

# Queuing and Statistical Analysis of Freeway Bottleneck Formation

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**Abstract:** A modified approach to treat traffic flow parameters (flow, density, and speed) has been introduced in this paper. A queuing analysis has been conducted on traffic flow data on Interstate 94 in the Minneapolis–St. Paul metro area. A methodology has been developed to calibrate loop detector count data. Corrected flow data has been subjected to analysis using queuing analysis to compute densities and speeds on freeway sections. Statistical analysis identifies “active bottleneck” locations on freeways and sections where bottlenecks occur because of disturbances caused by downstream bottlenecks propagating backwards in the form of shockwaves. A sample of 6 days on Interstate 94 was considered for the analysis. Our analysis reveals that the same section cannot always be characterized as a “bottleneck” location; at some times it is active and at others it is subject to downstream bottlenecks. Traffic flow characteristics change and that leads to changing situations on each freeway section.

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## Introduction

This paper has its roots in the Ramp Meter shut-off experiment that was conducted in the Twin Cities of Minneapolis and St. Paul during the fall of 2000. Along with determining the effectiveness of ramp metering in controlling freeway traffic, this experiment provided an opportunity to study traffic flow characteristics under both metered and unmetered conditions. This was made possible by the availability of a considerable amount of high quality loop detector data collected both during the period preceding the experiment and during the experiment. Our research applies queuing analysis to traffic data collected from loop detectors during unmetered conditions to determine the location of bottlenecks. It thus extends previous research by Cassidy and others (Cassidy 2001; Cassidy and Bertini 2001; Cassidy et al. 2002).

The paper also attempts to provide an alternate way of predicting active bottlenecks using queuing and statistical analyses. The importance of accurately predicting active bottlenecks can be hardly overemphasized. Recent studies by Aggemang-Duah and Hall (1991) show how determination of freeway capacity hinges on the location of active bottleneck. Active bottlenecks show traits of not being affected by traffic conditions downstream (Daganzo 1997) and that once an active bottleneck is formed, both

“rabbits and slugs” involved in the bottleneck move at the same low speed with a drastic drop in queue discharge rate (Cassidy and Chung 2002). Active bottlenecks can be predicted based on real-time analysis of data generated by loop detectors (Bertini 2003).

Inductive loop detectors (ILD)s typically measure flow (the number of vehicles that pass it in some time period) and occupancy (the percentage of time for which the ILD is occupied in that time period). But the data provided by such ILDs are often fraught with errors. ILDs undercount or overcount depending on freeway traffic conditions. Moreover, the accuracy and consistency of detector data depend strongly on their installation and calibration procedures. A loop detector with percentage accuracy within 5% is considered a “good” one (MDOT 2002). For a freeway with a daily output in the range of tens of thousands of vehicles, the 5% error can amount to hundreds or thousands of vehicles per day. In addition, an underlying assumption required to measure speed when only single loop detectors are available is that effective vehicle lengths are uniform (Jia et al. 2001). This research presents a queuing analysis of freeway traffic that does not rely on effective vehicle lengths following earlier theory (Lawson et al. 1997).

The next section describes the data used in the analysis and the method for balancing counts across detector stations. Then queuing analysis is linked with the fundamental diagram of traffic flow. A typology of traffic conditions and rules for detecting active bottlenecks are provided. Then our results are presented, which apply the rules to identify active bottlenecks, sections that are subject to downstream bottlenecks, and free-flowing sections. The paper concludes by noting that at different times, the same section may be either an active bottleneck or subject to one, consistent with the definition provided by Daganzo (1997).

## Data

The data used consisted primarily of 30-s flows and occupancies on freeways, aggregated to 5-min periods, to be consistent with

