

Finite Element Method Package  
of  
Learner Programs  
(FEMPAC)

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Finite Element Method Package

of

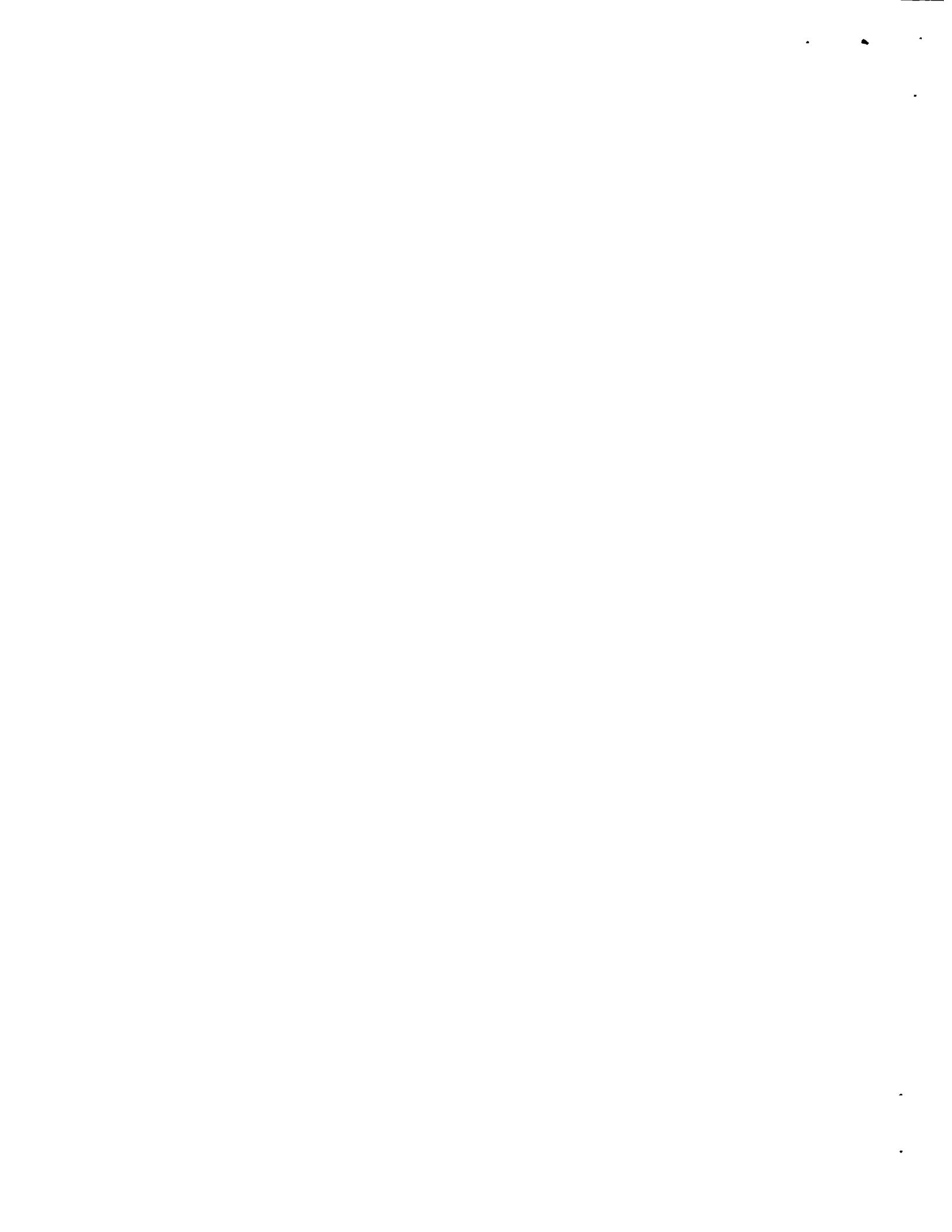
Learner Programs

(FEMPAC)

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Developed in cooperation with  
University Computer Center  
University of Minnesota

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## PREFACE

The purpose of FEMPAC is to facilitate the solution of learner-level problems by the finite element method student. In order to accomplish this purpose the program inputs and options available have been kept as simple as possible. The package is structured such that no programming experience should be necessary for its use.

The purpose of FEMPAC is not to make possible solution of highly complex or large problems, although it is structured as efficiently as possible and has considerable capability for large problems. The principal virtue is the variety of continuum and structural-type problems which can be solved through this package.

This version of FEMPAC is not considered final or complete. Work will continue to improve, refine, and add capabilities to the system.

This User's Manual is an attempt to present as concisely as possible the purposes and procedures for the problems and the necessary inputs. Extensive use is made of example problems illustrated with both input and output examples.



## INTRODUCTION

A person with a finite element problem is especially eager to have a dependable computer package requiring a minimum of concern for system details outside the scope of his/her primary interest. Most commercial finite element programs are directed at the solution of very large problems stemming from corporate or industry design needs. The software supplied by computer manufacturers and others is proprietary and very expensive. More often than not, their documentation is not designed for a novice user. The typical learner-user of the finite element method in a university environment has small- or medium-scale problems and is not inclined to spend a great deal of time learning about programming and computer systems for problem solution.

The objectives in designing a Finite Element Method PACKage (FEMPAC) of learner programs have been to stress ease of use, variety in problem types which can be solved, commonality of input between problem types, and minimization of computer experience needed.

The programs in this package are not highly sophisticated programs. They are not intended to compete with commercially available programs for detailed research and design. These programs are intended to solve learner-level problems in a variety of application areas. This package is intended to minimize the amount of classroom time required to explain use of the programs. This package should be useful not only in finite element method instruction, but also in other engineering courses where problem solutions are needed.

Many of the programs used in this text are modifications of those given by Segerlind (1976).<sup>1</sup> The Segerlind text is highly recommended for those desiring to gain a basic understanding of the finite element method.

<sup>1/</sup>Segerlind, L.J., 1976. Applied Finite Element Analysis, John Wiley & Sons, Inc., New York.





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## 1. PACKAGE DESCRIPTION

### 1.1 DESCRIPTION OF AVAILABLE PROGRAMS

This package is built up of two types of programs. The first type is support programs which are intended to facilitate development and display of input data, and display and analysis of results. The second type is application programs, each intended to solve problems in one specific application area. Detailed description of options available, input required, and sample applications are included for each program in the following chapters of this manual.

Support programs include automatic grid generation for programs using the simple three-node linear triangular element. The program has an option for renumbering nodes to minimize storage requirements. Grid plotting on the line printer is also available for programs using the simple linear triangular element with or without automatic grid generation. Result plotting is available for both nodal values and element resultants for pertinent variables within each application program. More detailed description of grid and result plotting are given in Chapters 4 and 5 respectively.

Application programs available and support options available for each are summarized in Table 1.1. More detail on each application program is given in Chapter 3 of this manual.

### 1.2 FLOW CHART OF PACKAGE

The following flow chart describes the general flow during execution of FEMPAC. Decisions are made by the program based on the Main Control Parameters input by the user. These control parameters are described in Section 1.4.

### 1.3 DESCRIPTION OF INPUT METHOD - FORMAT FREE INPUT

The main obstacle encountered in applying this package is communicating the correct information to the computer. In computer programming, information is transferred from external devices (the card reader, normally) to internal storage by means of read statements. The transfer of information into the computer is called INPUT, a term that appears frequently in this manual.

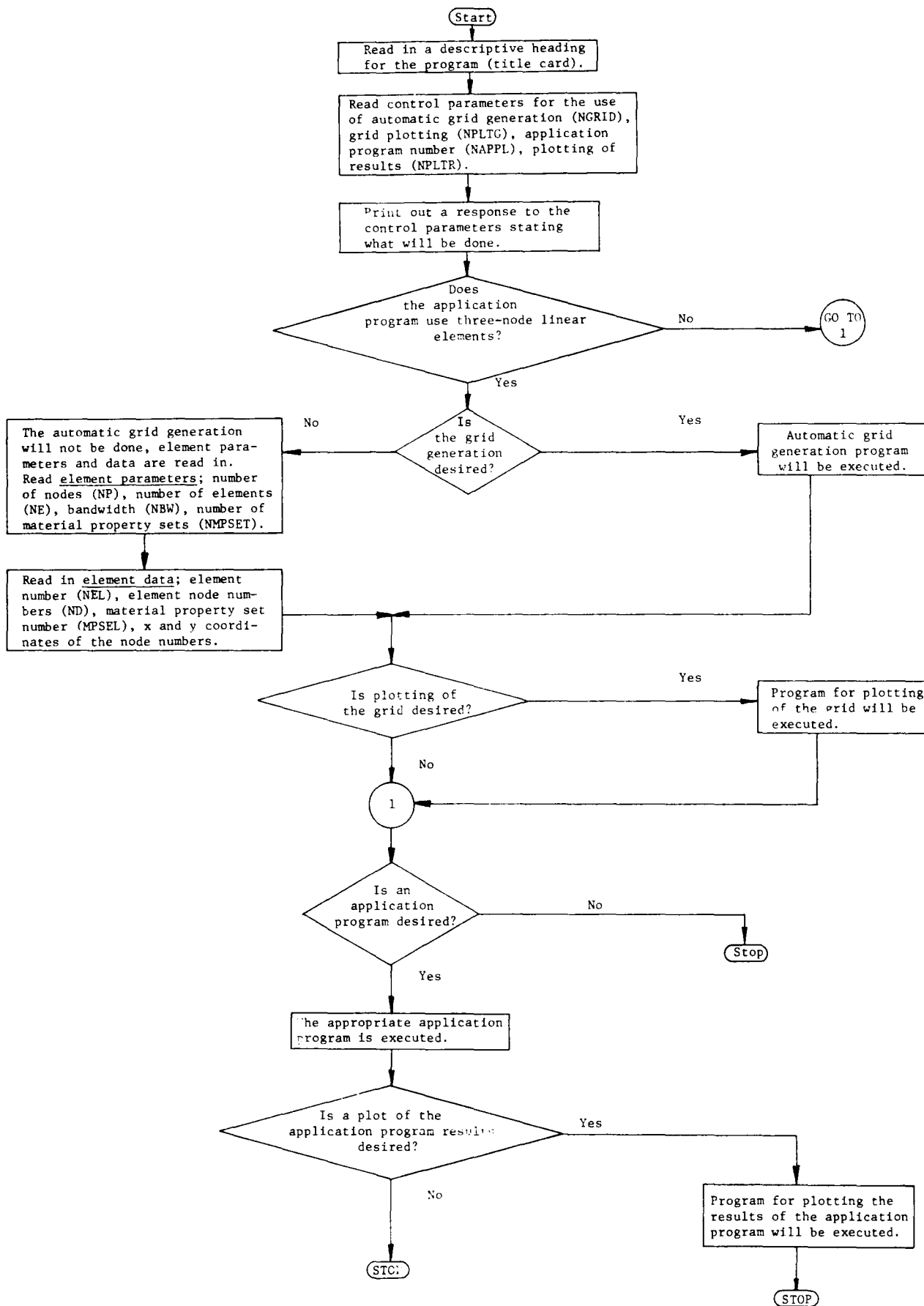
There are two possible modes of input available for this package. The first is FORMATTED INPUT. Formatted information is prepared on a card in a format specified by a format statement in the computer program and can not be deviated from. This form of input is not used here, but alternative read statements for its implementation are included in the programs as comments. A specified format makes the preparation of input data an error-prone, space-counting affair. The second mode of input eliminates these problems by making data input format free, hence it is called FORMAT-FREE INPUT. The input values are separated only by a delimiter which normally is a comma or blank space. One disadvantage in using this type of input is the need for extra zero values on a card to terminate data input.

Table 1.1

## Programs Available in FEMPAC

Program	Application Prog. No. (NAPPL)	Capabilities	Element Type	Available Support Options		
				Grid Generation	Grid Plotting	Result Plotting
2-D Elasticity	1	Two-dimensional plane stress and plane strain problems with optional thermal stresses.	linear triangle	Yes	Yes	Yes
2-D Heat Transfer	2	Two-dimensional steady state heat transfer with surface convection or specified boundary temperatures and internal heat sources.	linear triangle	Yes	Yes	Yes
2-D Ground Water Flow	3	Two-dimensional steady state field problem with sources and/or sinks, seepage can be included for ground water flow.	linear triangle	Yes	Yes	Yes
Torsion of Non-circular Shaft	4	Calculates shear stresses in non-circular shaft for given torsional input.	linear triangle	Yes	Yes	Yes
2-D Transient Heat Transfer	5	Two-dimensional transient heat transfer with surface convection or specified boundary temperatures and internal heat generation.	linear triangle	Yes	Yes	Yes
Truss	6	Simple trusses.	linear element	No	No	No

Flow Chart for FEMPAC

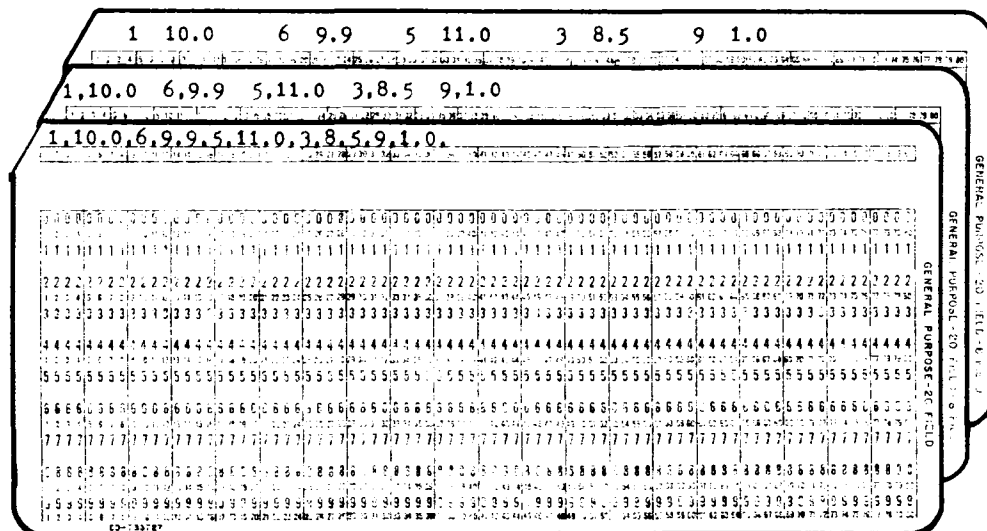


Both modes of input require an input list in the read statements of the program. The input list specifies the variable names or arrays to which the data items are to be transferred and the order of their transmission. Thus, care must be taken to insure that input values are in the correct order on the data cards, so that the values will be stored in locations to which the corresponding variables refer.

Format-free input is very versatile, allowing the users to space the input data to their advantage in terms of readability and/or space used. The following are some conventional rules governing format-free input. The delimiters divide the card up into what are known as fields. Either a blank or a comma following the value are commonly used as delimiters. Leading blanks in a field and blanks following the last delimiter on a card are ignored. A value cannot be continued from one card to another since each card is considered individually. This also implies that the last value on the card doesn't need a delimiter after it.

Two types of variables used in the programs are real and integer. Usually the input values must be of the same type as the variables to which they refer. This is why there is a need to include the decimal points in some input values even though they are whole numbers.

Examples: The following three cards show possible ways of inputting the numbers 1, 10.0, 6, 9.9, 5, 11.0, 3, 8.5, 9, 1.0 under format-free input.



#### 1.4 DEFINITION OF MAIN CONTROL PARAMETERS

Flow of the program and options executed is controlled by parameters on the main control parameters card. The control parameters by the order in which they appear on the main parameter card are:

NGRID - Controls use of automatic grid generation option.

NGRID = 0 implies auto-grid generation will not be used and that the user will input the element data for the problem. The user should see Section 1.6 or the appropriate application program section for description of the element data.

NGRID = 1 implies the grid generation option will be executed. The user should see Chapter 2 for description of the input necessary for auto-grid generation.

NPLTG - Controls option of plotting of grid. The grid plot program produces a line printer plot of the nodal locations. Chapter 4 describes and shows an example of the output available through this option.

NPLTG = 0 No plot is produced.

NPLTG = 1 Plot is produced.

NAPPL - Controls the selection of the appropriate application program. Table 1.1 gives a summary of the application programs available. The user is referred to the appropriate section of Chapter 3 for description of the necessary input for each application program.

NAPPL = 0 implies that no application program will be executed. This is a useful option for the user making a preliminary run of grid generation for determination of boundary nodes and locations of nodes for development of boundary value and/or initial value inputs required for the application programs.

#### Summary of Application Programs

<u>NAPPL</u>	<u>Program</u>
0	No program executed
1	Elasticity
2	Heat transfer
3	Ground water flow
4	Torsion
5	Transient heat transfer
6	Truss analysis

NPLTR - Controls option of plotting of results. The result plotting program produces line printer plots of pertinent variables. Chapter 5 describes and shows an example of the output available through this option.

NPLTR = 0 No plots are produced.

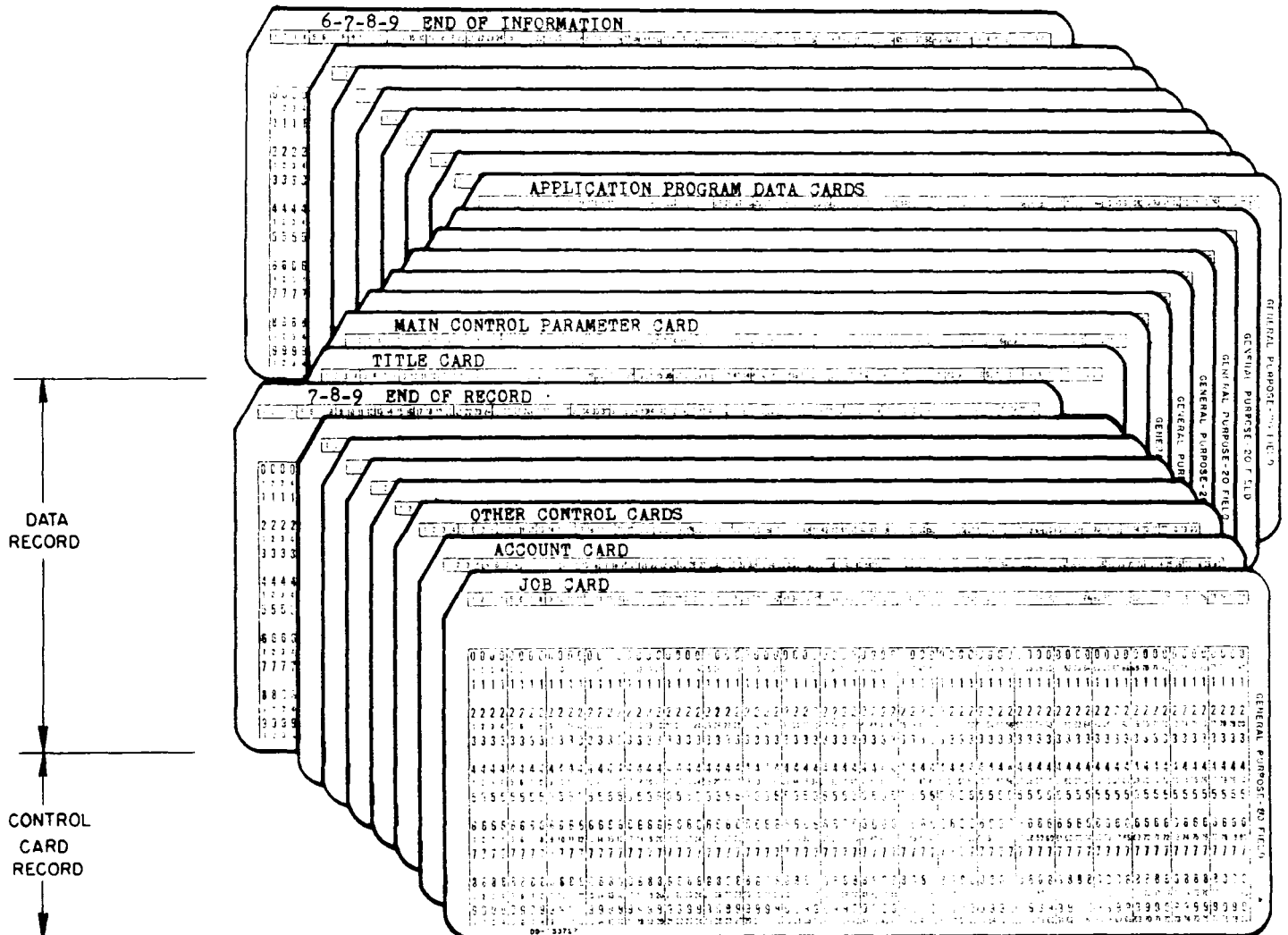
NPLTR = 1 Plots are produced.

Sample Main Control Parameter Card

1, 1, 2, 0

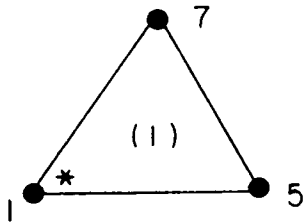
would imply NGRID = 1, auto-grid generation to be executed; NPLTG = 1 grid will be plotted; NAPPL = 2 - application program 2; heat transfer, will be executed; NPLTR = 0 - results plotting will not be executed.

### 1.5 SCHEMATIC OF DECK STRUCTURE FOR FEMPAC





## 1.6 ELEMENT DATA AND CONTROL PARAMETERS FOR ELEMENT DATA FOR THREE-NODE LINEAR TRIANGULAR ELEMENT.



The purpose of this section is to describe the element data and control parameters for element data for programs which use the three-node linear triangular element. Description of element data for other element types will be given in the application programs where they are used. For all programs in this package, the element data establishes the element number, node numbers for the element, the material properties set for the element, and the coordinates of the nodes for each element making up the region under study.

### 1.6.1 Element Data for Linear Triangular Element

The element data for programs using linear triangular elements can either be input for each element by the user or can be generated through the automatic grid generation program.

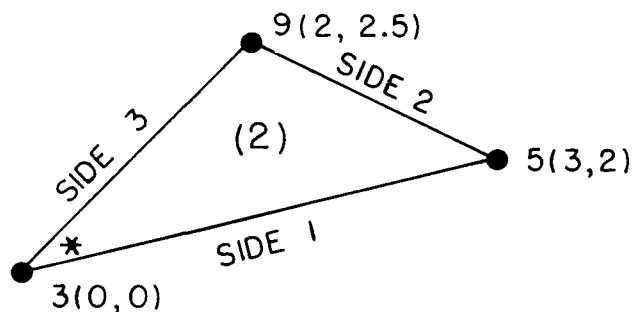
Element data for each element is input in the following order:

- (1) NE - element number
- (2) ND - three node numbers counterclockwise from first node.
- (3) MPSEL - material property set number for the element.
- (4) X1, Y1, X2, Y2, X3, Y3 - X and Y coordinates counterclockwise from first node.

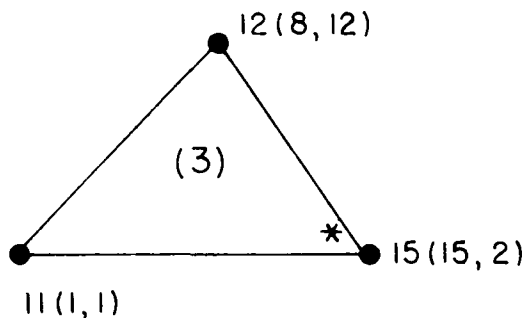
Node numbers are input in order counterclockwise from an arbitrarily selected starting node. It is convenient to record which node is being used for the first node by using an asterisk (\*) at that node. Nodal coordinates are also input following a counterclockwise pattern: X1, Y1 for starting node; X2, Y2 for next node counterclockwise on the element; X3, Y3 for the final node. When element side numbering is necessary, as in heat transfer application, the sides are numbered counterclockwise from the starting node with side one being the side between the starting node and the second node.

The parameter MPSEL is used to select the appropriate set of material properties from the material properties table established.

Sample Elements:



NE	ND			MPSEL	X1	Y1	X2	Y2	X3	Y3
2	3	5	9	1	0.0	0.0	3.0	2.0	2.0	2.5



NE	ND			MPSEL	X1	Y1	X2	Y2	X3	Y3
3	15	12	11	1	15.0	2.0	8.0	12.0	1.0	1.0

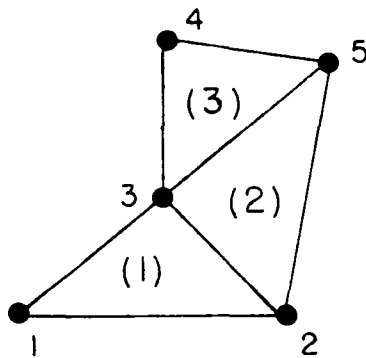
### 1.6.2 Control Parameters for Element Data

Control parameters for element data for programs using linear triangular elements establishes the size of the problem and the number of material properties sets to be used. The element data control parameters for these cases include, by order of input:

- NE - number of elements
- NP - number of nodes
- NBW - bandwidth\* = maximum difference in node numbers within any element + 1.
- NMPSET - number of material properties sets to be input

If automatic grid generation is used, these parameters are input to the application program by the grid generation program. If the user inputs the element data, a card including the Control Parameters for Element Data must be input with the parameters listed above.

Sample bandwidth determination



$$\text{Bandwidth} = (5-2) + 1 = 4$$

### 1.7 MATERIAL PROPERTIES TABLES

A common procedure has been established for input of material properties for all application programs in this package. A table of sets of material properties is established and stored. Material properties for a region or for an element are then selected by referring to the set number of the desired set of material properties. Table 1.2 shows what properties are included in a material properties sets for each application program. The table is sized for five parameters in each material properties set. Some applications do not require five parameters. Zero values must be used to specify the unused parameters in those material properties sets in application programs not requiring all five parameters. Up to twenty sets of material properties may be input. Each set must be input on a separate card. The number of material properties sets to be input is controlled by the parameter NMPSET which is input either through the automatic grid generation program or within the element data control parameters. Sets are numbered by order of their input.

---

\*Note this is bandwidth assuming one degree of freedom (unknown) at a node. Application programs automatically adjust bandwidth for number of unknowns at a node.

