain their reasoning and redirect.	Calling students in a specific order according to what they do Selecting students who made specific combinations	Call on students as I see fit throughout the story Go in order of 15, 20, 50 cents and have student with correct combinations share first and if students made incorrect combinations have one share second	Connect with prior knowledge, counting by 10s and try to make connections each time penny, nickel, or dimes are mentioned in the test. Sharing these combinations as a whole group will connect with the content objectives.
Samantha	Possible difficulties Specific solution strategies	Circulate the class to observe students' work	Looking for different solution strategies—no mention of specific strategies	Has a specific way of calling out students to share	Has specific connections to make during the summarize stage
Jocelyn	I anticipate that the students may bring the following strategies to the discussion: Specific thing students might like	Circulating to see what students are working on. Checking for specific solution strategies..	I will look for the following: Use of a standard algorithm to solve a problem, Use of visuals to solve a problem, Correct use of the Partial Quotient Model	Calling students who used different solution strategies in specific order	We can connect these methods to approaches that students may use when solving word problems that incorporate subtraction or multiplication as well
Morgan	How might the students answer this question? What	Look for specific response—those who	Students with different strategies	Based on teacher's discretion	How are all the responses connected?

	misconceptions might this question illustrate	can verbalize their responses Look for different strategies	Students who can verbalize their responses	How can they build on one another	
Rachel	Possible difficulties Possible way of thinking	Circulate and ask students to explain their work	Write names of how students are doing in the class: understanding, or need support Select students with different methods. No specific method is stated	Will call students based on her criteria	Students' ideas will be connected
Phoebe	Possible difficulties students may have with the work	Circulate the class and observe students' work. Provide support where necessary	As I am walking around, I will take note of students' responses that seem to understand the concept of probability	I will ask several students to share their discoveries they made during the lesson to the whole class.	As students are sharing, I will ask if anyone else had similar results.

Note: This is a continuation of Table 60

Summary of the five practices

Anticipating. While some preservice teachers found it challenging to specify students' possible solution strategies they might bring to the class, others were able to anticipate possible solutions strategies students might use to answer specific questions. The preservice teachers who could not anticipate specific things students might do in the class indicated that they were not sure about how students were going to solve the tasks they were going to be given. However, these preservice teachers appeared to understand that there were going to be some level of difficulties and successes in the lessons they were going to teach.

Monitoring. All the preservice teachers monitored students as the students engaged in mathematical tasks. This was done by circulating in the class while students worked in order to observe and monitor students' work. Generally, all the students reported that their monitoring centered on finding out what strategies students were using and also checking for students' difficulties.

Selecting. While students circulated in the class to monitor students' work, they took notes of students' work they wanted to share with the whole class during the summarize stage of their lessons. In their lesson plans, some of the preservice teachers were able to indicate specific types of students work they would select. For instance, Jocelyn indicated specific solutions strategies she would be looking for during her teaching. Teachers ability to identify the type of solutions paths they were looking for can enhance their teaching because they will know the exact things to look for as they monitor students' work. If these specific solution strategies are not seen, that can inform the teacher about instructional decisions to take. For instance, if those strategies are important that the teacher wants all students to learn, if those strategies are not seen in students' work, then the teacher will know that there is the need for that strategy to be taught.

Being able to state the type of work to select is however, not an easy task for beginning teachers. This is because some teachers find it difficult to know students' work that would be exhibited during the teaching and learning. The majority of the preservice teachers were not able to specifically state in their lesson plans, the type of students' work they would want to select.

Sequencing. Although all the preservice teachers knew how they were going to sequence students' work for whole-class discussions, it was difficult for some of them to specifically state in their lesson plans how they would sequence students' work. The difficulty seems to be attributed to the fact that because preservice teachers found it difficult to know what students would actually do in the class, they could not know write this in their lesson plan. If teachers are able to sequence students' work meaningfully, it is likely that they will be able to make the necessary connections in order to facilitate students' learning.

Connecting. Similar to selection and sequencing, most of the preservice teachers could not indicated how they would make specific connections. Only Samantha indicated specific mathematical concepts she would help her students learn during the summarize stage. Considering the preservice teachers' explanation of making connections, it appeared that some of them conceptualized making connections differently from how Stein et al., (2008) conceptualized the practice. Some of the preservice teachers could not explicitly explain how they will assist students to build on their ideas to make the necessary connections to achieve the learning goals.

