HOT or Not: Driver Elasticity to Price and Alternative Pricing Strategies on the MnPASS HOT Lanes

A THESIS

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Abstract

The Minnesota Department of Transportation (MnDOT) has added MnPASS High Occupancy Toll (HOT) lanes on two freeway corridors in the Twin Cities. While not the first HOT lanes in the country, the MnPASS lanes are the first implementation of road pricing in Minnesota and possess a dynamic pricing schedule. Tolls charged to single occupancy vehicles (SOVs) are adjusted every three minutes according to HOT lane vehicle density. Given the infancy of systems like MnPASS, questions remain about drivers responses to toll prices. Three field experiments were conducted on the corridors during which prices were changed. Data from the field experiments as well as two years of toll and traffic data were analyzed to measure driver responses to pricing changes. Driver elasticity to price was positive with magnitudes less than 1.0. This positive relationship between price and demand is in contrast with the previously held belief that raising the price would discourage demand. In addition, drivers consistently paid between approximately \$60-120 per hour of travel time savings, much higher than MnDOT's value of time (VOT) of \$15/hr. Reasons for this include the value drivers place on reliability, a misperception about the actual time savings and that MnPASS users have a greater VOT than the average driver. Four alternative pricing strategies are then proposed. These pricing strategies were tested using a HOT lane choice model based on previous research. The share of transponder owning SOVs using the MnPASS lane was measured against price producing positive elasticity values at lower prices and negative elasticity values at higher prices. MnPASS lane usage rises with price at lower tolls due to the increased time savings benefit but is eventually outweighed by the price, causing the lane share to decrease at higher tolls.

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Chapter 1

Introduction

Since 1992, the Minneapolis - St. Paul metropolitan area has used managed lanes to increase person throughput during peak periods [\(Doan,](#page-95-0) [2013\)](#page-95-0). With limited capacity and excess demand, speeds slow during the morning and afternoon commute. I-394 stretches from the western suburbs into downtown Minneapolis. The freeway originally contained high occupancy vehicle (HOV) lanes, including a two lane, barrier separated, reversible section. This section runs along approximately 1/3rd of the freeway's length. The remaining section contained one concurrent, double white line separated HOV lane running in each direction. In 2005, the Minnesota Department of Transportation (MnDOT) converted the HOV lanes on I-394 to high occupancy toll (HOT) lanes.

While the HOV lanes benefited carpools, motorcycles, and buses, single occupant vehicle (SOVs) drivers complained of their underutilization. In order to maintain the carpooling/transit incentive while utilizing the lanes to a greater extent, MnDOT explored the concept of HOT lanes. HOT lanes are a form of congestion pricing. They are tolled lanes (on otherwise untolled roads) which give a free or discounted trip for HOV users, and are thus optional. Other forms of congestion pricing may charge for use of all lanes.

Before the MnPASS lanes, no forms of road pricing had been adopted in Minnesota. Because the general purpose lanes remain free of charge, HOT lanes presented a more politically feasible option than other forms of congestion pricing [\(Fielding and Klein,](#page-95-1) [1993\)](#page-95-1). The MnPASS lanes would remain free of charge for carpools, buses, and motorcycles and the toll would regulate SOV use [\(Turnbull,](#page-97-0) [2008\)](#page-97-0). The toll helps ensure a high level of service (LOS) and provides some additional revenue [\(Konishi and Mun,](#page-96-0) [2010\)](#page-96-0). Level of service is an A-F scale defined in the Highway Capacity Manual with A representing free flow speeds and F breakdown [\(Transportation Research Board,](#page-97-1) [2000\)](#page-97-1).

Support for HOT lanes appears across various income levels, household sizes and educational levels [\(Munnich and Buckeye,](#page-96-1) [2007;](#page-96-1) [Burris et al.,](#page-94-1) [2007\)](#page-94-1). In addition, support tends to increase after implementation and is higher among areas with existing tolled roads [\(Finkleman et al.,](#page-95-2) [2011;](#page-95-2) [Burris et al.,](#page-94-1) [2007\)](#page-94-1). [Safirova et al.](#page-97-2) [\(2003\)](#page-97-2) believes that while HOT lanes benefit all income groups, they more greatly benefit the wealthy. [Mowday](#page-96-2) [\(2006\)](#page-96-2), on the other hand, believes HOT lanes are equitable due to users paying directly for use of the road. [Finkleman et al.](#page-95-2) [\(2011\)](#page-95-2) remarks that older, non-retired individuals and those new to their location support tolling more than others. While retired individuals may object to tolling due to their fixed income, [Burris and Pendyala](#page-94-2) [\(2002\)](#page-94-2) suggest that the retired and others on flexible schedules can more easily adjust their trips to avoid tolls and congestion.

While the idea seems to appease all sides of the debate, concerns arose, especially by those already using the HOV lanes [\(Burris et al.,](#page-94-1) [2007\)](#page-94-1). Transit proponents feared that the LOS in the HOT lanes would degrade [\(Turnbull,](#page-97-0) [2008\)](#page-97-0). [Turnbull](#page-97-0) [\(2008\)](#page-97-0) and [Burris](#page-94-3) [and Xu](#page-94-3) [\(2006\)](#page-94-3) analyzed the potential mode shift from transit to SOV. All cases resulted in either a statistically insignificant change or small enough change not to affect LOS. [Munnich](#page-96-1) [and Buckeye](#page-96-1) [\(2007\)](#page-96-1) observed a similar conservation of LOS on I-394 in Minneapolis after the conversion from HOV to HOT. In an analysis of the HOT lane conversion on I-85 in Atlanta, [Kall et al.](#page-95-3) [\(2009\)](#page-95-3) determined no statistically significant change in emissions levels due to mode shifting from the conversion. [Dahlgren](#page-95-4) [\(2002\)](#page-95-4) adds that lower emissions may result from HOT lanes due to reduced GP congestion.

In order to guarantee that HOVs could continue to use the HOT lanes at a high LOS, MnDOT adopted a dynamic pricing system. Similar systems had been adopted on several HOT lanes around the country, but none with such frequent price changes. The toll price for SOVs is displayed at various plazas along the corridor. Loop detectors monitor the density in the HOT lanes. As density in the HOT lanes rises, so too does the toll price. As congestion clears and density decreases, the price lowers again. Dynamic pricing, in theory, allows MnDOT to control the amount of SOV traffic in the HOT lanes and maintain a high LOS. This thesis reexamines that assumption.

Although I-394 was not the first HOT lane corridor in the country, few before had implemented a dynamic pricing scheme with such frequent pricing changes (every three minutes). The MnPASS Express Lanes, as the HOT lanes are called in Minnesota, have been running since 2005. In 2009, MnDOT added MnPASS lanes to the I-35W corridor. One MnPASS lane runs in each direction, separated by a double white line. In the southbound direction, the lane begins at 42nd Street South in Minneapolis and continues to the southern suburb of Lakeville. The northbound lane begins in Lakeville and continues to 38th Street South in Minneapolis where it becomes a priced dynamic shoulder lane (PDSL). The shoulder lane continues to downtown Minneapolis [\(MnDOT,](#page-96-3) [2013a\)](#page-96-3). The success of the lanes has created interest for expansion to other metro freeways [\(Cambridge Systematics,](#page-94-4) [2010\)](#page-94-4).

Given that dynamically priced HOT lanes is a relatively new concept, questions exist how optimal the current MnPASS pricing algorithm is at maximizing throughput while maintaining free flow speeds. The current algorithm operates by raising prices as the density in the MnPASS lanes rises. The assumption is that higher prices will dissuade usage and lower prices will entice users. Through this fluctuation in price, demand in the MnPASS lanes can be regulated and breakdown prevented.

This thesis first analyzes driving behavior, specifically looking at how much drivers pay for time savings and and their elasticity to change in price. By better understanding drivers responses to price, changes can be made to the pricing algorithm to better control the amount of demand. Current assumptions about drivers responses to pricing changes will be examined. Driving behavior is analyzed by looking at changes to price in demand using various data sources and methods.

Alternative pricing strategies are proposed which would allow MnPASS operators to more simply determine toll priced based on traffic density. The pricing strategies use simple functions to determine toll based on density. Unlike the current pricing algorithm, there is no need for various data tables. The current density (HOT and/or GP) directly determines the price. Three of the strategies account for GP density and tie the toll more closely to the time savings value provided by the MnPASS lanes. Each pricing strategy is presented including graphs comparing prices to the current algorithm and the respective HOT and GP densities.

These pricing strategies are then tested using a lane choice model based on previous research. The lane choice model is first calibrated. The calibrated lane choice model is then used to test each of the pricing strategies. Multiple iterations are run in which the share of transponder owning SOVs using the MnPASS lane is measured. The pricing coefficients are altered between each iteration. Finally, the HOT lane share is graphed versus price for each pricing strategy and demand elasticity to price is determined.

Data sources for these analyses include loop detectors, logs of price and density measurements from MnPASS as well as logs of individual MnPASS subscribers transponder data. The methods and results as well as their implications are discussed in the following sections.

The following table defines variables used throughout this thesis.

Variable	Definition	Introduced
B	The baseline period for the field experiment	Chapter 4
\mathcal{C}	The control period for the field experiment	Chapter 4
D	A measure of demand for the MnPASS lane (density, S_{MnPASS})	Chapter 3
Ε	The field experiment period	Chapter 4
HOT_{share}	The percentage of transponder owning SOVs which use the MnPASS lane	Chapter 6
Κ	Traffic density in vehicles/mile/lane	Chapter 2
L	SOV use of the MnPASS lane	Chapter 7
$\mathbf P$	Toll price in USD	Chapter 2
Q	Traffic flow	Chapter 3
R	Radio transponder ownership	Chapter 7
S_{MnPASS}	The percentage of freeway throughput using the MnPASS lane	Chapter 3
t	Time of MnPASS algorithm pricing changes	Chapter 3
T	The expected travel time along the corridor for the respective lane type	Chapter 7
$\Delta\mathrm{T}$	The difference in travel time between the HOT and GP lanes in minutes	Chapter 7
V	The expected travel time variability for the respective lane type	Chapter 7
ΔV	The difference in travel time variance in minutes	Chapter 7

Table 1.1: Variable Definitions

Chapter 2

Background

2.0.1 Frequency of Use

Each paying MnPASS user has a transponder, which communicates with detectors along the corridor to determine a user's entry and exit point and charge accordingly. The time of entry, amount charged and entry and exit plazas is recorded for each trip. This log was used to determine how frequently MnPASS subscribers pay to use the lanes. Subscribers are charged \$1.50 per month for leasing the transponder. The frequency of use analysis includes all subscribers throughout 2011 and 2012 and averages their use over the two-year period. It is not limited to those subscribers whose lease remains active over the entire two-year period.

The frequency of use analysis focused on personal and business accounts separately. No data were provided by MnPASS to specifically determine which accounts are business and which are personal. Therefore, the assumption was made that accounts with more than two transponders were business accounts, while those with one or two were personal. There are likely some personal accounts with more than two transponders and some business accounts with fewer than three, however two transponders was selected as a reasonable limit for most personal accounts. Personal and business accounts were separated based on the assumption that drivers with business accounts are less sensitive to price, because they are not charged the toll personally. This is true regardless of trip purpose. Individual accounts make up around 76 % of all MnPASS accounts. Unless explicitly stated, analysis throughout this thesis includes both business and personal accounts due to the inability to distinguish the two using loop detector data.