Table 1.2. Material Property Sets

Application Program	Material property (1)	Material property (2)	Material property (3)	Material property (4)	Material property (5)
2-D Stress analysis	Young's modules (EM)	Poisson's ratio (PR)	Coef. of thermal expansion (ALPHA)	Equilibrium temperature of body (TEMP)	Thickness of body (T)
2-D Heat transfer	Conductivity X - direction (KXX)	Conductivity Y - direction (KYY)	Convection coefficient (H)	Fluid temperature (TINF)	N/A (0.0)
Groundwater	Transmissivity X - direction (TXX)	Transmissivity Y - direction (TYY)	N/A (0.0)	N/A (0.0)	N/A (0.0)
Torsion	Shear modules (G)	Shaft length (SL)	Number axis of sym. *2 (PCT)	N/A (0.0)	N/A (0.0)
2-D Transient heat transfer	Conductivity X - direction (KXX)	Conductivity Y - direction (KYY)	Convection coefficient (H)	Fluid temperature (TINF)	Specific heat*density (CRHO)
Simple truss	Elastic modulus (EM)	Coefficient of thermal expansion (ALPHA)	Cross-section area (AREA)	N/A (0.0)	N/A (0.0)

Consider an example of input of two material properties sets for 2-D Heat Transfer.

Set 1	Set 2
KXX = 350.0	KXX = 600.0
KYY = 300.0	KYY = 500.0
H = 0.1	H = 0.5
TINF = 50.0	TINF = 60.0

The material properties input would appear as

Set 2	600.0, 500.0, 0.50, 60.0, 0.0
Set 1	350.0, 300.0, 0.1, 50.0, 0.0

## 2. AUTOMATIC GRID GENERATION PROGRAM

### 2.1 PURPOSE

This program can be used to automatically generate element data for application programs which use only the three-node linear triangular elements (see Table 1.1). Incorrect element data is one of the major sources of error when running finite element programs. Preparation of element data is time-consuming and tedious without the aid of a program such as this.

This program is intended to help the finite element user by reduction of time and errors in element data preparation. The output of this program, element data (element numbers, node numbers, material properties index, nodal coordinates), is printed out, can be graphically displayed, is automatically stored for use by an application and plotting programs, and can be punched on cards.<sup>1</sup>

This program is capable of generating up to 300 elements.

### 2.2 INPUT REGION DESCRIPTION

The input region for the program is the quadratic quadrilateral. This region is quite versatile; it can be used as rectangle, general quadrilateral, or as a triangle (see Fig. 2.1). Two sides of the quadrilateral are used to define one side of a triangular region.

The eight nodes that define a region are numbered counterclockwise from an arbitrarily-selected corner node. The sides are also numbered counterclockwise from the same starting point. (It is convenient to mark the starting point with an \*.) See Fig. 2.1 for sample regions.

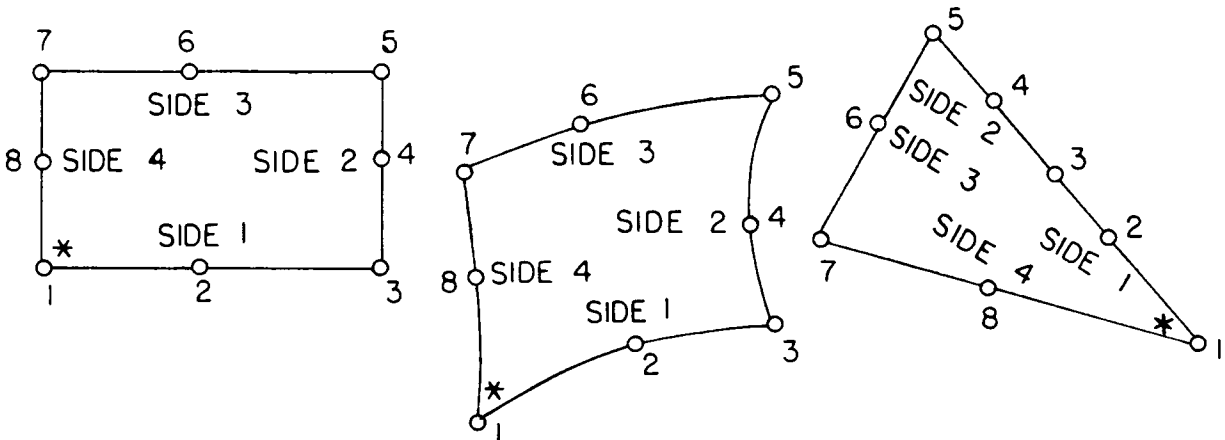


Fig. 2.1. Quadratic Quadrilateral for Grid Generation Input.

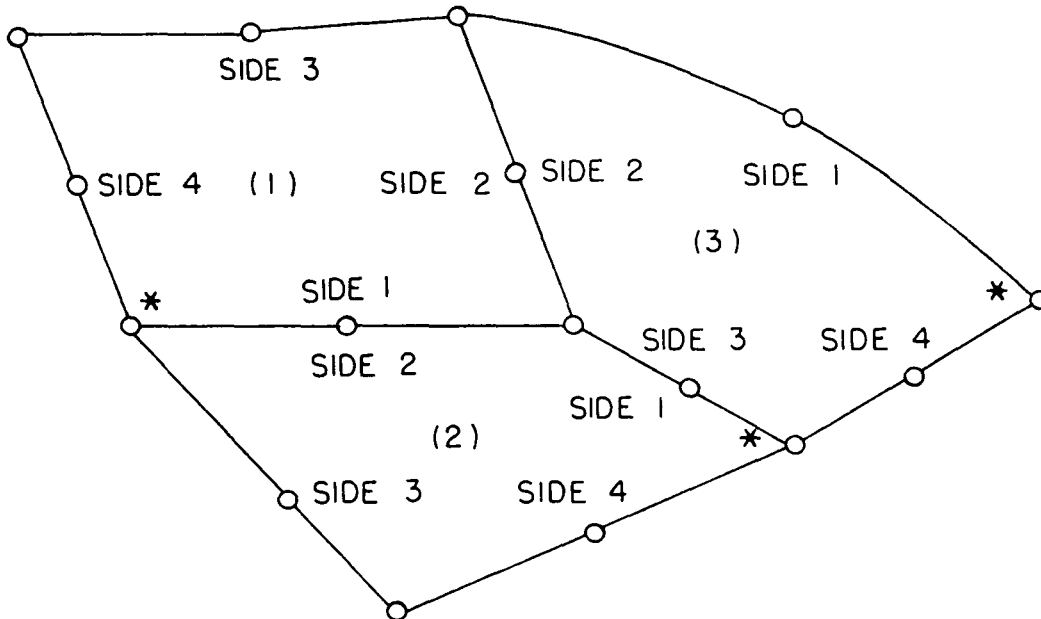
---

<sup>1/</sup> Further information on grid generation and more details on program operation can be found in Segerlind, 1976. Applied Finite Element Analysis, John Wiley & Sons, Inc., New York.

### 2.3 REGION CONNECTIVITY

Generally modeling the body or domain requires several input quadrilateral regions connected to one another along one or more sides. Having common boundaries between two regions requires that certain information be provided to insure that the nodes on the common boundaries have the same numbers regardless of which region is being considered. This is done by inputting "connectivity data." The connectivity data conveys to the computer how the regions are connected.

The connectivity data for a single region consists of four numbers, one for each side. Each value is the number of the region connected to a particular side. The determination of the connectivity data is probably best illustrated through an example as in the region shown below.



The region numbers, (1) through (3), and starting nodes have been assigned. Therefore, the side number for each region is known. The connectivity data for the 3-region body is as follows:

Region	Side			
	1	2	3	4
(1)	2	3	0	0
(2)	3	1	0	0
(3)	0	1	2	0

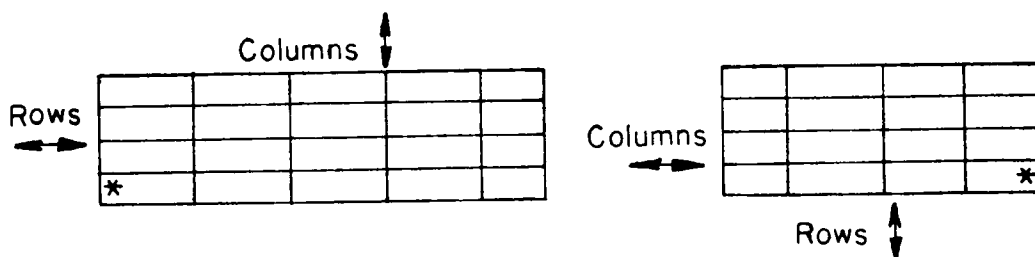
The first line of the data reads, side 1 of region 1 is connected to region 2, and side 2 of region 1 is connected to region 3. The zero indicates sides which are not connected to another region.

## 2.4 MATERIAL PROPERTIES SETS

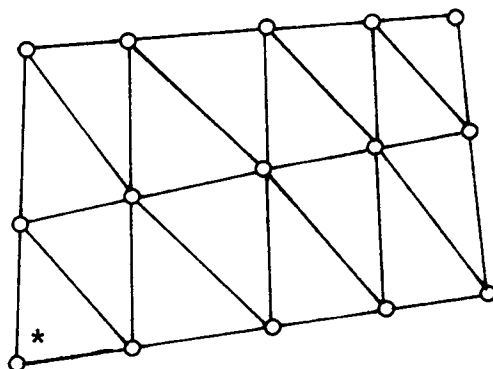
A material properties set consists of five numbers specifying the material properties needed for the application program to be used. Up to twenty material properties sets may be used. Each input region for grid generation is given one of the material properties sets. Unused numbers within a set should be input as zeros. Table 1.2 shows the material properties sets for each of the application programs. Each application program write-up also discusses the material properties sets for its specific application.

## 2.5. REGION DATA

A region data card is read for each region. The information on the card includes the input region number (NRG), the material property set desired (MPS), the number of rows of nodes (NROWS) and columns of nodes (NCOL) desired and the 8 node numbers used to define the quadrilateral. Rows of nodes and columns of nodes are defined relative to the starting node as shown below:



The program will subdivide the region into  $(NROWS - 1) * (NCOL - 1) * 2$  elements. For example, for  $NROWS=3$  and  $NCOL=5$ , the region would be subdivided into 16 elements as shown below.



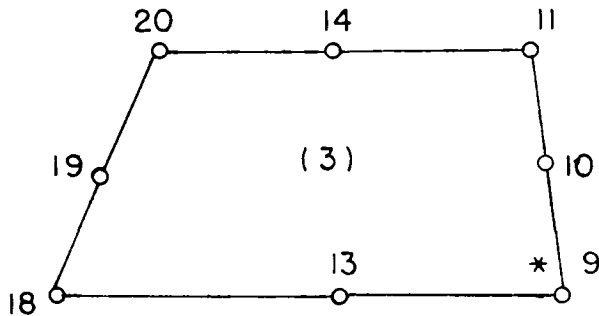
Number of rows of nodes (NROWS) and columns of nodes (NCOL) are limited to a maximum of twelve for any one input region. Total number of elements generated for all input regions is limited to three hundred elements.



Care must be taken in specifying row and columns so that continuity is maintained between regions, i.e., adjoining regions must have the same number of nodes on the common boundary.



Assume the region shown below was extracted from a body being discretized. The region data for the region is shown, assuming that a subdivision of 4 rows of nodes and 3 columns of nodes is desired.



NRG-Region Number	MPS-matl. Prop. set.	NROWS-4 rows of nodes	NCOL-3 cols. of nodes	Node number ccw from starting node							
3,	1,	4,	3,	9,	10,	11,	14,	20,	19,	18,	13

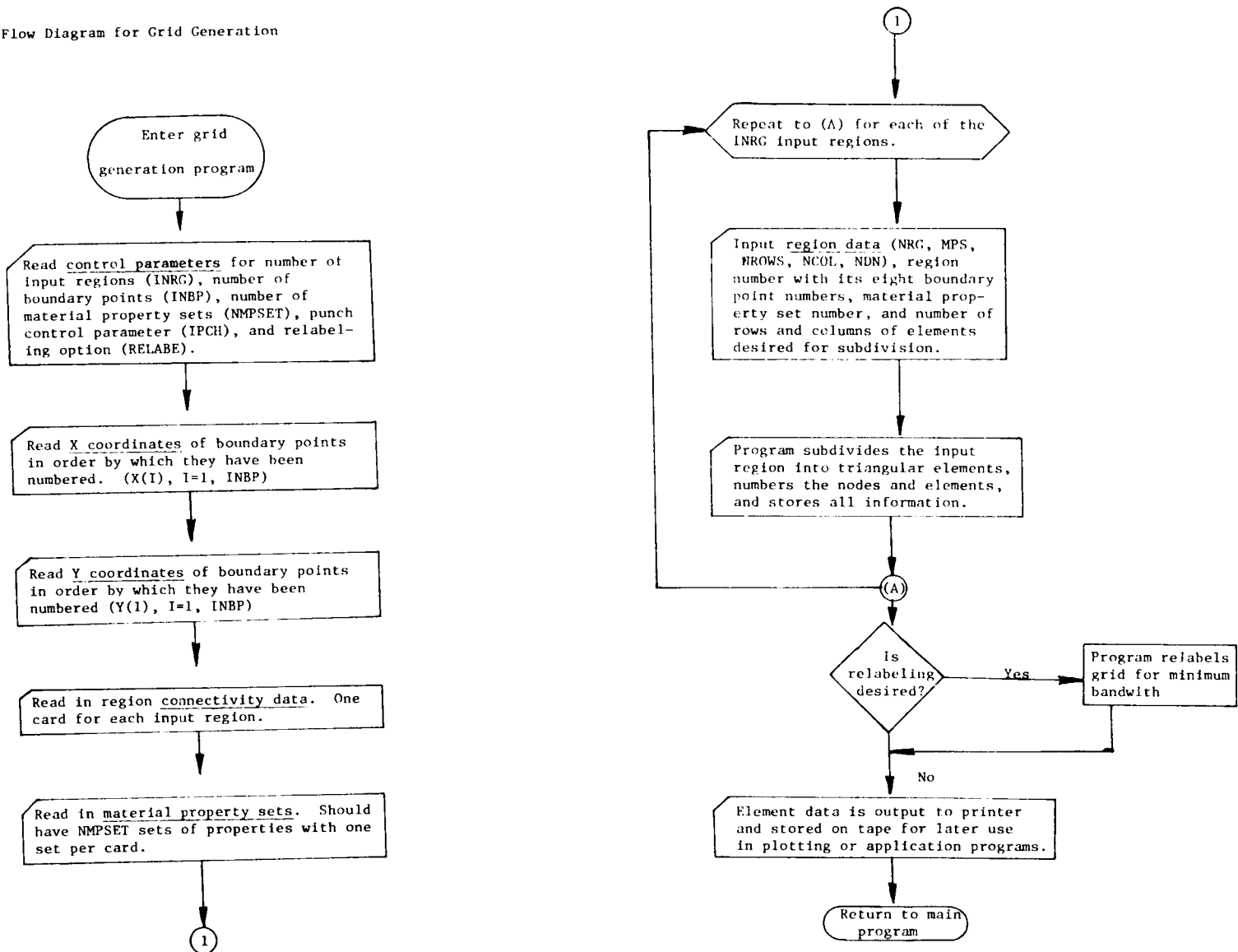
## 2.6 CONTROL PARAMETERS

Control parameters read in the grid program consist of 1) the number of input regions to be used (INRG), number of boundary points for defining the region (INBP), number of material properties sets to be input (NMPSET), punch control option (IPCH=0 no punch, =1 punch), and relabeling option control (RELABEL=0 relabel, =1 no relabel).

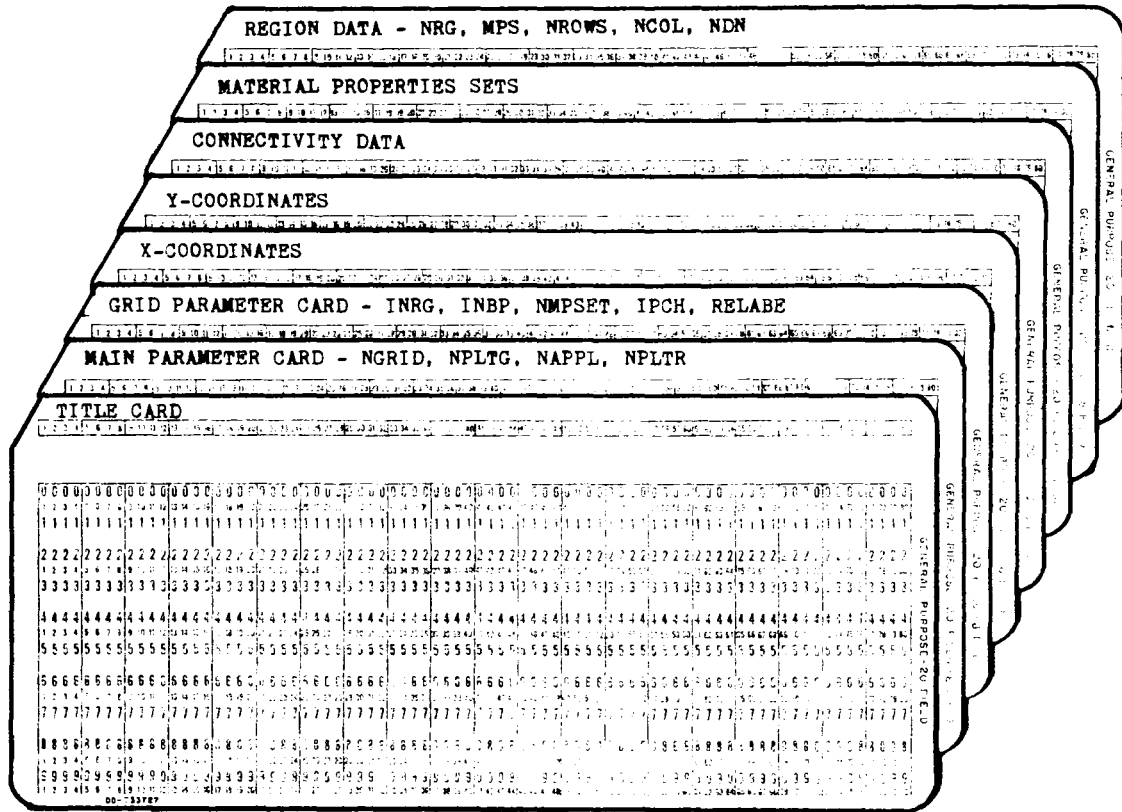
Sample Grid Parameter Card

INRG - No. of Regions	INBP - No. Bndry. Points	NMPSET - No. Matl. Sets	IPCH	RELABEL
3,	16,	2,	0,	0

2.7 General Flow Diagram for Grid Generation



2.8 SUMMARY OF INPUT AND ORDER OF INPUT



2.9 DEFINITION AND LIMITS FOR INPUT VARIABLES

- INBP = total number of boundary nodes  
 $\leq 100$
- INRG = total number of input regions to be subdivided  
 $\leq 20$
- IPCH = control for punched out of element data  
 IF IPCH = 0 no punch  
 = 1 punch output given
- JT = array of connectivity data  
 = JT(INRG,4)

JT(A,B)	Side number B			
	1	2	3	4
1	X			
Region No. A	2			
	3			
	⋮			
	INRG			

Side B of region number A is connected to region number X.  
 (See connectivity of data description and example problem).

MTPROP = Material properties arranged by sets of five values  
    < 20 sets (see material properties description Section 1.7).

MPS = Material properties set number corresponding to a particular region

NCOL = Number of columns of nodes for subdivision  
    < 12

NDN(I) = 8 node numbers used to define an input region, I = 1 to 8

NMPSET = Number of material properties sets  
    < 20

NRG = Region number

NROWS = Number rows of nodes for subdivision  
    < 12

RELABE = Control parameter for relabel for minimum bandwidth  
    IF RELABE = 1 the nodes are not relabeled  
    = 0 the nodes are relabeled

XP,YP = The global coordinators of input boundary nodes

## 2.10 SAMPLE PROBLEM

The region shown in Fig. 2.2(a) is to be discretized (divided into elements) using automatic grid generation. Figure 2.2(b) shows the body divided into three quadratic quadrilaterals for input to the grid program. Each boundary node has been numbered and the starting node for each element has been designated. Only grid generation is desired for this problem. Figure 2.3 shows the input used for this problem. It is assumed that regions (1) and (3) are to be divided into 3 rows and 2 columns of nodes, and that region (2) is to be divided into 3 rows and 3 columns of nodes.

Output of the grid generation program is shown in Figure 2.4 followed by a sketch of the discretized region, Figure 2.5.



Fig. 2.4 Output for Grid Generation Example

GRID GENERATION EXAMPLE

THE GRID WILL BE AUTOMATICALLY GENERATED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	0
2	1.00	0
3	2.00	0
4	3.00	0
5	4.00	0
6	5.00	0
7	6.00	0
8	5.50	.50
9	5.00	1.00
10	4.50	1.50
11	4.00	2.00
12	4.00	1.00
13	3.00	2.00
14	2.00	2.00
15	2.00	1.00
16	1.00	2.00
17	0	2.00
18	1.00	1.00

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	2	0	0
2	0	3	0	1
3	0	0	0	2

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
1	0	0	0	0	0

REGION DATA

REGION	ROWS	COLS	MATERIAL		INPUT REGION NODE NO.						
			SET								
1	3	2	1	1	2	3	15	14	16	17	18
2	3	3	1	3	4	5	12	11	13	14	15
3	3	2	1	5	6	7	8	9	10	11	12

NODE NUMBERS OF SUBDIVIDED REGION

1	1	3
	2	4
	5	6
2	3	7 10
	4	8 11
	6	9 12
3	10	13
	11	14
	12	15

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	2 4 3	1.0000	1.0000	2.0000	1.0000	2.0000	2.0000
2	2 3 1	1.0000	1.0000	2.0000	2.0000	0	2.0000
3	5 6 2	0	0	2.0000	0	1.0000	1.0000
4	6 4 2	2.0000	0	2.0000	1.0000	1.0000	1.0000

ELEMENT DATA, REGION 2 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
5	4 8 7	2.0000	1.0000	3.0000	1.0000	3.0000	2.0000
6	4 7 3	2.0000	1.0000	3.0000	2.0000	2.0000	2.0000
7	8 11 10	3.0000	1.0000	4.0000	1.0000	4.0000	2.0000
8	8 10 7	3.0000	1.0000	4.0000	2.0000	3.0000	2.0000
9	6 9 8	2.0000	0	3.0000	0	3.0000	1.0000
10	6 8 4	2.0000	0	3.0000	1.0000	2.0000	1.0000
11	9 12 11	3.0000	0	4.0000	0	4.0000	1.0000
12	9 11 8	3.0000	0	4.0000	1.0000	3.0000	1.0000

ELEMENT DATA, REGION 3 MAT. PROP. SET = 1

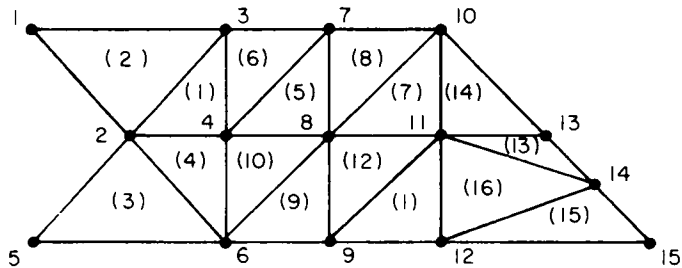
NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
13	11 14 13	4.0000	1.0000	5.5000	.5000	5.0000	1.0000
14	11 13 10	4.0000	1.0000	5.0000	1.0000	4.0000	2.0000
15	12 15 14	4.0000	0	6.0000	0	5.5000	.5000
16	12 14 11	4.0000	0	5.5000	.5000	4.0000	1.0000

NUMBER OF ELEMENTS = NE = 16

NUMBER OF NODES = NP = 15

BANDWIDTH = NBW = 5

Fig. 2.5 Discretized Region from Grid Generation



### 3. APPLICATION PROGRAMS

#### 3.1 TWO-DIMENSIONAL ELASTICITY

##### 3.1.1 Introduction

This program can be used to analyze two-dimensional bodies subjected to boundary forces and/or displacements. The user must specify if the problem under consideration is a plane stress case or a plane strain case. The program uses simple linear triangular elements only. Automatic grid generation can be used with the program. Thermal stresses may be included by inputting temperature values for the elements.

##### 3.1.2 Material Properties Description

The material properties sets for this program include, in the following order

EM - Elastic Modulus  
PR - Poisson's ratio  
ALPHA - Coefficient of thermal expansion  
TEMP - Initial steady-state, temperature for body  
T - Thickness of the element, must equal 1.0 for plane stress case

Any compatible set of units may be used. Up to twenty sets of material properties may be input.

##### 3.1.3 Control Parameters for Two-Dimensional Elasticity

Control parameters read in during the application program section for 2-D elasticity are

- 1) Control for problem type (ITYPE)  
ITYPE = 0 - plane stress  
= 1 - plane strain
- 2) Control parameter for input of element temperatures (ITEMP)  
ITEMP = 0 - no temperature input  
= 1 - temperature input for each element. See Section 3.1.4.

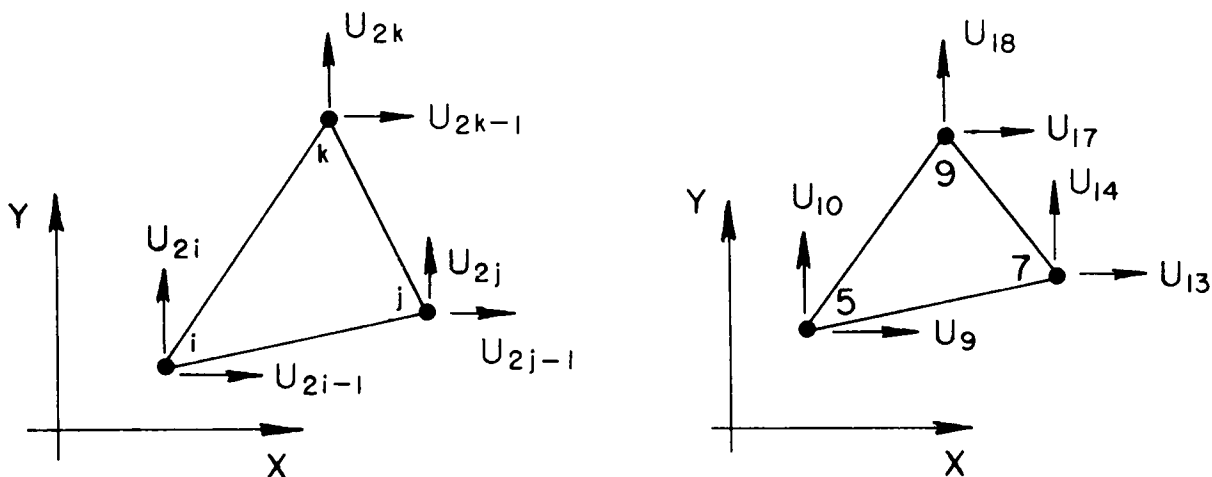
##### 3.1.4 Element Temperatures

Average temperature for each element may be input for calculation of thermal stresses. This requires specifying the control parameter ITEMP = 1 and input the average element temperatures ordered by element number. The temperature difference between the initial steady-state temperature (TEMP) and the average element temperature is used to calculate the thermal stress.

