Granular Impact Cratering by Liquid Drops: An Analogy to Asteroid Strikes

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Introduction
When a granular material is impacted by a sphere, its surface deforms like a liquid yet it preserves a circular crater like a solid. We investigate liquid-drop impact dynamics on granular surface and monitor the morphology of resulting impact craters. Surprisingly, we find that despite the enormous energy and length difference, granular impact cratering by liquid drops follows the same energy scaling and reproduces the same crater morphology as that of asteroid impact crater. We integrate the physical insight from planetary sciences, the liquid marble model from fluid mechanics, and the concept of jamming transition from granular physics into a simple theoretical framework that quantitatively describes all of the main features of liquid-drop imprints in granular media.

Methods
We release a stationary water drop of diameter \(D = 1.4 \pm 4.6\) mm from a height \(h\). The drop falls vertically in air onto a granular bed comprising \(d = 45-250\) mm glass beads with volume fraction \(\phi = 0.60\). We vary \(h\) from 1.8 mm up to 12 m. A Photron SA-X2 camera was used for high-speed imaging of drop impact dynamics. The morphology of impact craters was measured using a high precision laser profilometer. The camera and the profilometer were further combined to monitor the depth of crater during impacts. We also performed one set of experiments at one-tenth of the atmospheric pressure to test possible effects of ambient air on the dynamics of liquid-drop impact cratering.

Results
Surprisingly, the 0.17 scaling is quantitatively similar to the Schmidt–Holsapple (S-H) scaling from hypervelocity impact cratering associated with asteroid strikes.

\[
D_{\text{crater}} = g^{-0.17}D^{0.83}D_{\text{drop}}^{-0.34} (\rho_{\text{sand}} - \rho_{\text{water}})^{-0.17}D_{\text{water}}^{0.32}E_{\text{impact}}^{0.17}
\]

1. Liquid Impact Model for Crater Diameter:
   - Impact Energy Conversion
   - Contribution of Eject Energy
   \[E_{\text{impact}} = fE_{\text{impact}}\]
   \[E_{\text{impact}} = \rho_{\text{sand}} V_{\text{crater}} gh
\]
   \[f = \frac{\pi D_{\text{drop}}^2}{\pi D_{\text{crater}}^2}
\]
   - Spherical Cap Approximation
   \[h_{\text{crater}} = \frac{c_{\alpha}}{6}(3-2\alpha)D_{\text{crater}}
\]

\[D_{\text{crater}} = \left(\frac{\pi}{6}\right)^{-\frac{1}{2}} \left(3-2\alpha\right) \left(\frac{\rho_{\text{sand}}}{\rho_{\text{water}}}\right)^{\frac{1}{2}} \left(\rho_{\text{sand}} - \rho_{\text{water}}\right)^{\frac{1}{4}} \left(\frac{D_{\text{drop}}^2}{D_{\text{crater}}^2}\right)^{\frac{1}{2}}
\]

Prefactor=1.74

2. Liquid Marble Theory and Imbition Model for Granular Residue:
   - Low Energy Regime
   \[N = \frac{\pi D_{\text{drop}}^2}{\pi D_{\text{crater}}^2} \frac{D_{\text{crater}}^2}{d_{\text{crater}}^2}
\]
   - Spherical Crater Maintenance Criteria,
   \[\kappa = \frac{\text{capillary length}}{\text{depth}}
\]
   \[D_{\text{marble}} << \kappa^{-1} \rightarrow \text{Spherical}
\]
   \[D_{\text{marble}} > \kappa^{-1} \rightarrow \text{Puddle}
\]
   \[D_{\text{marble}} = \left(6 \frac{V_{\text{water}}}{\rho_{\text{water}}}\right)^{1/3}
\]
   \[D_{\text{imbition}} \leq \left[\frac{C_1 (D_{\text{imbition}}^3 - 3.02 \rho_{\text{sand}}) \rho_{\text{water}}}{D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} D_{\text{imbition}}^3 + 4.8 \rho_{\text{water}} \right]^{1/3}
\]

High Energy Regime

Washburn-Lucas Equation for Imbition

\[V_{\text{imb}} = \frac{\pi}{2} \left(\frac{C_1}{D_{\text{imbition}}^2}\right)^{1/2} \left(\frac{2}{\rho_{\text{water}}}\right)^{1/2} \frac{1}{C_2}
\]

Modify parameters and yield:

\[V_{\text{imb}} = 0.0584 (\mu_{\text{imb}}) \frac{D_{\text{imbition}}^{-1/2}}{D_{\text{imbition}}^{1/2}}
\]

Imbition-Jamming Model:

\[\frac{V_{\text{imb}}}{V_{\text{imb}} - V_{\text{imb}} = \frac{\pi D_{\text{imbition}}^2}{\pi D_{\text{imbition}}^2 - V_{\text{imb}}}}
\]

Conclusion
When a liquid drop impacts on a granular surface, the impact energy is converted into the surface energy of the deformed drop, the internal energy of liquid and particles, and the kinetic energy of the spreading lamella and ejected particles. The process is notoriously complicated, involving high Reynolds hydrodynamics, shock compression in the impinging drop, fast granular flows, and capillary interactions between fluid and granular particles. Given the complexity, it is surprising that the simple model presented here can quantitatively capture the morphology of liquid-drop impact craters over a large range of impact energy.

Moreover, our study reveals a quantitative similarity between raindrop impact cratering and asteroid impact cratering in terms of both the energy scaling and the aspect ratio of their impact craters. Compared with extensively studied low-speed solid sphere impact cratering, liquid-drop impact cratering provides a better analogy to high-energy asteroid impact cratering.

Apparently, one should be very cautious when drawing a close link between the two processes. Nevertheless, the remarkable similarity between the two processes indicates that they may share common mechanisms that are worth further investigation.

Selected Reference

Acknowledgement
This work is funded by the University of Minnesota Undergraduate Research Opportunity Program (UROP)