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This report offers an economic analysis of the impact of road pricing. Optimal pricing of congested roads would produce substantial revenues and efficiency gains. However, the direct effect of road pricing would be to make most drivers worse off, particularly those with low incomes. In the Twin Cities, pricing all congested roads optimally would generate $1.50-$1.75 in revenues for each dollar of additional costs to travelers. The revenues offer a source of potential funding to compensate those who lose while leaving appreciable toll revenues for highway improvements and other public purposes. The authors believe that unless such toll-revenue redistribution occurs, opposition to road pricing will be substantial.

Researchers calculated network equilibria for a variety of congestion-pricing and analyzed these potential income-distributional effects. They allowed the demand for travel to be price-sensitive and for drivers to differ in the valuations they place on time. Pricing all congested roads optimally would increase total travel costs by 18-42 percent, depending on the elasticity of demand for travel. With unit-elastic demand, pricing would increase travel costs by 31 percent and 5 percent for, respectively, the lowest and highest income groups examined.

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Congestion Costs and Congestion Pricing for the Twin Cities

Final Report

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Peak-period urban travelers both experience congestion and create it. In making travel decisions, most consider the time and money costs they will experience; few consider the costs they impose on others by adding to the level of congestion. These imposed costs can be very large. Our results using Tranplan, a widely used transportation-planning package, are summarized in the figure. The line labeled *AC* (for average cost") depicts the cost a traveler directly experiences at alternative volume/capacity ratios. The line labeled *MC* (for marginal cost") adds to *AC* congestion

Demand   **MC**

Total ECAF = $252,830/day
Total ECDC = $388,285/day

B   AC

The "Gap":
Max CD = $0.21/mi   Max AB = $0.62/mi
Avg CD = $0.09/mi   Avg AB = $0.26/mi

E
F
G

25% max decline
12% avg decline

Dollars/Mile

0.0   0.2   0.4   0.6   0.8   1.0   1.2
Volume/Capacity Ratio

Morning Peak-Hour Congestion Costs, Tolls and Traffic Reductions:
Full-Price Elasticity = -1, All Congested Roads Tolled,
Time Value = $12.50 per Vehicle Hour

costs the traveler imposes on occupants of other vehicles. We call the vertical distance between the two curves *the gap*. If the average vehicle's occupants value their travel time at $12.50 an hour, their directly incurred time cost is 37¢ a vehicle mile while the gap, the costs they impose on all other travelers, averages 26¢ a mile. On the most congested 10-mile stretch

of freeway, the gap is 62¢ a vehicle mile. On a few scattered road links, it exceeds $5 a mile.

Presently available electronic technologies would allow tolling all 1,200-1,500 miles of congested roads in the Twin Cities without the enormous cost and inconvenience of toll booths and toll collectors. Available data suggest that a 1% increase in the time-plus-money cost of trips would leads to about a 1% reduction in the rate at which they are taken. If so, tolling all of these roads would reduce traffic volumes by about 12% on average and by about 25% on the most heavily congested stretches of freeway. On these stretches, congestion tolls would average about 21¢ a vehicle mile. On the average road, tolls would be about 9¢ a mile.

Very well-off drivers and present mass-transit users would benefit from the faster trips that reduced congestion would provide and from the increased service frequency that would result from diverting auto travelers to transit. However, imposing congestion tolls would make most travelers worse off; they would lose an aggregate of about $250,000 during the morning peak hour and about $1,000,000 daily from the higher time-plus-money prices of the trips they continue to take and from foregoing the relatively low-value trips they would no longer make at their new, higher prices.

At the same time, however, optimal congestion tolls would eliminate these low-value trips and would result in utilizing the road network more efficiently. As a result, toll revenues would exceed the direct losses congestion pricing would impose on travelers. If, to repeat, a 1% increase in the time-plus-money price of an auto trip would result in about a 1% reduction in the rate at which they are taken, total toll collections would be about $390,000 per weekday-morning peak hour and about $1,500,000 for the day as a whole. *To emphasize, tolling would yield $1.50-1.75 in revenue for each dollar of costs incurred by the average traveler. Thus, tolling would make it possible to compensate losers fully with substantial money left over. Because most would be made worse off, gaining support for congestion pricing from a majority of Twin Cities peak-period travelers would require coupling tolls with a plan for distributing toll revenues that would benefit them more than the tolls would cost them.*

A second transportation planning package, Emme/2, permits examining the effects of

congestion pricing on different income groups. We worked with four:

Income Bracket	Average Annual Household Income	Fraction of All Households	Travel-Time Value ($/hour)	Number of Trips (1,000s)	Time Cost ($1,000)
< $35,000	$25,900	36.8%	5.40	104.9	161.8
$35-55,000	$44,900	28.8%	11.25	213.4	724.5
$55-75,000	$65,000	16.2%	16.25	117.4	584.7
> $75,000	$87,520	18.2%	21.88	82.3	563.7
Totals		100.0%	12.88	518.0	2,034.7

If peak-hour auto-travel rates are completely independent of travel costs, low income travelers would have the worst of all worlds. Seeking uncongested routes to avoid tolls would result in their trips becoming so circuitous that they would be burdened not only by tolls, but also by spending more time on the road than they would in the absence of tolls; congestion pricing would increase their travel costs by 93% as opposed to 24% and 42% for the high-income group and all travelers respectively.

If congestion pricing is to be applied only to part of a congested road network, it would be inefficient to charge those who use its tolled portions close to the costs they impose on each other. Doing so would lead to inefficiently great diversion to the network's untolled portion. We have restricted attention to the effects of setting charges on each tolled link equal to the same fraction of the difference between that link's AC and MC curves, to use the figure's notation. In the Twin Cities, aggregate benefits do not vary greatly for fractions in the 20-40% range. With such tolls, travel-time savings largely compensate higher-income groups for the tolls they pay; for them, tolls paid are appreciably greater than their net losses from congestion pricing. Not so for the lowest-income group, however. Tolls are less than their losses; tolls induce them to take more circuitous trips which results in travel that is more costly to them in both dollars and time.

Obviously, tolling only expressways would shift traffic from them to the surface road network. Surprisingly, pricing all congested roads would also result in greater traffic

reductions on expressways than on surface streets. Specifically, expressway and non-expressway vehicle miles would respectively decline by 19% and 8% if a 1% increase in the time-plus-money cost of a trip would result in a 1% reduction in auto trips. With only expressways tolled at 25% of the difference between *MC* and *AC*, expressway vehicle miles would decline by 8% while arterial travel would increase by 3%.

In no place of which we are aware has a spontaneous ground swell arisen for congestion pricing. In San Francisco and the Twin Cities, at least, claiming either enhanced efficiency or solving a funds shortage generated little enthusiasm for the concept. It may still be possible, however, to sell congestion pricing by emphasizing an important implication of its efficiency: getting something for nothing. While our calculations suggest that the immediate effect of congestion pricing will be to make all but a small fraction of the population worse off, they also suggest that tolling the entire road network would generate $1.50-$1.75 in revenue for each dollar travelers would lose. This being the case, using electronic technologies that can reduce collection costs to a modest fraction of total revenues, there ought to be ways of compensating losers while leaving substantial resources to finance reduced real-estate, fuel, and other taxes as well as transportation projects. Finding such distribution schemes should be a primary emphasis of congestion-pricing research.

Chapter 1: Introduction

As peak-period urban travelers, we not only experience congestion, we create it. In deciding when, where, and how to travel, we take into account the time and money costs we will experience. However, few of us consider the costs our travel decisions impose on others by adding to congestion. These costs are very large on some roads. On the Twin Cities' most congested major stretch of expressway -- I-35W between I-35E and I-494 -- we estimate that each mile each northbound auto travels during the morning peak hour costs all other travelers about 62¢. For all morning-peak auto trips, the average cost each vehicle imposes on all others is about 26¢ a mile.

