Continuous chiral symmetry breaking in a calculable regime

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Chiral symmetry breaking in QCD

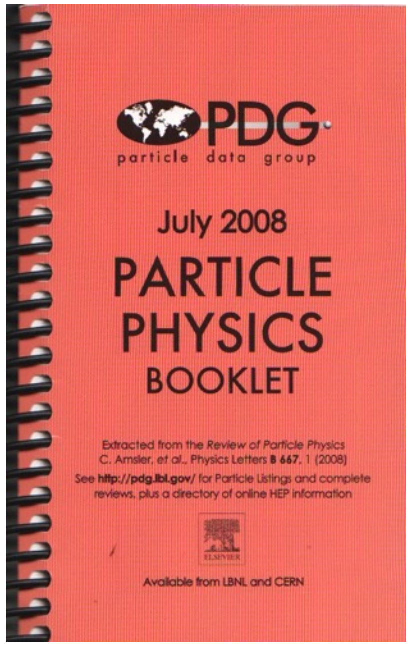
At the microscopic level, QCD has an approximate chiral symmetry

$$SU(N_F)_L \times SU(N_F)_R : \quad \psi_L \to L\psi_L, \; \psi_R \to R\psi_R$$

This symmetry is spontaneously broken in the vacuum:

$$SU(N_F)_L \times SU(N_F)_R \to SU(N_F)_V$$

Vital for QCD phenomenology! How do we know it?



actual experiments

numerical experiments

Chiral symmetry breaking in QCD

Theoretical understanding of chiral symmetry breaking (χ-SB) mostly based on inspirational phenomenological models of χ-SB in QFT:

Nambu-Jona-Lasinio models

Truncated Schwinger-Dyson equation models

Instanton/'dyon' liquid models

…

In all these constructions χ-SB happens at strong coupling, outside of regime where quantum effects are under systematic control.

Folk belief: χ-SB is fundamentally strongly-coupled, can't happen in weakly-coupled settings. So can't do any better!

We found a setting with **calculable** χ-SB using adiabatic compactification idea.

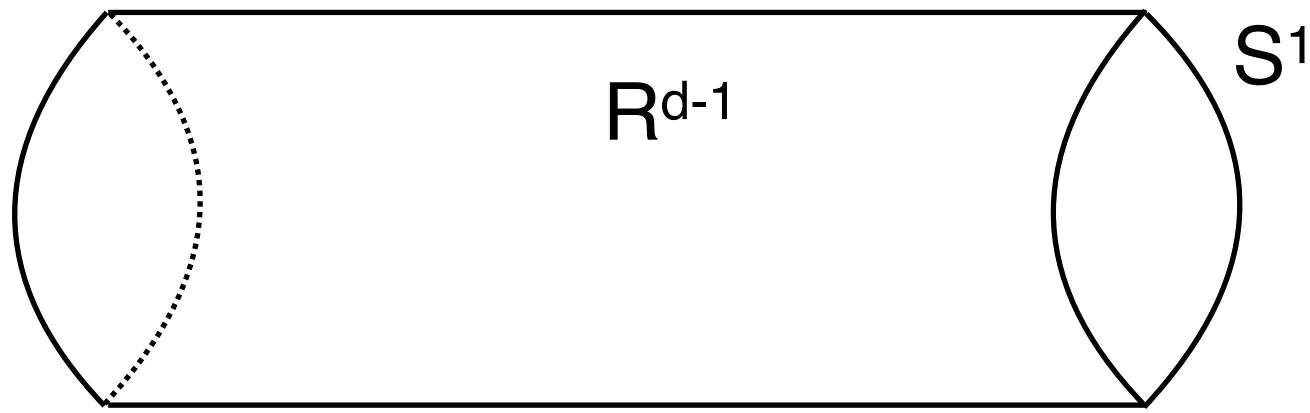
This talk: $N_c > 2$, $0 \leq N_F \leq N_c$, $\theta = 0$, N_c finite, fermions are fundamental rep.

