Essays in Public Economics

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Christopher Phelan

July, 2013

Acknowledgements

This dissertation could not be completed without the support of my advisor, Christopher Phelan to whom I am extremely thankful. His encouragements, comments, suggestions, thoughts, availability and support during the course of my doctoral studies tremendously helped my graduate school experience. His critical comments have shaped my understanding of economics. He has made it possible for me to access the LPS Analytics' database provided by the Federal Reserve Bank of Minneapolis. He has also given me access to supercomputers at the Minnesota Supercomputing Institute of the University of Minnesota. The computations in this dissertation required to execute a Message-Passing-Interface parallel code using 800 processors for thirteen hours. The computational resources were provided by the Minnesota Supercomputing Institute.

I would like to thank Kjetil Storesletten for his guidance in my research. He has always followed my progress and suggested several ways of improving my research, teaching me to always reach out for the best no matter how much effort and sacrifice is involved. I am also very grateful to Victor Rios Rull for his attention and suggestions. His comments and teachings have furthered my understanding of Economics modeling.

I thank my family for their love, assistance and support. I am grateful to my brother Max Pokam for teaching me computation and for guiding me in my life and in my studies. I am also very thankful to my parents and to all my other brothers and sisters for encouraging me at all times. Their belief in my work motivated me to successfully complete these studies.

I would like to thank Caty Bach, Simran Sahi, Kara Kersteter and my colleagues in the Economics department for their encouragements and support during my time in graduate school.

Dedication

To my sister Denise Pouokam who brought me up. She went above and beyond the possible to reach out for the best in me.

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Chapter 1

Introduction

This dissertation consists of two essays of public economics. In the first essay, I build a new and rich quantitative model of unsecured and secured debt to study the impact of the 2005 bankruptcy reform law on the foreclosure crisis during the great recession. In the second essay, I use a game theoretical model to show how political institutions shape prospects of economic growth.

In October 2005, a bankruptcy reform law was implemented with the intent to reduce the proportion of bankrupts among U.S. households. Prior to the bankruptcy reform law, U.S. households in bankruptcy had to repay their unsecured debts only up to the amount of their non-exempt home equity. The 2005 bankruptcy reform law imposed that above-median income earners repay their unsecured debts in bankruptcy up to the maximum between their non-exempt home equity and 5 years of "disposable income"(income over necessary living expenses). After the bankruptcy reform had fully taken place, housing prices fell steeply in most U.S. states, with certain states like the state of Nevada facing a percentage fall above 50% between 2007 and 2009. For the U.S. economy overall, the average price for U.S. single-family houses fell by 26% between the first quarter of 2007 and the last quarter of 2009. The U.S. average foreclosure rate rose from 0.16% in the period 2002-2004 to 0.9% in the period $2007-2009$. The first essay investigates the impact of the 2005 bankruptcy reform law on this surge in the foreclosure rate. To assess the quantitative impact of the bankruptcy reform law on the foreclosure crisis, I model a life-cycle economy in which households face idiosyncratic income and expense risks; they access homeownership by entering into

30-year fixed interest rate mortgage contracts with a "piggyback lien " option; they smooth consumption by borrowing in a second mortgage market and in an unsecured credit market; and they discharge unsecured debts through a bankruptcy system that mimics key features of both Chapter 7 and Chapter 13 of the U.S. bankruptcy code. I model the U.S. economy as a recourse economy in which following a mortgage default, households are liable for the fraction of the mortgage loan that is not covered by the proceeds of the foreclosure sale with a probability that is estimated using foreclosure data from the LPS Analytics Inc. database. I find that the bankruptcy reform did not significantly affect the foreclosure rate, but it moderately lowered the foreclosure rate by raising the opportunity cost of a bad credit record, thereby making households less likely to default simultaneously on mortgage contracts and on unsecured credit contracts.

The second essay is motivated by the observation that the standard neoclassical growth model fails to explain why during the last decade South-East Asian countries appeared to be catching up with leading economies while Sub-Saharan African and Latin American countries did not. While good performances from South-East Asian countries support the standard neoclassical growth model, economic stagnation in Sub-Saharan Africa and Latin America calls for a different theory that is capable of explaining both growth miracles and growth tragedies. The second essay of this dissertation shows that a high degree of patience in the preferences of citizens and politicians and the ability of citizens to replace a politician in power are key ingredients for economic growth. I characterize the necessary conditions for growth to occur in a context where technological progress is available and free, but requires the approval of self-interested politicians to be adopted. In the model proposed, when politicians in power have a low discount factor, they find it optimal to stop technological progress in exchange for static rewards that the representative citizen does not control. The study predicts that everything else equal, economies that are the most likely to grow are those with the strongest political institutions: the lowest probabilities of occurrence of a coup d'etat and the lowest probabilities of falling in an absorbing state of dictatorship. Consistently with empirical facts on growth, the relationship predicted between dictatorship and economic growth by the model is a non-linear one: given a probability of falling in the state of dictatorship, the occurrence of growth depends on the discount factor of citizens. The essay also shows that even when the economy is already growing as a dictatorship, a one-shot transition to democracy is still desirable to citizens as it reduces the payoffs that are necessary to provide dynamic incentives to politicians in power.

Chapter 2

Bankruptcy Reform and Foreclosure Crisis During The Great Recession

Introduction

Between the first quarter of 2007 and the last quarter of 2009, the average price of U.S. single-family houses as measured by the Housing Price Index[1] of the Federal Housing Finance Agency fell by 26%. Mortgage markets responded aggressively as the foreclosure rate subsequently rose from 0.16% in 2002-2004 to 0.9% in $2007 - 2009$. Almost concomitant with the 2007 housing bust was the introduction of a new bankruptcy law that was implemented in October 2005 and that reduced the amount of unsecured debts that high income earners may discharge in bankruptcy. In fact, Prior to the 2005 bankruptcy reform law, U.S. households in bankruptcy had to repay their unsecured debts only up to the amount of their non-exempt home equity. The 2005 bankruptcy reform law imposed that above-median income earners repay their unsecured debts in bankruptcy up to the maximum between their non-exempt home equity and 5 years

[1] I use the expanded Housing Price Index of the Federal Housing Agency which includes sales price information from Fannie Mae and Freddie Mac mortgages as well as transactions records for houses with mortgages endorsed by the Federal Housing Agency and county recorder data licensed from DataQuick Information Systems.

of disposable income (defined as the difference between the household's income and its necessary living expenses).

This paper investigates the impact of the 2005 bankruptcy reform on the foreclosure crisis. Specifically, I ask if the foreclosure crisis would have been milder or harsher, had the bankruptcy reform not taken place prior to the housing bust and drop in average earnings that occurred during the great recession. I find that the bankruptcy reform did not worsen the foreclosure crisis but instead it moderately lowered the foreclosure rate. This result is in stark contrast with the suggestion of Li, White & Zhu (2009) that the bankruptcy reform has induced a higher foreclose rate by making it more difficult for households to discharge unsecured debts in bankruptcy in order to relax their budget constraints and continue making mortgage payments.

To assess the quantitative impact of the bankruptcy reform law on the foreclosure crisis, I model a life-cycle economy in which households face idiosyncratic income and expense risks; they access homeownership by entering into 30-year fixed interest rate mortgage contracts with a "piggyback lien " option; they smooth consumption by borrowing in a second mortgage market and in an unsecured credit market; and they discharge unsecured debts through a bankruptcy system that mimics key features of both Chapter 7 and Chapter 13 of the U.S. bankruptcy code. I model the U.S. economy as a recourse economy in which following a mortgage default, households are liable for the fraction of the mortgage loan that is not covered by the proceeds of the foreclosure sale with a probability that is estimated using foreclosure data from the LPS Analytics Inc. database. The risk-free rate is assumed exogenous to the model but interest rates in mortgage markets and in unsecured credit markets reflect endogenous default premia.

My model assumes the existence of a linear technology that transforms consumption goods into housing capital as in Corbae & Quintin (2011). Housing prices are hence exogenous, allowing me to simulate the housing bust of the great recession as a partially anticipated shock to the relative productivities of housing and production capital. I calibrate the model to jointly match several moments of the U.S. economy in the period 2002-2004 characterized by the old bankruptcy code and high housing prices, and in the period 2007-2009 characterized by low housing prices, lower average earnings, a lengthier foreclosure process and the post-reform bankruptcy code.

U.S. states are divided into 40 recourse and 11 non-recourse states. In non-recourse states, households walk away from their homes not liable for any unpaid mortgage loan. In recourse states, a mortgage lender may request a “deficiency judgment” which makes the mortgagor liable for the amount of the mortgage loan that is not covered by the proceeds of the foreclosure sale. Deficiency judgments are costly to obtain to the mortgage lender in states in which mortgage lenders are required to go through a judicial foreclosure process in order to request them. Of the 40 U.S. recourse states, 18 states require that lenders go through a judicial foreclosure process in order to request a deficiency judgement. Ghent and Kudlyaky (2011) show that in recourse states, mortgage default typically terminates into a short-sale or a voluntary conveyance rather than a foreclosure. The methodology of this paper views U.S states as “partially” recourse states in the sense that after a mortgage default, households are liable for the mortgage loans in excess of the proceeds of the foreclosure sale only with an estimated probability of a deficiency judgment. In the model, I make the assumption that deficiency judgments are obtained only in states in which the foreclosure process is always a judicial one. Given that assumption, I estimate the probability of a deficiency judgment in the model as a weighted average of the U.S. fraction of mortgage defaults that started in $2007-2009$ and that terminated as a foreclosure among mortgage defaults that started during the same period and terminated either as a foreclosure, a short-sale or a voluntary conveyance.

My model succeeds at explaining 85% of the foreclosure rate in the U.S. economy over the period 2007-2009. When the great recession is wired in the baseline model with the introduction of a 26% negative shock to housing prices and a 10% negative shock to labor productivity, my results show that the 2005 bankruptcy reform did not significantly affect the foreclosure crisis. Specifically, among owners of small size houses, my model predicts a foreclosure rate of 0.79% in the pre-reform world and 0.76% in the post-reform world. For owners of large size houses, the predicted foreclosure rate is 0.82% in the pre-reform world and 0.77% in the post-reform world.

The mechanism driving the results is as follows. High income earners with moderately negative home equity which in the great recession world with the old bankruptcy code would have chosen to simultaneously go though bankruptcy and foreclosure are instead likely to choose not to go through foreclosure in the post-reform world. This is so because given the new bankruptcy code, high income earners are typically unable to discharge most of their unsecured debts in bankruptcy and are hence most likely to find themselves trapped with large unsecured debts and a bad credit record, had they chosen the foreclosure option. When their home equity is not too negative, the opportunity cost of a bad credit record is sufficiently high to discourage them from the foreclosure option, especially given that the reform makes borrowing in the unsecured credit market particularly cheaper for high income earners. For this reason, during the great recession, the bankruptcy reform not only contributed toward a smaller number of bankruptcies, but it also contributed toward a smaller number of foreclosures.

To understand why the predicted impact of the reform on the foreclosure crisis is small it should be noted that in the model homeowners for whom the value of the foreclosure option could be significantly affected by the bankruptcy reform are high income earners that have significantly negative equity and that find bankruptcy appealing because they either have large unsecured debts or are facing a deficiency judgment. However, in the model, only a small fraction of households have unsecured debts sufficiently large to trigger a bankruptcy. Also, independently of the bankruptcy code, households facing a deficiency judgment in the model never choose the foreclosure option when they possess a significant amount of liquid assets as it is the case for most high income earners.

I next explain why the mechanism suggested by Li, White & Zhu (2009) did not cause the bankruptcy reform to contribute to the surge of the foreclosure rate during the great recession. Households which file for bankruptcy and choose to keep their houses rather than foregoing them to bankruptcy trustees are primarily high income earners for whom the law binds. For this reason, consistently with the suggestion of Li, White & Zhu (2009), my model predicts that the reform has reduced the number of homeowners which file for bankruptcy in order to discharge their unsecured debts and use their relaxed budgets to continue making mortgage payments and keep their houses. However, contrary to the suggestion of Li, White & Zhu (2009), the foreclosure option is not the next best alternative once the law makes the bankruptcy option unattractive for those who before the reform would have sought relief in bankruptcy in order to maintain the ownership of their homes. In fact, households which would go through bankruptcy with the intention of keeping their houses typically have large unsecured debts but home equity that is either positive or not too negative. When bankruptcy becomes less generous because of the new law, these households become more likely to either sell their houses, or to continue making payments mortgage payments rather than going through foreclosure and suffering the costs of bad credit while still burdened with large unsecured debts that they can no longer discharge in bankruptcy. This is precisely the reason why the reform did not lead to a higher foreclosure rate as suggested by Li, White & Zhu (2009).

The study the most closely related to the current work is that of Mitman (2011) which studies an infinite horizon model where unsecured and mortgage lending markets are linked through the bankruptcy system. The work of Mitman (2011) shows that cross-state variations in homestead exemptions and foreclosure laws explain a significant share of the variations observed in bankruptcy and foreclosure rates. Mitman (2011) finds that in the absence of a shock to housing prices, the bankruptcy reform increases welfare, but leads to a higher foreclosure rate and a higher bankruptcy rate as it induces households to take on more unsecured debts (given that unsecured borrowing becomes cheaper after the reform), and to leverage their mortgages more highly (as the reform reduces the amount of home equity that can effectively be sheltered away from a bankruptcy trustee). During the transition period following the introduction of the new bankruptcy law in its model, Mitman (2011) finds that foreclosure rates increase for four years before stabilizing to new levels higher than pre-reform levels. From these findings, Mitman (2011) concludes that the bankruptcy reform led to a higher foreclosure rate as was suggested by Li, White & Zhu (2009). My paper shows the importance of taking account of large and unexpected shocks to home equity by demonstrating that at bad times, homeowners in credit markets behave differently than they do in periods of constant or rising housing prices. Using a new and rich quantitative model of households' debts, I show that given unexpected large shocks to their home equity, homeowners in the great recession world with the new bankruptcy reform have abandoned their houses in foreclosure slightly less often than they would have done it if the bankruptcy reform had not taken place prior to the crisis.

In addition to offering a different approach for the understanding of the foreclosure crisis, my paper also proposes a model that is different from that of Mitman (2011) in three fundamental ways. First, Mitman (2011) models first mortgages as one-period bond contracts while I model first mortgages as 30-year period fixed interest rate contracts with an initial downpayment requirement that equals the average downpayment requirement of 23% in the U.S. between 2002 and 2004. Modeling first mortgages as 30-year period contracts with a fixed-interest rate realistically gives households with advantageous first mortgage interest rates an incentive not to default on their mortgage contracts either non-strategically when they go through financial hardship, or strategically when they are hit with a negative shock to their home equity. The initial downpayment requirement is necessary to properly account for the implications of a 26% drop in housing prices as observed during the great recession, on the resulting distribution of negative home equity among households.

Second, my model allows for a mechanism by which households file for bankruptcy in order to free up their budget constraint with the goal of holding onto a first mortgage contract with a particulary attractive interest rate as suggested by Li, White & Zhu (2009). This mechanism is nonexistent in Mitman (2011) which models first mortgages as one-period bonds.

Third, my paper models second mortgages and treats them as unsecured debts in bankruptcy in the event that the household's first mortgage debt exceeds the market value of the household's house. This feature of the model captures the fact that in the U.S. when a household's first mortgage debt exceeds its house's market value, junior liens are stripped off in Chapter 13 and as a result, second mortgages are treated like unsecured debts in such case. For this reason, when housing prices are unexpectedly shocked negatively, second mortgages have important implications for both the probability that a homeowner would go through foreclosure (because a homeowner with a second mortgage is more likely to face negative equity when housing prices fall) and the probability that a homeowner would go through bankruptcy (because second mortgages are treated like unsecured debts whenever the first mortgage is under water). Second mortgages are not present in Mitman (2011).

The rest of the paper is organized as follows. Section 2.1 presents empirical facts on household default from the LPS Applied Analytics's database. Section 2.2 describes the U.S. bankruptcy code and explains how it is modeled in the paper. The complete model is described in section 2.3. Section 2.4 explains the calibration strategy. Results are given in section 2.5 and concluding remarks in section 2.6.

2.1 Empirical Facts on Household Default

In this section, I present key facts related to household default and supported with evidence from the LPS Applied Analytics's residential mortgage loans database, which is the largest residential mortgage loan-level database available at the national level. The dataset used comprises $38,289,370$ loans all classified as "first-mortgage" loans and secured by a real estate property occupied by the borrower as either a primary or a secondary residence. These loans represent about two-ninth of all first-mortgage loans in the U.S. residential mortgage servicing market. The dataset is nationally representative, as it covers all major mortgage loans servicers and properties over all U.S. states. Each loan in the database entered it for the first time at some point between April 1992 and April 2012, and was followed throughout on a monthly basis, except if the mortgage contract had been terminated. I define foreclosure on a property as the occurrence of a liquidation sale or a repossession of the real estate by the mortgage lender following one or several months of delinquency. The foreclosure start date is defined as the date of the first foreclosure flag since the loan stopped being current. The foreclosure start date therefore does not necessarily coincide with the date of the first missed payment. The foreclosure end date is defined as the date of liquidation sale or real estate repossession.

Findings

Facts #1: Mortgage loans secured by properties involved in a foreclosure-start during the period 1992-2011 were continuously delinquent for a median of 420 days from the date of the first missed mortgage payment to the date of the liquidation sale or repossession by the mortgage lender. In other words, borrowers were never current on their loans for a median of 420 days prior to the end of the foreclosure process. This first fact points to the importance of free-renting in a household's decision to forego its house in foreclosure. The median number of days of free-rent associated with the foreclosure process rose from 320 days for loans that started foreclosure in the period 2002-2004, to a median of 506 days for the loans that started foreclosure during the period 2007-2009.

Facts #2: Among mortgage loans that were registered in bankruptcy some time between 1992 and 2009, 33% entered bankruptcy after being continuously delinquent for a minimum of 90 days. However, among these loans which entered bankruptcy after a minimum of 90 days of delinquency, only 56% had terminated in a foreclosure sale or real estate repossession by April 2012. The remaining 44% were saved from the occurrence of a foreclosure after bankruptcy filing. I interpret this fact as evidence of the existence of a mechanism by which households file for bankruptcy in order to stop or prevent an imminent foreclosure.

Facts #3: In the pool of loans that were delinquent for a minimum of 90 days prior to a registration in bankruptcy during the period 1992 and 2009, but that had not terminated in foreclosure by April 2012, the average number of days of continuous delinquency prior to registration in bankruptcy was 302 days.

Facts #4: In certain U.S. states, in the event of a foreclosure, households are charged for the difference between the mortgage balance and the sale price of the real estate. This difference is referred to as a deficiency. States in which deficiency liens are applicable are known as recourse states. States in which deficiency liens are not applicable are known as non-recourse states.

In the sample, 10% of loans associated with both bankruptcy and foreclosure entered bankruptcy within 3 years prior to their foreclosure start dates; 38% entered bankruptcy in the year of their foreclosure start; 48% entered bankruptcy within 1 to 3 years after their foreclosure start date, and 87% were registered in their most recent bankruptcy either in the year of their foreclosure start date or at a later date. Very similar proportions are obtained when the sample is divided between recourse-state loans and non-recourse state loans.

In fact, among loans that were associated with both bankruptcy and foreclosure in non-recourse states, 38% were registered in their most recent bankruptcy in the year of their foreclosure start, and 88% were registered in their most recent bankruptcy either in the year of their foreclosure start date or at a later date (as opposed to being registered in bankruptcy prior to the foreclosure start). The fact that bankruptcy filing typically follows (rather than precedes) a foreclosure start in non-recourse states seems to indicate that financially distressed households first seek relief in foreclosure and then file for bankruptcy only in the last resort. From the significantly high proportion of loans that go into bankruptcy in the year of their foreclosure start date, it appears that bankruptcy and foreclosure are closely related events for households in financial distress.

Finally, in recourse states, 23% of loans in foreclosure were also registered in bankruptcy at some point in time, compared to an equivalent proportion of 22% for loans associated with foreclosure in non-recourse states. This implies an insignificant variation between recourse and non-recourse states in terms of registration in bankruptcy for loans associated with a foreclosure. In theory, one would expect a much higher proportion of loans associated with a foreclosure to be also associated with bankruptcy filing in recourse states, insofar as households facing a foreclosure in recourse states (compared to those in non-recourse states) have additional incentives to file for bankruptcy as they would wish to discharge the unsecured debts caused by deficiency liens in foreclosure. I interpret this last finding as evidence of an ineffective recourse technology in practice. The recourse technology is understood as the means by which a mortgage lender may sue the mortgagor for the difference between the mortgage debts and the foreclosure sale price.

Summary Statistics

In tables 2.1 and 2.2, column (1) gives the median number of days of delinquency prior to foreclosure for loans that entered foreclosure in the year of interest. Column (2) shows the means of the number of days of delinquency prior to bankruptcy filing for loans that in the relevant year entered bankruptcy after being delinquent for a minimum of 90 days and that had not started foreclosure by the end of April 2012. Column (3) gives the average downpayment at origination for each year. Column (4) shows the mean net-interest rate at origination for loans originated in a given year. The net-interest rate is defined as the difference between the gross interest rate and the service-fee rate charged for securitized transactions. [2] . Table 2.1 shows these statistics across all U.S. states while Table 2.2 displays them for U.S. states separated into recourse and non-recourse groups.

[2] Table B.2 in the appendix summarizes the same statistics for U.S. states separated into high-homestead and low-homestead groups. A state qualifies as a high-homestead state if the ratio of its homestead exemption over its 2001- median income is higher than 1. Among all 51 states, 40 are recourse states and 24 are high-homestead states

Table 2.1: LPS summary statistics

	(1)	(2)	(3)	(4)
2002	330	246	21.5%	5.82%
2003	330	247	23.8%	5.16%
2004	300	238	24.2%	5.38%
2005	360	256	24.8%	5.49%
2006	360	304	26.1%	5.12%
2007	420	297	27.7%	6.07%
2008	600	284	28.3%	5.72%
2009	600	290	28.8%	4.77%
2010	540	351	28.03%	4.46%

Table 2.2: Summary Statistics Across the Recourse and Non-Recourse Groups

(a) Recourse States

	(1)	(2)	(3)	(4)
2002	360	248	20.8%	5.85%
2003	330	253	22.5%	5.18%
2004	330	242	22.8%	5.41%
2005	360	257	23.4%	5.52%
2006	360	304	24.4%	6.16%
2007	450	298	26.3%	6.11%
2008	630	287	26.9%	5.75%
2009	660	296	26.8%	4.78%
2010	540	349	26.3%	4.46%

(b) Non-Recourse States

	(1)	(2)	(3)	(4)
2002	300	240	23.2%	5.76%
2003	300	224	26.2%	5.13%
2004	270	223	27.3%	5.31%
2005	300	248	27.7%	5.41%
2006	330	290	30.1%	6.01%
2007	330	294	31.3%	5.95%
2008	510	269	32.4%	5.64%
2009	480	276	33.8%	4.73%
2010	420	359	31.8%	4.45%

During the period 2002-2009, average net-interest rates at origination associated with fixed interest mortgages were consistently higher in recourse states than in non-recourse states. However, the average downpayment at origination on first mortgages is consistently higher in recourse states, suggesting that mortgage lenders use both the downpayment requirement and the interest rate to simultaneously adjust for an individual's specific risk and for the aggregate risk of a fall in housing prices. Because a foreclosure is more costly to mortgage lenders when it occurs in a non-recourse state relatively to when it occurs in a recourse state, mortgage lenders then tend to require larger downpayments in the former case. The median number of days in foreclosure is higher in the period 2007-2009 relative to the period 2002-2009 in both the recourse and non-recourse groups. Also, this statistic is consistently higher in the recourse case, even though the magnitude of the difference is not too large [3] .

Foreclosure Rates

Table 2.3 and 2.4 display the average foreclosure rate by recourse and homestead categories for 3 pools of states: a first pool that includes all U.S. states (All-States pool), a second pool that is referred to as Subgroup 1 and that restricts to U.S. states in which average housing prices dropped by a percentage between 20% and 30% from 2007 to 2009, and a third pool (Subgroup 2) made by restricting to those states in Subgroup 1 for whom the 2009 unemployment rate was below the 2009 unemployment rate in the US[4] . In Subgroup 1, the average percentage change in housing prices is -25% for the recourse subset and -26% for the non-recourse subset. In Subgroup 2, the numbers are -25% and -25%, respectively. Statistics in Table 2.3 and 2.4 show that foreclosure rate tends to be higher for recourse states in the All-State pool where states are included without any restriction. However, in Subgroup 2, that restricts to states with comparable unemployment rates and comparable percentage changes in housing prices, the foreclosure rate appears to be slightly higher for the non-recourse subset. In Subgroup 2, the foreclosure rate is also slightly higher for the Low-Homestead subset, relative to the High Homestead subset.

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[3] The fact that the number of days of free-rent due to a lengthy foreclosure process is higher in recourse states seems due to the fact that foreclosure in recourse states tend to be judicial foreclosures which are lengthier by nature.

[4] See the Appendix for the list of states in each pool.

Table 2.3: Foreclosure Rates In the Recourse and Non-Recourse Groups

(a) ALL-States

	R	NR
2002	0.32%	0.22%
2003	0.30%	0.17%
2004	0.30%	0.15%
2005	0.30%	0.12%
2006	0.43%	0.16%
2007	0.58%	0.36%
2008	0.9%	0.82%
2009	1.15%	1.14%

(b) Subgroup 1

	R	NR
2002	0.28%	0.30%
2003	0.25%	0.24%
2004	0.25%	0.19%
2005	0.26%	0.12%
2006	0.38%	0.16%
2007	0.53%	0.27%
2008	0.82%	0.56%
2009	1.14%	0.93%

(c) Subgroup 2

	R	NR
2002	0.18%	0.27%
2003	0.13%	0.22%
2004	0.11%	0.17%
2005	0.09%	0.13%
2006	0.12%	0.19%
2007	0.29%	0.3%
2008	0.58%	0.59%
2009	1.0%	0.9%

Table 2.4: Foreclosure Rates In the Low and High Homestead Groups

(a) ALL-States

	L	H
2002	0.36%	0.24%
2003	0.33%	0.20%
2004	0.33%	0.20%
2005	0.34%	0.18%
2006	0.47%	0.26%
2007	0.59%	0.47%
2008	0.84%	0.92%
2009	1.4%	1.6%

(b) Subgroup 1

	L	H
2002	0.30%	0.19%
2003	0.28%	0.16%
2004	0.29%	0.12%
2005	0.29%	0.10%
2006	0.41%	0.16%
2007	0.56%	0.30%
2008	0.87%	0.54%
2009	1.50%	0.9%

(c) Subgroup 2

	L	H
2002	0.22%	0.19%
2003	0.15%	0.16%
2004	0.14%	0.12%
2005	0.10%	0.10%
2006	0.11%	0.16%
2007	0.28%	0.3%
2008	0.62%	0.54%
2009	1.10%	0.9%

2.2 U.S. Bankruptcy Code: Real World and Model

The U.S. bankruptcy system allows households to use either Chapter 7 or Chapter 13 to discharge their unsecured debts. Prior to the 2005 bankruptcy reform, households in bankruptcy were required to avail only their non-exempt home equity for the repayment of their unsecured debts, no matter whether they filed for Chapter 7 or Chapter 13. Non-exempt home equity is defined as the difference between the household's home equity and the amount of homestead exemption in the household's state of residency. Before the reform, in Chapter 7, the household's house would be liquidated whenever it carried non-exempt equity that should be used toward repaying the household's unsecured debts. In Chapter 13 on the other hand, a household filing could either abandon its house, or could instead choose to keep it and repay its unsecured debts from future income through a 3-to-5 year Chapter 13 repayment plan. In that case, the filer would propose a Chapter 13 bankruptcy repayment plan that the bankruptcy court would approve only if over the lifetime of the plan, the household would have repaid its unsecured debts by at least the amount of non-exempt home equity listed at the time of bankruptcy filing.

The 2005 U.S. bankruptcy reform law introduced an income means-test that forces households which in the past six months prior to bankruptcy filing earned above their state median income to file under Chapter 13. The reform did not changed the amount of unsecured debts dischargeable in Chapter 7. However, the reform significantly limited the amount of unsecured debts that could be discharged in Chapter 13. Specifically, after the reform, bankruptcy filers are required to avail the maximum between their non-exempt home equity and five years of their current disposable income toward the repayment of their unsecured debts. Disposable income in the bankruptcy code is defined as the difference between the household's wage income and the household's necessary living expenses. Households in bankruptcy after the reform may still choose between keeping or foregoing their houses in bankruptcy. The key difference between the pre-reform and post-reform worlds is that U.S. households filing after the reform whether or not they choose to keep their houses must avail the maximum between their non-exempt home equity and five years of disposable income for the repayment of their unsecured debts, while only non-exempt home equity sufficed before the reform.

In the model, I use median income as a proxy for necessary living expenses. This allows me to simplify both the pre-reform and post-reform bankruptcy codes as a single option filing menu for non-owners, but a two-option filing menu for homeowners who choose to either keep or toss out their houses when filing. In the model as in the real world, before the reform households in bankruptcy would be requested to avail at least their non-exempt home equity toward the repayment of their unsecured debts. In the model, after the reform, given the use of median income as a proxy for non-disposable income, unsecured creditors in bankruptcy may receive as much as the maximum between the filer's non-exempt home equity and 5 years of the difference between the filer's current income and the median income. The means-test is then well accounted for in the model, given that the change in the bankruptcy code only binds for above-median income earners.

In the U.S. bankruptcy code, Chapter 13 is particularly attractive to homeowners for whom the first mortgage is underwater, given that junior liens are stripped off in Chapter 13 whenever the house value is smaller than the first mortgage debt. For instance, a household which owes a second mortgage debt and has its first mortgage under water will see its second mortgage debt discharged in the same fashion as its unsecured debts, while the household may continue to make mortgage payments in order to keep its house after bankruptcy filing. This unique feature of Chapter 13 existed before the reform and was not affected by the reform. In my model, I explicitly account for this provision of the U.S. bankruptcy code by treating second mortgage debts as unsecured debts in the case that the filer's house value is smaller that the first mortgage debts and the filer chooses to keep its house rather than foregoing it when filing for bankruptcy.

2.3 The model

The model economy is populated with households, first mortgage lenders, second mortgage lenders, unsecured credit lenders, and a housing sector. The housing sector consists of a housing firm which transforms physical capital into housing capital, and a rental agency. Mortgage lenders, unsecured lenders, the housing firm, and the rental agency are risk neutral and may borrow or lend at risk free rate r which is exogenous to the economy.

