

Optimal Bus Scheduling Considering Operating Costs and Emissions: A Multiple  
Objective, Mixed Integer Programming Framework

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## **Abstract**

Traditional vehicle scheduling problems primarily focus on minimizing operating costs, and few of them consider the environmental impacts of the fleet operation. This study develops a framework that optimizes bus assignments to routes with the objective of minimizing both operating costs and the environmental impacts of emissions. The optimization model is applied in a case study of Metro Transit in Minneapolis/St Paul area. The results show a set of tradeoff relationships between operating costs and emissions. The optimized vehicle assignments generated by the model can significantly reduce both the operating costs and emissions of the current fleet. It is also found that hybrid electric buses were underused by Metro Transit in 2013 and should be assigned to service more often. The analysis can also provide useful supporting information for strategic decisions such as vehicle replacement and purchase.

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## **List of Supplementary Materials**

### **Input\_Matrices.xls**

This Excel file is the source for all the include files used in the GAMS program. A GAMS program that is representative of those used in this study is listed in the appendix. In order to run the GAMS program presented in the appendix, you will need to convert all the worksheets into csv files and store them into GAMS project directory (gamsdir\projdir).

### **GAMS\_lst\_file.pdf**

This file presents the GAMS output lst file after executing the GAMS code in the appendix. This file includes the original GAMS code, display of input matrices, model analysis, model statistics, solution report, execution and display of output tables.

## **1. Introduction**

For the past 30 years, the transportation sector has experienced steady growth and is currently responsible for nearly 60% of the world total oil demand (Atabani et al, 2011). In the U.S., the transportation sector is the second largest energy consuming sector after the industrial sector and contributed to 29% of the total energy consumption in 2009 (Atabani et al, 2011). Road transport vehicles, which include automobiles, and medium and heavy-duty vehicles, represent 81% of transportation energy demand in the world (Atabani et al, 2011). The number of cars on the road in the world is expected to double in the next 20 years from one billion to two billion (IICT, 2010).

Emissions from the combustion of fossil fuels contribute to both global climate change and local air pollution (Bollen et al., 2009). The transportation sector in the U.S. is the second largest source of greenhouse gas emissions and contributes about 25% of global CO<sub>2</sub> emissions (Atabani et al, 2011). Passenger cars, and heavy, medium and light duty trucks are responsible for nearly 83 percent of transportation-related greenhouse gas emissions (US DOT, 2012). As the main source of air pollution, emissions from fossil fuel combustion contains nitrogen oxides, sulfur oxides and suspended particular matters, which impose serious threat to public health (Hedberg et al. 2005). Although heavy-duty diesel vehicles constitute a small portion of the on-road vehicles, they generate more than 45% of the nitrogen oxides and 75% of the particulate matter generated by all vehicles (U.S. EPA, 2003). Therefore, reducing emissions in public transportation sector can provide great benefit to public health and environmental sustainability.

Encouraging the use of environmentally friendly vehicles and improving the efficiency of public transportation systems are effective approaches to reducing greenhouse gas emissions and improving air quality (Geng et al., 2013). However, only about 2% of total passenger miles traveled in urban areas use public transit (US DOT, 2012). The Metropolitan Council in Minnesota has set the goal of doubling transit ridership to about 147 million rides by 2030 (Metropolitan Council, 2010). In order to promote bus transit to compete with automobiles, key features including scheduling,

passenger comfort, fuel efficiency and environmental friendliness should be improved. An increasing amount of research focuses on the optimal vehicle scheduling with the objective to minimize costs and emissions. However, only a limited number of studies consider the environmental impacts associated with the operation of public transportation systems. The purpose of this study is to develop a framework that determines the efficient deployment of buses considering operating costs and environmental outcomes of public transportation systems. The conceptual model will be presented and applied using data from the Metro Transit bus system in Minneapolis/St Paul area.

Section 2 of the thesis reviews literature in the related field. Section 3 presents the integer programming model of bus assignment, while Section 4 describes a case study for Twin-City Metro Transit. Section 5 outlines conclusions and potential extensions of the research.

## **2. Literature Review**

Research on transit vehicle scheduling usually focuses on the minimization of capital and operating costs. Banihashemi and Haghani (2000) formulate a multiple-depot vehicle scheduling problem as an integer-programming model and minimize a combination of capital costs and total deadhead costs. Deadhead costs are costs of operating buses while not in service to passengers, typically between depots or service facilities and the routes of service. The model imposes a restriction on route time for realistic operation considerations such as limited fuel supply when buses are in operation. The authors compare the optimal results generated by the model with the bus schedule used by the transit administration in Baltimore, Maryland, and find that the proposed model reduces the total cost by 5.77%. Ceder (2011) discusses an optimization framework to address the vehicle scheduling problem while considering the characteristics of different trips (urban, inter-city, etc.) and the vehicle type required for the particular trip. The problem is formulated as a cost-flow network problem where each trip is a node and an arc connects two trips that can be linked in a time sequence. Levels of operating costs and

deadheading costs (cost of switching routes) are assigned to each arc. The objective is to minimize total operating plus deadheading costs by changing the numbers of different types of buses.

In recent years, there is an increasing interest in incorporating the environmental impacts of bus operation into vehicle scheduling models. Dessouky et al. (2003) discuss a method for combining routing and scheduling of fleet vehicles in an optimization model where environmental impacts were included in the objective function. Through simulation within a demand-responsive transit context, the authors show that environmental performance can be improved substantially using heterogeneous fleets at various loading levels. Li and Head (2009) create a bus scheduling problem to minimize operating costs and vehicle emissions, under the constraints of a limited budget to purchase new buses and a timetable of bus trips. The authors develop a time-space network to optimize the vehicle movements needed to cover all routes on a timetable, and include emission constraints and penalties in its formulations. Stasko and Gao (2010) propose an integer programming model that focuses on long-term vehicle purchase decisions. The model minimizes operational costs plus penalties for emissions, given capital budget constraints. It allows for changes in aggregate vehicle task assignments while developing a vehicle purchase and retrofit strategy for multiple time periods. Figliozzi (2010) formulates and solves a vehicle routing problem which minimizes emissions and fuel consumption as part of a generalized cost function. Departure times and travel speeds are decision variables in the problem. The author treats the level of emissions amount as a function of the speed profile from the departure to reaching the destination, since congestion has a great impact on vehicle emissions and fuel efficiency. Gouge et al. (2013) use a nonlinear programming model to generate optimal bus assignments, which minimize operating costs, health impacts and climate impacts of the bus emissions. The climate impact of emissions is measured by the global warming commitment, while the public health impact is measured by the intake of primary PM<sub>2.5</sub> exhaust emission inhaled by the population within 5,000 meters of the bus routes. This study specifically explores how heterogeneity in the emission levels of different technologies and the exposure potential of bus routes can be exploited by vehicle

assignment. It also discusses the implications of applying cost-benefit analysis to evaluate tradeoffs between conflicting objectives.

Despite the ample literature focusing on the vehicle scheduling problem, little has been done to incorporate the demand for public transportation into the bus scheduling problem. In addition, previous studies have not considered the heterogeneity of vehicle performance on different routes. This paper proposes a mixed-integer, multiple-objective programming framework for a vehicle scheduling problem that minimizes operating costs and the social costs of emissions. Heterogeneous vehicle performance on different routes and the dynamic nature of passenger demand are taken into account. The model is optimized given input market conditions and fixed fleet resources, as well as passenger demand and service requirements. The decision variables include the number of buses by type assigned to each route in each time period, and the modes of operation for each bus type, route and time period.

### **3. Model Formulation**

A general mathematical programming model of the bus scheduling problem is presented in this section. The purpose of this model is to assign buses from a public transportation fleet to a set of defined routes and time table in a way that optimizes an objective function subject to a set of spatial, temporal and operational constraints. Although this study is motivated by Twin-City Metro Transit, the basic model is designed to be applicable to a wide range of vehicle scheduling problems.

The basic framework can be viewed as a production problem which considers the resources invested in the bus operation as inputs and the service attributes provided as outputs. The concept of “operating mode” reflects different ways to operate a bus which determine input usage and output levels. A solution to the problem identifies the number of buses assigned to each route and the way to operate the buses. Tradeoff relationships among different objectives, such as operating costs and emissions, can be established by changing their relative weights in the objective function. The potential of the vehicle

scheduling model will be explored and demonstrated using the dataset from Twin City Metro Transit.

The complete model formulation follows in Sections 3.1 – 3.4.

