AN ESTIMATE FOR THE NONSTOCHASTIC EIGENVALUES OF DOUBLY STOCHASTIC MATRICES

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AN ESTIMATE FOR THE NONSTOCHASTIC EIGENVALUES OF DOUBLY STOCHASTIC MATRICES

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Abstract. We introduce a new measure of irreducibility of a doubly stochastic matrix and find the best estimate from below for the distance of nonstochastic eigenvalues from one. The estimate is used to obtain a Cheeger-type inequality for the second Laplacean eigenvalue of a weighted graph.

1. Introduction. A *doubly stochastic* matrix (i.e. a square elementwise nonnegative matrix with all row- and column-sums equal to one) A has the property that 1 is its simple eigenvalue if and only if A is irreducible. This fact allows a quantitative strengthening when some measure of irreducibility is used. In [2], the number

$$\mu(A) = \min_{M \subset N, \phi \neq M \neq N} \sum_{i \in M} \sum_{k \in N \setminus M} a_{ik}$$

(for an $n \times n$ matrix $A - (a_{ik})$ and $N = \{1, 2, \cdots, , n\}$) was used and an estimate

$$|1 - \lambda| \geqq 2\left(1 - \cos\frac{\pi}{n}\right)\mu(A)$$

was proved for eigenvalues λ of A, $\lambda \neq 1$. It was also shown that $2\left(1 - \cos\frac{\pi}{n}\right)$ is the best multiplicative constant for each $n \geqq 2$.

With the same notation, we shall introduce here the *averaged measure of irreducibility* of A as

$$\alpha(A) = \min_{M \subset N, \phi \neq M \neq N} \sum_{i \in M} \sum_{k \in N \setminus M} \frac{a_{ik}}{|M|(n - |M|)}. \tag{1}$$

In the whole paper, we shall call *nonstochastic* eigenvalue of A any eigenvalue of A left after deleting one eigenvalue 1.

2. Results. We shall prove the following

THEOREM 1. *Let λ be any nonstochastic eigenvalue of a doubly stochastic matrix A. Then*

$$|1 - \lambda| \geq 2\alpha(A) \tag{2}$$

and the multiplicative constant 2 is the best possible for any fixed order of A.

Proof. We shall first prove three propositions.

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PROPOSITION 1. *Let $S = (s_{ik})$ be a symmetric (real or complex) $n \times n$ matrix with the eigenvalues λ, μ resp. corresponding to eigenvectors $x = (x_k)$, $y = (y_k)$, resp. Then for every nonvoid proper subset M of N,*

$$(3) \qquad (\lambda - \mu) \sum_{k \in M} x_k y_k = \sum_{i \in M} \sum_{k \in N \setminus M} a_{ik}(x_k y_i - x_i y_k).$$

Proof. *W.l.o.g.*, we can assume that $M = \{1, 2, \ldots, m\}$ and that A is partitioned as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{pmatrix},$$

where A_{11} is $m \times m$. If

$$x = \begin{pmatrix} x^{(1)} \\ x^{(2)} \end{pmatrix}, y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \end{pmatrix}$$

is the conformal partitioning of x and y, we have

$$\begin{aligned} A_{11}x^{(1)} + A_{12}x^{(2)} &= \lambda x^{(1)}, \\ A_{12}^T x^{(1)} + A_{22}x^{(2)} &= \lambda x^{(2)}, \\ A_{11}y^{(1)} + A_{12}y^{(2)} &= \mu y^{(1)}, \\ A_{12}^T y^{(1)} + A_{22}y^{(2)} &= \mu y^{(2)}. \end{aligned}$$

Premultiply the first equality by $y^{(1)T}$ and subtract the third equality premultiplied by $x^{(1)T}$. We obtain, because of the symmetry of A_{11},

$$y^{(1)T} A_{12} x^{(2)} - x^{(1)T} A_{12} y^{(2)} = (\lambda - \mu) y^{(1)T} x^{(1)}$$

which is (3).

