Development and Application of the Network Weight Matrix to Predict Traffic Flow for Congested and Uncongested Conditions

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ABSTRACT
To capture a more realistic spatial dependence between traffic links, we introduce two distinct network weight matrices to replace spatial weight matrices used in traffic forecasting methods. The first stands on the notion of betweenness centrality and link vulnerability in traffic networks. To derive this matrix, we assume all traffic flow is assigned to the shortest path, and thereby we used Dijkstra's algorithm to find the shortest path. The other relies on flow rate change in traffic links. For forming this matrix, we employed user equilibrium assignment and the method of successive averages (MSA) algorithm to solve the network. The components of the network weight matrices are a function not simply of adjacency, but of network topology, network structure, and demand configuration. We tested and compared the network weight matrices in different traffic conditions using Nguyen-Dupuis network. The results led to a clear and unshakable conclusion that spatial weight matrices are unable to capture the realistic spatial dependence between traffic links in a network. Not only do they overlook the competitive nature of traffic links, but they also ignore the role of network topology and demand configuration. In contrast, the flow-weighted betweenness method significantly operates better than unweighted betweenness to measure realistic spatial dependence between traffic links, particularly in congested traffic conditions. The results disclosed that this superiority is more than 2 times in congested flow situations. However, forming this matrix requires considerable computational effort and information. If the network is uncongested the network weight matrix stemming from betweenness centrality is sufficient.

Keywords: Spatial Weight Matrix; Spatial Econometrics; Traffic Flow; Vulnerability; Betweenness Centrality

INTRODUCTION

For decades, transportation analysts tackled short-term forecasting of traffic conditions, while focusing on time series approaches (1). Following the emergence of spatial analysis in traffic studies, a growing interest has aimed to embed spatial approaches in forecasting methods. At the core of the spatial approach is the belief that traffic links in a network have spatial dependence. As far as spatial dependence is concerned, traffic links on a network are spatially related, and the intensity of this association declines with distance. Capturing this dependence has essentially remained untouched since its inception at the birth of spatial weight matrices. While now acknowledged in transportation science, its roots are found in geography and spatial econometrics.

Although embedding the spatial component in forecasting methods acts as a catalyst, its functioning is hindered by the constraints of spatial weight matrices. The positivity of components in spatial weight matrices postulates that traffic links have a positive spatial dependency. In essence, this hypothesis is necessary to represent complementary (upstream and downstream) traffic links. The complementary nature demonstrates traffic streams are alike to fluid streams, and thereby vehicles observed at upstream at one time point will be observed at downstream at a later time point. For simple single facility corridors, this may be sufficient. On the flip side of the coin is the competitive nature of traffic links. The competitive nature acknowledges the close similarity between traffic streams and fluid streams. It demonstrates competitive links bear a significant proportion of diverted vehicles, when one of them is saturated or closed. Short-term forecasting of traffic conditions was initially confined to scrutinizing complementary links. In consequence the competitive nature of traffic links has been overlooked in the spatial weight matrix configuration.

We introduce two network weight matrices to fill the lacuna under the umbrella of network econometrics. These matrices are a function not simply of adjacent traffic links, but of network infrastructure, topology, and demand matrices. They have the potential of superseding the spatial weight matrices in traffic flow forecasting, and we suspect other network applications.

Having this introduction, the remainder of the paper is divided into seven parts. First, we review the traffic forecasting methods embedding spatial components, along with spatial weight matrices used in traffic analysis. Second, we discuss the concepts of betweenness centrality and vulnerability, as they are fundamental to derive network weight matrices. Third, we introduce two distinct network weight matrices. Fourth, for pedagogical purposes, we delve into the deriving process of network weight matrices for a toy network. Fifth, we introduce a general functional form of the network econometrics model, which is followed by validity assessment of network weight matrices in different traffic conditions. We finally conclude by broaching a number of arguments and suggestions for future studies.

TRAFFIC FORECASTING: A SPATIAL ANALYSIS PERSPECTIVE

"Everything is related to everything else, but near things are more related than distant things" (2). Tobler's "law" opened a new gateway and the "near" and "related" concept has spread to broader research disciplines. The footprint is seen in transportation science. Utilizing the "near" and "related" concept in traffic forecasting methods has had an active history. However, it is still as crude as it is perplexing. This concept emerged from two strands of thought in two discrete time spans. The origin of the first strand dates back to 1984, in which Okutani and Stephanedes (3) used the traffic flow on both the study link and its feeder links to predict traffic flow during the day. This so-called state-space approach subsequently burgeoned and developed in the literature. While this strand of traffic forecasting interested transportation scientists, the contemporary theoretical movement is equally drawing attention, if not more so. This strand was noticed by Kamarianakis and Prastacos (4), who tested the performance of the space-time autoregressive integrated moving average (STARIMA) model for relative velocity forecasting on major arterials of Athens, Greece. Since then, a whole host of studies have been carried out trying to test the space-time model class in forecasting traffic conditions. Comparable in concept to the state-space approach, the space-time approach benefits from the relationship between traffic links to augment the forecasting performance. Table 1 summarizes research conducted in these strands. Studies are chosen such that they cover a range of study locations, study years, and method of analysis.

The state-space approach acknowledges the positive dependency between upstream and downstream traffic links by weighting the adjacent links. This weight is either equal among adjacent links or is a function of distance. The weighted information of adjacent traffic links is then embedded in the modeling framework as independent variables. A fundamental challenge of this approach is the definition of "nearness" and distance metric. In spatial models, however, this dependency is captured by the spatial weight matrix borrowed from the field of spatial science.

A spatial weight matrix for the set of L, is a $l \times l$ matrix, where its components, W_{ij}, regularly satisfy three major rules:

1. $W_{ij} \geq 0$,
2. $W_{ii} = 0$, and
3. $\sum_{j=1}^{l} W_{ij} = 1$, for all $i = 1, 2, ..., l$ (5).

The weight matrix W_{ij}, defines the relative weight of spatial dependence between traffic links. While the spatial weight matrix can capture the self-influence of traffic link i upon itself, the matrix is typically considered to have a zero diagonal matrix. Non-diagonal elements are determined by a number of theoretical methods, which akin to state-space model, their roots are in the definition of "nearness."