Figure [2.1](#page-15-1) below depicts the number of MnPASS subscribers in 2011 and 2012 and the breakdown based on frequency of use during the morning peak period (weekdays/year). The data are divided into accounts which had two or fewer transponders (individual accounts) and accounts with at least 3 transponders (business accounts). The data sets were fitted

with exponential decay functions. The functions, their equation and respective r^2 values are displayed.

The results indicate that most users do not use the MnPASS lanes every weekday for their commute, but rather select various days to use the lanes. The number for trips among different frequencies of users is fairly constant for individual users. For business accounts, the number of trips declines steadily with frequency.

Figure 2.1: Frequency of Use - 2011 & 2012

Figure 2.2: Where y represents the number of MnPASS users and x the number of weekdays per year a user paid to use the lanes

Data taken over all weekdays in 2011 and 2012

Trips include any paid use of the MnPASS lanes

2.0.2 Current Operation

Figures [2.3](#page-16-0) and [2.4](#page-16-1) display the MnPASS entry and exit points along I-394 and I-35W. Outside of these points, drivers are not supposed to enter or exit the MnPASS lanes. Double white lines separate the lanes except during the entry and exit points, during which the lines are dashed. Plazas 1003, 3005, and 3012, which are referenced later in the analysis, are labled in the figures. The hours of operations are summarized in Table [2.1](#page-17-0) below.

Figure 2.3: MnPASS Entry and Exit Points on I-394

MnPASS (http://www.mnpass.org/)

Figure 2.4: MnPASS Entry and Exit Points on I-35W

MnPASS (http://www.mnpass.org/)

Corridor	Direction	Section	Start Time	End Time
$I - 394$	EB	I-494 to Hwy 100	6:00	10:00
I-394	EB	Hwy 100 to Downtown Minneapolis	6:00	13:00
I-394	WВ	Hwy 100 to $I-494$	14:00	19:00
I-394	WВ	Downtown Minneapolis to Hwy 100	14:00	5:00
$I-35W$	N _B	Crystal Lake Road to Hwy 62	6:00	10:00
$1-35W$	NB	Hwy 62 to Downtown Minneapolis	6:00, 15:00	10:00, 19:00
$1-35W$	SB	42nd St to I-494	6:00, 15:00	10:00, 19:00
$I-35W$	SB.	I-494 to Hwy 13	15:00	19:00

Table 2.1: Hours of Operation

Prices during operation times range from a minimum of \$0.25 to a maximum \$8.00. I-394 and I-35W are each divided into multiple sections with prices posted for use of each segment. The maximum price applies to use of each section individually, as well as use of all sections.

Prices are adjusted every three minutes based density levels measured in the MnPASS lanes only. Traffic levels in the GP lanes does not influence price. Loop detector counts are taken every 30 seconds. These counts are used to calculate the density in the MnPASS lanes at various plazas along the corridor. Each plaza consists of a multiple parallel detectors, one for each lane. Density measurements are averaged over the last 6 minute period in order to smooth out fluctuations. Drivers are charged based on the maximum density downstream of their entrance point. Densities upstream do not influence the paid price. Price is dictated by the magnitude of density as well as the change in density over the previous 6 minutes. A rise in density creates an increase in price. Table [6.3](#page-62-0) displays the pricing plan, which regulates the price based on density level. Minimums and maximums for a given LOS must be maintained. Table [6.4](#page-63-0) indicates the changes in price caused by a change in density.

Level of Service Min K Max K Min Rate $(\$)$				Default Rate $(\$)$	Max Rate $(\$)$
А	$\overline{0}$	11	0.25	0.25	0.50
B	12	18	0.50	0.50	1.50
C	19	31	1.50	1.50	2.50
D	32	42	2.50	3.00	3.50
E	43	49	3.50	5.00	5.00
$_{\rm F}$	50	50	5.00	8.00	8.00

Table 2.2: Pricing Plan for Normal Operation of MnPASS Lanes (both I-35W and I-394)

Density in veh/mi/ln; Prices in \$

Table 2.3: Price Changes Based on Changes in Density - Used for all pricing plans

Density $\Delta 1$ $\Delta 2$ $\Delta 3$ $\Delta 4$ $\Delta 5$ $\Delta 6$				
$0-18$			$$0.25$ $$0.25$ $$0.25$ $$0.25$ $$0.25$ $$0.25$ $$0.25$	
	$19 +$ \$0.25 \$0.50 \$0.75 \$1.00 \$1.25 \$1.50			

Density in veh/mi/ln

Price increases between \$0.25 and \$1.50 with a change in density between 1 and 6 veh/mi/ln

2.0.3 Average Tolls and Time Savings

The average payment for time savings on the MnPASS lanes was calculated for I-394 and I-35W for both the morning and afternoon peak periods. Two years of toll and loop detector data (2011 & 2012) were gathered in order to compute the average time savings for using the MnPASS lanes and the average paid toll. Pricing data came directly from logs provided by the MnPASS operators. Prices were provided for each plaza along the corridor, every 3 minutes. These represent posted prices and not individually paid prices. Average toll prices were computed by weighing the posted prices by the number of users experiencing a given price (reported MnPASS lane density). The average paid toll price also assumes use of the entire MnPASS corridor. Averages are calculated over the entire paid MnPASS periods 6:00-10:00 & 14:00-19:00 for I-394 and 6:00-10:00 & 15:00-19:00 for I-35W.

Time savings was calculated using loop detector data from the MnPASS and GP lanes. Commute times for the GP and MnPASS lanes were calculated assuming use of the entire corridor. The time savings assumes drivers are entering downtown Minneapolis during the morning commute. It does not account for time savings as a result of avoiding the queue to I-94 eastbound or other similar circumstances. The MnPASS corridor stretches approximately 12.4 miles (19.96 km) on I-394 and 16 miles (25.75 km) on I-35W. Calibrated field lengths were used for the detectors, which provide occupancy data every 30 seconds [\(MnDOT,](#page-96-4) [2013\)](#page-96-4). From the occupancy data, average speeds were calculated for each series of detectors along the corridor. Speeds were averaged over a three-minute time period, corresponding to the frequency of pricing changes.

In order to reduce extraneous speeds caused by varying vehicle sizes or detector reading inaccuracies, two filtering methods were applied. First, speeds exceeding 75 mph (120.7 $km \cdot h^{-1}$) were eliminated. Speed limits along the MnPASS corridors are most commonly 55 or 60 mph (88.5 or 96.6 km $\cdot h^{-1}$), with a stretch of I-35W at 65 mph (104.6 km $\cdot h^{-1}$) near the southern edge of the system. Second, interval speeds calculated from a single vehicle were eliminated to reduce the likelihood of inaccurate speed measurements (caused by very large or very small vehicles). A low speed threshold was not applied given that any non-negative speed was possible. Negative speeds were, however, naturally eliminated if they existed.

Travel times for the MnPASS and GP lanes were then calculated using the speeds from each detector series and computing the time required to traverse the entire MnPASS corridor length. Calculations were carried out for I-394 and I-35W over the entire morning and afternoon price enforced periods and averaged. Travel time savings were the differences in commute times between the MnPASS and GP lanes. Like the average prices, time savings was weighted based on density. Willingness to pay was computed using the weighted averages of time savings and toll price. Therefore, although data were averaged over the entire peak period, heavier demand periods were given greater weight. The resulting values are discussed later.

Table [2.4](#page-21-0) displays willingness to pay values from several previous studies. These studies were selected because they represent similar HOT lane facilities to MnPASS. [Burris et al.](#page-94-5) [\(2012\)](#page-94-5) also includes values for a study on the I-394 MnPASS lanes. The values from this study represent the average toll prices paid and the respective average time savings. This differs from willingness to pay, because it is not known what users would be willing to pay. These values simply represent what users pay and the resulting time savings they gain as a result.

Average weighted toll prices and time savings during the morning and afternoon peak periods on I-394 and I-35W are displayed in Table [2.5.](#page-22-1) Averages are weighted based on the number of users experiencing the price or time savings. The average toll price for the peak periods ranges from \$1.37 to \$2.91. The minimum and maximum tolls are \$0.25 and \$8.00 respectively. Average time savings for MnPASS users ranges from less than a minute (0.78 min) on I-394 in the afternoon to 2.87 minutes on I-35W in the morning peak. With the I-394 corridor running 12.4 miles (20 km), MnPASS users experienced 8.1 seconds/mile (5.0 $\sec(km)$ average time savings in the morning and 3.8 seconds/mile $(2.4 \sec(km))$ in the afternoon. The MnPASS lanes on I-35W stretch 16 miles (25.7 km), providing 10.8 second/mile (6.7 second/km) average time savings during the morning commute and 4.8 seconds/mile (2.98 second/km) in the afternoon. These values allow for better direct comparison of the time savings between I-394 and I-35W.

The average time savings and toll price values yielded price paid for time savings values from \$60.77 to \$124.10 per hour. These values are much higher than typical values of time (VOT). MnDOT, for example, uses a VOT of \$15.60 [\(MnDOT,](#page-96-5) [2013b\)](#page-96-5). Burris et al. (2012) found similarly high values of time on I-394, \$73/hr during the morning commute and \$116/hr in the afternoon. [Steimetz and Brownstone](#page-97-3) [\(2005\)](#page-97-3) discuss wide ranging VOT values and methods for better handling noisy data.

There are several possible explanations for the high VOT. First, it is expected that users of HOT lanes have a higher than average VOT, as most travelers do not use the lanes. Furthermore, both individual and business accounts make up the toll paying users. The higher VOT for businesses raises the overall VOT value. The second reason is distorted driver perception. As shown by [Ghosh](#page-95-5) [\(2001\)](#page-95-5) and [Yan](#page-97-4) [\(2002\)](#page-97-4), drivers have a distorted perception of reality and likely perceive their time savings to be greater than reality. MnPASS users probably do not realize how minimal their time savings is on average [\(Parthasarathi et al.,](#page-96-6) [2012\)](#page-96-6). A third factor is that the VOT includes value of reliability (VOR), which represents the monetary value placed on reduced travel variability [\(Carrion and Levinson,](#page-95-6) [2012b\)](#page-95-6). VOR is difficult to separately quantify, particularly in dynamic pricing experiments where there is a strong correlation between price and reliability [\(Brownstone and Small,](#page-94-6) [2005\)](#page-94-6). Studies have placed the reliability ratio (VOR/VOT) anywhere between 0.10 and 2.8 [\(Car](#page-95-7)[rion and Levinson,](#page-95-7) [2012a](#page-95-7)[,b\)](#page-95-6). The MnPASS lanes provide consistent travel time with a very small likelihood of breakdown. Therefore, some of the VOT is likely due to the increased reliability provide by the lanes. The MnPASS lanes provide other intangibles which are also important. The more consistent traffic flow makes driving in the MnPASS lanes safer and less stressful. Consistent driving speeds yield better gas mileage. Finally, MnPASS users may take advantage of queue jumps provided by the lanes. Users traveling WB on I-394 and headed south on Hwy 100, can bypass the queue that often forms. Likewise, morning commuters heading east on I-394 can avoid the queue to enter I-94 eastbound. All of these are important benefits provided by the lanes which influence the price drivers are willing to pay for the MnPASS lanes.