### 3.1. Sets

$\theta_T$ : set of operating periods ( $t$ )

$\theta_R$ : set of routes ( $r$ )

$\theta_B$ : set of bus types ( $i$ )

$\theta_M$ : set of operating modes ( $j$ )

$\theta_I$ : set of inputs and resources ( $k$ )

$\theta_{RB}(\theta_R \times \theta_B)$ : set of bus types mapping to routes ( $i, r$ )

$\theta_H$ : set of service/demand attributes ( $h$ )

$\theta_Q$ : set of performance measures ( $q$ )

### 3.2. Parameters

$\delta$ : percentage of operating time in each time period (between 0 and 1)

$AIS_{trik}$ : average input  $k$  usage for bus type  $i$  at time period  $t$  on route  $r$

$AIO_{trijk}$ : average input  $k$  usage for bus type  $i$  at time period  $t$  on route  $r$  with operation mode  $j$

$AYS_{trih}$ : average capacity of service  $h$  for bus type  $i$  on route  $r$  at time period  $t$

$AYO_{trijh}$ : average capacity of service  $h$  for bus type  $i$  on route  $r$  at time period  $t$  with operation mode  $j$

$CS_{triq}$ : performance measure  $q$  of operating bus type  $i$  on route  $r$  at time period  $t$

$CO_{trijq}$ : performance measure  $q$  of operating bus type  $i$  on route  $r$  at time period  $t$  with operation mode  $j$

$CM_{tkq}$ : unit of performance measure  $q$  produced from input  $k$  at time period  $t$

$P_q$ : price or relative weight of performance measure  $q$  in the objective function

### 3.3. Variables

$XS_{tri}$ : number of type  $i$  buses assigned to route  $r$  during time period  $t$

$XO_{trij}$ : number of type  $i$  buses assigned to route  $r$  during time period  $t$  under operation model  $j$

$Z_{tk}$ : supply of input  $k$  in time period  $t$

$YD_{trh}$ : demand of service attribute  $h$  on route  $r$  during time period  $t$

$QP_q$ : level of performance measure  $q$

### 3.4. Optimization Problem

Minimizing:

$$OBJ = F[QP] = \sum_{q \in \theta_Q} P_q QP_q \quad [1]$$

Subject to:

$$\sum_{j \in \theta_M} XO_{trij} = \delta * XS_{tri} \quad t \in \theta_T, r \in \theta_R, i \in \theta_{RB} \quad [2]$$

$$\sum_{r \in \theta_R} \sum_{i \in \theta_B} AIS_{trik} XS_{tri} + \sum_{r \in \theta_{RB}} \sum_{i \in \theta_{RB}} \sum_{j \in \theta_M} AIO_{trij} XO_{trij} \leq Z_{tk} \quad t \in \theta_T, k \in \theta_I \quad [3]$$

$$\sum_{i \in \theta_{RB}} AYS_{trih} XS_{tri} + \sum_{i \in \theta_{RB}} \sum_{k \in \theta_I} AYO_{trijh} XO_{trij} \geq YD_{trh} \quad t \in \theta_T, r \in \theta_R, h \in \theta_H \quad [4]$$

$$\sum_{t \in \theta_T} \sum_{r \in \theta_{RB}} \sum_{i \in \theta_{RB}} \left[ CS_{triq} XS_{tri} + \sum_{j \in \theta_M} CM_{triq} XO_{trij} \right] + \sum_{t \in \theta_T} \sum_{k \in \theta_I} CM_{tkq} Z_{tk} = QP_q \quad [5]$$

$$q \in \theta_Q$$

$$XS, XO \geq 0; \bar{Z}_{min} \leq Z \leq \bar{Z}_{max}; \bar{YD}_{min} \leq YD \leq \bar{YD}_{max}; \bar{QP}_{min} \leq QP \leq \bar{QP}_{max}; XS$$

Integer.

The objective, given by expression [1], is to minimize the combination of operating costs and penalties for emissions. By adjusting the relative price  $P_q$ , the objective function places weights on the operating costs and each pollutant, which creates different optimization scenarios. The relative prices are not necessarily constant and can vary by time and spatial locations. For instance, the social cost of emissions may increase as they accumulate. This reflects that fact that when air pollution concentration is already very high, adding additional pollutants would yield higher health damages to the public than in the case where air pollution concentration is relatively low. Also, the environmental impact of emissions is usually higher when the buses are operating in heavy-traffic regions with high population density, because more people are exposed to the health hazard of air pollution.

Expression [2] establishes the relation between bus assignment and operating mode variables. The sum of total service time under all operating modes should equal to the total service time supplied by assignment variables. The multiplier  $\delta$  is used to adjust the time lost during transfer or deadheading between each time period. The value of  $\delta$  is between 0 and 1 because only a portion of the time period is used for service operation. Expression [3] requires the usage of each input to be less than or equal to the total endowment supplied in each time period. This constraint include both multi-period and non-multi-period inputs. Multi-period input's endowment or price changes over time, while non-multi-period input does not. One example of a multi-period input is the number of 2010 hybrid buses available for service at a particular time period. Fuel supply is a non-multi-period input which is consumed during any service period. Additional route constraints can be applied implicitly through the mapping of bus types to routes. For instance, in order to prevent high pollution in crowded areas, there are certain routes that can only be run by hybrid buses during rush hours each day. In this case,  $\theta_{RB}$  defines a set of feasible bus-route combinations where only certain bus types can be assigned to certain routes. Other types of constraints can also be applied in this form. For example, there can be an upper limit for the total number of hours a bus can operate within a day, or an upper limit for the total amount of pollution a bus can emit during a trip on a route. Heterogeneous vehicle performance, such as fuel economy, is reflected in the value of

cost parameters  $AIS_{trik}$  and  $AIO_{trij}$ . Expression [4] requires the service capacity of the bus fleet to meet certain standards. For example, there should be enough seats for people on the bus at all times. However, this might be a relatively high standard since the passenger demand for bus service fluctuates over time and passengers can also stand on the bus. Therefore, a probability can be added to the model so that a bus does not have to meet all passenger demand at each stop at all time. For example, there is 90% chance that all passengers waiting at each bus stop will have seats on the bus. In addition to seat availability, service frequency constraint can be used to reflect the level of service quality. Expression [5] defines the performance measure of the fleet, which can be the total fuel usage or total CO<sub>2</sub> emission within a day.

The model forms a mixed integer problem because the choice variable is the number of buses assigned in a time period. It can be either linear (MIP) or non-linear (MINLP) problem, depending on the formulations of the equations. The model is solved using OSICPLEX solver in General Algebraic Modeling System (GAMS) mathematical programming software. The appendix provides the GAMS code for solving the model with the objective of minimizing total cost.

#### **4. A Case Study of Metro Transit in Minneapolis/St. Paul, Minnesota**

This case study uses data from Metro Transit, the primary public transportation operator in the Minneapolis-Saint Paul area in Minnesota. Metro Transit operated 912 buses on 128 routes in 2013, including 60 urban local routes, 60 express routes, six suburban local route, one light rail route and one commuter rail. The current fleet includes of 570 forty-foot diesel buses, 132 hybrid-electric buses, 41 coach buses and 169 sixty-foot articulated buses. In 2013, Metro Transit received 322.5 million dollars of revenue, of which 48% came from motor vehicle sales taxes, 28.4% came from fares, and the rest came from local, state and federal funding. The total expense was approximately equal to its revenue. 71% percent of expense went to salaries and benefits, 14.2% went to the consumption of fuel, materials and supplies (Metro Transit 2013 Facts).

#### 4.1. Data Sources and Coefficient Estimates

Data related to operating costs are provided by the technical support group at Metro Transit. The original dataset includes the daily consumption of fuel and diesel exhaust fluid (DEF), and the maintenance labor cost and replaced parts cost for every bus on every day from January 1, 2008 to December 31, 2013. The odometer readings at fuel refilling time are also included in this dataset, supporting the calculation of average fuel and DEF consumption rates per mile in 2013. Because maintenance operations usually bring long-run benefits to vehicles, average maintenance cost per mile is calculated using the data from 2008 to 2013.

Fuel price fluctuates through the year and is influenced by the changes of biodiesel blend in the fuel. Typically, the more biodiesel in the fuel, the lower the fuel price is because of the government subsidies on biodiesel (Metro Transit, 2013). The fraction of biodiesel used is largely influenced by weather conditions, because high biodiesel fuel content does not work well in cold weather. Nonetheless, biodiesel is a source of green renewable energy which produces less carbon emissions and is largely from locally grown crops in Minnesota (Metro Transit, 2013). The pricing information on different types of biodiesel blends and the dates of blend changes are provided by Metro Transit.

In this case study, buses are categorized based on engine size, engine manufacturer, the number of seats and the year of emission certification. The year of emission certification is usually the same as the model year of the bus, but there are a few exceptions. For example, some articulated and coach buses are equipped with older emission certified engines than what the model year implies. Hybrid-electric buses are equipped with 6.7 liter engines, while standard diesel, coach and articulated buses usually have 8.9 or 10.8 liter engines.

