Minneapolis, May 12, 2016

Pentagon OPE and scattering amplitudes

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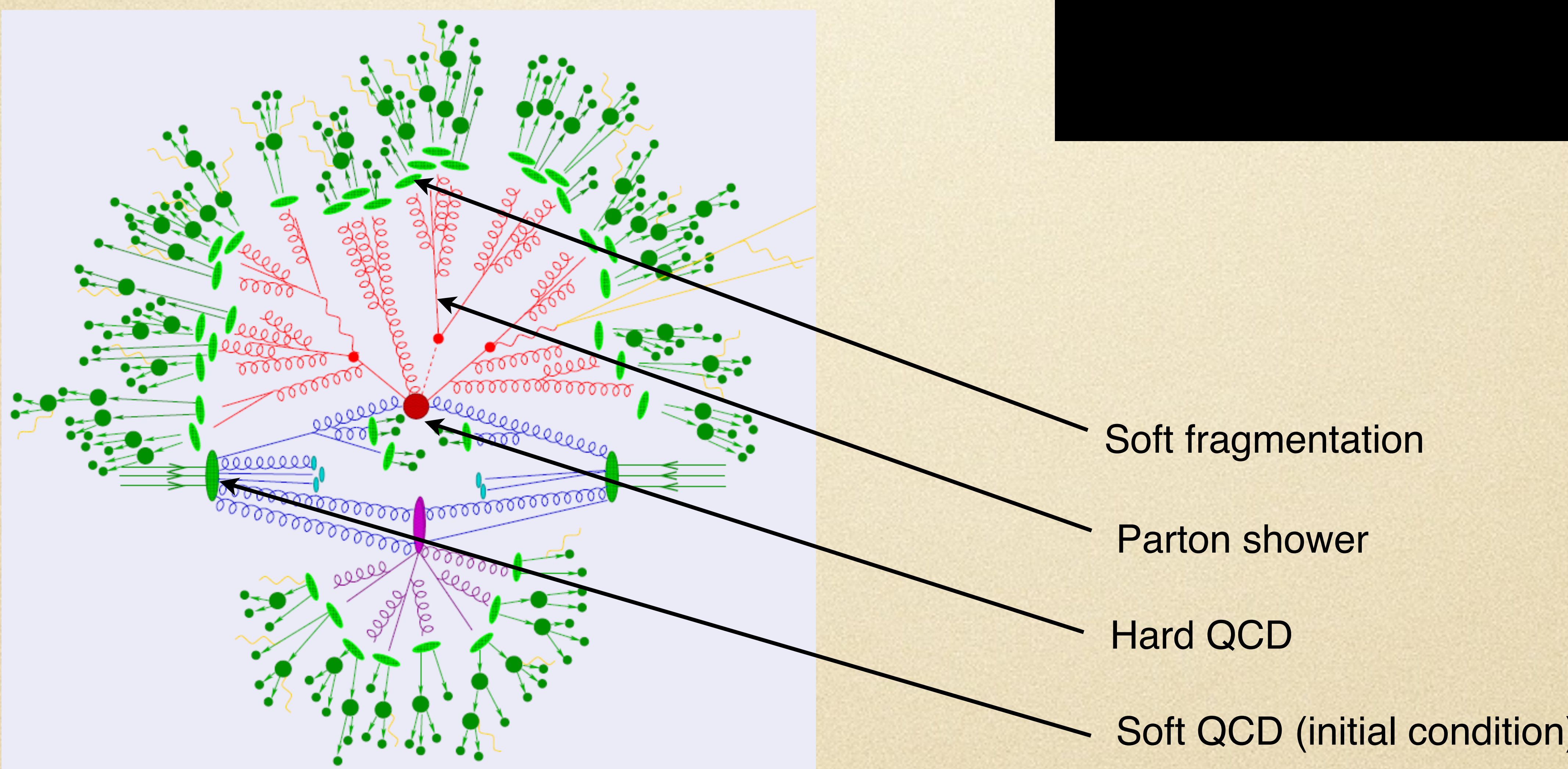
ASU

Scattering events @ LHC

- ✔ Specific feature of proton colliders – lots of produced quarks and gluons in the final state leading
- ✔ Identification of New Physics signals requires detailed understanding of scattering amplitudes for many particles
- ✔ The clouds of soft interactions obscure hard QCD amplitudes at their core, but also make physics much richer



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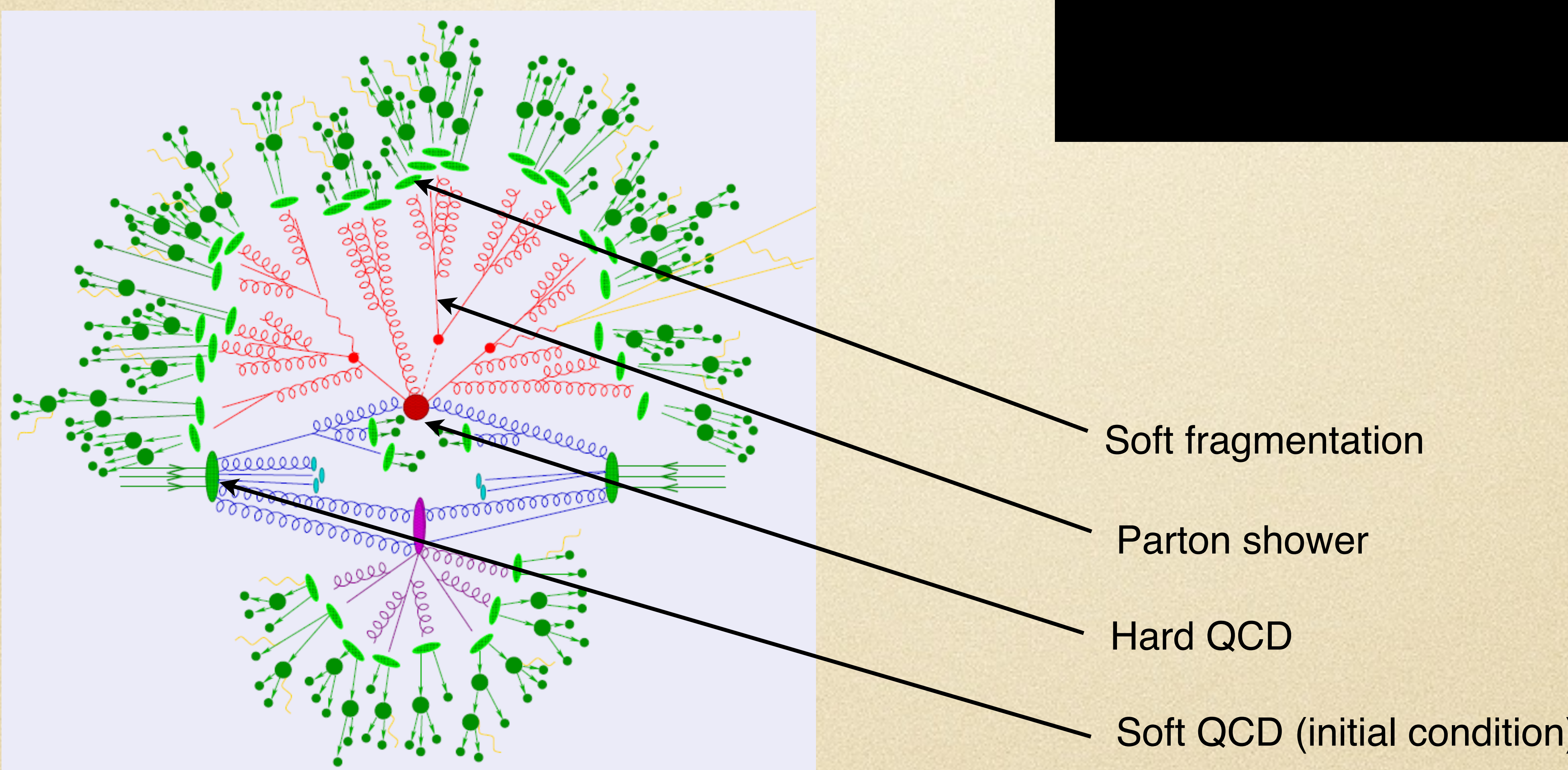
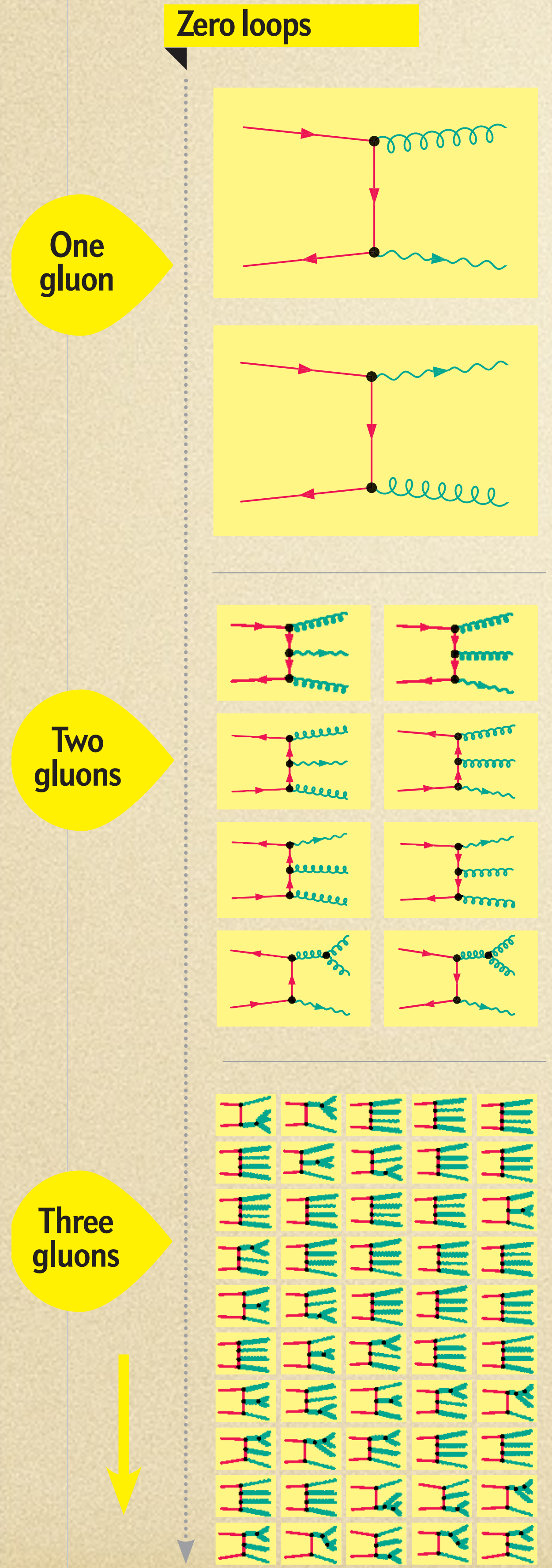