##### 3.1.5 Vector Components

Two-dimensional elasticity is complicated by having a vector unknown, displacement. For both the plane strain and plane stress cases the displacement is expressed as the sum of the component in the X and Y coordinate directions. The figures below demonstrate how the components at each node are expressed. At each node two degrees of freedom (global degrees of freedom) are used. They are two times the node number for the Y-component and two times node number minus one for the X-component.



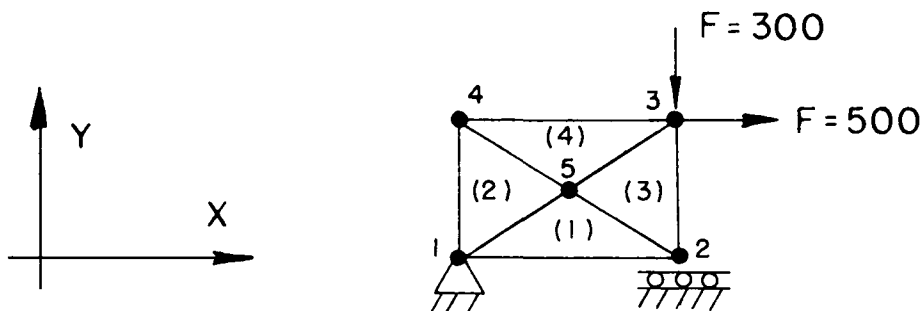


Input of force values and known nodal displacements are also keyed to the appropriate global degree of freedom.

### 3.1.6 Boundary Values

Boundary conditions are input in two sections. The first is nodal forces. The second is known displacements. Both are input by the global degrees of freedom at each node and the value of the vector component parallel to the direction of that degree of freedom.<sup>1</sup> Each card must have six sets consisting of global degree of freedom and associated values. If a complete card is not needed, the remaining sets are specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zeros (12 zeros) must follow to terminate that section of data input.

For the simple region shown below



the nodal forces would be

6, -300.0, 5, 500.0, 0, 0.0, 0, 0.0, 0, 0.0, 0, 0.0
---

<sup>1</sup>Global degrees of freedom at each node refers to nodes after grid generation when automatic grid generation is used. A preliminary run of auto-grid generation may be necessary to determine nodes for boundary conditions.

and the known nodal values (displacements).

1, 0.0, 2, 0.0, 4, 0.0, 0, 0.0, 0, 0.0, 0, 0.0

Note sign convention has been established such that forces and displacements in the positive coordinate direction are assumed positive. Those in the opposite direction are assumed negative.

### 3.1.7 Input Description

Table 3.1.1 shows the inputs required for two possible cases using 2-D elasticity. The first, auto-grid case, assumes the use of the automatic grid generation program. The second, non-auto-grid case, assumes the user will input the element data from cards.

### 3.1.8 Sample Problem

The body for a sample plane stresses problem is shown in Fig. 3.1.1. The body is a thin plate fixed at one end and subjected to a pressure force along a portion of the top side. Fig. 3.1.2 shows the region subdivided for automatic grid generation. Fig. 3.1.3 shows the input using automatic grid generation for the problem. Figure 3.1.4 shows the grid generation by the program. Figure 3.1.5 shows the output of the program.

Figure 3.1.6 shows the input for the same grid as shown in Figure 3.1.4 but not using automatic grid generation.

Figure 3.1.1 Example Problem 2-D Elasticity-Plane Stress

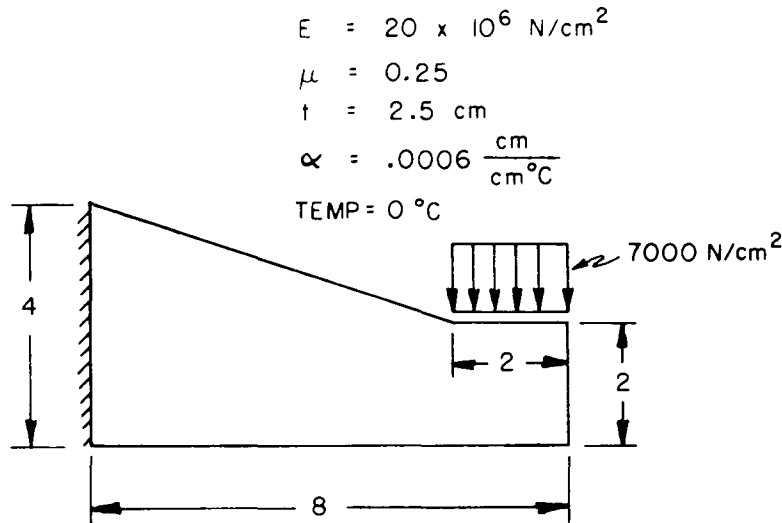


Table 3.1.1 Program Input for 2-D Elasticity

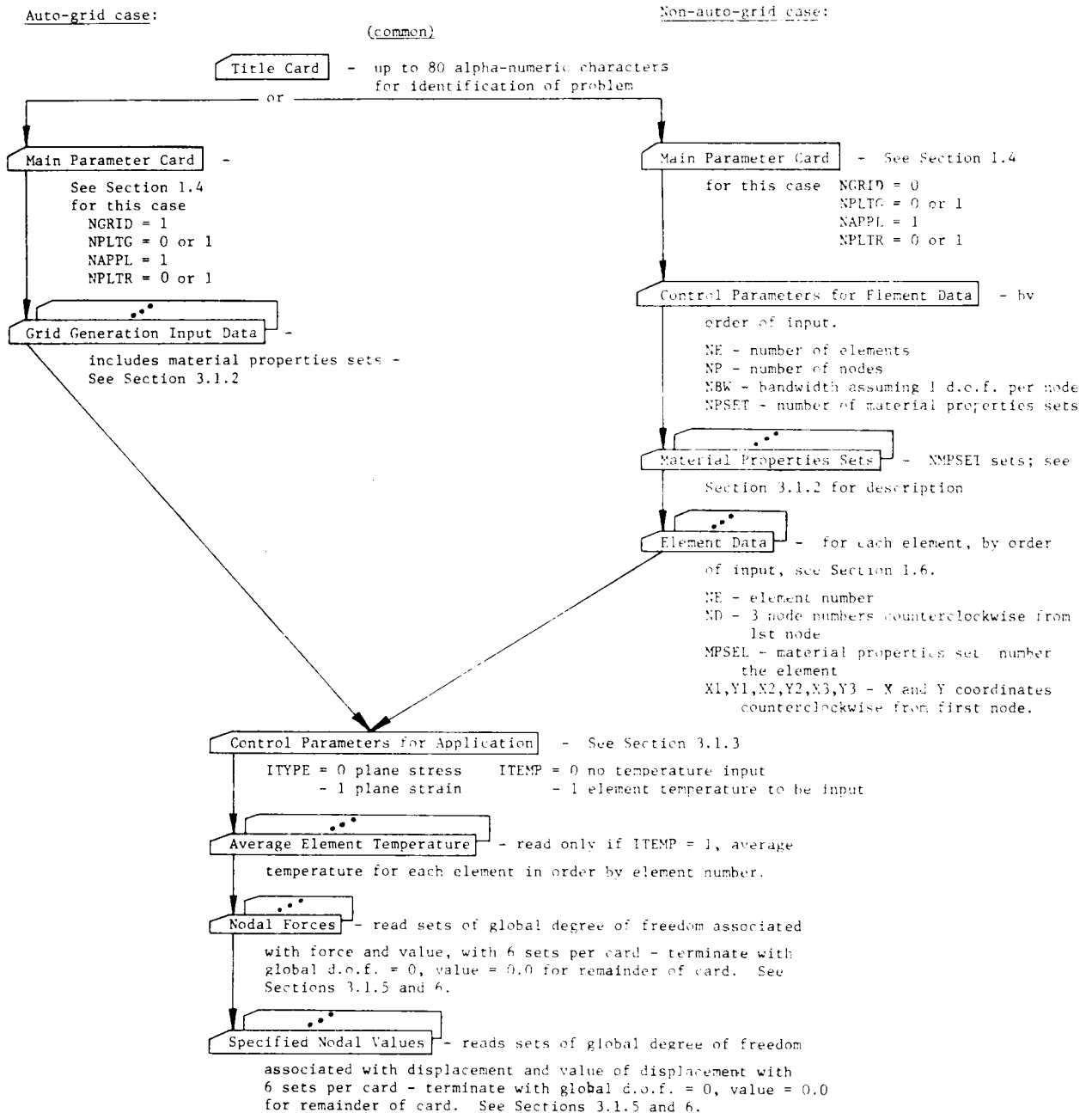


Fig. 3.1.2 Regions for Auto-Grid Generation

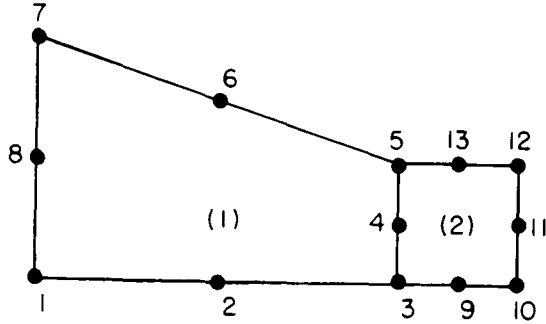


Fig. 3.1.3 Sample Input with Auto-Grid Generation

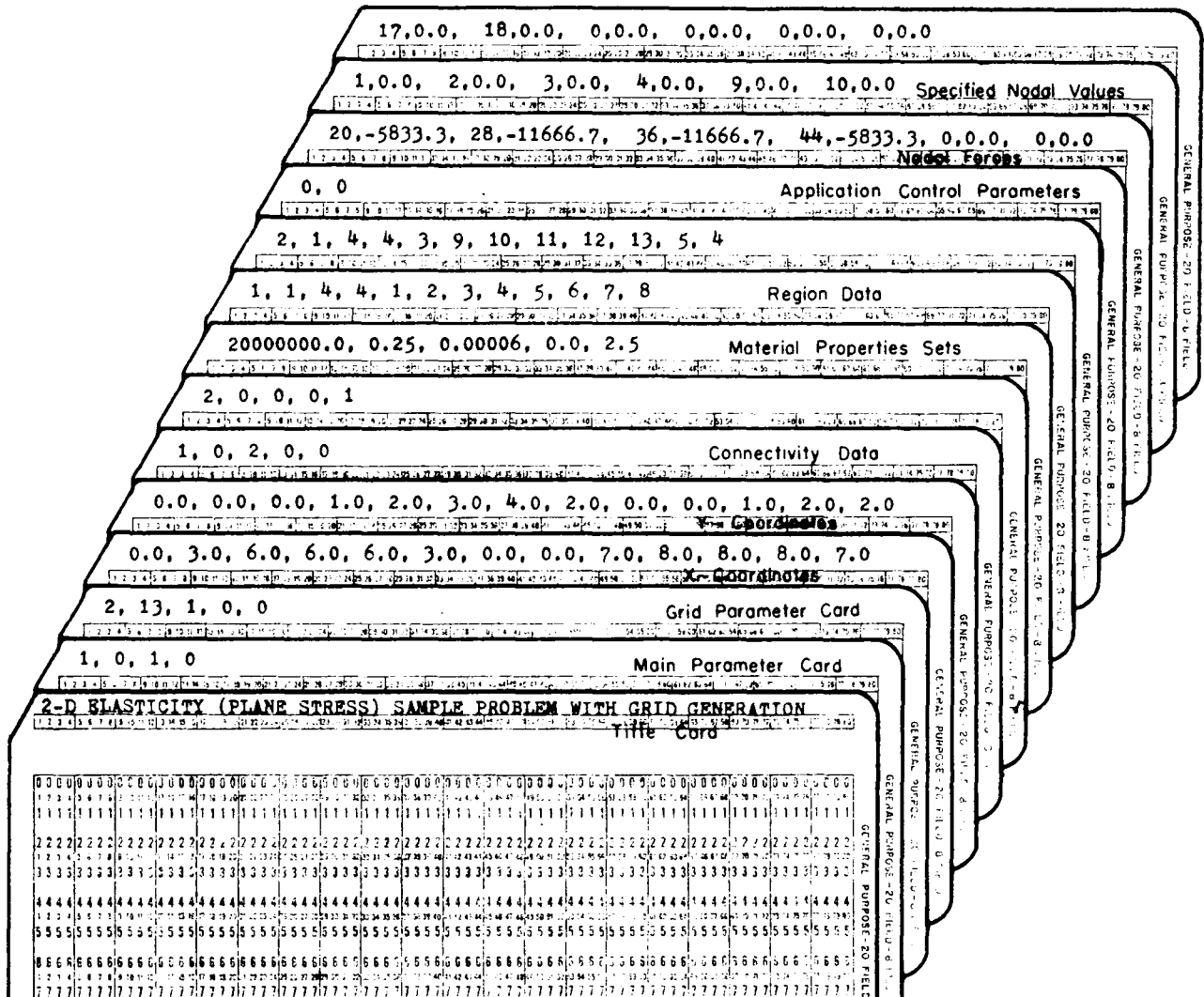


Fig. 3.1.4 Grid Generated for Elasticity Problem

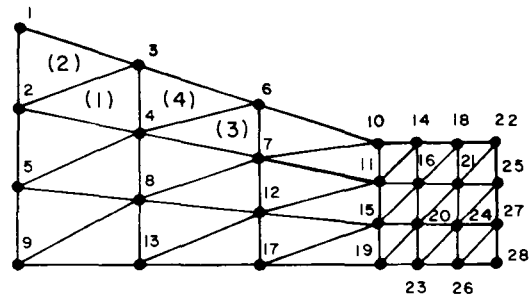


Fig. 3.1.5 Output for Elasticity Example with Grid Generation

2-D ELASTICITY (PLANE STRESS) SAMPLE PROBLEM WITH GRID GENERATION

THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM STRESS ANALYSIS WILL BE SOLVED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	0
2	3.00	0
3	6.00	0
4	6.00	1.00
5	6.00	2.00
6	3.00	3.00
7	0	4.00
8	0	2.00
9	7.00	0
10	8.00	0
11	8.00	1.00
12	8.00	2.00
13	7.00	2.00

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	2	0	0
2	0	0	0	1

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
	EM	PR	ALPHA	TEMP	T
1	.2000E+08	.2500E+00	.6000E-04	0	.2500E+01

REGION DATA

REGION	ROWS	COLS	MATERIAL			INPUT REGION NODE NO.							
			SET			1	2	3	4	5	6	7	8
1	4	4	1			1	2	3	4	5	6	7	8
2	4	4	1			3	9	10	11	12	13	5	4

NODE NUMBERS OF SUBDIVIDED REGION

1
1 3 6 10
2 4 7 11
5 8 12 15
9 13 17 19
2
10 14 18 22
11 16 21 25
15 20 24 27
19 23 26 28

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	2 4 3	0	2.6667	2.0000	2.2222	2.0000	3.3333
2	2 3 1	0	2.6667	2.0000	3.3333	0	4.0000
3	4 7 6	2.0000	2.2222	4.0000	1.7778	4.0000	2.6667
4	4 6 3	2.0000	2.2222	4.0000	2.6667	2.0000	3.3333
5	7 11 10	4.0000	1.7778	6.0000	1.3333	6.0000	2.0000
6	7 10 6	4.0000	1.7778	6.0000	2.0000	4.0000	2.6667
7	5 8 4	0	1.3333	2.0000	1.1111	2.0000	2.2222
8	5 4 2	0	1.3333	2.0000	2.2222	0	2.6667
9	8 12 7	2.0000	1.1111	4.0000	.8889	4.0000	1.7778
10	8 7 4	2.0000	1.1111	4.0000	1.7778	2.0000	2.2222
11	12 15 11	4.0000	.8889	6.0000	.6667	6.0000	1.3333
12	12 11 7	4.0000	.8889	6.0000	1.3333	4.0000	1.7778
13	9 13 8	0	0	2.0000	0	2.0000	1.1111
14	9 8 5	0	0	2.0000	1.1111	0	1.3333
15	13 17 12	2.0000	0	4.0000	0	4.0000	.8889
16	13 12 8	2.0000	0	4.0000	.8889	2.0000	1.1111
17	17 19 15	4.0000	0	6.0000	0	6.0000	.6667
18	17 15 12	4.0000	0	6.0000	.6667	4.0000	.8889

ELEMENT DATA, REGION 2 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
19	11 16 14	6.0000	1.3333	6.6667	1.3333	6.6667	2.0000
20	11 14 10	6.0000	1.3333	6.6667	2.0000	6.0000	2.0000
21	16 21 18	6.6667	1.3333	7.3333	1.3333	7.3333	2.0000
22	16 18 14	6.6667	1.3333	7.3333	2.0000	6.6667	2.0000
23	21 25 22	7.3333	1.3333	8.0000	1.3333	8.0000	2.0000
24	21 22 18	7.3333	1.3333	8.0000	2.0000	7.3333	2.0000
25	15 20 16	6.0000	.6667	6.6667	.6667	6.6667	1.3333
26	15 16 11	6.0000	.6667	6.6667	1.3333	6.0000	1.3333
27	20 24 21	6.6667	.6667	7.3333	.6667	7.3333	1.3333
28	20 21 16	6.6667	.6667	7.3333	1.3333	6.6667	1.3333
29	24 27 25	7.3333	.6667	8.0000	.6667	8.0000	1.3333
30	24 25 21	7.3333	.6667	8.0000	1.3333	7.3333	1.3333
31	19 23 20	6.0000	0	6.6667	0	6.6667	.6667
32	19 20 15	6.0000	0	6.6667	.6667	6.0000	.6667
33	23 26 24	6.6667	0	7.3333	0	7.3333	.6667
34	23 24 20	6.6667	0	7.3333	.6667	6.6667	.6667
35	26 28 27	7.3333	0	8.0000	0	8.0000	.6667
36	26 27 24	7.3333	0	8.0000	.6667	7.3333	.6667

NUMBER OF ELEMENTS = NE = 36

NUMBER OF NODES = NP = 28

BANDWIDTH = NBW = 6

BOUNDARY VALUES

NODAL FORCES

20	-.58333E+04	28	-.11667E+05	36	-.11667E+05	44	-.58333E+04
----	-------------	----	-------------	----	-------------	----	-------------

PRESCRIBED NODAL VALUES

1	0	2	0	3	0	4	0	9	0	10	0
17	0	18	0								

NODAL VALUES

1	0	2	0	3	0	4	0	5	.19287E-02	6	-.22636E-02
7	.67833E-03	8	-.21403E-02	9	0	10	0	11	.29784E-02	12	-.75620E-02
13	.45845E-03	14	-.75529E-02	15	-.88486E-03	16	-.21883E-02	17	0	18	0
19	.22085E-02	20	-.16345E-01	21	-.71078E-03	22	-.16278E-01	23	-.21960E-02	24	-.76640E-02
25	-.27681E-02	26	-.24751E-02	27	.25978E-02	28	-.20011E-01	29	-.35855E-02	30	-.16320E-01
31	-.61771E-03	32	-.19840E-01	33	-.50472E-02	34	-.78803E-02	35	.27642E-02	36	-.23543E-01
37	-.65080E-02	38	-.16445E-01	39	-.36910E-02	40	-.19778E-01	41	-.53880E-03	42	-.23352E-01
43	.28298E-02	44	-.26920E-01	45	-.68493E-02	46	-.19819E-01	47	-.37280E-02	48	-.23256E-01
49	-.48820E-03	50	-.26717E-01	51	-.69783E-02	52	-.23250E-01	53	-.37235E-02	54	-.26609E-01
55	-.69979E-02	56	-.26589E-01								



### 3.2 TWO-DIMENSIONAL HEAT TRANSFER

#### 3.2.1 Introduction

This program calculates the temperature distribution in two-dimensional bodies subjected to either prescribed boundary temperatures or surface convections. The body may have internal heat sources or sinks. The program uses only simple linear triangular elements. Automatic grid generation can be used with this program.

#### 3.2.2 Material Properties Description

The material property sets for this program include, in the following order,

- KXX - Thermal conductivity in the X-direction
- KYY - Thermal conductivity in the Y-direction
- H - Surface convection coefficient
- TINF - Fluid temperature at a distance from the convection surfaces
- 1 Dummy Variable - = 0.0.

Any compatible set of units may be used. Up to twenty sets of material properties may be input.

#### 3.2.3 Convection Data

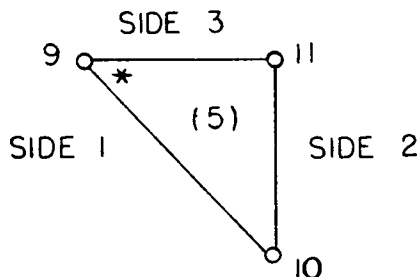
Data establishing sides of elements that are surfaces with convection may be input. The element number and the number of the side or sides having convection must be input for each element which has surface convection. Sides are numbered 1, 2, and 3, counterclockwise from the first input node for the element.

A maximum of two sides may have convection, therefore, the data is input as:

element no., 1<sup>st</sup> side for convection, 2<sup>nd</sup> side for convection

If only one side has convection, a zero value is entered in the second side for convection location.

Assuming that element 5 shown below has surface convection on side 2 and side 3,



\*designates first input node



the convection data appears as

5, 2, 3

This data set must be terminated with a card with element number and side numbers equal to zero, i.e.,

0, 0, 0

IF NO CONVECTION OCCURS for the problem, a card with zero values,

0, 0, 0 must be input for the convection data.

Convection data must be ordered by element numbers.

### 3.2.4 Boundary Values

Boundary conditions are input in two sections. First, the nodal forces which in this case would be line heat sources or sinks (heated added considered positive). Second, specified nodal values which would be known nodal temperatures.

Nodal forces and nodal values are both input in sets of node number and associate value with six sets per card.<sup>1</sup> If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zero values (12 zeros) must follow to terminate that section of input data.

Consider the following example if the temperature at node 5, 7, 9, 12, and 14 were all known to be 80.0 degrees, the specified nodal values would read

5, 80.0, 7, 80.0, 9, 80.0, 12, 80.0, 14, 80.0, 0, 0.0

### 3.2.5 Input Description

Table 3.2.1 shows the inputs required for the two possible cases using 2-D Heat Transfer. The first case, auto-grid case, assumes use of the automatic grid generation program. The second case, non-auto-grid case, assumes the user will input the element data from cards. The element data must be ordered by element number for this program.

---

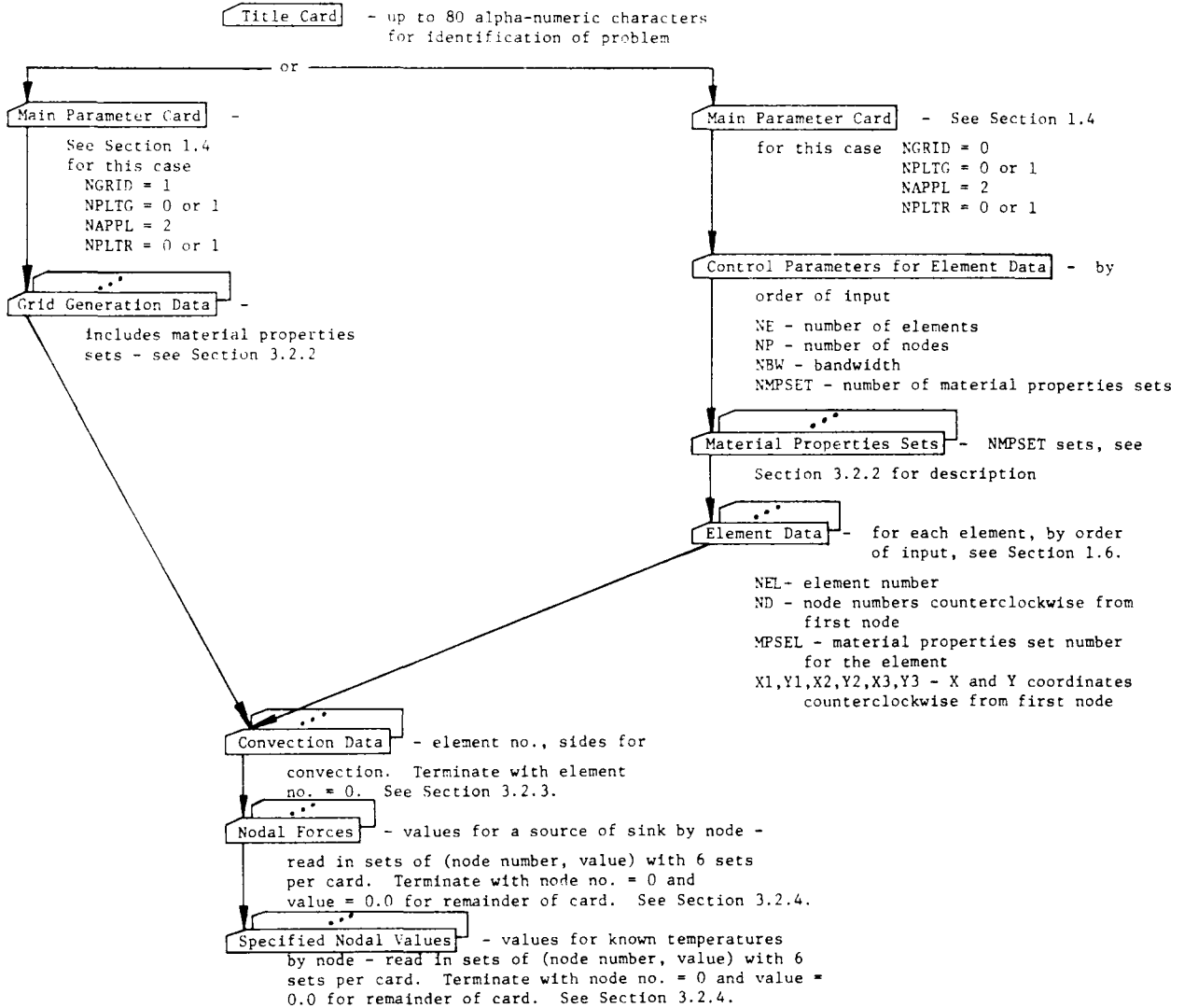
<sup>1/</sup>A preliminary run of the auto-grid generation may be necessary to determine nodes for boundary conditions.