Passenger information is from the dataset collected by the Automatic People Counting system. This system counts the number of people getting on and off a bus at each stop and calculates the number of the people remaining on the bus after each stop. The dataset also provides the time of each stop, total trip distance and the service bus ID, where a trip is defined as the service work performed on a route with one direction

(east/west or north/south bound). In order to keep the size of the dataset at a manageable level, only data on the buses from one garage (Heywood) are used. In order to characterize the passenger demand at different times of the day, each trip is assigned to an hour of the day based on the average time point in each trip. Passenger demand for each trip is measured by the maximum number of passengers remaining after leaving bus stop among all the stops made during the trip. Passenger demand for each hour is the sum of maximum demand of all trips in this hour. Trip frequency within each hour is calculated based on the actual trip schedules posted on the Metro Transit website.

A key assumption in this case study is that vehicles perform differently on different routes due to unique route characteristics, such as the number of bus stops and traffic lights, as well as the general traffic conditions. Therefore, fuel economy should be assessed at the route level. However, the data provided by Metro Transit only include the total amount of fuel added every day and the miles traveled between each refills. In fact, a bus can run on different routes during this time period. In addition, the deadheading distance, where a bus does not perform any service, is also included in the total distance traveled between the refills. In order to accurately calculate the fuel economy on each route for each bus type, a matching method of fuel usage to routes is used. The trip distance on each route is recorded in the automatic people counting dataset and the total distance traveled is in the fuel usage dataset. The ratio between of actual service distance and total traveling distance can be calculated. When the ratio is below 1.5, the data entry is regarded as invalid, because this usually implies this bus either runs on multiple routes or has very long deadheading distance, which makes the calculation of fuel economy per route inaccurate. In addition, any data entry of miles per gallon greater than 50 is regarded as invalid too, since such large numbers usually are caused by recording errors.

This matching method is necessary to determine the route-specific fuel economy for each bus type. However, it also generates missing data entries for many bus-route combinations because the dataset fails to provide the fuel consumption for the particular bus types on the particular routes. In order to make sure the dataset includes a sufficient number of bus types and routes, an estimation procedure is developed to fill in these missing values. First of all, the average fuel consumption per mile for each bus type is

calculated. Secondly, the existing fuel consumption data for a certain bus-route combination is identified. Then, the fuel consumption per mile ratio between the bus type without data and the bus type with data is calculated and used to multiply the existing fuel consumption data with a certain bus-route combination. Thus, the fuel consumption data for a missing bus-route combination is estimated.

According to Kittelson et al. (2013), the level of greenhouse gas emission depends on three factors: rate of fuel consumption, carbon content and activity. The equation to calculate CO<sub>2</sub> emission is provided as below:

$$\text{CO}_2 \text{ Emissions} = \left( \frac{\text{Liters}}{\text{Kilometer}} \right) \times \left( \frac{\text{Carbon}}{\text{Liter}} \right) \times (\text{Vehicle Kilometers Traveled})$$

Because the carbon content is relatively constant in the diesel fuel used by Metro Transit fleet, CO<sub>2</sub> emission becomes a function of the rate of fuel use and the distance traveled by the bus. The data for both parameters have been provided by Metro Transit, and CO<sub>2</sub> emission is estimated to be 10.28 grams/gallon of fuel.

Due to the limited empirical data on emission levels, the emission standards set by the California Environmental Protection Agency Air Resource Board (CARB) are used to measure the air pollutants' emission coefficients in this study. This is because the major bus engine manufacturers in cooperation with Metro Transit are required to comply with CARB emission standards. These standards are imposed on various vehicle emissions, including particulate matter (PM<sub>2.5</sub>), nitrogen oxides (NO<sub>x</sub>) and carbon monoxide (CO). The emission rates are defined in grams per unit of brake horsepower hour (g/bhp-hr), which measures the amount of air pollutant emitted during an hour of operation per horsepower load (Li and Head, 2009). The standards are set based on the engine manufacturer, engine size and year of manufacture. Clark et al. (2002) found that the factors affecting the heavy-duty diesel-powered vehicle emissions include vehicle class and weight, driving cycle, vehicle use, fuel type, engine exhaust after-treatment, vehicle age and terrain traveled. A test from the Bi-State Development Agency in St. Louis, MO, showed that there is no definite trend of increasing or decreasing in emissions as vehicle mileage accumulates. Therefore, CARB emission standards are used in the model to measure the level of air pollution without significant adjustments.

Even though CARB sets emission standards for various air pollutants, this case study will only focus on nitrogen oxides (NO<sub>x</sub>). Carbon monoxide (CO) is usually the result of inefficient fuel combustion. In the calculation of CO<sub>2</sub> emissions, we have assumed that all the carbon in the fuel is fully converted to CO<sub>2</sub> form. Including CO would further complicate the calculation of CO<sub>2</sub> emissions. Similarly, the formation of PM<sub>2.5</sub> usually involves complex chemical reactions among many air pollutants, including NO<sub>x</sub> (Hodan and Barnard, 2004). Therefore, including PM<sub>2.5</sub> will risk the chance of double counting the environmental impact of NO<sub>x</sub> emission. The equation to calculate the amount of NO<sub>x</sub> emission per gallon of fuel is provided by Andrew Kotz and William Northrop in the Department of Mechanical Engineering at University of Minnesota, and it is shown as the follow:

$$\frac{gNO_x}{mile} = N * (D - x * (D - B)) * \eta_{th} * \frac{1}{MPG}$$

Where:

$N$ = gNO <sub>x</sub> /Bhp-hr	CARB emission standard for NO <sub>x</sub>
$D$ = Bhp-hr/gal-Diesel	Diesel fuel energy content per gallon
$B$ = Bhp-hr/gal-Biodiesel	Biodiesel fuel energy content per gallon
$x$ = Volume fraction of biodiesel	The fraction varies by season. The summer level is 20%, while the winter level is 2.5%.
$MPG$ = Miles per gallon of fuel	Determined by total miles divided by total fuel
$G$ = Gallons Fuel	From Fuel consumption
$\eta_{th}$ = Thermal Efficiency	% of energy in fuel tuned into useful work

#### 4.2. Descriptive Statistics

The analysis uses a modest sized representative fleet from Metro Transit in Minneapolis/St. Paul area in 2013. The representative fleet is composed of ten types of buses, including articulated diesel, standard forty-foot diesel and hybrid forty-foot diesel buses. The representative fleet composition is presented in Table 1. 23 out of 60 local-

urban routes are included in the model, so the vehicle endowment for each bus type in the representative fleet are scaled down to 38% of the original numbers of buses available in the current Metro Transit fleet.

Table 1. Representative Fleet Composition

Engine type ID	# of Buses	Emission Year	Engine Size	Vehicle type	Engine make	# of Seats
B06	27	2003	10.8L	SD 40ft	Cummins	43
B14	17	2007	10.8L	AD	Cummins	58
B15	30	2007	6.7L	H 40ft	Cummins	38
B16	40	2007	8.9L	SD 40ft	Cummins	38
B17	20	2007	8.9L	AD	Cummins	58
B21	29	2010	6.7L	H 40ft	Cummins	38
B22	1	2010	6.7L	H 40ft	Cummins	40
B23	86	2010	8.9L	SD 40ft	Cummins	38
B26	1	2013	6.7L	H 40ft	Cummins	40
B28	58	2013	8.9L	SD 40ft	Cummins	38

Note: SD represents “Standard Diesel”, AD represents “Articulated Diesel”, H represents “Hybrid”

Vehicle fuel cost and maintenance cost per mile are presented in Figure 1. Maintenance cost includes the labor cost of mechanic technicians and the cost of replaced parts on buses. Fuel cost per mile is calculated by dividing the total fuel cost by the total mileage in 2013. Maintenance cost per mile is calculated by dividing the total maintenance cost by the total mileage from 2008 to 2013. Inflation is adjusted using CPI index from Bureau of Labor Statistics.