2.3.1 Households

Demographics and Preferences

Households live at most J periods. Staring with $\psi_1 = 1$, in every period, $\forall j \geq 2$, households of age $j-1$ survive to age j with probability ψ_j, while those of age J die with certainty at the end of the period. Every period, a constant measure of age one households is born so that population is kept constant. Households discount the future at rate $\beta \in (0,1)$. Households value non-durable consumption good $c > 0$ and housing services $h > 0$. Households'preferences over c and h are given by $u(c,h)$. The expected discounted lifetime utility of an age 1 household is then given by: $E\sum_{j=1}^{J} \beta^{j-1}\psi_{j+1}\, u(c_j,h_j)$.

2.3.2 The Housing Sector

The housing firm uses a linear technology that transforms one unit of production capital into A units of housing capital available in the form of houses. Houses come in four possible sizes. A house of size $i \in \{1,2,3,4\}$ does not depreciate and produces housing services h_i in every period. One unit of housing capital hence trades for $\frac{1}{A}$ units of physical capital. The rental agency purchases houses of all sizes from the housing firm and rents them to households which are then entitled to the housing services provided by these houses.

Households may rent all houses, but may purchase only houses of sizes 3 and 4. When selling their homes to buy a home of the same size (refinancing), households incur no transaction cost on the sale, but a per unit transaction cost τ_c on the new purchase. For all other housing transactions, households incur per unit transaction cost τ_s when selling a house, and per unit transaction cost τ_c when purchasing one if they are age $j < J$. Housing transactions initiated by the oldest retirees do not involve transaction costs. As renters, households pay a rent to the intermediary per unit of housing service provided. To purchase a house, households pay a downpayment that amount to fixed fraction μ of the value of the house, and then borrow the remaining fraction $(1-\mu)$ from a first mortgage lender. Households may if they wish take a second mortgage loan that amounts for fraction μ_1 of the value of the house, where $\mu_1 < \mu$ so that finally they would pay only fraction $\mu - \mu_1$ of the value of the house out of pockets. When they choose to do so, the home being purchased is said to have a "piggyback" lien. The terms of mortgage contracts will be specified later.

2.3.3 Housing Boom and Bust

The model economy starts in a boom characterized by an initial productivity level of housing capital $A = A_0$. Then, in every future period when $A = A_0$, agents anticipate that A may change from A_0 to $A_1 < A_0$ in the next period with probability γ and will remain equal to A_1 forever whenever it changes. In other words, agents anticipate with probability γ that the economy will fall in an absorbing state of low housing prices. In the rest of the paper, I will use Z to denote the aggregate state of the economy, with $Z = B$ if housing prices are high ($A = A_0$) and $Z = B$ if housing prices are low ($A = A_1$). For all $Z \in \{B, R\}$, I will use $P(Z)$ and $Rent(Z)$ to denote the per-unit housing price and rent that prevail given aggregate state Z.

2.3.4 Credit Markets

First Mortgage Contracts

In order to purchase a house of size $i \in \{3, 4\}$ given aggregate state Z, households must pay $\mu P(Z)h_i$ out of pockets and must borrow $(1-\mu)P(Z)h_i$ from a first mortgage lender at an interest rate r_m that depends on the household's characteristics. Loans offered by first mortgage lenders are to be reimbursed in at most T periods.

A first mortgage contract is hence indexed by the 4-tuple (i, r_m, τ, θ), where i the size of the house securing the contract, r_m is the associated fixed interest rate, $\tau \in \{1, \cdots, T+1\}$ is such that $T + 1 - \tau$ is the number of periods left for the loan to be repaid in full, and $\theta \in \{0, 1\}$ indicates whether the securing house was purchased when housing prices where either high (in which case $\theta = 0$) or if it was purchased when housing prices were low (in which case $\theta = 1$). Later on, r_m will be specified to come from a pricing function that allows first mortgage lenders to charge a fixed interest rate that depends on the size of the house purchased, on the aggregate state of the economy, and on the characteristics of the home buyer.

In each of the τ terms of a first mortgage contract indexed by (i, r_m, τ, θ), the household is asked to either make a constant payment M to the mortgage lender, or to repay the current balance $\ell(i, r_m, \tau, \theta)$ owed in full. The period mortgage payment $M = Mortgage(i, r_m, \theta)$ is given by:

$$Mortgage(i, r_m, \theta) = (1-\mu)Ph\frac{r_m}{1-(1+r_m)^{-T}},$$

where

$$P = \mathbb{1}_{\{\theta=0\}}P(B) + \mathbb{1}_{\{\theta=1\}}P(R).$$

The law of motion of ℓ is given by:

$$\begin{aligned} \ell(i, r_m, 1, \theta) &= (1-\mu)Ph_i \\ \ell(i, r_m, \tau+1, \theta) &= \left(\ell\left(i, r_m, \tau, \theta\right) - Mortgage\left(i, r_m, \theta\right)\right).(1+r_m), \forall \;\; \tau \;\; s.t. \;\; 1 \leq \tau \leq T-1 \\ \ell(i, r_m, T+1, \theta) &= 0 \end{aligned}$$

Second Mortgage Constracts or Home Equity Loans

A Homeowner in first mortgage contract indexed by (i, r_m, τ, θ) and which owns a house of size i worth $P(Z)h_i$ may borrow against it by purchasing a home equity bond of face value b' from a second mortgage lender, where $b' \in \mathbb{B} \equiv \{\mu_1 \times P(B)h_i,\ \mu_2 \times P(B)h_i\}$ and where $0 \leq b' \leq P(Z)h_i - \ell(i, r_m, \tau, \theta)$. If the household purchases a home equity bond at per unit price q_s, then the household's period resources will be increased by $q_s \times b'$ and the household will have to promise to repay b' to the second mortgage lender in the following period. Later on, q_s will be specified to come from a pricing function that allows second mortgage lenders to charge a price that depends on the size of the loan, on the aggregate state of the economy, and on the characteristics of the borrower.

Unsecured Credit Contracts

Households borrow in the unsecured credit market by purchasing an unsecured bond with face value $a' \in \mathbb{R}$ from an unsecured lender. Given $a' < 0$, a household which purchases an unsecured bond with face value $a' < 0$ at per unit price q_u increases its current period resources by $q_u \times (-a')$ units of the consumption good. In return, this household promises to repay $(-a')$ units of consumption tomorrow. On the other hand, when $a' > 0$, the household charged a per unit price q_u sees its period resources decreased by $q_u \times a'$ and in return is promised to receive a' units of consumption tomorrow. Later on, q_u will be specified to come from a pricing function that allows unsecured lenders to charge a price that depends on the size of the loan, on the aggregate state of the economy, and on the characteristics of the borrower.

Contract Termination Upon the Death of the Household

In the event of the death of a homeowner, its house of size i is sold for a net value $P(Z)h_i$ and the proceeds of the sale are used to repay the first mortgage lender only. I assume that second Mortgage lenders and unsecured lenders do not obtain any compensation upon the death of the borrower. In the event of the death of a household with positive unsecured bond holdings, the unsecured lender keeps the household's savings[5] .

Voluntary Default

Households are not committed to any credit market contract. They may jointly default on first and second mortgage contracts, and may default independently on unsecured credit contracts. Specifically, households may default in three possible ways:

- they may default on mortgage contracts only **(or walk-away from their home without going though bankruptcy)**
- they may jointly default on all mortgage contracts and on unsecured credit contracts **(bankruptcy option for tossers)**
- they may default on an unsecured credit contract only **(bankruptcy option for keepers)**.

Defaulting on an unsecured credit contract or on a second mortgage contract consists in not repaying the amount due to the unsecured credit lender or to the second mortgage lender. In a given period, a homeowner faced with $\zeta \in [0, 1]$ voluntarily defaults on a

[5] Not allowing mortgage lenders to share a household's liquid assets upon its death simplifies computations tremendously. Given this separation assumption, allowing second mortgage lenders to share the proceeds of the sale of a deceased household's house may create a profitable arbitrage condition. This is the case because under the assumption that second mortgage lenders share the proceeds of the sale of a deceased household's house , when the probability of a fall of housing prices is very small, interest rate premia on home equity loans are close to zero, and households find it profitable to take on second mortgages and invest the resources as liquid savings by purchasing an unsecured bond. This is especially true for older households with relatively low survival rates.

first mortgage contract (i, r_m, τ, θ) by repaying only $(1-\zeta) \times Mortgage(i, r_m, \tau, \theta)$ to the mortgage lender in that period. A homeowner which voluntarily defaults on its mortgage contracts continues to enjoy the housing services provided by the house during the period of default, but loses the house at the end of the period to a risk neutral intermediary which avails $\frac{(1-\phi)E(p)h_i}{1+r}$ to be used to repay the first mortgage lender in first priority and the second mortgage lender in second priority,

where given aggregate state Z,

$$E(p) = \begin{cases} (1-\gamma)\frac{1}{A_0} + \gamma\frac{1}{A_1} & \text{if Z=B} \\ \frac{1}{A_1} & \text{if Z=R.} \end{cases}$$

A household's eligibility for borrowing on any credit markets is conditioned by its credit record. Households are assumed born with a clean credit record. However, following a mortgage default without bankruptcy filing, the household's credit record is tarnished with a foreclosure flag that disappears in any subsequent period with probability λ. Following bankruptcy filing, the household's credit record is tarnished with a bankruptcy flag that disappears in any subsequent period with probability λ. Only households with no flag in their credit record may borrow in any market.

2.3.5 Households' Idiosyncratic Shocks and Timing

All households face expense shocks and housing luck shocks. However, only active households (households aged $j < J_r$) face income shocks. Specifically, in each period, active households draw a persistent income shock ξ and a transitory income shock ϵ. All households individually and independently draw an expense shock $x \in \{0, \chi_1, \chi_2\}$, and a housing luck shock $\zeta \in \{\zeta_1, \zeta_2\}$. $\forall i \in \{1, 2\}$, households draw expense shock χ_i with probability $p_{\chi,i}$. Similarly, they draw housing luck shock ζ_i with probability $p_{\zeta,i}$, $\forall i \in \{1, 2\}$.

The income process

Every period t, a household of age $j \in \{1, \cdots, J_r\}$ receives endowments in the form of a persistent income shock ξ_t and a transitory income shock ϵ_t, where $\xi_t = \rho\xi_{t-1} + \upsilon_t$, $\upsilon \sim N(0, \sigma_\upsilon^2)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$. Households retire at mandatory retirement age

$J_r < J$. Wage income at date t for a household of age $j < J_r$ is given by $y_t = \vartheta_j \ u_t$, where ϑ_j is the labor efficiency of individuals of age j and $u_t = exp(\xi_t + \epsilon_t)$.

After retirement, households of age $j \geq J_r$ receive pension income y_{ss} defined by $y_{ss} = Pens + \varsigma.exp(\xi_{J_r-1})$, where ς and $Pens$ are constant parameters, and ξ_{J_r-1} is the households' persistent shock in the pre-retirement age [6] . The model economy's median income is denoted by y_{med}.

Timing

The timing in a period is the following. At the beginning of the period, households first draw idiosyncratic shocks. Then, the housing and credit markets open simultaneously. Next, households which did not borrow on any credit market choose whether or not to default and select among the default options. Thereafter, households make mortgage and rents payments. Then, defaulters which have chosen the bankruptcy option get their current and future period cash-in-hand (or liquid asset) position readjusted according to the bankruptcy code. Consumption occurs and a new period starts.

$\forall i\{1, 2\}$, households facing housing luck shock ζ_i make only fraction $(1-\zeta_i)$ of their period first mortgage payments in the event of a mortgage default. In the event of a mortgage default, households are liable for mortgage loans not covered by the proceeds of the foreclosure sale if and only they drew $\zeta = \zeta_2$ at the beginning of the period.

Households are assumed born as non-owners with a clean credit record and with second mortgage and unsecured bonds of face values equal to zero. Afterwards, the credit status, ownership status and assets positions of the household evolve depending on choices made. let $\eta \in \{0, 1, 2\}$ represent a household's credit status: $\eta = 0$ if the household currently has a clean credit record and, $\eta = 1$ if the household only has a foreclosure flag, and $\eta = 2$ if the household either has a bankruptcy flag or has both bankruptcy and foreclosure flags. Let $o \in \{0, 1\}$ determine the household's ownership status: $o = 0$ if the household starts the period as a non-owner and $o = 1$ if it starts as a homeowner. Also, let θ indicate if a homeowner's house was bought during the boom or during the recession: $\theta = 0$ if the household's house was bought in the boom and $\theta = 1$ otherwise. Then, for a household of age j which has drawn idiosyncratic shocks ξ, ϵ, x, and ζ, the idiosyncratic state vector is given by :

[6] Pens is the part of pension income that is independent of earnings during the active life.

$$\{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (i, M, \tau, \theta)\},$$

where a is the household's beginning-of-period unsecured bond holding, b is the household's beginning-of-period second mortgage bond, M is the household's period mortgage payment, i is the size of the household's house, and τ is such that $1 \leq \tau \leq T+1$ and $(T+1-\tau)$ is the number of terms left in the first mortgage contract. Because (i, M, τ, θ) is irrelevant for non-owners, I will restrict to $(i, M, \tau, \theta) = (3, 0, 1, 0)$ when the household starts a period as a non-owner.

In what follows, y will be used to denote the household's before-tax wage income, and $\overline{y}$ will be used to denote the household's after-tax wage income specified in section 2.3.8. Also, ℓ_0 will be used to denote a homeowner' current first mortgage debt. That is, for a homeowner in a first mortgage contract indexed with (i, r_m, τ, θ),

$$\ell_0 = \ell(i, r_m, \tau, \theta).$$

2.3.6 Description of the Bankruptcy Code

Let E be the amount of homestead exemption that characterizes the economy. Consider a homeowner in idiosyncratic state $\{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (i, M, \tau, \theta)\}$ when the aggregate state is Z. Bankruptcy costs homeowners δ units of consumption in the period of bankruptcy filing. Unsecured debt rolled over in bankruptcy is subject to interest rate r_b.

Bankruptcy for Homeowners

Define:

$$\begin{cases} \textit{Home Equity} & = \max\{0, (\frac{(1-\phi).E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M) - b)\} \\ \textit{Exempt Equity} & = \min\{E, \textit{Home Equity}\} \\ \textit{Non Exempt Equity} & = \max\{0, \textit{Home Equity} - \textit{Exempt Equity}\}. \end{cases}$$

Now, let's define by $\hat{A}_{toss}(Z, s)$ the before-bankruptcy adjusted cash-in-hand position of the household at the time of bankruptcy filing given that the household has also chosen to default on its mortgage payments.

Then,

$$\hat{A}_{toss}(Z,s) = \begin{cases} a - x & \text{if } \zeta = \zeta_1 \\ a - x + a_1 + a_2 & \text{if } \zeta = \zeta_2, \end{cases}$$

where a_1 and a_2 in absolute values are the mortgage debts still owed to the first and second mortgage lenders after exhaustion of the expected discounted sale proceeds in a recourse world:

$$\begin{cases} a_1 & = \min\{0, \frac{(1-\phi)E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M)\} \\ a_2 & = \min\{0, \max\{0, \frac{(1-\phi)E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M)\} - b\}. \end{cases}$$

Bankruptcy if the household tosses its house away

If the household tosses its house away while filing for bankruptcy, then it will start the future period with after-bankruptcy adjusted cash-in-hand $D_{toss}(Z,s)$. In the period of bankruptcy filing, the household's consumption is given by:

$$c = \overline{y} - \delta - (1-\zeta)M + (\textit{Exempt Equity} + \max\{0, \hat{A}_{toss}(Z,s) + \textit{Non Exempt Equity}\} + D_{toss}(Z,s)).$$

The after-bankruptcy adjusted cash-in-hand $D_{toss}(Z,s)$ is the key variable that differentiates a pre-bankruptcy reform world from a post-bankruptcy reform world for a homeowner which chooses to abandon its house when filing for bankruptcy. In order to define it, let's first define *Creditor Hit* to be the loss incurred by unsecured creditors after all non-exempt proceeds have been exhausted toward repaying them:

$$\textit{Creditor Hit} = \min\{0, \hat{A}_{toss}(Z,s) + \textit{Non Exempt Equity}\}.$$

Then,

$$D_{toss}(Z,s) + \frac{D_{toss}(Z,s)}{(1+r_b)} = \textit{Additional Payment},$$

where *Additional Payment*

$$= \begin{cases} 0 & \text{pre-bankruptcy reform} \\ -\min\{\text{-}\textit{Creditor Hit}, \textit{Ability to Pay}\} & \text{post-bankruptcy reform}, \end{cases}$$

and

$$\textit{Ability to pay} = \max\{0, \kappa \max\{0, y - y_{med}\} - \textit{Non Exempt Equity}\}.$$

Bankruptcy if the household keeps its house

When the household chooses to keep its house while filing for bankruptcy, its before-bankruptcy adjusted cash-in-hand is given by:

$$\hat{A}_{keep}(Z,s) = \begin{cases} a - x - b & \text{if } ph_i - (\ell_0 - M) < 0 \\ a - x & \text{if } ph_i - (\ell_0 - M) \geq 0. \end{cases}$$

In other words, the second mortgage is considered an unsecured debt for a household which has a first mortgage under water and which chooses to keep its house while filing for bankruptcy. In this case, the household starts the following period with after-bankruptcy adjusted cash-in-hand $D_{keep}(Z,s)$ and the household's current consumption is given by:

$$c = \begin{cases} \overline{y} - \delta - M + \max\{0, \hat{A}_{keep}(Z,s)\} + D_{keep}(Z,s) & \text{if } P(Z)h_i - (\ell_0 - M) < 0 \\ \overline{y} - \delta - M - b + \max\{0, \hat{A}_{keep}(Z,s)\} + D_{keep}(Z,s) & \text{if } P(Z)h_i - (\ell_0 - M) \geq 0. \end{cases}$$

where

$$D_{keep}(Z,s) + \frac{D_{keep}(Z,s)}{(1+r_b)} = -\min\{-\min\{0, \hat{A}_{keep}(Z,s)\}, \textit{Ability to Pay for keepers}\}.$$

To define *Ability to Pay for keepers*, let

$$\textit{Virtual Proceeds} = \max\{0, \max\{0, (1-\phi)ph_i - \ell_0 - b\} - E\}.$$

Then,

Ability to Pay for keepers

$$= \begin{cases} \textit{Virtual Proceeds} & \text{pre-bankruptcy reform} \\ \max\{\textit{Virtual Proceeds}, \max\{0, \kappa(y - y_{med})\}\} & \text{post-bankruptcy reform.} \end{cases}$$

Bankruptcy for Non-Owners

Beginning-of-period non-owners may only rent when they file for bankruptcy. A non-owner which chooses to rent a house of size i and file for bankruptcy will start the following period with after-bankruptcy adjusted cash-in-hand position $D_{no}(Z,s)$. In the period of bankruptcy filing, the non-owner's consumption is given by:

$$c = \overline{y} - \delta - Rent(Z)h_i + \max\{0, a-x\} + D_{no}(Z,s),$$

where

$$D_{no}(Z,s) + \frac{D_{no}(Z,s)}{(1+r_b)} = -\min\{-\min\{0, a-x\}, Ability\ to\ Pay\ for\ non\text{-}owners\},$$

and

$$Ability\ to\ Pay\ for\ non\text{-}owners$$
$$= \begin{cases} 0 & \text{pre-bankruptcy reform (Z=B)} \\ \max\{0, \kappa(y - y_{med})\} & \text{post-bankruptcy reform (Z=R)}\ . \end{cases}$$

2.3.7 Households' Choices

I now turn to define the households' choices given idiosyncratic state

$$\{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (\theta, i, M, \tau)\}$$

. If the household is of age J (oldest age), then it may only rent. A household of age J that starts the period as a homeowner hence has to sell its house at no transaction cost and rent.

Choices of Homeowners of age $j < J$

Consider a homeowner of age $j < J$ in a first mortgage contract (i, M, τ, θ) with an unsecured bond holding a, a second mortgage bond b, and credit status η.

Case of the homeowner with $\eta = 0$:

The choices available to the clean homeowner can then be summarized as follows:

- sell the house and either rent or purchase a new house, repay all debts **(selling option)**
- repay b, $max(0, -a)$, and M **(stay current)**.
- repay $max(0, -a)$, pay only fraction $(1 - \zeta)$ of M, and default on b **(walk-away without bankruptcy filing)**
- default on b and $max(0, -a)$, and pay only fraction $(1-\zeta)$ of M, and **(bankruptcy option for tossers)**
- repay M and b, but default on $max(0, -a)$ **(bankruptcy option for keepers)**.

In the event of a default without bankruptcy, the homeowner chooses next period unsecured bond holdings $a' \geq 0$ and second mortgage bond holdings $b' = 0$. In the event of bankruptcy filing, the face value of the next period unsecured bond a' is settled by the bankruptcy code, and next period second mortgage bond holding $b' = 0$. Because households may only borrow when they start the period with $\eta = 0$, it turns out that homeowners cannot start a period with $\eta = 1$ (given that a homeowner loses its house after a mortgage default). Neither can they start a period simultaneously with $\eta = 2$ and with second mortgage bond holdings that have a nonzero face value.

Case of the homeowner with $\eta = 2$:

The choices available to a flagged homeowner are as follows:

- sell the house and rent a new house, repay all debts **(selling option)**
- repay $max(0, -a)$, and M **(stay current)**
- repay $max(0, -a)$, pay only fraction $(1-\zeta)$ of M **(walk-away without bankruptcy filing)**.

Choices of non-owners

A non-owner starting the period with ($\eta = 1$ or $\eta = 2$) (flagged non-owners) may only rent and purchase an unsecured bond holding with face value $a' \geq 0$. A Non-owner of age $j < J$ starting the period with $\eta = 0$ (clean non-owners) and with an unsecured bond of face value a, may :

- repay $max(0, -a)$ and either rent or buy a house,
- or default on $max(0, -a)$ and rent during the period **(bankruptcy option for non-owers)**.

Involuntary Default

Households for whom all the available choices lead to an empty budget set default involuntarily. Beginning-of-period non-owners rent a house of size 1 during a period of involuntary default. In the event of an involuntary default for a homeowner, the house is sold and the homeowner must rent a house of size 1. The proceeds of the sale are used to repay the first mortgage lender, then the second mortgage lender and finally the unsecured lender. Involuntary default leads to a fresh-start in the following period as a non-owner with a bankruptcy flag and with an unsecured bond holding of face value zero.

2.3.8 The Tax Code

Consider a household in state $\{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (\theta, i, M, \tau)\}$ with current period wage income y. Now, let:

$$m = \begin{cases} 0 & \text{if the household rents} \\ Mortgage(i', r'_m, \mathbb{1}_{\{Z=R\}}), & \text{if the household enters new contract } (i' r'_m, 1, \mathbb{1}_{\{Z=R\}}) \\ 0 & \text{if } \tau > T \text{ and the household does not sell} \\ M & \text{if } \tau \leq T \text{ and the household stays current} \\ (1-\zeta)M & \text{if } \tau \leq T \text{ and default on first mortgage.} \end{cases}$$

Also, let $\overline{\ell} = \begin{cases} \ell(i, r_m, \tau, \theta) & \text{if } \tau \geq 2 \text{ and the household stays current} \\ \ell(i', r'_m, 1, \mathbb{1}_{\{Z=R\}}) & \text{if the household purchases a new house.} \end{cases}$

Let $A = y + max(0, a) \times (1 - Q(j))$. Then, the household's taxable income is given by:

$$y_d = \begin{cases} A - b \times (1 - Q(j)) - r_m \times (\overline{\ell} - m) & \text{if homeowner does not sell} \\ A - b \times (1 - Q(j)) - r_{m'} \times (\overline{\ell} - m) & \text{if homeowner sells and buys} \\ A - b \times (1 - Q(j)) & \text{if homeowner sells and rents} \\ A & \text{if non-owner rents} \\ A - r_{m'} \times (\overline{\ell} - m) & \text{if non-owner buys} \end{cases}$$

where $\forall j$,

$$Q(j) = \frac{\psi_j}{1 + r}.$$

A household's with taxable income y_d has after-tax income given by:

$$\overline{y} = \max(0, y_d) \times (1 - \tau_\omega),$$

where τ_ω is the economy's constant marginal tax rate. Hence, the tax code features interest-deductibility for both first and second mortgages[7] .

2.3.9 The household's problem

I now formally define the problem of a household in idiosyncratic state

$$s = \{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (\theta, i, M, \tau)\}$$

[7] Later, it is shown that given the assumption that unsecured lenders keep the positive cash-in-hand position of a deceased household, the zero-profit condition implies that for $a' > 0$, the per unit price of a cash-in-hand bond with face value a' sold to a household of age j is $Q(j+1)$. Hence, if $r_s(j)$ is the savings interest rate offered to a household of age $j-1$, then $(1 - Q(j)) = \frac{r_s(j)}{1+r_s(j)}$, so that $a \times \frac{r_s(j)}{1+r_s(j)}$ is the net interest pocketed by a household of age j starting the period with positive unsecured bond position a. Because prices on second mortgage bonds turn out to be larger than those on an unsecured bond with equal positive face value by a factor that accounts for the household's default probability, (1-Q(j)) is used as a cap on the interest-deductibility of second mortgages. This allows to capture U.S. tax incentives to borrow on second mortgage markets, without the additional burden of carrying prices of second mortgage bonds as state variables .

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Age J households face the following problems.

Case 1: non-owner of age j=J :

$$V_Z(s) = \max_{i' \in \{1,2,3,4\}} u(c, h_{i'}) \text{ s.t. } c = \overline{y} + a - x - Rent(Z)h_{i'}$$

Case 2: owner of age j=J :

$$V_Z(s) = \max_{i' \in \{1,2,3,4\}} u(c, h_{i'}) \text{ s.t. } c = \overline{y} + a - x - b - Rent(Z)h_{i'} + (P(Z)h_i - \ell(i, r_m, \tau, \theta))$$

For the purpose of defining the problem of a household of age $j < J$, let ω denote the non-stochastic component of a household's idiosyncratic state vector: $\omega = \{j, \eta, a, b, o, (\theta, i, M, \tau)\}$. As before, let the economy's aggregate state be given by Z such that $Z = B$ if housing prices are high, and $Z = R$ if housing prices are low.

Now, $\forall\ Z \in \{B, R\}$, let V_Z denote the household's value function given aggregate state Z. Let denote by s' the household's state vector in the next period. Define $\omega' = \{j', \eta', a', b', o', (\theta', i', M', \tau')\}$. $\forall \xi$, and $\forall\ Z \in \{B, R\}$. Now, for $V = (V_B, V_S)$ define operator $E_{Z|\xi}$ by:

$$\begin{cases} (E_{R|\xi}V)(\omega') = & \int V_R(s')\ d\Phi(\Delta'|\xi) \\ (E_{B|\xi}V)(\omega') = & \left[\int V_R(s')\ d\Phi(\Delta'|\xi)\right] \times (1-\gamma) + \left[\int V_B(s')\ d\Phi(\Delta'|\xi)\right] \times \gamma \end{cases}$$

where $\Delta' = (\epsilon', \xi', x', \zeta')$ and

$$d\Phi(\Delta'|\xi) = Pr(\epsilon') \times Pr(\xi'|\xi) \times Pr(x') \times Pr(\zeta').$$

$\forall\ (Z, s)$ I define $Q_u(Z, \xi, \omega')$ to be the per unit price of an unsecured bond of face value a' offered to a household with current persistent shock ξ and future non-stochastic state component ω' when the aggregate state of the economy is Z. Similarly, $Q_s(Z, \xi, \omega')$ is the per unit price of a second mortgage bond of face value b' offered to such household. In the same way, $R_m(Z, \xi, \omega')$ denotes the interest rate offered on a first mortgage contract to a household that will start the next period with future non-stochastic state component ω' when the current state of the economy is Z and the household's current persistent income shock is ξ. Let parameter *stig* be the utility loss associated with bankruptcy filing. Also, let parameters δ be the pecuniary cost associated with bankruptcy filing.

Case 1: Clean homeowners: ($\eta = 0$):

$$V_Z(s) = \max\{V_Z^{sell,rent}(s), V_Z^{sell,buy}(s), V_Z^{current}(s), V_Z^{walk}, V_Z^{bk,toss}, V_Z^{bk,keep}\},$$

where:

1.

$$\begin{aligned}
V_Z^{sell,rent}(s) &= \max_{i' \in \{1,2,3,4\}, a'} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') \\
\text{s.t. } c &= \overline{y} + a - b - x - Q_u(Z,\xi,\omega')a' + (1-\tau_s)P(Z)h_i - \ell_0 - Rent(Z)h_{i'} \\
\omega' &= \{j+1, 0, a', 0, 0, (0,3,0,0)\} \\
\ell_0 &= \ell(i, r_m, \tau, \theta)
\end{aligned}$$

2.

$$\begin{aligned}
V_Z^{sell,buy}(s) &= \max_{i' \in \{3,4\}, a'} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') \\
\text{s.t. } c &= \overline{y} + a - b - x - Q_u(Z,\xi,\omega')a' + (1-\tau_s)P(Z)h_i - \ell_0 \\
&- (\tau_b + \mu)P(Z)h_{i'} - m \\
\omega' &= \{j+1, 0, a', 0, 1, (\theta', i', m, 2)\} \\
\ell_0 &= \ell(i, r_m, \tau, \theta) \text{ and } m = Mortgage(i, R_m(Z,\xi,\omega'), \theta') \\
\theta' &= \mathbb{1}_{\{Z=R\}}
\end{aligned}$$

3.

$$\begin{aligned}
V_Z^{current}(s) &= \max_{a',b'} u(c, h_i) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') \\
\text{s.t. } c &= \overline{y} + a - b - x - Q_u(Z,\xi,\omega')a' + Q_s(Z,\xi,\omega')b' - M \\
\omega' &= \{j+1, 0, a', b', 1, (\theta, i, M, \min(\tau+1, T+1))\} \\
0 &\leq b' \leq P(Z).h_i - \ell_0
\end{aligned}$$

4.

$$\begin{aligned}
V_Z^{walk}(s) &= \max_{a'\geq 0} u(c,h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') \\
\text{s.t. } c &= \overline{y} + a - x - Q_u(Z,\xi,\omega')a' - (1-\zeta)M + B \\
B &= (\mathbb{1}_{\{\zeta=\zeta_2\}} \times A + \mathbb{1}_{\{\zeta=\zeta_1\}} \times max(0,A)) \\
A &= \frac{E(P)(1-\phi)h_i}{1+r} - (\ell_0 - (1-\zeta)M) - b \\
\ell_0 &= \ell(i,r_m,\tau,\theta) \\
\omega' &= \{j+1,1,a',0,0,(0,3,0,0)\}
\end{aligned}$$

5.

$$\begin{aligned}
V_Z^{bk,toss}(s) &= u(c,h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') - stig \\
\text{s.t. } c &= \overline{y} - (1-\zeta)M(i,r_m) + (H_e + a_{new} + D_{toss}(\zeta,Z,s) - \delta \\
a_{new} &= \max\{0, \hat{A}_{toss}(\zeta,Z,s) + H_n\} \\
\ell_0 &= \ell(i,r_m,\tau,\theta) \\
H &= \max\{0, (\frac{(1-\phi).E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M(i,r_m)) - b)\} \\
H_e &= \min\{E,H\} \\
H_n &= H - H_e \\
a' &= D_{toss}(\zeta,Z,s) \\
\omega' &= \{j+1,2,a',0,0,(0,3,0,0)\}
\end{aligned}$$

6.

$$\begin{aligned}
V_Z^{bk,keep}(s) &= u(c,h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') - stig \\
\text{s.t. } c &= \overline{y} - M + a_{new} - \delta \quad \text{if} \quad P(Z)h_i - (\ell_0 + b) < 0 \\
c &= \overline{y} - M - b + a_{new} - \delta \quad \text{if} \quad P(Z)h_i - (\ell_0 + b) \geq 0 \\
a_{new} &= \max\{0, \hat{A}_{keep}(\zeta,Z,s)\} + D_{keep}(\zeta,Z,s) \\
\ell_0 &= \ell(i,r_m,\tau,\theta) \\
a' &= D_{keep}(\zeta,Z,s) \\
\omega' &= \{j+1,2,a',0,1,(\theta,i,M,\min(\tau+1,T+1))\}
\end{aligned}$$

$\hat{A}_{toss}(\zeta, Z, s)$ and $\hat{A}_{keep}(\zeta, Z, s)$ respectively are the before-bankruptcy adjusted cash-in-hand position as defined in section 2.3.6, for the cases when the household chooses to toss its house out and when it chooses to keep it. Similarly, $D_{toss}(\zeta, Z, s)$ and $D_{toss}(\zeta, Z, s)$ respectively are the after-bankruptcy adjusted cash-in-hand positions of the household if it chooses to toss or save its house in bankruptcy, as defined in section 2.3.6.