Reference	Willingness to Pay	Notes
Brownstone et al. (2003)	\$30/hr	I-15 in San Diego
Burris et al. (2012)	\$73/hr & \$116/hr	I-394 Morning & Afternoon
	\$49/hr & \$54/hr	I-15 Morning & Afternoon
Devarasetty et al. (2012)	\$51/hr	I-10 (Katy Freeway)

Table 2.4: Willingness to Pay from Literature

			Avg. Price (\overline{P}) Avg Time Savings (min) Cost/Time Savings $(\frac{1}{8}$ /hr)
I-394 Morning	2.579	1.673	92.49
I-394 Afternoon	1.369	0.777	105.70
I-35W Morning	2.909	2.872	60.77
I-35W Afternoon	2.533	1.224	124.10

Table 2.5: Average Toll Prices and Time Savings - 2011 & 2012

I-394 Morning: 6:00-10:00

I-394 Afternoon: 14:00-19:00

I-35W Morning: 6:00-10:00

I-35WAfternoon: 15:00-19:00

Data taken over all weekdays in 2011 and 2012

2.0.4 Economic Theory - Demand Curve of Toll Roads

Most goods are ordinary goods following the downward sloping demand curve where quantity consumed decreases as price rises as seen in Figure [2.5a](#page-24-0) [\(Beggs,](#page-94-7) [2010\)](#page-94-7).

Some luxury goods, on the other hand, may see an increase in consumption as price rises (at least for certain prices). Figure [2.5b](#page-24-0) represents this phenomenon [\(Beggs,](#page-94-7) [2010\)](#page-94-7). As reported in the [Miami Herald](#page-96-7) [\(2010\)](#page-96-7), drivers on I-95 may increase consumption of the toll lane as price rises. Drivers see the toll price as a signal of congestion in the untolled lanes and use of the tolled lanes increases. Therefore, a higher price leads to greater consumption. Does this mean toll roads have an upward sloping demand curve like a Veblen Good?

[Beggs](#page-94-7) [\(2010\)](#page-94-7) believes this is, in fact, not the case. In moving up the demand curve in Figure [2.5b](#page-24-0), the assumption is that all other factors are held constant. In the HOT lane case, this assumption breaks down. The belief of drivers is that the higher price indicates greater congestion and increased time savings. Therefore, drivers are assessing their willingness to pay for two different goods with different amounts of time savings. If time savings is held constant, HOT lanes follow a typical downward sloping demand curve where quantity decreases with an increase in price. [Beggs](#page-94-7) [\(2010\)](#page-94-7) suggests that perhaps, what is really happening as price increases, is that the demand curve is shifting to the right as seen in Figure [2.5c](#page-24-0). Drivers regard the higher priced HOT lane as a different good (one which provides greater time savings), for which they have a different demand curve.

[Beggs](#page-94-7) [\(2010\)](#page-94-7) demonstrates this by noting that if price were held constant, but time savings increased, then quantity consumed would increase to Q_3 on the right shifted demand curve. Therefore, HOT lanes are likely not Veblen Goods, but rather ordinary goods represented by different demand curves based on their properties (i.e. time savings).

Figure 2.5: Demand Curves

[\(Beggs,](#page-94-7) [2010\)](#page-94-7)

Part I

Driver Elasticity to Price

Chapter 3

Aggregate Analysis: Methods and Results

Understanding the elasticity to price of MnPASS drivers is important to determine an optimal pricing plan. Very inelastic behavior would mean large price changes would do very little to change the demand of the MnPASS lanes. This would cause difficulty in regulating MnPASS demand. Very elastic behavior, on the other hand, would mean large changes to demand from a small price change. This could lead to erratic changes in demand from small toll fluctuations.

Equally important to the magnitude of elasticity is the positive or negative relationship between price and demand. Does MnPASS demand increase or decrease with an increase in price? The assumption until now how has been that MnPASS lanes are a simple ordinary good, meaning an increase in price corresponds to a decrease in demand. However, as discussed by [Beggs](#page-94-7) [\(2010\)](#page-94-7), this is not always the case for HOT lanes, which may see increases in demand corresponding to higher prices.

Prices for elasticity calculations came directly from the MnPASS system logs. The MnPASS logs store posted prices and their corresponding density levels.

Demand was measured using several methods. MnPASS logs store the calculated densities, which determine price. These densities measure HOVs and SOVs in the HOT lane. Transponder data provides demand at an individual level. Transponder logs only record paying SOVs. Loop detector data were also used in order to calculate the lane share percentage of the MnPASS lanes as well as the vehicle flow for the SOV and HOV usage section. The MnPASS lane share percentage includes all MnPASS lane users and is not limited to paying SOVs. It measures the percentage of overall flow using the MnPASS lane. This helps control against overall fluctuations in traffic due to various externalities since it accounts for general purpose lane volumes as well as MnPASS. Holidays, poor weather days and other known anomalies, however, were excluded from all analyses in order to maintain more consistent data.

$$
S_{MnPASS} = \frac{Q_{MnPASS}}{Q_{MnPASS} + \Sigma Q_{GP}}
$$
(3.1)

Where S_{MnPASS} denotes MnPASS lane share percentage. Q represents flow in the respective lane type. Flow for the general purpose lanes is the sum of all general purpose lanes.

Table [3.1](#page-27-0) displays driver elasticity to price results from several previous papers. Several of the studies come from an analysis by [Burris](#page-94-8) [\(2003\)](#page-94-8). All values are negative and smaller in magnitude than -1.0. Elasticity results using various methods are displayed and discussed in the following sections.

Odeck & Brthen (2008) -0.45 & -0.82 Short-run and Long-run

Table 3.1: Driver Elasticity to Price from Literature

Several studies taken from [Burris](#page-94-8) [\(2003\)](#page-94-8)

Two years of MnPASS demand and pricing data (2011 and 2012) were gathered to examine aggregate demand responses to changes in price. Average price and demand (density and MnPASS lane share %) were plotted every 3 minutes throughout the peak period. The data is taken from the critical plazas discussed earlier. The prices and densities correspond to the logs from the MnPASS system. S_{MnPASS} is calculated from loop detector data.

The MnPASS pricing algorithm operates by changing price at time, $[t+3:t+6]$, according to changes in demand between $[t:t+3]$ (which is also averaged with the change in the previous 3 minutes). Price is responding to demand. In order to measure driver elasticity to price,

it is necessary to examine changes to demand following changes in price. We are interested in how demand is responding to changes in price and not the other way around. Elasticity was, therefore, calculated using the change of price between $[t:t+3]$ minutes, and demand change between $[t+3:t+6]$.

The elasticity equation:

$$
\varepsilon_t = \frac{\frac{D_{t+6} - D_{t+3}}{D_{t+3}}}{\frac{P_{t+3} - P_t}{P_t}}
$$
\n
$$
\tag{3.2}
$$

Where D represents demand (density or S_{MnPASS}), P represents price and epsilon the resulting elasticity.

Two years (2011 & 2012) of price, density and S_{MnPASS} data for the I-394 and I-35W morning peak periods are plotted in Figure [3.1](#page-29-0) and Figure [3.2.](#page-30-0) Afternoon peak data are displayed in Figure [3.3](#page-31-0) and Figure [3.4.](#page-32-0) Points are plotted every 3 minutes to correspond with the price changes. Twelve minute moving averages were used to smooth the data. The error bars represent one standard deviation in each direction. With the exception of 6 weeks during 2011 and 2012, the pricing plan for the lanes remained constant. Any changes which occurred were similar to the field experiment described earlier.

Figure 3.1: I-394 Morning Peak Period Average Price and Density for 2011 and 2012

I-394 Morning: 6:00-10:00 Data taken over all weekdays in 2011 and 2012Data from Plaza 1003, between Hwy 169 and Louisiana Ave on EB I-394

Figure 3.2: I-35W Morning Peak Period Average Price and Density for 2011 and 2012

I-35W Morning: 6:00-10:00 Data taken over all weekdays in 2011 and 2012Data from Plaza 3012 near 46th Street on NB I-35W

Figure 3.3: I-394 Afternoon Peak Period Average Price and Density for 2011 and 2012

I-394 Afternoon: 14:00-19:00 Data taken over all weekdays in 2011 and 2012Data from Plaza 2003, between Louisiana Ave and Hwy 169 on WB I-394

Figure 3.4: I-35W Afternoon Peak Period Average Price and Density for 2011 and 2012

I-35W Afternoon: 15:00-19:00 Data taken over all weekdays in 2011 and 2012Data from Plaza 4009 near 98th Street on SB I-35W

	Density	Elasticity (Density)		
	Average	Average	Median	Std Dev
I-394 Morning (1003)	23.75	$0.8005*$	0.6164	0.8387
I-394 Afternoon (2003)	14.05	$0.4885*$	0.6170	1.018
I-35W Morning (3005)	25.99	$0.7448*$	0.7331	0.9176
I-35W Morning (3012)	22.64	$0.8400*$	0.7813	0.3804
I-35W Afternoon (4009)	19.99	$0.6320*$	0.6117	1.140
I-35W Afternoon (4011)	15.28	0.4880	0.4487	2.332
		Elasticity (S_{MnPASS})		
	S_{MnPASS}			
	Average	Average	Median	Std Dev
I-394 Morning (1003)	19.82	$0.7010*$	0.6487	0.7754
I-394 Afternoon (2003)	12.69	$0.4638*$	0.3818	1.129
I-35W Morning (3005)	22.37	0.1775	0.3911	1.124
I-35W Morning (3012)	13.55	$0.6491*$	0.5936	0.5044
I-35W Afternoon (4009)	23.75	$0.3943*$	0.2964	0.7842

Table 3.2: Elasticity Values Using Density and MnPASS Lane Share: 2011 & 2012 Aggregate

* Significant at 0.05 significance level

Plaza in parentheses

Density in units veh/mi/ln

 S_{MnPASS} is percent of overall flow using the MnPASS lane I-394 Morning: 6:00-10:00 I-394 Afternoon: 14:00-19:00 I-35W Morning: 6:00-10:00 I-35W Afternoon: 15:00-19:00 Data taken over all weekdays in 2011 and 2012 Plaza 1003, 2003 lanes: 1 HOT, 2 GP, 1 Auxilliary Plaza 3005, 4009 lanes: 1 HOT, 2 GP Plaza 4011 lanes: 1 HOT, 3 GP Plaza 3012 lanes: 1 HOT, 4 GP

All statistically significant elasticity values from the aggregate data are positive and between 0.3392 and 0.8400.

The MnPASS pricing algorithm operates by changing price to match changes in demand (raising price with increasing demand). This analysis, however, looks at changes to demand immediately following pricing changes, in order to examine the response of demand to price. Overall, the analysis revealed that demand (both density and S_{MnPASS}) typically increased immediately following a price increase and vice versa.

Chapter 4

Field Experiment Analysis: Methods and Results

Several field experiments were conducted between October 2012 and January 2013. Drivers were never made aware of any changes to the pricing plan.