TABLE 1 : Summary of selected studies on spatial network analysis

Study	Location	Scale	Method	Variable	Dependency
Spatial Weight Matrix Approach					
Cheng et al. (6)	London	Arterial	Moran's I	Time	Spatial weight
Zou et al. (7)	China	Arterial	Moran's I	Speed	Spatial weight
Min et al. (8)	China	Arterial	GSTARIMA	Flow	Spatial weight
Ma et al. (9)	China	Freeway	Moran's I	Speed	Spatial weight
Yang et al. (10)	China	Arterial	Moran's I	Speed	Spatial weight
Kamarianakis and Prastacos (4)	Greece	Arterial	STARIMA	Flow	Spatial weight
State-space Approach					
Van Lint (11)	The Netherlands	Freeway	Neural network	Time	Adjacent upstream and downstream
Stathopoulos and Karlaftis(12)	Greece	Arterial	Kalman filtering	Flow	Adjacent upstream
Van Lint et al. (13)	The Netherlands	Freeway	Neural network	Time	Adjacent upstream and downstream
Whittaker et al. (14)	The Netherlands	Motorway	Kalman filtering	Flow	Adjacent
Okutani and Stephanedes (3)	Japan	Arterial	Kalman filtering	Flow	Adjacent

In the remainder of this section, we delve into synthesizing spatial weight matrices used in traffic forecasting models, as their close connection with the contribution of the current research. In traffic analysis, the components of spatial weight matrices are typically determined by two approaches: adjacency weights and distance weights. The former assumes the spatial dependence only exists between adjacent traffic links, and the amount of this dependency is equal for all adjacent traffic links. This commonly leads to a spatial matrix with binary elements, in which zero and one values demonstrate spatial independence and spatial dependence, respectively. The latter seeks more realistic spatial dependence between traffic links. Unlike the adjacency weights, the distance weights approach assumes the spatial dependency is a function not simply of adjacency, but of the distance. Subsequently, manifold methods have been introduced to explore the effective distance threshold in traffic networks. We categorize them in two general methods:

1. **Radial distance weights**: This method considers a distance threshold around the studied traffic links. The binary spatial weight components are then measured by Equation 1, where d is a critical distance and there is no spatial dependence beyond this threshold.

$$W_{ij} = \begin{cases} 1 & 0 \leq d_{ij} \leq d \\ 0 & d_{ij} > d \end{cases} \tag{1}$$

   This is a popular method among transportation analysts, which is known as d^{th} order neighbors. Cheng et al. (6), for instance, employ the first-order neighbor matrix to explore the spatio-temporal autocorrelation structure of road networks of London, England. Likewise, Kamarianakis and Prastacos(4) explore the spatial dependence between the relative velocity of traffic links in the city of Athens, Greece. They use the first- and second-order neighbors matrix to embed a spatial component in traffic forecasting models.

2. **Power distance weights**: Unlike radial distance weights method, this method assumes the spatial dependence reduces by distance as per Equation 2. In this equation, δ represents any positive value, commonly equals one or two.

$$W_{ij} = d_{ij}^{-\delta} \tag{2}$$

   Despite the prevalence of this method in spatial science and its widespread acceptance among transportation analysts, to the best of our knowledge, there is no study using this method to measure the spatial components in transportation science.

The methods of deriving spatial weight matrices used in traffic analysis have room to grow:

- The spatial weight matrix prejudges spatial dependence between traffic links.
- The spatial weight matrix overlooks the competitive nature of traffic links in a road network.
- The spatial weight matrix is a fixed matrix for a network structure.

These drawbacks motivated the authors to introduce two distinct network weight matrices to capture the realistic spatial dependence between traffic links. Subsequently, we test whether embedding the network weight matrices in traffic forecasting models improves the accuracy of predictions.

NETWORK TOPOLOGY AND STRUCTURE

Network topology stands on the foundation of the graph theory. A graph $G = (N, L)$ is a collection of n nodes ($N = \{1, 2, \cdots, n\}$), which are interconnected by l links ($L = \{1, 2, \cdots, l\}$). The topology of a network is the arrangement and connectivity of links and nodes, which is represented by an adjacency matrix. The adjacency matrix is a square matrix, which its components demonstrate whether two nodes of m and n are attached by a link. Contingent on the type of connecting links, a network is classified as directed or undirected and weighted or unweighted. An undirected network encompasses a two-way connecting links, resulting in a symmetric adjacency matrix. While, one-way connecting links produces a directed network, in which the adjacency matrix is asymmetrical. A weighted network comprises of weighted links that signifies some distinguishing trait such as cost, capacity, and length. For an unweighted network, however, all links have the same weight, producing a binary adjacency matrix.

Having to do with network topology, manifold measures are utilized to characterize the topology, and thereby its ability to withstand link failures. Among the measures, we intend to briefly introduce the concepts of link betweenness and vulnerability, as they are fundamental to understand the deriving process of network weight matrices. In the following subsections, we expound on these concepts.

Betweenness

In transportation networks, not all links are equivalent. Traffic flow on a network is highly concentrated on relatively few links. Hence, a fundamental question from the network science perspective is: How do you examine and measure the importance of a link in a network? The concept of centrality originating from social network science has been used to determine the importance of traffic links. The two widely used link centrality measures are degree and betweenness.

Freeman (15) first introduced the betweenness centrality measurement (hereafter Betweenness) for a node in a network. The betweenness of node n, by Freeman's definition, is the ratio of the shortest paths between each pair of nodes that pass through the given node n to all the shortest paths between nodes pairs. Girvan and Newman (16) generalized this definition for measuring betweenness of a link. In a network with N nodes, the betweenness of link i is formally defined by Equation 3. In this equation, S_{od} stands for the number of shortest paths between nodes m and n, and S_{min} represents the number of shortest paths between nodes m and n that pass through link i.

$$B(i) = \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} \frac{S_{min}}{S_{mn}} \tag{3}$$

The definition of betweenness is rooted in the shortest path assumption. It assumes traffic flow is passed from one node to another only along the shortest path. In most traffic networks, however, traffic flow does not stream only across shortest paths. To advance the betweenness index, Freeman et al. (17) proposed the flow betweenness measurement for a node, which is generalizable to a link. The flow betweenness of link i measures the amount of flow through this link when the maximum flow is transmitted between a pair of nodes, averaged over all pairs of nodes.