This is where congestion pricing comes in. Electronic technologies exist -- indeed, are presently in use -- that would allow tolling all 1,200-1,500 miles of congested roads in the Twin Cities without the enormous cost and inconvenience of toll booths and toll collectors. The optimum northbound peak-hour toll -- the toll that would maximize the net benefits that I-35W south of I-494 would deliver -- is about 21¢ a mile in the morning, we estimate. Imposing this toll would reduce auto travel on this stretch of road by about 25%. Optimal tolls averaging about 9¢ a mile would cut auto travel by about 12% on the entire road network. With congestion pricing, car pools and mass transit would become more popular. Trips not requiring peak-period departures or arrivals would be rescheduled. Businesses and schools would be pressed by employees and parents to shift opening and closing hours away from peak periods. Trip speeds would increase. Pressures to expand our road network would decline. Toll revenues of $1-1.5 million a day would be generated.

Regardless of how toll revenues are used, the very well-off -- those with $80,000-plus annual incomes -- would gain more from saving time than they would lose from paying tolls. Largely at the opposite end of the income distribution, present mass-transit users would benefit from the faster trips that reduced congestion would provide and from the increased service frequency that would result from diverting auto travelers to transit. For most of us in the middle of the income distribution, however, the direct effect of imposing congestion tolls

would be to make us worse off. Furthermore, the lower our incomes, the proportionately worse off tolling would make us. But tolling would improve traffic flows and would force off the road trips valued at less than their costs to society. For these reasons, toll revenues would exceed the costs that congestion tolling would impose on travelers. Specifically, we estimate that tolls would yield $1.50-1.75 in revenue for each dollar of costs they would impose on the average traveler. This being the case, toll revenues should suffice to compensate fully with cash those who lose directly from congestion pricing with substantial money left over for road and transit improvements, fuel and real-estate tax reductions, and even LRT.

The economic analysis that underlies congestion pricing has long been part of the economics literature; 1995 marks the 75th anniversary of recognition in this literature of its value. In the first edition of his masterwork, *The Economics of Welfare*, the great British economist, A. C. Pigou, maintained that, in producing commodities for which increasing output requires increasingly more intense utilization of particular scarce resources, unit costs increase with increases in aggregate output. He contended that unregulated markets would over-invest in producing such commodities. He supported this contention with an analogy between two industries and two roads that connect the same two towns:

> Suppose there are two roads ABD and ACD both leading from A to D. If left to itself, traffic would be so distributed that the trouble involved in driving a "representative" cart along each of the two roads would be equal. But, in some circumstances, it would be possible, by shifting a few carts from route B to route C, greatly to lessen the trouble of driving those still left on B. while only slightly increasing the trouble of driving along C. In these circumstances a rightly chosen measure of differential taxation and against road B would create an "artificial" situation superior to the "natural" one. [Pigou (1920), 194]

The situation Pigou described characterizes present-day peak-hour travel in the Twin Cities. Shifting a modest amount of traffic from expressways to arterials would reduce the travel time of the remaining expressway travelers by appreciably more than it would increase the travel time of those diverted plus those already on the arterials to which traffic is diverted. A great American economist, Frank Knight (1924), responded, in effect, that the resource misallocation which Pigou ascribed to malfunctioning "natural" market mechanisms actually resulted from the absence of property rights in fixed assets, the two roads in this case. Pigou's "artifi-

cial" tax would yield the same result as would ownership of roads by self-seeking individuals in competitive markets. Taxation and ownership both produce the differential returns on investments in the two roads that is essential for utilizing them efficiently. Congestion pricing would accomplish just this sort of efficient utilization.

The economics literature did not make much of Knight's insight until shortly after World War II. To cite just a few post-war landmarks, Beckmann, *et al.* (1956) showed that the problem of finding an equilibrium allocation of trips to a road network can be transformed into a constrained optimization problem formally similar to (albeit much more complicated than) the one that business firms solve in maximizing the profits they derive from their production and sales activities or that households solve in allocating their incomes among consumption activities. Walters (1961) cast road congestion in a framework very similar to that of the demand and supply schedules that crowd elementary economics textbooks and did seminal econometric work to develop relationships between traffic levels and road-user costs. Mohring and Harwitz (1962) showed that, given constant returns to scale[1], simultaneous optimization of investment in a road and of the charges for using it would yield results formally identical to long-run equilibrium in a competitive market.

The prevailing view on this subject today can be put in these terms: In a competitively organized gizmo industry, market forces assure that the price of gizmos equals their short-run marginal costs -- the costs of the additional variable inputs required to produce one more gizmo. The resulting revenues not only pay for the variable inputs gizmo production requires but also yield accounting profits which reward those who provide the services of the fixed or capital inputs -- e.g., buildings, machines, patented technology -- used in producing gizmos. If these rewards exceed the costs of providing fixed input services, their owners earn "economic" profits. That gizmo-industry accounting profits exceed capital-service costs induces current gizmo producers and others to invest in new gizmo-production capacity. Entry continues until, in long-run equilibrium, economic profits drop to zero, i.e., accounting profits just cover the costs of capital services thereby ending the incentive to add new capacity.

If they were imposed, congestion tolls would play the same role for roads as quasi-rents do for the gizmo industry; they would reward highway authorities for providing the services of fixed inputs to the trip-production process. Suppose that a road authority simultaneously charges optimal congestion tolls and optimizes the capacity of a road, in the sense that the last dollar spent adding to capacity produces benefits with a present value to users of $1 in the form of time-cost and operating-cost savings. Suppose also that the road is characterized by constant returns to scale -- an approximately realistic assumption for urban road networks (see, e.g., Keeler and Small (1977), Kraus (1981), and Small (1992, 94-99)). Then tolls equal to the difference between the short-run marginal and the average-variable congestion costs of trips would generate revenues just sufficient to cover the road's capital costs.[2]

The basic conceptual difference between gizmos and roads, then, is that, with gizmos, manufacturers' sales revenues cover the costs of their fixed and variable inputs while, with roads, cash payments to the road authority cover mainly fixed costs; the variable costs of trips -- vehicle-operating and travel-time costs -- are the responsibility of travelers themselves who provide at least one vital input, their own time, to the trip production process.

It would, perhaps, be useful to put this proposition in the context of Figure 1, a diagram with counterparts that are standard ingredients of elementary economics texts. For either the gizmo or the road case, the "demand" schedule indicates the number of gizmos or trips that would be bought or taken at alternative prices[3] for them. Profit-maximizing gizmo producers in competitive markets would adjust output to equate the additional costs they incur in producing the last unit of output in any given time period with what they can sell that unit for, an amount over which none of them has appreciable control. This marginal-cost price -- *VC* in Figure 1 -- not only covers the cost (*AB*) of the variable inputs used in producing the last gizmo sold but also provides an accounting profit or rent (*BC*) to the last unit's producer. Area *BCDE* in Figure 1 equals the total rents/accounting profits earned by all gizmo producers. Again, if these rents just equal the costs of providing the services of the industry's fixed capital equipment for a time period, the industry is in long-run equilibrium; no one has an incentive to

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Figure 1
Equilibrium in Manufacturing & Congestion-Priced Roads

