CAQCD 2016: Worldsheet-Induced Corrections to the Holographic Veneziano Amplitude

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Based on: A. Armoni (1509.03077)
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1. A reminder of the set-up

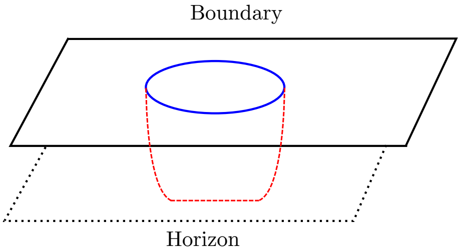


Figure: A representation of the set-up

We notice that in a particular type of string background, we can force a string hanging from the boundary of the space to lie mostly flat on the horizon of the space. The Veneziano amplitude is neatly reproduced by the flat string 4-point amplitude, so construct a holographic dual of a QCD-like theory to try and derive this from non-perturbative effects computed in string theory.

- Write large *Nc* QCD path integral as a sum over all shapes of Wilson loops
- Relate this sum over Wilson loops to a sum over string worldsheets hanging from the boundary via holography
- In the correct limit can argue that the main contribution is the flat open string 4-point amplitude $\left\langle \exp\left(\sum_{i=1}^{4} k^i \cdot X(\sigma_i)\right)\right\rangle$, i.e. the famous Veneziano amplitude.
- Used Witten's geometry of backreacted D4 branes compactified on a circle. Not dual to pure YM but has been argued to possess similar key ingredients, namely exhibiting confinement and a mass gap.
- Due to the complexity of the theory, necessary to make somewhat crude assumptions about the string's behaviour as it falls in the geometry, and to send the position of this model's horizon to very close to the boundary.

Q: What kind of information (if any) can be gleaned by allowing curvature effects around the horizon to affect the worldsheet sigma model? Can string theory predict deviations from the high-energy Veneziano regime for the four point amplitude?

2. Preparing the worldsheet field theory

- First express string partition function in a set of coordinates that allow for a field-theoretical treatment of coordinate fluctuations around the horizon. But there is a coordinate singularity, some thought required!

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(dx^2 + \mathrm{d}\tau^2\left(1 - \frac{U_{\mathrm{KK}}^3}{U^3}\right)\right) + \left(\frac{R}{U}\right)^{3/2}\left(\left(\frac{1}{1 - \frac{U_{\mathrm{KK}}^3}{U^3}}\right)dU^2 + U^2 d\Omega_4\right)$$

- A good change of coordinates needs to make this metric regular around the origin, but also to have a regular and non-vanishing Jacobian, because the parametrisation-invariant NLSM measure is $[DX] = \det(G)DX$.

- Solution: Kruskal-like procedure. Crucially, compute Tortoise coordinate, $\int G_{UU} dU$. Very impractical to do this here given the form of the integrand. But, doable order by order in an expansion around U_{KK} assuming small fluctuations.
- Writing $U = U_{KK}(1 + \frac{u^2}{U_{KK}^2})$ we expand the offending parts of the metric in orders of U_{KK}, then change variables from (u, τ) to Kruskal-like coordinates (Y, Z). This naturally preserves shift symmetry in τ i.e. a $U(1)$ global symmetry.
- With a metric expanded to first order the exact change is

$$\frac{Y^2 + Z^2}{2U_{KK}^2} = \frac{u^2}{2U_{KK}^2} \exp(\frac{u^2}{2U_{KK}^2}) \tag{1}$$

Relation can be inverted and new metric expanded to first order to obtain an effective Lagrangian for the fluctuations.

- Defining $\lambda = \frac{U_{KK}}{R}$ and the doublet $\Upsilon = (Y, Z)$, the (bosonic) Lagrangian in these coordinates is then

$$L = \lambda^{3/2}\left(1 + \frac{3\Upsilon^2}{2U_{KK}^2}\right)\partial_\alpha X^\mu \partial^\alpha X_\mu + \frac{4}{3\lambda^{3/2}}\partial_\alpha \Upsilon \cdot \partial^\alpha \Upsilon + \dots \tag{2}$$

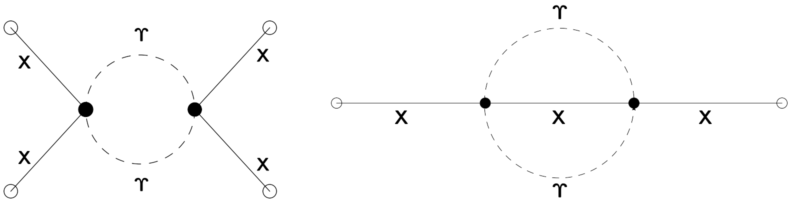
- But the integration measure also has to change, as it is proportional to $\det(G)$: by this change of coordinates

$$\det(G) = U_{KK}^8 \frac{16}{9\lambda^3}\left(1 + \frac{6\Upsilon^2}{U_{KK}^2} + \dots\right) \tag{3}$$

This can be exponentiated to give a small mass to the new radial field.

