Using Measures of Mathematics to Predict Response to Supplemental Intervention

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Dedication

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Abstract

The method for correctly identifying and intervening with students who are not meeting grade level expectations has varied. Historically, an approach relying on underlying cognitive characteristics or processing skills was used. This approach, referred to as an Aptitude-by-Treatment Interaction (ATI) was criticized for not fully capturing student needs or explaining intervention effectiveness (Cronbach & Snow, 1977; Kearns & Fuchs, 2013). Alternatively, a framework called a Skill-by-Treatment Interaction (STI) relies on matching interventions based on measurable and alterable skills (Burns, Codding, Boice, & Lukito, 2010). Preliminary research in the area of mathematics suggests that the STI approach may be useful in identifying specific subskill needs, such as conceptual understanding or computational fluency, for students (Burns, 2011).

The purpose of the current study was to better understand the relationship between mathematics assessment and intervention design. Specifically, the study examined the link between specific skill assessments of conceptual understanding, computational fluency, and application and word problem solving with a conceptually-based or computation-based intervention. Participants were 46 third and fourth grade students attending a suburban elementary school in the upper Midwestern United States. All participating students received a conceptually-based and computation-based intervention, the order of which was counterbalanced, for two weeks, respectively.

Students were assessed using measures of conceptual understanding, computational fluency, and application and word problem solving. Results indicated that gains in computation and application and word problem solving were best predicted by

students' pretest performance on the same measure, regardless of intervention. Interestingly, gains in computational fluency following a computation-based intervention were predicted by students' prior conceptual understanding. Pretest performance on the conceptual understanding and computational fluency measures were used post hoc to analyze groups of students based on identified need. Students' identified need did not account for a significant proportion of the variance following intervention.

The current results were contextualized within previous research and potential implications for practice were discussed. Specifically, the results of the study were discussed in terms of their contribution to (1) the role of and relationships between essential knowledge bases comprising mathematical proficiency, and (2) how the current study might inform frameworks for matching assessment data to intervention. Lastly, limitations to the study and future directions for research were outlined.

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CHAPTER 1

INTRODUCTION

Elementary and secondary students in the United States ranked 9th and 12th for fourth and eighth grades, respectively, on the 2011 Trends in International Mathematics and Science Study (Mullis, Martin, Foy, & Arora, 2012). Moreover, the 2011 National Assessment of Educational Progress (NAEP) reported that only 43% of fourth graders and 36% of eighth graders in the United States are proficient in mathematics (National Center for Education Statistics, 2013).

Given the relatively low mathematics performance for students in this country, mathematics is becoming more frequently researched. However, the focus of the research has been divided among assessment (e.g., Foegen, Jiban, & Deno, 2007), instruction (e.g., Gersten et al., 2009), and across disciplines (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Jitendra, DiPipi, & Perron-Jones, 2002; Matthews & Rittle-Johnson, 2009). Mathematical proficiency is a multi-dimensional construct that includes (a) procedural fluency (i.e., computational fluency), (b) conceptual understanding, (c) the ability to formulate and mentally represent problems, (d) mathematical reasoning, and (e) the ability to correctly apply math to daily activities (Kilpatrick, Swafford, & Finell, 2001). Further, the National Mathematics Advisory Panel (NMAP; 2008) concluded that

> Debates regarding the relative importance of conceptual knowledge, procedural skills (e.g., the standard algorithms), and the commitment of addition, subtraction, multiplication, and division facts to long-term memory are misguided. These capabilities are mutually supportive, each facilitating learning of the others. Conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations together support effective and efficient problem solving. (p. 26)

Statement of the Problem

Struggling students who are identified early and provided with supplemental academic supports experience improved reading or mathematics outcomes (Fuchs, Fuchs, & Compton, 2012a). However, despite a school's best efforts, there will be a subset of students for whom evidence-based interventions implemented with fidelity are not immediately effective. It has been suggested that close to 4% of all students receiving well-designed mathematics tutoring, do not respond to the intervention, which makes the need for further research critical (Fuchs, Fuchs, & Compton, 2012b).

Previous efforts to explain the rate at which students do not respond to generally effective interventions focused on underlying shared cognitive characteristics associated with a potential learning disability that required more intensive programming. Such a hypothesis suggested that these attributes were innate, easily assessed with measures of cognitive processing or aptitude, and were difficult to remediate with standard interventions (Dean & Burns, 2002). However, relying on measures of aptitude required educators to make higher inference decisions based on a model (i.e., Aptitude-by-Treatment Interaction) that was criticized for not fully capturing individual differences in intervention effectiveness (Cronbach & Snow, 1977; Kearns & Fuchs, 2013).

A competing hypothesis involves more directly assessing students' academic skills in order to make more direct decisions about current level of need and selecting supplemental interventions. A Skill-by-Treatment Interaction (STI) approach engages educators in a process of selecting interventions based on student functioning in a specific skill (Burns, Codding, Boice, & Lukito, 2010). For example, a child who struggles with conceptual understanding for math would be taught the underlying principles, but those who understand the concept and struggle with mathematical procedures would receive an intervention that focuses on successfully completing written problems (Burns, 2011). Among several advantages to this approach for intervention design is that it frames student difficulties in terms of alterable variables for which an educator can impact through varying preventative and intervening approaches from an ecological perspective. However, previous research using skill measures to predict student response have relied only on measures of procedural fluency (Codding et al., 2007).

Study Purpose

The purpose of this study is to broaden the research base around using data to inform mathematics intervention. Specifically, the study examined the relationship between specific skill assessments of conceptual understanding, computational fluency, and application and word problem solving with a conceptually-based or computation-based intervention. First, students completed a battery of mathematics assessments targeting their conceptual understanding, computational fluency, and application and word problem solving skills. Next, students received conceptually- and computation-based interventions, the order of which was counterbalanced across groups. Posttest assessments were administered after each intervention (i.e., conceptually- or computation-based). Student gains on a computational fluency measure and an application and word problem solving measure served as the outcome variables for the study.

Significance of the Study

This study is intended to inform research and practice related to mathematics assessment and intervention. Mathematics proficiency is a multi-dimensional domain that demands attention on a bevy of skills. Students are required to demonstrate conceptual understanding, computational fluency, and word problem solving skills in order to meet proficiency standards in the classroom and on state accountability tests. For this reason, early identification and intervention of targeted mathematical skills is critical. Thus, this study aims to help clarify which data can be used to effectively and efficiently target interventions.

Research Questions

The following research questions guided the current study:

1. To what extent does pretest data differentially predict intervention response to conceptual intervention on fluency measure.
2. To what extent does pretest data differentially predict intervention response to computation intervention on fluency measure.
3. To what extent does pretest data differentially predict intervention response to conceptual intervention on application measure.
4. To what extent does pretest data differentially predict intervention response to computation intervention on application measure.
5. To what extent does need predict response to a conceptual intervention?

6. To what extent does need predict response to a fluency intervention?

Definitions

Curriculum-based Measurement – Mathematics (CBM-M): An assessment approach that allows for brief, repeated administration of skill-specific domains, including reading, spelling, writing, and mathematics (Deno, 1985). CBM-M applies CBM principles to the area of mathematics by assessing specific mathematic skills such as computation and concepts and application.

Conceptual Understanding: A person's knowledge of the underlying principles of a specific domain, the interrelationship with other domains, and the development of representations for that domain (Greeno, 1978; Rittle-Johnson & Alabali, 1999). Conceptual understanding is sometimes referred to as conceptual knowledge.

Computational fluency: A person's proficient use of algorithmic operations in order to complete a mathematical task (Byrnes & Wasik, 1991; Geary, 1993; Hiebert & Lefevre, 1986; Nesher, 1986; Rittle-Johnson, Siegler, & Alibali, 2001). Computational fluency is sometimes referred to as procedural fluency or procedural knowledge.

Application and Word Problem Solving: A person's proficient use of mathematical concepts and algorithms to solve authentic, real-world problems.

Mathematics Difficulty (MD): A broad construct that represents students who perform below a cut-off of approximately the 25-35th percentile on a standardized math assessment (Mazzocco, 2007).

Skill-by-treatment interaction: A paradigm that targets interventions based on student skill performance rather than on underlying aptitudes (Burns, VanDerHeyden, & Zaslofsky, 2015).

Delimitations

The following limitations were placed on the study:

(a) Study participants were limited to 3rd and 4th grade students at risk for mathematics difficulty from one suburban school in the Midwestern United States.

(b) The interventions only targeted two (i.e., conceptual understanding and computation proficiency) of the five strands identified for mathematical proficiency by the National Research Council (Kilpatrick, Swafford, & Finell, 2001).

(c) Interventions occurred during four weeks of the spring semester of one academic year.

Organization of the Dissertation

This dissertation is organized around four additional chapters. Chapter 2 provides an overview of the literature relevant to (a) using data in frameworks of intervention, (b) mathematics development and assessment, and (c) targeting interventions in mathematics. Chapter 3 outlines the methodology used in the current study. It describes the characteristics of the 46 participants, the measures used for screening and assessing gains, the interventions that were used and the implementation procedures, and the data analysis. Chapter 4 reports the results for each research question and includes several tables to aid in interpreting the data. Chapter 5 discusses the results within the context of previous research. It also discusses the results in terms of implications for practice, mathematics assessment and intervention, and future research. The chapter concludes with limitations for interpreting the current data.

CHAPTER 2

LITERATURE REVIEW

Chapter two outlines relevant literature in mathematical development, assessment, and intervention. It is organized into three sections. The first section provides a discussion of the research base around mathematical development, with a focus on conceptual understanding and computational fluency, specifically as it relates to the current study. The section also includes brief examples of potential intervention techniques for each of the described mathematical domains. The next section examines current assessment practices broadly in the subject of mathematics and then more specifically as it relates to conceptual understanding and computational fluency. Finally, the last section discusses the importance of using assessment data to inform intervention decisions. The chapter concludes with a description of the critical importance of a continued focus on research examining the relationship between assessment, intervention, and the potential frameworks for informing instructional decisions.

Mathematical Development

Mathematical proficiency is unarguably a critical component of modern education with links to improved educational and employment outcomes (Cavanagh, 2006). However, the United States is trailing other developing countries (Mullis, Martin, Gonzalez, & Chrostowski, 2004) with less than half of our students nationally demonstrating proficiency in mathematics (U.S. Department of Education, 2011).

The National Research Council (NRC; Kilpatrick, Swafford, & Findell, 2001) identified the five strands of mathematical proficiency as (a) conceptual understanding,

(b) procedural fluency (i.e., computational fluency), (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition. The relationship between procedural fluency and conceptual understanding is complex. Developmental and cognitive psychologists agree with this assertion, noting that the development of conceptual understanding and procedural fluency is an iterative process (Rittle-Johnson & Koedinger, 2009).

There is some debate regarding which of these skills develop first (Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Siegler, 1998). Some researchers have hypothesized that procedural knowledge developments first and leads to the development of conceptual understanding. Rittle-Johnson, Siegler, and Alibali (2001) coin these theories *procedures-first* theories. Similarly, Rittle-Johnson and colleagues (2001) refer to *concepts-first* theories when describing the presumption that conceptual knowledge is the first to develop. In the end, an iterative model of development has been offered, whereby procedural and conceptual knowledge are described as developing simultaneously and through a recurring process. Below I will discuss conceptual knowledge and computational fluency, and interventions for each.

Conceptual knowledge. The National Council for Teachers of Mathematics (NCTM, 2006) identified conceptual understanding as one of the guiding principles for mathematics curriculum and assessment. They state that learning with understanding by activating and building upon prior knowledge is essential. Rittle-Johnson and Alibali (1999) define conceptual knowledge as "explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain" (p. 175). As is discussed by the NRC (2001) and in the NCTM standards (2006), conceptual understanding is important for mathematical achievement.

While there is widespread agreement that conceptual understanding is critical for reaching mathematical proficiency, the function that conceptual understanding plays in mathematics proficiency is not well understood. Developmental psychologists have suggested that conceptual understanding is an "integrated knowledge of important principles that can be flexibly applied to new tasks" (Rittle-Johnson & Koedinger, 2005; p. 317). This definition speaks more to the application (i.e., transfer) of mathematical concepts than the understanding of them. Conceptual understanding has also been reported to mediate "the understanding of arithmetic operations and laws pertaining to them," (Delazer, 2003, p. 402), and to provide "an integrated and functional grasp of mathematical ideas" (Kilpatrick et al., 2001, p. 118). Still others have suggested that conceptual understanding and procedural fluency are not mutually exclusive (Anderson, 1989; Siegler, 1988). For example, when students first learn basic math facts, they are problems to be solved. Siegler (1988) argues that it is not until after students learn how to solve the problem and understand the underlying structure that they can become automatic with the skill. Siegler's (1988) argument underscores the importance of assessing and explicitly teaching both procedural skills and conceptual understanding for new mathematical constructs.

Computational Fluency. Computational fluency, which is sometimes referred to as procedural fluency or procedural knowledge, in mathematics education has been traditionally defined as a students' proficient use of algorithmic operations in order to complete a mathematical task (Byrnes & Wasik, 1991; Geary, 1993; Hiebert & Lefevre, 1986; Nesher, 1986; Rittle-Johnson, Siegler, & Alibali, 2001).

Mathematical difficulties often manifest themselves first through procedural or computational errors. Moreover, research has demonstrated that students with a specific learning disability in the area of mathematics (MLD) often utilize immature and inefficient strategies for recalling math facts, leading to computational and procedural errors (Geary, 1990; Geary, 1993). Together, these issues make computational fluency an important component in understanding a student's mathematical thinking.