Vulnerability

The performance of traffic links depend on one another in a traffic network. This high level of interdependency has the potential of cascading failure. As a consequence, traffic network analysts are endeavoring to fathom:

- What are the most critical links of a network?
- How do and to what extent degradation of traffic links affect network performance?
- How to assess the risk of transportation systems?

The concept of vulnerability tackles to answer these questions. Broadly defined, vulnerability refers to system performance following insecure conditions. More precisely, Berdica (18) defines vulnerability as "a susceptibility to incidents that can result in considerable reductions in road network serviceability." In transportation science, the concept of vulnerability is largely used to assess network performance resulting from degradation or disruption of nodes and links in a traffic network. Ducruet et al. (19) scrutinizes the vulnerability of nodes in liner shipping networks. They conclude a node is vulnerable when it possesses low centrality and connection, but high dependency. Jenelius et al. (20) measure the vulnerability of links in road networks as a change in travel cost stemming from a link failure. Taylor and D'Este (21) use the notion of accessibility, the ease of reaching valued destinations, to define node and link vulnerability. By their definition, a node or link is vulnerable if its removal remarkably diminishes the accessibility of the road network.

Following the recapitulation of vulnerability concept in transportation literature and in parallel with existing definitions, we intend to define link betweenness vulnerability as follows:

"The betweenness vulnerability of the link is determined by the change in the betweenness of other links upon the elimination of the link."

We use this definition to extract the realistic spatial dependence between traffic links and to form components of network weight matrices.

NETWORK WEIGHT MATRIX

In this section, we derive the network weight matrix. We use topological and structural properties of the network in order to determine the realistic spatial dependence between traffic links. Concretely speaking, we borrow two concepts of betweenness and betweenness vulnerability from the network science to introduce two distinct network weight matrices. One relies on the betweenness, and the other is inspired by the definition of flow betweenness.

Unlike the spatial weight matrix, which is unable to capture the competitive nature of links, the network weight matrices have the potential to deal with both competitive and complementary nature of traffic links. To shed some light on the notion of competitive and complementary links in a network, we depict simple two-link networks in Figure 1. One represents two paths, while the other consists of a path between its origin and destination. By definition, link i is complementary to link j, as an increase in the cost of link i not only decreases the flow of link i, but it also diminishes the flow of link j. However, link i and link k are competitive, as an increase in the cost of link i decreases the flow of link i, but increases the flow of link k.

Levinson and Karamalaputi (22) propose an algorithm, which physically detects competitive or parallel links in a road network. The algorithm acknowledges four attributes: (1) Angular difference between two links, (2) Perpendicular distance, (3) Sum of the distance between the start

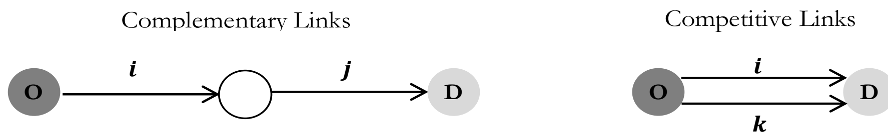

FIGURE 1 : A schematic example for complementary and competitive nature of traffic links

and end nodes of two links, and (4) The ratio of lengths of two links. However, we let the data speak for themselves. We postulate three major hypotheses to form network weight matrices:

1. The spatial dependence of competitive links is negative, and more vulnerable competitive links have more spatial dependence weight in a network.
2. The spatial dependence of complementary links is positive, and more vulnerable complementary links have a greater spatial dependence weight in a network.
3. The components of network weight matrices are a function not simply of adjacency, but of the demand configuration and network topology.

To measure the components of the network weight matrix, we propose two distinct, yet comparable approaches depicted in Figure 2. The roots of both algorithms are in the concept of link vulnerability. In the first approach, $C_B(l, G(N,L))$ is betweenness centrality of link l in network $G(N,L)$, and $C_B(l, G(N, L-\{j\}))$ is betweenness centrality of link l following the elimination of link j from network $G(N,L)$. The change in betweenness centrality of link l, stemming from the elimination of link j, actualizes the w_{ij} component of the unweighted betweenness network weight matrix. We label this approach "unweighted betweenness," as all links are equally weighted to one. Although the second and the first approaches are alike in the concept of link vulnerability, the second approach assigns different weights to each link. The assigned weight to each link is equal to the traffic flow that passes through the link. Appropriately, we label this approach "flow-weighted betweenness." In this approach, $Q_B(l, G(N,L))$ is the flow of link l in network $G(N,L)$, and $Q_B(l, G(N, L-\{j\}))$ is the flow of link l following the elimination of link j from network $G(N,L)$. The change in flow of link l due to the removal of link j, forms the w_{ij} component of the flow-weighted betweenness network weight matrix.

NETWORK WEIGHT MATRIX CALCULATION: A TOY NETWORK PROBLEM

For pedagogical purposes, we derive the network weight matrices for a simple example. The toy network, depicted in Figure 3, consists of four nodes and five links. We extract the network weight matrices introduced in the preceding section for two disparate OD demand. It reveals the effects of demand configuration on spatial dependence between traffic links.

We adopt the standard Bureau of Public Roads (BPR) link performance function as per Equation 4. In this equation, t_i is the link travel time, t_i^0 stands for free-flow link travel time, v_i represents the assigned traffic volume, and c_i is the capacity of link i.