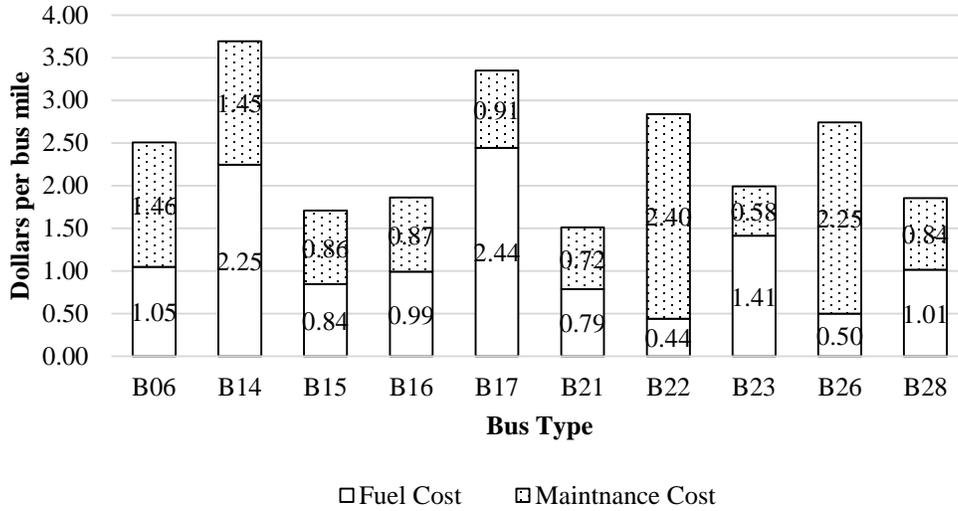


Figure 1. Fuel Cost and Maintenance Cost Per Mile (in 2013 USD) of All Vehicle Types in the Representative Fleet

Table 2. Emission Rate for Each Bus Type (grams/gallon of fuel)

Engine Type ID	NOx
B06	34.98
B14	43.72
B15	37.14
B16	25.36
B17	34.10
B21	8.44
B22	8.44
B23	5.77
B26	3.38
B28	3.50

Table 2 shows the emission rates of nitrogen oxides (NOx) for each vehicle type based on the emission standards set by California Air Resources Board. NOx is a major source of local air pollution and it is mainly from fossil fuel combustion. NOx and the products of its chemical reactions with other air pollutants can cause serious threat to

human health. According to the EPA Integrated Science Assessment (2008), short term exposure to NO<sub>x</sub> can cause increased respiratory and cardio-vascular hospital admission, while long term exposure to NO<sub>x</sub> can lead to decreased lung function. Health and environmental cost of NO<sub>x</sub> emission are calculated by Goodkind and Polasky (2013) in 2010 USD, and these numbers is adjusted for inflation and used in the analysis. Table 3 shows the health and environmental costs per ton of pollutant emissions in Minnesota urban counties.

Table 3. Health and Environmental Costs Per Ton of Pollutant Emissions in Minnesota Urban Counties (2013 USD)

Pollutant	Median (5th-95th percentile)
CO <sub>2</sub>	\$38.46 (\$11.75-\$58.76)
NO <sub>x</sub>	\$3525.51 (\$3,205.00-\$3,632.34)

#### 4.3. Model Assumptions and Validation

For the mathematical programming analysis, the following assumptions are made. Firstly, all buses operate up to 20 hours between 5am and midnight in a typical weekday. Secondly, the fuel economy of each bus type depends on the specific characteristics of the routes, such as traffic conditions. Hence, fuel consumption per mile is calculated for each bus type-route combination. Thirdly, it is assumed that the bus service should meet at least 90% of the maximum passenger demand at each time period and should be at least as frequent as the actual bus service based on the current time schedule.

Model validation is an important step of model building because it provides evidence of the model's credibility and accuracy. It is to demonstrate that the model can reasonably represent the actual system and provide enough credibility to meet the objectives of the analysis. According to McCarl and Apland (1986), two types of validation are usually applied to linear programming models: validation by construct and validation by results. The former requires "sensible techniques" inspired by real world observations to be used in the model construction, while the latter involves systematically

comparing model results with real world observations. The validation process in this study focuses on comparing the output values and system behavior to the observed data in the real world. Instead of comparing the model outputs directly to the observed statistics, the validation is done by comparing the optimal model results with the ones calculated by the model using the observed bus assignments in real life. In this way, the model results are compared to the actual assignments under the same assumptions.

Data on the actual vehicle assignments in 2013 is collected and applied in the optimization model to get the average numbers of seats supplied by the bus operation per hour, daily cumulative bus miles, daily fuel usage, cost values and emission levels. Because of the changing biodiesel blend fraction, fuel price and passenger demand over a year, four representative months are selected for model validation. Table 4 shows the percentage deviation of the optimal model results from the observed assignment results. The optimal results are calculated with the objective to minimize total operating costs plus total emission costs.

Table 4. Percentage Deviations of Optimal Model Results from Observed Assignment Results

Month	Average # of Seats Supplied/hour	Daily Bus Miles	Operating Cost	Total Emission Cost	Total Cost	CO <sub>2</sub> Emission	NO <sub>x</sub> Emission
January	-4.7%	0.0%	-7.0%	-11.4%	-7.3%	-8.1%	-29.5%
April	-4.2%	0.1%	-6.7%	-10.1%	-6.9%	-7.0%	-26.4%
July	-4.5%	0.1%	-6.9%	-9.7%	-7.1%	-7.0%	-25.3%
October	-5.0%	0.1%	-8.3%	-11.0 %	-8.4%	-8.3%	-25. 6%

Based on the comparisons presented in Table 4, the model is found to be well behaved with deviations in reasonable ranges. The average numbers of seats supplied per hour in the model outcomes are only around 5% lower than the observed values in all four months. The daily bus miles in the model results are almost the same as actual bus miles observed. Since seats supplied and bus miles are important indicators of the supply-

demand relation between the transit agency and passengers, the small deviations mean the model can well characterize the supply-demand dynamics of the transit system. However, the total cost with the observed assignments is 6.9-8.4% higher than the minimum total cost generated by the model. Both CO<sub>2</sub> and NO<sub>x</sub> emissions in the actual assignment case are also significantly higher than the emissions levels generated by the model. Because there are little differences in numbers of seats supplied and daily bus miles, the relatively large differences in the results for costs and emission levels might be caused by the inefficient allocation of vehicles in real life. This hypothesis is tested in the next section 4.4.

#### 4.4. Result Analysis

##### 4.1.1. Multiple Objectives and Tradeoff Frontiers

The bus-scheduling problem is solved for a representative weekday. Input coefficients are averaged over four months, each of which represents a season of the year. The results are calculated using these four-month average input coefficients. Table 5 shows the results of the optimal vehicle assignments given different objectives. The first column includes the objectives of the minimization problems. The other columns give the values of the costs and levels of emissions for an average weekday in 2013 USD. Total operating cost is the sum of fuel cost, maintenance cost and DEF cost. Total emission cost includes the social costs of CO<sub>2</sub> and NO<sub>x</sub> emissions. These results show that the bus scheduling can have significant impacts on vehicle emissions and operating costs. The amount of NO<sub>x</sub> is 371% higher in the minimum total cost scenario than it is in the minimum NO<sub>x</sub> emission scenario. The results also show great impacts on total CO<sub>2</sub> emission, with a difference of 2.5 tons of CO<sub>2</sub> emission between the minimum total cost result and minimum CO<sub>2</sub> emission result. The effect on operating cost is significant both in relative and absolute terms. The operating cost with minimum NO<sub>x</sub> emission is 23.65% higher than the minimum operating cost, which is a \$6358 cost difference on an average weekday.

Table 5. Optimization Results with Different Objectives (in 2013 USD)

Objective of Minimization	Operating Cost (\$/day)	Total Cost (\$/day)	Total Emission Cost (\$/day)	CO <sub>2</sub> Emission (kg/day)	NO <sub>x</sub> Emission (kg/day)
Total Cost	26,879	28,742	1,864	42,299	67
Operating Cost	26,879	28,778	1,899	42,309	77
Total Emission Cost	28,381	30,108	1,727	40,620	47
CO <sub>2</sub> Emission	28,140	29,970	1,830	39,694	86
NO <sub>x</sub> Emission	33,237	35,355	2,117	53,164	21

These extreme scenarios provide a useful characterization of the potential benefits that can be achieved from bus scheduling optimization, however it might not be socially optimal and feasible to operate the fleet in these ways. In order to visualize the tradeoff relationships between these objectives and identify the Pareto optimal solution given a budget constraint, tradeoff frontiers are plotted by changing the relative weights of different objectives. Specifically, a GAMS program is developed to assign a series of weights to one objective (such as operating costs) while keeping the weight of the other objective (such as emissions) constant. The GAMS program solves the model every time when a new weight is assigned to one of the objectives and the optimal values of the sub-objectives are combinations on the efficient frontier. Figure 2 shows a downward sloping tradeoff curve between total emission cost and total operating cost. The convex shape of the curve implies an increasing marginal cost of reducing emissions, that is, the marginal cost of reducing emissions increases as total emissions go down.<sup>1</sup> Specifically, when total emissions are high, the slope of the curve is steep, which means it only takes a small increase in operating cost to get a big decrease in total emissions. When the total emissions are low, the slope of the curve is flat, and the same reduction in emissions would lead to a big increase in operating cost. The two ends of the curve represent two extreme scenarios: the minimum operating cost solution and the minimum emission cost solution, respectively. As the weight on total emissions cost in the objective function

<sup>1</sup> Because this model is a mixed integer linear model, the frontiers are piecewise linear functions, appearing as smooth curves due to the large number of solutions and basis changes.

increases, the optimal solution moves along the tradeoff curve, approaching the minimum emission cost solution on the right end. However, this strict trade-off relationship only occurs on the frontier. Operating cost-emissions combinations above the frontier are inefficient, because for those combinations, it is possible to reduce emissions without increasing operating costs, and vice versa. In the case of Metro Transit, the efficiency of the current transit system can be improved by changing vehicle assignments if its current cost-emission combination is above the trade-off frontier.