The first field experiment took place on I-394 between October 8, 2012 and November 2, 2012. During this period, prices for all MnPASS lanes on I-394 were altered according to the revised pricing schedules displayed below. No changes were made to the delta values in Table [6.4.](#page-63-0) This same delta table was used for all field experiments. Prices during the first week were altered by raising the density threshold at each level of service by 2 points (Table [4.3\)](#page-37-0). Prices during the remaining three weeks were altered by raising the density thresholds by 20% instead of a fixed 2 point increase (Table [4.4\)](#page-37-1). The intent was to lower the average price to MnPASS users. The altered pricing plans for this field experiment, however, were based on a pricing plan dating back to 2005 (Table [4.2\)](#page-36-1). The actual pricing plan in place on I-394 before beginning the field experiment is displayed in Table [6.3.](#page-62-0) The structure of the 2005 plan is different. There is an additional C- LOS and the prices for the various LOS are different than the current pricing plan. The intent was to keep the minimum, maximum and default pricing values the same. However, because of this different pricing structure, pricing changes were not consistent across different density levels. Prices at lower densities were decreased as intended, but prices at higher densities were actually increased. This difference can be seen in figures discussed in the results section. The error in implementing this different pricing structure was later discovered and repaired. The second I-394 field experiment did not experience this same problem.
Level of Service Min K Max K Min Rate $(\$)$				Default Rate $(\$)$	Max Rate $(\$)$
А	Ω	11	0.25	0.25	0.50
B	12	18	0.50	0.50	1.50
C	19	31	1.50	1.50	2.50
D	32	42	2.50	3.00	3.50
E	43	49	3.50	5.00	5.00
$_{\rm F}$	50	50	5.00	8.00	8.00

Table 4.1: Pricing Plan for Normal Operation of MnPASS Lanes (both I-35W and I-394)

Level of Service Min K Max K Min Rate $(\$)$				Default Rate $(\$)$	Max Rate $(\$)$
А	$\overline{0}$	11	0.25	0.25	0.25
B	11	18	0.25	0.25	0.50
\mathcal{C}	18	26	0.50	1.50	2.50
C_{Ξ}	26	29	2.50	3.50	4.00
D	29	35	4.00	5.00	6.00
E	35	45	6.00	7.00	8.00
F	45	50	8.00	8.00	8.00

Table 4.2: Original Pricing Plan from 2005 for I-394

Density in veh/mi/ln; Prices in \$

Level of Service			Min K Max K Min Rate $(\$)$	Default Rate $(\$)$	Max Rate (\$)
A	Ω	13	0.25	0.25	0.25
B	14	20	0.25	0.25	0.50
\mathcal{C}	21	28	0.50	1.50	2.50
C -	29	31	2.50	3.50	4.00
D	32	37	4.00	5.00	6.00
E	38	50	6.00	7.00	8.00

Table 4.3: Modified pricing plan for first week of first field experiment on I-394 2012-10-8 to 2012-10-12

Table 4.4: Modified pricing plan for weeks 2-4 of first field experiment on I-394 2012-10-15 to 2012-11-2

Level of Service Min K Max K Min Rate $(\$)$				Default Rate $(\$)$	Max Rate $(\$)$
А	Ω	13	0.25	0.25	0.25
B	14	22	0.25	0.25	0.50
\mathcal{C}	23	31	0.50	1.50	2.50
$C-$	32	35	2.50	3.50	4.00
D	36	42	4.00	5.00	6.00
E	43	50	6.00	7.00	8.00

Density in veh/mi/ln; Prices in \$

The second field experiment took place on I-35W between October 29,2012 and November 23, 2012. During this period, prices were altered according to Table [4.3.](#page-45-0) Again, no changes were made to the delta values in Table [6.4.](#page-63-0) By raising the density threshold's by 20% at each level of service, price was effectively lowered. The same pricing plan was used for the entire field experiment. Several days were excluded from the analysis due to incomplete loop detector data for those dates. The corresponding dates from the baseline period were also excluded in order to stay consistent.

Level of Service Min K Max K Min Rate $(\$)$				Default Rate $(\$)$	Max Rate (\$)
А	θ	13	0.25	0.25	0.50
B	14	22	0.50	0.50	1.50
\mathcal{C}	23	37	1.50	1.50	2.50
Ð	38	50	2.50	3.00	8.00

Table 4.5: Modified pricing plan for I-35W field experiment 2012-10-29 to 2012-11-23

The third field experiment was conducted on I-394 lasting five weeks. The experiment consisted of changes to the pricing plan displayed in Table [4.6](#page-38-0) and took place in December 7-21, 2012 and January 7-25, 2013. No changes were made to Table [6.4,](#page-63-0) displayed above. The holiday season at the end of December and beginning of January was excluded. The density thresholds at which prices changed were lowered during this experiment, effectively increasing price. The change was estimated to increase the average price by around 15%. All other operations of the pricing algorithm were left the same. After the experiment, prices were reverted to their previous levels.

Table 4.6: Modified pricing plan for second field experiment on I-394 2012-12-10 to 2012-12-21 & 2012-1-7 to 2013-1-25

				Level of Service Min K Max K Min Rate $(\$)$ Default Rate $(\$)$	Max Rate $(\$)$
A	$\overline{0}$	9	0.25	0.25	0.50
B	10	15	0.50	0.50	1.50
\mathcal{C}	16	25	1.50	1.50	2.50
D	26	34	2.50	3.00	3.50
Ε	35	39	3.50	5.00	5.00
F	40	50	5.00	8.00	8.00

Density in veh/mi/ln; Prices in \$

The field experiments were analyzed by comparing to the same days on year prior. For

example, if the experiment began on the first Monday in October, that same Monday the year before was used as the start date. In order to account for changes occurring between 2011 and 2012, a control period was analyzed. The control period usually consists of one month prior to the field experiment. The changes in the control period between 2011 and 2012 can then be compared to the changes between the baseline period and the field experiment. The control periods and baseline period all contained the same pricing plan. This helps determine which changes are caused by the changes to the pricing plan and helps eliminate other confounds such as fuel prices and employment. [MinnesotaGasPrices.com](#page-96-0) [\(2013\)](#page-96-0) reveals, however, that average fuel prices in Minnesota between 2011 and 2012 are within \$0.50. Anomalies such as holidays and poor weather days were removed from analysis. In addition, no changes to express transit service on the corridors were made during the analysis period [\(Metropolitain Council,](#page-96-1) [2013\)](#page-96-1)

Price and demand data from the field experiments were taken from specific plazas along the corridor. The selected points represent plazas which typically have the maximum density compared to upstream plazas. Therefore, the density at these critical plazas (as they will be referred to) is often responsible for the posted prices upstream. Data for I-394 used price and demand measurements from plaza 1003 in the eastbound direction and plaza 2003 westbound. These plazas include the section of I-394 between Hwy 169 and Louisiana Ave. The corresponding HOT loop detectors used for the analysis include 5453 for eastbound and 5460 for westbound. These loop detectors are located within the respective plazas near Winnetka Ave S.

On I-35W, both plazas 3006 and 3013 in the northbound direction, along with 4009 and 4011 southbound were analyzed. Plaza 3006 includes the area around Black Dog Road and 3013 includes the section of south Minneapolis between 42nd Street and 26th Street. Plazas 4009 and 4011 are located near 98th Street S and Cliff Road respectively. The corresponding HOT loop detectors used were 525 (106th Street) and 6792 (38th Street) in the northbound direction and 1000 (98th Street) and 1008 (Black Dog Rd) in the southbound direction. The general purpose loop detectors used correspond to those listed on the MnDOT All Detector Report in parallel with the listed HOT detectors. The results discussed come from these critical points.

Driver elasticity for the field experiments was calculated by comparing price and demand to a baseline period. Average price and demand every three minutes throughout the peak period was calculated as well as the overall weighted average price and density. This was done for each week of the field experiments as well as same period one year prior. Data corresponds to the critical plazas discussed earlier. Prices and densities for this analysis come from the MnPASS system logs. S_{MnPASS} is calculated from loop detector data. Elasticity was calculated twice. Once by looking at the changes in price and demand between the two periods for every three-minute period. Elasticity values were then calculated for each 3 minute period and averaged to yield an average of elasticities. The other method compared the overall weighted prices and densities for the two periods. This yielded an elasticity of averages measurement. This same procedure was done for a control period, comparing 2011 and 2012 one month before each field experiment. The control periods utilized the same pricing plan as the baseline period. The final elasticity for the field experiments was the net change occurring between the baseline and field experiment, subtracting out any changes between 2011 and 2012 in the control.

Average of Elasticities

For Field Experiment

$$
\varepsilon_E = \frac{\frac{D_{E,2012,t} - D_{B,2011,t}}{D_{B,2011,t}}}{\frac{P_{E,2012,t} - P_{B,2011,t}}{P_{B,2011,t}}}
$$
(4.1)

For Control

$$
\varepsilon_C = \frac{\frac{D_{C,2012,t} - D_{C,2011,t}}{D_{C,2011,t}}}{\frac{P_{C,2012,t} - P_{C,2011,t}}{P_{C,2011,t}}}
$$
(4.2)

Elasticity of Averages

$$
\varepsilon_{avg} = \frac{\frac{\overline{D}_{E,2012} - \overline{D}_{B,2011}}{\overline{D}_{B,2011}} - \frac{\overline{D}_{C,2012} - \overline{D}_{C,2011}}{\overline{D}_{C,2011}}}{\frac{\overline{P}_{E,2012} - \overline{P}_{B,2011}}{\overline{P}_{B,2011}} - \frac{\overline{P}_{C,2012} - \overline{P}_{C,2011}}{\overline{P}_{C,2011}}}
$$
(4.3)

Where the subscript E denotes the field experiment and the subscript B denotes the baseline period. The control period is noted by subscript C and each period is marked with its respective year. D represents demand (density or S_{MnPASS}), P represents price and ε the resulting elasticity.

The following figures display changes in price and density for the third field experiment and its control. Twelve minute moving averages were used to smooth the data. The error bars represent one standard deviation in each direction.

Figure [4.1](#page-42-0) shows the changes in price and density during the morning peak period for the first field experiment on I-394 occurring between 2012-10-8 to 2012-11-2. As discussed earlier, the pricing plan during the field experiment followed a different structure than the baseline leading to a lower price at lower densities and a higher price at higher densities. The elasticity analysis focuses on the time period between 7:00-9:00. Over this time period, price during the field experiment experienced an increase on average when compared to the baseline. Density during the field experiment was higher than the baseline across the entire morning peak period, including the 7:00-9:00 elasticity analysis period.

Figure [4.2](#page-43-0) represents the control period for the first experiment. September 2011 (2011- 9-5 to 2011-10-7) is compared to September 2012 (2012-9-3 to 2012-10-5). This period represents five weeks leading up to the field experiment. In the shoulder peak, the two periods follow a fairly similar pattern, but 2012 has a much longer lasting peak. This led to an overall average price and density increase between 2011 and 2012 in the control.

Figure 4.1: Price and Density vs. Time - I-394 Field Experiment: 2012-10-8 to 2012-11-2

FE: 2012-10-8 to 2012-11-2 Baseline: 2011-10-10 - 2011-11-4Plaza 1003, between Hwy 169 and Louisiana Ave on EB I-394; Weekdays only

Figure 4.2: Price and Density vs. Time - I-394 Control: September 2011 & 2012

2012-9-3 to 2012-10-52011-9-5 to 2011-10-7

Plaza 1003, between Hwy 169 and Louisiana Ave on EB I-394; Weekdays only

Data in Figure [4.3](#page-45-0) show the average price and density levels during the morning peak period on I-35W for the baseline and field experiment periods. The field experiment took place between 2012-10-29 to 2012-11-23. The baseline period includes the same days as the field experiment, but one year prior.

The control period compared October 2011 (2012-10-3 to 2012-10-28) with October 2012 (2012-10-1 to 2012-10-26) to observe changes from one year to the next, shown in Figure [4.4.](#page-46-0) This period represents four weeks preceding the field experiment. The pricing plans during the control periods in the same, however, a significant increase demand can be seen in 2012. This increase in demand led to an increase in prices in October 2012.