$$t_i = t_i^0[1 + \alpha(\frac{v_i}{c_i})^\beta] \quad (4)$$

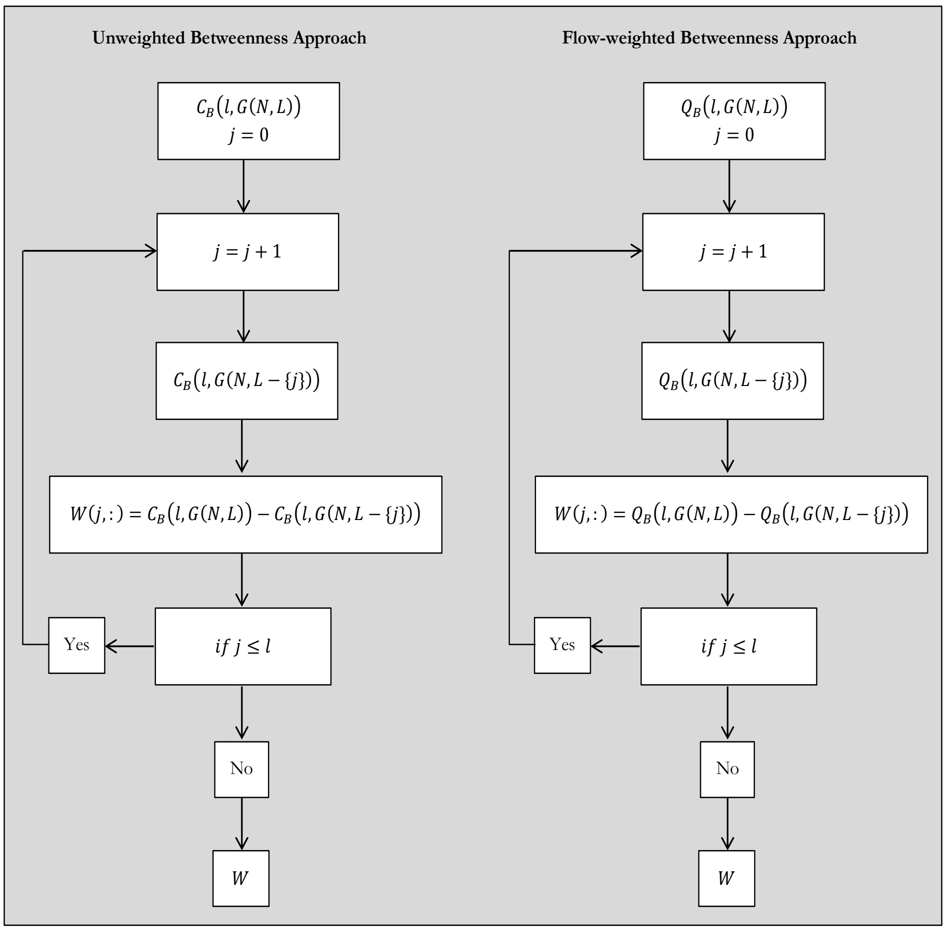

FIGURE 2 : The flowchart of network weight matrix measurement

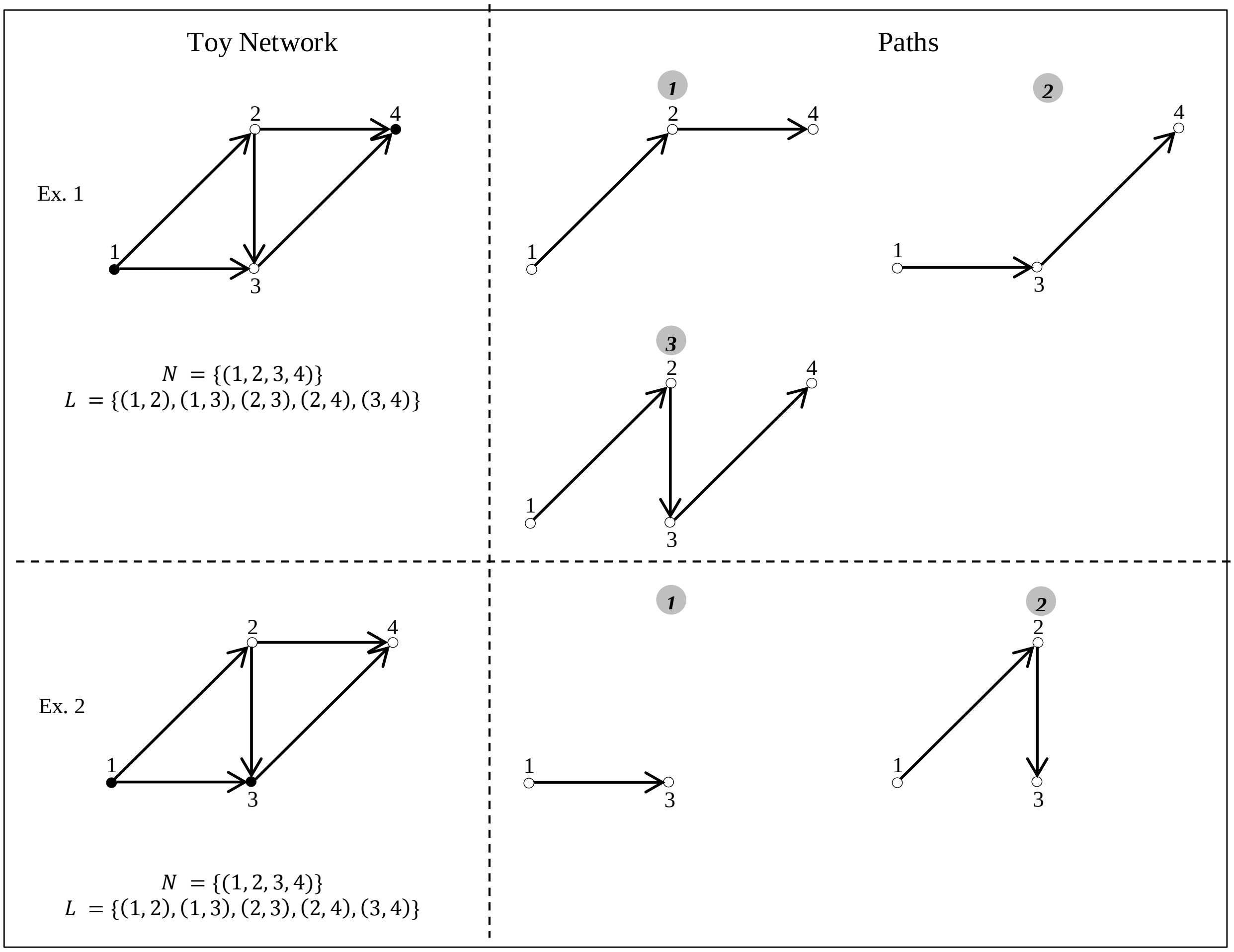

FIGURE 3 : A toy network and its link path graph

We outline the parameter values of each link in Table 2, which is extracted from Suwansirikul et al. (23). However, we slightly changed the free-flow link travel times of links (1,2) and (3,4) to have a unique shortest path between given OD pairs. In the following examples, we point out the derivation of network weight matrices in detail.

