The primordial CMB 4-point function

Kendrick Smith (Perimeter)
Minnesota, January 2015

Main references:

Smith, Senatore & Zaldarriaga (to appear in a few days)
Planck 2014 NG paper (to appear last week of January)

Outline

1. Primordial NG: what signals should we look for?

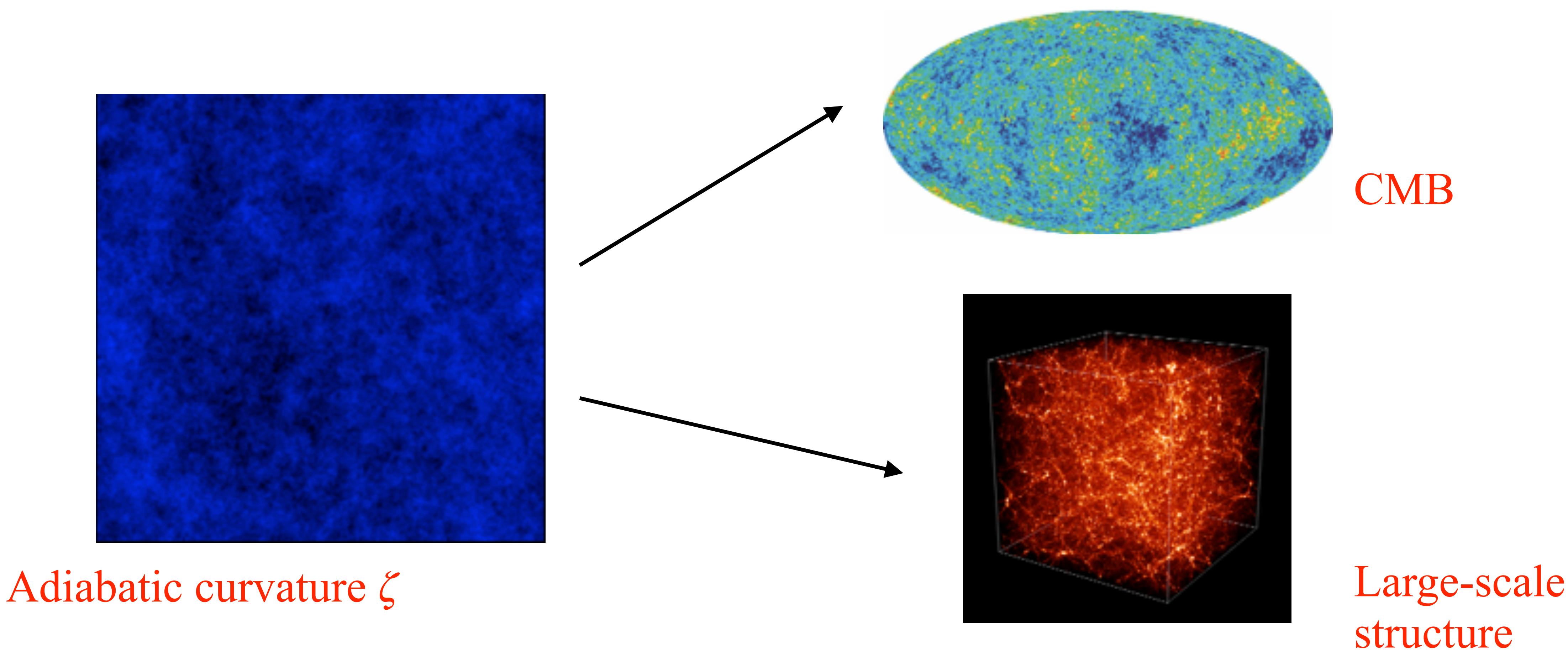
2. Data analysis challenges

3. WMAP/Planck results and interpretation

Cosmological initial conditions

Within current observational errors, initial conditions are:

- Adiabatic scalar: The initial conditions are completely determined by the (3D) adiabatic curvature $\zeta(x)$
- Gaussian: The statistics of the adiabatic curvature $\zeta(x)$ are completely determined by the power spectrum $P_\zeta(k)$
- Power law: $(k^3/2\pi^2)P_\zeta(k) = A_\zeta(k/k_0)^{n_s-1}$



Primordial non-Gaussianity

Searching for deviations from non-Gaussian statistics probes field content of inflation and interactions between fields

There are many possible signals to search for (partial list: $f_{NL}^{\rm loc}, f_{NL}^{\rm equil}, f_{NL}^{\rm orth}, g_{NL}^{\rm loc}, \tau_{NL}, g_{NL}^{\dot\sigma^4}, \cdots$). Each of these parameters is the coefficient of either a 3-point or 4-point function and can be further classified as "local" or "nonlocal".