Case 2: Flagged homeowners ($\eta = 2$):

$$V_Z(s) = \max\{V_Z^{sell,rent}(s), V_Z^{current}(s), V_Z^{walk}(s)\},$$

with $V_Z^{sell,rent}$, $V_Z^{current}$ as defined earlier, and with V_Z^{walk} for $\eta = 2$ defined as:

$$\begin{aligned}
V_Z^{walk}(s) &= \max_{a' \geq 0} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') \\
\text{s.t. } c &= \overline{y} + a - x - Q_u(Z, \xi, \omega')a' - (1-\zeta)M + B \\
B &= (\mathbb{1}_{\{\zeta=\zeta_2\}} \times A + \mathbb{1}_{\{\zeta=\zeta_1\}} \times max(0, A)) \\
A &= \frac{E(P)(1-\phi)h_i}{1+r} - (\ell_0 - (1-\zeta)M) - b \\
\ell_0 &= \ell(i, r_m, \tau, \theta) \\
\omega' &= \{j+1, 2, a', 0, 0, (0, 3, 0, 0)\}
\end{aligned}$$

Note that a household would not start as a homeowner with $\eta = 1$. This is so because $\eta = 1$ in the current period implies that the household defaulted on mortgages in the previous period and did not file for bankruptcy to discharge unsecured debts. However, when a household defaults on mortgages but not on unsecured debts, it is assumed that the household loses its house at the end of the period and starts the following period as a non-owner.

Case 3: Clean non-owners ($\eta = 0$):

$$V_Z(s) = \max\{V_Z^{rent}(s), V_Z^{buy}(s), V_Z^{def}(s)\},$$

where

1.

$$\begin{aligned} V_Z^{rent}(s) &= \max_{i' \in \{1,2,3,4\}, a'} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi} V)(\omega') \\ \text{s.t. } c &= \overline{y} + a - b - x - Q_u(Z, \xi, \omega')a' - Rent(Z)h_{i'} \\ \omega' &= \{j+1, 0, a', 0, 0, (0, 3, 0, 0)\} \end{aligned}$$

2.

$$\begin{aligned} V_Z^{buy}(s) &= \max_{i' \in \{3,4\}, a'} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi} V)(\omega') \\ \text{s.t. } c &= \overline{y} + a - b - x - Q_u(Z, \xi, \omega')a' - (\tau_b + \mu)P(Z)h_{i'} - m \\ \omega' &= \{j+1, 0, a', 0, 1, (\theta', i', m, 2)\} \\ m &= Mortgage(i, R_m(Z, \xi, \omega'), \theta') \\ \theta' &= \mathbb{1}_{\{Z=R\}}. \end{aligned}$$

3.

$$\begin{aligned} V_Z^{def}(s) &= \max_{i' \in \{1,2,3,4\}, a' \geq 0} u(c, h_{i'}) + \beta\psi(j+1) \times (E_{Z|\xi} V)(\omega') - stig \\ \text{s.t. } c &= \overline{y} + a - x - Rent(Z)h_{i'} + D_{no}(Z, s) - \delta \\ a' &= D_{no}(Z, s) \\ \omega' &= \{j+1, 2, a', 0, 0, (0, 3, 0, 0)\}. \end{aligned}$$

In the above equation, $D_{no}(s)$ is the after-bankruptcy adjusted cash-in-hand position of a non-owner in bankruptcy as defined in section 2.3.6.

Case 4: Flagged non-owners: ($\eta = 1$ **or** $\eta = 2$)

Flagged non-owners may not borrow on any market and have value function given by:

$$V_Z(s) = V_Z^{rent}(s) \text{ if } \eta = 2 \text{ and } V_Z(s) = \max\{V_Z^{rent}(s), V_Z^{def}(s)\} \text{ if } \eta = 1 \ .$$

Problem of an involuntary defaulter

Case 1: The involuntary defaulter is a non-owner

$$\begin{aligned} V_Z(s) &= u(c, h_1) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') - stig \\ \text{s.t. } c &= \overline{y} - Rent(Z)h_1 \\ \omega' &= \{j+1, 2, a', 0, 0, (0, 3, 0, 0)\}. \end{aligned}$$

Case 2: The involuntary defaulter is a homeowner

When a homeowner goes through involuntary default, its house is sold at the beginning of the period. However, its before-bankruptcy cash-in-hand position is adjusted in a similar way as for households which choose to toss their houses when filing for bankruptcy. Involuntary defaulters start the following period with an unsecured bond of face value equal to zero. Specifically, let a_1 and a_2 equal the negatives of the mortgage debts still owed to the first and second mortgage lenders after exhaustion of the expected discounted sale proceeds following an involuntary default in a recourse world:

Then, let

$$\begin{cases} a_1 & = \min\{0, (1-\phi)P(Z)h_i - \ell_0\} \\ a_2 & = \min\{0, \max\{0, (1-\phi)P(Z)h_i - \ell_0\} - b\}. \end{cases}$$

Then, let

$$\hat{a} = \begin{cases} a - x & \text{if } \zeta = \zeta_1 \\ a - x + a_1 + a_2 & \text{if } \zeta = \zeta_2. \end{cases}$$

The value function of an involuntary defaulter which is a homeowner is then given by:

$$\begin{aligned}
V_Z(s) &= u(c,h_1) + \beta\psi(j+1) \times (E_{Z|\xi}V)(\omega') - stig \\
\text{s.t. } c &= \overline{y} - Rent(Z)h_1 + H_e + a_{new} \\
a_{new} &= \max\{0, \hat{a} + H_n\} \\
H &= \max\{0, ((1-\phi)P(Z)h_i - \ell_0 - b)\} \\
\ell_0 &= \ell(i, r_m, \tau, \theta), H_e = \min\{E, H\}, H_n = H - H_e, \text{ and } a' = 0 \\
\omega' &= \{j+1, 2, a', 0, 0, (0, 3, 0, 0)\}
\end{aligned}$$

2.3.10 Profit Functions of Lenders

If the household starts the period as a homeowner, let $W_{m_1,Z}(s)$ denote the expected discounted flows to the first mortgage lender which has loaned to the household and let $W_{m_2,Z}(s)$ denote the expected discounted flows to the second mortgage lender. Let $W_{u,Z}(s)$ denote the expected discounted flows to the unsecured lender from whom the household has purchased its current unsecured bond. Finally, let $W_{m_1} = (W_{m_1,B}, W_{m_1,R})$, $W_{m_2} = (W_{m_2,B}, W_{m_2,R})$, and $W_u = (W_{u,B}, W_{u,R})$. $W_{m_1,Z}$, $W_{m_1,Z}$, and $W_{m_1,Z}$ are derived in the appendix. Let $\Omega'(Z,s)$ be the household's choice for tomorrow's non-stochastic component of the individual's state vector given current state (Z,s). Finally, let $g_b(\Omega'(Z,s))$ and $g_a(\Omega'(Z,s))$ respectively be the face values of the second mortgage and unsecured bonds associated with $\Omega'(Z,s)$. Then, expected profits to a first mortgage lender on a first mortgage contract $(i', R_m(Z,\xi,\Omega'(Z,s)), 1, \mathbb{1}_{\{Z=R\}})$ offered to a household in state (Z,s) that will start tomorrow with a non-stochastic state component $\Omega'(Z,s)$ are given by:

$$\begin{aligned}
\Pi_{m_1}(Z,\xi,\Omega'(Z,s)) = &-(1-\mu)P(Z)h_{i'} + \text{Mortgage}(i', R_m(Z,\xi,\Omega'(Z,s)), \mathbb{1}_{\{Z=R\}}) \\
&+ \frac{(E_{Z|\xi}W_{m_1})(\Omega'(Z,s))}{1+r}.
\end{aligned}$$

Expected profits to a second mortgage lender which lends at per unit price $Q_s(Z,\xi,\Omega'(Z,s))$ to a household that will start the following period with a non-stochastic state component $\Omega'(Z,s)$ are given by:

$$\Pi_{m_2}(Z,\xi,\Omega'(Z,s)) = -Q_s(Z,\xi,\Omega'(Z,s)) \times g_b(\Omega'(Z,s)) + \frac{(E_{Z|\xi}Wm_2)(\Omega'(Z,s))}{1+r}.$$

Expected profits to an unsecured lender which lends at per unit price $Q_u(Z,\xi,\Omega'(Z,s))$ to a household that will start the following period with a non-stochastic state component $\Omega'(Z,s)$ are given by:

$$\Pi_u(Z,\xi,\Omega'(Z,s)) = Q_u(Z,\xi,\Omega'(Z,s)) \times g_a(\Omega'(Z,s)) + \psi_{j+1}\frac{(E_{Z|\xi}Wu)(\Omega'(Z,s))}{1+r}.$$

2.3.11 Equilibrium Conditions

The economy is in equilibrium when:

1. Households optimize
2. The rental agency makes zero expected profits:

$$\begin{cases} Rent(R) = P(R) \times \frac{r}{1+r} \\ Rent(B) = P(B) \times \frac{\gamma+r}{1+r} - P(R) \times \frac{\gamma}{1+r} \end{cases}$$

3. Pricing functions R_m,Q_s, and Q_u are such that first and second mortgage lenders and unsecured lenders make zero expected profits on each loan offered

An equilibrium is a steady state equilibrium if there is a zero mass of beginning-of-period homeowners in state s with $\theta \neq \mathbb{1}_{\{Z=R\}}$. In the rest of the paper, I study steady state boom equilibria which are steady state equilibria with $Z = B$ such that after having started with high housing prices, the economy remains an economy of high housing prices in spite of agents' expectations for a potential fall of housing prices. In this sense, a steady-state equilibrium will be understood as an equilibrium in which housing prices are high and all beginning-of-period homeowners have purchased their houses at a time of high housing prices.

2.4 Calibration

Birth age and retirement age respectively correspond to real world ages of 25 and 64. Households die with certainty at age $J = 79$. Survival probabilities $\{\psi_j\}_{j=2,\cdots,J}$ are derived from the 2000 United States Life Tables 2000 from the National Center for Health Statistics. The functional form of households' utility function of households is given by:

$$u(c,h) = \frac{\left(c^{\mu_c}\left(\left(1+\varpi \times \mathbb{1}_{\{o=1\}}\right) \times h\right)^{1-\mu_c}\right)^{1-\sigma}}{1-\sigma},$$

where ϖ denotes the homeownership premium.

The weighted average of the ratio (homestead exemption/state's 2002 median income) among U.S states is 1.81. I match this number by setting the amount of homestead exemption E equal to 1.81 $\times$ median income. The foreclosure discount rate ϕ is set to 0.25, so that houses in foreclosure sale for about 25% of their values. Campbell, Giglio, and Pathak (2011) documents that the foreclosure discount rate was on average 27% in the period 1987-2009 for houses located in the state of Massachusetts. Using a nationally broader dataset of single-family 30-year fixed rate mortgages originated between 1995 and 1999 combined with the Office of Federal Housing Enterprise Oversight (OFHEO) repeat sales index, Pennington-Cross (2006) finds that foreclosed properties appreciate on average 22% less than the metropolitan area average appreciation rate. I hence target a foreclosure discount rate of 25% which is the average of the discount rates found in the two studies.

The model period corresponds to 3 years. Following Livshits, Macgee & Tertilit (2008), the annual risk-free rate is set equal to the 4% average return on capital reported by McGrattan and Prescott (2000). Housing transaction costs τ_b and τ_s are taken from Sommer et al. (2010) and are set as $\tau_b = 0.025$ and $\tau_s = 0.07$. A first mortgage contract hence lasts for at most $T = 10$ periods (or 30 years). The average downpayment μ is set equal to 0.23, which is the average downpayment on fixed interest residential first mortgage contracts over the period 2002-2004 obtained from the LPS Applied Analytics's database. μ_1 is set equal to 0.13 such that at the time of purchase, home buyers do not borrow more than 90% of the value of the house when aggregating first and second mortgage loans secured by the house. Kenneth Brevoort, Robert Avery, and

Glenn Canner (2006) find that in 2005, about 21.5% of first-lien conventional home-purchase loans on owner-occupied site-built homes involved a piggyback. μ_2 is set equal to 0.4 in order to target 21.5% as the fraction of households that in purchase their homes with a piggyback lien in the steady state. A_1 is set equal to 0.6 so as to match an average first mortgage balance at origination equal to 1.05×3 years of average wage income over the period 2002-2004 as found from the LPS Analytics's Inc. database. A_0 is set from $\frac{1}{A(0)} = 0.45 \times \frac{1}{A_1}$ so that $Z = R$ corresponds to the aggregate state of the economy in which housing prices have fallen from their level when $Z = B$ by 55%, which is the percentage by which average housing prices fell from 2007 to 2009 in the state of Nevada. The state of Nevada is the U.S. state which experienced the greatest housing price fall over that period.The probability γ of an aggregate housing price fall is set equal to 0.14 in order to match an average interest rate of 5.38% for first mortgages originated over the period 2002-2004 in the U.S. economy as I document it from the LPS database.

From the PSID database, I find that the median home was valued at $\$150,000$ in 2005. The average value of homes owned during the same period and for which the value fell below the median was \$82842 or 0.55×3 years of average household's wage income. Similarly, for homes valued above the median, the average value was \$332356 or 2.21×3 years of average household's wage income. Given $A_1 = 0.6$, I hence set $h_3 = 0.33$ and $h_4 = 1.33$ so that $P(B)h_3 = 0.55 \times 3$ years of average wage income and $P(B)h_4 = 2.21 \times 3$ years of average wage income.

The probability λ of a reversal of bad credit into good credit is set to 0.3, so that the average duration $\frac{1}{\lambda}$ of a bankruptcy flag on a household's credit record is 10 years, or $\frac{10}{3}$ model period. Given a model period of 3 years, fraction κ of disposable income (disposable income is defined as the difference between wage income and average income) assigned to the repayment of unsecured debts in bankruptcy in the post-reform world is set equal to 1.67, so that after the bankruptcy reform unsecured creditors may be repaid by as much as 5 times the difference between the household's annual income and the annual median income.

The fraction of a period mortgage payment that a household may default upon is set as ($\zeta_1 = 0, \zeta_2 = 0.3$) in the steady state world and ($\zeta_1 = 0, \zeta_2 = 0.53$) in the great recession world. Given a model period of 3 years, 0.3 and 0.53 are respectively equivalent to 330 days and 580 days of free-renting in the event of a default, which are the median number of days of free-renting in the periods 2002-2004 and 2007-2009 found on average for recourse states using the LPS Analytics's Inc database. Recall the assumption that a household becomes liable for the fraction of the mortgage loan that is not covered by the proceeds of a foreclosure sale if and only if it is faced with $\zeta = \zeta_2$. I hence interpret $p_{\zeta,2}$ as the probability of a deficiency judgment and I estimate it as a weighted average of the fraction of mortgage defaults that started in $2007 - 2009$ and that terminated as a foreclosure among mortgage defaults that started during the same period and terminated either as a foreclosure, a short-sale or a voluntary conveyance in the U.S. economy. Under the assumption that deficiency judgments are obtained only in recourse states in which the foreclosure process is always a judicial one, I estimate the probability of a deficiency judgment to be $p_{\zeta,2} = 0.11$.

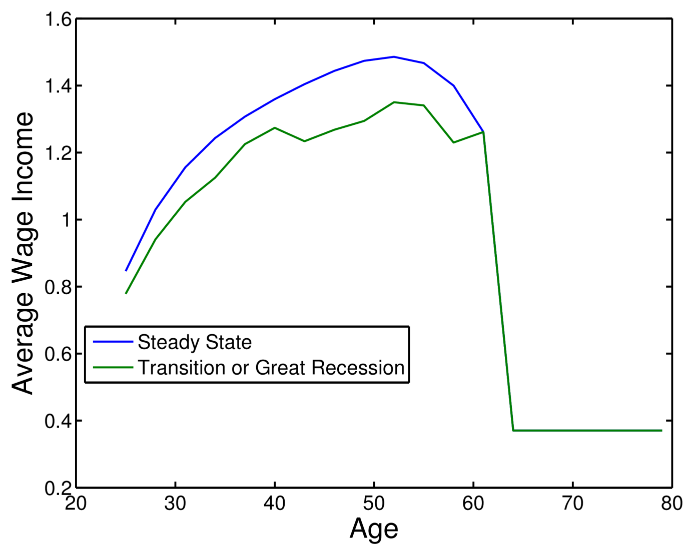
Filing cost δ is set equal to 2% of 3-year of average income and amounts to the median filing cost for Chapter 13 filers documented by the Government Accountability Office in 2008. Borrowing interest rates in the unsecured and second mortgage markets are bounded above by a value $r_{max} = 26\%$ which is chosen to match the bankruptcy rate among homeowners. The interest rate r_b applied to unsecured debt rolled over in bankruptcy is set equal to r_{max}.

Over the period 2002-2004, the average bankruptcy rate was 0.73%. Sullivan, Warren, and Westbrook (2000) document from a 1991 consumer bankruptcy survey that in 1991, about 50% of bankrupts were homeowners. Assuming the proportion of homeowners among bankrupts did not change between 1991 and 2004, and given a homeownership rate of 65% in 2002-2005, I target a bankruptcy rate of 0.56% among homeowners. The utility cost associated with bankruptcy filing $stig$ is set equal to 0.1 so as to match an average income of bankrupt homeowners equal to 0.61 of the average income of homeowners as documented by Sullivan, Warren, and Westbrook (2000) document from their 1991 consumer bankruptcy survey.

In the Transition world of the great recession which is obtained from the baseline model by shocking housing prices down by 26%, and by shocking average labor productivities as shown in figure, it is assumed that households predict that housing prices and labor productivities will rise back to their values in the baseline model with probability γ_{GR} which is calibrated to equal 0.17 so as to match a bankruptcy rate of 0.53% among homeowners in the in the period 2007-2009.[8] .

The Income Process

Figure 2.1: Labor Efficiency Profile $\{\vartheta_j\}_{j=1,\cdots,J_r}$



The autocorrolelation coefficient ρ and the variance σ_v^2 of persistent shock ξ have tri-annual values of 0.863 and 0.363, respectively, which correspond to the respective annual [9] values of 0.95 and 0.168 reported by Storesletten, Telmer & Yaron (2004). The variance σ_ϵ^2 of the transitory income shock ϵ has its tri-annual value set equal to 0.765, which corresponds to the annual value of 0.255 reported by Storesletten, Telmer & Yaron (2004).

For retirement's income, *Pens* and ς are set equal to 0.05 and 0.33 so that the

[8] Early in 2005, households surged to file for bankruptcy in anticipation of the implementation of the bankruptcy reform in October 2005. As a consequence, the bankruptcy rate turned to be very high in 2005, but then very low in 2006, before gradually rising to its 2009 level. As a consequence, I use the bankruptcy rate of 0.63% in 2009 as an estimate of the bankruptcy rate for the period 2007-2009 in order to limit transition effects. Katherine Porter (2012) documents from a 2007 consumer bankruptcy project that in 2007, 54.5% of bankrupt were homeowners. Given a homeownership rate of 65% in 2007, the bankruptcy rate among homeowners over the period $2007-2009$ is hence estimated equal to 0.53%.

[9] if ρ_1 and $\sigma_{v,1}^2$ are used to denote annual values, then tri-annual values are obtained as $\rho = \rho_1^3$ and $\sigma_v^2 = \sigma_{v,1}^2(\rho_1^4 + \rho_1^2 + 1)$. The tri-annual variance of the transitory income shock is found in a similar way.

ratio of the average pension of retirees over the average wage income, and the ratio of the standard deviation of pension income over average wage income are 0.37 and 0.27 respectively, as found from the CPS data for the period 2002-2004. The persistent and transitory income processes are discredited into 5 and 3-state Markov Process, respectively, using the Method of Tauchen (1986) as described in Adda and Cooper (2003).

The Expense Shock Process

The expense shock process is derived from the estimations of Livshits, Macgee & Tertilit (2008) for the period 1996-1997. They estimate a tri-annual divorce probability of 3.73% and a proportion 1.5% of unplanned pregnancy over a three year period. They determine the value of a divorce shock and of an unplanned pregnancy over three years to be $\$36,558$ and $\$24,000$ respectively. These two shocks assumed independent amount to a probability 5.23% of a $32956 expense shock. Following Livshits, Macgee & Tertilit (2008), I combine these two shocks with their estimated proportion 1.874% of households that received a medical shock of a magnitude close to $32956 or 0.26 $\times$ three year average wage income over their period of study. [10] This gives an estimate $p_{\chi,1} = 0.071$ for the expense shock probability, and $\chi_1 = 0.26$ after normalizing average wage income to 1. χ_2 is set equal to 0.82 and $p_{\chi,2} = 0.0048$ as estimated for the right tail of the medical shock distribution by Livshits, Macgee & Tertilit (2008).

Parameter values are summarized in Tables 2.5 and 2.6. Table 2.5 summarizes independently determined parameters, while Table 2.6 summarizes jointly determined parameters and displays the calibration targets.

[10] Using the 1996 and 1997 waves of the Medical Expenditure Panel Survey and aggregate data from the US Health Care Financing Administration, Livshits, Macgee & Tertilit (2008) estimate that over the period 1996-1997, 1.874% of households receive a medical expense shock of a magnitude close to $32956.

Table 2.5: Parameters Independently Determined

downpayment	23%
piggyback fraction	13%
annual risk-free saving rate	4%
Mortg.default Fraction $(\zeta_2, p_{\zeta,2})$	$(0.3, 0.11)$
expense shock $(\chi_1, p_{\chi,1})$	(0.26 Units,0.071)
expense shock $(\chi_2, p_{\chi,2})$	(0.82 Units,0.0048)
Transaction cost on selling	7%
Transaction cost on buying	2.5%
Probability λ	0.3
Filing Cost δ	0.02
σ	2
κ (post-reform fraction of disposable income assigned to repay unsecured debts in bankruptcy)	1.67
Foreclosure discount rate ϕ	0.25
Homestead Exemption E	0.69

Table 2.6: Parameters Jointly Determined

Parameter	Data Target	Data Value	Model Value
$\beta = 0.89$	Unsecured Debt/GDP	7.97%	6.04%
$\mu_c = 0.70$	Housing share of expenditures	19%	27%
$\tau_\omega = 18\%$	Gvm Expenditures/Output	19%	20%
$A_1 = 0.6$	Avg. Loan at Orig.	1.05 units[11]	1.08
$A_0 = 1.33$	$\frac{1}{A_0} = 0.45 \times \frac{1}{A_1}$		
$h_1 = 0.0001$	Rent/Income ratio below $5th$ pctl of income distribution	53%	75%
$h_2 = 0.10$	Rent/Income ratio below $30th$ pctl of income distribution	43%	60%
$h_3 = 0.33$	Avg home value below median home value	0.55%	0.55%
$h_4 = 1.3$	Avg home value above median home value	2.21%	2.21%
$\varpi = 1.015$	Ownership rate	65%	65%
$\mu_2 = 0.4$	Fraction of home purchases with piggyback liens	21.5%	20%
$r_{max} = 35.7\%$	Bankruptcy rate among homeowners in Baseline	0.56%	0.39%
$c_{max} = 0.33$	Avg Back End Debt to Income Ratio for home buyers	0.33	0.41
stig $= 0.1$	Avg. Inc of Bankrupt homeowners/Avg. Inc of homeowners	0.61	0.32
$\gamma = 0.15$	Average Int. rate on first mortgages	5.38%	5.27%
$\gamma_{GR} = 0.23$	Bankruptcy rate among homeowners during GR	0.53%	0.35%

2.5 Results

2.5.1 Steady State

In the steady-state of the model's non recourse world, 44% of homeowners own a small size property (property of size 3) and the remaining 56% own a large size property (property of size 4). In the steady-state of the model's non recourse world, the proportion of households which file for bankruptcy and choose to keep their houses is 0.55%, compared to a rate of 0.21% for Chapter 13 filers in the U.S. between 2002 and 2004. The foreclosure rate is defined as the proportion of homeowners with a positive loan-to-value ratio and which choose to either walk away from their houses or which have negative equity and choose to abandon their houses while filing for bankruptcy[12]
.

In the steady state , 5.17% of households borrow on the unsecured market, and the average unsecured debt is 6.04% of the model economy's average wage income. The fractions borrowing on the unsecured credit market are 8.52% for households living as renters, 3.2% for households living as homeowners and 3.45% for home buyers. The average loan-to-value ratio is 70% among homeowners. The average home equity equals 1.13× the median income.

2.5.2 Transition Analysis

The quantitative experiment carried next aims at assessing the impact of the 2005 bankruptcy reform law on the foreclosure crisis once housing prices started to fall in 2007. The experiment is carried over in two steps in both the recourse and non-recourse model' artificial worlds. First, four permanent features are added to the steady-state to define the post-reform transition world: a 26% drop of housing prices, a 10% downward shift of the labor efficiency profile as shown in figure 2.1, a lengthening of the foreclosure process through an increase of the median number of days of free-rent in foreclosure (from 330 days to 440 days to 580 days), and a shift to the new bankruptcy code. In other words, the model's world is permanently changed to a new world where housing prices are lower, earnings are lower, the foreclosure process is longer, the bankruptcy code looks

[12] Homeowners with positive equity which file for bankruptcy and choose to abandon their houses are hence not counted among the population of homeowners experiencing a foreclosure.

like the U.S. bankruptcy code after October 2005, and in each period households expect with probability γ_{GR} that in the next period, housing prices and earnings will go back to their pre-recession levels and nothing will ever change again. Afterward in a second step, the pre-reform transition world is studied. The pre-reform transition differs from the post-reform transition world in that it uses the old bankruptcy code and hence only has three features that differentiates it from the steady-state world: lower housing prices, lower earnings and a lengthier foreclosure process. The strategy is then to compare the outcomes of the pre-reform and post-reform transition economies in order to conclude on the impact of the bankruptcy reform on the foreclosure crisis during the transition implied by the great recession.

Transition Analysis

The model predicts a foreclosure rate of 0.77% during the first period of transition into the great recession world with the new bankruptcy code . This amounts to 85% of the average foreclosure rate of 0.9% observed in the U.S. in the period 2007-2009, as documented in the empirical section. In the transition of the post-reform world,the foreclosure rate is 76% among owners of small size properties, 77% among owners of large size properties.

During the first period of transition, the proportion of homeowners which simultaneously go through bankruptcy and foreclosure appears over predicted. This proportion is 32% for the model's non-recourse world, while from the LPS Applied Analytics's database, only 22% of residential fixed rate mortgages associated with a foreclosure start had also been in bankruptcy within a three year period of the foreclosure start date. In the model's first period of transition, the average wage income of homeowners which file for bankruptcy and choose to keep their houses equals 42% of the median income of the steady-state economy, while in the steady-state world, it amounts to 97% of the median income of the steady state economy. Hence, in the post-reform great recession world compared to the steady state world, households that use bankruptcy to discharge their unsecured debts while maintaining their ownership status are poorer. This happens because in the post-reform great recession world, the bankruptcy code is less generous to indebted households with high earnings. The proportion of homeowners which choose to file for bankruptcy and to keep their homes when doing so increases slightly from 0.55% in the steady-state world to 0.56% in the post-reform great recession world. Households are induced to choose to file for bankruptcy while keeping their homes more often in the transition world because in the model as in the U.S. economy, homeowners for whom the first mortgage balance is under water obtain a discharge of their second mortgage loan in bankruptcy if they choose to keep their homes.

According to the Corelogic Report on Negative Equity for the fourth quarter of 2009, 24% of residential properties had negative home equity and 17% had negative home equity that in absolute value exceeded 10% of the house value. In the model's transition world 23% have negative equity and 14% have negative home equity that in absolute value exceeded 10% of the house value.

Comparing Transition in the Pre and Post-Reform Worlds

Table 2.7 summarizes results for the model's transition economy in the pre and post-bankruptcy reform cases. The unit of measure in the table is the average wage income of the steady state economy. Bankrupt tossers are referred to as the homeowners which jointly choose foreclosure and bankruptcy. Bankrupt keepers are homeowners which choose to file for bankruptcy and choose to keep their homes. Walkers are homeowners which walk away from their homes and do not file for bankruptcy. Current homeowners are homeowners which choose to continue to make mortgage payments in order to maintain ownership of their homes. In the table, net liquid asset position for a household is defined as $a - x$, where a is the face value of the beginning-of-period unsecured bond, and x is the size of the expense shock drawn at the beginning of the period.