Summary

The preservice teachers used resources from the learning community to set learning goals, set learning tasks and orchestrate class discussions around the learning tasks to support students' learning. In setting the learning goals, preservice teachers were highly influenced by their cooperating teachers and their practicing school mathematics curriculum. While these resources dictated the mathematics content to teach, preservice teachers appeared to draw from their conceptual tools gleaned from their mathematics methods class as well as their personal decisions to structure the learning goals that were set. In seeking to address the learning goals, preservice teachers designed a mixture of low-and high cognitive demand tasks to address the learning goals. In setting the learning tasks, preservice teachers used their conceptual tools from their university methods course and their personal decisions to set learning tasks. All the preservice teachers had varying successes for enacting the five practices for orchestrating class discussions discussed in Stein et al., (2008). The results showed that most of the preservice teachers found it difficult to anticipate specific strategies students might use in the classroom. Other challenging practices for the preservice teachers to write in their lesson plans were "selecting," "sequencing," and "connecting." Based on the findings, some conclusions and implications are presented in chapter 5.

Chapter 5

Summary, Conclusions, Implication

The field of teacher education has been undergoing several transitions to seek productive ways of preparing teachers based on core practices to better prepare preservice teachers to learn how to apply their knowledge for teaching (Ball & Forzani, 2009; Lampert, 2010). Preparing preservice teachers to teach using the core practices is an attempt to centralize preservice teachers' learning to teach directly on the practical work of teaching rather than focusing on theoretical foundations that may not help preservice teachers acquire the skills and knowledge necessary for the practical work of classroom teaching (Forzani, 2014). By their very nature, core practices detail the routine actions that compel teachers to make in-the-moment decisions during teaching.

Currently, not much is known about the ways in which preservice teachers who are learning to teach use resources from their learning communities enact core practices during their students teaching experiences. As the field of teacher education leans more towards core practices, there is the need to develop a firm foundation to guide teacher education programs. Along this line of thinking, Forzani (2014), argues that if the current work on core practices “is to gain momentum and significantly influence teacher education practice, some more permanent infrastructure for teaching and learning core practice-based methods will need to be built, and the common view that teaching cannot be specified or taught repudiated" (p. 366). Building on this notion, and to address the gap in the literature, which is lack of knowledge about the resources preservice draw on to plan and enact core practices, the purpose of this study was to find out how preservice teachers enrolled in a standards-based mathematics methods class engaged in three core practices: setting mathematical learning goals, designing learning tasks, and facilitating classroom discussions around the learning tasks. In looking closely at how preservice teachers engaged in these three core practices the study revealed how they made use of the resources from the learning communities to enact these teaching practices.

Summary

A multiple case study design was used to investigate three core practices and uncover how resources from the different learning communities impacted decisions related to implementing the core practices. The sampling procedure occurred in two stages: the first involved all the 33 preservice teachers who enrolled in the elementary mathematics methods class in Fall 2015. The second stage of the sampling was a purposeful sampling technique to select six preservice teachers.

Multiple instruments were used to collect data including written survey, interviews, lesson plans, field notes from the mathematics methods course and students teaching observations, and audio recordings. The first interview was to follow up with preservice teachers' (the participants) survey responses. The second interview was to follow up with the preservice teachers' lesson plan they prepared for their students teaching. This interview was given before the preservice teachers taught their lesson. In the case of Morgan, however, the second interview was given after she had taught the first day's lesson. This was because Morgan completed her lesson plan close to her implementation and could not meet with the researchers before her first day of teaching.

All the six preservice teachers were observed during their three-day students teaching in local elementary schools.

The analysis of the data enabled the researcher to discuss which aspects of the learning communities impacted preservice teachers' enactment of the three core practices. In addition to discussing the influence of the learning community on the preservice teachers decision-making process, the characteristics of the preservice teachers' learning goals and learning tasks were also discussed. The data analyses were carried out for each single case and then a cross-case analysis was conducted to discuss the common themes that emerged among the cases. The following research questions were used to guide the study:

1. In what ways do preservice teachers use resources form the learning community to set mathematical learning goals from a given state standard to support elementary school students' mathematics learning?
2. In what ways do the learning community and preservice teachers' experiences influence the learning tasks preservice teachers design and use to support students' learning to achieve the stated learning goals? What are the characteristics of the learning tasks?
3. In what ways did the mathematics methods class influenced preservice teachers' enactment of the five practices for orchestrating whole-class discussions around the learning tasks to support students' learning of the goals?

In answering the first research question, three data sources were analyzed, namely, lesson plans, interviews, and field notes. The learning goals from the lesson plans were analyzed using a rubric (see Appendix B) to assess the clarity of the learning goals and how the learning goals addressed the benchmark on the state standard. The interviews were used to find out how preservice teachers used resources from the learning community and their personal decisions to formulate learning goals. In order to answer the second research question, the interview responses again provided information about the resources that were used to design the mathematical tasks. All the tasks that were used during preservice teachers' teaching were analyzed and categorized into cognitive demands using Stein et al., (1998) framework. The third research question was asked to find out how preservice teachers enacted the five practices (anticipating, monitoring, selecting, sequencing, and connecting) for orchestrating whole-class discussion. Preservice teachers' lesson plans and field note were analyzed to find out how each of the five practices were enacted.

