Essays on Macroeconomics

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Jingyi Jiang

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Varadarajan V. Chari

August, 2015

Acknowledgements

I thank my advisor V. V. Chari for his excellent guidance. I also thank the members of my committee: Christopher Phelan, Juliana Salomao, and Kei-Mu Yi for their invaluable help. I thank Zhang Qiong for providing the data used in Chapter 1. I would like to thank my colleagues in the Economics department for their encouragements and support during my time in graduate school.

Dedication

For my parents.

Abstract

Productivity Uncertainty, Learning, and Firm Leverage This paper studies the role of learning about firm productivity in accounting for the variation of firm leverage and capital growth rate with firm age. Using comprehensive firm-level data in China, I find average firm leverage increases then decreases with firm age. Also, I find that new entrants' capital stock grows significantly higher than that of incumbents. I analyze whether information frictions and learning, in addition to financial frictions, are important in rationalizing these two features in the data. My answer is: yes. In the theoretical part, I first present a model with financial frictions that arise from costly equity issuance and default risks in a full information environment. I then extend the model to include information frictions and learning where firms learn about their productivity through production. I calibrate both models to match salient features in the China data, including the difference in capital growth rate between new entrants and incumbents. I find that the model with learning accounts well for the hump-pattern of the age-leverage profile in the data, whereas the model with full information overpredicts the leverage of young firms. When parametrized to match the leverage-age pattern, the model with full information underestimates the capital growth rate of new entrants relative to the incumbents. Lastly I show in a counterfactual exercise that equity issuance costs are a key source of financial friction that shapes the hump-shaped age-leverage pattern in the model with learning.

The Great Recession through the Lens of Neoclassical Growth Theory This paper uses a standard growth model to examine the role of taxes and productivity growth in shaping market hours in ten OECD countries during the 2007 recession. The calibration and simulation exercises in this paper identify productivity slow-down as the main force that discouraged aggregate hours in these countries during the recession. In most countries, small decline in labor income taxes partially compensated for slower productivity growth. Compared to the data series, model-generated hours exhibit smaller decline from 2007:Q4 to 2009:Q4 in Austria and the U.S., and larger decline in Finland, France, and the U.K. This

result suggests that forces other than productivity growth and taxes played an important role in driving market hours in these countries during the recent recession.

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Chapter 1

Productivity Uncertainty, Learning, and Firm Leverage

1.1 Introduction

Recent studies on financial markets have presented ample evidence on the prevalence of capital market imperfections in China (See [1], [2], [3]). As China moves closer to the technological frontier, financial frictions may have a more pronounced impact on China's growth rate (see [4]). The growth and survival of firms are dependent of their ability to obtain external funds from financial markets. Therefore, an understanding of how financial constraints affect firms growth and financing behaviors sheds light on how financial markets influence the real economy. This paper approaches this topic by examining cross-sectional features of firm capital growth rate and debt financing in the firm-level data from China and rationalizing these features within the framework of a heterogeneous firm model.

My data source is the 2004-2007 China Industrial Enterprise Database (CIED), which reports detailed income statement and balance sheet information for industrial firms with 5 million RMB in annual sales. I find that average firm leverage (total liabilities/ total assets) increases then decreases with firm age. This non-monotonic age effect on firm leverage is robust after controlling for firm assets. Furthermore, I find that new entrants' capital stock grows significantly faster than that of incumbents.

I attempt to rationalize these empirical findings in an environment without age-dependent shocks. My starting point is a model with financial frictions that make external funds more costly than internal funds. Given the prevalence of capital market imperfections in China, modeling financial frictions is empirically relevant. Also, in a frictionless economy where Modigliani and Miller assumptions hold, the value of a firm is independent of its financial structure. Models of such economy would not have implications on cross-sectional financing patterns. I also introduce information frictions and learning into a model of financial frictions for both theoretical and empirical reasons. From the theoretical perspective, learning can generate endogenous links between firm investment and firm age, and between borrowing cost and firm age by affecting the beliefs on firms' productivity and expected returns on investment. From the empirical perspective, [5] provide suggestive evidence on substantial uncertainty over firm productivity at the micro level. They also find a significant amount of learning occurring from private, or internal, sources within the firm. Private learning is quantitatively important in improving allocative efficiency.

In order to assess the role of financial frictions and learning in accounting for the variation of firm leverage with firm age, I study two models: the first one is a dynamic stochastic model with heterogeneous firms in a full information environment, where financial frictions arise from costly equity issuance and default risks. I then extend this model to include information frictions and learning where firms learn about their productivity through production. I calibrate both models to firm-level data from China and find that the model with learning accounts well for the hump pattern in the age-leverage profile, whereas the model with full information overpredicts the leverage of young firms. This calibration exercise thus suggests that the data prefers the model with both financial frictions and learning. When calibrated to match the average leverage of new entrants, the full information model underestimates the capital growth rate of new entrants relative to that of the incumbents.

I now turn to a more detailed description of the models. In both models, firms use capital to produce output and their technology is decreasing returns to scale. This assumption implies the existence of optimal firm size. Firms must pay a fixed cost to operate and have costs of adjusting their capital stocks. Given these costs, this model generates firm entry and exit so that selection affects firm dynamics. Firms make decisions on capital investment, financing structure, equity payout/issuance, entry, and exit to maximize discounted dividend payout to their shareholders. Debt schedules are firm specific and reflect a firm's default probability. Firms' productivity consists of a persistent component and a transitory component. Information structure on firm productivity is the key difference between the two models that I study. In the full information environment, the persistent component of firm productivity is observable to the public. In the learning environment, potential new entrants observe an initial signal about their persistent productivity. At the production stage only realized productivity is observable to both lenders and firms, but not the persistent or transitory component. Therefore, at each point in time lenders and firms share the same information set that consists of the initial signal and subsequent realized productivity shocks. They update their beliefs over future productivity distribution through Bayesian learning.

In both models, all firms start with the same initial equity that is small relative to the efficient size of productive firms. Hence, young productive firms face higher returns on investment and have a strong incentive to borrow to finance their capital expansion. As they converge to their optimal scale, they delay dividend payout and accumulate equity to lower the risk of default or costly equity issuance in the future. Equity accumulation implies a negative age effect on firm leverage. In the learning model, productive new entrants underestimate their productivity and face lower borrowing limits because initial signals are uninformative. After experiencing good shocks, productive firms become more optimistic about their productivity, and project higher returns on investment. The growth in their capital investment exceeds the growth of their equity. Hence, as they grow towards their efficient sizes, they borrow more to finance capital investment. At the same time, firms that are hit by bad shocks become more pessimistic and may choose to exit. This selection effect is more pronounced in the learning model than in the full information model because unproductive firms would not enter if their initial types are known at the beginning. Learning thus drives the positive age effect on firm leverage among young firms through the increasing demand for capital of productive firms and the exiting of unproductive firms.

1.1.1 Related Literature

This paper is part of a large literature on firm dynamics. Empirical studies on firm dynamics have documented rich growth and financing patterns across firms of different sizes and maturities. [6] provide a summary of empirical regularities in the U.S. [7] examine firms' financing patterns in G7 countries, [8] study capital structure in 10 developing countries, and [9] document the variation of firm leverage with firm size in 27 European countries. Firms' financing patterns in China are less studied due to data limitations. Two exceptions are [10], who document highly concentrated bond and equity issuance activities among publicly listed firms in China, and [3], who present findings on firm-level credit constraints in China. I establish the age and size dependence of firm growth and firm financing behaviors in a comprehensive dataset that include both private and public firms in a panel data set.

The theoretical framework in this paper is related to the literature that studies the role of financial market development in shaping cross-sectional firm dynamics. I model financial frictions in the form of default risks similar to [6], who develop a model where financial frictions arise from limited commitment in debt contracts. They show that these frictions combined with persistent productivity shocks can potentially rationalize the simultaneous dependence of firm growth and financing choices on firm size and firm age in the U.S. In particular, under the assumption that new entrants enter with high persistent shock, their model can generate negative size and age effect on firm growth and leverage. [9] use a similar approach to model financial frictions and they focus on explaining how the differences in financial development can explain different size-growth and size-leverage patterns across countries. In my quantitative exercises, I use the difference in capital growth rate between new entrants and incumbents to discipline my models. I find that the model with financial frictions in a learning environment outperforms the model with full information in accounting for the age-leverage pattern in the China data.

The modeling of financial frictions in this paper is also related the literature in corporate finance on the capital structure of firms. [11] develop a similar framework of external financing and estimate debt and equity financing costs for public firms in the U.S. [12] find that default risks can rationalize the cross-sectional relationship between firm leverage and stock returns. [13] and [14] use similar models to explain the cyclical features of firm financing across firms. While these papers focus on the size effect on firm financing, I attempt to explain the variations along the age dimension.

The structure of firm learning is a reminiscent of [15], in which firms are subject to productivity shocks drawn from a distribution with unknown mean but known variance.

Selection is generated through entry and exit as firms learn about their average productivity. I incorporate this style of learning into a model of financial frictions. I emphasize learning as an important mechanism in explaining the variation of firm capital structure with firm age. [16] also studies how the interaction between information frictions and limited enforcement of financial contracts affects resource allocation and TFP. He highlights information frictions as a source of misallocation and the role of financial frictions in exacerbating this effect. Different from his work, I solve a dynamic model with learning and emphasize its role in accounting for cross-sectional features in the data. [5] devise a novel empirical strategy that uses a combination of firm-level production and stock market data to pin down productivity uncertainty and learning from private and public signals in the economy. Applying this methodology to the U.S., China, and India, they find information frictions generate substantial productivity and output losses. Here I argue for the importance of information frictions and learning in rationalizing age-leverage variations.

This paper also contributes to recent studies on Chinese economy that analyze the comprehensive firm-level data in CIED. Many of these studies focus on understanding how reallocation of resources affects aggregate productivity and macroeconomic growth in China. For example, [17] estimate the distribution of marginal products of capital and labor within a narrowly defined industry by fitting a production function to firm level data. They find that the gaps in this distribution can explain a large part of the differences in manufacturing TFP between developing economies such as China and India and the U.S. [18] develop a model of China's economic transition where high productive firms with limited access to financial market gradually outgrow low productive but financially integrated firms. Their model has predictions on reallocation within the manufacturing sector that is consistent with features in the CIED data. [19] present the first panel analysis of firm-level TFP estimates for China's manufacturing sector. [20] find that the effect of financial frictions on aggregate TFP through the misallocation channel is weak in a quantitative model calibrated to China. To my knowledge, this paper is the first to examine cross-sectional firm financing patterns in this data set.

The rest of the paper is organized as follows. Section 2 introduces the model with default risks in a full information environment. Section 3 presents the model with default risks in an environment with information friction and learning. Section 4 discusses our calibration strategies. Section 5 presents main quantitative results. Section 6 examines the role of financial frictions in counterfactual exercises. Section 7 concludes.

1.2 Dynamic Model with Default Risks and Full Information

In this section, I describe the dynamic model with default risk and full information. Time is discrete and there is no aggregate uncertainty. This is a small open economy with an exogenously determined risk-free interest rate.

1.2.1 Agents and Technology

There is a large number of risk-neutral lenders and a constant measure of firms: each period exiting firms are replaced by potential new entrants. Firms have access to a decreasing returns to scale technology that produces gross output with capital: $y = \exp(z)k^{\alpha}$, where $\alpha < 1$ implies that given a stochastic state, there exists an optimal firm size, and log productivity $z = x + \epsilon$ consists of a persistent component x that follows an AR(1) process

$$x' = \rho x + \nu, \quad \nu \sim N(0, \sigma_{\nu})$$

and a transitory component $\epsilon \sim N(-\frac{1}{2}\sigma_{\epsilon}^2, \sigma_{\epsilon})$. Shocks to the persistent and transitory components of productivity are i.i.d and independent of each other. In each period, the realization of the persistent shock x and the i.i.d shock ϵ are observable to both firms and lenders.

1.2.2 Firm Problems

Each period a firm maximizes the discounted dividend payout to their shareholders by making decisions on debt repayment, investment, and financing plans. Figure 1.1 summarizes the timeline for incumbents in the model: at the beginning of each period, productivity shocks are realized. An incumbent firm produces output using capital k, and receives gross revenue $y = \exp(z)k^{\alpha}$ less fixed production cost η. Next the firm decides whether or not to repay its debt. If the firm defaults, the firm value becomes zero and lenders receive a recovery value by liquidating firm assets. I allow firms to roll over there debt. That is, firms can pay back their old debts after obtaining a new loan. If the firm does not default, lenders provide the firm with a new set of debt schedules that are dependent of today's persistent productivity and its capital installment tomorrow: $\Omega(\cdot; k', x) : b' \to qb'$. After the firm repays its debt, it chooses its capital for the next period k' and finance the investment and associated adjustment cost using a combination of internal funds, external debt b', and equity issuance e by incurring a linear cost γ in e.

Now I provide a recursive formulation for an incumbent firm. Let $V(k, b, x, z)$ denote the post-production value of an incumbent firm with capital k, liability b, persistent productivity x, and transitory productivity $z - x$. This value is the maximum between the value of repayment and zero, the value of default.

$$V(k, b, x, z) = \max\{V^{ND}(k, b, x, z), V^D = 0\} \tag{1.1}$$

The repayment value can be represented by its current period dividend payout and the expected continuation value. Non-defaulting firms maximize their repayment value by choosing capital stock k' and external debt b' next period, taking into consideration their expected productivity next period. Both decisions affect firms' dividend payout this period. In particular, firms' choice of capital stock k' determines the investment $k' - (1 - \delta)k$, associated adjustment cost $\phi(k' - k)^2/k$, as well as borrowing cost $q(k', \cdot)$. Firms' debt choice determines how much dividend the firm pays out or equity it needs to raise.

$$V^{ND}(k, b, x, z) = \max_{e, k', (qb', b') \in \Omega} (1 + \gamma \mathbb{I}_{e<0})e + \beta E_{x', z'}[V(k', b', x', z')] \tag{1.2}$$

subject to

$$e = \underbrace{\exp(z)k^\alpha + (1 - \delta)k - b - \eta}_{\text{internal funds}} + qb' - k' - \phi(k' - k)^2/k \tag{1.3}$$

Now I turn to the problem of a potential entrant. At the beginning of each period, a potential new entrant endowed with equity e_0 receives a signal that reveals its initial persistent productivity x_0. Upon observing the signal, the potential entrant must decide whether to start operating next period. If so, it also needs to decide today the amount of capital stock with which it wants to start production tomorrow. They can finance the gap between its initial capital stock and initial equity e_0 by borrowing from a lender and equity issuance. The value of a new entrant after observing signal x is given by

$$V^e(x) = \max_{e, k', (qb', b') \in \Omega} (1 + \gamma \mathbb{I}_{e<0})e + \beta E_{x', z'} V(k', b', x', z') \tag{1.4}$$

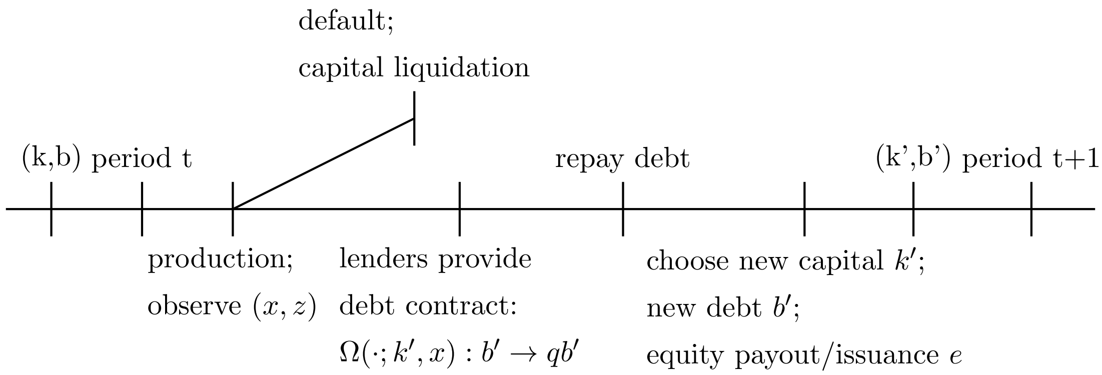


Figure 1.1: Timing for model with full information

subject to

$$e = e_0 + qb' - k'. \tag{1.5}$$

1.2.3 Lender's Problem and Debt Schedule

In this subsection I lay out the specifics of debt schedules derived from the lenders' problem. There are a large number of perfectly competitive, risk neutral lenders. Risk-free interest rate R is exogenously given. A firm can raise funds qb' in the current period by promising to pay back the face value of the bond b' next period if it does not default. Each debt contract $(b', qb') \in \Omega(k', x)$ maps a bond of face value b' to its present value qb'. Next period, if a firm value becomes negative, it defaults on the debt and exits the market. I follow [11] by assuming that in the event of a default, lenders can only recover a fraction of the defaulting firm's profits and remaining assets. Given the firm's capital choice k' and productivity $\exp(z')$ next period, its recovery value is given by

$$RC(k', z') = (1 - \lambda)[\exp(z')k'^{\alpha} + (1 - \delta)k'] \tag{1.6}$$

where λ denotes the linear bankruptcy cost. Each debt contract $(b', qb') \in \Omega(k', x)$ must therefore satisfy the following break-even condition:

$$\begin{aligned} &[b'/q(k', b', x)] \int (1 - d(k', b', x', z'))\Phi(dx' \times dz'; x) \\ &+ \int d(k', b', x', z')[(1 - \lambda)(\exp(z')k'^{\alpha} + (1 - \delta)k')]\Phi(dx' \times dz'; x) = Rb' \end{aligned}. \tag{1.7}$$

and

$$d(k', b', x', z') = \begin{cases} 1 & \text{if } V^{ND}(k', b', x', z') < 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.8}$$

Here the probability of default depends on the realization of persistent shock today, capital stocks that the firm holds next period, and the amount of debt that it needs to repay.

1.2.4 Optimality Conditions

I derive the optimality conditions for the incumbent's problem described in equation 1.2 after debt repayment. Let χ denote the Lagrange multiplier on the budget constraint in equation 1.3. The first order conditions with respect to equity today e, capital installment tomorrow k', and debt repayment tomorrow b' are given by

$$\begin{aligned} &[\text{equity}] && (1 + \gamma \mathbb{I}_{e<0}) + \chi = 0 \\ &[\text{capital}] && \beta E_{x',z'} V_{k'}(k', b', x', z') + \chi(1 - \tfrac{\partial q(b',k',x)}{\partial k'} b' + \tfrac{\partial}{\partial k'} \phi(k' - k)^2/k) = 0 \\ &[\text{debt}] && \beta E_{x',z'} V_{b'}(k', b', x', z') + \chi(q(b', k', x) + \tfrac{\partial q(b',k',x)}{\partial b'} b') = 0 \end{aligned} \tag{1.9}$$

The two envelope conditions are:

$$V_k(k, b, x, z) = \chi \left(\alpha \exp(z) k^{\alpha - 1} + (1 - \delta) - \frac{\partial}{\partial k} \phi(k' - k)^2/k \right) \tag{1.10}$$

$$V_b(k, b, x, z) = -\chi \tag{1.11}$$

Combining the first order conditions and envelope conditions, I can summarize the optimality condition at an interior solution as follows:

$$(1 + \gamma \mathbb{I}_{e<0}) + \chi = 0 \tag{1.12}$$

$$\begin{aligned} &(1 - \frac{\partial q(b', k', x)}{\partial k'} b' + \frac{\partial}{\partial k'} \phi(k' - k)^2/k) \\ &= \beta E_{x',z'} \frac{\chi'}{\chi} \left(\alpha \exp(z') k'^{\alpha - 1} + (1 - \delta) - \frac{\partial}{\partial k'} \phi(k'' - k')^2/k' \right) \end{aligned} \tag{1.13}$$

$$(q(b', k', x) + \frac{\partial q(b', k', x)}{\partial b'} b') = \beta E_{x',z'} \frac{\chi'}{\chi} \tag{1.14}$$

1.2.5 Equilibrium

I restrict the attention to the stationary recursive equilibrium. Given a risk-free rate R, a *stationary recursive equilibrium* consists of

- value functions for incumbents $V(k,b,x,z)$ and new entrants $V^e(x)$
- incumbents' policy functions for default $g_d(k,b,x,z)$, debt financing $g_b(k,b,x,z)$, capital investment $g_k(k,b,x,z)$, and equity payout/issuance $g_e(k,b,x,z)$
- new entrants' policy functions for debt financing $g_b^e(x)$, capital investment $g_k^e(x)$, and dividend payout $g_e^e(x)$
- debt schedule: $q(k',b',x)$
- measures of incumbents $\Gamma(k,b,x,z)$ and new entrants $\Gamma^e(x)$

such that

- given the schedule of loan contracts offered, the policy and value functions solve firms' optimization problem;
- loan contracts reflect the firm's default probabilities such that with every contract lenders break even in expectation;
- measure of firms $\Gamma(k,b,x,z)$ is consistent with firms' policy functions:

  Let M denote the measure of potential new entrants

$$M = \int \mathbb{I}_{g_k^e(x)=0} d\Gamma^e(x) + \int \mathbb{I}_{g_k(k,b,x,z)=0} d\Gamma(k,b,x,z) \tag{1.15}$$

$$\Gamma^e(x) = Mf(x) \tag{1.16}$$

  where $f(x)$ is the p.d.f of stationary distribution of persistent shock x.

$$\begin{aligned} &\Gamma(k',b',x',z') \\ &= \int \mathbb{I}_{g_k(k,b,x,z)>0} Q((k,b,x,z),(k',b',x',z'))\varphi(z'-x')h(x'/x)d\Gamma(k,b,x,z)+ \\ &M \int \mathbb{I}_{g_k^e(x)>0} Q^e(s,(k',b',x',z'))\varphi(z'-x')h(x'/x)d\Gamma^e(x) \end{aligned} \tag{1.17}$$

where $Q(\cdot)$ denotes a transition function for incumbents that maps current states into future states, and $Q((k,b,x,z),(k',b',x',z'))$ is 1 if the incumbent's optimal choice is (k',b') and 0 otherwise. The transition function for new entrants Q^e is defined similarly. Transitional probability from x to x' is given by $h(x'/x)$ and the probability of z' conditional on x' is equivalent to the probability of transitory shock $\epsilon' = z' - x'$ given by $\varphi(z' - x')$.

