ENGAGING IN R&D AND THE EMERGENCE OF EXPECTED NON CONVEX-TECHNOLOGIES

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1. Introduction

Consider a producer who is able to produce certain outputs from certain inputs by means of a known technology. The producer has also the option of diverting some of his resources to research and development activities with the hope of discovering a more efficient technology (or a better selling product). In deciding whether to engage in R&D at all, and if so , how intensely, the decision maker has to consider that real resources (capital, manpower, and even time) used up in R&D cannot be utilized to produce according to the good old production function, and that there is no certainty in R&D activities yielding positive results.

If the probability distribution of the success of the research and development activities (as a function of the magnitude of the resources invested in those efforts) is known, then it is possible to compute the expected output which will be produced with the remaining resources (as inputs) after some resources have been used up for R&D. The resulting expected production function will be, generally speaking, non-concave (i.e., the technology is non-convex) even if everything in sight is classical, exhibiting decreasing returns to scale, no set-up cost, etc.

In this note we will show, by means of simple examples, how such phenomena occur. We will consider the case of one input and one output only. (One can think of the output as the selling value of the product if one wishes to incorporate the possibility of the R&D effort resulting in a higher quality product.) In section 2 we consider a model where the research and development effort consists of making a sequence of experiments having a fixed cost c and a fixed probability of success p, while in section 3 a model where a continuous increase in the outlays for R&D improves monotonically the odds for a favorable outcome is discussed.

The mathematical formalism can be interpreted also as pertaining to utility functions (rather than to production). One can think of an individual presently deriving, under certain given circumstances, a utility level from his wealth. Our individual might wish to use part of his wealth in order to experiment with a radical change in his lifestyle (e.g., move to a different region or country) which might enhance his enjoyment of life. If the numerical values represent Van-Neumann Morgenstenn utilities, then one could use expected values for the emerging utility function of wealth, generating a non-concave utility function. A less dramatic example might involve the use of resources to shop for a store which sells for less items from which utility is derived.

I am very much indebted to W. Zame for several interesting discussions.

2. Fixed costs, fixed probabilities experiments.

Let $x = f_o(q)$ be a known concave, monotone increasing function mapping the non-negative half-line into itself with $f_o(o) = o$. Here q is interpreted as the input and x is the produced output. It is known that a better technology $f_1(q)$ exists (i.e., $f_1(q) > f_o(q)$ for all $q > o$), and if amount c of real inputs is invested in experimentation, then the production function f_1 can be discovered with probability p. Here c and p are positive real numbers, $p < 1$, and both c and p are independent of the number of experiments carried out so far. The function f_1 itself (the possible outcome of the R&D effort) is known in advance.

If $q \geqslant c$ is available, then one can use c units of input to experiment. If the experiment is successful, then one can utilize the remaining $q - c$ units to produce $f_1(q-c)$; otherwise, $f_o(q-c)$ units of output can be produced. The expected output $E_1(q)$ is thus

$$E_1(q) = p\ f_1(q-c) + (1-p)\ f_o(q-c).$$

More generally, denoting by E_n the expected production function if a sequence of n experiments is planned (obviously, one stops experimenting after a success), we find that

(2.1) $$E_n(q) = p \sum_{j=0}^{n-1} (1-p)^j f_1(q-(j+1)c) + (1-p)^n f_0(q-nc).$$

Hence

(2.2)

$$E_{n+1}(q)-E_n(q) = (1-p)^n [pf_1(q-(n+1)c) + (1-p) f_0(q-(n+1)c) - f_0(q-nc)]$$

Note that the right hand side of (2.2) is equal to $(1-p)^n$ times the expected gain from performing the n-th experiment, given that the first n-1 experiments failed. Thus $E_{n+1}(q) > E_n(q)$ iff this expected gain is positive.

Example 2.1: Set $f_i(q) = a_i q$, $i = 1, 1, a, > a_0 > 0$. Then

$$E_{n+1}(q)>E_n(q) \text{ iff } [p a_1 + (1-p) a_0] (q - (n+1)c) > a_0(q-nc).$$

An elementary computation shows that

(2.3) $$E_{n+1}(q) > E_n(q) \text{ iff } q > c\,[n+1 + \frac{a_0}{p(a_1-a_0)}]$$

It follows that in order for a research and development program to be worthwhile, not only do you need the required resources ($q > nc$), but you have got to have the resources necessary for profitable exploitation of the results of the program. Note also that if $E_{n+1}(q) > E_n(q)$ for some q and n, then $E_{n+1}(q^1) > E_n(q^1)$ for all $q^1 > q$ and $E_{jv1}(q) > E_j(q)$ for all $j < n$, and that the higher the probability of success or the technological advantage $(a_1 - a_0)$, the more attractive is the investment in research and development.