Overall, the learning goals that were formulated were clear, and demonstrated the mathematical knowledge students were required to demonstrate. There were few exceptions where learning goals were broad and did not clearly indicate the specific mathematical knowledge students were required to demonstrate.

Results show that in setting mathematical learning goals, preservice teachers focused more on addressing the content in the school mathematics curriculum rather than the state standards. Preservice teachers for the most part adhered to their cooperating teachers' schedule in order to teach assigned content stipulated by the district's mathematics curriculum. This finding was not surprising given the testing demands to cover all the content before students take their test. Preservice teachers had limited flexibility in setting goals that they would have liked to teach. As a result, resources related to the University methods course had limited influence on how learning goals were set. In addition, teaching to the schedule made some of the preservice teachers formulate more learning goals than should be addressed with the given time.

Findings from the second research question revealed that preservice teachers' belief that mathematics teaching and learning should be inquiry-based in which students explore mathematical knowledge, underpinned preservice teachers' mathematical tasks selection process. Preservice teachers' mathematics methods course served as a strong tool in shaping their thinking about teaching and learning of mathematics. The preservice teachers developed conceptual tools from the methods course which enabled them to use engaging tasks and multiple representations to support students learning. The preservice teacher designed their tasks, or in some cases adapted tasks from different sources to help them meet the needs of their students. Unlike setting learning goals where the cooperating teachers and the school mathematics curriculum dictated what mathematics content to be taught, in designing learning tasks, the preservice teachers drew heavily from eliciting and using students thinking, and the use of multiple representations to design and adapt learning tasks. The learning tasks served different purposes. While some were low-cognitive demand tasks, others were high-cognitive demand tasks. This finding indicated that where preservice teachers had the opportunity to apply their knowledge about teaching, they were able to draw from the University methods class meaningfully to either design their own tasks, or adapt tasks to support students' learning. It was interesting to see how preservice teachers were able to use ideas from the Cognitively

Guided Instruction and the Lesh Translation Model but could not use theoretical ideas discussed in the methods class. It appeared that structural nature of the Cognitively Guided Instruction and the Lesh Translation Model as well as the use of the five practices for orchestrating whole-class discussion, enabled preservice teachers to use these ideas as opposed to the less structural ideas (like Number Talks) discussed in the methods class.

Findings from the third research question showed that in enacting the five practices for orchestrating effective whole-class discussion, preservice teachers encountered some difficulties in incorporating the five practices in their lesson plans. The results further showed that preservice teachers' difficulty in incorporating the five practices in their lesson plans were due to the fact that they did not practice these five practices before their student teaching. To them, although they discussed the five practices, and also saw the practices being enacted on them by the methods course professor, they most likely needed to see the five practices being enacted in an elementary classroom before being expected to successfully implement them on their own. It is possible that because preservice teachers did not have enough time time to implement the five practices at the University, they could not master the practices before their student teaching. Some preservice teachers also indicated that they did not know that the five practices were important to be incorporated in the lesson plans.

The findings also revealed that while students knew what they wanted to do in the classroom, it was difficult for them to write some of the practices in their lesson plans. For instance, "selecting" and "sequencing" were difficult for preservice teachers to write in their lesson plans because they could not anticipate specific strategies, or solution paths students were going to use in the class. This finding was not surprising, it supports the idea that novice teachers need more time to enact each of the five practices around a given task (Stein et al., 2008). According to Stein et al., (2008), preservice teachers become more experienced with enacting the five practices if they implement these practices in parts and for a long time. Instead of engaging in all the five practices for the first time, preservice teachers could have been asked to focus on possible anticipation and monitoring students' work to better understand possible strategies students may use for a given task before they engage in selecting and sequencing possible solution strategies and finally helping students to make connections.

Implications

In light of the findings from this study, a number of implications for teacher development around core practices arise. Firstly, because preservice teachers' setting of mathematical learning goals are influenced mainly by the practicing school mathematics curriculum and their cooperating teachers, teacher educators should not only introduce preservice teachers to school districts' mathematics curricula, but should also establish strong collaboration with practicing schools. In preparing preservice teachers to write learning goals, it is important to introduce preservice teachers to mathematics curricula used by different school districts so that they will be able to adapt learning goals from these curriculum materials whenever necessary.