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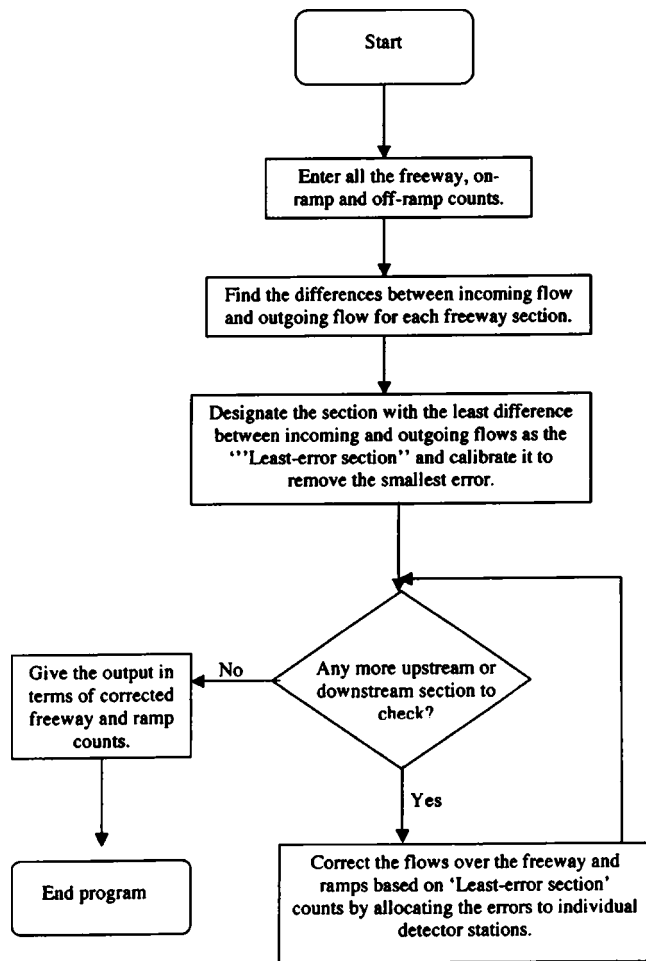


Fig. 3. Flowchart depicting count balancing process

After the corrected flows are obtained, the differences between the original counts and the new corrected counts are allocated to the individual detectors again according to the original flow proportions and to each 5 min time period. Mathematically,

$$C_{at} = \frac{Q_{at}}{Q_A} \times C_A \quad (4)$$

where  $Q_a$ =cumulative 24 h flow over detector a;  $Q_A$ =cumulative (24 h) flow over detector station, A, which includes all a detectors;  $C_A$ =correction needed for station, A; and  $C_{at}$ =correction needed in detector a count at time t.

Table 1. Correction of Flows on Individual Freeway Sections

Interstate 94 Section between	Daily differences		Mean hourly differences		Percentage reduction
	Before	After	Before	After	
CR 152 and Xerxes Avenue	369	1	21	2.12	89.90
Xerxes Avenue and Shingle Creek Parkway	319	1	19.63	3.30	83.16
Shingle Creek Parkway and Dupont Avenue	35	2	5.75	4.05	29.50
I-694 and 57th Avenue	38	1	9.25	6.71	27.47
57th Avenue and 49th Avenue	52	1	4.75	1.41	70.25
49th Avenue and 42nd Avenue	30	1	6	4.08	31.97

Note: Daily differences do not reduce to zero due to rounding error.

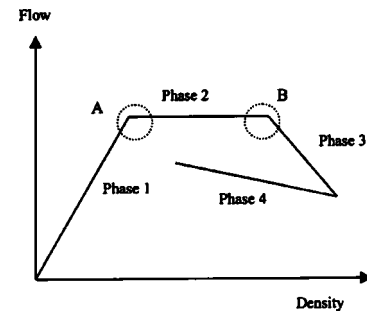


Fig. 4. Traffic phase diagram

### Quantification of Error Reduction

The reduction in errors in detector counts is illustrated in Table 1, where six freeway sections are laid out and the differences in daily and mean hourly flows are calculated. We observe that the cumulative daily differences between flows coming into any freeway section and flows going out of the same freeway section almost vanishes. There are also large percentage reductions in mean hourly flow differences between input and output flows for each freeway section. We expect differences in hourly input and output flows as long as there are vehicles stored on the freeway section.

### Theory and Methodology

It is hypothesized that if freeway traffic is allowed to behave without any restrictions imposed by ramp meters, it displays four phases in the flow-density curve as shown in Fig. 4.

- Phase 1 is the uncongested phase when there is no influence of the increasing density on the speeds of the vehicles. The speed does not drop with the introduction of newer vehicles onto the freeway.
- Phase 2 finds that the freeway cannot sustain the speed with injection of newer vehicles into the traffic stream. The density increases while speed falls, maintaining the flow.
- Phase 3 shows decreased speed and decreased flows. Very low speeds cause the queue discharge to drop at an active bottleneck, or a queue from a downstream bottleneck may be constraining the flow.
- Phase 4 is the recovery phase. During this phase, the density of traffic starts decreasing and speed starts increasing.