Set now

$$F(q) = \max_n E_n(q) \tag{2.4}$$

$(E_0(q) = f_0(q))$. Then $F(q)$ denotes the expected production obtainable from pursuing an optimal experimentation policy. It follows from (2.3) that

$$F(q) = E_n(q) \text{ for } c[n+1+\frac{a_0}{p(a_1-a_0)}] \geq q > c[n+\frac{a_0}{P(a_1-a_0)}] \tag{2.5}$$

and

$$F(q) = a_0 q \quad \text{for} \quad q \leq c\,[1+\frac{a_0}{p(a_1-a_0)}].$$

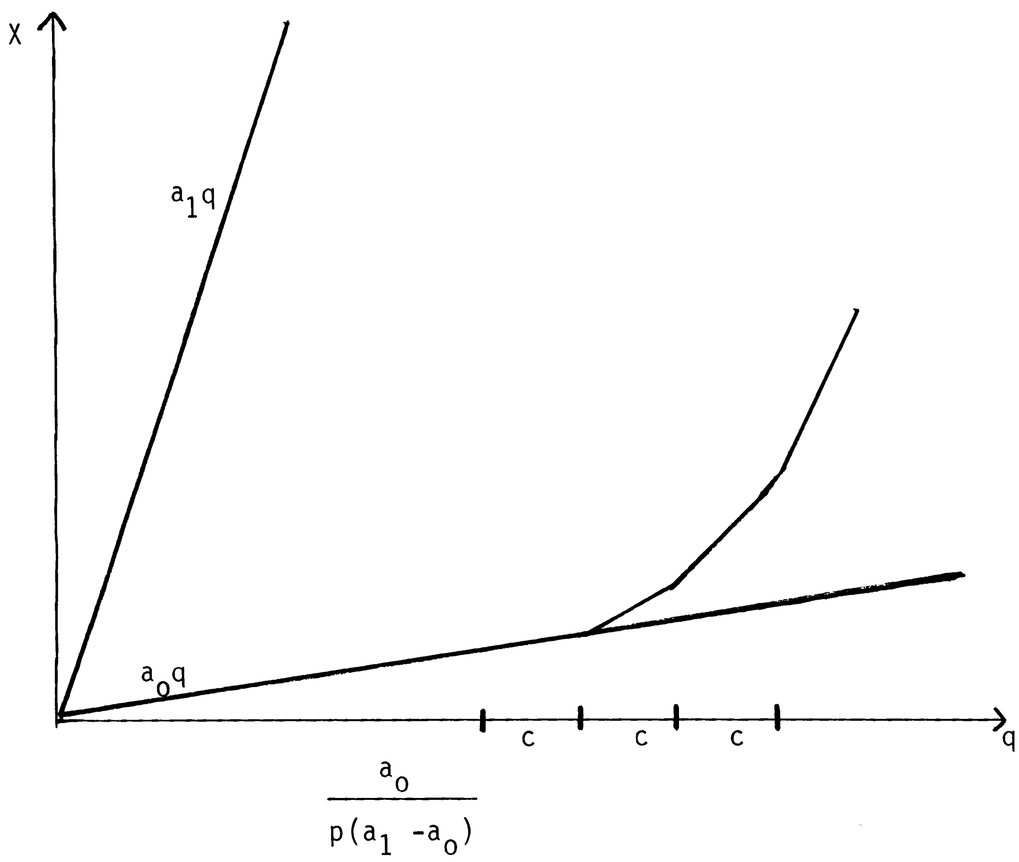
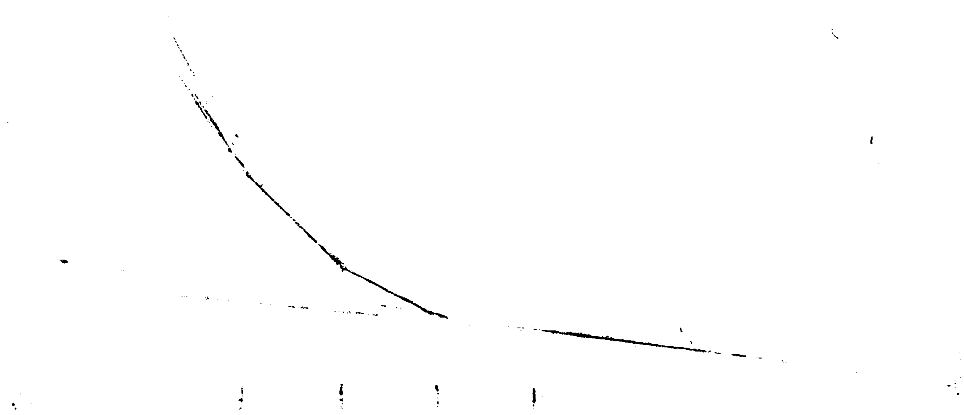


