EXISTENCE AND NON-EXISTENCE FOR QUASI-LINEAR ELLIPTIC EQUATIONS WITH THE P-LAPLACIAN INVOLVING CRITICAL SOBOLEV EXPONENTS

By

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1. Introduction

In this paper we consider the following quasi-linear elliptic problem:

$$
\text{(I)} \begin{cases} -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(u), \quad u > 0, & \text{in } \Omega \quad (1.1) \\ u = 0 & \text{on } \partial\Omega, \quad (1.2) \end{cases}
$$

where Ω is a smooth, bounded domain in $\mathbf{R}^N$, $1 < p < N$ and $f(u)$ is a given function with

$$
f(0) = 0, \quad f(s) > 0 \text{ if } s > 0.
$$

The differential operator $-\operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is commonly referred to as the p-Laplacian and denoted by $-\Delta_p u$. If $p = 2$ it reduces to the ordinary Laplacian. Problem (I) has been studied by many authors (see for instance [E], [GP], [GV], [PS] and references given there). It turns out that several phenomena occuring for Problem (I) when $p = 2$ and $N > 2$, also manifest themselves for general values of p, $1 < p < N$. Thus we observe that if $f(u)$ is given by a power of u:

$$
f(u) = u^q,
$$

then there exists a critical Sobolev exponent

$$
q^* = \frac{(p-1)N + p}{N - p}
$$

and the following results hold:

(a) if $1 < q < q^*$, then there always exists a solution of Problem (I) for any bounded domain Ω.

(b) if $q \geq q^*$, then there does not exist a solution of Problem (I) for any star-shaped domain Ω.

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One proves Result (a) by means of variational methods as in [AR], using the fact that the embedding

$$I : W_0^{1,p}(\Omega) \to L^{q+1}(\Omega) \tag{1.3}$$

is compact if $1 < q < q^*$. However, if $q \geq q^*$ this is no longer true.

Result (b) follows from a generalization of the Pohozaev Identity ([E], [GV], [PS]). It reads:

If u is a solution of Problem (I), then it must satisfy

$$N \int_\Omega F(u) - \frac{N-p}{p} \int_\Omega f(u)u = \frac{p-1}{p} \int_{\partial\Omega} |\nabla u|^p (x, n), \tag{1.4}$$

where $F(u)$ is $\int_0^u f(s)\,ds$ and n the outward pointing normal on $\partial\Omega$.

From [V], [T] it follows that $\frac{\partial u}{\partial n} < 0$ on $\partial\Omega$. The requirement that Ω is star-shaped is equivalent to $(x, n) > 0$. Hence the right-hand-side of (1.4) is positive. Upon substitution of $f(u) = u^q$ Result (b) is deduced.

It is well known that if one adds a lower order term to the critical power, one can have solutions of Problem (I) even on star-shaped domains. The first result in this direction was discovered by Brezis and Nirenberg.

Let Ω be a ball in $\mathbf{R}^N$ $(N > 2)$ and

$$p = 2 \qquad \text{and} \qquad f(u) = u^{(N+2)/(N-2)} + \lambda u, \tag{1.5}$$

where $(N+2)/(N-2)$ is the critical Sobolev exponent for $p = 2$.

THEOREM 1.1. [BN] *If $p = 2$, then*

(a) if $N = 3$, Problem (I) has a solution if and only if $\mu_1/4 < \lambda < \mu_1$,

(b) if $N \geq 4$, Problem (I) has a solution if and only if $0 < \lambda < \mu_1$,

where μ_1 is the first eigenvalue of $(-\Delta)$ with the Dirichlet boundary condition.

The dimensions $2 < N < 4$ are called the critical dimensions of Problem (I). In this paper we shall discuss the phenomenon of critical dimensions for general values of p, $1 < p < N$ and throughout we shall assume

$$\Omega = B_R = \{x \in \mathbf{R}^N : |x| < R\}.$$

The function $f(u)$ given in (1.5) is generalized in the following way:

$$f(u) = u^{q^*} + \lambda u^{p-1} \tag{1.6}$$

We choose the lower order term to be u^{p-1}, because it is of the same order as the differential operator. Connected to it is the 'half-linear' eigenvalue problem:

$$\text{(E1)} \begin{cases} -\Delta_p \psi = \mu \psi^{p-1}, \quad \psi > 0 & \text{in } B_R \\ \psi = 0 & \text{on } \partial B_R. \end{cases}$$

It is called 'half-linear' because the governing equation is invariant under multiplication by a constant.