Computational fluency is tied to specific problem types and therefore is not widely generalizable (Rittle-Johnson, Siegler, & Alibali, 2001). Moreover, focusing solely on meaningless computation drill and practice may perpetuate the issue of students with learning disabilities as passive learners and fail to address gaps in their conceptual understanding (Baroody & Hume, 1991; Jitendra, DiPipi, Perron-Jones, 2002; Parmar et al., 1994; Torgesen, 1982). However, the nature of computational fluency lends itself well to assessment and intervention. It is easily observable and measurable as a discrete skill. Researchers and educators both focus much of their teaching and intervention efforts on the development of computational fluency.

Automaticity with basic skills is also widely mentioned within the mathematics literature (e.g., Woodward, 2006) as a critical component of academic success within mathematics. Automaticity of basic skills plays a large role in computational fluency. If a student is able to quickly and accurately retrieve a math fact, he or she is better able to access working memory resources, reducing cognitive load (Delazer et al., 2005).

Further, this automaticity is particularly important in mental computation, estimation, and approximation skills (Woodward, 2006).

Intervention heuristic for mathematics proficiency. Regardless of the order in which mathematical skills may develop, there are several important features to consider when designing an effective mathematics intervention. Based on several meta-analyses, the following key components are considered crucial: (a) visual and graphic depictions, (b) systematic and explicit instruction, (c) student think-alouds, (d) peer-assisted learning, (e) formative assessment data provided to teachers, and (f) formative assessment data provided directly to students (Baker, Gersten, & Lee, 2002; Gersten, Chard, Jayanthi, Baker, & Lee, 2006). Baker, Gersten, and Lee (2002) summarized 17 mathematics intervention studies. Effect sizes were calculated based on the category of intervention. Among the categories identified, meta-analytic results indicated that providing data to students (d = .71), peer-assisted learning (d = .62), and explicit instruction (d = .65) netted the largest and statistically significant effects. Other categories for which effects were reported, but for whom effect size was not significantly greater than zero, included providing instructional recommendations to teachers (d = .51), teacher-facilitated instruction and practice (d = -.04), and concrete feedback to parents (d = .43).

Gersten et al. (2006) also examined the instructional components of 30 mathematics studies. Very large and statistically significant effects were noted for explicit instruction (g = 1.22), the use of heuristics (g = 1.56), student verbalizations (g = 1.04), and cross-age peer tutoring (g = 1.02). Meta-analytic results also indicated statistically significant effects for using visuals representations (g = 0.46), progressing from easier to more difficult or complex problems, sequencing lessons based a specific concept or problem type (*g* = 0.82), providing ongoing data and feedback to teachers on students' performance (*g* = 0.23), providing performance feedback to students on their performance using graphs (*g* = 0.21).

Additionally, recommendations for assisting students struggling with mathematics published by the Institute of Education Sciences (IES) advises educators to also include instruction on solving word problems, dedicate time each session for computational fluency practice, and include motivational strategies (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). Together, these recommendations suggest that mathematics development is a multidimensional and underscores the importance of addressing each of these essential knowledge bases.

Development of skills, including mathematical skills, follows a continuum beginning with number sense and culminating with the successful integration and application of previously learned skills to novel problems. Haring and Eaton (1978) describe the progression which students acquire academic skills as the Learning Hierarchy. The four stages of the Learning Hierarchy are *acquisition, fluency, generalization,* and *adaptation.* During the acquisition phase, students are learning a new skill and demonstrate slow and highly inaccurate performance. Modeling, explicit instruction, and immediate feedback are required to improve the accuracy of responses. As accuracy increases, response rate, or fluency, often remains low. Instruction is, therefore, aimed at improving the rate of accurate responding. Generalization occurs next when students begin to apply newly acquired skills fluently and with high accuracy to novel contexts and materials. Generalization can be programmed in a variety of ways (see Stokes & Baer, 1977 for examples). Finally, students demonstrate adaptation by spontaneously utilizing the underlying principles to new situations (Haring & Eaton, 1978).

The learning hierarchy can be a useful framework in which to think about academic and behavior learning. There has been some preliminary research around how conceptual understanding and computational fluency may fit within the learning hierarchy, particularly as it relates to identifying and selecting interventions (Burns, 2011; Burns et al., 2010). However, there has been little empirical evidence to support the proposed relationship and learning progression for specific mathematical knowledge bases. For example, Burns (2011) hypothesized that students' computational fluency may support students' generalization to word problems. These hypotheses have not been directly tested and thus require additional research. Other researchers have debated the order in which conceptual understanding and computational fluency develop (Byrnes & Wasik, 1991; Rittle-Johnson & Alabali, 1999). Additional research is needed in order to better understand the learning progression of students' mathematical development within the context of the learning hierarchy.

Synthesis. Mathematical proficiency is a multi-dimensional construct that includes conceptual understanding, computational fluency, strategic competence, adaptive reasoning, and productive disposition. Much of the current educational research in mathematics has focused on the constructs of conceptual understanding, computational proficiency, and application and word problem solving. These skills and the progression in which they develop are not well understood. However, they are believed to be intertwined with the improvement in one area supporting and advancing the skill in another (e.g., Baroody, 1985; Byrnes & Wasik, 1991; Delazer et al., 2005; Hiebert & Wearne, 1996; Rittle-Johnson et al., 2001). The learning hierarchy (Haring & Eaton, 1978) could serve as a useful framework for conceptualizing these relationships and serve as a theoretical base from which to think about assessment and intervention. Because of the limited research base surrounding the learning hierarchy for mathematics specifically, additional research directly testing the relationships between different mathematics knowledge bases and the learning hierarchy is warranted. Thus, continuing to examine the complex relationship of each of these domains in students' mathematical understanding will aid educators' efforts in supporting students' mathematical development and success.

Mathematical Assessment

There are four purposes for assessment: screening, instructional decision-making, progress monitoring, and program evaluation (Salvia, Ysseldyke, & Bolt, 2010). Each of these assessment activities plays an important role in resource allocation and determining student need for intervention. Formative assessment is the "systematic evaluation in the process of curriculum construction, teaching, and learning for the purposes of improving any of these three processes" (Bloom, Hastings, & Madaus, 1971, p. 117). More specifically, formative assessment should suggest specific objectives and/or items that need to be taught and how to best teach them (Clarke, Doabler, & Nelson, 2014; Stiggins, 2005). Formative assessment for mathematics is especially important because teachers need to understand the needs of individual learners and the instructional implications of those needs for mathematics (Chard, Ketterlin-Geller, & Jitendra, 2008; Clarke, Doabler, & Nelson, 2014; NCTM, 2000; 2006; NMAP, 2008).

Assessment of mathematical skill can take many forms. There are standardized, norm-referenced achievement batteries, such as the Woodcock-Johnson III Tests of Achievement (WJ-III ACH; Woodcock, McGrew, & Mather, 2001; 2007), the KeyMath3 Diagnostic Assessment (Connolly, A. J., 2007), and the Stanford Achievement Test-10th Edition (SAT-10; Pearson Assessment, 2003). Each test is often part of a larger battery of achievement tests, which can be time consuming to administer, score, and interpret. The cumbersome nature of these assessment batteries, limit their utility within education, particularly as part of a prevention-based framework.

Formative assessment in mathematics is often achieved with alternatives to norm-referenced batteries such as curriculum-based measurement (CBM; Deno, 1985) and curriculum-based assessment (CBA; Gickling & Havertape, 1981). CBM and CBA share similar qualities, but their fundamental purpose differs in that CBM is well suited to assess the effectiveness of current instructional practices and monitor student growth through repeated measures and CBA is used to identify specific skill deficits and potential mismatch between the classroom curriculum and individual student level (MacQuarrie, Burns, & Campbell, 1999; Shapiro, 2012; VanDerHeyden & Burns, 2005).

The use of both CBM and CBA has been widely researched for reading and mathematics with several commercially produced systems available (e.g., AIMSweb, 2006; EdFormation, 2005), web-based systems (e.g., www.mathfactscafe.com), or by sampling local curriculum. VanDerHeyden and Burns (2005) suggest that using CBM and CBA together offers a most comprehensive method for assessing student skill level and monitoring progress within a specific skill set and skill level (e.g., addition with answers 0-9).

Assessing Conceptual Understanding. Currently, many of the commercially available CBMs for mathematics are largely procedural in nature and are designed to assess computational skills (Helwig, Andersen, & Tindal, 2002). The National Center on Response to Intervention (NCRTI; 2012) identifies several CBM systems available for math, including those for computational fluency and conceptual understanding. For example, the AIMSweb system has a measure, which is designed to look specifically at students' computation skills (M-COMP; Pearson, 2010). Problems are presented vertically and include a mix of operation and number types, such as addition, subtraction, multiplication, and division of whole and rational numbers. Probe content varies depending on grade level and also includes algebraic expressions for middle-school students (Pearson, 2010).

Concept-oriented CBMs (e.g., easyCBM [Riverside, 2012]; AIMSweb [Pearson, 2010]), conversely, are designed to assess conceptual understanding and application of mathematical domains. Research on concept-oriented assessment is limited, but these data have been shown to be significant predictors of statewide assessment outcomes (Shapiro, Keller, Lutz, Santoro, & Hintze, 2006). Commercially prepared concept-oriented probes such as *Monitoring Basic Skills Progress-Math Concepts and Applications* (Fuchs, Hamlett, & Fuchs, 1999) and *Yearly Progress Pro* (McGraw-Hill

Digital Learning, 2002) cover a wide array of mathematical domains including time, money, fractions, and word problems.

While a mixed-skill probe may serve as a good indicator of overall mathematical achievement (i.e., general outcome measure; GOM), it may be less useful for designing specific interventions (Burns, 2011; Burns & Klingbeil, 2009) perhaps because of the breadth of content assessed and the primary focus on skill application rather than on underlying mathematical concepts. Moreover, students' understanding of mathematical concepts may be masked by their computational fluency, or vice versa. For example, a student may correctly answer the problem "What is 5x7?" with an answer of 35. When asked to explain how they reached their answer through an interview or by illustrating their answer, the student may provide a response that suggests that they have only superficial knowledge of the problem. Conversely, a student may answer the problem "What is 5x7?" with an answer of 30. But, when asked to explain their answer, may indicate that it is five groups of seven or that five times seven is the same as 30 divided by five, which they know to be seven. Here, the student has demonstrated an understanding of the meaning of the problem (e.g., five groups of seven), though has lesser skill in retrieving a correct answer (i.e., 35).

Both scenarios described above demonstrate that targeted assessment of each computational fluency and conceptual understanding is necessary in order to adequately allocate intervention resources. Burns et al. (2010) presented meta-analytic data that supported the need to directly assess student difficulty when determining mathematics interventions with a skill-by-treatment interaction approach. Thus, measuring students' application (i.e., transfer) of mathematical skills, rather than procedural fluency or conceptual understanding directly, requires a relatively high-inference decision and increases the risk of inappropriate instructional placement as compared to directly assessing the relevant constructs.

Discrimination tasks can also be used to assess students' understanding (Anderson, 1989; Geary, Bow-Thomas, & Yao, 1992; Greeno, 1978; Stokes & Baer, 1977) of mathematical concepts by asking students to judge whether or not items are correctly completed (Bisanz & LeFevre, 1992; Briars & Siegler, 1984; Canobi, 2004; Canobi, Reeve, & Pattison, 1998; Cowan, Dowker, Christakis, & Bailey, 1996). An example of this approach to conceptual understanding assessment can be accomplished by providing three examples of the same mathematical equation and asking students to circle the correct one (e.g., $3 + 7 = 12$, $4 + 7 = 10$, and $3 + 7 = 10$), or providing a list of randomly ordered correct and incorrect equations and asking them to write or circle "true" for the correct ones and "false" for the incorrect items (Beatty & Moss, 2007).

Burns (2011) proposed a model that extended the skill-by-treatment assessment approach to address conceptual and procedural knowledge by applying Beatty and Moss's (2007) model of providing a list of randomly ordered correct and incorrect equations and asking students to circle "true" for the correct ones and "false" for the incorrect items. Burns assessed student skills in procedural fluency with single-skill CBM for math (e.g., single-digit multiplication) and conceptual understanding with a series of correct and incorrect equations from which the student had to select the correct equations. After completing the assessments, a mismatched intervention was implemented to contraindicate student need, which was followed by a matched intervention according to the student's skill deficit. Student performance increased substantially (percentage of nonoverlapping data [PND] = 100%) when the matched intervention was implemented, while minimal effect was demonstrated for the mismatched intervention (PND=16.5%).

Assessing Computational fluency. Computation skills and fluency with basic facts is another important component comprising mathematical proficiency (Kilpatrick, Swafford, & Finell, 2001). As such, it is critical to assess students' skill related to such tasks. Computational fluency has also been shown to be a predictor of performance on state accountability tests and other high-stakes assessments (Nelson, Parker, & Zaslofsky, 2015; Shapiro, Keller, Lutz, Santoro, & Hintze, 2006), which make it a good indicator of student risk (Clarke, Doabler, & Nelson, 2014).

Computational fluency is often measured using CBMs, because such measures are brief and technically sound (Christ, Sculin, Tolbize, & Jiban, 2008; Deno, 1985; Fuchs, 2004). CBM computation probes can be single-skill (e.g., multiplication) or multi-skill (e.g., multiplication and division) in format and are timed. Students are instructed to work as quickly as they can without making mistakes and to attempt each problem. After the time has elapsed, the probe is scored and student responses are converted to a digits correct per minute metric. DCPM is thought to be more sensitive to change than measuring the number of correct answers (Hosp et al., 2007). For example, if the answer to an addition problem is 384, the total possible digits correct are 3. If a child answered 374, the digits correct would equal 2; although this answer is incorrect it shows a greater understanding of addition than a response of say 500 (DC = 0). Digits correct are also scored in the critical processes of a problem with placeholders counting as correct digits.