PROPOSITION 2. *The $n \times n$ matrix $Z = (z_{ik})$ corresponding to the quadratic form*

$$\sum_{k=1}^{n-1} k(n-k)(x_k - x_{k+1})^2$$

has 2 as its second smallest eigenvalue. Also,

$$(4) \qquad 2 = \min_{x \neq 0, \sum x_k = 0} \left(\sum_{k=1}^{n-1} k(n-k)(x_k - x_{k+1})^2 \Big/ \sum_{k=1}^{n} x_k^2 \right).$$

Proof. The matrix Z is a singular tridiagonal positive semidefinite matrix with the eigenvector e corresponding to the smallest eigenvalue 0. It is then easily checked that the vector $y = (y_k)$ with $y_k = n + 1 - 2k$, $k = 1, 2, \ldots, n$ satisfies

$$Zy = 2y.$$

By a well known property of the signs of the coordinates of the eigenvector y (cf. [4], Corr. 2,5), 2 is the second smallest eigenvalue of Z.

PROPOSITION 3. *Let $A = (a_{ik})$ be an $n \times n$ doubly stochastic matrix, $S = \frac{1}{2}(A + A^T)$. Let $\lambda_2(S)$ denote the second largest eigenvalue of S. Then for any nonstochastic eigenvalue λ of A,*

$$\text{Re } \lambda \leqq \lambda_2(S). \tag{5}$$

Also, for the averaged measures of irreducibility of A and S,

$$\alpha(A) = \alpha(S). \tag{6}$$

Proof. If $\lambda = 1$, A is reducible and $\lambda_2(S) = 1$ as well. If $\lambda \neq 1$, let $Ay = \lambda y$, $y \neq 0$. Then the (Hermitian) inner product $(y, e) = 0$. Further,

$$(Ay, y) = \lambda(y, y)$$

so that

$$\begin{aligned}(A^T y, y) &= (y, Ay) \\ &= \overline{(Ay, y)} \\ &= \bar{\lambda}(y, y).\end{aligned}$$

Consequently,

$$(Sy, y) = \text{Re } \lambda(y, y).$$

Thus

$$\begin{aligned}\lambda_2(S) &= \max_{x \neq 0, (x, e) = 0} \frac{(Sx, x)}{(x, x)} \\ &\geqq \frac{(Sy, y)}{(y, y)} \\ &= \text{Re } \lambda.\end{aligned}$$

To prove (6), let M be any non void proper subset of N. Then,

$$\begin{aligned}\sum_{i \in M} \sum_{k \in N \setminus M} a_{ik} &= \sum_{i \in M} \sum_{k \in N} a_{ik} - \sum_{i,k \in M} a_{ik} \\ &= |M| - \sum_{i,k \in M} a_{ik} \\ &= |M| - \left(|M| - \sum_{i \in N \setminus M} \sum_{k \in M} a_{ik} \right) \\ &= \sum_{i \in N \setminus M} \sum_{k \in M} a_{ik} \\ &= \sum_{i \in M, k \in N \setminus M} a_{ki}.\end{aligned}$$

Thus, for $S = (s_{ik})$,

$$\sum_{i \in M, k \in N \setminus M} s_{ik} = \sum_{i \in M, k \in N \setminus M} a_{ik}$$

and (6) follows.

Returning to the proof of the theorem, let $S = (s_{ik})$ be the matrix $\frac{1}{2}(A + A^T)$, $y = (y_k)$ a real eigenvector corresponding to the second largest eigenvalue $\tilde{\lambda}$ of S and such that $(y, e) = 0$. *W.l.o.g.*, we can suppose that $y_1 \geqq y_2 \geqq \cdots \geqq y_n$. Since $\sum y_k = 0$ and $y_1 > 0$, it follows that

$$\sum_{j=1}^{k} y_j > 0 \text{ for } k = 1, \ldots, n-1.$$

By Proposition 1,

$$(7) \qquad (1 - \tilde{\lambda}) \sum_{i=1}^{t} y_i = \sum_{i=1}^{t} \sum_{k=t+1}^{n} s_{ik}(y_i - y_k).$$