SRMC

Demand

SRAC

D

C

B

E

A

Dollars per Unit/Trip

Traffic Volume/Output per Time Period
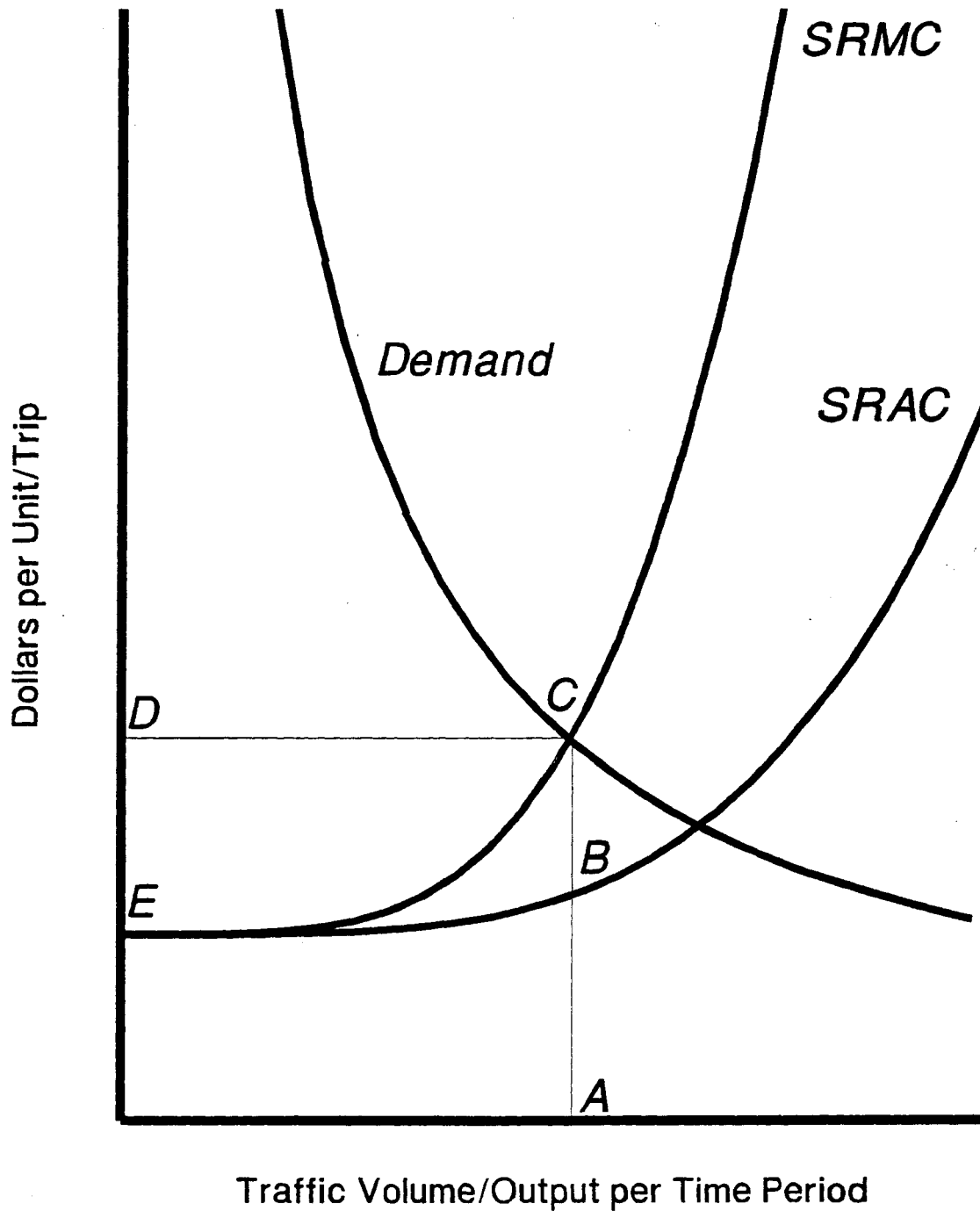
increase or decrease its output rate.

In the road case, the marginal-cost price of a trip consists of the value of the time and other resources supplied by the trip taker -- *AB* in Figure 1 -- plus a toll (*BC*) equal to the cost each traveler imposes on all other travelers by adding to the level of congestion. Area *BCDE* equals total toll collections. As with gizmos, area *BCDE* equals the costs -- interest on the capital invested in the road *plus* depreciation *plus* those costs of maintaining it that are not related to its use -- of providing these services. If providing these services is subject to constant returns to scale, an optimizing road authority would have no incentive to change its capacity; either expansion or contraction would reduce its aggregate net benefits.

In a nutshell, then, in the long run, optimal investing and pricing in a road network has much in common with the economist's concept of long-run equilibrium in a competitive market. With both, price and short- and long-run marginal costs would all be equal. In the competitive market case, revenues would cover the costs of variable inputs and would provide a return to the industry's investors that would leave them content to maintain (but, unless demand changes, not expand) their investments. In the road case, the variable costs of road use are borne by users. Given constant returns to scale, marginal cost-based tolls would cover the costs of roads and yield a normal return on the capital invested in them; congestion-based road pricing is consistent with self-supporting road networks.

Chapter 2: Some Low-Tech Economic Theory

The Economics of Travel Behavior: We are uneasy with the way in which most statistical analyses of travel behavior conceptualize the prices that influence it. To suggest why, consider the following simple model of consumer income allocation and mode choice. A household gets utility from consuming a general-purpose commodity, stuff (S), conveniently priced at $1 a unit. They also derive utility from what happens at the ends of the T trips a week requiring t minutes each that its members take downtown. Travel itself is unpleasant, however. The household cannot spend more than its income on stuff and on trips which cost F (for "Fare") each. It wants to allocate its limited income to these two commodities in a way that maximizes its utility, a function $U = U(Stuff, Trips, travel time)$ or, in symbols, $U = U(S, T, tT)$. It turns out (see Appendix A) that the household would treat the price of a trip as the dollar outlay, F, it requires plus the value it attaches to the time it takes, Vt. That is, the household would adjust its consumption of the two commodities so that last trip it takes would yield the same utility as $F + Vt$ units of stuff. V (with dimension $/minute) is the amount of stuff the household would be willing to give up to save an hour of travel time, i.e., the amount the household would be willing to pay to make its round trip downtown instantaneous.

One step further: Suppose that the household can choose between two modes for a trip, auto and bus. The bus fare, F_b, is lower than F_a, the money cost of an auto trip, but auto travel time, t_a, is lower than travel time by bus, t_b. The difference between the fares for the two trips divided by the difference between the times they take, $(F_a - F_b)/(t_b - t_a)$, can be interpreted as the price of saving time by taking the fast mode. If V exceeds this price, the household uses an auto for its trips downtown and, otherwise, the bus.

The lion's share of the three-dozen-or-so published studies of the values travelers attach to their travel time are based on this sort of choice. Studies of mode choice in commuting dominate this group. The study on which we rely in estimating congestion costs and tolls is Thomas Lisco's 1967-vintage University of Chicago doctoral dissertation, *The Value of Commuters' Travel Time--A Study in Urban Transportation.* With results translated into