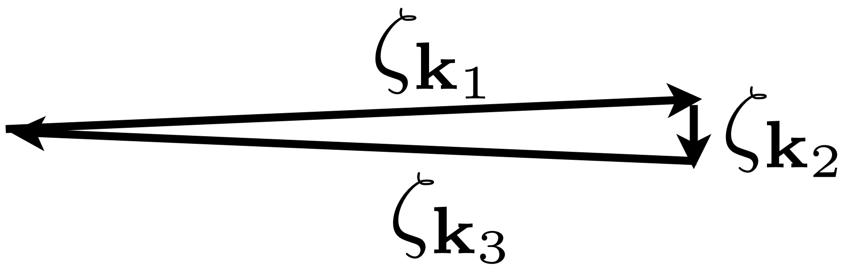
	Local	Nonlocal
3-point	$f_{NL}^{\rm loc}$	$f_{NL}^{\rm equil}, f_{NL}^{\rm orth}$
4-point	$g_{NL}^{\rm loc}, \tau_{NL}$	$g_{NL}^{\dot\sigma^4}, g_{NL}^{(\partial\sigma)^4}$

Local non-Gaussianity

"Local" = general term for non-Gaussianity generated by nonlinear processes which are local in real space. E.g.

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + f_{NL}^{\mathrm{loc}}\zeta_G(\mathbf{x})^2 \qquad \text{where } \zeta_G = \text{Gaussian}$$

$\Rightarrow$ "squeezed" 3-point function



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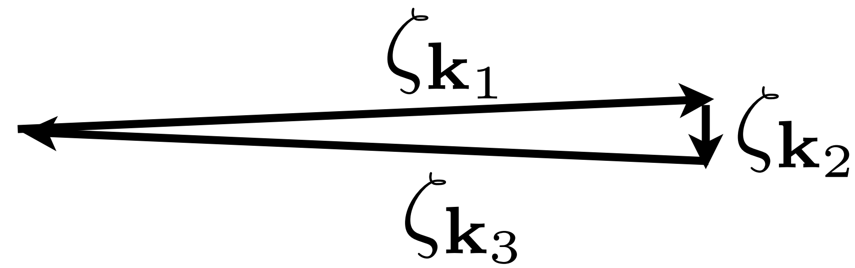
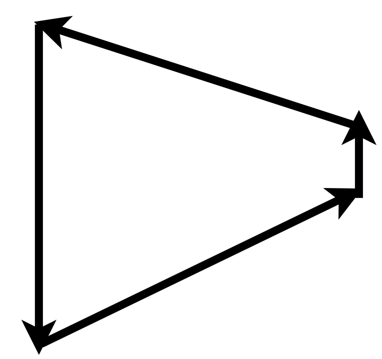


$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + g_{NL}^{\mathrm{loc}} \zeta_G(\mathbf{x})^3$$

$\Rightarrow$ "squeezed" 4-point function



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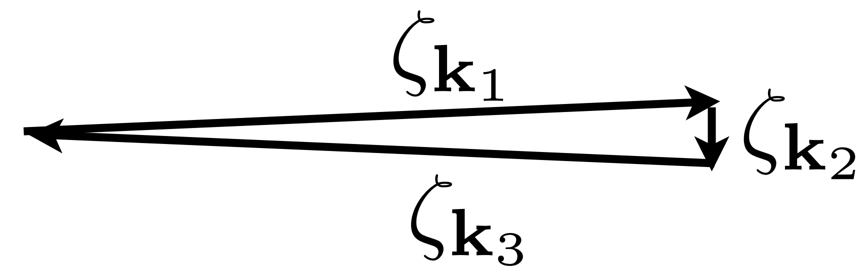