Figure 1



The function $F(q)$ can be regarded as the production function resulting from R&D efforts. Note that $F(q)$ is <u>not</u> concave. Rather, it is piecewise linear with the slope converging monotonically to a_1 - the slope of the best possible production function. (Thus $F(q)$ is convex.)

Example 2.2: Set $f_i(q) = a_1 \sqrt{q}$, $i = 1,2$, $a, > a_o > 0$. Here

$$(2.7) \qquad E_n(q) > E_n(q) \quad \text{iff } [pa_1 + (1-p)a_o] \sqrt{q-(n+1)c} > a_o \sqrt{q-nc}$$

Hence

$$(2.8) \qquad E_{n+1}(q) > E_n(q) \quad \text{iff} \quad q > c[n+1 + \frac{1}{2p(a_1/a_o-1) + p^2(a_1/a_o-1)^2}]$$

The qualitative features here are the same as in example 2.1, except that $F(q)$ is piecewise parabolic (and is neither convex nor concave).

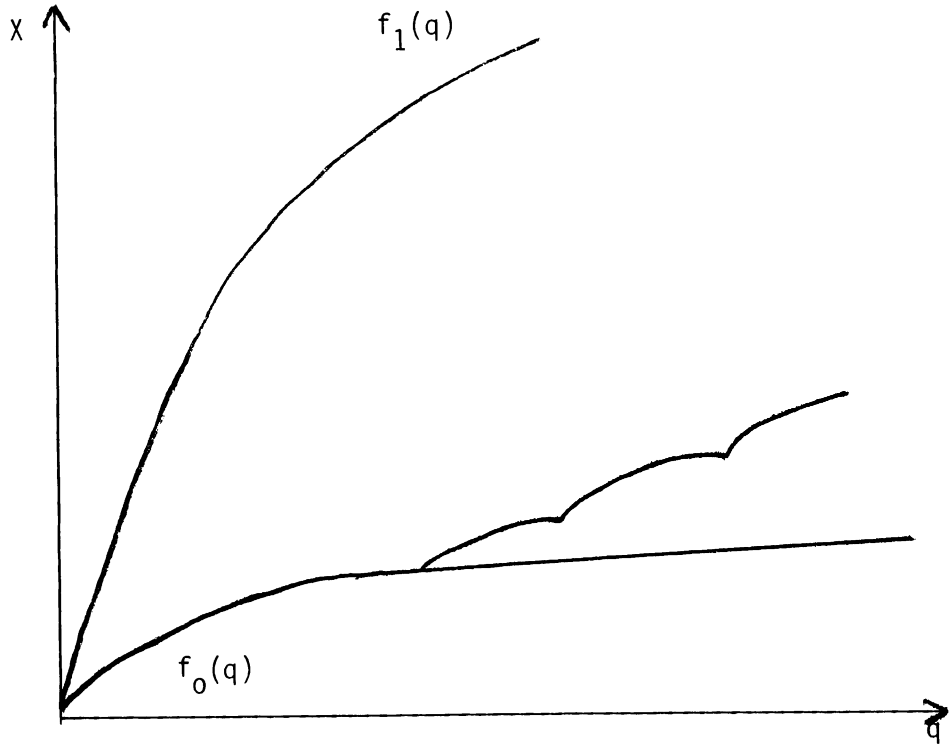


Figure 2

Example 2.3: We assume that $f_1(t)$, $f_2\,(t)$ are differentiable for $t < o$ and $f_1'(t) > f_o'(t)$ for all $t < o$ $(f_o(o) = f_1\,(1) = 0)$. As before, f_i are concave and monotone increasing. Set $D(q) = p\,f_1(q-c) + (1-p)\,f_o(q-c) - f_o(q)$. It follows from (2.2) that

