# Topology and Correlation in Quantum Materials with Strong Spin-Orbit Coupling II

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TPI Summer School, University of Minnesota
June16, 2016

# Outline

General Intro: Iridates

Brief Introduction:
Topological Phases

Hyper-Honeycomb Iridates
$\beta$-$Li_2IrO_3$ $\gamma$-$Li_2IrO_3$

Pyrochlore Iridates (bulk and film)
$R_2Ir_2O_7$

Hyper-kagome Iridate:
$Na_4Ir_3O_8$

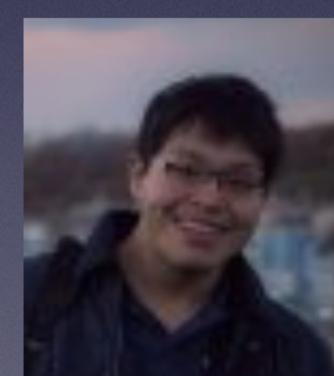
# Hyper-Honeycomb Iridates
# β-$Li_2IrO_3$ γ-$Li_2IrO_3$

Eric Lee
(Toronto)

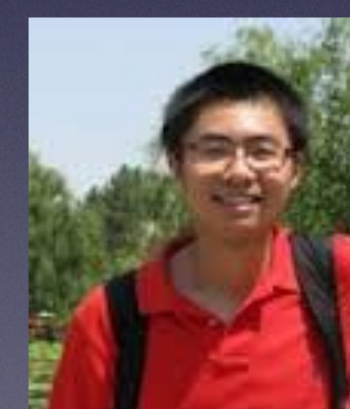
Robert
Schaffer
(Toronto)

Jeff Rau
(Waterloo)

Heung-Sik
Kim
(Toronto)



Yuan-Ming
Lu
(OSU)



# Realization of Kitaev Quantum Spin Liquid ?

2D Honeycomb

α- $Na_2IrO_3$  α- $Li_2IrO_3$
(H. Takagi, P. Gegenwart)

α- $RuCl_3$ (S. Nagler, Y. J. Kim)

3D Hyper-Honeycomb

β- $Li_2IrO_3$
(H. Takagi, P. Gegenwart)

3D Stripy-Honeycomb

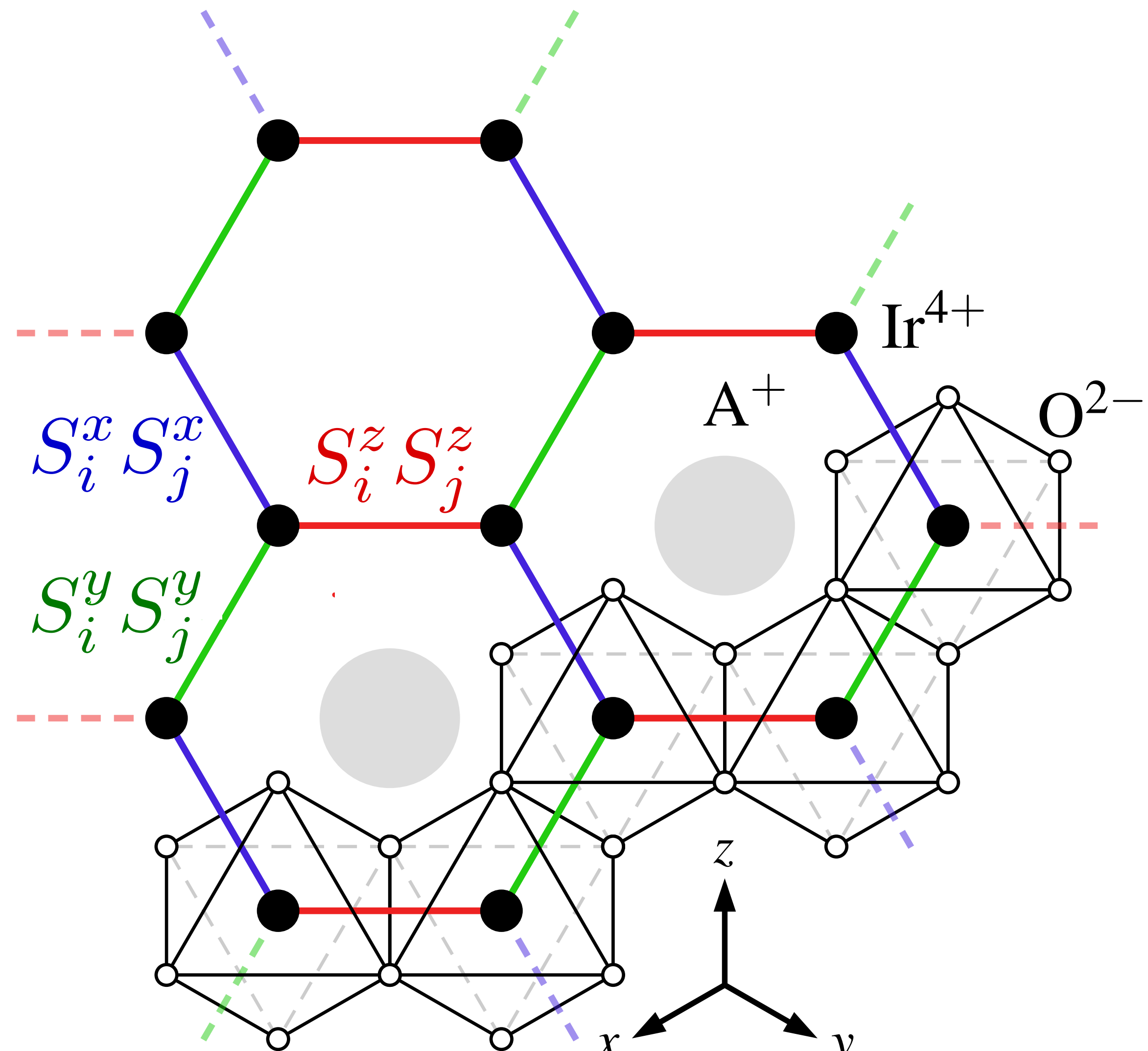
γ- $Li_2IrO_3$
(J. Analytis, R. Coldea)

# α-$A_2IrO_3$ Kitaev Interaction ?

$$H = \sum_{\langle ij\rangle \in \gamma} -K S_i^\gamma S_j^\gamma \qquad \gamma = x, y, z$$

G. Jackeli and G. Khaliullin,
PRL 102, 256403 (2009)

J. Chaloupka, G. Jackeli
and G. Khaliullin,
PRL 105, 027204 (2010)




## $J_{eff}=1/2$

$$|\uparrow_j\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_s\rangle + |yz,\downarrow_s\rangle + |xy,\uparrow_s\rangle)$$

$$|\downarrow_j\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_s\rangle - |yz,\uparrow_s\rangle + |xy,\downarrow_s\rangle)$$

Strong Spin-Orbit Coupling leads to Spin-Orbit entangled pseudo-spin basis (Kramers Doublet)

# $\alpha$-$A_2IrO_3$ Kitaev Interaction ?

$$H = \sum_{\langle ij \rangle \in \gamma} -K S_i^\gamma S_j^\gamma$$

$$\gamma = x, y, z$$

G. Jackeli and G. Khaliullin,
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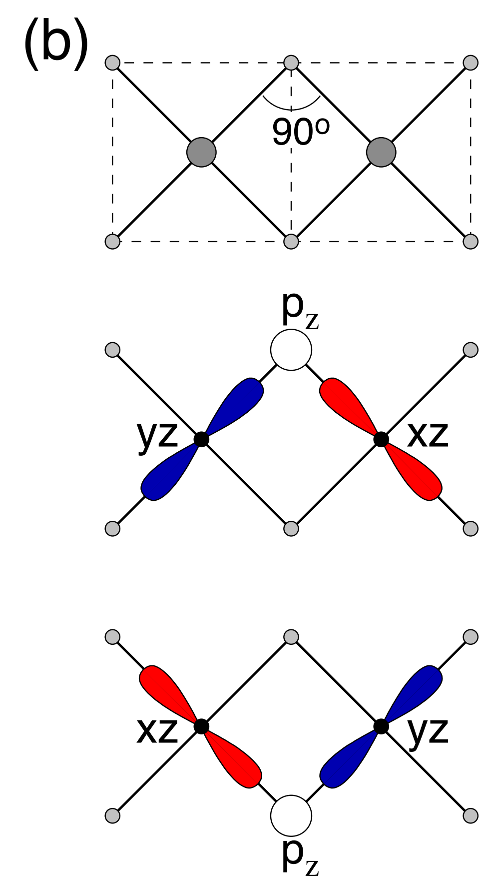



Edge-Sharing Oxygen Octahedra

Including Hund's coupling and projecting to $J_{eff}$=1/2 manifold

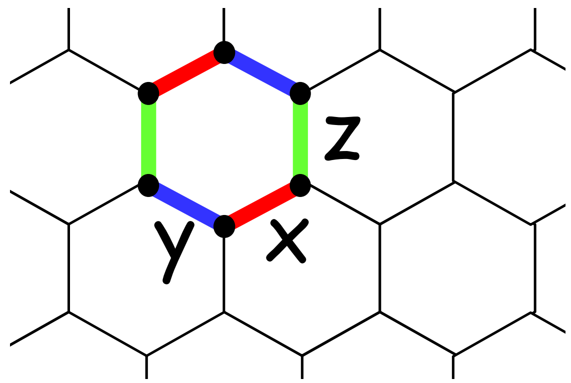
## $J_{eff}$=1/2

$$|\uparrow_j\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_s\rangle + |yz,\downarrow_s\rangle + |xy,\uparrow_s\rangle)$$

$$|\downarrow_j\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_s\rangle - |yz,\uparrow_s\rangle + |xy,\downarrow_s\rangle)$$

Strong Spin-Orbit Coupling leads to Spin-Orbit entangled pseudo-spin basis (Kramers Doublet)

# Kitaev Model on Honeycomb Lattice: Exact Solution




$$\mathcal{H}_\mathrm{K} = -\sum_{\alpha-links} S_i^\alpha S_j^\alpha$$

A. Kitaev (2006)

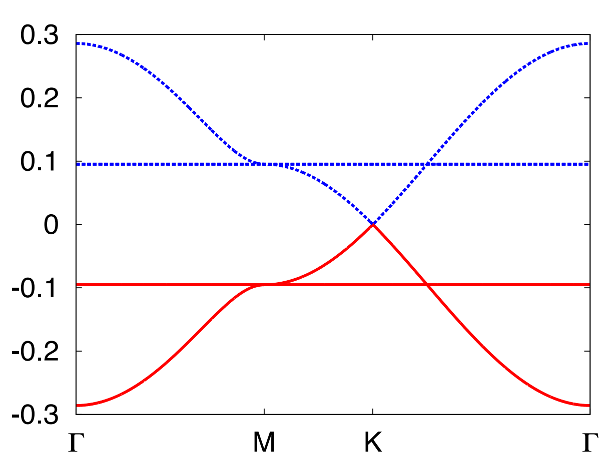
$$S_i^\alpha = i b_i^\alpha c \qquad \{b_i^x, b_i^y, b_i^z, c\}$$

Four Majorana Fermions

$$\mathcal{H}_\mathrm{K} = \frac{i}{2} \sum_{\alpha-\mathrm{links}} u_{ij}^\alpha c_i c_j \qquad (\text{where } u_{ij}^\alpha = i b_i^\alpha b_j^\alpha),$$

$\mathcal{W}_P = \prod_{\mathrm{loop}} u_{ij}^\alpha$ commute with the Hamiltonian $\Longrightarrow$ $\mathcal{W}_P = \pm 1$

Ground state is in the zero-flux sector $u_{ij}^\alpha = +1 \; (\forall \langle ij \rangle)$




Majorana Fermions with
Dirac Dispersion

# Kitaev Spin Liquid as a $Z_2$ Spin Liquid

Making connection to Slave-fermion approach (more conventional)

$$f_{i\uparrow} = \frac{1}{\sqrt{2}}(c_i + i b_i^z) \qquad S_i^a = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^a f_{i\beta}$$

$$f_{i\downarrow} = \frac{i}{\sqrt{2}}(b_i^x + i b_i^y)$$

with $\sum_\alpha f_{i\alpha}^\dagger f_{i\alpha} = 1$

$$H = \sum_{\langle ij \rangle \in a} \{ f_{i\alpha}^\dagger [T^a]_{\alpha\beta}^{ij} f_{j\beta} + f_{i\alpha} [\Delta^a]_{\alpha\beta}^{ij} f_{j\beta} \}$$

exactly one particle per site (insulator)

If $\Delta^a = 0$ $\quad f_{i\alpha} \longrightarrow f_{i\alpha} e^{i\theta_i} \quad [T^a]_{ij} = |[T^a]_{ij}| e^{i a_{ij}}$

$$a_{ij} \longrightarrow a_{ij} + \theta_i - \theta_j$$

U(1)

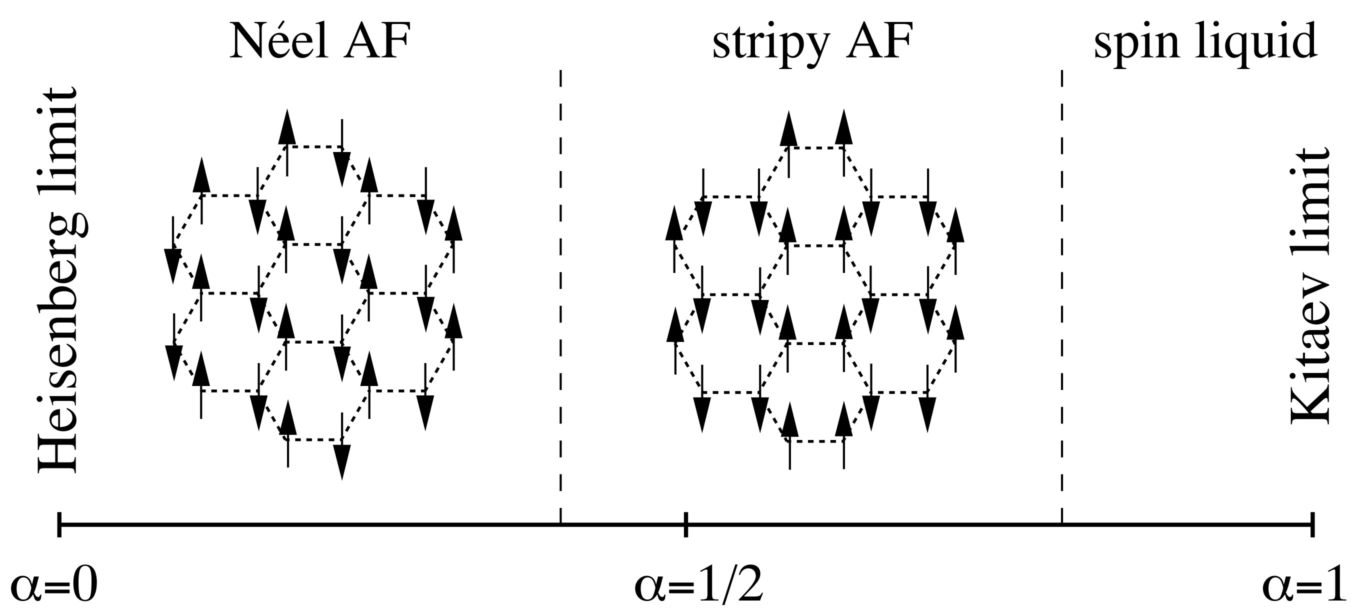
In general $\quad f_{i\alpha} \to f_{i\alpha} s_i \qquad s_i = \pm 1$

$$[T^a]_{ij}, \quad [\Delta^a]_{ij} \to s_i [T^a]_{ij} s_j, \quad s_i [\Delta^a]_{ij} s_j$$

$Z_2$

J. Chaloupka, G. Jackeli and G. Khaliullin,
PRL 105, 027204 (2010)

$$H_{\mathrm{HK}} = (1-\alpha)\sum_{\langle i,j\rangle}\vec{\sigma}_i\cdot\vec{\sigma}_j - 2\alpha\sum_{\gamma-\mathrm{links}}\sigma_i^\gamma\sigma_j^\gamma$$

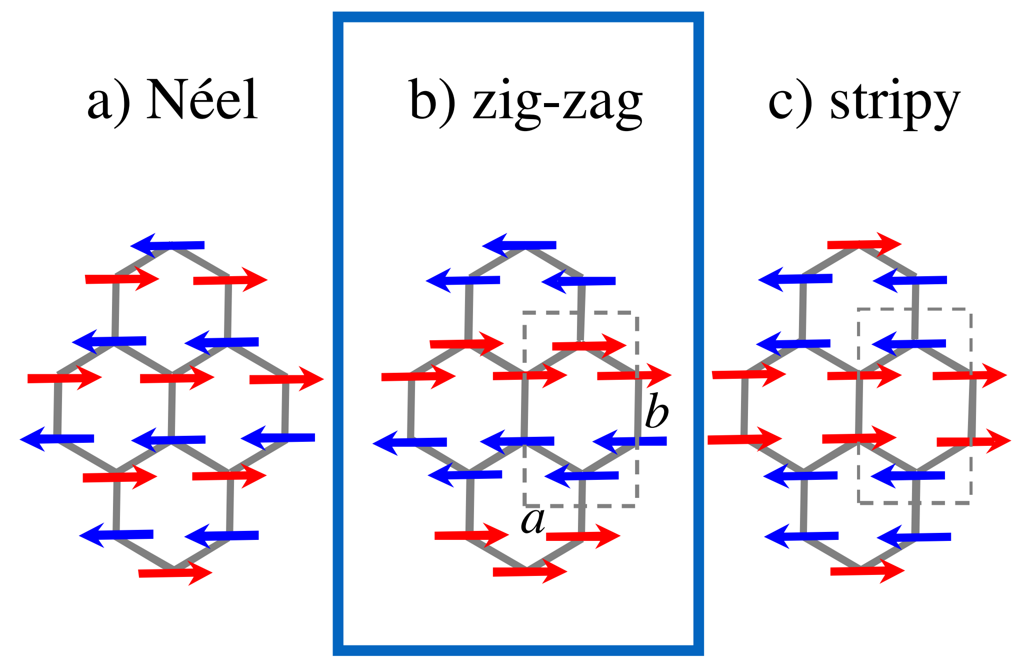



Quantum Spin Liquid robust
to small Heisenberg interaction

Radu Coldea
(Neutron Scattering)