Adiabatic compactification

Unsal, Yaffe, Shifman, … 2008-onward

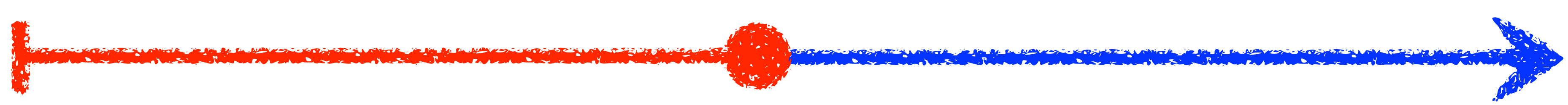
Compactify asymptotically-free 4D QFT to $R^3 \times S^1$

When S^1 size $L \ll \Lambda^{-1}$, theory becomes ≈ weakly-coupled



With thermal BCs, L = 1/T

Trouble: small-L and large-L theories separated by phase transition



small L	large L
no confinement, no χ-SB	confinement, χ-SB
(Free energy)/$N_c^2 \sim 1$	(Free energy)/$N_c^2 \sim 0$

Adiabatic compactification

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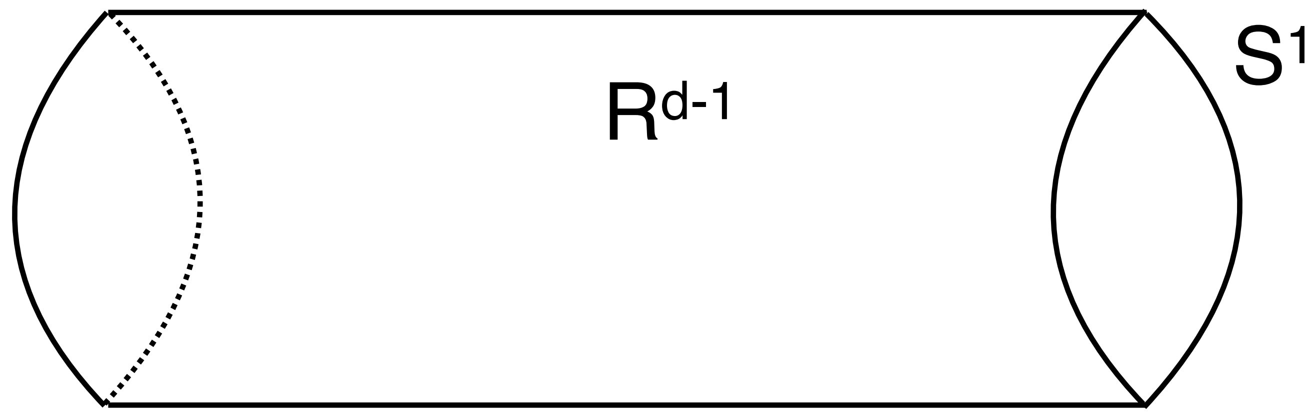


Need to find a situation where instead we get

small L	large L
confinement, χ-SB	confinement, χ-SB
(Free energy)/$N_c^2 \sim 0$	(Free energy)/$N_c^2 \sim 0$

Adiabatic compactification

Unsal, Yaffe, Shifman, … 2008-onward

Idea: 'deform' theory by something that doesn't matter at larger L, but makes L dependence smoother.

At large L, QCD-like theories are in confined phase; approximately vanishing Polyakov loop/ Z_{Nc} center symmetry.

$$\Omega = e^{i\int_{S_1} A_4},$$

$$\langle \mathrm{Tr}\ \Omega^n \rangle = 0$$

To keep center symmetry at small L, can add massive adjoint fermions with periodic BCs, $\Lambda \ll m \ll 1/L$

Or add appropriate double-trace deformation $\delta S = \int d^4x\, L^{-4} \Sigma_n\, [a(n)\ \mathrm{tr}\ |\Omega^n|^2]$

For e.g., pure YM, resulting small L theory has been shown to develop a mass gap, finite string tension, and so on.

Boundary conditions

In theory with quarks, must choose BCs:

$$\psi(x_4 + L, \vec{x}) = \Omega_F \Omega_Q \psi(x_4, \vec{x}), \;\; \Omega_F \in SU(N_f), \Omega_Q \in U(1)_Q$$

Not important for large L spectrum, but matters at small L!

