Essays on Market Incompleteness

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Dedication

I would like to dedicate this thesis to my wife Flávia, for all of her love and support.

Abstract

This thesis studies incomplete market economies. First from a normative perspective in Chapter 1, then from a positive one in Chapter 2.

Chapter 1 studies optimal taxation in an environment where heterogeneous households face uninsurable idiosyncratic risk. To do this, we formulate a Ramsey problem in a standard infinite horizon incomplete markets model. We solve numerically for the optimal path of proportional capital and labor income taxes, (possibly negative) lump-sum transfers, and government debt. The solution maximizes welfare along the transition between an initial steady state, calibrated to replicate key features of the US economy, and an endogenously determined final steady state. We find that in the optimal (utilitarian) policy: (i) capital income taxes are front-loaded hitting the imposed upper bound of 100 percent for 33 years before decreasing to 45 percent in the long-run; (ii) labor income taxes are reduced to less than half of their initial level, from 28 percent to about 13 percent in the long-run; and (iii) the government accumulates assets over time reducing the debt-to-output ratio from 63 percent to -17 percent in the long-run. Relative to keeping fiscal instruments at their initial levels, this leads to an average welfare gain equivalent to a permanent 4.9 percent increase in consumption. Even though non-distortive lump-sum taxes are available, the optimal plan has positive capital and labor taxes. Such taxes reduce the proportions of uncertain and unequal labor and capital incomes in total income, increasing welfare by providing insurance and redistribution. We are able to quantify these welfare effects. We also show that calculating the entire transition path (as opposed to considering steady states only) is quantitatively important. Implementing the policy that maximizes welfare in steady state leads to a welfare *loss* of 6.4 percent once transitory effects are accounted for.

The main determinants of credit limits are the rules that govern the ability of households to default on their loans and the risks that they are exposed to. Chapter 2 investigates the quantitative relevance of these determinants using a version of the incomplete markets life cycle model in which agents are allowed to default on their debt holdings by declaring bankruptcy. I document that credit limits are positively correlated with households' income levels. I then show that the introduction of profile heterogeneity in

the households' income processes increases the correlation between income and credit limits. This fact is consistent with the theoretical results established in a simple example. I also show that proportional income punishments or a threshold level of income, such that agents are only allowed to declare bankruptcy only for income levels bellow that threshold, can also be used to generate such a positive correlation. Finally, the main calibration results suggest an important qualification about heterogeneous income profile models: the lower levels of uncertainty implied by these models lead to a severe underestimation of the number of bankruptcy filings.

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Chapter 1

Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks

1.1 Introduction

How and to what extent should governments tax capital and labor income if they care about individual income inequality and risk? We want to provide a quantitative answer to this question. We, therefore, need a model that is able to generate realistic levels of income inequality and uninsurable risk. Our approach in this paper is to numerically solve a Ramsey problem in a quantitative general equilibrium model with heterogenous agents and uninsurable idiosyncratic risk - from now on referred to as the standard incomplete markets (SIM) model[1].

The SIM model has been used extensively for positive analysis and been relatively successful at matching some basic facts about inequality and uncertainty[2]. In this environment agents face uncertainty with respect to their individual labor productivity

[1] This type of model was originally developed and analyzed by Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994).

[2] Our calibration strategy is similar to the ones in Domeij and Heathcote (2004) and Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

which they cannot directly insure against (only a risk-free asset is available). Depending on their productivity realizations they make different savings choices which leads to endogenous wealth inequality. As a result, on top of the usual concern about not distorting agents decisions, a (utilitarian) Ramsey planner has two additional objectives: to redistribute resources across agents, and to provide insurance against their idiosyncratic productivity risk.

The study of optimal fiscal policy in the SIM model has focused, so far, on the maximization of steady state welfare[3]. In contrast, we allow policy to be *time varying* and the welfare function to depend on the associated *transition* path. We calibrate the initial steady state to replicate several aspects of the US economy; in particular the fiscal policy, the distribution of wealth, and statistical properties of the individual labor income process. The final steady state is, then, endogenously determined by the path of fiscal policy. The Ramsey planner finances an exogenous stream of government expenditures with four instruments: proportional capital and labor income taxes, (possibly negative) lump-sum transfers, and government debt.

Labor and capital income taxes are distortive, however, they can be used to provide insurance and redistribution. The only uncertainty that agents face, in our environment, is with respect to their labor productivities[4]. Hence, labor income is the only risky part of the agents' income. By taxing labor income and rebating the extra revenue via lump-sum, the planner can reduce the proportion of the agents' income that is uncertain and effectively provide insurance. On the other hand, capital income is particularly unequal, since the inequality of individual asset holdings is high, and by taxing capital the planner can reduce the proportion of unequal income in total income and, this way, provide redistribution. The effect of government debt is more subtle. Increasing government debt the government crowds out capital which affects prices indirectly, in particular reducing wages and increasing interest rates which leads to a less uncertain

[3]See, for instance, Aiyagari and McGrattan (1998), Conesa, Kitao, and Krueger (2009), and Nakajima (2010).

[4]Panousi and Reis (2012) and Evans (2014) focus instead on investment risk. One justification for our focus on labor income risk is the fact that it is a bigger share of the total income for most agents in the economy. The bottom 80 percent in the distribution of net worth have a a share of labor income above 77 percent, in the 2007 SCF.

but more unequal distribution of income. The optimal fiscal policy weighs all these effects against each other.

We find that capital income taxes should be front-loaded hitting the imposed upper bound of 100 percent for 33 years then decreases to 45 percent in the long-run. Labor income taxes are reduced to less than half of their initial level, from 28 percent to about 13 percent in the long-run. The ratio of lump-sum transfers to output is reduced to about a half of its initial level of 8 percent and the government accumulates assets over time; the debt-to-output ratio decreases from 63 percent to -17 percent in the long-run. Relative to keeping fiscal instruments at their initial levels, this leads to a welfare gain equivalent to a permanent 4.9 percent increase in consumption.

Unlike the Ramsey problem solved for representative-agent economies, in this paper we do not rule out lump-sum taxation. The optimal levels of distortive taxation are, therefore, derived rather than imposed. Even though lump-sum taxes are available, the planner chooses to tax both capital and labor income at positive rates, rebating the associated revenue via lump-sum *transfers*. Relative to a system that obtains all revenue via lump-sum taxes, such a tax system changes the composition of agents' after-tax income, reducing the proportions associated with uncertain and unequal labor and capital incomes and increasing the proportion of certain and equal transfer income; providing insurance and redistribution. To clarify this point and to understand exactly how the optimal policy reacts to changes in uncertainty and inequality we provide an analytic characterization of the solution to the Ramsey problem in a simple two-period version of the SIM model.

We decompose the average welfare gains of 4.9 percent associated with implementing the optimal policy into three parts: (i) 3.7 percent come from the more efficient allocation of aggregate resources due to the reduction of the distortions of agents' decisions; (ii) 4.9 percent come from redistribution - the reduction in ex-ante inequality; and (iii) -3.7 percent come from the reduction in insurance - there is more uncertainty about individual consumption and labor streams under the optimal policy. The optimal policy implies an overall increase of capital taxes and a reduction of labor taxes. The net effect on the distortions of agents' savings and labor supply decisions is positive.

The higher capital taxes decrease the proportion of the agents' income associated with the highly unequal asset income and lead to the redistributional gains. Finally, a lower labor income tax leads to a higher proportion of the agents' income to come from the uncertain labor income, thus the negative insurance effect.

We show that disregarding transitory welfare effects can be severely misleading. To make this point we compute the stationary fiscal policy that maximizes welfare in the final steady state, which leads to a 9.8 percent greater steady state welfare than the initial steady state. However, once transitory effects are considered, implementing this policy leads to a welfare *loss* of 6.4 percent relative to keeping the initial fiscal policy. Relative to the fiscal policy that maximizes welfare over transition it leads to a welfare loss of 11.3 percent.

In order to illustrate the role of market incompleteness in our findings, we develop the following build-up. We start from the representative agent economy and sequentially introduce heterogeneity in initial assets; different (but constant and certain) individual productivity levels; and, finally, uninsurable idiosyncratic productivity risk which adds up to the SIM model. At each intermediate step, building on the work of Werning (2007), we analytically characterize and then numerically compute the optimal fiscal policy over transition identifying the effect of adding each feature. In particular, we show that the planner will choose to keep capital taxes at the upper bound in the initial periods if there is asset heterogeneity, before reducing it to zero. Productivity heterogeneity rationalizes positive (and virtually constant) labor taxes. The key qualitative difference of the solution once uninsurable idiosyncratic productivity risk is introduced is that long-run capital income taxes are set to a positive level. Rationales for this result already exist in the literature and are discussed in the next section. To our knowledge, however, the level of the optimal long-run capital taxes in the SIM model had not been obtained before.

Finally, we present robustness exercises with respect to the welfare function and the calibration of the labor income process. Our benchmark results are for the utilitarian welfare function which implies a particular social choice with respect to the equality versus efficiency trade-off. We introduce a parameter in the welfare function that allows for different choices, in particular for the planner to completely ignore equality concerns. The long-run levels of capital and labor taxes are surprisingly resistant to changes in this parameter. What does change significantly, however, is how long the capital tax is maintained at the upper bound; the more the planner "cares" about inequality the more years it keeps those taxes at the upper bound. With respect to different calibrations of the labor income process, the magnitudes of the taxes are affected, but the qualitative features are maintained.

Related Literature

This paper is related to several strands of literature. First, it is related to the literature on the steady state optimal fiscal policy in the SIM model. In an influential paper, Conesa, Kitao, and Krueger (2009) solve for the tax system that maximizes steady state welfare in an overlapping generations SIM model. Their result includes an optimal long-run capital income tax of 36 percent. It is important to note that though this result is similar to ours the reasons behind it are different. They diagnose that their optimal capital tax level follows from the planner's inability to condition taxes on age, and the fact that a positive capital tax can mimic age-conditioned taxes in a welfare improving way (see Erosa and Gervais (2002)). This mechanism is not present in our analysis since we abstract from life-cycle issues.

Aiyagari (1995) and Chamley (2001) provide rationales for positive long-run capital taxes in environments similar to ours. Aiyagari (1995)'s logic depends on the planner choosing the path of government expenditure (appearing separably in the agent's utility function). The associated Euler equation implies the modified golden rule level of capital which can only be achieved by taxing savings; the planner does not have precautionary motives while the agents do. In our environment positive long-run capital taxes are preserved with exogenous governmental spending. Chamley (2001) shows, in a partial equilibrium version of the SIM model, that enough periods in the future every agent has the same probability of being in each of the possible individual (asset/productivity) states. It is, therefore, Pareto improving to transfer from the consumption-rich to the consumption-poor in the long-run. If the correlation of asset holdings with consumption is positive, this transfer can be achieved by a positive capital tax rebated via lump-sum.

In short, an agent's asset level in the long-run is a good proxy for how lucky she has been; hence, taxing it is a good way to provide insurance in the long-run. In recent work, Dávila, Hong, Krusell, and Ríos-Rull (2012) solve the problem of a planner that is restricted to satisfy agents' budget constraints, but is allowed to choose the savings of each agent. If the consumption-poor's share of labor income is higher than the average, increasing the aggregate capital stock relative to the undistorted equilibrium can improve welfare through its indirect effect on wages and interest rates. In our setup, the Ramsey planner taxes capital to affect after tax interest rates directly and achieves the same goal.

Another important work on fiscal policy in the SIM model is Aiyagari and McGrattan (1998), who search for the level of debt-to-output that maximizes steady state welfare. Interestingly, they find that the optimal level is very close to the pre recession level of around 67 percent. The fact that they abstract from the transitional dynamics makes the result even more remarkable: the government could chose its level of asset without having to finance it over time, it could, for instance choose to have enough assets to finance all its expenditures and yet it chooses to remain in debt. By holding debt, the government crowds out capital increasing interest rates and decreasing wages. This effectively provides insurance since the proportion of uncertain labor income out of total income is reduced. This benefit is what drives the choice of the government to hold debt. However, there is another effect associated with such a policy; it increases inequality (the proportion of the unequal asset income out of total income increases). This negative effect is not particularly important in Aiyagari and McGrattan (1998) because their calibration focuses on matching labor income processes which leads to an underestimation of wealth inequality. Winter and Roehrs (2014) replicate their experiment with a calibration that targets wealth inequality statistics and find the opposite result, i.e. the government chooses to hold high levels of assets. Our calibration procedure is closer to that of Winter and Roehrs (2014), which elucidates our result that the Ramsey planner chooses to accumulate assets over time.

Heathcote, Storesletten, and Violante (2014) and Gottardi, Kajii, and Nakajima (2014b) characterize the optimal fiscal policy in stylized versions of the SIM model. Their approaches lead to elegant and insightful closed-form solutions. The environment and Ramsey problem in Gottardi, Kajii, and Nakajima (2014b) is similar to ours except for the simplifications that yield tractability; i.e. exogenous labor supply, the absence of borrowing constraints, and i.i.d. shocks to human capital accumulation. Heathcote, Storesletten, and Violante (2014), on the other hand, focus on different, though related, questions. By abstracting from capital accumulation, they are able to retain tractability in a model with progressive taxation, partial insurance, endogenous government expenditure and skill choices (with imperfect substitution between skill types). This leads to several interesting dimensions that, in our paper, we abstract from. However, the simplifications in these models do not allow them to match some aspects of the data which we find to be important for the determination of the optimal tax system. In particular, the model in Heathcote, Storesletten, and Violante (2014) implies no wealth inequality (wealth is zero for all agents). Our calibration strategy allows us to match the distribution of wealth in the US.

We also contribute to the literature highlighting the importance of transition for policy prescriptions in incomplete markets models. Domeij and Heathcote (2004) use the SIM model to evaluate the implementation of a zero capital income tax policy taking into account the transitional welfare effects. They conclude that such a reform would be detrimental to welfare due to its transitory effect on inequality. Krueger and Ludwig (2013), Poschke, Kaymak, and Bakis (2012), and Winter and Roehrs (2014) also conduct experiments in this spirit. Acikgoz (2013) claims that the optimal long-run fiscal policy is independent of initial conditions and the transition towards it. He, then, studies the properties of fiscal policy in the long-run, but is silent about the optimal transition path which is the focus of this paper.

There is an extensive literature that studies the Ramsey problem in complete market economies; see Chari and Kehoe (1999) for a survey. The most well known result for the deterministic subset of these economies is due to Judd (1985) and Chamley (1986); capital taxes should converge to zero in the long run. Among others, Jones, Manuelli, and Rossi (1997) and Atkeson, Chari, and Kehoe (1999), show this result is robust to a relaxation of a number of assumptions. As was described above we make an effort to relate our main results to the results in this literature.

The New Dynamic Public Finance literature takes an alternative approach to answer our initial question. It focuses on the design of a mechanism that would allow the planner to extract information about the agents' unobservable productivities efficiently. It assumes tax instruments are unrestricted and in this sense it dominates the Ramsey approach in terms of generality, since the latter ignores the information extraction problem[5] and imposes ad-hoc linearity restrictions on the tax system. One of the main results steaming from this literature is the inverse Euler equation; see Golosov, Kocherlakota, and Tsyvinski (2003). Farhi and Werning (2012) show that starting from the allocations from the steady state of an undistorted SIM model and applying perturbations to implement the inverse Euler equation leads to small welfare gains, of the order of 0.2 percent. Moreover, it is difficult to solve the private information problem in dynamic economies with persistent shocks. Farhi and Werning (2013) and Troshkin, Tsyvinski, and Golosov (2010) have made advancements in this direction in partial equilibrium settings and find that restrictions to linear taxes lead to small welfare losses. Our view is that, even if only as a benchmark to more elaborate tax systems, it is useful to understand the properties of a simpler optimal linear tax system in a quantitative general equilibrium environment.

The rest of the paper is organized as follows. Section 1.2 illustrates the main mechanism behind our results in a two-period economy. Section 1.3 describes the infinite horizon model, sets up the Ramsey problem and discusses our solution technique. Section 1.4 describes the calibration. Section 1.5 presents the main results of the paper. Section 1.6 presents the build-up from the complete market economy results to our main results. Section 1.7 provides results for alternative welfare functions and calibrations and Section 1.8 concludes.

1.2 Mechanism: Two-Period Economy

In the SIM model, there are two dimensions of heterogeneity: productivity and wealth. Agents have different levels of productivity which follow an exogenous random process. In addition, markets are incomplete and only a risk-free asset exists. Therefore, the

[5] The Ramsey planner is also unable to observe productivity levels, it is not allowed to condition taxes on them.

idiosyncratic productivity risk cannot be diversified away. It follows that the history of shocks, affects the amount of wealth accumulated by each agent and there is an endogenously determined distribution of wealth.

In a two-period economy, it is possible to evaluate how each dimension of heterogeneity affects the optimal tax system. Since there is no previous history of shocks the initial wealth inequality can be set exogenously. In this section, we characterize, under some assumptions about preferences, the optimal tax system when the government has access to linear labor and capital income taxes, and (possibly negative) lump-sum transfers. First, we assume agents have the same level of wealth but face an idiosyncratic productivity shock - we call this the *uncertainty economy* . Then, we shut down uncertainty and introduce ex-ante wealth inequality - this is referred to as the *inequality economy* . Next we consider the case in which there is uncertainty *and* inequality and discuss the relationship with the infinite horizon problem.

1.2.1 Uncertainty economy

Consider an economy with a measure one of ex-ante identical agents who live for two periods. Suppose they have time-additive, von Neumann-Morgenstern utility functions. Denote the period utility function by $u(c, n)$ where c and n are the levels of consumption and labor supplied. Assume u satisfies the usual conditions and denote the discount factor by β. In the first period each agent is endowed with ω units of the consumption good which can be either consumed or invested into a risk-free asset, a, and supplies $\bar{n}$ units of labor inelastically.

In period 2, consumers receive income from the asset they saved in period 1 and from labor. Labor is supplied endogenously by each agent in period 2 and the individual labor productivity, e, is random and can take two values: e_L with probability π and $e_H > e_L$ with probability $1 - \pi$, with the normalization $\pi e_L + (1 - \pi) e_H = 1$. Due to the independence of shocks across consumers a law of large numbers operates so that in period 2 the fraction of agents with e_L is π and with e_H is $(1 - \pi)$. Letting n_i be the labor supply of an agent with productivity e_i, it follows that the aggregate labor supply is $N = \pi e_L n_L + (1 - \pi) e_H n_H$.

The planner needs to finance an expenditure of G in period 2. It has three instruments available: labor and capital income taxes, τ^n and τ^k, and lump-sum transfers T which can be positive or negative. Let w be the wage rate and r the interest rate. The total period 2 income of an agent with productivity e_i is, therefore, $(1-\tau^n)\, we_i n_i + \left(1 + \left(1-\tau^k\right) r\right) a + T$. In period 2, output is produced using capital, K, and labor and a constant-returns-to-scale neoclassical production function $f(K, N)$. We assume that $f(\cdot)$ is net of depreciation.

Definition 1 *A* ***tax distorted competitive equilibrium*** *is a vector* $(K, n_L, n_H, r, w;$ τ^n, τ^k, $T)$ *such that*

1. (K, n_L, n_H) *solves*

$$\max_{a, n_L, n_H} u(\omega - a, \bar{n}) + \beta E\left[u(c_i, n_i)\right]$$
$$s.t.\ c_i = (1-\tau^n)\, we_i n_i + \left(1 + \left(1-\tau^k\right) r\right) a + T;$$

2. $r = f_K(K, N)$, $w = f_N(K, N)$, *where* $N = \pi e_L n_L + (1-\pi)\, e_H n_H$*;*

3. *and,* $\tau^n wN + \tau^k rK = G + T$.

The Ramsey problem is to choose τ^n, τ^k, and T to maximize welfare. Since agents are ex-ante identical there is no ambiguity about which welfare function to use, it is the expected utility of the agents. If there is no risk, i.e. $e_L = e_H$, the agents are also ex-post identical and the usual representative agent result applies: since negative lump-sum transfers are available, it is optimal to obtain all revenue via this undistortive instrument and set $\tau^n = \tau^k = 0$.

In order to provide a sharp characterization of the optimal tax system we make the following assumption discussed below[6].

[6] In a similar two-period environment, Gottardi et al. (2014a) characterize the solution to Ramsey problem without Assumption A. However, they impose an alternative assumption about endogenous variables which are satisfied under Assumption A. Further, this assumption allows us to provide a sharper characterization of the optimal tax system (besides the signs of taxes we also characterize the levels).

Assumption 1 *No income effects on labor supply and constant Frisch elasticity, κ, i.e.*

$$u_{cn} - u_{cc}\frac{u_n}{u_c} = 0, \quad \text{and} \quad \frac{u_{cc}u_n}{n\left(u_{cc}u_{nn} - u_{cn}^2\right)} = \kappa.$$

We pursue a variational approach. Suppose $\left(K, n_L, n_H, r, w; \tau^n, \tau^k, T\right)$ is a tax distorted equilibrium[7]. We consider a small variation on the tax system $\left(d\tau^n, d\tau^k, dT\right)$, such that all the equilibrium conditions are satisfied. Then, evaluate the effect of such a variation on welfare, taking as given the optimal decision rules of the agents. Using this method we establish the following proposition (derivations and proofs are in Appendix A.1).

Proposition 1 *In the uncertainty economy, if u satisfies Assumption A, then, the optimal tax system is such that $\tau^k = 0$,*

$$\tau^n = \frac{(\nu - 1)\,\pi(1-\pi)\left(e_H n_H - e_L n_L\right)}{(\nu - 1)\,\pi(1-\pi)\left(e_H n_H - e_L n_L\right) + \kappa N\left(\pi\nu + (1-\pi)\right)} > 0, \tag{1.2.1}$$

where $\nu \equiv \frac{u_c(c_L, n_L)}{u_c(c_H, n_H)}$, and $T < 0$ balances the budget.

Notice that the planner could choose to finance G with T but chooses a positive distortive labor income tax instead. The revenue from labor taxation is rebated via lump-sum transfers and the proportion of the agents' income that comes from the uncertain labor income is reduced. Hence, this tax system effectively provides insurance to the agents. Why not provide full insurance by taxing away all the labor income? This is exactly what would happen if labor were supplied inelastically. In fact, notice that in this case $\kappa = 0$ and equation (1.2.1) implies $\tau^n = 1$. However, with an endogenous labor supply the planner has to balance two objectives: minimize distortions to agents' decisions and provide insurance. This balance is explicit in equation (1.2.1) seeing as a higher κ implies a lower τ^n. That is, the more responsive labor supply is to changes in labor taxes the more distortive these taxes are and the planner chooses a lower labor tax. In the limit, if $\kappa \to \infty$ it will be optimal to set $\tau^n = 0$.

With income effects on labor supply, distortions of the savings decision would spill

[7] Since the equilibrium does not exist for $\tau^n \geq 1$ or $\tau^k \geq (1+r)/r$, we impose the restrictions that $\tau^n < 1$ and $\tau^k < (1+r)/r$.

over to the labor supply decision and vice-versa. Thus, it could be optimal, for instance, to choose τ^k so as to mitigate the distortion imposed by a positive τ^n. This complex relationship complicates the analysis considerably. Assumption 1 unties this relationship and as a result it is optimal to set $\tau^k = 0$.

Next, suppose that $e_L = 1 - \epsilon^{unc}/\pi$ and $e_H = 1 + \epsilon^{unc}/(1-\pi)$, so that ϵ^{unc} is a mean preserving spread on the productivity levels. It is easy to see that if $\epsilon^{unc} = 0$ equation (1.2.1) implies that $\tau^n = 0$. The effect of an increase in ϵ^{unc} on the optimal τ^n is not as obvious since the right hand side of equation (1.2.1) contains endogenous variables. An application of the implicit function theorem, however, clarifies that as long as $\partial\nu/\partial\epsilon^{unc} > 0$ and $\partial\nu/\partial\tau^n < 0$, it follows that $\partial\tau^n/\partial\epsilon^{unc} > 0$, i.e. the optimal labor income tax is increasing in the level of risk in the economy. Under standard calibrations, the equilibrium ratio of marginal utilities, ν, is in fact increasing in the level of risk ($\partial\nu/\partial\epsilon^{unc} > 0$) and decreasing in the labor income tax ($\partial\nu/\partial\tau^n < 0$), as an example see section 1.2.3.

1.2.2 Inequality economy

Consider the environment described above only without uncertainty and with initial wealth inequality. That is, suppose the productivity levels do not vary between agents, i.e. $e_L = e_H = 1$, and that ω can take two values: ω_L for a proportion p of the agents and $\omega_H > \omega_L$ for the rest, with $\bar{\omega} \equiv p\omega_L + (1-p)\,\omega_H$.

Definition 2 *A* ***tax distorted competitive equilibrium*** *is* $(a_L,\ a_H,\ n_L,\ n_H,\ r,\ w;\ \tau^n,\ \tau^k,\ T)$ *such that*

1. *For* $i \in \{L, H\}$, (a_i, n_i) *solves*

$$\max_{a_i, n_i} u\left(\omega_i - a_i, \bar{n}\right) + \beta u\left(c_i, n_i\right), \quad s.t.\ c_i = \left(1-\tau^n\right) w n_i + \left(1 + \left(1 - \tau^k\right) r\right) a_i + T;$$

2. $r = f_K(K, N)$, $w = f_N(K, N)$, *where* $K = pa_L + (1-p)\,a_H$ *and* $N = pn_L + (1-p)\,n_H$;

3. *and,* $\tau^n wN + \tau^k rK = G + T$.

In this economy the concept of optimality is no longer unambiguous. Since agents are different ex-ante, a decision must be made with respect to the social welfare function. In what follows, by optimal we mean the one that maximizes $W \equiv pU_L + (1-p)\,U_H$; the utilitarian welfare function. The following proposition follows.

Proposition 2 *In the inequality economy, if u satisfies Assumption A and has CARA is GHH as in equation (1.4.1), then the optimal tax system is such that $\tau^n = 0$,*

$$\tau^k = \frac{\left(\frac{1+r}{r}\right)(\nu-1)\,p(1-p)\,(\omega_H-\omega_L)}{(\nu-1)\,p(1-p)\,(\omega_H-\omega_L)+\frac{\rho}{\psi}\,(p\nu+(1-p))} > 0, \tag{1.2.2}$$

where $\rho \equiv \frac{2+(1-\tau^k)r}{2+r}$ for CARA, $\rho \equiv \frac{1+\beta^{-\frac{1}{\sigma}}(1+(1-\tau^k)r)^{\frac{\sigma-1}{\sigma}}}{1+r+\beta^{\frac{1}{\sigma}}(1+(1-\tau^k)r)^{\frac{1}{\sigma}}}$ for GHH, and ψ is the level of absolute risk aversion[8]*. $T<0$ balances the budget.*

The planner chooses a positive capital income tax which distorts savings decisions but allows for redistribution between agents. The ex-ante wealth inequality is exogenously given. However, agents with different wealth levels in the first period will save different amounts and have different asset levels in the second period. This endogenously generated asset inequality is the one the tax system is able to affect. A positive capital tax rebated via lump-sum transfers directly reduces the proportion of the agents' income that will be dependent on unequal asset income achieving the desired redistribution which implies a reduction of consumption inequality.[9]

[8] The level of absolute risk aversion is endogenous is the GHH case.

[9] A related result was established in Dávila et al. (2005). They show that the competitive equilibrium allocation in the SIM model is constrained inefficient. That is, the incomplete market structure itself induces outcomes that could be improved upon if consumers merely acted differently; if they used the same set of markets but departed from purely self-interested optimization. The constrained inefficiency results from a pecuniary externality. The savings and labor supply decisions of the agents affects the wage and interest rates and, therefore, the uncertainty and inequality in the economy. These effects are not internalized by the agents and inefficiency follows. Notice that the planner's problem in their environment is significantly different from the Ramsey problem described here. There the planner affects allocations directly and prices indirectly whereas the Ramsey planner affects (after tax) prices directly and allocations indirectly. In the inequality economy, for instance, Dávila et al. (2005) show that there is underaccumulation of capital. A higher level of capital would decrease interest rates and increase wages, reducing inequality. A naive extrapolation of this logic would suggest that capital taxes should be negative so as to encourage savings. This logic, however, does not take into account the more relevant direct effect of the tax system on after tax prices. Proposition 2 shows that the opposite is true: capital taxes should be positive.

One of the key elements of equation (1.2.2) is the inverse of the coefficient of absolute risk aversion, $1/\psi$, which is proportional to the agents' intertemporal elasticity of substitution. This elasticity indicates the responsiveness of savings to changes in τ^k. Hence, the higher this elasticity is the lower is the optimal τ^k, since providing redistribution becomes more costly. The $\tau^n = 0$ result is again associated with Assumption 1.