Some comments about the path integration:

- Compute the string 4-point function $\left\langle \exp\left(\sum_{i=1}^{4} k^i \cdot X(\sigma_i)\right)\right\rangle$, by the usual trick of writing this as a current $\mathcal{J}(\sigma) = \sum_{i=1}^{4} k^i \cdot X(\sigma)\delta(\sigma - \sigma_i)$.
- Then, sufficient to compute partition function with a non-zero current. Introduces a 1-leg vertex in the Feynman rules (ending a propagator with a Fourier kernel), care taken for loop order vs. expansion order:



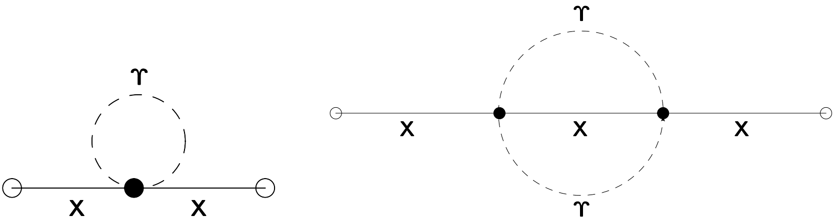
- Despite worldsheet having supersymmetry, ignore superpartners. They communicate less directly with the X fields than Υ does.

3. Computing and analysing the correction

- 2D QFTs have technical particularities, notably related to regularisation. Propagator is UV divergent in two dimensions. We use analytic regularisation: introducing an arbitrary mass scale μ,

$$\left[\frac{1}{p^2+m^2}\right]_{AR} = \lim_{x\to 0}\frac{d}{dx}\left(x\mu^x \frac{1}{(p^2+m^2)^{1+x}}\right) \tag{4}$$

- This also method also works in the massless case, dealing with the IR divergence identically.
- With correct variant of $\overline{MS}$, difficulties are dealt with automatically.



Two different situations:

- First diagram has no momentum transfer, integral factorises into independent terms, in our regularisation scheme the "bubble" is finite and body subsumes to a propagator.
- $\rightarrow$ Finite correction to the effective string tension, anyway set to right value *a posteriori*
- Second diagram has momentum transfer, therefore is much more interesting. However, keeping the Υ propagators massive makes it very difficult to compute even sub-integrals within the bigger computation.
- $\rightarrow$ Since mass parametrically small (same order as coupling) and massless diagram well-defined, justified to compute the latter for broadest behaviour.

- The 4-point function we compute is then summed over all possible positions of the 4 points, which subsumes to one Beta-type integral to leading order as shown previously. With the addition of the new diagram, we obtain (schematically)

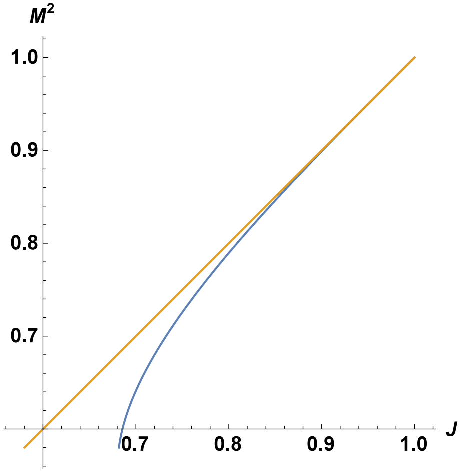
$$\mathcal{A}(s,t) = \int_0^1 dz z^{s-1}(1-z)^{t-1}\left(1-\rho\left(s\log^3(z)+t\log^3(1-z)\right)\right)$$
$$= \left(1-\rho\left(\left(\frac{\partial}{\partial s}\right)^3+\left(\frac{\partial}{\partial t}\right)^3\right)\right)B(s,t) \quad (5)$$

- Now analyse the behaviour of this amplitude in order to find an expression to tie J and M^2. Several ways of obtaining the linear Regge trajectory out of the standard Beta function, depending on which regimes one investigates.

By taking the $s \gg t$ limit of our result we get an approximate behaviour for the Regge trajectory to be (in normalised units)

$$\alpha(s) = s^{1-\rho \log^2(s)} \tag{6}$$

Plotting this behaviour we find a surprisingly natural behaviour:



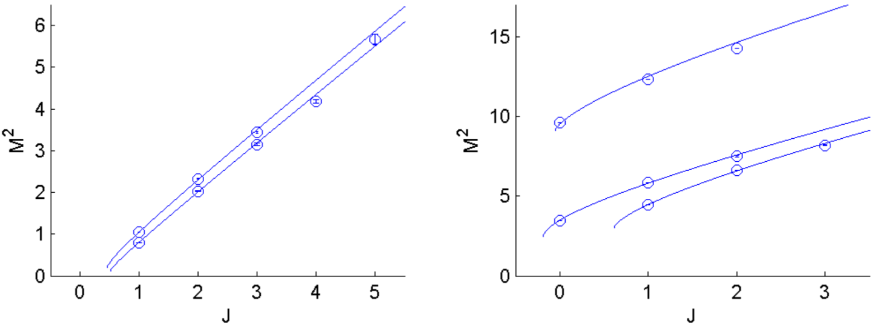


Figure: Source: 1602.00704, J. Sonnenschein

Future work

- Diversifying backgrounds: is the $\Upsilon^2 \partial X \partial X$ term generic among such constructions?
- Refining the computation: many approximations in the derivation, perhaps other backgrounds are more tractable.
- (Eventually) compare with experimental data once a better handle on the process is achieved.
- Numerical solutions to the problem may prove a better tactic if they can be implemented.