TABLE 2 : Parameter values of the toy network

Link	t_i^0	c_i	α	β
(1,2)	3.9	40.0	0.15	4.0
(1,3)	6.0	40.0	0.15	4.0
(2,3)	2.0	60.0	0.15	4.0
(2,4)	5.0	40.0	0.15	4.0
(3,4)	3.1	40.0	0.15	4.0

Example 1: The first instance passes 100 vehicles from node 1 to node 4 through three different paths. For the sake of understanding, we separately explain the process of deriving network weight matrices for two (unweighted) betweenness and two flow-weighted betweenness approaches in this example.

Having to do with betweenness approach, we are able to calculate betweenness based on either the free-flow travel time or the ultimate link travel time derived from Equation 3 for each scenario. For simplicity, we employ the free-flow travel time to measure betweenness in this example. However, we later test both cases for evaluating network weight matrices. To find the shortest path, we employed Dijkstra's algorithm. We represent the results in Table 3. For a network with all links scenario, looking at the betweenness indicates that the value of links (1,2) and (2,4) equal one, as path 1 is the shortest path between 1-4 OD pair. Following the removal of link (1,2), path 1 is no longer the shortest path and path 2 absorbs the traffic, as it is the only path in the network. Consequently, the values of links (1,3) and (3,4), which belong to path 2, take on the value of one. Differentiation of the betweenness of links when the network possesses all links and when link (1,2) is removed results in revealing the spatial dependence between link (1,2) and other links. The derived values form the first row of the network weight matrix depicted in Figure 4.The values disclose both complimentary and competitive nature of traffic links.

As expected, link (1,2) is intensely competitive with links (1,3) and (3,4), as removing link (1,2) shifts the traffic flow to link (1,3) and (3,4), which is the only path in the network. However, links (1,3) and (3,4) have do not any spatial impact on link (1,2), and thereby their corresponding components in the network weight matrix are zero. This is illustrated by the hypothesis of shortest path selection, which is the backbone of betweenness measurement. Link (1,2) belongs to the shortest path 1, which is selected by all users. Removal of link (1,3) does not change the path of flow in the network, and as a result the betweenness of the links is similar to the network with all links scenario. It is the shortcoming of betweenness measurement, which assumes all network users choose the shortest path.

Link (1,2) is complementary to link (2,4), as its removal paralyzes link (2,4). However, as shown in Figure 3, there is no spatial influence from link (2,4) on link (1,2). This is illuminated by two main reasons: (1) the studied links are directed and traffic link (1,2) is upstream of traffic link (3,4). Hence, flow streams from link (1,2) to link (3,4), and in the free-flow condition link (3,4) does not have any spatial influence on link (1,2). While in the congested situation, we might imagine the shockwave stemming from link (3,4) affects link (1,2). However, it is neither the case in this example nor is it measurable by betweenness index.

To implement the flow-weighted betweenness approach, we use the stochastic user equilibrium and the method of successive averages (MSA) solution algorithm to assign vehicular trip rates to the network (24). The results of the assignment are depicted in Table 3. Removal of link (1,2) paralyzes path 1 and path 3, and consequently the flow on links (1,2), (2,3), and (2,4) equal zero. Calculating the change rate in traffic flow of each link following the link (1,2) removal results in revealing spatial dependence between link (1,2) and other traffic links. The first row of network weight matrix depicted in Figure 4 discloses this dependency. For illustration, the components of the first row are calculated as follows:

$W_{(1,2)(1,2)} = 52 - 0 = 52$

$W_{(1,2)(1,3)} = 48 - 100 = -52$

$W_{(1,2)(2,3)} = 6 - 0 = 6$

$W_{(1,2)(2,4)} = 46 - 0 = 46$

$W_{(1,2)(3,4)} = 54 - 100 = -46$

The values of the network weight matrix formed by flow-weighted betweenness approach alike the betweenness approach reveal both complimentary and competitive nature of traffic links.

Link (1,2) is directly competitive with links (1,3) and (3,4), in line with our hypotheses and results of the betweenness approach. In contrast with betweenness approach, the flow-weighted betweenness approach acknowledges the reciprocal spatial dependence between links (1,2) and both link (1,3) and link (3,4). The reason is the flow-weighted betweenness approach is not simply a function of shortest path, but of user equilibrium assignment. However, the unweighted betweenness approach stands on the foundation of all-or-nothing assumption. Interestingly, looking at the second row of the network weight matrix indicates that link (1,3) is competitive with links (2,3) and (2,4), but with different magnitudes. Although link (1,3) is highly correlated with link (2,3), there is a low correlation between link (1,3) and (2,4). It is empirically, true as a significant amount of flow shifts to link (2,3) by removing link (1,3) in comparison with the network with all links scenario. However, traffic flow of link (2,4) does not witness a remarkable change.

Unlike the unweighted betweenness approach, link (1,2) is complementary not only to link (2,4), but also to link (2,3). It is not surprising, given link (1,2) is a feeder of both links. However, there is no reciprocal spatial dependence between links (1,2) and (2,4), since traffic links are directed in this example and link (1,2) is the only feeder of link (2,4). When a link absorbs traffic from more than one feeder, the reciprocal spatial dependence shows up in network weight matrix. The instance of such dependence is link (3,4), which is fed by links (1,3) and (2,3). Consequently, not just links (1,3) and (2,3) spatially affect link (3,4), but they are affected by link (3,4) as well.

Comparing two network weight matrices demonstrates that the network weight matrix built on the flow-weighted betweenness approach captures the more realistic spatial dependence between links than the unweighted betweenness approach. We hence hypothesize that the flow-weighted betweenness approach performs better than unweighted betweenness approach, particularly in congested traffic conditions. We later test this hypothesis.