Since for $i \in \{1, \ldots, t\}$, $k \in \{t+1, \ldots, n\}$

$$y_i - y_k \geqq y_t - y_{t+1},$$

we have from (7) for $t = 1, \ldots, n-1$

$$(1 - \tilde{\lambda}) \sum_{i=1}^{t} y_i \geqq \left(\sum_{i=1}^{t} \sum_{k=t+1}^{n} \frac{s_{ik}}{t(n-t)} \right) (y_t - y_{t+1}) t(n-t)$$

so that

$$(1 - \tilde{\lambda}) \sum_{i=1}^{t} y_i \geqq \alpha(S) t(n-t)(y_t - y_{t+1}).$$

Multiply the t-th inequality by $(y_t - y_{t+1})$ and add them, together with

$$(1 - \tilde{\lambda}) y_n \sum_{i=1}^{n} y_i = 0.$$

We obtain

$$(1 - \tilde{\lambda}) \sum_{i=1}^{n} y_i^2 \geqq \alpha(S) \sum_{t=1}^{n-1} t(n-t)(y_t - y_{t+1})^2.$$

By (4),

$$(8) \qquad 1 - \tilde{\lambda} \geqq 2\alpha(S).$$

Consequently, (5), (8) and (6) yield that for any nonstochastic eigenvalue λ of A

$$\begin{aligned}|1-\lambda| &\geqq |1-\text{Re }\lambda| \\ &= 1-\text{Re }\lambda \\ &\geqq 1-\tilde{\lambda} \\ &\geqq 2\alpha(S) \\ &= 2\alpha(A).\end{aligned}$$

To show that 2 is the best multiplicative constant for each n, define $R = n(n+1)$ and the doubly stochastic $n \times n$ matrix $A = (a_{ik})$ by

$$\begin{aligned}a_{11} &= 1 - \frac{n-1}{R}, \\ a_{nn} &= 1 - \frac{n-1}{R}, \\ a_{kk} &= 1 - \frac{k(n-k)}{R} - \frac{(k-1)(n-k+1)}{R}, k = 2, \cdots, n-1, \\ a_{k,k+1} &= a_{k+1,k} = \frac{k(n-k)}{R}, k = 2, \ldots, n-1, \\ a_{pq} &= 0 \text{ in all other cases }.\end{aligned}$$

The second largest eigenvalue of this symmetric matrix A is $1 - \frac{2}{R}$. Let us show that $\alpha(A) = \frac{1}{R}$. Let the minimum in (1) be attained for a subset M of N. We can assume that $1 \in M$ since otherwise we go over to $N \setminus M$. Let thus exactly the first $s_1 \geqq 1$ indices from N belong to M, next t_1 indices to $N \setminus M$, further s_2 indices again to M etc.

Thus (we do not specify the final numbers)

$$\begin{aligned}|M| &= s_1 + s_2 + \cdots, \\ |N \setminus M| &= t_1 + t_2 + \cdots\end{aligned}$$

We shall show that

$$\sum_{i \in M} \sum_{k \in N \setminus M} a_{ik} \geq \frac{1}{R} \sum s_i \sum t_j.$$

Indeed, the left-hand side is (only the pairs of consecutive pairs of indices belonging to different sets M, $N \setminus M$ count)

$$\frac{1}{R}[s_1(t_1 + s_2 + t_1 + \cdots) + (s_1 + t_1)(s_2 + t_2 + \cdots) + (s_1 + t_1 + s_2)(t_2 + s_3 + \cdots)\cdots];$$

this is at least $\frac{1}{R} \sum s_i \sum t_j$ since every term $\frac{1}{R} s_i t_j$ appears in the above expression at least once: for $j < i$ in the term the second factor of which begins with s_i, for $j \geqq i$ in the term the first factor of which terminates with s_i.