Experience with 2D sigma models: some choices of BCs allow smoother small L limit than others.

AC, Dorigoni, Dunne, Unsal

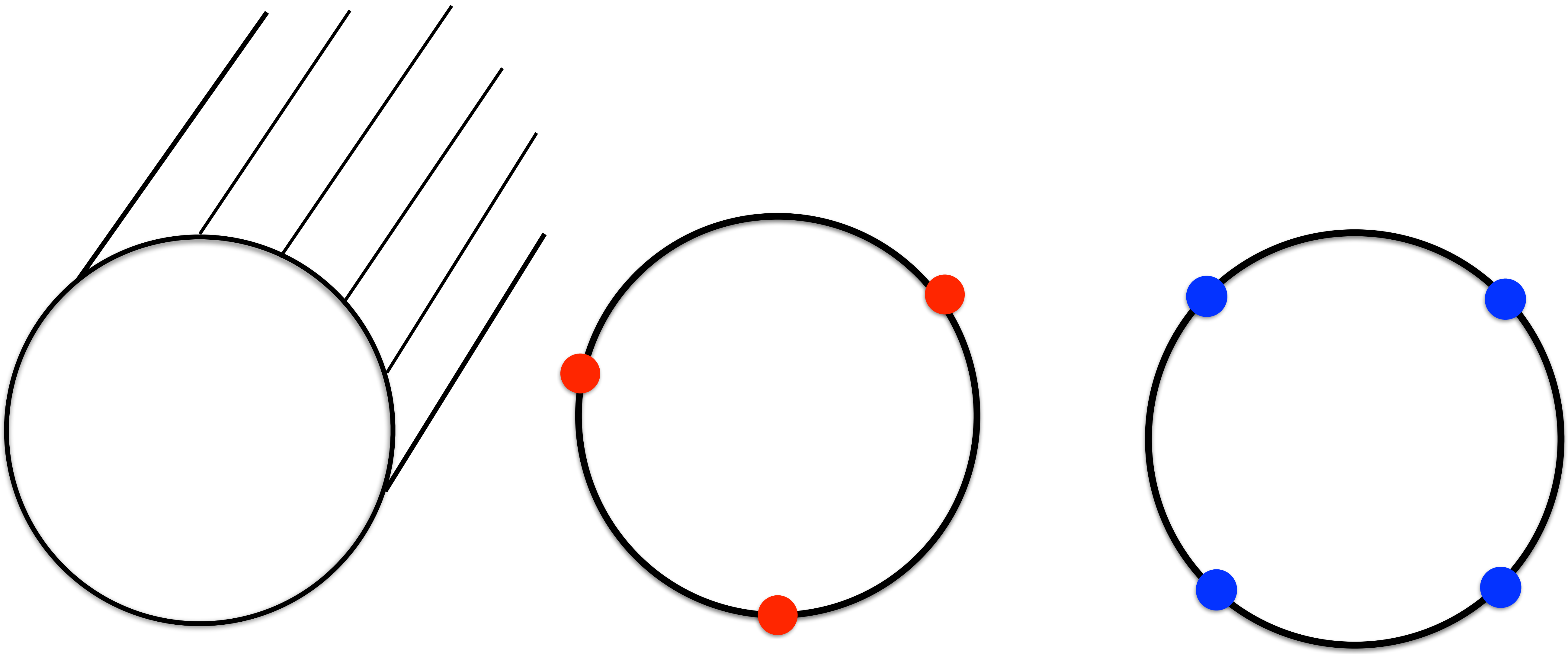
Can think of Ω_F, Ω_Q as background gauge field holonomies

Inspired by 2D examples, explore result of taking flavor-center-symmetric $SU(N_F)$ background holonomies:

$$\Omega_F = \mathrm{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i (N_f - 1)/N_f})$$

$$\text{Preserves } U(1)_L^{N_f - 1} \times U(1)_R^{N_f - 1} \in SU(N_f)_L \times SU(N_f)_R$$

Three circles



Compactification circle

Eigenvalue circle for background flavor holonomy Ω_F

Eigenvalue circle for dynamical color holonomy Ω

Large L expectations

Background holonomies/twisted BCs are equivalent to **imaginary** \`isospin' chemical potentials $\tilde{\mu} \sim 1/L$