$\alpha$-$Na_2IrO_3$

Zig-Zag Order




Monoclinic Distortions

Strong Trigonal Distortions

# Discovery of Three dimensional "Honeycomb" lattice

β- $Li_2IrO_3$

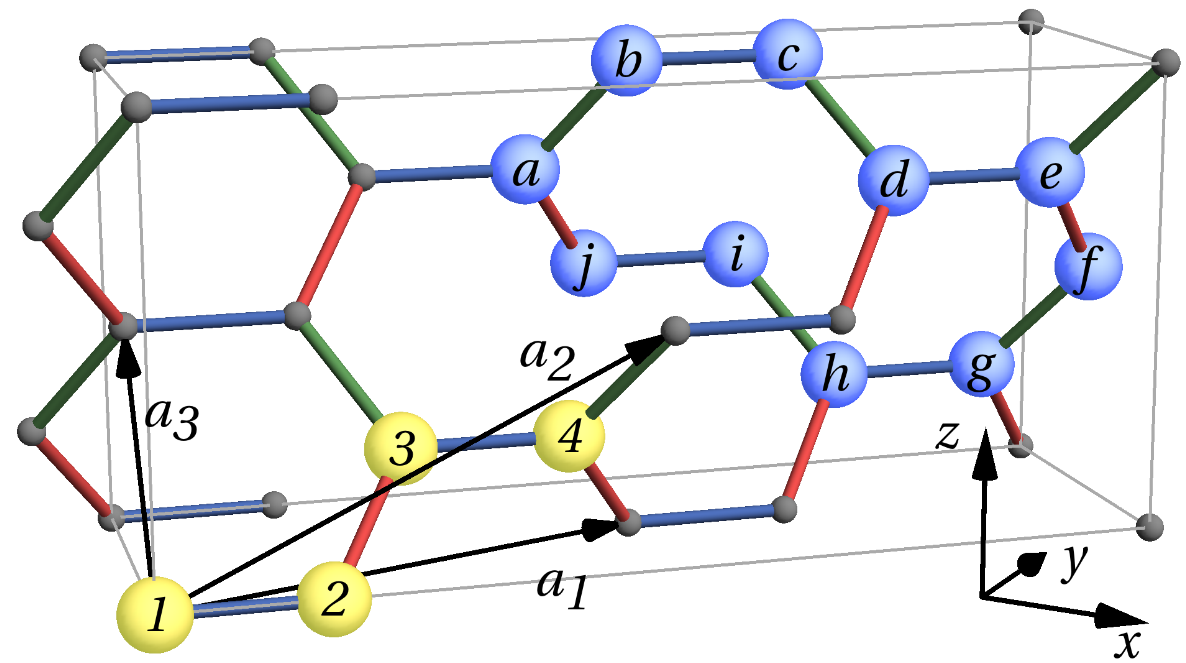
arXiv:1403.3296

H. Takagi

arXiv:1408.0246

P. Gegenwart

Hyper-Honeycomb



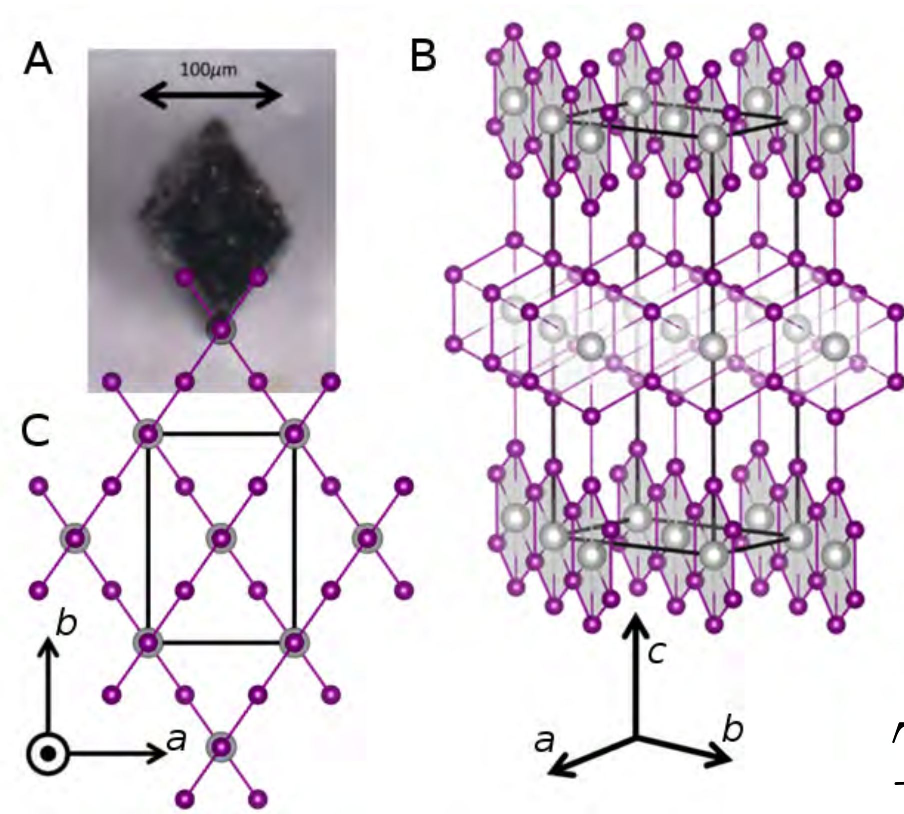

γ- $Li_2IrO_3$

arXiv:1402.3254

James Analytis

Radu Coldea

Stripy-Honeycomb




$T_c = 38K$

# Discovery of Three dimensional “Honeycomb” lattice

$\beta$- $Li_2IrO_3$

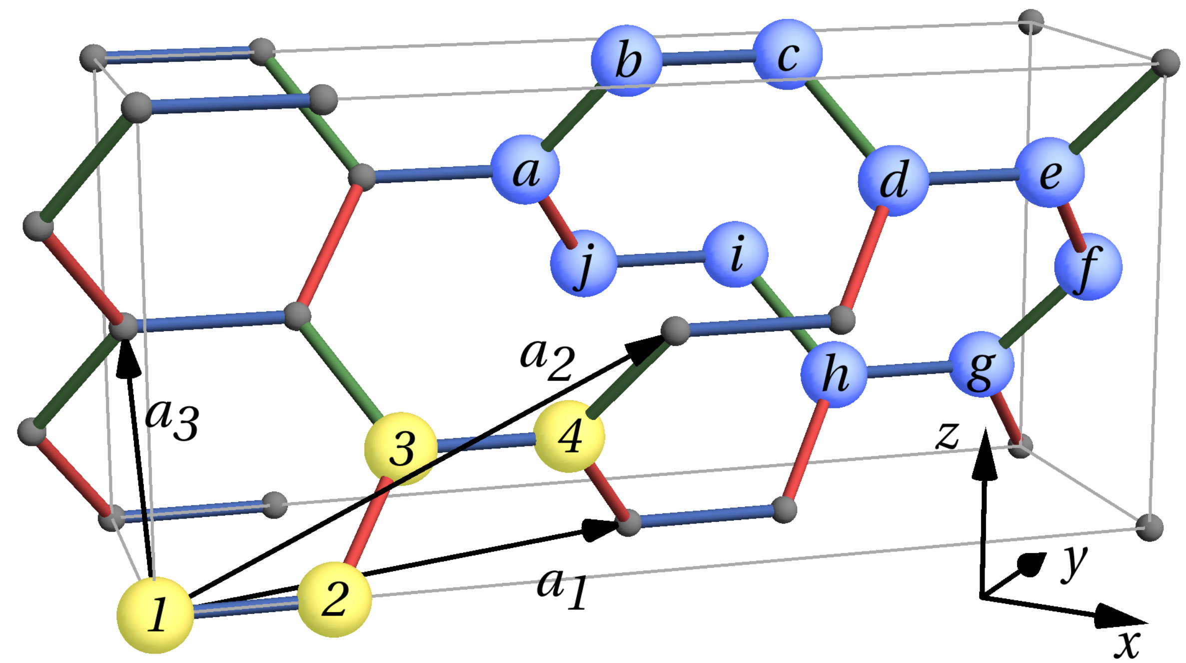
arXiv:1403.3296

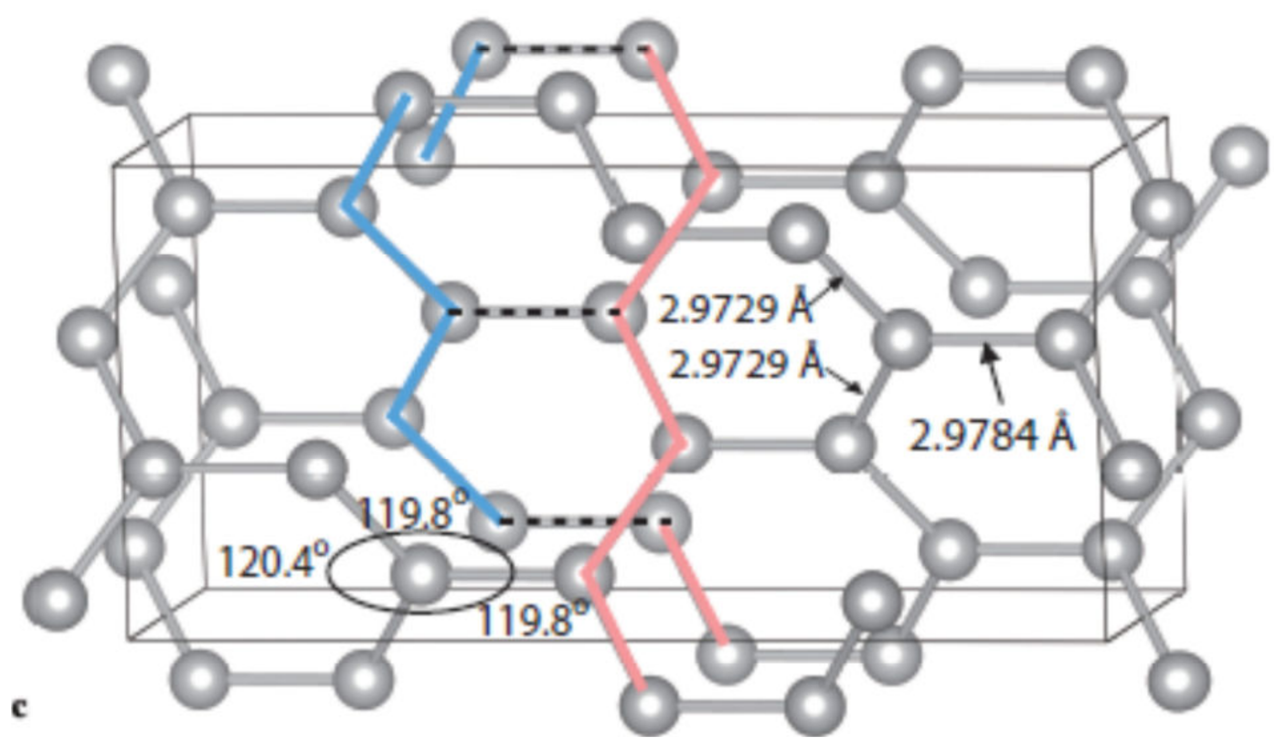
H. Takagi

arXiv:1408.0246

P. Gegenwart

Hyper-Honeycomb

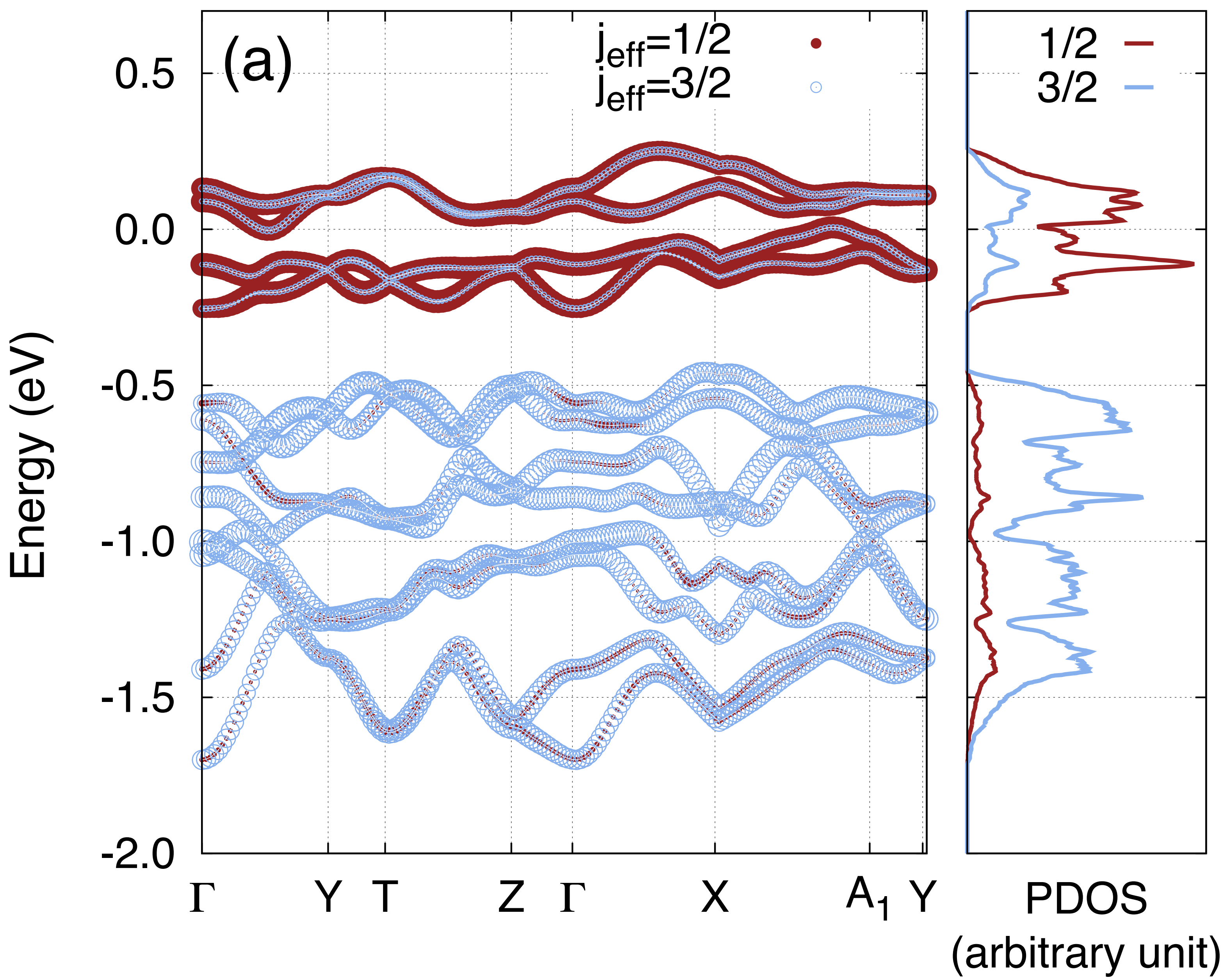



Close to ideal structure

# Is $J_{eff}$=1/2 picture valid ?

## β- $Li_2IrO_3$



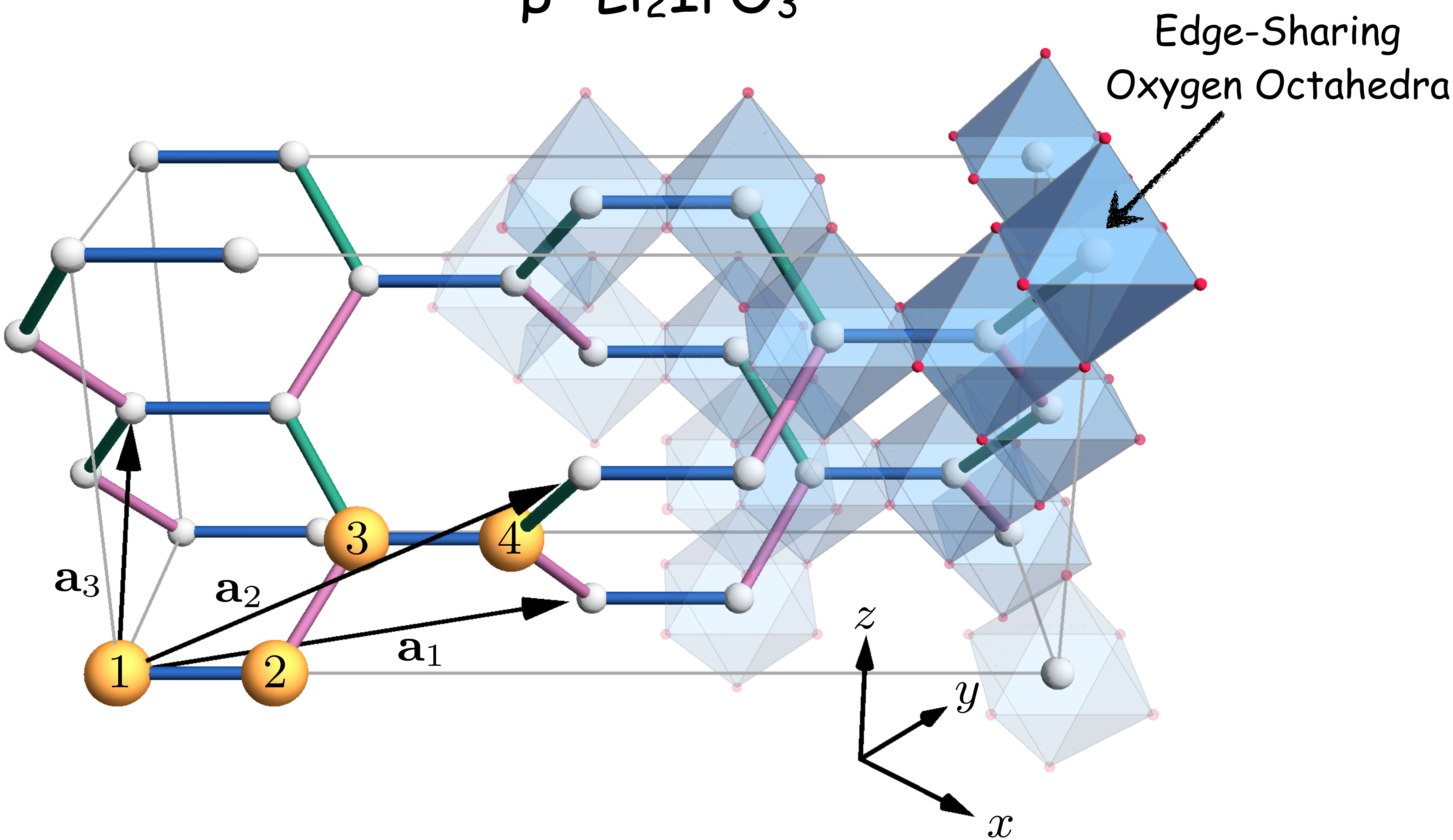

E. K.-H. Lee,
S. Bhattacharjee,
K. Hwang, H.-S. Kim,
H. Jin, Y. B. Kim,
arXiv:1402.2654 (2014)

H.-S.Kim, E. K.-H. Lee,
Y. B. Kim,
arXiv:1502.00006 (2015)

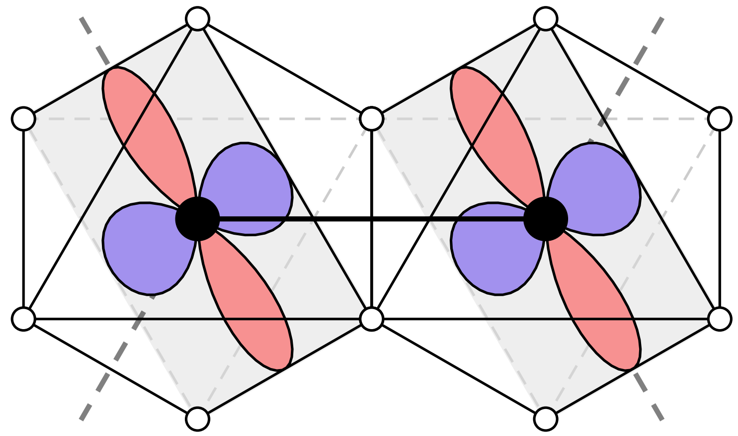
Ab Initio
(OpenMx, PBE-GGA)

# Edge-sharing Oxygen Octahedra Structure

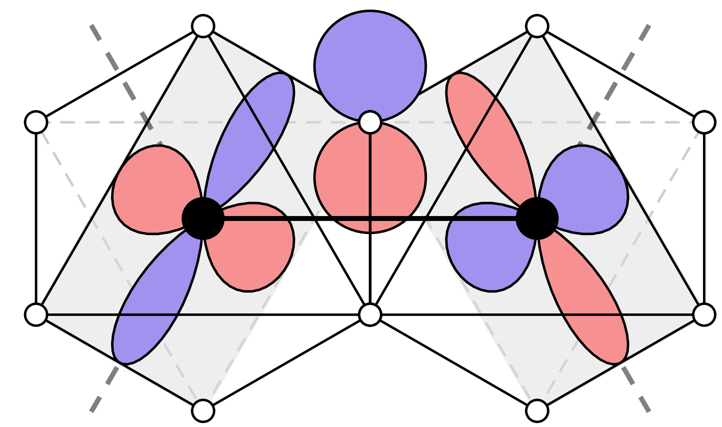
## β- $Li_2IrO_3$



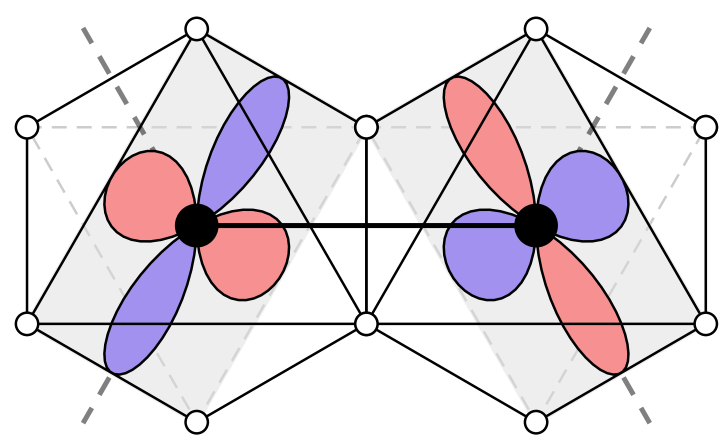
# Strong Coupling Limit: Localized Pseudo-Spin Model



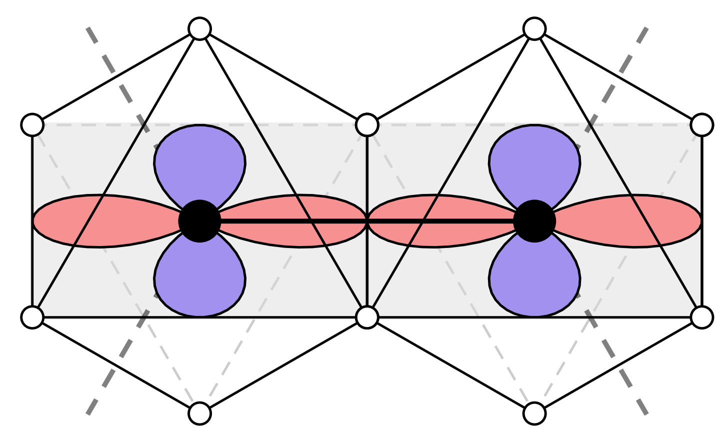
(a) Ir-Ir overlap for $t_1$



(b) Ir-O-Ir overlap for $t_2$



(c) Ir-Ir overlap for $t_2$



(d) Ir-Ir overlap for $t_3$

$$t_1 = \frac{t_{dd\pi} + t_{dd\delta}}{2}, \quad t_2 = \frac{t_{pd\pi}^2}{\Delta_{pd}} + \frac{t_{dd\pi} - t_{dd\delta}}{2}, \quad t_3 = \frac{3t_{dd\sigma} + t_{dd\delta}}{4},$$

# Strong Coupling Limit: Localized Pseudo-Spin Model

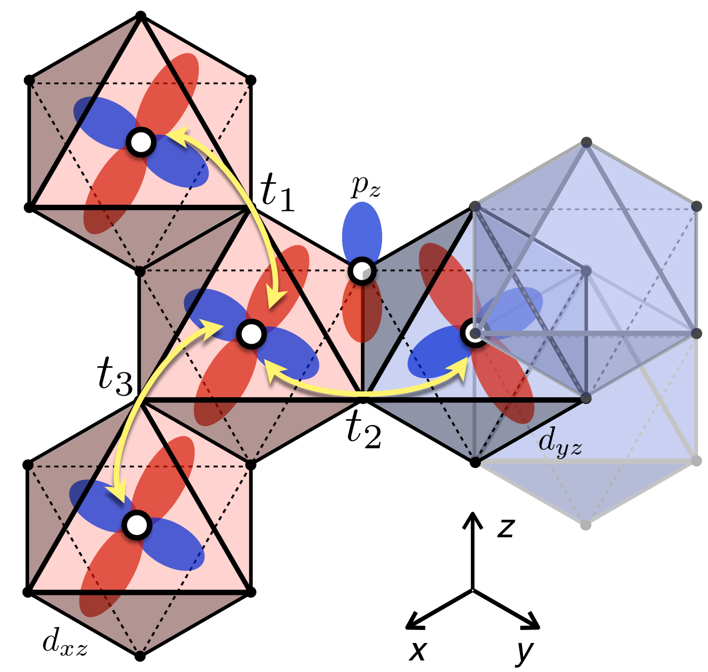
$$H = \sum_{\langle ij \rangle \in \alpha\beta(\gamma)} \left[ J \vec{S}_i \cdot \vec{S}_j + K S_i^\gamma S_j^\gamma + \boxed{\Gamma \left( S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \right) } \right]$$

e.g. In the limit of $U, J_H \gg \lambda \gg t$

$$J = \frac{4}{27} \left[ \frac{6t_1(t_1 + 2t_3)}{U - 3J_H} + \frac{2(t_1 - t_3)^2}{U - J_H} + \frac{(2t_1 + t_3)^2}{U + 2J_H} \right]$$

$$K = \frac{8J_H}{9} \left[ \frac{(t_1 - t_3)^2 - 3t_2^2}{(U - 3J_H)(U - J_H)} \right],$$

$$\boxed{\Gamma = \frac{16J_H}{9} \left[ \frac{t_2(t_1 - t_3)}{(U - 3J_H)(U - J_H)} \right].}$$




$$t_1 = \frac{t_{dd\pi} + t_{dd\delta}}{2}, \quad t_2 = \frac{t_{pd\pi}^2}{\Delta_{pd}} + \frac{t_{dd\pi} - t_{dd\delta}}{2}, \quad t_3 = \frac{3t_{dd\sigma} + t_{dd\delta}}{4},$$

# Kitaev Model: Exact Solution

$$\mathcal{H}_{\rm K} = - \sum_{\alpha-links} S_i^\alpha S_j^\alpha$$

$S_i^\alpha = i b_i^\alpha c$ $\quad \{b_i^x, b_i^y, b_i^z, c\}$ **Four Majorana Fermions**

$$\mathcal{H}_{\rm K} = \frac{i}{2} \sum_{\alpha-\text{links}} u_{ij}^\alpha c_i c_j \qquad (\text{where } \; u_{ij}^\alpha = i b_i^\alpha b_j^\alpha),$$