Feynman toolkit



Many legs …

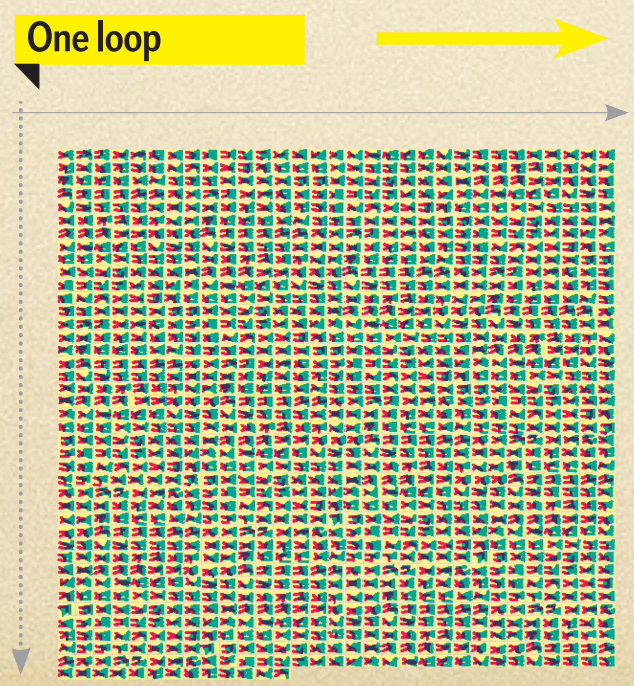
Number of external gluons	4	5	6	7	8	9	10
Number of ‘tree’ diagrams	4	25	220	2485	34300	559405	10525900

✔ Number of diagrams grows factorially for large number of external gluons

✔ If one spends 1 second for each diagram then the computation of 10 gluon amplitude will take 121 days!

✔ Little hope to get exact analytical solution ...

The story gets worse when one includes perturbative loop corrections! Loops represent one of the quintessential features of quantum theory: virtual particles.

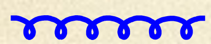


- Feynman diagrams are not optimized for the processes with many particles involved
- Important to find more efficient methods making use of hidden symmetries

The harmonic oscillator of 21st century

Maximally supersymmetric Yang-Mills theory (MSYM)

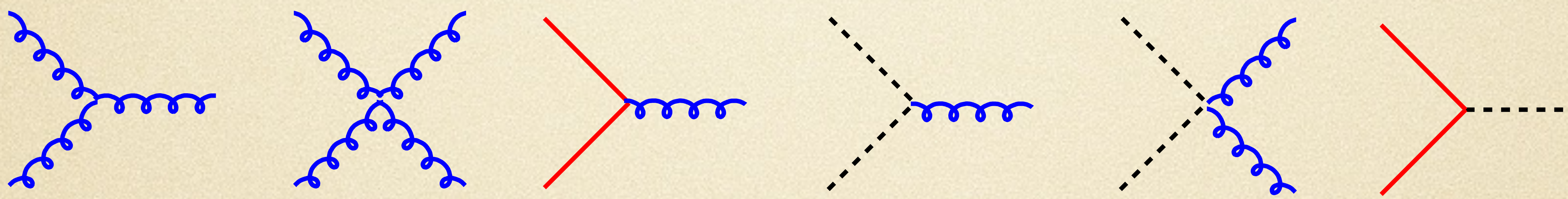
- ✔ Particle content:

 massless spin 1 gluon (= the same as in QCD)

 4 massless spin 1/2 gluinos (= cousin of the quarks)

 6 massless spin 0 scalars

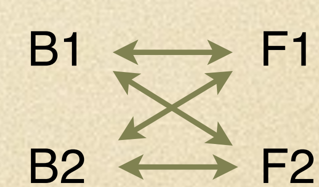
- ✔ Interaction between particles:



- ✔ All proportional to same dimensionless coupling λ and related to each other by supersymmetry

  - ✔ Most (super)symmetric theory possible (without gravity)

Supersymmetry: boson $\longleftrightarrow$ fermion   Many supersymmetries:



  - ✔ Uniquely specified by local internal symmetry group - e.g. number of colors N_c for $SU(N_c)$
  - ✔ Exactly scale-invariant field theory for any coupling λ (Green functions are powers of distances)

Why is it interesting?

- ✔ Four-dimensional gauge theory with extended spectrum of physical states/symmetries
- ✔ An excellent testing ground for QCD in the perturbative regime relevant for collider physics
- ✔ Is equivalent to QCD at *tree level* and serves as one (most complicated) piece of QCD *all-loop* computation
- ✔ Why $\mathcal{N}=4$ SYM theory is fascinating?
  - ✗ *At weak coupling*,
    - the number of contributing Feynman diagrams is *MUCH* bigger compared to QCD
    - ... but the final answer is *MUCH* simpler
  - ✗ *At strong coupling*, the conjectured gauge/string duality (AdS/CFT correspondence)

    Strongly coupled planar $\mathcal{N}=4$ SYM $\Longleftrightarrow$ *Weakly coupled 'dual' string theory on* $\mathrm{AdS}_5 \times \mathrm{S}^5$
- ✔ Final goal (dream):

Maximally supesymmetric Yang-Mills theory is an example of the four-dimensional gauge theory that can be/ should be/ would be solved exactly for arbitrary value of the coupling constant!!!