Table 3.2.1 Program Input for 2-D Heat Transfer

Auto-grid case:

(common)

Non-auto-grid case:



### 3.2.6 Sample Problem

The body shown in Figure 3.2.1 has three sides exposed to air for convection and the curved surface at a constant temperature of  $200^{\circ}$ . Due to symmetry only half of the body will be subdivided for study. Figure 3.2.2 shows the region for automatic grid generation. Figure 3.2.3 shows the input using automatic grid generation for the problem. Figure 3.2.4 shows the grid generated for the problem. Figure 3.2.5 shows the output of the problem.

Figure 3.2.6 shows the input for the same grid as shown in Figure 3.2.4 but not using automatic grid generation.

Fig. 3.2.1 Sample Problem for Heat Transfer

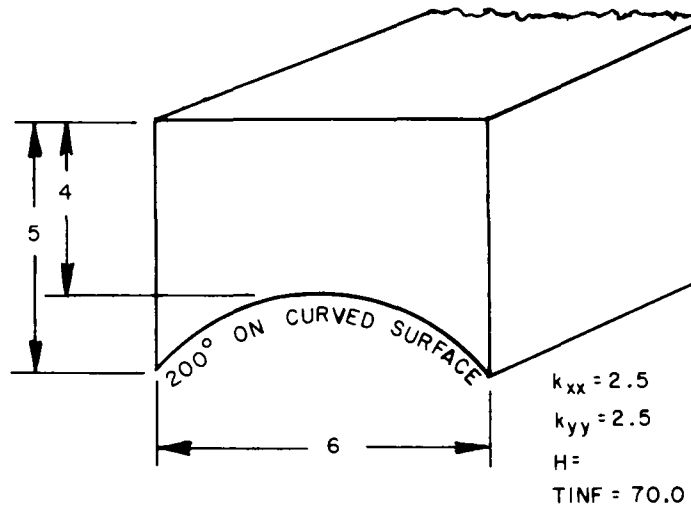


Fig. 3.2.2 Region for Automatic Grid Generation

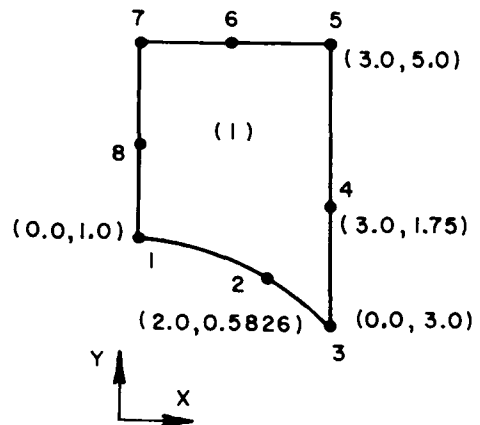


Fig. 3.2.3 Input for Heat Transfer Example with Grid Generation

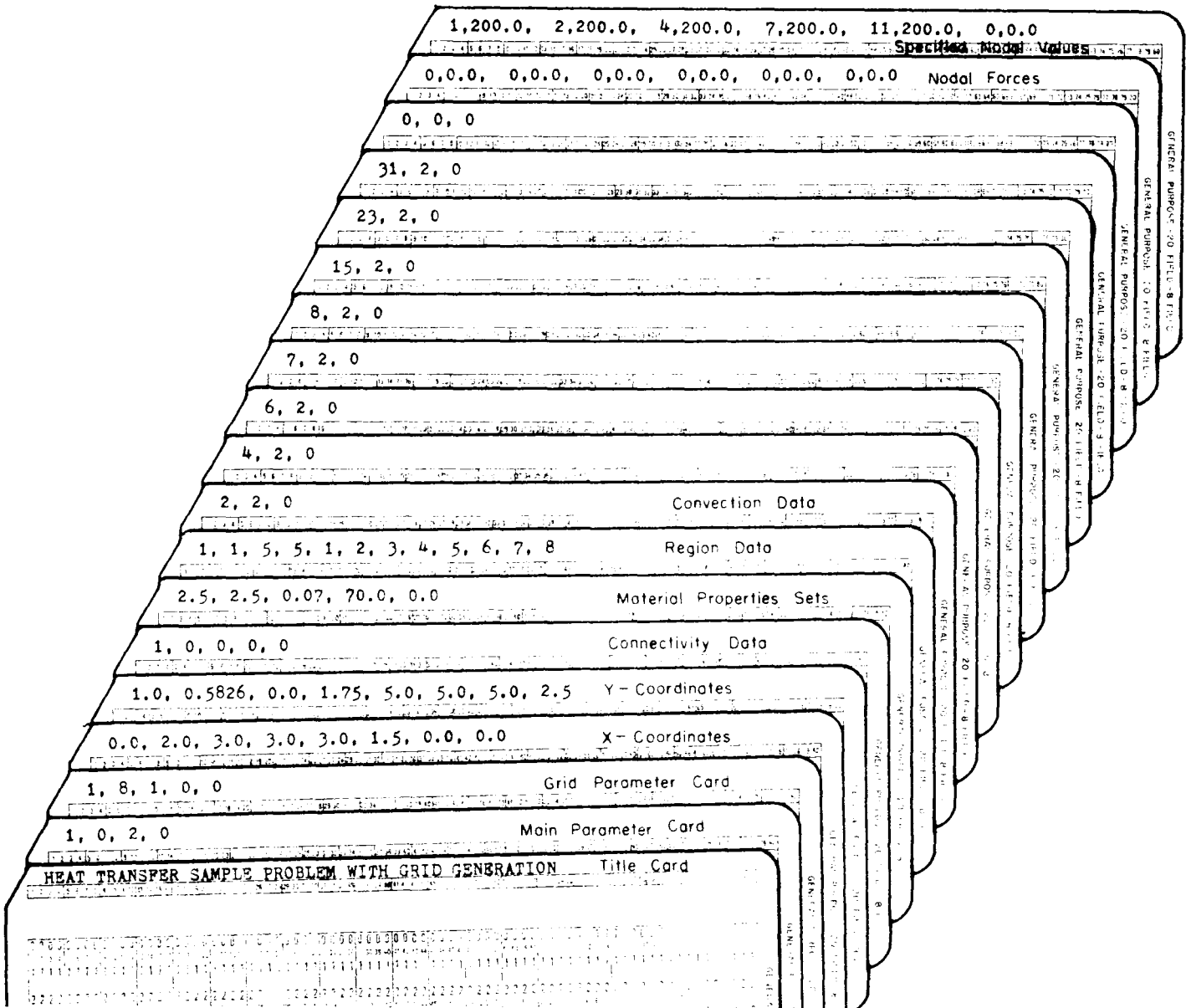


Fig. 3.2.4 Grid Generated for Heat Transfer Example

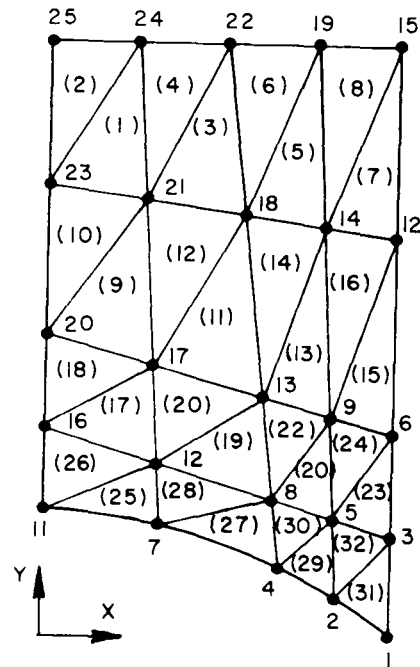


Fig. 3.2.5 Output for Heat Transfer Example with Grid Generation

HEAT TRANSFER SAMPLE PROBLEM WITH GRID GENERATION

APPLICATION PROGRAM HEAT TRANSFER WILL BE SOLVED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	1.00
2	2.00	.58
3	3.00	0
4	3.00	1.75
5	3.00	5.00
6	1.50	5.00
7	0	5.00
8	0	2.50

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	0	0	0

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
	KXX	KYY	H	TINF	N/A
1	.2500E+01	.2500E+01	.7000E-01	.7000E+02	0

REGION DATA

REGION	ROWS	COLS	MATERIAL		INPUT REGION NODE NO.							
			SET		1	2	3	4	5	6	7	8
1	5	5	1		1	2	3	4	5	6	7	8

NODE NUMBERS OF SUBDIVIDED REGION

1	25	24	22	19	15
	23	21	18	14	10
	20	17	13	9	6
	16	12	8	5	3
	11	7	4	2	1

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEI.	NODE NUMBERS				X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	23	21	24	24	0	3.6250	.8438	3.5311	.7500	5.0000
2	23	24	25	25	0	3.6250	.7500	5.0000	0	5.0000
3	21	18	22	22	.8438	3.5311	1.6250	3.4269	1.5000	5.0000
4	21	22	24	24	.8438	3.5311	1.5000	5.0000	.7500	5.0000

5	18	14	19	1.6250	3.4269	2.3438	3.3124	2.2500	5.0000
6	18	19	22	1.6250	3.4269	2.2500	5.0000	1.5000	5.0000
7	14	10	15	2.3438	3.3124	3.0000	3.1875	3.0000	5.0000
8	14	15	19	2.3438	3.3124	3.0000	5.0000	2.2500	5.0000
9	20	17	21	0	2.5000	.9375	2.3435	.8438	3.5311
10	20	21	23	0	2.5000	.8438	3.5311	0	3.6250
11	17	13	18	.9375	2.3435	1.7500	2.1663	1.6250	3.4269
12	17	18	21	.9375	2.3435	1.6250	3.4269	.8438	3.5311
13	13	9	14	1.7500	2.1663	2.4375	1.9685	2.3438	3.3124
14	13	14	18	1.7500	2.1663	2.3438	3.3124	1.6250	3.4269
15	9	6	10	2.4375	1.9685	3.0000	1.7500	3.0000	3.1875
16	9	10	14	2.4375	1.9685	3.0000	3.1875	2.3438	3.3124
17	16	12	17	0	1.6250	1.0313	1.4371	.9375	2.3435
18	16	17	20	0	1.6250	.9375	2.3435	0	2.5000
19	12	8	13	1.0313	1.4371	1.8750	1.2182	1.7500	2.1663
20	12	13	17	1.0313	1.4371	1.7500	2.1663	.9375	2.3435
21	8	5	9	1.8750	1.2182	2.5313	.9683	2.4375	1.9685
22	8	9	13	1.8750	1.2182	2.4375	1.9685	1.7500	2.1663
23	5	3	6	2.5313	.9683	3.0000	.6875	3.0000	1.7500
24	5	6	9	2.5313	.9683	3.0000	1.7500	2.4375	1.9685
25	11	7	12	0	1.0000	1.1250	.8119	1.0313	1.4371
26	11	12	16	0	1.0000	1.0313	1.4371	0	1.6250
27	7	4	8	1.1250	.8119	2.0000	.5826	1.8750	1.2182
28	7	8	12	1.1250	.8119	1.9750	1.2182	1.0313	1.4371
29	4	2	5	2.0000	.5826	2.6250	.3119	2.5313	.9683
30	4	5	8	2.0000	.5826	2.5313	.9683	1.8750	1.2182
31	2	1	3	2.6250	.3119	3.0000	0	3.0000	.6875
32	2	3	5	2.6250	.3119	3.0000	.6875	2.5313	.9683

NUMBER OF ELEMENTS = NE = 32

NUMBER OF NODES = NP = 25

BANDWIDTH = NBW = 6

CONVECTION FROM SIDE 2 OF ELEMENT 2  
 CONVECTION FROM SIDE 2 OF ELEMENT 4  
 CONVECTION FROM SIDE 2 OF ELEMENT 6  
 CONVECTION FROM SIDE 2 OF ELEMENT 7  
 CONVECTION FROM SIDE 2 OF ELEMENT 8  
 CONVECTION FROM SIDE 2 OF ELEMENT 15  
 CONVECTION FROM SIDE 2 OF ELEMENT 23  
 CONVECTION FROM SIDE 2 OF ELEMENT 31

BOUNDARY VALUES

NODAL FORCES

PRESCRIBED NODAL VALUES

1 .20000E+03 2 .20000E+03 4 .20000E+03 7 .20000E+03 11 .20000E+03

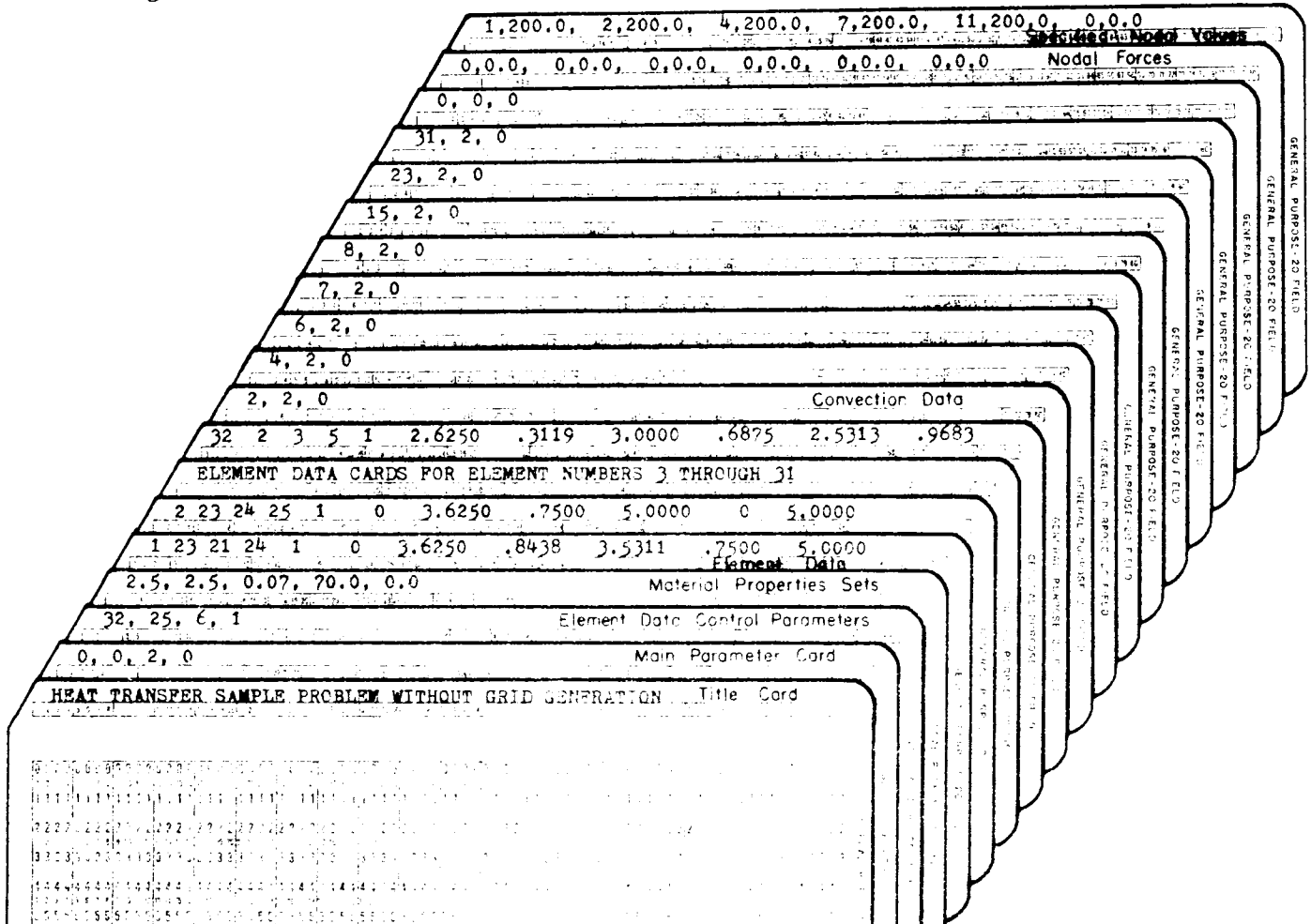
NODAL VALUES

1 .20000E+03 2 .20000E+03 3 .19597E+03 4 .20000E+03 5 .19561E+03 6 .18900E+03  
 7 .20000E+03 8 .19568E+03 9 .18943E+03 10 .18127E+03 11 .20000E+03 12 .19575E+03  
 13 .18991E+03 14 .18254E+03 15 .17438E+03 16 .19534E+03 17 .19016E+03 18 .18354E+03  
 19 .17635E+03 20 .18982E+03 21 .18411E+03 22 .17777E+03 23 .18408E+03 24 .17862E+03  
 25 .17892E+03

ELEMENT RESULTS

ELEMENT	GRAD(X)	GRAD(Y)	AVE TEMP
1	-.39042E+00	-.37570E+01	.18227E+03
2	-.38803E+00	-.37583E+01	.18054E+03
3	-.12292E+01	-.37666E+01	.18190E+03
4	-.11430E+01	-.38050E+01	.18017E+03
5	-.19859E+01	-.37803E+01	.18091E+03
6	-.18886E+01	-.38190E+01	.17922E+03
7	-.26626E+01	-.38023E+01	.17940E+03
8	-.26275E+01	-.38159E+01	.17776E+03
9	-.49315E+00	-.51352E+01	.18903E+03
10	-.53955E+00	-.50972E+01	.18600E+03
11	-.14407E+01	-.51962E+01	.18787E+03
12	-.14215E+01	-.52085E+01	.18533E+03
13	-.22182E+01	-.52769E+01	.18729E+03
14	-.22239E+01	-.52739E+01	.18533E+03
15	-.28468E+01	-.53771E+01	.18657E+03
16	-.29530E+01	-.53281E+01	.18441E+03
17	-.73732E+00	-.62446E+01	.19375E+03
18	-.68892E+00	-.63077E+01	.19177E+03
19	-.17205E+01	-.63140E+01	.19378E+03
20	-.16908E+01	-.63435E+01	.19194E+03
21	-.25514E+01	-.64206E+01	.19357E+03
22	-.25480E+01	-.64231E+01	.19167E+03
23	-.31668E+01	-.65557E+01	.19353E+03
24	-.32786E+01	-.64887E+01	.19135E+03
25	-.11657E+01	-.69735E+01	.19858E+03
26	-.95899E+00	-.74611E+01	.19703E+03
27	-.18782E+01	-.71656E+01	.19856E+03
28	-.19209E+01	-.70867E+01	.19714E+03
29	-.30870E+01	-.71287E+01	.19854E+03
30	-.29125E+01	-.73690E+01	.19710E+03
31	-.48801E+01	-.58664E+01	.19866E+03
32	-.35497E+01	-.71948E+01	.19719E+03

Fig. 3.2.6 Input for Heat Transfer Without Grid Generation



### 3.3 GROUNDWATER FLOW

#### 3.3.1 Introduction

This program can be used to analyze two-dimensional irrotational flow of ideal fluids which includes groundwater flow. The program description will be given in terms of the groundwater flow case. The program uses only simple linear triangular elements. Automatic grid generation can be used with this program.

#### 3.3.2 Material Properties Description

The material property sets for this program include, in the following order,

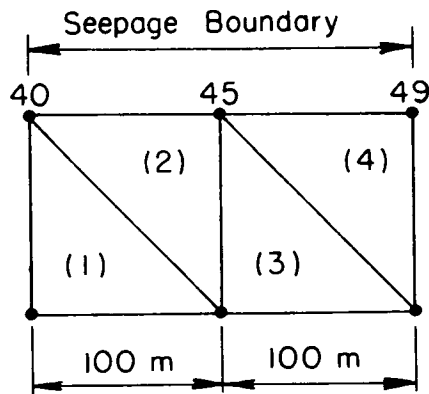
- TXX - transmissivity in the X direction
- TYY - transmissivity in the Y direction
- Dummy Variable 1 - = 0.0
- Dummy Variable 2 - = 0.0
- Dummy Variable 3 - = 0.0

Any compatible set of units may be used.

Up to twenty sets of material properties may be input.

#### 3.3.3 Seepage Input

Values for seepage into or out of the region under study are input as nodal forces on the boundary nodes. The seepage along the side of the element is calculated and then 1/2 allotted to each node. Seepage into the region is considered positive. Consider the example below:



Assume infiltration rate =  $0.30 \text{ m}^3/\text{day}/\text{m}$  into region.

$$\text{Seepage for each element} = 0.30 \text{ m}^3/\text{day}/\text{m} * 100\text{m}$$

$$= 30 \text{ m}^3/\text{day}$$

$$\text{Seepage by node} = \begin{array}{ll} 40 & 15 \text{ m}^3/\text{day} \end{array}$$

$$45 \quad 30 \text{ m}^3/\text{day}$$

$$49 \quad 15 \text{ m}^3/\text{day}$$

The seepage values would be input as nodal forces at nodes 40, 45, and 49.



### 3.3.4 Boundary Values

Boundary conditions are input in two sections. First, the nodal forces which for the groundwater case would be nodal values for seepage and pumping (fluid added to region assumed positive). Second, specified nodal values, which would be known nodal potentials.

Nodal forces and nodal values are both input by node number and associated value with six sets per card.<sup>1/</sup> Each card must have 6 sets of node number and associated value. If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zero values (12 zeros) must follow to terminate that section of input data. Nodal forces for the example shown in the previous section would appear as

```
40, 15.0, 45, 30.0, 49, 15.0 0,0.0, 0,0.0, 0,0.0
```

Consider the case where nodes 12, 15, 19, and 21 are known to have potential values of 100.0, 115.0, 115.0, and 114.0, respectively. The specified nodal values would appear as

```
12, 100.0, 15, 115.0, 19, 115.0, 21, 114.0 0, 0.0, 0, 0.0
```

### 3.3.5 Input Description

Table 3.3.1 shows the inputs required for the two possible cases using this program. The first case, auto-grid case, assumes use of the automatic grid generation program. The second case, non-auto-grid case, assumes the user will input the element data from cards.

### 3.3.6 Sample Problem

Figure 3.3.1 shows a region for study. Water infiltrates into the region from a stream on two sides. Heads are known at a number of locations on the boundary of the region. Linear interpolation will be used to obtain nodal values on the sections of the boundary with given potentials. Figure 3.3.2 shows the region divided for automatic grid generation. Figure 3.3.3 shows the input using automatic grid generation. Figure 3.3.4 shows the grid generated. Figure 3.3.5 shows the output of the program.

Figure 3.3.6 shows the input for the same grid as shown in Figure 3.3.4 but not using automatic grid generation. Figure 3.3.7 shows the output of the program.

<sup>1/</sup> A preliminary run of auto-grid generation may be necessary to determine nodes for boundary conditions.