Large L low-energy dynamics captured by chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\ \partial_\mu U \partial^\mu \partial^\mu U^\dagger \to \mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\ D_\mu U D^\mu U^\dagger$$

$$D_\mu = \partial_\mu + i[\mu_I \tau_3, \cdot]$$

$N_F = 2$ example

$$m_{\pi^0}^2 = m_\pi^2$$

$$m_{\pi^\pm}^2 = m_\pi^2 - \mu_I^2 \ \to \ m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

N_F-1 'pions' remain gapless, all others pick up positive gaps $E^2 \gtrsim 1/L^2$

If small L limit is smooth, should get N_F -1 gapless NGBs.

Small L limit in perturbation theory

At long distances ℓ >> N_c L ~ 1/m_W

$$SU(N_c) \to U(1)^{N_c - 1}$$

due to the center-symmetric background holonomy.

N_c - 1 Cartan gluons are gapless to all orders in perturbation theory thanks to an emergent symmetry

$$F^i_{\mu\nu} = g^2/(2\pi L)\epsilon_{\mu\nu\alpha}\partial^\alpha \sigma^i$$

$$S_\sigma = \int d^3x \, \frac{g^2}{8\pi^2 L}(\partial_\mu \vec{\sigma})^2.$$

Noether current for $[U(1)_J]^{N_c-1}$ shift symmetry conserved so long as there are no magnetic monopoles in theory.

Beyond perturbation theory

Unsal,
Yaffe, Shifman,
Poppitz,
Sulejmanpasic,
Zhitnitsky
...

Thanks to dynamical Abelianization of SU(N_c) gauge symmetry, BPST instanton fractionalizes into N_c constituents

$$\mathcal{M}_i \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_i \cdot \vec{\sigma}}$$

assuming no massless fermions.

Proliferation of these events — which carry magnetic charge — in Euclidean vacuum gives a gap to dual photons

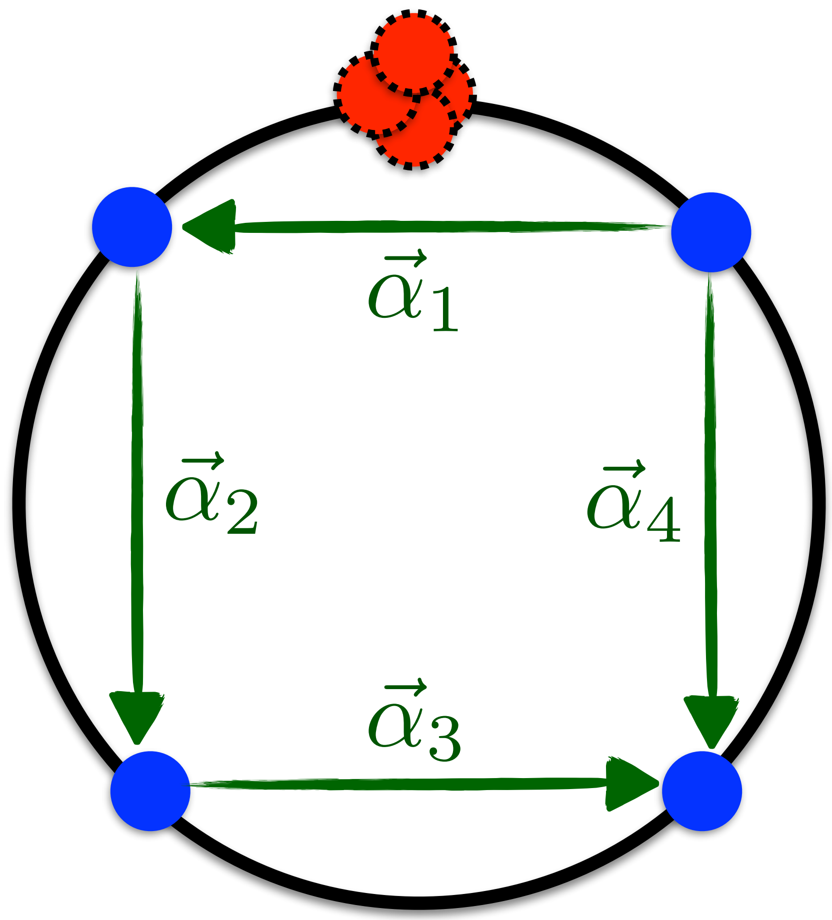
$$V(\vec{\sigma}) = m_W^3 e^{-\frac{8\pi^2}{g^2 N_c}} \sum_i \cos(\vec{\alpha}_i \cdot \vec{\sigma})$$