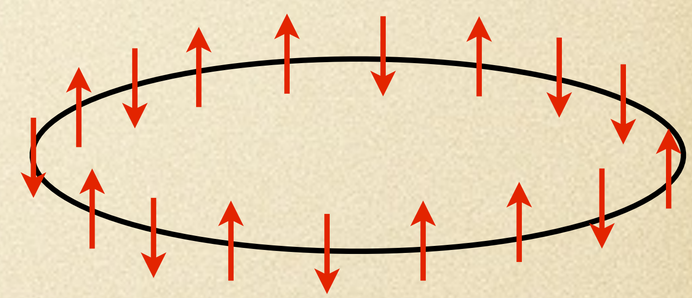
MSYM as a Heisenberg spin chain

- ✔ MYSM is an integrable theory! Implies that we can find energies of all composite "particles" in the theory exactly.
- ✔ Serves as a proof of the gauge/string duality conjecture!
  - ✔ Introduce "composite" particles

$$O_{s_1,s_2,\ldots,s_L} = \mathrm{tr}[(\partial^{s_1} X_1)(\partial^{s_2} X_2)\ldots(\partial^{s_L} X_L)]$$

  - ✔ Particles obey Schroedinger-like equation

$$\frac{d}{d\tau} O = H * O$$



  - ✔ The "Hamiltonian" can be mapped into a generalized Heisenberg magnet

    i.e., generalization of the spin-spin interaction $$H = -\sum_{j=1}^{L} \vec{\sigma}_j \vec{\sigma}_{j+1}$$

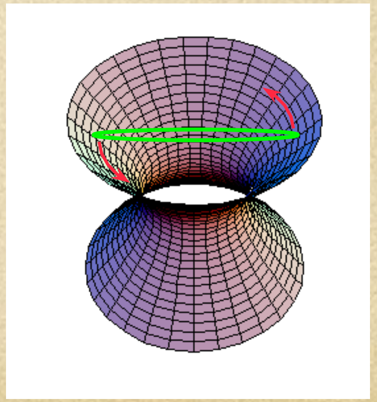
  - ✔ The eigenvalues of *H* are found exactly (via ABA) as a function of the coupling/quantum numbers:

$$E(s_i) = \ln(s_1 + \cdots + s_L)\Gamma(\lambda) + \sum_n E(u_n) + \ldots$$

AB, Gorsky, Korchemsky '06
Basso '09

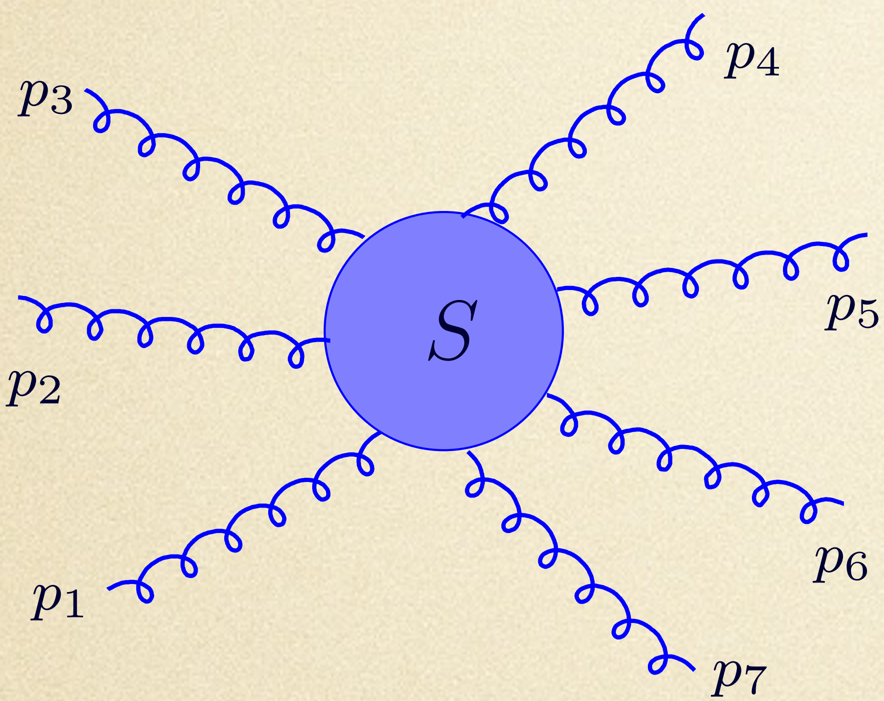
flux-tube vacuum — flux-tube excitations

  - ✔ Weak coupling $$\Gamma(\lambda) = 4\lambda - 8\zeta_2\lambda^2 + \ldots$$
  - ✔ Strong coupling $$\Gamma(\lambda) = 2\sqrt{\lambda} - 3\log 2/(2\pi) + \ldots$$



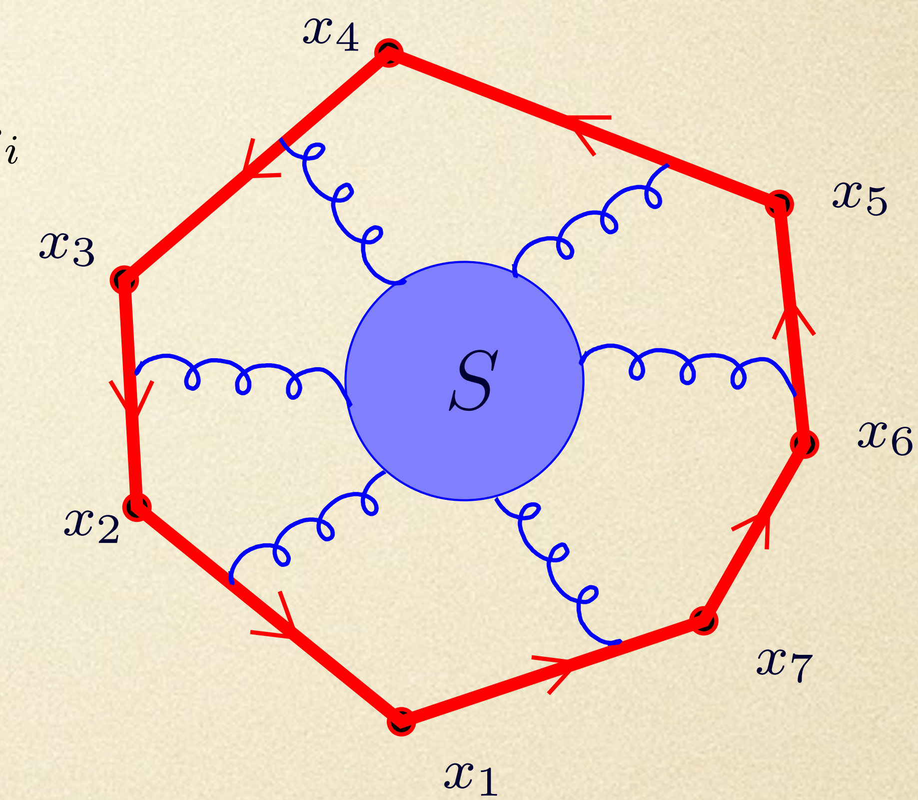
- ✔ **In fact, we know it at any coupling!!!**