The evolution of firm distribution depends on firms' decisions on borrowing and investing. I call firms that choose not to repay their debt in full "defaulting firms", i.e. $g_d(k,b,x,z) = 1$. I call firms that choose to repay their debt but have zero capital "exiting firms", i.e. $g_k(k,b,x,z) = 0$. Whenever an operating firm defaults or exits, it is replaced by a potential entrant such that the mass of firms stays constant.

1.3 Dynamic Model with Default Risks and Learning

In this section I describe the model with default risks in an environment with information frictions and learning. I assume that firms learn about their productivity level through operation in reminiscence of earlier works by [15]. In what follows I set up the model with learning and focus the discussion on how it differs from the model with full information.

1.3.1 Environment and Information Structure

As before, time is discrete and there is no aggregate uncertainty. The types of agents, firm technology, and idiosyncratic shock structure remain the same as in section 1.2. To recapitulate, there is a large number of risk neutral lenders and a constant measure of firms that have access to a decreasing return to scale technology: $y = \exp(z)k^\alpha$. Log productivity $z = x + \epsilon$ consists of a persistent component x that follows an AR(1) process

$$x' = \rho x + \nu, \quad \nu \sim N(0, \sigma_\nu)$$

and a transitory component $\epsilon \sim N(-\frac{1}{2}\sigma_\epsilon^2, \sigma_\epsilon)$. Shocks to the persistent and transitory components of productivity are i.i.d.

The information structure in this model differs from the model with full information. Specifically, each firm's realized productivity z is observable to the public. However, neither persistent nor transitory shocks are observable, but the distribution of these idiosyncratic shocks are known to the agents in the economy. At the beginning of period t, a firm and the lenders share the same prior belief over its current period persistent productivity x_t that is determined by its information set $\mathcal{F}(t-1) = \{s, z_1, ..., z_{t-1}\}$, which consists of a noisy signal of its initial productivity $s = x_0 + \epsilon_0, \quad \epsilon_0 \sim N(0, \sigma_s)$ and its subsequent productivities up until $t-1$. I assume agents in the economy update their belief using the Bayes' rule. Specifically, the prior belief of an incumbent belief can be summarized by the first two moments (μ_t, σ_t), where $\mu_t = E[x_t|\mathcal{F}(t-1)]$ and $\sigma_t^2 = Var[x_t|\mathcal{F}(t-1)]$. Prior beliefs evolve as follows:

$$\mu_{t+1} = E[x_{t+1}|\mathcal{F}(t)] = \rho \left[\frac{\sigma_t^{-2}\mu_t + \sigma_\epsilon^{-2}(z_t + \frac{1}{2}\sigma_\epsilon^2)}{\sigma_t^{-2} + \sigma_\epsilon^{-2}} \right]$$

$$\sigma_{t+1}^2 = Var[x_{t+1}|\mathcal{F}(t)] = \rho^2 \left(\frac{\sigma_t^2 \sigma_\epsilon^2}{\sigma_t^2 + \sigma_\epsilon^2} \right) + \sigma_\nu^2$$

1.3.2 Firm Problems

Each period a firm maximizes discounted dividend payout to their shareholders by making decisions on debt repayment, investment, and financing plans. Figure 1.2 summarizes the timing in this model and I highlight the key differences from previous timing in red: the firm and the lenders update their belief over the productivity distribution next period after observing today's productivity z. Lenders draw a new set of debt contracts that reflect the firm's default risks under the updated belief. The firm makes its decision on default, capital installment, and financing plans based on its updated belief.

Now I provide a recursive formulation for an incumbent firm. Upon observing productivity shock z, the value of an incumbent firm with capital k, liability b, prior belief (μ, σ) is $V(k, b, \mu, \sigma, z)$, which is the maximum between the value of repayment and zero, the value of default.

$$V(k, b, \mu, \sigma, z) = \max\{V^{ND}(k, b, \mu, \sigma, z), V^D = 0\} \tag{1.18}$$

As before, the repayment value can be represented by its current period dividend payout and the expected continuation value. Non-defaulting firms maximize their repayment value by choosing capital stock k' and external debt b' next period, conditional on their updated belief over expected productivity. The firm has the same budget constraint as before:

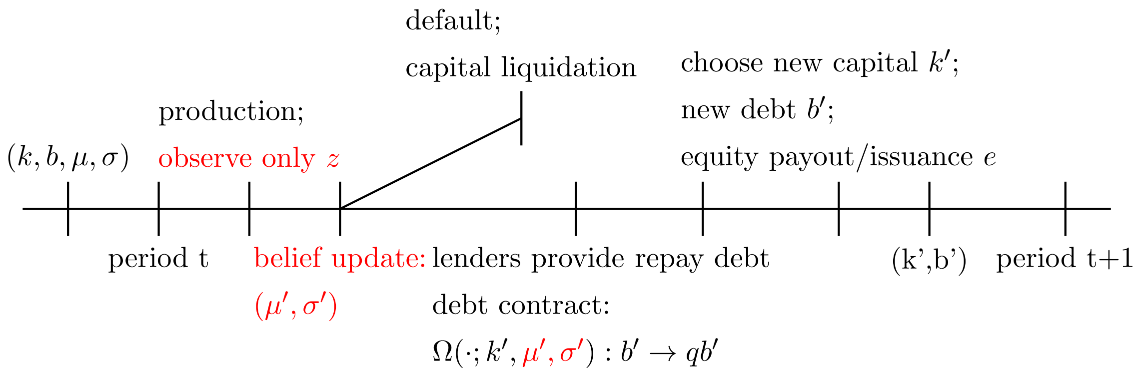


Figure 1.2: Timing for model with learning

capital installment k' and associated adjustment cost $\phi(k'-k)^2/k$ are financed by using a combination of internal funds $\exp(z)k^\alpha + (1-\delta)k - \eta - b$ and external financing.

$$V^{ND}(k, b, \mu, \sigma, z) = \max_{e, k', (qb', b') \in \Omega} (1 + \gamma \mathbb{I}_{e<0})e + \beta E_{z'}[V(k', b', \mu', \sigma', z')] \tag{1.19}$$

subject to

$$e = \underbrace{\exp(z)k^\alpha + (1-\delta)k - b - \eta}_{\text{internal funds}} + qb' - k' - \phi(k'-k)^2/k, \tag{1.20}$$

$$\mu'(z) = \rho \left[\frac{\sigma^{-2}\mu + \sigma_\epsilon^{-2}(z + \frac{1}{2}\sigma_\epsilon^2)}{\sigma^{-2} + \sigma_\epsilon^{-2}} \right]$$

$$\sigma'^2(z) = \rho^2 \left(\frac{\sigma^2 \sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2} \right) + \sigma_\nu^2$$

Now I turn to the entrant's problem. Each period potential new entrants endowed with initial equity e_0 draw a persistent productivity x_0. However, they only observe a noisy signal of $s = x_0 + \epsilon_0$, $\epsilon_0 \sim N(0, \sigma_s)$. Upon observing this signal, the potential entrant must decide whether to start operating next period. If so, it also needs to decide today the amount of capital stock with which it wants to start production tomorrow. A potential entrant can raise its initial capital stock by paying with its own equity e_0, borrowing from a lender, and issuing equity. The value of a new entrant after observing signal s is given by

$$V^e(s) = \max_{e, k', (qb', b') \in \Omega} (1 + \gamma \mathbb{I}_{e<0})e + \beta E_{z'} V(k', b', \mu'(s), \sigma'(s), z') \tag{1.21}$$

subject to

$$e = e_0 + qb' - k', \tag{1.22}$$

$$\mu'(s) = \rho \left[\frac{\sigma_s^{-2} s}{\sigma_\nu^{-2}(1-\rho^2) + \sigma_s^{-2}} \right]$$

$$\sigma'^2(s) = \rho^2 \left(\frac{\sigma_\nu^2 (1-\rho^2)^{-1} \sigma_s^2}{\sigma_\nu^2 (1-\rho^2)^{-1} + \sigma_s^2} \right) + \sigma_\nu^2$$

1.3.3 Lender's Problem and Debt Schedule

Debt schedules are derived from lender's problem as before. Lenders are risk-neutral and perfectly competitive. In equilibrium, their expected return on each loan equals the return on a risk-free investment. This expected return is calculated based on their belief on a firm's productivity. We assume that there is no asymmetric information between firms and lenders. In particular, at the time when debt contract is drawn, lenders and firms share the same belief over firms' productivity distribution next period. Each debt contract $(b', qb') \in \Omega(k', \mu', \sigma')$ must therefore satisfy the following break-even condition:

$$\begin{aligned} &[b'/q(k', b', \mu', \sigma')] \int (1 - d(k', b', \mu', \sigma', z'))\Phi(dz'; \mu', \sigma') \\ &+ \int d(k', b', \mu', \sigma', z')[(1-\lambda)(\exp(z')k'^{\alpha} + (1-\delta)k')]\Phi(dz'; \mu', \sigma') = Rb' \end{aligned} . \tag{1.23}$$

and

$$d(k', b', \mu', \sigma', z') = \begin{cases} 1 & \text{if } V^{ND}(k', b', \mu', \sigma', z') < 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.24}$$

1.3.4 Optimality Conditions and Equilibrium

The optimality conditions for the incumbent who solves the problem described in equation 1.19 are similar to the set of conditions laid out in equation 1.12-1.14. The only difference is that the expectations are taken with respect to updated priors.

I restrict the attention to the stationary recursive equilibrium. The evolution of prior (μ, σ) can be determined by the joint distribution of the beliefs (μ, σ, z). The evolution of firm productivity z is however determined by the true persistent productivity x. Therefore, I need to define measures of firms in the equilibrium in terms of joint distribution of belief and true persistent productivity.

Given a risk-free rate R, a *stationary recursive equilibrium* consists of

- value functions for incumbents $V(k, b, \mu, \sigma, z)$ and new entrants $V^e(s)$
- incumbents' policy functions for default $g_d(k, b, \mu, \sigma, z)$, debt financing $g_b(k, b, \mu, \sigma, z)$, capital investment $g_k(k, b, \mu, \sigma, z)$, and dividend payout $g_e(k, b, \mu, \sigma, z)$
- New entrants' policy functions for debt financing $g_b^e(s)$, capital investment $g_k^e(s)$, and dividend payout $g_e^e(s)$
- debt schedule: $q(k', b', \mu', \sigma')$
- measures of incumbents $\Gamma(k, b, \mu, \sigma, x, z)$ and new entrants $\Gamma^e(x, s)$

such that

- given the schedule of loan contracts offered, the policy and value functions solve firms' optimization problem;
- loan contracts reflect the firm's default probabilities such that with every contract lenders break even in expectation;
- measure of firms $\Gamma(k, b, \mu, \sigma, x, z)$ is consistent with firms' policy functions:

  Let M denote the measure of potential new entrants

$$M = \int \mathbb{I}_{g_k^e(x,s)=0} d\Gamma^e(x, s) + \int \mathbb{I}_{g_k(k,b,\mu,\sigma,z)=0} d\Gamma(k, b, \mu, \sigma, x, z) \tag{1.25}$$

$$\Gamma^e(x, s) = Mf(x, s) \tag{1.26}$$

  where $f(x, s)$ is the probability density of the joint distribution of true type x and signal s.

$$\begin{aligned} &\Gamma(k', b', \mu', \sigma', x', z') \\ &= \int \mathbb{I}_{g_k(k,b,\mu,\sigma,z)>0} Q((k, b, \mu, \sigma, x, z), (k', b', \mu', \sigma', x', z'))\varphi(z' - x') d\Gamma(k, b, \mu, \sigma, x, z) \\ &\quad + M \int \mathbb{I}_{g_k^e(x,s)>0} Q^e((x, s), (k', b', \mu', \sigma', x', z'))\varphi(z' - x') d\Gamma^e(x, s) \end{aligned} \tag{1.27}$$

where $Q(\cdot)$ denotes a transition function for incumbents that maps current states into future states, and $Q((k, b, \mu, \sigma, z), (k', b', \mu', \sigma', z'))$ is 1 if the incumbent's optimal choice is (k', b'), prior belief evolves to (μ', σ'), persistent productivity x evolves to x' and 0 otherwise. The transition function for new entrants Q^e is defined similarly. The probability of z' conditional on x' is equivalent to the probability of transitory shock $\epsilon' = z' - x'$ given by $\varphi(z' - x')$.

1.4 Calibration

In the quantitative exercise, I estimate the models with and without information friction to match a fixed set of moments in the data, and compare their predictions on the age-leverage pattern with the data. In this section, I present the calibration strategies. I first present parameters that are chosen independently of the model. Next I discuss the selection of moments to discipline parameters in the full information model. Finally I discuss one additional moment used in the estimation of the model with learning and compare calibration outcomes in both models.

Table 1.1 reports the values of parameters that are chosen independently of the model. I set the length of a period in the model to one year. The capital depreciation rate δ is set at 10% and the annual discount factor β is set to be 0.96, which are standard values for annual RBC models. The interest rate r is set at 4% per annum. Following [9], the decreasing returns to scale parameter α is set to 0.65 and equity issuance cost γ is set to 0.3. Following [13], I set linear bankruptcy cost λ to be 15%.[1] I choose a standard quadratic form for the capital adjustment cost: $\phi(k, k') = \phi(k' - k)^2/k$, and set ϕ to be 0.004, an intermediate value in the literature, which reports calibrated or estimated values from 0.043 ([21]) to 0.001 ([9]).

The firm's log productivity consists of a persistent component x and a transitory component ϵ. In particular, the persistent component x follows an AR(1) process governed by the persistence parameter ρ and the standard deviation σ_ν. In each period, there is a

[1] This value of bankruptcy cost is an intermediate value in the literature, which reports calibrated or estimated values from 0.1 ([11]) to 0.25 ([12]) from Compustat data. As a next step, I can estimate external financing costs using data on public firms in China. In the literature, covariance between investment and leverage and covariance between investment and equity issuance are used to discipline bankruptcy cost and equity issuance costs.

Table 1.1: Exogenously determined parameters

Parameter	Function	Value	Sources
Chosen parameter			
β	discount factor	.96	standard value
R	risk-free rate	1.04	[9]
α	decreasing return to scale	0.65	same above
δ	depreciation	0.1	same above
γ	equity issuance cost	0.3	[6]
λ	default loss	0.15	[13]
ϕ	capital adjustment cost	0.004	multiple studies

probability ξ that the firm's persistent productivity turns zero forever, which is akin to a death shock and the firm exits in this case. Transitory shocks are also normally distributed: $\epsilon \sim N(-\frac{1}{2}\sigma_\epsilon^2, \sigma_\epsilon)$. When solving the model, I discretize both the persistent and transitory components of productivity shocks into seven points, following [22].

In the model with full information, six parameters $\{\rho, \sigma_\nu, \sigma_\epsilon, \eta, e_0, \xi\}$ are jointly calibrated to match the following six moments in the data: autoregressive coefficient of log assets, coefficient of variation of log assets, average leverage of mature firms (firm age greater or equal to 10), default rate, difference in capital growth rate between new entrants (firm age less or equal to 2 year old) and incumbents, and average firm age. In the model, all these parameters affect all the target moments in a non-linear non-independent fashion. Nevertheless, each parameter impacts some more particular moments. Specifically, the persistent parameter ρ largely determines the persistence of log assets. The transitory shock variance σ_ϵ, and the fixed production cost η, affect average leverage and default probability. Higher transitory shock variance and larger fixed production cost η imply greater default risk and result in a lower leverage. Default is not directly observable in the data. I choose to target a 2.5% default rate, which is reported as the average default rate of publicly listed firms in China. Lower initial equity e_0 implies a larger fraction of new entrants who are financially constrained and suggests a higher capital growth rate of new entrants relative to incumbents. By targeting the difference in growth rate, I control for a trend in capital growth rate in the data as a consequence of China's fast TFP growth.

Table 1.2: Full Information Model: Parameters and Target Moments

Estimated parameter			
ρ	shock persistence		0.91
σ_ν	persistent shock variance		0.18
σ_ϵ	transitory shock variance		0.4
η	fixed production cost		8
k_0	initial equity		15
ξ	death shock		0.08
Target moments		Data	Model
autoregressive coefficient of log assets		0.87	0.87
coefficient of variation of log assets		0.21	0.22
average leverage (firm age≥10)		0.58	0.59
default rate		2.5%	2.5%
difference in capital growth rate between new entrants and incumbents		20pp	20pp
average firm age		9.9	10.5

As shown in Table 1.2, the calibration is successful in matching the target moments in the data. The estimated production fixed cost corresponds to 7% of average firm capital.

Now I turn to discuss the calibration of the model with information friction and learning. In addition to the six parameters estimated in the model with full information, I now have one more parameter σ_s that governs the initial signal variance. To jointly calibrate these seven parameters, I fix the six moments as before and match the difference in correlation between log capital and log output among new entrants and among incumbents as an additional target. If initial signals are not informative at all, then all new entrants enter with the same amount of capital, and the correlation between log capital and log output among new entrants would be zero. On the other hand, if initial signals are very informative, then productive firms start with large amount of capital and produce more output relative to unproductive firms. In this case, there should be little discrepancy in capital-output correlation between new entrants and incumbents. In the data, the correlation of log assets and log output is much higher among incumbents than new entrants,

Table 1.3: Learning Model: Parameters and Target Moments

Estimated parameter			
ρ	shock persistence		.91
σ_ν	persistent shock variance		.22
σ_ϵ	transitory shock variance		.4
η	fixed production cost		8.5
σ_s	initial signal variance		3
k_0	initial equity		52
ξ	death shock		.08
Target moments		Data	Model
autoregressive coefficient of log assets		0.87	0.88
coefficient of variation of log assets		0.21	0.21
average leverage (firm age≥10)		0.58	0.57
default rate		2.5%	2.8%
difference in correlation of log capital and log output between incumbents and new entrants)		0.24	0.22
difference in capital growth rate between new entrants and incumbents)		20pp	20pp
average firm age		9.9	9.9

and the calibration thus suggests a high initial signal variance. Table 1.3 summarizes the parameter values and the target moments in the data and in the model with learning. The estimated production fixed cost corresponds to 4% of average firm capital.

Now I compare the estimation results of two models and explain three main differences. First, compared to the full information model, I need to increase the variance of the persistent shock in order to match the coefficient of variation of log assets. With learning the dispersion of firm's belief on its persistent shock x is less than the dispersion of the persistent shock itself. Because capital installment is determined by the firm's prior belief in the model with learning, the distribution of log assets is smaller in the model with information frictions than in the model with full information under the same shock process. Second, the estimation suggests a larger initial signal variance relative to the variance of transitory shock. This is because in the data the correlation between log capital and log output is significantly lower among new entrants than that of incumbents. Third, the model with learning requires a higher initial equity level to match the differences in capital growth rate between new entrants and incumbents. If I set the initial equity to be the same in two models, then new entrants' capital stock relative to that of incumbents grows significantly faster in the model with learning than in the model with full information. In this case, middle firms will experience similar capital growth rate in both models. In the full information model, productive firms enter with large capital because they are able to finance it by borrowing from the lenders. In the learning model, the productive firms start with much smaller amount of capital and experience much faster capital growth than their counterparts in the full information model. Lenders in the learning model are unwilling to lend new entrants a large amount of money initially because of the lack of information, but are able to increase their debt limits as they become more optimistic about these firms' productivities. Unproductive firms do not enter in the full information model. In the learning model, unproductive firms enter but quickly drop out. Because of selection, the negative capital growth rate of unproductive firms is dominated by the capital expansion of productive firms in the learning model. Therefore, on average the capital stock of new entrants grows much faster in the learning model than in the full information model conditional on the initial equity level. This difference in the initial equity level between these two models implies that a greater fraction of new entrants are credit constrained in the full information model than in the learning model. These financially constrained firms enter inefficiently small and experience fast capital growth when they are young.