Massless fermions make things more subtle due to fermion zero modes on monopole-instantons

Fermion zero modes

van Baal + collaborators, 1999

Without Z_{N_f} twist, collective hopping phenomenon:



$N_c = N_F = 4$

$$\mathcal{M}_1 \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_1\cdot\vec{\sigma}} \ \det_{a,b}\left[\bar{\psi}_{L,a}\psi_{R,b}\right]$$

Invariant under $SU(N_F)_L x SU(N_F)_R x U(1)_Q$, but not $U(1)_A$

All $2N_f$ ‘instanton’ zero modes stick to a single monopole-instanton

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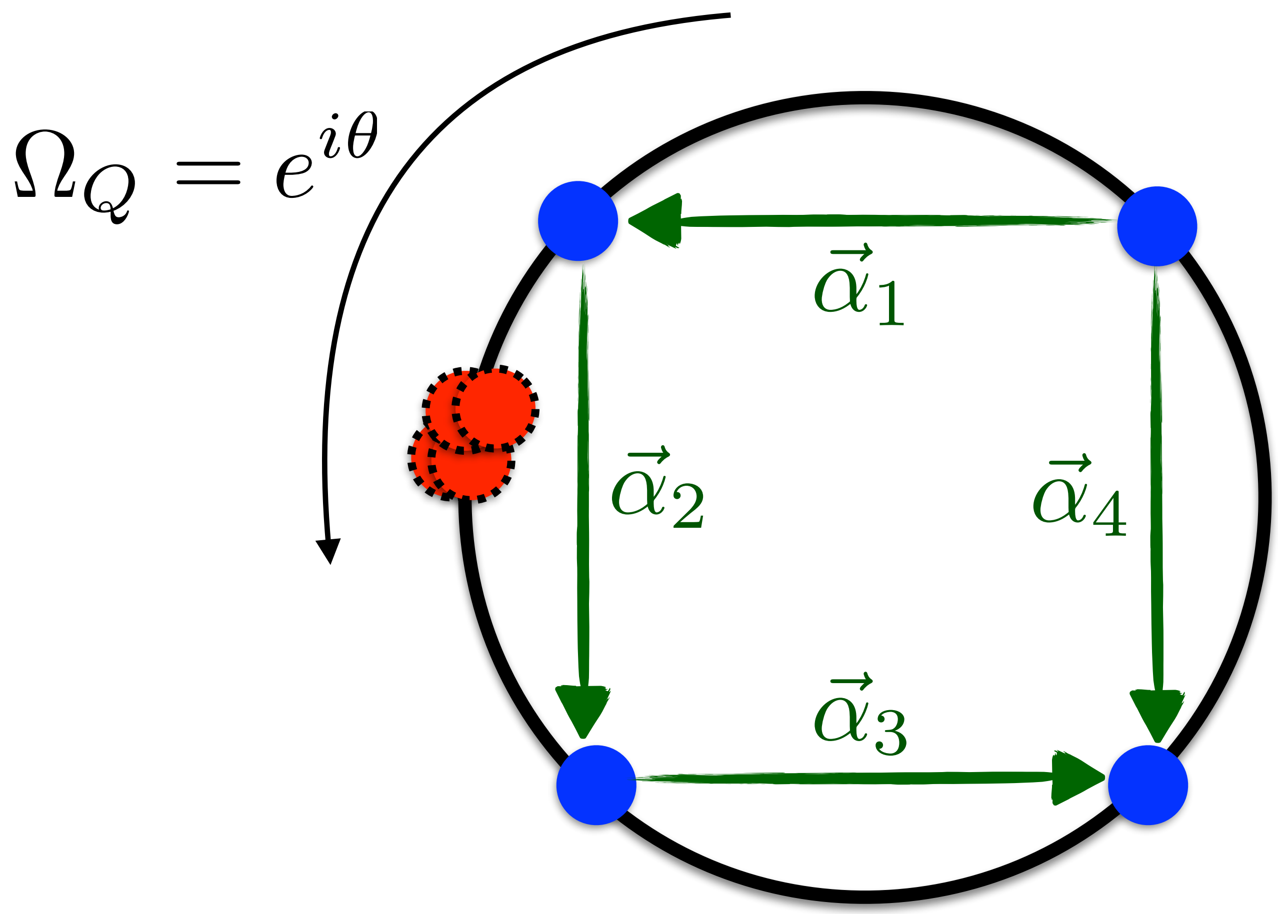