$\mathcal{W}_P = \prod_{\text{loop}} u_{ij}^\alpha$ commute with the Hamiltonian $\Longrightarrow$ $\mathcal{W}_P = \pm 1$

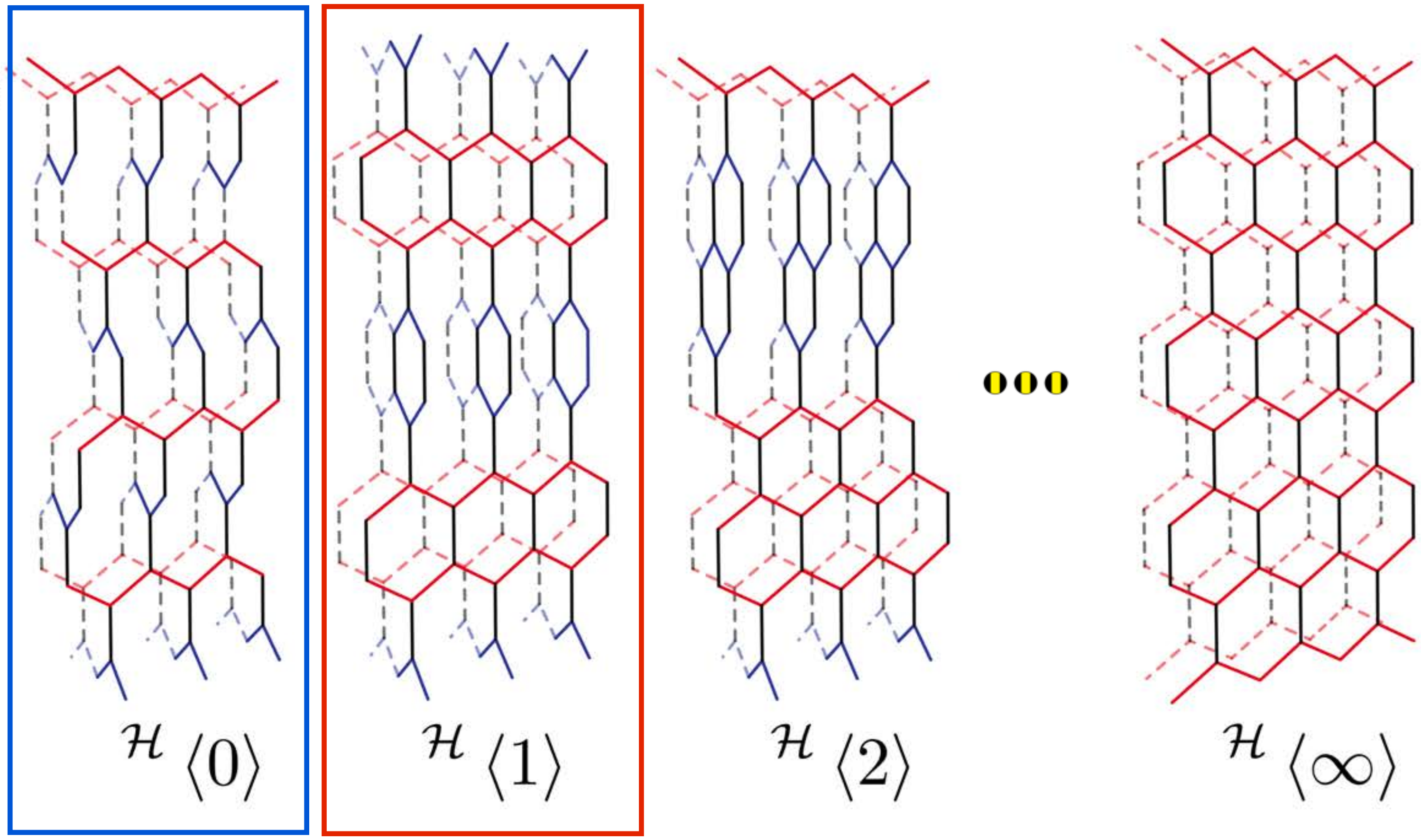
**Hyper-Honeycomb** S. Mandal and N. Surendran (2009)

Ground state is in the zero-flux sector $u_{ij}^\alpha = +1 \, (\forall \langle ij \rangle)$

**Stripy-Honeycomb** R. Schaffer, E. Lee, Y.-M. Lu, Y. B. Kim, PRL 2015

Ground state is in a π-flux sector some $\mathcal{W}_p = -1$

Hyper-Honeycomb Stripy-Honeycomb




$$\mathbf{H}_n^\Phi = \sum_k \vec{c}_{n,-k}^{\,T} H_{n,k}^\Phi \vec{c}_{n,k}$$

$$H_{n,k}^\Phi = \begin{bmatrix} 0 & -iD_{n,k}^\Phi \\ i\left(D_{n,k}^\Phi\right)^\dagger & 0 \end{bmatrix}$$

BDI Class

$\rightarrow$

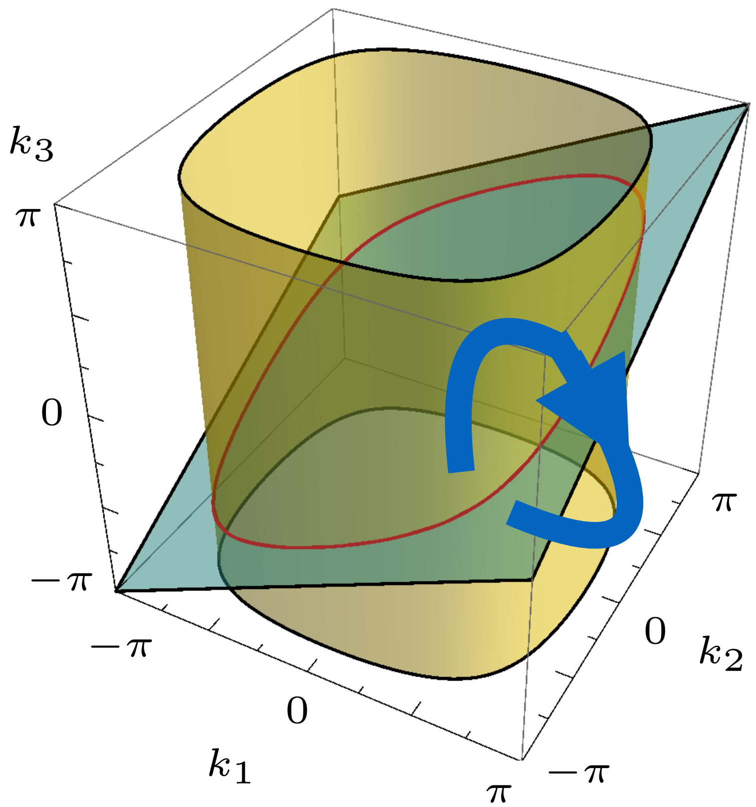
Gapless Modes determined by two equations

$$\mathrm{Re}\left[\det(D_k)\right] = 0$$
$$\mathrm{Im}\left[\det(D_k)\right] = 0$$

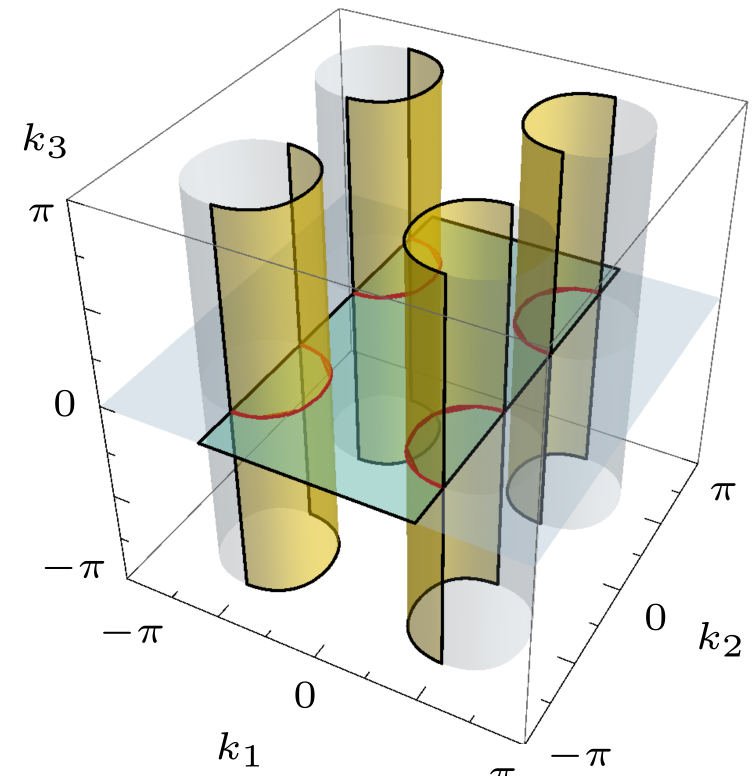
Nodal Line in 3D

robust to small short range interactions

## 0-FLUX SECTOR
### HYPERHONEYCOMB




## π-FLUX SECTOR
### STRIPYHONEYCOMB

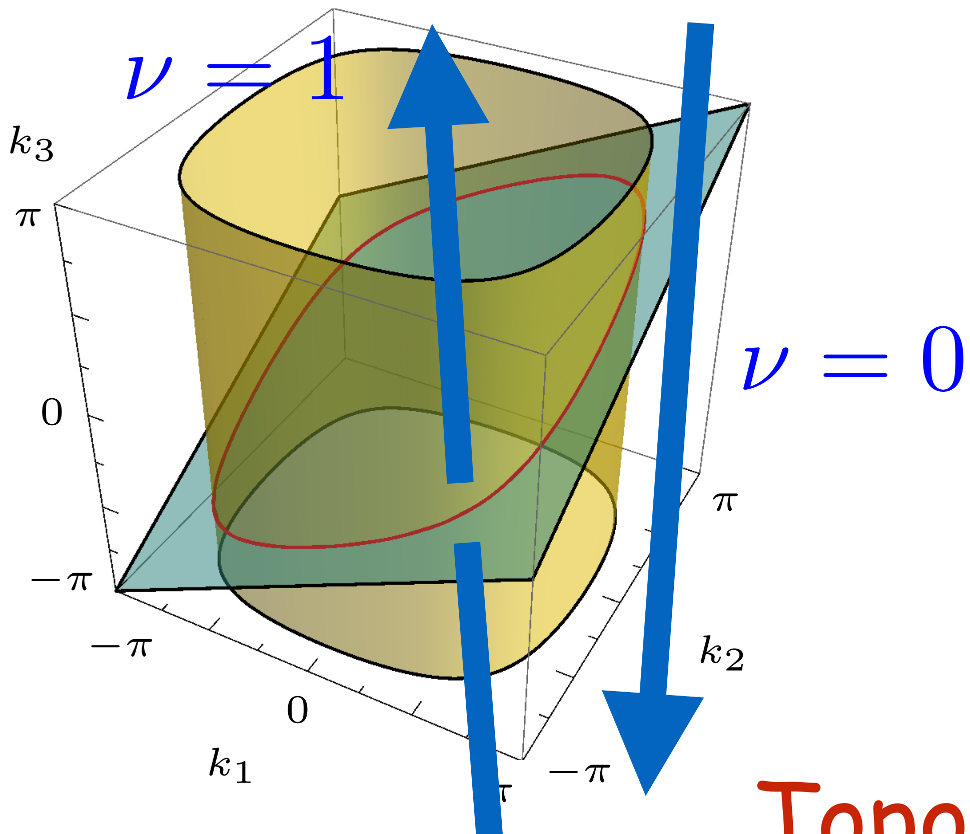



Topological
Spinon “Semi-metal”

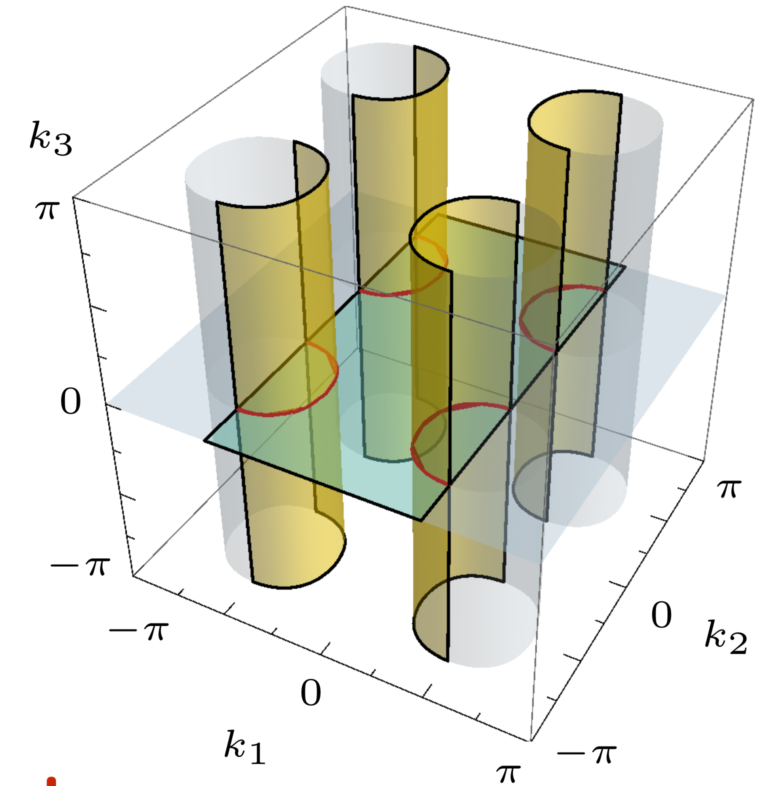
$$\nu = \frac{1}{4\pi i}\oint \mathrm{d}k \mathrm{Tr}[D_{\mathbf{k}}^{-1}\partial_k D_{\mathbf{k}} - (D^{\dagger})_{\mathbf{k}}^{-1}\partial_k D_{\mathbf{k}}^{\dagger}],$$ Topological Invariant

R. Schaffer, E. Lee, Y.-M. Lu, Y. B. Kim, PRL 2015

## 0-FLUX SECTOR
### HYPERHONEYCOMB




## $\pi$-FLUX SECTOR
### STRIPYHONEYCOMB



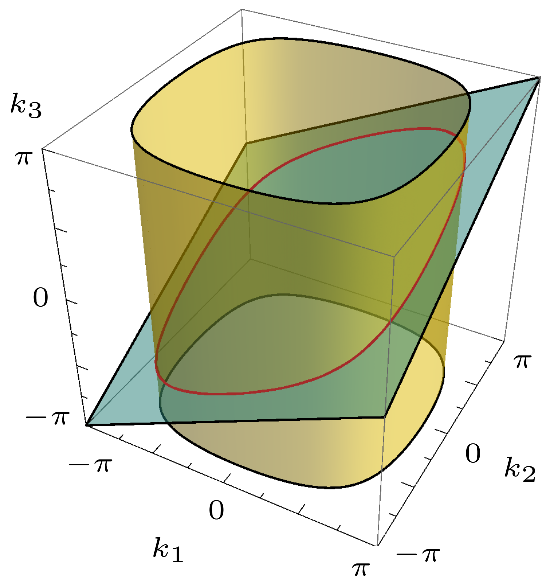

Topological
Spinon "Semi-metal"

$$\nu = \frac{1}{4\pi i} \oint \mathrm{d}k \mathrm{Tr}[D_{\mathbf{k}}^{-1} \partial_k D_{\mathbf{k}} - (D^\dagger)_{\mathbf{k}}^{-1} \partial_k D_{\mathbf{k}}^\dagger]$$

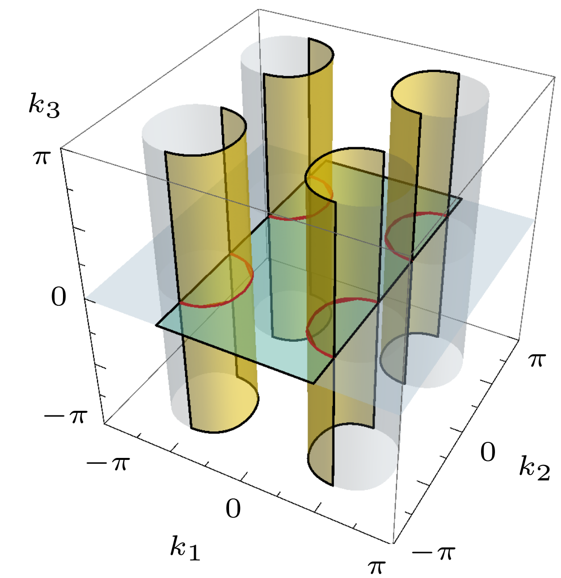
Topological Invariant

R. Schaffer, E. Lee, Y.-M. Lu, Y. B. Kim, PRL 2015

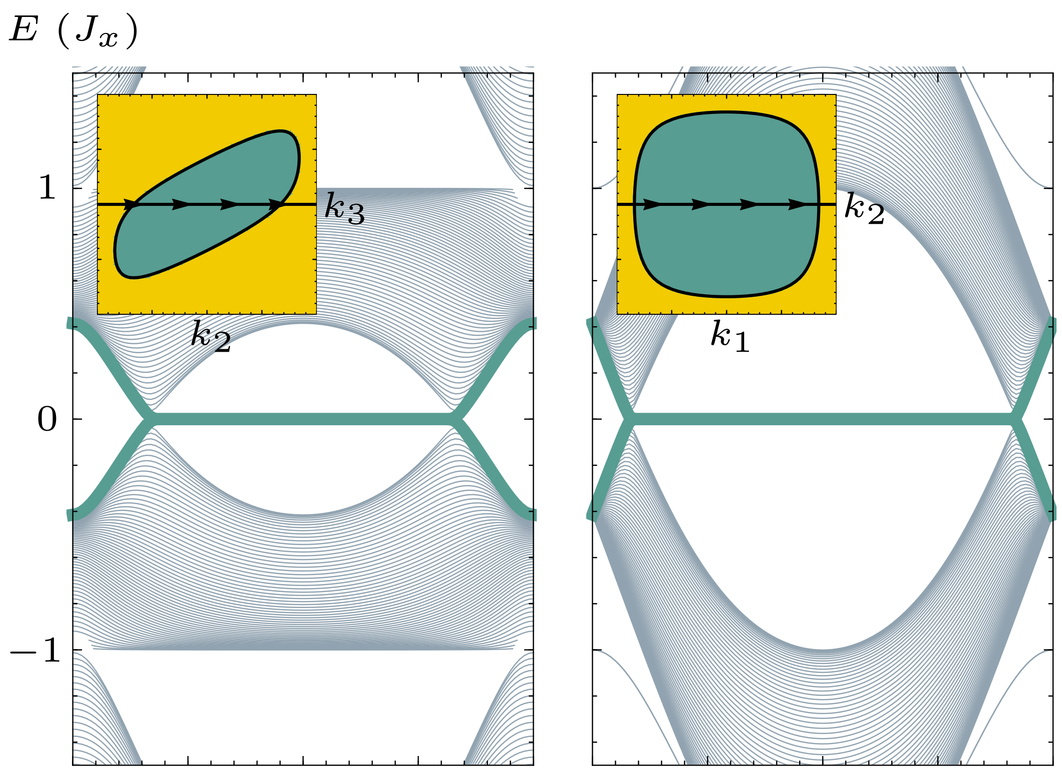
## 0-FLUX SECTOR
### HYPERHONEYCOMB




## π-FLUX SECTOR
### STRIPYHONEYCOMB

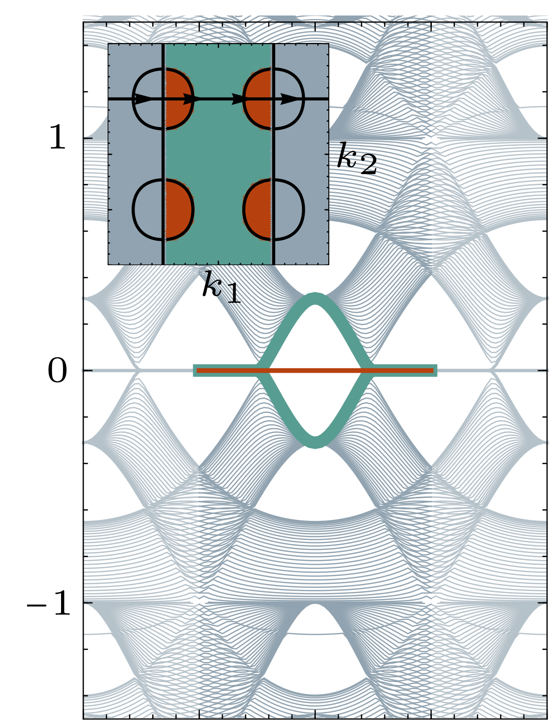



# Surface Majorana States




(d) (100) surface

(e) (001) surface



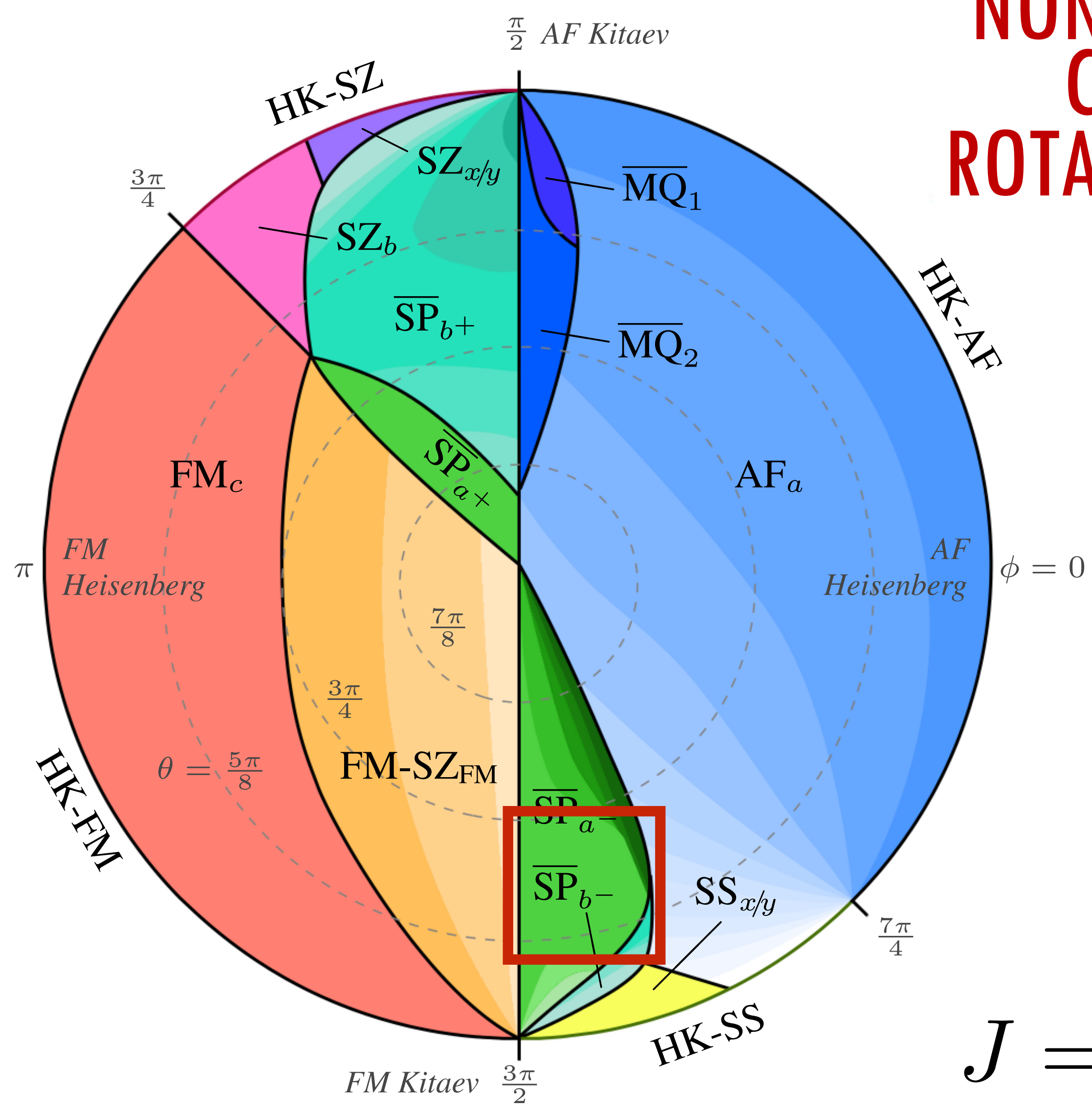

(g) (001) surface; $\delta = 1$

# Classical Model

Eric K.-H. Lee, Y. B. Kim (2014)