1.5 Quantitative Results

After solving the optimization problem given each state variable, I simulate 50,000 firms over 200 periods. At this point, the models deliver a cross-sectional distribution of firms, which is used to compute the model's statistics. In both models, firm size equals firms' capital K, output is measured as $\exp(z)k^{\alpha}$, the leverage of a firm equals to the ratio of outstanding debt to capital installed B/K.

In this section, I present the quantitative results for both models. I show that the calibrated model with learning is successful in matching the variation of leverage with age in the data, whereas the calibrated model without learning overpredicts the average leverage of young firms. I explain these quantitative results by discussing the interplay of model ingredients and the impacts of financial frictions and learning on firms' decisions on capital investment and debt financing.

1.5.1 Model Implications on Leverage

Figure 1.3 compares how the calibrated models with and without learning perform in regards to predicting the variation of average leverage with firm age. I can evaluate both models based on average leverage by age, because I do not target these moments in the calibration.

As depicted by the green line in both plots, new entrants on average have the lowest debt-to-asset ratio at about 0.45. Average leverage initially increases with firm age, peaks at 0.6 for firms that are 7 year old, and then it decreases slightly with age and stabilizes around 0.57 for firms that are more than 15 year old.

The blue line in the left panel plots average leverage generated by the learning model with the parametrization reported in table 1.3. It suggests that the learning model performs well in matching the age-leverage pattern in the data: average leverage in the simulation is close to the data for each age group. The predictions by the full information model are worse in comparison, as shown in the right panel. Average leverage in the simulated data produced by the full information model is much higher for young firms compared to the data. Also, while the full information model has a hump pattern, the peak of the hump is within a couple years, while in the data and in the learning model, the peak occurs after about six years.

Before I provide an elaborated analysis in the next two subsections, I briefly explain my results by addressing the following two questions. First, why do new entrants choose high leverage in the full information model than in the learning model? Recall that in our calibration, the absolute level of initial equity is significantly lower in the full information model than that in the learning model in order to match the difference of capital growth rate between new entrants and incumbents in the data. In the full information model, only middle and productive firms enter the market. These firms borrow more than their counterparts in the learning model, because they start off with less equity. Moreover, conditional on expected productivity, lenders are more willing to lend to firms when their types are known, because information frictions increases default risks.

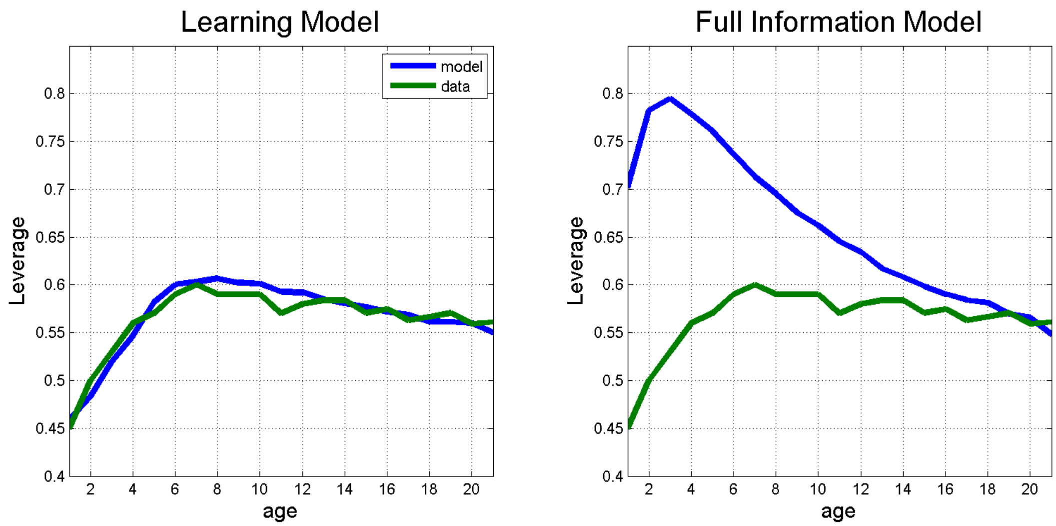
Second, why is age effect on leverage hump shaped in both models? In the full information model, all firms have incentives to accumulate equity to lower risks of default and costly equity issuance. Equity accumulation suggests a negative age effect on leverage. When firms are young, middle firms when hit by bad shocks are likely to default or exit. Since they have lower leverage relative to productive firms, selection generates a positive age effect on leverage among new entrants. The positive age effect on leverage for young firms is more significant in the learning model. There are two reasons for that. First, productive firms increases their leverage initially as they learn about their types and expand their capital. The growth of their capital demand exceeds the rate of their equity accumulation. Second, selection is more pronounced with learning. Unproductive firms would not have entered if their types were known. Therefore, learning and selection explains the increase of leverage with age among young firms in the learning model. As before, equity accumulation explains the decrease of leverage with age among mature firms, but its effect is milder in the learning model, because firms start off with higher equity.

1.5.2 Model Mechanism in the Full Information Model

In this section I explain the model mechanisms in the model with full information. I start with this model because this allows me to isolate the effects of financial frictions on firm's decision rules. In what follows, I first discuss how the assumptions on the technology affect firm size distribution. Next I describe how financial frictions affect firm growth and leverage.

I first discuss the main drivers of firm size distribution. First, decreasing returns to scale technology implies an efficient scale for each firm. Equation 1.13 shows that if equity issuance cost is zero and there is no capital adjustment cost, capital choice would satisfy the familiar optimality condition as in the standard neoclassical growth model: $\alpha E_z'[\exp(z')]k^{*\alpha-1} + 1 - \delta = 1/\beta$. Second, due to the presence of fixed production cost, firms with sufficiently low productivity choose to exit. Third, with adjustment costs and default risks, it takes several periods before firms attain their optimal sizes.

Now let us look at the equilibrium interest rates and how they vary with firm size,



Note: Average leverage by age implied by the learning model (left) and the full information model (right). In each plot, the blue line plots average leverage by firm age predicted by the baseline calibration and the green line plots the data counterpart.

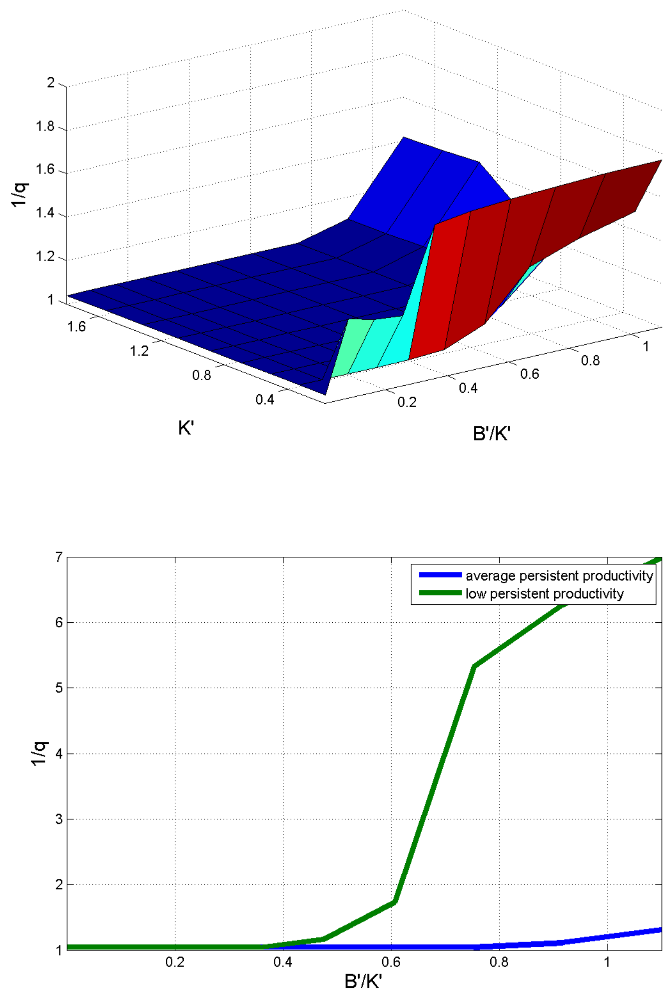
Figure 1.3: Average leverage by age

leverage, and expected productivity. The top panel of figure 1.4 plots interest rate schedules for an incumbent firm with average persistent productivity and average transitory shock, given its next period capital installment K' and leverage B'/K'. If firm leverage is less than 1, then conditional on leverage, borrowing costs decrease as next period's capital installment increases. In particular, firms whose choice of capital equals to its efficient scale next period can take a leverage position of 0.9 at risk free rate. On the other hand, if the same firms were to only operate at 40% of its efficient scale, they can only leverage up to 50% of their capital installment at risk free rate. A small choice of capital makes a firm more likely to default given the presence of fixed production cost and lowers its recovery value in the defaulting state. Conditional on capital installment, borrowing costs increase with leverage ratio. For example, if a firm chooses to operate at its efficient scale next period, the interest rate equals to the risk free rate when leverage is less than 0.9. This is because even with a very low transitory shock the firm can still fulfill its debt repayment with undepreciated capital. As leverage increases to 1 and above, interest rate increases rapidly. With high leverage the firm will need to issue equity in order to pay back debt when hit by low productivity shock. In this case, the firm may find it optimal to default, since equity issuance is costly.

In addition to firm capital and leverage, expected productivity also affects debt schedules. The bottom panel of figure 1.4 illustrates the debt schedules for two incumbents with low and average persistent productivity. When both firms choose capital installment equal to their corresponding efficient scales, the firm with low productivity can only borrow up to 37% at risk free rate, whereas the firm with average productivity can borrow up 78% at risk free rate. This is because the efficient scale for low productive firms is small and they are more likely to default given fixed production cost.

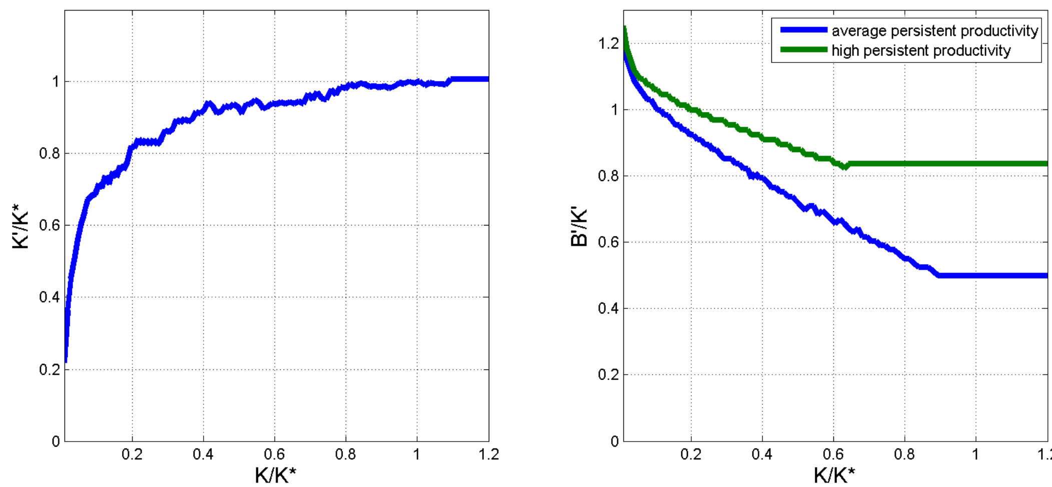
To illustrate how debt contracts affect firms' decision rules, I plot the policy functions for capital and leverage choices of a firm with average persistent shock and average leverage of 55% today in figure 1.5. With capital larger than 85% of the efficient scale, the firm chooses future capital stock close to the efficient scale, and holds a low level of debt of 50%. With capital less than 85% of the efficient scale, the firm invests in rapid capital expansion and finances it using external debt. In general, firms' policy function on future capital displays the following pattern: inefficiently small firms rapidly expand by taking on a large amount of debt and their growth rates slow down as they approach their efficient scales. As discussed above, a larger capital stock next period lowers default risks and borrowing cost today. Formally, the second term $\partial q(b', k', x)/\partial k'$ in the LHS of equation 1.13 is positive and the LHS is smaller relative to the case without default risk. Therefore, holding everything else unchanged, capital choice K' is greater in the case with default risk relative to the case where interest rate equals to risk free rate in all states.

The right panel of figure 1.5 shows the policy function for leverage for firms with average and high persistent shock and 55% leverage today. I highlight two features of the choice of leverage. First, conditional on average leverage today firms' choice on future leverage decreases with firm size initially, and then it stays at a constant level. When firms are insufficiently small, external financing is more costly than internal funds because of default risks and potential costly equity issuance. Inefficiently small firms would use up all internal funds for capital investment and finance the gap using external debt. As firms' internal funds grow, firms are less dependent on external financing. Conditional on leverage and



Note: The top panel plots the interest rate schedules for an incumbent firm with average persistent productivity x and transitory shock $\epsilon = 0$ as a function of its capital installment K' and leverage B'/K' next period. Values on the K' axis are relative to the efficient capital installment given x. The bottom panel plots the interest rate for incumbents with low and average persistent productivity and capital installment K' equal to the corresponding efficient scales.

Figure 1.4: Debt schedules



Notes: The left panel plots the optimal capital choice K' as a function of today's capital K for an incumbent with average persistent shock and current leverage B/K of 55%. The right panel plots the optimal leverage choice B'/K' as a function of today's capital K for two firms with average and high persistent shock, average transitory shock, and current leverage B/K of 55%. All values on the K-axis are relative to the corresponding efficient scales.

Figure 1.5: Policy functions

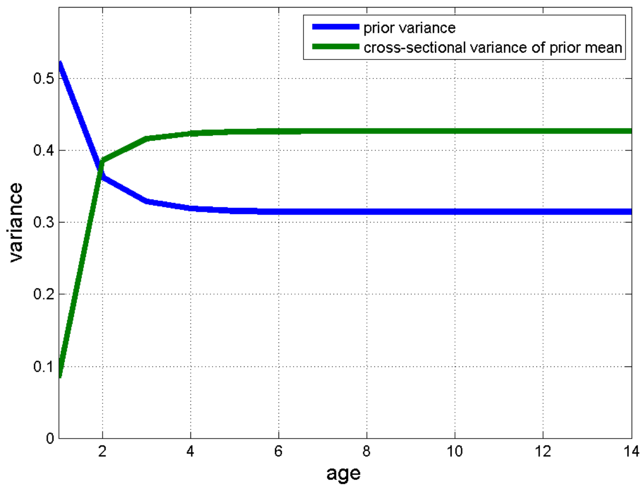
expected productivity, firms with larger capital today also have more internal funds and therefore less demand for external debt. This explains the downward sloping portion of the policy function. When firms' internal financing relative to total capital investment exceeds certain threshold, default and equity issuance in the next period become unlikely. When the wedge between external and internal financing costs disappears, firms stop saving for investment, start to pay out dividends, and maintain a low level of debt. This explains the constant portion of the policy function. The second feature of the policy function on leverage is that firms with higher productivity take higher leverage conditional on leverage and normalized firm size. Because of fixed production costs, unproductive firms, which are small in scale, are more likely to default or to rely on equity issuance if they take on large debt. These firms face lower borrowing limits and stronger precautionary saving motive to lower their leverage.

Now I explain the age-leverage prediction plotted in the right panel in figure 1.3 using the policy functions. New entrants with middle and high level of productivity enter with a small amount of equity. Both of these firms are financial constrained initially so that they enter insufficiently small relative to their optimal sizes. That is, new entrants are mostly distributed on the left half of the K-axis in the figure 1.5 with productive firms being closer to 0. As these firms become older, they invests in capital expansion and experience high capital growth rate, which is consistent with the data. As firms move towards the right of the K-axis, they accumulate more internal funds and they prefer internal funds to external funds to finance capital investment as long as external funds is more costly due to financial frictions. This explains the overall downward sloping pattern between age and leverage. The initial increase in leverage comes from selection: middle firms that are hit by bad shocks are likely to default or exit. Since exiting firms have lower leverage than the cross-sectional average, leverage increases with age for the initial couple years.

1.5.3 The Role of Learning

Now I turn to the learning model and discuss how learning affects debt schedule and firms' decision rules. I first describe the cross-sectional distribution of prior beliefs by firm age. I then discuss how prior mean and variance affects the debt schedule and firms' choice of debt. Combining these two arguments I can rationalize the quantitative results in figure 1.3.

As a result of Bayesian learning, mature firms have more precise belief about their productivity, and their expected persistent productivity has greater dispersion relative to that of new entrants. Figure 1.6 plots the variance in prior beliefs and the cross-sectional dispersion of average expected productivity as a function of firm age. Under the parametrization, prior variance asymptotically decreases from 0.52 for new entrants to 0.31 for mature firms. Much of the decrease happens within the first three periods when firms have less information. Since there are shocks to firms' persistent productivity, firms never fully learn their types; instead, the precision of their belief becomes constant after 9 periods. At the same time, the standard deviation of expected productivity increases from 0.1 for new entrants to 0.42 for mature firms. This is because we assume firms only know about the true distribution of persistent productivity but not their own type before they observe their initial signals. Conditional on observing good signals and experiencing good shocks, productive firms increases their projection for future productivity over the course of several periods. Conversely, conditional on observing bad signals and experiencing low shocks, unproductive firms become more and more pessimistic.

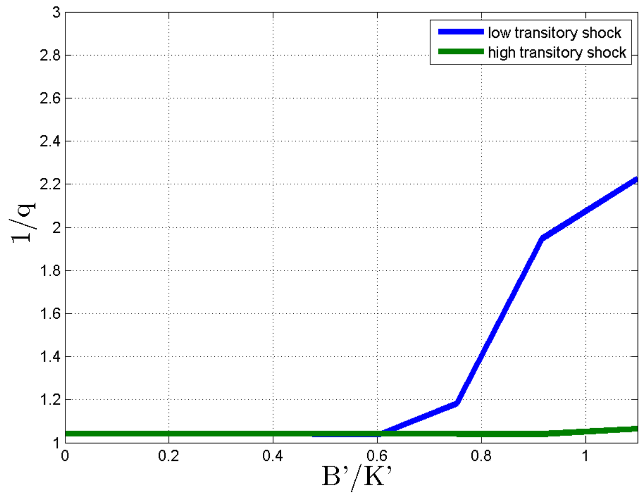


Notes: The blue line plots prior variance as a function of firm age. The green line plots the cross-sectional standard deviation of expected productivity by firm age.

Figure 1.6: Prior variance and dispersion of prior mean

Now that we have established the variation of prior belief (μ, σ) with firm age, we show how debt schedules and policy rules vary with prior beliefs. As the mean μ' increases, the firm is less likely to default and the borrowing cost $1/q(k', b', \mu', \sigma')$ decreases until it reaches the risk free rate. This is similar to how the debt schedule varies with expected productivity in the model with full information. Conditional on the mean, the realization of low productivity is less likely when the variance is smaller. Therefore, the default probability and the effective interest rate decreases with the prior variance σ'. In both cases, with lower borrowing cost, firms tend to choose higher leverage. To better illustrate the intuition, I solve a static version of my model analytically and show how equilibrium leverage varies with the productivity shock process in appendix A.1.1.

In the learning model, since initial signals are not very informative, new entrants enter



Notes: This figure plots the debt schedule for two 2-year-old firms with median expected productivity $\mu = 0$ and low and high transitory shocks $\epsilon_2 < \epsilon_6$. In both cases, their choices on capital installment equal to their corresponding efficient scale.

Figure 1.7: Transitory shock affects debt schedule in the model with learning

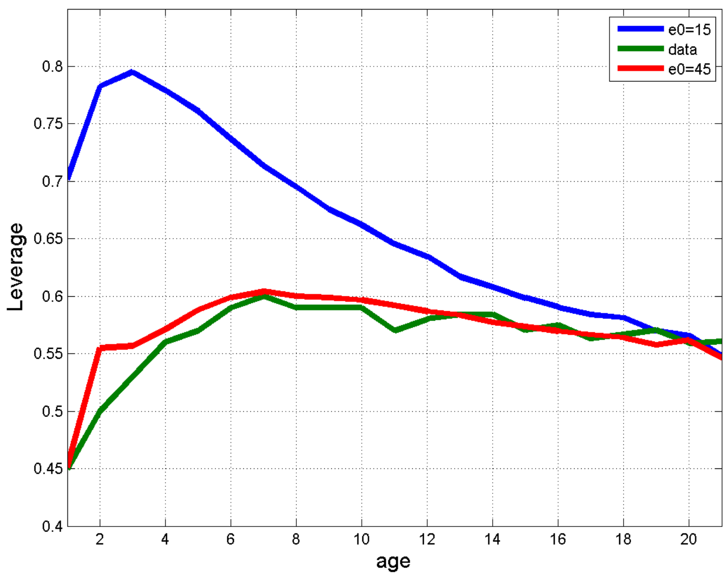
with similar level of capital stock and leverage. After experiencing good shocks, young firms become more optimistic about their future productivity and increase their demand for debt to finance capital expansion. The realization of both persistent and transitory shocks affects firms' debt limit. This is different from the full information model, where transitory shocks are observable and do not affect firms' borrowing costs. Figure 1.7 shows that after experiencing a low shock, a young firm can only borrow 45% of its next period capital at risk free rate, whereas another firm that shares the same prior after a high shock can borrow up to 90% of its capital installment at risk free rate next period. From the lender's perspective, a firm with low realized productivity is likely to be unproductive (low persistent productivity) and have small efficient operation scale. Such firm is more likely to default on the loan and higher interest rate reflects such increase in default risk.