Key findings are as follows. The reform led to a foreclosure rate that is slightly higher in the transition world with no significant effect in the case of small size properties. By reducing the amount of unsecured debts dischargeable for high income earners, the reform in the transition economy also leads to fewer homeowners choosing to file for bankruptcy while keeping their homes. In fact, in the pre-reform transition economy, 0.86% of homeowners choose this option, compared to 0.56% in the post-reform transition economy.

In the transition economy, the foreclosure rate is higher in the absence of the reform because given the reform, income rich homeowners that are unable to discharge significant unsecured debts in bankruptcy prefer to maintain a good credit status by avoiding a foreclosure whenever their home equity is not too negative. In fact, had these households decided that bankruptcy is not favorable because of the reform and had they chosen the foreclosure option instead, they would find themselves trapped simultaneously with large unsecured debts and bad credit. The opportunity cost of such action would be a restriction to credit markets which dampens their ability to smooth consumption by taking advantage of reform-induced lower interest rates in unsecured markets in the adverse event of bad income or expense shocks realizations. In the post-reform world relative to the pre-reform world, it hence takes home equity equity to be more significantly negative to induce income rich homeowners to walk away from their homes when they are unable to simultaneously discharge unsecured debts in bankruptcy. Because owners of small size properties are usually poorer and more indebted in unsecured markets than owners of large size properties, it turns out that small property owners are the least affected by the mechanism described.

Table 2.7: Transition Results for the Non-Recourse World

	Pre-Reform	Post-Reform
Fc rate	0.82%	0.77%
Fc among owners of small size prop.	0.79%	0.76%
Fc among owners of large size prop.	0.82%	0.77%
Prop. of walkers among those in fc	70%	80%
Average wage of walkers	0.23 units[13]	0.27 units
Net liquid asset position of walkers	0.49 units	0.48 units
Prop. of bankrupt tossers among those in fc	30%	20%
Average wage of bankrupt tossers	0.36 units	0.17 units
Net liquid asset of bankrupt tossers	−0.13 units	−0.16
Prop. of keepers among those with neg. home equity	0.53%	0.32%
Average wage of keepers with neg. home equity	0.40 units	0.32 units
Net liquid asset of keepers with neg/ home equity	−0.24 units	−0.22 units
Prop. of sellers among those with neg. home equity	1.82%	2.18%
Avg. Income of sellers with neg. home equity	0.18 units	0.18 units
Net liquid asset of sellers with neg. home equity	0.10 units	0.08 units
Prop. of current among those with neg. home equity	87%	88%
Average wage of current with neg. home equity	1.12 units	1.13 units
Net liquid asset of current with neg. home equity	2.32 units	2.33 units

2.6 Concluding Remarks

In the current paper, I have documented new facts on mortgage default from the LPS Applied Analytics's database. I have proposed a new and rich quantitative model that furthers our understanding of households' default decisions in mortgage and unsecured credit markets. I have used the model to demonstrate that:

1. households's decisions to forego their homes in foreclosure during the great recession was closely related to their indebtedness in unsecured credit markets

2. contrary to suggestions made in existing literature, the 2005 bankruptcy reform law increased the opportunity cost of bad credit and helped to mitigate the foreclosure crisis once housing prices fell in 2007.

Chapter 3

Political Economy Theory of Growth

Introduction

The standard neoclassical growth model predicts that developing economies will eventually catch up with leading economies. While good performances from Asian countries support the standard neoclassical growth model, economic stagnation in Sub-Saharan Africa and Latin America calls for a different theory that is capable of explaining both growth miracles and growth tragedies. In a context where technological progress is available and free but requires action from politicians to be adopted by the economy, this paper shows that a high degree of patience in the preferences of citizens and politicians and the ability of citizens to replace a politician in power are key ingredients for economic growth.

This study proposes a model where nature shifts the technological frontier forward every period. However, the economy can take advantage of that shift only after it has been approved by a politician in office. A politician in power approves (or adopts) technological progress by processing an investment good. This in turn propagates growth in the economy. A politician in power who chooses not to process the investment good for economic growth may consume it and increase his/her own welfare. When technological progress is adopted, it benefits both citizens and politicians. However, politicians are self-interested and approve growth only when they are sufficiently compensated to make up for not consuming the investment good. Even when they are paid the maximum resources available, impatient politicians still find it beneficial to consume the investment good and let the economy stagnate. This paper studies subgame perfect equilibria of the game between citizens and politicians and focuses on the properties of best subgame perfect equilibria which are subgame perfect equilibria that maximize payoffs of citizens. This methodology is common in the principal-agent literature and is also used by Acemoglu, Golosov and Tsyvinski (2008) and Miquel & Yared (2010) who study the properties of an optimal contract between an agent who has an advantage over the accomplishment of a task and a principal who is willing to delegate this task. By focusing on best subgame perfect equilibria, this paper departs from suboptimalities that may simply be resolved through renegotiation.

Given this basic setup laid out in section 1, this paper further analyzes the impact of political instability and dictatorship on economic growth in sections 2 and 3 respectively. Political instability is modeled as an event"coup d'etat" whose occurrence is associated with the politician in power being overthrown and replaced with a different politician randomly chosen by nature. Dictatorship is defined as a state in which citizens are deprived of their voting rights and the incumbent ruler (or politician in power) is expected to remain in power forever. Such a ruler is referred to as a dictator. An economy ruled by a dictator is referred to as a dictatorial economy. The paper compares performances of a continuum of economies each characterized by a given probability of falling in the state of dictatorship. In the model proposed, democracies are referred to as economies where the probability of falling in the state of dictatorship is null.

The introduction of political instability scales the effective discount factor of politicians down by the probability that a coup d'etat does not occur. The logic is as follows. Because a coup d'etat ends the term of office of an incumbent politician, its likelihood makes expected-utility maximizing politicians short-sighted and willing to consume the investment good and let the economy stagnate. The paper shows that once political instability is introduced, an economy that would otherwise grow will now grow only if the likelihood of a coup d'etat is not too large.

The introduction of a probability to fall in the state of dictatorship has a similar effect on economic growth. The theory predicts that democratic economies that do not grow will also not grow for any given probability of falling in the state of dictatorship.

On the other hand, dictatorial economies that are capable of growing will also grow for any given probability of falling in the state of dictatorship. For intermediate economies where agents are patient enough to allow for growth under democracy but not sufficiently patient to do so after the economy has become a dictatorship, the occurrence of growth depends on the probability of falling in the state of dictatorship. For this last class of economies, given identical preference parameters, economies that grow are those below a threshold probability of falling in the state of dictatorship. The leading mechanism behind this result is the following. In the best subgame perfect equilibria, rulers of democratic economies fear replacement while dictators do not, as they are guaranteed an eternal term of office. Dictators therefore have a higher bargaining power compared to rulers of non-dictatorial economies. In fact, the higher the probability of falling in the state of dictatorship, the higher the bargaining power of a ruler. It follows that everything else constant, compensations that make politicians willing to approve growth increase with the probability of falling in the state of dictatorship. In other words, given identical preference parameters, a dictatorial economy is more prone to stagnation than a democratic economy. At the same time, remunerations to politicians tend to increase with the probability of falling in the state of dictatorship. That is, citizens are better off as the economy shifts toward democracy since in this case, growth becomes more likely while payoffs to citizens naturally increase. Yet, as emphasized earlier, a dictatorial economy may perform better than a democratic economy if its agents are more patient than those of the latter. In short, the relationship between democracy and economic growth is a non-linear one.

An important limitation of the theory proposed in this paper is that it does not lend itself fully to empirical tests, given the lack of reliable data to estimate discount factors in poor countries. However, the theory fits the empirical evidence relatively well in several other dimensions.

First, using a worldwide data set on national leaders from 1945 to 2000, and restricting to years of transition where the end of the leader's rule was caused by death due to a natural cause or to an accident, Jones & Olken (2005) conclude that a one standard deviation change in leader quality leads to a growth change of 1.5 percentage points per year. Their finding suggests that the nature of a leader and perhaps his/her time preferences are key determinants of economic growth.

Second, the theory proposed in this paper predicts that impatient politicians ask for large compensations in order to approve growth. One should therefore expect that everything else equal, countries with the largest government consumption expenditures per capita are also those with the most impatient politicians and the smallest growth rates. This prediction is confirmed by the finding of Barro (1991) from cross-country regressions that the ratio of government consumption expenditure to GDP is inversely related to per-capita growth.

Third, this paper suggests that the frequency of coups d'etat is a negative determinant of growth. In a sample of 113 countries studied between 1950 and 1982, Alesina, Ozler, Roubini and Swagel (1996) find that the average frequencies of coups d'etat for Latin America, Africa, Asia and industrial countries were 0.079, 0.060, 0.037 and 0 respectively, while the sample average was 0.048. Over the period, the average annual growth rates for Latin America, Africa, Asia and industrial countries were 2.2%, 1.4%, 3.3% and 2.9% respectively. Using a structural equation system to control for simultaneity and reverse causality, they find strong evidence that a high frequency of coups d'etat deters economic growth. This result of strong causality from political instability to economic growth is also reported by Barro (1991) and Easterly & Rebelo(1993). Alesina, Ozler, Roubini and Swagel (1996) explain this result by arguing that a high level of political instability implies uncertain future policies which in turn encourage risk-averse economic agents to wait to take productive initiatives or to exit the economy by investing abroad. The current paper suggests a new channel through which political instability affects economy growth: by making it difficult to provide correct incentives to opportunistic politicians who become short-sighted as the economy becomes politically unstable.

Fourth, this paper predicts that democratization fosters economic growth. This result is validated by Persson and Tabellini (2006) who from cross-country regressions find that becoming a democracy accelerates growth by 0.75 percentage points.

Finally, the theory proposed in this paper predicts that the relationship between dictatorship and economic growth is a non-linear one. A dictatorship may grow or stagnate depending on how patient the dictator is. This implication is consistent with the empirical findings of Alesina, Ozler, Roubini and Swagel (1996) who conduct cross-country regressions and fail to identify a linear relationship between democracy and growth. Alesina, Ozler, Roubini and Swagel (1996) define democracy as a variable that takes on value 1 for countries with " free competitive general elections with more than one party running " , on value 2 for countries " with some forms of elections but with very severe limits in the competitiveness of such ballots " , and on value 3 for " countries in which leaders are not elected " . They find that over the period studied, the average value of democracy was 2.18, 2.82, 2, 32 and 1.07 for Latin America, Africa, Asia and industrial countries respectively and conclude that there is no obvious relationship between democracy and growth. Alesina, Ozler, Roubini and Swagel (1996) explain this finding by two observations. First, lobbyism encourages policy makers in democratic regimes to favor opportunistic policies that are detrimental to growth, while dictators free from competition may be less sensitive to it. However, because in some instances dictators may also need to be opportunistic if their survival is not secured, it is not clear which of democracy and dictatorship is more favorable to growth. Second, as emphasized by the authors:

> *"authoritarian regimes are not a homogenous lot: they include technocratic dictators and kleptocratic ones. While the apparent association of high economic growth with authoritarian regimes is suggested by the experience of several authoritarian technocratic regimes (such as those in Korea, Taiwan,Indonesia, Turkey, Chile and so on), it is as well evident that for each benevolent dictator, one can observe at least as many kleptocratic or inept authoritarian regimes whose rule led to systematic economic mismanagement and eventual political and economic collapse of their countries".*

This paper goes a step further to characterize kleptocratic and technocratic dictators while highlighting important mechanisms through which the nature of political institutions is linked to economic growth.

Brief Review of The Literature

There exists a large body of literature which seeks to understand divergences in economic performances across countries. Parente and Prescott (1999) explain these divergences by the existence of barriers such as unions which in some countries protect inefficient work practices at the firm level. Krusell and Rios-Rull (1996) propose a vintage human capital model where agents either accumulate skills related to existing or new technologies and become managers, or work as unskilled for their entire lifetimes. In their model, innovation shifts demand away from managers of current technologies toward managers of new and cheaper technologies, while raising the purchasing power of the unskilled. It results that the unskilled are in favor of innovation while the skilled are against it. In that model, every period, each agent votes for either laissez-faire (which allows the development of new technologies), or the prohibition of new technologies and the majority wins. The authors find that depending on the initial distribution of skills, the economy may converge to a long-term equilibrium with permanent growth or to a long-term equilibrium with economic stagnation.

Benhabib & Rustichini (1996) suggest a theory which predicts a positive relationship between the initial stock of physical capital of a country and its growth rate when the utility function of agents is sufficiently concave. In the model of Benhabib and Rustichini (1996), organized social groups independently choose consumption levels which are turned into actual consumptions according to a preestablished allocation rule. Residual output is then accumulated as capital for the following period. Benhabib Rustichini (1996) characterize subgame perfect equilibrium outcomes of this game between the social groups using trigger strategies which threaten to shift to an undesirable outcome after any deviation from a first best outcome featuring sustained growth. Their study finds that at high initial capital stocks where consumption levels are high and marginal utilities low, the social groups are not willing to deviate from the first best arrangement. However, at low initial capital stocks associated with low consumption levels and high marginal utilities, the social groups are more willing to deviate at the expense of future retaliation, especially when marginal productivity of capital is not very high at low wealth levels. The theory of Benhabib and Rustichini (1996) is consistent with the empirical finding of Fisher (1991) that investment rates in physical capital are positively correlated with income levels.

Agarwala (1983) finds that distortions in market prices explain a significant proportion of differences in economic performances among countries. Based on corruption indexes provided by non-governmental organizations that monitor countries, Mauro

(1995) reports that corruption is a negative determinant of economic growth. Easterly & Levine (1997) argue that ethnic diversity creates polarization and encourages opportunistic behaviors and find that it accounts for more than 28 % of the growth differential between the countries of Africa and East Asia. Alesina, Ozler, Roubini and Swagel (1996) find evidence that political instability negatively impacts economic growth. Barro (1991) concludes from cross-country regressions that among poor countries with the same initial GDP per capita, those that catch up the fastest are those with the highest initial levels of human capital per capita. He finds that the initial level of human capital, per capita government consumption expenditures, political instability (proxied by figures on revolutions, coups and political assassinations), price distortions (based on purchasing-power parity numbers for investment deflators) and the nature of the economic system (market versus not market-oriented) are important determinants of economic growth. Yet, the author reports that these factors together do not fully explain the relatively weak growth performances of countries in Sub-Saharan Africa and Latin America.

Despite existing evidence that political instability negatively affects economic growth, rigorous theoretical analyses on the subject remain scarce. Cukierman, Edwards, & Tabellini (1992) study the impact of political instability and polarization on seigniorage in a model economy with two political parties. They define political turnover to be governed by a Markov process whose transition probability measures political instability. They describe polarization among political parties as disagreement over the composition of a public good that the parties value. In their model, the political party in office in entrusted with choices of the current period level of seigniorage and tax rate (fiscal policy), as well as the next period efficiency level of the tax system (tax reform). They find that when the probability of political turnover or the degree of polarization is very high, it is optimal for the political party in office to choose an inefficient tax system in order to discourage future governments from collecting taxes and spending them on the goods that the party in office does not value. The authors succeed in validating these predictions with econometric regressions and conclude that highly polarized and politically unstable countries rely more heavily on seigniorage to finance public good consumption. Given that high inflation is negatively associated with economic growth (De Gregorio (1992)), the work of Cukierman, Edwards, & Tabellini (1992) may be understood as a theory of economic growth and political instability. Svensson(1998) uses a model similar to that of Cukierman, Edwards, & Tabellini (1992) to study the impact of polarization and political instability on the quality of the legal environment. In that model, a strong legal environment is associated with a high level of enforcement of property rights and a high level of private investment. Arguing that a high turnover rate prevents an incumbent government from fully internalizing the benefits of a legal reform, Svensson(1998) concludes that high political instability negatively affects private investment. Using cross-country regressions, Svensson(1998) finds that political instability has a negative and significant impact on private investment when quality of property rights is not accounted for. He also finds that quality of property rights has a positive and significant effect on private investment when political instability is not controlled for. However, when both political instability and quality of property rights are controlled for, the author finds that only the effect of quality of property rights remains significant. From these results, Svensson(1998) concludes that political instability affects private investment essentially by discouraging governments from reforming the legal system to improve the quality of property rights. In both papers however, the political process is exogenous and not sophisticated enough to capture the key mechanisms delivered by the current paper.

The methodology used in this paper builds on the work of Acemoglu, Golosov & Tsyvinski (2008) who study a principal-agent political economy model where citizens hire and pay politicians to implement the best outcome of the economy. They study a model with double-sided commitment and find that if politicians are more patient than citizens, then political-economy induced distortions will disappear in the long-term. The political-economy setup of this paper is essentially theirs. However, this paper differs fundamentally from theirs in three respects. First, the current study analyzes technology adoption and links it to growth, while their paper focuses on capital accumulation and does not relate it to growth. Second, this paper models political instability and dictatorship which are absent in theirs. Third, their paper assumes the existence of a commitment technology from which this paper abstracts. In the context of underdeveloped economies, the assumption of a commitment technology would be difficult to justify as it would deny the true nature of these economies which typically function under weak institutions.

3.1 The Basic Setup

This section studies the impact of patience in the preferences of economic agents on growth. The model economy is populated with identical citizens and identical politicians. Preferences and technology are first described. The game is then studied and conditions for growth are laid out in the section's main theorem.

3.1.1 The Model

The model economy is populated with identical citizens and an infinite number of identical politicians who belong to a set J. A representative citizen is assumed to make decisions on behalf of all other citizens. Each period, the representative citizen is endowed with 1/2 unit of a non-storable consumption good that can be readily consumed and 1/2 unit of a non-storable investment good that cannot be consumed before being processed. There exists two technologies for processing the investment good: it can be either transformed on a one-to-one basis into a non-storable consumption good that only politicians can consume, or it can be used to scale tomorrow's technological frontier forward for all economic agents. Politicians do not receive endowments. Only a politician in power possesses the technology for processing the investment good in either way. If no politician is hired to process the investment good, then the period 1/2 unit of endowment of investment good will be lost and the technology will stagnate.

3.1.2 Preferences

All politicians have common discount factor δ and all citizens have common discount factor β. S_t denotes the date t technological frontier. C_t^{HH} and C_t^P denote period t total consumptions for the representative citizen and an arbitrary politician. Later on, c_t^{HH} and c_t^P will be used to denote specific consumptions of the consumption good. For a politician in power, total consumption is the sum of consumptions of the investment good and the consumption good. Preferences are given by $\sum_{t=0}^{\infty} \beta^t \lambda^{S_t} U(C_t^{HH})$ for the representative citizen and $\sum_{t=0}^{\infty} \delta^t \lambda^{S_t} V(C_t^P)$ for a politician.

The following assumptions are made about preferences[1] :

A.1 U and V are continuous, strictly increasing, strictly concave and differentiable functions defined on $\mathbb{R}$

A.2 $\lambda > 1$

A.3 $\beta, \delta \in [0, \frac{1}{\lambda})$

A.3 $U(0) = V(0) = 0$ and $\lim_{c \to +\infty} V(c) = +\infty$.

3.1.3 Technology

θ_t is a variable which takes on value 1 when either the representative citizen chooses not to hire any politician at date t or the politician in power chooses to process the investment good into consumption for himself, and which takes on value 0 when the politician in power chooses to process the investment good for growth. Given S_0 given, the law of motion of the technological frontier is given by:

$$S_{t+1} = S_t + 1 - \theta_t.$$

Under functional forms $U(c) = \frac{c^{1-\epsilon_1}}{1-\epsilon_1}$ and $V(c) = \frac{c^{1-\epsilon_2}}{1-\epsilon_2}$, it is straightforward that $U(\hat{\lambda}^{S_t} c) = (\hat{\lambda}^{1-\epsilon_1})^{S_t} U(c)$ and $V(\hat{\lambda}^{S_t} c) = (\hat{\lambda}^{1-\epsilon_2})^{S_t} V(c)$. It follows that with constant relative risk aversion (CRRA) functional forms for U and V, given constant streams c^{HH} and c^P, the model period utility functions are $\lambda^{S_t} U(c^{HH}) \equiv U((\lambda^{\frac{1}{1-\epsilon_1}})^{S_t} c^{HH})$ and $\lambda^{S_t} V(c^P) \equiv V((\lambda^{\frac{1}{1-\epsilon_2}})^{S_t} c^P)$. That is, in a stationary model economy with constant streams c^{HH} and c^P and with CRRA functional forms, $(\lambda^{\frac{1}{1-\epsilon_1}})^{1-\theta_t}$ and $(\lambda^{\frac{1}{1-\epsilon_2}})^{1-\theta_t}$ may be interpreted as the gross growth rates of consumption between dates t and $t+1$ for the citizens and the politicians respectively. If U and V are not CRRA utility functions, then for constant streams c^{HH} and c^P, $\lambda^{S_t} U(c^{HH})$ and $\lambda^{S_t} U(c^{HH})$ should be interpreted to reflect a taste for quality as in the quality-ladder model of Grossman and Helpman (1991). In this latter case, between any dates t and $t+1$, quality jumps by factor $\lambda^{1-\theta_t}$ which defines the gross growth rate of the economy.

[1] Assumption **A.3** is not crucial for future results. It is made for convenience only. All the results of this paper continue to hold when U and V are restricted to be defined on the set of positive real numbers only.

3.1.4 Timing of the Stage Game

The economy starts date 0 with politician $\iota_0 \in J$ in power and with technological frontier S_0. Then, after receiving endowments of the consumption and investment goods, the representative citizen decides which share of the consumption good to give to politician ι_0. Specifically, the representative citizen chooses an allocation $\{c_0^{HH}, c_0^P\}$ of the endowment of the consumption good s.t: $c_0^{HH} + c_0^P = 1/2$. Thereafter, politician ι_0 moves and chooses the value of θ_0, by deciding whether to consume the investment good or to process it for economic growth. Consumption occurs afterward, the representative citizen consuming c_0^{HH} and arbitrary politician $\iota \in J$ consuming $c_0^P + \frac{\theta_0}{2}$ if in power and 0 if not in power. That is, total consumptions at date 0 are $C_0^{HH} = c_0^{HH}$ for the representative citizen and

$$C_0^P = \begin{cases} c_0^P + \frac{\theta_0}{2} & \text{for a politician in power} \\ 0 & \text{for a politician not in power.} \end{cases}$$

The representative citizen then moves once again, this time choosing whether or not to replace politician ι_0. Specifically, the representative citizen chooses ι_1 in $J \cup \{\varnothing\}$ with ι_1 potentially equal to ι_0 or $\varnothing$. $\iota_1 = \varnothing$ implies that there will be no politician in power at date 1. In this case, θ_1 is necessarily equal to 1. The economy evolves in a similar way in period 1 and in all future periods. Any date t starts with states $\iota_t \in J \cup \{\varnothing\}$ and S_t that denote the politician in power and the technological frontier respectively.

3.1.5 Relevant Histories, Strategies and Equilibrium Concept

Let $h_{c,1}^t$, h_P^t, $h_{c,2}^t$, denote the date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. Histories evolve as follows: $h_{c,1}^0 = \{\iota_0, S_0\}$, $h_P^0 = \{h_{c,1}^0, c_0^{HH}, c_0^P\}$, $\quad h_{c,2}^0 = \{h_P^0, \theta_0, S_1\}$, $\quad h_{c,1}^1 = \{h_{c,2}^0, \iota_1\} \cdots$

Let $H_{c,1}^t|h^t$, $H_P^t|h^t$, $H_{c,2}^t|h^t$, denote the sets of all possible histories that the representative citizen in the first stage, the politician in power in the second stage and the representative citizen in the third stage may reach starting from some history h^t with states $\{S_t, \iota_t\}$. Also, define:

$$\Lambda \equiv \{(c_t^H, c_t^P) \in [0, 1/2]^2 \;\; s.t. \;\; c_t^H + c_t^P = 1\}. \tag{3.1}$$

Then, in the continuation game that follows h^t, players'strategies for the representative citizen in the first stage, the politician in power in the second stage and the representative citizen in the third stage are the following measurable functions:

$$\sigma_{c,1}|h^t : H^t_{c,1}|h^t \mapsto \Lambda \tag{3.2a}$$

$$\sigma_P|h^t : H^t_P|h^t \mapsto \{0;1\} \tag{3.2b}$$

$$\sigma_{c,2}|h^t : H^t_{c,2}|h^t \mapsto J \cup \{\varnothing\} \tag{3.2c}$$

Let $\Sigma_{c,1}|h^t$, $\Sigma_{c,2}|h^t$, $\Sigma_P|h^t$ denote the sets of all continuation strategies after history h^t for the respective players. Given history h^t with states $\{S_t, \iota_t\}$, continuation strategies $\sigma_{c,1}|h^t$, $\sigma_{c,2}|h^t$ and $\sigma_P|h^t$ induce a sequences of allocations in the natural way. Let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ be a continuation strategy profile that follows history h^t with states $\{S_t, \iota_t\}$ and that induces the sequence of allocations $\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau+1}\}^{\infty}_{\tau=0}$. Resulting payoffs from $\sigma|h^t$ to the citizens and to politician ι_t are respectively given by:

$$\Phi_c(\sigma|h^t, S_t, \iota_t) = \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \tag{3.3a}$$

$$\Phi_{\iota_t}(\sigma|h^t_P, S_t, \iota_t) = \sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c^P_{t+\tau} + \frac{\theta_{t+\tau}}{2}] * \mathbf{1}_{\{\iota_{t+\tau}=\iota_t\}}. \tag{3.3b}$$

Definition 1 *A Strategy profile $\sigma|\{S_0, \iota_0\}$ is a subgame perfect equilibrium (SPE) of the game if $\forall h^t$ with states $\{S_t, \iota_t\}$, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma, \sigma_P|h^t, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_{c,1}|h^t \tag{3.4a}$$

$$\Phi_{\iota_t}(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_P|h^t \tag{3.4b}$$

$$\Phi_c(\sigma|h^t) \geq \Phi_{c,2}(\sigma_{c,1}|h^t, \sigma_P|h^t, \gamma), \forall \gamma \in \Sigma_{c,2}|h^t. \tag{3.4c}$$

Definition 2 *A sequence of allocations $\{c^{HH}_\tau, c^P_\tau, \theta_\tau, \iota_\tau\}^{\infty}_{\tau=0}$ is feasible if it satisfies: $c^{HH}_\tau + c^P_\tau = 1/2,\ \ \theta_\tau \in \{0;1\}$ and $\iota_\tau \in J \cup \{\varnothing\}, \forall \tau \geq 0$.*

Lemma 3 *A feasible sequence of allocations* $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$ *is an SPE sequence iff*

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0 \tag{3.5a}$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] * 1_{\iota_{t+\tau}=\iota_t} \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0, \tag{3.5b}$$

S_0 given and $S_{t+1} = S_t + 1 - \theta_t$.

Proof. ($\Rightarrow$) Suppose $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$ is the sequence of allocations induced by some arbitrary strategy profile σ. The goal is to show that conditions 3.5(a) and 3.5(b) above hold. Let $\gamma_{c,1}|h^t$ be the strategy of the representative citizen that calls for choosing $\hat{c}_{t+\tau}^{HH} = \frac{1}{2}\ \forall\ \tau \geq 0$. Then, $\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t) \geq \lambda^{S_t}U(\frac{1}{2}) + \frac{\beta}{1-\beta}\lambda^{S_t}U(\frac{1}{2})$, where the first inequality comes from the fact that $\sigma|h^t$ is a continuation strategy and the last inequality comes from the definition of the deviation strategy. This shows that condition 3.5(a) holds. Now, define $\gamma_P|h^t$ as the strategy of a politician in power at date t that calls for choosing $\hat{\theta}_t = 1$ today and $\hat{\theta}_{t+\tau} = 1,\ \forall\ \tau > 0$, conditional on being in power at date $t+\tau$. By a similar argument to the one above, $\Phi_P(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma_P|h^t, \sigma_{c,2}|h^t) \geq \lambda^{S_t}V[(c_t^{HH} + \frac{1}{2})]$, so that condition 3.5(b) holds as well.

($\Leftarrow$) Now, suppose 3.5(a) and 3.5(b) hold for given sequence $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$. It needs to be shown that there exists some SPE strategy profile that induces the sequence of allocations $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$. For this purpose, let's define a trigger strategy profile σ' with the following properties. If all players have so far followed script $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$, then σ' calls for continuing to follow the script. If any player has ever deviated in the past, then the representative citizen and all politicians are called to move to the worst outcome of the game forever: for all future periods, the representative citizen never hires any politician in power and consumes the total endowment of the consumption good; any politician ever hired always chooses to consume the investment good and let the technology stagnate. Clearly, σ' induces $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^\infty$ on path. It is therefore left to show that σ' is an SPE strategy profile.

Let h^t denote a date t history with states S_t and ι_t, for some $t \geq 0$. Note that by 3.5(a), $\Phi_c(\sigma'|h^t) \equiv \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2)$. But $\frac{\lambda^{S_t}}{1-\beta} U(1/2)$ corresponds to the payoff to the first stage representative citizen when he/she chooses his/her best deviation ($c_t^{HH} = \frac{1}{2}$) at date t while all other players continue to follow the trigger strategy profile $\sigma^{'}$. Hence, in the first stage of the game, the representative citizen never finds it profitable to deviate unilaterally from $\sigma^{'}$. Similarly, by 3.5(b), $\Phi_P(\sigma^{'}|h^t)$ $= \sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] * 1_{\iota_{t+\tau}=\iota_t} \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}]$, where the last term represents the payoff to the politician in power when he/she chooses his/her best deviation at date t ($\theta_t = 1$) while the representative citizen continues to follow the trigger strategy profile $\sigma^{'}$. It follows that the politician in power never finds it profitable to deviate unilaterally from $\sigma^{'}$.

To end the proof, it is left to check that the representative citizen does not want to unilaterally deviate from $\sigma^{'}|h^t$ in the third stage of the period game. The payoff to the representative citizen in the third stage of the game if he/she chooses politician $\hat{\iota}_t$ in power instead of politician ι_t as indicated by script $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is $\lambda^{S_t} U(c_t^{HH}) + \frac{\beta}{1-\beta}\lambda^{S_t} U(\frac{1}{2})$. But, by 3.5(a), $\Phi_c(\sigma^{'}|h^t) = \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2) \geq \lambda^{S_t} U(c_t^{HH}) + \frac{\beta}{1-\beta}\lambda^{S_t} U(\frac{1}{2})$. This implies that the third stage-representative citizen does not want to unilaterally deviate from $\sigma^{'}|h^t$ at date t, concluding the proof that $\sigma^{'}|h^t$ is an SPE strategy profile. This ends the proof of lemma 3.