$N_c = N_F = 4$

$$\mathcal{M}_2 \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_2\cdot\vec{\sigma}} \det_{a,b}\left[\bar{\psi}_{L,a}\psi_{R,b}\right]$$

Invariant under $SU(N_F)_L \times SU(N_F)_R \times U(1)_Q$, but not $U(1)_A$

All $2N_f$ 'instanton' zero modes stick to a single monopole-instanton

Fermion zero modes

Localization of all $2N_F$ fermion zero modes means
3D EFT is a sort of weakly-coupled 3D NJL model

$$S \sim \int d^3x \left(\sum_a \bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} e^{i\vec{\alpha}_1 \cdot \vec{\sigma}} \det_{a,b} \bar{\psi}_a \psi_b \right)$$

Known NOT to produce χ-SB, except at strong coupling,
where it's out of systematic control.

So if we set $\Omega_F = 1$, there must be a chiral transition
between small L and large L regimes of deformed QCD!

Then for $N_F > 1$, small and large L regimes not smoothly connected.

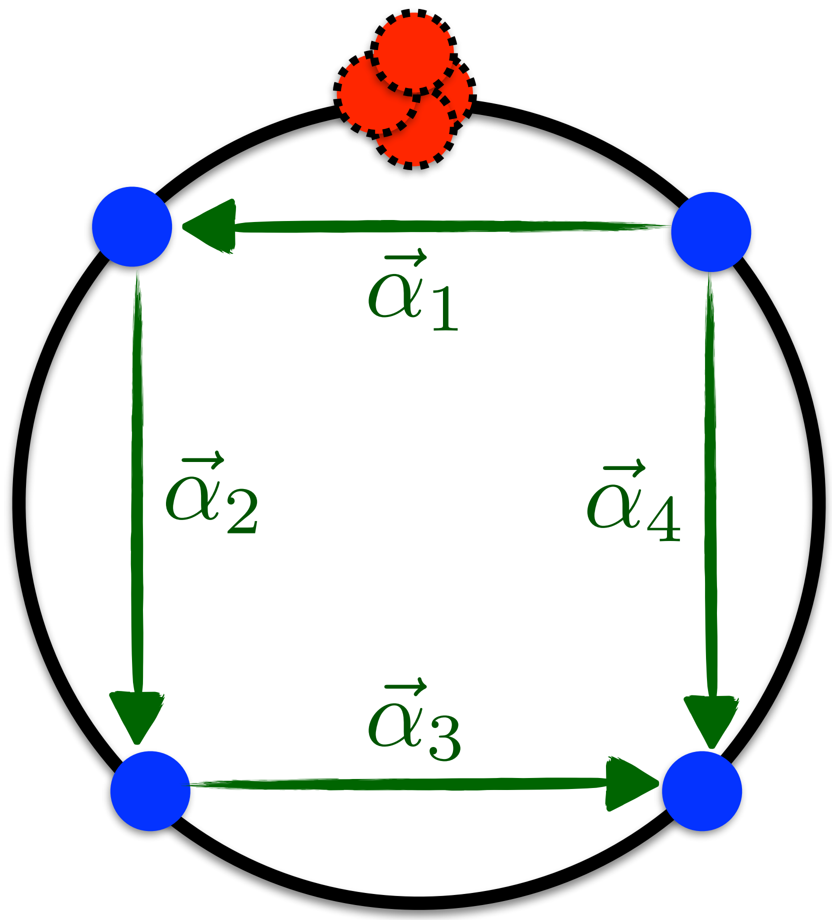
Shifman+Unsal
2009

Fermion zero modes

With $\mathbb{Z}_{N_f}$ twist

AC, Schafer, Unsal, 2016

using index theorem of Poppitz+Unsal 2008



$N_C = N_F = 4$

Fermion zero modes

With $\mathbb{Z}_{N_f}$ twist

AC, Schafer, Unsal, 2016

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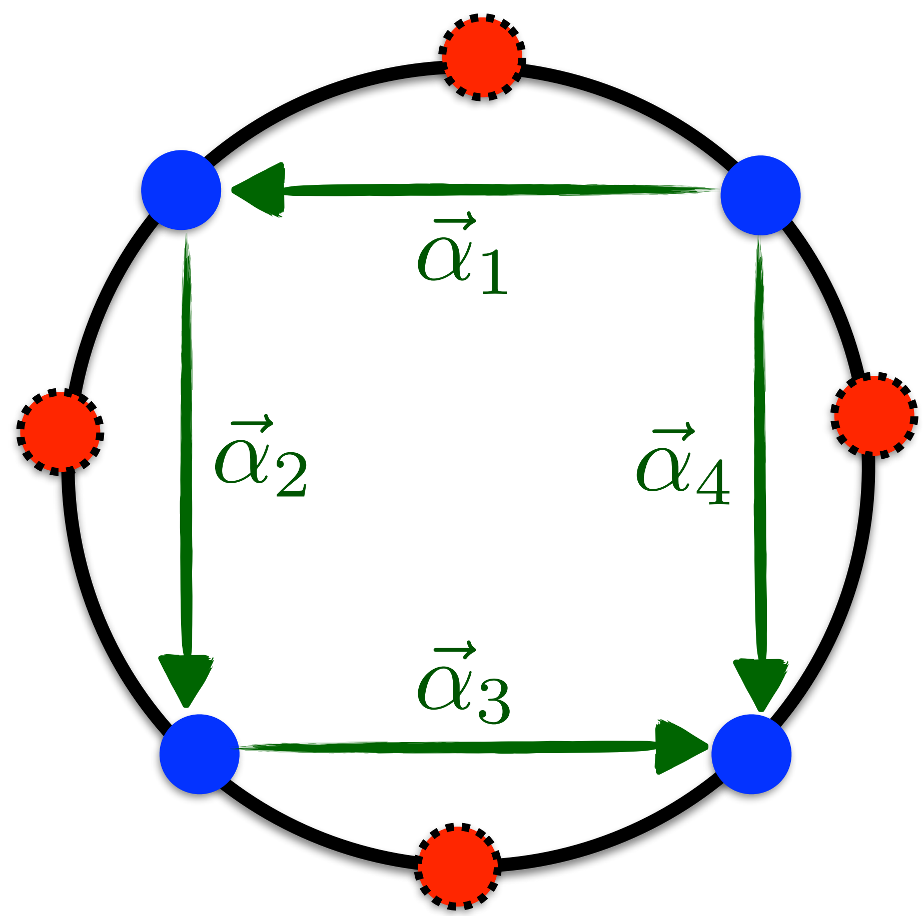
$N_c = N_F = 4$



$$\mathcal{M}_i = e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} (\bar{\psi}_{L,i}\psi_{R,i}), \qquad i = 1, \ldots N_f$$

In fact this drives chiral symmetry breaking!

Broken and unbroken symmetries

Before taking into account NP effects,
symmetry for gluons and fermions is

$$[U(1)_J]^{N_c-1} \times U(1)_V^{N_F-1} \times U(1)_A^{N_F-1} \times U(1)_Q$$

Symmetry only in
perturbation theory,
not sacred.