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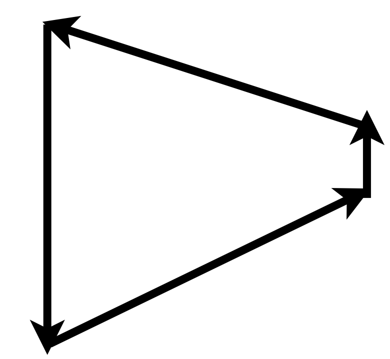
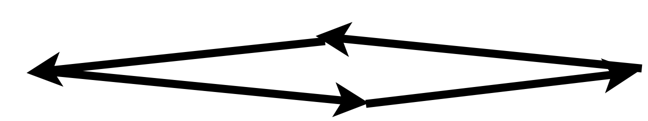


$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \tau_{NL}\zeta_G(\mathbf{x})\sigma_G(\mathbf{x})$$

$\Rightarrow$ "collapsed" 4-point function

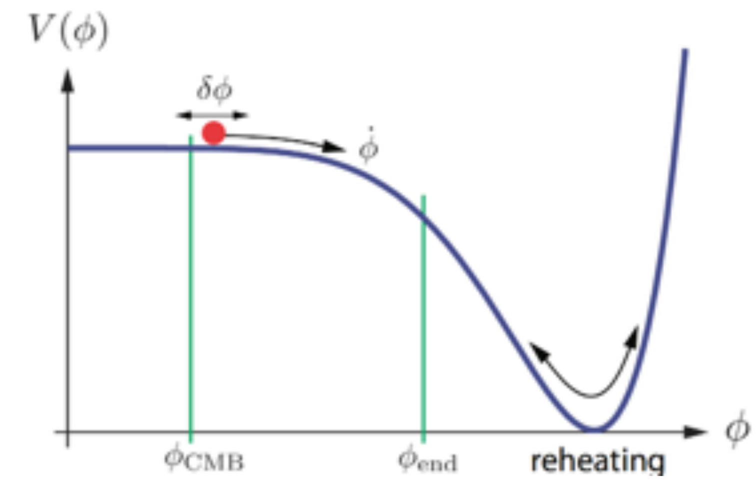


"Nonlocal" non-Gaussianity

"Nonlocal" = General term for non-Gaussianity generated by quantum-mechanical processes at horizon crossing during inflation. To illustrate by example, consider two models:

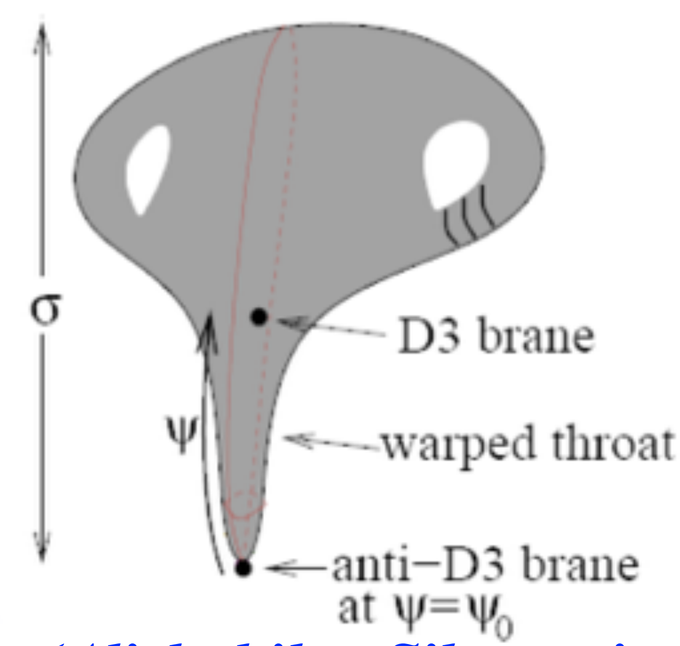
1. Single-field slow-roll inflation

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$



2. DBI inflation

$$S = \frac{1}{g_s} \int d^4x \sqrt{-g} \left(\frac{\sqrt{1 + f(\phi)(\partial\phi)^2}}{f(\phi)} - V(\phi) \right)$$



(Alishahiha, Silverstein & Tong)

“Non-local” non-Gaussianity

Both slow-roll and DBI inflation can give (n_s,r) values which are consistent with CMB constraints.