TABLE 3 : Weight matrix calculation for Example 1

Network Scenario	Unweighted Betweenness					Flow-Weighted Betweenness				
	(1,2)	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
All links	1	0	0	1	0	52	48	6	46	54
Without (1,2)	0	1	0	0	1	0	100	0	0	100
Without (1,3)	1	0	0	1	0	100	0	52.38	47.62	52.38
Without (2,3)	1	0	0	1	1	50	50	0	50	50
Without (2,4)	1	0	1	0	1	52	48	52	0	100
Without (3,4)	1	0	0	1	0	100	0	0	100	0

Unweighted Betweenness

$W =$

Links	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
(1,2)	1	-1	0	1	-1
(1,3)	0	0	0	0	0
(2,3)	0	0	0	0	0
(2,4)	0	0	-1	1	-1
(3,4)	0	0	0	0	0

Flow-weighted Betweenness

$W =$

Links	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
(1,2)	52	-52	6	46	-46
(1,3)	-48	48	-46.38	-1.62	1.62
(2,3)	2	-2	6	-4	4
(2,4)	0	0	-46	46	-46
(3,4)	-48	48	6	-54	54

FIGURE 4 : Network weight matrices for Example 1

Example 2: Similar to Example 1, this instance passes 100 vehicles, but from node 1 to node 2 through two different paths. We depict the results of flow-weighted and unweighted betweenness for different scenarios in Table 4. We also represent the corresponding network weight matrices in Figure 5. The process of deriving network weight matrices for unweighted and flow-weighted betweenness is akin to Example 1. We hence eschew digging into the derivation, and instead emphasize the dissimilarity between network weight matrices in two examples stemming from the change in demand configuration.

Not surprisingly, the corresponding components to link (2,4) and (3,4) in network weight matrices are zero, as they pass no flow from node 1 to node 2. In this example path 1 competes with path 2, and thereby links (1,2) and (2,3) are competitive with link (1,3). The negative sign of the components discloses this competitive nature. Comparing network weight matrices in two examples emphasizes the remarkable role of link (2,3) in 1-2 OD configuration. It is indeed true, as path 2 is the only substitute for path 1. The change in value of the components of the network weight matrices in two examples reveals the role of demand configuration in spatial dependence between traffic links. The spatial dependence between traffic links is not only related to the topology of the network, but it is also defined by the demand configuration in traffic networks.

USE OF NETWORK ECONOMETRICS TO ESTIMATE TRAFFIC FLOW

Let us consider again the toy network in Figure 3, when traffic demand generates from node 1 and attracts to node 4. The traffic flow of a particular link (2,3) is a function of travel cost on this link. Pertaining to the spatial dependence between links, the travel cost on the upstream and downstream links (1,2) and (3,4), which feed or absorb flows, could have significant impacts on flow of link (2,3). All else equal, parallel links (1,3) and (2,4) could also significantly affect the flow of link

TABLE 4 : Weight matrix calculation for Example 2

Network Scenario	Unweighted Betweenness					Flow-weighted Betweenness				
	(1,2)	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
All links	1	0	1	0	0	51.85	48.15	51.85	0	0
Without (1,2)	0	1	0	0	0	0	100	0	0	0
Without (1,3)	1	0	1	0	0	100	0	100	0	0
Without (2,3)	0	1	0	0	0	0	100	0	0	0
Without (2,4)	1	0	1	0	0	51.85	48.15	51.85	0	0
Without (3,4)	1	0	1	0	0	51.85	48.15	51.85	0	0

Unweighted Betweenness

$W =$

Links	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
(1,2)	1	-1	1	0	0
(1,3)	0	0	0	0	0
(2,3)	1	-1	1	0	0
(2,4)	0	0	0	0	0
(3,4)	0	0	0	0	0

Flow-weighted Betweenness

$W =$

Links	*(1,2)*	*(1,3)*	*(2,3)*	*(2,4)*	*(3,4)*
(1,2)	51.85	-51.85	51.85	0	0
(1,3)	-48.15	48.15	-48.15	0	0
(2,3)	51.85	-51.85	51.85	0	0
(2,4)	0	0	0	0	0
(3,4)	0	0	0	0	0

FIGURE 5 : Network weight matrices for Example 2

(2,3), although in the opposite direction by diverting trips. Pertaining to the temporal dependence, the observations of travel cost on lag time could also drive the traffic flow by influencing the expectation of travelers on travel cost. In addition, the traffic flow of a lag time is regarded as a continuation of current condition of traffic flow. Accordingly, the traffic flow in a traffic network is estimated by a network econometrics model as follows:

$$\nabla^h q_t = \gamma + \sum_{c=0}^{C}\sum_{z=0}^{Z} \psi_{cz} W_z \nabla^h q_{t-c} + \varphi_t W X_t + \sum_{c=0}^{C}\sum_{z=0}^{Z} \theta_{ck} W_z \varepsilon_{t-c} + \varepsilon_t \tag{5}$$

In Equation 5, X_t is a vector of explanatory variables such as capacity and speed limit of traffic links. The parameters of h, C, and Z are non-negative integers and stand for degree of differencing, order of the autoregressive, and order of the moving-average, respectively. W_z is a $l \times l$ network weight matrix for temporal lag z. Finally, ε_t denotes a normally distributed of error terms.

The network econometrics model has distinctive characteristics:

- This model has the potential to achieve better results when network topology changes or data is missing.
- This model accounts for demand uncertainty by developing a more comprehensive solution that is less likely to fail under extreme events. Consequently, it reduces the variance of costs.
- The network econometrics model is statistical, and based on initial demands, network structure, shortest path, and user equilibrium assumptions, and predicts flows rather than

the movement of individual vehicles. It does not incorporate physical models such as hydrodynamics or car-following. This model allows data to speak directly and play a more decisive role in traffic forecasting models

- The network econometrics model includes a network weight matrix, which captures realistic spatial dependence between traffic links.

In the following section, we focus on a specific functional form of the network econometrics model, in which traffic flow is just a function of travel cost on the current state. This aims to evaluate the excellence of the network weight matrices in line with the main contribution of the current research.

NETWORK WEIGHT MATRIX VERIFICATION

In order to exhibit validity of the network weight matrices and comparing the efficacy of each of which, we adopt the Nguyen and Dupuis (25) network depicted in Figure 6. The network consists of 13 nodes, 19 directed links and 4 OD pairs. The characteristics of each link are represented in Table 5, which are extracted from Xu et al. (26). We test the network weight matrices for three different demand scenarios:

1. $q_{12} = 20$, $q_{13} = 40$, $q_{42} = 30$, and $q_{43} = 10$,
2. $q_{12} = 400$, $q_{13} = 800$, $q_{42} = 600$, and $q_{43} = 200$, and
3. $q_{12} = 800$, $q_{13} = 1,600$, $q_{42} = 1,200$, and $q_{43} = 400$.