Figure 2: Travel-Time Value v. Income
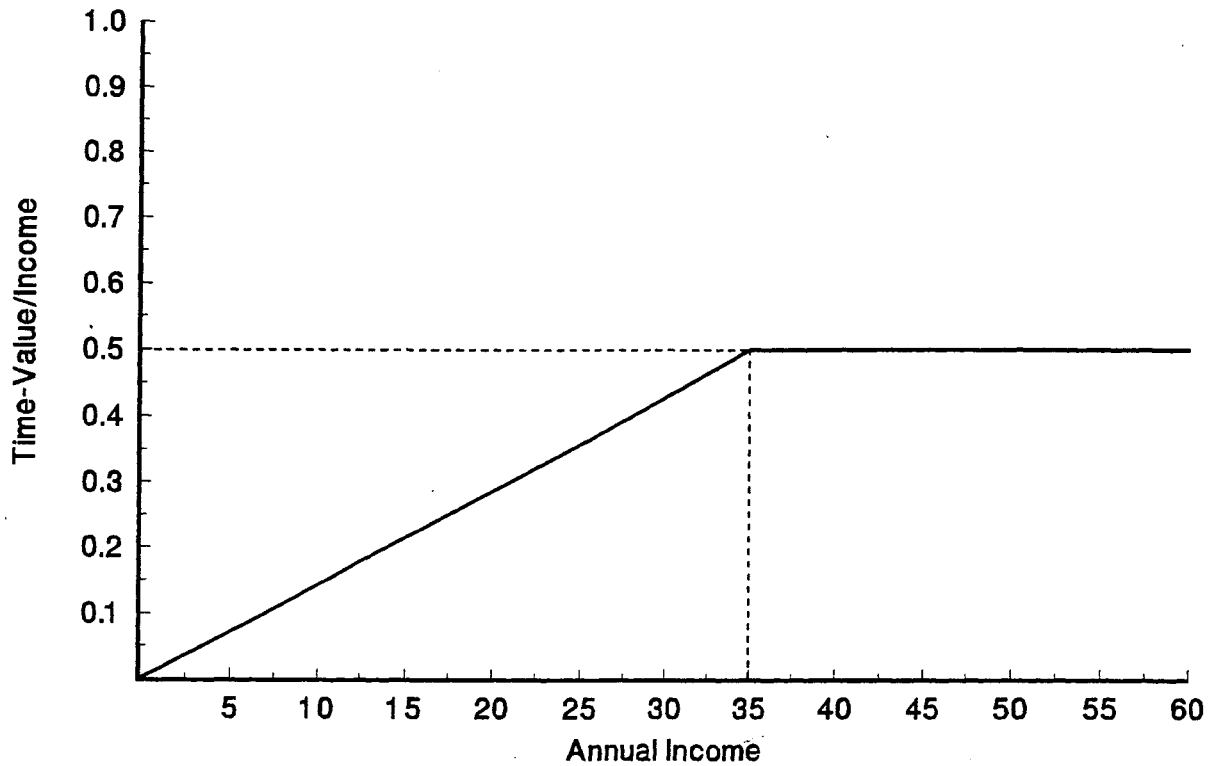
present-day price levels, the Lisco finding that we regard most highly is depicted in Figure 2. It indicates that commuters with incomes in excess of $35,000 a year value their travel time at half their hourly equivalent wage rates, on average. For those with yearly incomes of less than $35,000, the value of travel time *as a fraction of income*, V/I, increases linearly from zero for someone with no income to 50% at an income of $35,000.

Consider individuals who earn $40,000 by working 2,000 hours a year at $20 an hour. Lisco's analysis suggests that they would, on average be willing to pay $10 an hour to save travel time. Consider, alternatively, individuals who earn $17,500 (half of $35,000) a year -- $8.75 an hour. Lisco's analysis suggests that they would be willing to pay only $8.75/4 = $2.19 to save an hour of travel time.

Implicit in our relying in this fashion on Lisco's findings is the assumption that travel time is travel time. That is, we assume -- explicitly, now -- that the price of saving time at which a traveler is indifferent between auto and bus is the same as the traveler would be

willing to pay to save a minute of auto or transit travel time. Findings in the literature conflict on this matter. In a cleverly designed revealed-preference study, Calfee and Winston (1995) found that, on average, long-distance commuters would be willing to pay only about 20% of their hourly wage rates to shift their auto trips to less congested roads on which journey times are lower. The highest income group studied in the Calfee-Winston sample appeared, if anything, to be willing to pay a smaller fraction of income to save auto travel time than did members of somewhat lower-income groups.

Michael Beesley's (1965) classic study of Ministry of Transport employees in London came to quite different conclusions. For a substantial fraction of Beesley's respondents, the primary opportunity to trade money for reduced travel time involved two mass-transit services, not mass transit and auto. Clerical Officers, the lowest-salaried group (about $1,820 a year at prices and exchange rates in effect in the early 1960s), valued travel time for both transit/transit and auto/transit trades at about 31% of their wage rates. For Executive Officers, the next group up ($2,380, on average), the fraction was about 36% for transit/transit trades and about 43% for auto/transit trades. Finally, for a small group with yearly salaries above $6,200, the fraction ranged between 42-52% for transit/transit trades and 45-55% for auto/transit.

Optimizing Road Capacity: This conflicting evidence points to a very serious problem: We really know very little about what travelers are willing to pay to save travel time. The little we do know derives almost entirely from the choices wage earners make between auto and mass transit for commuting journeys. We know next to nothing about what wage earners are willing to pay to save time on non-work trips. We also know next to nothing about the relationship between the incomes of wage-earning members of a household and the amounts that its non-wage earning members are willing to pay to save travel time. Such information is essential in planning optimal road networks and prices for their use.

Why? Analysis summarized in Appendix A indicates that a highway authority which can price its roads efficiently and which desires to maximize "social welfare" -- some function of the utility levels of the households of which society is made up -- should take these

consumer valuations of travel time at face value. That is, if the average value users of a highway attach to their travel time is $12.50 an hour, the authority should also value travel-time savings at $12.50 an hour in deciding how much capacity to allocate to the highway. It should expand capacity to the point where the last dollar spent on expansion yields benefits with a present value of $1 in savings of traveler-supplied inputs to the trip production process. These inputs include travel time valued at S12.50 an hour.

This rule for optimizing highway capacity applies, it should quickly be emphasized, only when roads are optimally priced. Road pricing -- worldwide -- is far from optimal. When pricing in a market is screwed up, the rules for (constrained) optimization and for measuring the benefits of the market's operation differ from -- and are considerably more complicated than -- those of relevance in the world of universal price-equals-marginal-cost of which economists are so fond. With the aid of Figure 3, let us illustrate just with optimization rules: Suppose that supplying road services is subject to constant returns to scale and that input costs are independent of scale. Then the long-run average-variable costs -- the costs travelers incur directly in producing trips -- and the long-run marginal costs -- these directly borne costs plus those that a highway authority would incur in supplying the services of highway capital -- would be horizontal lines such as *LRMC* and *SRAVC* in Figure 3. The downward-sloping line labeled *Demand* indicates the number of trips that would be taken at alternative full (i.e., fare plus time-value) prices for them. At a full price equal to *LRMC*, travelers would take *CA* trips an hour. At that price, long-run efficiency would dictate providing that amount of capacity for which demand, long-run marginal cost, and short-run marginal cost intersect at the same point, *A*. The capacity associated with line $SRMC_1$ does this. $SRAVC_1$, is the short-run average variable cost associated with this short-run marginal cost schedule -- the cost of the user-supplied inputs for alternative travel rates given the optimal capacity level. With this average variable cost schedule, the optimal hourly travel rate would occur only if a congestion toll of *AB* is imposed on each trip.