■

Lemma 4 *If an SPE sequence involves a replacement of the initial politician, then there exists another SPE sequence with no replacement that yields the same payoffs to citizens.*

Proof.

Let $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ be an SPE sequence that calls for one or several replacements of politicians in power. Now, consider the new sequence of allocations $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_0\}_{\tau=0}^{\infty}$ which differs from the initial sequence only in that it maintains initial politician ι_0 in power forever. Clearly, at any date, this new sequence yields the same payoff to the representative citizen as the initial sequence. It is therefore left to show that the new sequence is also an SPE sequence. Clearly, because $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is an SPE sequence, the previous lemma implies that $\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0.$

The previous lemma also implies that $\sum_{\tau=0}^{\infty}\delta^{\tau}\lambda^{S_{t+\tau}}V[c^{P}_{t+\tau}+\frac{\theta_{t+\tau}}{2}]*1_{\iota_{t+\tau}=\iota_t}\geq\lambda^{S_t}V[c^{P}_{t}+\frac{1}{2}],\forall t\geq 0$, which in turn implies $\sum_{\tau=0}^{\infty}\delta^{\tau}\lambda^{S_{t+\tau}}V[c^{P}_{t+\tau}+\frac{\theta_{t+\tau}}{2}]\geq\lambda^{S_t}V[c^{P}_{t}+\frac{1}{2}],\forall t\geq 0$. It therefore follows from lemma 4 that the new sequence of allocations is also an SPE sequence. This ends the proof of the lemma. ■

In what follows, best SPE sequences are referred to as SPE sequences that among all SPE sequences , yield the highest payoff to citizens. Lemma 4 implies that to characterize payoffs induced by such sequences, it is legitimate to restrict attention to SPE sequences that involve no replacement of the initial politician. Therefore, by lemma 3, the best SPE problem of this economy may be written as:

$$\max_{\{\{c^{HH}_{\tau},c^{P}_{\tau}\}\in\Lambda,\theta_{\tau}\in\{0;1\}\}_{\tau=0}^{\infty}}\sum_{\tau=0}^{\infty}\beta^{\tau}\lambda^{S_{\tau}}U[c^{HH}_{\tau}]$$

s.t

$$\sum_{\tau=0}^{\infty}\beta^{\tau}\lambda^{S_{t+\tau}}U[c^{HH}_{t+\tau}]\geq\frac{\lambda^{S_t}}{1-\beta}U(1/2),\forall t\geq 0 \tag{3.6a}$$

$$\sum_{\tau=0}^{\infty}\delta^{\tau}\lambda^{S_{t+\tau}}V[c^{P}_{t+\tau}+\frac{\theta_{t+\tau}}{2}]\geq\lambda^{S_t}V[c^{P}_{t}+\frac{1}{2}],\forall t\geq 0 \tag{3.6b}$$

$$S_0 \quad \text{given} \quad , S_{t+1}=S_t+1-\theta_t. \tag{3.6c}$$

Lemma 5 *There exists a best SPE sequence that is stationary:*

$$\{c^{HH}_{\tau},c^{P}_{\tau},\theta_{\tau}\}=\{c^{*HH}_{\tau},c^{*P}_{\tau},\theta^{*}_{\tau}\},\forall\tau\geq 0$$

.

Proof.

The proof immediately follows from observing that the best SPE problem has a stationary structure in the sense that in any solution, citizens receive the same discounted sum of future utilities, in all periods. Details are given in the appendix . ■

Define $\phi^* = \max_{c_P\in\mathbb{R}\ ,\theta\in\{0;1\}} \dfrac{U(\frac{1}{2}-c_P)}{1-\beta\lambda^{1-\theta}}$ s.t $\dfrac{V[c_P+\frac{\theta}{2}]}{1-\delta\lambda^{1-\theta}} \geq V[c_P+\dfrac{1}{2}]$ and let argmax ϕ denote $\{c^*_{HH}, c^*_P, \theta^*\}$ such that $\phi^* = \frac{U(\frac{1}{2}-c^*_P)}{1-\beta\lambda^{1-\theta^*}}$ and $c^*_{HH} = \frac{1}{2} - c^*_P$. Then, the stationary best SPE sequence $\{c^*_{HH}, c^*_P, \theta^*\}$ is characterized by:

$$\{c^*_{HH}, c^*_P, \theta^*\} = \begin{cases} \text{argmax } \phi & \text{if } \Phi^* \geq \frac{U(1/2)}{1-\beta} \\ \{1/2, 0, 1\} & \text{if } \Phi^* < \frac{U(1/2)}{1-\beta}. \end{cases}$$

Theorem 6 *There exists* $\underline{\delta} < \frac{1}{\lambda}$ *such that for* $\delta \leq \underline{\delta}$*, the economy does not grow. For* $\delta > \underline{\delta}$*, there exists* $\beta^*(\delta)$ *such that the economy grows if and only if* $\beta > \beta^*(\delta)$*. Moreover,* $\beta^*(\delta)$ *is a decreasing function of* δ*.*

Proof.

Define function h by

$$h(c_P) = \frac{V(c_P)}{V(c_P+\frac{1}{2})}. \tag{3.7}$$

Since V is continuous, so is h. In fact, h is strictly increasing and therefore invertible as a continuous bijection. To see why, observe that the derivative of h is given by:

$$h^{'}(c_P) = \frac{V^{'}(c_P)V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P)}{V(c_P+\frac{1}{2})^2}.$$

But by strict concavity of V, $V^{'}(c_P+\frac{1}{2}) \leq V^{'}(c_P)$. Therefore, $V^{'}(c_P)V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) \geq V^{'}(c_P+\frac{1}{2})V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) > 0$ follows from the fact that V is strictly increasing. Hence, h is strictly increasing and invertible as a continuous bijection of $[0, \frac{1}{2}]$ onto $[0, \frac{V(\frac{1}{2})}{V(1)}]$. Now, Define $\underline{\delta}$ by $h^{-1}(1-\underline{\delta}\lambda) = \frac{1}{2}$ or $\frac{V(\frac{1}{2})}{V(1)} = 1-\underline{\delta}\lambda$. Clearly, $1-\underline{\delta}\lambda \in [0, \frac{1}{2}]$ implies $\underline{\delta} < \frac{1}{\lambda}$.

<u>Case 1</u>: $\delta \in [0, \underline{\delta}]$

$h(\frac{1}{2}) = \frac{V(\frac{1}{2})}{V(1)} = 1-\underline{\delta}\lambda$ implies that for all $\delta \in [0, \underline{\delta}]$ a politician in power will always choose $\theta = 1$, even when he is paid the total available resource $c_P = \frac{1}{2}$ in all periods. Therefore, when $\delta \in [0, \underline{\delta}]$, politicians are extremely impatient and would not allow for technological growth, no matter how large a fraction of total resources they receive. In this case, it is optimal for citizens to set $c_P = 0$ in all periods. Hence, when $\delta \in [0, \underline{\delta}]$, $c_P^* = 0$ and $\theta^* = 1$ in all periods: the economy never grows.

Case 2: $\delta \in (\underline{\delta}, 1/\lambda)$

$\delta \in (\underline{\delta}, 1/\lambda)$ and $h(0) = 0 \Rightarrow h^{-1}(1 - \delta\lambda) \in (0, 1/2)$.

Now define function g by $g(\beta) = \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [0, 1/\lambda)$. g is strictly decreasing with values in $(0, 1]$ for $\beta \in [0, 1/\lambda)$. Because g is also continuous, it is invertible. Now, let's define $\beta^*(\delta) = g^{-1}(\frac{U[\frac{1}{2} - h^{-1}(1-\delta\lambda)]}{U(1/2)})$. Clearly, $\beta^*(\delta) \in (g^{-1}(1), g^{-1}(0)) = (0, \frac{1}{\lambda})$. Note that $U[\frac{1}{2} - h^{-1}(1-\delta\lambda)] \geq \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [\beta^*(\delta), 1/\lambda)$, while $U[\frac{1}{2} - h^{-1}(1-\delta\lambda)] < \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [0, \beta^*(\delta))$. This implies, $\frac{U[\frac{1}{2} - h^{-1}(1-\delta\lambda)]}{1-\beta\lambda} > \frac{U[1/2]}{1-\beta}$ $\forall\, \beta \in (\beta^*(\delta), 1/\lambda)$ and $\frac{U[\frac{1}{2} - h^{-1}(1-\delta\lambda)]}{1-\beta\lambda} \leq \frac{U[1/2]}{1-\beta}$ $\forall\, \beta \in [0, \beta^*(\delta)]$. Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in (\beta^*(\delta), 1/\lambda)$, then $c_P^* = h^{-1}(1 - \delta\lambda) \in (0, 1/2)$ and $\theta^* = 0$ (the economy always grows).

If instead $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta)]$, then $\frac{U[\frac{1}{2} - h^{-1}(1-\delta\lambda)]}{1-\beta} \leq \frac{U[1/2]}{1-\beta\lambda}$: the representative citizen fails to achieve his/her outside option value $\frac{U[1/2]}{1-\beta}$ by giving the politician in power just enough to satisfy the politician's sustainability constraint. Now, recall that $h^{-1}(1 - \delta\lambda) \in (0, 1/2)$ is the unique stationary payment to the politician which makes the sustainability constraint hold with equality. It follows that in this case, for any scheme with a stationary and sustainable payment to the politician, the lifetime utility of the representative citizen will always be strictly less than $\frac{U[1/2]}{1-\beta}$. Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta)]$, then $c_P^* = 0$ and $\theta^* = 1$ (the economy never grows).

■

Corollary 7 *Suppose politicians and citizens have common discount factor ρ. Then, there exists $\tilde{\rho}^{\xi=0}$ such that the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$.*

Proof.

For an economy with common discount factor ρ for citizens and politicians, the theorem states that there exists $\underline{\rho} < \frac{1}{\lambda}$ such that if $\rho \leq \underline{\rho}$, then the economy stagnates. Now, suppose $\rho > \underline{\rho}$. Recall that $\frac{U[\frac{1}{2} - h^{-1}(1-\rho\lambda)]}{U[1/2]}$ is continuous and strictly decreasing in ρ while $\frac{1-\rho\lambda}{1-\rho}$ is continuous and strictly decreasing in ρ. Moreover, $\frac{U[\frac{1}{2} - h^{-1}(1-\underline{\rho}\lambda)]}{U[1/2]} = 0 <$

$\frac{1-\underline{\rho}\lambda}{1-\underline{\rho}}$, and $\frac{U[\frac{1}{2}-h^{-1}(1-\frac{1}{\lambda}\lambda)]}{U[1/2]} = 1 > \frac{1-\frac{1}{\lambda}\lambda}{1-\frac{1}{\lambda}}$. Therefore, there exists a unique $\tilde{\rho}^{\xi=0} \in (\underline{\rho}, \frac{1}{\lambda})$ such that $\frac{U[\frac{1}{2}-h^{-1}(1-\tilde{\rho}^{\xi=0}\lambda)]}{1-\tilde{\rho}^{\xi=0}\lambda} = \frac{U[1/2]}{1-\tilde{\rho}^{\xi=0}}$. Given $\rho > \underline{\rho}$, the theorem implies that the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$. Therefore, for any $\rho \in [0, \frac{1}{\lambda})$, the economy grows iff $\rho > \max\{\underline{\rho}, \tilde{\rho}^{\xi=0}\} = \tilde{\rho}^{\xi=0}$. That is , for any $\rho \in [0, \frac{1}{\lambda})$, the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$.
■

3.2 Coups d'Etat and Effective Discounting

This section analyzes the impact of political instability on economic growth. Assumptions made on preferences and technology in the previous section are maintained . However, the economy considered in this section differs from the previous section's economy in two respects. First, citizens and politicians now have a common discount factor ρ. Second, citizens do no longer have full control over the choice of the future period politician in power. Specifically, it is assumed that at the end of every period, before the representative citizen chooses the future period politician, a coup d'etat might occur with probability c. When a coup d'etat occurs, the representative citizen loses his voting right for the period and the politician in power is replaced with a different politician randomly chosen by nature. It is also assumed that both the representative citizen and nature may not choose a politician previously overthrown by a coup d'etat as the future period politician.

The timing is as follows. The economy starts date 0 with technological frontier S_0 and politician ι_0 in power. After receiving endowments of the consumption and investment goods, the representative citizen gives politician ι_0 a fraction of the endowment of the consumption good. Politician ι_0 then chooses whether to consume the investment good or to process it for growth. Thereafter, the representative citizen and the politician in power consume their respective allocations. Nature then draws ω in $\{0, 1\}$, where $\omega = 1$ is the event of a coup d'etat which has probability c and $\omega = 0$ is the complementary event. If $\omega = 0$, then the representative citizen chooses the next period politician in $J \cup \{\varnothing\}$ and a new period starts. If $\omega = 1$, then nature draws a new politician in $J - \{\iota_0\}$ and a new period starts with the politician drawn by nature as the politician in power. At the end of any future period t during which a coup d'etat has not occurred, the representative citizen chooses politician ι_{t+1} in $(J - \Pi_0^t) \cup \{\varnothing\}$, where Π_0^t is the collection of politicians overthrown by a coup d'etat from date 0 to date t. At the end of any period t during which a coup d'etat has occurred, nature draws politician ι_{t+1} from $J - \Pi_0^t$. The timing for all future periods is similar to that of period 0.

3.2.1 Relevant Histories, Strategies and Equilibrium Concept

As before, let $h_{c,1}^t$, $h_{c,2}^t$, h_P^t denote the date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. Histories evolve as follows: $h_{c,1}^0 = \{\iota_0, S_0\}$, $h_P^0 = \{h_{c,1}^0, c_0^{HH}, c_0^P\}$, $h_{c,2}^0 = \{h_P^0, \theta_0, S_1\}$, $h_{c,1}^1 = \{h_{c,2}^0, \iota_1\}$ $h_{c,2}^1 = \{h_P^1, \theta_1, S_2, \omega_1\}$ ...

The difference between these histories and those in section 3.1 is that every period, the information set of the stage 3 representative citizen is now augmented with ω. As in section 3.1, let $H_{c,1}^t|h^t$, $H_{c,2}^t|h^t$ and $H_P^t|h^t$ denote the set of histories that follow some history h^t. Also let $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$ define strategies for the representative citizen in stage 1, the politician in power in stage 2 and the representative citizen in stage 3 of the period game. Let $\Sigma_{c,1}|h^t, \Sigma_P|h^t$ and $\Sigma_{c,2}|h^t$ denote the sets of all such strategies.

Define $\omega_{-1} = 0$ and $\omega_s^t = \{\omega_s, ...\omega_t\}$. Let h^t denote some history ending with states ι_t, S_t and ω_{-1}^{t-1}. Let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ be a strategy profile which induces the sequence of allocations $\mathcal{A}_t \equiv \{c_{t+\tau}^{HH}(.), c_{t+\tau}^P(.), \theta_{t+\tau}(.), \iota_{t+1+\tau}(.)\}_{\tau=0}^{\infty}$. As before, for any history $\omega_{-1}^{t+\tau-1}$, the law of motion of the technological frontier is: $S_t(\omega_{-1}^{t-2}) \equiv S_t$ and $S_{t+\tau+1}(\omega_{-1}^{t+\tau-1}) = S_{t+\tau}(\omega_{-1}^{t+\tau-2}) + 1 - \theta_{t+\tau}(\omega_{-1}^{t+\tau-1})$.

Define

$$\kappa(\omega_{t-1}^{t+\tau-1}) \equiv \inf\{t \leq s \leq t+\tau-1 : \omega_s = 1\}.$$

Payoffs from $\sigma|h^t$ to the representative citizen and to politician in power ι_t are respectively given by $\Phi_c(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \equiv \Gamma_c(\mathcal{A}_t)$ and $\Phi_{\iota_t}(\sigma|(h_P^t, S_t, \iota_t, \omega_{-1}^{t-1})) \equiv \tilde{\Gamma}_P(\mathcal{A}_t)$, where

$$\Gamma_c(\mathcal{A}_t) \equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1}} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} U[c_{t+\tau}^{HH}(\omega_{-1}^{t+\tau-1})] * \text{ prob } (\omega_{t-1}^{t+\tau-1})$$

$$\tilde{\Gamma}_P(\mathcal{A}_t)$$
$$\equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1} | \kappa(\omega_{t-1}^{t+\tau-1}) > t+\tau-1} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} V[c_{t+\tau}^{P}(\omega_{-1}^{t+\tau-1}) + \frac{\theta_{t+\tau}(\omega_{-1}^{t+\tau-1})}{2}]$$
$$* \text{ prob } (\omega_{t-1}^{t+\tau-1}) * \mathbf{1}_{\{\iota_{t+\tau(\omega_{t-1}^{t+\tau-1})} = \iota_t\}}.$$

In the above formula, payoff-relevant histories for a politician in power are histories during which he is not overthrown by a coup d'etat. When a politician is not overthrown by a coup d'etat, he gets to consume only if the strategy profile calls for the representative citizen to keep him in power. This is reflected in $\mathbf{1}_{\{\iota_{t+\tau+1(\omega_{t-1}^{t+\tau-1})} = \iota_t\}}$ which is an indicator function that takes on value 1 if the strategy profile calls for the representative citizen to choose politician ι_t as their period $t+\tau$ politician in power given history $\omega_{t-1}^{t+\tau-1}$. Clearly, if $\omega_{t+\tau-1} = 1$ for some $\tau \geq 1$, then $\mathbf{1}_{\{\iota_{t+\tau(\omega_{t-1}^{t+\tau-1})} = \iota_t\}} = 0$ since the representative citizen does not get to choose the next period politician at the end of a period during which a coup d'etat has occurred.

Definition 8 *A Strategy profile $\sigma|\{S_0, \iota_0\}$ is a subgame perfect equilibrium (SPE) of the game if $\forall h^t$ with states $\{S_t, \iota_t, \omega_0^{t-1}\}$, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_c(\gamma, \sigma_P|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \sigma_{c,2}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})), \;\; \forall \gamma \in \Sigma_{c,1}|h^t \tag{3.8a}$$

$$\Phi_{\iota_t}(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_P(\sigma_{c,1}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \gamma, \sigma_{c,2}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})), \;\; \forall \gamma \in \Sigma_P|h^t \tag{3.8b}$$

$$\Phi_c(\sigma|h^t, (S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_{c,2}(\sigma_{c,1}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \sigma_P|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \gamma), \;\; \forall \gamma \in \Sigma_{c,2}|h^t. \tag{3.8c}$$

Definition 9 *Given some history ω_{-1}^{t-2}, a sequence of allocations*

$$\{c_{t+\tau}^{HH}(.), c_{t+\tau}^{P}(.), \theta_{t+\tau}(.), \iota_{t+1+\tau}(.)\}_{\tau=0}^{\infty}$$

is feasible if $\forall\tau \geq 0$, $\forall\omega_{t-1}^{t+\tau-1}$, it satisfies:

$$\{c_{t+\tau}^{HH}(\omega_{-1}^{t+\tau-1}), c_{t+\tau}^{P}(\omega_{-1}^{t+\tau-1})\} \in \Lambda, \quad \textit{where } \Lambda \textit{ defined by equation (3.1)}$$

$$\theta_{t+\tau}(\omega_{-1}^{t+\tau-1}) \in \{0; 1\}$$

$$\iota_{t+1+\tau}(\omega_{-1}^{t+\tau}) \in \begin{cases} (J - \Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau})) \cup \{\varnothing\} & \textit{if } \omega_{t+\tau} = 0 \\ J - \Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau}) & \textit{if } \omega_{t+\tau} = 1, \end{cases}$$

where $\Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau}) = \{\iota_{t+s+1}(\omega_{t-1}^{t+s})$ if $\omega_{t+s} = 1$ for $s \in \{0, \ldots, \tau\}\}$. That is, if history $\omega_{t-1}^{t+\tau}$ occurs, then, according to the sequence of allocations, $\Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau})$ will be the collection of politicians overthrown by a coup d'etat between between dates t and $t + \tau$.

An argument analogue to the one used to prove lemma 3 implies that a feasible sequence of allocations $\mathcal{A}_0 \equiv \{c_\tau^{HH}(.), c_\tau^{P}(.), \theta_\tau(.), \iota_\tau(.)\}_{\tau=0}^{\infty}$ which induces continuation sequences $\{\mathcal{A}_t, t \geq 0\}$ is an SPE sequence iff $\forall t \geq 0, \forall\omega_{t-1}^{t-1}$,

$$\Gamma_c(\mathcal{A}_t) \geq \frac{\lambda^{S_t(\omega_{-1}^{t-2})}}{1-\rho} U(1/2), \tag{3.10}$$

and

$$\Gamma_P(\mathcal{A}_t) \geq \lambda^{S_t(\omega_{-1}^{t-2})} V[c_t^P(\omega_{-1}^{t-1}) + \frac{1}{2}]. \tag{3.11}$$

Also, as in lemma 4, it is easy to check that if an SPE sequence involves a replacement of the initial politician in this economy, then there is another SPE sequence with no replacement that achieves the same payoff to the representative citizen.

Now, define

$$\Gamma_P(\mathcal{A}_t) \equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1} | \kappa(\omega_{t-1}^{t+\tau-1}) > t+\tau-1} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} V[c_{t+\tau}^P(\omega_{-1}^{t+\tau-1}) + \frac{\theta_{t+\tau}(\omega_{-1}^{t+\tau-1})}{2}]$$

$$* \text{ prob } (\omega_{t-1}^{t+\tau-1}).$$

For a sequence of allocations $\mathcal{A}_0 \equiv \{c_\tau^{HH}(.), c_\tau^P(.), \theta_\tau(.), \iota_{1+\tau}(.)\}_{\tau=0}^{\infty}$, let $\mathcal{A}_t$ denote the natural continuation of $\mathcal{A}_0$ for any $t \geq 0$. Then, for the class of economies of interest, the best SPE problem is characterized as:

$$\max_{\mathcal{A}_0 \text{ feasible}} \Gamma_c(\mathcal{A}_0)$$

s.t

$$\Gamma_c(\mathcal{A}_t) \geq \frac{\lambda^{S_t(\omega_{-1}^{t-2})}}{1-\rho} U(1/2) \qquad , \forall t \geq 0,\ \forall \omega_{t-1}^{t-1} \tag{3.12a}$$

$$\Gamma_P(\mathcal{A}_t) \geq \lambda^{S_t(\omega_{-1}^{t-2})} V[c_t^P(\omega_{-1}^{t-1}) + \frac{1}{2}],\ \forall t \geq 0,\ \forall \omega_{t-1}^{t-1}. \tag{3.12b}$$

The objective function of the above problem is homogenous of degree 0 in λ^{S_0}. Moreover, for any period t, λ^{S_t} may be simplified from both sides of the constraint inequalities. It is also easy to check that at any date and given any history, the best SPE problem has the following recursive formulation:

$$\lambda^S \Psi_{HH} = \max U(c^{HH}) + \rho\lambda^{1-\theta} * \lambda^S \Psi_{HH}$$

s.t

$$U(c^{HH}) + \rho\lambda^{1-\theta} * \lambda^S \Psi_{HH} \geq \lambda^S \frac{U(\frac{1}{2})}{1-\rho} \tag{3.13a}$$

$$\lambda^S \Psi_P \equiv V[c^P + \frac{\theta}{2}] + \rho\lambda^{1-\theta}\lambda^S \Psi_P * \ (1-c) \geq \lambda^S V[c^P + \frac{1}{2}]. \tag{3.13b}$$

Now, define

$$\phi^{**} = \max_{c_P \in \mathbb{R}\ \theta \in \{0;1\}} \frac{U(\frac{1}{2} - c_P)}{1-\rho\lambda^{1-\theta}} \text{ s.t } \frac{V[c_P + \frac{\theta}{2}]}{1-\rho(1-c)\lambda^{1-\theta}} \geq V[c_P + \frac{1}{2}] \ ,$$

and let argmax ϕ' denote $\{c_{HH}^*, c_P^*, \theta^*\}$ such that $\phi^{**} = \frac{U(\frac{1}{2}-c_P^*)}{1-\rho\lambda^{1-\theta^*}}$ and $c_{HH}^* = \frac{1}{2} - c_P^*$

.

Then, the best SPE problem above has a stationary solution $\{c_{HH}^*, c_P^*, \theta^*\}$ defined by:

$$\{c_{HH}^*, c_P^*, \theta^*\} = \begin{cases} \text{argmax } \phi' & \text{if } \Phi^{**} \geq \frac{U(1/2)}{1-\rho} \\ \{1/2, 0, 1\} & \text{if } \Phi^{**} < \frac{U(1/2)}{1-\rho}. \end{cases}$$

The best SPE problem of this economy is therefore identical to that of the economy of the previous section for $\delta = \rho(1-c)$ and $\beta = \rho$. That is, introducing the possibility of a coup d'etat scales the effective discount factor of politicians down by the probability of the event that a coup d'etat does not occur. It follows that if citizens and politicians are sufficiently patient to allow for growth in the basic setup with no political instability, then growth will occurs in this setup iff the probability of a coup d'etat is not too large. This is stated in the next theorem.

Theorem 10 *Suppose $\lambda > 1$. Let $\tilde{\rho}^{\xi=0}$ be defined as in corollary 7. If $\rho \in (\tilde{\rho}^{\xi=0}, \frac{1}{\lambda})$, then, there exists $c^* \in [0,1]$ such that the economy grows iff the probability of coup d'etat c is such that $c < c^*$. If $\rho \in [0, \tilde{\rho}^{\xi=0}]$, then the economy does not grow for any value of c in $[0,1]$.*

Proof. First, suppose $\rho \in (\tilde{\rho}^{\xi=0}, \frac{1}{\lambda})$. Define $\overline{c}$ by $\frac{V(\frac{1}{2})}{V(1)} = 1 - \rho(1-\overline{c})\lambda$. Then, $\frac{V(\frac{1}{2})}{V(1)} < 1 \Rightarrow \rho(1-\overline{c})\lambda > 0 \Rightarrow \overline{c} < 1$.

First suppose $c > \overline{c}$. Then, $\frac{V(\frac{1}{2})}{V(1)} < 1 - \rho(1-c)\lambda$ and it follows that in this case, the politician would not let growth occur even if he was offered the maximal resources available. It is therefore optimal for citizens to set $c_P^* = 0$ and the economy necessarily stagnates in this case.

Now, suppose $c \leq \overline{c}$. It follows that $\frac{V(\frac{1}{2})}{V(1)} \geq 1 - \rho(1-c)\lambda$ while $0 = \frac{V(0)}{V(\frac{1}{2})} < 1 - \rho(1-c)\lambda$. Hence, given $c \leq \overline{c}$, $h^{-1}(1-\rho(1-c)\lambda) \in (0, \frac{1}{2}]$, where h is the function defined by equation (3.7). Moreover, by the monotonicity argument laid out in section 3.1, $\frac{U[\frac{1}{2}-h^{-1}(1-\tilde{\rho}^{\xi=0}\lambda)]}{1-\tilde{\rho}^{\xi=0}\lambda} = \frac{U[1/2]}{1-\tilde{\rho}^{\xi=0}}$ and $\rho \geq \tilde{\rho}^{\xi=0}$ imply $\frac{U[\frac{1}{2}-h^{-1}(1-\rho\lambda)]}{1-\rho\lambda} \geq \frac{U[1/2]}{1-\rho}$ or $\frac{U[\frac{1}{2}-h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} \geq \frac{U[1/2]}{1-\rho}$ for $c = 0$. However, as argued above, $h^{-1}(1-\rho(1-\overline{c})\lambda) = \frac{1}{2}$ and therefore

$$\frac{U[\frac{1}{2} - h^{-1}(1-\rho(1-\overline{c})\lambda)]}{1-\rho\lambda} < \frac{U[1/2]}{1-\rho}$$

. Given that $h^{-1}(1-\rho(1-c)$ is continuous and strictly increasing in c for fixed ρ, it follows from the continuous value theorem that there exits $c^* \in [0, \overline{c})$ such that $\frac{U[\frac{1}{2}-h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} > \frac{U[1/2]}{1-\rho}$ iff $c < c^*$. Hence for fixed ρ, the economy grows iff $c < \min\{\overline{c}, c^*\} = c^*$. This ends the proof of the theorem.

Now, suppose $\rho \in [0, \tilde{\rho}^{\xi=0}]$. Then, $\frac{U[\frac{1}{2}-h^{-1}(1-\rho\lambda)]}{1-\rho\lambda} \leq \frac{U[1/2]}{1-\rho}$ and therefore, $\frac{U[\frac{1}{2}-h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} \leq$

$\frac{U[1/2]}{1-\rho}, \forall c \in [0,1]$. It follows that in this case, the economy does not grow for any probability c.

■

3.3 Dictatorship and Economic Growth

This section analyzes the impact of dictatorship on economic growth. Assumptions on preferences and technology made in section 1 are maintained. As in section 2, citizens and politicians are assumed to have a common discount factor ρ. Also as in section 2, citizens do not have full control over the choice of the politician in power. However, in this section, the loss of voting rights is caused by a different friction. In section 2, the loss of voting rights was temporary and succeeded the occurrence of a coup d'etat. In this section, the loss of voting rights is permanent and occurs after a politician in power has become a dictator. Specifically, this section assumes that at the end of every period, before citizens vote, the politician in power draws a dictatorship ticket $d \in \{0,1\}$. If the politician draws $d = 1$, then the representative citizen is not allowed to choose the next period's politician. In this case, the politician who made the draw stays in power forever. If the politician draws $d = 0$, then the representative citizen chooses the next period's politician in power and a new period will start. A politician who has drawn $d = 1$ is referred to as a dictator. The economy is called a dictatorship or a dictatorial economy when the politician in power is a dictator. An economy with $\xi = 0$ is referred to as a democracy or a democratic economy. Conditional on not being a dictator, the probability that a politician draws $d = 1$ is assumed equal to $\xi \in [0,1]$ in all periods. If $d_t = 1$ in period t, then with probability 1, $d_{t+s} = 1, \forall s \geq 0$.

The timing of the game in a non-dictatorial economy is as follows. The economy starts date 0 with politician ι_0. First, the representative citizen receives endowments of the consumption and investment goods and decides on the allocation of the consumption good between the politician in power and citizens. The politician in power is then entrusted with the investment good and chooses whether to consume it or to process it to shift the technological frontier. Afterward, the citizens and the politician in power consume their respective allocations. The politician then draws d in $\{0,1\}$. If $d = 0$, then the representative citizen chooses the next period's politician and a new period starts.