In addition, teacher educators should establish strong collaboration with cooperating teachers in the practicing schools so that the cooperating teachers and teacher educators can work together to support preservice teachers. With effective collaboration, both teacher educators and cooperating teachers will have better understanding of how the University methods course and student teaching can be structured to complement each other to help preservice teachers.

Secondly, preservice teachers should also be prepared to write their own mathematical tasks to meet specific learning goals. Given a mathematical learning goal from a given benchmark in a given standard, preservice teachers should be able to design both high-cognitive and low-cognitive demand tasks that will address the given goal(s). In addition to designing learning tasks, preservice teachers should also be supported to adapt mathematical tasks to the needs of their students. In helping preservice teachers learn to design, or adapt learning tasks, it is important that preservice teachers draw from the understandings of student thinking, Cognitively Guided Instruction and the Lesh Translation Model in order to set learning tasks which would support students' inquiry-based learning. It is also important for teacher educators to stress the importance of using multiple modes of representations in teaching mathematics. In view of this, preservice teachers should be encouraged to set learning tasks that would require students to express their mathematical understandings in multiple presentations.

Thirdly, in helping preservice teachers learn about the five practices for orchestrating whole class discussions, preservice teachers need to have practical experiences with each of the five practices. Learning about those practices and discussing them in class will not be sufficient for preservice teachers to enact these practices during teaching practice. Preservice teachers should be given the chance to practice what they are required to do on the field (Grossman et al, 2009). Given a mathematical goal, preservice teachers should be supported to learn what they should anticipate from students, the kind of monitoring they will do during the teaching and learning process, the specific strategies they will be looking for in the class, how the selected strategies will be sequenced, and how selected ideas would be connected to meet learning goals. As indicated by Stein et al., (2008), preservice teachers should be give several opportunities to practice each of the five components of leading whole-class discussions. In doing this however, preservice teachers' discussions facilitation should be directed on specific learning tasks aimed at a desired learning goal.

Recommendations for Future Research

To contribute to the literature on preparing teachers to teach using core practice, this study investigated three core practices, namely, setting learning goals, designing tasks to meet the goals, and orchestrating whole-class discussions around the tasks to meet the learning goals. The learning to teach in a community framework by Hammerness et al., (2005) enabled the researcher to investigate the aspect of the learning community that are used by preservice teachers and how those resources are used to enact the three core practices. Although findings from this study will shed more light on how preservice teachers enact these core practices, the following suggestions for future research are offered.

First, in order to understand how the learning community impact preservice teaching in learning to enact core practices, the researcher suggest that studies that will investigate other core practices should be carried out in the context of mathematics methods courses. Because preservice teachers would eventually become full teachers who would enact all eight core practices outlined in the *Principles to Action* book, it is also suggested that studies on how inserviece teachers enact core practices should be conducted. By studying how in-service teachers engage in core practices would shed more light on how teachers enact core practices in general.

Secondly, in order to ascertain how the core practices impacts students' learning, future studies should seek to investigate the impact of the core practices on students' performance. Findings from such a study may help the research community to better understand how the core practices impact students' achievement.

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Appendix A: Survey for Preservice Teachers

You are kindly requested to respond to the following items to the best of your abilities. There are no rights or wrong answers and your honest responses are very much appreciated.

Please write your class number here ....

Mathematics Content and Pedagogy Classes

1. What advanced mathematics classes did you take in high school? What are the math content classes you have taken **after high school** ? *(Please include all classes you have taken at the University of Minnesota or any college you attended before coming to this institution*)

Advanced classes in high school

Math content classes taken after high school

2. What were your strengths and challenges in MTHE 3101 (Mathematics and Pedagogy for Elementary Teachers I)?

3. What were your strengths and challenges in MTHE 3102 (Mathematics and Pedagogy for Elementary Teachers II)?

4. What do you want to learn about Teaching Mathematics in CI 5822 (Teaching Mathematics in Elementary School)?

5. Explain how CI 5822 (Teaching Mathematics in Elementary School) is important to you?

6. Explain your personal goals for CI 5822 (Teaching Mathematics in Elementary School)

7. What goals do you have for yourself as a math teacher in K-6 grades?

Beliefs about the Nature of Math

8. ***To what extent do you agree or disagree with the following beliefs about the <u>nature of mathematics? Please Circle one of the options provided.</u>***

Mathematics is a collection of rules and procedures that prescribe how to solve a problem.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

Mathematics involves the remembering of definitions, formulas, facts, and procedures.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

Mathematics involves creativity and new ideas.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

In mathematics many things can be discovered and tried out by oneself.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

When solving mathematics tasks you need to know the correct procedure else you will be lost.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

Mathematics problems can be solved correctly in many ways.
Strongly Agree   Agree   Somewhat Agree   Disagree   Strongly Disagree

Rationale for Teaching Mathematics

9. Explain the reasons that made you decide to be an elementary mathematics teacher.

Appendix B: Rubrics for Learning Goals

Learning Goals should:

- Be specific to show the mathematics students will learn. The goals should not be vague; it should be explicit.
- Indicate how students will demonstrate their understanding (What should students do to show that they have attained the goal? What representation(s)—verbal, symbols, pictorial, or real life example, etc should the students use to communicate their understanding?)