Where the optimal solution lies on the tradeoff curve depends on the relative weights that transit agencies place on the operating cost and emissions. In Table 5, the optimal solution for minimizing total cost, where the operating cost and the emission cost are weighted equally, is very close to the solution for minimizing operating costs. In this case, the marginal cost and the marginal benefit of reducing emissions are equal when operating cost is close to its minimum level.

Figure 2 also shows the cost of the actual fleet compared to the efficient frontier. Since the results for the four representative months are very similar, the graphs only show the model results using the four-month average input and output coefficients. It shows that the cost with actual vehicle assignments is above and far away from the frontier, which means there is significant room to reduce both emissions and operating cost from their current levels. Specifically, when the optimal vehicle assignment with minimum total cost is used, the daily operating cost can be brought down by 2049.48 dollars per day and NOx emission can be reduced by 41%.

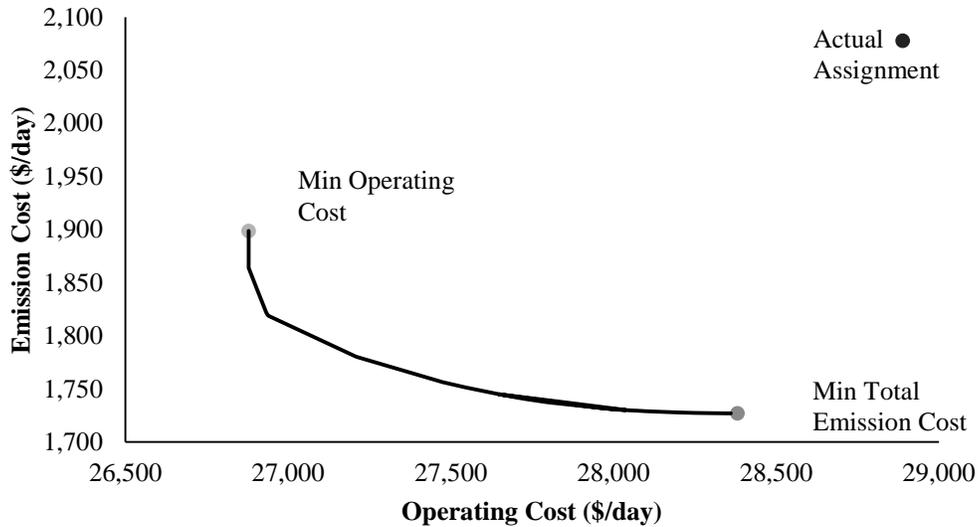


Figure 2. Tradeoff Frontier between Operating Cost and Total Emission Cost

In order to examine the impacts of reducing individual pollutants on the operating cost and emission performance of the bus fleet, tradeoff relationships between the operating cost and emissions are shown in Figure 3-4. The tradeoff frontiers between operating cost and both types of emissions are piece-wise linear and downward slopping curves with increasing opportunity costs of reducing emissions. The operating cost-emission combinations generated by the actual assignments locate above and faraway from both the tradeoff frontiers, which suggests that the actual bus assignments generate more emissions and higher operating cost than the optimal assignment results. Specifically, the CO<sub>2</sub> emission of the actual fleet is 15% higher than the minimum CO<sub>2</sub> emission, while the NO<sub>x</sub> emission level of the actual fleet is 4.4 times of the minimum level. The operating cost of the actual fleet is 7% higher than the minimum operating cost. These numbers confirm that there is great potential to reduce both types of emissions without increasing operating cost.

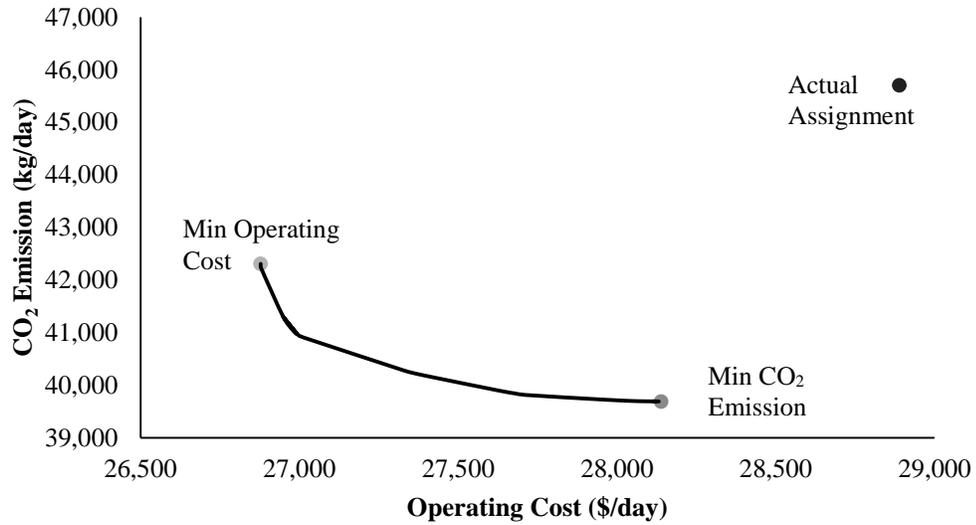


Figure 3. Tradeoff Frontier between Operating Cost and CO<sub>2</sub> Emission

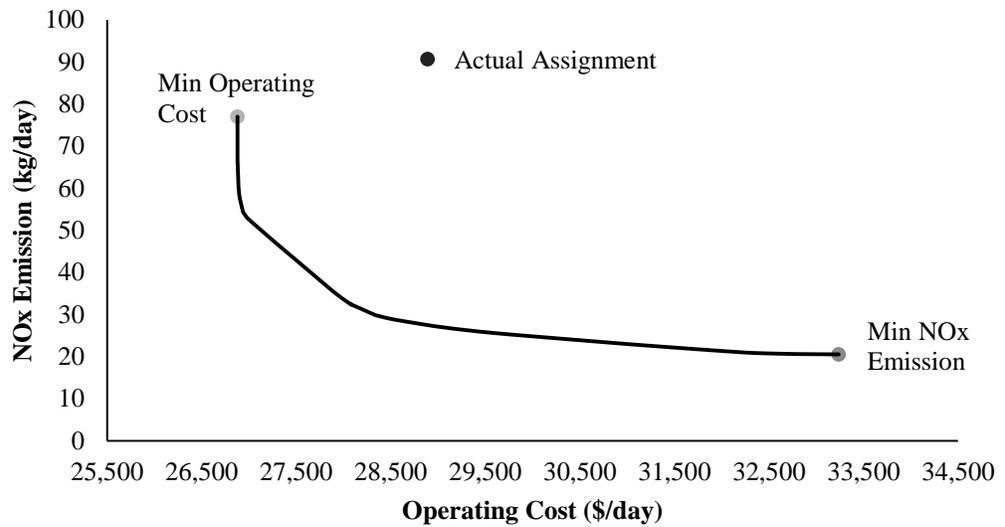


Figure 4. Tradeoff Frontier between Operating Costs and NO<sub>x</sub> Emission

#### 4.4.2. Fleet Composition Analysis

As the major choice variables in the model, vehicle assignments provide important information on how the optimal results are achieved and the cost-emission performance

of each bus type. Table 6 shows the assignment results of each bus type with different objectives in an average weekday. The differences among these fleet compositions highlight the characteristics of each bus type. Percentage of usage is the ratio between the number of assignments for a bus type and its maximum number of assignments available.