We denote the first eigenvalue of Problem (E1) by

$$\mu = \mu_1. \tag{1.7}$$

The following theorems were proved in [GV] and respectively in [E] and [GV]:

THEOREM 1.2. *If $1 < p < N$, then Problem (I) does not admit any solution if $\lambda \geq \mu_1$.*

THEOREM 1.3. *If $N \geq p^2$, then there exists a solution of Problem (I) for every $\lambda \in (0, \mu_1)$.*

Conversely, in [APS] it is shown that:

THEOREM 1.4. *If $p < N < p^2$, then there exists a number $\bar{\mu} > 0$ such that if $\lambda \in [0, \bar{\mu})$ Problem (I) does not admit any radially symmetric solution.*

Thus for general values of p the critical dimensions become:

$$p < N < p^2.$$

In this paper we shall give a precise characterization of the number $\bar{\mu}$, when Ω is a ball and the solutions of Problem (I) are required to be radially symmetric. To do this we introduce a second, degenerate half-linear eigenvalue problem, defined on $B_R - \{0\}$:

$$\text{(E2)} \begin{cases} -\Delta_p \varphi = \mu \varphi^{p-1}, \quad \varphi > 0 & \text{in } B_R - \{0\} & (1.8) \\ \varphi = 0 & \text{on } \partial B_R & (1.9) \\ \varphi(x) - \dfrac{\nu^\nu}{|x|^\nu} \to 0 & \text{as } x \to 0, & (1.10) \end{cases}$$

where $\nu = (N-p)/(p-1)$. Problem (E2) has a radial solution exactly when $p < N < p^2$ as is proved in [KP]. We denote the eigenvalue associated to the first radially symmetric eigenfunction by:

$$\mu = \mu^*. \tag{1.11}$$

We shall prove

THEOREM A. *If $p < N < p^2$ and Ω is the ball B_R, then there exists a radially symmetric solution of Problem (I) if and only if*

$$\lambda \in (\mu^*, \mu_1).$$

REMARK 1.5. If $p = 2$ the number μ^* is identical to $\mu_1/4$, the constant occuring in Theorem 1.1.

REMARK 1.6. The result of Theorem A was predicted by numerical computations carried out by Budd and Egnell [BuE].

2. Proof of Theorem A

In this section we shall establish existence and non-existence results for the following problem:

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = u^{q^*} + \lambda u^{p-1}, \quad u > 0, & \text{in } B_R \qquad (2.1) \\ u = 0 & \text{on } \partial B_R, \qquad (2.2) \end{cases}$$

when

$$p < N < p^2.$$

Since we require the solutions to be radially symmetric, we can rewrite (2.2) into an ODE. To this ODE we apply the following transformation:

$$t = \left(\frac{\nu}{|x|}\right)^{\nu}, \qquad y(t) = u(|x|), \tag{2.3}$$

with

$$\nu = \frac{N-p}{p-1}. \tag{2.4}$$

This leads to the following problem

$$
\text{(II)} \begin{cases} (|y'|^{p-2}y')' + t^{-k}(y^{pl-1} + \lambda y^{p-1}) = 0, \quad y(t) > 0 \text{ on } (T, \infty), & (2.5) \\ \lim_{t\to\infty} y'(t) = 0, \quad y(T) = 0, & (2.6) \end{cases}
$$

where

$$
k = p\frac{N-1}{N-p}, \qquad l = \frac{k-1}{p-1}, \tag{2.7}
$$

$$
q^* = pl - 1 \quad \text{and} \quad T = \left(\frac{\nu}{R}\right)^{\nu}. \tag{2.8}
$$

Since y is concave on (T, ∞), the boundary condition (2.6) implies that $y' > 0$ on $[T, \infty)$, whence we can leave out the modulus signs in $(|y'|^{p-2}y')$. Remark that in this setting the conditions $N > p$ and $p < N < p^2$ become respectively

$$
k > p \quad \text{and} \quad k > p + 1.
$$

We shall prove the following theorem:

THEOREM 2.1. *Suppose $k > p+1$. Then Problem (II) admits a solution if and only if*

$$
\mu^* < \lambda < \mu_1,
$$

where μ_1 and μ^* are given by (1.7) and (1.8).