The following essential ideas were obtained from the Minnesota State Mathematics Framework. The Framework identifies the essential mathematics ideas for the benchmarks in the state standards.

Each of the preservice teachers' selected benchmarks was assessed based on the essential ideas associated with the benchmark.

Essential ideas for Anna's benchmark

What students should know and be able to do related to these benchmarks:

- Identify pennies, nickels, and dimes.
- Know the value of a penny, a nickel and a dime.
- Count groups of pennies.
- Count groups of nickels.
- Count groups of dimes.
- Count a group of pennies, nickels and dimes up to a dollar.
  Use the cents symbol-¢.

Essential ideas for Samantha's benchmark

- What students should know and be able to do related to the skip counting benchmark:
-
- Recognize patterns in numbers and apply the pattern to predict what comes next in a number pattern.
- Describe the rule for a given number pattern.
- Use a given rule to extend or complete a number pattern.
- Use patterns to solve problems.

Essential ideas for Jocelyn's benchmark

What students should know and be able to do related to the division fact benchmark:

- Readily derive the multiplication and division facts (factors including but not limited to 1-9).

Essential ideas for Morgan's benchmark

What students should know and be able to do related to the rounding and estimations benchmarks:

- Round four and five-digit numbers to the nearest 10,000, 1,000, 100 and 10.
- Use rounding to estimate sums and differences.
- Apply place value understanding when adding and subtracting multi-digit numbers.
- Students develop meaningful strategies to subtract across zero in three- and four-digit numbers.
- Solve real-world and mathematical problems involving multi-digit addition and subtraction using a variety of strategies. These strategies include assessing reasonableness of results, use of technology and the relationship between addition and subtraction, place value understanding when using the standard algorithm.
- Use multiple strategies to compose and decompose numbers flexibly in problem solving situations.
- Demonstrate an understanding of the relationship between addition and subtraction and using one of these operations to check for reasonableness of the answer.
- Compose and decompose numbers. For example, adding 184 and 37 could include 180 and 30, then adding 4 and 7 or 184 + 37 = (180 + 4) + (20 + 10 +7) = 180 + 20 + 10 + 4 +7. Expand their knowledge of "making 10" to include making multiples of 10 or 100 or 1,000 to add and subtract more efficiently.

Essential ideas for Rachel's benchmark

What students should know and be able to do related to the place value benchmark

- Read and write whole numbers up to 100,000.
- Represent whole numbers up to 100,000 using words, pictures, numerals, expressions with operations, and number lines.
- Using base ten materials, represent numbers from 1000 to 100,000 using ten thousands, thousands, hundreds, tens and ones.
- Represent numbers from 1000 to 100,000 in written form. For example, 4,756 can

be represented as 4000 + 700 + 50 + 6 or as 4700 + 56, or as 47 hundreds and 56 ones, or 45 hundreds and 25 tens and 6 ones, or four thousand seven hundred fifty-six.

Identify value and place in large numbers, for example, in the number 4,756, 7 is

Essential ideas for Phoebe's benchmark

What students should know and be able to do related to the division fact benchmark

- Use words such as shorter, longer/taller, and about the same to compare the length of objects. Use this knowledge to order two or more objects according to length.
- Use words such as larger, smaller, and about the same to compare the size of objects. Use this knowledge to order two or more objects according to size.
- Use words such as heavier, lighter, and about the same to compare weight of objects. Use this knowledge to order two or more objects according to weight.

Describe the relative position of an object using above, below, between and next

Appendix C: Rubric for Learning Tasks

Learning Tasks would be assessed whether they meet the following criteria:

- Linked directly to the goal. That is, the tasks are aimed at helping students acquire the skills and knowledge embedded in the goal.

Learning Tasks would also be classified according to Stein, Grover, and Henningsen (1996)'s criteria:

Lower-level Demands Tasks

- Memorization of facts—students recall mathematical facts without making use of any connection or using any mathematical idea in a novel setting.