As young firms expect higher productivity in the future, they also face a relaxed borrowing limit and borrow more as they expand towards an updated efficient production scale. When the growth of their capital investment exceeds the rate of their equity accumulation, they increase leverage to expand their production scale. Therefore, productive firms experience faster asset growth and increase their leverage when they are young. On the other hand, firms with low persistent productivity are likely to experience bad shocks. After experiencing bad shocks, these firms may choose to default on their debt payment or exit the market. .[2] The leverage of middle firms stay constant in the learning model. This is because the initial equity level in the learning model is sufficient for middle firms to start with their optimal size by borrowing at risk free rate. Therefore, in the cross section the variation of firms dynamics with age is mostly driven by productive firms. Including learning in the model generates a much more significant positive age effect on leverage for young firms. As firms become older, the importance of new information and the learning effect subside. At the same time, productive firms accumulate more equity and maintain a low level of leverage. Thus, equity accumulation explains the negative age effect on leverage, as in the full information model.

1.5.4 Parameter Identification

In this subsection, I show that an alternative parametrization of the full information model can perform reasonably well in terms of matching the age-leverage in the data. However, the full information model under such parametrization underestimates the capital growth rate of new entrants relative to incumbents. In this exercise, we re-estimate the size of initial equity in the model with full information to match average leverage of new entrants in the data. This estimation implies an initial equity level of 45, which is around 45% of average firm assets and three times as large as the estimates in the benchmark calibration. Figure 1.8 plots its implication on age-leverage variation and we keep the prediction by the benchmark calibration and data counterpart for comparison. As we see, this model matches very well average leverage for firms that are four year or older, and slightly over-predicts average leverage for firms that are two or three year old. Under this parametrization, new entrants with middle productivity are no longer credit constrained and they enter with

[2] This selection effect is also present in the case with full information: young firms are much likely to default and exit. However, the selection effect is much more pronounced in the case with learning, since some new entrants under full information would not have started operation under full information.

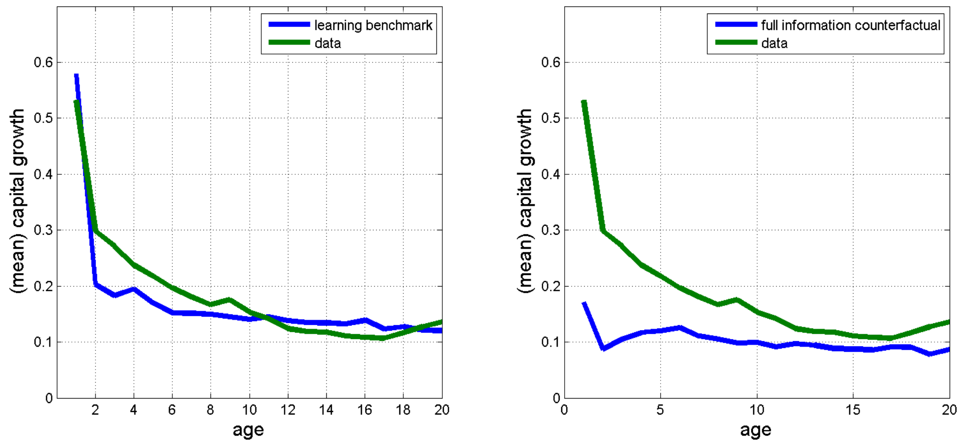


Note: Alternative parametrization of the full information model. The blue line plots average leverage by firm age in the full information model under the benchmark calibration, the red line plots the prediction under the parametrization with higher initial equity level, and the green line plots the data counterpart.

Figure 1.8: Average leverage by firm age in the full information model: different initial equity levels

optimal capital. [3] That is, many new entrants are distributed on the right half of the K-axis in figure 1.5. Since these firms enter with sufficient equity as safety buffer against future default risks or costly equity issuance, they may choose to distributes any excessive internal funds through dividend payout during initial periods. When they are hit by a bad shock, they may scale back in operation and increase their leverage. This dynamics of middle firms drives the increasing portion of the age-leverage pattern. However, this calibration under-estimates the relative capital growth rate of new entrants as shown in figure 1.9 : it suggests that the average capital growth rate among new entrants is 5 percentage points higher than that of the incumbents, whereas the difference is 20 percentage points in the data. This is because middle firms enter with assets close to their efficient scale and

[3] Low productive firms do not enter.



Note: Average capital growth rate by age implied by model with information frictions and learning under benchmark calibration (left) and model with full information and higher initial equity (right). In each plot, the blue line plots average capital growth rate by firm age in the model simulation and the green line plots the data counterpart.

Figure 1.9: Average capital growth rate by firm age

their growth dynamics are similar to that of mature firms with similar productivity level. Therefore, a higher initial equity level implies a lower capital growth rate of new entrants. To the extent that we focus on differential capital growth rates and leverage patterns along the age dimension, the data prefers the learning model.

1.5.5 Regression Analysis

[6] show that financial frictions and persistent shocks can generate simultaneous dependence of firm dynamics on firm size and age. In a similar spirit, I now show that the learning model in this paper also generates a simultaneous dependence of firm leverage on firm size and age that is consistent with the data. Specifically, we run the following regressions for firms that are less or equal to 6 year old and firms that are great than 6 year old using China data and the simulated data from the model with learning:

$$B_i/K_i = \beta_0 + \beta_1 ln(K_i) + \beta_2 age_i + \epsilon_i,$$

Table 1.4: Leverage regression on firm size and firm age

	data		model with learning	
	firm age ≤ 6	firm age > 6	firm age ≤ 6	firm age >6
size	0.015***	0.013***	0.226***	0.129***
	(0.0006)	(0.0008)	(0.0009)	(0.001)
age	0.008***	-0.002***	0.015***	-0.003***
	(0.0004)	(0.0003)	(0.0006)	(.0001)

Standard errors in parentheses; *** $p<0.01$

where i denotes the firm index.[4] The regression results are reported in table 1.4. As in the data, conditional on firm size, age has a hump-shaped effect on firm leverage. As explained above, the positive correlation for young firms is driven by the increasing demand for external funds of productive firms to finance their capital expansion as their priors become more optimistic. The negative correlation for mature firms is driven by firms' precautionary saving motive: firms would like to accumulate enough equity buffer and not exhaust their borrowing capacities to avoid costly equity issuance or default in the future. Conditional on age, firm size is positively correlated with firm leverage in both regressions. Within my framework, firm size is a proxy for productivity. Young productive firms become bigger almost immediately after experiencing good shocks. Since all firms start out with the same amount of initial equity, productive firms will have more demand for debt to finance their capital investment. Among mature firms, larger firms tend to be more productive, so they also face a lower risk of default or of incurring equity issuance costs. Therefore, they are willing to take higher leverage as they approach their efficient scale of production.

1.6 Counterfactual: Alleviating Financial Frictions

Equity issuance cost and bankruptcy cost are two sources of financial frictions in my model. In this section I compare firm size, and leverage with and without these costs in the model

[4] In the regression using China data we also control for year, province, and industry fixed effect.

with full information and the model with learning. I find that, first and foremost, without equity issuance costs both models cannot generate the hump-shaped age-leverage pattern, whereas without bankruptcy costs the hump shape remains in both models. In addition, in both models eliminating equity issuance costs leads to lower productivity cutoffs for entry and exit, smaller average firm size, and higher average leverage. These changes are more pronounced in the model with full information than in the model with learning.

The top left panels in figure 1.10 plots average log assets with and without equity issuance cost in the full information model. First, average firm size is much bigger for new entrants in the case without equity issuance cost relative to the benchmark. Productive new entrants choose to enter at a larger scale and issue more equity to finance their initial capital investment. Average firm size only increases slightly with firm age because of selection and capital adjustment costs. Second, average firm size is smaller in the case without equity issuance costs for firms that are more than four years old. When equity financing is costless, firm value is much higher controlling for firm equity and productivity. This implies a lower productivity cutoff for entry and exit and a larger fraction of low productive firms in the economy. This compositional change explains the effect of equity issuance cost on average firm size for mature firms.

Firms also choose a higher level of leverage as shown in the top right panel in figure 1.10. Lowering equity issuance cost positively affects both supply and demand for debt. With $\beta R < 1$ debt financing at risk-free rate is still preferred to equity financing even without equity issuance cost.[5] Furthermore, when hit by a bad shock, firms are more willing to issue equity to sustain its operation and less likely to default. In this sense, cheap equity creates a safety buffer for debt financing and results in a higher borrowing limit at risk-free rate. On the demand side, cheap equity also dampens firms' precautionary saving motive. With lower default risk and cheap equity financing options, firms have less incentive to accumulate equity and greater demand for debt financing. In this case, average leverage increases slightly with firm age.

The bottom panels in figure 1.10 plot results of the same counterfactual exercise in the learning model. Average firm size is smaller without equity issuance cost and this is driven by the same compositional effect as before. However, average size for new entrants

[5] An alternative way for the model to generate this pecking order is to include tax benefits to debt financing and assume $\beta R = 1$.

remains unchanged. In the calibrated learning model, initial signals are uninformative and therefore all new entrants have similar priors and choose to enter with capital installment level similar to the efficient scale of an average firm. When equity issuance is costly, new entrants use up all their initial equity to finance its initial capital installment and borrow the rest. When equity issuance becomes costless, in the event of bad shocks firms are willing to issue equity to repay debt. Therefore, even with a small amount of internal funds, default risks are low and so are the external financing costs. In this case, firms may prefer external funds to internal funds. They may choose to pay out dividends to shareholders before they start operating and finance a larger fraction of initial capital with debt. Productive firms expand their production scale as they learn about their own productivity. This learning effect drives much of the increase of average firm size with firm age.

As in the case with full information, age-leverage profile flattens out without equity issuance costs in the model with learning. The upward sloping portion of the age-leverage relationship disappears because new entrants already enter with high leverage. The downward sloping portion disappears because of firms' diminishing incentive to accumulate equity. Average leverage for mature firms is around 0.78, which is lower than the 0.87 in the case without learning. Given the information structure in the model, firms are never fully certain of their types. This additional uncertainty increases default probability and implies a lower average leverage in the learning model relative to the full information model.

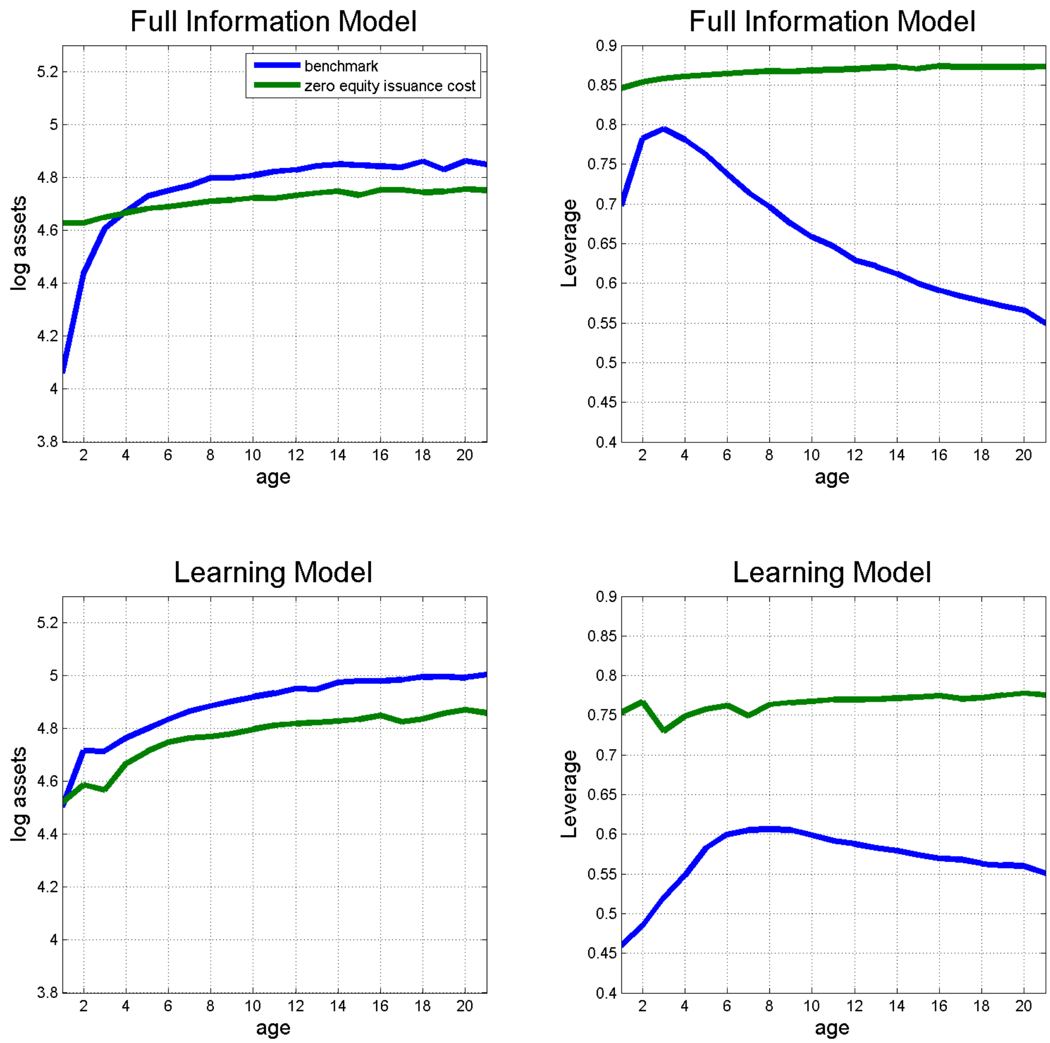
Now I briefly discuss what happens if there is no bankruptcy cost, i.e. $\lambda = 0$. Figure 1.11 shows that eliminating bankruptcy cost do not qualitatively change how average firm size or leverage varies with firm age. In regards to firm size, average firm size does not decrease in this case. Unlike the exercise of eliminating equity issuance costs, bankruptcy costs in the calibrated models do not play an important role in affecting the entry and exit margin. Eliminating such costs mostly affect small surviving firms with high leverage so that these firms can borrow more and operate in a larger scale. This effect explains the small increase in firm size and leverage in both models.

1.7 Conclusion

This paper studies the role of learning about firm productivity in a model of financial frictions, and argues that learning allows the model to generate outcomes closer to the China data. I first develop a model in which firms can default on their debt and equity issuance is costly in an environment with full information. I then extend the model to include information frictions and learning where firms learn about their productivity through production. I calibrate both models to match salient features in the China data and evaluate the models based on their implications on untargetted moments of age-leverage variations. I find that the learning model accounts well for the average leverage by firm age in the data, whereas the full information model overpredicts the leverage of young firms. When calibrated to match the leverage of new entrants in the data, the full information model underestimates the capital growth rate of new entrants relative to that of the incumbents.

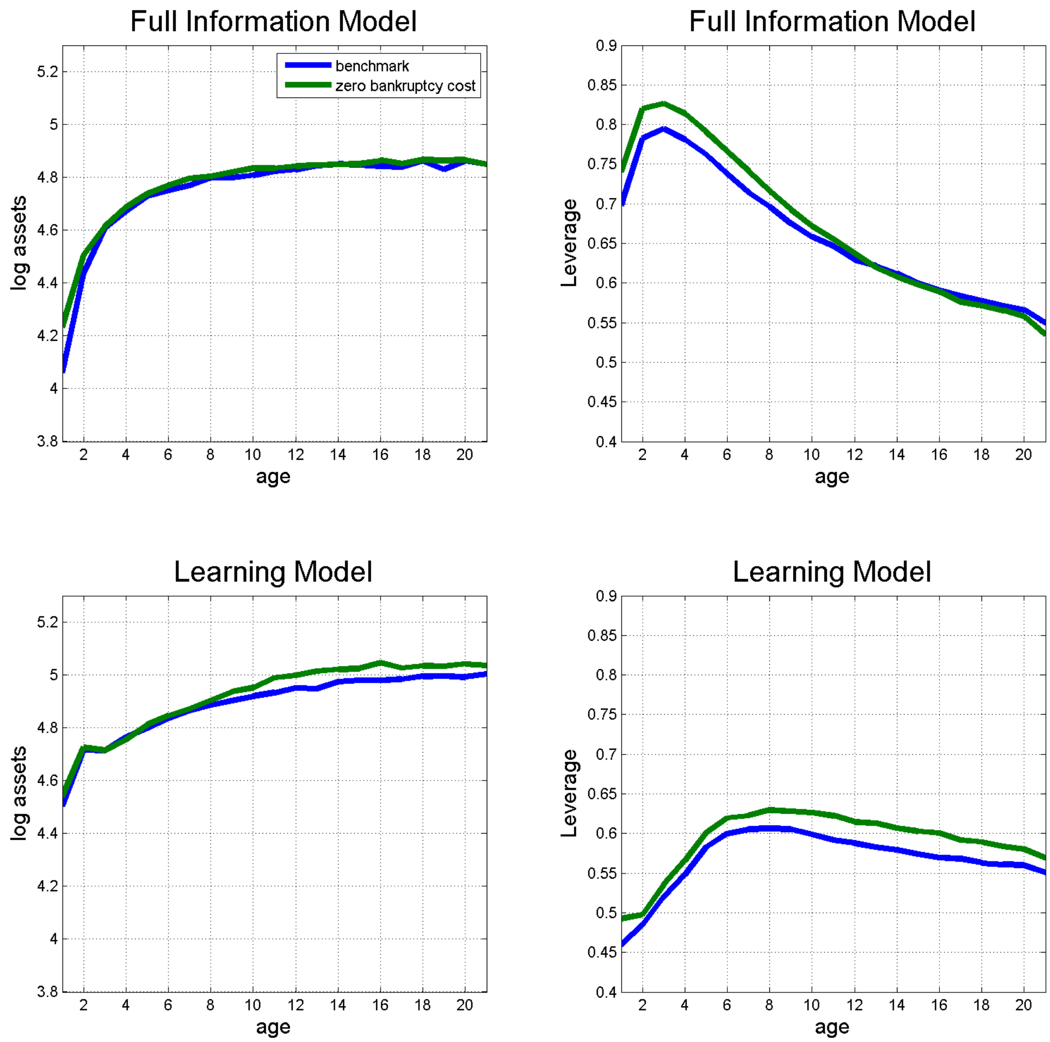
In both models, productive firms have incentives to accumulate equity and reduce their needs for external debt because of the existence of financial frictions and their precautionary saving motives. This explains the downward sloping age-leverage relationship for mature firms. In the model with learning, all new entrants choose relatively low leverage because they are uninformed about their type and face greater default risk. Productive firms become more optimistic after experiencing good shocks. The growth of capital investment of productive firms exceeds their equity accumulation. Therefore, they increase leverage to finance capital expansion. On the other hand, unproductive firms after experiencing bad shocks may default or exit. Learning thus explains the upward sloping age-leverage relationship for young firms. In our counterfactual exercises, I find that equity issuance costs are a key source of financial frictions that generates the hump pattern between age and leverage.

By combining firm-level data and a heterogeneous firm model of financial frictions, I highlight the importance of learning to understand the growth and financing patterns in the cross section of firms with different maturity. An interesting extension would be to introduce aggregate fluctuations in the model. This extension would be a useful framework in analyzing how financial frictions and information frictions affect the cyclical features of cross-sectional firm dynamics.



Note: Average log assets (left) and average leverage (right) by firm age in the full information model (top) and the learning model (bottom). In each plot, the blue line plots model implications with 30% equity issuance cost and the green line plots model implications with zero equity issuance costs.

Figure 1.10: Average log assets and average leverage by firm age with and without equity issuance cost



Note: Average log assets (left) and average leverage (right) by firm age in the full information model (top) and the learning model (bottom). In each plot, the blue line plots model implications with 15% bankruptcy loss and the green line plots model implications with zero bankruptcy losses.

Figure 1.11: Average log assets and average leverage by firm age without bankruptcy cost

Chapter 2

The Great Recession through the Lens of Neoclassical Growth Theory

2.1 Introduction

During the Great Recession from 2007 to 2009, output in many OECD countries declined by a similar magnitude, but market hours experienced much more severe decline in the U.S. than hours in Europe. According to OECD Quarterly National Accounts, in 2007:Q4 - 2009:Q4, output per capita declined by 7.3% in the U.S., 7.2% in France, 6.5% in Germany, 10.5% in Italy, and 9.4% in the U.K. In contrast, aggregate hours per adult in the U.S. declined by 8.4% over the same period of time, much more than the contraction in these other countries: 2.2% in France, 0.4% in Germany, 5.9% in Italy, and 3.3% in the U.K. This paper studies whether differences in total factor productivity (TFP) growth and taxation can account for this cross-country variation in market hours during the Great Recession.