3-point function can discriminate the two:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \approx 0 \qquad \text{(single-field slow-roll)}$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \approx f_{NL}^{\text{equil}} \frac{1}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3} \qquad \text{(DBI)}$$

Non-local NG: EFT of inflation

π = Goldstone boson of spontaneously broken time translations

1-1 correspondence between operators in S_π and f_{NL}-like parameters
(Degree-N operator generates N-point CMB correlation function)

$$S_\pi = \int d^4x \sqrt{-g}(-\dot{H} M_{\rm pl}^2) \left[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2}\dot{\pi}^3 + \frac{1-c_s^2}{c_s^2}\frac{\dot{\pi}(\partial_i\pi)^2}{a^2}\right.$$

Leading (in derivative expansion)
cubic interactions

These terms generate two 3-point signals $f_{NL}^{\rm equil}, f_{NL}^{\rm orth}$
(Senatore, KMS & Zaldarriaga 2009)

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$$+ \frac{A}{c_s^2} \dot{\pi}^3 + \frac{1 - c_s^2}{c_s^2} \frac{\dot{\pi} (\partial_i \pi)^2}{a^2}$$

Leading (in derivative expansion) cubic interactions

$$+ B \dot{\pi}^4 + C \dot{\pi}^2 (\partial_i \pi)^2 + D (\partial_i \pi)^2 (\partial_j \pi)^2$$

Leading quartic interactions

These terms generate three 4-point signals, in principle independent. However, a Fisher matrix analysis shows that there is one large correlation among the three 4-point functions, so we propose two observables $g_{NL}^{\dot{\sigma}^4}, g_{NL}^{\dot{\sigma}^2 (\partial\sigma)^2}$ (KMS, Senatore &Zaldarriaga 2015)

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Leading quartic interactions

$$+\frac{1}{2}\dot\sigma^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{m^2\sigma^2}{2} + \rho\dot\pi\sigma + E\sigma^3$$

Quasi single-field inflation (Chen & Wang 2009, Baumann & Green 2011)

These terms generate interesting four-point signals, but currently unconstrained due to technical difficulties to be explained shortly

Non-local NG: EFT of inflation

π = Goldstone boson of spontaneously broken time translations

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(Degree-N operator generates N-point CMB correlation function)

$$S_\pi = \int d^4x \sqrt{-g}(-\dot{H} M_{\rm pl}^2) \Bigg[\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2}$$

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$$+ \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}(\partial_i\sigma)^2 - \frac{m^2\sigma^2}{2} + \rho\dot{\pi}\sigma + E\sigma^3$$

Quasi single-field inflation

$$+ \cdots \Bigg]$$

more possibilities I won't mention in this talk!
(resonant models, higher-derivative, etc.)

Non-Gaussian models

In summary, primordial non-Gaussianity is a many-parameter space; each parameter corresponds roughly to one interaction term in the inflationary action, or one physical process

Measuring these parameters constrains the inflationary action, in the same sense that measuring cross sections in a collider experiment constrains the action.

	Local	Nonlocal
3-point	$f_{NL}^{\rm loc}$	$f_{NL}^{\rm equil}, f_{NL}^{\rm orth}$
4-point	$g_{NL}^{\rm loc}, \tau_{NL}$	$g_{NL}^{\dot\sigma^4}, g_{NL}^{(\partial\sigma)^4}$

Next: Data analysis challenges

Data analysis challenges

General problem: we want to estimate the amplitude of a specified curvature N-point function $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_N} \rangle$, given an observation of the CMB multipoles $a_{\ell m}$

Conceptually, we are trying to do the following:

Curvature N-point function $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_N} \rangle$

↓

CMB N-point function $\langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle$