During the field experiment, prices were decreased by raising density thresholds. Prices between the baseline and field experiment are displayed in Figure 9. Increases to price in the field experiment are due to an increase in demand similar to what can be seen in the control. Both graphs show a demand increase between the 2011 period and 2012, however, in Figure [4.3,](#page-45-0) the price increase between 2011 and 2012 is less dramatic than during the control in Figure [4.4.](#page-46-0) Therefore, the changes to the pricing plan had the expected effect of decreasing price compared to what it would have been if no change was made.

Figure 4.3: Price and Density vs. Time - I-35W Field Experiment: 2012-10-29 to 2012-11-23

FE: 2012-10-29 to 2012-11-23 Baseline: 2011-10-31 to 2011-11-25Plaza 3012, near 46th Street on NB I-35W; Weekdays only

Figure 4.4: Price and Density vs. Time - I-35W Control: October 2011 & 2012

2012-10-1 to 2012-10-26 2011-10-3 to 2011-10-28Plaza 3012, near 46th Street on NB I-35W; Weekdays only

Data in Figure [4.5](#page-48-0) show the average price and density levels during the morning peak period on I-394. The field experiment includes 2 weeks in December 2012 (12/7-12/21) and 3 weeks in January 2013 (1/7-1/25). The baseline period includes the same days as the field experiment, but one year prior. Prices were increased during the field experiment by lowering density thresholds. Average paid prices throughout the morning peak period were consistently higher during the 5 week experiment.

Figure [4.6](#page-49-0) represents the control period which compares November 2011 (2011-11-18 to 2011-12-9) and November 2012 (2012-11-16 to 2012-12-7). This period represent 3 weeks preceding the field experiment. The first two weeks in November could not be used in the control because the pricing plan during these weeks in 2012 was set to match the plan from 2005 in Table [4.2](#page-36-0) instead of the baseline plan in Table [6.3.](#page-62-0) The resulting changes in the control were relatively small compared to the changes seen in Figure 11 between the baseline and field experiment.

Figure [4.5](#page-48-0) reveals that the MnPASS lanes saw a consistent increase in density throughout the peak period during the field experiment. Although less than the price increase, density at nearly every time segment during the analyzed periods was higher. This led to the positive elasticity results displayed in Tables [4.8](#page-53-0) and [4.9.](#page-54-0)

Figure 4.5: Price and Density vs. Time - I-394 Field Experiment: 2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-25

2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-252011-12-12 to 2011-12-23 & 2012-1-9 to 2012-1-27

Plaza 1003, between Hwy 169 and Louisiana Ave on EB I-394; Weekdays only

Figure 4.6: Price and Density vs. Time - I-394 Control: November 2011 & 2012

2011-11-18 to 2011-12-92012-11-16 to 2012-12-7

Plaza 1003, between Hwy 169 and Louisiana Ave on EB I-394; Weekdays only

Table [4.7](#page-52-0) displays weighted averages of price and density for the baseline, field experiment and control periods. A net change between the baseline and field experiment, including changes in the control, are also displayed. The number of lanes corresponding to the S_{MnPASS} are displayed below the table. Average general purpose lane speeds are included as another measure of change between the periods. Elasticity was calculated using both density and S_{MnPASS} as a measure of demand. Table [4.8](#page-53-0) shows the elasticity values calculated from the weighted averages in Table [4.7.](#page-52-0) Results in Table [4.9](#page-54-0) include the mean, median and standard deviation of elasticity values for every three minutes between 7:00-9:00.

The first field experiment on I-394 resulted in statistically significant changes in price and density. The control period also resulted in significant changes in price and density between 2011 and 2012. There was no statistically significant change in the average GP speed. Overall, there was a net increase in price, density and S_{MnPASS} . Although the intention was to decrease the price, the varied structure of the pricing plan for this field experiment led to higher prices and higher densities. The averages were taken between 7:00- 9:00. Over this time period, price was primarily higher during the field experiment than the baseline. The increase in both price and demand led to positive elasticity of averages values for the first field experiment. The mean, median and standard deviation of individual elasticity measurements displayed in [4.9](#page-54-0) reveal no statistically significant difference for the field experiment. There was a high standard deviation of the individual measurements. There was, however, a statistically significant positive elasticity measured in the control period between 2011 and 2012.

Values for the I-35W field experiment were separated into measurements from plaza 3005 and plaza 3012. At both plazas, there was a statistically significant increase in price, density and S_{MnPASS} between 2011 and 2012. There was no statistically significant change in the average GP speed. The price increases between the baseline and field experiment were less pronounced due to the "price decrease" caused by increasing the density thresholds. This led to a net price decrease in price in both plazas. In all cases except S_{MnPASS} on plaza 3012, demand also saw a net decrease when including the control period. This resulted in nearly all positive elasticity results in Table [4.8.](#page-53-0) Similarly to the first field experiment,

high standard deviation values in Table [4.9](#page-54-0) resulted in no statistically significant average elasticity measurements between the baseline and field experiment. The control, however, saw statstically significant increases between 2011 and 2012. There was no statistically significant change in the average GP speed.

The third field experiment saw statistically significant increases in price and density both between the baseline and field experiment. The control period only saw a significant change in S_{MnPASS} between 2011 and 2012. There was no statistically significant change in the average GP speed. The net values were all positive, resulting in positive elasticity values in Table [4.9.](#page-54-0) The average of individual elasticity measurements were also positive and statistically significant between the baseline and field experiment for both density and S_{MnPASS} . Unlike the other field experiments, price, density and S_{MnPASS} for this experiment saw consistent increases across all time periods and density levels. This can be seen in Figure [4.5.](#page-48-0) This consistency led to steady elasticity results and the small standard deviation values. Another indication of consistency are the similar mean and median values.

				Baseline Field Experiment % Change Control % Change Net % Change	
(1) Plaza 1003					
Price	2.024	2.418	$19.45*$	$16.09*$	3.353
Density	25.31	27.50	$10.54*$	$9.657*$	0.885
S_{MnPASS}	20.76	21.50	3.566	1.627	1.939
GP_{speed}	91.5	93.8	2.5	0.9	1.6
(2) Plaza 3005					
Price	2.010	2.229	$10.88*$	$68.75*$	-57.87
Density	24.98	30.92	23.79*	$37.41*$	-13.62
S_{MnPASS}	22.36	24.13	$7.871*$	$16.17*$	-8.301
${\cal G}P_{speed}$	90.1	89.3	-0.9	2.2	-3.1
Plaza 3012					
Price	1.71	1.882	9.717	38.04^{\ast}	-28.33
Density	21.74	25.78	$18.61*$	$22.45*$	-3.840
S_{MnPASS}	13.36	15.56	$16.49*$	$12.02*$	4.471
${\cal G}P_{speed}$	$87.6\,$	85.8	-2.1	-0.8	-1.3
(3) Plaza 1003					
Price	2.192	3.044	$38.84*$	-2.569	41.41
Density	26.03	28.07	$7.830*$	-6.381	14.21
S_{MnPASS}	20.9	20.99	2.980	$-8.217*$	11.20
GP_{speed}	91.9	88.0	-4.24	-4.04	0.20

Table 4.7: Weighted Averages of Descriptive Statistics

* Significant at 0.05 significance level

Time of Day: 7:00-9:00

Density in units veh/mi/ln

Speed in km/h

 S_{MnPASS} is percent of overall flow using the MnPASS lane

(1) I394: FE: 2012-10-8 to 2012-11-2, Base: 2011-10-10 to 2011-11-4, Control: September 2011 and 2012 (2) I35W: FE: 2012-10-29 to 2012-11-23, Base: 2011-10-31 to 2011-11-25, Control: October 2011 and 2012 (3) I394: FE 2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-25, Base: 2011-12-12 to 2011-12-23 & 2012- 1-9 to 2012-1-27, Control: November 2011 and 2012

Plaza 1003 lanes: 1 HOT, 2 GP, 1 Auxiliary

Plaza 3005 lanes: 1 HOT, 2 GP

Plaza 3012 lanes: 1 HOT, 4 GP

		Demand Measure Without Control Net (with control)
(1) Plaza 1003		
Density	0.5421	.2641
S_{MnPASS}	0.1829	.5784
(2)		
Plaza 3005		
Density	2.186	0.2354
S_{MnPASS}	0.7234	0.1435
Plaza 3012		
Density	1.915	0.1356
S_{MnPASS}	1.697	-0.1578
(3) Plaza 1003		
Density	0.2016	0.3431
S_{MnPASS}	0.0767	0.2704

Table 4.8: Field Experiment Elasticity of Averages

Time of Day: 7:00-9:00

(1) I394: FE: 2012-10-8 to 2012-11-2, Base: 2011-10-10 to 2011-11-4, Control: September 2011 and 2012 (2) I35W: FE: 2012-10-29 to 2012-11-23, Base: 2011-10-31 to 2011-11-25, Control: October 2011 and 2012

(3) I394: FE 2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-25, Base: 2011-12-12 to 2011-12-23 & 2012-1-9 to 2012-1-27, Control: November 2011 and 2012

Table 4.9: Field Experiment Average of Elasticities

Demand Measure	Mean	Median	Std Dev
(1) Plaza 1003			
Density (FE)	-0.9719	0.1245	7.385
S_{MnPASS} (FE)	-1.192	-0.0719	7.920
Density (Control)	$0.5058*$	0.4613	0.8900
S_{MnPASS} (Control)	$0.1377*$	0.0495	0.3914
(2)			
Plaza 3005			
Density (FE)	-2.769	-0.2377	18.05
S_{MnPASS} (FE)	-1.624	-0.2695	9.520
Density (Control)	$0.6654*$	0.5440	0.0236
S_{MnPASS} (Control)	$0.3131*$	0.2836	0.1752
Plaza 3012			
Density (FE)	-2.581	0.7562	22.44
S_{MnPASS} (FE)	-2.8290	0.4052	22.29
Density (Control)	$0.6925*$	0.6035	0.2870
S_{MnPASS} (Control)	$0.4522*$	0.3965	0.3129
(3) Plaza 1003			
Density (FE)	$0.2110*$	0.2307	0.0874
S_{MnPASS} (FE)	$0.0981*$	0.1011	0.0755
Density (Control)	1.016	1.159	3.148
S_{MnPASS} (Control)	0.8144	0.9299	2.447

* Significant at 0.05 significance level

Time of Day: 7:00-9:00

(1) I394: FE: 2012-10-8 to 2012-11-2, Base: 2011-10-10 to 2011-11-4, Control: September 2011 and 2012 (2) I35W: FE: 2012-10-29 to 2012-11-23, Base: 2011-10-31 to 2011-11-25, Control: October 2011 and 2012

(3) I394: FE 2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-25, Base: 2011-12-12 to 2011-12-23 & 2012-1-9 to 2012-1-27, Control: November 2011 and 2012

Loop detector data were used to determine the total number of MnPASS lanes users

 $(HOV + SOV)$ along the two corridors. Counts were gathered for the critical plaza(s) on each corridor using loop detector 5453 for eastbound I-394 and 5460 for westbound. On I-35W, loop detectors 525 and 6792 in the northbound direction were used and 1000 and 1008 in the southbound direction. The transponder logs record the starting and ending plaza for paying SOVs, along with their starting time and paid toll. The assumption was made that drivers do not exit the MnPASS lane between their starting and ending plaza. Therefore, a paying SOV is counted at each plaza between their starting and ending plaza. If the critical plaza lies between the starting and ending plaza, the vehicle is counted as a paying SOV. Cross-referencing these two data sources, independent counts for SOV and HOV can be determined. SOV in this case excludes business account which are those accounts with more than two transponders.