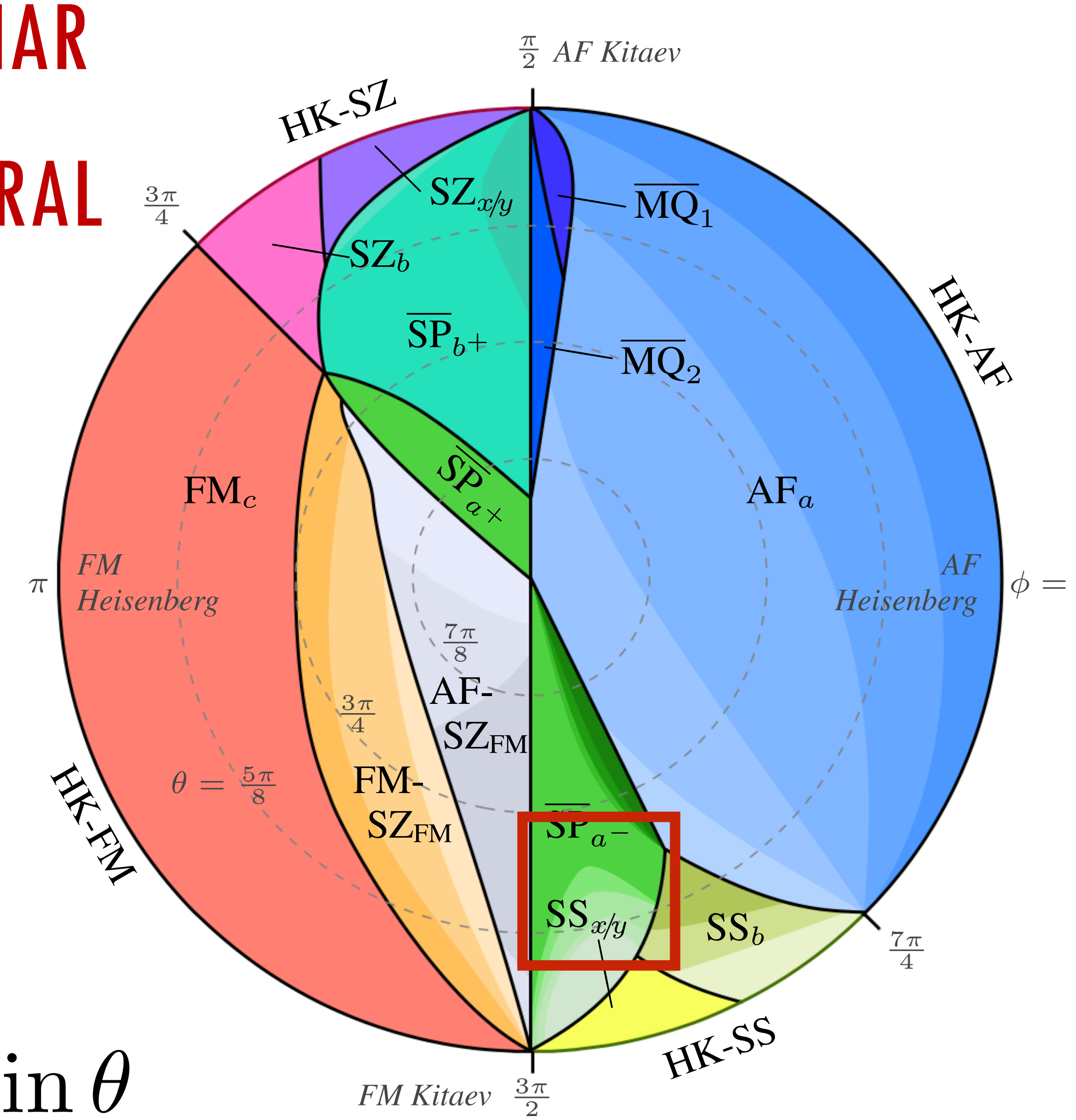
$$H = \sum_{\langle ij\rangle \in \alpha(\beta\gamma)} J S_i \cdot S_j + K S_i^\alpha S_j^\alpha + \Gamma^\alpha (S_i^\beta S_j^\gamma + S_i^\gamma S_j^\beta)$$

INCOMMENSURATE NON-COPLANAR COUNTER-ROTATING SPIRAL




HYPERHONEYCOMB

$$J = \cos\phi \sin\theta$$
$$K = \sin\phi \sin\theta$$
$$\Gamma = \cos\theta$$



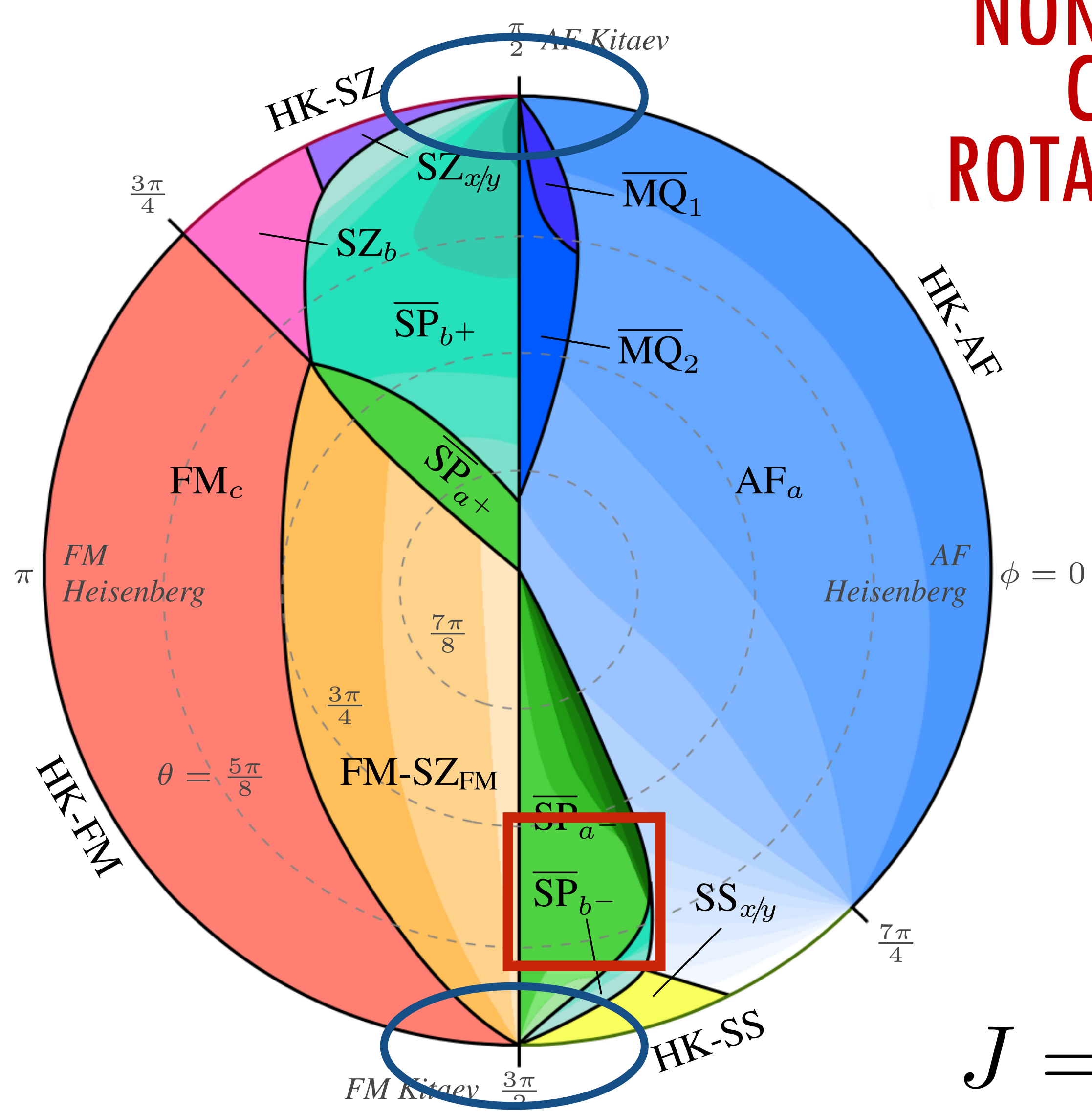

STRIPYHONEYCOMB

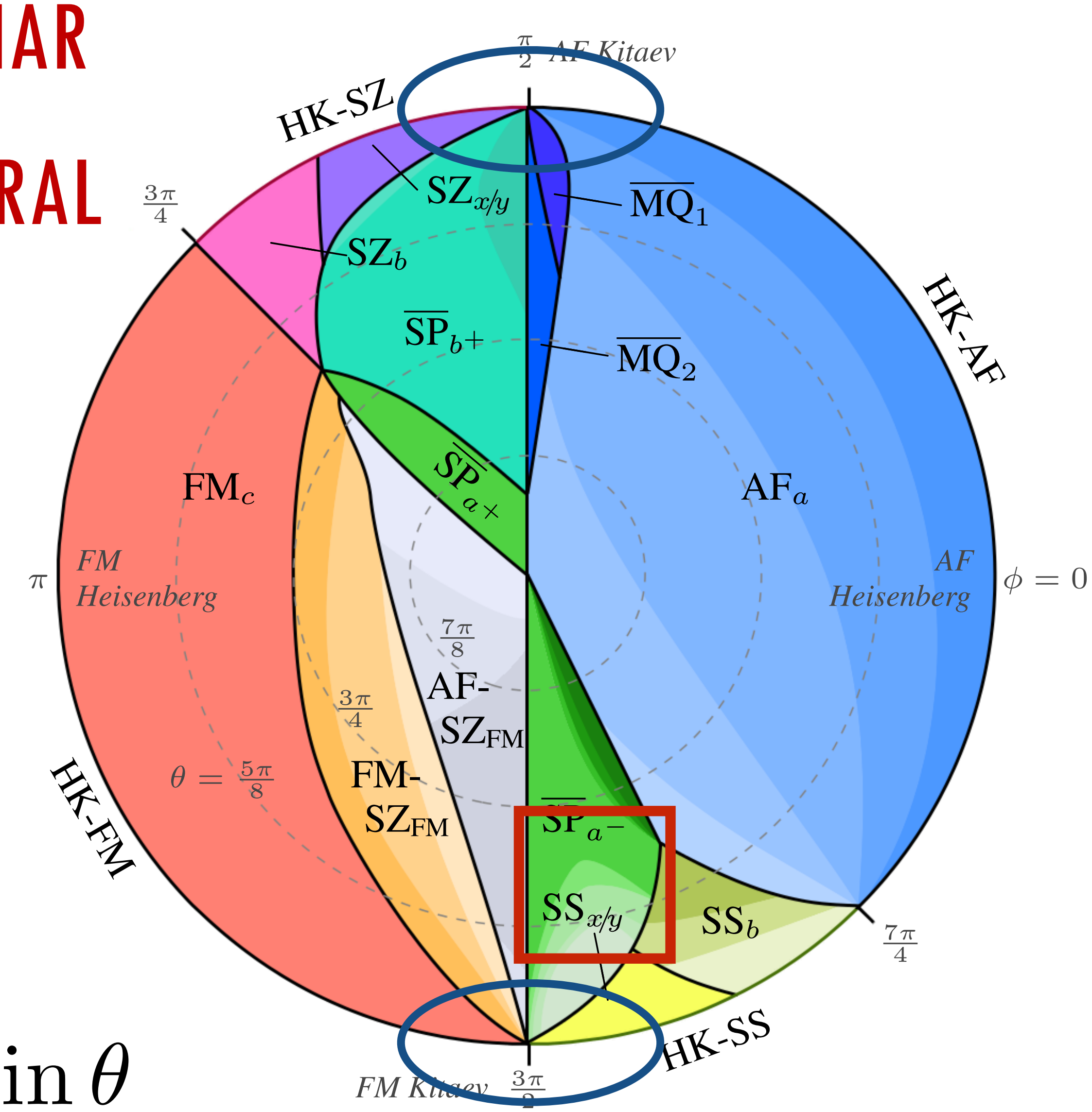
# Classical Model

Eric K.-H. Lee, Y. B. Kim (2014)

$$H = \sum_{\langle ij \rangle \in \alpha(\beta\gamma)} J S_i \cdot S_j + K S_i^\alpha S_j^\alpha + \Gamma^\alpha (S_i^\beta S_j^\gamma + S_i^\gamma S_j^\beta)$$

INCOMMENSURATE
NON-COPLANAR
COUNTER-
ROTATING SPIRAL




HYPERHONEYCOMB

$$J = \cos\phi \sin\theta$$
$$K = \sin\phi \sin\theta$$
$$\Gamma = \cos\theta$$

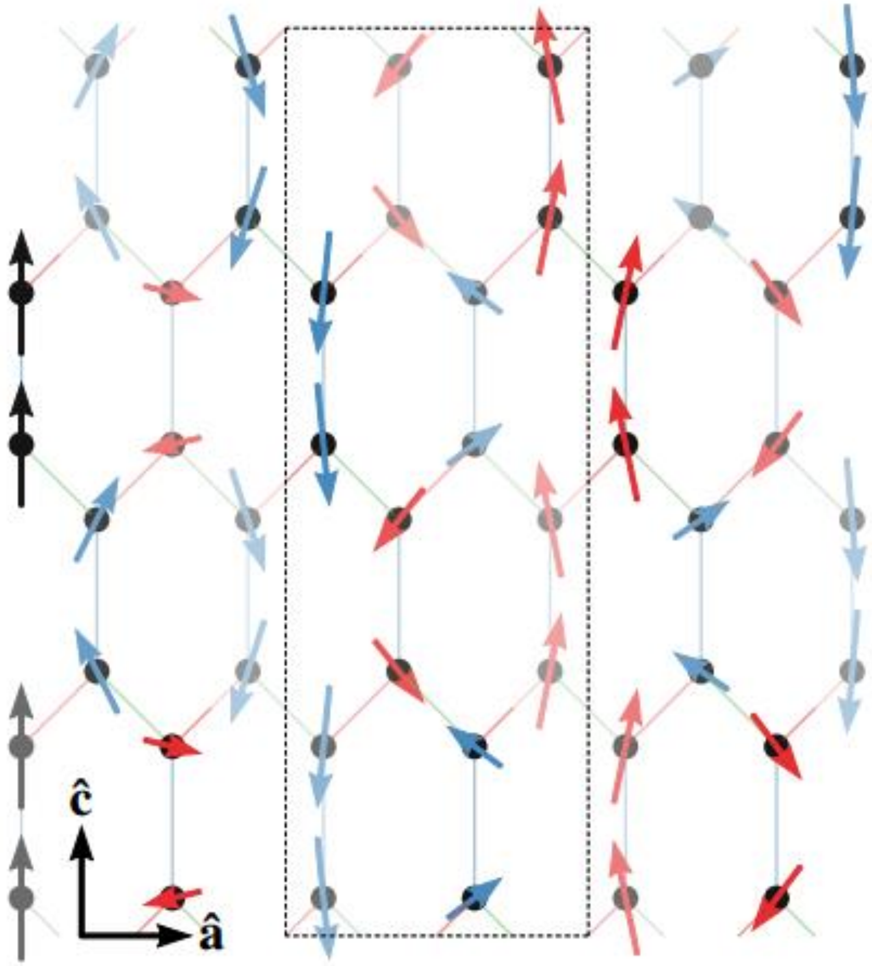
STRIPYHONEYCOMB

$\beta$-$Li_2IrO_3$

# Counter-rotating Incommensurate Non-Coplanar Spiral Orders

EXPERIMENT

Radu Coldea,
P. Gegenwart (2014)




$$\mathbf{Q}_{\text{exp}} = (0.57, 0, 0)$$

$$(iA_a, iC_b, F_c)$$

$$F = [+, +, +, +]$$
$$C = [+, +, -, -]$$
$$G = [+, -, +, -]$$
$$A = [+, -, -, +]$$

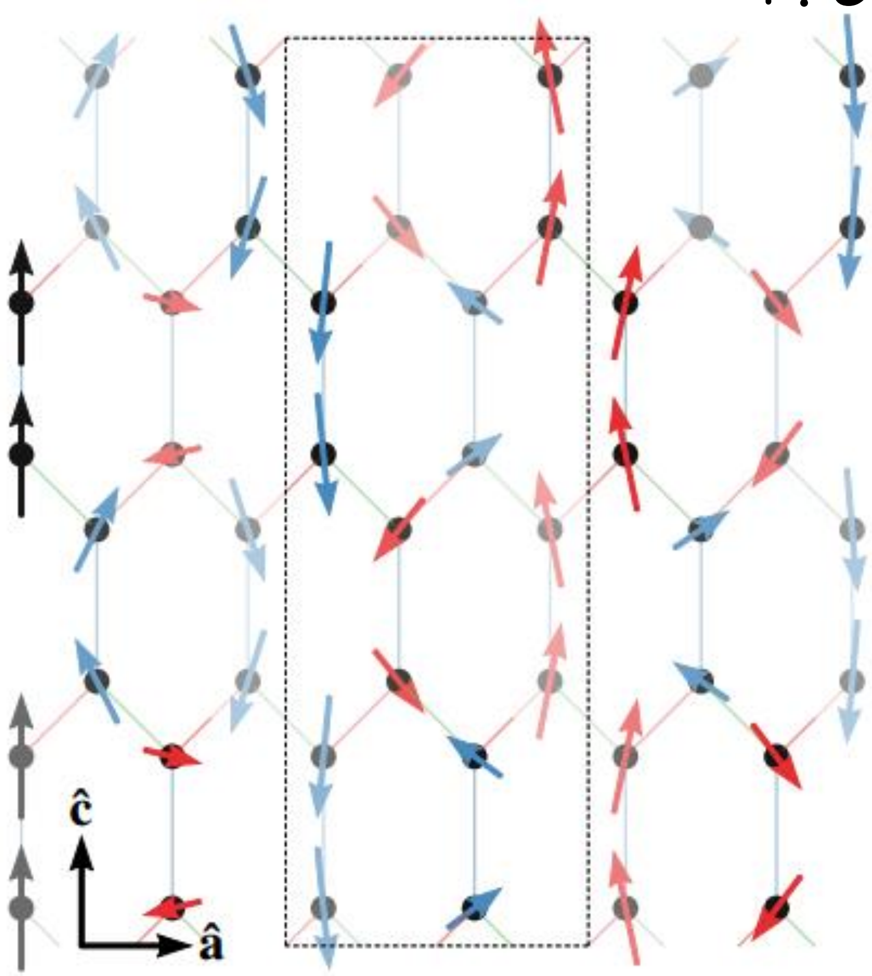
| Irreducible Representation | Basis Vectors |
|---|---|
| $\Gamma_1$ | $F_x, G_y, A_z$ |
| $\Gamma_2$ | $C_x, A_y, G_z$ |
| $\Gamma_3$ | $G_x, F_y, C_z$ |
| $\Gamma_4$ | $A_x, C_y, F_z$ |

$\beta$-$Li_2IrO_3$

# Counter-rotating Incommensurate Non-Coplanar Spiral Orders

Radu Coldea, P. Gegenwart (2014)