The second scenario, which was used by Nguyen and Dupuis (25), gives a semi-congested traffic condition. In this scenario, links 6, 8, 12, 15, and 17 have not reached their capacity. The first and third demand scenarios are designed to assess the network weight matrices in uncongested and congested traffic regimes, respectively.

TABLE 5 : Link characteristics of the Nguyen-Dupuis network

Link	t_i^0	c_i	α	β	Link	t_i^0	c_i	α	β
1	7	300	0.15	4	11	9	500	0.15	4
2	9	200	0.15	4	12	10	550	0.15	4
3	9	200	0.15	4	13	9	200	0.15	4
4	12	200	0.15	4	14	6	400	0.15	4
5	3	350	0.15	4	15	9	300	0.15	4
6	9	400	0.15	4	16	8	300	0.15	4
7	5	500	0.15	4	17	7	200	0.15	4
8	13	250	0.15	4	18	14	300	0.15	4
9	5	250	0.15	4	19	11	200	0.15	4
10	9	300	0.15	4					

To test the network weight matrices, we develop five distinct models. The functional form of the models assumes a simple linear relationship between exogenous and endogenous variables. We are aware that this is a naive assumption. However, this does not jeopardize our results, as we aim to judge whether and to what extent the network weight matrices have the potential of advancing the traffic flow forecasting. The first model simply considers a linear relationship between traffic flow in each link and its corresponding travel cost. The other models capture both direct

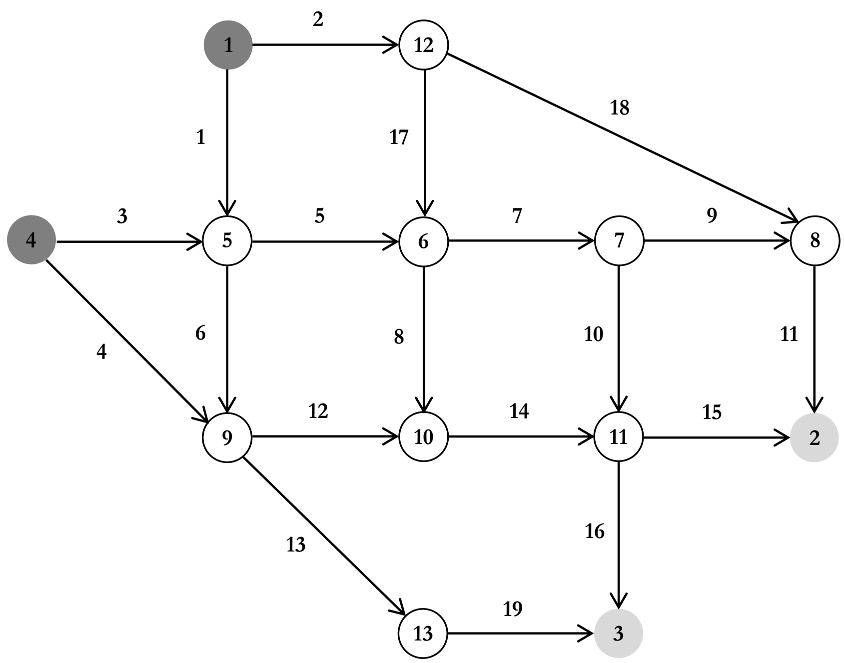

FIGURE 6 : The Nguyen-Dupuis network

and spatial relationship between traffic flow and travel cost. The models are unique in the method of measuring spatial dependence between traffic links. We depict the information of the models in Table 6. For estimation, we use the ordinary least squares (OLS) estimation method, which is the best linear unbiased estimator (BLUE) of the coefficients for both bivariate linear regression and spatial cross-regressive models. We summarize the results of the models in Table 7. The student's t-test for exogenous variables is reported in parentheses.

TABLE 6 : Specification of models used in this study

Models	Formula	Method of Spatial Dependence Measurement
Model 1	$q_t = \gamma + \mu_t X_t + \varepsilon_t$	No spatial component
Model 2	$q_t = \gamma + \mu_t X_t + \varphi_t W X_t + \varepsilon_t$	Spatial Weight matrix: First-order neighbor
Model 3	$q_t = \gamma + \varphi_t W X_t + \varepsilon_t$	Network weight matrix: unweighted betweenness based on t_i^0
Model 4	$q_t = \gamma + \varphi_t W X_t + \varepsilon_t$	Network weight matrix: unweighted betweenness based on t_i
Model 5	$q_t = \gamma + \varphi_t W X_t + \varepsilon_t$	Network weight matrix: flow-weighted betweenness

As for the significance of variables, the spatial component of Model 5 is significant constantly. It demonstrates that the network weight matrix deriving from the flow-weighted betweenness approach is able to capture the spatial dependence significantly in all traffic conditions. It is also true for Model 4. The spatial component of Mode1 2, however, is not statistically significant. This discloses that the traditional spatial weight matrix is unable to measure the realistic spatial dependence between traffic links. Finally, the cost component of Model 1 statistically defines the traffic flow in free-flow traffic condition. When the traffic condition transits from free-flow to congested flow, there is no significant linear correlation between traffic cost and traffic flow. This is generalizable to the coefficient of spatial component in Model 3. It was expected, as the network weight matrix used in Model 3 derived from the free-flow cost function, and thereby performs better in the free-flow traffic condition.

TABLE 7 : Results of the models in different demand scenarios

Models		Scenario 1	Scenario 2	Scenario 3
Model 1	*Constant*	87.90 (4.29)	417.29 (3.01)	793.01 (4.07)
	Coefficient	-6.94 (-3.16)	1.44 (0.21)	2.66 (0.61)
	R^2	0.37	0.002	0.02
Model 2	*Constant*	99.62 (3.77)	423.96 (2.49)	804.47 (3.32)
	Coefficient	-7.48 (-3.18)	1.40 (0.20)	2.67 (0.59)
	Coefficient	-0.60 (-0.72)	-0.26 (-0.07)	-0.27 (-0.08)
	R^2	0.39	0.003	0.02
Model 3	*Constant*	1.62 (0.25)	428.51 (8.41)	903.57 (10.14)
	Coefficient	-9.41 (-5.11)	-41.99 (-1.07)	-7.21 (-0.25)
	R^2	0.60	0.06	0.003
Model 4	*Constant*	2.05 (0.31)	358.52 (6.49)	778.89 (7.81)
	Coefficient	-8.69 (-5.03)	-78.07 (-2.49)	-49.04 (-1.99)
	R^2	0.59	0.26	0.18
Model 5	*Constant*	8.59 (4.63)	356.54 (10.06)	808.95 (10.63)
	Coefficient	-0.05 (-18.09)	-4.29 (-5.21)	-4.52 (-3.11)
	R^2	0.95	0.61	0.36