To evaluate the importance of TFP growth and taxes in accounting for changes in market hours in the Great Recession, I conduct business cycle accounting for the period using the framework developed in [23]. Specifically, I write down a neoclassical growth model with productivity growth and taxes on labor and capital income. I then calibrate the preference parameters in this model to match allocations in each country in 2007:Q4.

Holding calibrated parameters constant across time, I introduce country-specific taxes and productivity into the model to simulate market hours in ten OECD countries. I find that for Canada, Germany, Italy, Sweden, and Spain, changes in TFP growth and taxes can account for the changes in market hours during the 2007 recession. In Austria and the U.S., labor market distortions other than taxation prevented market hours from recovering in 2009 despite faster TFP growth and constant taxes on labor income. In Finland, France and the U.K., forces other than labor taxes largely offset the negative effects of slower TFP growth on market hours during the recession.

I choose to focus on productivity growth and labor taxes because these two factors are identified as two main forces in shaping the long-run evolution of market hours in OECD countries. As a recent study on the topic, [24] compares market hours per adult at two points in time (the early 1970s and the mid-1990s) in G-7 countries. He finds that the changes in labor wedge, which measures the inequality between households' marginal substitution between leisure and consumption, and marginal return to market work, are consistent with changes in actual labor income taxes. He comes to the conclusion that differences in labor income taxes can account for changes in labor supplies from early 1970s to mid 1990s in G-7 countries. [25] builds upon Prescott's finding to explain structural changes across different sectors in the American and European economies. Specifically, he builds a model that includes home production and also allows for different technology growth rates across sectors. He calibrates the model so as to match time allocation across sectors in the United States in 1956 and 2000. He uses the calibrated model to simulate time allocation in Europe. His calibration exercise shows that the difference in time allocations between Europe and the United States in 1956 can be explained by the fact that Europe lags the United States in terms of technology and has a somewhat higher tax rate. Higher taxes also account for the bulk of the observed differences in aggregate labor input and sectoral labor allocation between these two economies in 2000. Lastly, the most recent [**?**] uses a growth model augmented with home production and subsistence consumption to study the forces shaping hours worked in OECD countries from 1960-2004. She finds that changing labor income tax rates is the primary force that drives changes in market hours, and productivity catch-up is an important secondary force. This paper builds on this line of literature and applies their framework to study a business cycle episode. In particular, this paper contributes to the existing cross-country study on labor market by showing that changes in productivity growth were the most important factor in accounting for hour contraction, and that labor income taxes had strong mitigating effects during the recession.

This paper shares its focus of study and several findings with a recent work by [26]. They show that in the 2007-2009 global recession, the deviation in labor wedge is much smaller in many European countries than the deviation in the U.S. In addition to confirming his findings for the U.S., this paper points out that changes in taxes cannot account for the large deviation in the U.S. labor wedge. Furthermore, I solve the model and simulate aggregate hours for different scenarios. In particular, I show how hours would have evolved in OECD countries, if these countries had only experienced changes in productivity growth and taxes, and if one of these factors had stayed constant. This exercise isolates the effects of taxes from exogenous productivity changes, and reveals that in most countries small changes in labor income taxes played an important role in mitigating negative effects of productivity growth on labor market in the recession.

The approach adopted in this paper is similar to the one in [27], which also studies the recent recession episode through the lens of a growth model. His paper focuses on evaluating how the expansion of means-tested subsidies in 2007-2009 affected the U.S. macroeconomic aggregates. He finds that the changes in the U.S. replacement rate as the only impulse to the model can generate time series of labor usage, consumption, and output close to the U.S. data. In addition to sharing his methodology, my paper also extends Mulligan's work and considers the impact of expanded subsidies in the context of changing productivity growth and taxes. I find that in this case the model overpredicts the decline in U.S. hours by a considerable margin.

The paper is organized as follows. Section 2 lays out the baseline model for the accounting exercise. Section 3 describes the data and the calibration procedure. Section 4 uses the calibrated model to simulate hours in ten OECD countries. Section 5 discusses two extensions of the baseline model, unemployment subsidies and home production, and evaluates their respective implications for aggregate hours. Section 6 concludes.

2.2 Business Cycle Accounting

I consider a standard, representative agent neoclassical growth model. In each period, firms hire labor and rent capital from households to produce output using a Cobb-Douglas production technology. The production function is given by

$$y_t = k_t^{\theta}((1+g)^t A_t h_t)^{1-\theta},$$

where A_t captures the time-varying productivity and the variable g denotes the constant growth rate of labor augmented technology. Firms choose $\{k_t, h_t\}_{t=0}^{\infty}$ to maximize profits $y_t - r_t k_t - w_t h_t$.

Households are endowed with an initial capital stock of k_0. They take prices $\{w_t, r_t\}_{t=0}^{\infty}$ and taxes $\{\tilde{\tau}_t^c, \tilde{\tau}_t^x, \tilde{\tau}_t^k, \tilde{\tau}_t^h\}_{t=0}^{\infty}$ as given, and choose consumption, investment, and labor supply $\{c_t,\ x_t,\ h_t\}_{t=0}^{\infty}$ to maximize their discounted utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

subject to the budget constraint

$$c_t(1+\tilde{\tau}_t^c) + x_t(1+\tilde{\tau}_t^x) \leq r_t k_t(1-\tilde{\tau}_t^k) + w_t h_t(1-\tau^{ss}-\tau^{inc}) + T_t,$$

and the law of motion for capital

$$(1+g)k_{t+1} = x_t + (1-\delta)k_t.$$

Notice that the taxes on labor income consist of the payroll tax τ^{ss}, and the average *marginal* income tax τ^{inc}. In all equations above, I have detrended all per-capita variables by productivity growth $(1+g)^t$. Preferences over consumption and leisure can be represented by

$$u(c_t, h_t) = \log(c_t) + \frac{a}{1-\frac{1}{\gamma}}(1-h_t)^{1-\frac{1}{\gamma}}, \tag{2.1}$$

where a captures the marginal rate of substitution between consumption and leisure, and the parameter γ denotes the intertemporal elasticity of substitution for leisure (IES), which governs the sensitivity of changes in labor supply with respect to changes in wage.

Government transfers all tax revenues back to household via lump-sum rebate

$$T_t = \tilde{\tau}_t^c c_{mt} + \tilde{\tau}_t^x x_t + \tilde{\tau}_t^h w_t h_{mt} + \tilde{\tau}_t^k r_t k_t.$$

Necessary conditions of a competitive equilibrium are given by two first order conditions and a resource constraint

$$\frac{ac_t}{(1-h_t)^{\frac{1}{\gamma}}} = (1-\tau_{ht})(1-\theta)\frac{y_t}{h_t} \tag{2.2}$$

$$\frac{c_{t+1}}{\beta c_t} = (1+\tau^{cx}_{t+1})\left((1-\tau^k_{t+1})\theta\frac{y_t}{k_t} + 1 - \delta\right) \tag{2.3}$$

$$c_t + x_t = y_t \tag{2.4}$$

Equation 2.2 is the static first order condition for the optimal allocation of time, and it states that the marginal rate of substitution between market consumption and leisure equals to the tax distorted return of market work. The tax that distorts households' intratemporal condition is the effective tax on labor income, which includes taxes on consumption expenditure $\tilde{\tau}^c$, the payroll tax τ^{ss}, and the average *marginal* income tax τ^{inc}:

$$\tau^h_t = \frac{\tau^{ss} + \tau^{inc} + \tilde{\tau}^c_t}{1+\tilde{\tau}^c_t}.$$

Notice that the effective tax on labor income increases with each component. In particular, an increase in consumption taxes also raises the effective tax on labor income, because consumption taxes increase the real price of consumption goods and effectively lowers the marginal return of market work in terms of real consumption. Equation 2.3 is the Euler equation for investment, which illustrates the equality between the marginal rate of substitution between market consumption in period t and market consumption in period $t+1$ and the tax-adjusted return to capital. Tax distortions are captured by the following two terms. The first is the effective tax on capital income, which depends on both taxes on capital income and taxes on investment expenditure

$$\tau^k_{t+1} = \frac{\tilde{\tau}^k_{t+1} + \tilde{\tau}^x_{t+1}}{1+\tilde{\tau}^x_{t+1}}.$$

The second tax distortion is

$$\tau^{cx} = \frac{1+\tilde{\tau}^c_t}{1+\tilde{\tau}^c_{t+1}}\frac{1+\tilde{\tau}^x_{t+1}}{1+\tilde{\tau}^x_t} - 1.$$

Similar to consumption taxes, taxes on investment expenditure lowers the return on capital by making capital investment more expensive. Notice that the second tax distortion τ^{cx} distorts marginal rate of return to capital only when taxes on consumption expenditure $1+\tilde{\tau}_t^c$ and taxes on investment expenditure $1+\tilde{\tau}_t^x$ grow at different rates. Suppose that investment taxes grow faster than consumption taxes. Then investment goods become more expensive relative to consumption goods over time, and this reduces the real return of capital investment.

2.3 Data and calibration

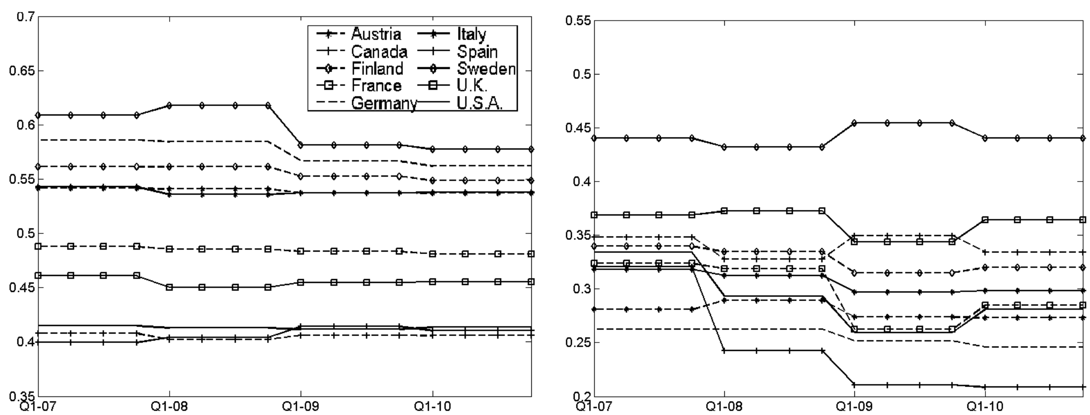
Here I present data used in the simulation exercises in later sections. I construct relevant tax series and estimate productivity series from data for the following ten OECD countries: Austria, Canada, Finland, France, Germany, Italy, Spain, Sweden, the United Kingdom, and the United States. During the Great Recession, all countries experienced slower productivity growth relative to trend and very little change in effect tax on labor income.

2.3.1 Taxes series

All tax series are reported at annual frequency and I make the assumption that tax rates in every country remain constant for all quarters within a year. For each country, taxes on capital income $\tilde{\tau}^k$, consumption expenditures $\tilde{\tau}^c$, and investment expenditures $\tilde{\tau}^x$ are taken from [28]. These taxes are reported as average tax rate, which is calculated by dividing tax revenue generated from each tax category with the relevant income or expenditure reported in the national accounts in each country. Tax distortions on labor income in the U.S. come from two sources. The payroll tax τ^{ss} comes from [28] and the average marginal income tax τ^{inc} is taken from the National Bureau of Economic Research (NBER) TAXSIM program. For other countries, taxes on income and social security combined are taken as the all-in tax rates, calculated as the combined central and sub-central government income tax plus employee social security contribution, as a percentage of gross wage earnings, from OECD Taxing Wages. The all-in tax rates are reported for a single person without dependent in four income classes (67%, 100%, 133%, 167% of the average wage) in each OECD country.

Since total income generated by each income class is not available, I cannot compute the weights necessary for computing average marginal income tax rate. Instead of taking a weighted average, I take the mean of marginal tax rates faced by four income classes to be the sum of average marginal tax on labor income and payroll taxes, corresponding to $\tau^{inc} + \tau^{ss}$ in the model.

Figures 1(a) and 1(b) show the effective labor and capital taxes series for all countries between 2007:Q1-2010:Q4. For all countries and years, τ_t^{cx} is effectively zero so this series is not shown. For most countries, tax distortion on labor income changed less than 1 % over this period. In Sweden, the effective labor income tax decreased from 61.7% in 2008 to 57.7% in 2010. Germany also saw a 2.5% decrease in labor income tax from 58.7% in 2008 to 56.2% in 2010. Comparing figures 1(a) to 1(b) shows that change in capital taxes is much greater than that labor taxes. For instance, capital taxes in Spain decreased from 30% to 20%.



(a) Labor income (b) Capital income

Figure 2.1: Effective tax rate, OECD countries

2.3.2 Productivity series

Series for A_{mt} in each country are computed as follows

$$A_t = \left(\frac{Y_t}{K_t^{\theta} H_{mt}^{1-\theta}} \right)^{\frac{1}{1-\theta}} / (1+g)^t$$

where Y_t and K_t are per capita measures of real output and capital stock. Aggregate output and investment come from OECD Quarterly National Account, and they are reported in constant 2005 US dollars. Aggregate capital series are constructed from investment series using perpetual inventory methods. I divide aggregate real output and investment by working age population between 15-64 as in [26] to obtain per capita measures of these two series. Total market hours also come from [26], which measure aggregate hours worked in OECD countries at quarterly frequency. Figure 2.2 shows that all countries experienced slower productivity growth relative to trend during the Great Recession. In the United States, the detrended productivity dropped 5% in 2008, but it recovered by 3% in 2009. Compared to the US, other countries experienced steeper decline in productivity in 2008 and smaller or no recovery in 2009.

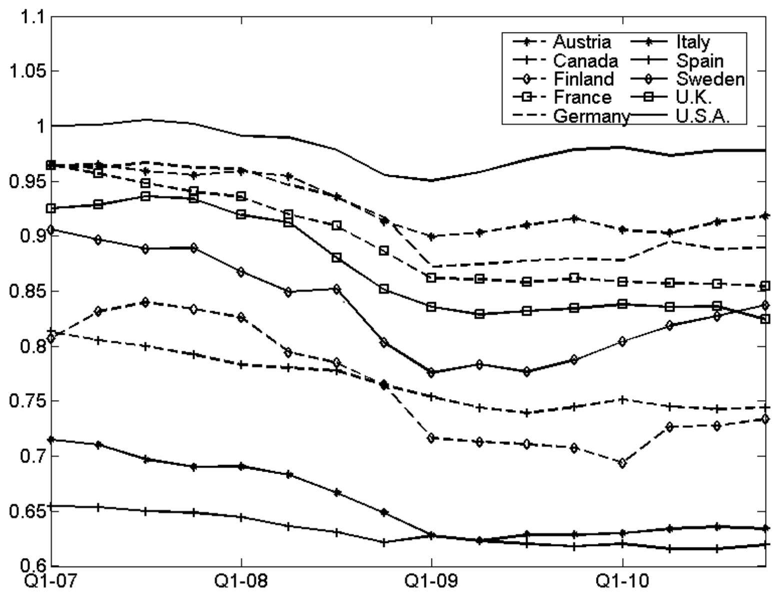


Figure 2.2: Productivity, OECD countries

2.3.3 Parameter calibration

The model is calibrated to match aggregate hours per adult and investment-output ratio in 2007:Q4 in each country. Specifically, the parameters θ, δ, g, and γ are set to be the same across all countries. On the contrary, a and β are country specific and they are chosen so that the series generated by the model match corresponding targets in each country. One can interpret this calibration strategy as assuming preferences are different across countries. A better interpretation would be that there exist country-specific frictions that distort household's and firm's first order conditions. In other words, these frictions manifest as labor wedge and investment wedge in the equilibrium. By allowing a and β to vary across countries, I can capture the level effect of these frictions that are not specifically modeled here. At the same time, by holding a and β constant over time, I can isolate the effects of productivity growth and taxation from other frictions.

The four parameters that are common across all countries are taken from literature without solving the model. I follow [29] for the capital share $\theta = 0.3$. I follow [26] for the depreciation rate $\delta = 0.0175$. I set g to be 0.45% in order to match the average growth rate of A_t over the period 2003-2010. For the benchmark case, I set the IES parameter $\gamma = 1$, which implies a relatively large elasticity of labor supply as suggested in [30]. [31], on the other hand, suggests that a reasonable estimate for the IES is around 0.5. I also report results for this smaller elasticity $\gamma = 0.5$ in a appendix for comparison.

Two preference parameters a, β are calibrated to match aggregate hours per adult and investment-output ratio in 2007:Q4 in each country. It is standard practice when calibrating this type of model to assume the country is in a steady state. However, in this case productivity exhibits significant changes in 2007-2009 in all countries studied, and aggregate per adult experienced sharp decline during the same period of time. These two issues necessitate a modification of the standard calibration procedure. Instead of assuming a country is in the steady state in 2007, I assume that it begins converging to a steady state after 2010:Q4. Specifically, I assume that both market productivity growth and taxes stay at the level in 2010:Q4 after that. Given the asymptotic behavior of the model, I can then solve for the transitional dynamics over the period 2007:Q1-2010:Q4. My calibration strategy is to require that the model matches several moments in the data in 2007:Q4, even though the economy is not in a steady state.

2.4 Simulation Results

In this section I use calibrated models to simulate series for ten OECD countries in my sample. Each country is assumed to have the same parameters γ, θ, δ, and g. For each country, I calibrated a and β as discussed above. I then introduce country specific tax and productivity series to the model and evaluate their importance in driving market hours in the Great Recession by comparing the hours generated by the model and the series in the data.

2.4.1 Implication for hours

Figure 2.3 displays time series of market hours predicted by the model and the data series between 2007:Q1-2010:Q4. Three results are worth highlighting. First, models calibrated to the U.S. and Austria underpredict hour decline in 2007:Q4-2009:Q4, mainly because they fail to generate decreasing hours in 2009. Second, calibrated models predict the right level of hours changes for Canada, Germany, Italy, Spain, and Sweden, although the fluctuation in model-generated series is more dramatic than that in the data series. Lastly, calibrated models predict much steeper contraction in hours in Finland, France, and the U.K.

For the U.S., the model calibrated so as to match hours in 2007:Q4 generates time path of hours that closely resembles the data from 2007:Q3 to 2008:Q4. In other words, changes in productivity and taxes can account for all changes in hours in this period of time. However, in the second half of the recession, the model predicts a recovery of 4% over the course of 2009, whereas the data shows a continued drop of 4%. Therefore, the model generates about 1% overall decline in hours from 2007:Q4 to 2009:Q4 in contrast to the 8.4% drop in the data. This finding suggests that forces other than taxes started affecting the U.S. labor market since the end of 2008 and prevented hours from recovering in spite of the pick up in productivity growth in the U.S.

For Austria, the model-generated series are much more sensitive to changes in productivity growth than the data. Specifically, the model predicts a 5% decline in 2008 in response to the change productivity of similar magnitude, whereas the data only shows 3% drop in hours in 2008. However, while the hours continued to drop in 2009, the model-generated series exhibits a dramatic recovery in 2009. Therefore, the model under predicts

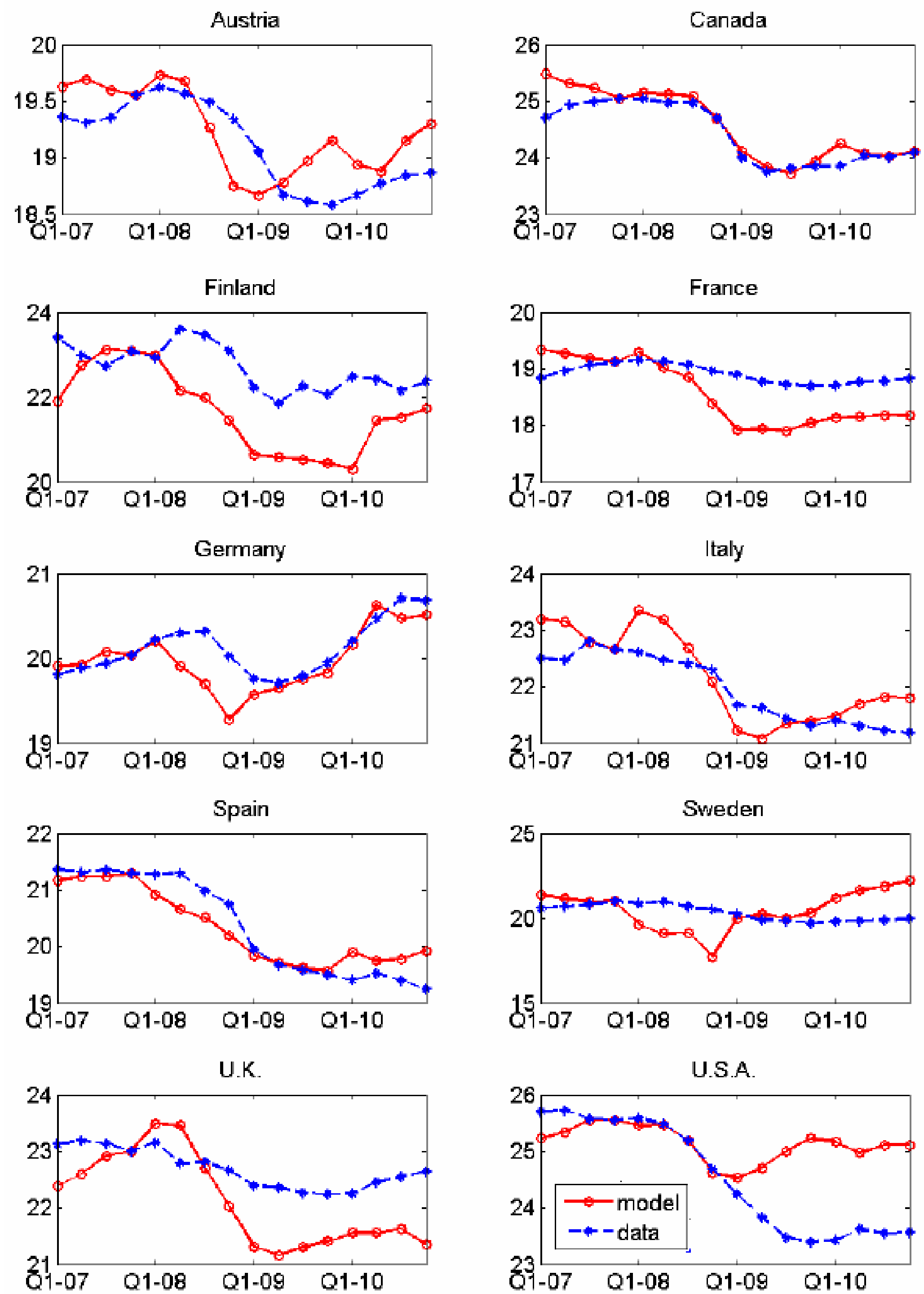


Figure 2.3: Market hours per week 2007:1-2010:4

the overall decline in hours in Austria from 2007-2009. Similar to the U.S., the simulation exercise for Austria also suggests forces other than taxes delayed hours from responding to the recovery in productivity growth.