Again, marginal-cost pricing is the rare exception for road services. Suppose, to

Figure 3: Second-Best Highway Investment
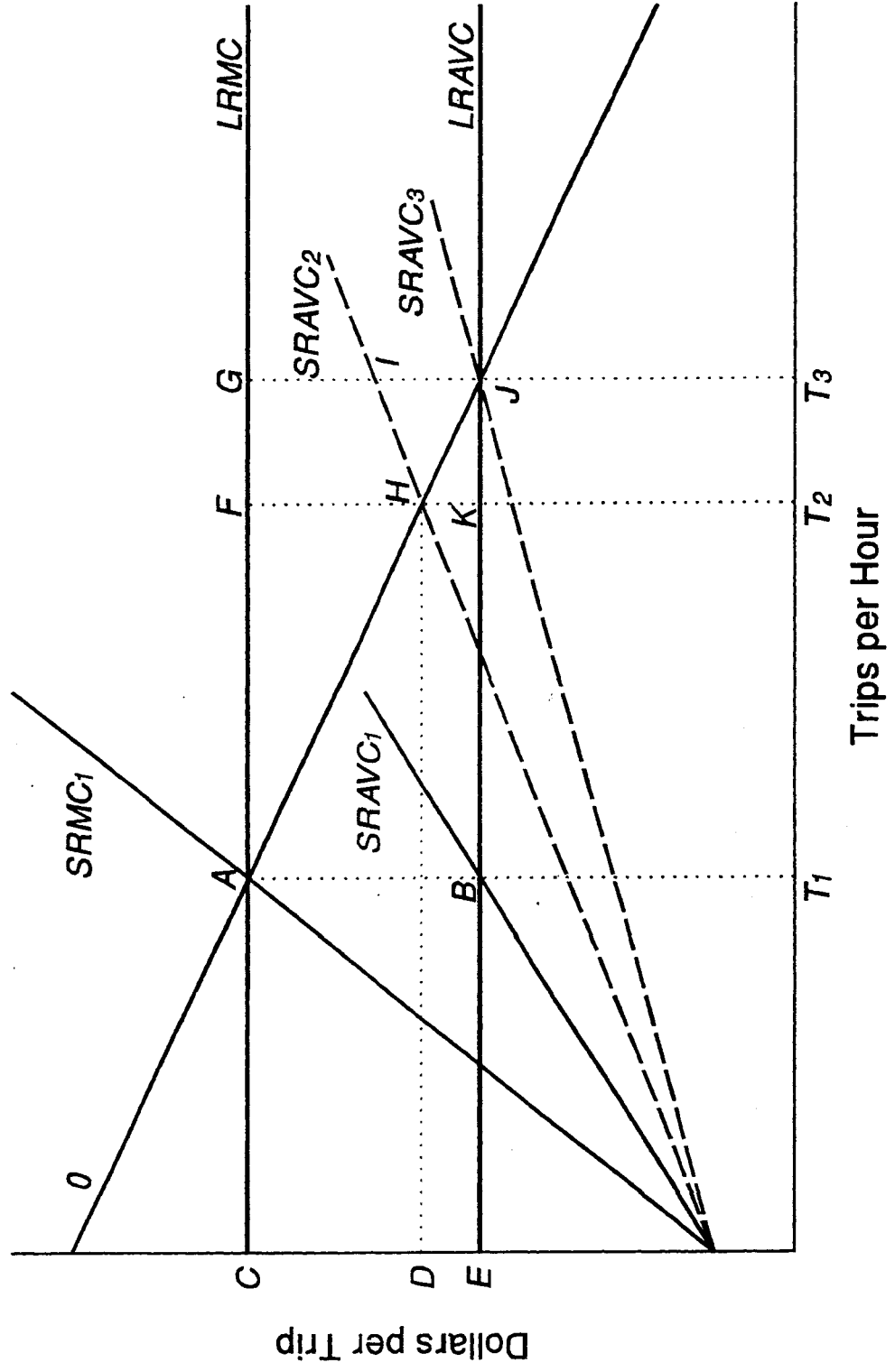
simplify the exposition, that no fuel or ton-mile or tire or other excises are imposed which vary with road use. Our impression is that, in the absence of highway-authority budget constraints, standard highway benefit/cost analysis would call for providing a level of road capacity that would minimize the total highway-authority plus user costs of whatever number of trips is taken. Achieving this objective requires providing the road capacity associated with the dashed line, $SRAVC_3$. With this cost schedule, CG trips would be taken and the total user plus highway-authority costs of these trips would be a minimum, the area $OCGT_3$. Although this equilibrium minimizes the costs of the CG hourly trips that are taken, it also results in travelers taking AG trips an hour that are valued at less than the full social costs of producing them.

Demand schedules tell us not only the rate at which commodities will be purchased at alternative prices for them but also provide measures of the values consumers attach to these purchases; that X gizmos a week would be purchased at a price of Y tells us that the purchaser of the Xth gizmo places essentially the same value on it as on the most desirable collection of other commodities that he or she would buy with Y dollars. The aggregate value of the CG hourly trips that would be taken only at a price less than their cost is T_1AJT_3; their aggregate cost is T_1GT_3. Cost exceeds value, therefore, by an hourly total of AGJ.

Suppose that the road which would minimize the costs of producing CG trips is not built but, rather, a somewhat smaller one -- that associated with short-run average variable cost $SRAVC_2$. With this road, the equilibrium hourly travel rate would be CF, the number of trips at which *Demand* and $SRAVC_2$ intersect. In this equilibrium, each of the CF trips still taken would cost DE more -- an aggregate increase of $DHKE$ -- than if the road associated with $SRAVC_3$ had been built. At the same time, however, this road would result in FG fewer trips per hour that are valued at less than the social costs of producing them. Area $FGJH$ is the reduction in the amount by which aggregate hourly trip values would fall short of aggregate hourly trips costs if road $SRAVC_2$, rather than road $SRAVC_3$ is built. Efficiency (subject to the constraint that user charges which vary with road use cannot be imposed) would call for settling on that road for which a small decrease in size would balance the loss from increasing

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user-borne costs (i.e., from increasing area *DHKE*) against the gain from discouraging trips valued at less than their costs (i.e., from increasing area *FGJH*).

The proposition suggested by Figure 3 can be put in more general terms: *If, for whatever the reasons, the money component of the full price of a trip must deviate from that required to minimize the social costs of the equilibrium travel rate, constrained efficiency would call for a deviation in the opposite direction of the time component of the full price. A particularly important implication of this finding: Urban roads cost much more per unit of capacity than do rural roads. For this reason, efficient urban roads would be designed for substantially higher volume/capacity ratios and, hence, would have substantially higher congestion tolls than would efficient rural roads. If we are forced to rely on the same fuel tax to support both road types, constrained efficiency would call for a tax somewhat greater than that which would be optimal for rural roads taken by themselves but somewhat lower than that which would be optimal for urban roads taken by themselves. At the same time, constrained efficiency would call for building what are, in a sense, "inefficiently small" urban roads and "inefficiently large" rural roads.*

The Technology of Congestion: A commonly invoked rule of safe road behavior is that, to allow sufficient time to react to unexpected events, drivers should stay three seconds behind the vehicles they follow. If all travelers follow this rule and all would travel at 60 mph (*i.e.*, would take one minute to travel a mile) on an otherwise unused expressway, there would result the relationships between the instantaneous ratio of actual traffic volumes to "ideal" capacity (we take it to be about 2,000 vehicles per lane-hour on an expressway) and the average (AC_I) and marginal (MC_I) travel times per mile that are given by the solid curves in Figure 4. Curve AC_I depicts the travel-time costs that individual travelers directly experience; curve MC_I, includes these costs plus those that each vehicle operator imposes on others by adding to congestion. Curve AC_I illustrates a commonly observed phenomenon of urban-expressway travel: maximum traffic flow occurs at about 30 mph. When speed is above 30 mph, a lower speed results in an increased traffic flow. In the top, backward-bending portion of AC_I it takes more