Table 3.3.1 Program Input for Groundwater

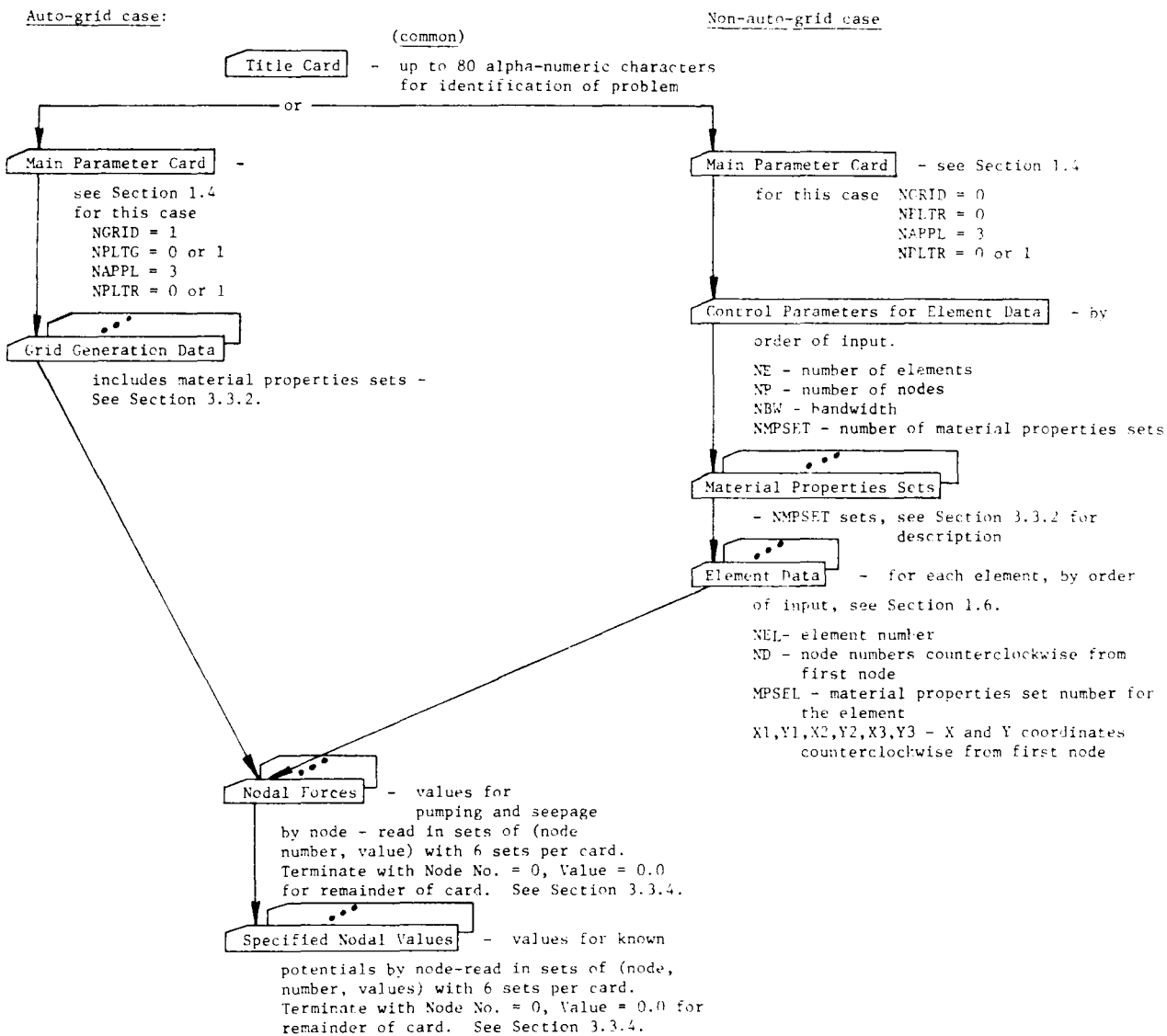


Fig. 3.3.1 Sample Problem for Groundwater Flow

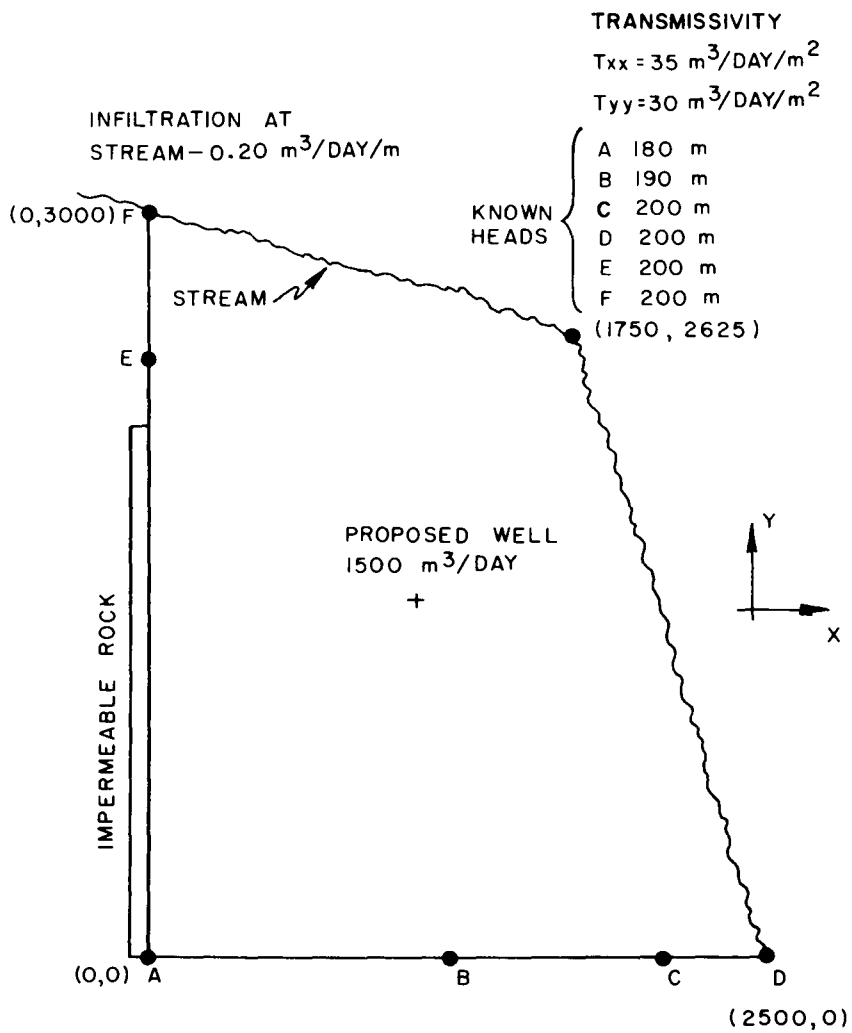


Fig. 3.3.2 Region for Grid Generation

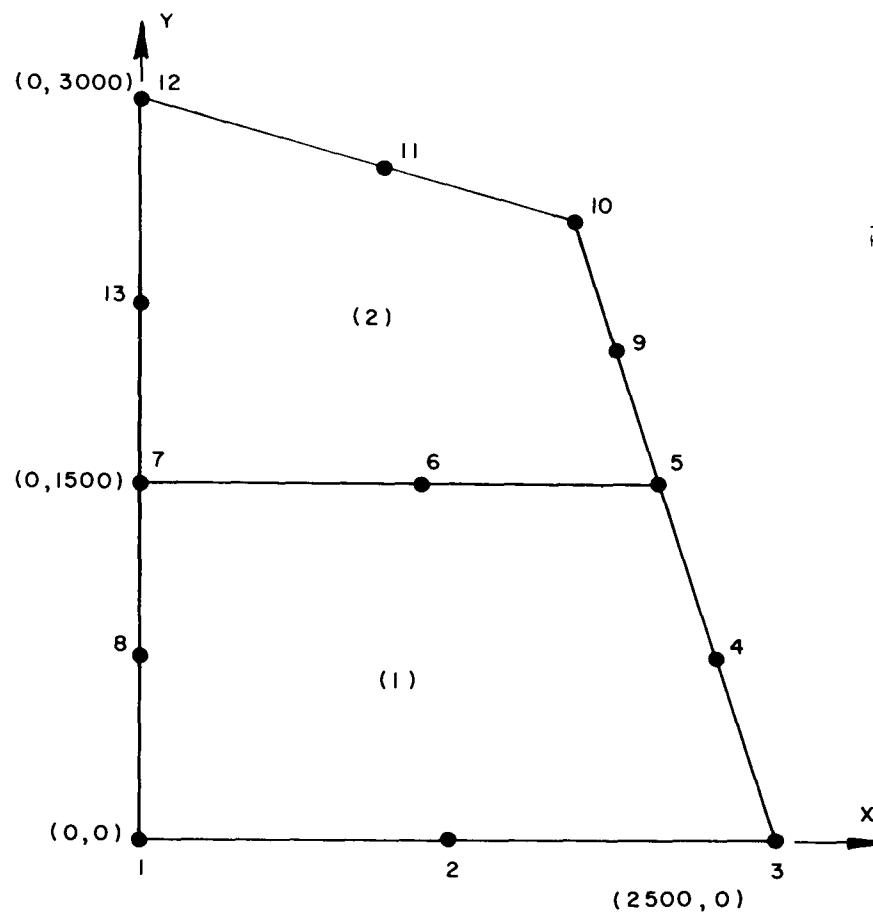


Fig. 3.3.3 Input for Groundwater Problem with Grid Generation

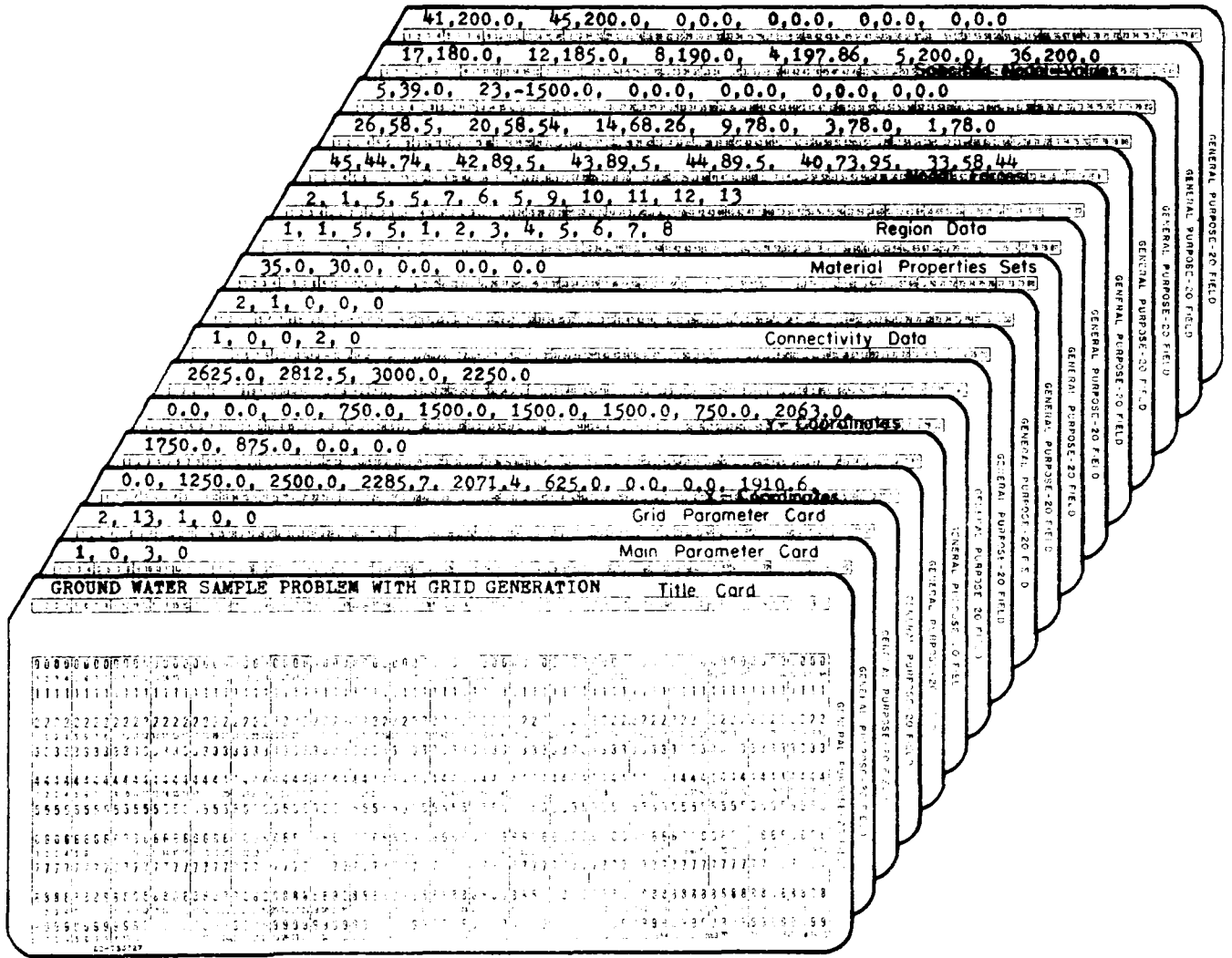


Fig. 3.3.4 Grid Generated Groundwater Example

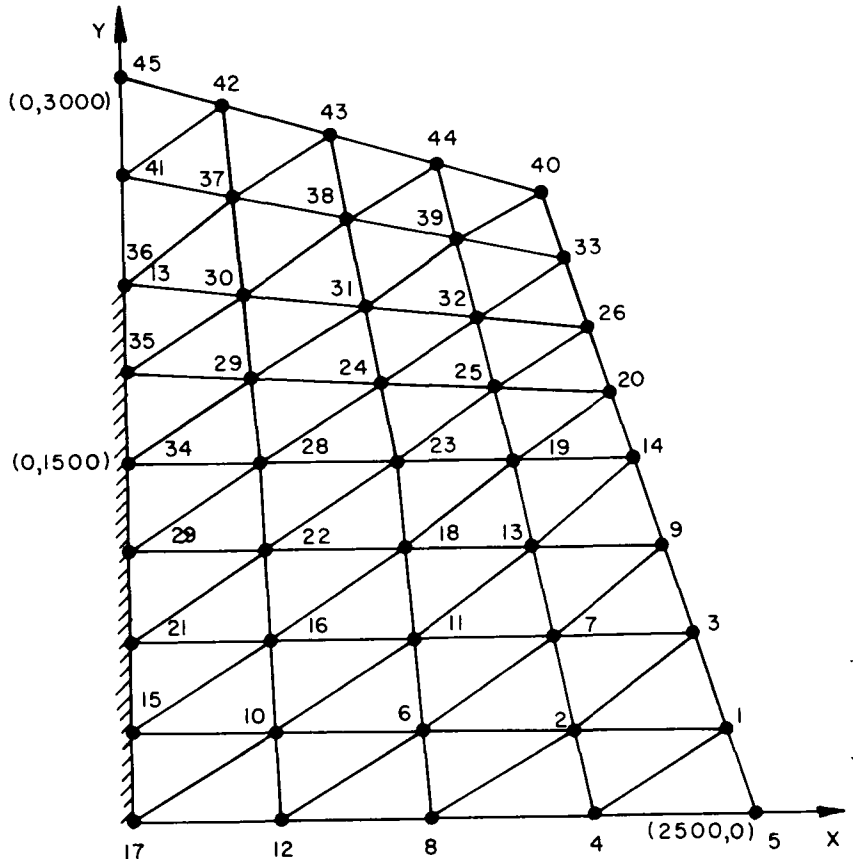


Fig. 3.3.5 Output for Groundwater Example

GROUND WATER SAMPLE PROBLEM WITH GRID GENERATION

THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM GROUNDWATER FLOW WILL BE SOLVED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	0
2	1250.00	0
3	2500.00	0
4	2285.70	750.00
5	2071.40	1500.00
6	625.00	1500.00
7	0	1500.00
8	0	750.00
9	1910.60	2063.00
10	1750.00	2625.00
11	875.00	2812.50
12	0	3000.00
13	0	2250.00

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	0	2	0
2	1	0	0	0

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
	TXX	TYX	N/A	N/A	N/A
1	.3500E+02	.3000E+02	0	0	0

REGION DATA

REGION	ROWS	COLS	MATERIAL			INPUT REGION NODE NO.							
			SET										
1	5	5	1	1	2	3	4	5	6	7	8		
2	5	5	1	7	6	5	9	10	11	12	13		

NODE NUMBERS OF SUBDIVIDED REGION

1					
	34	28	23	19	14
	27	22	18	13	9
	21	16	11	7	3
	15	10	6	2	1
	17	12	8	4	5
2					
	45	42	43	44	40
	41	37	38	39	33
	36	30	31	32	26
	35	29	24	25	20
	34	28	23	19	14

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEL	NODE	NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	27	22 28	0	1125.0000	313.6188	1125.0000	209.8250	1500.0000
2	27	28 34	0	1125.0000	209.8250	1500.0000	0	1500.0000
3	22	18 23	313.6188	1125.0000	781.2500	1125.0000	625.0000	1500.0000
4	22	23 28	313.6188	1125.0000	625.0000	1500.0000	209.8250	1500.0000
5	18	13 19	781.2500	1125.0000	1402.8937	1125.0000	1245.5250	1500.0000
6	18	19 23	781.2500	1125.0000	1245.5250	1500.0000	625.0000	1500.0000
7	13	9 14	1402.8937	1125.0000	2178.5500	1125.0000	2071.4000	1500.0000
8	13	14 19	1402.8937	1125.0000	2071.4000	1500.0000	1245.5250	1500.0000
9	21	16 22	0	750.0000	417.4125	750.0000	313.6188	1125.0000
10	21	22 27	0	750.0000	313.6188	1125.0000	0	1125.0000
11	16	11 18	417.4125	750.0000	937.5000	750.0000	781.2500	1125.0000
12	16	18 22	417.4125	750.0000	781.2500	1125.0000	313.6188	1125.0000
13	11	7 13	937.5000	750.0000	1560.2625	750.0000	1402.8937	1125.0000
14	11	13 18	937.5000	750.0000	1402.8937	1125.0000	781.2500	1125.0000
15	7	3 9	1560.2625	750.0000	2285.7000	750.0000	2178.5500	1125.0000
16	7	9 13	1560.2625	750.0000	2178.5500	1125.0000	1402.8937	1125.0000
17	15	10 16	0	375.0000	521.2062	375.0000	417.4125	750.0000
18	15	16 21	0	375.0000	417.4125	750.0000	0	750.0000
19	10	6 11	521.2062	375.0000	1093.7500	375.0000	937.5000	750.0000
20	10	11 16	521.2062	375.0000	937.5000	750.0000	417.4125	750.0000
21	6	2 7	1093.7500	375.0000	1717.6312	375.0000	1560.2625	750.0000
22	6	7 11	1093.7500	375.0000	1560.2625	750.0000	937.5000	750.0000
23	2	1 3	1717.6312	375.0000	2392.8500	375.0000	2285.7000	750.0000
24	2	3 7	1717.6312	375.0000	2285.7000	750.0000	1560.2625	750.0000
25	17	12 10	0	0	625.0000	0	521.2062	375.0000
26	17	10 15	0	0	521.2062	375.0000	0	375.0000
27	12	8 6	625.0000	0	1250.0000	0	1093.7500	375.0000
28	12	6 10	625.0000	0	1093.7500	375.0000	521.2062	375.0000
29	8	4 2	1250.0000	0	1875.0000	0	1717.6312	375.0000
30	8	2 6	1250.0000	0	1717.6312	375.0000	1093.7500	375.0000
31	4	5 1	1875.0000	0	2500.0000	0	2392.8500	375.0000
32	4	1 2	1875.0000	0	2392.8500	375.0000	1717.6312	375.0000

ELEMENT DATA, REGION 2 MAT. PROP. SET = 1

NEL	NODE	NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
33	41	37 42	0	2625.0000	350.5625	2554.7813	437.5000	2906.2500
34	41	42 45	0	2625.0000	437.5000	2906.2500	0	3000.0000
35	37	38 43	380.5625	2554.7813	812.4625	2484.5625	875.0000	2812.5000
36	37	43 42	380.5625	2554.7813	875.0000	2812.5000	437.5000	2906.2500
37	38	39 44	812.4625	2484.5625	1295.7000	2414.3438	1312.5000	2718.7500
38	38	44 43	812.4625	2484.5625	1312.5000	2718.7500	875.0000	2812.5000
39	39	33 40	1295.7000	2414.3438	1830.2750	2344.1250	1750.0000	2625.0000
40	39	40 44	1295.7000	2414.3438	1750.0000	2625.0000	1312.5000	2718.7500
41	36	30 37	0	2250.0000	323.6375	2203.2500	380.5625	2554.7813
42	36	37 41	0	2250.0000	380.5625	2554.7813	0	2625.0000
43	30	31 37	323.6375	2203.2500	749.9500	2156.5000	380.5625	2554.7813
44	31	38 37	749.9500	2156.5000	812.4625	2484.5625	380.5625	2554.7813
45	31	32 39	749.9500	2156.5000	1278.9375	2109.7500	1295.7000	2414.3438
46	31	39 38	749.9500	2156.5000	1295.7000	2414.3438	812.4625	2484.5625
47	32	26 33	1278.9375	2109.7500	1910.6000	2063.0000	1830.2750	2344.1250
48	32	33 39	1278.9375	2109.7500	1830.2750	2344.1250	1295.7000	2414.3438
49	35	29 30	0	1875.0000	266.7250	1851.6563	323.6375	2203.2500
50	35	30 36	0	1875.0000	323.6375	2203.2500	0	2250.0000
51	29	24 30	266.7250	1851.6563	687.4625	1828.3125	323.6375	2203.2500
52	24	31 30	687.4625	1828.3125	749.9500	2156.5000	323.6375	2203.2500
53	24	25 31	687.4625	1828.3125	1262.2125	1804.9688	749.9500	2156.5000
54	25	32 31	1262.2125	1804.9688	1278.9375	2109.7500	749.9500	2156.5000
55	25	20 26	1262.2125	1804.9688	1990.9750	1781.6250	1910.6000	2063.0000
56	25	26 32	1262.2125	1804.9688	1910.6000	2063.0000	1278.9375	2109.7500
57	34	28 35	0	1500.0000	209.8250	1500.0000	0	1875.0000
58	28	29 35	209.8250	1500.0000	266.7250	1851.6563	0	1875.0000
59	28	23 29	209.8250	1500.0000	625.0000	1500.0000	266.7250	1851.6563
60	23	24 29	625.0000	1500.0000	687.4625	1828.3125	266.7250	1851.6563
61	23	19 24	625.0000	1500.0000	1245.5250	1500.0000	687.4625	1828.3125
62	19	25 24	1245.5250	1500.0000	1262.2125	1804.9688	687.4625	1828.3125
63	19	14 20	1245.5250	1500.0000	2071.4000	1500.0000	1990.9750	1781.6250
64	19	20 25	1245.5250	1500.0000	1990.9750	1781.6250	1262.2125	1804.9688

NUMBER OF ELEMENTS = NE = 64

NUMBER OF NODES = NP = 45

BANDWIDTH = NBW = 9

BOUNDARY VALUES

NODAL FORCES

45	.44740E+02	42	.89500E+02	43	.89500E+02	44	.89500E+02	40	.73950E+02	33	.58440E+02
26	.58500E+02	20	.58540E+02	14	.68260E+02	9	.78000E+02	3	.78000E+02	1	.78000E+02
5	.39000E+02	23	-.15000E+04								

PRESCRIBED NODAL VALUES

17	.18000E+03	12	.18500E+03	8	.19000E+03	4	.19786E+03	5	.20000E+03	36	.20000E+03
41	.20000E+03	45	.20000E+03								

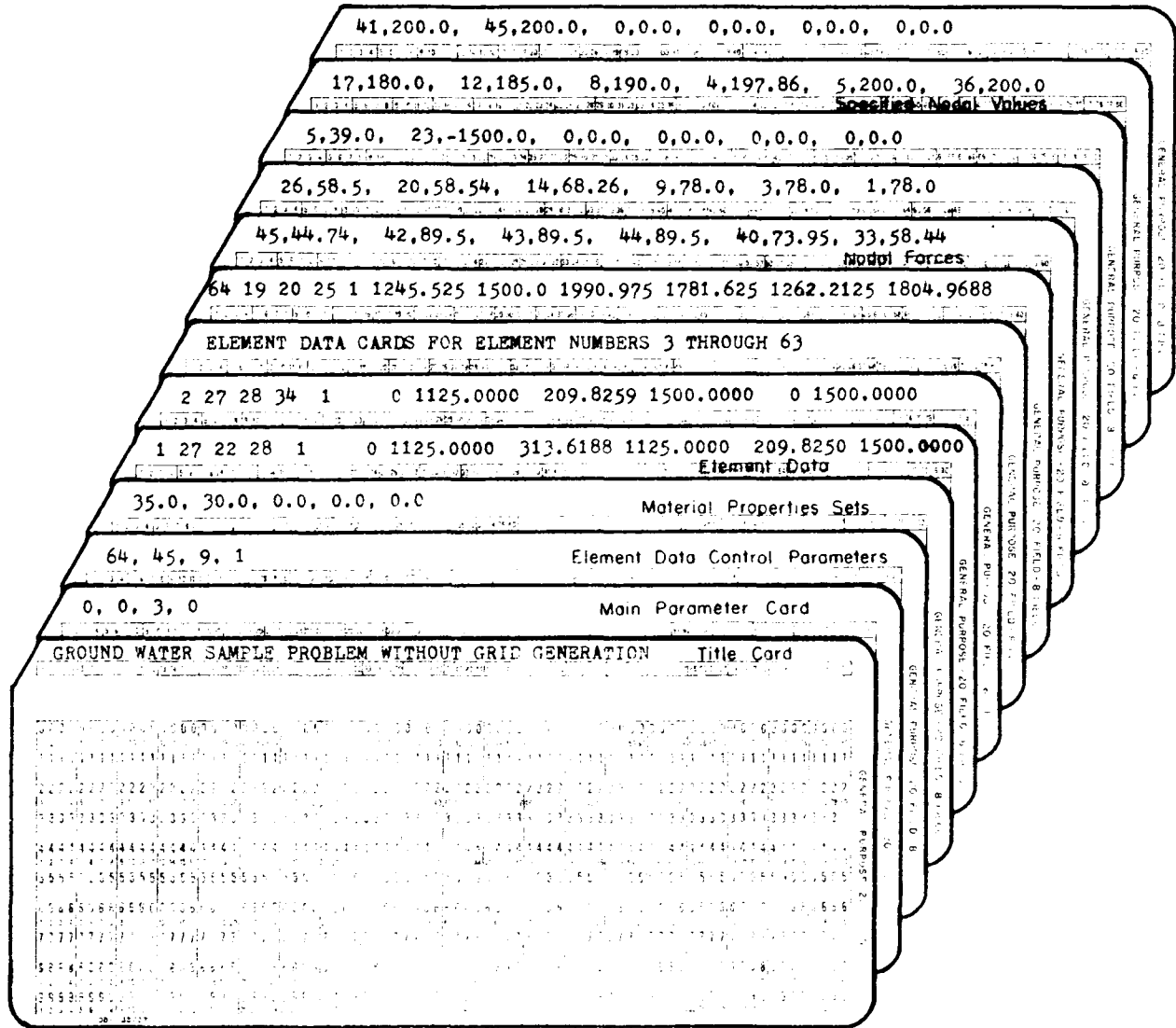
NODAL VALUES

1	.19860E+03	2	.19382E+03	3	.19654E+03	4	.19786E+03	5	.20000E+03	6	.18779E+03
7	.19086E+03	8	.19000E+03	9	.19485E+03	10	.19380E+03	11	.18488E+03	12	.19500E+03
13	.18831E+03	14	.19395E+03	15	.18179E+03	16	.18202E+03	17	.18000E+03	18	.18021E+03
19	.18643E+03	20	.19407E+03	21	.18134E+03	22	.17943E+03	23	.16957E+03	24	.18235E+03
25	.18875E+03	26	.19494E+03	27	.17985E+03	28	.17904E+03	29	.18538E+03	30	.19333E+03
31	.18989E+03	32	.19174E+03	33	.19629E+03	34	.17998E+03	35	.18713E+03	36	.20000E+03
37	.19685E+03	38	.19434E+03	39	.19454E+03	40	.19775E+03	41	.20000E+03	42	.19917E+03
43	.19716E+03	44	.19682E+03	45	.20000E+03						

ELEMENT VELOCITY COMPONENTS

ELEMENT	VEL(X)	VEL(Y)
1	.46946E-01	.42475E-01
2	.15711E+00	-.10358E-01
3	-.58564E-01	.83084E+00
4	.79857E+00	.22079E+00
5	-.45614E+00	-.13340E-01
6	-.95121E+00	.51203E+00
7	-.29485E+00	-.42912E-03
8	-.31874E+00	.35081E-01
9	-.56860E-01	.19381E+00
10	.46946E-01	.11940E+00
11	-.19203E+00	.30440E+00
12	-.58564E-01	.19340E+00
13	-.33629E+00	.82581E-01
14	-.45614E+00	.21007E+00
15	-.27416E+00	.58246E-01
16	-.29485E+00	.97486E-01
17	-.13464E+00	.10995E+00
18	-.56860E-01	.35734E-01
19	-.24404E+00	.14582E+00
20	-.19203E+00	.96334E-01
21	-.33838E+00	.11510E+00
22	-.33629E+00	.11288E+00
23	-.24764E+00	.10377E+00
24	-.27416E+00	.13820E+00
25	-.28000E+00	.29929E-01
26	-.13464E+00	-.14324E+00
27	-.28000E+00	.76988E-01
28	-.24404E+00	.38460E-01
29	-.44016E+00	.16493E+00
30	-.33838E+00	.50140E-01
31	-.11984E+00	.82916E-01
32	-.24764E+00	.23418E+00
33	.24017E+00	-.23125E+00
34	.66731E-01	0
35	.14941E+00	-.28199E+00
36	.10734E+00	-.21281E+00
37	-.51751E-01	-.22221E+00
38	-.35611E-01	-.25175E+00
39	-.14416E+00	-.19099E+00
40	-.12926E+00	-.21854E+00
41	.65583E+00	-.39159E+00
42	.28995E+00	0
43	.23932E+00	-.33378E+00
44	.12187E+00	-.42714E+00
45	-.14992E+00	-.26878E+00
46	-.80879E-01	-.39403E+00
47	-.19405E+00	-.10184E+00
48	-.15604E+00	-.26849E+00
49	.15756E+00	-.69939E+00
50	.54831E+00	-.10296E+01
51	.20703E+00	-.70625E+00
52	.18983E+00	-.72056E+00
53	-.41948E+00	-.62112E+00
54	-.15177E+00	-.28673E+00
55	-.26121E+00	-.15693E+00
56	-.20204E+00	-.28437E+00
57	.15711E+00	-.57193E+00
58	.17127E+00	-.56514E+00
59	.79857E+00	-.65214E+00
60	.17531E+00	-.11964E+01
61	-.95121E+00	-.10127E+01
62	-.39998E+00	-.20958E+00
63	-.31874E+00	-.90455E-01
64	-.26342E+00	-.21598E+00

Fig. 3.3.6 Input for Groundwater Example Without Grid Generation





### 3.4 TORSION OF NON-CIRCULAR SHAFT

#### 3.4.1 Introduction

This program calculates the shear stresses in a non-circular shaft for a specified torque. The program is based on using a stress function,  $\phi$ , which has a zero value on the surface of the body and for which

$$\frac{\partial \phi}{\partial y} = \tau_{zx} \quad \text{and} \quad \frac{\partial \phi}{\partial x} = \tau_{xy} \quad \frac{1}{}$$

Therefore all nodal values on the surface of the body must be specified as zero. The program uses simple linear triangular elements only. Automatic grid generation can be used with the program.