If $d = 1$, then the economy becomes a dictatorship and a new period starts. The timing in a dictatorship is the following: the representative citizen receives the endowments and allocates the consumption good between the dictator and the citizens. The dictator then decides whether or not to shift the technological frontier. Consumption occurs and a new period starts.

3.3.1 Relevant Histories, Strategies and Equilibrium Concept

As before, let $h^t_{c,1}$, $h^t_{c,2}$, h^t_P denote date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. The evolution of histories is conditioned on the realization of the dictatorship ticket in the following fashion:

$$h^0_{c,1} = \{\iota_0, S_0\},\ \ h^0_P = \{h^0_{c,1}, c^{HH}_0, c^P_0\},\ \ h^0_{c,2} = \{h^0_P, \theta_0, S_1, d_0\}$$
$$h^1_{c,1}|(d_0 = 0) = \{h^0_{c,2}, \iota_1\},\ h^1_P|(d_0 = 0) = \{h^1_{c,1}, c^{HH}_1, c^P_1\},\ h^1_{c,2}|(d_0 = 0) = \{h^1_P, \theta_1, S_2, d_1\}$$
$$h^1_{c,1}|(d_0 = 1) = \{h^0_{c,2}\},\ h^1_P|(d_0 = 1) = \{h^1_{c,1}, c^{HH}_1, c^P_1\}$$
$$h^1_{c,2}|(d_0 = 1) = \{h^1_P, \theta_1, S_2, d_1\}$$
$$h^2_{c,1}|(d_0 = 0 \text{ and } d_1 = 0) = \{h^1_{c,2}, \iota_2\},\ \ h^1_P|(d_0 = 0 \text{ and } d_1 = 0) = \{h^1_{c,1}, c^{HH}_2, c^P_2\},$$
$$h^1_{c,2}|(d_0 = 0 \text{ and } d_1 = 0) = \{h^1_P, \theta_2, S_3, d_2\}$$
$$h^2_{c,1}|(d_0 = 0 \text{ and } d_1 = 1) = \{h^1_{c,2}\},\ \ h^1_P|(d_0 = 0 \text{ and } d_1 = 1) = \{h^1_{c,1}, c^{HH}_2, c^P_2\},$$
$$h^1_{c,2}|(d_0 = 0 \text{ and } d_1 = 1) = \{h^1_P \theta_2, S_3\}$$
$$h^2_{c,1}|(d_0 = 1) = \{h^1_{c,2}\},\ \ h^1_P|(d_0 = 1) = \{h^1_{c,1}, c^{HH}_2, c^P_2\}, h^1_{c,2}|(d_0 = 1) = \{h^1_P, \theta_2, S_3\},\ \ \ldots$$

More generally, let's define $d_{-1} = 0$ and $d^t = \{d_0, ...d_t\}$ and

$$\kappa(d^t) = \ \inf\ \{-1 \leq s \leq t : d_s = 1\}. \tag{3.15}$$

Also, let's define by $\pi(d^t|d^{t-1})$ the probability that the economy reaches $d^t \equiv (d^{t-1}, d_t)$ given current state d^{t-1}.

Then,

$$\pi(d^t|d^{t-1}) = \begin{cases} 1 \text{ if } d_t = 1 \text{ and } 0 \text{ if } d_t = 0, & \text{given } \ \kappa(d^{t-1}) \leq t-1 \\ \xi \text{ if } d_t = 1 \text{ and } 1-\xi \text{ if } d_t = 0, & \text{given } \ \kappa(d^{t-1}) > t-1. \end{cases}$$

Relevant histories in a democratic economy evolve as:

$$\begin{cases} h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1}>t-1)) = \{\iota_t, S_t\} \\ h^t_P|(d^{t-1}|\kappa(d^{t-1}>t-1)) = \{h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1}>t-1)), c^{HH}_t, c^P_t\} \\ h^t_{c,2}|(d^{t-1}|\kappa(d^{t-1}>t-1)) = \{h^t_P|(d^{t-1}|\kappa(d^{t-1}>t-1)), \theta_t, S_{t+1}, d_t\}, \end{cases}$$

while in a dictatorship, relevant histories are given by:

$$\begin{cases} h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1}\le t-1)) = \{\iota_{\kappa(d^{t-1})}, S_t\}\} \\ h^t_P|(d^{t-1}|\kappa(d^{t-1}\le t-1)) = \{h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1}\le t-1)), c^{HH}_t, c^P_t. \end{cases}$$

As in the previous sections, let $H^t_{c,1}|h^t$, $H^t_{c,2}|h^t$ and $H^t_P|h^t$ denote the set of histories that follow some history h^t. Also let $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$ define strategies for the representative citizen in stage 1, the politician in power in stage 2 and the representative citizen in stage 3 of the period t game. Let $\Sigma_{c,1}|h^t, \Sigma_P|h^t$ and $\Sigma_{c,2}|h^t$ denote the sets of all such strategies.

For any history h^t and strategies $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$, let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ denote the corresponding strategy profile. Let $h^t|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) < t-1)$ denote a history h^t which ends with technological frontier S_t and politician in power ι_t who is a dictator. Similarly, let $h^t|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \geq t-1)$ denote a history h^t which ends with technological frontier S_t and politician in power ι_t who is not a dictator. Because dictatorship is an absorbing state , a strategy profile $\sigma|(h^t, S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \leq t-1)$ induces a sequence of allocations $\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau}\}^\infty_{\tau=0}$ which is not a function of future draws of the dictatorship tickets. A strategy profile $\sigma|(h^t, S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) > t-1)$ induces a sequence of allocations $\{c^{HH}_{t+\tau}(.), c^P_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}^\infty_{\tau=0}$, where $\{c^{HH}_{t+\tau}(.), c^P_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)$ are functions of histories $d^{t+\tau}$. Define $d_{-1} = 0$ and:

$$\mathcal{A}(d^{t+\tau-1}) \equiv (c^{HH}_{t+\tau}(d^{t+\tau-1}), c^P_{t+\tau}(d^{t+\tau-1}), \theta_{t+\tau}(d^{t+\tau-1})) \tag{3.16a}$$

$$\Omega^{HH}(\mathcal{A}(d^{t+\tau-1})) \equiv \lambda^{\tau-\theta_t(d^{t+\tau-1})-\theta_{t+1}(d^{t-1})-\ldots-\theta_{t+\tau-1}(d^{t+\tau-2})} U[c^{HH}_{t+\tau}(d^{t+\tau-1})] \tag{3.16b}$$

$$\Omega^P(\mathcal{A}(d^{t+\tau-1})) \equiv \lambda^{\tau-\theta_t(d^{t-1})-\theta_{t+1}-\ldots-\theta_{t+\tau-1}(d^{t+\tau-2})} V[c^P_{t+\tau}(d^{t+\tau-1}) + \frac{\theta_{t+\tau}(d^{t+\tau-1})}{2}]. \tag{3.16c}$$

For all $t \geq 0$, I also define:

$$\sigma^D|(S_t, \iota_t, d^{t-1}) = \sigma|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \leq t-1)$$
$$\sigma^{ND}|(S_t, \iota_t, d^{t-1}) = \sigma|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) > t-1).$$

Then, resulting payoffs to the representative citizen and to politician ι_t in power from a strategy profile $\sigma^D|(S_t, \iota_t, d^{t-1})$ which induces a sequence of allocations

$$\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau}\}_{\tau=0}^{\infty}$$

are:

$$\Phi_c(\sigma^D|(S_t, \iota_t, d^{t-1})) = \sum_{\tau=0}^{\infty} \rho^\tau \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \tag{3.17}$$

and

$$\Phi_{\iota_t}(\sigma^D|(S_t, \iota_t, d^{t-1})) = \sum_{\tau=0}^{\infty} \rho^\tau \lambda^{S_{t+\tau}} V[c^P_{t+\tau} + \frac{\theta_{t+\tau}}{2}]. \tag{3.18}$$

Similarly, resulting payoffs to the representative citizen and to the politician in power from a strategy profile strategy profile $\sigma^{ND}|(S_t, \iota_t, d^{t-1})$ with induced sequence of allocations $\{\mathcal{A}(d^{t+\tau-1})\}_{\tau=0}^{\infty}$ are given by:

$$\Phi_{HH}(\sigma^{ND}|(S_t, \iota_t, d^{t-1})) = \lambda^{S_t} \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^{HH}(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}) \tag{3.19}$$

and

$$\Phi_{\iota_t}(\sigma^{ND}|(S_t, \iota_t, d^{t-1})) = \lambda^{S_t} \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^P(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}) * \mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}. \tag{3.20}$$

For a politician in power in a non-dictatorial economy, $\mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}$ in the expression of payoffs to the politician reflects the fact that a politician who has not yet drawn a dictatorship ticket will remain in power only if the strategy profile calls for the representative citizen to keep him in power. Note that $\mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}$ is absent from the expression of payoffs to a dictator given that a dictator is always guaranteed to remain in power.

Definition 11 *A Strategy profile* $\sigma|\{S_0, \iota_0\}$ *is a subgame perfect equilibrium (SPE) of the game if* $\forall h^t$ *with states* $\{S_t, \iota_t, d^{t-1}\}$*, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma, \sigma_P|h^t, \sigma_{c,2}|h^t), \forall\gamma \in \Sigma_{c,1}|h^t \tag{3.21}$$

$$\Phi_{\iota_t}(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma, \sigma_{c,2}|h^t), \forall\gamma \in \Sigma_P|h^t \tag{3.22}$$

$$\Phi_{c,2}(\sigma|h^t) \geq \Phi_{c,2}(\sigma_{c,1}|h^t, \sigma_P|h^t, \gamma), \forall\gamma \in \Sigma_{c,2}|h^t. \tag{3.23}$$

Definition 12 *Given* $d_{-1} = 0$*, a sequence of allocations* $\{c_\tau^{HH}(.), c_\tau^P(.), \theta_\tau(.), \iota_\tau(.)\}_{\tau=0}^{\infty}$ *is feasible if* $\forall\tau \geq 0$*. and* $\forall d^{\tau-1}$*, it satisfies:*

$c_\tau^{HH}(d^{\tau-1}) + c_\tau^P(d^{\tau-1}) = 1/2,$
$\theta_\tau(d^{\tau-1}) \in \{0; 1\},$
$\iota_\tau(d^{\tau-1}) \in J \cup \{\varnothing\}$ *if* $\kappa(d^{\tau-1}) > \tau - 1$ *and* $\iota_\tau(d^{\tau-1}) = \iota_{\kappa(d^{\tau-1})}(d^{\kappa(d^{\tau-1})}),$
if $\kappa(d^{\tau-1}) \leq \tau - 1$*, where* $d^{\kappa(d^{\tau-1})}$ *is the truncation of* $d^{\tau-1}$ *at date* $\kappa(d^{\tau-1})$*.*

The next lemma characterizes SPE sequences by supporting them with trigger strategies that threaten to shift to the worst outcome of the economy whenever a deviation occurs. The worst outcome of this economy is a situation in which the politician in power always receives $c_P = 0$ whether or not he is a dictator, always stops growth by choosing $\theta = 0$ whether or not he is a dictator, and always gets fired whenever not a dictator.

Let's define

$$u_b^* = \frac{U(1/2)}{1-\rho}, \tag{3.25}$$

and

$$v_b^* = \frac{V(1/2)}{1-\rho}. \tag{3.26}$$

Lemma 13 *A feasible sequence of allocations* $\{c^{HH}_{t+\tau}(.), c^{P}_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}_{\tau=0}^{\infty}$ *from some strategy profile* $\sigma|(S_t, \iota_t, d^{t-1})$ *is an SPE sequence iff* $\forall s \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$ *and* $S_{t+s}(d^{t+s-2}) = S_t + s - \theta_t - - - \theta_{t+s-1}(d^{t+s-2})$,

$$\Phi_c(\sigma^{ND}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})}U_b^* \tag{3.27a}$$

$$\Phi_c(\sigma^{D}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})}U_b^* \tag{3.27b}$$

$$\begin{aligned} &\Phi_{\iota_t}(\sigma^{ND}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \\ &\geq \lambda^{S_{t+s}(d^{t+s-2})}V[c^P_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho\xi\lambda^{S_{t+s}(d^{t+s-2})}v_b^* \end{aligned} \tag{3.27c}$$

$$\begin{aligned} &\Phi_{\iota_t}(\sigma^{D}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \\ &\geq \lambda^{S_{t+s}(d^{t+s-2})}V[c^P_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho\lambda^{S_{t+s}(d^{t+s-2})}v_b^*. \end{aligned} \tag{3.27d}$$

Proof. The proof is similar to that of lemma 3. Let's first show necessity of (3.27a) to (3.27d). Suppose $\sigma|(S_t, \iota_t, d^{t-1})$ induces the sequence of allocations

$$c^P_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}_{t=0}^{\infty}.$$

Now, let's suppose that after some date $t+s$ with history d^{t+s-1}, the representative citizen in stage 1 decides to deviate from $\sigma|(S_t, \iota_t, d^{t-1})$ by choosing the allocation $\{\hat{c}^{HH}_{t+s+\tau}(d^{t+s+\tau-1}), \hat{c}^{P}_{t+s+\tau}(d^{t+s+\tau-1})\} = \{1/2, 0\}$ for all $\tau \geq 0$, and $\forall d^{t+s+\tau-1}$. Assuming all other players follow $\sigma|(S_t, \iota_t, d^{t-1})$, the payoff to the representative citizen from this deviation is:

$$\sum_{\tau=0}^{\infty}\rho^{\tau}\sum_{d^{t+\tau+s-1}\succeq d^{t+s-1}}\lambda^{S_{t+s+\tau}(d^{t+s+\tau-1})}\pi(d^{t+s+\tau-1}|d^{t+s-1})U(1/2)$$

$\geq \lambda^{S_{t+s}(d^{t+s-1})}\frac{U(1/2)}{1-\rho} \equiv \lambda^{S_{t+s}(d^{t+s-1})}u_b^*$. Now because $\sigma|(S_t, \iota_t, d^t)$ is an SPE strategy profile, it holds that $\Phi_c(\sigma^j|(d^{t+s-1}, S_{t+s}(d^{t+s-1}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-1})}u_b^*$, $j \in \{ND, D\}$ implying (3.27a) and (3.27b). To show necessity of (3.27c) and (3.27d), let's suppose a politician ι_{t+s} in power at date $t+s$ following some history d^{t+s-1} decides to deviate by choosing $\hat{\theta}_{t+s+\tau}(d^{t+s+\tau-1}) = 1$ for all $\tau \geq 0$ and for all d^{t+s-1}, conditional on being in power in $t+s+\tau$. The payoff to the politician if he deviates unilaterally in this fashion is:

$$\sum_{\tau=0}^{\infty} \sum_{d^{t+\tau+s-1} \succeq d^{t+s-1}} \rho^{\tau} \lambda^{S_{t+s+\tau}(d^{t+s-1})} V[c_{t+\tau}^{P}(d^{t+\tau-1}) + \frac{1}{2}]$$

$$* \pi(d^{t+s+\tau-1}|d^{t+s-1}) * \mathbf{1}_{\{\iota_{t+s+\tau}(d^{t+s+\tau-1})=\iota_{t+s}\}}$$

$$\geq \lambda^{S_{t+s}(d^{t+s-1})} V[c_{t+s}^{P}(d^{t+s-1}) + \frac{1}{2}] + \rho \lambda^{S_{t+s}(d^{t+s-1})} v_b^*$$

$$\geq \lambda^{S_{t+s}(d^{t+s-1})} V[c_{t+s}^{P}(d^{t+s-1}) + \frac{1}{2}] + \rho \xi \lambda^{S_{t+s}(d^{t+s-1})} v_b^*.$$

This proves necessity of (3.27c) and (3.27d). To show sufficiency, let's first suppose (3.27a) to (3.27d) hold for some sequence of allocations

$$\{\mathcal{A}(.)\}_{\tau=0}^{\infty} \equiv \{c_{t+\tau}^{HH}(.), c_{t+\tau}^{HH}(.), c_{t+\tau}^{P}(.), \theta_{t+\tau}(.)\}_{\tau=0}^{\infty}.$$

It needs to be shown that there exists an SPE that induces this sequence. For this end, let's consider the following trigger strategy: if all players have always followed $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$, then the player called to play continues to follow; if any player has ever deviated, then the strategy calls to move to the worst outcome: the representative citizen always chooses $c^P = 0$, politicians in power always choose $\theta = 1$, and the representative citizen always fires any politician in power who is not a dictator. Because the right hand sides of (3.27a) to (3.27d) represent payoffs from best deviations given this trigger strategy profile, citizens in the first stage and politicians in power never want to unilaterally deviate from the trigger strategy profile. It is then left to check that the representative citizen in third stage does not want to deviate as well. Under the defined trigger strategy, if the representative citizen deviates at date $t+s$ after some history d^{t+s-1} by choosing a politician in power different from what indicated by $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$, then the citizens will receive payoff $\lambda^{S_{t+s}(d^{t+s-1})} U(c_{t+s}^{H}(d^{t+s-1})) + \frac{\rho}{1-\rho} \lambda^{S_{t+s}(d^{t+s-1})} U(1/2)$ which by (3.27c) and (3.27d) is smaller than the citizens'payoff under $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$. The representative citizen therefore does not want to deviate from $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$ in the third stage as well. Hence, the defined trigger strategy profile is an SPE which induces the sequence $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$. This ends the proof of the lemma.

■

In what follows, best SPE sequences are referred to as SPE-induced allocations that maximize payoffs to the representative citizen. A straightforward argument analogue to the one used to prove lemma 4 implies that citizens do not strictly improve their payoffs by replacing a politician in power. The best SPE problem of this economy will therefore without loss of generality restrict to best SPE sequences which involve no replacement of the initial politician. Let $\mathcal{A}(d^{t+\tau}) \equiv \{c^{HH}_{t+\tau}(d^{t+\tau}), c^{P}_{t+\tau}(d^{t+\tau}), \theta_{t+\tau}(d^{t+\tau})\}$ and let Ω^{HH} and Ω^{P} be given by equations (3.16c) and (3.16d). Define:

$$E^i(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \equiv \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^i(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}), i \in \{HH, P\}. \tag{3.29}$$

The Best SPE problem of this economy is then given by:

$$\max_{\{c^{HH}_{t+\tau}(.), c^{P}_{t+\tau}(.), \theta_{t+\tau}(.)\}_{\tau=0}^{\infty} \text{feasible}} \lambda^{S_t} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \text{ s.t. } \forall \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} \frac{U(1/2)}{1-\rho},$$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho \xi v_b^*,$$

if $\kappa(d^{t+s-1}) > t+s-1$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho v_b^*,$$

if $\kappa(d^{t+s-1}) \leq t+s-1$,

S_t given , $S_{t+\tau+1}(d^{t+\tau-1}) = S_{t+\tau}(d^{t+\tau-2}) + 1 - \theta_{t+\tau}(d^{t+s-1})$, which simplifies to:

$$\max_{\{c^{HH}_{t+\tau}(.),c^{P}_{t+\tau}(.),\theta_{t+\tau}(.)\}_{\tau=0}^{\infty}\text{feasible}} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \text{ s.t. } \forall \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$$
$$E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \frac{U(1/2)}{1-\rho},$$
$$E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho\xi v_b^*, \text{ if } \kappa(d^{t+s-1}) > t+s-1$$
$$E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho v_b^*, \text{if } \kappa(d^{t+s-1}) \leq t+s-1,$$
$$S_t \text{ given }, S_{t+\tau+1}(d^{t+\tau-1}) = S_{t+\tau}(d^{t+\tau-2}) + 1 - \theta_{t+\tau}(d^{t+s-1}).$$

Because dictatorship is an absorbing state for the economy, lemma 13 then implies that at any date t, the best SPE problem of an economy that has become a dictatorship is:

$$\max_{\{c^{HH}_{t+\tau},c^{P}_{t+\tau},\theta_{s+\tau}\}_{\tau=0}^{\infty} \text{ feasible}} \sum_{\tau=0}^{\infty} \rho^{\tau}\lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \text{ s.t.}$$
$$\sum_{\tau=0}^{\infty} \rho^{\tau}\lambda^{S_{t+s+\tau}} U[c^{HH}_{t+s+\tau}] \geq \lambda^{S_{t+s}} U_b^*, \forall s \geq 0$$
$$\sum_{\tau=0}^{\infty} \rho^{\tau}\lambda^{S_{t+s+\tau}} V[c^{P}_{t+s+\tau} + \frac{\theta_{t+s+\tau}}{2}] \geq \lambda^{S_{t+s}} V[c^{P}_{t+s} + \frac{1}{2}] + \rho v_b^*, \forall s \geq 0$$
$$S_t \text{ given }, S_{t+\tau+1} = S_{t+\tau} + 1 - \theta_{t+\tau}, \forall \tau \geq 0.$$

By an argument identical to the outline in the proof of lemma 5, the above problem is stationary and reduces to:

$$\Phi_D^c = \max\{u_b^*, \max_{\{c_D^{HH},c_D^P\}} \frac{U(c^D_{HH})}{1-\rho\lambda} s.t. \frac{V(c^P_D)}{1-\rho\lambda} \geq V[c^P_D + \frac{1}{2}] + \rho v_b^*\}.$$

Let's now define real function h_D by:

$$h^D_\rho(c) = \frac{V(c)}{V[c+\frac{1}{2}] + \rho v_b^*}. \tag{3.33}$$

For given ρ and λ, $h^D_\rho(0) = 0$ and $h^D_\rho(c)$ converges to 1 as c goes to ∞. Moreover, $h^D_\rho(.)$ is continuous and strictly increasing in c. Hence because for any λ and for any

$\rho \in [0, \frac{1}{\lambda})$, $1 - \rho\lambda \in (0, 1]$, there exists a unique $c(\rho) \in (0, +\infty]$ implicitly defined by

$$h^D(c(\rho)) = 1 - \rho\lambda. \tag{3.34}$$

It should be noted that at this point no restriction is imposed ρ to guarantee that $c(\rho)$ is smaller than $\frac{1}{2}$.

The following series of lemma establishes several results that will be useful in proving the first theorem of this section.

Lemma 14 *Suppose* $\rho \in [0, \frac{1}{\lambda})$. *Then,* $\frac{V(c)}{1-\rho\lambda} - \rho\frac{V(\frac{1}{2})}{1-\rho}$ *is strictly increasing in* ρ *iff* $V(c) > \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$.

Proof. Let $\lambda > 1$ be given. For fixed c , define $l(\rho) = \frac{V(c)}{1-\rho\lambda} - \rho\frac{V(\frac{1}{2})}{1-\rho}$. Then, $l^{'}(\rho) = \frac{\lambda V(c)}{(1-\rho)^2} - \frac{V(\frac{1}{2})}{(1-\rho)^2}$. Therefore, $l^{'}(\rho) > 0$ iff $V(c) > \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. This ends the proof of the lemma. ■

Lemma 15 *Suppose* c *is such that* $V(c) \leq \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. *Then,* $\frac{V(c)}{1-\rho\lambda} < \frac{V(\frac{1}{1})}{1-\rho}$.

Proof.

Suppose $V(c) \leq \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. Then, $V(c) \leq \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2} < V(\frac{1}{2})\frac{1-\rho\lambda}{1-\rho}$ since $\frac{1}{\lambda} < 1$ and $\frac{(1-\rho\lambda)^2}{(1-\rho)^2} < 1$, as $\lambda > 1$ and $\rho \in [0, \frac{1}{2})$. This in turn implies $\frac{V(c)}{1-\rho\lambda} < \frac{V(\frac{1}{1})}{1-\rho}$. ■

Lemma 16 *For any* $\rho \in [0, \frac{1}{\lambda})$, *let* $c(\rho)$ *be defined by equation (3.34). Suppose* $\rho_2 > \rho_1$. *Then,* $c_{\rho(1)} > c_{\rho(2)}$.

Proof. Consider $\rho_2 > \rho_1$ and $c_{\rho(i)}, i \in \{1, 2\}$ defined by $h^D(c(\rho)) = 1 - \rho\lambda$ or $\frac{V(c(\rho_i))}{1-\rho_i\lambda} - \rho_i\frac{V(\frac{1}{2})}{1-\rho_i} = V(c(\rho_i) + \frac{1}{2})$. First observe that this implies $c(\rho_i) > 0, i \in \{1, 2\}$. Therefore, $\frac{V(c(\rho_1))}{1-\rho_1\lambda} - \rho_1\frac{V(\frac{1}{2})}{1-\rho_1} = V(c(\rho_1) + \frac{1}{2})$ implies $\frac{V(c(\rho_1))}{1-\rho_1\lambda} > \frac{V(\frac{1}{2})}{1-\rho_1}$. Then, lemma 15 implies $V(c) > \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho_1\lambda)^2}{(1-\rho_1)^2}$. Lemma 14 then implies that $\frac{V(c(\rho_1))}{1-\rho_1\lambda} - \rho_1\frac{V(\frac{1}{2})}{1-\rho_1} < \frac{V(c(\rho_1))}{1-\rho_2\lambda} - \rho_2\frac{V(\frac{1}{2})}{1-\rho_2}$, as $\rho_2 > \rho_1$. Hence, $V(c(\rho_1) + \frac{1}{2}) < \frac{V(c(\rho_1))}{1-\rho_2\lambda} - \rho_2\frac{V(\frac{1}{2})}{1-\rho_2}$, or $\frac{V(c(\rho_1))}{1-\rho_2\lambda} - V(c(\rho_1) + \frac{1}{2}) > \rho_2\frac{V(\frac{1}{2})}{1-\rho_2}$. Therefore, $\frac{V(c(\rho_2))}{1-\rho_2\lambda} - V(c(\rho_2) + \frac{1}{2}) = \rho_2\frac{V(\frac{1}{2})}{1-\rho_2}$ implies $c_{\rho(1)} > c_{\rho(2)}$, since from concavity of V, $\frac{V(c)}{1-\rho\lambda} - V(c + \frac{1}{2})$ is strictly increasing in c for any given ρ and λ. ■

Lemma 17 *Let $c(\rho)$ be uniquely defined on $(0, \frac{1}{\lambda})$ by $\frac{V(c(\rho))}{1-\rho\lambda} - \rho\frac{V(\frac{1}{2})}{1-\rho} = V(c(\rho) + \frac{1}{2})$. Then, $c(.)$ is a continuous function of ρ. Moreover, $c((0, \frac{1}{\lambda})) = (0 + \infty)$.*

Proof.

Lemma (16) has established that $c(.)$ is a monotonic function. To show that $c(.)$ is also continuous, it then suffices to show that $\{c(\rho_n)\}$ converges to $c(\rho)$ whenever $\{\rho_n\}$ is a monotonic sequence in $(0, \frac{1}{\lambda})$ that converges to $\rho \in [0, \frac{1}{\lambda}]$.

First consider the case $\rho \in (0, \frac{1}{\lambda})$. Clearly, for such ρ, there exists a unique $c(\rho)$ such that $\frac{V(c(\rho))}{V(c(\rho)+\frac{1}{2})+\frac{\rho V(\frac{1}{2})}{1-\rho}} = 1-\rho\lambda$. Therefore, $\frac{V(c(\rho_n))}{V(c(\rho_n)+\frac{1}{2})+\frac{\rho V(\frac{1}{2})}{1-\rho}}$ converges to $\frac{V(c(\rho))}{V(c(\rho)+\frac{1}{2})+\frac{\rho V(\frac{1}{2})}{1-\rho}}$ as ρ_n converges to ρ. Now, let's define function $m(.)$ by $m(c) = \frac{V(c)}{V(c+\frac{1}{2})+\frac{\rho V(\frac{1}{2})}{1-\rho}}$. Then, $m(c(\rho_n))$ converges to $m(c(\rho))$.

Let's first assume that $\{\rho_n\}$ is an increasing sequence that converges to ρ. In this case, $\{c(\rho_n)\}$ is a decreasing and bounded sequence which therefore converges to some real number y^- such that $m(y^-) = m(c(\rho))$. This in turn implies that $y^- = c(\rho)$, since m is a strictly increasing function.

Now, let's suppose that $\{\rho_n\}$ is a sequence that decreases to ρ. Then, $\{c(\rho_n)\}$ is monotonic and converges to some real number y^+ such that $m(y^+) = m(c(\rho))$. Hence, m strictly increasing implies $y^- = c(\rho)$. Therefore, $y^- = y^+ = c(\rho)$ which concludes the proof that $c(.)$ is a continuous function.

Now, suppose $\rho = \frac{1}{\lambda}$. Then, $m(c(\rho_n))$ converges to the same limit as $1 - \rho_n\lambda$ which itself converges to 0. Hence, $m(c(\rho_n))$ converges to 0 which is in fact the value of $m(0)$. Therefore, when $\{\rho_n\}$ is an increasing sequence, $\{c(\rho_n)\}$ is a bounded and decreasing sequence that converges to some y such that $m(y) = m(0)$. Hence, by strict monotonicity of m, $\{c(\rho_n)\}$ converges to 0. But by definition of $c(.)$, $\lim_{x\uparrow\frac{1}{\lambda}} c(x) = 0$. Hence, for any sequence $\{\rho_n\}$ that increases to $\frac{1}{\lambda}$, $\{c(\rho_n)\}$ converges to $0 = \lim_{x\uparrow\frac{1}{\lambda}} c(x)$. This shows that $c(.)$ is left-continuous at $\frac{1}{\lambda}$.

Finally, suppose $\rho = 0$. It is left to show that for any sequence $\{\rho_n\}$ that decreases to ρ, $\{c(\rho_n)\}$ converges to $\lim_{x\downarrow 0} c(x)$. First, note that $\lim_{x\downarrow 0} c(x) = +\infty$. Note also that $\lim_{c\downarrow+\infty} m(c) = 1$. Now, suppose $\{\rho_n\}$ is a sequence that decreases to ρ. Then, $\{c(\rho_n)\}$ is a bounded and increasing sequence that converges to some z such that $\lim_{c\downarrow z} m(c) =$

$1 = \lim_{c \downarrow +\infty} m(c)$. Hence, $\{c(\rho_n)\}$ converges to $+\infty = \lim_{x \downarrow 0} c(x)$. Therefore, $c(.)$ is right-continuous at 0. This ends the proof that $c(.)$ is continuous on $(0, \frac{1}{\lambda})$. Finally, because $c(.)$ is monotonic, it follows that $c((0, \frac{1}{\lambda})) = (0 + \infty)$.

■

Theorem 18 *There exists $\rho^* \in (0, 1)$ such that if $\rho \leq \rho^*$, then in the best SPE, the economy stagnates whenever it becomes a dictatorship; if $\rho > \rho^*$, then in the best SPE the economy grows permanently whenever it becomes a dictatorship.*

Proof.