Models calibrated to Canada, Germany, Italy, Spain, and Sweden predict changes in hours that are very similar to what is seen in the data in 2007:Q4-2009:Q4. For Canada, the model-generate series closely resemble the data throughout the entire sample period, except for early 2007. In other words, changes in productivity growth and taxes can account for all the movements in Canadian hours in the recession. For the other four European countries, calibrated models over-predict the decline in hours in response to the productivity slowdown in 2008. The models also predict a bigger recovery in hours in 2009, or a smaller decline in case of Spain, so that the simulated series mostly line up with the data series at the end of 2009.

Calibrated models for Finland, France, and the U.K. predict much more severe decline in market hours than what the data series show. All three countries experienced larger deviation in productivity during the recession relative to the U.S. Since there was no significant changes in effective labor taxes, the dispartiy between the model predictions and the data implies that there are forces that drive market hours in these countries that are not captures in the model.

In the appendix, I show the results from the same calibration exercise with an IES parameter γ equal to 0.5. With a lower IES, hours becomes less sensitive to changes in wage and therefore to changes in productivity growth and taxes. Indeed, by comparing the results against the baseline calibration, I find that model-generated series exhibit less dramatic fluctuations for all countries with IES equal to 0.5. However, main results from the baseline calibration still hold.

2.4.2 Decomposition

The baseline model includes three forces that potentially influence market hours: effective labor income taxes, effective capital income taxes, and productivity change. To evaluate their each individual contribution to hours, I follow the methodology developed in [23] and simulate the baseline model with each force eliminated. The results of these simulations are reported in table 2.1.

Table 2.1: Change in hours from 2007:Q4-2009:Q4

Country	*Data*	*Baseline*	$\bar{\tau}_h$	$\bar{\tau}_k$	$\bar{A}$
Austria	4.9%	2.0%	3.3%	2.3%	-1.6%
Canada	4.8%	4.4%	4.7%	4.4%	-0.5%
Finland	4.4%	11.4%	13.7%	11.5%	-2.5%
France	2.2%	5.6%	7.0%	5.8%	-1.5%
Germany	0.4%	1.0%	7.4%	1.4%	-6.9%
Italy	5.9%	5.5%	7.2%	5.7%	-1.9%
Spain	8.4%	8.1%	4.2%	8.5%	3.7%
Sweden	6.1%	3.2%	12.9%	3.3%	-10.3%
U.K.	3.3%	6.9%	8.5%	6.8%	-1.5%
U.S.A.	8.5%	1.3%	2.1%	1.0%	-0.5%

The first two columns of table 2.1 display the percent decline in hours in the data and generated by the baseline model with all forces present. The next column, labeled $\bar{\tau}_h$, shows this change generated by the model with capital taxes and productivity varying as in the baseline case, but labor income tax rates held constant at their 2007:Q4 levels. The fourth column, labeled $\bar{\tau}_k$, display changes in hours when capital taxes are held constant, labor taxes and productivity vary. Notice that holding labor income tax rate constant generates change in hours that are meaningfully different from the predictions of the baseline case. In particular, decline in hours with fixed labor income taxes is three times greater than the ones with varying labor tax rate in Germany and Sweden. The drop in hours would have been only half of what is seen in the data, if labor income taxes stayed constant in 2007-2009. Given that labor tax rate changed by less than 5 % in these countries, this dramatic difference in hours suggests that market hours are very sensitive to after-tax return to work in the model. In contrast, in spite of relatively bigger change in capital taxes, holding capital tax rate constant has very little impact on model predictions of the change in market hours. Even in Spain where capital tax rate dropped from 30 to 20 %, the impact of this change on market hours is very limited.

Labor taxes and capital taxes affect market hours through different mechanisms. Labor taxes directly distort household's labor supply decision by lowering the after-tax return to work. Capital taxes indirectly affects hours by reducing the real return to investment and resulting in a lower capital-to-labor ratio in the equilibrium. The simulation exercises demonstrate that hours are much more responsive to changes in labor taxes than they are to changes in capital taxes.

The last column displays changes in hours generated by the model with constant productivity series A_t for all countries. Changes in taxes alone would have generated expansion in aggregate hours in all countries other than Spain from 2007:Q4 - 2009:Q4. Take Italy for example. If its productivity stayed on trend since 2007, we would expect to see a 2% expansion in hours instead of a 5% decline. Spain is the only country in the sample, which saw an increase in labor income tax in the recession, and this change in taxes can account for more than half of the drop in hours. This decomposition shows that productivity decline was the primary force that drove the decline in hours in the Great Recession, and its negative impact on hours was dampened substantially by small changes in labor income taxes.

2.4.3 Investment and output

The primary focus of this paper is to study the impact of taxes and productivity on hours in the Great Recession. However, given the dynamic nature of the model, it is possible to evaluate their influence on investment and output over the same period. Figures 2.4 and 2.5 display the investment series and output series generated by the model and the data with the initial level normalized to 1 for all countries.

In general, changes in investment generated by the model is much more dramatic than what is observed in the data. As a consequence, the model also predicts steeper output decline than the data shows. This discrepancy is likely due to the assumption that households have perfect foresight over tax and productivity series. If they cannot predict the arrival of negative productivity shock in the next period, they will invest more in the current period and cut investment in the low productivity period. Similarly, if households cannot foresee the recovery in productivity, they will not increase investment until they experience high productivity. A model with smoother expectation about productivity growth and taxes would generate investment and output series that more closely resemble the data.

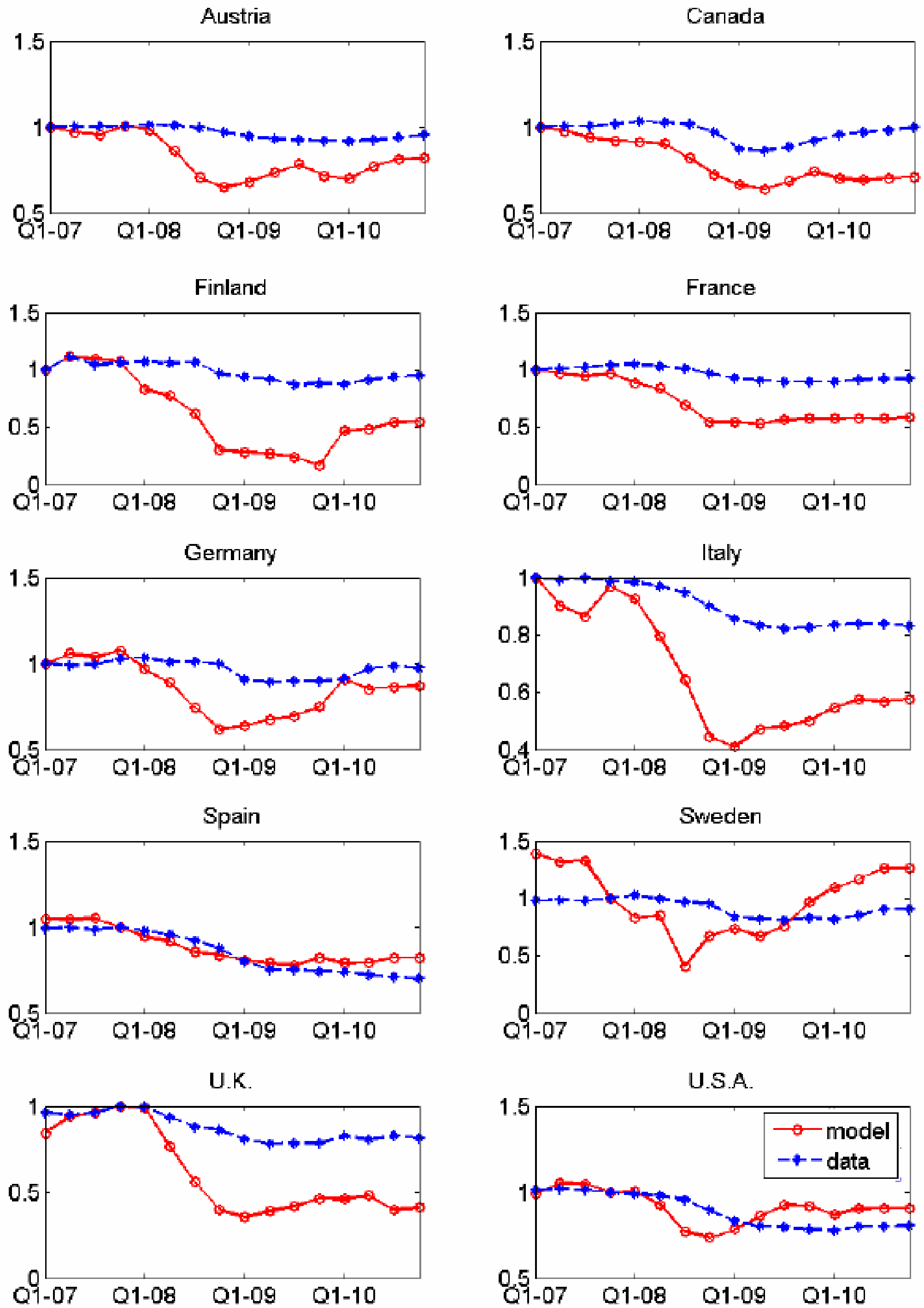


Figure 2.4: Change in investment 2007:1-2010:4

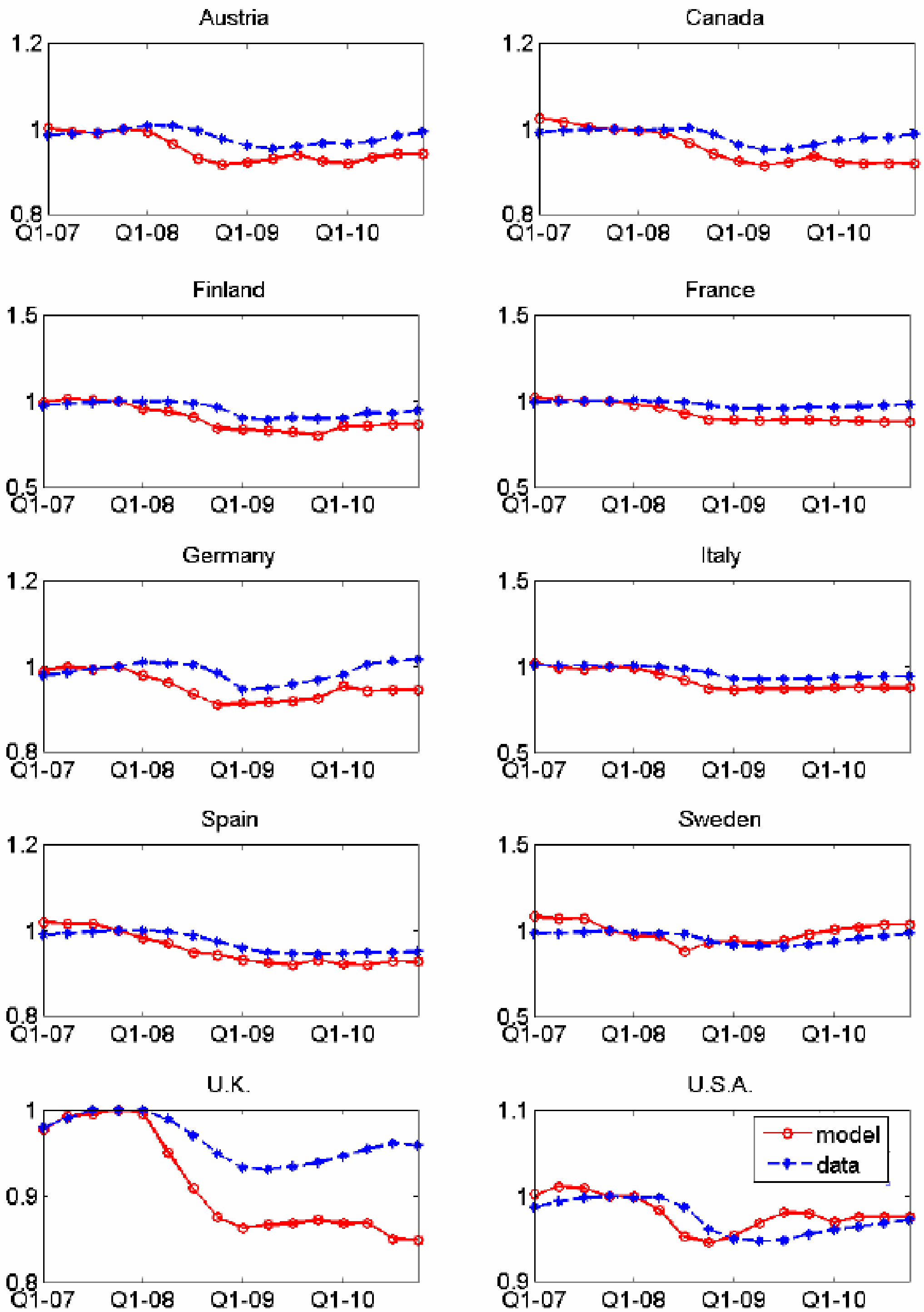


Figure 2.5: Change in output 2007:1-2010:4

2.5 Discussion

2.5.1 Replacement rate

In this section, I address the question on whether the extension of unemployment benefits can explain the drop in market hours in the U.S. in 2009, despite the recovery in productivity and constant tax distortion on labor income. [27] shows that inflation-adjusted means-tested subsidies per eligible person have increases sharply since the end of 2007. These subsidies are allocated on the basis of household income and employment status. For someone who is receiving such subsidies, the cost of employment would include losing eligibility to certain unemployment benefits. As these subsidies become more generous, the returns to employment become lower. Therefore, means-tested subsidies distort household labor supply decisions in a similar way as labor income taxes. I now incorporate means-tested subsidies into household budget constraint in the spirit of [27] and study the effect of expanding means-tested subsidies on market hours with the presence of productivity and tax changes.

Consider the following modification to the baseline model. A household is partitioned into two types: "prime" members who have a probability $p_t > 0$ of receiving a means-tested subsidy b_t in the event they are not working at time t, and "others" who are never eligible for means-tested subsidies. Household preferences are the same as in equation 2.1. Household faces the following budget constraint,

$$c_t(1+\tilde{\tau}_t^c) + x_t(1+\tilde{\tau}_t^x) = r_t^* k_t(1-\tilde{\tau}_t^k) + w_t^* h_t(1-\tilde{\tau}_t^h) + \tau_t^r w_t^*(1-h_t) + T_t,$$

where $\tau_t^r = \frac{p_t b_t}{w_t}$ is the expected pre-tax replacement rate from the time t subsidy. Government balances budget each period:

$$\tau_t^r w_t^*(1-h_t^*) + T_t^* = \tilde{\tau}_t^c c_t^* + \tilde{\tau}_t^x x_t^* + \tilde{\tau}_t^h w_t^* h_t^* + \tilde{\tau}^k r_t^* k_t^*.$$

Equilibrium allocation can still be characterized by equations 2.2-2.4, with the only difference that the effective tax on labor income now includes the average replacement τ_t^r, in addition to taxes on consumption expenditure, the payroll tax, and the average marginal income tax

$$\tau_t^h = \frac{\tau^{ss} + \tau^{inc} + \tau_t^r + \tilde{\tau}_t^c}{1+\tilde{\tau}_t^c}.$$

According to the estimation in [27], the average replacement rate increased from 12 or 13 % before the recession to over 25 % by late 2009.[1] Next I simulate the model incorporating this change in replacement rate. Specifically, I assume that the replacement rate τ_t^r grows linearly from 13 % in 2007:Q4 to 26 % in 2009:Q4, and that it remains at this higher level thereafter. I add this replacement rate series to the effective tax on labor income and repeat the calibration exercise in section 3. Figure 2.6 shows the hours generated by the model with IES equal to 1 and 0.5. The model generates a 40% decline in hours with IES equal to 1, and a 20% drop with IES equal to 0.5. Compared to the data, the drop in hours predicted by the model is several times bigger than the 8 % decline in the data. The 13% increase in the average replacement rate implies a spike in labor income tax of the same magnitude in 2008-2009. The decomposition exercise in the previous section reveals that hours are very responsive to small changes in labor income tax rate and this explains the excessive contraction of hours in the model as a consequence of a sudden spike in the replacement rate. [2]

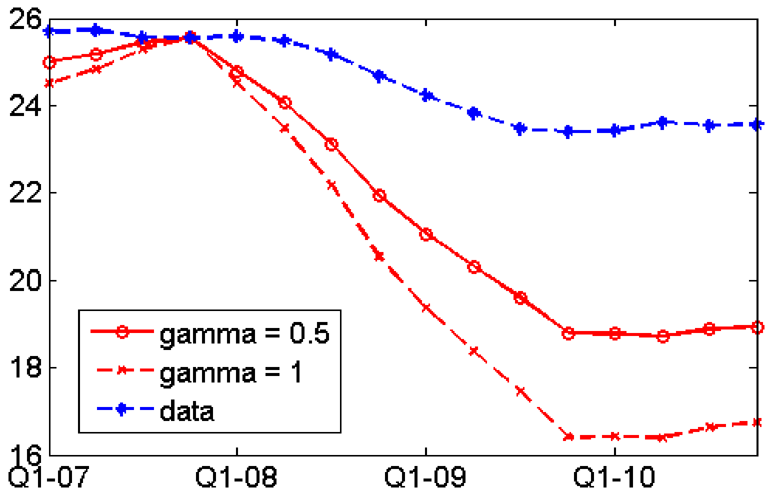
One possible explanation of this calibration result could be that the replacement rate did not increase as much as 13% , and that hours responded less to changes in unemployment subsidies than to changes in labor income taxes. Another possible explanation is that there exist forces that largely offset the negative impact of expanding social benefits on labor market during the recession. In both cases, the rising of replacement rate can be still quantitatively important in explaining the large drop in the U.S. hours, especially in the second half of the recession.

Assuming the average replacement rate stayed constant at 13% until the end of 2008, I find that if this rate increased to 16-18 % by the end of 2009, the model would generate market hours closely resemble the data. Many other labor market distortions such as search frictions and rigid wages could have similar effects as rising replacement rate. The above

[1] In [27], the average replacement rate is calculated by dividing the combined subsidies by the product of the numbers of all non-working individual aged 25-64 and a $700 per week estimate of the median earnings. Combined subsidies include unemployment insurance, "home retention actions", consumer loan charge-offs by commercial banks, and other means-tested government transfer program including food stamps, SSI, state and local family assistance, general assistance, and energy assistance.

[2] [27] conducts a similar exercise using a different utility function. His simulation result shows that the doubling of replacement from 13% to 26% as the only impulse to a standard growth model can generate 10% decline in hours. I replicate his finding in the appendix and show that the magnitude of hour decline is sensitive to elasticity of labor supply as well as the level of labor income taxes.

Figure 2.6: The U.S. hours with replacement rate



accounting exercise is also useful for thinking about the importance of these distortions. A model incorporating these distortions would be able to account for changes in the U.S. hours during the Great Recession if these distortions manifest into a 3-5% increase in labor wedge in 2009.