The half-linear eigenvalue Problem (E2) plays a major rôle in the proof of Theorem 2.1. Using the transformation (2.3) we rewrite equation (1.8) of Problem (E2) and the boundary conditions (1.9):

$$
\beta(t) = \varphi(x), \quad \beta(T) = \varphi(R) = 0,
$$

where T is given by (2.8). The 'initial' condition of φ at the origin is weakened to the following condition at infinity: $\lim_{t\to\infty} \beta'(t) = 1$ and we arrive at:

$$
\text{(III)} \begin{cases} ((\beta')^{p-1})' + \lambda t^{-k}\beta^{p-1} = 0, \quad \beta(t) > 0 \text{ on } (T, \infty), & (2.9) \\ \beta(T) = 0 \quad \lim_{t\to\infty} \beta'(t) \ = 1. & (2.10) \end{cases}
$$

Problem (III) has a solution for every $\lambda \in [0, \mu_1)$ precisely when $k > p+1$. The asymptotic expansion of $\beta(t)$ for t large is given by

$$
\beta(t) = t + a(\lambda) + O(t^{-k+p+1}). \tag{2.11}
$$

The proof of these results can be found in [KP, Section 5] . The function $a(\lambda)$ has the following properties:

LEMMA 2.2. *We have*

$$(i) \qquad a(\lambda) = -T + O(\lambda) \quad \text{as } \lambda \to 0$$

$$(ii) \qquad \text{if } \lambda_1 > \lambda_2, \text{ then } a(\lambda_1) > a(\lambda_2)$$

$$(iii) \qquad a(\mu^*) = 0,$$

Observe that if $\lambda = \mu^*$, then the corresponding solution $\beta(t)$ is equal to the first radial eigenfunction $\varphi_1(|x|)$ of Problem (E2).

We shall show that the sign of $a(\lambda)$ determines whether or not Problem (II) admits a solution. If $0 \leq \lambda \leq \mu^*$, then $a(\lambda) \leq 0$ and there does not exist any solution whilst for $\mu^* < \lambda < \mu_1$, $a(\lambda) > 0$ and a solution does exist.

To prove this we introduce the functional $H(v)$ defined for functions v which satisfy the following problem:

$$\begin{cases} ((v')^{p-1})' + t^{-k}g(v) = 0, & v(t) > 0 \text{ on } (T, \infty), \\ v(T) = 0. \end{cases}$$

It is given by

$$H(v) = t(v')^p - v(v')^{p-1} + \frac{p}{p-1}t^{1-k}G(v),$$

where $G(v) = \int_0^v g(s)\,ds$ and its derivative with respect to t is

$$\frac{d}{dt}H(v) = -t^{-k}\{plG(v) - g(v)v\}.$$

We are going to compare $H(v)$ when $v = y$ and when v is a multiple of β. The first step is to establish the negative result stated in Theorem 2.1. By Theorem 1.2 [GV], we only have to prove that there can not be a solution when λ lies in the interval $[0, \mu^*]$.

LEMMA 2.3. *If $k > p+1$ and $\lambda \in [0, \mu^*]$, then Problem (II) does not have a solution.*

PROOF. Arguing by contradiction, we suppose that we do have a solution $y(t)$ for a certain λ in the interval $[0, \mu^*]$. Then we multiply $\beta(t)$ by a constant $\theta_1 > 0$ such that

$$\theta_1 \beta'(T) = y'(T)$$

and set $\bar{\beta}(t) = \theta_1 \beta(t)$.

PROPOSITION 2.4. *The functions $y(t)$ and $\bar{\beta}(t)$ intersect an even number of times on (T, ∞).*

PROOF. Since $\lim_{t \to \infty} y(t) < \infty$ while $\bar{\beta}(t) \sim \theta_1 t$ for t large, $y(t) < \bar{\beta}(t)$ for t sufficiently large. Hence we need to show that $y(t) < \bar{\beta}(t)$ in a right neighbourhood $(T, T+\delta)$ of T. Suppose to the contrary that $y(t) \geq \bar{\beta}(t)$ in $(T, T+\delta)$. Then if one integrates (2.5) and (2.9) over (T, t) one finds for every t in this interval:

$$\begin{aligned} y'(t)^{p-1} &= y'(T)^{p-1} - \int_T^t s^{-k}(y^{pl} + \lambda y^{p-1})\, ds \\ &< \bar{\beta}'(T)^{p-1} - \int_T^t s^{-k} \lambda \bar{\beta}^{p-1}\, ds = \bar{\beta}'(t)^{p-1}, \end{aligned}$$

which, since $y(T) = \bar{\beta}(T) = 0$ yields a contradiction.