Vehicle counts from the field experiment as well as the baseline period were gathered. The tolls paid by SOVs were used to find the average price paid for each period. The changes in price and SOV vehicle counts were used to determine the elasticity to price of paying SOVs. Elasticity for HOVs as well as total elasticity were also calculated.

One month before each field experiment were compared to the same period in 2011. The pricing plan used during the two periods was the same and also matched the prices during the baseline period. Elasticity results were calculated using the net change in price and vehicle counts, subtracting any changes occurring between 2011 and 2012 in the control period.

Elasticity for SOVs and HOVs follows the same format as Equation [4.3,](#page-40-0) where demand is replaced with flow (veh/hour).

HOV and SOV vehicle counts for the MnPASS lanes during the three field experiments were measured at the respective critical plazas. SOV counts are for individual accounts and exclude business accounts or those with more than two tranponders tied to one account. The values are converted to flow (vehicles/hours) and are displayed in Table [4.10.](#page-56-0) Average prices can be found in Table [4.7.](#page-52-0)

Using the change in vehicle flow and the average price change between the two periods, elasticity values were calculated and are displayed below in Table [4.10.](#page-56-0)

Flow in Vehicles/Hour (Q)						
	Baseline (B)	Field (E)	Δ (%)	Control Δ (%)	Net Δ (%)	Elasticity
(1) Plaza 1003						
Total HOT	1083	1111	2.581	1.211	1.370	0.4086
HOV	665	636	-4.458	-8.391	3.673	1.095
SOV	416	475	14.29	16.62	-2.333	-0.6958
(2)						
Plaza 3005						
Total HOT	1043	1167	11.96	16.75	-4.791	0.0828
HOV	738	808	9.606	11.09	-1.481	0.0256
SOV	305	359	17.66	32.64	-14.97	0.2587
Plaza 3012						
Total HOT	905	1071	18.30	19.33	-1.033	0.0365
HOV	678	789	16.40	16.19	0.2101	-0.0074
SOV	227	281	23.96	25.84	-1.882	0.0664
(3) Plaza 1003						
Total HOT	817	821	0.4092	-4.108	4.517	0.1091
HOV	442	412	-6.779	-9.412	2.633	0.0636
SOV	375	409	8.867	4.519	4.348	0.1071

Table 4.10: Field Experiment Elasticity of Average Vehicle Flow

Time of Day: 7:00-9:00

(1) I394: FE: 2012-10-8 to 2012-11-2, Base: 2011-10-10 to 2011-11-4, Control: September 2011 and 2012 (2) I35W: FE: 2012-10-29 to 2012-11-23, Base: 2011-10-31 to 2011-11-25, Control: October 2011 and 2012 (3) I394: FE 2012-12-10 to 2012-12-21 & 2013-1-7 to 2013-1-25, Base: 2011-12-12 to 2011-12-23 & 2012- 1-9 to 2012-1-27, Control: November2011 and 2012

Results of vehicle flow for the three field experiments tend to validate earlier results, with a few exceptions. Both field experiments on I-394 saw a total net increase in flow. Previous results showed net increases in density and S_{MnPASS} during these experiments. The first field experiment saw net increases in total flow and HOV. Although there was a net decline in SOV flow, there was a large increase seen in both the field experiment and control period. The observed net price change during the first field experiment was positive. This is due to an increase in demand as well as the increase in price from the pricing plan at higher density levels as explained earlier. With the modified pricing structure, it was expected that prices would be lower at lower densities and higher at higher densities. This complexity makes analysis of the first field experiment more difficult to discern.

Results from the I-35W field experiment resulted primarily in net decreases in flow. This corresponds to a net decrease in price. Changes were greater and more consistent at plaza 3005, compared to plaza 3012. Plaza 3005 saw net decreases in both SOV and HOV flow, while plaza 3012 saw a net decrease in SOV flow, but a very small net increase in HOV flow. Total flow at plaza 3012, however, decreased. Elasticity results are displayed in Table [4.10.](#page-56-0)

Results from the third field experiment were the most consistent with net increases in SOV and HOV flow. These increases corresponded with an increase in price. These led to the positive elasticity values in Table [4.10.](#page-56-0)

Chapter 5

Discussion

With the increasing interest in HOT lanes around the US, it is important to understand drivers' responses to varying toll prices. Specifically focusing on the MnPASS lanes on I-394 and I-35W in Minneapolis, this thesis found drivers paid between \$60 and \$124 per hour of travel time savings. Consistent with other studies, these values suggest drivers are paying for more than just travel time savings, but other factors such as reliability.

Analysis of driver elasticity using various methods yielded positive demand elasticity to price. Both SOVs and HOVs increased usage of the MnPASS lanes with higher prices. Statistically significant elasticities ranged between about $+0.03$ to $+0.85$. The increased demand resulting from higher prices (and decreased demand from lower prices) is likely a result of driver perception of the posted price. Drivers likely view the price as an indication of time savings and congestion, suggesting higher prices provide greater time savings. No travel times or congestion levels are made available to drivers entering MnPASS corridors, therefore, the MnPASS price may act as a signal of downstream congestion. Drivers must make a quick decision whether to use the MnPASS lanes and the posted price acts as one important factor. Other intangibles also influence a user's lane choice decision. In any case, drivers are consuming different goods when the toll varies, because time savings is not constant. These different goods represent different demand curves and not movement along one downward sloping demand curve [Beggs](#page-94-0) [\(2010\)](#page-94-0). Therefore, although price is higher, quantity consumed is also higher.

Part II

Alternative Pricing Strategies and Partial Equilibrium Analysis

Chapter 6

Alternative HOT Lane Pricing Strategies

6.1 Introduction

HOT lanes charge a toll to single occupancy vehicles (SOVs) for several reasons. The toll serves to raise revenue to cover operating costs and to regulate the demand of SOVs. HOT lanes around the country use different methods for determining the toll, however, all methods raise the toll price during more congested periods. The theory is, a higher toll price discourages demand and is used to maintain a high level of service (LOS) in the HOT lane(s). Part I of this thesis showed, however, that a higher price may act as a signal of downstream congestion, causing demand to increase to a point. This section will explore current HOT pricing strategies and propose some alternatives. These alternative strategies will be tested using a partial equilibrium analysis. This analysis uses a calibrated HOT lane choice model to determine the HOT lane share at various prices and determine demand elasticity to price.

6.2 Pricing on HOT Lanes

Table [6.1](#page-61-0) summarizes the tolling strategies of various HOT lanes around the United States. Several HOT lane systems base the toll on time of day, while others are dependent on HOT density or speed. Details of the MnPASS lanes' pricing system are outlined in the following section.

City		Highway System Open Date Length (miles) Toll Dependency		
Atlanta ²⁰	$I-85$	2011	16	HOT Density
Denver ¹³	$I-25$	2006	7	Time
H ouston ²³	$I-10$	2009	12	Time
Miami ¹⁹	$I-95$	2008, 2014	$8, 13$ (total)	HOT Density
Minneapolis ²⁹	$I-394$	2005	11	HOT Density
Orange County ¹	SR 91	2003	10	Time
San Diego 16	$I-15$	1998	12	Dynamic
S eattle ⁴⁰	SR 167	2008	9	HOT Speed
Washington, D.C. 38	$I-495$	2012	14	HOT Density

Table 6.1: HOT Lane Tolling Strategies

6.3 MnPASS Current Operation

This sections details the toll operation of the MnPASS lanes. Table [6.2](#page-61-1) below lists the hours of operation for the two MnPASS corridors.

Corridor	Direction	Section	Start Time	End Time
I-394	EB.	I-494 to Hwy 100	6:00	10:00
I-394	EB	Hwy 100 to Downtown Minneapolis	6:00	13:00
$I-394$	WВ	Hwy 100 to I-494	14:00	19:00
I-394	WВ	Downtown Minneapolis to Hwy 100	14:00	5:00
$1-35W$	N _B	Crystal Lake Road to Hwy 62	6:00	10:00
$1-35W$	N _B	Hwy 62 to Downtown Minneapolis	6:00, 15:00	10:00, 19:00
$I-35W$	SB	$42nd$ St to I-494	6:00, 15:00	10:00, 19:00
$1-35W$	SB	I-494 to Hwy 13	15:00	19:00

Table 6.2: Hours of Operation

Prices during operation times range from a minimum of \$0.25 to a maximum \$8.00. I-394 and I-35W are each divided into multiple sections with prices posted for use of each segment. The maximum price applies to use of each section individually, as well as use of all sections.

Prices are adjusted every three minutes based on density levels measured in the MnPASS lanes only. Traffic levels in the GP lanes do not influence price. Loop detector counts are taken every 30 seconds. These counts are used to calculate the density in the MnPASS lanes at various plazas along the corridor. Each plaza consists of several detectors in series. Density measurements are averaged over the last 6 minute period in order to smooth out fluctuations. Drivers are charged based on the maximum density downstream of their entrance point. Densities upstream do not influence the paid price. Price is dictated by the magnitude of density as well as the change in density over the previous 6 minutes. A rise in density creates an increase in price. Table [6.3](#page-62-0) displays the pricing plan, which regulates the price based on density level. Minimums and maximums for a given LOS must be maintained. Table [6.4](#page-63-0) indicates the changes in price caused by a change in density.

Density in veh/mi/ln; Prices in \$

Density $\Delta 1$ $\Delta 2$ $\Delta 3$ $\Delta 4$ $\Delta 5$ $\Delta 6$			
0.18 0.25 0.25 0.25 0.25 0.25 0.25 0.25			
$19 + 0.25$ 0.50 0.75 1.00 1.25 1.50			

Table 6.4: Price Changes Based on Change in Density

6.4 Alternative Pricing Strategies

The following pricing strategies are proposed alternatives to the current system used on the MnPASS HOT lanes. The continuous function is similar to the current pricing algorithm in that it relies strictly on HOT density for determining price, however, instead of relying on a series of tables, price is determined from a simple mathematical equation. The three value pricing strategies incorporate GP density and use the difference in density between the HOT and GP lanes to determine price. Details of the pricing strategies are outlined below.

In all cases, the prices are confined to several contraints to match the existing pricing algorithm. The minimum price is \$0.25, the maximum \$8.00 and all prices are rounded to the nearest \$0.25. The following equation represents the contraints which are applied after the unconstrained price is determined.

$$
P_{constrained} = Rnd(Min(Max(P_{unconstrained}, 0.25), 8.00), 0.25)
$$
\n
$$
(6.1)
$$

 $P_{unconstrained}$ may be defined several ways, as discussed below.

6.4.1 Continuous Function

Prices using this function are determined by:

$$
P_{continuous} = \alpha \ast K_{HOT}^{\beta} \tag{6.2}
$$

where P represents the price in USD and K the density in vehicles/mile/lane.

 K_{HOT} is found using the same method as the current algorithm (maximum downstream density averaged over last 6 minutes). α and β are constants which can be adjusted to achieve the desired curve. The current algorithm implementation relies on a table of values relating density to price, whereas the continuous function determines the toll based on a simple mathematical relationship between density and price.