#### 3.4.2 Material Properties Description

The material Property sets for this program include, in the following order

- G - the shear modulus (N/cm<sup>2</sup>)
- SL - the length of the shaft (cm)
- TORQUE - the applied torque (N-cm)
- PCT - twice the number of axes of symmetry. PCT is 1.0 when no symmetry exists
- Dummy Variable 1 - = 0.0

Units shown are only sample units. Any compatible sets of units English or metric may be used. Up to 20 sets may be input.

#### 3.4.3 Torque Input Card

The value of the torque applied to the shaft for which stresses are to be calculated is input in units compatible with those used for material properties. Torque value is input on a card. For example, if the torque value is 2.5 ft-lb, the torque input card appears as 

215.0
-------

 .

#### 3.4.4 Boundary Values

Boundary conditions are input in two sections. First, the nodal forces which for the torsion are nonexistent. Second, specified nodal values which would be specifying zero values for the stress function at the free surface nodal locations.

Nodal values are input by node numbers and associated value with six sets per card. <sup>2/</sup> Each card must have six sets of node number and value. If a complete card is not needed, the remaining set should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zeros (12 zeros) must follow to terminate that section of data input.

For the torsion problem, one card with 6 sets of zero values (12 zeros) would be input for nodal forces.

---

<sup>1/</sup> For more detailed discussion of this approach see Segerlind, Applied Finite Element Analysis, 1976, Wiley.

<sup>2/</sup> A preliminary run of auto-grid generation may be necessary to determine nodes for boundary conditions.

If nodes 3, 5, 7, and 9 are the surface nodes, the specified nodal values would be input as

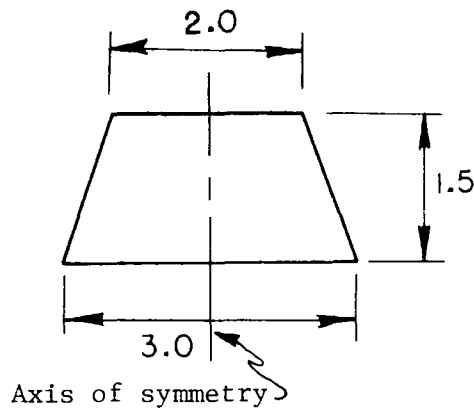
3, 0.0, 5, 0.0, 7, 0.0, 9, 0.0, 0, 0.0, 0, 0.0

### 3.4.5 Input Description

Table 3.4.1 shows the inputs required for the two possible cases using TORSION. The first case, auto-grid case, assumes the use of the automatic grid generation program. The second case, non-auto-grid case, assumes the user will input the element data from cards.

### 3.4.6 Sample problem

Consider the problem of torsion of a trapezoidal shaft as shown below. Note the shaft has one axis of symmetry, therefore, it is only necessary to consider one-half of the body.



#### Material Properties

G = 8000000.  
 SL = 100.  
 TORQUE = 196.6  
 PCT = 2

Regions for grid input

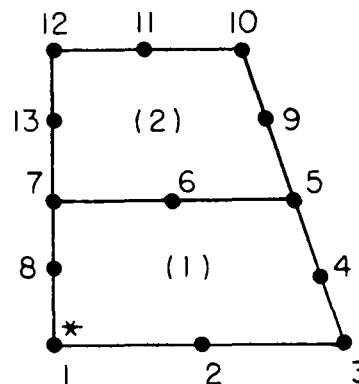


Figure 3.4.1 shows the input using grid generation for the sample problem. Figure 3.4.2 shows the grid generation by the program. Figure 3.4.3 shows the output of the program.

Figure 3.4.4 shows the input for the same grid as shown in Figure 3.4.2 but not using automatic grid generation.

Table 3.4.1 Program Input for Torsion

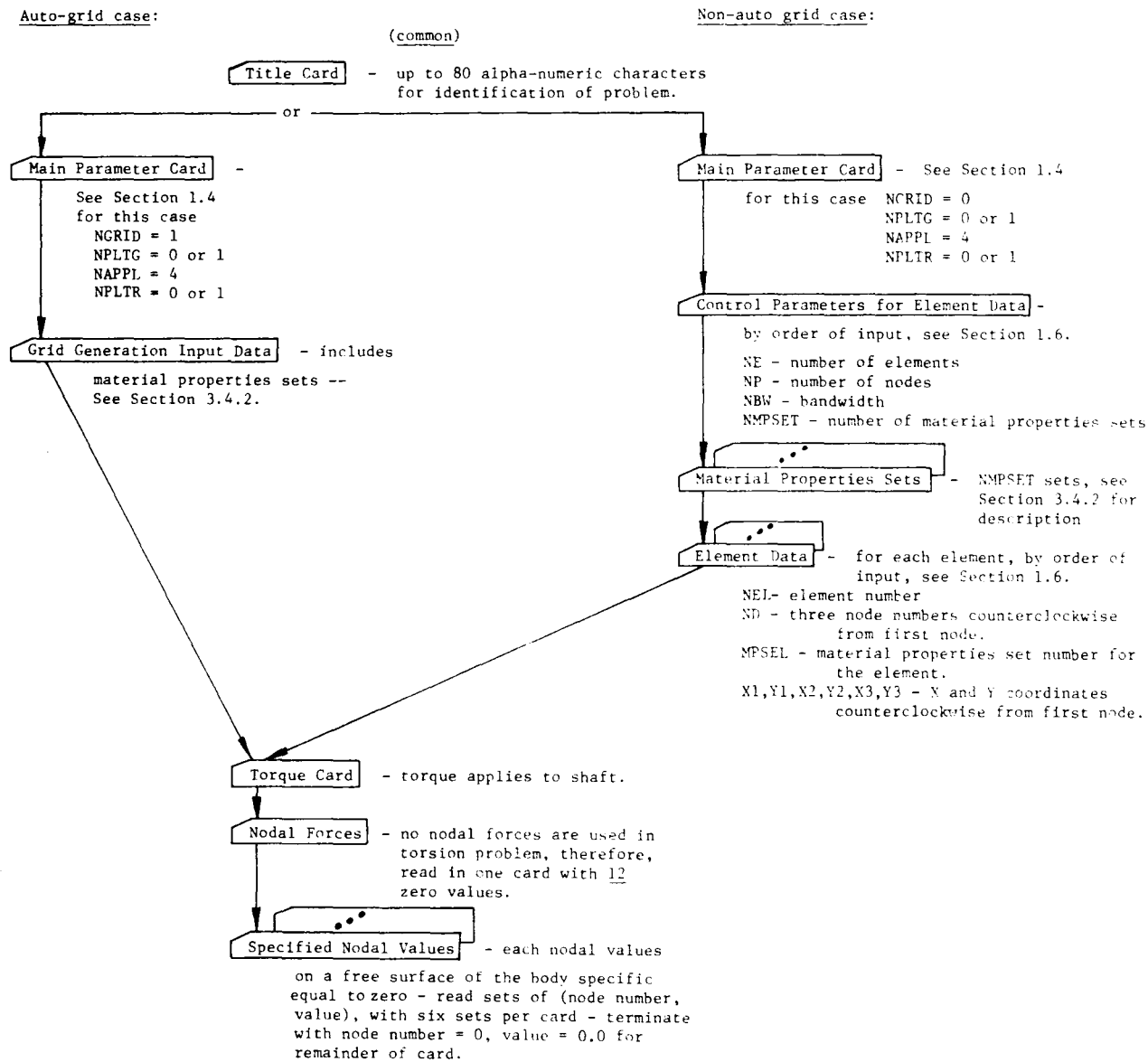


Fig. 3.4.1 Input for Torsion Problem with Grid Generation

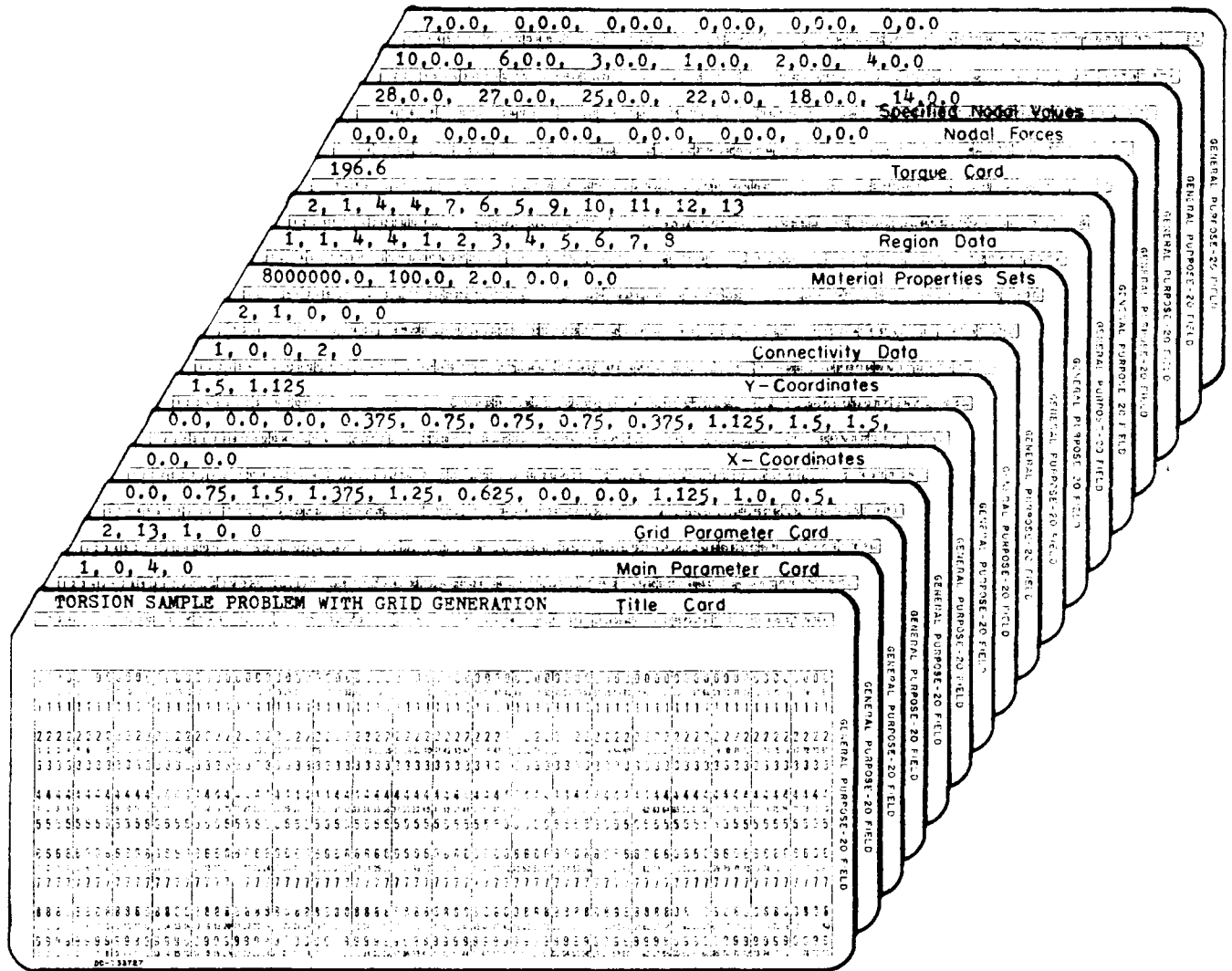
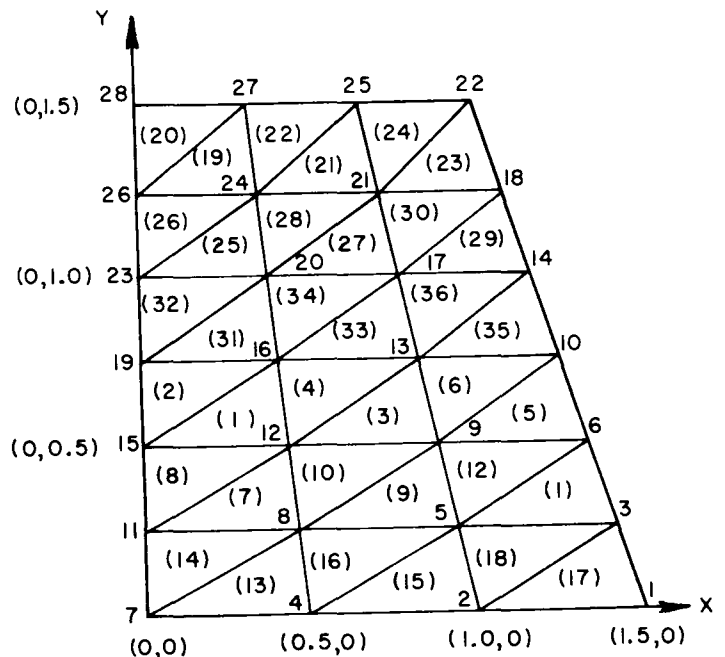


Fig. 3.4.2 Grid Generated for Torsion Example



NUMBER OF ELEMENT = NE = 36  
 NUMBER OF MODES = NP = 28  
 BANDWIDTH = NBW = 5

Fig. 3.4.3 Output for Torsion Example with Grid Generation

TORSION SAMPLE PROBLEM WITH GRID GENERATION

THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM TORSION(1) WILL BE SOLVED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	0
2	.75	0
3	1.50	0
4	1.38	.38
5	1.25	.75
6	.63	.75
7	0	.75
8	0	.38
9	1.13	1.13
10	1.00	1.50
11	.50	1.50
12	0	1.50
13	0	1.13

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	0	2	0
2	1	0	0	0

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
	G	SL	PCT	N/A	N/A
1	.8000E+07	.1000E+03	.2000E+01	0	0

REGION DATA

REGION	ROWS	MATERIAL			INPUT REGION NODE NO.							
		COLS	SET		1	2	3	4	5	6	7	8
1	4	4	1		1	2	3	4	5	6	7	8
2	4	4	1		7	6	5	9	10	11	12	13

NODE NUMBERS  
OF SUBDIVIDED REGION

1	19	16	13	10
	15	12	9	6
	11	8	5	3
	7	4	2	1
2	28	27	25	22
	26	24	21	18
	23	20	17	14
	19	16	13	10

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	15 12 16	0	.5000	.4444	.5000	.4167	.7500
2	15 16 19	0	.5000	.4167	.7500	0	.7500
3	12 9 13	.4444	.5000	.8889	.5000	.8333	.7500
4	12 13 16	.4444	.5000	.8333	.7500	.4167	.7500
5	9 6 10	.8889	.5000	1.3333	.5000	1.2500	.7500
6	9 10 13	.8889	.5000	1.2500	.7500	.8333	.7500
7	11 8 12	0	.2500	.4722	.2500	.4444	.5000
8	11 12 15	0	.2500	.4444	.5000	0	.5000
9	8 5 9	.4722	.2500	.9444	.2500	.8889	.5000
10	8 9 12	.4722	.2500	.8889	.5000	.4444	.5000
11	5 3 6	.9444	.2500	1.4167	.2500	1.3333	.5000
12	5 6 9	.9444	.2500	1.3333	.5000	.8889	.5000
13	7 4 8	0	0	.5000	0	.4722	.2500
14	7 8 11	0	0	.4722	.2500	0	.2500
15	4 2 5	.5000	0	1.0000	0	.9444	.2500
16	4 5 8	.5000	0	.9444	.2500	.4722	.2500
17	2 1 3	1.0000	0	1.5000	0	1.4167	.2500
18	2 3 5	1.0000	0	1.4167	.2500	.9444	.2500

ELEMENT DATA, REGION 2 MAT. PROP. SET = 1

NEL	NODE NUMBERS	X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
19	26 24 27	0	1.2500	.3611	1.2500	.3333	1.5000
20	26 27 28	0	1.2500	.3333	1.5000	0	1.5000
21	24 21 25	.3611	1.2500	.7222	1.2500	.6667	1.5000
22	24 25 27	.3611	1.2500	.6667	1.5000	.3333	1.5000
23	21 18 22	.7222	1.2500	1.0833	1.2500	1.0000	1.5000
24	21 22 25	.7222	1.2500	1.0000	1.5000	.6667	1.5000
25	23 20 24	0	1.0000	.3889	1.0000	.3611	1.2500
26	23 24 26	0	1.0000	.3611	1.2500	0	1.2500
27	20 17 21	.3889	1.0000	.7778	1.0000	.7222	1.2500
28	20 21 24	.3889	1.0000	.7222	1.2500	.3611	1.2500
29	17 14 18	.7778	1.0000	1.1667	1.0000	1.0833	1.2500
30	17 18 21	.7778	1.0000	1.0833	1.2500	.7222	1.2500
31	19 16 20	0	.7500	.4167	.7500	.3889	1.0000
32	19 20 23	0	.7500	.3889	1.0000	0	1.0000
33	16 13 17	.4167	.7500	.8333	.7500	.7778	1.0000
34	16 17 20	.4167	.7500	.7778	1.0000	.3889	1.0000
35	13 10 14	.8333	.7500	1.2500	.7500	1.1667	1.0000
36	13 14 17	.8333	.7500	1.1667	1.0000	.7778	1.0000

NUMBER OF ELEMENTS = NE = 36

NUMBER OF NODES = NP = 28

BANDWIDTH = NRW = 5

APPLIED TORQUE EQUALS 196.6000

BOUNDARY VALUES

NODAL FORCES

PRESCRIBED NODAL VALUES	VALUES
28	0 27 0 25 0 22 0 18 0 14 0
10	0 6 0 3 0 1 0 2 0 4 0
7	0

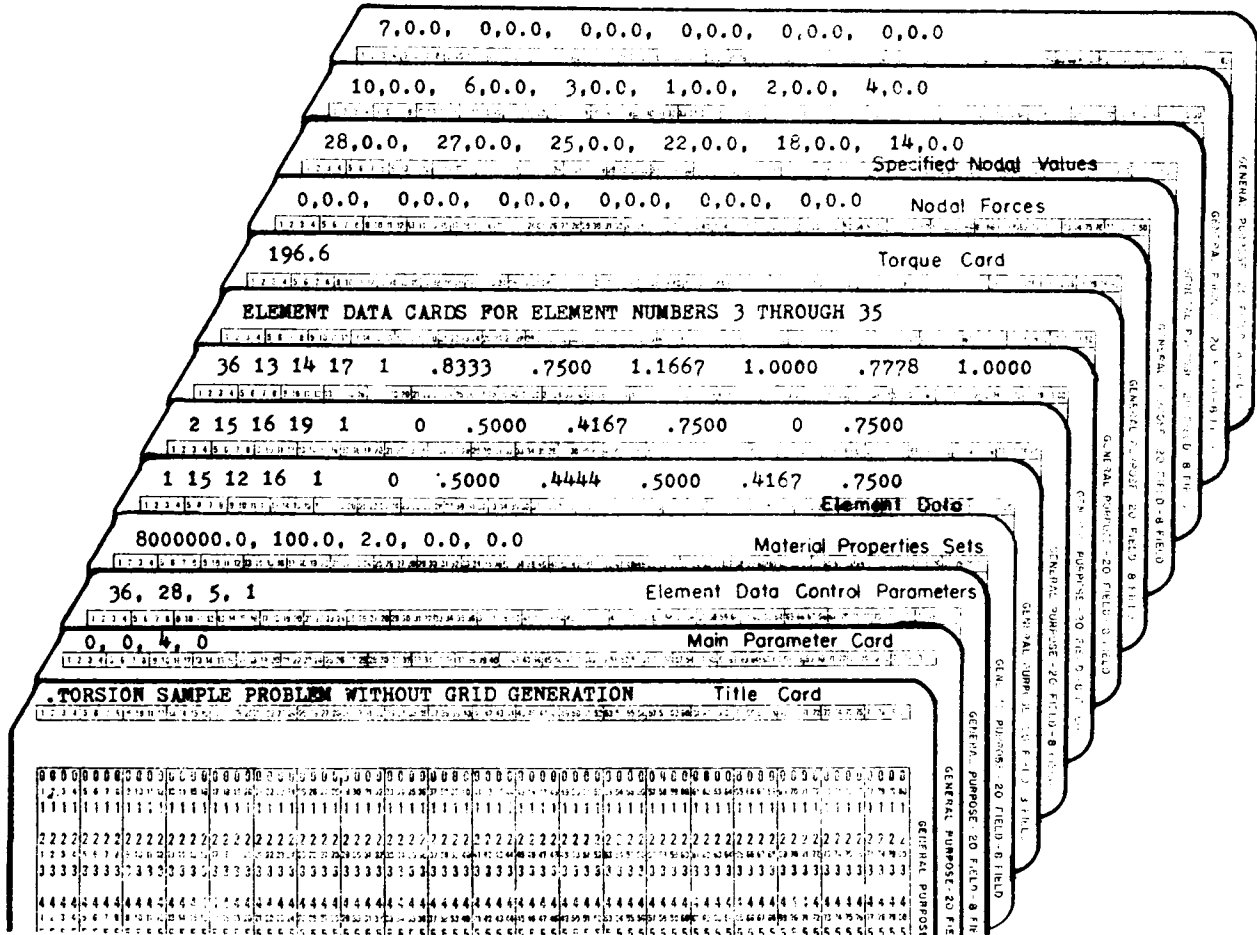
NODAL VALUES	VALUES
1	0 2 0 3 0 4 0 5 .21259E+02
6	0 7 0 8 .30494E+02 9 .33319E+02 10 0
11	.32788E+02 12 .48001E+02 13 .37604E+02 14 0 15 .51702E+02
16	.53745E+02 17 .34615E+02 18 0 19 .57761E+02 20 .48225E+02
21	.23463E+02 22 0 23 .51468E+02 24 .30911E+02 25 0
26	.32574E+02 27 0 28 0

ELEMENT STRESSES

ELEMENT	TAUZX	TAUZY	TAUMAX
1	.22048E+02	.83269E+01	.23568E+02
2	.24235E+02	.96394E+01	.26082E+02
3	.97980E+01	.33034E+02	.34456E+02
4	.18669E+02	.38737E+02	.43001E+02
5	-.24990E+02	.74969E+02	.79024E+02
6	-.29167E+01	.90250E+02	.90297E+02
7	.69490E+02	.48583E+01	.69660E+02
8	.75657E+02	.83269E+01	.76114E+02
9	.43896E+02	.19556E+02	.48055E+02
10	.66360E+02	.33034E+02	.74127E+02
11	-.15006E+02	.45019E+02	.47454E+02
12	.31582E+02	.74969E+02	.81349E+02
13	.12198E+03	0	.12198E+03
14	.13115E+03	.48583E+01	.13124E+03
15	.85036E+02	0	.85036E+02
16	.11980E+03	.19556E+02	.12139E+03
17	0	0	0
18	.75032E+02	.45019E+02	.87502E+02
19	-.12416E+03	.46054E+01	.12424E+03
20	-.13030E+03	0	.13030E+03
21	-.98435E+02	.20626E+02	.10057E+03
22	-.12364E+03	0	.12364E+03
23	-.21658E+02	.64974E+02	.68489E+02
24	-.93852E+02	0	.93852E+02
25	-.70181E+02	.83398E+01	.70675E+02
26	-.75575E+02	.46054E+01	.75715E+02
27	-.52386E+02	.34997E+02	.63000E+02
28	-.71546E+02	.20626E+02	.74460E+02
29	-.29670E+02	.89010E+02	.93825E+02
30	-.59047E+02	.64974E+02	.87797E+02
31	-.23150E+02	.96394E+01	.25077E+02
32	-.25172E+02	.83398E+01	.26517E+02
33	-.20565E+02	.38737E+02	.43897E+02
34	-.25967E+02	.34997E+02	.43578E+02
35	-.30083E+02	.90250E+02	.95132E+02
36	-.31737E+02	.89010E+02	.94499E+02

ANGLE OF TWIST FOR THE APPLIED TORQUE IS .09 DEGREES  
 MAXIMUM SHEAR STRESS FOR THE APPLIED TORQUE IS 131.2  
 MAXIMUM SHEAR STRESS IS LOCATED IN ELEMENT 14

Fig. 3.4.4 Input for Torsion Problem Without Grid Generation



### 3.5 TWO-DIMENSIONAL TRANSIENT HEAT TRANSFER

#### 3.5.1 Introduction

This program can be used to analyze two dimensional problems involving transient heat transfer with either prescribed boundary temperatures or surface convections. The body may have internal heat generation. The user must specify the initial temperature of each node. The time step, number of time steps, and frequency of printing of nodal temperatures can be controlled by the user. This program uses a finite element procedure for temperature distribution at any time step and a forward difference technique in time for solution of the transient problem.

The program uses only simple linear triangular elements. Automatic grid generation can be used with this program.

#### 3.5.2 Material Properties Description

The material properties sets for this program include, in the following order

- KXX - Conductivity in X-direction
- KYY - Conductivity in Y-direction
- H - Surface convection coefficient
- TINF - Fluid temperature at a distance from convection surface.
- CRHO - Product of specific heat and density

Any compatible set of units may be used. Up to twenty material properties sets may be input.

#### 3.5.3 Control Parameters for Two-Dimensional Transient Heat Transfer

Control parameters read in during the application program section are

- 1) NIT - Number of iterations or time steps
- 2) IWTR - Control for output of calculated nodal temperatures. Temperatures are printed every IWTR iterations. If IWTR = 4 temperature are printed every fourth iteration.
- 3) IPCH - Control for punched output of final nodal temperatures. 0 for no punch and 1 to punch. (These cards may be used for thermal stress analysis by 2-D elasticity.)
- 4) DT - Time increment or time step



Sample Application Control Parameter Card

NIT, No. of Iterations	IWTR Print Output Control	IPCH, Punch Output Control	DT, time step
10,	5,	0,	0.1

This example specifies 10 iterations with printing every 5 iterations, no punched output and a time step of 0.1 units.