- Using formulas, algorithms, or procedures without connection—Student are engaged in mathematical tasks that do not require them to make connections to other concepts or understanding

High-level Demands Tasks

- Using mathematical formulas, algorithm, and procedures with connection—Students use formulas, algorithms, and procedures and make connections to concepts, and understandings in order to solve mathematical tasks.
- Doing mathematics—Students do not have conventional procedures, they innovate, based on their conceptual understanding of math, to design their own approach to solve a problem. Students apply concepts in novel ways.

Appendix D: Classroom Observing Instrument

Date___________________________	School_________________________
Grade__________________________	Classroom Code_________________
Beginning Time_________________ ______	End Time_______________
Number of Students______________	Males_______Females___________
Lesson_________________________	Unit___________________________

Classroom Map

(Classroom map adopted from Clarkson, 2001)

Look for instances where preservice teachers use the five practices proposed by Stein et. al., (2008) to facilitate the class discussion. When using preservice lesson notes, find out instances in the lesson plan preservice teachers plan to use the five practices.

- Anticipating likely students' responses to cognitively demanding mathematics tasks. Teachers have an idea of students possible solutions, ideas, and strategies they might use in the class discussion
- Monitoring students' responses to the tasks. Teachers take notice of students' generated ideas, strategies and solutions and begin to decide which ideas get used for further discussion.
- Selecting particular students to present their mathematical responses for discussion. Once teacher have seen or heard students' generated ideas and solutions, teachers will decide which of those solution strategies get used for class discussions.
- Purposefully sequencing the student responses that will be displayed. When teachers decide on which solutions to use for class discussion, they will then

sequence them in such a way that will serve the intended aim of the lesson, usually, the goal of the lesson.

- Helping the class make mathematical connections between different students' responses and between students' responses and key ideas. Teachers being able to guide students to connect mathematical ideas meaningfully.

Time	**Discussion Code**	**Description of Discussion /Notes**	
	Anticipating students' possible ideas and solutions		
	Monitory students' solutions, strategies, and ideas		
	Purposeful Selecting solutions, ideas, and strategies to be shared		
	Sequencing students' solutions, strategies, ideas		
	Making Connection with students solutions/ideas		

- In what ways do preservice teacher orchestrate their discussions around the learning tasks in order to achieve the end goal?

Appendix E: Interview Protocol for Preservice Teachers

Interviewee: Nii Ansah Tackie
Date:
Time: 40 MINUTES

I have planned this interview to last no longer than 40 minutes. During this time, I am going to ask you some questions about your learning experiences in the mathematics methods class. Nothing you say here will be revealed to anyone in a way that you can be identified. To facilitate my note-taking, I will audio tape our conversation today. The recordings will be destroyed after it is transcribed. Please answer the specific questions as completely as possible. At the end of the interview I will allow you to share your thoughts about anything you feel were not addressed by any of the questions. Thank you for your agreeing to participate in this interview.
Please do you have any question at this time? If so you can ask.

Formulating mathematical learning goals

1. What are mathematical learning goals?
2. Describe how you set mathematical goals for a given lesson.
    - Probe: Describe the steps you use to set goals for two or more lessons.
3. What are the factors that influence the mathematical goal you set?
4. Describe the resources you use to set goals for your lessons.

Developing mathematical learning tasks

5. Explain what you understand by mathematical tasks.
6. What factors do you consider before designing mathematical tasks?
7. Describe the resources you draw from when designing mathematical learning tasks for a given lesson.
8. Describe what you do to design mathematical tasks.
9. Explain the factors you consider when designing mathematical tasks.

Facilitating mathematical discourse

10. From your point of view what would you consider to be an effective mathematics discussion?
11. Describe the recommendations you will give mathematics teachers who want to facilitate mathematics discussions effectively.

    - Probe: What are the differences and similarities of your recommendations to Stein (2008)'s five practices for facilitating mathematics discussion?

Personal, Formal and Informal resources

12. Describe all the resources you have used in learning to set learning goals, designing tasks, and leading effective discussion throughout this class.
    - Explain how each of the resources facilitated your learning.

Any final thoughts you would want to share?

Appendix F: Sample Lesson Plan

University of Minnesota Elementary Lesson Plan Template*[1]

Sample Mathematics Inquiry Lesson Plan

(To be used when student is observed)

Lesson Rationale

This lesson is based on the district's *Investigations* curriculum and was adapted based on readings from the class text: *Mathematics for Every Student -Responding to Diversity Grades PreK-5*. The lesson meets the State's standards 2.1.2.4 – 2.1.2.6. The lesson builds on students' number sense, place value ideas, and their skill with addition and subtraction with basic facts and extends their addition and subtraction work to 2-digit numbers > 20.