6.4.2 Unweighted Value Pricing

While the current pricing algorithm only evaluates the density in the HOT lane, this pricing strategy would compute price based on the difference in density between the GP and HOT lanes. The difference in density between the lane groups is correlated with a difference in time savings and therefore, the value provided by the HOT lane. Implementation of this pricing scheme (and subsequent strategies), will require the integration of GP density as a factor in determining price. GP density is averaged among parallel detectors. The maximum downstream GP density is then used to determine price, along with the maximum downstream HOT density.

$$
P_{Value_{unweighted}} = \gamma * [K_{GP} - K_{HOT}] \tag{6.3}
$$

6.4.3 $HOT_{weighted}$ Value Pricing

Differences in density between GP and HOT lanes do not correlate directly to travel speeds. Rather, there is a correlation with the magnitude of densities. For example, little speed difference exists between 10 and 20 vehicles/mi/ln, both likely experience free flow speeds. However, a greater speed difference exists at higher densities (between 40 and 50 veh/mi/ln). Therefore, it makes more sense to weight the density difference between the GP and HOT, based on the magnitude of density. This function weights the difference based on the magnitude of the HOT lane density. Similarly to the current algorithm, price will increase proportionally with HOT density.

$$
P_{ValueHOT_{weighted}} = \delta * [K_{GP} - K_{HOT}] * K_{HOT}
$$
\n(6.4)

6.4.4 $GP_{weighted}$ Value Pricing

This pricing strategy is weighted based on GP density instead of HOT density. If K_{GP} is much greater than K_{HOT} and K_{HOT} is very low, then the HOT weighted value pricing strategy would yield a low price even though there would be a significant value in using the HOT lane. By weighting based on K_{GP} , this strategy ties price more directly to the GP lane congestion and the actual time savings gained by using the HOT lane.

$$
P_{ValueGP_{weighted}} = \sigma * [K_{GP} - K_{HOT}] * K_{GP}
$$
\n(6.5)

Chapter 7

Partial Equilibrium Analysis

The partial equilibrium analysis involves using a fixed demand of SOVs with predefined commute times and locations to calibrate a lane choice model and eventually test alternative pricing strategies. The SOVs are equipped with transponders and can decide whether to use the MnPASS or GP lanes based on the toll and their expected travel time and reliability. The following sections outline the process.

7.1 Lane Choice Model

This HOT lane choice model extends work done by Carlos Carrion [\(Carrion,](#page-95-4) [2010\)](#page-95-4). The binomial logit model determines the probability of a vehicle using the HOT lane based on several independent variables. These variables include estimated travel times and travel time variability for both the HOT lane and the GP lanes, as well as the posted toll price. The lane choice model applies only to SOVs equipped with transponders. SOVs not equipped with transponders are not allowed to use the MnPASS lanes. A separate subscription choice model was developed to determine which vehicles are equipped with transponders. Details of this model are outlined in [Owen et al.](#page-96-4) [\(2013\)](#page-96-4).

7.1.1 Model Coefficients

Utility from [Carrion](#page-95-4) [\(2010\)](#page-95-4) is described as:

$$
U = f(T, V, P, A)
$$

where

T: Expected Travel Time The utility decreases with an increase in expected travel time, decreasing the probability of using the given lane type. Expected travel time is 58

measured in minutes.

V: Travel Time Variability Travel time variability in this model is defined as the 90th percentile - 50th percentile to correspond with [Carrion](#page-95-4) [\(2010\)](#page-95-4). This value is calculated separately for the HOT lane and GP lanes. Like expected travel time, an increase in variability decreases the probability of using that lane. Travel time variability is measured in minutes.

P: Expected Toll Price The expected toll variable is based on the dynamic message sign posted price. The price corresponds to a user's entry and exit points. This model assumes all drivers will exit in downtown Minneapolis. Therefore, the expected toll will vary only by entry point. Toll prices are in USD. The negative sign indicates a dissuasion from higher tolls, assuming all other factors remain constant.

A: Alternative Specific Constant In this model, the ASC was defaulted to zero and adjusted if necessary in the calibration.

7.2 Calibration of Lane Choice Model

While the model was previously calibrated in [\(Carrion,](#page-95-4) [2010\)](#page-95-4), the calibration relied on a very small sample size of vehicles and was therefore, recalibrated using the following methodology.

The lane choice model is calibrated by matching a set of simulated vehicles' HOT lane decisions to match historical data. The list of vehicles was generated from trip tables provided by the Metropolitan Council. All vehicles are SOVs traveling eastbound to downtown Minneapolis on I-394 between 6:00-10:00 AM. Each vehicle has an entrance ramp and time of entry into the system. The subscription choice model from [Owen et al.](#page-96-4) [\(2013\)](#page-96-4) is first applied to filter non-transponder owning SOVs. Each vehicle experiences various travel times based on the entrance ramp and time of entry. These travel times are the basis of the expected travel time and travel time reliability parameters of the lane choice model. Details of the calibration steps are outlined in the following sections.

The lane choice model coefficients are adjusted using a grid search technique. Default values for the coefficients were taken from [Carrion](#page-95-4) [\(2010\)](#page-95-4), with the exception of the alternative specific constant (ASC) which was set to zero. The grid search approach involves adjusting each of the coefficients separately, while keeping all others constant. The first coefficient is altered until the model achieves its best fit to the calibration target. This coefficient is then kept constant and the second coefficient is adjusted and so on until the fit can no longer be improved.

In this model, the ratio of expected travel time to travel time variability was kept constant and the ASC was defaulted to zero. The travel time coefficients were adjusted first, followed by the toll coefficient and ASC (if necessary). The ratio of expected travel time to travel time variability was kept constant due to the extensive literature research outlined in from [Carrion and Levinson](#page-95-5) [\(2012b\)](#page-95-5) in determining this value.

7.2.1 Travel History

Each vehicle builds a travel time history by experiencing MnPASS travel times along the corridor based on their entrance ramp and time of entry. All travel is along I-394 Eastbound to downtown Minneapolis. The travel times are calculated using loop detector data from each Wednesday of 2012 (except July 4 and December 26). This travel history determines a vehicles expected travel time (mean of travel history) and travel time variability (90th percentile minus 50th percentile.

7.2.2 Calibration Target

In order to calibrate the lane choice model, it is necessary to determine the probability that a transponder owning SOV will use the MnPASS lane.

Using Bayes' theorem:

$$
Pr(L|R) = Pr(R|L) * Pr(L)/Pr(R)
$$
\n(7.1)

 $Pr(R)$ is the probability of radio transponder ownership (from subscription choice model). $Pr(L)$ represents the probability of using the HOT lane among all SOVs. $Pr(R|L)$

is the probability of owning a transponder given use of the HOT lane. Since only SOVs are being considered, $Pr(R|L)$ is 1 (or 100%) assuming no illegal use of the HOT lane.

Pr(L) was calculated by finding the number of SOVs using the MnPASS lane and dividing by total number of vehicles using the corridor during the same time period. Total vehicle counts were gathered from loop detector data. The number of HOVs using the GP lanes is assumed to be zero. Counts of SOVs using the MnPASS lane come from transponder data which shows entry and exit plazas and entry time, along with paid toll price. By comparing the counts throughout morning peak period with the GP loop detector data, Pr(L) can be determined.

Pr(R) was calculated by correlating the subscription choice model in [Owen et al.](#page-96-4) [\(2013\)](#page-96-4) with subscription data for each transportation analysis zone (TAZ) along the corridor. Each vehicle's entrance ramp can be probabilistically correlated to surrounding TAZs. By then applying the subscription choice model to the total set of SOVs, a subset of transponder equipped SOVs is formed.

7.2.3 Calibration Day

In previous research conducted by the Minnesota Traffic Observatory (MTO), trip generation models and traffic simulations were calibrated to November 29, 2011. This day was selected because it was an average day with no weather or crash related problems along the MnPASS corridors. Due to the connection of this research to the calibrated simulation used in the MTO, this calendar day was selected for calibration of the lane choice model.

The $Pr(L)$ value from 11/29/2011 and $Pr(R)$, result in:

$$
Pr(L|R) = (100\%) * (11.8\%) / (17.3\%) = 68.1\% \tag{7.2}
$$

7.2.4 Price-Time Savings and Price-Reliability Models

Although the MnPASS toll price fluctuates based on HOT density, there is a direct correlation between the toll and the time savings the MnPASS lanes provide over the GP lanes. The higher the toll, the greater the time savings. This correlation is observed by users and explains the positive demand elasticity to price results in Part I of this thesis.

Using average toll prices and time savings data from 2012, a log relationship was fit. The bimodal relationship of the data meant two log functions were fit, one for congestion onset and one for the offset.

The relationship between price and time savings during congestion onset is represented by $\Delta T_{onset} = 1.2587ln(P) + 0.5527$ with an r^2 value of 0.892. The relationship during congestion offset is represented by $\Delta T_{offset} = 0.7953ln(P) + 1.2965$ with an r^2 value of 0.913. The congestion onset and offset data are significantly different at the α =0.001 level. The curves are displayed in Figure [7.1.](#page-72-0)

The increased travel time reliability of the MnPASS lanes is also proportional to the toll price. Again, two log functions were fit to the congestion onset and offset data.

The relationship between price and difference in time savings variance during congestion onset is represented by $\Delta V_{onset} = 1.1413ln(P) + 0.9566$ with an r^2 value of 0.942. The relationship between during congestion offset is represented by $\Delta V_{offset} = 0.926ln(P) +$ 1.6636 with an r^2 value of 0.9657. The congestion onset and offset data are significantly different at the α =0.001 level.

Table 7.1: Price-Time Savings and Price-Reliability Regression Results

		Time Savings vs Price	Time Variance Difference vs Price		
Variable	Onset	Offset	Onset	Offset	
Intercept	$0.5527(0.05972)$ ***	$1.2965(0.02912)$ ***	$0.9566(0.03867)$ ***	$1.6636(0.01664)$ ***	
log(P)	$1.2587(0.07732)$ ***	$0.7953(0.03743)$ ***	$1.1413(0.05006)$ ***	$0.926(0.02139)$ ***	
\boldsymbol{n}	40	40	40	40	
r^2	0.8923	0.913	0.942	0.9657	

(Standard error in parentheses)

Significance * 0.05, ** 0.01, *** 0.001

Time Savings and Time Variance Difference in minutes are the dependent variables, price in USD is the independent variable

Figure 7.1: Price-Time Savings Log Model

 $\Delta T_{onset} = 1.2587ln(P) + 0.5527 (r^2 = 0.8923)$ $\Delta T_{offset} = 0.7953ln(P) + 1.2965 (r^2 = 0.913)$ where ΔT is travel time savings in minutes and P is price in USD

Figure 7.2: Price-Reliability Model

 $\Delta V_{onset} = 1.1413ln(P) + 0.9566(r^2 = 0.942)$ $\Delta V_{offset} = 0.926ln(P) + 1.6636 (r^2 = 0.9657)$
where ΔV is time variance difference in minutes and P is price in USD

7.2.5 Calibration Process

The following flowchart displays the process of calibrating the lane choice model. The model coefficients are altered following a grid search technique until the resulting HOT lane share percentage matches the calibration target of 68.1%.

7.2.6 Resulting Coefficients

The lane choice parameters were for both congestion onset and offset. The resulting values are found in Table [7.2](#page-76-0) below.

Parameter	Carrion(2010)	Onset	Offset
Expected Travel Time	-0.672	-7.27	-10.7
Travel Time Variability	-0.228	-2.47	-3.63
HOT Lane Toll	-6.94	-6.94	-6.94
Alternative Specific Constant	-2.23	\cup	

Table 7.2: Lane Choice Model Parameters for Calibration

7.3 Testing of the Alternative Pricing Strategies

The calibrated HOT lane choice model was used to test the behavior of the alternative pricing strategies and how changing prices affect HOT_{share} , which is the share of transponder owning SOVs which use the MnPASS lane.

$$
HOT_{share} = \frac{\text{\# of transport}}{\text{Total \# of transport}} = \frac{\text{SOVs using the MnPASS lane(s)}}{\text{Total Hanes}} = Pr(L|R)
$$
\n(7.3)

Each pricing strategies' coefficients were incrementally adjusted and the process rerun to determine the resulting HOT_{share} . The average price and HOT_{share} were recorded for each iteration. The results were graphed and fit for each pricing strategy (congestion onset and offset). Table [7.3](#page-77-0) displays the regression results from fitting one pricing strategy using a first, second, third and fourth order polynomial function. The fourth degree polynomial functions for each scenario are displayed in [7.4](#page-77-1) and the figures below. The first four figures represent congestion onset, the latter four congestion offset.