Let's fix λ. By Lemma 17, there exists $\tilde{\rho} \in (0, \frac{1}{\lambda})$ such that $c(\tilde{\rho}) = \frac{1}{2}$ and $c(\rho) \geq \frac{1}{2}, \forall \rho \leq \tilde{\rho}$. Clearly then, for any $\rho \leq \tilde{\rho}$, the solution to the best SPE problem of the dictatorship is $c^P = 0$ and $\Psi^c_D = \frac{V(\frac{1}{2})}{1-\rho}$. That is, any dictatorial economy with $\rho \leq \tilde{\rho}$ does not grow.

Note that $c(.)$ maps $[\tilde{\rho}, \frac{1}{\lambda})$ into $(0, \frac{1}{2})$. As in the proof of theorem 6, let's now define real functions g and f_D by $g(\rho) = \frac{1-\rho\lambda}{1-\rho}$ and $f_D(\rho) = \frac{U(1/2-c(\rho))}{U(1/2)}$. g is continuous and strictly decreasing in ρ. By lemmas 17 and 16, f_D is continuous and strictly increasing in ρ. Moreover, by the definition of $c(\tilde{\rho})$, $\frac{V(c(\tilde{\rho}))}{V(c(\tilde{\rho})+\frac{1}{2})+\frac{\tilde{\rho}V(\frac{1}{2})}{1-\tilde{\rho}}} = 1 - \tilde{\rho}\lambda$ and $c(\tilde{\rho}) = \frac{1}{2}$, so that $g(\tilde{\rho}) = \frac{1-\tilde{\rho}\lambda}{1-\tilde{\rho}} = \frac{1}{1-\tilde{\rho}} \frac{V(\frac{1}{2})}{V(1)+\frac{\tilde{\rho}V(\frac{1}{2})}{1-\tilde{\rho}}} > 0 = f_D(\tilde{\rho})$. Also, $g(\frac{1}{\lambda}) = 0 < 1 = f_D(\frac{1}{\lambda})$. ($c(\frac{1}{\lambda}) = 0$ implies $f(\frac{1}{\lambda}) = 1$). Therefore, there exists $\rho^* \in (\tilde{\rho}, 1/\lambda)$ s.t. $g(\rho^*) = f_D(\rho^*)$. For such ρ^*, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} = \frac{U(1/2)}{1-\rho^*}$. For all $\rho \leq \rho^*$, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} leq \frac{U(1/2)}{1-\rho^*}$ and $c^P_D = 0$, $\theta_D = 1$: the economy does not grow. For all $\rho > \rho^*$, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} > \frac{U(1/2)}{1-\rho^*}$ and $c^P_D = c(\rho) \in (0, \frac{1}{2}]$, $\theta_D = 0$: the economy grows. Since it was argued earlier that the economy does not grow for $\rho < \tilde{\rho}$, this concludes the proof that a dictatorial economy grows iff $\rho > \rho^*$.

■

For given ρ and λ, let $\lambda^{S_t}\Psi^c_D$ and $\lambda^{S_t}\Psi^P_D$ denote payoffs to citizens and to the politician in power respectively in a dictatorial economy ending with state S_t. By an argument similar to that used to prove lemma 5 , the best SPE problem of an ξ- economy is stationary. It follows that payoffs to citizens in a best SPE starting with technological frontier S_t are given by $\lambda^{S_t}\Psi^c_{ND}$, where Ψ^c_{ND} is defined recursively as:

$$\Psi^c_{ND} = \max_{\{c^{HH},c^P\}} U(c^{HH}) + \rho\lambda[(1-\xi)\Psi^c_{ND} + \xi\Psi^c_D]$$

s.t.

$$U(c^{HH}) + \rho\lambda((1-\xi)\Psi^c_{ND} + \xi\Psi^c_D) \geq \frac{U(1/2)}{1-\rho} \tag{3.35}$$

$$\Psi^P_{ND} = V(c^P) + \rho\lambda((1-\xi)\Psi^P_{ND} + \xi\Psi^P_D) \tag{3.36}$$

$$\Psi^P_{ND} \geq V(c^P + \frac{1}{2}) + \rho\xi\frac{V(1/2)}{1-\rho}. \tag{3.37}$$

The above operator maps the bounded interval $[\frac{U(1/2)}{1-\rho}, \frac{U(1/2)}{1-\rho\lambda}]$ into itself, satisfies Blackwell's conditions for a contraction mapping with modulus $\rho\lambda$ and is therefore a contraction mapping. This proves the existence of Ψ^c_{ND} as the fixed point of the defined contraction mapping.

Equivalently, $\Psi^c_{ND} = max\{\hat{\Psi}^c_{ND}, \frac{U(1/2)}{1-\rho}\}$, where $\hat{\Psi}^c_{ND}$ may be defined recursively as:

$$\hat{\Psi}^c_{ND} = \max_{\{c^{HH},c^P\}} U(c^{HH}) + \rho\lambda[(1-\xi)\hat{\Psi}^c_{ND} + \xi\Psi^c_D]$$

s.t.

$$\frac{V(c^P) + \rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \geq V(c^P + \frac{1}{2}) + \rho\xi\frac{V(1/2)}{1-\rho}. \tag{3.38}$$

Because $\frac{V(c)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \geq V(c+\frac{1}{2})$ is strictly increasing in c for given ρ, λ, ξ, any solution $\{c^{HH}_\xi, c^P_\xi\}$ to the sustainability constraint of the politician in an $\xi-$economy must hold with equality, unless $c^P_\xi = 0$.

Lemma 19 *Let λ be given. Let ρ^* be the threshold level of ρ that conditions growth in a dictatorial economy. Suppose $\rho > \rho^*$. Define c^P_D by $\Psi^P_D = \frac{V(c^P_D)}{1-\rho\lambda} = V(c^P_D + \frac{1}{2}) + \rho v^*_b$. For any $\xi \in [0,1]$, define c^P_ξ implicitly by:*

$$\begin{cases} \frac{V(c^P_\xi)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} = V(c^P_\xi + \frac{1}{2}) + \rho\xi v^*_b \quad \textit{if} \quad \frac{V(0)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \leq V(0+\frac{1}{2}) + \rho\xi v^*_b \\ c^P_\xi = 0 \quad \textit{otherwise} \quad . \end{cases}$$

Then, $c^P_\xi < c^P_D$ for $\xi \in [0,1)$ and $c^P_1 = c^P_D$.

Proof.

Let's first show that $c_1^P = c_D^P$. By definition, $V(c_1^P) + \rho\lambda\Psi_D^P = V(c_1^P + \frac{1}{2}) + \rho v_b^*$, where $\Psi_D^P = \frac{V(c_D^P)}{1-\rho\lambda}$. Then, $V(c_1^P) + \rho\lambda\frac{V(c_D^P)}{1-\rho\lambda} - V(c_1^P + \frac{1}{2}) = \rho v_b^* = \frac{V(c_D^P)}{1-\rho\lambda} - V(c_D^P + \frac{1}{2})$, which implies $V(c_1^P) - V(c_1^P + \frac{1}{2}) = \frac{V(c_D^P)}{1-\rho\lambda}(1-\rho\lambda) - V(c_D^P + \frac{1}{2})$, or $V(c_1^P) - V(c_1^P + \frac{1}{2}) = V(c_D^P) - V(c_D^P + \frac{1}{2})$. But by strict concavity of V, $V(c) - V(c + \frac{1}{2})$ is strictly increasing in c. It follows that $c_1^P = c_D^P > 0$. Now, for $\xi \in [0,1]$, define $z(\xi) = \frac{V(c_D^P)+\rho\lambda\xi\frac{V(c_D^P)}{1-\rho\lambda} - \rho\xi v_b^*(1-\rho\lambda+\rho\lambda\xi)}{1-\rho\lambda+\rho\lambda\xi}$. It is straightforward to show that $z^{'}(\xi) * [-\rho V_b^*(1 - \rho\lambda + \rho\lambda\xi)] > 0$. That is, for all $\xi \in [0,1]$, $z^{'}(\xi) < 0$, given $\rho \geq \rho^* > 0$. But $z(1) = \frac{V(c_D^P)}{1-\rho\lambda} - \rho V_b^* = V(c_D^P + \frac{1}{2})$, where the last equality follows from the definition of c_D^P. Hence, $\forall\xi \in [0,1)$, $z(\xi) > V(c_D^P + \frac{1}{2})$, or $\frac{V(c_D^P)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda+\rho\lambda\xi} > V(c_D^P + \frac{1}{2}) + \rho\xi v_b^*$. If $\frac{V(0)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} \leq V(0 + \frac{1}{2}) + \rho\xi v_b^*$, then it follows that there exists a unique $c_\xi^P \in [0, c_D^P)$ s.t $\frac{V(c_\xi^P)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} = V(c_\xi^P + \frac{1}{2}) + \rho\xi v_b^*$, since $\frac{V(y)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda+\rho\lambda\xi} - V(y + \frac{1}{2})$ is strictly increasing in y for ρ and ξ given. Now, suppose that $\frac{V(0)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} > V(0 + \frac{1}{2}) + \rho\xi v_b^*$. Then, by definition, $c_\xi^P = 0$ and so, $c_\xi^P < c_D^P = c_1^P$. This ends the proof of the lemma. ■

Let $\tilde{\rho}^{\xi=0}$ be the value of ρ that conditions growth in a $0-$economy, as defined in corollary 7. Note that $c_D^P > c_0^P$ implies that $\tilde{\rho}^{\xi=0} < \rho^*$.

Theorem 20 *Let $\lambda > 1$ be given. Suppose $\rho > \rho^*$, where ρ^* is the threshold value of ρ that conditions growth in a dictatorial economy. Then, the $\xi-$economy grows for any $\xi \in [0,1]$.*

Proof. $\xi \in [0,1]$ be arbitrary. Let $\rho > \rho^*$. Then, from the previous lemma, $c_D^P > c_\xi^P$. But from the proof of theorem 18, $c_D^P \in (0,1]$. Hence, to show that the ξ-economy grows, it suffices to show that $\frac{U(\frac{1}{2}-c_\xi^P)+\rho\lambda\xi\Psi_D^c}{1-\rho\lambda(1-\xi)} > \frac{U(\frac{1}{2})}{1-\rho}$. Now, let $c_j^{HH} = \frac{1}{2} - c_j^P, j \in \{D, \xi\}$. Then, $c_\xi^{HH} > c_D^{HH}$ and therefore it suffices to show that $\frac{U(c_D^P)+\rho\lambda\xi\Psi_D^c}{1-\rho\lambda(1-\xi)} > \frac{U(\frac{1}{2})}{1-\rho}$, which given $\Psi_D^c = \frac{U(c_D^P)}{1-\rho\lambda}$, holds iff $\frac{U(c_D^P)}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. The last inequality is true by the proof of theorem 18, given $\rho > \rho^*$. Therefore, when $\rho > \rho^*$, the economy grows for any value of ξ. ■

The above theorem states that an economy that grows after becoming a dictatorship would also grow under any regime of ξ. It has also been shown that payoffs to the citizens continuously increase as the probability of falling into dictatorship decreases.

These results constitute a caveat against using paragons of growing dictatorial economies to advocate dictatorship as an ideal political regime. Dictatorial economies that grow owe it to technocratic rulers. An economy endowed with impatient politicians would be worse off if it were to shift toward dictatorship.

The rest of this section examines the implications for growth under different regimes of ξ, when the discount factor is not sufficient high for growth to occur under dictatorship. Specifically, it is asked whether an economy known to stagnate under dictatorship (i.e. if $\rho \leq \rho^*$) would grow at all for any value of ξ. The answer is that in this case, an ξ-economy grows if and only if ξ is not too large. To obtain this last result, it is assumed that λ is not too large. Specifically, the rest of this section assumes that $\lambda \leq 2$ to ensure that if $\frac{V(x)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(x+\frac{1}{2}) + \rho\xi v_b^*$ for some x, then $\frac{V(x)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^*$ is decreasing in ξ. Recall that $\Psi_D^P = v_b^*$ when $\rho \leq \rho^*$. Hence when $1 < \lambda \leq 2$, as ξ increases, the sustainability constraint becomes tighter as politicians tend to ask a higher share of the consumption good in order to let growth occur. When λ is too large (greater than 2), it is no longer guaranteed that $\frac{V(x)}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^* + \frac{\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)}$ is decreasing in ξ . In this case, it is not clear that optimal payments to politicians decrease with ξ, and so payoffs to citizens may increase or decrease as ξ increases. For the rest of this section, it is assumed that $1 < \lambda \leq 2$.

Lemma 21 *Suppose* $1 < \lambda \leq 2$. *Then,* $\forall \rho \in [0, \frac{1}{\lambda})$ *and* $\forall \xi \in [0,1]$, $\frac{(1-\rho+\rho\xi)(1-\rho\lambda+\rho\lambda\xi)-\rho\lambda\xi}{1-\rho\lambda+\rho\lambda\xi} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi][1-\rho\lambda+\rho\lambda\xi]}{1-\rho\lambda+\rho\lambda\xi}$.

Proof. It is easy to check that $\frac{(1-\rho+\rho\xi)(1-\rho\lambda+\rho\lambda\xi)-\rho\lambda\xi}{1-\rho\lambda+\rho\lambda\xi} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi][1-\rho\lambda+\rho\lambda\xi]}{1-\rho\lambda+\rho\lambda\xi}$ holds iff $2\rho(1-\xi) < \frac{1}{\lambda}$. Since $\rho(1-\xi) < 1$, $\lambda \leq 2$ guarantees $2\rho(1-\xi) < \frac{1}{\lambda}$, $\forall \rho \in [0, \frac{1}{\lambda})$ and $\forall \xi \in [0,1]$.

■

Lemma 22 *Let* $1 < \lambda \leq 2$. *Let* $\rho \in [0, \frac{1}{\lambda})$ *be such that* $\rho \leq \rho^*$, *where* ρ^* *is the value of* ρ *that conditions growth in a dictatorship. Let* $c_\xi^P \in [0, +\infty)$ *be defined implicitly by*

$$c_\xi^P = \begin{cases} 0 & \text{if } \frac{V(0)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} > V(0+\frac{1}{2}) + \rho\xi v_b^* \\ \frac{V(c_\xi^P)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c_\xi^P+\frac{1}{2}) + \rho\xi v_b^* & \text{otherwise.} \end{cases}$$

Then, for $\xi_2, \xi_1 \in [0,1]$ *such that* $\xi_2 > \xi_1$ *and* $c_{\xi_1}^P > 0$, *it holds that* $c_{\xi_2}^P > c_{\xi_1}^P > 0$.

Proof. Let $1 < \lambda \leq 2$. Fix $\rho \leq \rho^*$, so that payoffs to the politician in a dictatorship are given by $\Psi_D^P = v_b^*$. Fix $\xi \in [0,1]$. I first argue that c_ξ^P is well defined. Define function $l_\xi(.)$ by $l_\xi(c) = \frac{V(c)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - V(c+\frac{1}{2})$, for $c \in [0,+\infty]$. Clearly, $\lim_{x\to+\infty} l_\xi(x) = \lim_{x\to+\infty} V(x)[\frac{1}{1-\rho\lambda(1-\xi)} - 1] + \frac{\rho\lambda\xi V_b^*}{1-\rho\lambda(1-\xi)} = +\infty$. Therefore, if $\frac{V(0)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} \leq V(0+\frac{1}{2}) + \rho\xi v_b^*$, then there exists $c \in [0,+\infty)$ s.t. $\frac{V(c)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c+\frac{1}{2}) + \rho\xi v_b^*$. This proves that $c_\xi^P \in [0,+\infty)$ is well defined.

Now, consider $\xi_2 > \xi_1$ and such that $c_{\xi_1}^P > 0$. By definition, $c_{\xi_1}^P > 0$ implies that $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = V(c_{\xi_1}^P + \frac{1}{2}) + \rho\xi_1 v_b^*$. Define function l by $l(\xi) = \frac{V(c_{\xi_1}^P)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^*$, for $\xi \in [0,1]$. It is straightforward to show that for $\xi \in [0,1]$, $l'(\xi)$ has the same sign as $q(\xi)= \frac{\lambda*[v_b^*(1-\rho\lambda)-V(c_{\xi_1}^P)]}{[1-\rho\lambda(1-\xi)]^2} - v_b^*$. But $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} = V(c_{\xi_1}^P + \frac{1}{2}) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} \geq V(\frac{1}{2}) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = v_b^*(1-\rho) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = \frac{v_b^*[(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. That is, $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} \geq \frac{v_b^*[(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. But by lemma 3.3.1, $\lambda \leq 2$ implies $\frac{(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1}{1-\rho\lambda+\rho\lambda\xi_1} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi_1][1-\rho\lambda+\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. Therefore, $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} > v_b^* \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi_1]^2}{1-\rho\lambda+\rho\lambda\xi_1}$, which in turn implies that $q(\xi_1) < 0$. Hence, $l'(\xi_1) < 0$. Therefore, $\xi_2 > \xi_1$ implies $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} - \rho\xi_2 v_b^* < \frac{V(c_{\xi_1}^P)+\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} - \rho\xi_1 v_b^* = V(c_{\xi_1}^P + \frac{1}{2})$. That is, $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} < \rho\xi_2 v_b^* + V(c_{\xi_1}^P + \frac{1}{2})$. Hence, by strict monotonicity and continuity of l_{ξ_2} and by the fact that $\lim_{x\to+\infty} l_\xi(x) = +\infty$, there exists $c_{\xi_2}^P > c_{\xi_1}^P > 0$ such that $\frac{V(c_{\xi_2}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} = \rho\xi_2 v_b^* + V(c_{\xi_2}^P + \frac{1}{2})$. This concludes the proof. ■

Corollary 23 *Let Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$, where ρ^* is the value of ρ that conditions growth in a dictatorship. For all $\xi \in [0,1]$, let c_ξ^P be the solution to the best SPE problem of the $\xi-$economy. Then, for any ξ_1 and ξ_2 in [0,1] such that $\xi_2 > \xi_1$, it holds that $c_{\xi_2}^P > c_{\xi_1}^P > 0$.*

Proof. First note that in the best SPE problem of an ξ-economy, the sustainability constraint of the politician holds with equality if $c_\xi^P > 0$. To see why, suppose $c_\xi^P > 0$ and suppose that the sustainability constraint holds with strict inequality. Then, by decreasing the politician's consumption slightly, one improves citizens'payoffs without violating the politician's sustainability constraint. This contradicts that the initial allocation solves the best SPE problem. Now, because $0 = \frac{V(0)}{1-\rho\lambda} < V(\frac{1}{2})$, it follows that

$c_0^P > 0$ for the 0-economy. Then from the lemma, it follows that for, any ξ_1 and ξ_2 in [0,1] such that $\xi_2 > \xi_1$, $c_{\xi_2}^P > c_{\xi_1}^P > 0$. Note that a direct implication of the corollary is that $\forall \xi \in [0,1], \exists$ unique $c_\xi^P \in (0, +\infty)$ s.t $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi) + \frac{1}{2}) + \rho\xi v_b^*$. ■

Lemma 24 *Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$, where ρ^* is the value of ρ that conditions growth in a dictatorship. For all $\xi \in [0,1]$, let $c^P(\xi)$ be defined by: $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi) + \frac{1}{2}) + \rho\xi v_b^*$. Then, $c^P(.)$ is a continuous function of ξ.*

Proof. By the preceding corollary, $c^P(.)$ is a decreasing function of ξ. Hence, to show continuity, it suffices to show that for any monotonic sequence $\{\xi_n\}$ in $[0,1]$ that converges to some $\xi \in [0,1]$, $\{c^P(\xi_n)\}$ converges to $c^P(\xi)$. Let $\xi \in [0,1]$ be arbirtary. Let $\{\xi_n\}$ in $[0,1]$ be a monotonic sequence that converges to ξ. Then, $\{c^P(\xi_n)\}$ is a monotonic sequence that is bounded below by 0 and above by $c^P(1)$. Therefore, $\{c^P(\xi_n)\}$ converges to a unique y such that $\frac{V(y)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(y + \frac{1}{2}) + \rho\xi v_b^*$. Hence $\{c^P(\xi_n)\}$ converges to $c^P(\xi)$ and so $c^P(.)$ is a continuous function of ξ. ■

Theorem 25 *Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$, where ρ^* is the value of ρ that conditions growth in a dictatorship. Let $\tilde{\rho}^{\xi=0}$ be the value of ρ that conditions growth in the $\xi = 0-$economy. Then, there exists $\xi^*(\rho) \in [0,1]$ such that:*

1. *If $\rho \leq \tilde{\rho}^{\xi=0}$, then the $\xi-$economy does not grow, $\forall \xi \in [0,1]$.*

2. *If $\rho > \tilde{\rho}^{\xi=0}$, then the the $\xi-$economy grows iff $\xi < \xi^*(\rho)$.*

Proof. Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$. For any ξ in $[0,1]$, let $c^P(\xi)$ be defined by: $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi) + \frac{1}{2}) + \rho\xi v_b^*$ and define $c^{HH}(\xi) = \frac{1}{2} - c^P(\xi)$. Recall the $\xi-$economy grows iff $\frac{U(c^{HH}(\xi))+\rho\lambda\xi u_b^*}{1-\rho\lambda(1-\xi)} > u_b^*$, which holds iff $\frac{U(c^{HH}(\xi))}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. First suppose $\rho \leq \tilde{\rho}^{\xi=0}$. Then, the $0-$economy does not grow and $\frac{U(c^{HH}(0))}{1-\rho\lambda} \leq \frac{U(\frac{1}{2})}{1-\rho}$. But by lemma 22, $c^{HH}(\xi) < c^{HH}(0)$, $\forall \xi \in (0,1]$ and hence, $\frac{U(c^{HH}(\xi))}{1-\rho\lambda} < \frac{U(\frac{1}{2})}{1-\rho}$. That is, $\forall \xi \in [0,1]$, the economy never grows if $\rho \leq \tilde{\rho}^{\xi=0}$.

Now, suppose $\rho > \tilde{\rho}^{\xi=0}$, so that $\frac{U(c^{HH}(0))}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. Recall from lemma 19 that for $\xi = 1$, $c^{HH}(1) = c_D^{HH}$, and $\frac{U(c_D^{HH})}{1-\rho\lambda} \leq \frac{U(\frac{1}{2})}{1-\rho}$, given $\rho \leq \rho^*$. Therefore, since $c^{HH}(.)$ is continuous and strictly increasing in ξ, there exists $\xi^*(\rho) \in (0,1]$ such that the economy grows iff $\xi < \xi^*(\rho)$. This ends the proof of the theorem.

■ To summarize, if $\rho \leq \tilde{\rho}^{\xi=0}$, then the ξ-economy does not grow for any value of ξ. If $\tilde{\rho}^{\xi=0} < \rho \leq \rho^*$, then the economy grows iff $\xi \leq \xi^*(\rho)$. If instead $\rho > \rho^*$, then the economy always grows. That is, a democratic economy which does not grow will also not grow if it were to shift toward dictatorship. A dictatorial economy which grows will also grow after democratization. For intermediate economies where agents are patient enough to allow growth under democracy but not sufficiently patient to allow growth under dictatorship, there exists a threshold probability of falling into dictatorship such that the economy grows if and only if it is below that threshold probability.

Conclusion

This paper has characterized necessary conditions for growth when technological progress is available and free, but requires the approval of self-interested politicians to be adopted. When they have a low discount factor, politicians in power find it optimal to stop technological progress in exchange for static rewards that the representative citizen does not control. This paper has shown that among economies where agents have the same degree of time preferences, those that grow are those with the strongest political institutions: the lowest probabilities of occurrence of a coup d'etat and the lowest probabilities of falling in the state of dictatorship. It has also been proved that citizens of a dictatorial economy will always gain from a one-shot transition to democracy, even in the case where the economy is already growing as a dictatorship. In fact, an economy that grows as a dictatorship will also grow for any given probability of falling into the state of dictatorship. Moreover, the smaller the probability of falling in the state of dictatorship, the smaller the minimal compensation that makes a politician in power willing to approve growth in a given period. This in turn implies that payoffs to the citizens increase as the likelihood of falling in the state of dictatorship decreases. Understanding the emergence of political structures is a next step that will further our understanding of growth mechanisms. For this purpose, extending the current framework by endogenizing the choice of political regimes appears a promising avenue for future research. In particular, it would be interesting to construct a model where agents' beliefs lead to two possible equilibria: one with a low probability of coups d'etat (or a low probability of transiting to dictatorship) and economic growth and one with a high probability of coups d'etat (or a high probability of transiting to dictatorship) and economic stagnation. I plan to undertake this task in the future.

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Lemma 26 *Fixing* λ*,*

Let's define $\underline{\delta}$ *and* $\overline{\delta}$ *by:* $\frac{V(\frac{1}{2})}{V(1)} = 1 - \underline{\delta}\lambda$ *and* $0 = \frac{V(0)}{V(1/2)} = 1 - \overline{\delta}\lambda$

($\underline{\delta}$ *and* $\overline{\delta}$ *are well-defined since* $1 - \delta\lambda$ *is continuous and strictly decreasing as a function of* δ*, with values in* $[0, 1]$ *for* $\delta \in [0, \frac{1}{\lambda}]$*.)*

Then, for given λ*, there exists a function* f_λ *of* δ *from* $[0, \frac{1}{\lambda})$*to* $[0, 1]$ *s.t.:*

$$\begin{cases} f_\lambda(\delta) = 0, \quad \forall \delta \in [0, \underline{\delta}) \\ f_\lambda(\underline{\delta}) = \frac{U(0)}{U(1/2)} \\ f_\lambda \text{ strictly increasing and continuous in } [\underline{\delta}, \overline{\delta}] \text{ with values in } [\frac{U(0)}{U(1/2)}, 1] \\ f_\lambda(\delta) = 1, \forall \delta \in [\overline{\delta}, \frac{1}{\lambda}) \end{cases}$$

and s.t.

$\theta^* = 0 \; iff \; f_\lambda(\delta) \geq \frac{1-\beta\lambda}{1-\beta}$.

Appendix

Proof. of lemma 5

By lemma 3, best SPE sequence with no replacement of the initial politician are solutions to the SPE problem defined below as:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^\infty} \sum_{\tau=0}^\infty \beta^\tau \lambda^{S_\tau} U[c_\tau^{HH}]$$

s.t.

$$\sum_{\tau=0}^{\infty}\beta^{\tau}\lambda^{S_{t+\tau}}U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta}U(1/2), \forall t \geq 0 \tag{3.39}$$

$$\sum_{\tau=0}^{\infty}\delta^{\tau}\lambda^{S_{t+\tau}}V[c_{t+\tau}^{P} + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t}V[c_t^P + \frac{1}{2}], \forall t \geq 0 \tag{3.40}$$

S_0 given, $S_{t+1} = S_t + 1 - \theta_t$, and $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty}$ feasible.

After using the law of motion of the technological frontier and simplifying S_t from both sides of the inequality constraints After some algebra, the above problem simplifies to:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty}} \lambda^{S_0}[U[c_0^{HH}] + \sum_{\tau=1}^{\infty}\beta^{\tau}\lambda^{\tau-\theta_0-\ldots-\theta_{\tau-1}}U[c_\tau^{HH}]]$$

s.t.

$$\sum_{\tau=0}^{\infty}\beta^{\tau}\lambda^{\tau-\theta_t-\ldots-\theta_{t+\tau-1}}U[c_{t+\tau}^{HH}] \geq \frac{U(1/2)}{1-\beta}, \forall t \geq 0 \tag{3.41}$$

$$\sum_{\tau=0}^{\infty}\delta^{\tau}\lambda^{\tau-\theta_t-\ldots-\theta_{t+\tau-1}}V[c_{t+\tau}^{P} + \frac{\theta_{t+\tau}}{2}] \geq V[c_t^P + \frac{1}{2}], \forall t \geq 0, \tag{3.42}$$

S_0 given and $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty}$ feasible.

Now, recall that if $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty}$ is solution to the SPE problem, then for all $s \geq 0$, $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}$ must solve the date s SPE problem defined by:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}} \sum_{\tau=s}^{\infty}\beta^{\tau-s}\lambda^{S_\tau}U[c_\tau^{HH}]$$

s.t.

$$\sum_{\tau=s}^{\infty}\beta^{\tau-s}\lambda^{S_{t+\tau}}U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta}U(1/2), \forall t \geq 0 \tag{3.43}$$

$$\sum_{\tau=s}^{\infty}\delta^{\tau-s}\lambda^{S_{t+\tau}}V[c_{t+\tau}^{P} + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t}V[c_t^P + \frac{1}{2}], \forall t \geq 0 \tag{3.44}$$

S_s given, $S_{\tau+1} = S_\tau + 1 - \theta_\tau$, for all $\tau \geq s$ and $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}$ feasible.

After using the law of motion of the technological frontier and simplifying S_t from both sides of the inequality constraints date s SPE problem above simplifies to:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}} \lambda^{S_s}[U[c_s^{HH}] + \sum_{\tau=s+1}^{\infty} \beta^{\tau-s}\lambda^{\tau-\theta_0-\ldots-\theta_{\tau-1}}U[c_\tau^{HH}]]$$

s.t.

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{\tau-\theta_t-\ldots-\theta_{t+\tau-1}}U[c_{t+\tau}^{HH}] \geq \frac{U(1/2)}{1-\beta}, \forall t \geq 0 \quad (3.45)$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{\tau-\theta_t-\ldots-\theta_{t+\tau-1}}V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] \geq V[c_t^P + \frac{1}{2}], \forall t \geq 0, \quad (3.46)$$

S_s given, and $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}$ feasible.

■

$$\Psi_{ND}^{HH} = \max_{\{c^{HH}, c^P\}} U(c^{HH}) + \rho\lambda((1-\xi)\Psi_{ND}^c + \xi\Psi_D^c)$$

s.t.

$$U(c^{HH}) + \rho\lambda((1-\xi)\Psi_{ND}^c + \xi\Psi_D^c) \geq \frac{U(1/2)}{1-\rho} \quad (3.47)$$

$$\Psi_{ND}^P = V(c^P) + \rho\lambda((1-\xi)\Psi_{ND}^P + \xi\Psi_D^P) \quad (3.48)$$

$$\Psi_{ND}^P \geq V(c^P + \frac{1}{2}) + \rho\xi\frac{V(1/2)}{1-\rho}. \quad (3.49)$$

Appendix 2

Proof of stationarity for the coup d'etat economy.

Proof. of theorem 6

Define function h by $h(c_P) = \frac{V(c_P)}{V(c_P+\frac{1}{2})}$. Clearly, h is a continuous function. I now argue that h is strictly increasing and therefore invertible as a continuous bijection.