State Content Standards

Demonstrate mastery of addition and subtraction basic facts; add and subtract one- and two-digit numbers in real-world and mathematical problems.

2.1.2.3 Estimate sums and differences up to 100. (*For example*: Know that 23 + 48 is about 70).

2.12.4 Use mental strategies and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers. Strategies may include decomposition, expanded notation, and partial sums and differences. *(For example*: Using decomposition, 78 + 42, can be thought of as:

78 + 2 + 20 + 20 = 80 + 20 + 20 = 100 + 20 = 120 and using expanded notation, 34 - 21 can be thought of as:

30 + 4 – 20 – 1 = 30 – 20 + 4 – 1 = 10 + 3 = 13).

2.1.2.5 Solve real-world and mathematical addition and subtraction problems involving whole numbers with up to 2 digits.

2.1.2.6 Use addition and subtraction to create and obtain information from tables, bar graphs and tally charts.

Content Information (resources and research used to support lesson plan)

Investigations curriculum module: *Combining and Comparing: Addition and Subtraction grade 3* is the source for the lessons. This curriculum supports the view that students need to construct for themselves their own strategies for operating with whole numbers. This view is supported by the research from the Cognitive Guided Instruction (CGI) at the University of Wisconsin. CGI work shows students are capable of developing meaningful strategies for operating with whole numbers before learning any standard algorithms. Doing so allows students to participate in problem solving and meaningful mathematics. Research also supports that students learn mathematics by explaining their thinking, analyzing strategies, and justifying their solution strategies. I used readings from class text on "Mathematics for All Students" to provide ideas for supporting all students communication skills.

Previous Learning

Students have had experience with place value, building meaning for 2 and 3-digit numbers, identifying larger and smaller of pairs of numbers. They have also constructed their own thinking strategies for adding

[1] Adapted from the University of California Lesson Design Frame

and subtracting numbers less than 20. Both experiences will support this new work with adding and subtracting 2-digit numbers > 20. In future lessons, students will construct strategies for working with 3-digit numbers.

Content Objectives (Established Goals) (Label objectives C1, C2, C3…)

C1: Students will construct strategies for comparing two, two-digit numbers;
C2: Students will develop written records to communicate their strategies;
C3: Students will be able to verbally describe their strategies to compare two, two-digit numbers.

Academic Language Objectives (Label objectives L1, L2, L3…)

L1: Students will explain using simple past tense their strategies for comparing two numbers using math vocabulary like: estimate, tens and ones, how many more, difference.

Formative Assessment (Process)

As students work in their groups and participate in the lesson closure I will look for the following:

1. Were students willing to work with their partner to construct a strategy for comparing two amounts?
2. Were students able about to record their strategies for comparing two, 2-digit numbers and describe these strategies using appropriate academic language?
3. Were students able to use the empty number line to model their oral and concrete strategies for finding differences between two numbers?
4. Were students able to write subtraction number sentences for the data?

I will collect their group work, the recordings they made to compare two handfuls of beans or determine the difference to find out the range of student strategies. I will build on the different strategies in future lessons by deciding which ones are productive and beneficial for all students to learn how to use and which ones would help support students' understanding of the standard algorithm.

Co-Teaching Model

I will be the main instructor with the classroom teacher working closely with XXX who may need help with this lesson.

Provisions for Individual Differences

I will use different size beans for different pairs of students; larger beans for students who should be working with smaller 2-digit numbers and smaller beans for students who can work with larger 2- digit numbers (or 3-digit numbers).

I will be open to a wide range of strategies during the sharing time. This allows all students freedom to contribute, as I will be avoiding the idea that there is only "one" right way to solve the problem.

Resources

Students: Bag of beans for each group; 100s chart, grid paper, chart paper; markers; base ten blocks; paper cups available at the *materials* station for students to use as needed.

Teacher: Beans to model the activity during Launch phase of the lesson.

Management and Safety Issues

To avoid management issues, I will have clear expectations for what paired worked looks like and sounds like. Materials will be organized for easy distribution and collection (small baggies of beans; station with support materials available as needed). I will have clear expectations for how to present ideas: eye contact with audience, voice projection.

<u>Instructional Strategies and Learning Tasks to Support Learning</u> <u>(this is where you communicate what happens in the classroom – your instructional plan)</u>

Launch: How will you introduce the lesson and motivate the students?) List activities and key questions.
Explore: List activities, key questions, specific examples, and assessment strategies. Five practices here and partially in summarize (i.e. anticipate, monitor, select, sequence, connect).
Share/summarize: Summarize, review, lead-in for next lesson. Five practices here or partially in explore (i.e. anticipate, monitor, select, sequence, connect).