		Variable Model 1 (1st order) Model 2 (2nd order) Model 3 (3rd order) Model 4 (4th order)		
Intercept	$21.46(5.321)$ ***	$-0.9280(3.140)$	$-15.297(1.997)$ ***	$-25.884(0.9949)$ ***
\boldsymbol{P}	1.017(1.291)	$35.019(2.892)$ ***	$71.931(3.546)$ ***	$108.91(2.670)$ ***
P^2		$-4.5576(0.3788)$ ***	$-17.218(1.129)$ ***	$-40.396(1.516)$ ***
P^3			$1.0781(0.09485)$ ***	$5.7446(0.2956)$ ***
P^4				$-0.2941(0.01849)$ ***
\boldsymbol{n}	44	44	44	44
r^2	0.0146	0.7825	0.9486	0.9931

Table 7.3: Continuous Pricing Function Onset Regression Results

(Standard error in parentheses)

Significance * 0.05, ** 0.01, *** 0.001

HOTshare is dependent variable, P is price in USD

P is price in USD

Figure 7.3: Continuous Pricing Function - Onset

 $HOT_{share} =$ $=-0.2941P$ $^{4}+5.7446P$ $3 - 40.396P$ $^{2}+108.91P$ $\begin{split} HOT_{share} & = -0.2941P^{*} + 5.7446P^{5} - 40.396P^{2} + 108.91P - 25.884 \ r^{2} & = 0.9931 \end{split}$

Figure 7.4: Unweighted Value Pricing - Onset

 $HOT_{share} =$ $= -0.1555P$ $^{4}+3.6124P$ $3 - 31.812P$ $^{2}+106.14P$ $\begin{split} HOT_{share} & = -0.1555P^{*} + 3.6124P^{5} - 31.812P^{2} + 106.14P - 26.573 \ r^{2} & = 0.9909 \end{split}$

Figure 7.5: $HOT_{weighted}$ Value Pricing - Onset

 $\begin{split} HOT_{share} & = -0.3468P^4 + 6.3111P^3 - 40.349P^2 + 98.579P - 22.116 \ r^2 & = 0.9493 \end{split}$

Figure 7.6: $GP_{weighted}$ Value Pricing - Onset

 $\begin{split} HOT_{share} & = -0.1785P^{4} + 3.2471P^{3} - 22.649P^{2} + 66.301P - 13.515\ r^{2} & = 0.9604 \end{split}$

Figure 7.7: Continuous Pricing Function - Offset

 $HOT_{share} =$ = −0.2394*P*
′ $^{4} + 4.4049P$ $3 - 27.423P$ $\begin{split} HOT_{share} & = -0.2394P^4 + 4.4049P^3 - 27.423P^2 + 51.688P + 61.127 \ r^2 & = 0.9904 \end{split}$

Figure 7.8: Unweighted Value Pricing - Offset

 $HOT_{share} =$ $=-0.1284P$ 4 + 3.0066*P* $3 - 23.308P$ $\begin{split} HOT_{share} & = -0.1284P^4 + 3.0066P^3 - 23.308P^2 + 51.99P + 61.762 \ r^2 & = 0.9887 \end{split}$

Figure 7.9: $HOT_{weighted}$ Value Pricing - Offset

 $\begin{split} HOT_{share} & = -0.1652P^{4}+2.9438P^{3}-17.877P^{2}+29.923P+69.3 \ r^{2} & = 0.9801 \end{split}$

Figure 7.10: $GP_{weighted}$ Value Pricing - Offset

 $\begin{split} HOT_{share} & = -0.0546P^4 + 1.2879P^3 - 10.691P^2 + 22.483P + 68.142 \ r^2 & = 0.9905 \end{split}$

7.3.1 Elasticity

The functions above describe HOT_{share} as a function of toll price. The elasticity of HOT_{share} to price is determined by taking the derivative of the function and multiplying by the quotient of price divided by HOT_{share} .

$$
\varepsilon_{HOT_{share}(P)} = \frac{P * HOT'_{share}(P)}{HOT_{share}(P)} = \frac{dlnHOT_{share}(P)}{dlnP}
$$
\n(7.4)

[7.11](#page-87-0) and [7.12](#page-88-0) are graphs of elasticity as a function of price for the continuous function pricing strategy (onset and offset). The elasticity equations are displayed below each figure.

Figure 7.11: Continuous Pricing Function - Onset

 $\varepsilon_{HOT_{share}(P)} = \frac{P*(-1.176*4*P^3+17.23*P)}{HOT_{share}}$ $\frac{^{3}+17.23*P^{2}-80.79*P+108.9)}{HOT_{share}(P)}$ where p is price in USD

Figure 7.12: Continuous Pricing Function - Offset

 $\varepsilon_{HOT_{share}(P)} = \frac{P*(-0.9576*P^3+13.21*P)}{HOT_{share}}$ $\frac{3+13.21*P^2-54.85*P+51.69)}{HOT_{share}(P)}$ where p is price in USD

7.4 Discussion

All four pricing strategies show a similar pattern in the relationship between HOT_{share} and price. The maximum HOT_{share} during congestion onset is achieved between \$2 and \$3, whereas during congestion offset, the greatest HOT_{share} occurs between \$1 and \$2. In general, the HOT_{share} during congestion offset is greater than during the onset due to the greater time savings and reliability per dollar toll price as demonstrated previously in [7.1](#page-72-0) and [7.2.](#page-73-0)

The unweighted value pricing strategy achieves the highest maximum HOT_{share} of all pricing strategies, but the share rises and drops more quickly than the weighted value pricing strategies $(HOT_{weighted}$ and $GP_{weighted}$). Table [7.5](#page-89-0) shows the average price and HOT_{share} for each pricing strategy along with the standard deviation.

					Pricing Strategy Avg Price (\$) Std Dev Price (\$) Avg HOT_{share} (%) Std Dev HOT_{share} (%)	
			Onset	Offset	Onset	Offset
Continuous	2.93	2.93	54.6	24.4	31.6	24.7
Unweighted	3.19	3.20	54.1	24.5	36.9	32.6
HOT Weighted	3.65	2.93	49.4	24.5	31.6	19.3
GP Weighted	3.83	3.26	45.5	18.9	34.0	21.0

Table 7.5: Average HOT_{share} and Prices

Figures [7.3](#page-78-0) - [7.10](#page-85-0) show the rise and fall of the HOT_{share} as the toll increases (and therefore, time savings). The following chart outlines how changes in toll and time savings affect HOT_{share} and ultimately, elasticity to price.

At lower tolls, an increase in price results in a higher HOT_{share} (positive elasticity), whereas at higher tolls, an increase in price causes a decrease in HOT_{share} (negative elasticity). At lower tolls, the improved time savings and reliability outweigh the increase in toll. However, at higher tolls, the increase in toll outweighs greater time savings and reliability causing the HOT_{share} to decrease.

7.5 Conclusion

Part II of this thesis outlined four HOT lane pricing strategies which could serve as alternatives to the current MnPASS pricing system. The current system relies on a series of density and price tables to determine the toll based strictly on HOT lane density. The proposed alternatives determine the toll based on a simple mathematical function relating HOT lane density (and GP density in three of the strategies) to price. The three value pricing strategies use the difference in GP and HOT lane density to determine the toll. Due to the nonlinear relationship between density and time savings, two of the strategies are weighted by either HOT density or GP density. The $HOT_{weighted}$ strategy combines the value pricing concept with the current algorithm's direct correlation between HOT density and price. For this reason, this pricing strategy would provide the greatest improvement over the current pricing system while still maintaining some of the same logic. The continuous function, on the other hand, most closely resembles the current pricing system, but fails to account for the density in the GP lanes.

The behavior of the four alternative pricing strategies was tested using a fixed demand partial equilibrium analysis. Using a calibrated lane choice model, simulated vehicles made decisions on whether to use the MnPASS lane based on the toll and their anticipated time savings and improved travel time reliability. The HOT_{share} was determined at various price increments for each pricing system. These were plotted and fit with a fourth degree polynomial, the derivatives of which correlate to the elasticity to price. In all cases, demand elasticity to price was positive at lower tolls and negative at higher tolls. MnPASS users recognize the correlation between the toll price and the time savings and travel time reliability provided by the lanes. The toll price acts as a proxy of downstream congestion. At lower tolls, the travel time savings and reliability advantage outweighs the cost of the toll and HOT_{share} rises. However, at higher tolls, the cost of using the lane begins to outweigh the benefit and the HOT_{share} drops.

Chapter 8

Conclusion

With the increasing interest in HOT lanes around the US, it is important to understand drivers responses to varying toll prices. Specifically focusing on the MnPASS lanes on I-394 and I-35W in Minneapolis, this thesis found drivers paid between \$60 and \$124 per hour of travel time savings. Consistent with other studies, these values suggest drivers are paying for more than just travel time savings, but other factors such as reliability.

Analysis of driver elasticity using various methods yielded positive demand elasticity to price. Both SOVs and HOVs increased usage of the MnPASS lanes with higher prices. Statistically significant elasticities ranged between about $+0.03$ to $+0.85$. The increased demand resulting from higher prices (and decreased demand from lower prices) is likely a result of driver perception of the posted price. Drivers likely view the price as an indication of time savings and congestion, suggesting higher prices provide greater time savings. No travel times or congestion levels are made available to drivers entering MnPASS corridors, therefore, the MnPASS price may act as a signal of downstream congestion. Drivers must make a quick decision whether to use the MnPASS lanes and the posted price acts as one important factor. Other intangibles also influence a user's lane choice decision. In any case, drivers are consuming different goods when the toll varies, because time savings is not constant. These different goods represent different demand curves and not movement along one downward sloping demand curve [\(Beggs,](#page-94-0) [2010\)](#page-94-0). Therefore, although price is higher, quantity consumed is also higher.

Alternative pricing strategies were proposed which allow for determining tolls through simple mathematical functions instead of pricing tables. Three of the strategies incorporate the density of the GP lanes. These strategies more closely tie the toll price to the time savings benefit provided by the MnPASS lanes by looking at the difference in density between the HOT and GP lanes.

The partial equilibrium analysis used fixed demand data to calibrate a HOT lane choice

model developed by [\(Carrion,](#page-95-0) [2010\)](#page-95-0). The lane choice model was used to test the four alternative pricing strategies and see how HOT_{share} levels change with price. The correlation between toll price and time savings causes the HOT_{share} to rise as price is increased at lower toll values. However, at higher toll prices, the travel time benefits are outweighed by the increase in price causing the HOT_{share} to decrease. This behavior causes both positive and negative elasticity values. The positive elasticity results from Part I are likely the result of the average toll price being less than the"critical price" at which elasticity turn negative and HOT_{share} decreases. Therefore, an increase in the average toll caused an increase in demand and vice versa. As HOT lane engineers incorporate pricing strategies, it is important to understand this behavior and ensure that the average HOT density at the "critical price" is less than or equal to the maximum desired HOT density.

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