↓

CMB estimator

$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle \prod_{i=1}^{N} \tilde{a}_{\ell_i m_i} + \cdots$$

observed CMB multiples (appropriately filtered)

Data analysis challenges

In general, there are computational difficulties…

CMB three-point function: 4D oscillatory integral for each (ℓ_i, m_i)

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

$$\times \int dr\, dk_1\, dk_2\, dk_3 \left(\prod_{i=1}^{3} \frac{2k_i^2}{\pi} j_{\ell_i}(k_i r) \Delta_{\ell_i}(k_i) \right) \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$$

CMB transfer function (computed w/CAMB)

CMB estimator: number of terms in sum is $\mathcal{O}(\ell_{\max}^5)$

$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle\, \tilde{a}_{\ell_1 m_1} \tilde{a}_{\ell_2 m_2} \tilde{a}_{\ell_3 m_3} + \cdots$$

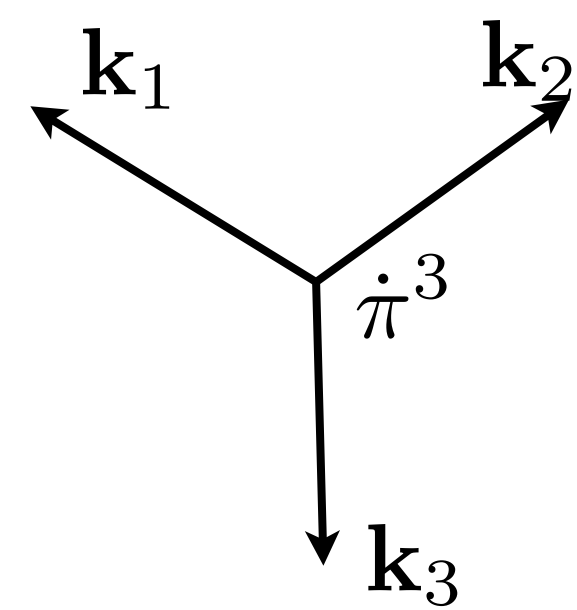
observed CMB multiples (appropriately filtered)

Data analysis challenges

… which can be solved by going back to the physics as follows.

First, take a step backwards by writing down the Feynman diagram which gives the N-point function $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_N} \rangle$, but leave integrals unevaluated. E.g. for DBI inflation:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \int_{-\infty}^{0} d\tau \, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1 k_2 k_3}$$