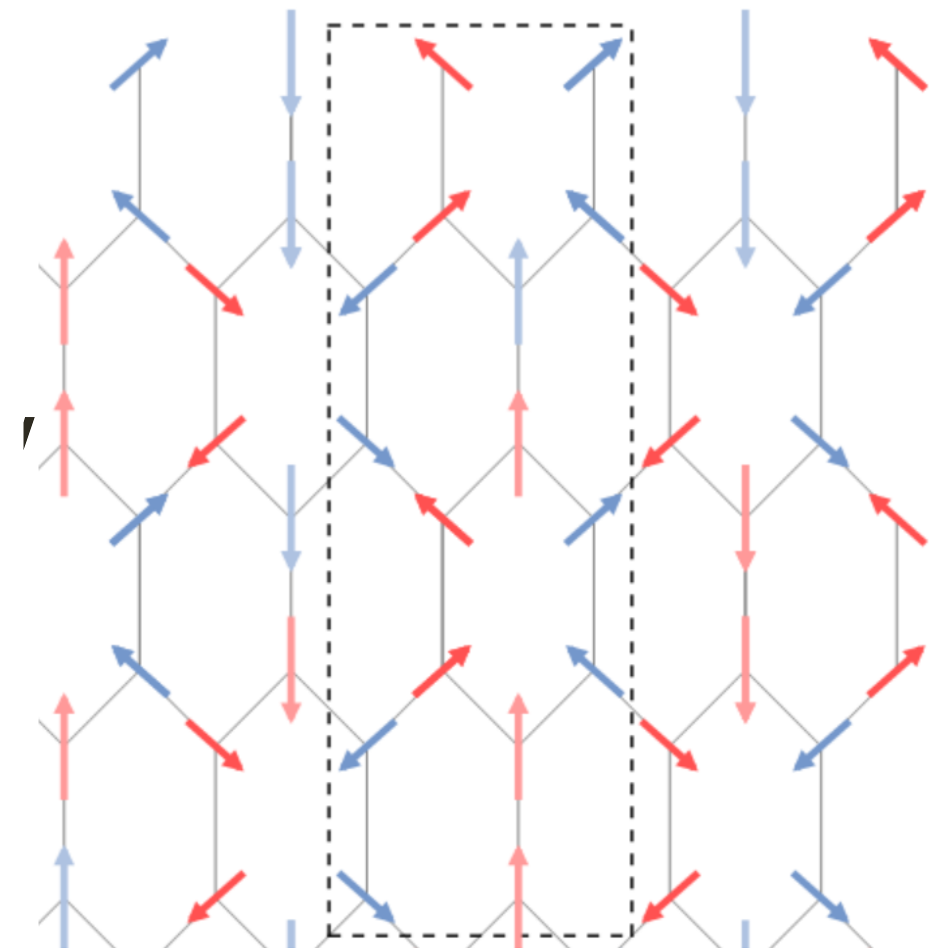
EXPERIMENT




$$\mathbf{Q}_{\text{exp}} = (0.57, 0, 0)$$

$$(iA_a, iC_b, F_c)$$

THEORY

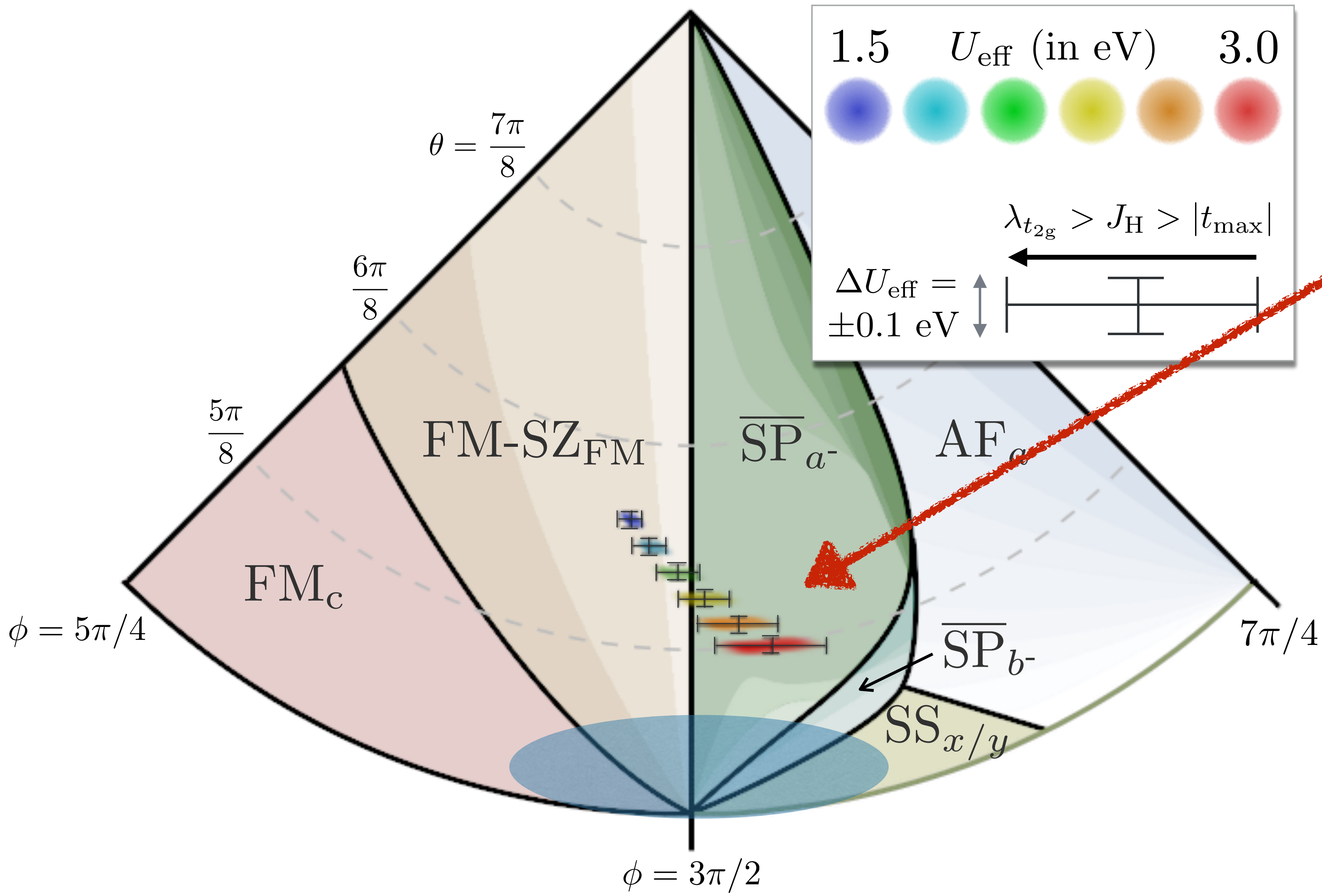


$$\boxed{\mathbf{Q} = (h'00) \quad 0.53 \lesssim h' \lesssim 0.80}$$

$$(iA_a, iC_b, F_c)$$

# β-$Li_2IrO_3$

$$H=\sum_{\langle ij\rangle\in\alpha\beta(\gamma)}\left[J\vec{S}_i\cdot\vec{S}_j+KS_i^{\gamma}S_j^{\gamma}+\Gamma\left(S_i^{\alpha}S_j^{\beta}+S_i^{\beta}S_j^{\alpha}\right)\right]$$




$(iA_a, iC_b, F_c)$

Counter-rotating
Incommensurate
Non-Coplanar
Spiral Order

Ab Initio
(OpenMx, PBE-GGA)

Heung-Sik Kim, Eric K.-H. Lee, Y. B. Kim (2015)

# What did we learn ?

This is a benchmark test for Kitaev interaction

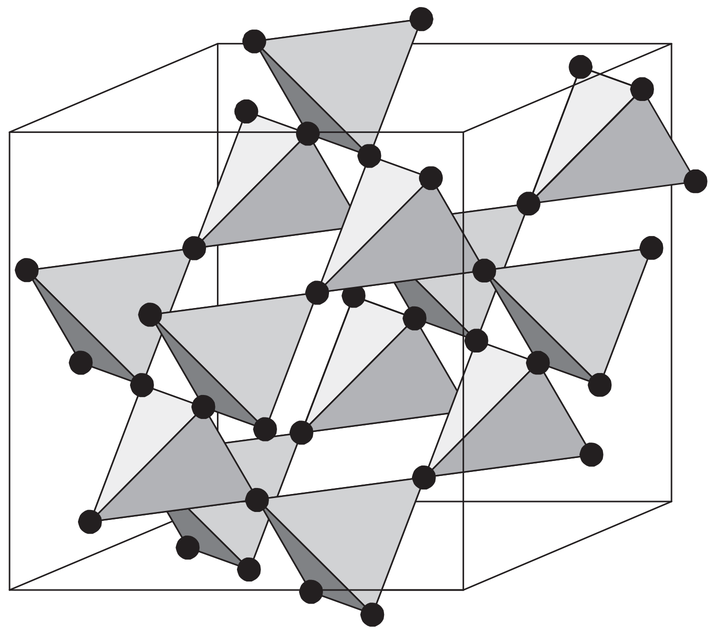
The Kitaev interaction may be dominant in the 3D materials,
irrespective of details: Highly Unusual Magnetic Order

High pressure (2.5GPa) on β (H. Takagi): Gapless Spin Liquid ?

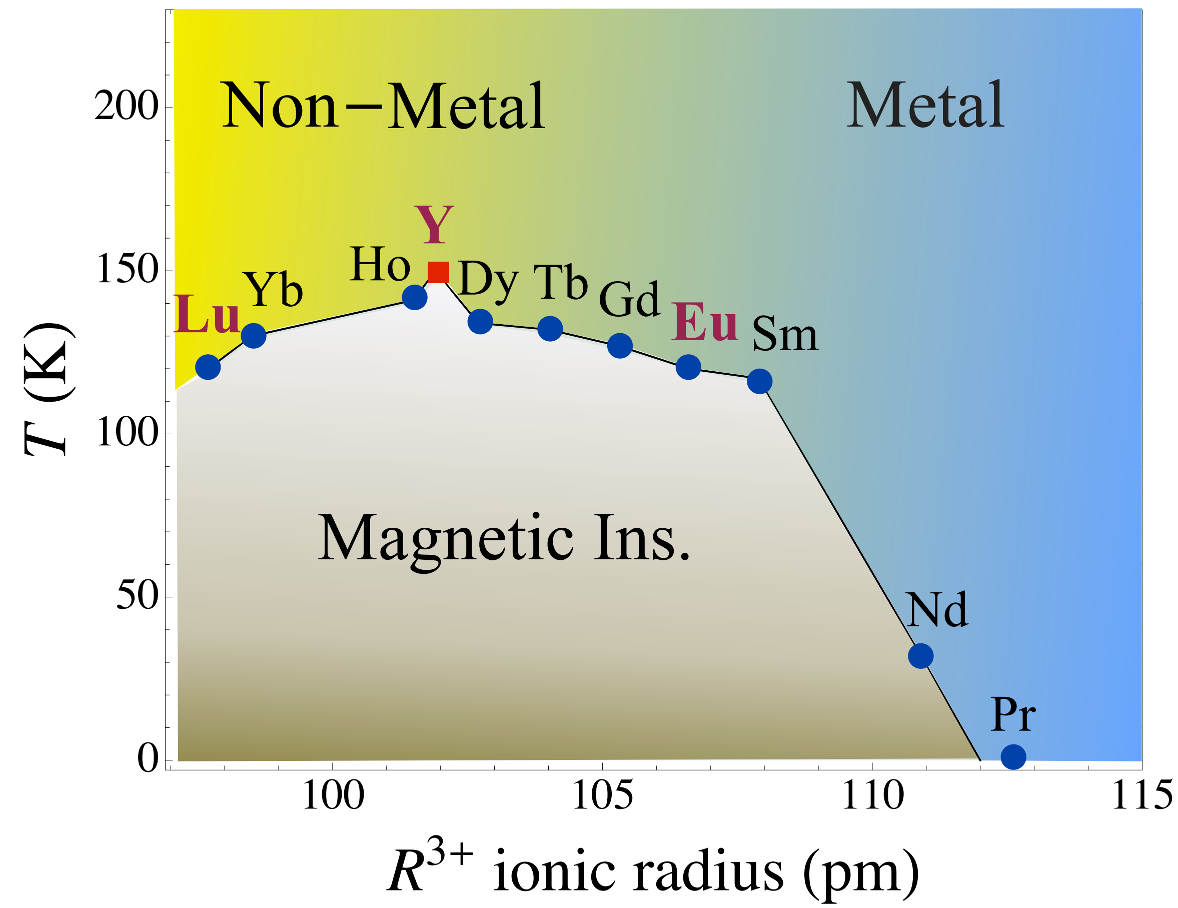
NMR and specific heat measurements (2015, unpublished)

# Pyrochlore Iridates: $A_2Ir_2O_7$

# Pyrochlore Iridates $R_2Ir_2O_7$



R = Y, Ho, Dy, Tb, Gd, Eu, Sm, Nd, Pr

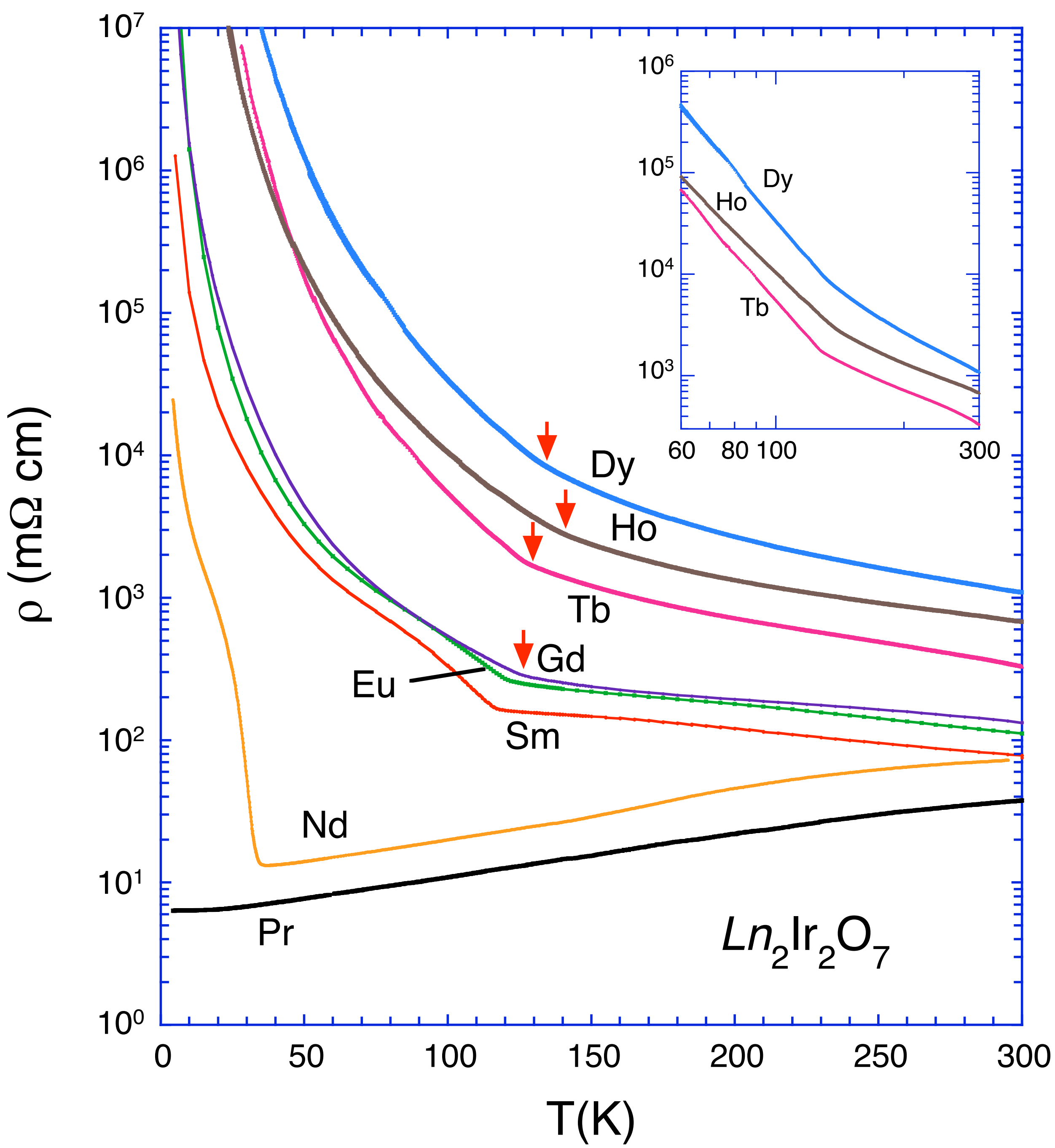



R=Y, Ln and Ir reside on the inter-penetrating two pyrochlore lattices (cubic, FCC Bravais lattice)

D. Yanagishima and Y. Maeno, JPSJ 70, 2880 (2001)

K. Matsuhira et al JPSJ 76, 043706 (2007)

# $A_2Ir_2O_7$ Metal to Insulator Transition ?




K. Matsuhira et al
JPSJ 76, 043706 (2007)

Also Earlier Data from
Y. Maeno's Group
(2001)

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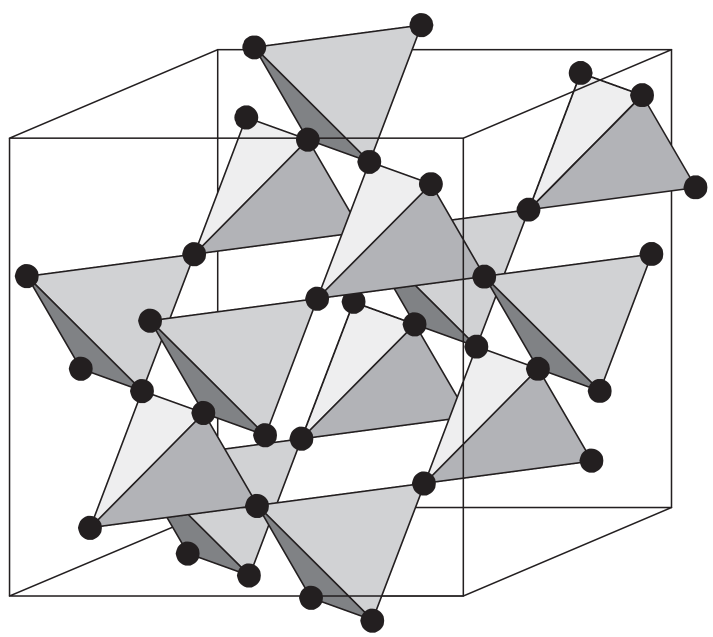

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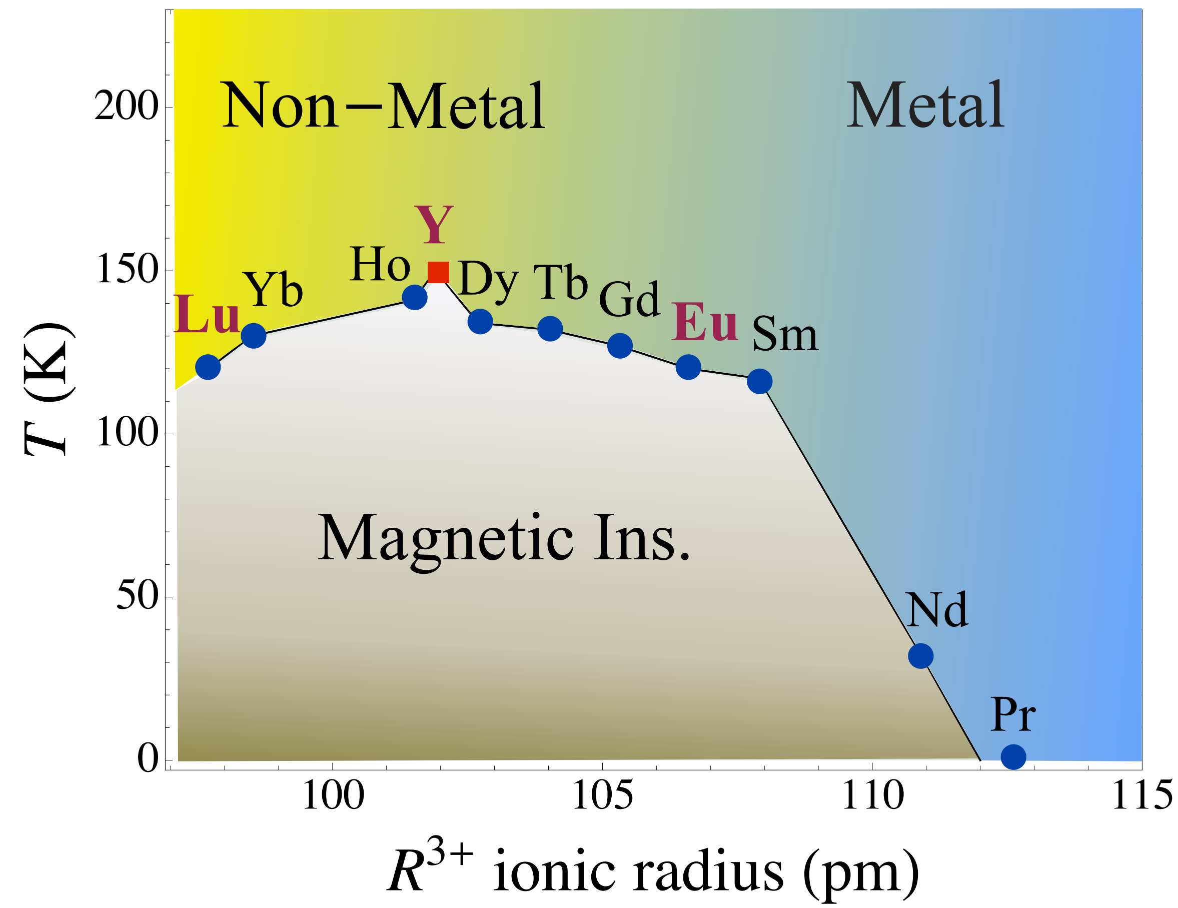
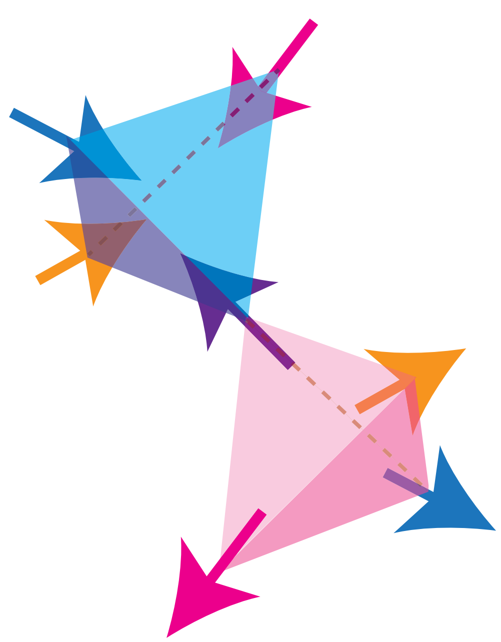