The circled region, A (in Fig. 4) is typically where we start observing "freeway breakdown." In other words, this region occurs when flow exceeds some critical capacity at a specific point

and there is a drop in speed. However, detectors may have a lag in identifying this threshold as they may be upstream or downstream of the critical point. In the circled region, B, flow drops are associated with very low speeds. The phenomenon is particularly evidenced by the formation of queues upstream of where the breakdown occurs and a low discharge rate of vehicles due to sustained low speeds. Upstream detector flows are governed by downstream bottlenecks if there are long queues.

To perform the queuing analysis, we need a way of identifying the occurrence and location of bottlenecks. This requires identification of bottlenecks forming over space and time on the freeway.

Conventional traffic flow analyses have used flow and occupancy data that are provided by freeway detectors. From measured occupancy ( $O$ ), applying the following empirical formula gives density ( $K$ ):

$$K = \frac{O}{L_{\text{eff}}} \quad (5)$$

where  $L_{\text{eff}}$ =effective length of the average vehicle, that is the length of the vehicle from the front to the end plus the length of the detector.

Occupancy estimates the percentage of time a detector was occupied by vehicles within a certain time span and there is no way to ascertain the integrity of a detector by looking at the occupancy of that detector and detectors upstream or downstream of it. Whereas, "flow," the other parameter generated by detectors, allows us a way to check for detector errors using flow conservation as previously described. Moreover, the conventional density calculation assumes average "vehicle length" that may not be true.

An advantage of applying queuing analysis to traffic flow is that we do not have to rely on occupancy data obtained from individual detectors. Rather, we can find the densities directly from the corrected flows. Assuming an empty section (starting in the middle of the night), we count each car that passes the start and end of the section. At time  $t$  we find the number of cars that are present on a freeway section by subtracting the cumulative number of cars that have passed the end of the freeway section

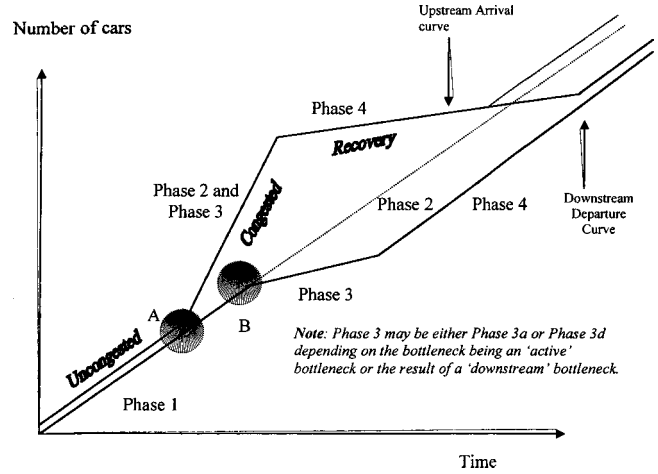


Fig. 5. Queuing input-output diagram: Relationship with phases

from the cumulative number of cars that passed the start of the freeway section. Mathematically

$$N = \sum_0^t Q_{f_{in}} + \sum_0^t Q_{r_{on}} - \sum_0^t Q_{f_{out}} - \sum_0^t Q_{r_{off}} \quad (6)$$

where  $N$ =number of cars in the freeway section after a time  $t$  from the start of counting. The normalized density ( $K_n$ ) in the section is given by

$$K_n = \frac{N}{n \cdot L} \quad (7)$$

where  $L$ =length of the section; and  $n$ =number of lanes in the section.

We repeatedly accumulate the vehicle flows over freeway sections. This leads to the drawback of applying queuing analysis to traffic flow, namely, errors tend to accumulate. However, starting with a clean set of data, and using the queuing diagram to reveal excess accumulation, ensures that such errors are small.