2.5.2 Home production

Both [25] and [29] find home production to be a key extension to a standard growth model in explaining the differences between the long-term evolution of the European and American market hours. In particular, [25] makes the assumption that individuals are able to produce good substitutes for many market services using a home production technology. This assumption is important in explaining why higher taxes in Europe have asymmetric effects across sectors, and why Europes market service sector has been consistently small relative to that of the U.S. [29] finds that the extension of home production is important in capturing the right direction of change in market hours from 1960-2004 for many OECD countries. In this section, I extend the baseline model to include home production and show that in contrast to earlier findings, home production model does not improve the model prediction of hour changes over the recent business cycle.

In this extension, firm production technology and government fiscal policies are the same as in the baseline model. In addition to market work (h_{mt}) and leisure, households can also allocate their time to home production (h_{nt}). Utility function is given by

$$U = \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \frac{a}{1-\frac{1}{\gamma}} (1 - h_{mt} - h_{nt})^{1-\frac{1}{\gamma}} \right) \tag{2.5}$$

where consumption c_t is an aggregate of market goods (c_{mt}) and home goods produced using technology linear in labor input:

$$c_t = (bc_m^{\epsilon} + (1-b)(A_{nt}h_{nt})^{\epsilon}))^{\frac{1}{\epsilon}} \tag{2.6}$$

Necessary conditions of a competitive equilibrium are summerized in the following first order conditions

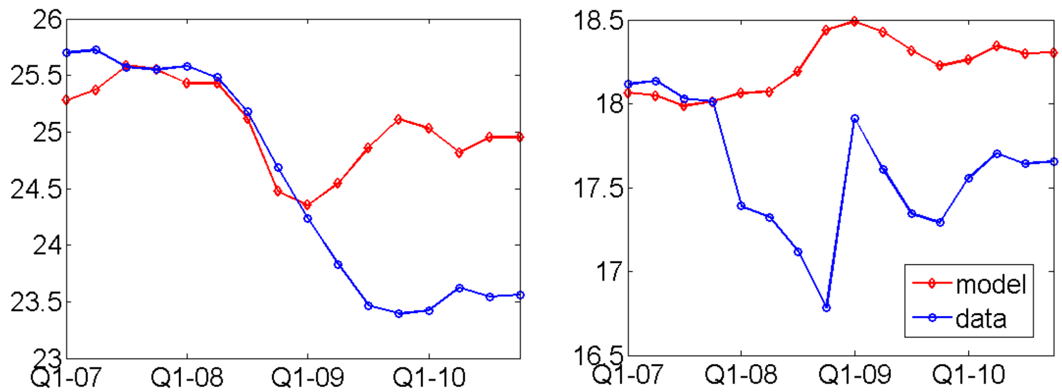
$$\frac{ac_t^{\epsilon} c_{mt}^{1-\epsilon}}{b(1 - h_{mt} - h_{nt})} = (1-\theta) A_{mt}^{1-\theta} \left(\frac{k_t}{h_{mt}} \right)^{\theta} (1 - \tau_t^h) \tag{2.7}$$

$$\frac{1-b}{b} \frac{c_{mt}^{1-\epsilon}}{A_{nt}^{-\epsilon} h_{nt}^{1-\epsilon}} = (1-\theta) A_{mt}^{1-\theta} \left(\frac{k_t}{h_{mt}} \right)^{\theta} (1 - \tau_t^h) \tag{2.8}$$

$$\frac{c_{t+1}^{\epsilon} c_{mt+1}^{1-\epsilon}}{\beta c_t^{\epsilon} c_{mt}^{1-\epsilon}} = (1 + \tau_{t+1}^{cx}) \left(\theta A_{mt+1}^{1-\theta} \left(\frac{k_{t+1}}{h_{mt+1}} \right)^{\theta-1} (1 - \tau_{t+1}^k) + 1 - \delta \right) \tag{2.9}$$

The parameters θ, δ, g, and γ are set to be the same as in the baseline model. I set $\epsilon = 0.5$ so that the elasticity of substitution between market goods and home production equal to 2. Three preference parameters a, b, β are calibrated to match output-capital ratio 0.13, market hour share 0.26, and home production share 0.18 in 2007:Q4 in the U.S.. The U.S. home production hours are taken from [32]. Since I do not have data on home production productivity or output, I assume that the home productivity grows at the constant rate g throughout the sample period.

Figures 7(a) and 7(b) show the U.S. hours in market and home sectors generated by the model and in the data. Extending the model to include home production does not change the model's prediction for market hours during the recession. This result suggests that the substitution effect generated by the difference between market and home technology growth is very small. More specifically, home production adds an extra margin to the household



(a) Market sector

(b) Home sector

Figure 2.7: Hours in the U.S.

labor supply decision. Both increase in labor taxes and growing differential between market and home technology growth would induce the household to substitute market goods with home goods. Since the U.S. labor income taxes stayed flat during the sample period, nearly all the movements in market hours generated by the model are driven by changes in productivity growth. The fact that the model predicts similar level of decline in market hours with or without home production suggests the technology differential generates very limited substitution effect. [3]

In terms of home production hours, the model fails to generate its decline during the recession. According to the data, home production hours, following similar pattern of market hours, also declined by 10 % between 2007:Q4-2008:Q4. The model on the contrary predicts a small increase in home production hours over this time period. In the model, I assume that home technology grows at the constant rate g. Intuitively, without any changes in tax distortion on labor income, slower productivity growth in the market sector results in a lower marginal return to market work relative to home production. Both substitution effect and wealth effect should lead to more time spent on home production. The fact that home production hours decline along with market productivity and market hours is

[3] Due to data limitation, I cannot replicate the same calibration for other OECD countries. Using the same parameters calibrated to the U.S. and country-specific productivity and tax series, I simulate hours for other OECD countries, and find that models with or without home production predict similar changes in market hours in other countries as well.

puzzling from the perspective of this model.

2.6 Conclusion

In this paper I use a standard neoclassical growth model to evaluate the impact of productivity growth and taxes on market hours for ten OECD countries in the 2007-2010 recession. The calibrated model with country-specific productivity growth and tax series can generate comparable contraction in market hours in Canada, Germany, Italy, Spain, and Sweden. The model overestimates the decline in hours in Finland, France, and the United Kingdom, and underestimates such decline in Austria and the United States. I find the decline in TFP growth to be the primary force that drove down market hours across all countries studied during the recession, but its negative impacts were substantially dampened by small changes in labor income taxes in most countries, especially in Germany and Sweden. Contrary to the important role that home production plays in generating long-run changes in labor input in OECD countries, an extension to include home production in the baseline model does not improve the model prediction for changes in hours over the recent business cycle.

This paper provides several avenues for future research. First, the baseline model fails to generate the contraction in the U.S. hours in 2009 in spite of recovering productivity growth. The expansion of means-tested subsidies could potentially distort the household's labor supply and prevent hours from recovering. In order to evaluate its quantitative importance, future research needs to construct accurate measures of unemployment benefits, and to estimate the elasticity of aggregate hours with respect to changes in these benefits in a framework that includes involuntary unemployment. Second, relative to the U.S., countries such as France, and the U.K. experienced a steeper decline in productivity growth, but smaller drop in hours. Taxes could still be important in accounting for this difference. In particular, the weighted average of marginal income taxes might have dropped more than the mean of tax schedule indicates, because many households moved down to lower tax brackets with shrinking income in the recession. More accurate measures of labor income taxes are needed to evaluate their role in generating different labor market responses to productivity growth across different economies. Third, many argue that the uncertainty over fiscal policy is important in driving macroeconomic aggregates in this recession. Conducting a similar accounting exercise with uncertainty and smoother explanation would offer insights into its quantitative importance. Last but not the least, suppose all these countries were hit by a common shock. Then questions remain about whether there exist certain distinctions between the European economies and the U.S. economy so that a common shock transmitted into decline in productivity in Europe and drop in hours in the U.S. in the recent recession.

Bibliography

[1] Yasheng Huang. *Selling China*. Cambridge University Press, 2003.

[2] Franklin Allen, Jun Qian, and Meijun Qian. Law, Finance, and Economic Growth in China. *Journal of Financial Economics*, 77(1):57 – 116, 2005.

[3] Sandra Poncet, Walter Steingress, and Hylke Vandenbussche. Financial Constraints in China: Firm-Level Evidence. *China Economic Review*, 21(3):411–422, September 2010.

[4] Timothy Kehoe and Kim Ruhl. Does Openness Generate Growth? Reconciling the Experiences of Mexico and China. *VoxEU*, 2011.

[5] Joel M. David, Hugo A. Hopenhayn, and Venky Venkateswaran. Information, misallocation and aggregate productivity. Working Paper 20340, National Bureau of Economic Research, July 2014.

[6] Thomas F. Cooley and Vincenzo Quadrini. Financial Markets and Firm Dynamics. *American Economic Review*, 91(5):1286–1310, 2001.

[7] Raghuram G. Rajan and Luigi Zingales. What Do We Know about Capital Structure? Some Evidence from International Data. *The Journal of Finance*, 50(5):1421–1460, 1995.

[8] Laurence Booth, Varouj Aivazian, Asli Demirguc-Kunt, and Vojislav Maksimovic. Capital Structures in Developing Countries. *The Journal of Finance*, 56(1):87–130, 2001.

[9] Cristina Arellano, Yan Bai, and Jing Zhang. Firm Dynamics and Financial Development . *Journal of Monetary Economics*, 59(6):533 – 549, 2012.

[10] Tatiana Didier and Sergio L. Schmukler. The Financing and Growth of Firms in China and India: Evidence from Capital Markets. *Journal of International Money and Finance*, 39(C):111–137, 2013.

[11] Christopher A. Hennessy and Toni M. Whited. How Costly Is External Financing? Evidence from a Structural Estimation. *The Journal of Finance*, 62(4):1705–1745, 2007.

[12] Joao F. Gomes and Lukas Schmid. Levered Returns. *The Journal of Finance*, 65(2):467–494, 2010.

[13] Francisco Covas and Wouter J. Den Haan. The Role of Debt and Equity Finance Over the Business Cycle. *The Economic Journal*, 122(565):1262–1286, 2012.

[14] Juliane Begenau and Juliana Salomao. Firm Financing over the Business Cycle. Working paper, Harvard Business School and University of Minnesota, 2014.

[15] Boyan Jovanovic. Selection and the Evolution of Industry. *Econometrica*, 50(3):649–70, May 1982.

[16] Joseph Steinberg. Information, Contract Enforcement, and Misallocation. Working paper, University of Minnesota, 2013.

[17] Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448, 2009, http://qje.oxfordjournals.org/content/124/4/1403.full.pdf+html.

[18] Zheng Song, Kjetil Storesletten, and Fabrizio Zilibotti. Growing Like China. *American Economic Review*, 101(1):196–233, 2011.

[19] Loren Brandt, Johannes Van Biesebroeck, and Yifan Zhang. Creative Accounting or Creative Destruction? Firm-level Productivity Growth in Chinese Manufacturing. *Journal of Development Economics*, 97(2):339 – 351, 2012.

[20] Virgiliu Midrigan and Daniel Yi Xu. Finance and Misallocation: Evidence from Plant-Level Data. *American Economic Review*, 104(2):422–58, 2014.

[21] Russell W. Cooper and John C. Haltiwanger. On the Nature of Capital Adjustment Costs. *The Review of Economic Studies*, 73(3):pp. 611–633, 2006.

[22] George Tauchen. Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economics Letters*, 20(2):177–181, 1986.

[23] V. V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. Business Cycle Accounting. *Econometrica*, 75(3):781–836, 2007.

[24] Edward C. Prescott. Why Do Americans Work So Much More Than Europeans? *Quarterly Review*, (Jul):2–13, 2004.

[25] Richard Rogerson. Structural Transformation and the Deterioration of European Labor Market Outcomes. *Journal of Political Economy*, 116(2):235–259, 04 2008.

[26] Lee E. Ohanian and Andrea Raffo. Aggregate Hours Worked in OECD Countries: New Measurement and Implications for Business Cycles . *Journal of Monetary Economics*, 59(1):40 – 56, 2012.

[27] Casey B. Mulligan. Means-Tested Subsidies and Economic Performance Since 2007. Working Paper 17445, National Bureau of Economic Research, September 2011.

[28] Cara McDaniel. Average Tax Rates on Consumption, Investment, Labor and Capital Income in the OECD 1950-2003, 2014.

[29] Cara McDaniel. Forces Shaping Hours Worked in the OECD, 1960-2004. *American Economic Journal: Macroeconomics*, 3(4):27–52, 2011.

[30] Richard Rogerson and Johanna Wallenius. Micro and Macro Elasticities in a Life Cycle Model With Taxes. *Journal of Economic Theory*, 144(6):2277–2292, November 2009.

[31] Raj Chetty. Bounds on Elasticities With Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply. *Econometrica*, 80(3):969–1018, 2012.

[32] Mark Aguiar, Erik Hurst, and Loukas Karabarbounis. Time Use during the Great Recession. *American Economic Review*, 103(5):1664–96, 2013.

Appendix A

A.1 Additional Materials for Chapter 1

A.1.1 Stylized Two-period Model

In this section I build a two-period model of productivity uncertainty and financial friction to illustrate how learning affects firms' choice of capital, growth, and exit. In order to aid the analytical characterization, I make simplifying assumptions on the equity endowment so that the contracting problem is essentially static. I show that learning alleviates the negative effect of financial frictions on allocation efficiency for mature firms.

Productivity uncertainty and learning I first describe how I model productivity uncertainty and learning. Each firm has the same amount of endowment e at the beginning of period 1. The firm chooses to invest k units of capital and finance it by taking out a loan $b = k - e$. Technology is decreasing returns to scale. The firm's resource post-production

$$\exp(x + \epsilon)k^{\alpha} + (1 - \delta)k - \eta, \tag{A.1}$$

includes output and undepreciated capital minus the fixed production cost, η. The firm's productivity consists of the permanent productivity x, and an i.i.d transitory shock ϵ drawn from a normal distribution $N(-\frac{1}{2}\sigma_{\epsilon}^2, \sigma_{\epsilon})$. I assume that both the firm and the lender do not know the firm's permanent productivity with certainty; instead, they have prior beliefs over productivity distribution $x \sim N(\mu_1 - \frac{1}{2}\sigma_1^2, \sigma_1)$, and update their beliefs after production takes place in each period using Bayes' rule. Here I assume that there is no information asymmetry between lenders and firms. After output zk^{α} is produced in the first period, both lenders and the firm adjust their posterior belief on permanent productivity, which can be characterized as a lognormal distribution with $x \sim N(\mu_2 - \frac{1}{2}\sigma_2^2, \sigma_2)$, where the updated mean μ_2 is

$$\mu_2 = E[x \mid z] = \frac{\sigma_1^{-2}}{\sigma_1^{-2} + \sigma_\epsilon^{-2}}\mu_1 + \frac{\sigma_\epsilon^{-2}}{\sigma_1^{-2} + \sigma_\epsilon^{-2}}(\log(z) + \frac{1}{2}\sigma_\epsilon^2) \tag{A.2}$$

and the updated variances σ_2^2 becomes

$$\sigma_2^2 = (\sigma_1^{-2} + \sigma_\epsilon^{-2})^{-1}. \tag{A.3}$$

This posterior belief affects whether the firm continues its operation, and also the debt schedule that lenders provide in the next period. I will highlight how learning interacts with selection and debt financing after I set up the contracting problem in the next subsection.

Contracting problem Consider an economy that lasts for two periods $t = 1, 2$, and is populated by a large number of firms and lenders. At the beginning of each period, each firm is endowed with e units of equity. Firms do not have access to a saving technology and they pay out all remaining resources after debt repayment as dividends. Lenders are risk-neutral and perfectly competitive, and therefore make zero profits in expectation from making one-period loans to firms. This is an open economy with exogenously given risk-free interest rate. I assume that markets are exogenously incomplete; that is, loan contracts are not contingent upon idiosyncratic states. Loans are risky because of the possibility of default.

The timing of events is as follows:

1. At the beginning of period 1, each firm receives an endowment of e units of equity. It chooses to invest k units in capital and to finance it using one-period loans.

2. After production takes place in period 1, the firm defaults if resources in the firm are not enough to pay back the amount due. In this case, lenders seize all firm resources (if any) minus bankruptcy costs. The firm's value becomes zero in the defaulting state.

3. On the other hand, if the firm has enough resources to repay the loan, it does so, and all remaining resources are paid out as dividend. The firm decides whether to exit

or to continue its operation in period 2 based on an updated belief on its permanent productivity.

4. At the beginning of period 2, an operating firm receives another endowment of e and a new set of debt contracts that reflect its adjusted default risk. The firm chooses a new loan amount from available debt contracts to finance its capital investment in period 2.

5. Production in period 2 takes place. Default rules and dividend payout policies are decided similarly as in period 1.

Debt contracts are not enforceable as firms can default on their debt. Debt contracts (b, rb) are firm specific and incorporate the firm specific default risk. Let $\Omega(e, \mu, \sigma)$ denote the set of debt contracts available to a firm with equity e and productivity distribution summarized by (μ, σ). Each contract $(b, rb) \in \Omega(e, \mu, \sigma)$ maps a current loan b to a repayment amount rb. If a firm defaults, lenders get

$$(1 - \lambda)z(e + b)^{\alpha} + (1 - \delta)(e + b) - \eta, \tag{A.4}$$

where λ represents bankruptcy costs, which are assumed to be a fraction of the output. Since lenders are risk neutral and perfectly competitive, each debt contract allows lenders to break even in expected value:

$$\int (1-d(rb, z))rb\Phi(dz) + \int d(rb, z)[(1-\lambda)z(e+b)^{\alpha} + (1-\delta)(e+b) - \eta]\Phi(dz) = Rb, \tag{A.5}$$

where $d(rb, z)$ represents a default state, and $\Phi(z)$ is the cdf of productivity based on prior beliefs.

At the beginning of each period, an operating firm maximizes its expected income by choosing a loan amount. The contracting problem is given by

$$W(e, \mu, \sigma) = \max_{(b,rb)} \int (1 - d(rb, z))[z(e + b)^{\alpha} + (1 - \delta)(e + b) - rb - \eta]\Phi(dz) \tag{A.6}$$

subject to equation (A.5) and the default rule

$$d(rb, z) = \begin{cases} 0 & \text{if } z(e + b)^{\alpha} + (1 - \delta)(e + b) - \eta \geq rb \\ 1 & \text{otherwise} \end{cases}$$

Lemma 1. *At each period t, there exists $\bar{\mu}_t$ such that if $\mu \leq \bar{\mu}_t$, then $W(e, \mu, \sigma) \leq 0$. In this case, the firm exits the market.*

Lemma 1 shows that because of the fixed production cost, there exists a cutoff threshold of expected productivity, below which firms exit. The decreasing returns to scale technology implies that the optimal operational scale is small for firms with low expected productivity. Small-scale production is unprofitable in the presence of a fixed cost.

Next we characterize the solution to the contracting problem following [13]. Notice that firms default if z is less than the default threshold $\bar{z}$, where $\bar{z}$ satisfies

$$\bar{z}k^{\alpha} + (1-\delta)k - \eta = r(k-e). \tag{A.7}$$

I can write the contracting problem given by equation (A.6) in terms of the default threshold $\bar{z}$ by substituting for repayment rb and $d(rb, z)$:

$$W(e, \mu, \sigma) = \max_{(b,\bar{z})} \underbrace{\int_{\bar{z}} (z - \bar{z})\Phi(dz)}_{F(\bar{z})}(e+b)^{\alpha} \tag{A.8}$$

subject to

$$\underbrace{(\int_{\bar{z}} \bar{z} + \int^{\bar{z}} (1-\lambda)z)}_{G(\bar{z})} k^{\alpha} + (1-\delta)k - \eta = Rb \tag{A.9}$$

Assumption 1. *The maximization problem has an interior solution.*[1]

Assumption 2. *At the default threshold $\bar{z}$, the prior belief on productivity satisfies*

$$\frac{\partial \frac{\Phi'(\bar{z})}{1-\Phi(\bar{z})}}{\partial \bar{z}} > 0. \tag{A.10}$$

Assumption 3. *Default probability is less than 1/2, that is, $\Phi(\bar{z}) < 0.5$.*

Assumption 1-2 ensures that default probability is strictly positive and firms face interest rate that is greater than the risk free rate. Assumption 3 says that default is a low probability event. In particular, more precise prior belief is associated with a lower default risk. The following proposition shows that in the equilibrium, debt and leverage increase

[1] *When firm equity is large relative to its expected productivity, the firm might find it optimal not to borrow at all.*

with prior mean μ, and decrease with prior variance σ. The intuition for this result is as follows. When the expected productivity increases, the firm's expected marginal return on capital increases and the firm's demand for capital also increases. Furthermore, firms are less likely to default. Lenders adjust borrowing cost to reflect this lower default probability. Both forces lead to a higher leverage ratio. Conditional on expected productivity, when the prior belief becomes more precise, default probability also becomes lower.[2] Firms tend to borrow more debt. That is, even if the firm receives an average shock so that the expected productivity in the posterior belief is very close to the prior mean, firms will take on more debt in the second period, because they face more favorable loan terms.