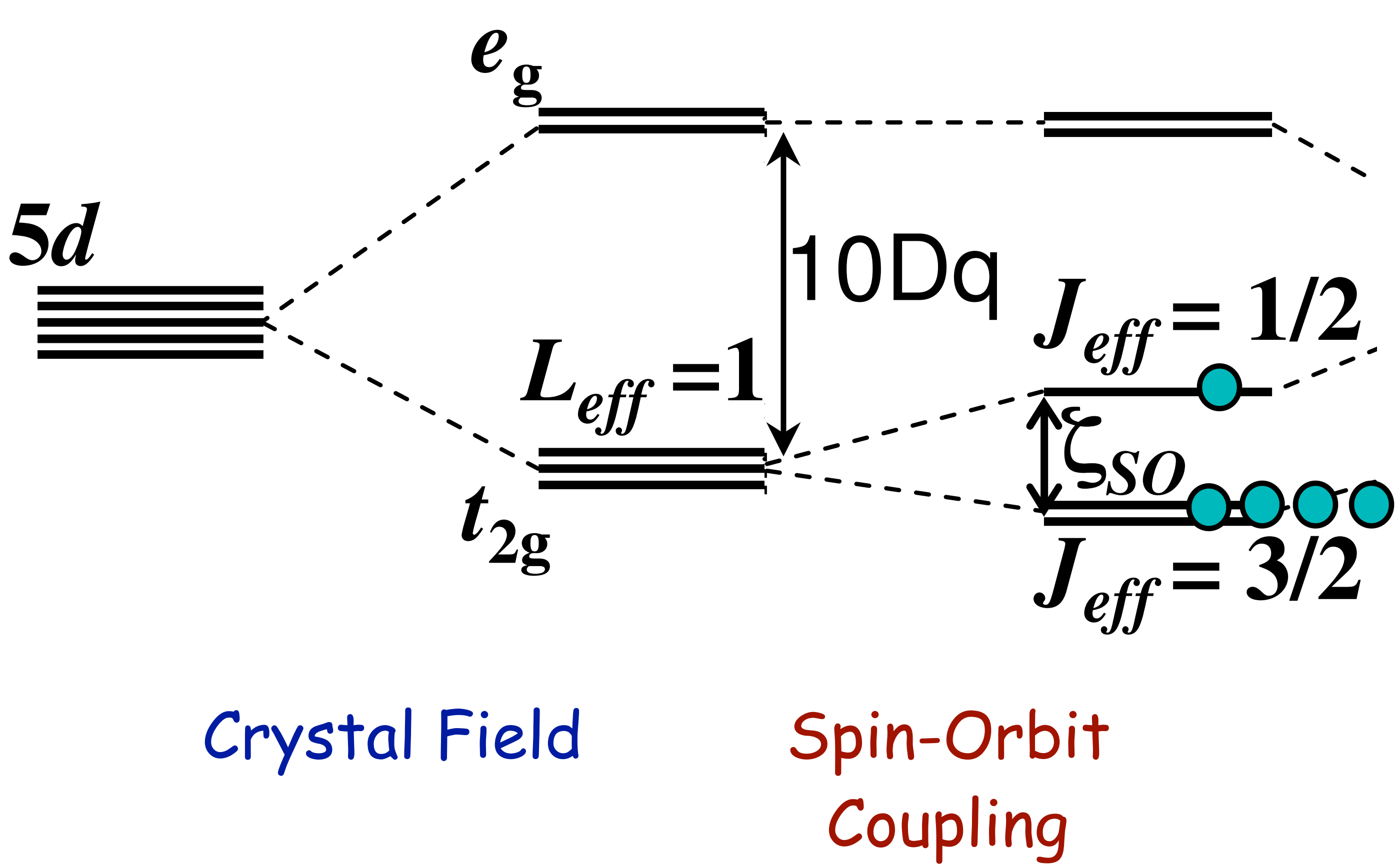



## All-in/All-out magnetic order

Satoru Nakatsuji
T. Arima
Y. Tokura

# 5d orbitals of $Ir^{4+}$: large spin-orbit coupling

$Ir^{4+}$ = [Xe] $4f^{14}$ $5d^5$




Construct
tight binding model
for $J_{eff}$=1/2

+

Hubbard U

$$|\uparrow_j\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_s\rangle + |yz,\downarrow_s\rangle + |xy,\uparrow_s\rangle)$$

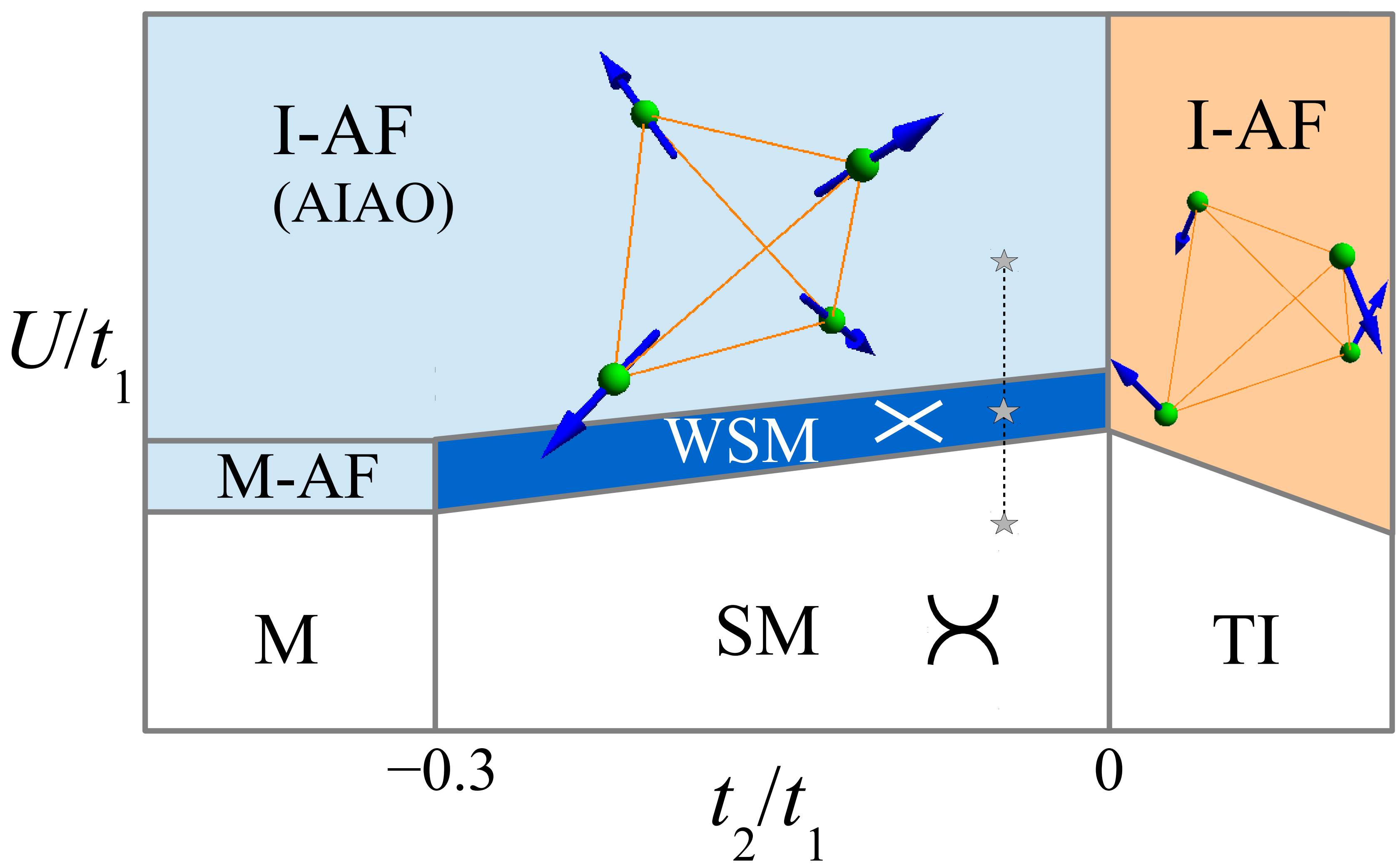
$$|\downarrow_j\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_s\rangle - |yz,\uparrow_s\rangle + |xy,\downarrow_s\rangle)$$

$$\mathcal{P}_{t2g}\mathbf{L}_{\ell=2}\mathcal{P}_{t2g} = -\mathbf{L}^{\mathrm{eff}}_{\ell=1}$$

B.J.Kim, T.W.Noh, G.Cao et al.
PRL (2008)

B.J.Kim, H.Takagi, et al,
Science (2009)

# Generic Phase Diagram

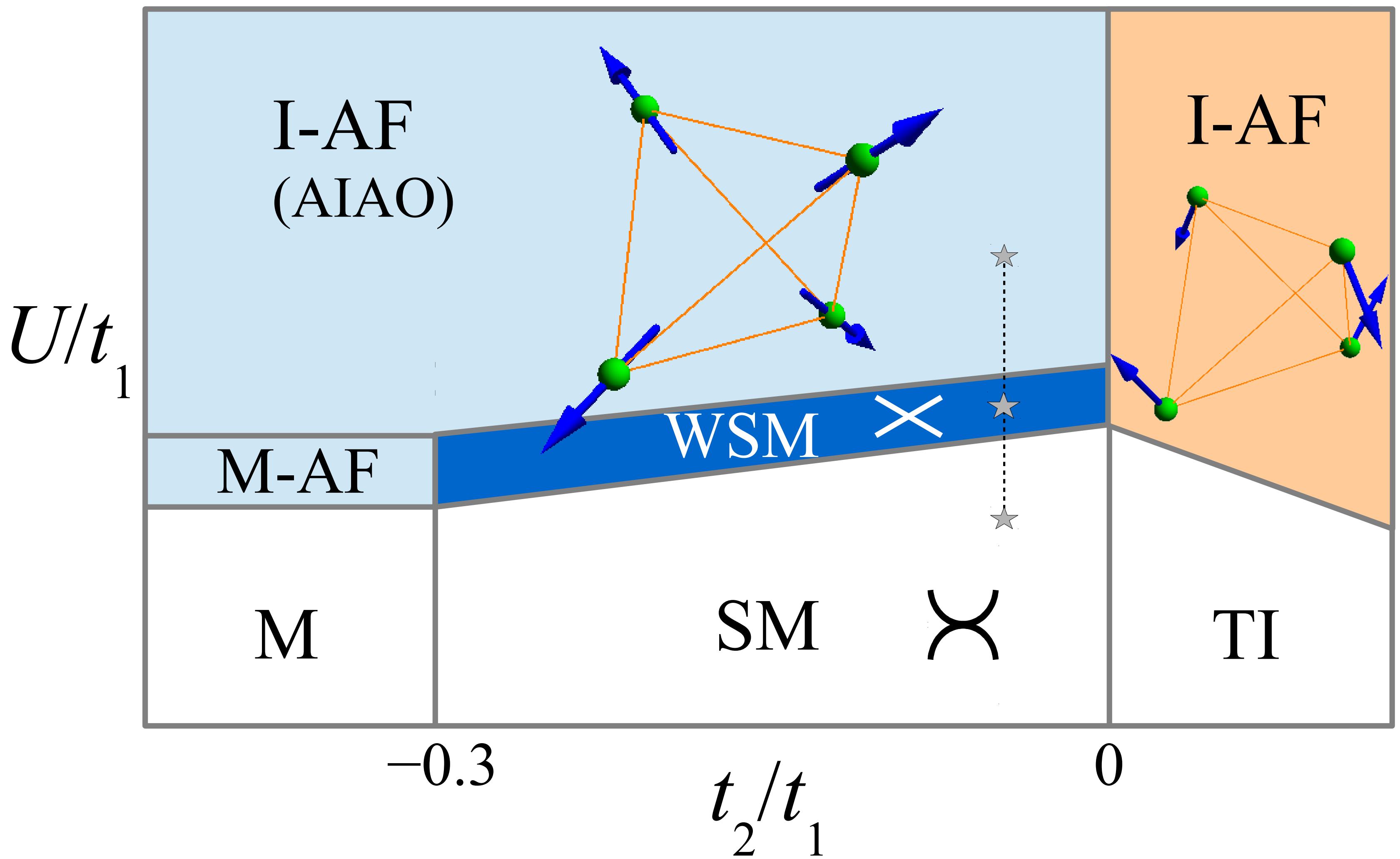



W. Witczak-Krempa,
Y. B. Kim
(2012)

W. Witczak-Krempa,
G. Chen
Y. B. Kim,
L. Balents
(2014)
Ann. Rev. Cond. Matt.

$$H_0 = \sum_{\langle i,j \rangle} c_i^\dagger (t_1 + i t_2 \boldsymbol{d}_{ij} \cdot \boldsymbol{\sigma}) c_j \qquad c_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger)$$ $J_{\rm eff}$ = 1/2 basis

# Generic Phase Diagram

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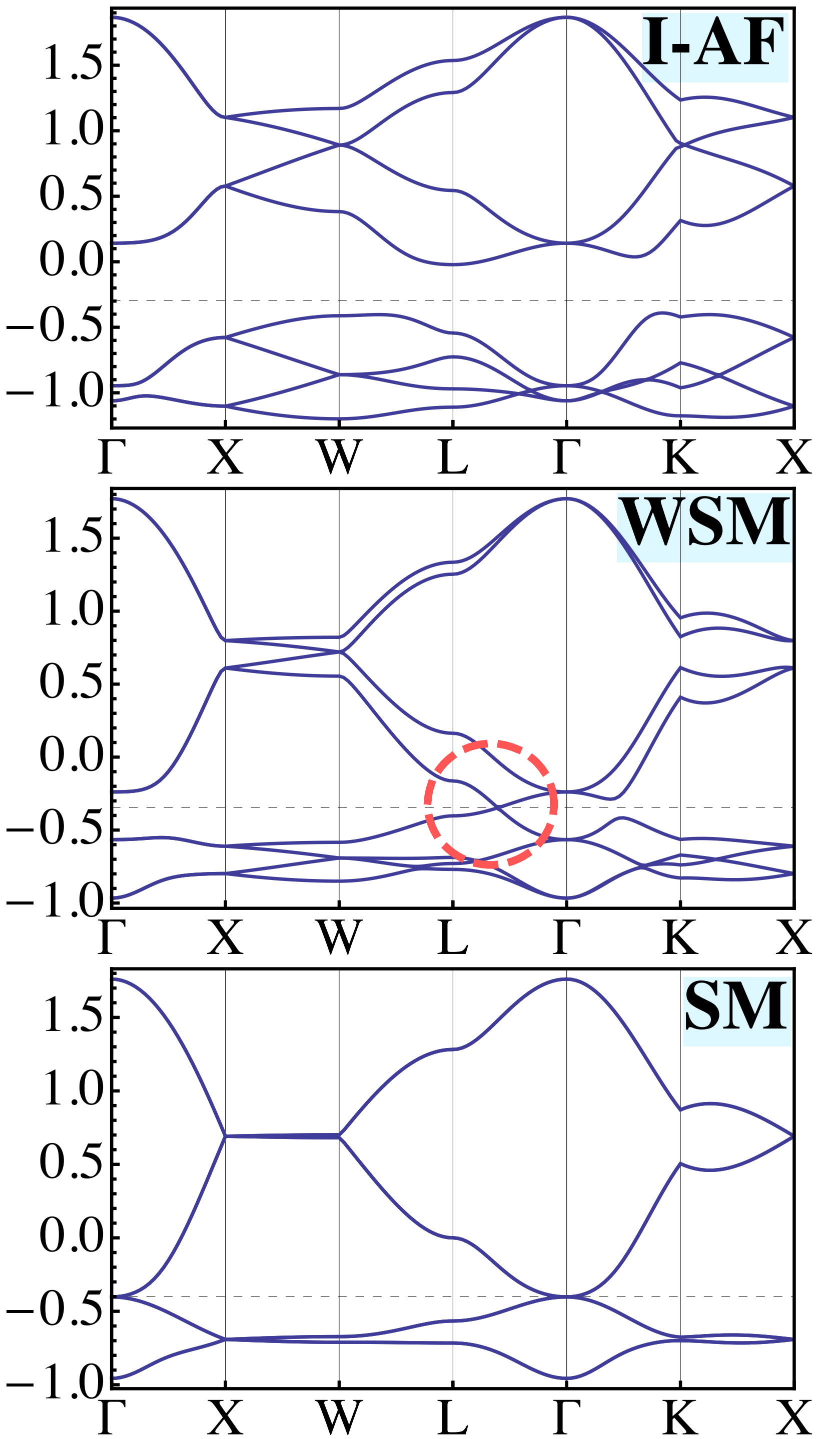
L. Balents
(2014)
Ann. Rev. Cond. Matt.

## WSM = Weyl Semi-Metal

Semi-metal with 3D Dirac points in the bulk

c.f. X. Wan, A. Turner, A. Vishwanath, S. Savrasov, PRB 83, 205101 (2011)

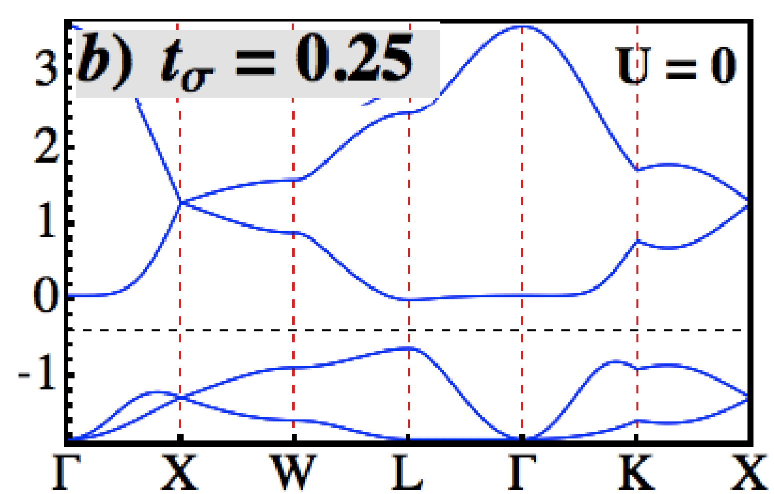
# Effect of Interaction: Hartree-Fock




Magnetic Insulator

Weyl
Semi-Metal

Quadratic
Band Touching
Semi-Metal




TI

W. Witczak-Krempa, Y. B. Kim, PRB 85, 045124 (2012)

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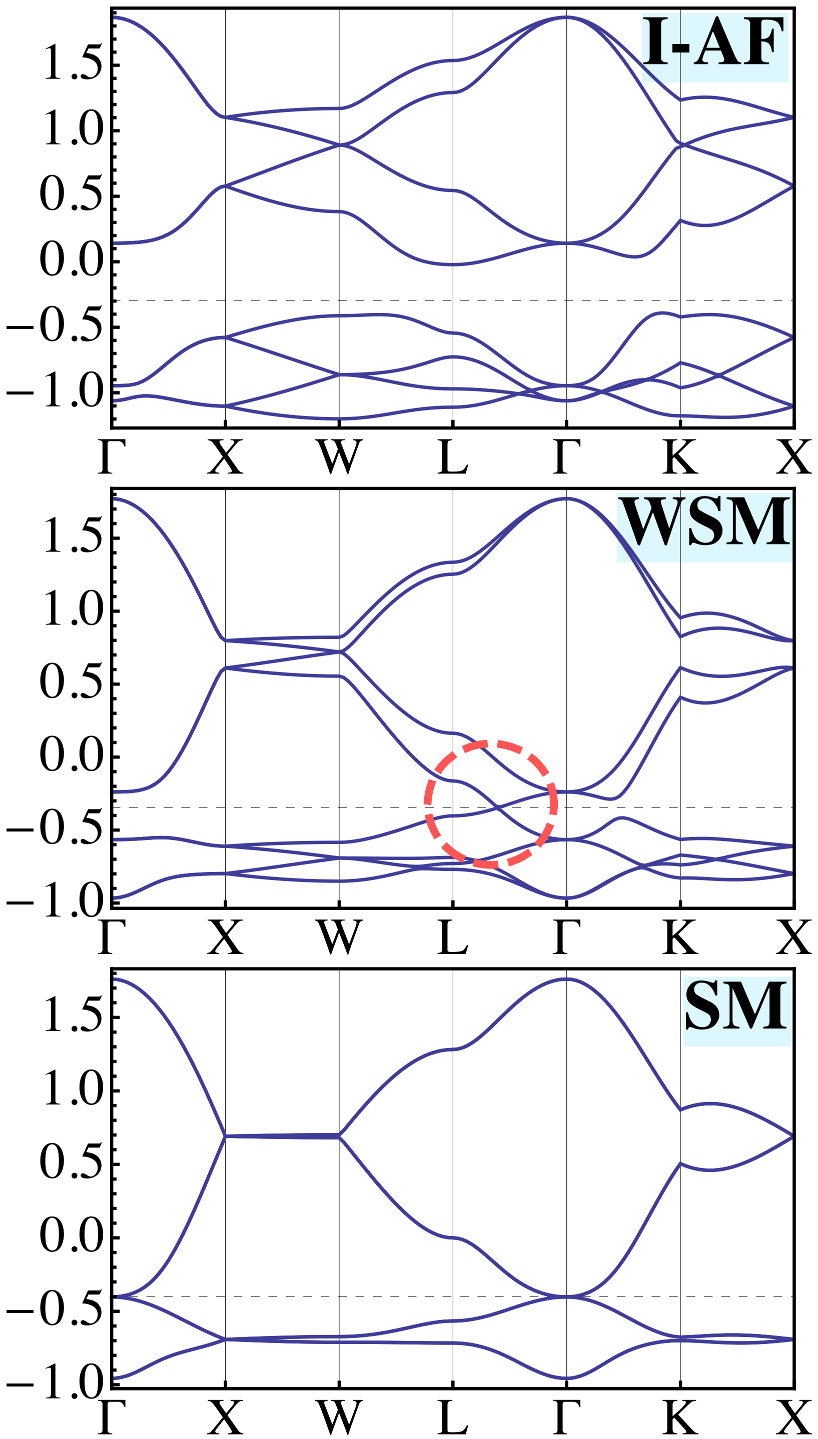


Magnetic Insulator

Weyl
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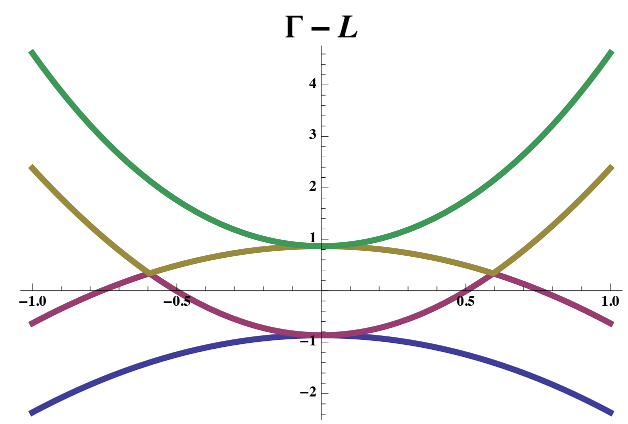