Now we introduce the function $\tilde{\beta}$, which is another multiple of β: If $\bar{\beta}$ and y intersect we denote the last two points larger than T at which they do by t_1 and t_2. Let θ_2 be

$$\theta_2 = \inf\{\theta > 1 : \theta \bar{\beta}(t) > y(t) \text{ on } [t_1, \infty)\}$$

and define

$$\tilde{\beta}(t) = \theta_2 \bar{\beta}(t).$$

Hence there exists a point $\tau \in (t_1, t_2)$ such that

$$\tilde{\beta}(t) > y(t), \quad \text{for all } t > \tau \tag{2.12}$$

$$\tilde{\beta}(\tau) = y(\tau), \quad \tilde{\beta}'(\tau) = y'(\tau). \tag{2.13}$$

If $y(t)$ and $\bar{\beta}(t)$ do not intersect for any $t > T$, we define $\tilde{\beta}(t) = \bar{\beta}(t)$, $\theta_2 = 1$ and $\tau = T$. Because $\tilde{\beta}(T) = y(T) = 0$ and $\tilde{\beta}'(T) = y'(T)$, (2.12) and (2.13) remain true. Hence the point τ lies in the interval $[T, \infty)$.

The function $\tilde{\beta}(t)$ thus constructed, satisfies the equation (2.9), while its expansion for t large is given by

$$\tilde{\beta}(t) = \theta_1\theta_2 t + \theta_1\theta_2 a(\lambda) + O(t^{p-k+1}).$$

The functional $H(v)$ becomes for y and $\tilde{\beta}$ respectively .

$$H(y)(t) = t(y')^p - y(y')^{p-1} + \frac{t^{1-k}}{k-1}(y^{pl} + \lambda l y^p),$$
$$\lim_{t\to\infty} H(y)(t) = 0, \quad \frac{d}{dt}H(y)(t) = -\lambda(l-1)t^{-k}y^p.$$

For $v = \tilde{\beta}$, we find

$$H(\tilde{\beta})(t) = t(\tilde{\beta}')^p - \beta(\tilde{\beta}')^{p-1} + \frac{t^{1-k}}{k-1}\lambda l \tilde{\beta}^p$$
$$\lim_{t\to\infty} H(\tilde{\beta})(t) = -(\theta_1\theta_2)^p a(\lambda), \quad \frac{d}{dt}H(\tilde{\beta})(t) = -\lambda(l-1)t^{-k}\tilde{\beta}^p.$$

This is the place where $a(\lambda)$ manifests itself. Since the derivatives $\frac{d}{dt}H(y)(t)$ and $\frac{d}{dt}H(\tilde{\beta})(t)$ are of the same form one can easily compare them. After integrating them over (τ, ∞) we find

$$\begin{aligned}
-\tau y'(\tau)^p + y(\tau)y'(\tau)^{p-1} - \frac{\tau^{1-k}}{k-1}\left(y(\tau)^{pl} + \lambda l y(\tau)^p\right) \\
= -\lambda(l-1)\int_\tau^\infty s^{-k} y^p \, ds \\
> -\lambda(l-1)\int_\tau^\infty s^{-k} \tilde{\beta}^p \, ds \\
= -(\theta_1\theta_2)^p a(\lambda) - \tau\tilde{\beta}'(\tau)^p + \tilde{\beta}(\tau)\tilde{\beta}'(\tau)^{p-1} - \lambda l \frac{\tau^{1-k}}{k-1}\tilde{\beta}(\tau)^p,
\end{aligned}$$

where we used (2.12). Finally substituting (2.13) we arrive at the inequality

$$\frac{\tau^{1-k}}{k-1} y(\tau)^{pl} < \theta_1\theta_2 a(\lambda).$$

However, since $y(\tau) \geq 0$ and $a(\lambda) \leq 0$ on $[0, \mu^*]$ this yields a contradiction and Lemma 2.3 is proved.