Assuming that $\omega_L = 1 - \epsilon^{ine}/p$ and $\omega_H = 1 - \epsilon^{ine}/(1-p)$. The effect of an increase in ϵ^{ine} on the optimal τ^k can again be found by applying the implicit function theorem on equation (1.2.2). It follows that, if $\partial\nu/\partial\epsilon^{ine} > 0$ and $\partial\nu/\partial\tau^k < 0$, then $\partial\tau^k/\partial\epsilon^{ine} > 0$; the optimal capital income tax is increasing in the level of inequality in the economy. Under the assumptions of Proposition 2 it is possible to show that this will always be the case.

1.2.3 Uncertainty and inequality

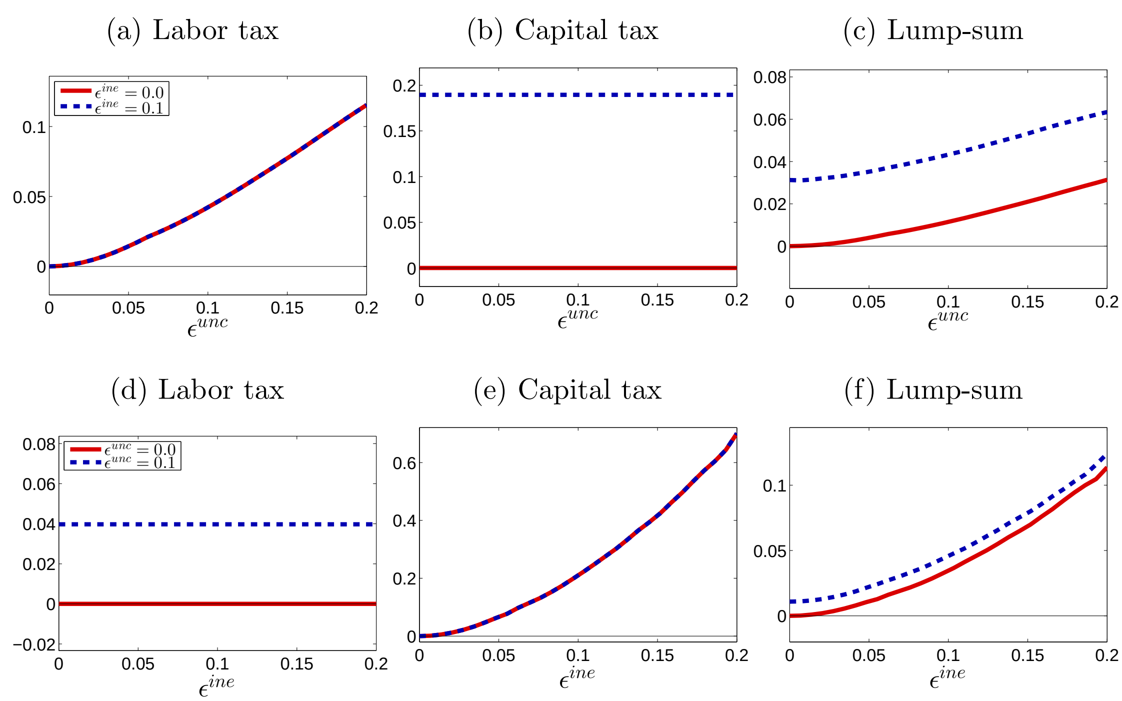
If both uncertainty and inequality are present, the optimal tax system has to balance three objectives: minimize distortions, provide insurance and redistribution. A reasonable conjecture is that under Assumption 1 the optimal tax system will be a convex combination of the ones in Propositions 1 and 2, that is, positive labor and capital income taxes with magnitudes associated with the levels of uncertainty and inequality in the economy. A more subtle extrapolation of the results above points to another interesting prediction associated with Assumption 1: the capital (labor) income taxes should be invariant with respect to the level of uncertainty (inequality). We corroborate these conjectures with a numerical example the results of which are in Figure 1.1[10].

The first row of Figure 1.1 shows the optimal tax system with the level of uncertainty (embodied by the parameter ϵ^{unc}) in the x-axis with two levels of inequality: $\epsilon^{ine} = 0$ (solid line) and $\epsilon^{ine} = 0.1$ (dashed line). The solid lines corroborate Proposition 1. The comparison between the dashed and the solid lines corroborates the conjectures made

[10]We use GHH preferences which satisfy Assumption 1. The most relevant interpretation of this two-period economy is that each period corresponds to half of the working life of a person. Accordingly, we set $\beta = 0.95^{20}$ and $\delta = 1 - 0.9^{20}$. Other parameters are set to satisfy the usual targets: $\sigma = 2$, $\kappa = 0.72$, $\chi = 6$, $\bar{n} = 0.3$, $\omega = 3.5$, $\pi = p = 0.5$, and $f(K,N) = K^\alpha N^{1-\alpha} - \delta K$ with $\alpha = 0.36$. G is set to 0, but any other feasible level would just shift the lump-sum transfers correspondingly.

above. The labor tax is increasing with the level of uncertainty and independent on the level of inequality whereas capital taxes increase with the level of inequality and are independent on level of risk. The second row of Figure 1.1 shows the results for the analogous experiment with ϵ^{ine} on the x-axis and $\epsilon^{unc} = 0$ (solid) and $\epsilon^{unc} = 0.1$ (dashed).

Figure 1.1: Optimal taxes in the presence of both uncertainty and inequality.



1.2.4 Relationship with infinite horizon problem

The two-period examples are useful to understand the key trade-offs faced by the Ramsey planner, since they allow for the exogenous setting of the levels of uncertainty (ex-post risk) and inequality (ex-ante risk). In the infinite horizon version of the SIM model, however, these concepts are inevitably intertwined. The characterization of the optimal tax system, therefore, becomes considerably more complex. Labor income taxes affect not only the level of uncertainty through the mechanism described above, but also the labor income inequality and the distribution of assets over time. An agent's asset level at a particular period depends not only on its initial value, but on the history of shocks this agent has experienced. Therefore, capital income taxation affects not only the ex-ante risk faced by the agent but also the ex-post. Nevertheless, these results are useful to understand some of the key features of the optimal fiscal policy in the infinite horizon model as will become clear in what follows.

1.3 The Infinite-Horizon Model

Time is discrete and infinite, indexed by t. There is a continuum of agents with standard preferences $E_0\left[\sum_t \beta^t u\left(c_t, n_t\right)\right]$ where c_t and n_t denote consumption and labor supplied in period t and u satisfies the usual conditions. Individual labor productivity, $e \in E$ where $E \equiv \{e_1, ..., e_L\}$, are i.i.d. across agents and follow a Markov process governed by Γ, a transition matrix[11]. Agents can only accumulate a risk-free asset, a. Let $A \equiv [\underline{a}, \infty)$ be the set of possible values for a and $S \equiv E \times A$. Individual agents are indexed by the a pair $(e, a) \in S$. Given a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$, labor income $\{\tau_t^n\}_{t=0}^{\infty}$, (positive) capital income $\{\tau_t^k\}_{t=0}^{\infty}$, and lump-sum transfers $\{T_t\}_{t=0}^{\infty}$, each household, at time t, chooses $c_t\left(a, e\right)$, $n_t\left(a, e\right)$, and $a_{t+1}\left(a, e\right)$ to solve

$$v_t(a, e) = \max u(c_t(a, e), n_t(a, e)) + \beta \sum_{e_{t+1} \in E} v_{t+1}(a_{t+1}(a, e), e_{t+1})\Gamma_{e, e_{t+1}}$$

subject to

$$\begin{aligned}(1 + \tau^c)c_t(a, e) + a_{t+1}(a, e) &= \left(1 - \tau_t^n\right) w_t e n_t(a, e) + (1 + (1 - I_{\{a \geq 0\}}\tau_t^k)r_t)a + T_t \\ a_{t+1}(a, e) &\geq \underline{a}.\end{aligned}$$

Note that value and policy functions are indexed by time, because policies $\{\tau_t^k, \tau_t^n, T_t\}_{t=0}^{\infty}$ and aggregate prices $\{r_t, w_t\}_{t=0}^{\infty}$ are time-varying. The consumption tax, τ^c, is a parameter[12]. Let $\{\lambda_t\}$ be a sequence of probability measures over the Borel sets $\mathcal{S}$ of S

[11] A law of large numbers operates so that the probability distribution over E at any date t is represented by a vector $p_t \in \mathbb{R}^L$ such that given an initial distribution p_0, $p_t = p_0\Gamma^t$. In our exercise we make sure that Γ is such that there exists a unique $p^* = \lim_{t\to\infty} p_t$. We normalize $\sum_i p_i^* e_i = 1$.

[12] We could potentially allow consumption taxes to also be chosen by the Ramsey planner and it is not without loss of generality that we impose this restriction. There are two reasons for this choice. The first is practical, we are already using the limit of the computational power available to us, and allowing

with λ_0 given. Since the path for taxes is known, there will be a deterministic path for prices and for $\{\lambda_t\}_{t=0}^{\infty}$. Hence, we do not need to keep track of the distribution as an additional state; time is a sufficient statistic.

Competitive firms own a constant-returns-to-scale technology $f(\cdot)$ that uses capital, K_t, and efficient units of labor, N_t, to produce output each period ($f(\cdot)$ denotes output net of depreciation - δ denotes the capital depreciation rate). A representative firm exists that solves the usual static problem. The government needs to finance an exogenous constant stream of expenditure, G, and lump-sum transfers with taxes on consumption, labor income, and (positive) capital income. It can also issue debt $\{B_{t+1}\}$ and, thus, has the following intertemporal budget constraint

$$G + r_t B_t = B_{t+1} - B_t + \tau^c C_t + \tau_t^n w_t N_t + \tau_t^k r_t \hat{A}_t - T_t, \tag{1.3.1}$$

where C_t is aggregate consumption and $\hat{A}_t$ is the tax base for the capital income tax.

Definition 3 *Given an initial distribution λ_0 and a policy $\pi \equiv \{\tau_t^k, \tau_t^n, T_t\}_{t=0}^{\infty}$, a **competitive equilibrium** is a sequence of value functions $\{v_t\}_{t=0}^{\infty}$, an allocation $X \equiv \{c_t, n_t, a_{t+1}, K_t, N_t, B_t\}_{t=0}^{\infty}$, a price system $P \equiv \{r_t, w_t\}_{t=0}^{\infty}$, and a sequence of distributions $\{\lambda_t\}_{t=0}^{\infty}$, such that for all t:*

1. *Given P and π, $c_t(a,e)$, $n_t(a,e)$, and $a_{t+1}(a,e)$ solve the household's problem and $v_t(a,e)$ is the respective value function;*
2. *Factor prices are set competitively,*

$$r_t = f_K(K_t, N_t), \quad w_t = f_N(K_t, N_t);$$

3. *The probability measure λ_t satisfies*

$$\lambda_{t+1} = \int_S Q_t\left((a,e), \mathcal{S}\right) d\lambda_t, \quad \forall \mathcal{S} \in S$$

for one more choice variable would increase it substantially. Second, for the US in particular capital and labor income taxes are chosen by the Federal Government while consumption taxes are chosen by the states, so this Ramsey problem can be understood as the one relevant for a Federal Government that takes consumption taxes as given. We need to add τ^c as a parameter for calibration purposes.

where Q_t is the transition probability measure;

4. *The government budget constraint, (1.3.1), holds and debt is bounded;*

5. *Markets clear,*

$$C_t + G_t + K_{t+1} - K_t = f\left(K_t, N_t\right), \quad K_t + B_t = \int_{A\times E} a_t(a,e)d\lambda_t.$$

1.3.1 The Ramsey Problem

We now turn to the problem of choosing the optimal tax policy in the economy described above. We assume that, in period 0, the government announces a commits to a sequence of future taxes $\{\tau_t^k, \tau_t^n, T_t\}_{t=1}^{\infty}$, taking period 0 taxes as given. We need the following definitions:

Definition 4 *Given λ_0, for every policy π* ***equilibrium allocation rules*** *$X\left(\pi\right)$ and* ***equilibrium price rules*** *$P\left(\pi\right)$ are such that π, $X\left(\pi\right)$, $P\left(\pi\right)$ and corresponding $\{v_t\}_{t=0}^{\infty}$ and $\{\lambda_t\}_{t=0}^{\infty}$ constitute a competitive equilibrium.*

Definition 5 *Given λ_0, τ_0^k, τ_0^n, T_0 and a welfare function $W\left(\pi\right)$, the* ***Ramsey problem*** *is to $\max_\pi W\left(\pi\right)$ such that $X\left(\pi\right)$ and $P\left(\pi\right)$ are equilibrium allocation and price rules.*

In our benchmark experiments we assume that the Ramsey planner maximizes the utilitarian welfare function: the ex-ante expected lifetime utility of a newborn agent who has its initial state, (a, e), chosen at random from the initial stationary distribution λ_0. The planner's objective is thus given by

$$W\left(\pi\right) = \int_S E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\left(a, e|\pi\right), n_t\left(a, e|\pi\right)\right) d\lambda_0.$$

In Section 1.7 we provide results for alternative welfare functions.

1.3.2 Solution method

We solve this problem numerically. Given an initial stationary equilibrium, for any policy π we can compute the transition to a new stationary equilibrium consistent with the policy[13] and calculate welfare $W(\pi)$. We then search for the policy π that maximizes $W(\pi)$. This is, however, a daunting task since it involves searching in the space of infinite sequences. In order to make it computationally feasible we impose the following ad-hoc constraints: that each path $\{\tau_t^k, \tau_t^n, T_t\}_{t=1}^{\infty}$ be smooth over time and become constant after a finite amount of periods. We denote the set of policies that satisfy these properties by Π_R. These conditions are restrictive, but they allow the problem to be solved and are flexible enough to characterize some of the key features of the optimal paths of taxes.

The statement about the ad-hoc constraints must be qualified. It is well know from the existing solutions to the Ramsey problem in complete markets economies that capital taxes should be front-loaded. We obtain similar results in Section 1.6. Hence, in defining the set Π_R we take this under consideration. That is, we allow capital taxes to hit the imposed upper bound of 100 percent for the first t^* periods, where a model period is equivalent to one calendar year. Importantly, t^* is endogenously chosen and is allowed to be zero, so the fact that the solution displays a capital tax at the upper bound for a positive amount of periods is not an assumption but a result. Other than this, we assume that the paths for $\left\{\tau_t^k\right\}_{t=t^*+1}^{\infty}$ and $\{\tau_t^n, T_t\}_{t=1}^{\infty}$ follow splines with nodes set at exogenously selected periods. The placement of the nodes is arbitrary, we started with a small number of them and sequentially added more until the solution converged. In the main experiment the planner was allowed to choose 17 variables in total: t^*, $\tau_{t^*+1}^k$, τ_{45}^k, τ_{60}^k, τ_{100}^k, τ_1^n, τ_{15}^n, $\tau_{t^*+1}^n$, τ_{45}^k, τ_{60}^k, τ_{100}^k, T_1, T_{15}, T_{t^*+1}, T_{45}, T_{60}, and T_{100}. In the intermediate periods the paths follow a spline function and after the final period they become constant at the last level. The choice of the periods 1, 15, 45, 60, and 100, were a result of the fact that for experiments with less nodes, the optimal t^* was always close to 30, hence we placed the nodes at the same distance from each other except for the last ones which are supposed to capture the long run levels[14].

Solving the problem described above is a particularly hard computational task. Effectively we are maximizing $W(\pi)$ on the domain $\pi \in \Pi_R$, where each element of Π_R can be defined by a vector with a finite number of elements (the nodes described above).

[13] As long as the taxes become constant at some point.

[14] If the solver chooses t^* close to one of these predetermined nodes the algorithm replaces that node for $t = 30$. For instance, if $t^* = 43$ the periods became 1, 15, 30, $t^* + 1$, 60, and 100.

We know very little about its properties; it is a multivariate function with potentially many kinks, irregularities and multiple local optima[15]. Thus, we need a powerful and thorough procedure to make sure we find the global optimum. We use a global optimization algorithm that randomly draws a very large number of policies in Π_R and computes the transition between the exogenously given initial stationary equilibrium and a final stationary equilibrium that depends on the policy. Then, we compute welfare $W(\pi)$ for each of those policies and select those that yield the highest levels of welfare. These selected policies are then clustered, similar policies placed in the same cluster. For each cluster we run an efficient derivative free local optimizer. The whole procedure is repeated depending on how many local optima have been found and a Bayesian stopping rule is used to figure out if enough global procedures have been run. A more detailed description of the algorithm can be found in Appendix A.4[16].

1.4 Calibration

We calibrate the initial stationary equilibrium of the model economy to replicate key properties of the US economy relevant for the shape of the optimal fiscal policy. Table 1.1 summarizes our parameters choices together with the targets we use to discipline their values and their model counterparts. We use data from the NIPA tables for the period between 1995 and 2007[17] and from the 2007 Survey of Consumer Finances (SCF).

[15] See Guvenen (2011) for a discussion of how to deal with such problems.

[16] The algorithm was parallelized for multiple cores. For each global iteration, we drew 131,072 policies and computed the transition and welfare for each of them. The number of transitions run for each cluster is endogenously determined by the local solver, on average it amounted to around 150 transitions to find each local maximum. A total of 8 global iterations were needed. We performed our analysis on the Itasca cluster at the Minnesota Supercomputing Institute using 1024 cores.

[17] We choose this time period to be consistent with the one used to pin down fiscal policy parameters which we take from Trabandt and Uhlig (2011).

Table 1.1: Benchmark Model Economy: Target Statistics and Parameters

Statistic	Target	Model	Parameter	Value
Preferences and Technology				
Intertemporal elasticity of substitution	0.50	0.50	σ	2.00*
Frisch elasticity	0.72	0.72	ν	0.72*
Average hours worked	0.30	0.30	χ	4.12
Capital to output	2.72	2.71	β	0.97
Capital income share	0.38	0.38	α	0.38*
Investment to output	0.27	0.27	δ	0.10
Borrowing Constraint				
Households with negative wealth (%)	18.6	19.1	$\underline{a}$	-0.04
Fiscal Policy				
Capital income tax (%)	36.0	36.0	τ_k	0.36*
Labor income tax (%)	28.0	28.0	τ_n	0.28*
Consumption tax (%)	5.0	5.0	τ_c	0.05*
Transfer to output (%)	8.0	8.0	T	0.08
debt-to-output (%)	63.0	63.0	G	0.15
Labor Productivity Process				
Wealth Gini index	0.82	0.81	e_1/e_2	0.62
Percentage of wealth in 1st quintile	-0.2	-0.2	e_3/e_2	3.89
Percentage of wealth in 4th quintile	11.2	10.2	Γ_{11}	0.94
Percentage of wealth in 5th quintile	83.4	83.4	Γ_{12}	0.05
Percentage of wealth in top 5%	60.3	60.8	Γ_{21}	0.01
Correlation btw wealth and labor income	0.29	0.29	Γ_{22}	0.92
Autocorrelation of labor income	0.90	0.90	Γ_{31}	0.01
Standard Deviation of labor income	0.20	0.20	Γ_{32}	0.04

Notes: Parameter values marked with (*) were set exogenously, all the others were endogenously and jointly determined.

1.4.1 Preferences and technology

We assume GHH preferences[18] with period utility given by

$$u(c,n) = \frac{1}{1-\sigma}\left(c - \chi \frac{n^{1+\frac{1}{\kappa}}}{1+\frac{1}{\kappa}}\right)^{1-\sigma}, \tag{1.4.1}$$

where σ is the coefficient of relative risk aversion, κ is the Frisch elasticity of labor supply and χ is the weight on the disutility of labor. These preferences exhibit no wealth effects on labor supply, which is consistent with microeconometric evidence showing these effects are in fact small[19]. Further, they imply that aggregate labor supply is independent of the distribution of wealth which is convenient for computing out of steady state allocations in our main experiment. We set the intertemporal elasticity of substitution to 0.5; the number frequently used in the literature (e.g. Dávila et al. (2012) and Conesa et al. (2009)). For the Frisch elasticity, κ, we rely on estimates from Heathcote et al. (2010) and use 0.72. This value is intended to capture both the intensive and the extensive margins of labor supply adjustment together with the typical existence of two earners within a household. It is also close to 0.82, the number reported by Chetty et al. (2011) in their meta-analysis of estimates for the Frisch elasticity using micro data. The value for χ is chosen so that average hours worked equals 0.3 of total available time endowment[20]. To pin down the discount factor, β, we target a capital to output ratio of 2.72, and the depreciation rate, δ, is set to match an investment to output ratio of 27 percent[21].

The aggregate technology is given by a Cobb-Douglas production function $Y = AK^{\alpha}N^{1-\alpha} + (1-\delta)K$ with capital share equal to α. The total factor productivity A is set to normalize output per capita, Y, to 1. The capital share parameter, α, is set to its empirical counterpart of 0.38.

[18] See Greenwood et al. (1988).

[19] See Holtz-Eakin et al. (1993), Imbens et al. (2001) and Chetty et al. (2012) for details.

[20] It is understood that in any general equilibrium model all parameters affect all equilibrium objects. For the presentation purposes, we associate a parameter with the variable it affects quantitatively most.

[21] Capital is defined as nonresidential and residential private fixed assets and purchases of consumer durables. Investment is defined in a consistent way.

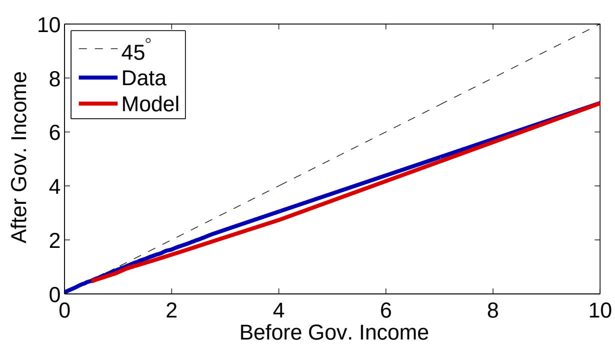
1.4.2 Borrowing Constraints

We discipline the borrowing constraint $\underline{a}$ using the percentage of households in debt (negative net worth). We target 18.6 percent following the findings of Wolff (2011) based on the 2007 SCF.

1.4.3 Fiscal policy

In order to set the tax rates in the initial stationary equilibrium we use the effective average tax rates computed by Trabandt and Uhlig (2011) from 1995 to 2007 and average them. The lump-sum transfers to output ratio is set to 8 percent and we discipline the government expenditure by imposing a debt to output ratio of 63 percent also following Trabandt and Uhlig (2011). The latter is close to the numbers used in the literature (e.g. Aiyagari and McGrattan (1998), Domeij and Heathcote (2004) or Winter and Roehrs (2014)). The calibrated value implies a government expenditure to output ratio of 15 percent, the data counterpart for the relevant period is approximately 18 percent. Further, we also approximate well the actual income tax schedule as can be seen in Figure 1.2.

Figure 1.2: Income tax schedule



Notes: The data was generously supplied by Heathcote et al. (2014) who used PSID and the TAXSIM program to compute it. The axis units are income relative to the mean.

1.4.4 Labor income process

The individual labor productivity levels e and transition probabilities in matrix Γ are chosen to match the US wealth distribution, statistical properties of the estimated labor income process and the correlation between wealth and labor income. There are three levels of labor productivity in our model. Since we normalize the average productivity to one we are left with two degrees of freedom. The transition matrix is 3×3. The fact that it is a probability matrix implies its rows add up to one, therefore we are left with an additional six degrees of freedom. Thus, we end up with eight parameters to choose

It is common to use the Tauchen method when calibrating the Markov process for productivities. This method imposes symmetry of the Markov matrix which further reduces the number of free parameters. Following Castañeda et al. (2003) we do not impose symmetry which allows us to target at the same time statistics from the labor income process and the individual wealth distribution.

To match the wealth distribution we target shares of wealth owned by the first, fourth and fifth quintile, the share of wealth owned by individuals in the top 5 percent and the Gini index. The targets are taken from the 2007 Survey of Consumer Finances[22]. We also target properties of individual labor income estimated as the AR(1) process, namely its autocorrelation and its standard deviation[23]. According to Domeij and Heathcote (2004), existing studies estimate the first order autocorrelation of (log) labor income to lie between 0.88 and 0.96 and the standard deviation (of the innovation term in the continuous representation) of 0.12 and 0.25. We calibrate the productivity process so that the Markov matrix and vector e imply an autocorrelation of (log) labor income of 0.9 and a standard deviation of 0.2[24] (in Section 1.7 we provide robustness results with respect to these choices). Finally, we target the correlation between wealth and labor income which is 0.29 in the 2007 SCF data. This way we discipline to some extent the labor income distribution using the wealth distribution that we match accurately. The resulting productivity vector, transition matrix and stationary distribution

[22] For a general overview of this data see Díaz-Giménez et al. (2011).

[23] Including transitory shocks would allow a better match to the labor income process. However, these types of shocks can, for the most part, be privately insured against (see Guvenen and Smith (2013)) so we chose to abstract from them to keep the model parsimonious.

[24] We follow Nakajima (2012) in choosing these targets. The targets are associated with labor income, wen, which includes the endogenous variables w and n. Therefore, to calibrate the parameters governing the individual productivity process, the model must be solved repeatedly until the targets are satisfied.

of productivities, λ_e^*, are

$$e = \begin{bmatrix} 0.79 \\ 1.27 \\ 4.94 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} .956 & .043 & .001 \\ .071 & .929 & .000 \\ .012 & .051 & .937 \end{bmatrix}, \quad \text{and} \quad \lambda_e^* = \begin{bmatrix} .616 \\ .377 \\ .007 \end{bmatrix}.$$

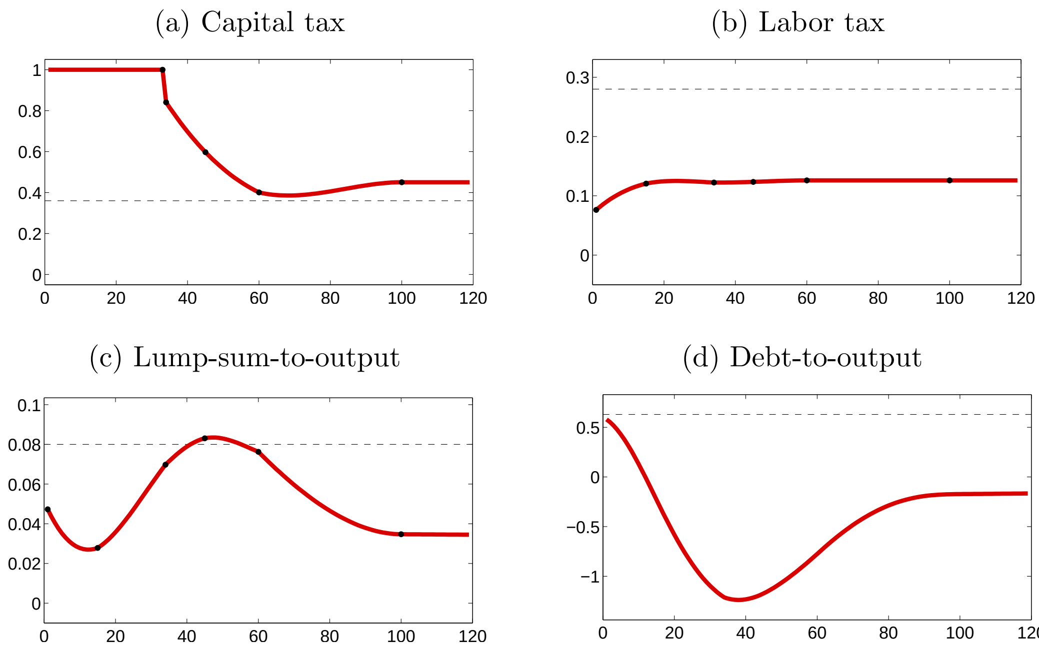
1.4.5 Model performance

Table A.1 presents statistics about the wealth and labor income distributions. We target five of the wealth distribution statistics, so it is not surprising that we match that distribution quite well. Table A.2 presents another crucial dimension along which our model is consistent with the data: income sources over the quintiles of wealth. The composition of income, specially of the consumption-poor agents, plays an important role in the determination of the optimal fiscal policy. The fraction of uncertain labor income determines the strength of the insurance motive and the fraction of the unequal asset income affects the redistributive motive. Our calibration delivers, without targeting, a good approximation of the income composition. Finally, we also match the consumption Gini which remained fairly constant around 0.27 in the period from 1995 to 2007 (see Krueger and Perri (2006)).

1.5 Main Results

The optimal paths for the fiscal policy instruments are portrayed in Figure 1.3. Capital taxes should be front-loaded hitting the upper bound for 33 initial periods then decrease to 45 percent in the long-run. Labor income taxes are substantially reduced to less than half of its initial level, from 28 percent to about 13 percent in the long-run. The ratio of lump-sum transfers to output decreases initially to about 3 percent, then increases back to its initial level of 8 percent before it starts converging to its final level of 3.5 percent. The government accumulates assets in the initial periods of high capital taxes reaching a level of debt-to-output of about -125 percent, which then converges to a final level of -17 percent. Relative to keeping fiscal instruments at their initial levels, this leads to a welfare gain equivalent to a permanent 4.9 percent increase in consumption.