Quadratic
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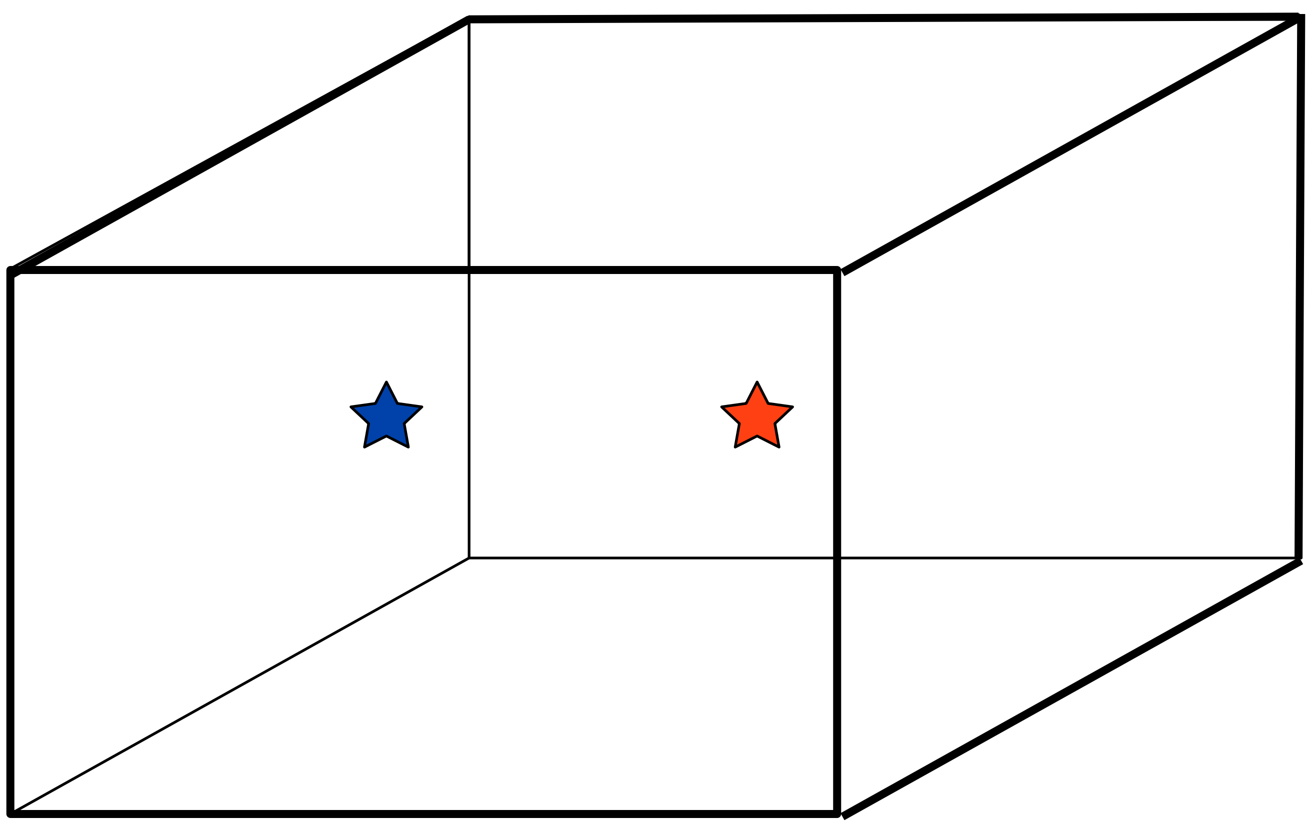
B. J. Yang, Y. B. Kim (2010)

W. Witczak-Krempa, Y. B. Kim, PRB 85, 045124 (2012)

# Weyl Semi-Metal

A pair of Weyl fermion points related by inversion; carry opposite chirality

$$\mathcal{H} = \sum_{i=1}^{3} \mathbf{v}_i \cdot \mathbf{k}\ \sigma_i \qquad c\ =\ \mathrm{sign}(\mathbf{v}_1\ \cdot\ \mathbf{v}_2\ \times\ \mathbf{v}_3) =\ \pm 1$$



Surface State:
Fermi arcs at
the surface BZ

X. Wan, A. Turner,
A. Vishwanath,
S. Y. Savrasov,
PRB 83, 205101 (2011)

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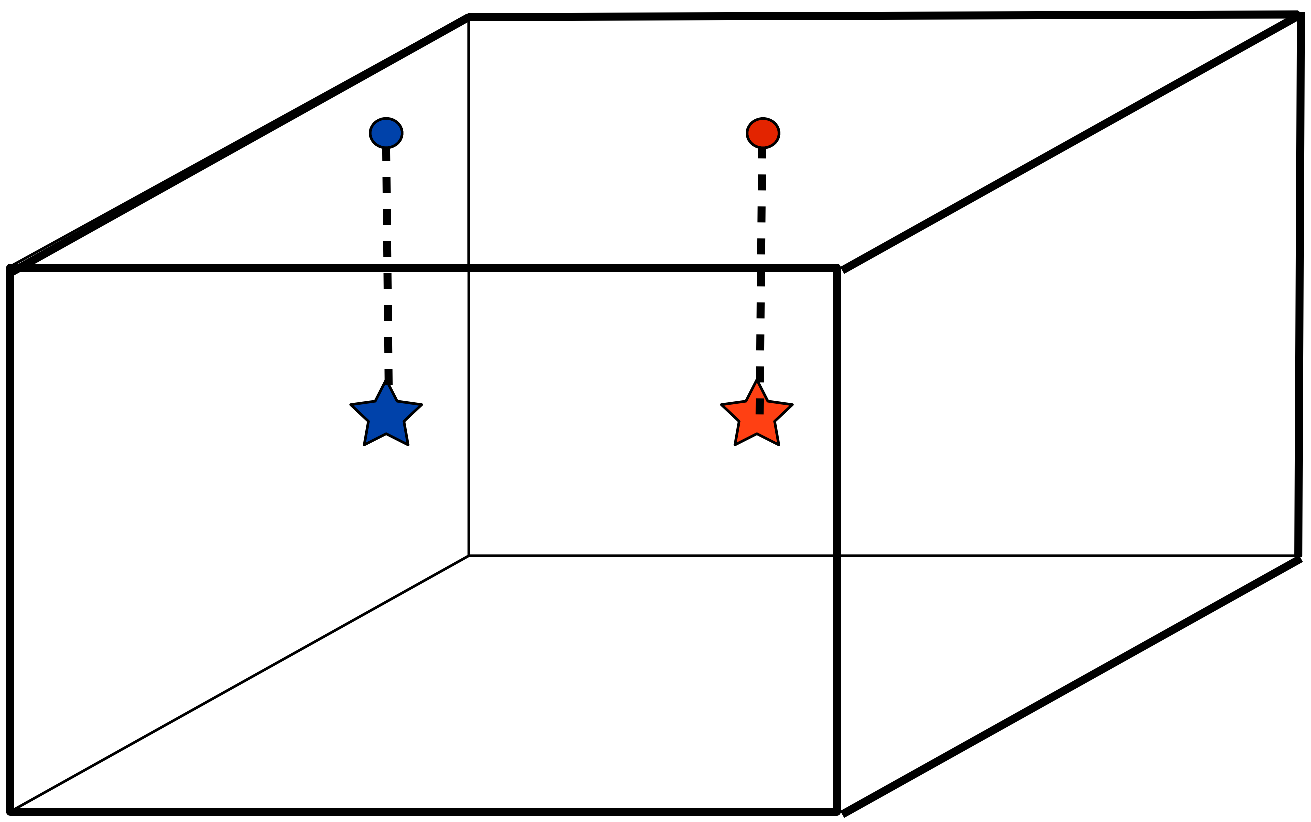

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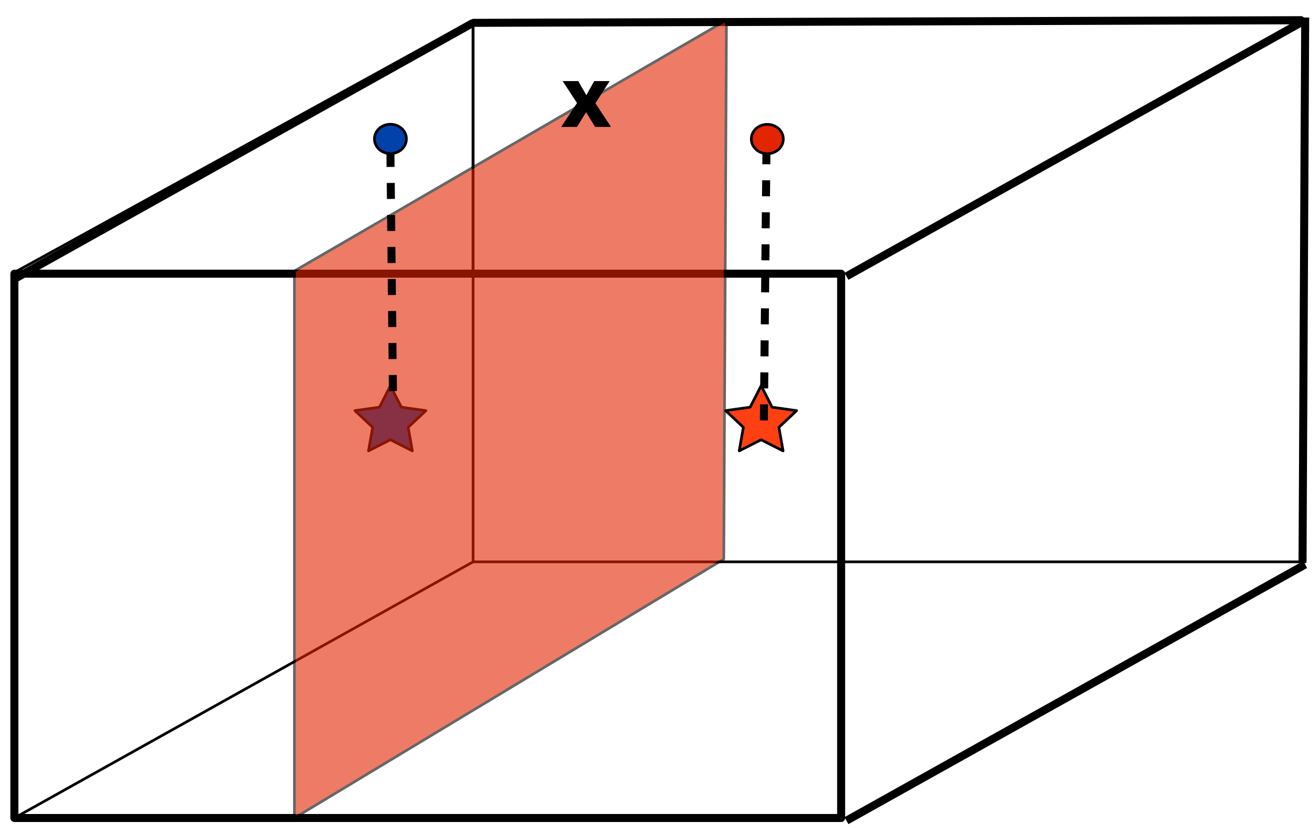


Integer Quantum Hall state: C=1

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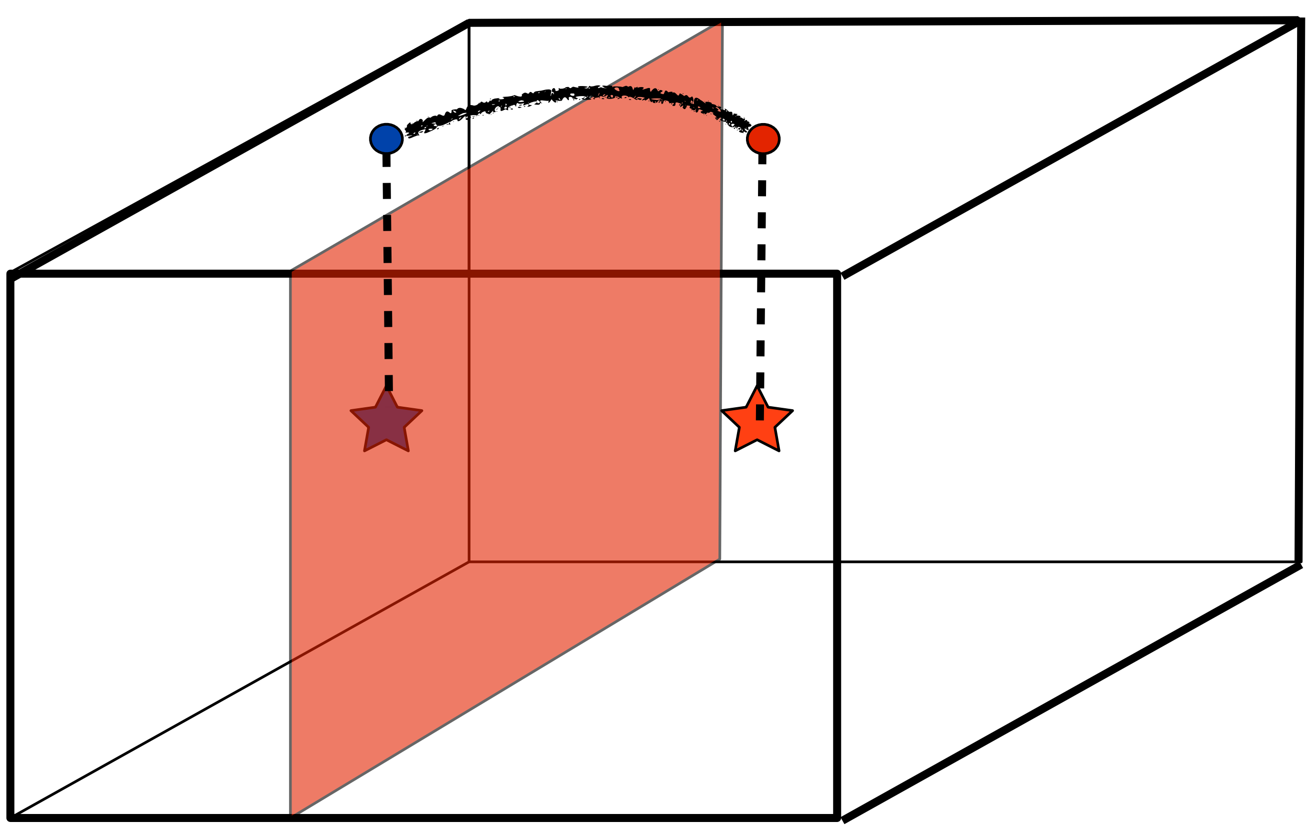

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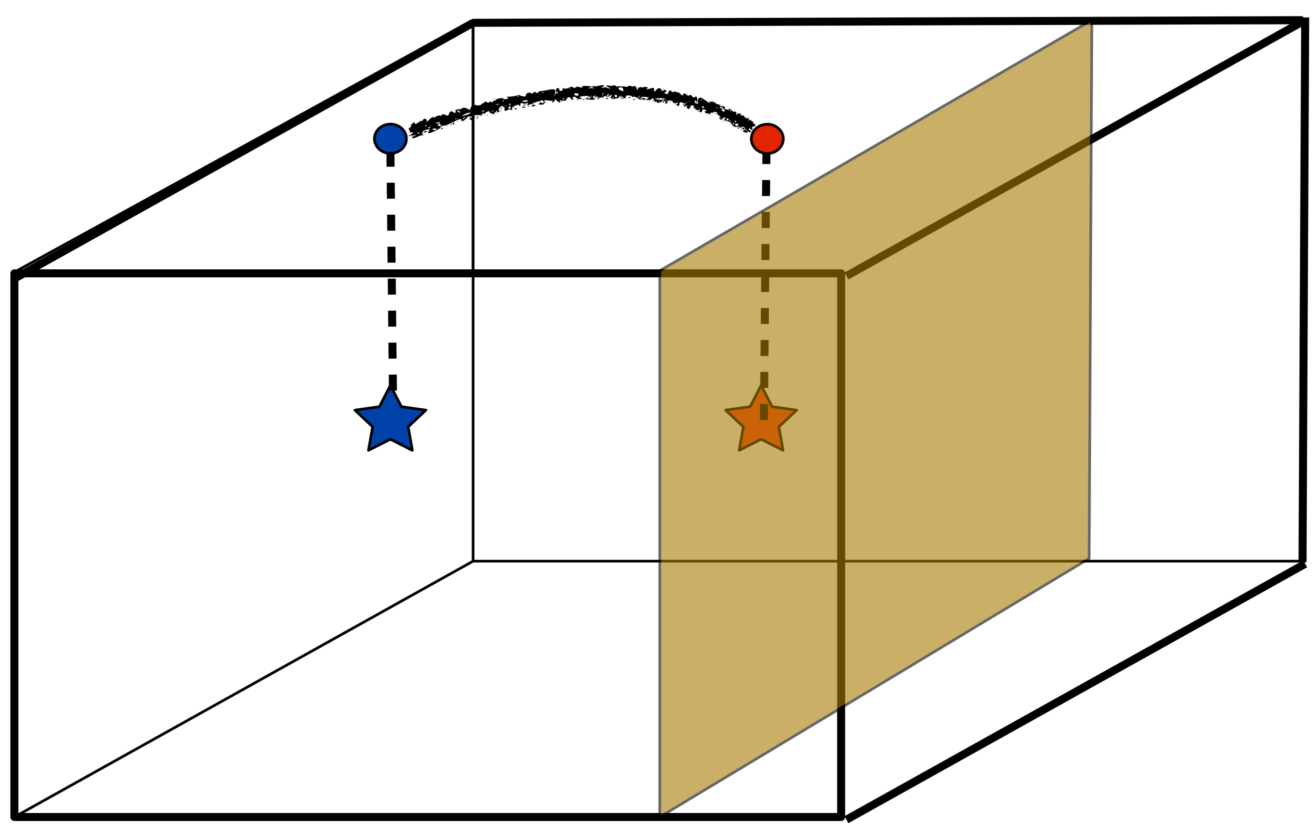


Trivial state: C=0

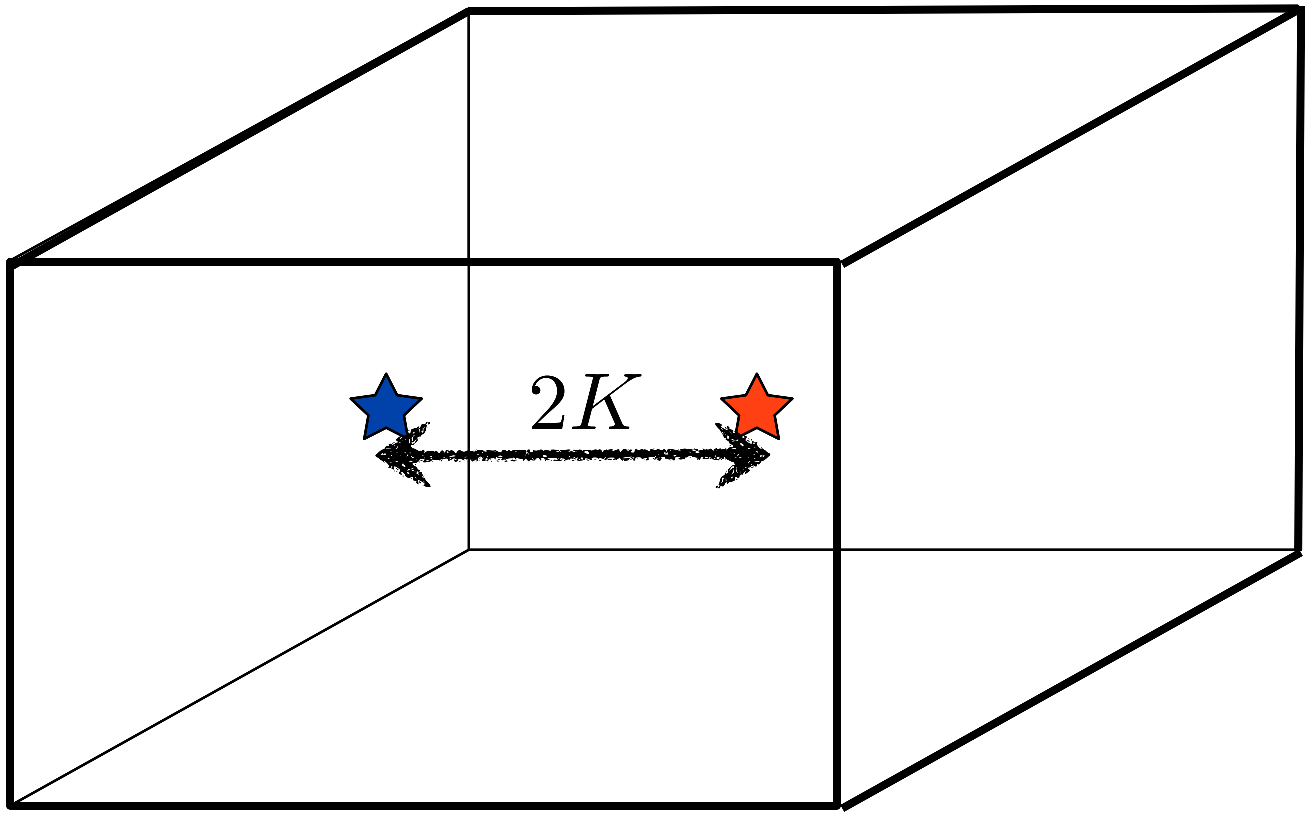
Surface State: Fermi arcs at the surface BZ

X. Wan, A. Turner, A. Vishwanath, S. Y. Savrasov, PRB 83, 205101 (2011)

# Weyl Semi-Metal

## Anomalous Hall effect

$$\mathcal{H} = \sum_{i=1}^{3} \mathbf{v}_i \cdot \mathbf{k}\ \sigma_i \qquad c = \mathrm{sign}(\mathbf{v}_1 \cdot \mathbf{v}_2 \times \mathbf{v}_3) = \pm 1$$




$$\sigma_H = \frac{e^2}{h} \times \frac{2K}{2\pi}$$

Cubic Symmetry

$$\Longrightarrow \quad \sigma_H = 0$$

# Minimal Hamiltonian: Luttinger Model

$\vec{k} \cdot \vec{p}$ expansion near $\Gamma$

$$\mathcal{H}_0(k) = \alpha_1 k^2 + \alpha_2 (\vec{k} \cdot \vec{J})^2 + \alpha_3 (k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)$$

$$= \frac{k^2}{2\tilde{M}_0} + \frac{(\frac{5}{4}k^2 - \vec{k} \cdot \vec{J})^2}{2m} - \frac{(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)}{2M_c}$$