With much of the early work in mathematics CBM examining computation, researchers turned their focus to assessment of concepts and applications, word problem solving, and algebra with a lesser focus on computational fluency (Foegen, Deno, & Jiban, 2007). However, there has been a renewed focus on computation (e.g., Methe, Briesch, Hulac, 2015). Notably, researchers are beginning to examine the instructional utility of assessing computation (Dennis, Calhoun, Olson, & Williams, 2014), its predictability on high stakes assessment (Good, Simmons, & Kameenui, 2001; Muyskens & Marston, 2002; Nelson et al., 2015), and as a measure of student growth (Deno, 2005; Keller-Margulis, Mercer, & Shapiro, 2014). Together, as an essential component to mathematical proficiency and its role in formative assessment, computational fluency continues to be an important skillset in which to assess.

Synthesis. Assessment is a critical component of instruction in general (Salvia et al., 2010) and formative assessment is especially important for mathematics (Clarke, Doabler, & Nelson, 2014; NCTM, 2000; 2006; National Mathematics Advisory Panel, 2008). Given the multidimensional nature of mathematics, it is essential to assess all relevant knowledge bases. It is important to discuss the relevancy of Haring and Eaton's (1978) learning hierarchy within the context of assessment. The learning hierarchy describes four stages of skill development beginning with acquisition and culminating in adaptation. In between, students gain proficiency through fluency building tasks and repeated practice, which can include discrimination and differentiation training.

Application or transfer of mathematical skills falls near the end of the learning hierarchy continuum so much as it requires students to use their previously learned and practiced skills for the purposes of problem-solving. Surely, most educators would agree that application is the ultimate goal of instruction. The issue with relying on tasks of application or transfer exclusively, which is the focus of many commercially available systems, is that important information about foundational skills may be missed. As such, it is important that educators select measures that can reliably assess each mathematical knowledge base in order to best inform instruction.

Linking Assessment to Intervention

A recommendation of the NCTM standards (2000) and a cornerstone of a prevention framework, such as RTI, is linking assessment results to empirically supported interventions. Educators are expected to utilize formative data when making instructional and behavioral programming decisions (Stecker & Fuchs, 2000; Kratochwill & Shernoff, 2004). Once these data are collected, they must be analyzed to evaluate whether students are making adequate progress (Shinn, 2002) and whether modifications to interventions are warranted (Stecker & Fuchs, 2000). Educators are held accountable for these programming decisions, where they must provide quantitative data on the student's lack of progress (Shinn, 2007).

Historically, educators have relied on assessment of student characteristics and intuitively appealing academic interventions that had limited empirical support to make programming decisions. Such a framework is referred to as an aptitude-by-treatment interaction (ATI) approach (Cronbach, 1957) and calls for educators to select instructional strategies for individual students based on measured characteristics of the students. For example, using an ATI approach, an educator would provide an intervention based on students' poor cognitive process, like spatial reasoning. The student in this example would receive an intervention purported to increase spatial reasoning ability, with the results intended to improve performance of all tasks that involve spatial reasoning. During the mid-twentieth century, it was popular to implement interventions that focused on various approaches to perceptual training, intervening with psycholinguistic skills (e.g., receptive and expressive language, association, and sequential memory), and matching the modality of instruction with the preferred learning style, all of which led to small effects (Kavale, 2007). The proponents of the original ATI framework recommended that such an approach to intervention selection be abandoned (Cronbach & Snow, 1977), because they concluded, after a career of related research, that cognitive abilities alone could not explain individual differences in intervention effectiveness (Kearns & Fuchs, 2013). Moreover, recent meta-analytic research of 23 studies that involved interventions for working memory found effect sizes for word reading and math that were close to zero; leading the researchers to conclude that, "there was no convincing evidence of the generalization of working memory training to other skills (nonverbal and verbal ability, inhibitory processes in attention, word decoding, and arithmetic)" (Melby-Lervag & Hulme, 2013, p. 270).

Recent research has suggested that a skill-by-treatment interaction (STI) might be a promising alternative approach to ATI for determining interventions for mathematics with a high likelihood for success (Burns, Codding, Boice, & Lukito, 2010). In an STI paradigm, interventionists assess skills to identify specific targets and to select an intervention aligned with students' skill deficits. However, a successful STI approach depends on using a validated intervention framework to guide selection of appropriate interventions.

Math proficiency is comprised of both conceptual understanding and procedural fluency (Kilpatrick, Swafford, & Finell, 2001), and the distinction between the two types of math understanding could provide the basis for a STI intervention approach for mathematics (Burns & Klingbeil, 2010). Conceptual knowledge is the understanding of the relationships that underlie mathematics problems, and procedural knowledge is the understanding of the rules and steps to actually solve the problems (Hiebert & Lefevre, 1986). It is somewhat unclear which type of knowledge develops first and the sequence may be specific to the domain or the individual (Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Wagner, 2001), but the two are clearly interrelated.

Timed drill, which is sometimes referred to as explicit timing, is an intervention intended to improve students' automaticity with basic facts, such as single-digit addition, subtraction, multiplication, and division (Miller, Hall, & Heward, 1995; Van Houten & Thompson, 1976; Woodward, 2006). The components of timed drill that make it effective for improving students' computational fluency include skill level materials (Burns, Codding, Boice, & Lukito, 2010), brief practice opportunities that incorporate modeling, feedback, and reinforcement (Fuchs et al., 2008; Rivera & Bryant, 1992), timed practice (Rivera & Bryant, 1992), and self-management of individual practice opportunities (McDougal & Brady, 1998). Moreover, timed drill is particularly effective for students transitioning from the *acquisition stage* to the *fluency stage* of the learning hierarchy (Burns, Codding, Boice, & Lukito, 2010; Haring & Eaton, 1978) because of its emphasis on speed in addition to accuracy of response.

Meta-analytic research examined 55 single-case design studies of interventions to improve mathematics computational fluency and found large effects (percent of all nonoverlapping data [PAND] = .87, ϕ = .50, Codding, Burns, & Lukito, 2010). The largest effects were for repeated practice of facts (PAND = 1.00, ϕ = .92) and for interventions that practiced mathematical facts with some aspect of modeling included (PAND = .91, ϕ = .71). Other interventions that resulted in a large effect included computer-assisted instruction with first-grade students (Fuchs et al., 2006), computer-based mathematics fact rehearsal (Nelson, Burns, Kanive, & Ysseldyke, 2013), strategy instruction combined with timed drills (Woodward, 2006).

Strategy instruction is often used to introduce and reinforce conceptual understanding of academic tasks. Several studies have identified strategy instruction as a more effective intervention method than drill-and-practice techniques, which primarily target computational fluency and rote learning of mathematics facts (Montague, 1997; Tournaki, 2003; Woodward, 2006). Another important aspect of using the strategy instruction technique for students with learning disabilities in mathematics is the use of direct and explicit instruction (Tournaki, 2003). Cognitive research also supports the effectiveness of strategy instruction in students' learning. Delazer and colleagues (2005) found that students' who learned algorithmic procedures using strategy instruction demonstrated higher levels of accuracy on trained problem types than students in the drill condition. Students who learned strategies for solving algorithms were more accurate in their attempts to solve transfer problems (Delazer et al., 2005).

Meta-analytic research found a negligible effect for conceptual mathematics interventions, and reported that they represented "a complex puzzle of findings, open to multiple interpretations" (Baker, Gersten, & Lee, 2002, p. 66). One potential reason for the complex findings for interventions that target conceptual understanding could be that many students who struggle with mathematics may understand the underlying concept but may require additional intervention for the procedure (Gersten, Jordan, & Flojo, 2005), and interventionists may need to determine if the student has developed conceptual understanding and procedural fluency in order to better target intervention efforts.

Burns (2011) measured conceptual understanding and found that students who lacked understanding of the underlying concepts responded better to interventions that explicitly taught the concepts, and students who demonstrated adequate conceptual understanding responded better to interventions that targeted procedural fluency. The Burns (2011) study used contra-indication in which the intervention that was not indicated by the data was implemented first, and the one that was indicated was implemented second. The mean percentage of nonoverlapping data (PND) for the indicated intervention was 100% (as compared to the data in the contra-indicated phase), but it was 16.5% for the data in the contra-indicated phase as compared to baseline. The study was implemented with two students, one who scored low on conceptual understanding (a second-grade student), and one who demonstrated adequate conceptual understanding but low procedural fluency (a fourth-grade student). Although the contra-indication design was novel, it was not implemented in a truly experimental manner because it used a multiple-baseline design with only two staggered implementation phases instead of three as recommended to demonstrate experimental control (Kennedy, 2005).

Burns et al. (in press) replicated that contra-indication design (e.g., given a procedural intervention first when they demonstrated a conceptual deficit) for several sessions followed by the prescribed intervention. Each student followed the expected pattern with relatively flat growth during the contra-indicated phase and larger slopes of growth during the prescribed intervention. The prescribed intervention was more effective than the contra-indicated intervention regardless of which intervention it was. The PND for the contra-indicated intervention was 11.76% across the three students as compared to baseline, which increased to 81.25% for the prescribed intervention as compared to the data from the contra-indication phase. The PND for the conceptual intervention phases across the students was 37.5% as compared to the preceding phase, and it was 52.9% for the procedural intervention as compared to the preceding phase. Thus, neither intervention was effective unless it was implemented as the prescribed intervention.

Both conceptual and procedural fluency correlate well with word problem solving, but mathematics skill level seems to affect the relationship. Kanive and Burns (2015) recently examined the relationship between conceptual knowledge, computational fluency, and word problem solving with 493 third-grade students. The correlation with word problem solving was $r = .50$, $p < .001$ for fact fluency and $r = .36$, $p < .001$ for the measure of conceptual understanding. Both measures were significantly related to word problem solving skills, but the data were further analyzed with a regression model that also included mathematics skill level, as determined by the Measures of Academic Progress for Math (MAP-M; Northwest Evaluation Association, 2003) and the interaction of the skill level with both measures. The five variables were significantly related to word problem solving, and the overall model was significant and accounted for a large percentage of the variance ($R^2 = .47$). The skill group accounted for a significant amount of variance of the word problem solving score, and the interaction between skill group and measures of procedural and conceptual knowledge was also significant.

The data in the Kanive and Burns (2015) study were divided into two groups based on scoring above or below the 25^{th} percentile on the MAP-M. Computational fluency accounted for 18% of the variance of the word problem solving scores for students above the 25^{th} percentile, and the conceptual understanding measure accounted for 6% of the variance in the word problem-solving data. However, among students below the 25^{th} percentile, the conceptual knowledge measured accounted for 40% ($r = .63$, $p < .05$) of the variance in the word problem solving, and the computational fluency measure resulted in a lower correlation ($r = .36$, $p > .05$) with the word problem-solving measure.

Students with mathematics difficulties frequently struggle to rapidly solve basic facts (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Hanich, Jordan, Kaplan, & Dick, 2001), which suggest that intervention efforts for some students may be more effective if they focus on procedural issues (e.g., practice completing basic facts to increase fluency). For other students, a conceptual intervention (e.g., representing problems and discussing the relationship between concepts, such as addition and multiplication) might be more promising because the students may not understand the underlying concepts. Therefore, determining which skill was the deficit area for individual students could improve student outcomes.

Synthesis

Linking assessment results to empirically supported interventions is critical for improving student outcomes. Following an STI framework allows educators to more directly assess student skills and thereby better match interventions to skill deficits. Preliminary results are encouraging that such an approach may be a good alternative to the debunked ATI approach (Burns, 2011; Burns, VanDerHeyden, & Zaslofsky, 2015). In addition to matching interventions to student skill deficits, it is also important to address the skill level of students, so that material is sufficiently challenging (Burns, 2002; Shapiro & Ager, 1992).

Summary and Research Questions

Linking assessment results to empirically supported interventions is critical for improving student outcomes. The current research base of mathematics assessment and intervention is developing with limited recommendations. For example, many chapters in *Best Practices in School Psychology* (5^{th} edition; Harrison & Thomas, 2014) outline recommendations for assessing and intervening with students struggling academically.

Among these chapters, interventions for literacy skills (i.e., reading and writing) accounted for three chapters (Daly, O'Connor, & Young, 2014; Joseph, 2014; Martinez, 2014) and assessment techniques specific to literacy accounted for an additional three chapters (Gravois & Nelson, 2014; Hosp & MacConnell, 2014; Malecki, 2014). Conversely, recommendations specific to mathematics were limited to three total chapters, inclusive of assessment and intervention (Clarke, Doabler, & Nelson, 2014; Zannou, Ketterlin-Geller, & Shivraj, 2014). Thus, there is a continued need to further examine the relationship between assessment, intervention, and potential frameworks for informing instructional decisions.

The following research questions guided the current study:

1. To what extent does pretest data differentially predict intervention response to conceptual intervention on fluency measure.
2. To what extent does pretest data differentially predict intervention response to computation intervention on fluency measure.
3. To what extent does pretest data differentially predict intervention response to conceptual intervention on application measure.
4. To what extent does pretest data differentially predict intervention response to computation intervention on application measure.
5. To what extent does need predict response to a conceptual intervention?
6. To what extent does need predict response to a fluency intervention?