Proposition 1. *Under Assumptions 1 and 2, leverage increases with expected productivity, that is,*

$$\frac{\partial b(e,\mu,\sigma)}{\partial \mu} > 0; \tag{A.11}$$

Under Assumptions 1-3, leverage increase with prior precision, that is,

$$\frac{\partial b(e,\mu,\sigma)}{\partial \sigma} < 0; \tag{A.12}$$

Proof. The contracting problem in the stylized model is given by

$$W(e,\mu,\sigma) = \max_{(b,\bar{z})} \underbrace{\int_{\bar{z}} (z-\bar{z})\Phi(dz)}_{F(\bar{z})}(e+b)^{\alpha} \tag{A.13}$$

subject to

$$\underbrace{(\int_{\bar{z}} \bar{z} + \int^{\bar{z}} (1-\lambda)z)}_{G(\bar{z})}(e+b)^{\alpha} + (1-\delta)(e+b) - \eta = Rb \tag{A.14}$$

where Φ is a cdf of lognormal distribution $logN(\mu - \frac{1}{2}\sigma^2, \sigma)$. First order conditions are

$$\alpha k^{\alpha-1}F(\bar{z}) = \psi(\alpha k^{\alpha-1}G(\bar{z}) + (1-\delta) - R) \tag{A.15}$$

$$k^{\alpha}F'(\bar{z}) = \psi(k^{\alpha})G'(\bar{z})) \tag{A.16}$$

$$\Rightarrow \frac{\alpha k^{\alpha-1}F(\bar{z})}{-\alpha k^{\alpha-1}G(\bar{z}) - (1-\delta) + R} = -\frac{F'(\bar{z})}{G'(\bar{z})} \tag{A.17}$$

[2] This is the case when firm's default probability is less than 0.5 under an imprecise prior belief.

The FOC and the productivity cutoff will determine equilibrium debt b and cutoff $\bar{z}$. Without loss of generosity, we set $\delta = 1$ and $\eta = 0$.

$$k^{\alpha} G(\bar{z}) = Rb \tag{A.18}$$

$$\frac{\alpha k^{\alpha-1} F(\bar{z})}{-\alpha k^{\alpha-1} G(\bar{z}) + R} = -\frac{F'(\bar{z})}{G'(\bar{z})} \tag{A.19}$$

$$\Rightarrow \frac{R}{\alpha k^{\alpha-1}} = G(\bar{z}) - F(\bar{z}) \frac{G'(\bar{z})}{F'(\bar{z})} \tag{A.20}$$

The following identities and derivatives will be used in the proof:

$$F(\bar{z}) = \int_{\bar{z}}^{\infty} z\Phi(dz) - (1 - \Phi(\bar{z}))\bar{z}$$

$$F'(\bar{z}) = -\bar{z}\Phi'(\bar{z}) - (1 - \Phi(\bar{z})) + \Phi'(\bar{z})\bar{z} = -(1 - \Phi(\bar{z}))$$

$$G(\bar{z}) = E_{\Phi}(z) - F(\bar{z}) - \lambda\Phi(\bar{z})$$

$$G'(\bar{z}) = -F'(\bar{z}) - \lambda\Phi'(\bar{z})$$

Therefore, FOC can be rewritten as

$$\begin{aligned} \frac{R}{\alpha k^{\alpha-1}} &= G(\bar{z}) - (\int_{\bar{z}}^{\infty} z\Phi(dz) - (1 - \Phi(\bar{z}))\bar{z})(-1 - \frac{\lambda\Phi'(\bar{z})}{F'(\bar{z})}) \\ &= G(\bar{z}) + F(\bar{z})(1 - \frac{\lambda\Phi'(\bar{z})}{1 - \Phi(\bar{z})}) \end{aligned} \tag{A.21}$$

Lemma 2. *Under Assumption 1, $G'(\bar{z}) > 0$.*

Proof. See [13]. □

A direct consequence of this lemma is that eq. (A.18) gives an implicit function $b(\bar{z})$ increasing in $\bar{z}$. In addition, assumption 1 and 2 ensures that the RHS of eq. (A.21) is decreasing in $\bar{z}$. To see this,

$$\begin{aligned} &\frac{d}{d\bar{z}} G(\bar{z}) + F(\bar{z})(1 - \frac{\lambda\Phi'(\bar{z})}{1 - \Phi(\bar{z})}) \\ &= G'(\bar{z}) + F'(\bar{z})(1 - \frac{\lambda\Phi'(\bar{z})}{1 - \Phi(\bar{z})}) - F(\bar{z}) \frac{d}{d\bar{z}} \frac{\Phi'(\bar{z})}{1 - \Phi(\bar{z})} \\ &= -F(\bar{z}) \frac{d}{d\bar{z}} \frac{\Phi'(\bar{z})}{1 - \Phi(\bar{z})} < 0 \end{aligned}$$

Now we move on to show how the optimal debt level $b(e,\mu,\sigma)$ varies with prior mean and variance.

Lemma 3. $F_\mu(\bar{z};\mu,\sigma) > 0$*,* $G_\mu(\bar{z};\mu,\sigma) > 0$*,* $\frac{\partial}{\partial\mu}\frac{\Phi'(\bar{z};\mu,\sigma)}{1-\Phi(\bar{z};\mu,\sigma)} > 0$

The first part of proposition 1 is a direct consequence of Lemma 3. An increase in prior mean will shift both the supply and demand curves for debt to the right and therefore equilibrium debt increases.

Lemma 4. $F_\sigma(\bar{z};\mu,\sigma) > 0$*, and* $G_\sigma(\bar{z};\mu,\sigma) < 0$*.*

Proof.

$$F(\bar{z};\sigma) = \int_{\bar{z}}^{\infty} z - \bar{z}\Phi(dz) = \int_0^{\infty}\omega\phi(\omega)d\omega = \mu - \int_{-\bar{z}}^{0}\omega\phi(\omega)d\omega,$$

where ϕ is a PDF of a log normal distribution $(\mu - \frac{1}{2}\sigma^2, \sigma)$.

$$\phi(\omega;\sigma) = P[\log(x) = \log((\bar{z}+\omega) - (\mu - \frac{1}{2}\sigma^2)/\sigma)],$$

where $\log(x) \sim N(0,1)$. Since

$$\frac{d}{d\sigma}\frac{\log(\bar{z}+\omega) - (\mu - \frac{1}{2}\sigma^2)}{\sigma} = \frac{(\mu + \frac{1}{2}\sigma^2) - \log(\bar{z}+\omega)}{\sigma^2} > 0$$

and

$$\log(\bar{z}+\omega) - (\mu - \frac{1}{2}\sigma^2) < 0,$$

$\phi(\omega;\sigma)$ increases in σ for all $\omega < 0$. Therefore, $F(\bar{z},\sigma)$ increases with σ.

$$\frac{dG_\sigma(\bar{z};\mu,\sigma)}{d\sigma} = -F_\sigma(\bar{z};\mu,\sigma) - \lambda\Phi_\sigma(\bar{z};\mu,\sigma) < 0$$

□

Lemma 5. $\frac{d}{d\sigma}\frac{\Phi'(\bar{z};\sigma)}{1-\Phi(\bar{z};\sigma)} > 0$*.*

Proof. Let $\bar{x} = \frac{\log(\bar{z}) - (\mu - \frac{1}{2}\sigma^2)}{\sigma}$, then $\Phi'(\bar{z};\sigma) = P[x = \bar{x}]$ and $\Phi'(\bar{z};\sigma) = P[x \leq \bar{x}]$. Therefore, $\frac{\Phi'(\bar{z};\sigma)}{1-\Phi(\bar{z};\sigma)} = \frac{\varphi'(\bar{x}(\sigma))}{1-\varphi(\bar{x}(\sigma))}$, where φ' and φ are the pdf and cdf of standard normal distribution. As shown above, $\frac{d}{d\sigma}\bar{x}(\sigma) > 0$ for $\log(\bar{z}) < \mu - \frac{1}{2}\sigma^2$, the monotone hazard condition for normal distribution gives $\frac{\Phi'(\bar{z};\sigma)}{1-\Phi(\bar{z};\sigma)}$ increasing in σ. □

Lemma 6. $G(\bar{z}) + F(\bar{z})(1 - \frac{\lambda\Phi'(\bar{z})}{1-\Phi(\bar{z})}) = \mu - \lambda\Phi(\bar{z}) - F(\bar{z})\frac{\Phi'(\bar{z})}{1-\Phi(\bar{z})}$ *decreases in* σ.

Combining Lemma 4-6, we obtain that an increase in prior variance shift both supply and demand curves for debt to the left. Therefore, equilibrium debt $b(e, \mu, \sigma)$ decreases. □

The proposition above highlights how learning can alleviate the negative impact of financial frictions on capital allocation. Without default risk, firms make investment choices so that their marginal returns on capital would equal the risk free interest rate ex ante. Because borrowers cannot commit to repay the loan, firms face a debt schedule that encodes that default risk, which increases their marginal costs of capital. This discrepancy between the debt schedule that firms face and the risk-free rate is a distortion on the capital margin. Conditional on equity and expected productivity, mature firms face a smaller distortion relative to young firms, because lenders are willing to offer a lower debt schedule as they become more certain of firm types and their default probabilities.

A.1.2 Empirical Appendix

In this appendix, I provide a description of the data sources and data cleaning procedures that I use to obtain calibration moments and data figures drawn in Chapter 1.

Data source: My data source is the China Industrial Enterprise Database, which reports detailed income statement and balance sheet information for industrial firms with 5 million RMB in annual sales in the following sectors: mining, manufacturing, utilities, and construction.[3] The benchmark calibration and results comes from a panel between 2004-2007.

Panel construction: Each firm in CIED is given a unique firm ID that is associated with its legal representative. If a firm has a new legal representative in a given year, it receives a new firm ID for that year. In order to form a larger panel of continuing firms, I consider two observations from different years as one continuing firm if two observations share the same firm ID or they share the same location, telephone number, and opening year.

Selection criteria: I restrict the sample to operating private firms that are less than 20 years old. I impose restriction on firm ownership, because previous studies such as [3] have found that *private* Chinese firms are credit constrained while *state-owned* firms and *foreign-owned* firms in China are not. I impose age restriction, because firm age cannot be reliably inferred from the dataset due to various changes in ownership and privatization. Firms less than 20 years old account for 95% of private firms in the entire sample. I drop firms with either leverage or capital growth at the top one percentile of the leverage or growth distribution of that year. I drop observations that are consequences of reporting errors: negative assets, negative fixed assets, negative sales, nonpositive employees, and starting year greater than reporting year. In the end, we have 286,045 firm-year observations for 116,227 firms. Out of these firms, we have roughly 45,000 firms with two observations, 17,000 firms with three observations, and 30,000 firms with four observations.

[3] See [19] for a detailed description of the database.

Variable definition: Firm age is reporting year less starting year. Firm size is measured as total assets. Leverage is total liability divided by total assets. Real capital is measured as total assets deflated by price index of investment goods. Capital growth is the percentage change of real capital from previous year.

A.1.3 Computation Algorithms

In this appendix, I provide a detailed description of the computation algorithms that solve the full information model and the learning model. In solving both models, I first discretize the state variables. I then solve the problem of incumbents using value function iteration and new entrants' problem using backward induction.

Algorithm to solve the full information model The details of the algorithm consist of the following steps:

1. Discretize productivity shocks. Log productivity z is the sum of a persistent component x that follows an AR(1) process and a transitory shock drawn from log normal distribution. Following [22], the persistent shock process and the transitory shock process are approximated with a Markov chain consisting of 7 points.

2. Create grids for capital k and leverage b/k. I use a clustered grid for capital stock. First, I construct a normalized capital grid $\vec{k}$ with 10 points between $[0, \bar{k}]$. Next, for each x_i, $i = 1, ..., 7$, I compute the efficient scale

$$K_i^* = \left(\frac{R + \delta - 1}{\alpha E[z'|x_i]}\right)^{\frac{1}{\alpha - 1}}$$

   Asset grid is the collection of $\vec{k} * K_i^*$, $i = 1, ..., 7$ with 70 points. Third, I construct a grid for leverage with 10 points between $[0, \bar{l}]$ and debt level is given by $b = l * k$.

3. Make initial guesses for incumbent value function V^c and debt schedule q.

4. Solve decision rules on each grid point by searching over grid points.

5. Update repayment value and debt schedule according to decision rules.

6. Repeat the previous two steps until the repayment value function and debt schedule converge.

7. Solve new entrants' problem given incumbents' value function.

Algorithm to solve the model with information frictions and learning Bayesian learning complicates the computation both in terms of solving the optimization problem and simulating the economy by introducing additional state variables. Specifically, we now need to keep track of both the mean and variance of prior belief when solving the model. Furthermore, we need to keep track of a firm's true type in addition to its belief when simulating the economy. The details of the algorithm consist of the following steps:

1. Use Bayes' rule to recursively compute prior variances σ_t and the standard deviation of prior mean σ_{μ_t}.

2. Find T such that learning is stationary, i.e. $\sigma_t = \sigma_T$, $\forall t > T$.

3. Discretize prior mean: we know that prior means are normally distributed with $\mu_t \sim N(0, \sigma_{\mu_t})$. For each $t = 1, ..., T$, I approximate this distribution with seven values μ_t^i using Tauchen method. For each prior belief (μ_t, σ_t), I approximate the realized productivity $z = x + \epsilon$ that is normally distributed with seven values.

4. Construct transition matrix for each (μ, σ, z). For each μ_t, σ_t, z_t, I calculate μ_{t+1} using Bayes' rule, and then the probability of μ_{t+1} hitting one of the seven grid points, say μ_{t+1}^i is set to be $(1/|\mu_{t+1}^i|)/(1/|\mu_{t+1}^i| + 1/|\mu_{t+1}^{i+1}|)$ if μ_{t+1} lies between μ_{t+1}^i and μ_{t+1}^{i+1} and zero otherwise. The probability of (μ_{t+1}^i, z_{t+1}) next period is the product of probability of hitting μ_{t+1}^i and the probability of z_{t+1} conditional on μ_{t+1}^i.

5. Compute $V^c(\mu, k, b; \sigma_T)$ on a grid for prior mean μ, capital k, debt b using value function iteration.

   (a) Create grids for capital k and leverage b/k. I use a clustered grid for capital stock. First, I construct a normalized capital grid $\vec{k}$ with 10 points between $[0, \bar{k}]$. Next, for each (μ, z) pair at time T, I compute the efficient scale

   $$K^*(\mu, z) = \left(\frac{R + \delta - 1}{\alpha E[z'|\mu, z]}\right)^{\frac{1}{\alpha - 1}}$$

   Asset grid is the collection of $\vec{k} * K^*(\mu, z)$ with 490 points. Third, I construct a grid for leverage with 10 points between $[0, \bar{l}]$ and debt level is given by $B = l * K$.

   (b) Make initial guesses for incumbent value function V^c and debt schedule q.

(c) Solve decision rules on each grid point by searching over grid points. Conditional expectations $E[z'|\mu, z]$ are computed as a matrix multiplication as described in step 4.

(d) Update repayment value and debt schedule according to decision rules.

(e) Repeat the previous two steps until the repayment value function and debt schedule converge.

6. Compute repayment value $V^c(\mu_t, \sigma_t, k_t, b_t, z_t)$ for $t = 1, ..., T-1$ via backward induction. Record decision rules for capital installment, leverage, and default decisions.

7. Solve new entrants' problem given incumbents' value function.

8. Simulation. The realization of z is determined by the true type of a firm instead of its prior belief. Therefore, I need to keep track of the true type of a firm so that the realized productivity shock is consistent with its true type. In particular, I discretized the persistent productivity into seven values. For each (μ, z, x) triplet, I calculate the probability of z conditional on μ by inverting the c.d.f. of a normal distribution $(x - .5\sigma_\epsilon^2, \sigma_\epsilon)$.

A.2 Additional Materials for Chapter 2

The Implications of Rising Replacement Rates for the US Hours Mulligan (2011) presents a neoclassical growth model augmented with unemployment subsidies. His model shows that changes in mean-tested subsides and the rise of replacement rates at the end of 2007 as the only impulse can generate time series for aggregate labor usage, consumption, investment, and real GDP that closely resemble the actual U.S. time series. In the exercise below, I show that his result is contingent on the level of tax distortion on labor income and the elasticity of labor supply.

In his model, families are partitioned into two groups: "prime" members who have a probability $p > 0$ of receiving a means-tested subsidy b_t in the event they are not working, and "others" who are never eligible for means-tested subsidies. Household maximizes his discounted utility $\sum_0^\infty \beta^t u_t$ with preferences given by

$$u_t = \frac{\sigma}{\sigma - 1} c_t^{\frac{\sigma-1}{\sigma}} - \gamma_n \frac{\eta}{\eta+1}(n_t - \beta m_t)^{\frac{\eta+1}{\eta}} - \gamma_m \frac{\eta}{\eta+1} m_t^{\frac{\eta+1}{\eta}},$$

subject to the budget constraint $n_t w_t + (\Gamma - n_t)\tau_t w_t + r_t k_t = c_t + k_{t+1} + (1-\delta)k_t + L_t$, where τ_t is the measured replacement rate $\frac{p_t b_t}{w_t}$ and L_t denotes date t lump sum taxes, and Γ is the prime worker time endowment. Firms maximize profits: $k_t^\theta (An_t)^{1-\theta} - w_t n_t - r_t k_t$. Equilibrium allocations are given by

$$k_t^\theta (A_t n_t)^{1-\theta} = c_t + k_{t+1} - (1-\delta)k_t$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\frac{1}{\sigma}} = \beta[\theta\left(\frac{An_t}{k_t}\right)^{1-\theta} + 1 - \delta]$$

$$n_t^{\frac{1}{\eta}} = \frac{1-\tau_t}{\gamma} \frac{(1-\theta)A^{1-\theta}}{c_t^{\frac{1}{\sigma}}} (\frac{k_t}{n_t})^{-\theta},$$

where in the last equation $\gamma > 0$ is a combination of preference parameters $\eta,\ \beta,\ \gamma_n,\ \gamma_m$.

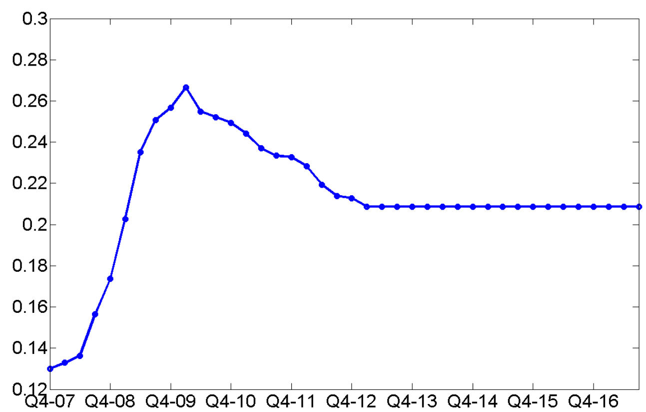
For calibration and simulation, I set the following parameter values as in Mulligan (2011): $\theta = 0.3$, $\sigma = 1.45$, $\delta = 0.019$, and $\beta = 0.993$. I then calibrate the parameters γ and A so that initial level of hours and capital are 1. I estimate replacement rate τ_r from BEA unemployment insurance (UI) data. Specifically, I construct time series of aggregate expenditure on UI and other social subsidies and compute its growth rate.[4] Using this

[4] The UI expenditure is from line 7 of BEA Table 3.12U. The other means-tested transfers are the sum of lines 21, 23, and 35-39.

growth rate, I impute the time path for the replacement rate from 2007:Q4 to 2013:Q2 by setting initial replacement rate equal to 13% in 2007:Q4. I also assume that the replacement rate stays at the level in 2013:Q2 from then on. Figure A.1 displays the time path for the replacement rate used in the simulation.[5]

Using this replacement rate series as the only impulse to the model, I conduct the following two simulation exercises. In the first experiment, I assume that there is no distortionary taxes on labor income in addition to the replacement rate. Figure A.2 displays simulated hours for three different elasticity of labor supply: $\eta = 0.3,\ 0.5,\ 1$ and normalized aggregate hours between 2007:Q4-2010:Q4. In this case, the model with an elasticity of labor supply between 0.5 and 1 generates comparable level of decline in labor usage as seen in the data. In the second simulation, I add 0.4 to the replacement rate, which reflects the average level of effective labor income taxes between 2007:Q4-2010:Q4.

Figure A.1: Time path for the replacement rate



[5] When constructing the replacement rate series, I do not take into account the changes in home retention act and consumer loan discharge, both of which are considered in Mulligan's paper. In addition, I make the assumption that the number of people who are eligible for UI has been constant from 2007 on. This assumption is clearly not supported by the data. However, Mulligan (2011b) points out that the increase in aggregate expenditures on means-tested subsidies is mostly due to changes in the amount of subsidies per eligible person as opposed to the rise of unemployed workers. By using the growth rate of total UI expenditure, I obtain a 13% rise in replacement rate over the two-year period after 2007:Q4, as suggested in his paper.

Figure A.2: Simulated market hours, replacement rate only