Figure 1.3: Optimal Fiscal Policy: Benchmark



Notes: Dashed line: initial stationary equilibrium; Solid line: optimal transition; The black dots are the choice variables: the spline nodes and t^*, the point at which the capital tax leaves the upper bound.

1.5.1 Aggregates

The aggregates associated with the implementation of the optimal policy are shown in Figure A.1. The capital level initially decreases by about 8 percent in the first 13 years, but then increases towards a final level 20 percent higher than the initial steady state. The increase might be surprising at a first glance given the higher capital taxes. First notice that, even if capital income taxes were set to 100 percent forever, there would still be precautionary incentives for the agents with relatively high productivity to save: if they receive a negative shock they can then consume their savings. The decrease in government debt also contributes substantially to this increase - an effect we explain further below in Section 1.5.4. Most importantly though, the level of aggregate labor increases by about 15 percent immediately after the policy change following the reduction in labor taxes, increasing the marginal productivity of capital.

The higher levels of capital and labor lead to higher levels of output and consumption, which increases by 15 and 20 percent respectively over the transition. The concomitant increase in average consumption and labor has ambiguous effects on the welfare of the average agent. Hence, we also plot in Figure A.1f what we call the average consumption-labor composite, defined below in equation (1.5.1), which is the more relevant measure for welfare. On impact the labor-consumption composite increases by 13 percent as the higher consumption levels (due to the initial reduction in savings) more than compensate for the higher supply of labor. It then decreases for some periods following the reduction in output and the increasing savings. In the long-run it returns to a level about 13 percent higher than the one in the initial steady state.

1.5.2 Distributional Effects

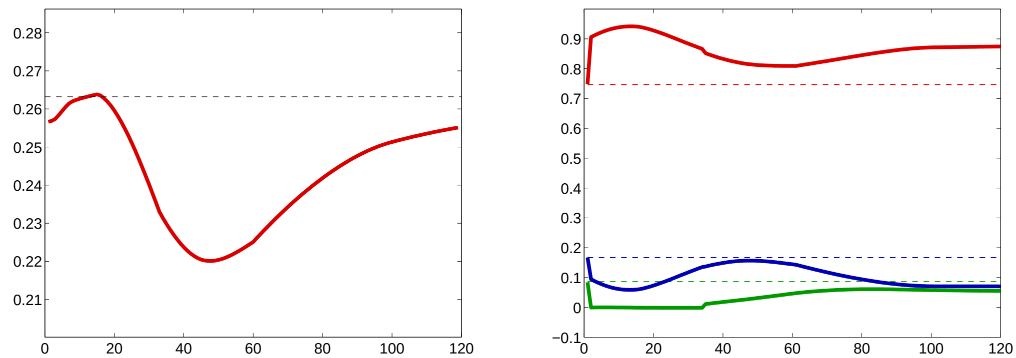
Movements in the levels do not provide a full picture of what results from the implementation of the optimal fiscal policy. It is also important to understand its effects on inequality and on the risk faced by the agents. Figure 1.4a plots the evolution of the Gini index for consumption[25]. Notice that, though it takes some time for the reduction to start, the consumption Gini is significantly reduced over the transition reaching a low about 16 percent lower than the initial level. As will become clear below, this reduction in inequality is behind most of the welfare gains associated with the optimal policy. Not surprisingly, such a change would be supported by most agents in the economy with the exception of the highly productive and, therefore, wealthier ones - see Table 1.2.

Figure 1.4b displays the evolution of the shares of labor, capital and transfer income out of total income. Importantly, notice that the share of labor income is significantly increased under the optimal policy. Since all the risk faced by agents in the SIM model is associated with their labor income, it turns out that they face more risk after the policy is implemented. This has an obvious negative effect on welfare which is, however, outweighed by the gains associated with the higher levels of consumption and the reduction in inequality it provides. The next sections will clarify some of these issues.

[25]Since labor supply is proportional to productivity levels, the inequality of hours is unaffected by the policy, it is in fact determined exogenously. Hence, here we can focus on consumption inequality.

Figure 1.4: Inequality measures

(a) Consumption Gini (b) Income shares



Notes (a) and (b): Dashed lines: initial stationary equilibrium; Solid lines: optimal transition. Notes (b): Red lines: labor income share; Blue lines: transfer income share; Green lines: asset income share

Table 1.2: Proportion in favor of reform

$e = L$	$e = M$	$e = H$	All
99.6	98.3	3.7	99.5

1.5.3 Welfare decomposition

Here we present a result that will be particularly helpful for understanding the properties of the optimal fiscal policy. First, let x_t be the individual consumption-labor composite (the term inside the utility function 1.4.1), that is

$$x_t \equiv c_t - \chi \frac{n_t^{1+\frac{1}{\kappa}}}{1+\frac{1}{\kappa}}, \tag{1.5.1}$$

and X_t denote its aggregate. The utilitarian welfare function can increase for three reasons. First, it will increase if the utility of the average agent, $U(\{X_t\})$, increases; we call this the *level effect*. Reductions in distortive taxes will achieve this goal by allocating resources more efficiently[26]. Second, since agents are risk averse, it increases

[26] This is the only relevant effect in a representative agent economy.

if the uncertainty about individual paths $\{x_t\}_{t=0}^{\infty}$ is reduced; we call this the *insurance effect*. By redistributing from the (ex-post) lucky to the (ex-post) unlucky, a tax reform can reduce the uncertainty faced by the agents. Finally, it will increase if the inequality across the certainty equivalents of the individual paths $\{x_t\}_{t=0}^{\infty}$, for agents with different initial (asset/productivity) states, is reduced; we call this the *redistribution effect*. By redistributing from the rich (ex-ante lucky) to the poor (ex-ante unlucky), the tax reform reduces the inequality between agents. In Appendix A.3 we give precise definitions for each of these effects and show how it is possible to measure them. Then, letting Δ be the average welfare gain, Δ_L the gains associated with the level effect, Δ_I with the insurance effect, and Δ_R with the redistribution effect, we prove the following proposition.

Proposition 3 *If preferences are GHH as in (1.4.1), then*

$$1 + \Delta = (1 + \Delta_L)(1 + \Delta_I)(1 + \Delta_R).$$

Hence, it is possible to decompose the average welfare gains into the components described above[27]. The results for this decomposition for our main results are in Table 1.3. Most of the welfare gains implied by the implementation of the optimal fiscal policy come from the reduction in ex-ante inequality (redistribution effect). The also substantial welfare gains associated with the reduction in distortions (level effect) is almost exactly offset by welfare losses due to the increase in uncertainty (insurance effect).

Table 1.3: Welfare decomposition

Average welfare gain Δ	Level effect Δ_L	Insurance effect Δ_I	Redistribution effect Δ_R
4.9	3.7	-3.7	4.9

[27]The welfare gains described above are in terms of consumption-labor composite units. The decomposition does not hold exactly in terms of consumption units. To keep our results comparable with others, we report the average welfare gains in terms of consumption units and normalize the numbers for Δ_L, Δ_I, and Δ_R accordingly.

1.5.4 Fixed instruments

In order to understand the role played by each instrument in the optimal fiscal policy, we ran experiments in which we hold each of them fixed and optimize only with respect to the others. Figures A.2, A.3, A.4, and A.5 display the solutions and Table 1.4 the welfare decomposition for each of these experiments.

Capital taxes

It is clear from the welfare decomposition in Table 1.4 that the path of capital taxes plays a crucial role in the redistributional gains associated with the unrestricted optimal policy. Restricting capital taxes to their initial level brings the redistribution effect from 4.9 percent to -0.2 percent. In line with the result in Proposition 2, the increase in capital taxes especially in the initial years leads to a strong redistribution effect as the proportion of unequal asset income is reduced (actually brought to zero in the first 33 years). Relative to the optimal policy, the restriction on capital taxes also leads to higher labor taxes (which explains the better insurance effect) and a lower accumulation of assets by the government.

Labor taxes

Fixing labor taxes at their initial level is particularly detrimental to the level effect. In the optimal policy labor taxes are reduced substantially and the labor supply distortions reduced accordingly. The redistributional gains are virtually unaffected whereas the insurance effect is improved, which is consistent with the result in Proposition 1 since the restriction implies higher labor taxes. The fact that the insurance effect is still negative might be surprising though. What is behind this effect is the role played by the accumulation of assets by the government which we explain bellow.

Lump-sum transfers

Restricting lump-sum transfers to its initial level doesn't affect the results as much as the other restrictions; the average welfare gains are reduced from 4.9 percent to 4.4 percent. Most of the losses come from the reduction in the level effect. The restriction leads to a higher overall level of transfers and, therefore, higher labor taxes relative to the unrestricted optimal policy whereas capital taxes are virtually unaffected. This leads to an overall higher level of distortions which explains the lower level effect.

Table 1.4: Welfare decomposition: Fixed instruments

	Δ	Δ_L	Δ_I	Δ_R
Fixed capital taxes	1.0	3.7	-2.5	-0.2
Fixed labor taxes	3.3	0.0	-1.6	4.8
Fixed lump-sum	4.4	1.8	-2.5	5.1
Fixed debt	4.0	3.8	-3.2	3.2
Benchmark	4.9	3.7	-3.7	4.9

Government debt

In the absence of borrowing constraints an increase in government debt is innocuous, in response agents simply adjust their savings one-to-one and the Ricardian equivalence holds. In the SIM model, however, agents face borrowing constraints (which are binding for some of them). The Ricardian equivalence breaks down and in response to an increase in government debt aggregate savings increase by less than one-to-one. Since the asset market must clear (i.e. $A_t = K_t + B_t$), it follows that capital must decrease as a result. Hence, increases in government debt crowd out capital while decreases crowd in capital[28].

In order to understand why the government accumulates assets in the optimal policy it is important to look at its effect on equilibrium prices[29]. A lower amount of government debt leads to a higher level of capital which reduces interest rates and increases wages. Hence, besides the positive level effect associated with the higher levels

[28] See Aiyagari and McGrattan (1998) and Winter and Roehrs (2014) for an extensive discussion os this issue.

[29] The fact that the government accumulates assets does not imply that it becomes the owner of part of the capital stock. Agents own the capital, but on average owe the government (in the form of IOU contracts) more than the value of their capital holdings.

of capital such a policy also affects the insurance and redistribution effects. It effectively reduces the proportion of the agents' income associated with the unequal asset income and increases the proportion associated with uncertain labor income. The result is a positive redistribution effect and a negative insurance effect. Thus, when government debt-to-output is held fixed the redistributional gains are reduced from 4.9 percent to 3.2 percent while the insurance loss is reduced from -3.7 percent to -3.2 percent. This also clarifies why the planner chooses to accumulate assets when the instrument is not restricted: the welfare gains associated with the resulting redistribution outweigh the losses from the increased uncertainty.

1.5.5 Transitory effects

In this section we first compute the optimal fiscal policy ignoring transitory welfare effects. A comparison with our benchmark results allows us to measure the importance of accounting for these transitory effects. If the difference was small this would be a validation of experiments of this kind performed in the literature. It turns out, however, that the results are remarkably different. A better option, is to solve for the optimal policy with constant instruments accounting for transitory welfare effects. The welfare loss associated with holding the instruments constant, however, is still significant. The results are summarized in Tables 1.5 and 1.6.

Table 1.5: Final Stationary Equilibrium: Transitory Effects

	τ^h	τ^k	T/Y	B/Y	K	H	r	w
Initial equilibrium	28.0	36.0	8.0	63.0	1.65	0.33	4.1	1.14
Stat. equil.	18.0	-	3.7	-326.1	4.01	0.44	0.0	1.45
Stat. equil. fixed debt	4.7	-5.2	-5.4	63.0	2.84	0.43	1.9	1.26
Constant policy	7.6	73.7	3.5	49.8	1.31	0.36	7.1	1.01
Benchmark	12.6	45.1	3.5	-16.9	2.00	0.38	3.7	1.16

Notes: The values of τ^h, τ^k, T/Y, B/Y, and r are in percentage points.

Table 1.6: Welfare Decomposition: Transitory Effects

	Δ	Δ_L	Δ_I	Δ_R
Stat. equil.	24.7	19.6	-4.6	9.3
Stat. equil. fixed debt	9.8	18.8	-5.2	-2.6
Constant policy	3.3	3.4	-3.0	3.0
Benchmark	4.9	3.7	-3.7	4.9

Stationary equilibrium policy

Here the the planner chooses stationary levels of all four fiscal policy instruments to maximize welfare in the final steady state. In particular, the planner can choose any level of government debt without incurring in the transitional costs associated with it. It chooses a debt-to-output ratio of -326 percent. At this level the amount of capital that is crowded in is close to the golden rule level, that is, such that interest rates (net of depreciation) equal to zero. Thus, taxing capital income in this scenario has no relevant effect and we actually find multiple solutions with different levels of capital taxes which is why we do not display that number in Table 1.5. The average welfare gains associated with this policy are of 24.7 percent, that is, agents would be willing to pay this percentage of their consumption in order to be born in the stationary equilibrium of an economy that has this policy instead of the initial stationary equilibrium. However, these welfare gains ignore the transitory effects, it is as if the economy jumped immediately to a new steady state in with the government has a large amount of assets without incurring in the costs associated with accumulating it.

A more reasonable experiment, which is closer to the one studied by Conesa et al. (2009), is to restrict the level of debt-to-output ratio to remain at its initial level. When this is the case, the planner reduces labor taxes and capital taxes substantially obtaining most of the necessary revenue via lump-sum taxes. This has detrimental insurance and redistribution effects, but the associated level effect more than makes up for it. The policy leads to a welfare gain of 9.8 percent relative to the initial steady state when transitory effects are ignored. However, once transitory effects are considered, implementing this policy leads to a welfare *loss* of 6.4 percent. Hence, ignoring transitory effects can be severely misleading. Importantly, the transitory distributional effects of the policy and the costs associated with the accumulation of capital (or assets by the government) are ignored.

Transition with constant policy

Here we consider the problem of finding the *constant* optimal fiscal policy that maximizes the same welfare function we use in our benchmark experiment, in which transitory effects are accounted for. We present a comparison with the benchmark results in Figures A.7 and A.6. The level of capital taxes is close to average between the upper bound of 100 percent and the final capital tax in the benchmark experiment. Labor taxes are reduced from a long-run level of 12.6 percent to 7.6 percent and lump-sum transfers converge much faster to the final level of 3.5 percent. The main difference in the fiscal policy instruments is the fact that with a constant policy the government is not able to accumulate assets via higher initial capital taxes. The debt-to-output ratio remains close to the initial level[30]. As a result of the higher long-run capital tax and relatively higher debt-to-output ratio, capital decreases by about 20 percent in the long-run whereas it *increases* by approximately the same amount in the benchmark experiment. The associated higher interest rates and lower wages lead to the reduction in the redistributional gains and reduces the insurance losses associated with the lower labor tax. This policy leads to an average welfare gain of 3.3 percent whereas the time varying policy increases welfare by 4.9 percent. That is, the restriction to constant policies leads a welfare loss of 1.6 percent.

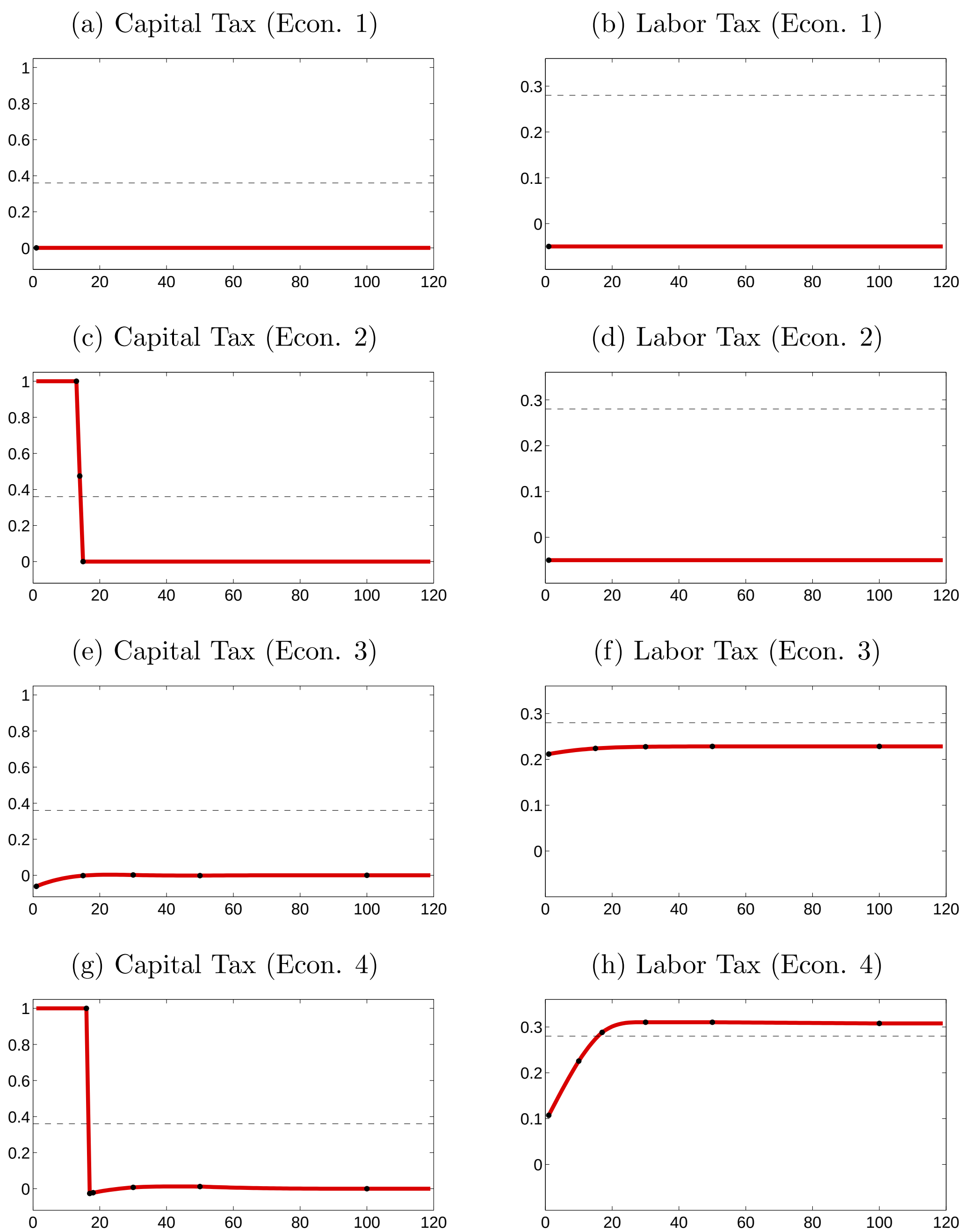
1.6 Complete Market Economies

To our knowledge, this paper is the first to solve the Ramsey problem in the SIM environment. In order to provide further insight and relate it to other results in the literature, we provide a build up to our benchmark result. First, we start from the representative agent economy (Economy 1) and introduce heterogeneity only in initial assets (Economy 2), heterogeneity only in individual productivity levels (constant and certain) (Economy 3), and heterogeneity both in initial assets and in individual productivity levels (Economy 4). Introducing idiosyncratic productivity shocks and borrowing constraints brings us back to the SIM model. At each step, we analyze the optimal fiscal policy identifying the effect of each feature.

In what follows we examine the optimal fiscal policy in Economies 1-4. Their formal environments can be quickly described by starting from the SIM environment delineated above. Economy 4 is the SIM economy with transition matrix, Γ, set to the identity matrix. and borrowing constraints replaced by no-Ponzi conditions. Then, we obtain Economy 3 by setting initial asset levels to its average, Economy 2 by setting the productivity levels to its average, $e = 1$, and Economy 1 by equalizing both initial assets and levels of productivity. Figure 1.5 contains the numerical results.

[30]We do not restrict debt-to-output ratio to be constant in this experiment.

Figure 1.5: Optimal Taxes: Complete Market Economies



Notes: Dashed line: initial taxes; Solid line: optimal taxes.

1.6.1 Economy 1: representative agent

To avoid a trivial solution, the usual Ramsey problem in the representative agent economy does not consider lump-sum transfers to be an available instrument. Since in this paper we do, the solution is, in fact, very simple. It is optimal to obtain all revenue via lump-sum taxes and set capital and labor income taxes so as not to distort any of the agent's decisions. This amounts to $\tau_t^k = 0$ and $\tau_t^n = -\tau^c$ for all $t \geq 1$. Since consumption taxes are exogenously set to a constant level, zero capital taxes leaves savings decisions undistorted and labor taxes equal to minus the consumption tax ensures labor supply decisions are not distorted as well. In this setup the Ricardian equivalence holds so that the path for lump-sum taxes and debt are indeterminate: there is no lesson to be learned from this model about the timing of lump-sum taxes or the path of government debt. This will also be the case in Economies 2, 3 and 4.

1.6.2 Economy 2: add heterogeneity in initial assets

Introducing heterogeneity in the initial level of assets we can diagnose the effect of this particular feature on the Ramsey policies by comparing it to the representative agent ones. We extend the procedure introduced by Werning (2007)[31] to characterize the optimal policies for this and the next two economies. We describe them in a proposition leaving the proof to Appendix A.2.

Proposition 4 *There exists a finite integer $t^* \geq 1$ such that the optimal*[32] *tax system is given by $\tau_t^k = 1$ for $1 \leq t < t^*$ and $\tau_t^k = 0$ for all $t > t^*$; and $\tau_t^n = -\tau^c$ for all $t \geq 1$.*

Once again, there is no reason to distort labor decisions since labor income is certain and the same for all agents. However, the paths for capital taxes and lump-sum transfer do differ from the representative agent ones. Proposition 2 provides a rationale for taxing capital in this case; since agents have different initial asset levels, capital taxes can be used to provide redistribution. This fact together with the fact that capital taxes are

[31] Werning (2007) solves for separable and balance growth path utility functions. Besides solving for GHH preferences we also impose the upper bound on capital income taxes and remove the possibility of time zero taxation.

[32] All propositions in this section are valid for any set of welfare weights, not only the Utilitarian ones. The associated numerical results do assume a Utilitarian welfare function though.

zero in the long-run determine the optimal path for capital taxes[33]. Capital taxes are positive and front-loaded, hitting the upper bound in the initial periods subsequently being driven to zero. The extra revenue obtained via capital taxation is redistributed via lump-sum transfers (or a reduction in lump-sum taxes relative to the representative agent level). It is important to reemphasize that since lump-sum transfers are an unrestricted instrument, there is no reason to tax capital in the initial periods other than for redistributive motives.

In order to have a sense of the magnitudes of t^* and the increase in lump-sum transfers, we apply the same procedure to the one we used to solve for the optimal tax system in the benchmark economy. All we need to do is choose the initial distribution of assets. The stationary distribution of assets in this economy is indeterminate[34], hence, we can choose any one we want. To keep the results comparable we choose the initial stationary distribution from the benchmark experiment [35].

1.6.3 Economy 3: add heterogeneity in productivity levels

It turns out that the Ramsey policies for this economy are a bit more complex. Let Φ, Ψ, and Ω^n be constants (defined in Appendix A.2) and define

$$\Theta_t \equiv \frac{C_t}{\Omega^n \chi \frac{\kappa}{1+\kappa} N_t^{\frac{1+\kappa}{\kappa}}} - 1.$$

The following proposition can be established.

[33] Straub and Werning (2014) show that capital taxes can be positive in environments similar to this. The reason why their logic does not apply here is the fact that the planner has lump-sum taxes as an available instrument. In particular, the proof of Proposition 4 does not impose convergence of any Lagrange multipliers.

[34] For the preferences chosen above, consumption is linear on, and labor supply is independent of the individual asset level. It follows that the equilibrium levels of aggregates are independent of the asset distribution and equal to the representative agent ones (see Chatterjee (1994)). In a steady state, $\beta\left(1+\left(1-\tau^k\right)r\right)=1$ and, therefore, every agent will keep its asset level constant.

[35] In fact, a rescaling of it since the steady state aggregate level of assets is different when there is no idiosyncratic risk (since there is no precautionary savings).

Proposition 5 *Assuming capital taxes are bounded only by the positivity of gross interest rates, the optimal labor tax, τ_t^n, can be written as a function of Θ_t given by*

$$\tau_t^n(\Theta_t) = \frac{(1+\tau^c)\,\Psi\Theta_t}{\Phi\Theta_t + \Psi(\sigma+\Theta_t)} - \tau^c, \quad \textit{for } t \geq 1, \tag{1.6.1}$$

with sensitivity

$$\Theta_t \frac{d\tau_t^n(\Theta_t)}{d\Theta_t} = \frac{\sigma\left(\tau_t^n(\Theta_t)+\tau^c\right)^2}{(1+\tau^c)\,\Theta_t}. \tag{1.6.2}$$

It is optimal to set the capital-income tax rate according to

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{\tau_t^n+\tau^c}{\tau_{t+1}^n+\tau^c}\frac{1-\tau_{t+1}^n}{1-\tau_t^n}, \quad \textit{for } t \geq 1. \tag{1.6.3}$$

Since labor income in unequal, there is a reason to tax it, in order to provide redistribution. Optimal labor taxes are not constant over time since they depend on Θ_t. If they were constant, however, equation (1.6.3) would imply $\tau_t^k = 0$ for all $t \geq 2$. Thus, capital taxes will fluctuate around zero to the extent that labor taxes vary over time. We disregard the upper bound on capital taxes, $\tau_{t+1}^k \leq 1$, because it would complicate the result even further and in a non-interesting way. It could be that the bound is violated if the variation of Θ_t between t and $t+1$ is large enough. However, as discussed below, quantitatively this is unlikely.

To obtain a numerical solution we set the productivity levels to the ones in the benchmark economy and apply the same procedure. To have a sense of the magnitude of the sensitivity of τ_t^n to Θ_t we plug the initial stationary equilibrium numbers ($\tau^n = 0.221$, $\tau^c = 0.046$, $\sigma = 2$, and $\Theta \approx 2$) into equation (1.6.2). This implies a sensitivity of 0.06, i.e. a 1 percent increase in Θ_t changes the tax rate by 0.06 of a percentage point, from 0.221 to 0.2209. We can then calculate the path of Θ_t, which we plot in Figure A.8. Notice that the volatility of Θ_t over time is unsubstantial. It follows that the optimal labor taxes are virtually constant and capital taxes virtually zero.

In any case, the fact that capital is taxed at all seems to be inconsistent with the logic put forward so far. It is not, when labor taxes vary over time they distort the savings decision, capital taxes are then set to "undo" this distortion. The analogous is not the case in Economy 2 because of the absence of income effects on labor supply;

distortions of the savings decision do not affect the labor supply.

1.6.4 Economy 4: add heterogeneity in both

The result for this economy is a combination of the last two.

Proposition 6 *There exists a finite integer $t^* \geq 1$ such that the optimal tax system is given by $\tau_t^k = 1$ for $1 \leq t < t^*$, τ_t^k follows equation (1.6.3) for $t > t^*$; τ_t^n evolves according to equation (1.6.3) for $1 \leq t < t^*$; and τ_t^n is determined by equation (1.6.1) for all $t \geq t^*$.*

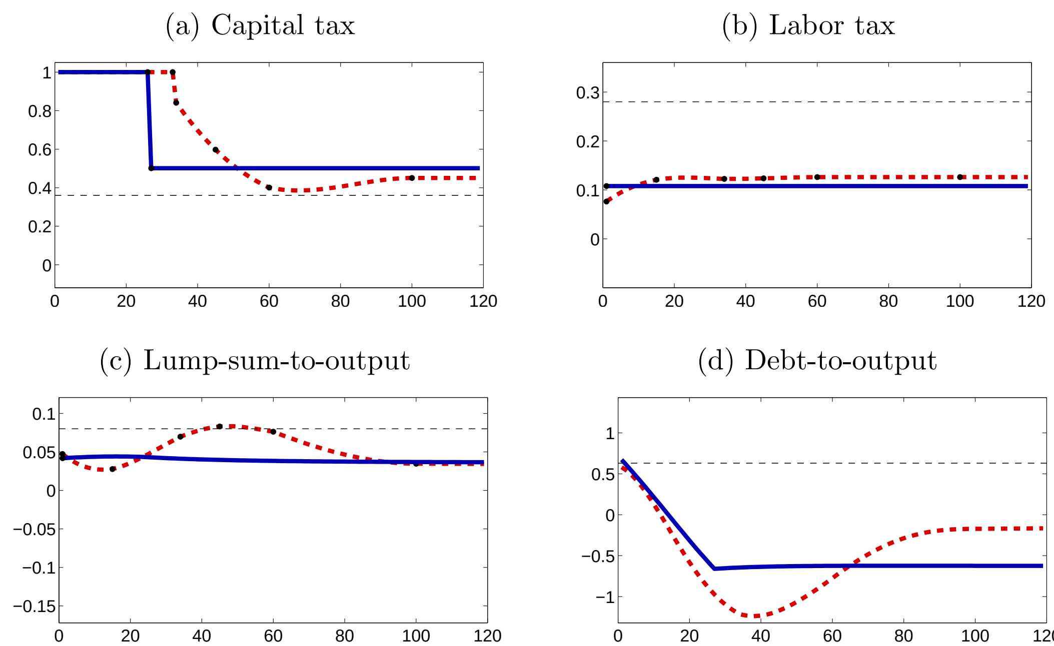
Optimal capital taxes are very similar to Economy 2 and for the same reasons. Labor taxes are determined by the same equation as in Economy 3 for $t \geq t^*$. In initial period, $1 \leq t < t^*$, while capital taxes are at the upper bound, $R_t = 1 < R_t^*$ and, therefore, equation (1.6.3) implies that labor taxes should be increasing. Lump-sum transfers are higher than the in Economies 2 and 3 since they are used to redistribute the capital *and* labor tax revenue.[36]

1.7 Robustness

Figure 1.6 shows that the solution with 4 nodes (t^*,$\tau_{t^*+1}^k$,τ_1^n, and T_1) produces a reasonable approximation for the benchmark solution, at least with respect to its basic features. In this section, we make use of this fact, and present results for alternative welfare functions and for different calibrations of the labor income process using these 4 nodes.