Dual description of scattering amplitudes



$p_i \equiv x_{i+1} - x_i$

$\Longleftrightarrow$



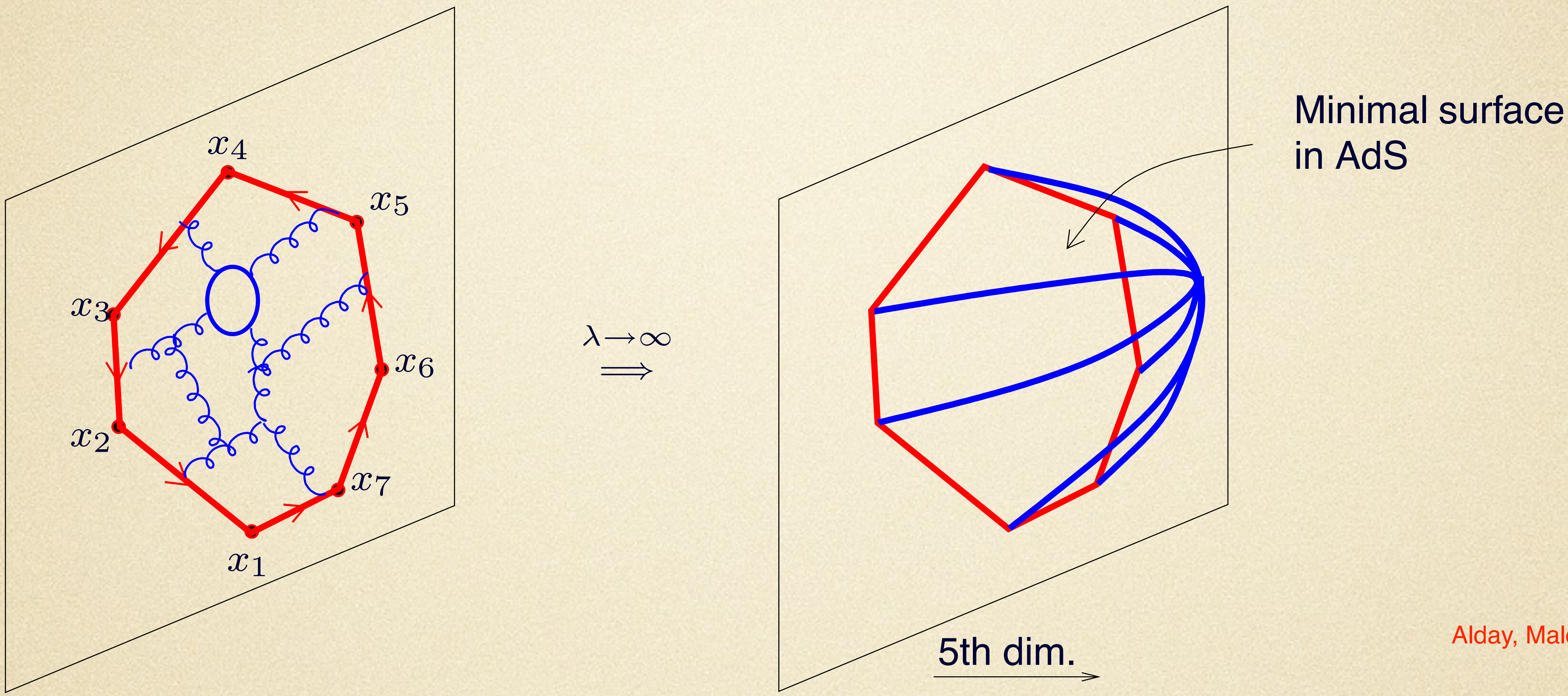
Alday, Maldacena '07
Drummond, Henn, Korchemsky, Sokatchev '07

- ✔ Circulation of nonabelian gauge field along a null polygonal contour in dual coordinate space

$$W_N = P \exp\left(ig \oint_{C_N} dx_\mu A_\mu(x) \right)$$

- ✔ Generalizes to all amplitudes (i.e., all external states) by promoting the loop into superspace
- ✔ Weak coupling expansion in $\mathcal{N}=4$ SYM (back to Feynman diagrams): duality works perfectly!
- ✔ How to compute the Wilson loop at strong coupling? Gauge/string duality!

Strong coupling



Alday, Maldacena '07

The Wilson loop at strong coupling from gauge/string duality

$$\text{Wilson loop} \sim \exp\left(-\sqrt{\lambda} \times \text{Area}\right)$$

✔ Defined by the area of minimal surface in anti-de Sitter space

✔ The surface ends at the AdS boundary on a polygon given by a sequence of gluon momenta

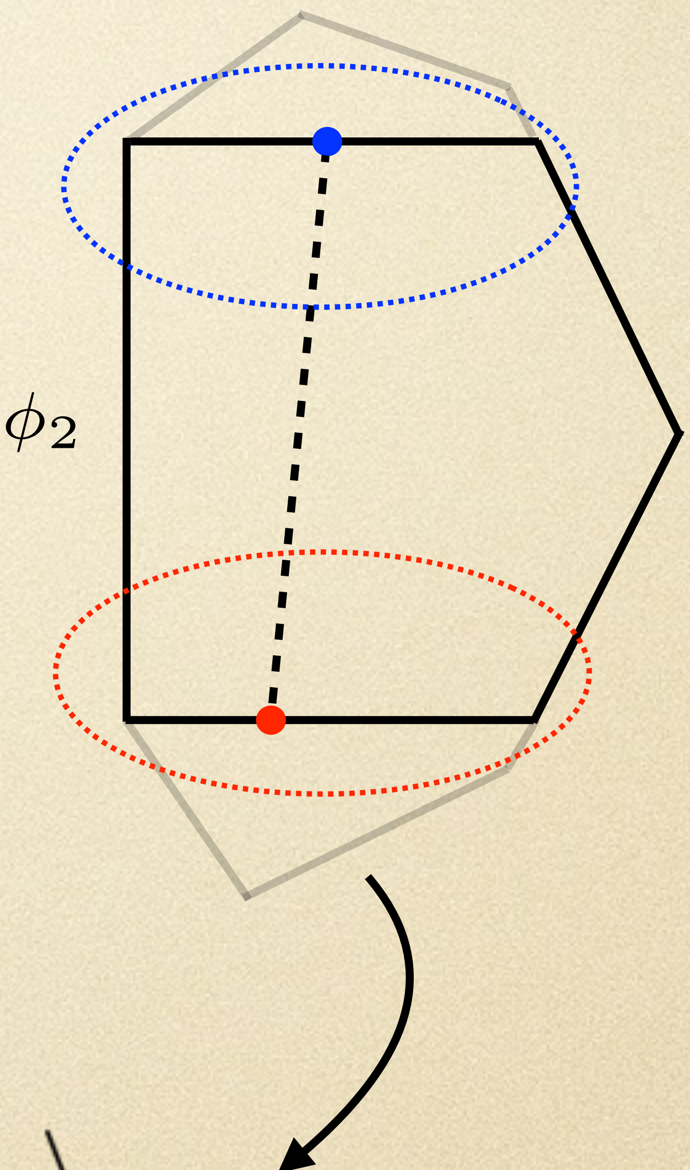
Explicit solution (of the Plateau's problem) for the surface of an arbitrary polygon is extremely complicated (not known to date). However, it can be bypassed by formulating TBA directly for the area!

Towards Wilson loop @ finite coupling

Consider an example…

Conformal remainder of the heptagon is a function of 6 conformal cross ratios $(\tau_i, \sigma_i, \phi_i), \; i = 1, 2$

$$\overset{\tau_i \to \infty}{=} \mathrm{e}^{-\tau_1-\tau_2}\mathrm{e}^{-i\phi_1-i\phi_2}$$