CHAPTER 3

METHOD

Setting and Participants

All study procedures were completed during the spring semester of the academic year with approval from the University of Minnesota Institutional Review Board and informed consent and assent from participants. Grades 3 and 4 of the participating school, located in a suburban Midwestern city, agreed to participate in a 4 week mathematics intervention where students received a conceptually-based and computation-based intervention for 2 weeks each. The principal investigator and several research assistants collected screening, pretest, and posttest data, which provided the data for the current study.

Setting

All participating students were enrolled in a suburban school located in the upper Midwestern United States, which served 480 students in kindergarten through fifth grade. The school served a diverse student population with 55% of students identified as Caucasian, 16% of students identified as Black, 7% of students identified as Hispanic, 21% of students identified as Asian/Pacific Islander, and <1% identified as Native American/Alaska Native. Additionally, 25% of students met eligibility for a free or reduced-price lunch. Third- and fourth-grade students from a total of eight classrooms were targeted for participation because this is the age at which number sense should be firmly established for most students and instruction focuses primarily on higher order skills such as multi-digit multiplication and division, manipulation of fractions, and geometry (Common Core State Standards Initiative, 2010). A total of 79% of the third-grade students and 87.5% of the fourth-grade students in the participating school met or exceeded standards on the 2013 state accountability test for mathematics.

Participants

Students were eligible to participate in the current study if they were at-risk for mathematics difficulty. At-risk status was based on performance on the mid-year administration of the Optional Local Purpose Assessment (OLPA; Minnesota Department of Education, 2013) for mathematics. The purpose of the OLPA is to predict how students will score on the state accountability test for Minnesota. Students were considered at risk for mathematics difficulty (MD) if they scored in the Partially Meets or Does Not Meet proficiency range on the OLPA. Similar cutoff criteria have been used to screen at-risk learners from general education classrooms in early reading or mathematics research (e.g., Clarke, Smolkowski, Baker, Hank, Doabler, & Chard, 2011; Kaminski, Cummings, Powell-Smith, & Good, 2008).

Based on the eligibility criteria, 50 students were identified and recruited to participate. An *a priori* power analysis indicated that a sample size of 50 would be needed to achieve adequate power for a regression analysis with an alpha level of .05. Of the 50 students initially identified, one student did not provide consent, two students were no longer enrolled in the school, and one student was disqualified by his classroom teacher because of scheduling conflicts with his special education programming. The final sample for the study included 46 third (n = 24) and fourth grade (n = 22) students identified as at-risk for MD.

The final sample was racially and ethnically diverse with 11% of students identifying as Asian/Pacific Islander, 28% as Black, 13% as Hispanic, and 48% as White. Twenty percent of participating students were English Language Learners and 17% of students received special education services. Moreover, 52% of students were male and 48% of students were female. Table 1 provides demographic information for participating students by overall sample and need group.

Assignment to conditions. The 46 students comprising the final sample were randomly assigned to one of two conditions. Students in the first condition received a conceptually based intervention and then a computation-based intervention. Students in the second condition received a computation-based intervention and then a conceptually based intervention. Students within each condition were then randomly assigned within their grade to instructional groups of four to five students. Students were randomly assigned to condition in order to reduce selection bias and ensure that differences across groups were not due to assigned condition, but rather because of intervention received.

Table 1. *Demographic Characteristics for the Overall Sample*

	Overall Sample			
		%	OLPA winter	
	n	%	*M*	*SD*
Gender				
Male	24	52	390.27	50.53

Female	22	48	374.48	50.93
Race/Ethnicity				
Asian/Pacific Islander	5	11	396.00	56.56
Black	13	28	391.85	43.49
Hispanic	6	13	355.60	47.92
American Indian	0	0	--	--
White	22	48	379.41	54.74
Grade				
3	24	52	334.09	8.31
4	22	48	432.50	12.05
Special education status				
Yes	8	17	362.00	48.41
No	38	83	390.97	49.45
English language learner				
Yes	9	20	351.50	44.98
No	37	80	385.92	50.95
Total	46	100	382.20	50.78

Interventionists and assessors

The interventionists and assessors were six female graduate students from an educational psychology program. The interventionists and assessors were racially and ethnically diverse with four (66%) identifying as White, 1 (17%) as Hispanic, and 1

(17%) as Asian/Pacific Islander. The principal investigator trained all interventionists to criterion prior to meeting with students. Interventionists needed to demonstrate 100% accuracy with intervention and assessment procedures based on an implementation fidelity checklist, which was also used to assess implementation fidelity throughout the current study. The principal researcher met with all interventionists to review intervention and assessment protocols, provide a model of procedures, and observe interventionists' implementation of all components of the study procedures. Three of the interventionists, including the principal researcher, were responsible for implementing the interventions. One interventionist was primarily responsible for delivering the conceptually-based intervention, one interventionist was primarily responsible for delivering the computation-based intervention, and the principal researcher delivered both interventions while monitoring overall implementation. Three additional interventionists assessed implementation fidelity during intervention sessions using a procedures checklist and aided with administration of study assessments at pre- and posttest periods.

Measures

Several measures were administered during the current study. Participants completed two screening measures and two outcome measures. Each measure will be described below.

Screening Measures

Two screening measures were administered to all participating students. The first screening measure, the OLPA, was used to determine at-risk status and eligibility for study participation. The second screener, the Single-Digit Multiplication Conceptual Understanding Assessment was used to help determine students' need for either a conceptually- or computation-based intervention.

Optional Local Purpose Assessment (OLPA). The winter administration of the OLPA (Minnesota Department of Education, 2013) was used for screening. The OLPA is a group-administered standardized measure of mathematics skills that was developed with item-response theory to predict how students will score on the state accountability test for Minnesota. The computer adaptive measure is administered online with a series of multiple-choice items. Items from the test equally represent four strands of (a) numbers and operations, (b) algebra, (c) geometry and measurement, and (d) data analysis and probability. The data are converted to grade-based standard scores that range from 300 to 399 for third grade, and 400 to 499 for fourth grade. Scores of 350 and 450 represent proficiency in third and fourth grade, respectively. The mean OLPA Math score was 334.09 (SD = 8.31) for the third-grade participants and 432.50 (SD = 12.05) for the fourth-grade students in the study. A total of 40% (n = 22) of the students in grades 3 and 4 at the participating school scored in the Meets or Exceeds proficiency range on the OLPA, with one student (1.8%) exceeding the proficiency range. Therefore, a majority of the students' (n = 33, 60%) OLPA score fell in the Partially Meets or Does Not Meet proficiency range. Table 1 further summarizes OLPA data for the current sample. OLPA data correlate with the state test of accountability for Minnesota at r = .84. Although no reliability data are reported, the standard error of measure ranges from 4 to 9 for third grade, and from 4 to approximately 11 for fourth grade.

Single-digit multiplication conceptual understanding assessment. Conceptual understanding of single-digit multiplication was assessed using a researcher-designed measure based on Buschman (2003; Appendix A). The conceptual measure is an individually-administered, untimed test. One single-digit multiplication problem is displayed at the top of the page. Students were instructed to draw a visual model (e.g., picture, figure, diagram) to accompany the problem and to solve the problem using the visual model. Then, students were interviewed about how they approached the problem (Appendix B).

Following administration, student responses to the interview questions were rated according to the following criteria, (a) counts with understanding, (b) understands the number sign including relevant, formal mathematics language, (c) understands the facts of multiplication of whole numbers, (d) correctly uses the visual model (i.e., there was a correct relationship between the visual model and the problem solution), (e) uses an identifiable strategy, and (f) answers the problem correctly (Buschman, 2003). Each question was scored with a four-point rubric based on Van de Walle, Karp, and Bay-Williams (2010; Appendix C). A score of 1 meant that the student demonstrated unsatisfactory knowledge of the item, a 2 meant partial demonstration of the item, a 3 indicated adequate understanding to accomplish the objective, and a 4 indicated full accomplishment of the item.

Previous research (Burns, Kanive, Zaslofsky, Jitendra, & Coolong-Chaffin, 2014) reported internal reliability of .87 for the six items. Internal consistency for the present sample was r = .76. Inter-scorer agreement assessed by a second scorer independently scoring 30% of the protocols was 100.

Outcome Measures

CBM-Math Computation (CBM-M). Students' computational fluency was assessed pre- and post-intervention with a single-skill curriculum-based measure of mathematics (CBM-M) that assesses single-digit multiplication. Each probe consisted of 40 single-digit multiplication problems, printed vertically, in eight rows of five. The worksheets were created by the researcher using an online CBM-M generator (www.mathfactcafe.com). Student responses were converted to a digits correct per minute (DCPM) metric in order to assess students' fluency with single-digit multiplication combinations. Students were instructed to begin working in the upper left corner with the first problem, continue working across the page, on to the next row, and continue on the back until the assessor told them to stop. Students were encouraged to do their best work and to attempt each problem. After reading the standardized directions, the assessor started a timer and allowed students to work for 2 mins before telling them to put down their pencils and stop working.

Data obtained from single-skill mathematics assessments have demonstrated high reliability (Burns, VanDerHeyden, & Jiban, 2006) and are dependable for criterion-referenced decisions regarding that skill (Hintze, Christ, & Keller, 2002). Inter-scorer agreement assessed by a second scorer independently scoring 30% of the protocols was 98, 100, and 100 at pretest, posttest 1, and posttest 2, respectively.

Application measure of word problem solving. To examine application of mathematics knowledge and competence on word problems, students completed an untimed measure of word problem solving skills. The application measure was administered at pretest, posttest 1, and posttest 2. Items for the assessment were selected from several standardized assessment subtests categorized as measures of application skills, including (a) Math Concepts and Applications subtests of the Kaufman Test of Educational Achievement, Second Edition (KTEA-II; Kaufman & Kaufman, 2004), (b) Application subtest of the Key Math Revised (Connolly, 2000), (c) Math Reasoning subtest of the Wechsler Individual Achievement Test, Second Edition (WIAT-II; Psychological Corporation, 2001), (d) Practical Applications subtest of the Comprehensive Math Abilities Test (CMAT; Hresko, Schlieve, Herron, & Sherbenau, 2003), (e) Diagnostic Achievement Battery, Third Edition (DAB-III; Newcomer, 2001), and (f) Applied Problems subtest of the Woodcock Johnson Achievement Test, Third Edition (WJ-Ach III; Woodcock, McGrew, & Mather,2001).

The test was untimed and group administered. All problems were read aloud for students. Students were required to apply simple computation skills to solve the problems, which included single-digit multiplication of numbers 2-9. Responses were scored for both correct problem representation and correct answer, with 2 possible points per problem for a total of 38 points on 19 items. Internal consistency for the present sample was r = .933, r = .954 for the first posttest sample, and r = .968 for the second posttest sample. Inter-scorer agreement assessed by a second scorer independently scoring 30% of the protocols was 99, 98, and 100 at pretest, posttest 1, and posttest 2, respectively.

Procedure

All participating students completed the Optional Local Purpose Assessment (OLPA; Minnesota Department of Education, 2013) prior to intervention, as well as the following assessments: (a) curriculum-based measurement (CBM-M) for computation of single-digit multiplication, (b) a single-digit multiplication conceptual understanding assessment (Burns, 2011), and (c) Application of word problem solving. Each measure has published reliability and validity coefficients within the acceptable range for screening and research purposes (Salvia, Ysseldyke, & Bolt, 2010). Students also completed two posttest measures (CBM-M and application of word problem solving) following the first intervention phase and three posttest measures after the second intervention phase (CBM-M, application of word problem solving, and single-digit multiplication conceptual understanding assessment).

All participating students were randomly assigned to one of two intervention conditions: (a) conceptual intervention, then computation intervention or (b) computation intervention, then conceptual intervention. Students received each intervention three times per week for two weeks for a total of 12 sessions over 4 weeks. Following the first two weeks of intervention, students were given the first battery of posttests, which included CBM-M and application of word problem solving. Then, students switched intervention groups so that they received the alternate intervention (conceptual or computation) for the remaining two weeks. Finally, at the end of week four, all students received the second battery of posttests, which included CBM-M, application of word problem solving, and single-digit multiplication conceptual understanding assessment.

Intervention

Participating students met in a small group with the principal researcher or an interventionist, an educational psychology graduate student trained in study procedures, three times per week for 4 weeks for a total of 12 sessions. Intervention groups focused on single-digit multiplication combinations involving the numbers six, seven, and eight (e.g., 0…6x9; 0…7x9; and 0…8x9), because these problem sets require the most practice before mastery (Nelson, Burns, Kanive, & Ysseldyke, 2013). Multiplication combinations were randomly assigned to condition in order to minimize practice effects across conditions. Specific interventions are described below.

Conceptually-based. Students in the conceptual group met in small groups of 4-5 students three times per week for 2 weeks with each session lasting approximately 15 minutes. The conceptual intervention consisted of multiple activities, which were explicitly taught following an explicit instructional approach of model-lead-test, with the responsibility of problem completion gradually released to the student. An explicit approach was chosen because it has been described as a critical component of effective interventions for students struggling with academic skills (Burns, VanDerHeyden, & Zaslofsky, 2015; Vaughn, Gersten, & Chard, 2000). Namely, the researcher modeled the problem-solving process, then the student was asked to complete the problem with support and feedback, and finally the student completed the problem independently. The researcher also modeled mathematical language and the problem-solving process, as meta-cognitive skills have been shown to be important for effective problem solving (Montague, 1997; Rosenzweig, Krawec, & Montague, 2011; Gersten et al., 2009).

The randomly assigned multiplication combinations were taught with tasks designed to build and strengthen students' conceptual understanding through repeated practice, feedback, and making explicit underlying schema and connections to previously learned mathematics concepts. Interventionists followed a scripted lesson in order to limit the variability in lesson implementation (Appendix D). The equal groups problem type was targeted, because it is one of the most common multiplication problem types found in elementary school mathematics (Van de Walle et al., 2010; Greer, 1992). Equal groups problems are sometimes referred to as repeated addition where there are a number of sets that together make a product (whole).