[36] Bhandari et al. (2013) solve recursively for Ramsey policies in an economy similar to Economy 4 with aggregate risk.

Figure 1.6: Optimal Fiscal Policy with 4 nodes



Notes: Dashed thin line: initial stationary equilibrium; Dashed thick line: optimal transition with 17 nodes (benchmark); Solid line: optimal transition with 4 nodes.

1.7.1 Welfare function

All the results presented so far used the same social welfare function: the utilitarian one, which places equal Pareto weights on each agent. This implies a particular social preference with respect to the equality versus efficiency trade-off. Here we consider different welfare functions that rationalize different preferences about this trade-off. With this in mind we propose the following function

$$W^{\hat{\sigma}} = \left(\int \bar{x} \left(a_0, e_0 \right)^{1-\hat{\sigma}} d\lambda_0 \right)^{\frac{1}{1-\hat{\sigma}}},$$

where λ_0 is the initial distribution of individual states (a_0, e_0), $\bar{x}$ denotes the individual certainty equivalents of labor-consumption composite (given a particular initial state (a_0, e_0)), and, following Benabou (2002), we call $\hat{\sigma}$ the planner's degree of inequality aversion. First notice that if $\hat{\sigma} = \sigma$ (the agents' degree of risk aversion), maximizing

W^{σ} is equivalent to maximizing the utilitarian welfare function [37]. If $\hat{\sigma} = 0$, then maximizing W^0 is equivalent to maximizing $(1 + \Delta_L)(1 + \Delta_I)$, that is, the planner has no redistributive concerns and focuses instead in the reduction of distortions and the provision of insurance[38]. Finally, as $\hat{\sigma} \to \infty$ the welfare function approaches $W^{\infty} = \min(\bar{x}(a_0, e_0))$. Hence, by choosing different levels for $\hat{\sigma}$ we can place different weights on the equality versus efficiency trade-off, from the extreme of completely ignoring equality ($\hat{\sigma} = 0$), passing through the utilitarian welfare function ($\hat{\sigma} = \sigma$), and in the limit reaching the Rawlsian welfare function ($\hat{\sigma} \to \infty$). Table 1.7 displays the results for different levels of $\hat{\sigma}$.

Table 1.7: Robustness: Welfare Function

	t^*	τ^k	τ^n	T/Y	B/Y	Δ	Δ_L	Δ_I	Δ_R
$\hat{\sigma} = 0$	0	34.7	12.2	0.0	79.8	0.58	5.32	-2.74	-1.80
$\hat{\sigma} = 1$	19	49.9	10.1	2.9	-36.4	4.56	3.73	-3.83	4.81
$\hat{\sigma} = 2^*$	26	49.7	10.8	3.6	-62.5	4.64	2.97	-3.84	5.68
$\hat{\sigma} = 3$	29	49.8	10.4	3.5	-76.8	4.64	2.90	-4.01	5.94
$\hat{\sigma} = 4$	30	48.9	11.5	4.1	-76.0	4.61	2.52	-3.78	6.05
$\hat{\sigma} = 5$	32	49.2	11.3	4.0	-84.2	4.59	2.45	-3.88	6.21

Notes: (*) When $\hat{\sigma} = 2 = \sigma$ the welfare function is utilitarian, this is the solution plotted in Figure 1.6. The values for T/Y and B/Y are the ones from the final steady state. For the welfare decomposition we use the utilitarian welfare function for comparability.

When $\hat{\sigma} = 0$ the planner has no redistributive motive and, accordingly, $t^* = 0$ which is consistent with the results displayed above, in particular in Section 1.6. The benchmark result that capital taxes should be held fixed at the upper bound for the initial periods is inherently linked to the redistributive motive of the planner. It follows that higher $\hat{\sigma}$ imply higher t^*'s (lower lump-sum-to-output ratios and higher debt-to-output

[37] Notice that $\left(\int \bar{x}(a_0, e_0)^{1-\sigma} d\lambda_0\right)^{\frac{1}{1-\sigma}}$ is a monotonic transformation of $\int \frac{\bar{x}(a_0,e_0)}{1-\sigma}^{1-\sigma} d\lambda_0$, which is equivalent to the utilitarian welfare function.

[38] This result can be established following a similar procedure to the one used in proof of Proposition 3. The online appendix contains the proof.

ratios). Otherwise, overall, specially for $\hat{\sigma} \geq 1$, the results do not change significantly with changes in $\hat{\sigma}$. In particular, the final levels of capital and labor taxes are remarkably similar.

1.7.2 Labor income process

The labor income process (summarized by the Markov matrix, Γ, and the vector of productivity levels, e) is a key determinant of the amount of uncertainty and inequality faced by agents in the economy. These parameters are a discrete approximation for a continuous process for labor income, $li_t \equiv we_t n_t$, that is

$$\log(li_{t+1}) = \rho \log(li_t) + \varepsilon, \quad \text{where } \varepsilon \sim N\left(0, \sigma_\varepsilon^2\right).$$

In our benchmark calibration we target $\rho = 0.9$ and $\sigma_\varepsilon = 0.2$. Given the importance of these choices for our results and the lack of consensus in the literature about them (see Section 1.4.4 for a discussion), we provide here the results for alternative numbers for ρ and σ_ε. For each of these we recalibrate the economy modifying only the corresponding target, Table 1.8 contains the results.

Table 1.8: Robustness: Labor income process

	t^*	τ^k	τ^n	T/Y	B/Y	Δ	Δ_L	Δ_I	Δ_R
$\rho = 0.85$	24	34.8	4.8	0.0	-100.2	5.43	4.81	-3.72	4.48
$\rho = 0.95$	21	42.8	11.5	3.7	-49.5	3.91	3.63	-3.35	3.74
$\sigma_\varepsilon = 0.15$	28	28.1	4.9	0.1	-126.3	5.64	4.59	-4.09	5.31
$\sigma_\varepsilon = 0.25$	34	57.8	11.6	4.7	-75.9	4.52	2.51	-4.29	6.52
Benchmark	26	49.7	10.8	3.6	-62.5	4.64	2.97	-3.84	5.68

Notes: The values for T/Y and B/Y are the ones from the final steady state.

As one would expect, the magnitudes of the results do change considerably given changes in these important parameters. However, reassuringly, the qualitative features of the fiscal policy instruments and of where the welfare gains come from is not substantially affected.

1.8 Conclusion

In this paper we quantitatively characterize the solution to the Ramsey problem in the standard incomplete market model. We find that even though the planner has the ability to obtain all revenue via undistortive lump-sum taxes, it chooses instead to tax capital income heavily and labor income to a lesser extent. Moreover, we show that it is beneficial for the government to accumulate assets over time. With a welfare decomposition we diagnose that, relative to the current US tax system, this policy leads to an overall reduction of the distortions of agent's decisions, to a substantial amount of redistribution and to a reduction in the amount of insurance provided by the government. Importantly, we also show that disregarding the transitory dynamics and focusing only on steady states can lead to severely misleading results.

Finally, we do not view our results as a final answer to our initial question: to what extent should governments use fiscal policy instruments to provide redistribution and insurance? Instead, we understand it as a contribution to the debate. The model we use abstracts from important aspects of reality, as any useful model must, and we miss some important dimensions. For instance, in the model studied above an agent's productivity is entirely a matter of luck, it would be interesting to understand the effects of allowing for human capital accumulation. We also assume the government has the ability to fully commit to future policies, relaxing this assumption could lead to interesting insights.

Chapter 2

A Study of Unsecured Credit Limits and its Determinants

2.1 Introduction

Credit limits are an important determinant of household's consumption and savings behavior[1]. In turn, what determines credit limits are the rules that govern the ability of households to default on their loans and the risks that they are exposed to. If households could default without any constraint or punishment they would not be able to borrow at all. On the other hand, if defaulting was never allowed and households faced no risk, they would be able to borrow up to the present value of their future income. When households are exposed to income or other types of risk and default possibilities and punishments are less stylized this calculation becomes more involved.

This paper investigates the quantitative relevance of the key determinants of unsecured consumer credit limits in the US[2]. I use a version of the incomplete markets life cycle model in which agents are allowed to default on their debt holdings by declaring bankruptcy. It follows that credit limits in the model are endogenously determined. The parameters that determine the punishments for declaring bankruptcy and in which

[1]See, for instance, Scheinkman and Weiss (1986), Deaton (1991), and Carroll (2001).

[2]Limits on secured credit are mostly determined by the value of the asset that a household is borrowing against. This paper abstracts from this type of borrowing altogether focusing instead on unsecured credit limits.

circumstances the households are able to do it have important effects on credit limits. However, they are somewhat subjective and, therefore, difficult to discipline directly. In the literature they are usually calibrated to match the data's debt-to-income ratio. This procedure leads to a wide range of results. An important contribution of this paper is to suggest a new disciplining target and explain how it can help identify the bankruptcy parameters. The new target is the relationship between income levels and credit limits. More precisely, I document, using data for the Survey of Consumer Finances, that a 1 percent increase in income leads to a 0.7 increase in credit limits (controlling for other individual characteristics).

The household's income process is not subject to calibration in this paper. Instead, I take them as given, estimated directly from data. I argue, however, that the type of income process that households face, in particular whether the process allows for profile heterogeneity or not, has important effects on credit limits. By profile heterogeneity I mean households having different income growth rates which they learn about over their lives. To understand this, suppose, for instance, that the only punishment for declaring bankruptcy is that the household is not allowed to borrow for a number of periods. If the households face the same income process (with a persistent and a transitory component), then following a positive income shock reversion to the mean implies that income is expected to fall (or increase less) in future periods and, therefore, the household expects to want to save more (or borrow less) than if it had received a negative income shock. It follows that positive income shocks reduce the relevance of the punishment and, on average households with higher income (who accumulated positive income shocks) are more likely to declare bankruptcy and, therefore, face lower credit limits, which contradicts the pattern observed in the data.

I provide a simple example that allows me to clarify this point. Then, I use it to show how the introduction of profile heterogeneity is able to mitigate this problem. I, then, move to analyzing the effects of adding additional punishments. One additional punishment commonly used in the literature is that households lose a proportion of their income when they choose to declare bankruptcy. This implies that agents with higher income levels face harsher punishments and are, therefore, less likely to default and face higher credit limits.

I argue that an important feature of the bankruptcy law, which is usually disregarded is the fact that not everyone who files is granted bankruptcy. The decision is ultimately made by a judge who will evaluate whether or not it is "feasible" for the household to make its payments. I model this by including a forward looking constraint on the ability to declare bankruptcy: households that while paying back their loans are still able to maintain a level of consumption above a certain threshold in the future will not be granted bankruptcy. It should be evident that such a rule could also generate a positive relationship between income and credit limits since agents with high enough expected future income (usually those with current high income) are simply not allowed to default on their loans.

I, then, perform comparative statics experiments to understand and quantify the effects on credit limit of movements in the proportional income costs to bankruptcy and threshold levels of income, in particular on its correlation with income. I find that for all bankruptcy systems analyzed the correlation between credit limits and income is higher under the assumption of heterogeneity of income profiles. Higher proportional income punishments lead to a higher correlation as well, and the model matches the data in this respect with relatively small levels of proportional income costs, 2.6 percent without and 1.4 percent with profile heterogeneity (over 10 years). These numbers are small relative to others obtained in the literature; for instance, Livshits et al. (2007) estimate this number to be 35.5 percent (over 3 years) while Chatterjee et al. (2007) find it to be 3.5 percent (over 10 years). The wide range of estimates gives more reason to the search for additional disciplining targets. The calibrated threshold income levels, are 24 percent of average income with profile heterogeneity and 30 percent without it.

Finally, I calibrate the bankruptcy parameters targeting key statistics about unsecured consumer credit and bankruptcy filings[3] and find that it performs relatively well with one important caveat: the HIP model underestimates the number of bankruptcy filings by a factor of 10. This is an important failure and can be viewed as a problem for the HIP literature. One of the main findings in this literature is that once profile

[3]Namely, the wealth-to-income ratio, the debt-to-income ratio, the percentage of households who declare bankruptcy, and the percentage of those that declare for reasons other than shocks to income.

heterogeneity is allowed the models are able to generate the variance in income of households with significantly less risk. A large part of this variance is shown to be due to differences in income profiles which the households know a lot about. With these levels of risk the model severally underestimates the number of bankruptcy filings.

The paper is organized as follows. In Section 2.2 I present a simple example that allows me to explain the main mechanisms behind the results. Section 2.3 describes the quantitative version of the model. I describe and discuss the parametrization of the model in Section 2.4. In Section 2.5 I present the comparative statics experiment. Section 2.6 contains the comparison between model results and data and Section 2.7 concludes. All proofs are given in the Appendix.

2.2 Mechanism

In this section I discuss three ways to obtain, with a standard incomplete markets model, the positive correlation between income and credit limits observed in the data (see Section 2.6.1). Somewhat surprisingly, if all households face the same income process with a persistent component and the only punishment for declaring bankruptcy is the loss of the ability to borrow, the model is likely to predict a negative correlation. I start by introducing a simple example that allows me to clarify this point. Then, I show that if the households face profile heterogeneity with potentially different growth rates of income, which they learn about over time, the model is more likely to predict a positive correlation between income and credit limits. An alternative way to achieve the same goal, one that is frequently used in the literature (e.g. Chatterjee et al. (2007), Livshits et al. (2007), Gordon (2015)), is to impose an extra punishment to declaring bankruptcy. Namely that, in addition to losing the ability to borrow, households also lose a proportion of their income. A third option is to restrict the ability to declare bankruptcy of households with higher income levels. After the discussion of these three alternatives I study their quantitative relevance in a less stylized version of the model.

2.2.1 Example

Consider an economy with a continuum of ex-ante identical households who live for three periods: 1, 2 and 3. Suppose they share time-separable von Neumann-Morgenstern preferences with period utility $u(c)$, where c denotes their consumption level, and let β denote their discount factor. For tractability, in this section, I assume that $u(c) = -\exp(-\alpha c)$, where α is the level constant absolute risk aversion, and that $\beta(1+r) = 1$, where r is the exogenous risk-free interest rate.

In each period, households receive an exogenous random income of y_t, which follows $y_t^i = \theta^i t + z_t^i$. The value of the trend θ^i is initially unknown and independently drawn from $N\left(0, \sigma_\theta^2\right)$, and z_t^i is a persistent shock, with $z_{t+1}^i = \rho z_t^i + \eta_{t+1}^i$, $\eta_{t+1}^i \sim N\left(0, \sigma_\eta^2\right)$, $\rho \in (0,1)$, and $z_0^i = 0$.[4] In each period, households observe their level of income y_t^i, but not its components $\theta^i t$ and z_t^i. Beliefs about θ^i and z_t^i are updated following each observation of y_t^i according to Bayes rule.

In periods 1 and 2, households are only allowed to trade an asset, a_{t+1}, its price denoted by q_{t+1}. Trading occurs between households and financial intermediaries who have access to an outside market where they can borrow or lend at the risk-free rate of r. There is no information asymmetry. Financial intermediaries observe income levels and form beliefs in the same way households do.

In period 1, households can, given $q_2(a_2, y_1)$, borrow or lend without any constraints. In period 2 they are allowed to declare bankruptcy, in which case any amount of debt that they hold is discharged. Then, households can once again choose how much to borrow or save, except that households who have declared bankruptcy are not allowed to borrow. In period 3, households simply consume their cash-in-hand $(y_3 + a_3)$; bankruptcy is not allowed in this period. In each period trading occurs after the observation of income levels.

There is perfect competition between financial intermediaries, so that their trading

[4] The fact that y_t^i is not necessarilly positive is irrelevant for the utility function considered in this section.

with households occurs at actuarially fair prices, that is

$$q_2(a_2, y_1) = \frac{1 - \pi(a_2, y_1)}{1 + r}, \quad \text{and} \quad q_3 = 1/(1 + r). \tag{2.2.1}$$

where $\pi(a_2, y_1)$ is the probability that a household who borrowed $-a_2$ and whose income was y_1 in period 1 declares bankruptcy in period 2. Evidently, $\pi(a_2, y_1) = 0$ if $a_2 \geq 0$.

Thus, at the end of period 2 a household who *has not* declared bankruptcy ($b = 0$) chooses $a_3^0(a_2, y_1, y_2)$ to solve

$$V^0(a_2, y_1, y_2) = \max_{a_3} u\left(y_2 + a_2 - \frac{a_3}{1 + r}\right) + \beta E\left[u(y_3 + a_3) | y_1, y_2\right], \tag{2.2.2}$$

whereas a household who *has* declared bankruptcy ($b = 1$) chooses $a_3^1(y_1, y_2)$ to solve

$$V^1(y_1, y_2) = \max_{a_3 \geq 0} u\left(y_2 - \frac{a_3}{1 + r}\right) + \beta E\left[u(y_3 + a_3) | y_1, y_2\right]. \tag{2.2.3}$$

At the beginning of period 2 the household chooses $b(a_2, y_1, y_2)$ to solve

$$V(a_2, y_1, y_2) = \max_{b \in \{0,1\}} (1 - b) V^0(a_2, y_1, y_2) + bV^1(y_1, y_2). \tag{2.2.4}$$

Finally, in period 1 the household chooses $a_2(y_1)$ to solve

$$\max_{a_2} u(y_1 - q_2(a_2, y_1) a_2) + \beta E\left[V(a_2, y_1, y_2) | y_1\right].$$

Definition 6 *An equilibrium is policy functions $a_2(y_1)$, $b(a_2, y_1, y_2)$, $a_3^0(a_2, y_1, y_2)$, and $a_3^1(y_1, y_2)$, and prices $q_2(a_2, y_1)$ and q_3, such that: (i) the policy functions solve the household's problem, and (ii) $q_2(a_2, y_1)$ and q_3 are given by equation (2.2.1) with $\pi(a_2, y_1) = E\left[b(a_2, y_1, y_2) | y_1\right]$.*

I am interested in the correlation between income shocks and credit limits in period 1, that is, the effect that a change in y_1 has on how much the household is able to borrow. Accordingly, I must define precisely what credit limits are in this environment. If the household promises to pay $-a_2$ in period 2, it receives $-q_2(a_2, y_1) a_2$ in period 1. Hence, the natural definition for credit limits is the following.

Definition 7 *The credit limit* $\omega(y_1)$*, faced by a household with income* y_1 *in period* 1*, is given by*

$$\omega(y_1) \equiv \max_{a_2}\left(-q_2(a_2, y_1)\, a_2\right).$$

Households pay back their loans only if the punishment for declaring bankruptcy, namely the loss of the ability to borrow in period 2, outweighs the cost of making the payment. In particular, if the household in period 1 is certain that it will want to save in period 2, it wold default on any outstanding debt in period 2, and, for that reason, would not be able to borrow at all in period 1. The stronger the expected desire to borrow in period 2, the more relevant the punishment becomes and the more the household is able to borrow in period 1.

2.2.2 Negative Correlation without Profile Heterogeneity

If the households face only a persistent income shock (i.e. $\sigma_\theta^2 = 0$), then, credit limits are *decreasing* with income. In order to see this, first notice that the conditional expectations become $E\left[y_2^i \mid y_1^i\right] = \rho y_1^i$ and $E\left[y_3^i \mid y_1^i\right] = \rho^2 y_1^i$. Consider two households, L and H, who receive $y_1^H > y_1^L$ in period 1. It follows that

$$E\left[y_3^H - y_2^H \mid y_1^H\right] = \rho(\rho - 1)\, y_1^H < \rho(\rho - 1)\, y_1^L = E\left[y_3^L - y_2^L \mid y_1^L\right],$$

so that, given the information that the households have in period 1, household H expects its income to increase less (or fall more) between periods 2 and 3, than household L. Therefore, household H is likely to want to borrow less (or save more) than household L. It follows that, in expectation, the punishment is less relevant for household H, and it faces a lower credit limit in period 1 as a result, that is $\omega\left(y_1^H\right) < \omega\left(y_1^L\right)$. In fact, the following proposition can be established.

Proposition 7 *If* $\sigma_\theta = 0$*, then credit limits are decreasing with income, i.e.* $\omega'(y_1) \leq 0$ *for all* y_1*.*

The proof for this and the following propositions can be found in Appendix **??**.[5]

[5]Notice that the CARA assumption is partially responsible for this result. These preferences allow me to abstract from involuntary bankruptcy (when the available income is so low that the agent is not

2.2.3 Positive Correlation with Profile Heterogeneity

Introducing profile heterogeneity allows the model to predict the positive correlation between income levels and credit limits observed in the data without introducing any additional punishments for declaring bankruptcy. When $\sigma_\theta > 0$, a high income level could be due to a persistent shock, z_t^i, or to high level of the trend component of income, θ^i. Whereas when $\sigma_\theta = 0$ a higher income level necessarily leads to a lower incentive to borrow, when $\sigma_\theta > 0$ this is not the case. Observing a higher income level is, in this case, indicative of a higher θ^i which could (depending on the level of σ_θ relative to σ_η) lead to the expectation that income levels will continue to increase and, therefore, to a stronger incentive to borrow next period. It, then, follows that a higher y_1 implies, in expectation, a higher relevance for the punishment for declaring bankruptcy, and households who observe a higher y_1 should face higher credit limits. The following proposition corroborates this logic.

Proposition 8 *Credit limits are increasing with income, i.e.* $\omega'(y_1) \geq 0$ *for all* y_1*, if and only if* $\sigma_\theta > \rho(1-\rho)\sigma_\eta$.

Abstracting from the correlation between income and credit limits, this result shows that the introduction of profile heterogeneity can have dramatic effects on the incentives to declare bankruptcy. The quantitative importance of this effect is the subject of the main experiment in this paper.

2.2.4 Positive Correlation with Additional Punishment

If when declaring bankruptcy the household, in addition to losing the ability to borrow, also loses a proportion γ of their income, then it is possible to obtain a positive correlation even if $\sigma_\theta = 0$. The only necessary change to the setup above is to redefine the

able to make payments which is never the case here). If the utility function displays CRRA instead, it can be shown that the lower a households income is, the higher the probability of involuntary bankruptcy which leads credit limits to be increasing with income for very low levels, and only after a certain point do they become increasing.

value function under bankruptcy, that is to substitute equation (2.2.3) by

$$\tilde{V}^1(y_1, y_2) = \max_{a_3 \geq 0} u\left((1-\gamma)\, y_2 - \frac{a_3}{1+r}\right) + \beta E\left[u\left((1-\gamma)\, y_3 + a_3\right)|y_1, y_2\right]$$

where I have assumed that the proportion of income is extracted in periods 2 and 3, though this is not necessary for the result in the following proposition.

Proposition 9 *If $\sigma_\theta = 0$, credit limits are increasing with income, i.e. $\omega'(y_1) \geq 0$ for all y_1, if and only if $\gamma \geq (1-\rho)/(1+\beta)$.*

The intuition for this result is simple. If the households lose a proportion of their income when declaring bankruptcy, the households with higher income levels lose more in absolute terms. As a result, the higher a household's income level is, the more it has to pay to declare bankruptcy, and higher credit limits follow.

It is not easy to quantify directly the parameter γ. It is supposed to capture the extra costs to a household that declares bankruptcy, aside from the loss of the ability to borrow[6]. What makes it difficult to measure γ is the fact that these costs are arguably subjective, tantamount to "stigma". One way to measure γ would be to ask individuals who have declared bankruptcy how much they would be willing to pay in order for their bankruptcy record to become observable only to creditors. However, to my knowledge such a survey has not been conducted, so that we must identify γ indirectly. In the less stylized version of the model that follows, I perform comparative statics experiments with γ in order to obtain a better understanding of its effects.

A similar assumption, also commonly used in the literature (Zame (1993), Athreya (2008), Athreya et al. (2012)), is to impose a fixed utility cost to declaring bankruptcy. This assumption has similar implications, since households with higher income levels have lower marginal utility levels and, therefore, a constant utility cost implies a higher income cost to households with higher income levels.

[6] It is important to note that this is not a feature of the bankruptcy law. In fact, an agent is allowed to borrow after declaring bankruptcy and there are banks that specialize in providing this type of credit. However, (Chapter 7) bankruptcy stays on ones record for 10 years and this has a significant impact on an individual's credit rating and on her ability to borrow.

2.2.5 Positive Correlation with Restrictions on Ability to Declare Bankruptcy

This is perhaps the most straight-forward way to obtain a positive correlation between income levels and credit limits. If households cannot declare bankruptcy if their income levels are above a certain threshold, for instance, it is quite evident that higher income levels lead to lower probabilities of bankruptcy and, therefore, to higher credit limits.

Moreover, the inability to declare bankruptcy of some individuals is a salient feature of the bankruptcy law. Consider, for instance, an individual who had a significant amount of unsecured debt and suddenly gets a big promotion. This individual is likely to never again need to borrow, in which case it would benefit from declaring bankruptcy. The reason this situation is not observed in the data is because such an individual would *not be allowed* to declare bankruptcy.[7] A judge ultimately decides whether or not to grant bankruptcy to an individual. This decision is subjective, however it has important implications to limits on unsecured credit and should not be ignored.

In order to incorporate this characteristic of the bankruptcy system into the model at hand, I assume that households can only declare bankruptcy when paying back their current debts leaves them with a utility level below an exogenously set threshold. This amounts to replacing equation (2.2.4) with

$$\tilde{V}\left(a_2, y_1, y_2\right) = \begin{cases} V^0\left(a_2, y_1, y_2\right) & \text{, if } V^0\left(a_2, y_1, y_2\right) \geq \underline{V} \\ \max\left\{V^0\left(a_2, y_1, y_2\right), V^1\left(y_1, y_2\right)\right\} & \text{, if } V^0\left(a_2, y_1, y_2\right) < \underline{V} \end{cases}$$

where $\underline{V}$ is a cutoff utility level. If by paying back its debt a household is able to obtain a utility above $\underline{V}$, i.e. if $V^0\left(a_2, y_1, y_2\right) \geq \underline{V}$, then it is not allowed to declare bankruptcy. Otherwise, bankruptcy is allowed.

For any relevant choice of $\underline{V}$, however, this setup implies that $\tilde{V}\left(a_2, y_1, y_2\right)$ is not monotonic or continuous in a_2. If $V^1\left(y_1, y_2\right) \leq \underline{V}$ for all $\left(y_1, y_2\right)$, then, the introduction of this threshold is innocuous, since the households would only wish to declare bankruptcy in the region that they are, in fact, allowed to, that is $V^0\left(a_2, y_1, y_2\right) <$

[7] See Gordon (2014) for a normative investigation of who should be allowed to declare bankruptcy.

$V^1(y_1, y_2)$ implies $V^0(a_2, y_1, y_2) < \underline{V}$. Hence, if this addition to the model is to have any effect it must be that $V^1(y_1, y_2) > \underline{V}$ for some (y_1, y_2). Thus, consider a triple $(\hat{a}_2, \hat{y}_1, \hat{y}_2)$ such that $V^1(\hat{y}_1, \hat{y}_2) > \underline{V}$ and $V^0(\hat{a}_2, \hat{y}_1, \hat{y}_2) = \underline{V}$. It follows that, $\lim_{a_2 \to \hat{a}_2^-} \tilde{V}(a_2, \hat{y}_1, \hat{y}_2) = V^1(\hat{y}_1, \hat{y}_2) > \underline{V} = V^0(\hat{a}_2, \hat{y}_1, \hat{y}_2) = \lim_{a_2 \to \hat{a}_2^+} \tilde{V}(a_2, \hat{y}_1, \hat{y}_2)$; in some states the households would be better off if they had more debt so as to be able to declare bankruptcy. Hence, the policy function $a_2(y_1)$ would not be a continuous function of y_1. If the household expects $V^0(a_2, y_1, y_2)$ to be close enough to $\underline{V}$, it would be better off by borrowing up to its credit limit in order to increase its chances of being able to declare bankruptcy in the following period.