$$E_{n+1}(q) - E_n(q) \;>\; \text{iff}\;\; D(q-nc) \;>0 \tag{2.9}$$

By assumptions,

$$D'(q) = p\,[f_1'(q-c) - f_o'(q-c) + [f_o'(q-c) - f_o'(q)\,] > 0 \tag{2.10}$$

Hence $D(\overline{q}) > 0$ for some $\overline{q}$ implies $E_{n+1}\,(q) > E_n\,(q)$ for all $q > \overline{q} + nc$. It also follows from (2.9) that if $E_{n+1}(q) > E_n(q)$ then $E_n(q-c) > E_{n-1}\,(q-c)$, so that $E_n(q) > E_{n-1}(q)$. Thus we get the same phenomena as in examples 2.1 and 2.2.

Remarks: If $f_0 = q^{\alpha}$,$f_1 = q^{\beta}$ and $\beta > \alpha$, then the assumptions are not satisfied for all q, but are satisfied for q sufficiently large.

If the graphs of f_0 and f_1 intersect (besides at the origin) at more than one point, it might happen that only a producer whose endowment q is at a certain range will find it profitable to engage in R&D, the bigger (and smaller) ones being in regions when $E_1(q) < E_0(q)$ (this might happen if D(q) is no longer monotone).

3. A continuous model.

We present here a simple model in which the amount invested in research and development can vary continuously. Variations of data in the model can cause wide variations in propensities to engage in R&D.

Let $x = f(q)$ be a known production function. The effect of experimenting is to discover a way of producing $af(q)$, with $a \geqslant 1$, according to a distribution function having $g(c)$ as its expectation, where $g(c)$ is an increasing function of the amount c of input invested in R&D.

Hence, given a fixed amount q of input, the expected amount $E(c)$ of output which can be produced is given by

$$E(c) = g(c)\, f(q-c) \tag{3.1}$$

Looking (as usual in such investigations) for an interior maximum, we are led to the following equation satisfied by the optimal research investment c^*:

$$E'(c^*) = 0 \tag{3.2}$$

or

$$\frac{g'(c^*)}{g(c^*)} = \frac{f'q-c^*)}{f\ (q-c^*)} \tag{3.3}$$

From now on we assume, for simplicity, that $f(q) = q^{\alpha}$ for a certain real positive α, and investigate the effect of choosing various functional forms for g (it is clear that we could have used other choices as well). Then (3.3) reads

$$\frac{g'(c^*)}{g(c^*)} = \frac{1}{\alpha(q-c^*)} \tag{3.4}$$

Example 3.1: Let $h(c) = \ln c$. Then (3.4) implies that

$$\frac{1}{c^*\ln c^*} = \frac{1}{\alpha(q-c^*)} . \tag{3.5}$$

In this case c^* is much smaller than q, with c^*/q tending to zero as $q \to \infty$.

Example 3.2: Let $g(c) = c$. Then (3.4) implies that

$$c^* = \frac{\alpha}{1+\alpha}\, q \tag{3.6}$$

The same result applies if $h(c) = \beta c$ for a real constant β.

Example 3.3: Let $h(c) = e^{c}$. Then (3.4) implies that

$$c^* = q - \frac{1}{\alpha} \tag{3.7}$$

The intensity of research activity is usually defined as the ratio between the R & D effort and the total volume of the economic activity. In our model the research intensity is equal to c^*/q. In example 3.1, the intensity goes to zero when size goes to infinity; in example 3.2, the intensity is independent of size; while is example 3.3, the intensity goes to 1 as the size increases. As a matter of fact, the expectation of "windfall profits" causes the decision maker to invest almost everything in research and development (3.7).

Remarks: It was assumed implicitly in sections 2 and 3 that the decision making producer is risk neutral and evaluates projects by their expected outcome only. This assumption is commonly made and is presumably valid for big firms owned by a large number of shareholders who split their investments among many companies. On the other hand, a risk-loving small company (maybe it has no other choice?) can invest heavily in R&D and succeed in a small fraction of the cases. Note also that it is assumed here that patent rights belong exclusively to the developer, but is probably true that in the long run, with enough small firms being engaged in R&D and a number of them being eventually successful, the well known production curve will shift upward from $f_0(q)$.