The equal groups problem type was taught using models, such as equal sets (i.e., model of multiplication using same sized groups, which can be solved using repeated addition) and arrays (i.e., model of multiplication using rows and columns). Models, when used without context can be helpful in teaching the meaning of an operation and can be particularly useful when taught within a real-world context (Van de Walle et al., 2010). Students were explicitly taught how to use equal sets and arrays in order to solve multiplication problems. In addition, they were asked to generate written addition and multiplication equations of their findings (e.g., 8x3=8+8+8) to make the connection between multiplication and addition clearer.

Students worked on representing multiplication combinations with no context provided in order to focus on the meaning and associated symbolism of the operation

(Van de Walle et al., 2010). For example, the problem 8x4=_____ was written at the top of a white board. Then, the interventionist lead a discussion around the meaning of the sentence (i.e., it is 8 groups/sets of 4 objects in each group/set). Next, the interventionist drew a representation of the sentence using equal sets or an array and connected the model to addition (e.g., 8x4 is the same as 8+8+8+8). Finally, the interventionist explained that 8x4 is the same as 8+8+8+8. In the "we do" stage of the lesson, the interventionist wrote a new problem on the top of a white board and engaged the group in a discussion around the meaning of the problem and possible ways to represent it using a model. During the "we do" stage, students had the opportunity to work on the problem as a group and explain their problem-solving to each other and with support from the interventionist. Finally, during the "you do" stage of the lesson, students worked independently on new problems while the interventionist provided corrective feedback, as needed. Students came together as a group after each problem to describe their work.

Throughout each intervention session, students were asked to describe their problem-solving process, thinking, and reasoning through group talk alouds and were challenged to check for the accuracy in their solutions in order to help build their meta-cognitive skills (Gersten et al., 2009). Interventionists also checked for students' understanding by asking students to construct a story situation using the presented multiplication combination.

Computation-based. Students in the computation group met in groups of 12-13 three times per week for two weeks with each session lasting approximately 15 minutes. The computation groups engaged in a timed trial intervention focused on increasing students' fluency with single-digit multiplication combinations (Burns, Riley-Tillman, & VanDerHeyden, 2012). Students were given worksheet packets (Appendix E). Page one presented a practice set of eleven targeted mathematics combinations in a randomized order with each of the targeted combinations appearing one time. Page two of the packet included additional problems of the targeted combinations in a randomized order. Each targeted combination appeared at least three times with some appearing four times for a total of 36 problems. Page three was an answer key for page two.

Students were instructed to write their name and date at the top of the first page. Next, students completed all of the targeted combinations on the first page (i.e., the practice set) with corrective feedback. The practice set was untimed. After students completed all of the practice set problems, the interventionist reviewed the answers to each problem and students made corrections, as needed. Then, a timer was set for two minutes. Students were instructed to turn to page 2 of their packets and to solve as many combinations as they could in two minutes. When the timer rang, students scored their work and wrote the number of correctly completed problems at the top of their paper. Students corrected any errors they made during the timed drill. Then, students graphed their score on a progress-monitoring chart (Appendix F). The interventionist reviewed students' scores and praised students for beating their previous score and encouraged students to continue to do their best work.

Core Classroom Mathematics Program

All participating students also received mathematics instruction in their general education classrooms for 60 mins each day, as well as 30 mins of differentiated instruction during a Response-to-Intervention block. The primary focus of students' daily mathematics instruction could be categorized in four strands: (a) number sense and computation, (b) geometry and measurement, (c) patterns, functions, and algebra, and (d) data, statistics, and probability.

The mathematics curriculum used in the third and fourth grade classrooms was called enVisionMATH, which is published by Pearson Education, Inc. (2012). The objective of enVisionMATH is to enhance students' development of concepts through inquiry-based and small group activities with a focus on reasoning and modeling. The curriculum includes frequent and ongoing assessment so that teachers can better differentiate instruction for each student. The Institute for Education Sciences lists enVisionMATH as having potentially positive effects on students' mathematics achievement based on a review conducted through the What Works Clearinghouse (U. S. Department of Education, 2013).

Research Design and Data Analysis

Research Design

The research questions were answered using a within groups crossover design (Figure 1; Shadish, Cook, & Campbell, 2002). All participants were randomly assigned to one of two conditions and then administered the same battery of pretests, as described above. Following the administration of either a conceptually-based intervention or a computation-based intervention, participating students completed a battery of posttest measures and then switched to the alternate intervention. In other words, students receiving the conceptually-based intervention first, received the computation-based intervention second and students receiving the computation-based intervention first received the conceptually-based intervention second. After the second intervention, all students completed another battery of posttest measures.

Figure 1. Research design.

R	O_{Pretest}	$X_{\text{Concepts-based Intervention}}$	$O_{\text{Posttest 1}}$	$X_{\text{Procedures-based Intervention}}$	$O_{\text{Posttest 2}}$
R	O_{Pretest}	$X_{\text{Procedures-based Intervention}}$	$O_{\text{Posttest 1}}$	$X_{\text{Concepts-based Intervention}}$	$O_{\text{Posttest 2}}$

Data Analysis

The data for the present study were analyzed in several stages. First, a one-way analysis of variance was conducted for each of the outcome measures to examine any differences at pretest across the two intervention groups. A one-way analysis of variance across the two groups on the conceptual understanding assessment was not significant (F (1, 44) = 0.83, p = .37), was nonsignificant (F (1, 44) = 0.84, p = .37) on the application word problem solving measure for the two groups, and was nonsignificant (F (1, 44) = 0.13, p = .72) on the CBM-M fluency measure for the two groups. Thus, the two groups had equivalent mathematics skills before beginning the intervention. Similarly, a one-way analysis of variance resulted in a nonsignificant F (1, 44) = 0.397, p = .532 effect across the two grades on the conceptual understanding assessment, nonsignificant F (1, 44) = 2.448, p = .125 effects on the application word problem solving measure for the two grades, and nonsignificant F (1, 44) = 1.938, p = .171 effects on the CBM-M fluency measure for the two grades. Thus, the two grades had equivalent mathematics skills before beginning the intervention.

Although there were not significant differences between groups at pre-test or between grades, scores were standardized for ease of interpretation. Scores for the single-digit multiplication conceptual understanding assessment and each outcome measure (i.e., CBM-M fluency and application measure of word problem solving) were converted to grade-based z-scores, so that scores could be interpreted on the same scale across students. Students' scores were transformed to z-scores by first calculating the mean score (μ) and standard deviation (σ) of each measure by grade. Next, the grade-based mean was subtracted from an individual student score (x) and divided by the grade-based standard deviation. The following formula was used: $z = \frac{x-\mu}{\sigma}$

Next, pretest data were used to determine students' intervention need group (i.e., conceptually-based, computation-based, or not differentiated). Students were classified as needing a conceptually-based intervention if their pretest score on the conceptual understanding assessment was at or below 17 and their CBM-M fluency assessment was at or above 25 (n = 14). A cutoff score of 17 was used for the conceptual understanding assessment because there were 6 items in all, with a score of 3 or better on each item indicating adequate understanding based on the rubric described above. Therefore, 18 or higher represented 90%, which previous research suggested was a strong criterion for instructional level of mathematics skills (Burns, 2004) and indicated adequate understanding in order to accomplish the objective. A score of 17 or less indicated that a student needed a conceptual intervention. A cutoff score of 25 DCPM was used for the

CBM-M fluency pretest based on instructional level recommendations (Burns, VanDerHeyden, & Jiban, 2006). Students were classified as needing a computation-based intervention if their pretest score on the CBM-M fluency assessment was at or below 25 and their conceptual understanding assessment score was at or above 18 (n = 22). Finally, students whose pretest scores on the conceptual understanding assessment and CBM-M fluency assessment were both below the criteria or were both above the criteria were classified as having a mathematics need that could not be differentiated by the pretest data (n = 10). Student demographic characteristics for the need groups are summarized in Table 2.

Table 2

Demographic Characteristics by Need Group

	Need Group					
	Conceptual		Computation		Not differentiated	
	n	%	*n*	%	*n*	%
Gender						
Male	9	64	11	50	4	40
Female	5	36	11	50	6	60
Race/Ethnicity						
Asian/Pacific Islander	0	0	3	14	2	20

Black	4	29	6	27	3	30
Hispanic	2	14	4	18	0	0
American Indian	0	0	0	0	0	0
White	8	57	9	41	5	50
Grade						
3	10	71	10	45	4	40
4	4	29	12	55	6	60
Special education status						
Yes	0	0	6	27	2	20
No	14	100	16	73	8	80
English language learner						
Yes	3	21	6	27	0	0
No	11	79	16	73	10	100
Total	14	100	22	100	10	100

Regression Models

A series of linear regression models were fitted to answer the research questions.

1. To what extent does pretest data differentially predict intervention response to conceptual intervention on fluency measure.

$$\Delta_{fluency\ conceptual\ intervention} = b_0 + b_1 x_{CBM\text{-}M\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{application\ pre} + e$$

2. To what extent does pretest data differentially predict intervention response to computation intervention on fluency measure.

$$\Delta_{fluency\ computation\ intervention} = b_0 + b_1 x_{CBM\text{-}M\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{application\ pre} + e$$

3. To what extent does pretest data differentially predict intervention response to conceptual intervention on application measure.

$$\Delta_{fluency\ conceptual\ intervention} = b_0 + b_1 x_{application\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{CBM\text{-}M\ pre} + e$$

4. To what extent does pretest data differentially predict intervention response to computation intervention on application measure.

$$\Delta_{fluency\ computation\ intervention} = b_0 + b_1 x_{application\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{CBM\text{-}M\ pre} + e$$

5. To what extent does need predict response to a conceptual intervention?

$$\Delta_{application\ conceptual\ intervention} = b_0 + b_1 x_{need\ group} + e$$

$$\Delta_{fluency\ conceptual\ intervention} = b_0 + b_1 x_{need\ group} + e$$

6. To what extent does need predict response to a fluency intervention?

$$\Delta_{application\ computation\ intervention} = b_0 + b_1 x_{need\ group} + e$$

$$\Delta_{fluency\ computation\ intervention} = b_0 + b_1 x_{need\ group} + e$$

Fidelity of Implementation

Implementation fidelity was assessed for each intervention by an independent observer. Thirty percent of the conceptually-based intervention sessions were assessed for fidelity using a 10-item checklist of procedures. The total number of items correctly implemented was divided by the total number of items and multiplied by 100. Fidelity of implementation for the conceptually-based intervention was 98.33% (range of 90-100%).

Similarly, 30% of the computation-based intervention sessions were observed. An independent observer rated the implementation using an 11-item checklist of procedures. The total number of items correctly implemented was divided by the total number of items and multiplied by 100. Fidelity of implementation for the computation-based intervention was 100%.

CHAPTER 4

RESULTS

Purpose and Research Questions

The purpose of this chapter is to provide the results of the current study. The chapter begins with a review of the research questions and proceeds to review the data pertaining to each question. The research questions that guided the study were:

1. To what extent does pretest data differentially predict intervention response to conceptual intervention on fluency measure.
2. To what extent does pretest data differentially predict intervention response to computation intervention on fluency measure.
3. To what extent does pretest data differentially predict intervention response to conceptual intervention on application measure.
4. To what extent does pretest data differentially predict intervention response to computation intervention on application measure.
5. To what extent does need predict response to a conceptual intervention?
6. To what extent does need predict response to a fluency intervention?

Descriptive Analysis

Correlations among measures are reported in Table 3. Statistically significant bivariate correlations were found among outcome measures. Correlations were strongest between test administrations of the same measure (e.g., CBM-M Fluency pretest and CBM-M Fluency posttest 1). Table 4 summarizes the descriptive characteristics of each predicator and outcome measure, including skew and kurtosis of each distribution. Distributions for each measure were positively skewed and leptokurtic with the exception

Table 3. *Correlations among measures* ***

	1	2	3	4	5	6	7
1. Conceptual Measure Pre		.25	.48*	.43*	.38*	.36*	.49*
2. CBM-M Fluency Pre	.25		.45*	.88*	.34*	.83*	.40*
3. Applications Measure Pre	.48*	.45*		.53*	.72*	.49*	.63*
4. CBM-M Fluency Post 1	.43*	.88*	.53*		.45*	.86*	.53*
5. Applications Measure Post 1	.38*	.34*	.72*	.45*		.37*	.67*
6. CBM-M Fluency Post 2	.36*	.83*	.49*	.86**	.37*		.49*
7. Applications Measure Post 2	.49*	.40*	.63*	.53*	.67*	.49*	

Note. CBM-M = curriculum-based measure for mathematics.

* $p < .05$

of the single-digit multiplication conceptual understanding assessment (i.e., conceptual measure), which demonstrated a negative skew and normal kurtosis. This deviation from normality may have occurred because of the low achieving nature of the sample with student performance, which was clustered around the lower end of the range. The negative skew of the conceptual measure may be an issue of ceiling effects and exposure to the standards-based curriculum used in the classroom, which emphasizes students' conceptual understanding. Thus, students in the sample may have had increased exposure to mathematical concepts and limited exposure to and practice with word problem solving and multiplication fluency.

Average student gains across the CBM-M fluency and application and word problem solving measures following the conceptually- and computation-based interventions, respectively, were summarized and presented in Table 5. On average, students gained nearly 4 DCPM on the CBM-M fluency measure following the conceptually-based intervention. Similarly, students gained nearly 4 DCPM on the CBM-M fluency measure following the computation-based intervention. Similar results were found for gains in application and word problem solving. On average, students gained -0.65 points on the application and word problem solving measure following the conceptually-based intervention and 0.79 points following the computation-based intervention.