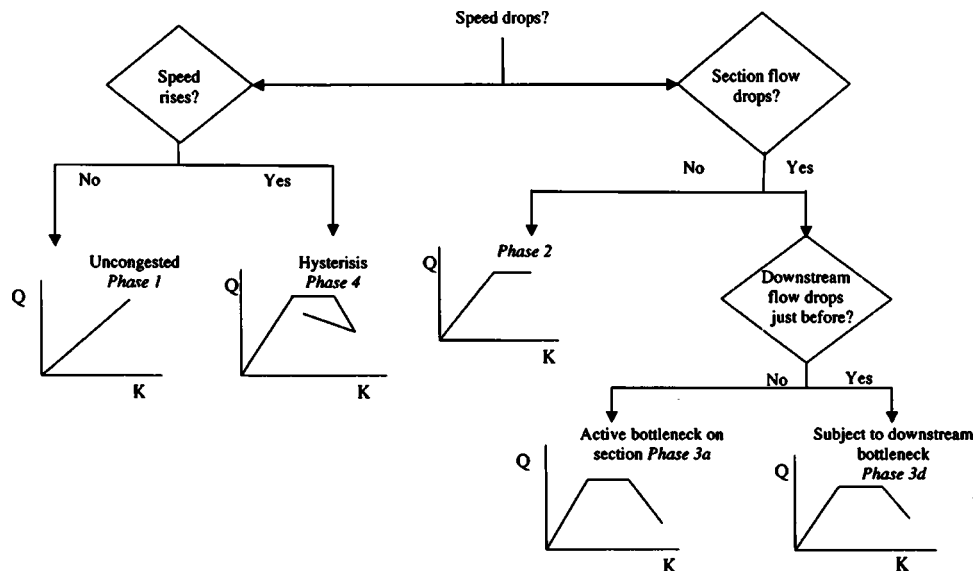


Fig. 6. Active bottleneck identification typology

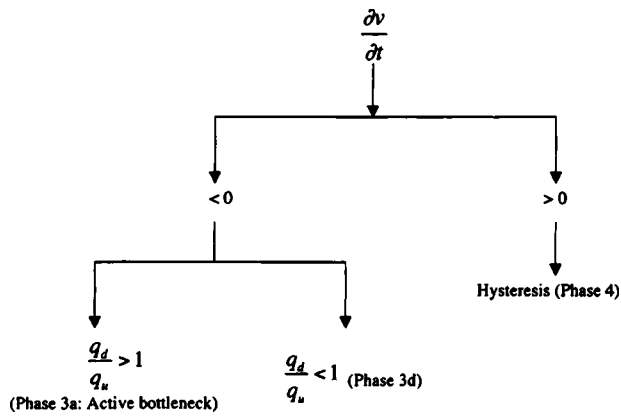


Fig. 7. Identification of phases

### Relationship between Queuing Diagram and Fundamental Diagram of Traffic Flow

Looking at Fig. 4, we see that while cars move at free-flow speed in *Phase 1*, the density increases but flow remains uncongested. After a critical density is reached (Point A), we move to *Phase 2*, speeds drop but flow does not. When we move over to *Phase 3* (Point B), both speed and flow drop and bottleneck formation takes place. This bottleneck may be a result of a *downstream* bottleneck or the section may itself be the *active* bottleneck. Last comes *Phase 4* wherein speeds recover and hysteresis takes place.

These relationships can also be observed with respect to the queuing diagram as shown in Fig. 5. It is easy to understand how the queuing diagram can explain the occurrence of phases. So long as the arrival rate is less than the service rate, the queue is uncongested, which is *Phase 1*. Congestion takes place when density increases beyond the critical density and that is when speed drops and bottleneck formation may also take place representing *Phase 3*. Hysteresis (*Phase 4*) is seen when speed recovers and

Table 2. Regression Results (Speed over Density) for Detector 52 (Broadway Avenue) on I-94

Variable	Dependent variable: speed		
	Hypothesis	Coefficient	$P >  Z $
Constant	+S	47.89	1.96E-14
Density	NS	0.862	0.58

Note: Significance based on 90% confidence interval; number of observations=43; and  $R$  squared=0.0075.

the queue dissipates. Important to note are the different arrival and departure curves. It has been claimed earlier that speed drops at the bottleneck to such an extent that the bottleneck cannot sustain the maximum departure rate. That shows up in the departure curve that drops. However, this is only true in case of bottleneck formation at the section or downstream (*Phase 3a* and *Phase 3d*, respectively). If it is *Phase 2*, then the departure curve remains the same (shown by a dotted line). Following this is the hysteresis in *Phase 4*. Also note the separation between departure and arrival curves even before the start of *Phase 2* and after *Phase 4*, which reflects the vehicles that are already present in the section prior to and after the effects of queuing are over. The lateral shift in the departure curve occurs because once flow falls, it can only go back to the original departure rate and not beyond it. If bottleneck formation does not take place in the departure region, the arrival curve does not have to “recover” as much as it has to otherwise. The dotted line in the *Phase 4* region again shows this.