LEMMA 2.5 *If $\lambda \in (\mu^*, \mu_1)$ and $k > p+1$, then Problem (II) admits a solution.*

PROOF. The proof of this Lemma is essentially contained in [KP]. For completeness we sketch it here using the results deduced in [KP].

Consider the subcritical problem

$$(\mathrm{II}_\varepsilon)\begin{cases} ((y')^{p-1})' + t^{-k}(y^{pl-1-\varepsilon} + \lambda y^{p-1}) = 0, & y(t) > 0 \text{ on } (T,\infty), \\ y(T) = 0, \quad \lim_{t\to\infty} y'(t) = 0, & \end{cases}$$

Provided that $\varepsilon < p(l-1)$, Problem $(\mathrm{II}_\varepsilon)$ admits a solution $y_\varepsilon(t)$ as long as $0 < \lambda < \mu_1$. We shall show that if $\lambda \in (\mu^*, \mu_1)$, then $y_\varepsilon(t)$ converges to a solution of Problem (II) as $\varepsilon \searrow 0$.

Suppose to the contrary that Problem (II) does not admit a solution. In [KP; Section 5] it is proved that then necessarily $\gamma_\varepsilon = \lim_{t\to\infty} y_\varepsilon(t)$ blows up:

$$\lim_{\varepsilon\searrow 0} \gamma_\varepsilon = \infty.$$

We use this number γ_ε to rescale $y_\varepsilon(t)$; Set

$$\eta_\varepsilon(t) = \gamma_\varepsilon^\omega y_\varepsilon(t),$$

where $\omega = 1 - \theta/(p-1)$ and $\theta = \varepsilon/(l-1)$.

The following lemma is small adaptation of Lemma 6.1 in [KP].

LEMMA 2.6. *If*

$$\lim_{\varepsilon\searrow 0} \gamma_\varepsilon = \infty.$$

Then we have

$$\gamma_\varepsilon^{-\theta} \int_T^\infty y_\varepsilon(t)^{pl-\varepsilon}\, dt \to K$$

and

$$\eta_\varepsilon(t) \to k_1^{1/(p-1)} \beta(t) \quad \text{as } \varepsilon \searrow 0,$$

uniformly on compact sets in $[T, \infty)$, where

$$K = k_1 \frac{k-1}{k-p} \frac{\Gamma\left(\frac{l}{k-p}\right)\Gamma\left(\frac{k-1}{k-p}\right)}{\Gamma\left(\frac{pl}{k-p}\right)}$$

and

$$k_1 = (k-1)^{1/(l-1)}.$$

Thus, we see that if Problem (II) has no solution, then the rescaled function η_ε converges to a solution of the degenerate, half-linear eigenvalue Problem (III). However, we shall see that if we compare the functionals $H(\eta_\varepsilon)$ and $H(\beta)$, this leads to a contradiction when $\lambda \in (\mu^*, \mu_1)$. It turns out that $a(\lambda)$ then has the wrong sign. The functional $H(\eta_\varepsilon)$ is given by

$$\begin{aligned} H(\eta_\varepsilon)(t) &= \gamma_\varepsilon^{p\omega} H(y_\varepsilon)(t) \\ &= t(\eta_\varepsilon')^p - \eta_\varepsilon(\eta_\varepsilon')^{p-1} + \frac{t^{1-k}}{k-1}(\gamma_\varepsilon^{p\omega} y_\varepsilon^{pl-\varepsilon} + \lambda l \eta_\varepsilon^p), \end{aligned}$$

and

$$\begin{aligned} &\frac{d}{dt} H(\eta_\varepsilon)(t) = -t^{-k}\left(\frac{\varepsilon\gamma_\varepsilon^{p\omega}}{pl-\varepsilon} y_\varepsilon^{pl-\varepsilon} + \lambda(l-1)\eta_\varepsilon^p\right) \qquad (2.14) \\ &\lim_{t\to\infty} H(\eta_\varepsilon)(t) = 0. \end{aligned}$$

Integrating (2.14) over (T, ∞) yields

$$T\eta_\varepsilon'(T)^p = \frac{\varepsilon\gamma_\varepsilon^{p\omega}}{pl-\varepsilon}\int_T^\infty s^{-k} y_\varepsilon^{pl-\varepsilon}\, ds + \lambda(l-1)\int_T^\infty s^{-k}\eta_\varepsilon^p\, ds. \qquad (2.15)$$