3.5.4 Input of Convection and Internal Heat Generation.

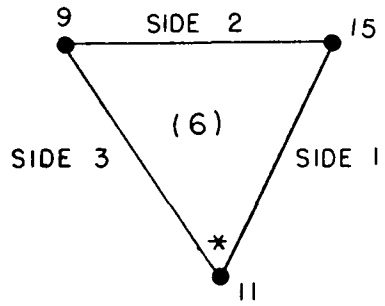
Data establishing sides of elements that are surfaces with convection and elements with internal heat generation must be input. The element number, the number of the side or sides having convection, and the internal heat generation per unit volume must be input for each element having convection and/or internal heat generation. Sides are numbered 1, 2, and 3 counter-clockwise from the first input node for the element. Heat added to the body is considered positive.

A maximum of two sides may have convection, therefore, the data input as:

NELC - element no.,	ISIDE (1) - 1st side for convection	ISIDE (2) - 2nd side for convection	QG - internal heat generation per unit volume
---------------------	-------------------------------------	-------------------------------------	---

If only one side has convection, a zero value is entered in the second side for convection location.

Assuming that element (6) shown below has surface convection on sides 2 and 3, and no internal heat generation,

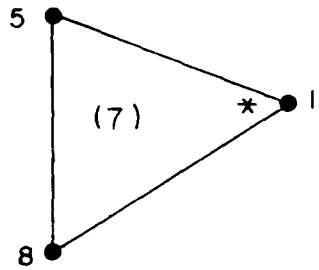


\*designates first input node

the convection and heat generation data appears as

6,	2,	3,	0.0
----	----	----	-----

Assuming element (7) shown below has convection on side 2 only and an internal heat generation of 10.5 units/unit volume,



\*designates first input node

the convection and heat generation data appears as 

7,	2,	0,	10.5
----	----	----	------

 .

This data set must be terminated with a card with element number, side number, and heat generation all equal to zero, i.e. 

0,	0,	0,	0
----	----	----	---

 .

IF NO CONVECTION OR INTERNAL HEAT GENERATION OCCURS for the problem, a card with zero values 

0,	0,	0,	0
----	----	----	---

 , must be input for the convection and internal heat generation data.

This data must be ordered by element numbers with one card per element.

### 3.5.5 Initial Nodal Temperatures

Input of the initial temperature value for each node must be ordered by node numbers. For example, if a body had 52 nodes, fifty-two values for initial temperature must be input.

### 3.5.6 Fixed Nodal Values

Nodal numbers for those nodes which are to remain at their initially prescribed values must be input. Node numbers are input in sets of ten per card. If a full set of ten is not required, zero values should be used to complete the set of 10. For example, if nodes 12, 14, 18, 19, 23, and 26 are to be held at the initial temperature, the Fixed Nodal Values would read

12,	14,	18,	19,	23,	26,	0,	0,	0,	0
-----	-----	-----	-----	-----	-----	----	----	----	---

 .

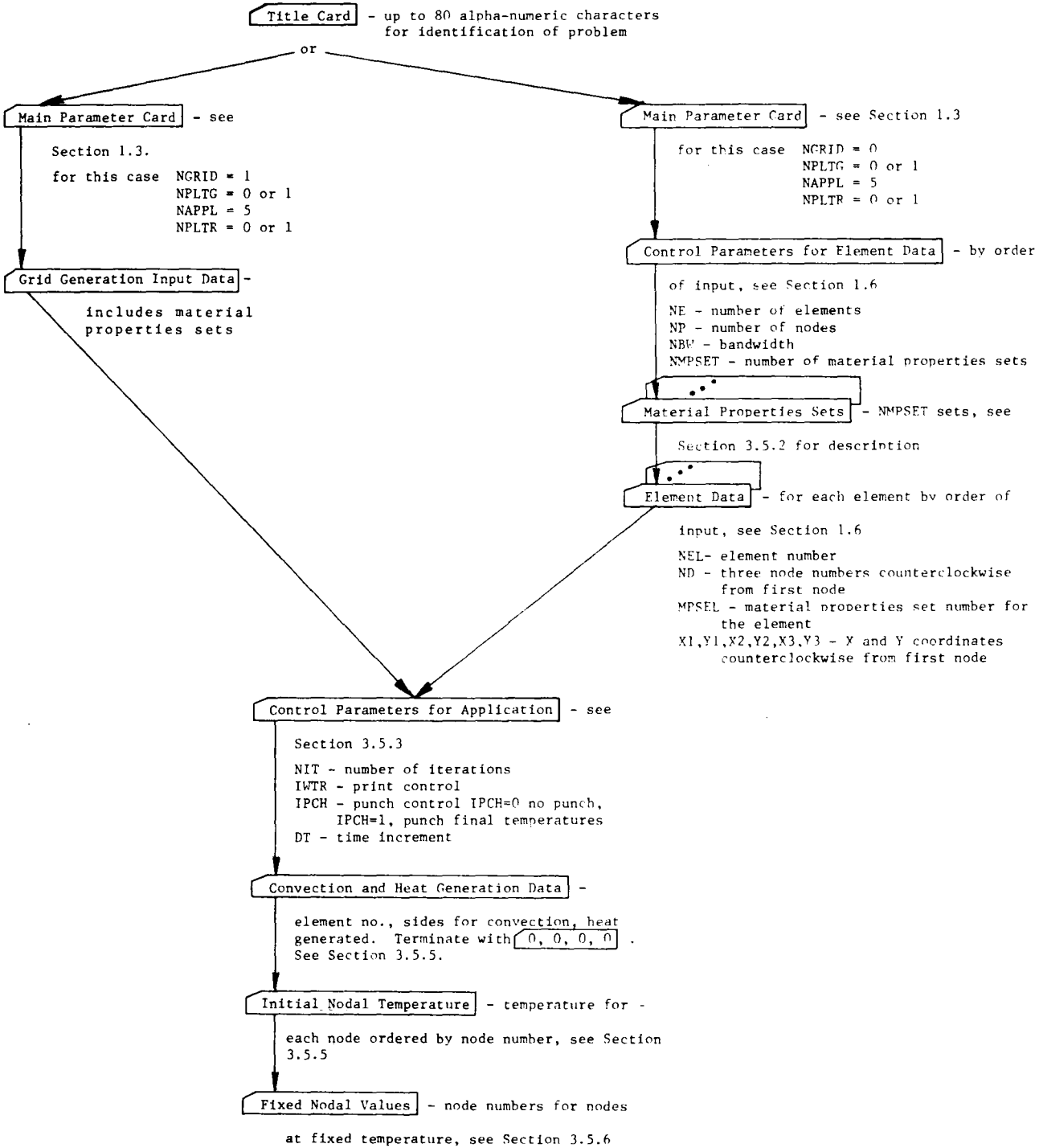
If an even multiple of ten values occurs or if no nodal values are to be fixed, a card with ten zero values must be input to terminate that section of data input.

Table 3.5.1 Program Input for 2-D Transient Heat Transfer

Auto-grid case:

Non-auto-grid case:

(common)



### 3.5.7 Input Description

Table 3.5.1 shows the inputs and order of input required for the two possible cases using 2-D transient heat transfer. The first case, auto-grid case, assumes use of the automatic grid generation program. The second case, non-auto-grid case, assumes the users will input the element data from cards. The element data must be ordered by element number for this program.

### 3.5.8 Sample Problem

Assume the body shown below is inserted through an insulated wall. The body was at an initial temperature of 20°C. Assume at time t=0 the left side is subjected to convection as shown and that the right side stays at 20°C. Determine temperature change in the body with time.

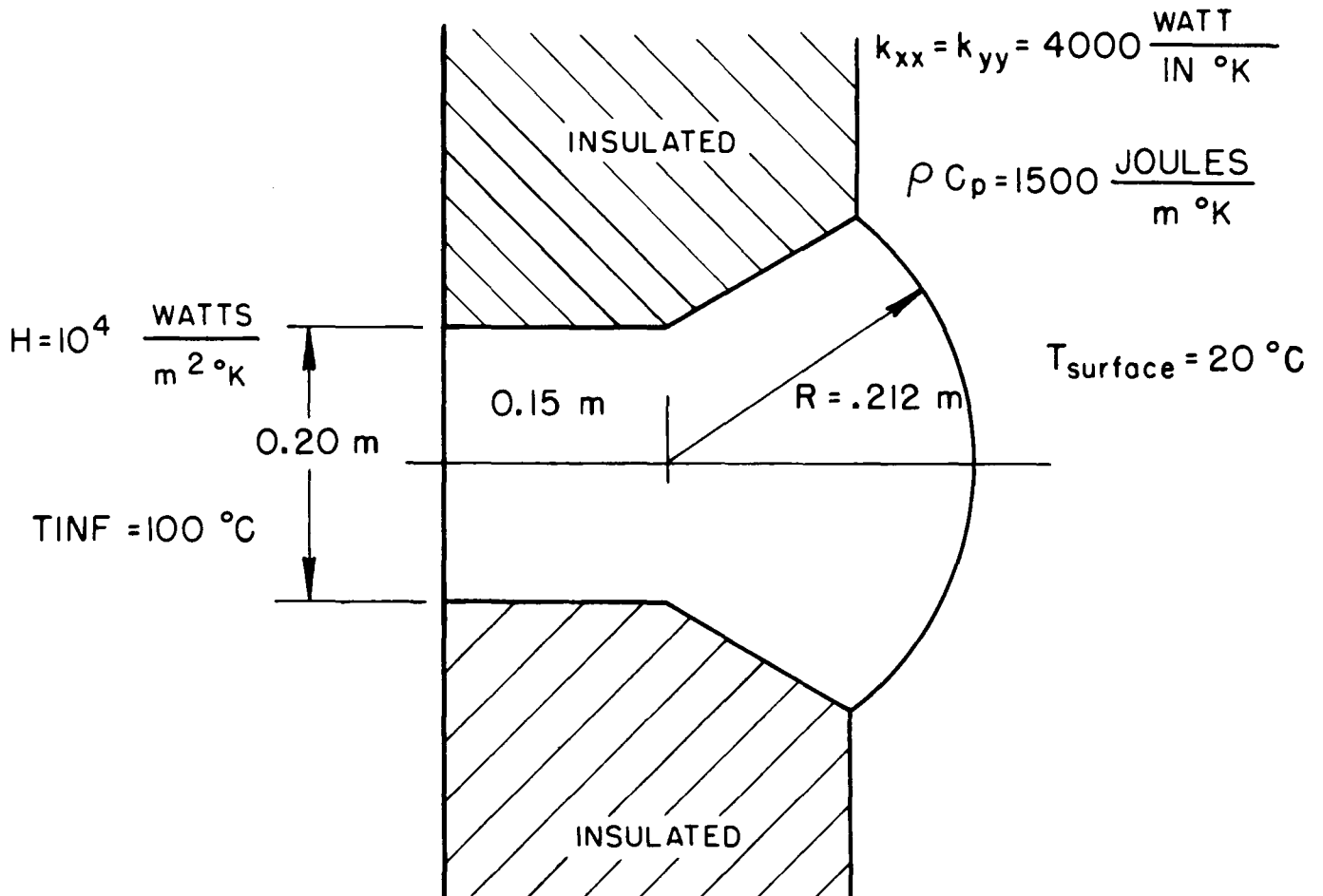




Figure 3.5.2 Output for Sample Transient Heat Transfer

TRANSIENT HEAT TRANSFER EXAMPLE WITH GRID GENERATION

THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM TRANSIENT HEAT TRANSFER WILL BE SOLVED

GLOBAL COORDINATES

NUMBER	X COORD	Y COORD
1	0	0
2	.07	0
3	.15	0
4	.26	0
5	.36	0
6	0	.05
7	.15	.05
8	.35	.08
9	0	.10
10	.07	.10
11	.15	.10
12	.28	.13
13	.30	.15

CONNECTIVITY DATA

REGION	SIDE			
	1	2	3	4
1	0	2	0	0
2	0	0	0	1

MATERIAL PROPERTIES

SET	MAT PROP 1	MAT PROP 2	MAT PROP 3	MAT PROP 4	MAT PROP 5
	KXX	KYY	H	TINF	CRHO
1	.4000E+04	.4000E+04	.1000E+05	.1000E+03	.1500E+04

REGION DATA

REGION	ROWC	COLS	MATERIAL		INPUT REGION NODE NO.							
			SET		1	2	3	7	11	10	9	6
1	5	5	1		1	2	3	7	11	10	9	6
2	5	5	1		3	4	5	8	13	12	11	7

NODE NUMBERS  
OF SUBDIVIDED REGION

1	1	3	6	10	15
	2	4	7	11	16
	5	8	12	17	20
	9	13	18	24	29
	14	19	25	32	37
2	15	21	28	36	45
	16	20	26	33	41
	22	23	27	34	42
	29	30	31	35	43
	37	38	39	40	44

ELEMENT DATA, REGION 1 MAT. PROP. SET = 1

NEL	NODE NUMBERS			X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
1	2	4	3	0	.0750	.0300	.0750	.0300	.1000
2	2	3	1	0	.0750	.0300	.1000	0	.1000
3	4	7	6	.0300	.0750	.0650	.0750	.0650	.1000
4	4	6	3	.0300	.0750	.0650	.1000	.0300	.1000
5	7	11	10	.0650	.0750	.1050	.0750	.1050	.1000
6	7	10	6	.0650	.0750	.1050	.1000	.0650	.1000
7	11	16	15	.1050	.0750	.1500	.0750	.1500	.1000
8	11	15	10	.1050	.0750	.1500	.1000	.1050	.1000
9	5	8	4	0	.0500	.0300	.0500	.0300	.0750
10	5	4	2	0	.0500	.0300	.0750	0	.0750
11	8	12	7	.0300	.0500	.0650	.0500	.0650	.0750
12	8	7	4	.0300	.0500	.0650	.0750	.0300	.0750
13	12	17	11	.0650	.0500	.1050	.0500	.1050	.0750
14	12	11	7	.0650	.0500	.1050	.0750	.0650	.0750
15	17	22	16	.1050	.0500	.1500	.0500	.1500	.0750
16	17	16	11	.1050	.0500	.1500	.0750	.1050	.0750
17	9	13	8	0	.0250	.0300	.0250	.0300	.0500
18	9	8	5	0	.0250	.0300	.0500	0	.0500
19	13	18	12	.0300	.0250	.0650	.0250	.0650	.0500
20	13	12	8	.0300	.0250	.0650	.0500	.0300	.0500
21	18	24	17	.0650	.0250	.1050	.0250	.1050	.0500
22	18	17	12	.0650	.0250	.1050	.0500	.0650	.0500
23	24	29	22	.1050	.0250	.1500	.0250	.1500	.0500
24	24	22	17	.1050	.0250	.1500	.0500	.1050	.0500
25	14	19	13	0	0	.0300	0	.0300	.0250
26	14	13	9	0	0	.0300	.0250	0	.0250
27	19	25	18	.0300	0	.0650	0	.0650	.0250
28	19	18	13	.0300	0	.0650	.0250	.0300	.0250
29	25	32	24	.0650	0	.1050	0	.1050	.0250
30	25	24	18	.0650	0	.1050	.0250	.0650	.0250
31	32	37	29	.1050	0	.1500	0	.1500	.0250
32	32	29	24	.1050	0	.1500	.0250	.1050	.0250

ELEMENT DATA, REGION 2 MAT. PROP. SET = 1

NEL	NODE NUMBERS			X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
33	16	20	15	.1500	.0750	.2223	.0855	.1500	.1000
34	20	21	15	.2223	.0855	.2250	.1125	.1500	.1000
35	20	26	21	.2223	.0855	.2759	.0960	.2250	.1125
36	26	28	21	.2759	.0960	.2750	.1250	.2250	.1125
37	26	33	28	.2759	.0960	.3107	.1065	.2750	.1250
38	33	36	28	.3107	.1065	.3000	.1375	.2750	.1250
39	33	41	36	.3107	.1065	.3268	.1170	.3000	.1375
40	41	45	36	.3268	.1170	.3000	.1500	.3000	.1375
41	22	23	16	.1500	.0500	.2178	.0577	.1500	.0750
42	23	20	16	.2178	.0577	.2223	.0855	.1500	.0750
43	23	27	20	.2178	.0577	.2730	.0655	.2223	.0855
44	27	26	20	.2730	.0655	.2759	.0960	.2223	.0855
45	27	34	26	.2730	.0655	.3157	.0732	.2759	.0960
46	34	33	26	.3157	.0732	.3107	.1065	.2759	.0960
47	34	42	33	.3157	.0732	.3460	.0810	.3107	.1065
48	42	41	33	.3460	.0810	.3268	.1170	.3107	.1065
49	29	30	22	.1500	.0250	.2113	.0292	.1500	.0500
50	30	23	22	.2113	.0292	.2178	.0577	.1500	.0500
51	30	31	23	.2113	.0292	.2664	.0335	.2178	.0577
52	31	27	23	.2664	.0335	.2730	.0655	.2178	.0577
53	31	35	27	.2664	.0335	.3152	.0377	.2730	.0655
54	35	34	27	.3152	.0377	.3157	.0732	.2730	.0655
55	35	43	42	.3152	.0377	.3577	.0420	.3460	.0810
56	35	42	34	.3152	.0377	.3460	.0810	.3157	.0732
57	37	38	29	.1500	0	.2030	0	.1500	.0250
58	38	30	29	.2030	0	.2113	.0292	.1500	.0250
59	38	39	30	.2030	0	.2560	0	.2113	.0292
60	39	31	30	.2560	0	.2664	.0335	.2113	.0292
61	39	40	31	.2560	0	.3090	0	.2664	.0335
62	40	35	31	.3090	0	.3152	.0377	.2664	.0335
63	40	44	35	.3090	0	.3620	0	.3152	.0377
64	44	43	35	.3620	0	.3577	.0420	.3152	.0377

NUMBER OF ELEMENTS = NE = 64

NUMBER OF NODES = NP = 45

BANDWIDTH = NBW = 10

CONVECTION FROM SIDE 3 OF ELEMENT 2  
CONVECTION FROM SIDE 3 OF ELEMENT 10  
CONVECTION FROM SIDE 3 OF ELEMENT 18  
CONVECTION FROM SIDE 3 OF ELEMENT 26

TRANSIENT ANALYSIS

NO. ITERATION 20		TIME INCREMENT .00100					
TIME = 0							
1	.200000E+02	2	.200000E+02	3	.200000E+02	4	.200000E+02
5	.200000E+02	6	.200000E+02	7	.200000E+02	8	.200000E+02
9	.200000E+02	10	.200000E+02	11	.200000E+02	12	.200000E+02
13	.200000E+02	14	.200000E+02	15	.200000E+02	16	.200000E+02
17	.200000E+02	18	.200000E+02	19	.200000E+02	20	.200000E+02
21	.200000E+02	22	.200000E+02	23	.200000E+02	24	.200000E+02
25	.200000E+02	26	.200000E+02	27	.200000E+02	28	.200000E+02
29	.200000E+02	30	.200000E+02	31	.200000E+02	32	.200000E+02
33	.200000E+02	34	.200000E+02	35	.200000E+02	36	.200000E+02
37	.200000E+02	38	.200000E+02	39	.200000E+02	40	.200000E+02
41	.200000E+02	42	.200000E+02	43	.200000E+02	44	.200000E+02
45	.200000E+02						

TIME = .0045							
1	.313277E+02	2	.313008E+02	3	.289222E+02	4	.289143E+02
5	.312846E+02	6	.266307E+02	7	.266295E+02	8	.289055E+02
9	.312683E+02	10	.246362E+02	11	.246424E+02	12	.266281E+02
13	.288966E+02	14	.312412E+02	15	.230622E+02	16	.231062E+02
17	.246495E+02	18	.266266E+02	19	.288837E+02	20	.220904E+02
21	.221332E+02	22	.231294E+02	23	.220522E+02	24	.245558E+02
25	.265251E+02	26	.223542E+02	27	.219102E+02	28	.230153E+02
29	.231486E+02	30	.220703E+02	31	.216957E+02	32	.245601E+02
33	.226511E+02	34	.218410E+02	35	.214433E+02	36	.252889E+02
37	.231743E+02	38	.221630E+02	39	.216868E+02	40	.213727E+02
41	.200000E+02	42	.200000E+02	43	.200000E+02	44	.200000E+02
45	.200000E+02						

TIME = .0095							
1	.357253E+02	2	.357078E+02	3	.334254E+02	4	.334201E+02
5	.356774E+02	6	.310812E+02	7	.310806E+02	8	.334146E+02
9	.356870E+02	10	.288301E+02	11	.288351E+02	12	.310309E+02
13	.334091E+02	14	.356594E+02	15	.267759E+02	16	.268411E+02
17	.288484E+02	18	.310809E+02	19	.334037E+02	20	.249598E+02
21	.249283E+02	22	.268770E+02	23	.249980E+02	24	.288566E+02
25	.310797E+02	26	.246685E+02	27	.242424E+02	28	.253207E+02
29	.269017E+02	30	.251035E+02	31	.240834E+02	32	.288604E+02
33	.246771E+02	34	.237456E+02	35	.232738E+02	36	.275552E+02
37	.269262E+02	38	.252987E+02	39	.241881E+02	40	.232839E+02
41	.200000E+02	42	.200000E+02	43	.200000E+02	44	.200000E+02
45	.200000E+02						

TIME = .0145							
1	.389134E+02	2	.389002E+02	3	.367065E+02	4	.367027E+02
5	.388927E+02	6	.343897E+02	7	.343899E+02	8	.366991E+02
9	.388351E+02	10	.320691E+02	11	.320776E+02	12	.343514E+02
13	.366953E+02	14	.388718E+02	15	.298112E+02	16	.298867E+02
17	.320899E+02	18	.343925E+02	19	.306913E+02	20	.275093E+02
21	.274285E+02	22	.299287E+02	23	.275971E+02	24	.320092E+02
25	.343919E+02	26	.268050E+02	27	.263953E+02	28	.274593E+02
29	.299553E+02	30	.277580E+02	31	.262923E+02	32	.321031E+02
33	.265588E+02	34	.255308E+02	35	.250044E+02	36	.295112E+02
37	.299772E+02	38	.280157E+02	39	.264910E+02	40	.250553E+02
41	.200000E+02	42	.200000E+02	43	.200000E+02	44	.200000E+02
45	.200000E+02						

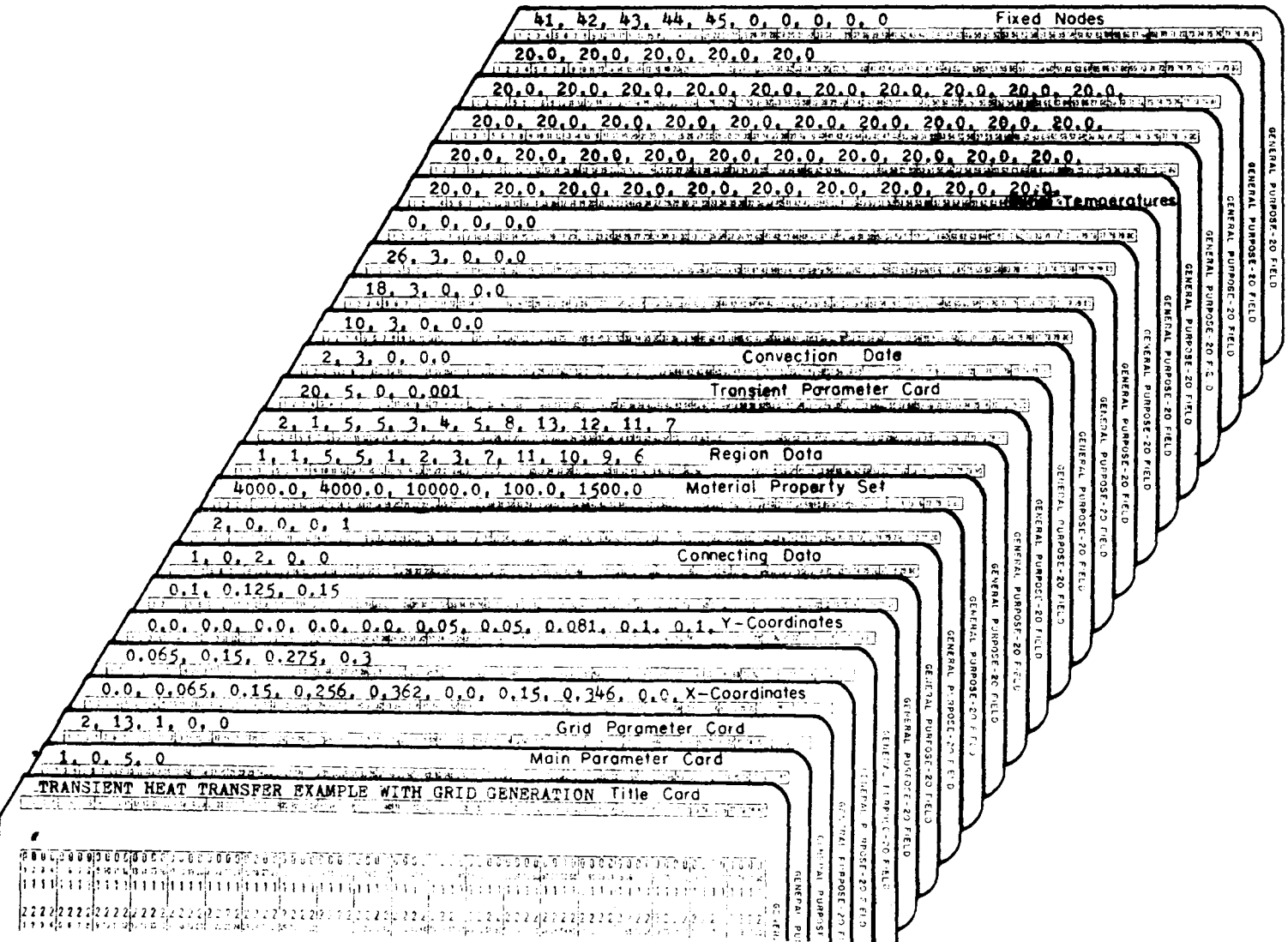
TIME = .0195							
1	.414057E+02	2	.413953E+02	3	.392746E+02	4	.392717E+02
5	.413897E+02	6	.369918E+02	7	.369926E+02	8	.392693E+02
9	.413840E+02	10	.346391E+02	11	.346497E+02	12	.369850E+02
13	.392663E+02	14	.413736E+02	15	.322571E+02	16	.323394E+02
17	.346632E+02	18	.369959E+02	19	.302631E+02	20	.298137E+02
21	.294959E+02	22	.323852E+02	23	.297370E+02	24	.346735E+02
25	.369958E+02	26	.280908E+02	27	.281050E+02	28	.298380E+02
29	.324127E+02	30	.292070E+02	31	.281293E+02	32	.347727E+02
33	.281353E+02	34	.270295E+02	35	.264561E+02	36	.313243E+02
37	.324323E+02	38	.302405E+02	39	.293080E+02	40	.265559E+02
41	.200000E+02	42	.200000E+02	43	.200000E+02	44	.200000E+02
45	.200000E+02						