### Active Bottleneck Identification

We apply a systematic procedure in order to identify active bottlenecks. This framework is illustrated in Fig. 6.

The following steps describe the process of finding active bottlenecks:

1. Obtain the speeds over detectors from the flow-density-speed relationship ( $V=Q/K_n$ ), with flow and density calculated from queuing analysis.

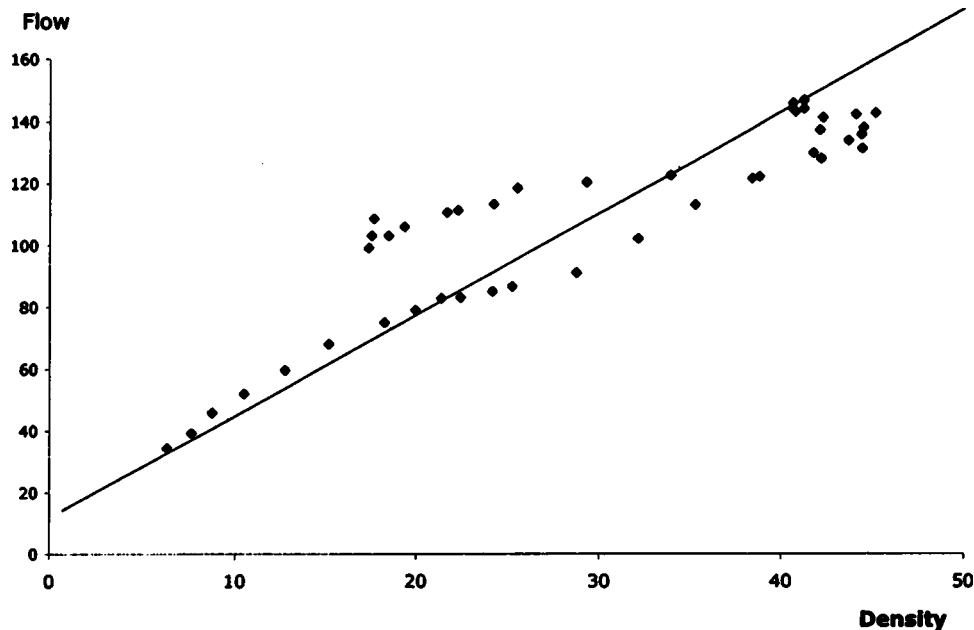


Fig. 8. Flow-density relationship for 52 (Lowry Avenue)

- Calculate five-interval moving averages of flow, density, and speed, each interval being 5 min long. This is done to smooth the transitions of each traffic flow characteristic.
- Select a representative detector out of any station. However, consistency was maintained by selecting a middle lane detector. The subsequent detectors downstream would be selected from the same lane.
- Select two periods to analyze: morning peak (beginning at 5:30 a.m. and continuing until 10:30 a.m.) and evening peak period (beginning at 1:30 p.m. and continuing until 6 p.m.). During the intermediate periods, data indicate that no queuing is taking place. Activity patterns are seen most distinctly during the morning and afternoon peaks.
- Conduct statistical analysis of flow, density, and speed relationships to identify active bottlenecks for each selected detector.
- Plot flows and speeds over time for each successive detector going upstream to downstream to support the results of the statistical analyses.

### Statistical Determination of Active Bottleneck Formation

The typology developed in Fig. 6 has been implemented statistically and is detailed here.

#### Phase 1

Neither flow nor speed drops on a freeway section as density increases. Vehicles move at free-flow speeds.

$$\frac{\partial V_a}{\partial K_a} = 0 \pm \varepsilon_1 \quad (8)$$

To operationalize this, we regress speed as a function of density and test the significance of density. Statistically it is represented as

**Table 3.** Regression Results (Flow over Density, for Speeds below Free-Flow Speed) for Detector 83, near 42nd Avenue on I-94 on November 2, 2000

Variable	Dependent variable: flow		
	Hypothesis	Coefficient	$P >  Z $
Constant	+S	68.23	1.96E-14
Density	NS	0.127	0.58

Note: Significance based on 90% confidence interval; number of observations=49; and  $R$  squared=0.011.

$$V_a = \alpha_0 + \alpha_1 \cdot K_a \quad (9)$$

where  $\varepsilon_1$ =allowable error;  $V_a$ =speed over detector  $a$ ; and  $K_a$ =density over detector  $a$ .