As for the fit of the models, we compare the R^2 measure of goodness of fit for all models in whole scenarios. We show the results in Figure 7. Model 5 performs far better than the other models in all traffic conditions. There is not a significant difference between the performance of model Mode 2, which embeds the spatial weight matrix, and Model 1. In the uncongested traffic condition, Model 3 and Model 1 reach the same result, and both of them perform 1.6 times better than Model 1. Model 5 performs 1.5 times better than Models 3 and 4, and thereby 2.5 times better than Model 1. In the semi-congested and congested traffic conditions, Model 3 lose its ability to capture the realistic spatial dependence between traffic links, while it yet performs far better than both Model 1 and Model 2. Although the prediction power of Model 4 and Model 5 declines in the semi-congested and congested traffic conditions, their relative power increases significantly. In the congested traffic condition, for example, the R^2 measure of Model 4 and Model 5 is 8.7 and 16.7 times of Model 1, respectively. The results lead inexorably to the conclusion that the network weight matrices more realistically measure spatial dependency between traffic links. This results improves traffic flow forecasting models.

CLOSING REMARKS

Despite the fact that forecasting traffic conditions is sophisticated, it is still tractable and predictable with a deep understanding of relationships between traffic components. Under this conviction, continuous attempts have been made to analyze transportation networks and modeling traffic conditions. Correspondingly, two strands of methods have emerged, which embed the spatial component in traffic forecasting framework: state-space and spatio-temporal approaches. However, the evolution of spatial traffic forecasting models were mainly based on spatial weight matrices, which may not accurately reflect the spatial dependence between traffic links.

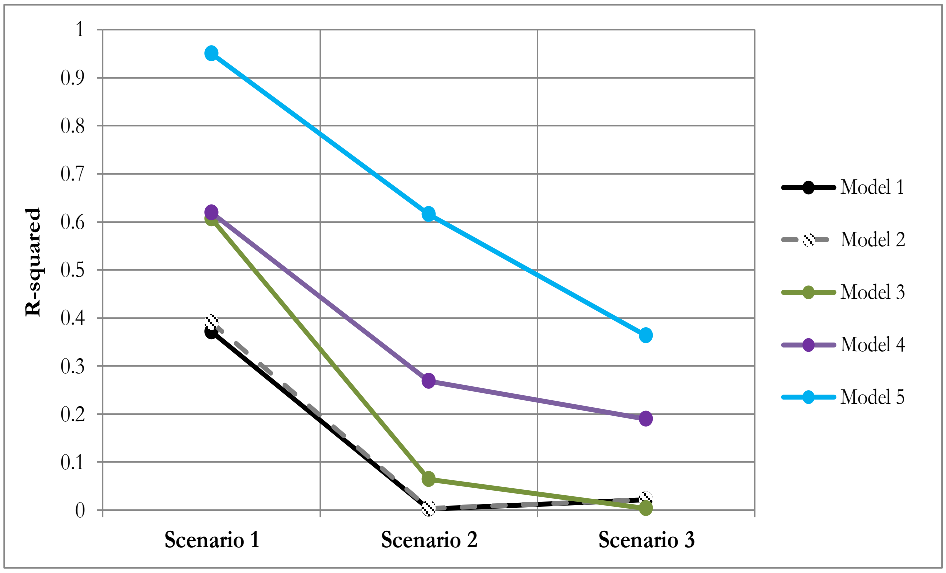

FIGURE 7 : The goodness of fit comparison of the models in all demand scenarios

We have introduced two distinct network weight matrices. The first is built on the notion of betweenness and link vulnerability in traffic networks. To derive this matrix, we assume all traffic flow is assigned to the shortest path, and thereby we use Dijkstra's algorithm to find the shortest path. The second relies on flow rate change in traffic links. For forming this matrix, we employed user equilibrium assignment and MSA algorithm to solve the network. This approach enabled us to capture more realistic traffic flow distribution, especially in the congested traffic conditions. Both network weight matrices acknowledge the network topology and demand configuration. If topological and hierarchical attributes correctly capture the substitutive effects on the network, we are able to better predict how traffic flow would redistribute on the network in cases of major network changes.

We have tested and compared the network weight matrices in different traffic conditions. Such a comparison exemplifies the capability of network weight matrices to advance traffic forecasting. The best performing implementations for capturing spatial dependence between traffic links are the network weight matrices. The models with network weight matrices perform better than both the model without spatial weight matrix and without the spatial component. This leads inexorably to the conclusion that traditional spatial weight matrices are unable to capture the realistic spatial dependence between traffic links. We also demonstrate that the network econometrics model encompassing the network weight matrix stemming from the flow-weighted betweenness approach performs far better than the other models, particularly in congested traffic conditions. However, forming this matrix requires considerable computational effort and information. If the network is in the uncongested state, we recommend the network weight matrix stemming from betweenness. Drilling down further, the key findings include:

- The spatial dependency that is captured by spatial weight matrix is unsuccessful in explaining the spatial relationship between traffic links.

- The network weight matrix deriving from betweenness method performs well in free-flow traffic conditions, and loses its ability in congested traffic condition. However, measuring betweenness by the ultimate travel cost instead of free-flow travel time enhances the capability of this matrix in congested traffic conditions.

- The assigned flow method significantly operates better than betweenness method to measure realistic spatial dependence between traffic links, particularly in congested traffic condition. The results disclose that this superiority is more than 2 times in congested flow situations.

This study has led to a clear and unshakable conclusion that traditional spatial weight matrices are unable to capture the realistic spatial dependence between traffic links in a network. Not only do they overlook the competitive nature of traffic links, but they also ignore the role of network topology and demand configuration in measuring the spatial dependence between traffic links. Neglecting these elements is not simply information loss. It has nontrivial impacts on concluding remarks and policy decisions. Although we believe this study is a valuable extension to the current literature, as a first step it requires additional research.

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