Thus

$$\alpha(A) \geqq \frac{1}{R}$$

but equality is attained for $s_1 = 1$ and $t_1 = n - 1$. It follows that equality in (2) can be attained for every n (and nonzero $\alpha(A)$). □

3. Application to weighted graphs. For a weighted undirected connected graph $G_C = (N, E)$ on n vertices $1, 2, \cdots, n$, we call *averaged minimal cut* the number

$$\gamma(G_C) = \min_{\emptyset \neq M \neq N} \sum_{i \in M, k \in N \setminus M} \frac{c_{ik}}{|M|(n - |M|)} \tag{9}$$

(c_{ik} is the nonnegative weight of the edge (i, k); if there is no edge, we set $c_{ik} = 0$).

We recall that the *Laplacean matrix* of such weighted graph G_C is the matrix $L(G_C)$ of the quadratic form

$$\sum_{\substack{i,k \in N \\ i < k}} c_{ik}(x_i - x_k)^2.$$

Its second smallest eigenvalue (the smallest is zero) $a(G_C)$ was called in [3] *algebraic connectivity* of G_C.

THEOREM 2. *For every weighted graph G_C, with at least two vertices, the algebraic connectivity $a(G_C)$ and the averaged minimal cut $\gamma(G_C)$ satisfy*

$$2\gamma(G_C) \leqq a(G_C) \leqq n\gamma(G_C); \tag{10}$$

the multiplicative constant 2 is the best for any (fixed) number of vertices of G_C, the multiplicative constant n is also the best.

Remark. (10) is a Cheeger-type inequality [1].

Proof. Let d_c be the maximum weighted degree of a vertex in G_C. Then the matrix

$$S = I - \frac{1}{d_c} L(G_C)$$

is doubly stochastic and symmetric. The eigenvalues $\sigma_1 \geqq \sigma_2 \geqq \cdots \geqq \sigma_n$ of S and the eigenvalues $\lambda_1 \leqq \lambda_2 \leqq \cdots \leqq \lambda_n$ of $L(G_C)$ are thus related by

$$\sigma_i = 1 - \frac{1}{d_c}\lambda_i, \quad i = 1, \cdots, n.$$

In particular,

$$\sigma_2 = 1 - \frac{1}{d_c} a(G_C).$$

By (1) and (9), the averaged minimal cut $\gamma(G_C)$ and the averaged measure of irreducibility $\alpha(S)$ of S satisfy

$$\alpha(S) = \frac{1}{d_c}\gamma(G_C).$$

By Theorem 1,

$$1 - \sigma_2 \geqq 2\alpha(S)$$

so that

$$\frac{1}{d_c}a(G_C) \geqq \frac{2}{d_c}\gamma(G_C),$$

i.e. the left-hand side of (10).

For each number n of vertices, equality here is attained for a weighted path the weight of each edge of which is chosen as the product of the number of vertices in the two branches.

To prove the right inequality in (10), let equality in (9) be attained for M_0, $\emptyset \neq M_0 \neq N$. We apply the formula [3]

$$a(G_C) = n\frac{\min \sum\limits_{i<k} c_{ik}(x_i - x_k)^2}{\sum\limits_{i<k}(x_i - x_k)^2}$$

to $x = (x_i)$, $x_i = 1$ for $i \in M_0$, $x_i = 0$ for $i \in N \setminus M_0$, thus obtaining

$$a(G_C) \leqq n\gamma(G_C).$$

Equality is attained for the complete graph.

In the conclusion, let us discuss relations between the average minimal cut $\gamma(G_C)$ and the usual minimal cut (edge-connectivity) $e(G_C)$ defined by

$$e(G_C) = \min_{\emptyset \neq M \neq N} \sum_{\substack{i \in M \\ k \in N \setminus M}} c_{ik}.$$

Since

$$\frac{8}{2n^2 + (-1)^n - 1} \leqq \frac{1}{|M|(n - |M|)} \leqq \frac{1}{n-1},$$

we have

(11) $$\frac{8e(G_C)}{2n^2 + (-1)^n - 1} \leqq \gamma(G_C) \leqq \frac{1}{n-1}e(G_C).$$

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