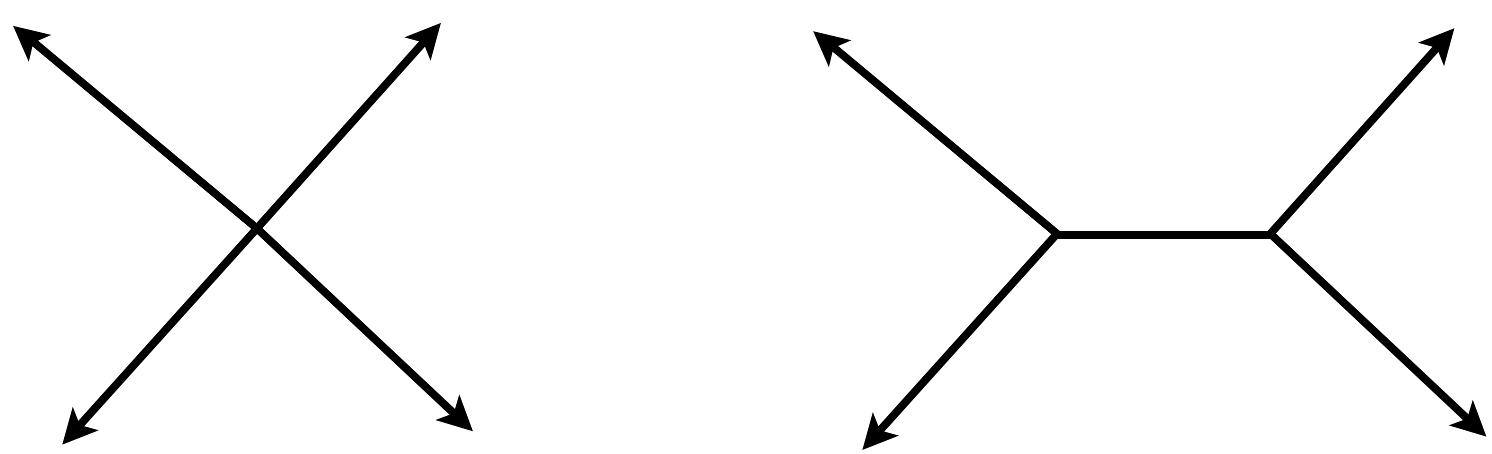
Data analysis challenges

If we write down the CMB estimator and bring the time integral to the outside, then everything factorizes, leading to reduced computational cost:

$$\mathcal{E} = \int \tau^2 d\tau \, d^3\mathbf{r} \left(\sum_{\ell m} \alpha_\ell(r,\tau) a_{\ell m} Y_{\ell m}(\hat{\mathbf{r}}) \right)^3$$

$$\text{where} \quad \alpha_\ell(r,\tau) = \int \frac{2k^2\, dk}{\pi} \frac{e^{k\tau}}{k} j_\ell(kr) \Delta_\ell(k)$$

Generalizes to any tree diagram, e.g. 4-point estimators:



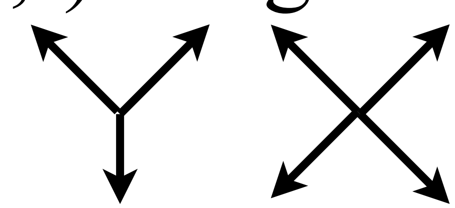
Data analysis challenges

$$\mathcal{E} = \int \tau^2 d\tau\, d^3\mathbf{r} \left(\sum_{\ell m} \alpha_\ell(r,\tau) a_{\ell m} Y_{\ell m}(\hat{\mathbf{r}}) \right)^3$$

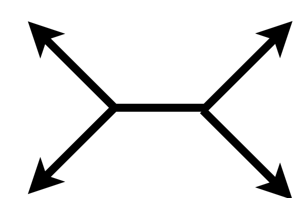
Computational cost is still marginal: number of spherical transforms is $O(N_{mc}\, N_{quad})$ where

N_{mc} = # of MC sims = $O(10^3)$

N_{quad} = # of quadrature points needed to do (τ, r) integrals
= $O(10^5)$ for cubic or quartic diagrams



= $O(10^9)$ for exchange diagrams!



This problem is currently preventing us from analyzing exchange-diagram 4-point functions.

Data analysis challenges

Sensitivity to modeling errors in the Gaussian part of the signal. Simple four-point estimators of the schematic form

$$\hat{g}_{NL} = (T_1 T_2 T_3 T_4) - \langle T_1 T_2 T_3 T_4 \rangle_{\rm Gaussian}$$

are disastrous since $\langle T_1 T_2 T_3 T_4 \rangle_{\rm Gaussian}$ is very sensitive to modeling of beams, noise, residual foregrounds, etc.

Must model instrument with fractional error $O(l_{max}^{-1}) \sim 0.1\%$!

Powerful fact: estimators with "(2+2)-point terms"

$$\hat{g}_{NL} = (T_1 T_2 T_3 T_4) - (T_1 T_2 \langle T_3 T_4 \rangle_{\rm Gaussian} + 5 \text{ perm.}) + 3 \langle T_1 T_2 T_3 T_4 \rangle_{\rm Gaussian}$$

have smaller error bars and are parametrically less sensitive to modeling errors. Modeling requirement is $O(l_{max}^{-1/2}) \sim$ (few %)

Data analysis challenges

Large guaranteed 4-point signals.

Gravitational lensing trispectrum is 40σ in Planck!

… but uncertainty in the expected signal is small enough that one can simply compute (by Monte Carlo) the lensing bias in a fiducial model and subtract it.