Table 4. *Descriptive characteristics of predictor and outcome variables*

	Pretest				Posttest 1				Posttest 2			
	M	*SD*	Skew	Kurtosis	*M*	*SD*	Skew	Kurtosis	*M*	*SD*	Skew	Kurtosis
Conceptual Measure	19.15	4.96	-1.33	0.98								
CBM-M Fluency	18.63	14.49	1.85	3.56	23.39	14.72	1.43	1.89	26.61	16.56	1.17	1.83
Applications Measure	21.88	9.61	-0.52	-0.53	20.30	10.20	-0.63	-0.77	21.54	10.18	-0.70	-0.63

Note: Scores are reported as raw scores. CBM-M = curriculum-based measure for mathematics.

Table 5. *Descriptive characteristics of student gains by group following each intervention*

		Conceptually-based Intervention				Computation-based Intervention			
		Δ Fluency		**Δ** Application		**Δ** Fluency		**Δ** Application	
	N	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Group 1	24	4.65	7.05	-0.65	8.06	3.06	8.83	0.79	10.13
Group 2	22	3.17	8.35	1.50	5.74	4.93	7.90	-2.76	6.74
Total	46	3.98	7.63	0.36	7.08	3.93	8.37	-0.87	8.80

Research Question 1

The first research question examined the predictability of pretest measures on students' performance on a fluency measure after receiving a conceptually-based intervention. In order to analyze this question, a linear regression model was fitted where students' pretest performance on a CBM-M fluency measure, students' pretest performance on a single-digit multiplication conceptual understanding assessment, and students' pretest performance on an application measure of word problem solving predicted the change in student performance on a CBM-M fluency measure. Each predictor variable was entered simultaneously. The following linear regression equation was used:

$$\Delta_{fluency\ conceptual\ intervention} = b_0 + b_1 x_{CBM\text{-}M\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{application\ pre} + e$$

The results (Table 5) of the linear regression analysis indicated that, together, students' pretest performance on CBM-M fluency, a single-digit multiplication conceptual understanding assessment, and a measure of application and word problem solving explained a nonsignificant 18.4% of the variance in students' change in CBM-M fluency scores following a conceptually-based intervention ($R^2 = .184$, $F(1,41) = 1.265$, $p = .267$). *** Further review of the results revealed that students' performance on the CBM-M fluency pretest measure alone significantly predicted the gain score on CBM-M fluency performance following a conceptually-based intervention ($\beta = -.512$, $p = .005$). Students' pretest performance on measures of conceptual understanding ($\beta = .192$, $p = .848$) and application and word problem solving ($\beta = .202$, $p = .267$) did not significantly predict gains on CBM-M fluency performance following a conceptually-based intervention.

Table 5. *Fluency gains after a conceptually-based intervention*

	b (SE)	*β*	*t*	*p*
CBM-M Fluency Pretest	-0.25 (0.08)	-.51	-2.99	.005*
Conceptual Measure Pretest	0.02 (0.09)	.03	0.19	.85
Applications Measure Pretest	0.10 (0.09)	.20	1.13	.27

Note. CBM-M = curriculum-based measure for mathematics.
*$p < .05$

Research Question 2

The second research question examined the predictability of pretest measures on students' performance on a fluency measure after receiving a computation-based intervention. In order to analyze this question, a linear regression model was fitted where students' pretest performance on a CBM-M fluency measure, students' pretest performance on a single-digit multiplication conceptual understanding assessment, and students' pretest performance on an application measure of word problem solving predicted the change in student performance on a CBM-M fluency measure. Each predictor variable was entered simultaneously. The following linear regression equation was used:

$$\Delta_{fluency\ computation\ intervention} = b_0 + b_1 x_{CBM\text{-}M\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{application\ pre} + e$$

The results (Table 6) of the linear regression analysis indicated that, together, students' pretest performance on CBM-M fluency, a single-digit multiplication conceptual understanding assessment, and a measure of application and word problem solving did not explain a statistically significant proportion of the variance in students' change in CBM-M fluency scores following a computation-based intervention (R^2 = .14, F (1, 41) = 0.337, p = .565). Further review of the results revealed that students' performance on the single-digit multiplication conceptual understanding pretest measure significantly alone predicted the gain score on CBM-M fluency performance following a computation-based intervention (β = .38, p = .02). Students' pretest performance on CBM-M fluency (β = -.206, p = .190) and application and word problem solving (β = -.093, p = .565) did not significantly predict gains on CBM-M fluency performance following a conceptually-based intervention.

Table 6. *Fluency gains after a computation-based intervention*

	b (SE)	*β*	*t*	*p*
CBM-M Fluency Pre	-0.11 (0.08)	-.21	-1.33	.19
Conceptual Measure Pre	0.19 (0.08)	.39	2.42	.02*
Applications Measure Pre	-0.05 (0.08)	-.09	-0.58	.57

Note. CBM-M = curriculum-based measure for mathematics.
*p < .05

Research Question 3

The third research question examined the predictability of pretest measures on students' performance on a measure of application and word problem solving after receiving a conceptually-based intervention. In order to analyze this question, a linear regression model was fitted where students' pretest performance on an application measure of word problem solving, students' pretest performance on a single-digit multiplication conceptual understanding assessment, and students' pretest performance on a CBM-M fluency measure predicted the change in student performance on a application and word problem solving measure. Each predictor variable was entered simultaneously. The following linear regression equation was used:

$$\Delta_{application\ conceptual\ intervention} = b_0 + b_1 x_{application\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{CBM\text{-}M\ pre} + e$$

The results (Table 7) of the linear regression analysis indicated that, together, students' pretest performance on a measure of application and word problem solving, a single-digit multiplication conceptual understanding assessment, and CBM-M fluency did not explain a statistically significant proportion of the variance in students' change in application and word problem solving scores following a conceptually-based intervention ($R^2 = .11$, $F(1, 41) = 0.042$, $p = .838$). Further review of the results revealed that students' performance on the application and word problem solving measure significantly predicted the gain score on application and word problem solving performance following a conceptually-based intervention ($\beta = -.386$, $p = .046$). Students' pretest performance on single-digit multiplication conceptual understanding pretest measure ($\beta = .15$, $p = .39$)

and CBM-M fluency (β = .04, p = .838) did not significantly predict gains on application and word problem performance following a conceptually-based intervention.

Table 7. *Application and word problem solving gains after a conceptually-based intervention*

	b (SE)	β	t	p
CBM-M Fluency Pre	0.02 (0.12)	.04	0.21	.84
Conceptual Measure Pre	0.10 (0.12)	.15	0.87	.39
Applications Measure Pre	-0.25 (0.12)	-.39	-2.06	.05*

Note: CBM-M = curriculum-based measure for mathematics.
*p < .05

Research Question 4

The fourth research question examined the predictability of pretest measures on students' performance on a measure of application and word problem solving after receiving a computation-based intervention. In order to analyze this question, a linear regression model was fitted where students' pretest performance on an application measure of word problem solving, students' pretest performance on a single-digit multiplication conceptual understanding assessment, and students' pretest performance on a CBM-M fluency measure predicted the change in student performance on an application and word problem solving measure. Each predictor variable was entered simultaneously. The following linear regression equation was used:

$$\Delta_{application\ computation\ intervention} = b_0 + b_1 x_{application\ pre} + b_2 x_{conceptual\ pre} + b_3 x_{CBM\text{-}M\ pre} + e$$

The results (Table 8) of the linear regression analysis indicated that, together, students' pretest performance on a measure of application and word problem solving, a single-digit multiplication conceptual understanding assessment, and CBM-M fluency did not explain a statistically significant proportion of the variance in students' change in application and word problem solving scores following a computation-based intervention (R^2 = .28, F (1, 41) = 1.239, p = .272). Further review of the results revealed that students' performance on the application and word problem solving measure significantly predicted the gain score on application and word problem solving performance following a conceptually-based intervention (β = -.588, p < .001). Students' pretest performance on single-digit multiplication conceptual understanding pretest measure (β = .204, p = .163) and CBM-M fluency (β = .157, p = .272) did not significantly predict gains on application and word problem performance following a computation-based intervention.

Table 8. *Application and word problem solving gains after a computation-based intervention*

	b (SE)	β	t	p
CBM-M Fluency Pre	0.14 (0.12)	.16	1.11	.27
Conceptual Measure Pre	0.17 (0.12)	.20	1.42	.16
Applications Measure Pre	-0.50 (0.12)	-.59	-4.03	<.001*

Note: CBM-M = curriculum-based measure for mathematics.
*p < .05

Research Question 5

The fifth research question examined the predictability of student need (i.e., conceptual, computation, or no differentiation) on gains in conceptual understanding and application from a conceptually-based intervention. In order to analyze this question, two linear regression models were fitted. The first regression equation modeled the extent to which students' need group assignment, which was determined based on performance on a single-digit multiplication conceptual understanding and CBM-M fluency screeners, predicted the change in student performance on an application and word problem solving measure following a conceptually-based intervention. The following linear regression equation was used in the first model:

$$\Delta_{application\ conceptual\ intervention} = b_0 + b_1 x_{need\ group} + e$$

Results (Table 9) from the first model indicated that students' need did not explain a statistically significant proportion of the variance in students' change in application and word problem solving scores following a conceptually-based intervention ($R^2 = .05$, $F\,(1, 43) = 2.061$, $p = .158$).

Table 9. *Predictability of need on gains in application and word problem solving after a conceptually-based intervention*

	b (SE)	β	t	p
Need group	-0.20 (0.14)	-.21	-1.44	.16

$^*p < .05$

The second regression equation modeled the extent to which students' need group assignment, which was determined based on performance on a single-digit multiplication conceptual understanding and CBM-M fluency screeners, predicted the change in student performance on a CBM-M fluency measure following a conceptually-based intervention. The following linear regression equation was used in the second model:

$$\Delta_{fluency\ conceptual\ intervention} = b_0 + b_1 x_{need\ group} + e$$

Results (Table 10) from the second model indicated that students' need did not explain a statistically significant proportion of the variance in students' change in CBM-M fluency scores following a conceptually-based intervention ($R^2 = .05$, $F(1, 43) = 2.318$, $p = .135$).

Table 10. *Predictability of need on gains in fluency after a conceptually-based intervention*

	b (SE)	β	t	p
Need group	-0.16 (0.11)	-.23	-1.52	.14

$^*p < .05$

Research Question 6

The sixth research question examined the predictability of student need (i.e., conceptual, computation, or no differentiation) on gains in computation and application from a computation-based intervention. In order to analyze this question, two linear regression models were fitted. The first regression equation modeled the extent to which students' need group assignment, which was determined based on performance on a single-digit multiplication conceptual understanding and CBM-M fluency screeners, predicted the change in student performance on an application and word problem solving measure following a computation-based intervention. The following linear regression equation was used in the first model:

$$\Delta_{application\ computation\ intervention} = b_0 + b_1 x_{need\ group} + e$$

Results (Table 11) from the first model indicated that students' need did not explain a statistically significant proportion of the variance in students' change in application and word problem solving scores following a conceptually-based intervention ($R^2 = .01$, $F\ (1, 43) = 0.164$, $p = .688$).

Table 11. *Predictability of need on gains in application and word problem solving after a computation-based intervention*

	b (SE)	*β*	*t*	*p*
Need group	0.07 (0.17)	.06	0.41	.69

**p* < .05

The second regression equation modeled the extent to which students' need group assignment, which was determined based on performance on a single-digit multiplication conceptual understanding and CBM-M fluency screeners, predicted the change in student performance on a CBM-M fluency measure following a computation-based intervention. The following linear regression equation was used in the second model:

$$\Delta_{fluency\ computation\ intervention} = b_0 + b_1 x_{need\ group} + e$$

Results (Table 12) from the second model indicated that students' need did not explain a statistically significant proportion of the variance in students' change in CBM-M fluency scores following a conceptually-based intervention (R^2 = .01, F (1, 43) = .461, p = .501).

Table 12. *Predictability of need on gains in fluency after a computation-based intervention*

	b (SE)	β	t	p
Need group	0.07 (0.10)	.10	0.68	.50

*p < .05

CHAPTER 5

DISCUSSION

Organization of the Chapter

The purpose of this chapter is to synthesize the results from the present study. It begins with a discussion of each research question. Next, the results are described within the context of previous research with implications for practice and intervention design. Finally, the chapter concludes with suggestions for future research and a review of study limitations.

Review of Study Purpose

Struggling students who are identified early and provided with supplemental academic supports experience improved reading or mathematics outcomes (Fuchs, Fuchs, & Compton, 2012a). The method for correctly identifying and intervening with students who are not meeting grade level expectations has varied. Historically, an approach relying on underlying cognitive characteristics or processing skills was used. This approach, referred to as an Aptitude-by-Treatment Interaction (ATI) was criticized for not fully capturing student needs or explaining intervention effectiveness (Cronbach & Snow, 1977; Kearns & Fuchs, 2013). Alternatively, a framework called a Skill-by-Treatment Interaction (STI) relies on matching interventions based on measurable and alterable skills (Burns, Codding, Boice, & Lukito, 2010). Preliminary research in the area of mathematics suggests that the STI approach may be useful in identifying specific subskill needs, such as conceptual understanding or computational fluency, for students (Burns, 2011).

The purpose of the current study was to better understand the relationship between mathematics assessment and intervention design. Specifically, the study examined the link between specific skill assessments of conceptual understanding, computational fluency, and application and word problem solving with a conceptually-based or computation-based intervention.