If  $\alpha_1$  is not statistically different from zero, we conclude that this is the uncongested Phase 1. Note as the number of observations increases, the likelihood of statistical significance increases, thus our test must get more rigorous.

#### Phase 2

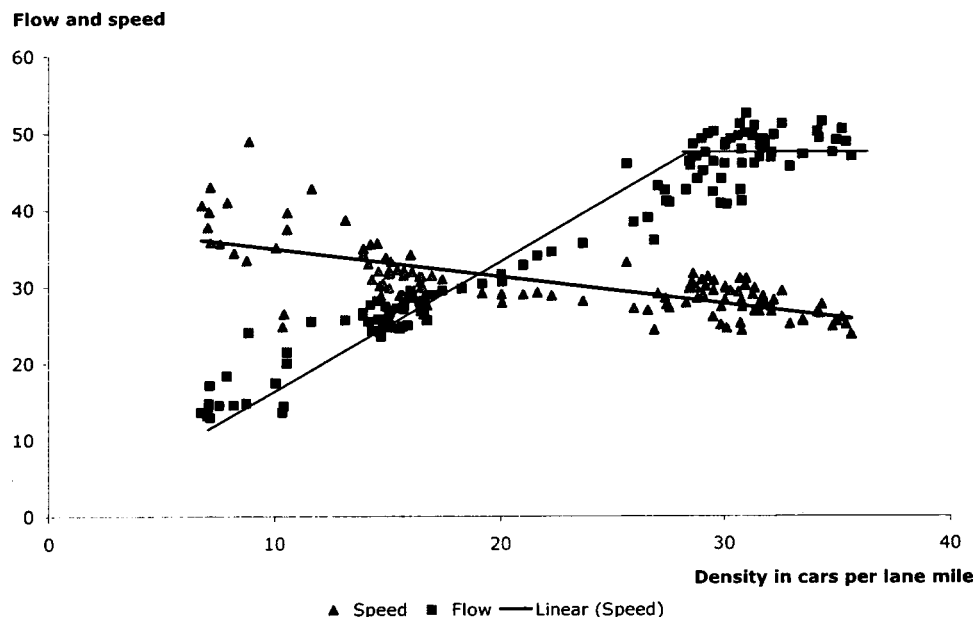
Speed drops on a section but flow does not. In this case, flows are maintained with occurrence of high densities but low speeds.

$$\frac{\partial Q_a}{\partial K_a} = 0 \pm \varepsilon_2 \quad (10)$$

From the analysis of change in speed above, we conclude that it is not Phase 1. To operationalize this measure, for such a detector, we separate the time intervals when speed drops by more than one standard deviation from the average speed over the whole period and regress the flows over densities for such time intervals. Statistically

$$Q_a = \beta_0 + \beta_1 \cdot K_a \quad (11)$$

where  $Q_a$ =flow over detector  $a$ , and  $K_a$ =density over detector  $a$ . And we test for statistical significance of  $\beta_1$ .



**Fig. 9.** Flow and speed varying by density on 83 (Dowling Avenue)

If  $\beta_1$  is not statistically different from zero, we have Phase 2.

If we encounter a significant  $\beta_1$  we know that it is not Phase 2. Then we test the variation of speed over time.

### Phase 3a

For a negative variation of speed over time, we test the variation of downstream flow over time and if this variation is positive, we conclude that higher flows are taking place downstream with time and the present section must be an *active* bottleneck. In this case, flow on the section drops and at the same time speed also drops, while flow downstream does not. Speeds drop to such an extent that vehicles cannot travel their own length in the service headway.

### Phase 3d

For a negative variation of speed over time, both flow and speed drop on the section but flows *downstream* drop just before. This is indicated by the negative derivative of downstream flow with respect to time. Thus the present section is subject to an active bottleneck downstream.