To do away with this type of behavior, in what follows I assume instead that

$$\tilde{V}(a_2, y_1, y_2) = \begin{cases} V^0(a_2, y_1, y_2) & , \text{if } V^0(a_2, y_1, y_2) \geq \underline{V} \\ \max\left\{V^0(a_2, y_1, y_2), \min\left\{V^1(y_1, y_2), \underline{V}\right\}\right\} & , \text{if } V^0(a_2, y_1, y_2) < \underline{V} \end{cases}$$

which implies that if the household is able to attain a utility level above $\underline{V}$ by declaring bankruptcy, that is if $V^1(\hat{y}_1, \hat{y}_2) > \underline{V}$, then its debt is only partially discharged, up to the point in which the household is indifferent between declaring bankruptcy and receiving $\underline{V}$.

2.3 Model

This section contains the description of the quantitative model used in the empirical analysis. Relative to the simpler model introduced in the previous section, the most significant changes are that utility is now assumed to be CRRA, the income process Log-Normal, and there are more periods including some in which the household is assumed to be retired. The economy consists of a continuum of $T = W + R$ overlapping generations of households, where T denotes the maximum life-span of a household, with W denoting the number of working-age years and R the number of retirement years. There are also a continuum of perfectly competitive financial intermediaries with which the agents trade discount bonds.

2.3.1 Preferences

Households only value consumption and are assumed to supply labor inelastically. during their working-age years. Preferences are represented by a time-separable utility function over sequences of consumption with CRRA period utility, $u(c) = c^{1-\sigma}/(1-\sigma)$. Households have a common discount factor denoted by β and face an age dependent probability of death. Let p_t denote the probability of death between the ages t and $t+1$ and define $\delta_t \equiv \beta(1-p_t)$. The problem of a household is to choose, over time, a feasible sequence of consumption levels $\{c_t\}_{t=1}^{T}$ to maximize their expected utility.

2.3.2 Income

Working Age

Here I set up an income process that encompasses the possibility of a standard common income profile and that of households having potentially different profiles. An individual of working-age faces an exogenous stochastic income process with

$$y_t^i = \underline{y} + \exp\left(g_t + \theta^i t + z_t^i + \varepsilon_t^i\right), \tag{2.3.1}$$

where $\underline{y}$ is a lower bound to y_t^i; g_t is a common life cycle component supposed to capture differences in the mean income for different ages; $\theta^i \sim N\left(0, \sigma_\theta^2\right)$ captures differences between individual income profile growth rates; z_t^i is a persistent shock with

$$z_t^i = \rho z_{t-1}^i + \eta_t, \tag{2.3.2}$$

$\eta_t^i \sim N\left(0, \sigma_\eta^2\right)$, and $z_0^i \sim N\left(0, \sigma_{z_0}^2\right)$; and $\varepsilon_t^i \sim N\left(0, \sigma_\varepsilon^2\right)$ is a transitory shock. More precisely, let θ^i be a result of two independent random draws, $\theta^i = \theta_k^i + \theta_u^i$, with $\theta_k^i \sim N\left(0, \sigma_{\theta_k}^2\right)$ and $\theta_u^i \sim N\left(0, \sigma_{\theta_u}^2\right)$. The first part, θ_k^i, is observed (known) by the household, and the second, θ_u^i, is not (unknown). It follows that $\sigma_\theta^2 = \sigma_{\theta_k}^2 + \sigma_{\theta_u}^2$. The proportion of the variance that is unknown, $\sigma_{\theta_u}^2/\sigma_\theta^2$, is an important determinant of the uncertainty associated with the income process. The data counterpart of y_t^i is supposed to be after-taxes/transfers earnings controlled for household characteristics not contained in the model.

Retirement

In reality, retirement benefits (US Social Security Administration's Old-Age Insurance Benefit System) are a function of the individual average life cycle income, $\bar{y}^i$. Introducing this feature in the model is not computationally practical as it implies another state variable. With this in mind I follow Guvenen and Smith (2014) and instead calculate retirement income in two parts. First I estimate a function $\bar{y}^i = \alpha_0 + \alpha_1 y^i_W \equiv \hat{y}^i$, using data simulated using equation (2.3.1). Then, denoting the cross-sectional average working-age income by $\bar{y}$, I calculate the retirement benefit, y_R, with the actual formula used by the US Social Security Administration,

$$y_R\left(\hat{y}^i\right) = \begin{cases} 0.9\hat{y}^i & \text{, if } \hat{y}^i \leq 0.3\bar{y} \\ 0.27\bar{y} + 0.32\left(\hat{y}^i - 0.3\bar{y}\right) & \text{, if } 0.3\bar{y} < \hat{y}^i \leq 2\bar{y} \\ 0.814\bar{y} + 0.15\left(\hat{y}^i - 2\bar{y}\right) & \text{, if } 2\bar{y} < \hat{y}^i \leq 4.1\bar{y} \\ 1.129\bar{y} & \text{, if } \hat{y}^i > 4.1\bar{y}, \end{cases} \tag{2.3.3}$$

where the estimated average income, $\hat{y}^i$, acts as a substitute for the actual individual average $\bar{y}^i$.

2.3.3 Bayesian Learning

If the households observed their output levels, y^i_t, *and* the stochastic component $z^i_t + \varepsilon^i_t$, then they would immediately, in period 1, learn their income profile growth rate, θ^i. In what follows, though, I assume that households observe only y^i_t and, based on these observations, form beliefs about its stochastic components. Households, then, learn about θ^i over time according to a Bayesian rule. In order to compute a household's Bayesian estimates at each period it is convenient to write the learning process as a Kalman filtering problem with state and observation equations given respectively by

$$\underbrace{\begin{bmatrix} \theta^i \\ z^i_t \end{bmatrix}}_{S^i_t} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} \theta^i \\ z^i_{t-1} \end{bmatrix}}_{S^i_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ \eta^i_t \end{bmatrix}}_{\nu^i_t}, \quad \text{and} \quad \tilde{y}^i_t = \underbrace{\begin{bmatrix} t & 1 \end{bmatrix}}_{H'_t} \underbrace{\begin{bmatrix} \theta^i \\ z^i_t \end{bmatrix}}_{S^i_t} + \varepsilon^i_t,$$

where S_t is a vector of hidden state variables and $\tilde{y}^i_t \equiv \ln\left(y^i_t - \underline{y}\right) - g_t$.

The system is assumed to start in period time $t = 0$ at which time the state vector S_1^i is regarded as a random variable. Let $\left(\hat{S}_{1|0}^i, P_{1|0}^i\right)$ be the Bayesian prior on S_1^i, that is $\hat{S}_{1|0}^i \equiv E_0^i\left[S_1^i\right]$ and $P_{1|0}^i \equiv E_0^i\left[\left(S_1^i - \hat{S}_1^i\right)\left(S_1^i - \hat{S}_1^i\right)'\right]$. Also, define $\hat{S}_{t|s}^i \equiv E_s^i\left[S_t^i\right]$ and $P_{t|s}^i \equiv E_s^i\left[\left(S_t^i - \hat{S}_{t|s}\right)\left(S_t^i - \hat{S}_{t|s}^i\right)'\right]$, where $E_s^i\left[\cdot\right] \equiv E\left[\cdot|\tilde{y}_s^i, ..., \tilde{y}_1^i, \hat{S}_{1|0}^i\right]$. Finally, let Q denote the covariance matrix of ν_t^i.

It follows that, given the Bayesian prior on S_1^i, we can calculate the evolution of mean beliefs using the following recursive formulas. The covariance matrices follow

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H_t\left(H_t'P_{t|t-1}H_t + \sigma_\varepsilon^2\right)^{-1}H_t'P_{t|t-1}, \text{ and } P_{t+1|t} = FP_{t|t}F' + Q. \tag{2.3.4}$$

Notice that $P_{t|t}^i$ and $P_{t|t-1}^i$ are independent of the observations of $\tilde{y}_t^i$ (which is why I removed the corresponding superscripts in these last equations). The one period ahead estimates about the hidden state variables can be computed using

$$\hat{S}_{t|t}^i = \hat{S}_{t|t-1}^i + P_{t|t-1}H_t\left(H_t'P_{t|t-1}H_t + \sigma_\varepsilon^2\right)^{-1}\left(\tilde{y}_t^i - H_t'\hat{S}_{t|t-1}^i\right), \text{ and } \hat{S}_{t+1|t}^i = F\hat{S}_{t|t}^i. \tag{2.3.5}$$

Then, it follows that

$$\tilde{y}_{t+1}^i|\hat{S}_{t|t}^i \sim N\left(H_{t+1}'F\hat{S}_{t|t}^i, H_{t+1}'P_{t+1|t}H_{t+1} + \sigma_\varepsilon^2\right).$$

Hence, in order built an estimate of next period's income it sufficient to keep track of two state variables, $\hat{S}_{t|t}^i = \left(\hat{\theta}_t^i, \hat{z}_t^i\right) = E_t^i\left[\left(\theta^i, z_t^i\right)\right]$; that is, the estimates for $\left(\theta^i, z_t^i\right)$ given the information the household has in the current period, t.

Notice that even if $\sigma_\theta^2 = 0$ (so that $\theta^i \equiv 0$), observing $\tilde{y}_t^i$ does not reveal the stochastic components z_t^i and ε_t^i. Thus, there exists a version of the model, not pursued in this paper, in which there is no profile heterogeneity but households still use Bayesian learning to identify over time the stochastic components of their income. In order to stay in contact with what has been done in the literature, in what follows I study a version of the model with learning and heterogeneous income profiles (HIP) and a version without learning or profile heterogeneity (RIP - for restricted income profiles). In the RIP model I assume that the households observe z_t^i and ε_t^i, thus, in order to estimate next period's income, it is sufficient to have only one state variable, z_t^i. From now on I will denote the state variable necessary for the one period ahead estimation of the next period income level by s_t, with $s_t = \left(\hat{\theta}_t^i, \hat{z}_t^i\right)$ for the HIP model and $s_t = z_t$ for the RIP model.

2.3.4 Market Arrangements

In each period t, a household can borrow or save by purchasing a single one-period discount bond with face value a_{t+1}. A debt contract, such that $a_{t+1} < 0$, implies that the household receives $-q_{t+1}(a_{t+1}, s_t)\, a_{t+1}$ units of consumption in period t in exchange for the promise to pay $-a_{t+1}$ units in period $t+1$, unless it declares bankruptcy. If $a_{t+1} > 0$, on the other hand, the contract implies a payment of $q_{t+1}(a_{t+1}, s_t)\, a_{t+1}$ in period t in exchange for a promise to receive a_{t+1} in period $t+1$. These contracts are signed between households and financial intermediaries who are allowed to price them differently depending on the size of the contract, a_{t+1}, and the households state variable s_t. Allowing, this price differentiation is necessary to avoid cross subsidization (see Chatterjee et al. (2007)) between households facing different risks of declaring bankruptcy in period $t+1$. Further, behind the dependency of q_{t+1} on s_t is the assumption that there is no information asymmetry between households and intermediaries[8].

2.3.5 Household's Decision

During their working life, that is for $t < W$, households decide whether to declare bankruptcy or not after observing the current period's income level. Let x_t denote the cash-in-hand that a household starts the period with. If it chooses not to declare bankruptcy, then the household solves

$$V_t^0(x_t, s_t) = \max_{a_{t+1}} \left\{ \begin{array}{l} u\left(x_t - q_{t+1}(a_{t+1}, s_t)\, a_{t+1}\right) + \\ \\ \delta_t E_t \left[\max \left\{ \begin{array}{c} V_{t+1}^0\left(a_{t+1} + \chi_{t+1} + y_{t+1}, s_{t+1}\right), \\ \\ \min\left\{ V_{t+1}^1\left(\Gamma(y_{t+1}), s_{t+1}\right), \underline{V}_{t+1} \right\} \end{array} \right\} \right] \end{array} \right\}, \tag{2.3.6}$$

[8] Chatterjee et al. (2008) study unsecured debt in an environment with private information.

where $\chi_{t+1} \leq 0$ denotes a potential expense shock that the household might incur in the next period and the function $\Gamma(\cdot)$ specifies the income of an agent who has declared bankruptcy,

$$\Gamma(y) = (1-\gamma)\, y - \gamma_0. \tag{2.3.7}$$

This function allows for a proportional and an absolute cost to income. The threshold value $\underline{V}_t$, analogous to the one introduced in Section 2.2.5, is defined as the value of of receiving a constant income equal to a proportion κ of average income for the rest of ones life, while being able to borrow or save at prices given by equation (2.3.13) bellow[9]. Further, it takes as given the price schedule $q_{t+1}(a_{t+1}, s_t)$ and the evolution of s_t, which follows equations (2.3.4) and (2.3.5) for the HIP model and (2.3.2) for the RIP model.

If, instead the household chooses to declare bankruptcy it will solve

$$V_t^1(x_t, s_t) = \max_{a_{t+1}\geq 0} \left\{ \begin{array}{l} u\left(x_t - q_{t+1}(a_{t+1}, s_t)\, a_{t+1}\right) + \\ \\ \delta_t E_t \left[\begin{array}{l} (1-\lambda)\max \left\{ \begin{array}{c} V_{t+1}^1\left(a_{t+1} + \chi_{t+1} + \Gamma(y_{t+1}), s_{t+1}\right), \\ \\ \min\left\{V_{t+1}^1\left(\Gamma(y_{t+1}), s_{t+1}\right), \underline{V}_{t+1}\right\} \end{array} \right\} \\ \\ +\lambda \max \left\{ \begin{array}{c} V_{t+1}^0\left(a_{t+1} + \chi_{t+1} + y_{t+1}, s_{t+1}\right), \\ \\ \min\left\{V_{t+1}^1\left(\Gamma(y_{t+1}), s_{t+1}\right), \underline{V}_{t+1}\right\} \end{array} \right\} \end{array} \right] \end{array} \right\}. \tag{2.3.8}$$

When the household declares bankruptcy, it stays on its record for a random amount of periods during which it is not allowed to sign debt contracts, thus the constraint $a_{t+1} \geq 0$. The parameter λ is the probability that a household has this record removed in the next period. Even though these households cannot accumulate debt, they still might need to declare bankruptcy if they receive an expense shock. The corresponding household's savings/borrowing policy functions are given by $a_{t+1}^0(x_t, s_t)$ and $a_{t+1}^1(x_t, s_t)$. Moreover, let $b_t(a_t + \chi_t, y_t, s_t)$ be the bankruptcy decision policy function

[9] I impose only a natural borrowing limit in the calculation of $\underline{V}_t$. This is a simplifying assumption that does not have an important assumption since $\beta(1+r)$ is close to 1 in the calibrations bellow.

associated with the second maximization in equation (2.3.6), that is

$$b_t\left(a_t+\chi_t,y_t,s_t\right)=\begin{cases}0 & \text{, if } V_t^0\left(a_t+\chi_t+y_t,s_t\right)\geq \min\left\{V_t^1\left(\Gamma\left(y_t\right),s_t\right),\underline{V}_t\right\}\\ \\ 1 & \text{, if } V_t^0\left(a_t+\chi_t+y_t,s_t\right)< \min\left\{V_t^1\left(\Gamma\left(y_t\right),s_t\right),\underline{V}_t\right\}\end{cases}$$

One could also define policy functions for the bankruptcy decisions in equation (2.3.8), however, these will not be necessary for defining the equilibrium of this economy. The reason being that any debt that the agent chooses not to pay as a result of declaring bankruptcy while excluded from credit markets must be due expense shocks. These expense shocks, however, are assumed to be a wasteful cost paid to no one inside the model[10].

The focus of this paper is on the income risk that household's face and its effect on bankruptcy decisions and credit limits. Since during retirement the households no longer face any income risk this period is only of interest to the extent that it affects households' behavior during their working life. With this in mind, I make the following simplifying assumptions. During retirement, for $t>W$ households are not allowed to borrow and cease to face expense shocks. Their decisions can be summarized by

$$V_t^{R0}\left(x_t\right)=\max_{a_{t+1}\geq 0}\left\{u\left(x_t-\left(\frac{1-p_t}{1+r}\right)a_{t+1}\right)+\delta_t V_{t+1}^{R0}\left(a_{t+1}+y_R\right)\right\}\tag{2.3.9}$$

if they enter the retirement period without a record of bankruptcy and

$$V_t^{R1}\left(x_t\right)=\begin{cases}\max_{a_{t+1}\geq 0}\left\{u\left(x_t-\left(\frac{1-p_t}{1+r}\right)a_{t+1}\right)+\delta_t V_{t+1}^{R1}\left(a_{t+1}+\Gamma\left(y_R\right)\right)\right\} & \text{, if } t<W+\tau\\ \\ \max_{a_{t+1}\geq 0}\left\{u\left(x_t-\left(\frac{1-p_t}{1+r}\right)a_{t+1}\right)+\delta_t V_{t+1}^{R0}\left(a_{t+1}+y_R\right)\right\} & \text{, if } t\geq W+\tau\end{cases}\tag{2.3.10}$$

otherwise. This means that, if a household enters retirement with bankruptcy on its record, its income will be reduced according to equation (2.3.7) for a fixed number of years, τ. Let $a_{t+1}^{R0}\left(x_t\right)$ and $a_{t+1}^{R1}\left(x_t\right)$ denote the policy functions associated the the savings decision during retirement.

[10]For a general equilibrium treatment of these costs see Chatterjee et al. (2007).

Finally, in the last period of working life, for $t = W$, agents solve

$$V_t^0(x_t, s_t) = \max_{a_{t+1}} \left\{ u\left(x_t - \left(\frac{1-p_t}{1+r} \right) a_{t+1} \right) + \delta_t V_{t+1}^{R0}(a_{t+1} + y_R(y_t)) \right\} \tag{2.3.11}$$

and

$$V_t^1(x_t, s_t) = \max_{a_{t+1} \geq 0} \left\{ u\left(x_t - \left(\frac{1-p_t}{1+r} \right) a_{t+1} \right) + \delta_t V_{t+1}^{R1}(a_{t+1} + \Gamma(y_R(y_t))) \right\} \tag{2.3.12}$$

where $y_R(\cdot)$ is given by equation (2.3.3).

2.3.6 Financial Intermediaries

Every financial intermediary has the unlimited ability to borrow or save at a fixed interest rate r. Perfect competition between them implies that they charge actuarially fair prices which amounts to

$$q_{t+1}(a_{t+1}, s_t) = \frac{1-p_t}{1+r}, \text{ if } a_{t+1} \geq 0; \tag{2.3.13}$$

$$q_{t+1}(a_{t+1}, s_t) = \frac{1-p_t}{1+r} E_t \left[1 - b_{t+1}(a_{t+1} + \chi_{t+1}, y_{t+1}, s_{t+1}) \right], \text{ if } a_{t+1} < 0, \tag{2.3.14}$$

where the numerator is the probability that a household survives until the next period and decides not to default given the face value of the contract, a_{t+1}, and the household's income state, s_t.

2.3.7 Equilibrium

Definition 8 *A recursive (partial) competitive equilibrium consists of value functions V_t^0 and V_t^1 for all $t \leq W$, and V_t^{R0} and V_t^{R1} for all $t > W$; policy functions a_{t+1}^0, a_{t+1}^1 and b_t for all $t \leq W$, and a_{t+1}^{R0} and a_{t+1}^{R1} for all $t > W$; and a pricing functions q_{t+1} for all $t < W$ such that*

1. *the value functions V_t^0 and V_t^1 satisfy the functional equations* (2.3.6), (2.3.8), (2.3.11), *and* (2.3.12), *and the policy functions a_{t+1}^0, a_{t+1}^1 and b_t are the solutions to the corresponding problems;*

2. *the value functions* V_t^{R0} *and* V_t^{R1} *satisfy the functional equations* (2.3.9)*, and* (2.3.10)*, and the policy functions* a_{t+1}^{R0} *and* a_{t+1}^{R1} *are the solutions to the corresponding problems;*

3. *the pricing functions* q_{t+1} *are given by equations* (2.3.13) *and* (2.3.14)*.*

2.3.8 Credit Limits

The definition of credit limits is analogous to Definition 7 in Section 2.2. In period t, if the household promises to pay $-a_{t+1}$ in period $t+1$, it receives $-q_{t+1}(a_{t+1}, s_t)\, a_{t+1}$ in period t. The credit limit is defined as the most that a household is allowed to borrow, that is:

Definition 9 *The credit limit* $\omega_t(s_t)$*, faced by a household with income state* s_t *in period* t*, is given by*

$$\omega_t(s_t) \equiv \max_{a_{t+1}} \left(-q_{t+1}(a_{t+1}, s_t)\, a_{t+1}\right).$$

2.4 Parametrization

The model period is one calendar year. Working life begins at age 25, and ends at age 65 ($W = 41$), and the retirement duration is 30 years ($R = 30$). Relative risk aversion is set to $\sigma = 2$ and the risk-free interest rate to $r = 1/0.96 - 1$. The lower bound income level, $\underline{y}$, is set to 5% of average working-age income.

The probability of a household having bankruptcy removed from its record is set to $\lambda = 0.1$ so that the average number of years a record is kept is 10 years. This is in line with the time period that the Fair Credit Reporting Act allows it to be maintained on an individual's record (see Figure B.1). For the same reason, τ, the number of retirement years that bankruptcy stays on a household's record when it enters retirement having such a record is set to 5 years.

For the common life cycle profile of log income, g_t, I feed into the model the empirical profile computed from the PSID. The age-dependent death probabilities p_t are taken from the National Vital Statistics Reports (Vol. 54, No. 14, April 19, 2006). For every calibration that follows, I choose the household's discount factor, β, to match an average wealth-to-output ratio of 4.9 (the value in the SCF sample).

Expense shocks are supposed to capture unexpected out-of-pocket medical bills, divorces, and unplanned pregnancies which are frequently cited as reasons for declaring bankruptcy. Livshits et al. (2007) used data from Medical Expenditure Panel Survey, the US Health Care Financing Administration, the American Hospital Association and the US Census Bureau to obtain reasonable estimates for these shocks. Following their procedure I allow χ_t to take three values summarized in Table 2.1.

Table 2.1: Expense Shocks Parameters

	No Shock	**Low Shock**	**High Shock**
% of Average Income	0.0	26.4	82.2
Probability	97.4	2.45	0.15

Notes: Livshits et al. (2007) report probabilities of the event occurring once in a three-year period, here the probabilities have been adjusted to be that of the event occurring in a year.

For the RIP and HIP income process parameters (summarized in Table 2.2) I use estimates from Guvenen (2009) and Guvenen and Smith (2014) respectively. The estimation of the RIP process is relatively standard in the literature. Identifying the parameters for the HIP process is significantly more involved. By merging consumption data from the Consumer Expenditure Survey and income data from the PSID, and using indirect inference, Guvenen and Smith (2014) obtain reliable estimates[11].

An important aspect of the calibration of the HIP income process is the determination of the prior about unobservables. The prior about z_1 is that it has mean 0 and variance σ_η^2, which follows from the assumption that z_0 is unobserved. For the prior about

[11] Guvenen and Smith (2014) allow for another dimension of profile heterogeneity which I have not included in equation (2.3.1), namely households can have different income profile intercepts besides having different profile growth rates. Including this, however, would lead to one more state variable and add a significant computational burden. Instead, I use their estimate for the intercept variance as the variance of z_0 (which they assume to be 0).

θ^i, I again make use of Guvenen and Smith (2014) who estimate that the prior variance of θ^i is only about 35.5 percent the population variance (that is , $\sigma^2_{\theta_u}/\sigma^2_{\theta} = 0.355$); households know relatively well what is the growth rate of their income when they enter the labor market (or by the time they reach the age of 25).

Table 2.2: Income Process Parameters

Model	ρ	σ_η	σ_ε	σ_{z_0}	σ_θ
RIP	0.988	0.122	0.247	0.241	-
HIP	0.768	0.196	0.008	0.265	0.0166

In some versions of the model analyzed below the credit limits that some agents face are unusually high. For instance, if κ is relatively low, an agent with a high income level will face virtually zero probability of default until the natural borrowing limit is reached. Thus the lower bound of the asset space, denoted by $\underline{a}$, must be chosen carefully. This parameter and the remaining parameters associated with the bankruptcy rules and punishments are the subject of the experiments in the next section.

2.5 Comparative Statics

Let ϕ denote the percentage increase in credit limits that results from a 1% in income (controlling for other individual characteristics). In the data ϕ is approximately 0.65 (see Section 2.6.1). However, if the only punishment for declaring bankruptcy is the loss of the ability to borrow and if anyone can declare bankruptcy, the model predicts ϕ close to 0. This is due to the following logic. Abstract from retirement or death probabilities, and consider an individual in the last period of its life, T. This individual would obviously not be able to borrow in period T, since she will not be alive to pay back any debt. Hence, in period $T-1$, the punishment of not being able to borrow is irrelevant since this will happen in any case. It follows that this individual will not be able to borrow in period $T-1$ either. By backwards induction it follows that credit limits will be 0 in every period[12]. Having a retirement period, death probabilities and an absolute punishment such as γ_0 do not mitigate this effect in a significant way.

So, in this section I study the quantitative effect of the two alternative solutions to this problem presented in Section 2.2, namely the addition of a proportional income punishment, γ, and a limit to the ability to declare bankruptcy of agents with high levels of income, controlled by the parameter κ. In both experiments I report the results for the RIP and HIP income processes.

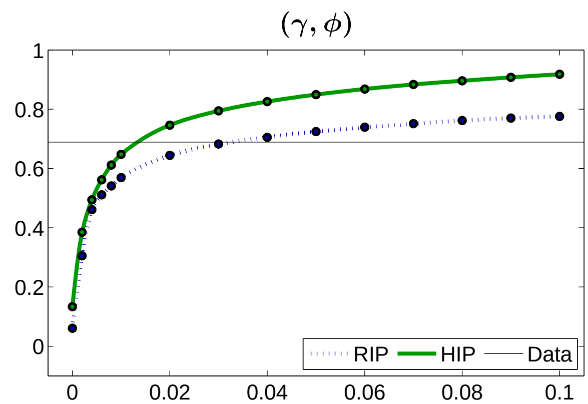
For these comparative static experiments, I need to set benchmark parameter values for $\underline{a}$ and γ_0. In the data the only 0.4% have credit limits above 2 times the average income level. Thus, I set $\underline{a}$, so that $-\underline{a}/\left(1+r\right)$ is equal to 2 times the average income in the model. The whole Chapter 7 bankruptcy process costs about 300 dollars in filing fees and an additional $1,500$ dollars in legal fees. I choose the absolute cost, γ_0, accordingly.

2.5.1 Proportional Income Punishment (changes in γ)

In this section I set $\kappa \to \infty$, so that anyone can declare bankruptcy and focus for the effect of changes in γ. Figure 2.1 shows the effect of changes in the proportional income punishment, γ, on the relationship between income and credit limits summarize by ϕ. The HIP model generates higher ϕ's than the RIP model for every level of γ which is consistent with the intuition from Section 2.2.3. Accordingly, the level of γ necessary for the model to generate the ϕ close to the data ($\phi^{data} \approx 0.65$) is significantly lower in the HIP model ($\gamma^{HIP*} \approx 0.014$) than in the RIP ($\gamma^{RIP*} \approx 0.026$). .

[12]See Bulow and Rogoff (1989) for a version of this argument for an infinitely lived agent.

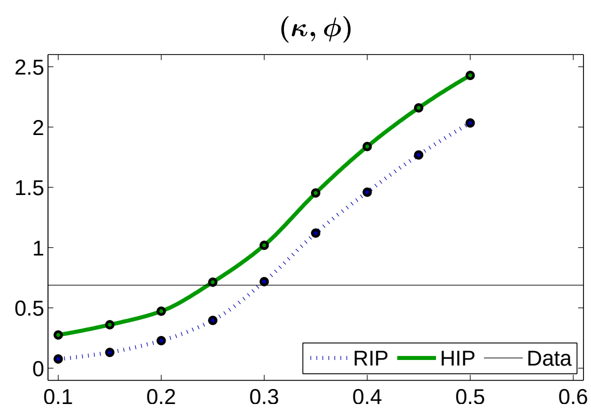
Figure 2.1



2.5.2 Restrictions on Ability to Declare Bankruptcy (changes in κ)

In this section γ is set to 0, and I focus on changes in κ. It is interesting to observe that the both curves have a positive slope in Figure 2.2. This is due to the lower bound on assets, $\underline{a}$. For low values of κ, most households face a borrowing limit in (or close to) the upper bound of $-\underline{a}/(1+r)$ and only for households with particularly low income levels do credit limits increase with income. For higher levels of κ the subset of income levels for which credit limits are below this upper bound increases and as a result the overall correlation between income and credit limits increases.

Figure 2.2



Similarly to the result in Figure 2.1 and for the same reason, the HIP model generates a higher ϕ than the RIP model for any given level of κ. However, since the slope of the curve in now positive, this leads to $\kappa^{HIP*} \approx 0.24 < 0.30 \approx \kappa^{RIP*}$.

2.6 Model versus Data

In this section I document key facts related to credit limits and bankruptcy filings and use them to calibrate the RIP and HIP versions of the model. Most of the data used in this section comes from merging the Survey of Consumer Finances (SCF) datasets from 2001 to 2010. The income processes estimated by Guvenen (2009) and Guvenen and Smith (2014) use a subset of the PSID data. In order for the SCF sample to be consistent with the one used to estimate the income processes I clean the data following the procedure described in Guvenen and Smith (2014)[13].