Table 6. Comparison of Fleet Compositions with Different Objectives

Panel a. Number of assignments per day for each bus type

Bus type \ Objective	B06	B14	B15	B16	B17	B21	B22	B23	B26	B28
Total cost	0	4	167	37	41	528	0	437	0	0
Operating cost	0	4	167	37	41	528	0	437	0	0
Emission cost	0	88	201	219	117	524	19	517	19	588
CO <sub>2</sub> emission	0	4	390	38	24	513	19	44	19	164
NOx emission	0	0	0	0	0	42	19	199	19	940

Panel b. Percentage of usage for each bus type (%)

Total cost	0	1.2	27.9	4.7	10.2	90.9	0	25.4	0	0
Operating cost	0	1.2	27.9	4.7	10.2	90.9	0	25.4	0	0
Emission cost	0	1.2	11.9	3.4	5.9	89.7	92.5	7.0	92.5	35.3
CO <sub>2</sub> emission	0	1.2	65.1	4.7	5.9	88.5	92.5	2.6	92.5	14.1
NOx emission	0	0	0	0	0	7.3	92.5	11.6	92.5	81.0

In the minimum operating cost scenario, bus B21 (2010 40ft hybrid bus) is used the most frequently, as a result of its relative low operating cost (shown in Figure 1). Bus B23 (2010 40ft standard diesel bus) and B15 (2007 40ft hybrid bus) are the second and third most used bus types in the minimum operating cost case. In the minimum total emission cost scenario, bus B22 (2010 40ft hybrid bus) and B26 (2010 40ft hybrid bus) are assigned to almost all of the working hours, even though there is only one bus available for each type. B21 (2010 40ft hybrid bus) and B15 (2007 40ft hybrid bus) are assigned most frequently in terms of the total numbers of assignments. B15 (2007 40ft hybrid bus) is not assigned at all in the minimum operating cost case, because it is a low-

emissions bus with a relatively high operating cost. However, B21 (2010 40ft hybrid bus) is among the most frequently assigned buses in all six scenarios, which makes it the most cost efficient and cleanest bus in the representative fleet. In terms of CO<sub>2</sub> emissions, B22 (2010 40ft hybrid bus) and B26 (2010 40ft hybrid bus) are proven to be the favorable choices to lower greenhouse gas emissions, though there are only two buses available in total. B21 (2010 40ft hybrid bus) and B15 (2007 40ft hybrid bus) are the most assigned bus types in the fleet in terms of the total numbers of assignment. However, B28 (2013 40ft standard diesel) is preferred if the objective is to minimize NO<sub>x</sub> emission. B22 (2010 40ft hybrid bus), B26 (2010 40ft hybrid bus) and B28 (2013 40ft standard diesel) are optimal when the objective is to minimize the environmental impact of bus operations, but these buses would not be assigned if the transit agency sought to minimize the total operating cost of bus services. This is due to the relatively high estimated maintenance cost for these buses.

Examining the fleet compositions under various optimal scenarios can provide insights on vehicle performance and how various types of buses should be used. By comparing the optimal vehicle assignments to actual vehicle assignments, methods to achieve optimality can be identified. Figure 5 compares the actual assignments with the optimal vehicle assignments generated by minimizing the total cost where the operating cost and the environmental cost of emissions are equally weighted. The x-axis represents the bus types and y-axis represents the percentage of assignments in an average weekday, which is calculated as the ratio between the number of assignments for a bus type and total number of assignments for the fleet. One unit of assignment means one bus gets assigned to a route in a time period. There are ten bus types available for deployment in the model, and only five of them get assigned to minimize total cost in an average weekday. Bus type B21 (2010 40ft hybrid bus) gets assigned most frequently, and bus type B23 (2010 40ft standard diesel bus) is the second most frequently assigned bus type, followed by B15 (2007 40ft hybrid bus). B16 is a 2007 standard 40ft diesel bus and B17 is a 2007 articulated diesel bus, and both types are equipped with an 8.9 liter engine. Both types get assigned equally infrequently. B14, a 2007 articulated bus with a 10.8 liter engine, has only a few assignments. B6 (2003 40ft standard diesel bus), B22 (2010 40ft

hybrid bus), B26 (2013 40ft hybrid bus) and B28 (2013 40ft standard diesel bus) are not assigned when total cost of operation is minimized.

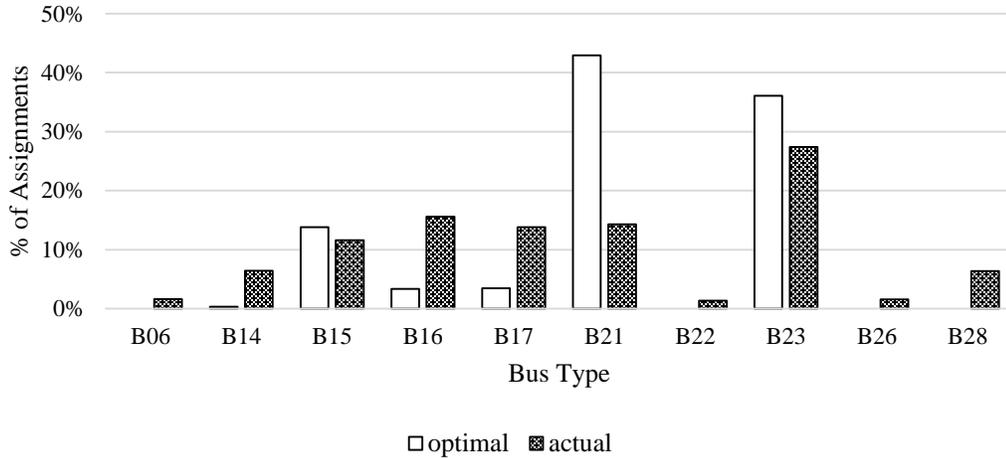


Figure 5. Optimal Vehicle Assignments and Actual Assignments

Figure 5 also shows the actual vehicle assignments implemented by Metro Transit in an average weekday in 2013. Bus type B23 (2010 standard 40ft diesel bus) most frequently. They are followed by B16 (2007 40ft standard diesel bus) and B21 (2010 40ft hybrid bus). 2007-built articulated diesel bus B17 gets 14% of the total assignments, followed by B15 (2007 40ft hybrid bus). B6 (2003 40ft standard diesel bus) and B26 (2013 40ft hybrid bus) are the least assigned bus types. B28 (2013 40ft standard diesel bus) only gets assigned in July and October, which may be due to the time required for purchasing and testing the new buses. According to Figure 1, B21 (2010 40ft hybrid bus) is the most cost efficient bus type with only \$1.51 operating cost per mile. B15 (2007 40ft hybrid diesel bus) is the second most cost efficient bus with \$1.70 operating cost per mile. However, B23 (2010 40ft standard diesel bus) is the second most assigned bus type in the optimal scenario and the most frequently assigned bus type in real life.

Figure 6 shows the comparison between the optimal and actual assignments by hybrid and standard diesel buses. Articulated buses are counted as standard diesel buses in this chart because they share the same type of diesel engine. In the optimal scenario, hybrid

buses get 57% of the total assignments, in contrast to only 29% of the total assignments in real life. Standard diesel buses are used far more frequently in real life than the optimal scenario where total cost of operation is minimized. This suggests that the cleaner hybrid buses are significantly under-used in real life even though they are available in the garages.

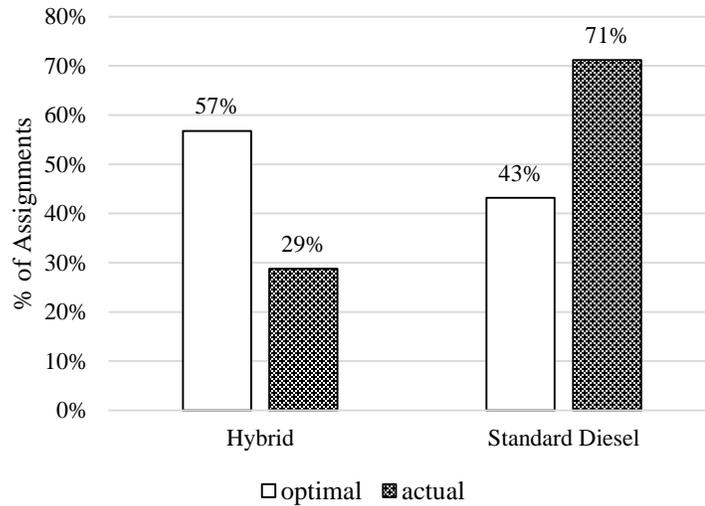


Figure 6. Comparison of Assignments by Hybrid and Standard Diesel Bus

#### 4.4.3. Strategic planning

Bus fleet purchases are frequently subsidized by Federal grants aimed at accelerating turnover, improving fuel economy, reducing noise, and lowering emissions. The Federal Transit Administration requires large transit buses purchased with Federal funds to operate for at least 12 years or 500,000 miles (Li and Head, 2009). Therefore, every year or every several years, transit agencies will make bus purchase and replacement decisions based on their budgets and existing fleets. The analysis of the optimal bus scheduling results can provide useful information on which bus type should be replaced and purchased. In order to measure the cost-emission performance of each bus type, cost parameters are calculated assuming the whole fleet is populated by a single bus type. Figure 7 shows the relative locations of the single bus type fleets in the operating cost-

emission cost space. The relative position of each bus type to the origin shows their cost and emission performance. As the point moves further away from the frontier, the corresponding bus type becomes dirtier and/or less cost efficient. In Figure 7, B21 (2010 40ft hybrid bus) is closest to the origin, which it has the best performance among all the bus types in the fleet in terms of cost efficiency and environmental friendliness. Hence, Metro Transit should increase the amount of B21 (2010 40ft hybrid bus) in its bus portfolio and deploy more to operation. B15 (2007 40ft hybrid bus) also has good cost and environmental performance and should also be considered in the future purchase decisions. B16 (2007 40ft standard diesel bus), B28 (2013 40ft standard diesel bus) and B23 (2010 40ft standard diesel bus) also have relative good cost-environmental performance. However, B6 (2003 40ft standard diesel bus), B17 (2007 articulated diesel bus), B22 (2010 40ft hybrid bus) and B26 (2013 40ft hybrid bus) are relatively far away from the frontier comparing to other buses, which means they have relatively high operating cost and emissions. B14 (2007 articulated diesel bus) is farthest away from the origin, which implies it is very environmental unfriendly and costly to operate. Hence, it should be considered in the replacement decision<sup>2</sup>.