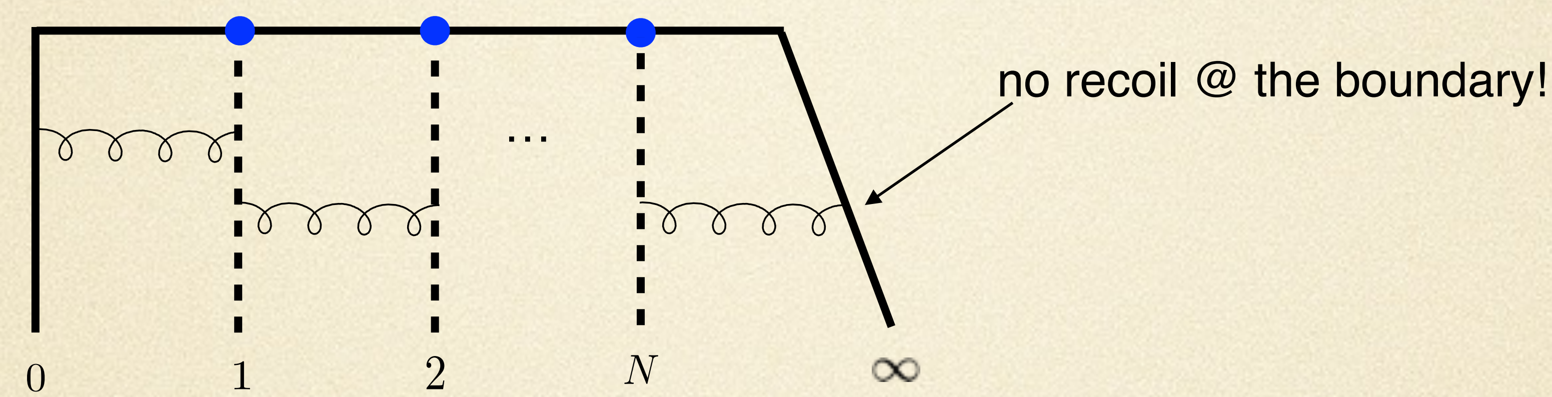
$$\langle O_{\Pi_{\mathbf{top}}} O_{\Pi_{\mathbf{bottom}}} \rangle$$

Correlation function of Pi-shaped Wilson lines with gauge field insertion.

Open spin chain for the flux tube

The correlation function can be computed by finding the eigenfunction of the flux-tube Hamiltonian

AB, Derkachov, Manashov '13



✔ Total (integrable) Hamiltonian is given by pair-wise form at leading order in coupling

$$H = H_{01} + H_{12} + \cdots + H_{N\infty}$$

✔ Diagonalization can be done with the help of SoV formalism (no ground state $\to$ no ABA)

$$H|\psi_N\rangle = E_N|\psi_N\rangle$$

✔ Integrability yields diffractionless spectrum

$$E_N = E(p_1) + \cdots + E(p_N) \qquad E(p) = \psi(s + ip/2) + \psi(s - ip/2) - 2\psi(1)$$

✔ Overlap of eigenfunctions in different conformal frames result in factorized form of the correlation function

$$\langle O_{\Pi_{\rm top}} O_{\Pi_{\rm bottom}} \rangle \;\to P(u_1, \ldots, u_N | v_1, \ldots, v_N) = \frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i>j} P(u_i|u_j) \prod_{i<j} P(v_i|v_j)}$$

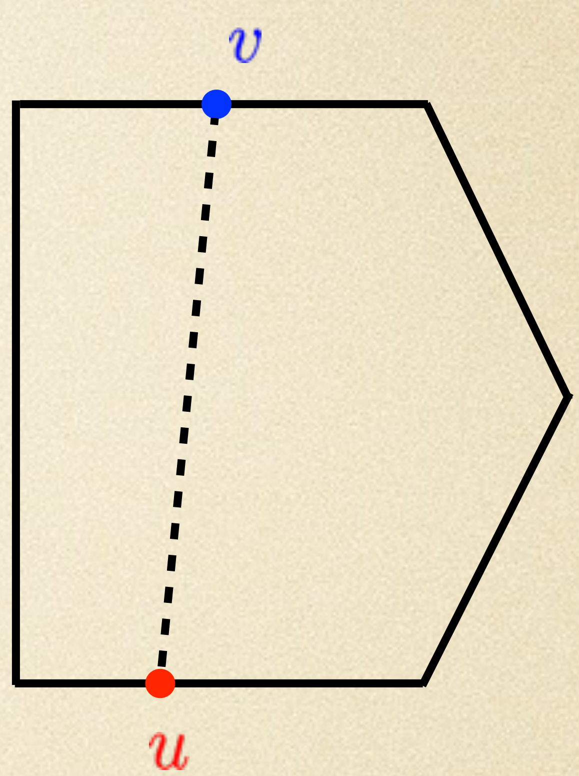
Rapidity is related to momentum as $u = p/2$

Pentagon transition

✔ Single-particle pentagon transition

$$P(u|v) = \langle \psi_v | T | \psi_u \rangle = \frac{\Gamma(2s)\Gamma(iu - iv)}{\Gamma(s+iu)\Gamma(s-iv)}$$

conformal transformation



✔ Obeys a “bootstrap” equation (not a Watson equation!)

$$\frac{P(u|v)}{P(v|u)} = S(u,v)$$

✔ The flux-tube scattering matrix

$$\psi \overset{\sigma_2 \gg \sigma_1}{=} \mathrm{e}^{2iu\sigma_1}\mathrm{e}^{2iv\sigma_2} + S(u,v)\mathrm{e}^{2iv\sigma_1}\mathrm{e}^{2iu\sigma_2}$$

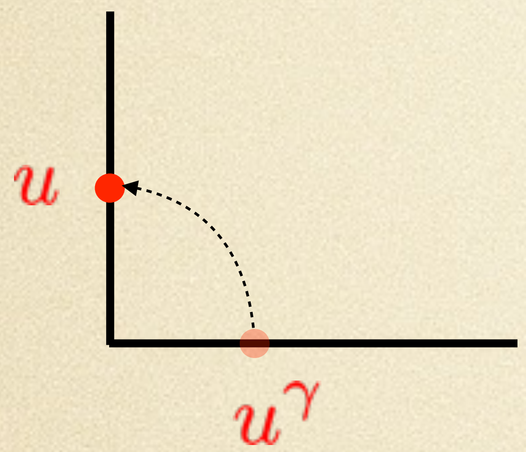
$$S(u,v) = \frac{\Gamma(iu-iv)\Gamma(s-iu)\Gamma(s+iv)}{\Gamma(iv-iu)\Gamma(s-iv)\Gamma(s+iu)}$$

✔ Consistency with Watson equation? Have to learn to move excitations from one side of the polygon to another!

Mirror transformation

✔ Analytic continuations in excitation's rapidity that interchanges space and time (i.e., double Wick rotation aka mirror transformation)