2.6.1 Credit Limits and Income

To measure (unsecured) credit limits I use credit card limits reported by the SCF[14]. This measure does not include some categories of unsecured debt such as unsecured personal loans. For that reason I do not use the absolute values of these limits for the analysis only its relationship with income. The underlying assumption is, therefore, that these other types of unsecured debt correlate with income in a similar way.

It is important to note that the type of credit contracts available in the model are somewhat different from the types observed in the data. In the model a household, depending on its current income state, s_t^i, faces a schedule of interest rates. This schedule generates a Laffer type curve, at some point promising to pay more in the next period (i.e. choosing a higher $-a_{t+1}$) lead to lower amount of credit (i.e. a lower $-q_{t+1}(a_{t+1}, s_t)\, a_{t+1}$). Credit limits in the model are defined as the top of this Laffer

[13] First I drop the households in the top 1% of net worth since the PSID is known for being a good representation of the bottom 99% (see Juster et al. (1999)). Then, I keep only married, working age, non-retired, non-disabled, and male-headed observations in order to preserve consistency with Guvenen and Smith (2014).

[14] More precisely, a household's credit limit is their answer to the question "What is the maximum amount you could borrow on (this account/ all of these accounts); that is, what is your total credit limit?" which is asked after inquires about each of the household's credit cards.

curve. In the data, on the other hand, credit card contracts usually offer a limit, independent of the loan size, at a pre-approved interest rate[15]. The difference, however, is not irreconcilable. First, the contracts being offered on the wrong side of the Laffer curve are never chosen in the model and could be ignored, I could have simply assumed that such contracts are not permitted without any consequences. Moreover, the "right" part of the schedule can be understood as the interest rates a household would face if it chooses to acquire additional credit cards until eventually it is denied[16].

The measure of income is after taxes and transfers earnings calculated using the SCF together with the NBER program TAXSIM. Regressing log-credit limits on log-income, a cubic in age and other household observables[17] leads to the following results (Figure B.2 shows the corresponding scatter plot).

Table 2.3: Credit Limits vs. Income

Dependent Variable: Log Credit Limit			
Log Income	0.736	0.695	0.689
	(0.013)	(0.012)	(0.012)
Cubic in Age	No	Yes	Yes
Other Observ.	No	No	Yes
Adjusted $\mathbf{R^2}$	0.149	0.179	0.200

Notes: Standard errors in parenthesis. Based on 29281 observations from the SCF 2001-2010.

2.6.2 Bankruptcy Abuse Prevention and Consumer Protection Act

Before moving on with the comparison between model and data it is important to notice a significant policy change that happened during the period under analysis. The Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 made

[15]For an analysis of credit limits that takes this fact explicitly into consideration see Mateos-Planas (2013).

[16]Around 84 percent of the SCF sample has at least one credit card, out of those 74 percent report have more than one, 40 percent more than two, and 7 percent more than five.

[17]Namely: family size, race, survey year, and cohort dummies.

several changes to bankruptcy law[18]. Most notably it imposed a restriction on the ability of some agents to declare bankruptcy under Chapter 7. If a household's monthly income averaged over the six months prior to filing is less than the median monthly income level in their state (adjusted for family size) then it can file. Otherwise, the household must fill out a form detailing its income and necessary expenses (including debt payments) which are used to calculate its disposable income. If their calculated disposable income is above 166 dollars a month, the household must file under Chapter 13 (which involves garnishment of future income).

This change in the law had important effects, on impact, in the proportion of bankruptcies filed under Chapter 7 (see Figure B.3). This proportion which used to average around 72% filings spiked in the months before the reform was signed into law to about 86%. Then, it dropped to 58% before virtually recovering to its initial level. This together with the fact that the distribution of the income of those who file for bankruptcy did not change in any significant way (see Figure B.4), lead me to conclude that this change would not affect the numerical experiments I perform in this paper which assumes a constant bankruptcy system throughout. The fact that the BAPCPA had relatively minor long run effects could, in fact, be used a supporting evidence of the modeling of how bankruptcy judges make their decisions. In short what the means test implies is that one can only file under Chapter 7 if it cannot support a minimal level of consumption that qualifies as necessary. The fact that the proportions households who file under Chapter 7 did not change significantly could be explained by the fact that a version of such a rule was already in place and that households who knew they would not be granted bankruptcy (and, therefore did not file) are the same who cannot file after BAPCPA.

2.6.3 Model Performance

In what follows I present results for the calibration of the RIP and HIP versions of the model models in which the discount factor, β, and the bankruptcy parameters γ_0, γ, κ, and $\underline{a}$ are calibrated to match ϕ and other important statistics related to unsecured credit. Table 2.4 summarizes the results.

[18] See White (2007) for a detailed discussion.

Table 2.4: Model Performance

Parameters			**Targets**			
	RIP	**HIP**		**Data**	**RIP**	**HIP**
β	0.967	0.968	**Wealth-to-Inc.**	4.90	4.91	4.90
$\boldsymbol{\gamma_0}/\overline{\mathbf{y}}$	0.005	0.001	ϕ	0.69	0.69	0.70
γ	0.010	0.029	**Debt-to-Inc.**	0.22	0.22	0.22
κ	0.452	0.437	**Bkr. Filings (%)**	0.46	0.39	**0.04**
$-\underline{\mathbf{a}}/(\mathbf{1}+\mathbf{r})\overline{\mathbf{y}}$	0.703	0.536	**Bkr. from Inc.**	0.53	0.59	0.49

Households in Debt

Since there is only one type of asset in the model I use the household's consolidated asset position, net-worth, as a measure of their asset level. This implies that only households with negative net-worth will be considered to be in debt It also implies that a negative net-worth is associated with unsecured debt. Though not perfect, this assumption is not as problematic as it might seem at first. Since, for instance, if a household has a mortgage with a principal above the market value of their house, it is reasonable to consider the difference to be unsecured debt. A household would lose their house if it chooses to default on such a loan, the value of the house would be automatically deducted from what it owes, some of the extra debt might be forgiven but usually most of it is assumed as unsecured debt (this procedure is known as "short payoff"). Using data from the SCF I calculate that about 5.2 percent of households have negative net-worth and that the average debt-to-income ratio of those households is close to 22.3 percent. Both versions of the model are able to match this target.

Bankruptcy Filings

Using data from the Administrative Office of the US Courts and of the US Census I compute the percentage of households who file for bankruptcy each year. Between 2001 and 2010 the average number of bankruptcies filed was $979,715$ while the average population (above age 20) was $212,435,033$, so about 0.456 percent of households filled for bankruptcy each year. The RIP model performs relatively well only slightly underestimating the number of filings. On the other hand, the HIP model is able to generate only about 10 percent of the number of bankruptcy filings.

This is an important failure and can be viewed as a problem for the HIP literature. One of the main findings in this literature is that once profile heterogeneity is allowed the models are able to generate the variance in income of households with significantly less risk. A large part of this variance is shown to be due to differences in income profiles which the households know a lot about. In fact, in this paper I am using the parameters from the simpler version of the model in Guvenen and Smith (2014). More elaborate versions that allow for private partial insurance imply even less risk. The problem highlighted here is that at these levels of risk the model severally underestimates the number of bankruptcy filings.

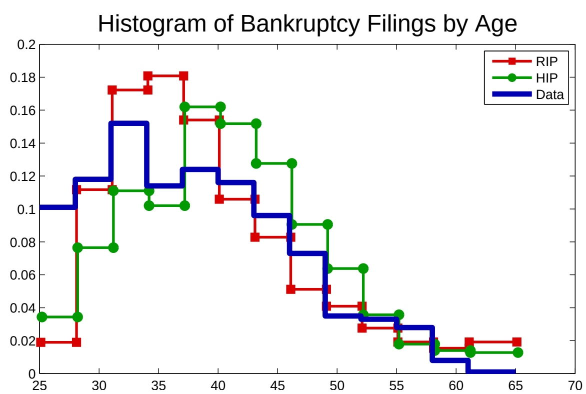
Reasons for Declaring Bankruptcy In 1996, the Panel Study of Income Dynamics (PSID), asked its respondents who had filed for bankruptcy if they used Chapter 7 or 13 and what was their main reason for doing so[19]. Out of those who filed Chapter 7, around 12.2 percent reported the main reason being job loss, 41.3 percent credit misuse, 14.3 marital disruption, 16.4 percent health-care bills, and 15.9 percent reported lawsuits/harassment. Following Chatterjee et al. (2007) I associate the first two reasons (job loss and credit misuse) with shocks in the income process and the following three (marital disruption, health-care bills, and lawsuits/harassment) with expense shocks. I assume that a bankruptcy filing in the model is due to an expense shock if the household receives such a shock concomitantly. The corresponding target is in the last row of Table 2.4. This data is based on only 189 households in the PSID who had filed Chapter 7 bankruptcy, so in the calibration procedure I used a weight for this target that was 1/3

[19] See Chakravarty and Rhee (1999) for a more detailed analysis of this dataset.

of the other weights. This lead to results that do not match the target exactly but are close enough.

Filings over the life cycle Figure 2.3 displays the pattern of bankruptcy filings over the life-cycle in the data and in the two versions of the model.

Figure 2.3



The models have similar performances in this untargeted dimension with most bankruptcy filings occurring between the ages of 25 and 50. The fact that households start their lives in the model at 25 years of age and with zero net worth reduces the ability of the model to generate enough fillings between the ages of 25 and 30; at these ages most households have not accumulated enough debt. Starting the model at an earlier age could be a way to fix this problem.

2.7 Conclusion

This paper investigates the quantitative relevance of the key determinants of credit limits. I find that the introduction of profile heterogeneity in the households' income processes increases the correlation between income and credit limits in all bankruptcy systems evaluated. This fact is consistent with the theoretical results established earlier in the paper (in Section 2.2). Relative to other results in the literature I find that calibrated proportional income punishments are quantitatively small with or without profile heterogeneity (1.4 percent and 2.6 percent respectively). A threshold level of income, such that agents are only allowed to declare bankruptcy for income levels bellow that threshold, is shown to be an alternative to such proportional punishments. I also show that credit limits and income levels are strongly related in the data, a fact that I argue can be used to further discipline bankruptcy models. Finally, the main calibration results suggest an important qualification about HIP models: the lower levels of uncertainty implied by these models lead to a severe underestimation of the number of bankruptcy filings.

The findings in this paper open new avenues of research that should be explored. The importance of profile heterogeneity for the results shows that patterns on credit limits could be useful in obtaining further empirical tests about this assumption and help refine the identification of the associated income process parameters. It would also be interesting to study these differences from a normative perspective, it is likely that the optimal bankruptcy system is significantly different depending on which process is being analyzed. The parameters that determine the bankruptcy system in the proposed model (i.e. the proportional income cost and the threshold level of income) are good candidates for such an experiment. Finally, this paper highlights the importance of the bankruptcy rules and punishments, it would be interesting if one could obtain more direct evidence on them.

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Appendix A

Appendix to Chapter 1

This appendix presents concise versions of the proofs. Extensive versions with more details are contained in a separate online appendix which can be found in our websites.

A.1 Proofs for two-period economies

A.1.1 Uncertainty economy

Define $\tau_R^k \equiv r\tau^k/(1+r)$. Six equations determine a tax distorted equilibrium (K, n_L, n_H, r, w; τ^n, τ_R^k, T) according to Definition 1: the first order conditions of the agent's problem (one intertemporal and two intratemporal), the first order conditions of the firm's problem

$$r = f_K(K,N), \quad \text{and} \quad w = f_N(K,N), \quad \text{where } N = \pi e_L n_L + (1-\pi) e_H n_H \quad \text{(A.1.1)}$$

and the government's budget constraint. Using equation (A.1.1) to substitute out for r and w we are left with a system of four equations that any vector $\left(K, n_L, n_H, \tau^n, \tau_R^k, T\right)$ of equilibrium values must satisfy. The two degrees of freedom are a result of the fact that the planner has three instruments $\left(\tau^n, \tau_R^k, T\right)$ that are restricted by one equation, the government's budget constraint. Defining welfare by

$$W \equiv u(\omega - K, \bar{n}) + \beta E\left[u\left((1-\tau^n) f_N(K,N) e_i n_i + \left(1-\tau_R^k\right) f_K(K,N) K + T\right), n_i\right]$$

and totally differentiating the four equilibrium equations together with this definition and making the appropriate simplifications using Assumption 1 we obtain the following equation (the algebra is tedious and, therefore, suppressed[1]):

$$dW = \Theta^n d\tau^n + \Theta^k d\tau_R^k,$$

where Θ^n and Θ^k are complicated functions of equilibrium variables[2].

Lemma 2 *Under Assumption 1, in equilibrium $n_H > n_L$ and $u_c(c_L, n_L) > u_c(c_H, n_H)$.*

The proof of this Lemma is contained in the online appendix.

Proof of Proposition 1. First notice that the optimal tax system must satisfy $\Theta^n = 0$ and $\Theta^k = 0$, otherwise there would exist variations in $\left(\tau^n, \tau_R^k\right) \in (-\infty, 1)^2$ that would increase welfare. $\Theta^k = 0$ simplifies to $\theta_1^k + \theta_2^k \tau^n + \theta_3^k \tau_R^k = 0$ where

$$\theta_1^k \equiv f_N f_{KN} N \left(V_c - U_c\right), \quad \theta_2^k \equiv f_N f_{KN} N \left((1+\kappa) U_c - V_c\right), \quad \text{and} \quad \theta_3^k \equiv f_K \left(f_N + f_{KN} K \kappa\right) U_c.$$

Solving this equation for τ_R^k, substituting it in $\Theta^n = 0$ and simplifying entails

$$V_c \left(1 - \tau^n\right) - U_c \left(1 - (1+\kappa) \tau^n\right) = 0.$$

[1]Mathematica codes that compute all the algebraic steps are available upon request.

[2]Here are the exact formulas:

$$\Theta^k \equiv \frac{f_K K U_c}{\Phi} \left\{ f_N f_{KN} N \left[(1-\tau^n)(V_c - U_c) + \tau^n \kappa U_c\right] + \tau_R^k f_K \left(f_N + f_{KN} K \kappa\right) U_c \right\}.$$

$$\Theta^n \equiv \frac{f_N N}{(1-\tau^n)\Phi} \{ \left(1 - \tau_R^k\right) f_K^2 f_N K \left[(1-\tau^n) \left(U_{cc} (U_c - V_c) + \tau_R^k (V_{cc} - U_{cc}) U_c \right) - \left(1 - \tau_R^k\right) \tau^n \kappa U_{cc} U_c \right]$$
$$+ f_N \left[(1-\tau^n)(V_c - U_c) + \tau^n \kappa U_c\right] \left[\left(1 - \tau_R^k\right) f_{KN} N U_c - K u_{cc}^0 \right] + \left(1 - \tau_R^k\right) \tau_R^k f_{KN} f_K K \kappa U_c^2 \}.$$

where

$$U_c \equiv \beta \left[\pi u_c(c_L, n_L) + (1-\pi) u_c(c_H, n_H)\right], \quad U_{cc} \equiv \beta \left[\pi u_{cc}(c_L, n_L) + (1-\pi) u_{cc}(c_H, n_H)\right],$$

$$V_c \equiv \beta \left[\pi u_c(c_L, n_L) \frac{e_L n_L}{N} + (1-\pi) u_c(c_H, n_H) \frac{e_H n_H}{N} \right],$$

$$V_{cc} \equiv \beta \left[\pi u_{cc}(c_L, n_L) \frac{e_L n_L}{N} + (1-\pi) u_{cc}(c_H, n_H) \frac{e_H n_H}{N} \right],$$

$$\Phi \equiv \left(1 - \tau_R^k\right) \left(f_K f_N f_{KN} K N \left((1-\tau^n)(V_{cc} - U_{cc}) + \tau^n \kappa U_{cc}\right) + \left(f_N + f_{KN} K \kappa\right) f_K^2 K U_{cc} - f_N f_{KN} N U_c \right)$$
$$+ \left(f_N + f_{KN} K \kappa\right) K u_{cc}^0.$$

Solving for τ^n we obtain equation (1.2.1) and substituting it back in the equation for τ_R^k we obtain $\tau_R^k = 0$; and, therefore, $\tau^k = 0$. This is the only pair $\left(\tau^n, \tau_R^k\right) \in (-\infty, 1)^2$ that solves the system $\Theta^n = 0$ and $\Theta^k = 0$. The fact that the optimal level of $\tau^n > 0$ follows from Lemma 2. ■

A.1.2 Inequality economy

The proof of Proposition 2 is entirely analogous and for that reason suppressed here. It can be found in the online appendix.

A.2 Proofs for complete market economies

The proofs follow straight-forwardly the approach introduced by Werning (2007). Hence, for details on the logic behind the procedure we refer the reader to that paper, here we focus mainly on the parts that comprise our value added. We depart from Werning (2007) in following ways: we use the GHH utility function (whereas he studies the separable and Cobb-Douglas cases), we do not allow the Ramsey planner to choose time zero policies and impose an upper bound of 1 for capital income taxes. These departures make the Ramsey planner's problem comparable to our benchmark experiment. The restriction on time zero policies is particularly important because it prevents the planner from confiscating the (potentially unequal) initial capital levels eliminating the corresponding redistribution motives.

Consider Economy 4 as described in Section 1.6. For simplicity, we assume that agents are divided into a finite number of types $i \in I$ of relative size π_i. Type i has an initial asset position of $a_{i,0}$ and a productivity level of e_i. Let p_t denote the price of the consumption good in period t in terms of period 0. Since markets are complete we can write down the present value budget constraint of the agent (remember that τ^c is a parameter),

$$\sum_{t=0}^{\infty} p_t \left(\left(1+\tau^c\right) c_{i,t} + a_{i,t+1} \right) \leq \sum_{t=0}^{\infty} p_t \left(\left(1-\tau_t^n\right) w_t e_i n_{i,t} + R_t a_{i,t} + T_t \right),$$

where $R_t \equiv 1 + \left(1 - \tau_t^k\right) r_t$. Rule out arbitrage opportunities by setting $p_t = R_{t+1} p_{t+1}$, and define $T \equiv \sum_{t=0}^{\infty} p_t T_t$. Then, the budget constraint simplifies to

$$\sum_{t=0}^{\infty} p_t \left((1+\tau^c) c_{i,t} - (1-\tau_t^n) w_t e_i n_{i,t} \right) \leq R_0 a_{i,0} + T. \tag{A.2.1}$$

Similarly, the government's budget constraint simplifies to

$$R_0 B_0 + T + \sum_t p_t G = \sum_t p_t \left(\tau^c C_t + \tau_t^n w_t N_t + \tau_t^k r_t K_t \right). \tag{A.2.2}$$

The resource constraint is given by

$$C_t + G + K_{t+1} = f(K_t, N_t), \quad \text{for all } t \geq 0. \tag{A.2.3}$$

Definition 10 *Given $\{a_{i,0}\}$, K_0, B_0 and $\left(\tau_0^n, \tau_0^k, T_0\right)$, a competitive equilibrium is a policy $\left\{\tau_t^n, \tau_t^k, T_t\right\}_{t=1}^{\infty}$, a price system $\{p_t, w_t, r_t\}_{t=0}^{\infty}$, and an allocation $\{c_{i,t}, n_{i,t}, K_{t+1}\}_{t=0}^{\infty}$, such that: (i) agents choose $\{c_{i,t}, n_{i,t}\}_{t=0}^{\infty}$ to maximize utility subject to budget constraint (A.2.1) taking policies and prices (that satisfy $p_t = R_{t+1} p_{t+1}$) as given; (ii) firms maximize profits; (iii) the government's budget constraint (A.2.2) holds; and (iv) markets clear: the resource constraints (A.2.3) hold.*

Given aggregate levels C_t and N_t, individual consumption and labor supply levels can be found by solving the following static subproblem

$$U^m(C_t, N_t; \varphi) \equiv \max_{c_{i,t}, n_{i,t}} \sum_i \pi_i \varphi_i u(c_{i,t}, n_{i,t})$$

$$\text{s.t.} \sum_i \pi_i c_{i,t} = C_t \quad \text{and} \quad \sum_i \pi_i e_i n_{i,t} = N_t$$

where u is given by equation (1.4.1), for some vector $\varphi \equiv \{\varphi_i\}$ of market weights $\varphi_i \geq 0$. Let $c_{i,t}^m(C_t, N_t; \varphi)$, and $n_{i,t}^m(C_t, N_t; \varphi)$ be the *argmax* of this problem. It can be shown that[3]

$$
\begin{aligned}
c_{i,t}^{m}\left(C_t, N_t; \varphi\right) &= \omega_i^c C_t + \chi \frac{\kappa}{1+\kappa}\left(\left(\omega_i^n\right)^{\frac{1+\kappa}{\kappa}} - \omega_i^c \Omega^n\right)\left(N_t\right)^{\frac{1+\kappa}{\kappa}} \\
n_{i,t}^{m}\left(C_t, N_t; \varphi\right) &= \omega_i^n e_i N_t \\
U^{m}\left(C_t, N_t; \varphi\right) &= \frac{\Omega^c}{1-\sigma}\left(C_t - \Omega^n \chi \frac{\kappa}{1+\kappa}\left(N_t\right)^{\frac{1+\kappa}{\kappa}}\right)^{1-\sigma}
\end{aligned}
$$

Then, implementability constraints can be written as

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t (U_C^m\left(C_t, N_t; \varphi\right) c_{i,t}^m\left(C_t, N_t; \varphi\right) + U_N^m\left(C_t, N_t; \varphi\right) n_{i,t}^m\left(C_t, N_t; \varphi\right)) \\
= U_C^m\left(C_0, N_0; \varphi\right)\left(\frac{R_0 a_{i,0} + T}{1+\tau^c}\right) \quad \text{for all } i \in I
\end{aligned}
\tag{A.2.4}
$$

Proposition 10 *An aggregate allocation $\{C_t, N_t, K_{t+1}\}_{t=0}^{\infty}$ can be supported by a competitive equilibrium if and only if the resource constraints (A.2.3) hold and there exist market weights φ and a lump-sum tax T so that the implementability conditions (A.2.4) hold for all $i \in I$. Individual allocations can then be computed using functions $c_{i,t}^m$ and $n_{i,t}^m$, prices and taxes can be computed using the usual equilibrium conditions.*

The Ramsey problem is that of choosing policies $\left\{\tau_t^n, \tau_t^k, T_t\right\}_{t=1}^{\infty}$, taking $\{a_{i,0}\}$, K_0, B_0 and $\left(\tau_0^n, \tau_0^k, T_0\right)$ as given, to maximize a weighted sum of the individual utilities,

$$
\sum_{t=0}^{\infty} \beta^t \pi_i \lambda_i u\left(c_{i,t}, n_{i,t}\right), \tag{A.2.5}
$$

where $\{\lambda_i\}$ are the welfare weights normalized so that $\sum_i \pi_i \lambda_i = 1$ with $\lambda_i \geq 0$, subject to allocations and policies being a part of a competitive equilibrium and $\tau_t^k \leq 1$ for all $t \geq 1$.

---

[3]Where constants are defined as follows:

$$
\omega_i^c \equiv \frac{\left(\varphi_i\right)^{\frac{1}{\sigma}}}{\sum_j \pi_j \left(\varphi_j\right)^{\frac{1}{\sigma}}}, \quad \omega_i^n \equiv \frac{\left(e_i\right)^{\kappa}}{\sum_j \pi_j \left(e_j\right)^{1+\kappa}}, \quad \Omega^c \equiv \left(\sum_i \pi_i \left(\varphi_i\right)^{\frac{1}{\sigma}}\right)^{\sigma}, \quad \text{and} \quad \Omega^n \equiv \left(\sum_j \pi_j \left(e_j\right)^{1+\kappa}\right)^{-\frac{1}{\kappa}}
$$

First notice that in equilibrium it must be that

$$U_C^m(t) = \beta\left(1 + \left(1 - \tau_{t+1}^k\right) r_{t+1}\right) U_C^m(t+1), \tag{A.2.6}$$

so that

$$U_C^m(t) \geq \beta U_C^m(t+1), \tag{A.2.7}$$

is equivalent to $\tau_{t+1}^k \leq 1$. Moreover, notice that τ_0^k and T_0 have not been substituted out in the implementability constraint. The fact that τ_0^n is given together with the equilibrium condition $(1 - \tau_0^n) w_0 = -U_N^m(0) / U_C^m(0)$ is equivalent to

$$N_0 = \bar{N}_0, \tag{A.2.8}$$

where $\bar{N}_0$ is defined implicitly as a function of variables given to the Ramsey planner,

$$(1 - \tau_0^n) f_N\left(K_0, \bar{N}_0\right) = \Omega^n \chi\left(\bar{N}_0\right)^{\frac{1}{\kappa}}.$$

Finally, we can use Proposition 10 to rewrite the Ramsey problem as that of choosing $\{C_t, N_t\}_{t=0}^{\infty}$, T, and φ to maximize ($A.2.5$) subject to ($A.2.3$) for all $t \geq 0$, ($A.2.4$) for all $i \in I$ with multiplier μ_i, ($A.2.7$) for all $t \geq 0$ with multiplier η_t, and ($A.2.8$). Equivalently, we can write it as that of solving the following auxiliary problem

$$\max_{\{C_t, N_t\}_{t=0}^{\infty}, T, \varphi} \sum_{t=0}^{\infty} \beta^t W\left(C_t, N_t; \varphi, \mu, \lambda\right) - U_C^m\left(C_0, N_0; \varphi\right) \sum_{i \in I} \mu_i \left(\frac{R_0 a_{i,0} + T}{1 + \tau^c}\right),$$

subject to ($A.2.3$) for all $t \geq 0$, ($A.2.7$) for all $t \geq 0$, and ($A.2.8$), where

$$\begin{aligned} W\left(C_t, N_t; \varphi, \mu, \lambda\right) \equiv \sum_i \pi_i \{ & \lambda_i u\left(c_{i,t}^m\left(C_t, N_t; \varphi\right), n_{i,t}^m\left(C_t, N_t; \varphi\right)\right) \\ & + \mu_i\left(U_C^m\left(C_t, N_t; \varphi\right) c_{i,t}^m\left(C_t, N_t; \varphi\right) + U_N^m\left(C_t, N_t; \varphi\right) n_{i,t}^m\left(C_t, N_t; \varphi\right)\right)\}. \end{aligned}$$

With some algebra it can be shown that[4]

$$W(C_t, N_t; \varphi, \mu, \lambda) =$$
$$\frac{1}{1-\sigma}\left(C_t - \Omega^n \chi \frac{\kappa}{1+\kappa}(N_t)^{\frac{1+\kappa}{\kappa}}\right)^{-\sigma}\left(\Phi C_t - (\Phi + (1-\sigma)\Psi)\Omega^n \chi \frac{\kappa}{1+\kappa}(N_t)^{\frac{1+\kappa}{\kappa}}\right) \tag{A.2.9}$$

Define $R_t^* \equiv 1 + r_t$ and

$$\eta_{-1} \equiv \frac{R_0}{\beta(1+\tau^c)} \sum_i \pi_i \mu_i a_{i,0},$$

and first order conditions (for the following proofs we need only necessary conditions) together with equilibrium conditions imply the following equations[5]

$$\sum_i \pi_i \mu_i = 0 \tag{A.2.10}$$

$$\frac{\tau_t^n + \tau^c}{1+\tau^c} = \frac{\Psi \Theta_t}{\Phi \Theta_t + \Psi(\sigma + \Theta_t) + \Upsilon_t \sigma(\beta \eta_{t-1} - \eta_t)}, \quad \text{for } t \geq 1 \tag{A.2.11}$$

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{\Phi \Theta_{t+1} + \Psi \sigma + \Upsilon_{t+1} \sigma(\beta \eta_t - \eta_{t+1})}{\Phi \Theta_t + \Psi \sigma + \Upsilon_t \sigma(\beta \eta_{t-1} - \eta_t)} \frac{\Theta_t}{\Theta_{t+1}}, \quad \text{for } t \geq 0 \tag{A.2.12}$$

Notice that $\Upsilon_t > 0$ and $\Theta_t > 0$, for all $t \geq 0$.

A.2.1 Economy 2

Lemma 3 *If $e_i = 1$ for all $i \in I$, then $\Psi = 0$ and $\Phi > 0$.*

---

[4] Where constants are defined as follows:

$$\Phi \equiv \sum_j \pi_j \left(\frac{\lambda_j}{\varphi_j} - \sigma \mu_j \omega_j^c\right), \quad \text{and} \quad \Psi \equiv \frac{\Omega^c}{\kappa} \sum_j \pi_j \mu_j e_j \omega_j^n.$$

[5] Where $\Upsilon_t \equiv \Omega^c / \Omega^n \chi \frac{\kappa}{1+\kappa} (N_t)^{\frac{1+\kappa}{\kappa}}$.