Figure 4
Relationships between Volume/Capacity Ratios and Travel Time
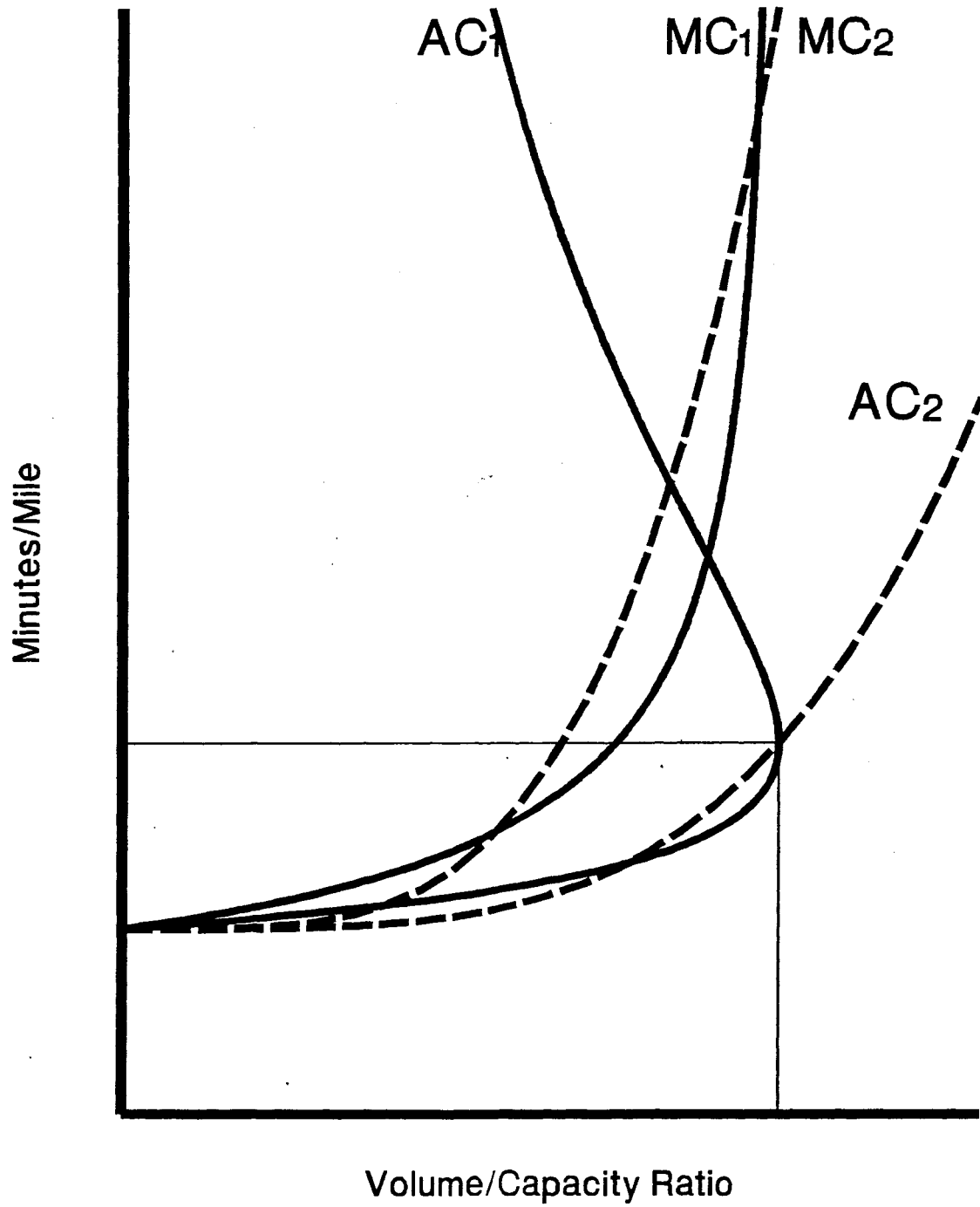
than two minutes to travel a mile. There, speed reductions *lower* traffic flows. We have all been there.

Peak-period travel does not take place at a constant rate but, rather, gradually increases to a peak, then decreases. Someone traveling at the peak of the peak period is more likely to experience the backward-bending portion of curve AC_1 than is someone who travels at the beginning or end of the peak. Still, both peak-of-the-peak and fringe-of-the-peak travelers do almost always get where they are going; a peak-of-the-peak trip just takes longer. We have, therefore, used marginal and average travel-time relationships similar to those given by the dashed curves, AC_2 and MC_2 in Figure 4 in deriving the toll estimates that are reported below. These curves are close kin of the "BPR curve" long enshrined in the highway literature to describe the relationship between traffic speed and traffic flow on a road.[4]

It was a common view among Twin Cities transportation planners of our acquaintance that imposing congestion tolls would push people around among roads and generate additional taxes but would not actually save any travel resources except those of travelers who are tolled off the roads. Indeed, the manual for Tranplan, the transportation-planning package that is most commonly used in the Twin Cities, maintains that, if each driver seeks a route which minimizes the time required by his or her trip, an equilibrium in which no traveler is able to find a faster route is one that minimizes total travel time.

Convincing people of this view's falsity is not easy but is essential if the benefits of congestion pricing are to be understood. The more efficient allocation of trips to the road network that congestion pricing would induce would be responsible for a substantial fraction of its benefits. It will, therefore, be useful for the discussion of later sections to examine this view carefully here by using as illustration a simple example that leads to a surprising conclusion: carelessly chosen road "improvements" can actually worsen travel flows.[5] In the hypothetical road network illustrated in Figure 5, travelers wish to go only from node N_1 to node N_4 via nodes N_2, N_3, or both. How they get back from N_4 to N_1 need not concern us. Travel between nodes N_1 and N_4, is on some combination of the five one-way roads, l_1, l_2, l_3, l_4, and l_5 that

Figure 5: A Braess Paradox Network
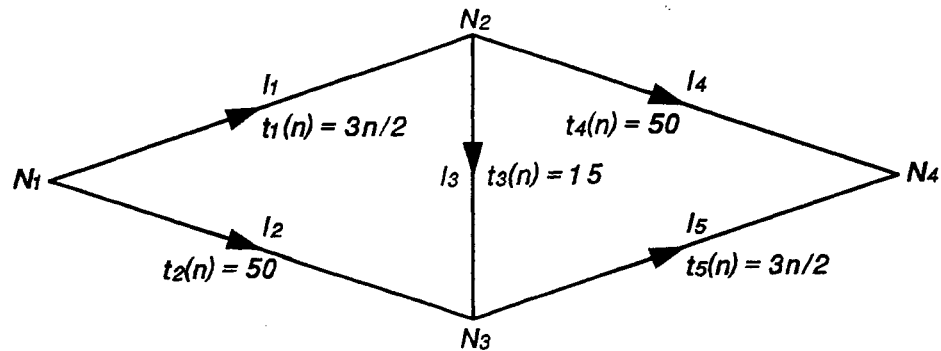
connect pairs of the four nodes. Travel on two roads -- l_1 and l_5 -- is subject to congestion; the greater is the rate at which trips are made on them, the slower traffic flows. Specifically, if n trips per hour are taken on l_1 or l_5, travel time on either is $3n/2$ minutes for each trip. Although circuitous, the remaining three roads, l_2, l_3, and l_4, are so wide that travel time on them is independent of traffic flow -- 50 minutes on l_2 and l_4, 15 minutes on l_3.

Suppose that 20 trips an hour are made between N_1 and N_4. They can be made using any of three routes l_1 and l_4, l_2 and l_5, and l_1, l_3, and l_5. Call these routes A, B, and C, respectively. All travelers desire to minimize the time *plus* toll costs of their trips. In general terms, an equilibrium assignment is one for which (a) taking other drivers' route choices as given, no driver can reduce travel costs by a change in route, and (b) the demand for trips between any pair of nodes equals the number of drivers on all routes who make that trip. When no tolls are levied, there is a unique equilibrium assignment: all 20 trips use route C. With this assignment, travel time is 75 minutes -- 15 minutes on link l_3 and 30 minutes on each of links l_1 and l_5. Routes A and B require 80 minutes, so no driver wishes to change routes.