### Phase 4

Positive variation of section speeds with time indicates hysteresis and speed recovery. The identification typology is shown in Fig. 7.

## Results

### Phase 1: No Bottleneck Formation, neither Speeds nor Flows Drop

Table 2 shows regression results of speed over density (Detector 52 near Broadway Avenue on Interstate 94 East Bound, November 3, 2000) which indicate that density is not a significant variable and so this location remains uncongested (Phase 1), where

**Table 4.** Phases and Phase-Switches for Three Representative Detectors

Detector	Time period	Number of occurrences			Number of phase switches <sup>a</sup>
		Phase 3a	Phase 3d	Phase 4	
600	1405 to 1755	22	—	26	23
83	1405 to 1755	—	24	24	23
666	0530 to 1000	21	9	21	26

<sup>a</sup>Number of phase switches is the number of times the freeway section transitions from one phase to another, switches being random across time.

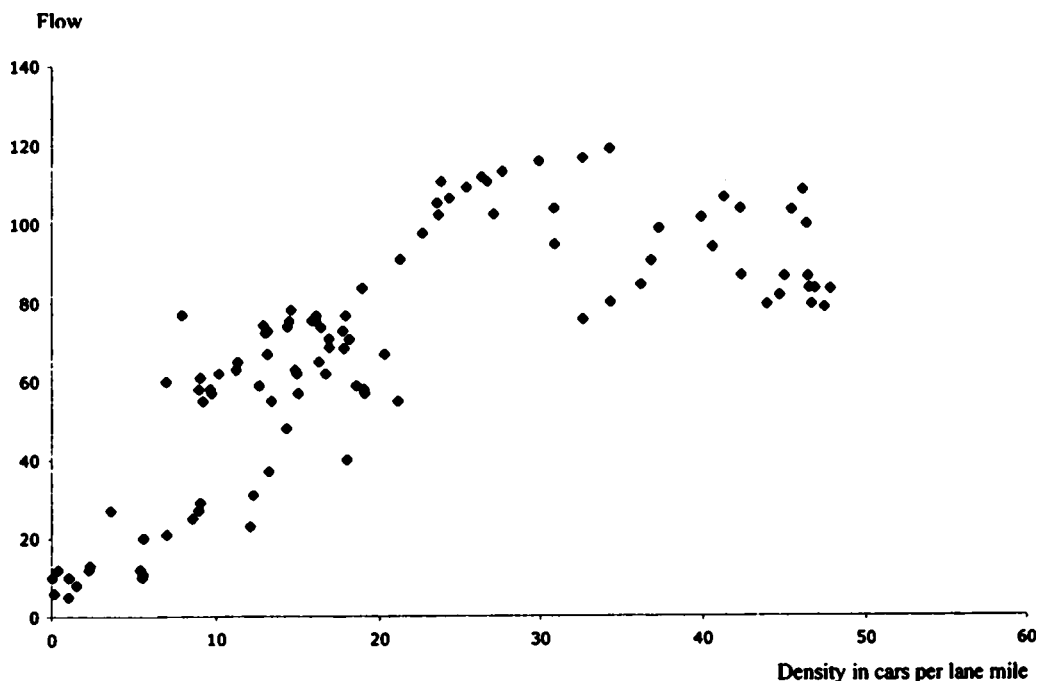
neither flow nor speed drops and no queuing or bottleneck formation/propagation takes place. This is corroborated by the flow-density relationship curve shown in Fig. 8.

### Phase 2: Speed Drops but Flow Does Not

When it is not Phase 1 for a detector, we find the standard deviation of speeds for that detector and weed out the data for it for which speeds do not fall by at least one standard deviation. Then for the rest of the data, we find the statistical significance of density while regressing flow over density. Results for 83 on Interstate 94 (42nd Avenue) are shown in Table 3. We see the insignificance of density in this table and conclude that it is Phase 2 and speed should drop while flow remains almost the same. This is corroborated by the examining the speed-flow-density relationship in Fig. 9.