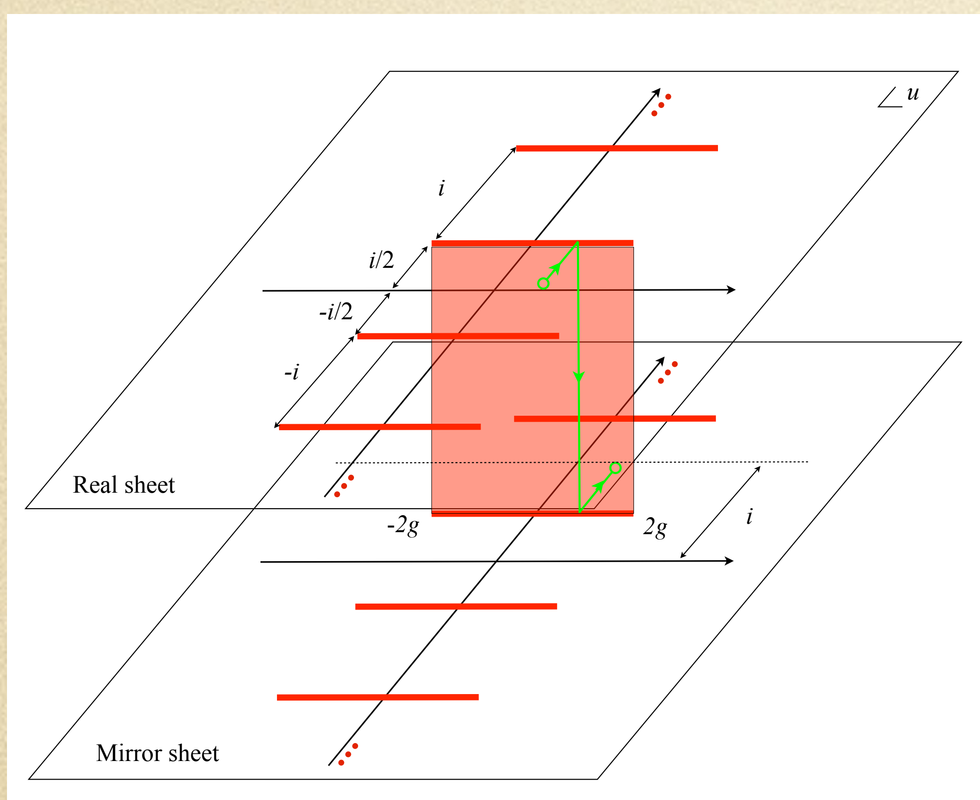
AB, Basso '11



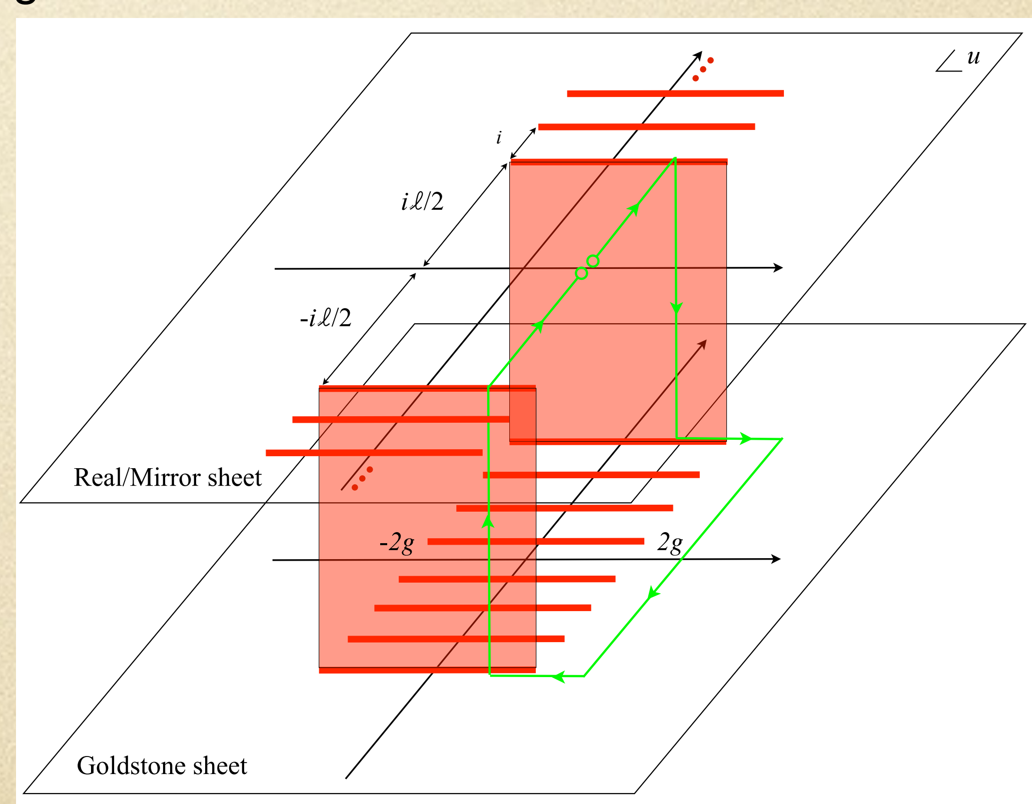
$$E(u^\gamma) = ip(u) \qquad p(u^\gamma) = iE(u)$$

✔ γ is non-perturbative in its origin since to reach the mirror kinematics one has to go through the cuts which open up only at finite coupling. E.g.,

scalars:



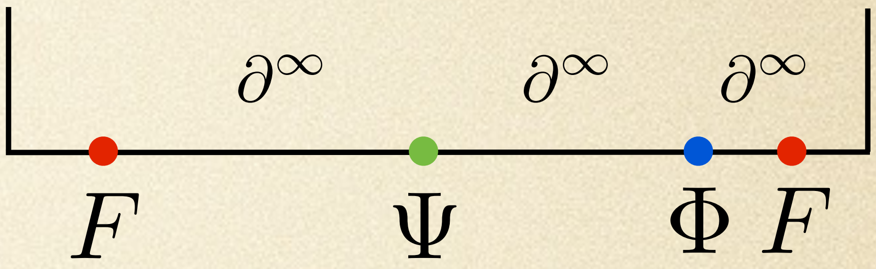
gluons:



Flux-tube eigenstates

✔ Multi-particle state $|\psi\rangle$:

adjoint fields inserted along the Wilson line, i.e., created on the flux-tube



✔ Spectral data:

Excitations exhibit diffractionless scattering and can be found using integrabillity.

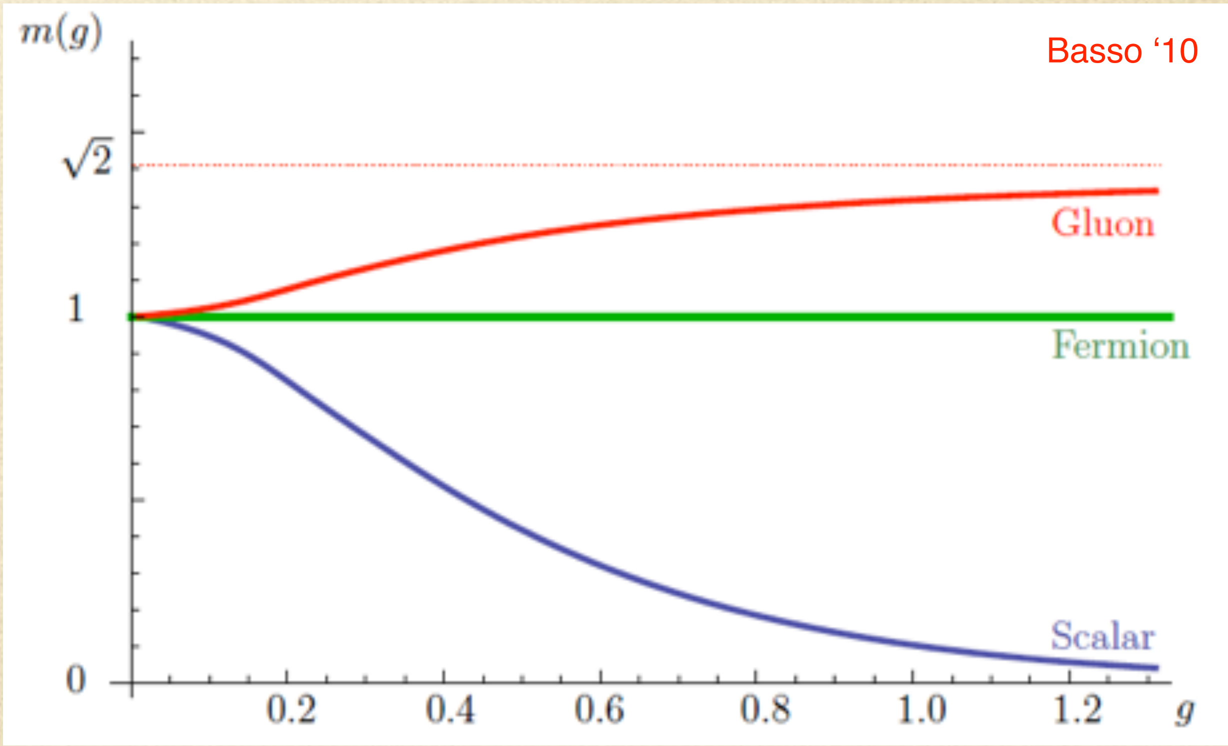
$$E_N(\psi) = E(u_1) + E(u_2) + \cdots + E(u_N) \qquad p_N(\psi) = p(u_1) + p(u_2) + \cdots + p(u_N)$$

✔ Weak coupling:

$$E(u) = 1 + g^2 \ldots$$
$$p(u) = 2u + g^2 \ldots$$

✔ Strong coupling:

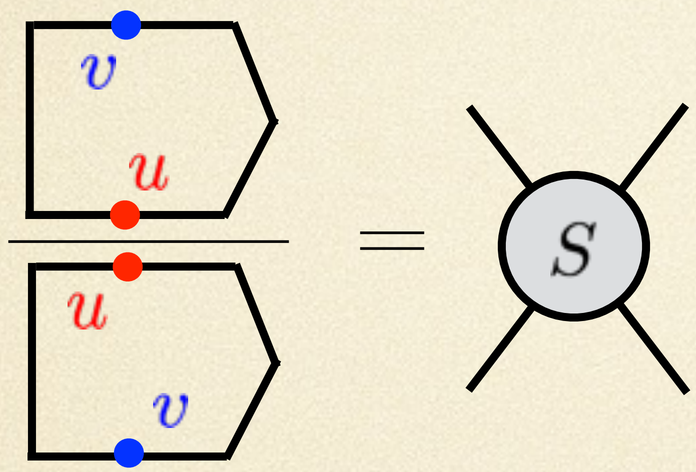
$$E(u) = m(g) \cosh \theta(u)$$
$$p(u) = m(g) \sinh \theta(u)$$



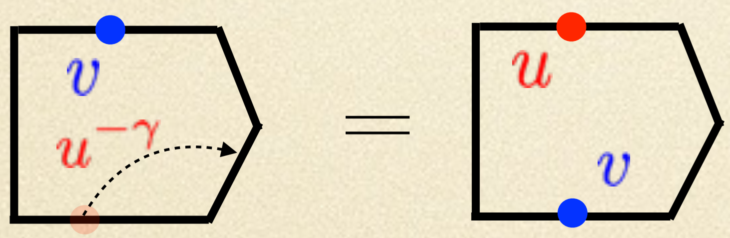
Pentagon bootstrap @ any coupling

✔ Watson-like equation:

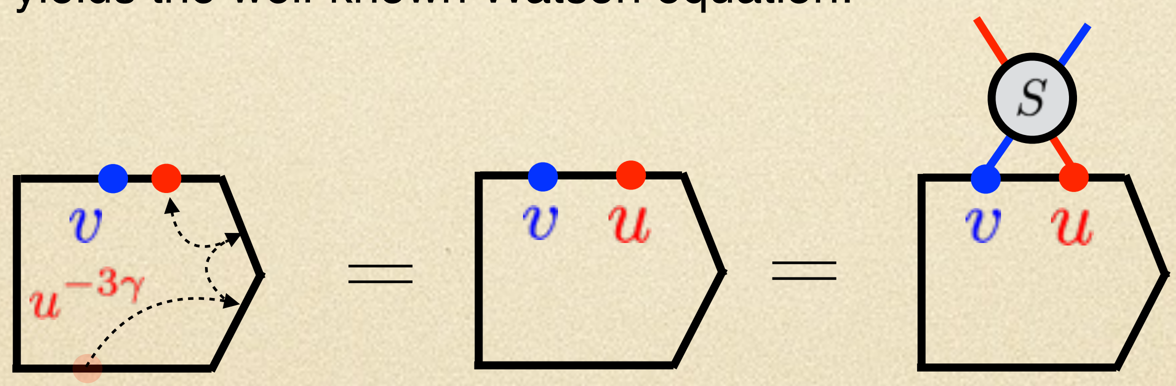
Basso, Sever, Vieira '13



✔ Mirror equation:



Consistency condition yields the well-known Watson equation:



Basso, Seve, Vieira, Caetano, Cordova '13-15
AB '13-14

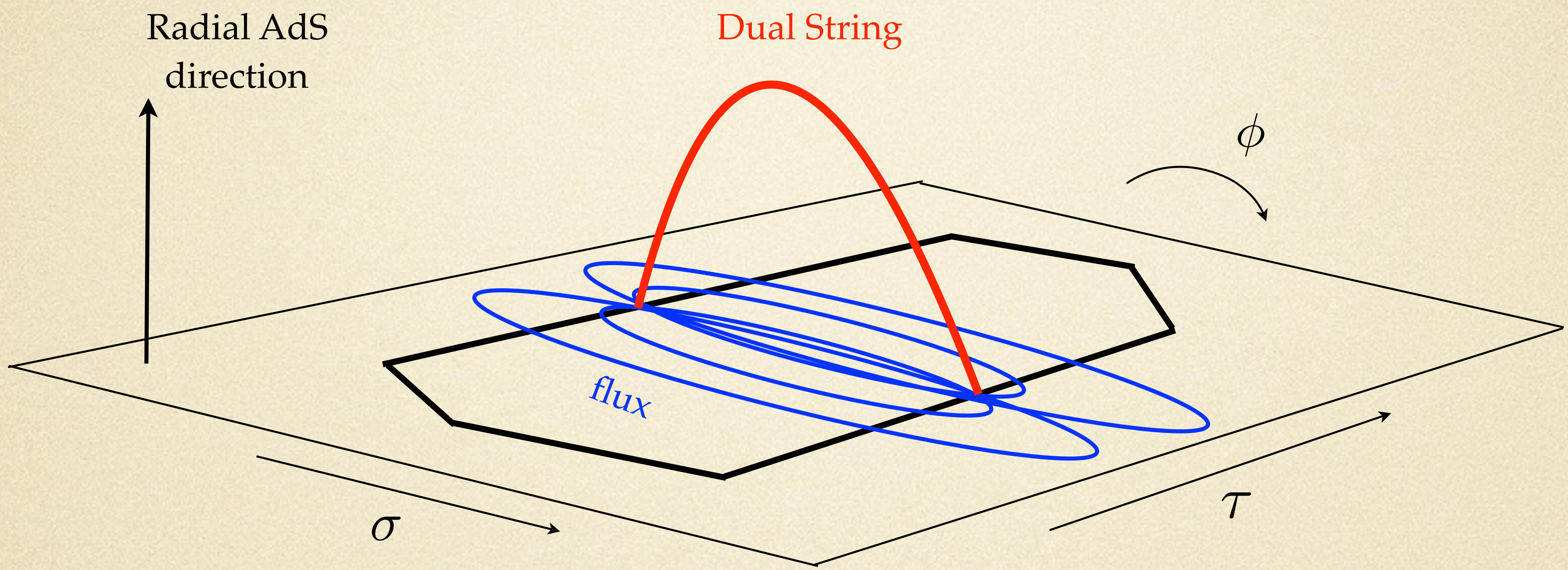
✔ Solution to bootstrap equations

$$P(u|v) = w(u,v)\left[\frac{S(u,v)}{S(u^{\gamma},v)}\right]^{1/2}$$

The most uncertain part of the solutions! Huge ambiguities! Once fixed at leading order, an all-order form is unique!