Residual secondaries (SZ, dusty galaxies, etc.) also have large 4-point functions

Need excellent foreground removal (template subtraction is good enough for WMAP, for Planck we need a fancy method like SMICA)

Current status

Blue = WMAP   Magenta = Planck 2013   Red = Planck 2015

	Local	Nonlocal
3-pt	$f_{NL}^{\rm loc} = 37.2 \pm 19.9$ $f_{NL}^{\rm loc} = 0.71 \pm 5.1$	$f_{NL}^{\rm equil} = 51 \pm 136$ $f_{NL}^{\rm orth} = -254 \pm 100$ $f_{NL}^{\rm equil} = -9.5 \pm 44$ $f_{NL}^{\rm orth} = -25 \pm 22$
4-pt	$g_{NL}^{\rm loc} = (-9.00 \pm 7.73) \times 10^4$ $\tau_{NL} < 2800 \; (95\% \; {\rm CL})$	$g_{NL}^{\dot\sigma^4} = (-2.11 \pm 1.74) \times 10^6$ $g_{NL}^{(\partial\sigma)^4} = (-1.10 \pm 3.82) \times 10^5$

Note normalization of these parameters!

3-point: f_{NL}~1 corresponds to dimensionless NG of order ~10^{-4}

4-point: g_{NL} or τ_{NL}~1 corresponds to NG of order ~10^{-8}

Interpretation

Non-Gaussianity at the $f_{NL} \sim 10^1$-10^2 level is easy to generate in non-Gaussian models of inflation, but to rule out qualitative classes of models, we need to get to $f_{NL} \sim 1$.

$$f_{NL}^{\rm loc} = 0.7 \pm 5.1$$
$$f_{NL}^{\rm equil} = -3 \pm 45$$
$$f_{NL}^{\rm orth} = -25 \pm 22$$

(Note: there is a “guaranteed signal” at $f_{NL} \sim 10^{-2}$ even in single-field slow-roll inflation)

We are getting close with Planck, and future E-mode polarization measurements will help by a factor ~few.

More futuristic large-scale structure measurements can do much better in principle (mode-counting 3D vs 2D) but practical challenges are currently unclear

Interpretation: local NG

There are theorems (Maldacena 2002, Creminelli & Zaldarriaga 2003) which show that single-field inflation cannot generate local NG of any type

$$f_{NL}^{\rm loc} = 0.71 \pm 5.1$$

$$g_{NL}^{\rm loc} = (-9.00 \pm 7.73) \times 10^4$$

$$\tau_{NL} < 2800 \ (95\% \ \mathrm{CL})$$

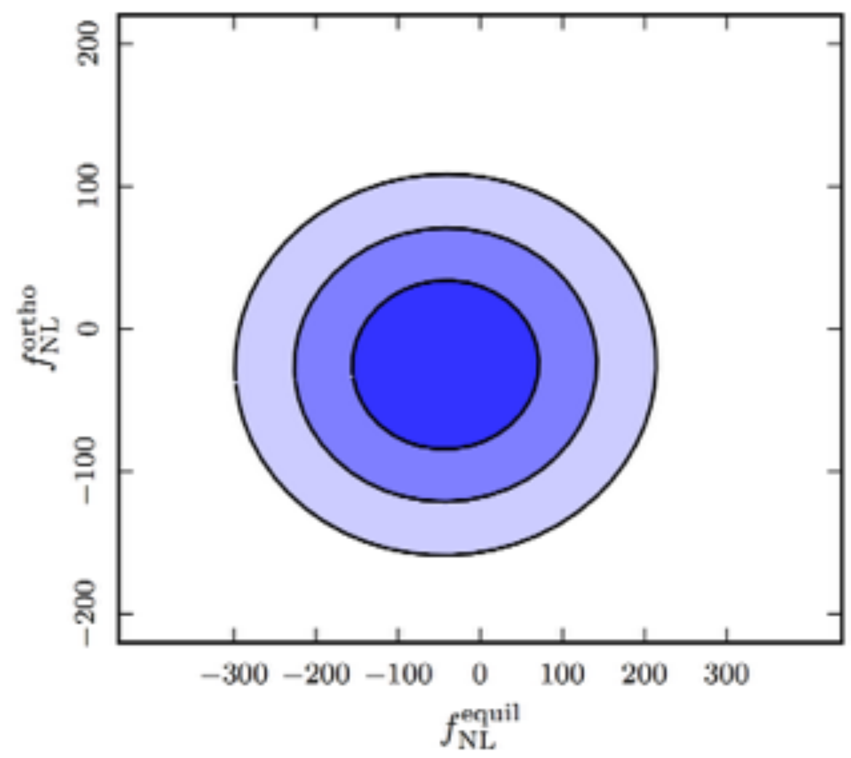
Conversely, local NG is somewhat generic in multifield models of inflation.