This equilibrium assignment is not optimal[6] in the sense that traffic assignments exist

that result in a cost less than 75 minutes for each trip. Specifically, 10 trips each along routes A and B taking 65 minutes each would minimize resource costs. Without tolls, this is not an equilibrium assignment; a single trip shifted to route C would take only 45 minutes. The optimal assignment would be an equilibrium if link l_3 is closed. This paradoxical result, due to Braess (1968), is that, given inefficient pricing, situations can exist in which eliminating part of a transportation network can make all travelers better off.

Closing l_3 or imposing a $100 toll per trip on its use would make all N_1-to-N_4 travelers better off but -- to add a new wrinkle to the system -- would seriously harm N_2-to-N_3 travelers whose trips impose no congestion costs on the system. Tolls could make the optimal assignment an equilibrium without harming N_2-to-N_3 travelers. By assumption, on network links l_1 and l_5, the travel-time increase each traveler imposes on all other travelers by adding to the level of congestion equals the travel time each traveler directly experiences, $3n/2$.[7] Suppose that each traveler values time at $1 a minute. With 10 travelers using each of routes A and B, congestion tolls and direct time costs would respectively be $15 and $65 a trip on each for a total cost of $80, $5 more than route C costs in the absence of tolls. This system's drivers would, therefore, oppose congestion tolls unless they could be convinced that toll revenues -- $300 an hour -- would be used in ways that benefit them by more than the $5-per-trip increase in their travel costs. Since the toll travelers pay is three times the net cost paying it imposes on each of them, it ought to be possible to devise an allocation scheme that makes all of them better off.

To extend these conclusions, imposing congestion tolls on urban roads would do much more than just enhance government revenues and reduce vehicular travel. Imposing tolls would increase the efficiency with which urban travel takes place. *At present, if travelers can take more than one route between two points, they tend to adjust their travel patterns so as to equalize travel time -- the* average *time cost of a trip -- among the routes they use. Equalizing* average *costs does not, in general, equalize* marginal *costs, a necessary condition for minimizing total costs. Suitably chosen congestion tolls would achieve this objective.* Imposing them

would, in general, both reduce the travel time required to take any given menu of trips and increase tax revenues to highway authorities or state or federal governments.

The relative and absolute sizes of government and user benefits depend on a number of factors. Two are particularly important: the greater is the current level of road congestion and the more sensitive are changes in travel behavior to price changes, the greater will be both total benefits and the users' share of them. "Price elasticity" is the measure of sensitivity to price change that economists most commonly use. If the demand for gizmos has a price elasticity of -1, a 1% increase in their price would lead to a 1% reduction in the rate at which they are consumed. Several studies have estimated toll, fuel-cost, or similar dollar-cost elasticities of demand for travel, *i.e.*, the percentage change in the number of trips taken that would result from a 1% change in some measure of the cost of a trip.[8] The cost measures used have included total operating costs, fuel costs, tolls, and fuel taxes. Dollar-cost elasticities in the -0.1 to -0.5 range have generally been found.

If, as the simple model sketched out in the first section of this chapter suggests, the *full* price of a trip, not just its cash component, governs trip-taking decisions, *full-price* rather than *dollar-cost* elasticity seems the more relevant concept. We have generally worked with full-price elasticities of -0.5 and -1.0. Applying a general principle, we can write,

(Full-price elasticity)·(cash component)/(full price) = (Cash-component elasticity)

Its time component generally counts for more than half of a trip's cost. If so, our assumptions about full-price elasticities are consistent with money-price-elasticity numbers that are half as large.

Chapter 3: Public Opposition to Congestion Pricing

Shortly after the Clean Air Act of 1970 was passed, Boston attempted to institute a comprehensive package of transportation control measures. It succeeded with some. Others proved much more difficult, particularly those dealing with area licensing and parking charges. In writing about the opposition such proposals face from the public, politicians, and the many institutions involved in implementing them, Howitt (1980) found public response to be

> more sensitive to the distribution and visibility of the impact of the policies than to the net benefits or costs.... What prevents the emergence of more active support for auto restraint policies is the almost total absence of individuals or firms that might receive immediate, direct, 'selective' benefits. The few such beneficiaries -- private taxi or transit firms, residents eager to exclude commuter parking and through traffic from their neighborhoods -- are not the base of a broad political coalition.

Referring back to Figure 1 suggests why this passage probably applies to congestion tolls. Imposing tolls would make the average auto user worse off. Tolls would induce some travelers not to take some trips on which, before tolls, they placed small net values. They lose as a result. While the trips they continue to take would require less time, for the average auto user, the sum of the time and the money costs of trips would increase.

The severity of these cost increases varies with travelers' income levels. The higher are their incomes and, with them, travel-time values, the smaller is the toll for a trip as a fraction of its full price. Indeed, in the Twin Cities, the reduction in travel time per mile that tolls yield would benefit some high-income travelers more than toll payments would cost them; on the most congested stretches of the Interstate System, we calculate that, if the incomes of the occupants of a car aggregate to more than about $80,000 a year, tolling would benefit them regardless of what is done with toll revenues.

Most people respond with opposition to government actions that reduce their welfare -- socially desirable though these actions may be. Their opposition may be particularly severe when they learn that others would benefit from policies that harm them. Many of these opponents could be won over by allocating toll revenues in ways that would benefit them sufficiently to offset their direct losses from tolls. *To emphasize, gaining the support of a majority of Twin Cities peak-period travelers for congestion tolls would require coupling the tolls with a*

plan that would benefit them more than the tolls would cost them. Since congestion pricing would yield toll revenues greater than the aggregate damage tolling would impose on travelers, it is certainly possible that such an allocation scheme could be devised. Using toll revenues for road or, perhaps, other transportation improvements is one possibility. Using some of them to reduce fuel and other taxes is a second. Using some to make direct cash grants to low-income households is a third. Using some to reduce real-estate taxes is yet a fourth.

Congestion pricing has been much more widely discussed in Britain than in North America. Here are brief economist-type arguments that respond to six reservations that, Jones reports, are particularly prominent there followed by two arguments that he does not mention but that have been heard in North America:

a. *Congestion pricing will not work; people will still drive:* People *do* respond to changes in transportation prices with changes in travel behavior. Here are just a few examples of responses to changes in these prices; many more can be cited. In 1988, the Massachusetts Port Authority (MassPort) briefly changed its landing fee structure at Logan International from $1.31 per thousand pounds of aircraft weight with a minimum fee of $25 to $88 *plus* $0.47 per thousand pounds. Several organizations and states successfully sued MassPort claiming that the new formula discriminated against small aircraft and was contrary to federal statute. The US Department of Transportation made it clear that, as long as MassPort continued in its "discriminatory" ways, it would receive no federal funds for airport capital improvements. Still, during the brief period during which the new price schedule was in effect, the number of commuter-aircraft flights to Logan from Burlington, VT declined from 22 to 12.

Several transit systems in the United States have experimented with surcharges and discounts during peak- and off-peak periods respectively. The results have varied from system to system and with characteristics of travelers and trips -- age, income, e.g., and trip length and purpose -- but, Cevero (1990) reports, the overall fare elasticity of demand -- the percentage change in demand resulting from a one percent fare change -- ranges between -0.22 and -0.33.