ELEMENT RESULTANTS

ELEMENT	GRAD(X)	GRAD(Y)	AVE TEMP				
1	-.70737E+02	.11297E+00	.39931E+02	33	-.37225E+02	-.32913E+01	.31405E+02
2	-.71037E+02	.41281E+00	.40692E+02	34	-.36702E+02	-.63404E+00	.30455E+02
3	-.65118E+02	-.30587E-01	.37752E+02	35	-.18614E+02	-.24945E+01	.29233E+02
4	-.65221E+02	.11297E+00	.36512E+02	36	-.10664E+02	.22025E+02	.29102E+02
5	-.58574E+02	-.42246E+00	.35427E+02	37	-.19621E+02	.21755E+02	.28654E+02
6	-.58819E+02	-.30587E-01	.36293E+02	38	.27308E+02	.11223E+03	.29556E+02
7	-.51339E+02	-.32913E+01	.33092E+02	39	-.46921E+03	-.9550E+02	.26437E+02
8	-.52933E+02	-.42246E+00	.33349E+02	40	-.11176E+04	-.20534E+03	.23775E+02
9	-.70679E+02	.96390E-01	.39977E+02	41	-.38878E+02	-.18325E+01	.31487E+02
10	-.70787E+02	.22541E+00	.40636E+02	42	-.37960E+02	.17726E+01	.30553E+02
11	-.64980E+02	-.97248E-01	.37752E+02	43	-.27927E+02	.12399E+00	.29192E+02
12	-.65118E+02	.96390E-01	.38511E+02	44	-.22046E+02	.15027E+02	.26793E+02
13	-.58297E+02	-.54070E+00	.35436E+02	45	-.30125E+02	.15723E+02	.27930E+02
14	-.58574E+02	-.97248E-01	.36212E+02	46	-.22076E+02	.27896E+02	.27918E+02
15	-.50621E+02	-.13325E+01	.33129E+02	47	-.23186E+03	-.20442E+01	.25055E+02
16	-.51339E+02	-.54070E+00	.33884E+02	48	-.37530E+03	-.20968E+03	.22712E+02
17	-.70574E+02	.10144E+00	.39973E+02	49	-.40287E+02	-.11006E+01	.31579E+02
18	-.70679E+02	.22777E+00	.40681E+02	50	-.33296E+02	.18266E+01	.30687E+02
19	-.64855E+02	-.72918E-01	.37753E+02	51	-.32877E+02	.37652E+00	.29268E+02
20	-.64980E+02	.10144E+00	.36510E+02	52	-.29042E+02	.20673E+01	.28687E+02
21	-.58090E+02	-.40287E+00	.35444E+02	53	-.35083E+02	.93192E+01	.27593E+02
22	-.58297E+02	-.72918E-01	.36213E+02	54	-.30278E+02	.16637E+02	.27227E+02
23	-.50234E+02	-.11006E+01	.33157E+02	55	-.14726E+03	-.44365E+02	.22152E+02
24	-.50621E+02	-.40287E+00	.33907E+02	56	-.23748E+03	.19916E+02	.24495E+02
25	-.70327E+02	.11915E+00	.39968E+02	57	-.41354E+02	-.78290E+00	.31695E+02
26	-.70574E+02	.41617E+00	.40675E+02	58	-.40443E+02	.11491E+01	.30864E+02
27	-.64773E+02	.44551E-02	.37752E+02	59	-.34755E+02	-.46455E+00	.29525E+02
28	-.64855E+02	.11915E+00	.39509E+02	60	-.33018E+02	.22041E+01	.28822E+02
29	-.57982E+02	-.15994E+00	.35446E+02	61	-.34756E+02	.27423E+01	.27654E+02
30	-.58090E+02	.44551E-02	.36222E+02	62	-.34540E+02	.30173E+01	.27047E+02
31	-.49888E+02	-.78259E+00	.33174E+02	63	-.12370E+03	.17631E+02	.24337E+02
32	-.50234E+02	-.15994E+00	.33921E+02	64	-.15017E+03	-.15196E+02	.22152E+02

Figure 3.5.3 Input for Transient Heat Transfer without Grid Generation



### 3.6 TRUSS

#### 3.6.1 Introduction

This program calculates the nodal displacements and member stresses for statistically determinate and indeterminate plane trusses. Temperature increase from the steady state conditions may be included. Material properties may be varied from member to member. The program uses only simple, one-dimensional elements (2 nodes/element). Automatic grid generation and grid plottings are not available for this program. Element data control parameters and the element data must be input by the user.

#### 3.6.2 Material Properties Description

The material properties sets for this program include, in the following order

- EM - Elastic modulus
- ALPHA - Coefficient of thermal expansion
- AREA - Cross-section area
- Dummy Variable 1 - = 0.0
- Dummy Variable 2 - = 0.0

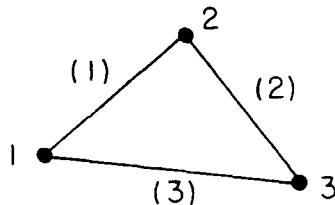
Any compatible set of units may be used. Up to twenty sets of material properties may be input.

#### 3.6.3 Simple Linear Element

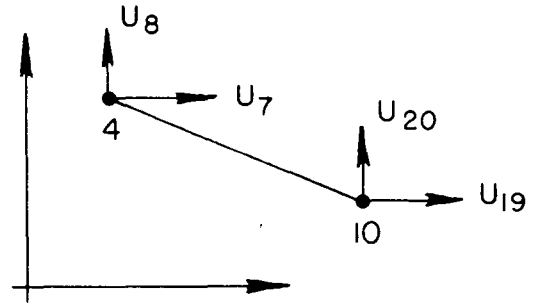
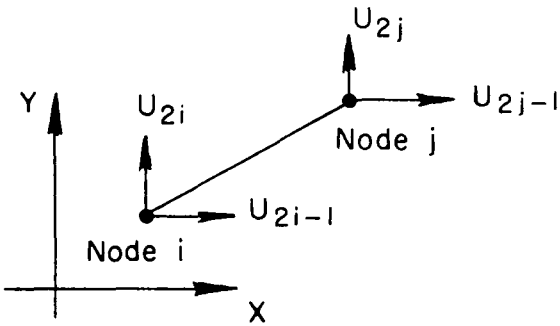
The element type used in this program consists of a 2 node linear element



Each node and each element must be numbered. The example below shows an assemblage of three elements:



Truss problems are complicated by having a vector unknown, displacement of nodes. The displacement is expressed as the sum of the components in the X and Y coordinate directions. The figures below demonstrate how the components at each node are expressed. At each node two degrees of freedom (global degrees of freedom) are used. They are two times the node number for the Y-component and two times the node number minus one for the X-component.



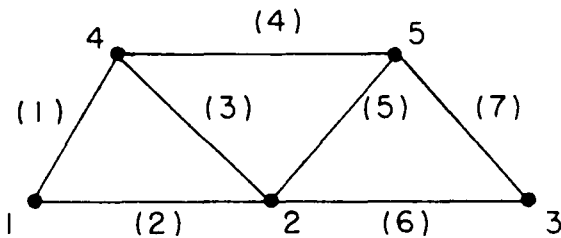
Input of force values and known nodal displacements are also input in vector components keyed to the global degree of freedom.

### 3.6.4 Element Data Control Parameters

Element data control parameters establish the size of the problem and the number of material properties sets to be used. Element data control parameters for Truss include, in order of input

- NE - number of elements
- NP - number of nodes
- NBW - bandwidth = (maximum difference in node numbers of any element + 1) \* 2
- NMPSET - number of material properties sets to be input.

Sample bandwidth determination



$$\text{Bandwidth} = ((4-1) + 1) * 2 = 8$$

### 3.6.5 Element Data

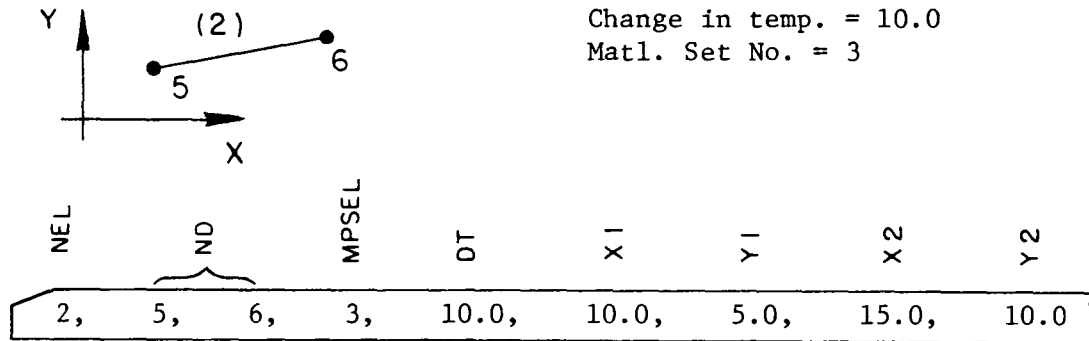
The element data establish the element numbers, node numbers for the elements, number of the material properties set for each element, the change in temperature for each element, and the X and Y coordinates of the nodes for the truss under study.

Element data for each element are input in the following order

- NEL - element number
- ND - node numbers
- MPSEL - material properties set number for the element
- DT - increase in temperature of the element from the steady state
- X1,Y1,X2,Y2 - X and Y coordinates for element nodes in same order as node numbers were input.

Element data must be input with one element per card.

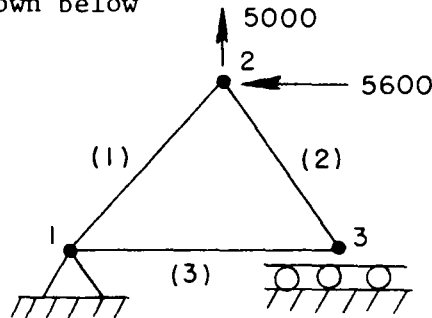
Sample element data card



3.6.6. Boundary Values

Boundary values are input in two sections. The first is nodal forces. The second is known displacements. Both are input by the global degrees of freedom at each node and the value of the vector component parallel to the direction of that degree of freedom. Each card must have six sets consisting of global degree of freedom and associated value. If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zeros (12 zeros) must follow to terminate that section of data input.

For the simple truss shown below



the nodal forces would be input as

3,	-5600.0,	4,	5000.0,	0,	0.0,	0,	0.0,	0,	0.0,	0,	0.0
----	----------	----	---------	----	------	----	------	----	------	----	-----

and the known nodal values (displacements)

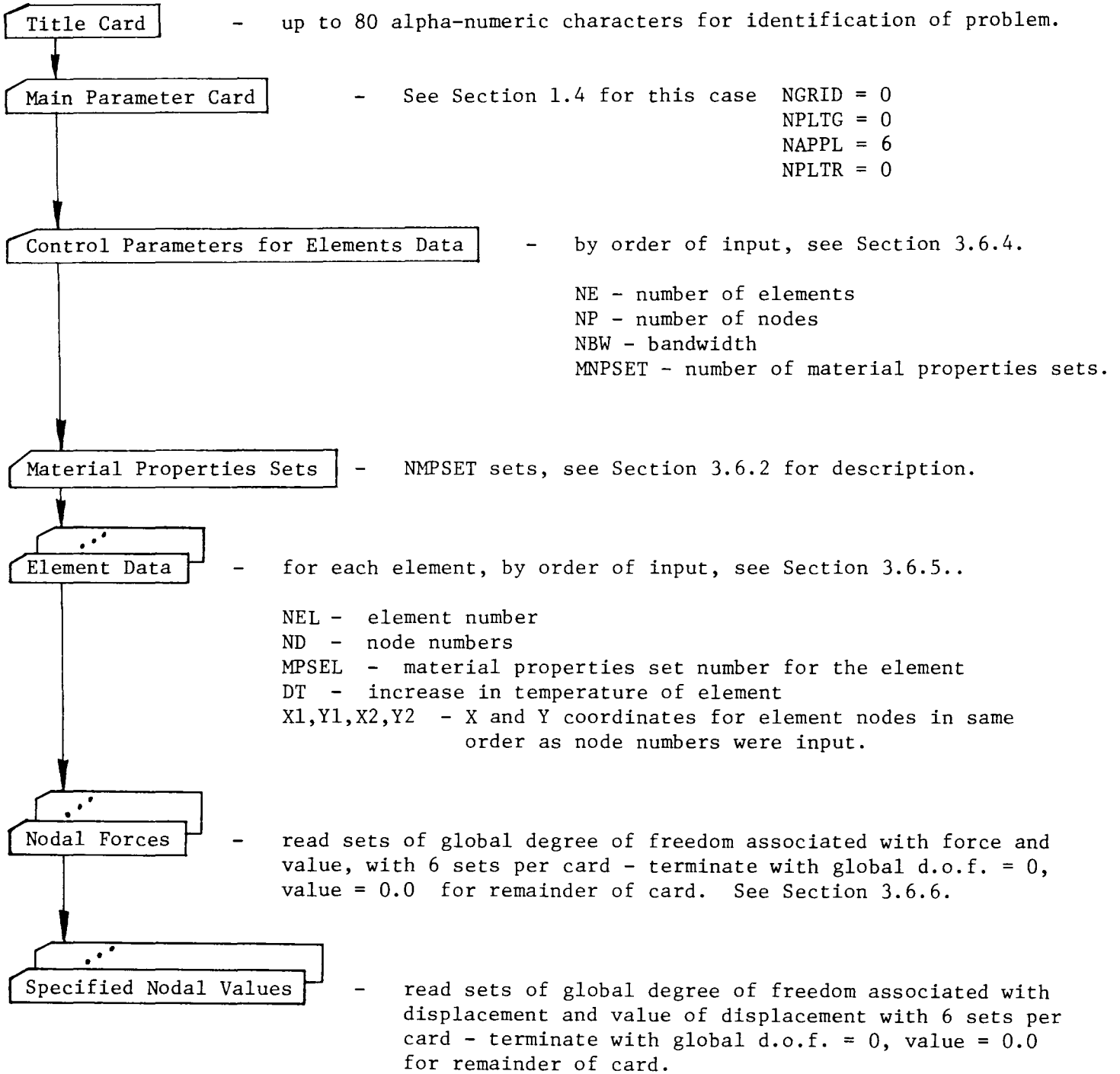
1,	0.0,	2,	0.0,	6,	0.0,	0,	0.0,	0,	0.0,	0,	0.0
----	------	----	------	----	------	----	------	----	------	----	-----

Note sign convention has been established such that forces and displacements in the positive coordinate direction are assumed positive. Those in the opposite direction are assumed negative.

3.6.7 Input description

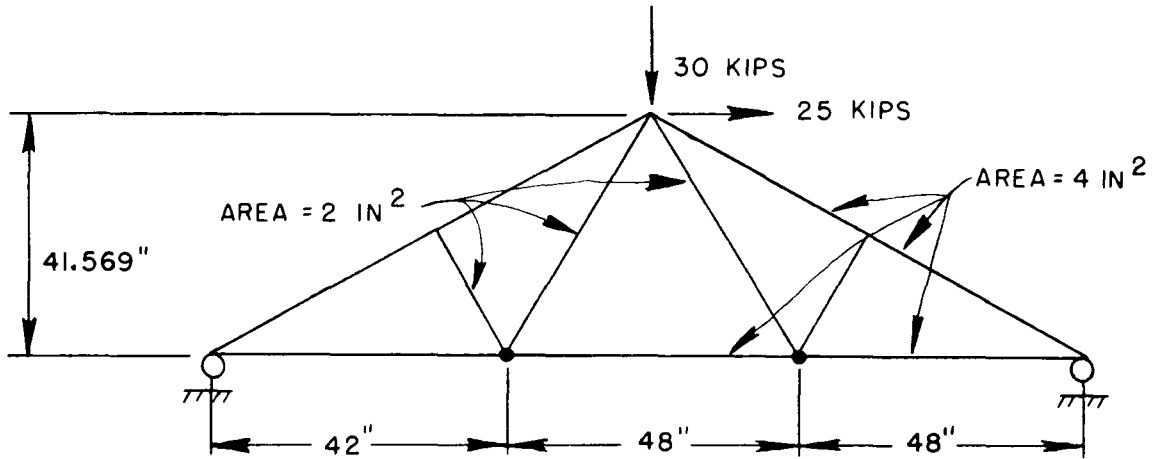
Table 3.6.1 shows the inputs required for the truss program.

Table 3.6.1 Program Input for Truss



### 3.6.8 Sample Problem

Consider the truss problem diagrammed below. The truss is assumed to be fixed at both ends.



The following figure shows the truss with elements and nodes labeled for input.

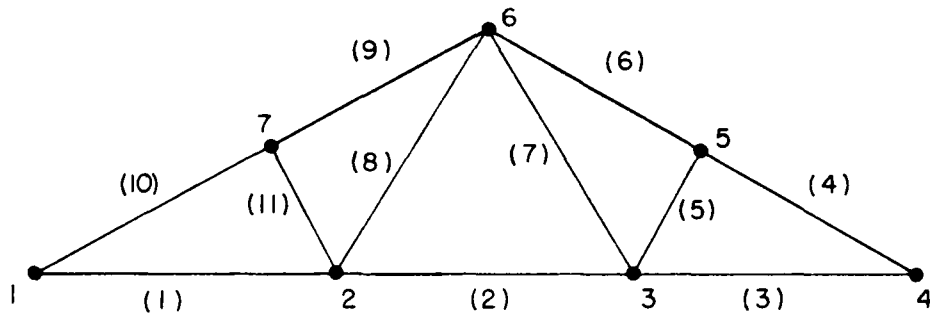


Figure 3.6.1 shows the input for this problem. Note that units must be consistent (i.e., if material properties are in inches, nodal distance must be in inches).

Figure 3.6.2 shows the output of the truss program for this problem.



Figure 3.6.2 Output for Truss Example Problem

BOUNDARY VALUES

NODAL FORCES

45	.44740E+02	42	.89500E+02	43	.89500E+02	44	.89500E+02	40	.73950E+02	33	.58440E+02
26	.58500E+02	20	.58540E+02	14	.68260E+02	9	.78000E+02	3	.78000E+02	1	.78000E+02
5	.39000E+02	23	-.15000E+04								

PRESCRIBED NODAL VALUES

17	.18000E+03	12	.19500E+03	8	.19000E+03	4	.19786E+03	5	.20000E+03	36	.20000E+03
41	.20000E+03	45	.20000E+03								

NODAL VALUES

1	.19860E+03	2	.19382E+03	3	.19654E+03	4	.19786E+03	5	.20000E+03	6	.18779E+03
7	.19086E+03	8	.19000E+03	9	.19485E+03	10	.18380E+03	11	.18488E+03	12	.18500E+03
13	.18831E+03	14	.19395E+03	15	.18179E+03	16	.18202E+03	17	.18000E+03	18	.18021E+03
19	.18643E+03	20	.19407E+03	21	.18134E+03	22	.17943E+03	23	.16957E+03	24	.19235E+03
25	.18875E+03	26	.19494E+03	27	.17985E+03	28	.17904E+03	29	.18538E+03	30	.19333E+03
31	.18989E+03	32	.19174E+03	33	.19629E+03	34	.17998E+03	35	.18713E+03	36	.20000E+03
37	.19685E+03	38	.19434E+03	39	.19454E+03	40	.19775E+03	41	.20000E+03	42	.19917E+03
43	.19716E+03	44	.19682E+03	45	.20000E+03						

ELEMENT VELOCITY COMPONENTS

ELEMENT	VEL(X)	VEL(Y)			
1	.46946E-01	.42475E-01			
2	.15711E+00	-.10358E-01			
3	-.58564E-01	.83084E+00			
4	.79857E+00	.22079E+00			
5	-.45614E+00	-.13340E-01			
6	-.95121E+00	.51203E+00			
7	-.29485E+00	-.42912E-03			
8	-.31874E+00	.36081E-01			
9	-.56860E-01	.19381E+00			
10	.46946E-01	.11940E+00			
11	-.19203E+00	.30440E+00			
12	-.58564E-01	.19340E+00			
13	-.33629E+00	.82581E-01			
14	-.45614E+00	.21007E+00			
15	-.27416E+00	.68246E-01			
16	-.29485E+00	.97486E-01			
17	-.13464E+00	.10995E+00			
18	-.56860E-01	.35734E-01			
19	-.24404E+00	.14582E+00			
20	-.19203E+00	.96334E-01			
21	-.33838E+00	.11510E+00			
22	-.33629E+00	.11288E+00			
23	-.24764E+00	.10377E+00			
24	-.27416E+00	.13820E+00			
25	-.28000E+00	.29929E-01			
26	-.13464E+00	-.14324E+00			
27	-.28000E+00	.76988E-01			
28	-.24404E+00	.38460E-01			
29	-.44016E+00	.16493E+00			
30	-.33838E+00	.56140E-01			
31	-.11984E+00	.82916E-01			
32	-.24764E+00	.23418E+00			
33	.24017E+00	-.23125E+00			
34	.66731E-01	0			
35	.14941E+00	-.28199E+00			
36	.10734E+00	-.21281E+00			
37	-.51751E-01	-.22221E+00			
38	-.35611E-01	-.25175E+00			
39	-.14416E+00	-.19099E+00			
40	-.12926E+00	-.21854E+00			
41	.65583E+00	-.39159E+00			
42	.28995E+00	0	58	.17127E+00	-.56514E+00
43	.23932E+00	-.33378E+00	59	.79857E+00	-.65214E+00
44	.12187E+00	-.42714E+00	60	.17531E+00	-.11964E+01
45	-.14992E+00	-.26878E+00	61	-.95121E+00	-.10127E+01
46	-.80879E-01	-.39403E+00	62	-.39998E+00	-.20958E+00
47	-.19405E+00	-.19184E+00	63	-.31874E+00	-.90455E-01
48	-.15604E+00	-.26849E+00	64	-.26342E+00	-.21598E+00
49	.15756E+00	-.69939E+00			
50	.54831E+00	-.10296E+01			
51	.20703E+00	-.70625E+00			
52	.18983E+00	-.72056E+00			
53	-.41948E+00	-.62112E+00			
54	-.15177E+00	-.28673E+00			
55	-.26121E+00	-.15693E+00			
56	-.20204E+00	-.28437E+00			
57	.15711E+00	-.57193E+00			



#### 4. GRID PLOTTING

Grid plotting is intended to aid the user in visualization of the grid developed and in development of boundary condition inputs for application programs using the three-node linear triangular element. The user needs only specify the parameter  $NPLTG = 1$  on the main control parameter card to obtain a grid plot. No additional input is required. Automatic grid generation is not required for use of the grid plotting program.

The program automatically produces a ten-inch square plot with an asterisk (\*) at each nodal location. The plot scale is determined by the element data and is printed at the top of each plot. The same scale is used in both the X and Y directions.

Figure 4.1 shows a sample plot. Figure 4.2 shows how the user, with the help of the element data printed out prior to the grid plot, might develop the plot to assist in determination of nodal values for boundary conditions.





## 5. RESULT PLOTTING

### 5.1 PURPOSE

Interpretation and use of the large amount of data which can be produced by a finite element program is often tedious and difficult. The purpose of the result plotting program is to assist in interpretation of pertinent output variables. The program produces line-printer plots of the same scale and size as those produced by the grid plotting program (ten inch square plot).

### 5.2 REQUIREMENT FOR USE

The user need only specify the parameter  $NPLTR = 1$  on the main control parameter card to obtain result plots. No additional input is required. Table 5.1 below shows the results which will be plotted for each application program.

Table 5.1 Result Plotting

<u>Application Program</u>	<u>NAPPL</u>	<u>Variables Plotted</u>	<u>Nodal or Element Values</u>
2-D Elasticity	1	Stress X-direction Stress Y-direction Shear Stress Maximum Principal Stress Minimum Principal Stress Principal Shear Stress	Element Element Element Element Element Element
2-D Heat Transfer	2	Temperatures	Nodal
2-D Groundwater	3	Potentials Velocity X-direction Velocity Y-direction	Nodal Element Element
Torsion	4	Shear Stress ZX Shear Stress ZY Maximum Shear Stress	Element Element Element
2-D Transient Heat Transfer	5	Temperatures - each time step at which values are printed.	Nodal
Truss	6	Result plotting not available.	

### 5.3 PLOTTING PROCEDURE AND SAMPLE PLOT

This program divides the range of the variable to be plotted into ten equal subranges which are labelled 0 through 9. Subrange values are printed at the right of each plot. Appropriate subrange labels are then plotted at nodal or element centriodal locations. Figure 5.1 shows a sample result plot.



Appendix 1. Sample Time and Core Memory Requirements

The table below gives execution time and core memory requirements for sample moderate size problems.

<u>Application Program</u>	<u>Number of Elements</u>	<u>Bandwidth</u>	<u>Time (sec)</u>		
			<u>w/Grid Generation w/All Plots</u>	<u>w/o Grid Generation w/o Plot</u>	<u>w/o Grid Generation w/o Plots</u>
2-D Elasticity	96	6	7.9	2.7*	2.0
2-D Heat Transfer	96	7	5.9	2.1*	1.4
2-D Groundwater	96	6	5.9	2.0*	1.3
Torsion	96	6	4.5	2.1*	1.4
2-D Transient Heat Transfer -100 iterations	108	9	14.1	6.1	5.4
Truss	23	10	---	---	0.4**

\*Minimum Core Memory = 46000

\*\*Minimum Core Memory = 21000

Appendix 2 - Control Cards for Using FEMPAC

The following control cards are necessary to obtain and execute this package. Details on those cards not listed completely can be obtained from appropriate University Computer Center Documentation or the KRONOS manual.

JOB card  
ACCOUNT card  
FETCH, FEMPAC.  
FEMPAC.  
Δ (7-8-9 card)  
    FEMPAC input cards.  
    ⋮  
□ (6-7-8 card)