Subgroup of anomaly-free symmetry

Must be respected by all
effective vertices in theory

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

At NP level, must understand symmetries preserved by

$$e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_k\cdot\vec{\sigma}}(\bar{\psi}_{L,k}\psi_{R,k})$$

$[U(1)_V]^{Nf-1} x U(1)_Q$ is obvious. What about axial transformations?

$$U(1)_A^{N_F-1} : \quad (\bar{\psi}_{L,k}\psi_{R,k}) \to e^{i\epsilon_k}(\bar{\psi}_{L,k}\psi_{R,k})$$

[ϵ_k have single linear constraint to account for $U(N_F)_A \to SU(N_F)_A$]

Monopole-instanton vertex naively not invariant?!

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

At NP level, must understand symmetries preserved by

$$e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_k \cdot \vec{\sigma}} (\bar{\psi}_{L,k} \psi_{R,k})$$

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$$U(1)_A^{N_F - 1} : \quad (\bar{\psi}_{L,k} \psi_{R,k}) \to e^{i\epsilon_k} (\bar{\psi}_{L,k} \psi_{R,k})$$

Monopole-instanton vertex invariance **requires**

$$\begin{aligned} (\bar{\psi}_{L,k} \psi_{R,k}) \quad &\to \quad e^{i\epsilon_k} (\bar{\psi}_{L,k} \psi_{R,k}), \\ e^{i\vec{\alpha}_k \cdot \vec{\sigma}} \quad &\to \quad e^{-i\epsilon_k} e^{i\vec{\alpha}_k \cdot \vec{\sigma}}. \end{aligned}$$

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

So monopole-instanton operators are of course invariant under

$$U(1)_V^{N_F-1} \times U(1)_A^{N_F-1} \times U(1)_Q$$

The "cost" is that N_F - 1 dual photons pick up an exact shift symmetry, coming from intertwining of topological and axial symmetries

They remain exactly massless, even at non-perturbative level.

All topological molecules have uncompensated fermi zero modes. No "magnetic bions" exist here.

Where is the promised chiral symmetry breaking?

Chiral symmetry breaking

AC, Schafer, Unsal, 2016

Gapless dual photons are precisely the “pions”

The dual photons transform under $[U(1)_A]^{N_f-1}$.
Giving them any VEV - including zero - breaks chiral symmetry.

It also immediately produces non-perturbative chiral-symmetry-breaking constituent quark masses, as expected from models:

$$S \sim \int d^3x \sum_a \left(\bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} e^{i\vec{\alpha}_a \cdot \vec{\sigma}} \bar{\psi}_a \psi_a \right)$$

$$\Downarrow$$

$$S \sim \int d^3x \sum_a \left(\bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} \bar{\psi}_a \psi_a \right)$$

First systematic derivation of constituent quark mass we’re aware of.

Chiral Lagrangian

AC, Schafer, Unsal, 2016

Turning on a small quark mass m_q gives $m_\pi \sim m_q^{1/2}$, since soaking up zero modes with quark mass insertion gives

$$S_{m_q} \sim \int d^3x \left[m_W^2 m_q e^{-\frac{8\pi^2}{\lambda}} e^{i\vec{\alpha}_k \cdot \vec{\sigma}} + \text{h.c.} \right]$$

Theory satisfies expected GMOR relation $m_\pi^2 f_\pi^2 = m_q \langle \bar{\psi}\psi \rangle$

In fact dual photon action can be written as

$$S_\sigma = L \int d^3x \left[\frac{f_\pi^2}{4} \text{Tr}\, \partial_\mu \Sigma' \partial^\mu \Sigma'^\dagger - c\, \text{Tr}\, (M_q^\dagger \Sigma' + \text{h.c.}) \right]$$

Σ' is usual chiral field restricted to maximal torus, as expected from large L. But at small L f_π is **calculable**:

$$f_\pi^2 = \left(\frac{g}{\pi L \sqrt{6}} \right)^2 = \frac{N_c \lambda m_W^2}{24\pi^4}$$

Conclusions

Found small L limit of QCD with systematically calculable χ-SB, at weak coupling and low monopole-instanton density

χ-SB driven by “condensation” of monopole-instantons, which induces a chiral condensate.

Pions mapped to dual photons, constituent quark masses come for free.

Supports continuity between large and small L

Open questions:

$N_f > N_c$? Other fermion representations? Other gauge groups?

χ-SB in a chiral gauge theory?

…

The End