Proof. If $e_i = 1$ for all $i \in I$, then it follows from the definition of Ψ that

$$\Psi = \frac{\Omega^c}{\varepsilon} \frac{\sum_j \pi_j \mu_j (e_j)^{1+\varepsilon}}{\sum_j \pi_j (e_j)^{1+\varepsilon}} = \frac{\Omega^c}{\varepsilon} \frac{\sum_j \pi_j \mu_j}{\sum_j \pi_j} = 0$$

where the last equality follows from equation ($A.2.10$). Next, notice that

$$u\left(c_{i,t}^m (C_t, N_t; \varphi), n_{i,t}^m (C_t, N_t; \varphi)\right) = \frac{(\omega_i^c)^{1-\sigma}}{1-\sigma} \left(C_t - \Omega^n \chi \frac{\kappa}{1+\kappa} (N_t)^{\frac{1+\kappa}{\kappa}}\right)^{1-\sigma}$$

and, therefore, the solution to the problem must satisfy $C_t > \Omega^n \chi \frac{\kappa}{1+\kappa} (N_t)^{\frac{1+\kappa}{\kappa}}$ for all $t \geq 0$. Otherwise, the objective function of the Ramsey problem would be $-\infty$. On the other hand, since $\Psi = 0$, it follows from equation ($A.2.9$) that

$$W(C_t, N_t; \varphi, \mu, \lambda) = \frac{\Phi}{1-\sigma} \left(C_t - \Omega^n \chi \frac{\kappa}{1+\kappa} (N_t)^{\frac{1+\kappa}{\kappa}}\right)^{1-\sigma}.$$

It follows that, if $\Phi \leq 0$, setting $C_0 = f\left(K_0, \bar{N}_0\right) - G$, and $C_t = N_t = 0$, for all $t \geq 1$ (so that $C_t = \Omega^n \chi \frac{\kappa}{1+\kappa} (N_t)^{\frac{1+\kappa}{\kappa}}$ for all $t \geq 1$) would maximize the objective function of the auxiliary problem while being feasible which is a contradiction. ■

Proof of Proposition 4. Using Lemma 3, from equation ($A.2.11$) it follows that

$$\tau_t^n = -\tau^c, \text{ for } t \geq 1.$$

Next, suppose $\eta_t = 0$, for all $t \geq 0$. Then, it follows from ($A.2.12$) that $\tau_1^k < 1$ if

$$-\frac{1}{\beta} \frac{\Phi \Theta_0}{\Upsilon_0 \sigma} \equiv P_1 < \eta_{-1} < M_1 \equiv \frac{1}{\beta} \frac{(R_1^* - 1) \Phi \Theta_0}{\Upsilon_0 \sigma},$$

and that $\tau_t^k = 0$ for $t \geq 2$. Hence, if $P_1 < \eta_{-1} < M_1$, the constraints will in fact never be binding. Now, suppose $\eta_t > 0$, for $t \leq t^* - 2$ and $\eta_t = 0$, for all $t \geq t^* - 1$, then it follows from ($A.2.12$) that $\tau_{t^*}^k < 1$ if

$$-\sum_{\tau=1}^{t^*} \frac{1}{\beta^\tau} \frac{\Phi \Theta_{\tau-1}}{\Upsilon_{\tau-1} \sigma} \equiv P_{t^*} < \eta_{-1} < M_{t^*} \equiv \sum_{\tau=1}^{t^*} \frac{1}{\beta^\tau} \frac{\left(\prod_{t=\tau}^{t^*} R_t^* - 1\right) \Phi \Theta_{\tau-1}}{\Upsilon_{\tau-1} \sigma},$$

and that $\tau_t^k = 0$ for $t \geq t^* + 1$. The result follows from the fact that η_{-1} is finite, $\lim_{t\to\infty} P_t = -\infty$ and $\lim_{t\to\infty} M_t = \infty$.

■

A.2.2 Economy 3

Proof of Proposition 5. In this economy there is no heterogeneity in initial levels of asset, i.e. $a_{i,0} = a_0$ for all $i \in I$. Then it follows that

$$\eta_{-1} = \frac{R_0}{\beta\left(1+\tau^c\right)} \sum_i \pi_i \mu_i a_{i,0} = \frac{R_0}{\beta\left(1+\tau^c\right)} a_0 \sum_i \pi_i \mu_i = 0$$

where the last equality follows from equation ($A.2.10$). Since here we assume that τ_t^k does not have to be bounded by 1, it follows that $\eta_t = 0$ for all $t \geq 1$. Then, equation (1.6.1) follows directly from equation ($A.2.11$), (1.6.2) from its derivative with respect to Θ_t, and (1.6.3) from equations ($A.2.11$) and ($A.2.12$). ■

A.2.3 Economy 4

Proof of Proposition 6. Equation (1.6.3) can be established for all $t \geq 1$, by substituting ($A.2.11$) into ($A.2.12$). The existence of a t^* such that $\eta_t > 0$, for $t < t^* - 1$ and $\eta_t = 0$, for all $t \geq t^* - 1$, follows from a very similar logic to the one used in the proof of Proposition 4, which we suppress here[6]. Hence, for $t \geq t^*$ we can obtain τ_t^n by using (1.6.1), which follows from ($A.2.11$) with $\eta_t = 1$. For the same time period τ_t^k can then be found by using (1.6.3). Now, having $\tau_{t^*}^n$ we can use the fact that $\tau_t^k = 1$ and (1.6.3) moving backwards to obtain τ_t^n for $t < t^*$. ■

[6]With

$$P_{t^*} \equiv -\sum_{\tau=1}^{t^*} \frac{1}{\beta^\tau} \frac{\Phi\Theta_{\tau-1} + \Psi\sigma}{\Upsilon_{\tau-1}\sigma}, \quad \text{and} \quad M_{t^*} \equiv \sum_{\tau=1}^{t^*} \frac{1}{\beta^\tau} \frac{\left(\prod_{t=\tau}^{t^*} R_t^* - 1\right)\Phi\Theta_{\tau-1} + \left(\frac{\Theta_{\tau-1}}{\Theta_{t^*}} \prod_{t=\tau}^{t^*} R_t^* - 1\right)\Psi\sigma}{\Upsilon_{\tau-1}\sigma}$$

A.3 Welfare decomposition

Let $v(x) \equiv u(c,n)$ where x is the consumption-labor composite defined in Section 1.5.3 and u is defined in (1.4.1). Consider a policy reform. Denote by $x_t^R(a_0, e^t)$ the equilibrium consumption-labor composite path of an agent with initial assets a_0 and history of productivities e^t if the reform is implemented. Let $x_t^{NR}(a_0, e^t)$ be the equilibrium path in case there is no reform. The average welfare gain, Δ, that results from implementing the reform is defined as the constant percentage increase to $x_t^{NR}(a_0, e^t)$ that equalizes the (utilitarian) welfare to the value associated with the reform, that is,

$$\int E_0\left[U\left((1+\Delta)\left\{x_t^{NR}\left(a_0, e^t\right)\right\}\right)\right] d\lambda_0\left(a_0, e_0\right) = \int E_0\left[U\left(\left\{x_t^R\left(a_0, e^t\right)\right\}\right)\right] d\lambda_0\left(a_0, e_0\right), \tag{A.3.1}$$

where λ_0 is the initial distribution over states (a_0, e_0) and

$$U\left(\left\{x_t\left(a_0, e^t\right)\right\}\right) \equiv \sum_{t=0}^{\infty} \beta^t v(x_t(a_0, e^t)) = \sum_{t=0}^{\infty} \beta^t u\left(c_t\left(a_0, e^t\right), n_t\left(a_0, e^t\right)\right).$$

Define

$$X_t^j \equiv \int x_t^j\left(a_0, e^t\right) d\lambda_t^j\left(a_0, e^t\right), \quad \text{for } j = R, NR.$$

to be the average level of x at each t. Then, the level effect, Δ_L, is

$$U\left((1+\Delta_L)\left\{X_t^{NR}\right\}\right) = U\left(\left\{X_t^R\right\}\right), \tag{A.3.2}$$

In order to define the other two components we need some previous definitions. Let $\bar{x}^j(a_0, e_0)$ denote the individual consumption-labor certainty equivalent,

$$U\left(\left\{\bar{x}^j\left(a_0, e_0\right)\right\}\right) = E_0\left[U\left(\left\{x_t^j\left(a_0, e^t\right)\right\}\right)\right], \quad \text{for } j = R, NR, \tag{A.3.3}$$

(notice that $\bar{x}^j(a_0, e_0)$ can be chosen to be constant) and let $\bar{X}^j$ be the aggregate consumption-labor certainty equivalent,

$$\bar{X}^j = \int \bar{x}^j\left(a_0, e_0\right) d\lambda\left(a_0, e_0\right), \quad \text{for } j = R, NR. \tag{A.3.4}$$

The insurance effect, Δ_I, is defined by

$$1+\Delta_I \equiv \frac{1-p_{unc}^R}{1-p_{unc}^{NR}}, \text{ where } \quad U\left(\left(1-p_{unc}^j\right)\left\{X_t^j\right\}\right) = U\left(\left\{\bar{X}^j\right\}\right), \tag{A.3.5}$$

and the redistribution effect, Δ_R, by

$$1+\Delta_R \equiv \frac{1-p_{ine}^R}{1-p_{ine}^{NR}}, \text{ where } \quad U\left(\left(1-p_{ine}^j\right)\left\{\bar{X}^j\right\}\right) = \int U\left(\left\{\bar{x}^j\left(a_0,e_0\right)\right\}\right) d\lambda\left(a_0,e_0\right). \tag{A.3.6}$$

The following proposition holds[7].

Proof of Proposition 3. First notice that $v(x) \equiv u(c,n)$ where u is the GHH utility function, defined in (1.4.1), satisfies the following regularity property: there exists a totally multiplicative function h : (i.e. $h(ab) = h(a)h(b)$, and $h(a/b) = h(a)/h(b)$) such that for any scalar α,

$$v(\alpha x) = h(\alpha)\, v(x). \tag{A.3.7}$$

[7] This result is similar to the one introduced by Benabou (2002) and used in Floden (2001).

Hence, suppressing the dependence on (a_0, e_0), we obtain:

$$\begin{aligned}
\int E_0 U\left(\left\{x_t^R\right\}\right) d\lambda_0^R &\overset{(A.3.3)}{=} \int U\left(\left\{\bar{x}^R\right\}\right) d\lambda_0^R \overset{(A.3.6)}{=} U\left(\left(1-p_{ine}^R\right)\left\{\bar{X}^R\right\}\right) \\
&\overset{(A.3.7)}{=} h\left(1-p_{ine}^R\right) U\left(\left\{\bar{X}^R\right\}\right) \overset{(A.3.5)}{=} h\left(1-p_{ine}^R\right) U\left(\left(1-p_{unc}^R\right)\left\{X_t^R\right\}\right) \\
&\overset{(A.3.7)}{=} h\left(\left(1-p_{ine}^R\right)\left(1-p_{unc}^R\right)\right) U\left(\left\{X_t^R\right\}\right) \\
&\overset{(A.3.2)}{=} h\left(\left(1-p_{ine}^R\right)\left(1-p_{unc}^R\right)\right) U\left(\left(1+\Delta_L\right)\left\{X_t^{NR}\right\}\right) \\
&\overset{(A.3.7)}{=} h\left(\left(1+\Delta_L\right)\left(1-p_{ine}^R\right)\left(1-p_{unc}^R\right)\right) U\left(\left\{X_t^{NR}\right\}\right) \\
&\overset{(A.3.7)}{=} h\left(\left(1+\Delta_L\right)\left(1-p_{ine}^R\right)\frac{\left(1-p_{unc}^R\right)}{\left(1-p_{unc}^{NR}\right)}\right) U\left(\left(1-p_{unc}^{NR}\right)\left\{X_t^{NR}\right\}\right) \\
&\overset{(A.3.5)}{=} h\left(\left(1+\Delta_L\right)\left(1+\Delta_I\right)\left(1-p_{ine}^R\right)\right) U\left(\left\{\bar{X}^{NR}\right\}\right) \\
&\overset{(A.3.7)}{=} h\left(\left(1+\Delta_L\right)\left(1+\Delta_I\right)\frac{\left(1-p_{ine}^R\right)}{\left(1-p_{ine}^{NR}\right)}\right) U\left(\left(1-p_{ine}^{NR}\right)\left\{\bar{X}^{NR}\right\}\right) \\
&\overset{(A.3.6)}{=} h\left(\left(1+\Delta_L\right)\left(1+\Delta_I\right)\left(1+\Delta_R\right)\right) \int U\left(\left\{\bar{x}^{NR}\right\}\right) d\lambda_0^{NR} \\
&\overset{(A.3.5)}{=} h\left(\left(1+\Delta_L\right)\left(1+\Delta_I\right)\left(1+\Delta_R\right)\right) \int E_0 U\left(\left\{x_t^{NR}\right\}\right) d\lambda_0^{NR} \\
&\overset{(A.3.7)}{=} \int E_0 U\left(\left(1+\Delta_R\right)\left(1+\Delta_I\right)\left(1+\Delta_L\right)\left\{x_t^{NR}\right\}\right) d\lambda_0^{NR}.
\end{aligned}$$

The result follows from the definition of Δ in equation ($A.3.1$). ■

A.4 Algorithms

Here we describe the algorithms used to obtain our results.

Algorithm for computing the transition between steady states [8]

1. Solve for the initial stationary equilibrium.

2. Assume the economy converges to a new stationary equilibrium in $\bar{t}$ periods and guess a sequence $K_2, ..., K_{\bar{t}-1}$.

[8] This is an extension of the procedure proposed by Domeij and Heathcote (2004). To solve for agent's decision rules we use the endogenous grid method (see Carroll (2006)).

3. Solve for the new tax on labor such that given $K_2, ..., K_{\bar{t}-1}$ and the path for the other taxes, government debt is unchanged between $\bar{t}-1$ and $\bar{t}$. Compute the associated path for the government debt, $B_1, ..., B_{\bar{t}-1}$ (for details see the Final Tax Computation section in the online appendix).

4. Solve for the final stationary equilibrium given final tax rates τ^k, τ^n, τ^c and T, and $B_{\bar{t}}$. Compute $K_{\bar{t}}$.

5. Solve for households savings decisions in transition.

6. Update the path of capital, i.e. take the initial stationary distribution over wealth and productivity and use the decision rules computed above to simulate the economy forward. Then, check for market clearing at each date and adjust $K_2, ..., K_{\bar{t}-1}$ appropriately.

7. If the new sequence for capital is the close to the old, we have found the equilibrium path. Otherwise go back to step 5.

8. Increase $\bar{t}$ until the solution stops changing.

Algorithm for global optimization[9]

1. Sample a large set X of points from a uniform distribution over the domain[10].

2. Evaluate the objective function for all points in X.

3. Select a reduced set X_r with the highest objective function values. Sort the elements of X_r into clusters and run a local[11] solver one time for each cluster[12].

4. Use a Bayesian stopping rule to determine whether or not the procedure should be repeated.

[9] This procedure is described in more detail in Kucherenko and Sytsko (2005).
[10] We used pseudo-random numbers from a Sobol sequence which give more efficient results.
[11] We used an open source local solver called BOBYQA.
[12] See Rinnooy Kan and Timmer (1987).

A.5 Tables and Figures

Table A.1: Distribution of wealth

	Bottom (%)	Quintiles					Top (%)	Gini
	0-5	1st	2nd	3rd	4th	5th	95-100	
Data	-0.1	-0.2	1.1	4.5	11.2	83.4	60.3	0.82
Model	-0.1	-0.2	1.5	4.7	10.2	83.4	60.8	0.81

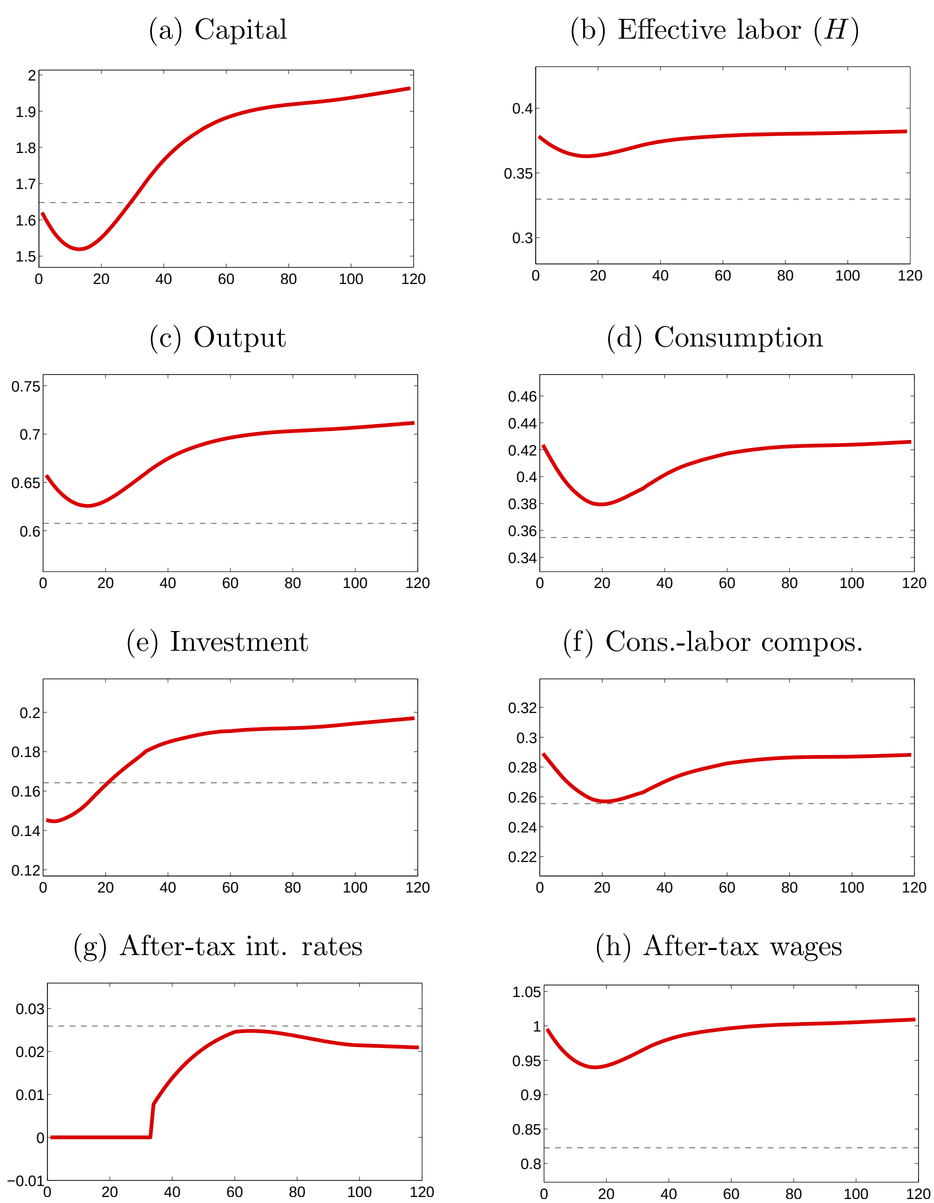
Notes: Data come from the 2007 Survey of the Consumer Finance.

Table A.2: Income sources by quintiles of wealth

Quintile	Model			Data		
	Labor	Asset	Transfer	Labor	Asset	Transfer
1st	83.7	-0.1	16.4	82.0	2.0	16.0
2nd	85.4	1.6	13.1	83.0	4.8	12.2
3rd	84.1	4.7	11.2	80.0	7.3	12.7
4th	81.4	8.6	10.0	77.6	10.3	12.1
5th	58.7	36.2	5.2	51.7	40.0	8.3

Notes: Table summarizes the pre-tax total income decomposition. We define the asset income as the sum of income from capital and business. Data come from the 2007 Survey of the Consumer Finance, the numbers are based on a summary by Díaz-Giménez et al. (2011).

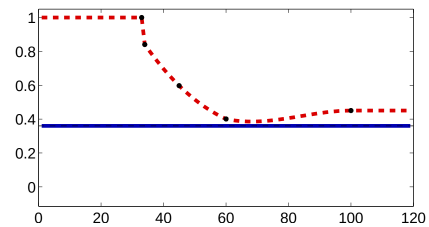
Figure A.1: Aggregates: Benchmark



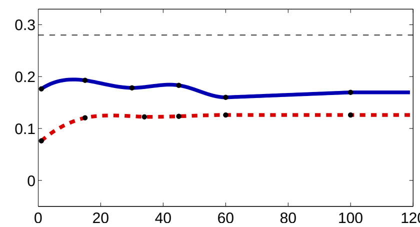
Notes: Dashed line: initial stationary equilibrium; Solid line: optimal transition.

Figure A.2: Optimal Fiscal Policy: Fixed Capital Taxes

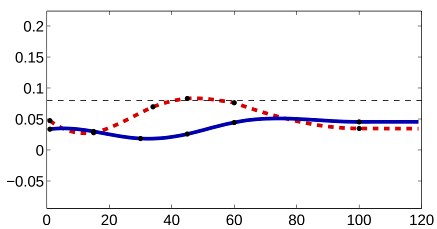
(a) Capital tax



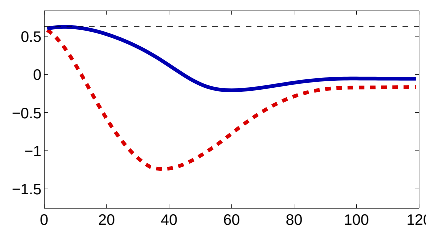
(b) Labor tax



(c) Lump-sum-to-output



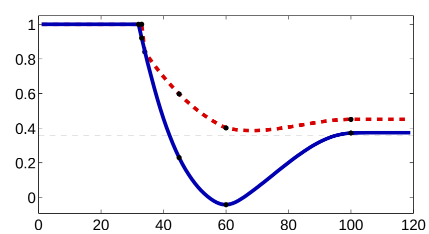
(d) Debt-to-output



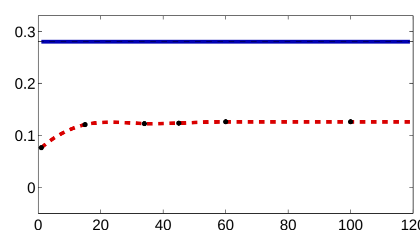
Notes: Dashed thin line: initial stationary equilibrium; Dashed thick line: optimal transition with unrestricted instruments (benchmark); Solid line: optimal transition with fixed capital taxes.

Figure A.3: Optimal Fiscal Policy: Fixed Labor Taxes

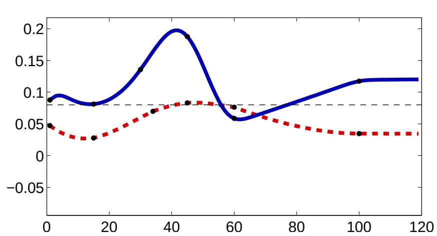
(a) Capital tax



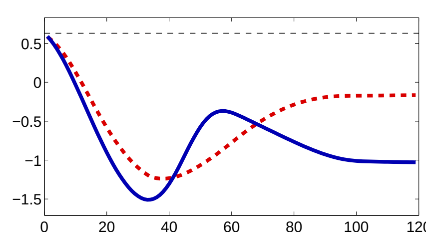
(b) Labor tax



(c) Lump-sum-to-output

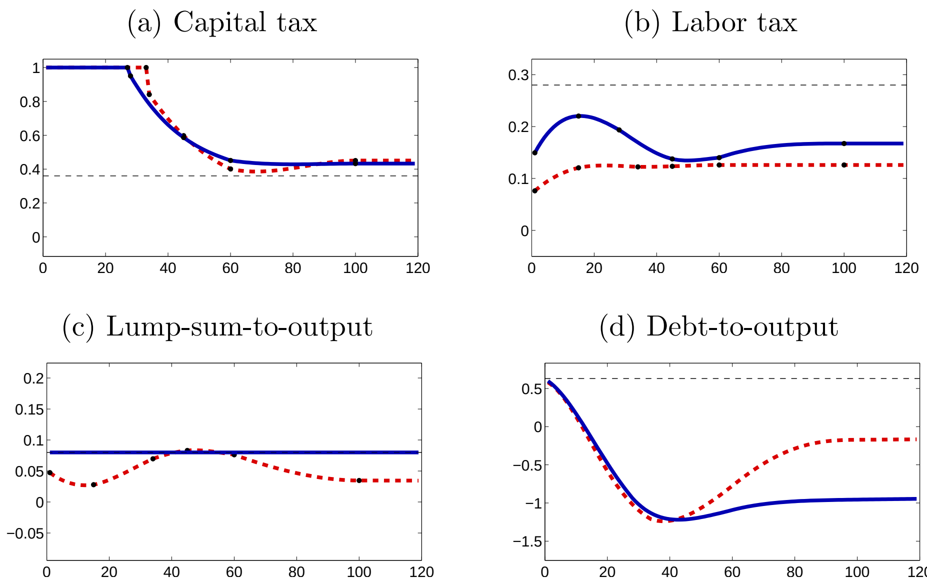


(d) Debt-to-output



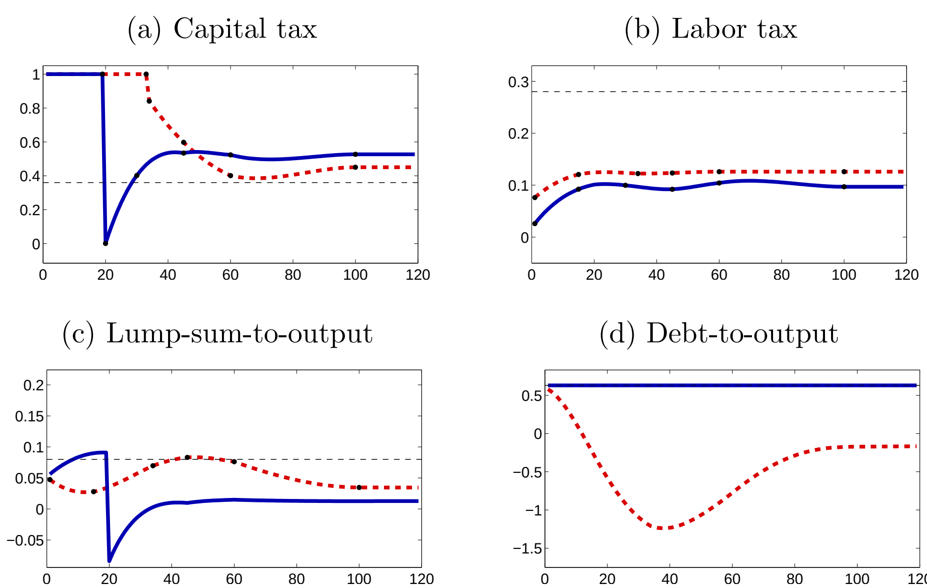
Notes: Dashed thin line: initial stationary equilibrium; Dashed thick line: optimal transition with unrestricted instruments (benchmark); Solid line: optimal transition with fixed labor taxes.

Figure A.4: Optimal Fiscal Policy: Lump-Sum Transfers to Output



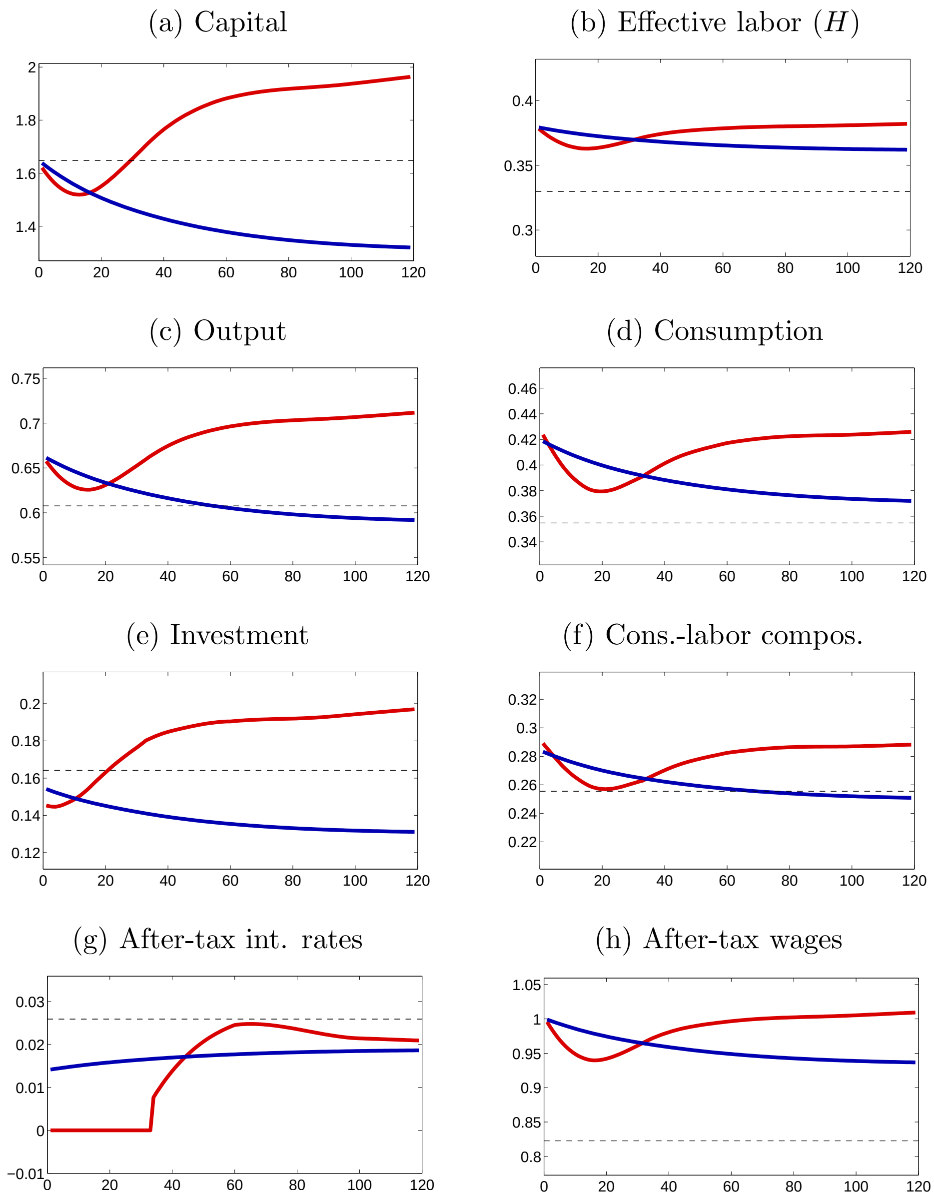
Notes: Dashed thin line: initial stationary equilibrium; Dashed thick line: optimal transition with unrestricted instruments (benchmark); Solid line: optimal transition with fixed lump-sum transfers to output ratio.