Wilson loop at finite coupling

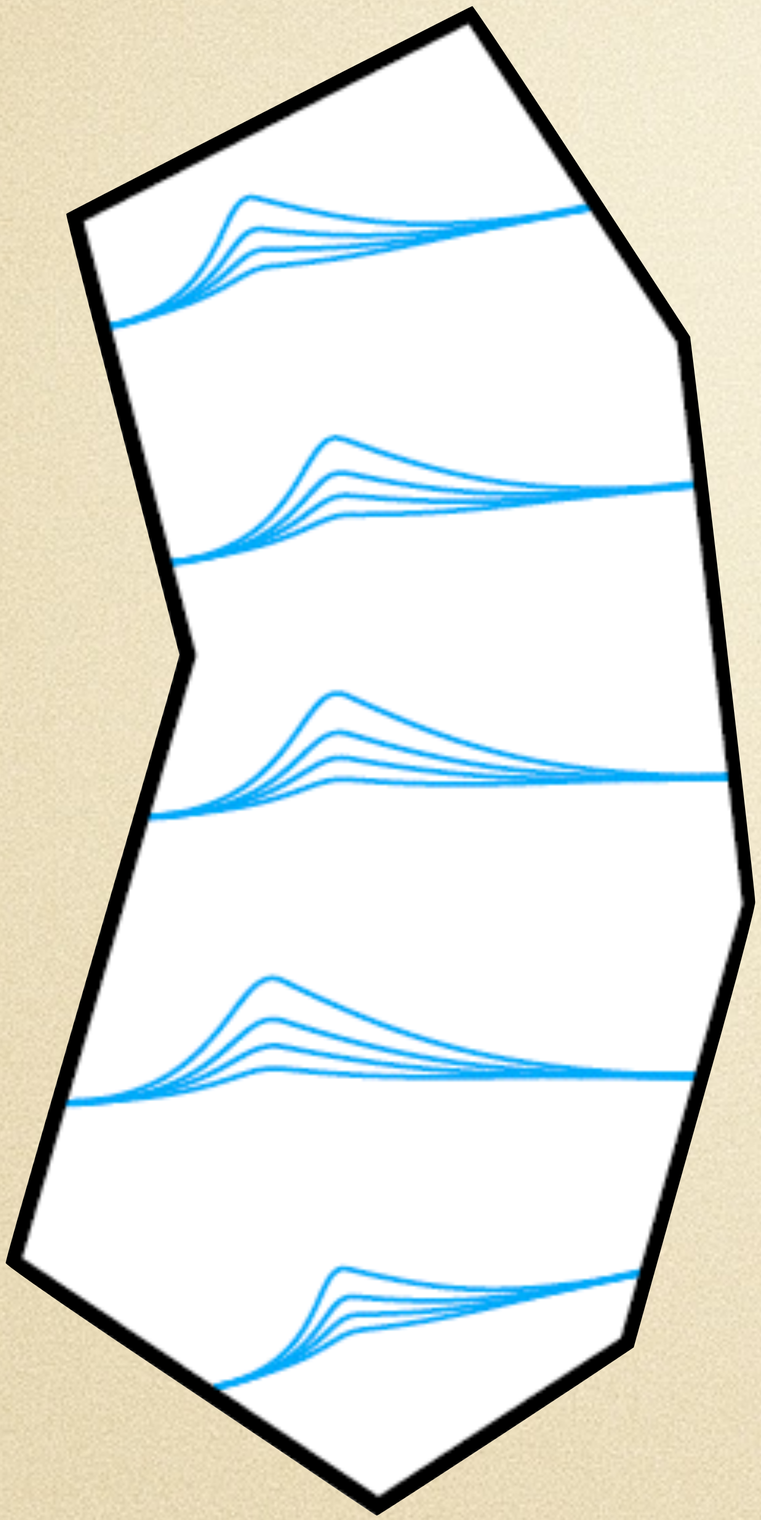
Alday, Gaiotto, Maldacena, Sever, Vieira '09



- Flux-tube is sourced by two light-like lines, i.e., chromo-charged particles propagating with speed of light
- Bottom/Top caps excite the flux-tube from its ground state, i.e., create/absorb flux-tube excitations
- Sum over all flux-tube states determines the Wilson loop/Scattering amplitude

$$\mathcal{W} = \sum_{\psi} C_{\rm bot}(\psi) \times \mathrm{e}^{-\tau E(\psi) + i\sigma p(\psi) + i\phi m(\psi)} C_{\rm top}(\psi)$$

Pentagon paradigm



✔ Tesselate a polygon into squares with light-like lines:

For n-side polygon, (n-3) middle squares (matches the number of external kinematical (geometrical) data)

✔ Two squares overlap in a pentagon, (n-4) of them

$$= \sum_{\psi_i} \left[\mathrm{e}^{-\tau_i E(\psi_i) + i\sigma_i p(\psi_i) + i m_i \phi(\psi_i)} \right]$$

$$\times P(0|\psi_1) P(\psi_i|\psi_2) P(\psi_2|\psi_3) P(\psi_3|0)$$

To calculate any amplitude at any value of the coupling on needs

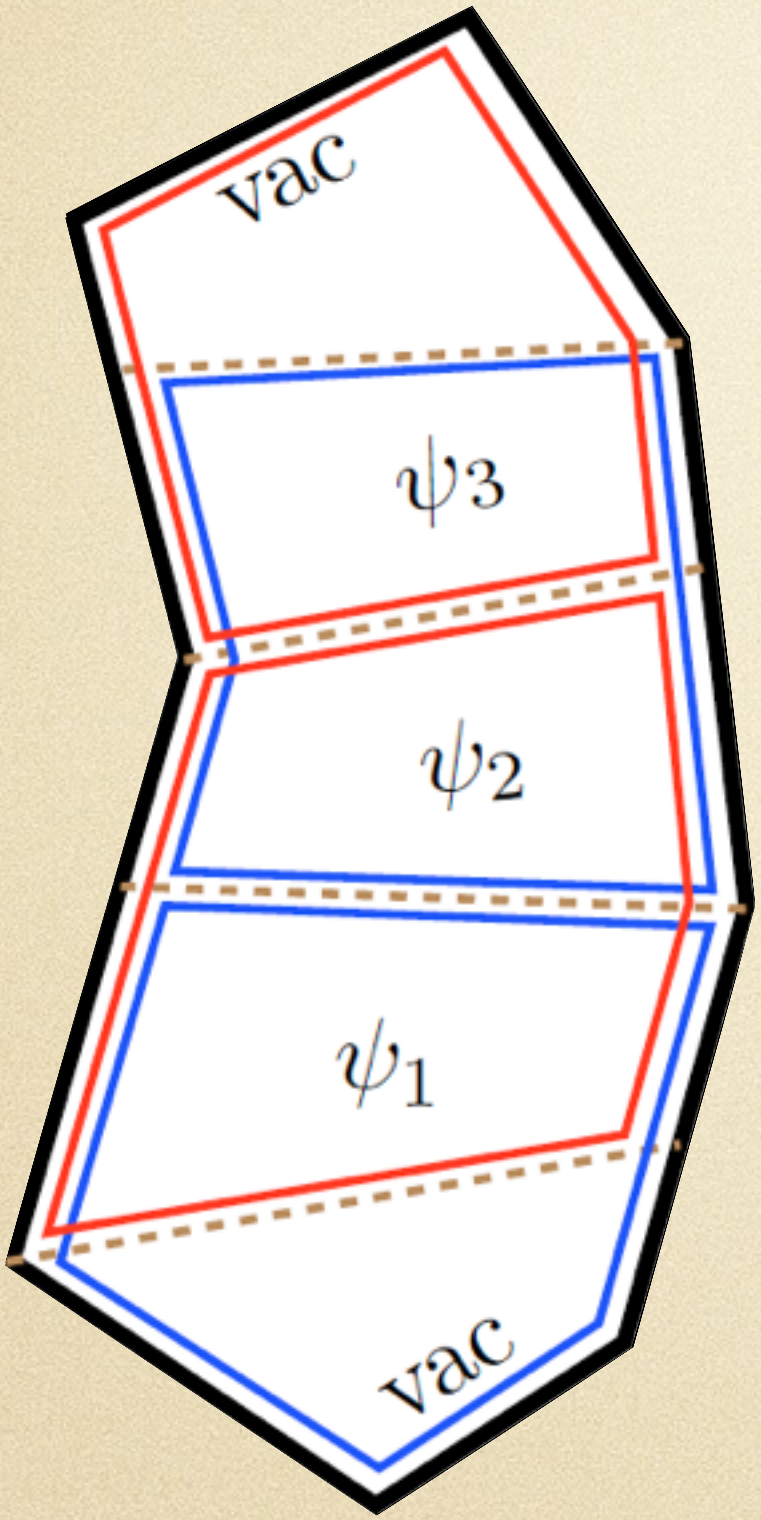
✔ The spectrum of flux-tube excitations (E, p)

✔ Pentagon transitions between all states

$$P(\psi_i|\psi_j) = \langle \psi_j | \widehat{\mathcal{P}} | \psi_i \rangle$$

This is the dynamical input! Geometry enters trivially through (τ, σ, ϕ)

Pentagon paradigm



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Verifications and predictions

- ✔ Bootstrap program explicitly verified against explicit calculations to four loops order by Dixon et al.

- ✔ At strong coupling it reproduces TBA and provides results beyond the minimal area approximation

- ✔ We are getting closer to solving the 4D (super)Yang-Mills theory exactly

- ✔ What lessons can be learned for QCD?

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Stay tuned!