### Phases 3a, 3d, and 4

Some detectors show a mix of Phases 3a, 3d, and 4. This indicates that, at the location of the detector, sometimes active bottleneck formation takes place, sometimes speed recovers and hysteresis takes place, and at other times, queue formation is a result of bottlenecks forming downstream. This is natural on a freeway with stochastic traffic flow. However often, only Phases 3a and 4



**Fig. 10.** Flow varying by density on 688 (49th/53rd Avenue)

**Table 5.** Phases for Individual Sections on I-94 for the Period from 1 November, 2000 to 6 November, 2000

Date(s)	Sections								
	Shingle Creek Parkway	Dupont Avenue	57th Avenue	49th/53rd Avenue	42nd Avenue 3a, 3d, 4	Dowling Avenue	Lowry Avenue	Broadway Avenue	Plymouth Avenue
Morning peak									
November 1, 2000	3a, 4	3a, 3d, 4	1	1	3a, 3d, 4	1	1	1	*
November 2, 2000	3a, 3d, 4	1	1	1	1	1	3d, 4	3a, 3d, 4	2
November 3, 2000	3a, 3d, 4	1	1	1	1	1	1	3a, 3d, 4	2
November 4, 2000	1	1	1	*	1	3d, 4	3d, 4	3a, 3d, 4	*
November 5, 2000	3a, 3d, 4	*	1	1	*	*	1	*	*
November 6, 2000	1	1	3d, 4	3,5	3d, 4	3d, 4	3d, 4	*	*
Evening peak									
November 1, 2000	2	3a, 4	1	1	3d, 4	1	1	2	*
November 2, 2000	2	2	1	1	2	1	2	2	2
November 3, 2000	*	2	2	3a, 3d, 4	3a, 3d, 4	3a, 3d, 4	1	2	2
November 4, 2000	1	1	3a, 3d, 4	3a, 3d, 4	3d, 4	3d, 4	3d, 4	3a, 4	2
November 5, 2000	3d, 4	2	1	1	2	3d, 4	2	2	*
November 6, 2000	3d, 4	2	1	1	3d, 4	1	1	*	*

Note: The asterisk (\*) fields are phases where either the detector was not functioning properly or the phase could not be ascertained due to nonavailability of data for downstream sections; and phases where only Phases 3d and 4 occur and not 3a and the ones where Phases 3a and 4 occur but not 3d.

take place (or only Phases 3d and 4 take place) which suggest that the section may be considered an active bottleneck (or subject to downstream bottleneck). There are, however, instances where all three phases occur on one section as will be illustrated here. Table 4 shows a section on I-94 near Dupont Avenue, which fluctuates between the states of active bottleneck and recovery. It also shows a section defined by 83 (Dowling Avenue), which is largely affected by downstream bottleneck, but is itself not an active bottleneck. We can also see the third situation (Detector 666, Shingle Creek Parkway) where, for the early part of the morning peak, the section behaved mostly as an active bottleneck; though during the later part, a downstream bottleneck was affecting it. This example shows how all three phases can coexist for a single freeway section. There are a few occasions when none of the three phases appears. This is because those times are when it is either Phase 1 or 2. Any one section cannot be labeled as an “active bottleneck” in generic terms for this reason. The  $Q$ - $K$  relationship in Fig. 10 illustrates this. The summary table which shows the bottleneck properties for each section on Interstate 94 for the period from November 1 to November 6 is included in Table 5.

## Summary and Conclusions

This paper analyzes traffic flow parameters (flow, density, and speed) over various freeway sections on the Twin Cities freeway network. An algorithm is developed to balance flows on any freeway section ensuring flow conservation. This algorithm searches for the region of least error and then calibrates the other freeway section based on this least-error section. Errors in flows given by detectors have been minimized by the calibration procedure. Queuing analysis has been extensively applied to predict and track bottleneck formation and to detect active bottleneck locations. A typology was developed to determine the location of active bottlenecks on the freeways based on statistical analysis and their correspondence to visual relationships between flow, density, and speed, this typology helps us identify the locations that are prone to bottleneck formation and those that are prone to

being affected by downstream bottlenecks. This typology was tested on data gathered from sections on Interstate 94. A sample of six days for I-94 for morning and evening peak periods shows that the same section cannot always be characterized as an “active bottleneck” location; at some times it is active and at others, it is subject to downstream bottlenecks. Traffic flow characteristics change and that leads to changing situations on each freeway section.

The methodology developed in this paper can be used to correct flawed loop detector data and find active bottleneck locations. Having done this, more intelligent ramp-metering algorithms can be developed that control ramp flows leading to such locations. This will improve freeway operations. As part of future work, more freeway sections can be examined over longer time periods.

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