Our results on local NG constrain parameter spaces in multifield models, but won't rule out qualitative classes of models until $f_{NL}{\sim}1$ is reached.

Interpretation: nonlocal NG

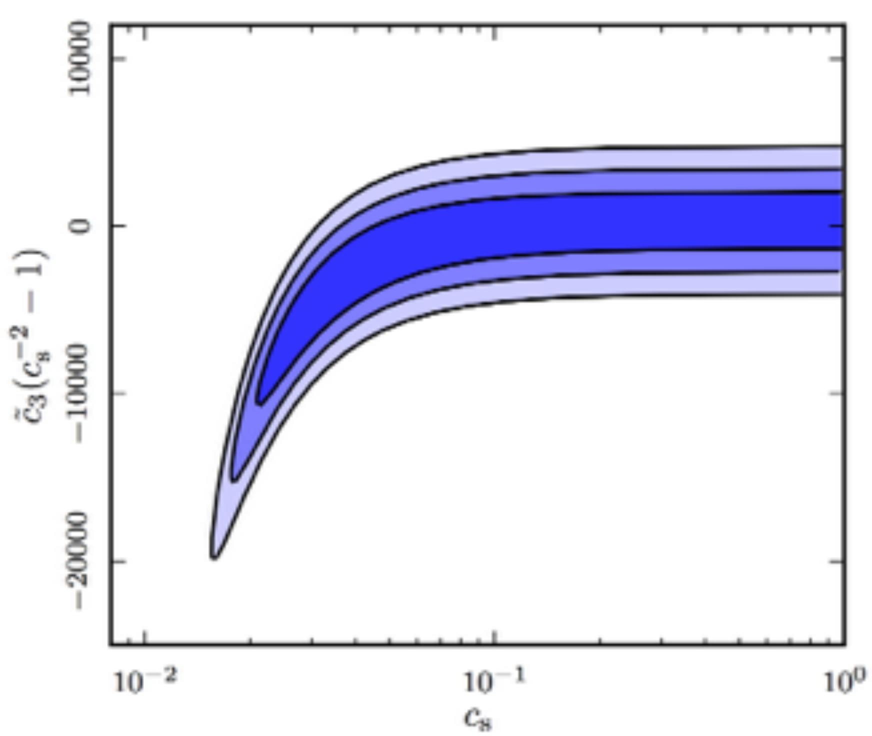
In single-field inflation, 3-point function is linked to the sound speed c_s

$$f_{NL}^{\rm equil} = 51 \pm 136$$
$$f_{NL}^{\rm orth} = -254 \pm 100$$



Four-point function constrains interactions which are allowed by symmetry and radiatively stable

$$g_{NL}^{\dot\sigma^4} = (-2.11 \pm 1.74) \times 10^6$$
$$g_{NL}^{(\partial\sigma)^4} = (-1.10 \pm 3.82) \times 10^5$$



Planck 2013

Conclusions

- Primordial NG is a multifaceted probe of the field content and interactions in inflation ("one parameter per Feynman diagram")

- 3-point and 4-point pipelines are now working in Planck, but can only do "contact" 4-pt case, not the "exchange" case

- Nondetection of local NG places strong constraints on multifield inflation but not particularly conclusive until $f_{NL}\sim1$

- Nondetection of nonlocal NG constrains sound speed during inflation and constrains interacting models such as DBI inflation

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$$f_{NL}^{\rm orth} = -254 \pm 100$$

$$g_{NL}^{\rm loc} = (-9.00 \pm 7.73) \times 10^4$$
$$\tau_{NL} < 2800 \text{ (95\% CL)}$$

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