Time	Objective Code (C1, C2 or L1, L2)	Learning Activities (What and How)	Purpose (Why)
9:00 – 9:10	C1 and C2	**<u>Launch</u>** • Show students a container of beans. Ask: How many beans do you think you could grab with one hand? • Have a student come up and grab a handful of beans. (Discuss rules for grabbing beans) • Count the beans. Ask students for different ways to count them (count by one's; count by two's; put into piles of tens and extras) • Record on the board the number of beans the student grabbed; now ask if students think if the student grabbed a handful of beans with her other hand, would the amount be more, less or the same. • Have the student do this; count out the beans. Record amount on board. Ask students which hand grabbed more. **Estimate** about how much more. • Explain that now students will do this same activity with a partner and come up with their own way to find out exactly which handful has more and how much more. • Explain that students will be sharing their strategies with the whole class at the end of the exploration.	Launch models what the students will do when they work with their partner during the explore time. This is the time to ensure students know what they have to do and what is expected of them.
	C1 and C2 L1	**<u>Explore</u>** Students are to work with a partner. Each student grabs a handful of beans with her right hand, counts the beans, and records information on her paper. Repeat for the left hand. Then students are to compare the two amounts for their hands and determine what the difference is. Together they are to come up with a strategy for comparing the two amounts and finding the difference. Explain to students that they are responsible for recording the data on paper so other students will understand their strategies. Also comment that	Here is the time students have for constructing their own strategy for comparing two numbers > 20. The teacher's role is to monitor

students can of course use manipulative models as part of their strategies, but they should record on paper what they did with those models. Point out the materials station with "stuff" they can use to help them compare the two numbers.

As the students work on the exploration I will:

- Circulate and ask students to show (and explain) how they compared the two amounts.
- Encourage students to jot down the way they solved the problem and draw pictures to show their solutions.
- Record names of students to present their strategies so a wide range of strategies is shared.
- Refer students to different materials as needed: 100's chart; base ten blocks; interlocking cubes

C3 and L1

Share/Summarize

Record class data on a chart like this one:

Right hand	Left hand	Difference
85	73	12
92	79	13
76	90	14

Ask students to share their ways of comparing the two amounts. I expect students to use multiple strategies and to recreate them using oral explanations, drawings, reenactments with manipulatives, and with symbols.

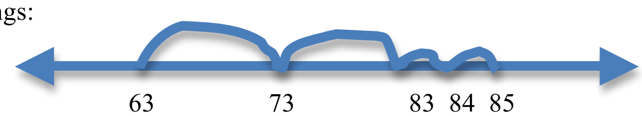
During the presentations I will invite other students to participate by describing similarities and differences among strategies presented and to ask questions about any strategy. During the presentations I will push for details to help students clarify their ideas and explanations.

Examples of possible strategies follow:
85-63

Oral explanation: " I say 63 and then count by tens: 73, 83. Then I count by ones: 84, 85. That's 20 plus 2, 22"

Manipulatives: Students shows 85 with base ten blocks. Takes off 6 tens and then 3 ones. This leaves 22 (2 tens and 2 ones).

Drawings:



their work, give hints, and reinforce productive strategies. Implement 5 Practices by monitoring, selecting and sequencing student strategies for use during Summary phase of lesson

Paired conversations around the task will allow EL students to prepare their ideas and practice the language they will use in the large group sharing

During summary phase students you select present their strategies in the order you determined. You task here is build connections across the different strategies.

Communicating your ideas is an important part of understanding mathematical

		Symbols: 85-63----- 80 – 60 = 20; 5 – 3 = 2 answer is 22 Symbols: 85- 63------85 - 60=25; 25 – 3 = 22 Select one of the ways modeled and has all students to compare their two handfuls using that strategy. Also have all students try to use the empty number line strategy. Write number sentences on the board for the student comparisons: 85 -63 = 22 **Extensions:** How many beans are in all if you combined both handfuls? Give students a chance to solve this problem. Share some strategies. Add a column to the original chart to show this amount. Ask: What are some number sentences you can write using the chart? Ask students to explain each sentence they give you by relating the numbers to the data. For example: 164 - 85 = 79 means "164 beans in all; if you take away the beans from the right hand (85) you will have the 79 beans you grabbed with the left hand.	ideas. Suggestions for monitoring large group discussion come from the class text and in particular the chapter: "ELL Learning from and Contributing to Mathematical Discussions" Nonlinguistic resources like drawings and manipulatives supplement oral explanations that otherwise are unclear. This strategy will benefit all students.

Reflection (To be completed at the end of each lesson)

Reflective Commentary (To be completed at the end of the Unit and derived from analysis of student work and teaching):