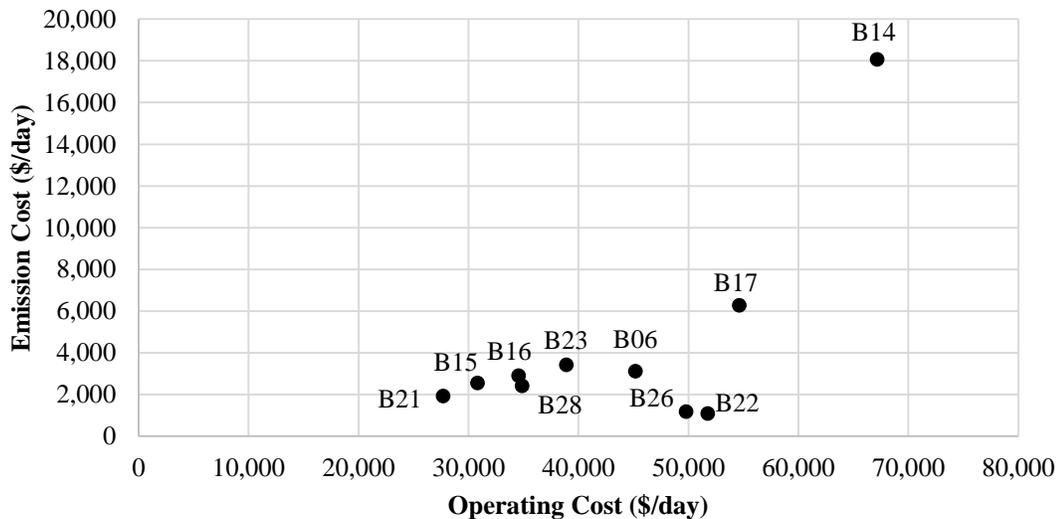


Figure 7. Comparison of Bus Performance by Type

<sup>2</sup> Optimal timing of bus replacement involves detailed estimates of changes in operating costs and emissions levels as buses age, versus the ownership and operating costs associated with new buses.

## 5. Conclusions and Discussion

Capital and operating costs are the primary concerns to transit agencies in the traditional bus scheduling problems. However, as the transportation sector becomes the major contributor to greenhouse gas emissions and other air pollution, the reduction of bus emissions has become an increasingly important goal for transit agencies when determining bus schedules and making purchase and replacement decisions. Meanwhile, bus engine manufacturers are improving the efficiency of conventional diesel buses and developing new technologies to use alternative power sources, including hybrid-diesel buses (Li and Head, 2009). Aimed to address both the cost and environmental concerns and to utilize the existing bus technologies, this paper develops a math programming model to optimize bus assignments with the objective of minimizing operating costs and the environmental outcomes of public transportation system, while considering the heterogeneity in vehicle performance, changing passenger demand and resource availability.

The optimization model is validated and applied to Metro Transit in the Minneapolis/St Paul area using 2013 data on bus performance and demand for service. The model results show that vehicle assignments can have a significant impact on the operating cost and environmental outcomes of the existing fleet. Optimization scenarios with different objectives are developed to demonstrate the capacity of the vehicle scheduling optimization model. Tradeoff frontiers between operating cost and emissions generated by the model are downward sloping, convex curves, reflecting the increasing marginal cost of reducing emissions. The performance of the actual fleet is compared to the efficient frontier for each scenario. The results suggest that there is great potential for the current fleet to reduce both emissions and operating cost. The analysis on the optimal and actual fleet compositions provide insight on how the optimal results can be achieved through the reassignment of the existing buses to routes. It is found that hybrid buses are significantly underused, especially for the 2010 40ft hybrid buses. By comparing the locations of single bus type fleets with respect to both operating costs and emissions, suggestions on purchase and replacement decisions can be supported. Based on the data

used in the analysis, the 2010 40ft hybrid bus has the lowest operating costs and emissions relative to the other buses in the fleet. Estimated operating costs and emissions rates were highest for the 2007 articulated bus, suggesting that it should be replaced soon even though it is not the oldest bus type in the fleet.

The model and the case analysis can be improved with additional data and resources. Firstly, this case analysis does not take spatial aspects of the emissions into account. Gouge et al. (2013) find that  $PM_{2.5}$  imposes greater threat to public health when it is emitted in regions with bigger population density. Local demographic data is used in their study to estimate the proportion of pollutant inhaled by the public around bus routes. If this heterogeneity in the social cost of emissions is considered, the externality of emissions would be more accurately represented. Secondly, the social welfare of passengers consuming the public transportation service is not included in the objective function. Instead, the passenger demand is reflected as the lower bounds in the model in terms of the number of seats and bus service frequency. In future research, greater analytical power can be achieved by including a benefits function to more accurately characterize passenger demand. Possible scenarios can include insufficient seats on the buses or that a bus cannot meet its time schedule. With a longer planning horizon, the consideration of passenger welfare from receiving bus service can affect the strategic planning of transit system, such as route design, bus stop selection and time table design. With additional data, it would be possible to address other transit costs not considered in this project. For example, this study does not consider the costs of driving the buses from their garages to the routes where they are used, referred to as “deadheading”. In fact, the duration and distance of deadheading constitutes a large part of bus operation. In future research, deadheading costs could be included in the assignment problem and reflected in the optimal assignment to routes as well as in the assignment of buses to garages.

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## Appendix: GAMS Program for Optimal Vehicle Assignments with Minimum Total Cost

This appendix provides the GAMS program for the baseline model presented in this study. In this model, there is one operating mode and the objective is to minimize total cost, which is the sum of operating costs and social costs of emissions. The program is structured in the following order: defining sets, importing data, declaration of variables, equations, model and result tables. OSICPLEX solver is used to solve this model in GAMS. The complete GAMS output lst file and the Excel file with input matrices are included as supplementary materials.

```
$TITLE OPTIMAL BUS ROUTING ASSIGNMENT
Option LIMCOL=0, LIMROW=0;
Option Solprint=off;
* this version of model doesn't have operating modes

***** DEFINING SETS *****
Set I Bus types /B06,B14,B15,B16,B17,B21,B22,B23,B26,B28/;

Set K Inputs /F fuel usage, M maintenance cost, DEF/;

Set P Parameters for inputs /MIN, MAX, PRICE/

Set E Emission /CO2,NO/;

Set R Routes /R003, R004, R005, R007, R009, R010, R014, R016, R018,
R019, R022, R025,R050, R061, R094, R250, R649, R652, R667, R672, R674,
R675, R766/;

Set T Time periods /T05, T06, T07, T08, T09, T10, T11, T12, T13, T14,
T15, T16, T17, T18, T19,T20, T21, T22, T23, T24/;

***** IMPORTING DATA *****
Table AMS(I,R,K)
$offlisting;
$ondelim
$include AMS.csv
$offdelim
$onlisting;
display AMS;
* fuel usage: gallon per mile; M, DEF: dollar per mile

Table EL(I,E) emission levels
$offlisting;
$ondelim
$include EL-avg.csv
$offdelim
$onlisting;
display EL;
* kilograms of emission/gallon of fuel
```

```
Set IR(I,R) Mapping buses to routes; IR(I,R)=YES;
IR(I,R)$(AMS(I,R,"F")>500) =NO;
```

```
Parameters D(R) Route distance (miles)
/R003 11.0283, R004 13.7994, R005 12.9161, R007 11.1495, R009 14.4419,
R010 11.9998, R014 13.6039, R016 10.8919, R018 8.7030, R019 8.0338,
R022 18.2136, R025 17.0215, R050 8.5292, R061 15.9828, R094 11.0846,
R250 18.3216, R649 7.9602, R652 13.99, R667 13.3804, R672 14.2926,
R674 25.3110, R675 20.2573, R766 18.1889/;
```

```
Parameters S(I) Number of passenger seats on each type of bus
/B06 43, B14 58, B15 38, B16 38, B17 58, B21 38, B22 40, B23 38,
B26 40, B28 38/;
```

```
Parameters Trip(R) Number of trips a bus can run within each time
period
/R003 1.1950, R004 0.9570, R005 0.9460, R007 1.0390, R009 1.0000,
R010 1.0340, R014 0.8960, R016 0.9960, R018 1.2890, R019 0.8960,
R022 0.7350, R025 0.8613, R050 1.3420, R061 0.8610, R094 1.7820,
R250 1.3990, R649 2.2120, R652 1.3219, R667 1.2628, R672 1.5040,
R674 0.9550, R675 0.9630, R766 1.3770/;
```

```
Scalar CQ_OPC Weight of operating costs in objective /1/;
```

```
Scalar CQ_TEC Weight of total emission costs in objective /1/;
```

```
Scalar delta Probability of meeting max demand /0.9/;
```

```
Table CN(T,K,P) Parameter table for inputs
$offlisting;
$ondelim
$include CN-avg.csv
$offdelim
$onlisting;
display CN;
* maximum and minimum level of inputs, input prices
```

```
Table MaxBus(T,I) Parameter table for inputs
$offlisting;
$ondelim
$include MaxBus.csv
$offdelim
$onlisting;
display MaxBus;
```

```
Table EMIS(E,P) Parameter table for emissions
$offlisting;
$ondelim
$include EMIS.csv
$offdelim
$onlisting;
display EMIS;
* emission price: dollar per kilogram
```

```

Table FRQ(R,T) Minimum frequency of bus service
$offlisting;
$ondelim
$include FRQ.csv
$offdelim
$onlisting;
display FRQ;