Figure A.5: Optimal Fiscal Policy: Fixed debt-to-output



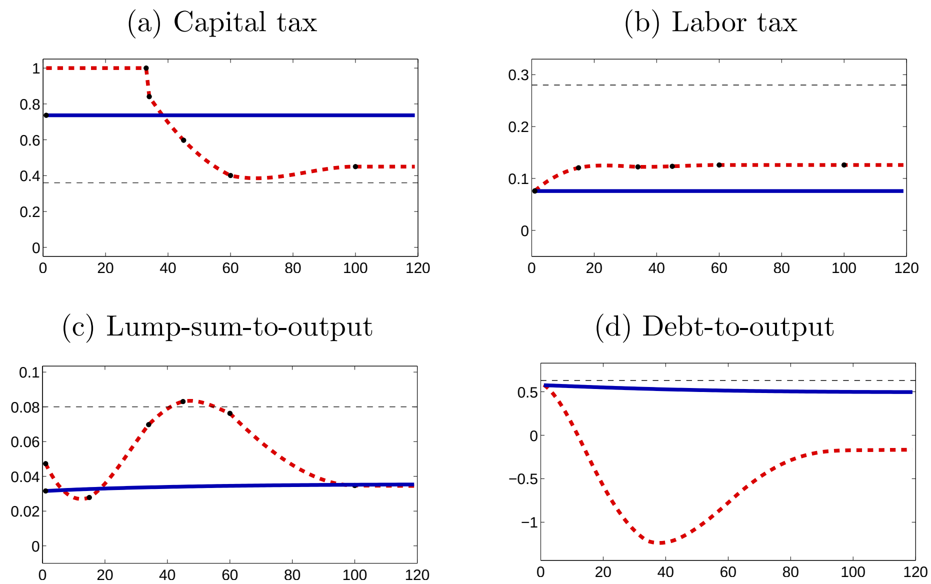
Notes: Dashed thin line: initial stationary equilibrium; Dashed thick line: optimal transition with unrestricted instruments (benchmark); Solid line: optimal transition with fixed debt-to-output ratio.

Figure A.6: Aggregates: Constant Policy



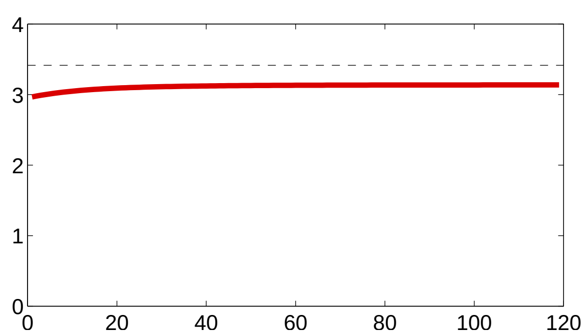
Notes: Dashed line: initial stationary equilibrium; Solid line: optimal transition.

Figure A.7: Optimal Fiscal Policy: Constant Policy



Notes: Dashed line: initial stationary equilibrium; Solid line: optimal transition; The black dots are the choice variables: the spline nodes and t^*, the point at which the capital tax leaves the upper bound.

Figure A.8: Economy 3: Θ_t



Appendix B

Appendix to Chapter 2

B.1 Proof of Example Propositions

First I will prove the following proposition which is stronger and encompasses both Propositions 7 and 8.

Proposition 11 *If $\sigma_\theta > \rho(1-\rho)\sigma_\eta$, then credit limits are increasing with income, i.e. $\omega'(y_1) \geq 0$ for all y_1; if $\sigma_\theta < \rho(1-\rho)\sigma_\eta$, then credit limits are decreasing with income, i.e. $\omega'(y_1) \leq 0$ for all y_1; and if $\sigma_\theta = \rho(1-\rho)\sigma_\eta$, then credit limits are independent of income, i.e. $\omega'(y_1) = 0$ for all y_1.*

Proof. First notice that the income process implies the following conditional distributions[1]

$$y_2 \mid y_1 \sim N\left(\nu_{21} y_1, \chi_2^2\right), \quad \text{and} \quad y_3 \mid (y_1, y_2) \sim N\left(\nu_{31} y_1 + \nu_{32} y_2, \chi_3^2\right),$$

[1] With

$$\begin{aligned}
\nu_{21} &\equiv \frac{2\sigma_\theta^2 + \rho\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2}, \quad \chi_2^2 \equiv \frac{\left(\left(\rho^2 - 4\rho + 5\right)\sigma_\theta^2 + \sigma_\eta^2\right)\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2}, \\
\nu_{31} &\equiv \frac{(1-\rho)^2 (3-2\rho)\sigma_\theta^2}{(\rho^2 - 4\rho + 5)\sigma_\theta^2 + \sigma_\eta^2}, \quad \nu_{32} \equiv \frac{\left(\rho^3 - 2\rho^2 - 2\rho + 6\right)\sigma_\theta^2 + \rho\sigma_\eta^2}{(\rho^2 - 4\rho + 5)\sigma_\theta^2 + \sigma_\eta^2}, \quad \text{and} \\
\chi_3^2 &\equiv \frac{\left(\left(5\rho^2 - 16\rho + 14\right)\sigma_\theta^2 + \sigma_\eta^2\right)\sigma_\eta^2}{(\rho^2 - 4\rho + 5)\sigma_\theta^2 + \sigma_\eta^2}
\end{aligned}$$

where ν_{21}, ν_{31}, and ν_{32} are strictly positive. The necessary and sufficient first order conditions of problems (2.2.2) and (2.2.3) lead to the following policy functions

$$a_3^0(a_2, y_1, y_2) = \frac{1+r}{2+r}\left((1-\nu_{32})\, y_2 - \nu_{31} y_1 + a_2 + \frac{\alpha\chi^2}{2}\right),$$

and

$$a_3^1(y_1, y_2) = \max\left\{a_3^0(0, y_1, y_2), 0\right\}.$$

It follows that

$$V^0(a_2, y_1, y_2) = -(1+\beta)\exp\left(-\frac{\alpha}{2+r}\left(\nu_{31} y_1 + (1+r+\nu_{32})\, y_2 + (1+r)\, a_2 - \frac{\alpha\chi_3^2}{2}\right)\right), \tag{B.1.1}$$

and that

$$\begin{aligned} V^1(y_1, y_2) \quad = \quad & -\exp\left(-\frac{\alpha}{2+r}\min\left\{\nu_{31} y_1 + (1+r+\nu_{32})\, y_2 - \frac{\alpha\chi_3^2}{2}, (2+r)\, y_2\right\}\right) \qquad \text{(B.1.2)} \\ & -\beta\exp\left(-\frac{\alpha}{2+r}\left(\max\left\{\begin{array}{c} \nu_{31} y_1 + (1+r+\nu_{32})\, y_2 - \frac{\alpha\chi_3^2}{2}, \\ (2+r)(\nu_{31} y_1 + \nu_{32} y_2) - (2+r)\frac{\alpha\chi_3^2}{2} \end{array}\right\}\right)\right). \end{aligned}$$

Next, define the threshold $a_2^*(y_1, y_2)$ to be such that

$$V^0(a_2^*(y_1, y_2), y_1, y_2) = V^1(y_1, y_2),$$

that is, $a_2^*(y_1, y_2)$ is the lowest level of asset (or highest amount of debt) that a household can start period 2 with and not declare bankruptcy[2]. If $a_2 < a_2^*(y_1, y_2)$, the household declares bankruptcy, and if $a_2 \geq a_2^*(y_1, y_2)$, it does not. Using equations ($B.1.1$) and ($B.1.2$) we can explicitly solve for $a_2^*(y_1, y_2)$, which implies

$$a_2^*(y_1, y_2) = \begin{cases} 0 & \text{, if } (1-\nu_{32})\, y_2 \geq \nu_{31} y_1 - \frac{\alpha\chi_3^2}{2} \\ \\ \Omega(y_1, y_2) & \text{, if } (1-\nu_{32})\, y_2 < \nu_{31} y_1 - \frac{\alpha\chi_3^2}{2} \end{cases} \tag{B.1.3}$$

[2] It is easy to see that $V^0(a_2, y_1, y_2)$ is increasing in a_2 so that $a_2^*(y_1, y_2)$ is uniquely defined.

where

$$\Omega(y_1,y_2) \equiv -\frac{\nu_{31}y_1-(1-\nu_{32})y_2-\frac{\alpha\chi_3^2}{2}}{1+r}$$
$$-\frac{2+r}{1+r}\frac{1}{\alpha}\ln\left(\frac{1+\beta\exp\left(-\alpha\left(\nu_{31}y_1-(1-\nu_{32})y_2-\frac{\alpha\chi_3^2}{2}\right)\right)}{1+\beta}\right).$$

For what follows, it will be useful to define

$$\hat{y}_2 \equiv \frac{\nu_{31}y_1}{(1-\nu_{32})}-\frac{\alpha\chi_3^2}{2(1-\nu_{32})},$$

and let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the cumulative distribution and density of a $N(0,1)$ random variable, respectively. Further, define $y_2^*(a_2,y_1)$ to be such that

$$a_2^*(y_1,y_2^*(a_2,y_1))=a_2,\ \text{ for all } a_2<0.$$

Given a_2 and y_1, $y_2^*(a_2,y_1)$ is the level of income in period 2 that would leave the household indifferent between declaring bankruptcy or not. Now, consider the following two (exhaustive) possibilities.

Case 1: $\sigma_\theta^2<\frac{(1-\rho)}{1+\rho(1-\rho)(2-\rho)}\sigma_\eta^2$, so that $\nu_{32}<1$.

From equation ($B.1.3$) it follows that

$$\frac{\partial a_2^*(y_1,y_2)}{\partial y_1} = \begin{cases} 0 & \text{, if } y_2\geq\hat{y}_2 \\ -\frac{\nu_{31}}{1+r}\Lambda(y_1,y_2)<0 & \text{, if } y_2<\hat{y}_2 \end{cases}$$
$$\frac{\partial a_2^*(y_1,y_2)}{\partial y_2} = \begin{cases} 0 & \text{, if } y_2\geq\hat{y}_2 \\ \frac{1-\nu_{32}}{1+r}\Lambda(y_1,y_2)>0 & \text{, if } y_2<\hat{y}_2 \end{cases}$$

where

$$\Lambda(y_1, y_2) \equiv \frac{1 - \exp\left(-\alpha\left(\nu_{31}y_1 - (1-\nu_{32})\,y_2 - \frac{\alpha\chi_3^2}{2}\right)\right)}{1 + \beta\exp\left(-\alpha\left(\nu_{31}y_1 - (1-\nu_{32})\,y_2 - \frac{\alpha\chi_3^2}{2}\right)\right)} > 0.$$

Consider an $a_2 < 0$ and notice that $y_2^*(a_2, y_1) < \hat{y}_2$. Thus, from the implicit function theorem it follows that

$$\frac{\partial y_2^*(a_2, y_1)}{\partial y_1} = \frac{\nu_{31}}{1-\nu_{32}}.$$

Equation (2.2.1) implies that

$$q_2(a_2, y_1) = \frac{1 - \Pr\left(y_2 > y_2^*(a_2, y_1) \mid y_1\right)}{1+r},$$

and, therefore

$$q_2(a_2, y_1) = \frac{\Phi\left(\frac{y_2^*(a_2,y_1) - \nu_{21}y_1}{\chi_2}\right)}{1+r},$$

which entails

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} = \frac{\phi\left(\frac{y_2^*(a_2,y_1) - \nu_{21}y_1}{\chi_2}\right)}{1+r}\,\frac{\frac{\nu_{31}}{1-\nu_{32}} - \nu_{21}}{\chi_2}.$$

Then, using the definitions of the conditional distribution parameters, it can be established[3] that

$$\begin{aligned}
\sigma_\theta^2 &< \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \frac{\partial q_2(a_2, y_1)}{\partial y_1} < 0,\\
\sigma_\theta^2 &= \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \frac{\partial q_2(a_2, y_1)}{\partial y_1} = 0,\\
\sigma_\theta^2 &> \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \frac{\partial q_2(a_2, y_1)}{\partial y_1} > 0.
\end{aligned}$$

Finally, recall that credit limits are defined as $\omega(y_1) = \max_{a_2}\left(-q_2(a_2, y_1)\,a_2\right)$, which implies the envelope condition

$$\omega'(y_1) = -\frac{\partial q_2(\tilde{a}_2(y_1), y_1)}{\partial y_1}\tilde{a}_2(y_1),$$

[3] Notice that $\rho(1-\rho) < \frac{(1-\rho)}{1+\rho(1-\rho)(2-\rho)}$.

where $\tilde{a}_2(y_1) = \arg\max_{a_2}(-q_2(a_2, y_1)\, a_2)$. Since $q_2(a_2, y_1) \geq 0$, it follows that $\tilde{a}_2(y_1) \leq 0$ and, therefore

$$\begin{aligned}
\sigma_\theta^2 &< \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \omega'(y_1) \leq 0, \\
\sigma_\theta^2 &= \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \omega'(y_1) = 0, \\
\sigma_\theta^2 &> \rho(1-\rho)\,\sigma_\eta^2 \Rightarrow \omega'(y_1) \geq 0.
\end{aligned}$$

Case 2: $\sigma_\theta^2 > \frac{(1-\rho)}{1+\rho(1-\rho)(2-\rho)}\sigma_\eta^2$, so that $\nu_{32} > 1$.

Analogously to Case 1, it can be established that

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} = -\frac{\phi\left(\frac{y_2^*(a_2,y_1)-\nu_{21}y_1}{\chi_2}\right)}{1+r}\frac{\frac{\nu_{31}}{1-\nu_{32}} - \nu_{21}}{\chi_2}$$

and it follows straight-forwardly that

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} > 0,$$

which, in turn, implies that $\omega'(y_1) \geq 0$ by the same argument used in Case 1.

Case 3: $\sigma_\theta^2 = \frac{(1-\rho)}{1+\rho(1-\rho)(2-\rho)}\sigma_\eta^2$, so that $\nu_{32} = 1$.

From equation ($B.1.3$) it follows that

$$a_2^*(y_1) = \begin{cases} 0 & \text{, if } y_1 \leq \frac{\alpha\chi_3^2}{2\nu_{31}} \\ \\ \Phi(y_1) & \text{, if } y_1 > \frac{\alpha\chi_3^2}{2\nu_{31}} \end{cases}$$

where

$$\Phi(y_1) \equiv -\frac{\nu_{31}y_1 - \frac{\alpha\chi_3^2}{2}}{1+r} - \frac{2+r}{1+r}\frac{1}{\alpha}\ln\left(\frac{1+\beta\exp\left(-\alpha\left(\nu_{31}y_1 - \frac{\alpha\chi_3^2}{2}\right)\right)}{1+\beta}\right).$$

Equation (2.2.1) implies that

$$q_2\left(a_2, y_1\right) = \begin{cases} 0 & \text{, if } a_2 < a_2^*\left(y_1\right) \\ \\ \frac{1}{1+r} & \text{, if } a_2 \geq a_2^*\left(y_1\right) \end{cases}$$

and, therefore, in effect

$$\omega\left(y_1\right) = -\frac{a_2^*\left(y_1\right)}{1+r}.$$

Finally, it follows that

$$\omega'\left(y_1\right) = \begin{cases} 0 & \text{, if } y_1 \leq \frac{\alpha\chi_3^2}{2\nu_{31}} \\ \\ -\frac{\Phi'\left(y_1\right)}{1+r} & \text{, if } y_1 > \frac{\alpha\chi_3^2}{2\nu_{31}} \end{cases}$$

where

$$\Phi'\left(y_1\right) = \frac{\nu_{31}}{1+r}\frac{\exp\left(-\alpha\left(\nu_{31}y_1 - \frac{\alpha\chi_3^2}{2}\right)\right) - 1}{1 + \beta\exp\left(-\alpha\left(\nu_{31}y_1 - \frac{\alpha\chi_3^2}{2}\right)\right)} < 0, \text{ if } y_1 > \frac{\alpha\chi_3^2}{2\nu_{31}}$$

which implies that $\omega'\left(y_1\right) \geq 0$. ■

Next, I will present the proof of Proposition 9.

Proof of Proposition 9. If $\sigma_\theta = 0$, then the conditional distributions become

$$y_2 \mid y_1 \sim N\left(\rho y_1, \sigma_\eta^2\right), \quad \text{and} \quad y_3 \mid \left(y_1, y_2\right) \sim N\left(\rho y_2, \sigma_\eta^2\right),$$

so that, in fact, $V^0\left(a_2, y_1, y_2\right)$ and $\tilde{V}^1\left(y_1, y_2\right)$ do not depend on y_1 (thus, this argument will be suppressed from now on). Then, following the same steps contained in the beginning of the previous proof and defining $a_2^*\left(y_2\right)$ to be such that

$$V^0\left(a_2^*\left(y_2\right), y_2\right) = \tilde{V}^1\left(y_2\right),$$

we obtain

$$a_2^*(y_2) = \begin{cases} -\frac{1+r+\rho}{1+r}\gamma y_2 & \text{, if } y_2 \geq -\frac{\alpha\sigma_\eta^2}{2(1-\gamma)(1-\rho)} \\ -\frac{1+r+\rho}{1+r}\gamma y_2 + \Omega(y_2) & \text{, if } y_2 < -\frac{\alpha\sigma_\eta^2}{2(1-\gamma)(1-\rho)} \end{cases}$$

where

$$\begin{aligned} \Omega(y_2) \quad \equiv \quad & \frac{(1-\rho)(1-\gamma)\,y_2 + \frac{\alpha\sigma_\eta^2}{2}}{1+r} \\ & -\frac{2+r}{1+r}\frac{1}{\alpha}\ln\left(\frac{1+\beta\exp\left(\alpha\left((1-\rho)(1-\gamma)\,y_2 + \frac{\alpha\sigma_\eta^2}{2}\right)\right)}{1+\beta}\right). \end{aligned}$$

It follows that

$$\frac{\partial a_2^*(y_2)}{\partial y_2} = \begin{cases} -\frac{1+r+\rho}{1+r}\gamma & \text{, if } y_2 \geq \hat{y}_2 \\ \\ -\frac{1+r+\rho}{1+r}\gamma + \frac{(1-\rho)(1-\gamma)}{1+r}\Lambda(y_2) & \text{, if } y_2 < \hat{y}_2 \end{cases}$$

where

$$\Lambda(y_2) \equiv \frac{1-\exp\left(\alpha\left((1-\rho)(1-\gamma)\,y_2 + \frac{\alpha\sigma_\eta^2}{2}\right)\right)}{1+\beta\exp\left(\alpha\left((1-\rho)(1-\gamma)\,y_2 + \frac{\alpha\sigma_\eta^2}{2}\right)\right)}.$$

Notice that

$$\max_{y_2 < -\frac{\alpha\sigma_\eta^2}{2(1-\gamma)(1-\rho)}} \Lambda(y_2) = 1,$$

so that

$$\frac{\partial a_2^*(y_2)}{\partial y_2} < 0 \text{ for all } y_2 \Longleftrightarrow \gamma \geq \frac{1-\rho}{1+\beta}.$$

Case 1 $\gamma \geq \frac{1-\rho}{1+\beta}$.

Define $y_2^*(a_2)$ to be such that

$$a_2^*(y_2^*(a_2)) = a_2, \text{ for all } a_2 < 0.$$

for any $a_2 < 0$. Equation (2.2.1), then, implies that

$$q_2(a_2, y_1) = \frac{1 - \Pr\left(y_2 < y_2^*(a_2) \mid y_1\right)}{1+r},$$

and, therefore

$$q_2(a_2, y_1) = \frac{1 - \Phi\left(\frac{y_2^*(a_2) - \rho y_1}{\sigma_\eta}\right)}{1+r},$$

which entails

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} = \frac{\phi\left(\frac{y_2^*(a_2) - \rho y_1}{\sigma_\eta}\right)}{1+r}\frac{\rho}{\sigma_\eta} > 0.$$

Hence, from the definition of credit limits we have that

$$\omega'(y_1) = -\frac{\partial q_2(\tilde{a}_2(y_1), y_1)}{\partial y_1}\tilde{a}_2(y_1),$$

where $\tilde{a}_2(y_1) = \arg\max_{a_2}(-q_2(a_2, y_1)a_2)$. Since $q_2(a_2, y_1) \geq 0$, it follows that $\tilde{a}_2(y_1) \leq 0$ and, therefore $\omega'(y_1) \geq 0$.

Case 2 $\gamma < \frac{1-\rho}{1+\beta}$.

In this case, there exists $\bar{y}_2$ such that $a_2^*(y_2)$ is increasing for $y_2 < \bar{y}_2$ and decreasing for $y_2 > \bar{y}_2$. Hence, there exists an interval $(\underline{a}_2, \bar{a}_2)$ such that, $a_2^*(y_2) = a_2$ has two solutions in y_2, for all $a_2 \in (\underline{a}_2, \bar{a}_2)$. For any $a_2 \in (\underline{a}_2, \bar{a}_2)$, denote the two solutions by $\underline{y}_2^*(a_2)$ and $\bar{y}_2^*(a_2)$ with $\underline{y}_2^*(a_2) < \bar{y}_2^*(a_2)$. It follows that,

$$q_2(a_2, y_1) = \frac{1 - \Pr\left(\underline{y}_2^*(a_2) < y_2 < \bar{y}_2^*(a_2) \mid y_1\right)}{1+r}, \text{ for all } a_2 \in (\underline{a}_2, \bar{a}_2),$$

which implies

$$q_2(a_2, y_1) = \frac{1 - \left(\Phi\left(\frac{\bar{y}_2^*(a_2) - \rho y_1}{\sigma_\eta}\right) - \Phi\left(\frac{\underline{y}_2^*(a_2) - \rho y_1}{\sigma_\eta}\right)\right)}{1+r}, \text{ for all } a_2 \in (\underline{a}_2, \bar{a}_2),$$

and therefore

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} = \frac{\phi\left(\frac{\bar{y}_2^*(a_2)-\rho y_1}{\sigma_\eta}\right) - \phi\left(\frac{\underline{y}_2^*(a_2)-\rho y_1}{\sigma_\eta}\right)}{1+r}\frac{\rho}{\sigma_\eta}, \text{ for all } a_2 \in (\underline{a}_2, \bar{a}_2),$$

so that, if y_1 is close to $\underline{y}_2^*(a_2)$ it follows that

$$\frac{\partial q_2(a_2, y_1)}{\partial y_1} < 0,$$

and, again, from the definition of $\omega(y_1)$ we obtain $\omega(y_1) < 0$ for some y_1. ■

B.2 Figures

Figure B.1

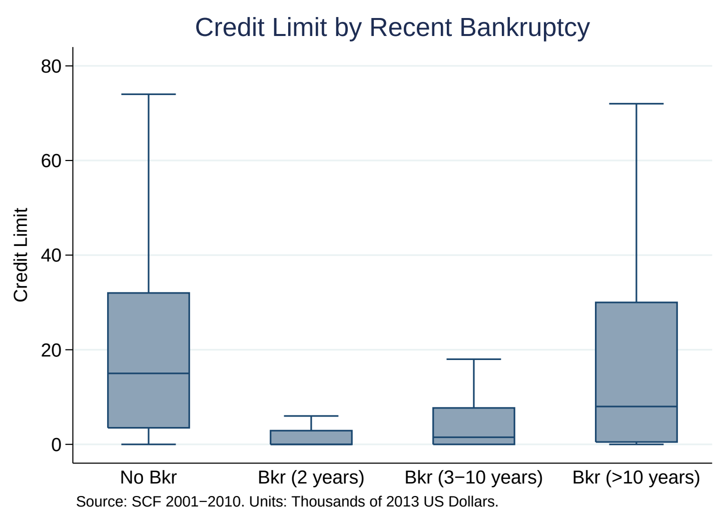


Figure B.2

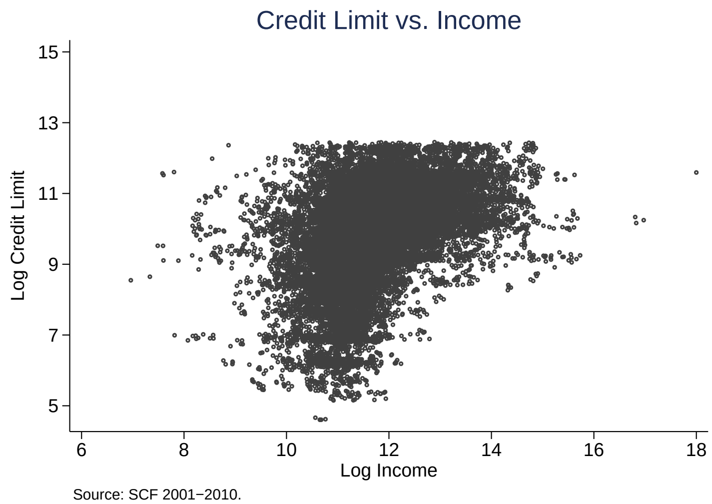


Figure B.3

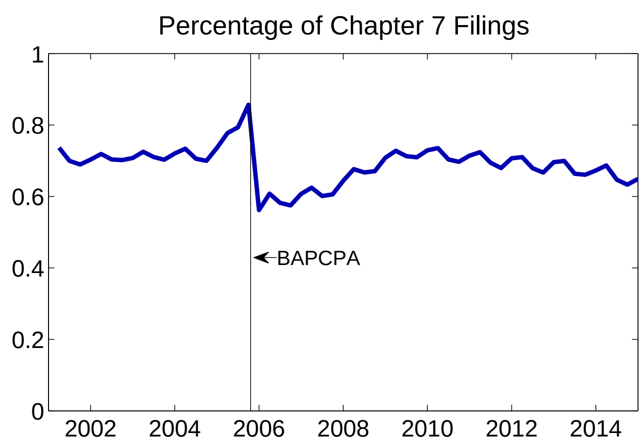
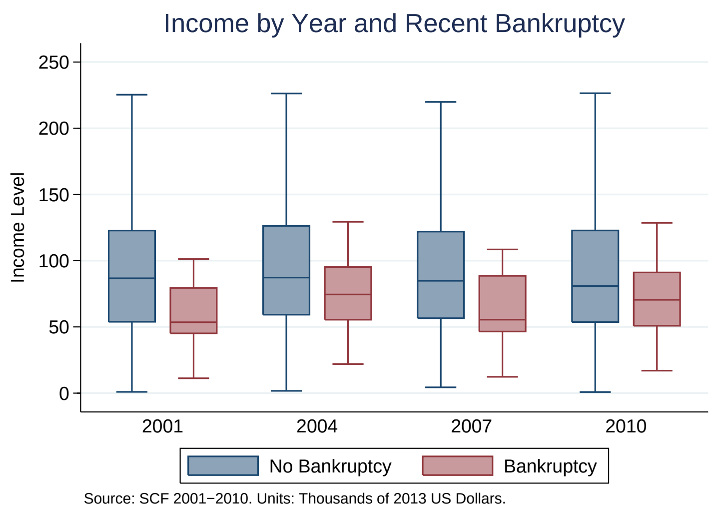


Figure B.4


Income by Year and Recent Bankruptcy
Income Level
250
200
150
100
50
0
2001
2004
2007
2010
No Bankruptcy
Bankruptcy
Source: SCF 2001−2010. Units: Thousands of 2013 US Dollars.