Using (2.11), we evaluate $H(\beta)(t)$ at infinity and find

$$\lim_{t\to\infty} H(\beta)(t) = -a(\lambda).$$

Hence in the same way as above we find for $\beta(t)$

$$T\beta'(T)^p = -a(\lambda) + \lambda(l-1)\int_T^\infty s^{-k}\beta^p\, ds. \qquad (2.16)$$

Taking the limit as $\varepsilon \searrow 0$ in (2.15) and then substituting (2.16), Lemma 2.6 induces that

$$\frac{\varepsilon}{pl-\varepsilon}\gamma_\varepsilon^{(p-\theta)/(p-1)} \to -k_1^{p/(p-1)} K^{-1} a(\lambda).$$

But this is impossible since $a(\lambda) > 0$ on (μ^*, μ_1). Hence γ_ε must remain bounded as $\varepsilon \searrow 0$. Therefore we conclude that our supposition that Problem (II) did not admit any solution was false and that a solution of Problem (II) does exist. Moreover this solution can be obtained as the limit of the solutions $y_\varepsilon(t)$ of the subcritical problem (II_ε) as $\varepsilon \searrow 0$. Since $y_\varepsilon(t) < \gamma_\varepsilon$ on $[T, \infty)$, it follows that $y_\varepsilon(t)$ is bounded uniformly on $[T, \infty]$ uniformly in ε, for ε sufficiently small. Thus we infer after applying a compactness argument that

$$y_\varepsilon(t) \to y(t) \quad \text{as } \varepsilon \searrow 0 \text{ uniformly on } [T, \infty).$$

where $y(t)$ is a solution of Problem (II).

References

[AP1] F.V. ATKINSON & L.A. PELETIER, Elliptic equations with nearly critical growth, *J. Diff. Equ.* **70**, (1987), 349-365.

[AP2] F.V. ATKINSON & L.A. PELETIER, Emder–Fowler equations involving critical exponents, *Nonlinear Anal. TMA* **10** (1986), 755-776.

[AP3] F.V. ATKINSON & L.A. PELETIER, Large solutions of elliptic equations involving critical exponents, *Asymptotic Anal.* **1** (1988), 139-160.

[APS] F.V. ATKINSON, L.A. PELETIER, & J. SERRIN, *Unpublished Notes.*

[AR] A. AMBROSETTI & P.H. RABINOWITZ, Dual variational methods in critical point theory and applications, *J. Funct. Anal.* **14** (1973), 349-381.

[BN] H. BREZIS & L. NIRENBERG, Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents, *Comm. Pure Appl. Math.* **XXXVI** (1983), 437-477.

[BuE] C. BUDD & H. EGNELL, *Private Communication* (1988).

[E] H. EGNELL, Existence and nonexistence results for m-Laplace equations involving critical Sobolev exponents, *Ph.D. Thesis Uppsala Univ.* (1987).

[GP] J.P. GARCIA AZORERO & I. PERAL ALONSO, Existence and nonuniqueness for the p-Laplacian: Nonlinear eigenvalues, *Comm. P.D.E.* **12** (1987), 1389-1430.

[GV] M. GUEDDA & L. VERON, Quasilinear elliptic equations involving critical Sobolev exponents, *Nonlinear Anal. TMA* **13** (1989), 879-902.

[KP] M.C. KNAAP & L.A. PELETIER, Quasilinear elliptic equations with nearly critical growth, *Comm. P.D.E.* **14** (1989), 1351-1383.

[P] S.I. POHOZAEV, Eigenfunctions of the equation $\Delta u + \lambda f(u) = 0$, *Soviet Math. Dokl.* **6** (1965), 1408-1411.

[PS] P. PUCCI & J. SERRIN, A general Variational Identity, *Indiana Univ. Math. J.* **35** (1986), 681-703.

[T] P. TOLKSDORF, On the Dirichlet problem for quasilinear elliptic equations in domains with conical boundary conditions, *Comm. P.D.E.* **8** (1983), 773-817.

[T] J.L. VAZQUEZ, A strong maximum principle for some quasilinear elliptc equations, *Appl. Math. Opt.* **12** (1984), 191-202.

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