Research Question 1: Gains on CBM-M fluency after a conceptually-based intervention

The first question examined the relationship between pretest measures and their predictability on student gains on a computational fluency measure following a conceptually-based intervention. Regression analyses revealed that, overall, pretest measures did not explain a statistically significant proportion of the variance in student gains on a CBM-M fluency measure. However, student performance at pretest on a CBM-M fluency measure significantly predicted performance on the same measure at posttest.

These results are not surprising, because CBM-M fluency scores between test administrations were highly correlated at .88 and .83 respectively. Moreover, the conceptually-based intervention focused largely on the underlying principles and relationships of multiplication and not on the quick retrieval of single-digit multiplication combinations. As such, students' prior skill and fluent retrieval of these combinations were expected to predict performance following intervention.

Research Question 2: Gains on CBM-M fluency after a computation-based intervention

The second question examined the relationship between pretest measures and their predictability on student gains on a computational fluency measure following a computation-based intervention. Regression analyses revealed that, overall, pretest measures did not explain a statistically significant proportion of the variance in students' gains on a CBM-M fluency measure. However, student performance at pretest on a measure of conceptual understanding of single-digit multiplication significantly predicted performance on CBM-M fluency at posttest, suggesting that students' conceptual understanding may impact their response to fluency-based interventions.

This finding is interesting, because of the debate surrounding which knowledge base develops first (Rittle-Johnson & Alabali, 1999). Given that students' initial performance on a conceptual understanding measure significantly predicted gains in computational fluency, it may suggest that grasping the underlying principles and relationships governing a construct, such as single-digit multiplication, may actually mediate the potential for improving their fluency skills.

Delazer and her colleagues (2005) noted similar findings in their work examining the role of strategy instruction in comparison to rote drill instruction on basic math combinations. Their results indicated that instruction delivery did not significantly impact student performance on reaction time (i.e., fluency), though students in the strategy instruction group outperformed students in the drill instruction group on measures of computation accuracy. Delazer and her colleagues (2005) suggested that a potential reason could be that students who learn to solve basic combinations via strategy instruction or by learning underlying concepts may demonstrate higher accuracy because they can rely on back-up strategies when direct retrieval of a fact fails. Similarly, in the current study, students who entered with stronger conceptual understanding may have been able to more accurately solve items on the CBM-M fluency probes, leading to higher fluency scores, because students were scored on the number of correct digits they answered during the timed assessment. Byrnes and Wasik (1991) found a similar relationship, wherein a dynamic interaction appeared to be at play between conceptual understanding and computation. It was suggested that conceptual understanding forms the bases from which procedures around computation are formed (Byrnes & Wasik, 1991).

Research Question 3: Gains on application and word problem solving after a conceptually-based intervention

The third research question examined the predictability of pretest measures on gains in students' application and word problem solving performance following a conceptually-based intervention. A regression analysis revealed that the battery of pretest assessments did not explain a statistically significant proportion of the variance in students' gains on the application and word problem solving measure. Students' performance at pretest on the same measure significantly predicted gains following the conceptually-based intervention.

These results are not surprising, because application and word problem solving scores between test administrations were highly correlated at .72 and .63 respectively.

Interestingly, the conceptual understanding and CBM-M fluency measures did not significantly contribute to the prediction of students' gains in application and word problem solving. The Instructional Hierarchy suggests that application tasks occur toward the end of the learning progression (Haring & Eaton, 1978), and thus, it would seem reasonable that students' prior conceptual understanding and computational fluency would positively contribute to students' performance on a measure of application and word problem solving. Moreover, there appears to be a clear relationship between conceptual understanding and application tasks, since both rely on organizing and recognizing patterns in prior knowledge in order to solve a problem (Jitendra, DiPipi, & Perron-Jones, 2002; Silver & Marshall, 1990). Jitendra, DiPipi, and Perron-Jones (2002) noted that incorporating concepts into word problem solving intervention may aid in the further development and application of such skills. Contrary to the findings of the current study, it stands to reason that assessment in both domains could account for unique variance in students' gains in application and word problem solving following a conceptually-based intervention.

Research Question 4: Gains on application and word problem solving after a computation-based intervention

The fourth question examined pretests measures as a predictor for gains on a measure of application and word problem solving following a computation-based intervention. A regression analysis revealed that the battery of pretest measures, collectively, did not significantly account for variance in students' gains on a measure of application and word problem solving. Further review of the results revealed that students' performance at pretest on the application and word problem solving measure significantly predicted their gains on the same measure.

Application and word problem solving scores between test administrations were highly correlated at .72 and .63 respectively. It was surprising that students' performance on measures of conceptual understanding and computational fluency did not significantly contribute to the model, particularly since these basic skills have been shown to important for more complex tasks, such as word problem solving (Vukovic & Sigel, 2010). Moreover, application and word problem solving is considered a capstone of student learning (Haring & Eaton, 1977) where students are required to draw upon their conceptual understanding, as well as, their computation skills in order to solve novel problems (Jitendra, DiPipi, & Perron-Jones, 2002; Silver & Marshall, 1990). As such, it was expected that students' prior knowledge of mathematical concepts and computation would predict their gains in application and word problem solving.

Research Questions 5 and 6: Predictability of need on response to conceptually- and computation-based interventions

The fifth question examined the role of identified student need on their response to a conceptually-based intervention. Need was determined *post hoc* using pretest scores on the conceptual understanding measure and CBM-M fluency measure to categorize students into one of three categorical groups, (a) conceptual need, (b) computation need, and (c) not differentiated. Students' identified need did not explain a statistically significant proportion of the variance in predicting gains on either outcome measure following the conceptually-based intervention. Additionally, students' identified need did not explain a statistically significant proportion of the variance in the predicting gains on either outcome measure following the computation-based intervention.

This direct assessment of the STI framework did not yield results consistent with those presented by Burns (2011). Namely, students' identified need did not appear to impact their response to the conceptual intervention. One possible explanation for these results is the limited sample size in each need group. There were 14 students in need of a conceptual intervention, 22 students in need of a computation intervention, and 10 student for whom need was not differentiated based on pretest performance. This limited sample size directly impact statistical power, and therefore, limits the confidence in which we can interpret these results. Further, a small sample size limits the generalizability of the results. That said, these findings contribute to the conversation of how to best link assessment data to intervention for mathematics.

Current research spans from the examination of varying assessment systems (Foegen, Jiban, & Deno, 2007) to intervention components (Gersten, Chard, Jayanthi, Baker, Morphy, & Flogo, 2009). A more limited subset of researchers in education is exploring frameworks for matching assessment to intervention in mathematics, particularly for those students who persistently underperform and struggle with mathematics. For example, Fuchs (2013) and colleagues have been examining shared cognitive characteristics from which to target intervention, in the same vein as ATI, as a way to explain individual differences in intervention response. Namely, Fuchs (2013) and Cirino et al. (2015) have been examining the role of working memory and language processing as potential mechanisms for which to explain learning difficulties.

Vukovic and Siegel (2010) also examined shared academic and cognitive characteristics of students with persistent mathematics difficulty. In their work, the researchers noted that persistently low or below average performance on mathematics measures did not consistently appear until third or fourth grade. They also indicated that there may be shared cognitive characteristics, such as phonological awareness, working memory, and processing speed, among students persistently perform below their same-age peers (Vukovic & Siegel, 2010).

The work of Fuchs and colleagues and Vukovic and colleagues is renewing the conversation around ATI. While persistently low performing students may share academic or cognitive characteristics, the question remains about how to best intervene and support these students. Historically, research has not supported the use of interventions aimed at underlying cognitive characteristics (Cronbach & Snow, 1977; Kavale, 2007; Kearns & Fuchs, 2013; Melby-Lervag & Hulme, 2013) leading to a push for a competing framework from which to identify interventions (Burns, Codding, Boice, & Lukito, 2010). While the findings from the current study are not consistent with previous research examining an STI approach, it furthers the conversation and provides a base from which to direct future research.

Implications

Potential Implications for Practice

The findings from the current study provide several practical implications. Educators are in need of an evidence-based and efficient method for identifying students at risk for mathematics difficulty and ways in which to intervene with them. Research efforts to support such decision making has been varied and included

One of the most interesting findings from the current study is the apparent relationship between conceptual understanding and computational fluency. Developmental and educational researchers have suggested that such a relationship is iterative (Byrnes & Wasik, 1991; Rittle-Johnson & Alabali, 1999) and development in one area supports further development in the other (Delazer, 2005). Mathematics educators have long emphasized the importance of developing conceptual understanding and flexible problem solving (Hiebert & Lefevre, 1986; Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000; National Mathematics Advisory Panel [NMAP], 2008; Rittle-Johnson & Koedinger, 2005). Recent adaptation of the Common Core State Standards (2010) has further underscored the importance of including conceptual understanding in instruction and intervention. Findings from the current study suggest that students' conceptual understanding may impact their performance on computational fluency. This is particularly interesting since it may also support the idea that conceptual understanding develops first in the learning progression (Haring & Eaton, 1977) with students' computation skills and fluency developing next. Given that the current study did not address this question directly, it is important to use caution when interpreting the results in this way.

Another important implication for practice from the current study is the idea of assessing different knowledge bases for the purposes of intervention. Findings here suggest that performance specific subskills, such as conceptual understanding, computational fluency, and application and word problem solving, are important for predicting student gains following intervention. Educators should consider using subskill specific measures for student who are identified as at risk for mathematics difficulty. Burns, Codding, Boice, and Lukito (2010) found in their meta-analyses of mathematics interventions that students made the most gains when interventions were appropriately matched to both their specific skill need and instructional level. While instructional level was not directly measured in the current study, it is something that educators should consider when designing intervention plans for students.

Implications for Theory

The current study has several implications for theory. First, the findings here contribute to the body of research surrounding the learning hierarchy, particularly as it relates to mathematics. Researchers have debated the order in which students acquire mathematical knowledge and skills. Rittle-Johnson and Alabali (1999), Byrnes and Wasik (1991), Delazer, 2005, and others have suggested that the development in one mathematical domain supports the development in others. Further, Burns (2011) has suggested that students acquire conceptual understanding followed by computational fluency, both of which, support students' development of application and word problem solving. The findings from the current study contribute to the theoretical development of the learning hierarchy model for mathematics proposed by Burns (2011). The findings from this study suggest that students' conceptual understanding may impact their computational fluency. Thus, may support the idea that conceptual understanding develops first followed by computational fluency. This question was not addressed directly, and therefore, it is important to pursue additional research to explore these findings further.

Another potential theoretical implication from the current study has to do with the Skill-by-Treatment Interaction (STI) framework for assessment and intervention. The emerging research on STI indicates that it is a promising alternative to ATI (Burns, Codding, Boice, & Lukito, 2010; Burns, VanDerHeyden, & Zaslofsky, 2014), however more research is needed. Findings form the current study did not corroborate previous findings, insomuch that students' need did not significantly predict gains in computational fluency or application and word problem solving. It is possible that the specific sub-skills targeted in the current study are not the most salient features for assessment or intervention. Additionally, it is possible that the way in which each knowledge base was assessed in this study did not have the sensitivity or specificity required to make reliable decisions about targeting interventions. Moving forward it will be important to examine and merge the theoretical bases for assessment and intervention in order to better refine the STI framework.

Directions for Future Research

The directions for future research from the current study are many. First, it seems that a continued examination of the role and relationship of each knowledge base would be appropriate. The findings from the current study provide some preliminary support for the idea that conceptual understanding may develop first, or at the very least, support students' computational fluency. Conceptual understanding and computational fluency are clearly linked (Rittle-Johnson et al., 2001) and some have argued against differentiating them (Wu, 1999). Mathematics instruction should include both components (NMAP, 2008), but targeting conceptual understanding or procedural fluency for mathematics interventions with struggling students has been shown to be an effective heuristic (Burns, 2011; Burns, 2013). Therefore, future researchers should further examine the relationship and distinctness of the two components of mathematics proficiency among students who struggle to learn mathematics and how well interventions can uniquely address each domain.

Future research could also examine the way in which each knowledge base is assessed. In the current study, approaches from previous research were selected, but other methods exist for measuring conceptual understanding, computation, and application and word problem solving. Perhaps other assessment approaches would lead to a better diagnostic method for identifying specific subskill deficits. For example, there have been several approaches for assessing students' conceptual understanding, including discrimination tasks (Anderson, 1989; Beatty & Moss, 2007; Canobi, 2004; Geary, Bow-Thomas, & Yao, 1992) and with semi-structured interviews (Hiebert & Wearne, 1996). Conceptual understanding is often assessed using measures of application, such as those found in commercially available CBM systems, which may be less useful for diagnostic purposes. Computational fluency is more straightforward, since it is generally considered a discrete skill. However, recent research has examined ways to make CBM-M fluency probes more psychometrically sound (Methe, Briesch, & Hulac, 2015). Similarly, there are other systems available for assessing students' application and word problem skills. Jitendra and colleagues (2007) have developed a CBM system specific to word problem solving. The bevy of assessment options may lead to error in diagnostic accuracy and local decision-making. Future research could replicate the current study using alternate methods for assessing each of knowledge base.

Future research may also extend the work of the current study to further examine the STI framework for linking assessment data to evidence-based intervention. It would be beneficial to extend the work of Burns (2011, 2013), particularly in the area of mathematics, for efficient and theoretically sound methods for identifying student need and matching an intervention to that need. The current study used subskill specific pretest data to inform intervention decisions. Future research could incorporate the work of Vukovic and Siegel (2010), Cirino et al. (2015), and Fuchs (2012) and investigate skill-specific alterable variables that may lend themselves well to intervention.