Because V is differentiable, so is h, with derivative given by:

$h^{'}(c_P) = \frac{V^{'}(c_P)V(c_P+\frac{1}{2})-V^{'}(c_P+\frac{1}{2})V(c_P)}{V(c_P+\frac{1}{2})^2}$.

Now, by strict concavity of V, $V^{'}(c_P+\frac{1}{2}) \leq V^{'}(c_P)$, such that $V^{'}(c_P)V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) \geq V^{'}(c_P+\frac{1}{2})V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) > 0$ since V is strictly increasing. It follows that h is strictly increasing and therefore invertible as a continuous bijection.

Define $\underline{\delta}$ by: $h(\frac{1}{2}) = \frac{V(\frac{1}{2})}{V(1)} = 1 - \underline{\delta}\lambda$.

Case 1: $\delta \in [0, \underline{\delta}]$

Then, $h(\frac{1}{2}) = \frac{V(\frac{1}{2})}{V(1)} = 1 - \underline{\delta}\lambda$ implies that for all $\delta \in [0, \underline{\delta}]$ the politician in power will always choose $\theta = 1$, even if he is being paid $c_P = \frac{1}{2}$ in all periods. That is, when $\delta \in [0, \underline{\delta}]$, politicians are extremely impatient and would not allow for technological growth, even if they were paid the total endowments available. It is therefore optimal for the representative citizen in this case to set $c_P = 0$ in all periods. So when $\delta \in [0, \underline{\delta}]$, $c_P^* = 0$ and the economy never grows: $\theta^* = 1$ in all periods.

Case 2: $\delta \in (\underline{\delta}, 1/\lambda)$

$\delta \in (\underline{\delta}, 1/\lambda) \Rightarrow h^{-1}(\frac{1}{1-\delta\lambda}) \in [0, 1/2]$. Now define function g by $g(\beta) = \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [0, 1/\lambda)$. Because g is strictly decreasing with values in $(0, 1]$ for $\beta \in [0, 1/\lambda)$, let's define $\beta^*(\delta)$ by $\frac{U[\frac{1}{2}-h^{-1}(\frac{1}{1-\delta\lambda})]}{U(1/2)} = g(\beta^*(\delta))$. Then, $U[\frac{1}{2} - h^{-1}(\frac{1}{1-\delta\lambda})] \leq \frac{1-\beta\lambda}{1-\beta}$ $\forall$ $\beta \in [0, \beta^*(\delta))$ and $U[\frac{1}{2} - h^{-1}(\frac{1}{1-\delta\lambda})] \geq \frac{1-\beta\lambda}{1-\beta}$ $\forall$ $\beta \in [\beta^*(\delta), 1/\lambda)$.

This implies, $\frac{U[\frac{1}{2}-h^{-1}(\frac{1}{1-\delta\lambda})]}{1-\beta} \leq \frac{U[1/2]}{1-\beta}$ $\forall$ $\beta \in [0, \beta^*(\delta))$ and $\frac{U[\frac{1}{2}-h^{-1}(\frac{1}{1-\delta\lambda})]}{1-\beta} \geq \frac{U[1/2]}{1-\beta}$ $\forall$ $\beta \in [\beta^*(\delta), 1/\lambda)$.

Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [\beta^*(\delta), 1/\lambda)$, then $c_P^* = h^{-1}(\frac{1}{1-\delta\lambda}) \in (0, 1/2)$ and $\theta^* = 0$ (the economy always grows).

If instead $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta))$, then $\frac{U[\frac{1}{2}-h^{-1}(\frac{1}{1-\delta\lambda})]}{1-\beta} < \frac{U[1/2]}{1-\beta}$: the representative citizen fails to achieve his outside option value $\frac{U[1/2]}{1-\beta}$ by giving the politician in power just enough to satisfy his sustainability constraint. Now, recall that $h^{-1}(\frac{1}{1-\delta\lambda}) \in (0, 1/2)$ is the unique stationary payment to the politician which makes the sustainability constraint hold with equality. So, in this case, for any scheme with a stationary and sustainable payment to the politician, the lifetime utility of the representative citizen will always be strictly less than $\frac{U[1/2]}{1-\beta}$. Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta))$, then $c_P^* = 0$ and $\theta^* = 1$ (the economy never grows).

Q.E.D.

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Appendix A

Political Economy Theory of Growth

Proof. of Lemma 5

By lemma 3, best SPE sequences with no replacement of the initial politician are solutions to the SPE problem defined as:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty} \text{ feasible}} \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_\tau} U[c_\tau^{HH}] \qquad s.t.$$
$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0$$
$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^{P} + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0$$

Now, observe that if $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^{\infty}$ is solution to the SPE problem, then for all $s \geq 0$, $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^{\infty}$ must solve the date s SPE problem defined by:

$$\max_{\{c^{HH}_{s+\tau}, c^{P}_{s+\tau}, \theta_{s+\tau}\}_{\tau=0}^{\infty} \text{ feasible}} \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{s+\tau}} U[c^{HH}_{s+\tau}]$$

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{s+t+\tau}} U[c^{HH}_{s+t+\tau}] \geq \frac{\lambda^{S_{s+t}}}{1-\beta} U(1/2), \forall t \geq 0$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{s+t+\tau}} V[c^{P}_{s+t+\tau} + \frac{\theta_{s+t+\tau}}{2}] \geq \lambda^{S_{s+t}} V[c^{P}_{s+t} + \frac{1}{2}], \forall t \geq 0$$

$$S_s \text{ given}, S_{s+\tau+1} = S_{s+\tau} + 1 - \theta_{s+\tau}, \forall \tau \geq 0.$$

After using the law of motion of the technological frontier and simplifying S_{s+t} from both sides of the inequality constraints, the above problem reduces to:

$$\max_{\{c^{HH}_{s+\tau}, c^{P}_{s+\tau}, a_{s+\tau}, \theta_{s+\tau}\}_{\tau=0}^{\infty}} \lambda^{S_s} [U[c^{HH}_s] + \sum_{\tau=1}^{\infty} \beta^\tau \lambda^{a_s + \ldots + a_{s+\tau-1}} U[c^{HH}_{s+\tau}]]$$

s.t.

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{a_{s+t} + \ldots + a_{s+t+\tau-1}} U[c^{HH}_{s+t+\tau}] \geq \frac{U(1/2)}{1-\beta}, \forall t \geq 0 \tag{A.3}$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{a_{s+t} + \ldots + a_{s+t+\tau-1}} V[c^{P}_{s+t+\tau} + \frac{1}{2} - \frac{a_{s+t+\tau}}{2}] \geq V[c^{P}_{s+t} + \frac{1}{2}], \forall t \geq 0, \tag{A.4}$$

$$\theta_{s+\tau} = 1 - a_{s+\tau}, \forall \tau \geq 0, \tag{A.5}$$

S_s given, $S_{s+\tau+1} = S_{s+\tau} + 1 - \theta_{s+\tau}$, for all $\tau \geq 0$ and $\{c^{HH}_{s+t+\tau}, c^{P}_{s+t+\tau}, \theta_{s+t+\tau}\}_{\tau=0}^{\infty}$ feasible.

■

Now, for a contradiction, suppose some date $s+1$ best SPE sequence $\{\hat{c}^{HH}_{s+1+\tau}, \hat{c}^{P}_{s+1+\tau}, \hat{\theta}_{s+1+\tau}\}_{\tau=0}^{\infty}$ yields strictly higher payoff to citizens than some date s best SPE sequence $\{c^{HH}_{s+\tau}, c^{P}_{s+\tau}, \theta_{s+\tau}\}_{\tau=0}^{\infty}$. Then, the new sequence $\{c^{*HH}_{s+\tau}, c^{*P}_{s+\tau}, \theta^{*}_{s+\tau}\}_{\tau=0}^{\infty} \equiv \{\hat{c}^{HH}_{s+1+\tau}, \hat{c}^{P}_{s+1+\tau}, \hat{\theta}_{s+1+\tau}\}_{\tau=0}^{\infty}$ satisfies the inequality constraints of the date s best SPE problem and yields strictly higher date s payoff to citizens, which contradicts that the original date s sequence was a best SPE sequence.

By a symmetric argument, no date s best SPE sequence yields a strictly higher payoff than a date $s+1$ best SPE sequence. Therefore, best SPE sequences in all periods yield the same payoff to citizens.

Now, define $A_\tau = \{c_\tau^{HH}, c_\tau^P, \theta_\tau\}$, for all $\tau \geq 0$. It therefore follows that there exists a best SPE sequence in which $\{A_\tau\}_{\tau=0}^{\infty} = \{A_\tau\}_{\tau=1}^{\infty} = \{A_\tau\}_{\tau=2}^{\infty} = ... = \{A_\tau\}_{\tau=s}^{\infty}$ ... implying $\{A_\tau\} = A^*$, $\forall \tau \geq 0$. That is, there exists a best SPE sequence that is stationary: $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\} = \{c_\tau^{*HH}, c_\tau^{*P}, \theta_\tau^*\}$, $\forall \tau \geq 0$.

Appendix B

Bankruptcy Reform and Foreclosure Crisis During The Great Recession

B.0.2 Classification of States

The recourse/non-recourse classification is from Ghent and Kudlyak (2011). A state is classified as a high homestead state if its homestead exemption is larger than its 2002 median income.

Table B.1: Classification of U.S. States

	If Recourse	If High Homestead
Alaska	NO	YES
Alabama	YES	NO
Arkansas	YES	NO
Arizona	NO	YES
California	NO	YES
Colorado	YES	YES
Connecticut	YES	YES
Delaware	YES	NO
Florida	YES	YES

Georgia	YES	NO
Hawaii	YES	NO
Iowa	NO	YES
Idaho	YES	YES
Illinois	YES	NO
Indiana	YES	NO
Kansas	YES	YES
Kentucky	YES	NO
Louisiana	YES	NO
Massachusetts	YES	YES
Maryland	YES	NO
Maine	YES	NO
Michigan	YES	NO
Minnesota	NO	YES
Missouri	YES	NO
Mississippi	YES	YES
Montana	NO	YES
North Carolina	NO	NO
North Dakota	NO	YES
Nebraska	YES	YES
New Hampshire	YES	YES
New Jersey	YES	NO
New Mexico	YES	NO
Nevada	YES	YES
New York	YES	YES
Ohio	YES	NO
Oklahoma	YES	NO
Oregon	YES	NO
Pennsylvania	YES	NO
Rhode Island	YES	YES
South Carolina	YES	YES
South Dakota	YES	NO

Tennessee	YES	NO
Texas	YES	YES
Utah	YES	NO
Virginia	YES	NO
Vermont	YES	YES
Washington	NO	YES
Wisconsin	NO	YES
West Virginia	YES	NO
Wyoming	YES	NO

B.0.3 Lists of States in Subgroups 1 and 2

Subgroup 1 is made of 12 recourse states (Illinois, Georgia, Ohio, Idaho, New Jersey, New Hampshire, Connecticut, Virginia, Massachusetts, Delaware, Hawaii and Utah.) and 3 non-recourse states (Oregon, Minnesota, and Washington). Subgroup 1 is made of 6 high homestead exemption states (Idaho, New Hampshire, Minnesota, Connecticut, Washington, and Massachusetts), and of 9 low homestead exemption states (Illinois, Oregon, Georgia, Ohio, New Jersey, Virginia, Delaware, Hawaii, Utah).

Subgroup 2 is made of 9 recourse states (Idaho, New Jersey, New Hampshire, Connecticut, Virginia, Massachusetts, Delaware, Hawaii and Utah.) and 2 non-recourse states (Minnesota, and Washington). Subgroup 2 is made of 6 high homestead exemption states[1] (Idaho, New Hampshire, Minnesota, Connecticut, Washington, and Massachusetts), and of 5 low homestead exemption states (New Jersey, Virginia, Delaware, Hawaii, Utah).

B.0.4 Summary Statistics for the High and Low Homestead Exemption Groups

The following table summarizes statistics for U.S. states separated into high-homestead and low-homestead groups. A state qualifies as a high-homestead state if the ratio of its homestead exemption over its 2001- median income is higher than 1. Among all

[1] All the high homestead states of subgroup 1 have an unemployment rate below the 2009 US average

51 states, 40 are recourse states and 24 are high-homestead states. Column (1) gives the median number of days of delinquency prior to foreclosure for loans that entered foreclosure in the year of interest. Column (2) shows the means of the number of days of delinquency prior to bankruptcy filing for loans that in the relevant year entered bankruptcy after being delinquent for a minimum of 90 days and that had not started foreclosure by the end of April 2012. Column (3) gives the average downpayment at origination for each year. Column (4) shows the mean net-interest rate at origination for loans originated in the year of interest. The net-interest rate is defined as the difference between the gross interest rate and the service-fee rate charged for securitized transactions.

Table B.2: Summary Statistics Across the High and Low-Homestead Groups

(a) High-Homestead States

	(1)	(2)	(3)	(4)
2002	385	238	21.8%	5.83%
2003	384	251	24.6%	5.17%
2004	401	230	25.5%	5.37%
2005	466	253	26.0%	5.46%
2006	511	285	27.6%	6.08%
2007	658	285	29.3%	6.02%
2008	737	267	31.3%	5.70%
2009	646	286	31.3%	4.77%
2010	496	358	29.5%	4.47%

(b) Low-Homestead States

	(1)	(2)	(3)	(4)
2002	449	253	21.2%	5.81%
2003	427	245	22.6%	5.16%
2004	444	243	22.3%	5.40%
2005	507	258	23.1%	5.51%
2006	511	311	24.1%	6.17%
2007	587	303	25.8%	6.12%
2008	674	295	25.3%	5.73%
2009	628	295	26.1%	4.76%
2010	501	345	26.2%	4.45%

B.0.5 Flow Values and Profit Functions of Lenders

As in the text, Z and s respectively denote the economy's aggregate state and a household's idiosyncratic state. Let $\Omega'(Z, s)$ be tomorrow's optimal non-stochastic component of the individual's state vector given current state (Z,s) as induced by optimal choices. If $s = \{j, \eta, a, b, \epsilon, \xi, x, \zeta, o, (\theta, i, M, \tau)\}$ and $\Omega'(Z, s) = \{j', 0, a', b', o', (\theta', i', M', \tau')\}$, then let $\hat{\Omega}'(Z, s) = \{j', \eta, a', b', o', (\theta', i', M', \tau')\}$.

$g_b(\Omega'(Z, s))$ denotes the face value of the second mortgage bond optimally purchased in the current period by a household in state (Z,s) and $g_a(\Omega'(Z,s))$ denotes the face value of the unsecured bond optimally purchased in the current period by a household in state (Z,s).

Let $\mathbb{1}_{\{inv\}}$ be an indicator function that takes on value 1 if the household defaults involuntarily, and that takes on value 0 otherwise.

If the household starts as a homeowner, then let $\mathbb{1}_{\{sell\}}$ equal 1 if a household's house is sold at the beginning of the period either voluntarily by the household, or upon the household's death, and equal 0 otherwise. Similarly, let $\mathbb{1}_{\{toss\}}$ indicate if a homeowner chooses to file for bankruptcy and to toss its house. Let $\mathbb{1}_{\{toss\}}$ indicate if a homeowner chooses to file for bankruptcy and to keep its house. Let $\mathbb{1}_{\{walk\}}$ indicate if a homeowner chooses to walk away without filing for bankruptcy. and let $\mathbb{1}_{\{curr\}}$ indicate if it chooses to stay current. For a non-owner, let $\mathbb{1}_{\{walk\}}$ indicate if it chooses to file for bankruptcy.

If the household starts as a non-owner, let $\mathbb{1}_{\{bkrupt\}}$ equal 1 if the household chooses to file for bankruptcy and equal 0 otherwise.

Upon a default, the household's after-bankruptcy adjusted cash-in-hand position is equally distributed among the household's unsecured creditors which are: the unsecured bond lender, the expense shock sector, and also the household's first and second mortgage lenders if the household was subject to a deficiency shock.

Let a_1^{toss} and a_2^{toss} equal the negatives of the mortgage debts still owed to the first and second mortgage lenders after exhaustion of the expected discounted sale proceeds after a household has tossed out its house a bankruptcy:

$$\begin{cases} a_1^{toss} & = \min\{0, \frac{(1-\phi)E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M)\} \text{ if } \zeta = \zeta_2 \\ a_1^{toss} & = 0 \text{ if } \zeta = \zeta_1 \\ a_2^{toss} & = \min\{0, \max\{0, \frac{(1-\phi)E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M)\} - b\} \text{ if } \zeta = \zeta_2 \\ a_2^{toss} & = 0 \text{ if } \zeta = \zeta_1. \end{cases}$$

Define

$$\begin{cases} \alpha_1^{toss} & = \frac{a_1^{toss}}{a_1^{toss}+a_2^{toss}+\min\{0,a\}-x} \\ \alpha_2^{toss} & = \frac{a_2^{toss}}{a_1^{toss}+a_2^{toss}+\min\{0,a\}-x} \\ \alpha_3^{toss} & = \frac{\min\{0,a\}}{a_1^{toss}+a_2^{toss}+\min\{0,a\}-x} \end{cases}$$

Also recalling that second mortgage loans are treated as unsecured debts in bankruptcy when the first mortgage is under water, define:

$$\begin{cases} a_2^{toss} & = -b \text{ if } P(Z) - \ell_0 < 0 \\ a_2^{toss} & = 0 \quad \text{if } P(Z) - \ell_0 \geq 0 \end{cases}$$

and

$$\begin{cases} \alpha_1^{keep} & = \frac{\min\{0,a\}}{a_2+\min\{0,a\}-x}. \\ \alpha_2^{keep} & = \frac{a_2}{a_2+\min\{0,a\}-x} \end{cases}$$

Finally, for involuntary defaulters, let

$$\begin{cases} a_1^{inv} & = \min\{0, (1-\phi)P(Z)h_i - \ell_0\} \text{ if } \zeta = \zeta_2 \\ a_1^{inv} & = 0 \text{ if } \zeta = \zeta_1 \\ a_2^{inv} & = \min\{0, \max\{0, (1-\phi)P(Z)h_i - \ell_0\} - b\} \text{ if } \zeta = \zeta_2 \\ a_2^{inv} & = 0 \text{ if } \zeta = \zeta_1. \end{cases}$$

Define

$$\begin{cases} \alpha_1^{inv} & = \frac{a_1^{inv}}{a_1^{inv}+a_2^{inv}-x} \\ \alpha_2^{inv} & = \frac{a_2^{inv}}{a_1^{inv}+a_2^{inv}-x} \\ \alpha & = \frac{\min\{0,a\}}{\min\{0,a\}-x}. \end{cases}$$

If the household starts the period as a homeowner, let $W_{m_1,Z}(s)$ denote the expected discounted flows to the first mortgage lender which has loaned to the household. Then,

$$\begin{aligned} W_{m_1,Z}(s) = & \mathbb{1}_{\{sell\}} \times \ell(i, r_m, \tau, \theta) \\ & + \mathbb{1}_{\{curr\}} \times W_{m_1,Z}^{curr}(s) \\ & + \mathbb{1}_{\{keep\}} \times W_{m_1,Z}^{keep}(s) \\ & + \mathbb{1}_{\{toss\}} \times W_{m_1,Z}^{toss}(s) \\ & + \mathbb{1}_{\{walk\}} \times W_{m_1,Z}^{walk}(s) \\ & + \mathbb{1}_{\{inv\}} \times W_{m_1,Z}^{inv}(s). \end{aligned}$$

I next define $W_{m_1,Z}^{curr}$, $W_{m_1,Z}^{toss}$, $W_{m_1,Z}^{keep}$, $W_{m_1,Z}^{walk}$, and $W_{m_1,Z}^{inv}$. Let $W_{m_1} = (W_{m_1,B}, W_{m_1,R})$.

Then,

$$W^{curr}_{m_1,Z}(s) = \begin{cases} M + \frac{(\lambda\times(E_{Z|\xi}W_{m_1})(\Omega'(Z,s))+(1-\lambda)\times(E_{Z|\xi}W_{m_1})(\hat{\Omega}'(Z,s)))}{1+r} & \text{if } \eta \neq 0 \\ M + \frac{(\lambda\times(E_{Z|\xi}W_{m_1})(\Omega'(Z,s))+(1-\lambda)\times(E_{Z|\xi}W_{m_1})(\hat{\Omega}'(Z,s)))}{1+r} & \text{if } \eta \neq 0 \end{cases}$$

and,

$$W^{keep}_{m_1,Z}(s) = M + \frac{(E_{Z|\xi}W_{m_1})(\Omega'(Z,s))}{1+r}.$$

$$\begin{aligned} W^{toss}_{m_1,Z} = {} & \mathbb{1}_{\{\zeta=\zeta_1\}} \times ((1-\zeta)M + \min\{\ell_0 - (1-\zeta)M, \frac{(1-\phi)E(p)h}{1+r}\}) \\ & + \mathbb{1}_{\{\zeta=\zeta_2\}} \times ((1-\zeta)M + \min\{\ell_0 - (1-\zeta)M, \frac{(1-\phi)E(p)h}{1+r}\}) \\ & + \mathbb{1}_{\{\zeta=\zeta_2\}} \times \min\{\max\{0, \ell_0 - (1-\zeta)M - \frac{(1-\phi)E(p)h}{1+r}\}, \max\{0,a\} \times \alpha_1^{toss}\} \\ & + \mathbb{1}_{\{\zeta=\zeta_2\}} \times (-D_{toss}(Z,s) \times \alpha_1^{toss}) \\ & \qquad \times (1 + \frac{1 - E_{Z|\xi}(\mathbb{1}_{\{inv\}}(B,.), \mathbb{1}_{\{inv\}}(R,.))(\Omega'(Z,s))}{1+r}) \end{aligned}$$

$$\begin{aligned} W^{walk}_{m_1,Z} = {} & \mathbb{1}_{\{\zeta=\zeta_1\}} \times ((1-\zeta)M + \min\{\ell_0 - (1-\zeta)M, \frac{(1-\phi)E(p)h}{1+r}\}) \\ & + \mathbb{1}_{\{\zeta=\zeta_2\}} \times \ell(i, r_m, \tau, \theta). \end{aligned}$$

$$\begin{aligned} W^{inv}_{m_1,Z} = {} & \mathbb{1}_{\{\zeta=\zeta_1\}} \times (\min\{\ell_0, (1-\phi)P(Z)h_i\}) \\ & + \mathbb{1}_{\{\zeta=\zeta_2\}} \times (\min\{\ell_0, (1-\phi)P(Z)h_i\}) \\ & + \min\{\max\{0, \ell_0 - (1-\phi)P(Z)h_i\}, \max\{0,a\}\alpha_1^{inv}\} \end{aligned}$$

In the period of home purchase, a household may only stay current in its mortgage contracts. The expected profits made on mortgage a contract $(i', R_m(Z,\xi,\Omega'(Z,s)), 1, \mathbb{1}_{\{Z=R\}})$ made to a household in state (Z,s) and starting tomorrow with a non-stochastic state component $\Omega'(Z,s)$ are hence given by:

$$\Pi_{m_1}(Z,\xi,\Omega'(Z,s)) = -(1-\mu)P(Z)h_{i'} + \text{Mortgage}(i', R_m(Z,\xi,\Omega'(Z,s)), \mathbb{1}_{\{Z=R\}}) + \frac{W_{m_1,Z}(s)}{1+r}.$$

Flow Values and Profit Function to Second Mortgage Lenders

If the household is a homeowner, let $W_{m_2,Z}(s)$ denote the expected discounted flows to the second mortgage lender. Then

$$\begin{aligned} W_{m_2,Z}(s) = & (1-\mathbb{1}_{\{keep\}}-\mathbb{1}_{\{toss\}}-\mathbb{1}_{\{walk\}}-\mathbb{1}_{\{inv\}})\times b \\ & +\mathbb{1}_{\{keep\}}\times W^{keep}_{m_2,Z}(s) \\ & +\mathbb{1}_{\{toss\}}\times W^{toss}_{m_2,Z}(s) \\ & +\mathbb{1}_{\{walk\}}\times W^{walk}_{m_2,Z}(s) \\ & +\mathbb{1}_{\{inv\}}\times W^{inv}_{m_2,Z}(s), \end{aligned}$$

where $W^{toss}_{m_2,Z}$, $W^{keep}_{m_2,Z}$, $W^{walk}_{m_2,Z}$, and $W^{inv}_{m_2,Z}$ are defined next.

$$\begin{aligned} W^{toss}_{m_2,Z} = & \mathbb{1}_{\{\zeta=\zeta_1\}}\times(\min\{b,\max\{0,\frac{(1-\phi)E(p)h}{1+r}-(\ell_0-(1-\zeta)M)\}\}) \\ & +\mathbb{1}_{\{\zeta=\zeta_2\}}\times(\min\{b,\max\{0,\frac{(1-\phi)E(p)h}{1+r}-(\ell_0-(1-\zeta)M)\}\}) \\ & +\mathbb{1}_{\{\zeta=\zeta_2\}}\times\min\{\max\{0,b-\max\{0,\frac{(1-\phi)E(p)h}{1+r} \\ & \quad -(\ell_0-(1-\zeta)M)\}\},\max\{0,a\}\times\alpha_2^{toss}\} \\ & +\mathbb{1}_{\{\zeta=\zeta_2\}}\times(-D_{toss}(Z,s)\times\alpha_2^{toss}) \\ & \quad \times(1+\frac{1-E_{Z|\xi}(\mathbb{1}_{\{inv\}}(B,.),\mathbb{1}_{\{inv\}}(R,.))(\Omega'(Z,s))}{1+r}) \end{aligned}$$

$$\begin{aligned} W^{keep}_{m_2,Z} = & \mathbb{1}_{\{P(Z)h-\ell_0\geq 0\}}\times b \\ & +\mathbb{1}_{\{P(Z)h-\ell_0<0\}}\times(-D_{keep}(Z,s)\times\alpha_2^{keep}) \\ & \quad \times(1+\frac{1-E_{Z|\xi}(\mathbb{1}_{\{inv\}}(B,.),\mathbb{1}_{\{inv\}}(R,.))(\Omega'(Z,s))}{1+r}) \end{aligned}$$

$$W^{walk}_{m_2,Z} = \mathbb{1}_{\{\zeta=\zeta_1\}} \times \min\{b, \max\{0, \frac{(1-\phi)E(p)(Z)h_i}{1+r} - (\ell_0 - (1-\zeta)M)\}\}$$
$$+ \mathbb{1}_{\{\zeta=\zeta_2\}} \times b$$

$$W^{inv}_{m_2,Z} = \mathbb{1}_{\{\zeta=\zeta_1\}} \times \min\{b, \max\{0, (1-\phi)P(Z)h_i - \ell_0\}\}$$
$$+ \mathbb{1}_{\{\zeta=\zeta_2\}} \times \min\{b, \max\{0, (1-\phi)P(Z)h_i - \ell_0\}\}$$
$$+ \mathbb{1}_{\{\zeta=\zeta_2\}} \times \min\{\max\{0, b - \max\{0, (1-\phi)P(Z)h - \ell_0\}\}, \max\{0, a\} \times \alpha^{inv}_2\}$$

Let $W_{m_2} = (W_{m_2,B}, W_{m_2,R})$. Then, the expected profits to a second mortgage lender which lends at per unit price $Q_s(Z, \xi, \Omega'(Z, s))$ to a household that will start the following period with a non-stochastic state component $\Omega'(Z, s)$ are given by:

$$\Pi_{m_2}(Z, \xi, \Omega'(Z, s)) = -Q_s(Z, \xi, \Omega'(Z, s)) \times g_b(\Omega'(Z, s)) + \frac{(E_{Z|\xi} W m_2)(\Omega'(Z, s))}{1+r}.$$

Flow Values and Profit Function to Unsecured Lenders

Let $W_{u,Z}(s)$ denote the expected discounted flows to the unsecured lender.

Case 1: The Household is a homeowner

Then,

$$W_{u,Z}(s) = -\max\{0, a\}$$
$$+ (1 - \mathbb{1}_{\{keep\}} - \mathbb{1}_{\{toss\}} - \mathbb{1}_{\{inv\}}) \times -\min\{0, a\}$$
$$+ \mathbb{1}_{\{keep\}} \times W^{keep}_{u,Z}(s) \times \mathbb{1}_{\{a<0\}}$$
$$+ \mathbb{1}_{\{toss\}} \times W^{toss}_{u,Z}(s) \times \mathbb{1}_{\{a<0\}}$$
$$+ \mathbb{1}_{\{inv\}} \times W^{inv}_{u,Z}(s) \times \mathbb{1}_{\{a<0\}},$$

where for $H = \max\{0, (\frac{(1-\phi).E(p)h_i}{1+r} - (\ell_0 - (1-\zeta)M(i, r_m)) - b)\}$ and for $H_n = H - \min\{E, H\}$,

$$W_{u,Z}^{toss}(s) = \alpha_3^{toss} \times \min\{-A_{toss}(Z,s), H_{ne}\}$$
$$- \alpha_3^{toss} \times (-D_{toss}(Z,s) + \frac{E_{Z|\xi}(W_u)(\Omega'(Z,s))}{1+r})$$
$$W_{u,Z}^{keep}(s) = \alpha_1^{keep} \times (-D_{keep}(Z,s) + \frac{E_{Z|\xi}(W_u)(\Omega'(Z,s))}{1+r})$$
$$W_{u,Z}^{inv}(s) = \min\{-a, \alpha \times \max\{0, (1-\phi)P(Z)h_i - E\}\}.$$

Case 2: The Household is a non-owner

Then,

$$W_{u,Z}(s) = -\max\{0,a\}$$
$$+ (1 - \mathbb{1}_{\{bkrupt\}} - \mathbb{1}_{\{inv\}}) \times -\min\{0,a\}$$
$$+ \mathbb{1}_{\{bkrupt\}} \times \frac{\min\{a,0\}}{\min\{a,0\} - x} \times \left(-D_{no}(Z,s) + \frac{E_{Z|\xi}(W_u)(\Omega'(Z,s))}{1+r}\right) \times \mathbb{1}_{\{a<0\}}.$$

Let $W_u = (W_{u,B}, W_{u,R})$. Then, the expected profits to an unsecured lender which lends at per unit price $Q_u(Z,\xi,\Omega'(Z,s))$ to a household that will start the following period with a non-stochastic state component $\Omega'(Z,s)$ are given by:

$$\Pi_u(Z,\xi,\Omega'(Z,s)) = Q_u(Z,\xi,\Omega'(Z,s)) \times g_a(\Omega'(Z,s)) + \Psi_{j+1}\frac{(E_{Z|\xi}Wu)(\Omega'(Z,s))}{1+r}.$$