```

```

Table CR(R,T) Capacity requirement (number of passengers) on each route
$offlisting;
$ondelim
$include APC-avg.csv
$offdelim
$onlisting;
display CR;

```

```

***** DELARATION OF VARIABLES *****
Variable X(I,R,T) Number of type I bus assigned to route R in period T,
Seat average number of seats, Y(T,K) Each input usage level,
TY(K) total performance measure, Fuel daily fuel,
Maint daily maint, OPC(T) Operating cost by time period,
BY(I,K), TOPC Total operating cost, TX(T,I),
Em(T,E) Total emission level, TM total milage per day,
EC(T) Emission cost by time, TEC total emissions cost,
TC Total cost, CO2L CO2 level, NOL NOx level;

```

```

Integer variable X(I,R,T);

```

```

***** EQUATIONS *****
Equations Perform(T,K), OperatingCost(T), MaxInput(T,K), MinInput(T,K),
TotalY(K), TotalBus(T,I), BusPerform(I,K), TotalOPC,
PasDemand(T,R), Freq(T,R), Emissionlevel(T,E),
Totalec, Miles, SeatNum, FuelNum, MaxEmis(T,E), MinEmis(T,E),
CO2level, NOxlevel, Totalec, MaintNum, HourBus(T,I),
EmissionCost(T), TotalCost;

```

```

* Define performance measures and operating cost
Perform(T,K) .. Sum(R, Sum(I$IR(I,R), AMS(I,R,K) * X(I,R,T) * D(R) *
Trip(R))) =E= Y(T,K);
BusPerform(I,K) .. Sum(R, Sum(T, AMS(I,R,K) * X(I,R,T) * D(R) * Trip(R) *
CN(T,K, "PRICE"))) =E= BY(I,K);
OperatingCost(T) .. Sum(K, Y(T,K) * CN(T,K, "PRICE")) =L= OPC(T);
TotalOPC .. Sum(T, OPC(T)) =E= TOPC;
MaxInput(T,K) .. Y(T,K) =L= CN(T,K, "MAX");
MinInput(T,K) .. Y(T,K) =G= CN(T,K, "MIN");
TotalY(K) .. Sum(T, Y(T,K)) =E= TY(K);

```

```

TotalBus(T,I) .. Sum(R$IR(I,R), X(I,R,T)) =L= MaxBus(T,I);
HourBus(T,I) .. Sum(R$IR(I,R), X(I,R,T)) =E= TX(T,I);
Miles .. Sum(I, Sum(R, Sum(T, X(I,R,T) * D(R) * Trip(R)))) =E= TM;

```

```

SeatNum..          Sum(T, Sum(R, Sum(I$IR(I, R), X(I, R, T) * Trip(R) * S(I))))
                   /20=E= Seat;
FuelNum..          Sum(T, Y(T, "F"))=E= Fuel;
MaintNum..         Sum(T, Y(T, "M"))=E= Maint;

```

```

* Service attribute 1: seats for passeners
PasDemand(T,R)..  Sum(I$IR(I, R), X(I, R, T) * Trip(R) * S(I))=G=
                   delta*CR(R, T);

```

```

* Service attribute 2: minimum frequency of bus service
Freq(T,R)..       Sum(I$IR(I, R), X(I, R, T) * Trip(R))=G= FRQ(R, T);

```

```

* Define emission levels and costs
Emissionlevel(T,E).. Sum(I, Sum(R$IR(I, R), X(I, R, T) * AMS(I, R, "F") *
                               EL(I, E) * D(R) * Trip(R))) =L= Em(T, E);
MaxEmis(T,E)..     Em(T, E)=L= EMIS(E, "MAX");
MinEmis(T,E)..     Em(T, E)=G= EMIS(E, "MIN");
CO2level..         Sum(T, Em(T, "CO2")) =E= CO2L;
NOxlevel..         Sum(T, Em(T, "NO")) =E= NOL;
EmissionCost(T).. EC(T) =E= Sum(E, EMIS(E, "PRICE") * Em(T, E));
TotalEC..          Sum(T, EC(T)) =E= TEC;

```

```

TotalCost..        CQ OPC*TOPC+CQ TEC*TEC =L= TC;

```

```

***** MODEL *****

```

```

** minimizing total costs
Model BusRouting1 /All/;
Solve BusRouting1 Using MIP Minimizing TC;

```

```

***** RESULTS TABLES *****

```

```

Set HDR Table headers /LOWER, LEVEL, UPPER/;

```

```

Parameter RTPPerform(T,K,HDR) Results table for performance measure
equations;

```

```

RTPPerform (T,K,"LOWER") = MinInput.LO(T,K);
RTPPerform (T,K,"LEVEL") = MaxInput.L(T,K);
RTPPerform(T,K,"UPPER") = MaxInput.UP(T,K);
Option RTPPerform:3:2:1;
Display RTPPerform;

```

```

Parameter RTTotalBus(T,I,HDR) Results table for bus usage equations;

```

```

RTTotalBus(T,I,"LOWER") = TotalBus.LO(T,I);
RTTotalBus(T,I,"LEVEL") = TotalBus.L(T,I);
RTTotalBus(T,I,"UPPER") = TotalBus.UP(T,I);
Option RTTotalBus:3:2:1;
Display RTTotalBus;

```

```

Parameter RTPasDemand(T,R,HDR) Results table for passenger demand
equations;

```

```
RTPasDemand(T,R,"LOWER") = PasDemand.LO(T,R);
RTPasDemand(T,R,"LEVEL") = PasDemand.L(T,R);
RTPasDemand(T,R,"UPPER") = PasDemand.UP(T,R);
Option RTPasDemand:3:2:1;
Display RTPasDemand;
```

```
Parameter RTFreq(T,R,HDR) Results table for trip frequency equations;
RTFreq(T,R,"LOWER") = Freq.LO(T,R);
RTFreq(T,R,"LEVEL") = Freq.L(T,R);
RTFreq(T,R,"UPPER") = Freq.UP(T,R);
Option RTFreq:3:2:1;
Display RTFreq;
```

```
Parameter RTX(I,T,R) Results table for assignment variable;
RTX(I,T,R) = 0;
RTX(I,T,R) = X.L(I,R,T);
Option RTX:0:1:1;
Display RTX;
```

```
Parameter RTY(T,K) Results table for performance measures by period;
RTY(T,K)=0;
RTY(T,K)=Y.L(T,K);
Option RTY:2:1:1;
Display RTY;
```

```
Parameter RTEm(T,E) Results table for emission levels in each period;
RTEm(T,E)=0;
RTEm(T,E)=Em.L(T,E);
Option RTEm:2:1:1; Display RTEm;
```

```
Parameter RTOPC(T) Operating cost by time period;
RTOPC(T)=0;
RTOPC(T)=OPC.L(T);
Option RTOPC:2; Display RTOPC;
```

```
Parameter RTEC(T) Emission cost by time;
RTEC(T)=0;
RTEC(T)=EC.L(T);
Option RTEC:2; Display RTEC;
```