Limitations

It is important to consider the findings of the current study within the context of its limitations. First, the conceptual understanding had several limitations within the current study. The current sample of students was relatively high performing, which may have created some issues around range restriction. The distribution of the conceptual understanding measure was negatively skewed, which may be an issue of ceiling effects and exposure to the standards-based curriculum used in the classroom, which emphasizes students' conceptual understanding. Thus, students in the sample may have had increased exposure to mathematical concepts and limited exposure to and practice with word problem solving and multiplication fluency. It is also possible that the conceptual measure may not have been sensitive enough to detect individual student differences.

Another limitation to the current study involved exposure to the classroom curriculum. The third grade students may have received as many as seven semesters of the standards-based curriculum, whereas fourth grade students may have received as many as nine semesters of the standards-based curriculum. The variability in exposure to the core curriculum is a potential threat to the current study's internal validity.

The current study occurred over four weeks during the spring semester of the academic year. It is possible that the length of the intervention period in the current study was not long enough to impact changes in students' skill level. Similarly, it is possible that the measures used were not sensitive enough to detect student growth after the intervention period. Finally, the total sample of the current study was 46 students. An a priori power analysis recommended a total sample of 50. Thus, it is possible that the smaller sample size of the current study was not sufficient for detecting statistically significant results.

Conclusion

Mathematical proficiency is unarguably a critical component of modern education with links to improved educational and employment outcomes (Cavanagh, 2006). Current statistics suggest that the United States is trailing other developing countries (Mullis, Martin, Gonzalez, & Chrostowski, 2004) with less than half of our students nationally demonstrating proficiency in mathematics (U.S. Department of Education, 2011).

Educators are faced with the challenge of identifying interventions to meet the needs of the diverse population of students with whom they work. Previous attempts at creating a framework for matching student need to evidence-based interventions (i.e., ATI) were largely unsuccessful (Cronbach & Snow, 1977; Kavale, 2007; Kearns & Fuchs, 2013; Melby-Lervag & Hulme, 2013). An alternate approach has emerged and appears promising, but has limited research to support its use (Burns, 2011; Burns, Codding, Boice, & Lukito, 2010). The current study sought to broaden this research base by examining the predictability of mathematics measures on student gains following intervention. The work in mathematics is far from complete and a continued focus on these critical skills will benefit students for generations to come.

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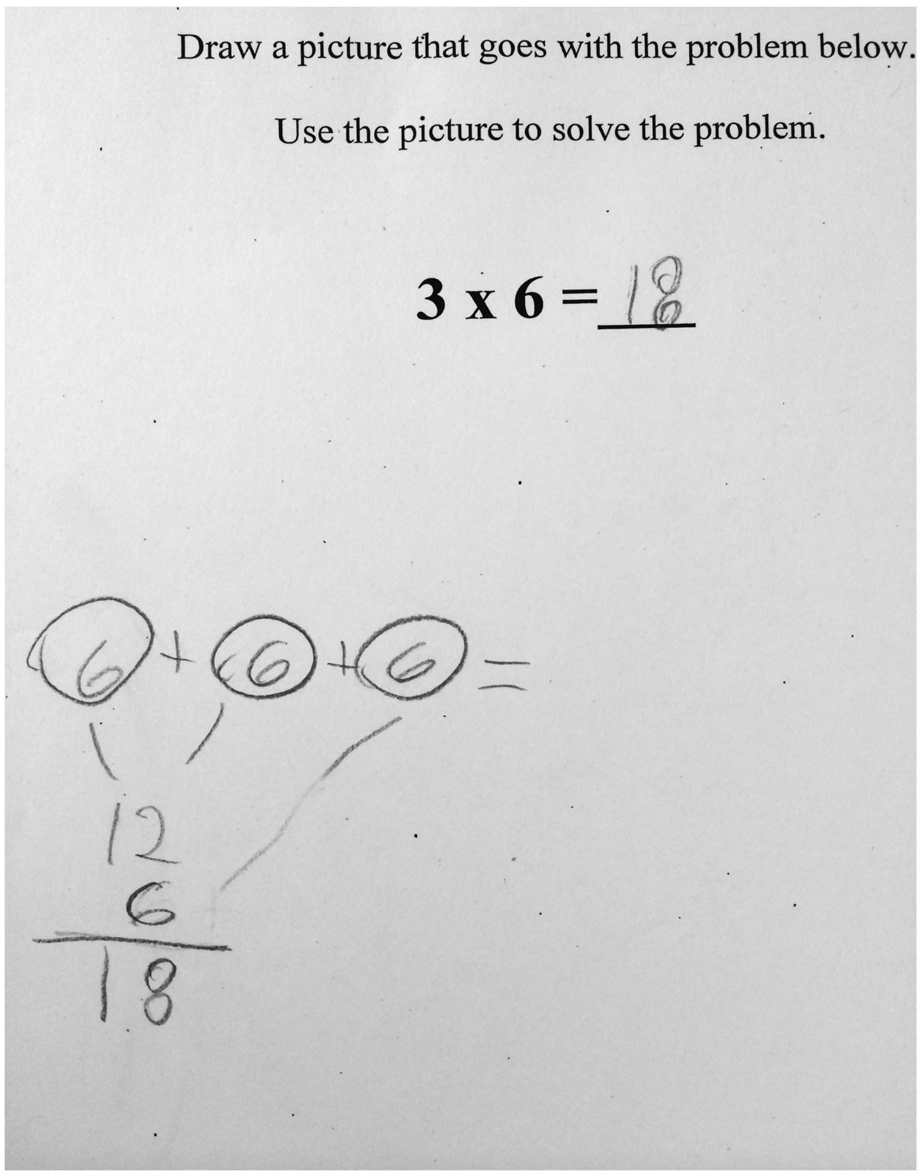
Appendices

Appendix A.

Single-digit Multiplication Conceptual Understanding Assessment



Draw a picture that goes with the problem below.

Use the picture to solve the problem.

3 x 6 = 18

6 + 6 + 6 =

12
6
18

Appendix B.

Single-digit Multiplication Conceptual Understanding Assessment: Interview Protocol

Ask the students the following questions and
write down their answer as close to verbatim as possible.

If needed, answers can be further probed with the following two follow-up questions:

Please tell me more about what you did so I can understand you better.
I never thought about it that way. Can you tell me more?

1. How did you figure this problem out?
2. How did you find the answer?
3. *(point to pictures/objects)* What do you mean and how did they help you solve the problem?
4. Tell me what you were thinking in your head when you were doing this.
5. How did you check your answer to see if it is correct?

Appendix C.

Single-digit Multiplication Conceptual Understanding Assessment: 4-point Rubric

CONCEPTUAL UNDERSTANDING ASSESSMENT

	Unsatisfactory Task is attempted, but little or no success	**Partial** Part of the items is demonstrated, but no evidence of understanding	**Proficient** Only minor errors and adequate understanding of the item to complete the task	**Excellent** Complete understanding to accomplish item.
1. Counts with understanding	1	2	3	4
2. Understands the number sign including relevant and formal math language/vocabulary	1	2	3	4
3. Understands the facts of adding/subtracting or multiplication/division of whole numbers	1	2	3	4
4. Correctly uses the visual model (i.e., there was a correct relationship between the diagram that the student created and the problem)	1	2	3	4
5. Uses an identifiable strategy	1	2	3	4
6. Answers the problem correctly	1	2	3	4

Total Score_________ of 24

Appendix D.

Conceptual Intervention: Sample Lesson Script.

Objective

Students will increase conceptual understanding of single-digit multiplication with embedded story situations.

Materials

- Base-10 manipulatives and counters
- Dry erase boards & markers
- Student folders

Error Correction Procedure

- Provide immediate corrective feedback on errors.
- Provide additional examples as needed.

Preview

Today, you will learn to solve multiplication problems. Solving multiplication problems can help you improve your math and help you understand how math applies to everyday life. You already know how to add and subtract. Now you will learn what multiplication means and how to solve multiplication problems.

We are going to read some problems. We will work together to solve the problems. First, I will read the problem and explain what it means. Then, I will show you how you can use manipulatives like these *(point to manipulatives)* to help solve the problem. Next, we will work together to solve a problem. Then, you will have a chance to practice solving more problems on your own. I will help you when you need it.

Vocabulary

- Groups/Sets
- Objects

Targeted Facts

6 x 5	8 x 4	7 x 4	8 x 5	8 x 9
6 x 2	6 x 3	6 x 9	8 x 2	6 x 4

Model ("I do")

TUTOR	STUDENTS
I am going to read you a problem and then I am going to show you how to solve it using these manipulatives. *(gesture to base-10 blocks)* 6 x 2 = ______ This number sentence says, "6 times 2" and it means "6 groups with 2 in each group." Now, we can use the base-10 blocks to build a model to show the number sentence. The "blocks" represent 1. First, I will use the marker to make the groups. The number sentence tells us that there are 6 groups. So, I will draw 6 circles. *(make 6 circles)* Next, I will use the "blocks" to make 2. I will put 2 in each group until all the circles are full. See? Now, I can use the model to find the solution to the problem. There are 2 in each group. *(write "2" under each circle)* We can add two 6 times to get the answer. *(write a "+" in between each "2" under the circles and connect it to the multiplication sentence by writing 6 x 2 = 2 + 2 + 2 + 2 + 2 + 2 and complete the computation)* Now, I can read the number sentence. "6	

times 2 equals 12." Read it with me. Good! So, the answer to our problem is 12. We can check our work by counting the "blocks." The "blocks" represent 1, so let's count by 1 to find how many are in the groups. *(count aloud until you reach 12; complete the number sentence with 12)* I can also think about this problem as part of a story. "My mom bought me 6 packages of chocolate. Each package has 2 pieces in it. How many pieces of chocolate do I have?" The number of packages represents the number of groups there are in this problem and the size of the group is 2, since there are 2 pieces of chocolate in each package. I will need to use multiplication to solve this problem. I can set it up like the problem we just completed so that it reads, "6x2=___" Then, I can solve it by drawing a model of 6 groups with 2 in each group.	"6 times 2 equals 12."

Guided Practice ("We do")

TUTOR	STUDENTS
Now, let's do another problem together. *(show written problem and read aloud)* 6 x 4 = _______	
Can someone tell me what this number sentence means?	"6 groups with 4 objects in each group."
Good! This number sentence says, "6	

groups with 4 in each group."	
Now, we can use the base-10 blocks to build a model to show the number sentence. Remember, the "blocks" represent 1.	
(pass out the base-10 blocks to the students; make sure that each student has enough to find the solution)	
Let's draw circles for each group. How many groups are there in this problem?	"6"
Good! Let's draw 6 circles	*(students will work on drawing 6 circles on their white boards)*
How many objects are in each group?	"4"
Great! So, let's use the "blocks" to make 4 and put 4 in each group until all the circles are full.	*(students will put 4 "blocks" into each circle)*
How can we use the model to solve the problem?	"We can use addition to solve it, like 4+4+4+4+4+4."
(Prompt for incorrect responses: "Could we use addition to solve the problem? What would that look like?")	
(write "4" and "+" under each circle)	
What will the number sentence look like now? Use the white boards to write the complete number sentence. (6x4=4+4+4+4+4+4)	*(students should write 6 x 4 = 4 + 4 + 4 + 4 + 4 + 4 on their white boards)*
Now, let's complete the number sentence. What is the answer?	"24"
Great! Let's read the complete number sentence together.	"6 times 4 equals 24"
Does our answer make sense? How do we	"We can count the 'blocks'."

know? How can we check our work? *(guide students in checking their work)* *Check for understanding: Ask students to construct a story situation for the presented problem. Ask students to identify which element of the story represents the groups and which element represents the size of the group (i.e., number of objects in the group).*	

Independent Practice ("You do")

TUTOR	STUDENTS
Now, it's your turn. Read the problem carefully and use the manipulatives to solve the problem. *(show written problem and read aloud)*	
6 x 9 = ________ *(monitor students work and provide prompts and corrective feedback as needed)*	*(students will work on creating 6 groups of 9 using circles and "blocks")*
It looks everyone is finished working. Let's talk about what you did. What did you do first?	I figured out that there were 6 groups with 9 in each group. So, I drew 6 circles to represent the groups and used the "blocks" to make 9 in each group.
Good! Then, how did you solve the problem?	I added 9 six times to get 54.
What will the complete number sentence say?	6 x 9 = 54
How did you know that you had found the right answer? *(repeat the independent practice steps above with new problems as time allows)*	I counted all the "blocks" and found 54

Check for understanding: Ask students to construct a story situation for the presented problem. Ask students to identify which element of the story represents the groups and which element represents the size of the group (i.e., number of objects in the group).

Appendix E.

Computation Intervention: Sample Timed Drill Worksheet.

Name: ______________________ Date: ____________ Score: ____________

6	6	6	6	6	6
x 5	x 8	x 1	x 6	x 0	x 4
6	6	6	6		
x 9	x 7	x 3	x 2		

6	6	6	6	6	6
x 5	x 8	x 1	x 6	x 0	x 4
6	6	6	6	6	6
x 9	x 7	x 3	x 2	x 2	x 7
6	6	6	6	6	6
x 8	x 3	x 2	x 1	x 0	x 8
6	6	6	6	6	6
x 9	x 2	x 6	x 4	x 2	x 7
6	6	6	6	6	6
x 8	x 8	x 3	x 1	x 0	x 8
6	6	6	6	6	6
x 9	x 7	x 3	x 2	x 2	x 7

Appendix F.

Computation Intervention: Sample Progress Monitoring Chart.