$\vec{J} = (J_x, J_y, J_z)$ are J=3/2, 4 x 4 matrices

Quadratic band touching of two doubly-degenerate bands

B. J. Yang, Y. B. Kim (2010)

# Time-Reversal Symmetry Breaking

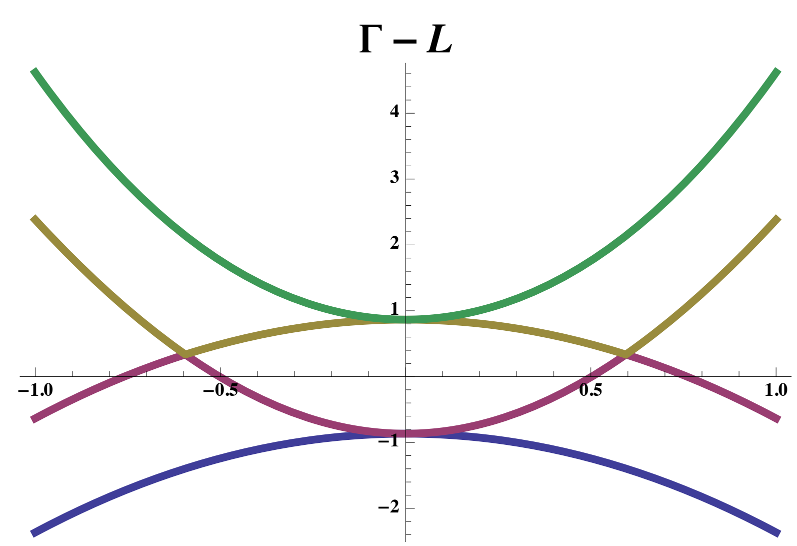
$$H_{\text{field}} = C_1(H_xJ_x + H_yJ_y + H_zJ_z) + C_2(H_xJ_x^3 + H_yJ_y^3 + H_zJ_z^3)$$

$$H_{\text{mag order}} = D_1(\{k_x, k_y\}V_z + \{k_y, k_z\}V_y + \{k_z, k_x\}V_y) + D_2(J_xJ_yJ_z + J_zJ_yJ_x)$$

$$V_x \equiv \{(J_y^2 - J_z^2)J_x\}, V_y \equiv \{(J_z^2 - J_x^2)J_y\}, V_z \equiv \{(J_x^2 - J_y^2)J_z\}$$

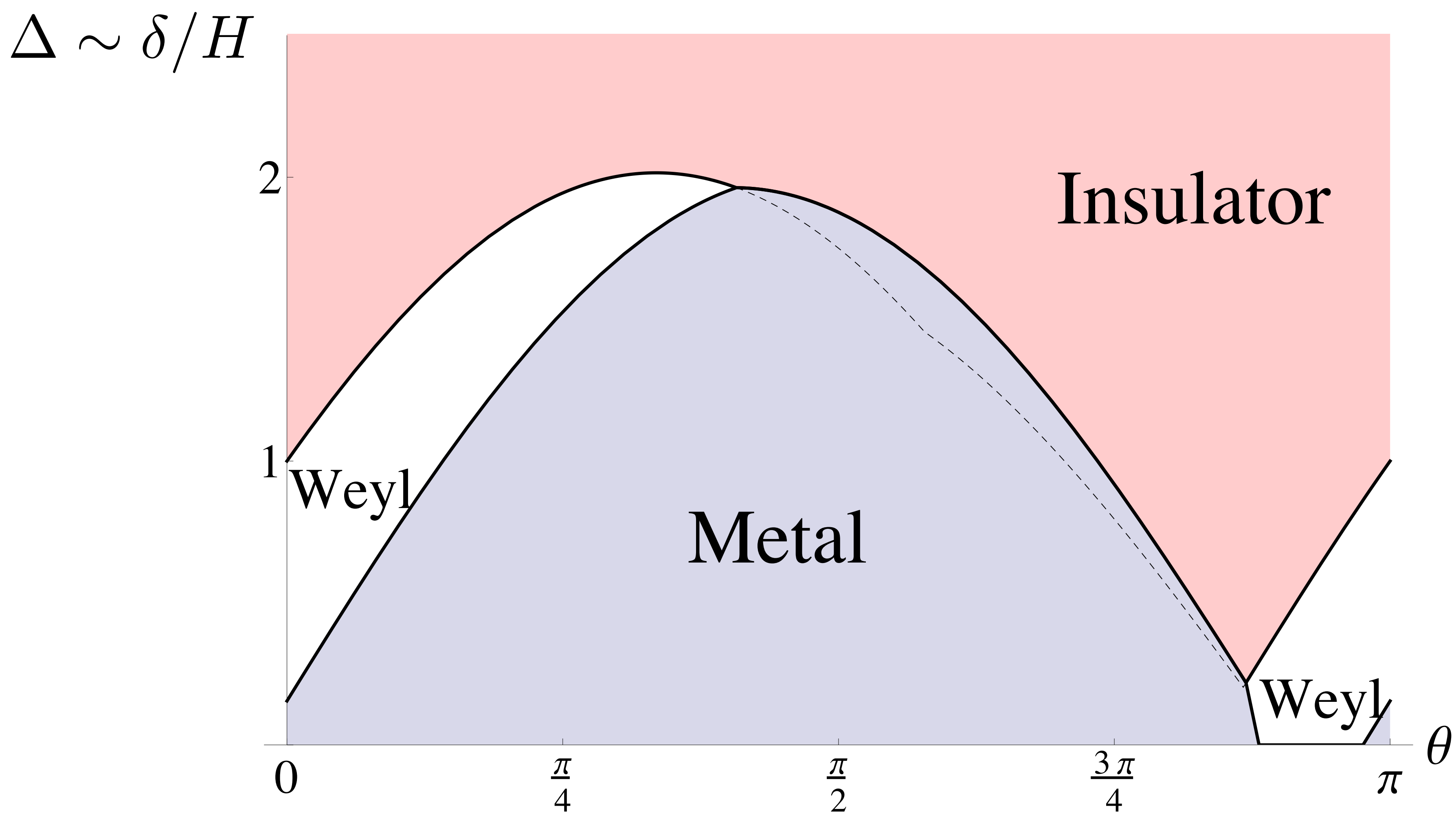
T. Hsieh, L. Fu (2012)    E. G. Moon, C. Xu, Y. B. Kim, L. Balents (2012)

Both types of perturbations can generate Weyl fermions



# Time-Reversal Symmetry Breaking and Strain

$$\mathcal{H}' = -\delta(J_z^2 - \frac{5}{4}) - H(\cos(\theta)J_z + \sin(\theta)J_z^3),$$




E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

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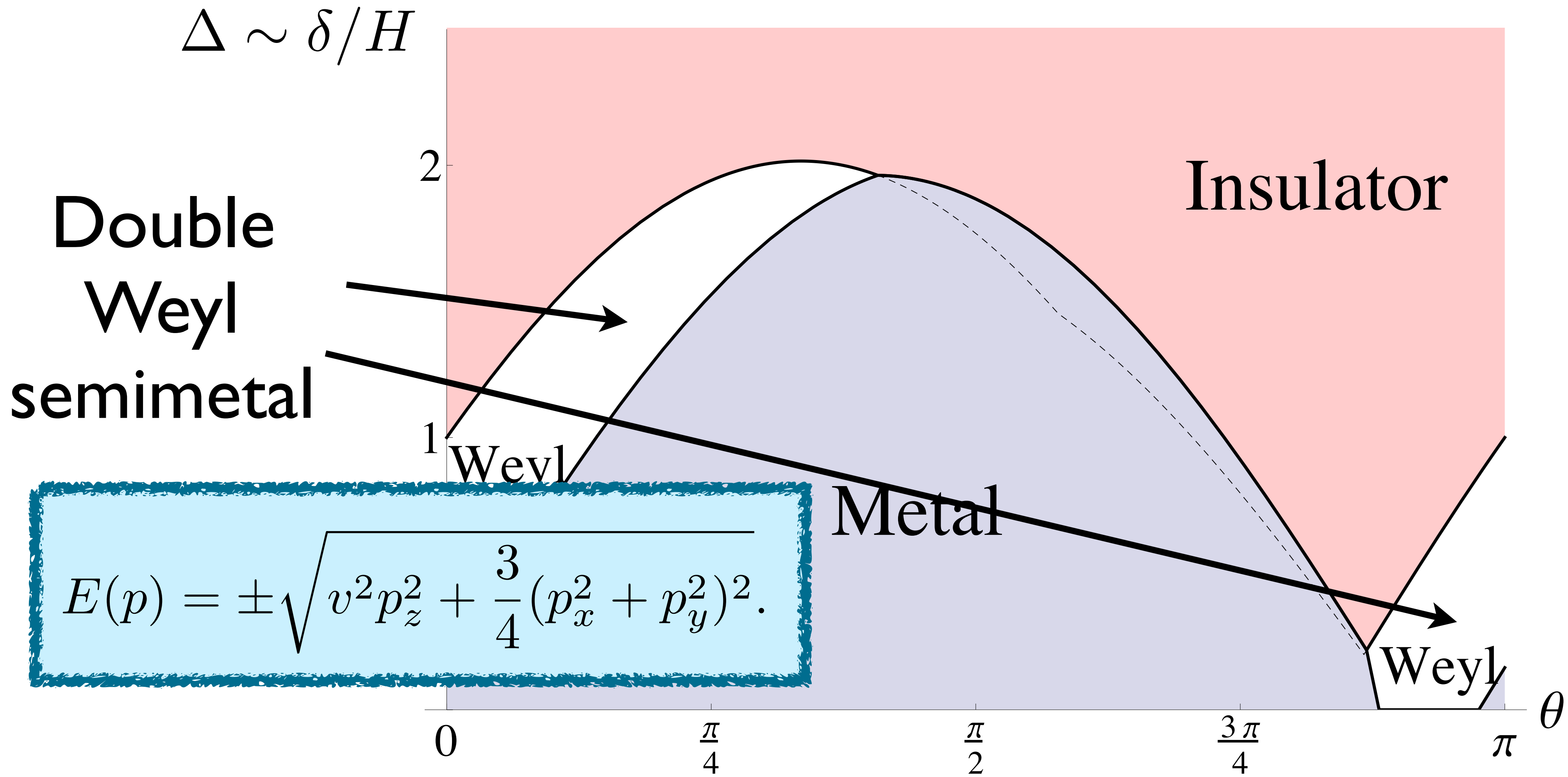


$$E(p) = \pm\sqrt{v^2 p_z^2 + \frac{3}{4}(p_x^2 + p_y^2)^2}.$$

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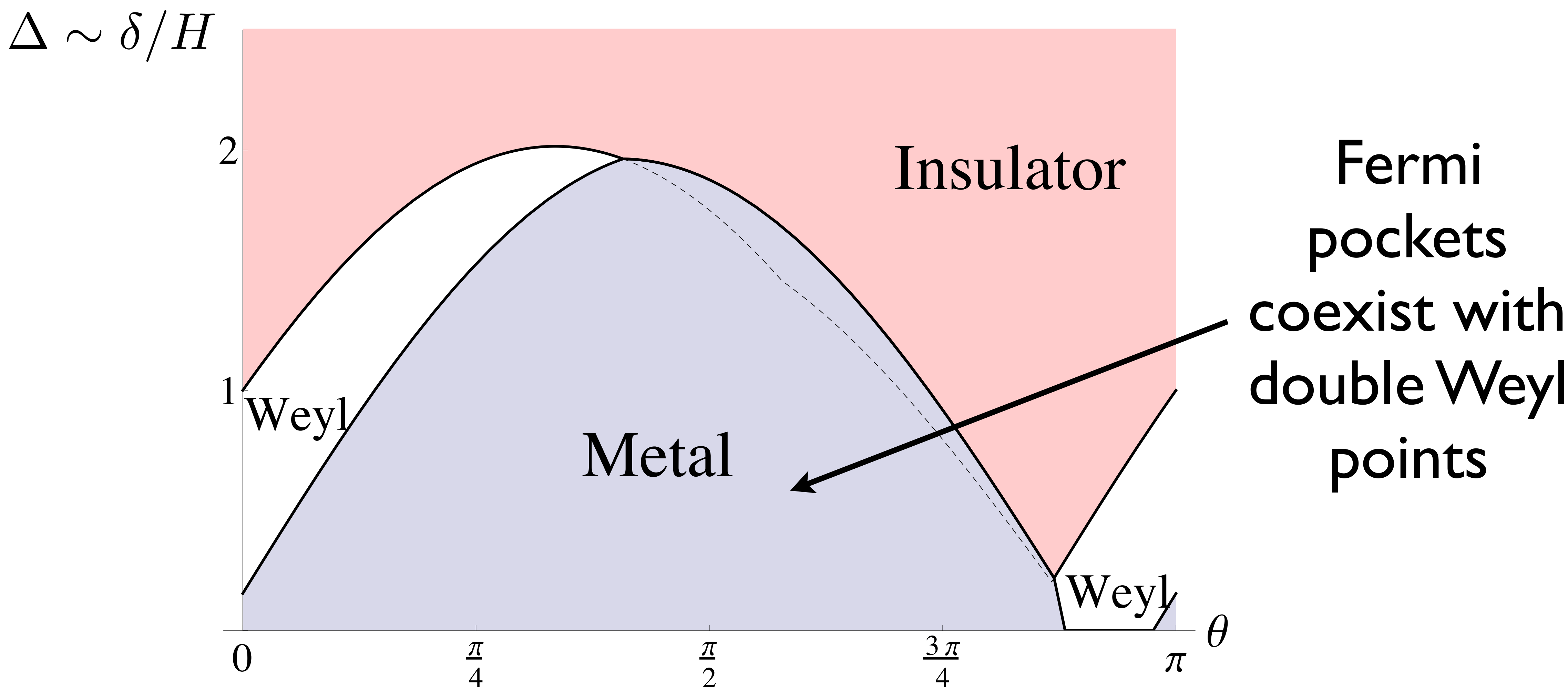



E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

# Double-Weyl Fermions

$$\mathcal{H}_2^{\pm} = \vec{b}_{\pm}(\vec{p}) \cdot \vec{\tau},$$

$$E(p) = \pm\sqrt{v^2 p_z^2 + \frac{3}{4}(p_x^2 + p_y^2)^2}.$$

$$\vec{b}_{\pm}(\vec{p}) = \begin{pmatrix} -\frac{\sqrt{3}}{2}(p_x^2 - p_y^2) \\ \sqrt{3}p_x p_y \\ \mp v p_z \end{pmatrix}$$

$$\boxed{\begin{aligned} \mathcal{B}_{\pm}^{\mu} &= \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}\hat{b}_{\pm} \cdot \partial_\nu \hat{b}_{\pm} \times \partial_\nu \hat{b}_{\pm}, \\ \partial_\mu \mathcal{B}^\mu(\vec{p}) &= \pm 2\delta(\vec{p}). \end{aligned}}$$

$$\sigma_{xy} = = 2 \times \frac{e^2}{h} \times \frac{2K}{2\pi}$$

Anomalous Hall effect

E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

# Long-range Coulomb interaction

Relevant in RG sense near the non-interacting limit

$\varepsilon = 4 - d$ expansion leads to non-trivial interacting (isotropic) fixed point

Non-Fermi Liquid scaling in physical quantities

See also Abrikosov + Beneslavskii (71), Abrikosov (74)

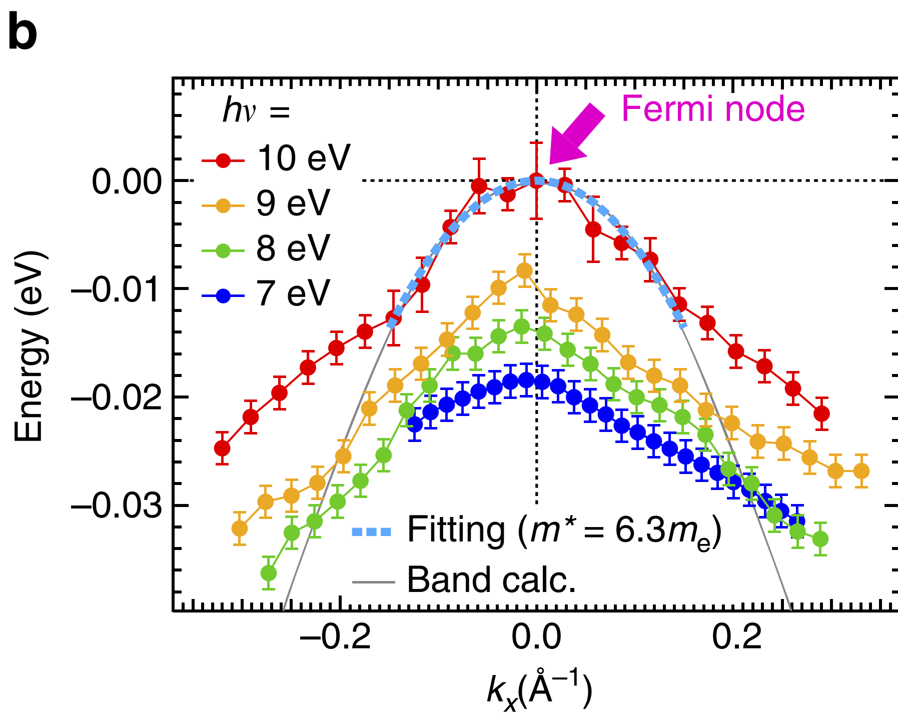
$$z \approx 1.8 \qquad C_v \sim T^{d/z} \approx T^{1.7} \qquad \chi \sim a + bT^{0.5}$$

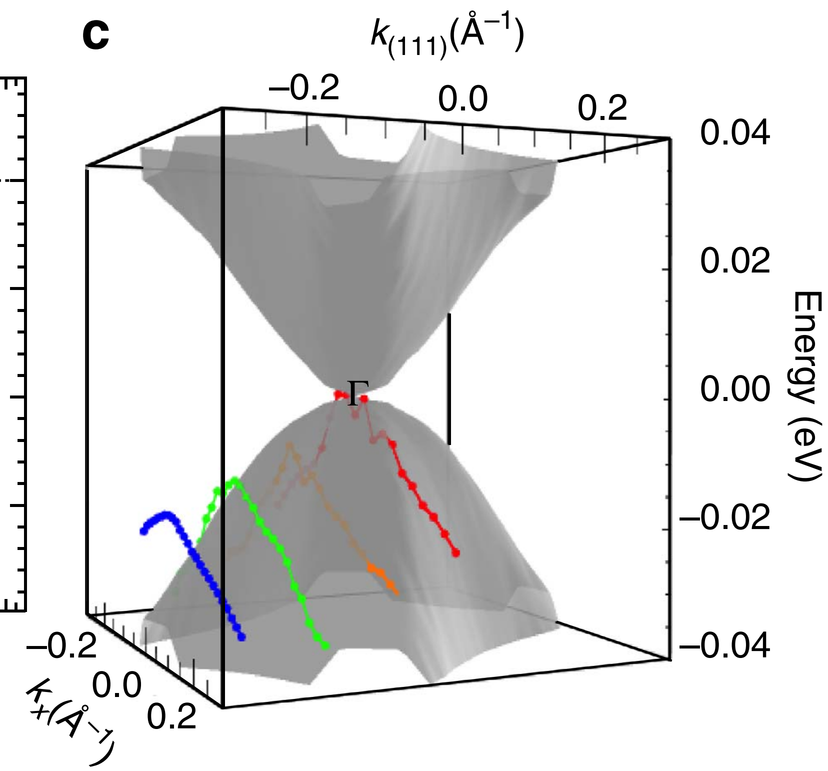
$$\chi_3 = \left.\frac{\partial^3 M}{\partial H^3}\right|_{H=0} \sim T^{-1.7} \qquad \sigma_{xy} \sim M^{0.51}$$

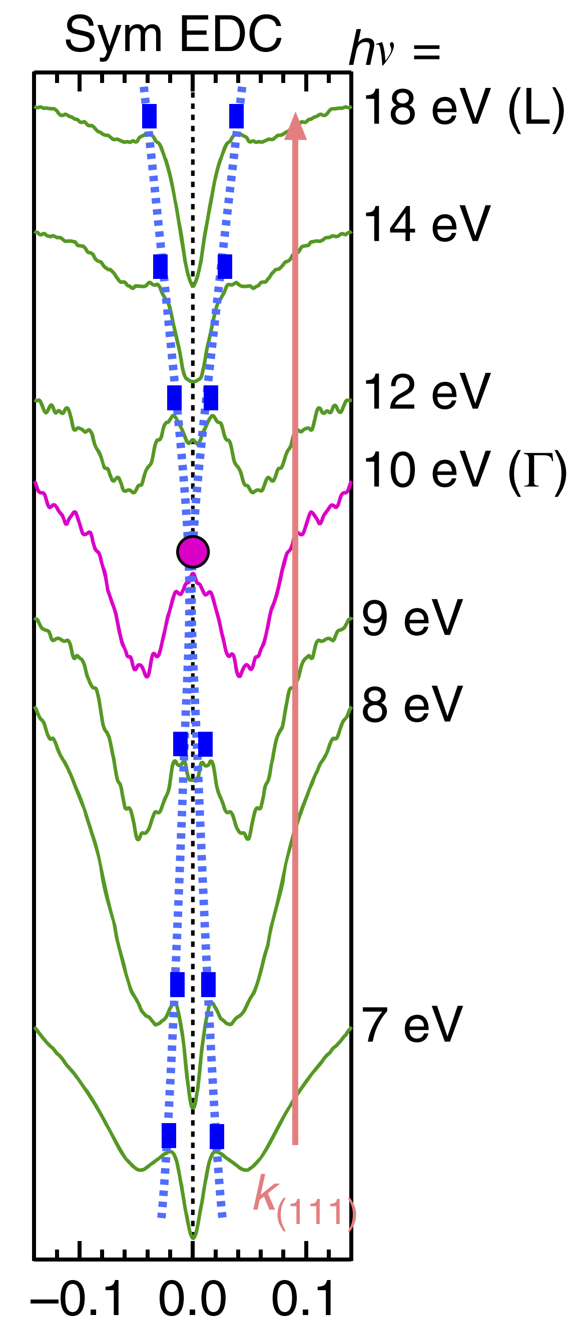
E. G. Moon, C. Xu, Y. B. Kim, L. Balents, arXiv:1212.1168

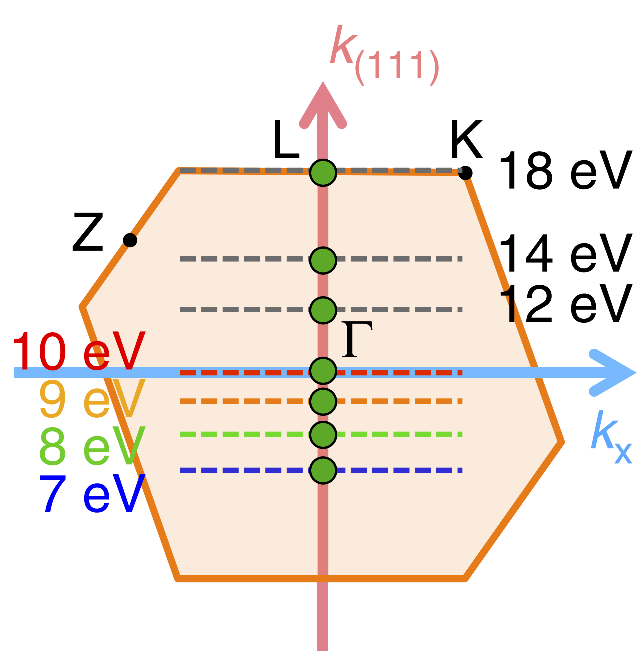
# ARPES experiment on $Pr_2Ir_2O_7$

## T. Kondo, S. Shin, S. Nakatsuji et al (2015)




Observation of
Quadratic Band Touching