Finally, we have the example of Singapore where a modest peak-period fee to enter its Central Area together with significant increases in parking charges dramatically reduced peak-period traffic congestion.

b. *The technology is not reliable*: Hong Kong's tests of an inductive-loop/transponder combination achieved accuracies in the 99.9-100% range a decade ago. Since then, rapid strides have been -- and are being -- made in a variety of technologies that could be used in sophisticated road-pricing applications. Automatic vehicle-identification (AVI) procedures are particularly advanced but smart-card technology that would permit in-vehicle record keeping for prepaid road-use accounts is not far behind. Except for paper licenses, fraudulent use of road pricing technologies is no more easy than fraudulent use of magnetically coded credit cards. Fraud resistance is being developed rapidly for both credit cards and AVI systems.

c. *Congestion pricing will invade privacy; it can provide information that would make it possible to trace unpopular individuals' vehicles continuously*: AVI technology is somewhat -- but only somewhat -- further developed and lower cost than are technologies that would allow in-vehicle accounting for road use. Using AVI technology is not necessarily inconsistent with privacy; Swiss banks are quite successful in keeping secret the identities of privacy desiring account holders. It would not be difficult to provide account numbers whose owners are known only to themselves but whose vehicles could be identified if used when account balances are negative.

d. *Congestion pricing would inflict significant harm on the poor*: That they are ill-fed, ill-clothed, ill-housed, and, perhaps, ill-transported is really not the basic problem that poor people have. Their fundamental problem is, rather, that they are poor. If we are genuinely concerned with how road pricing would affect their welfare, we should give them cash or marketable road scholarships. It would be the height of folly, however, to subsidize all of our private-passenger-vehicle road use in the supposed interests of helping them.

e. *There are boundary problems in dealing with congestion pricing, when does the peak begin and end?* In a manually policed system like that of Singapore which uses paper licenses,

human monitors are seriously limited in the number of combinations of license size, shape and color that they can distinguish reliably and speedily. The combination of this limitation and cordon-line pricing can lead to appreciable temporal and spatial boundary congestion; congestion around the periphery of Singapore's Central Area was so severe that its Area License Scheme left travel times to Central Area destinations unchanged despite free-flowing traffic in the Central Area itself. With electronic technology, however, charges can be graduated as finely in time or space as travelers are capable of comprehending. With narrower gradations, boundary congestion will become a smaller problem.

f. *No matter what "they" say, congestion pricing is just another way to collect taxes:* The services that government provide tend to be labor intensive. With increases in living standards, the relative costs of these services have increased. If living standards continue to improve, therefore, government revenue requirements will continue their secular increase. If only for this reason, the suspicion that congestion pricing is just another tax gimmick will be difficult to dispel. *It is, therefore, essential that a detailed proposal for using congestion-toll revenues accompany any serious proposal to institute them. The greater is the number of auto travelers who feel that proposed reductions in road-user taxes and other give-backs will make them better off, the more likely it is that congestion pricing will be accepted.*

Turning to objections to congestion pricing that have appeared in the North American literature and have not already been touched upon, consider:

g. *We have already paid for roads through fuel and other user taxes. Why should we have to pay for them again?* It is by no means certain that user fees cover road costs. Consider the following:

> Contrary to popular belief, drivers do not pay their own way through user fees. In the United States, gasoline taxes and other user fees account for roughly 60 percent of federal, state, and local spending on highways and roads. The remainder, $29 billion in 1989, comes from general funds, property taxes, and other sources. Another cost, "free" parking, has an estimated value of $85 billion per year. Additional expenses not covered by drivers such as for police and emergency services, traffic management, and routine street maintenance represent some $68 billion annually. When harder-to-quantify costs such as air pollution, traffic congestion, and road accidents are figured in, the total subsidy to drivers in the United States soars to an estimated $300 billion a year. [MacKenzie, et al. (1992)]

Even those who would cast these views aside as the rantings of hard core environmentalists cannot dispute that those who gain most from urban road expenditures -- peak-period travelers in the main direction of traffic flow -- have paid through user taxes only a small fraction of the costs of providing the benefits they have received. With important qualifications, road expansion makes sense as long as the present value of aggregate benefits to users exceed the costs of expansion. Peak-period users benefit much more from road expansion than do off-peak users but pay about the same in user taxes per vehicle mile for these benefits.

h. *We impose congestion on each other; why should we pay someone else for the harm we do to ourselves?* It is true that travelers pay for the congestion costs they experience. They do not, however, pay for the congestion costs they impose on others by adding to it. Congestion tolls would force them to recognize the external costs they impose by making them pay their cash equivalent. The logical recipients of these payments is not the users affected by congestion but, rather, the public at large whose expenditures on roads reduce the congestion costs that all drivers experience.

Thus far in this section, we have emphasized the concerns with "efficiency" -- roughly speaking, cost minimization -- that dominate the thinking of economists in discussing such policies as congestion pricing. Economists tend to ignore the income redistribution to which changes in social policy can give rise on grounds that altering the income distribution is a "normative" issue -- an issue that economists are not professionally qualified to decide but, rather, that they must leave to politicians and "decision makers."

Economists' discussions of efficiency have a way of leaving the general public glassy eyed; to them, it is the income distribution -- often, "What's in it for me?" -- that really matters. Distributional issues are particularly important in dealing with congestion pricing since its immediate effect would be to make most travelers worse off. In addressing similar serious concerns, economists are inclined to point out that increased efficiency would provide the wherewithal to make everyone better off (or, at least, to make some person or group better off without harming anyone else); they then leave it to politicians to decide just how to accomplish

"making everyone better off."

Often, accomplishing the income redistribution that would make everyone better off would require so much information as to be a practical impossibility. With road pricing, however, the potential gains are so great -- about $1.50 of toll revenues for each $1 of direct harm to the average traveler, we calculate -- that careful analysis of available information (*available*, note, not *ideally* available) should make it possible to come up with one or -- better still -- more schemes for allocating the gains from congestion pricing so that a substantial majority of us would be made better off. Meeting this objective must be high on the list of tasks to be accomplished if congestion pricing is not to be rejected out of hand by the public at large. We return to this subject in Chapter 6 where we describe the direct effects of congestion pricing on different income groups and our attempts (sadly to say, so far unsuccessful) to devise ways to allocate toll revenues to make (almost) everyone better off.

Chapter 4: Estimating Congestion Costs and Congestion Tolls -- Tranplan

The notion that cost-based prices are essential to efficiently allocate the limited resources that society has at its disposal is ingrained in economists trained in North America and Western Europe. If prices deviate substantially from costs, it is possible to reallocate resources in a fashion that would make some -- perhaps many -- people better off without harming any others. Road pricing -- world-wide, not just in the US -- is emphatically not "cost-based." Road travel, particularly in urban areas, and most particularly during peak travel hours is heavily subsidized. The subsidies come in part from the public at large in the form of general revenues used to finance police and other services provided to road users and in the form of environmental waste-disposal services for which road users don't presently pay. Subsidies also come -- possibly in even larger part -- from users themselves in the form of congestion costs that they impose on each other but for which they are not charged.

In the Twin Cities Metropolitan Area, metering entry at access ramps limits the frequency with which peak-period expressway travel enters the backward-bending part of the travel time-volume/capacity relationship shown in Figure 4. Time spent in long queues discourages expressway use, particularly for short peak-period trips. Use of high-occupancy vehicles (HOVs) is encouraged by allowing them to bypass queues of single-occupancy vehicles (SOVs) at a growing number of entry ramps and by providing them with reserved "diamond lanes" and heavily subsidized parking at the Minneapolis CBD end of one expressway.

How would the performance of the present system compare to that of a system which collects from each vehicle -- HOV or SOV -- an amount equal to the costs it imposes on all other vehicles by adding to the level of congestion? We began our search for an answer to this question by working with a package of computer programs, Tranplan, that is used locally for a variety of highway and transit forecasting projects by transportation planners at Mn/DOT, the Metropolitan Council, and several consulting firms.

Our analysis relies on the 1990 Travel Behavior Inventory (TBI), a survey in June-November 1990 of 9,746 households which took a total of approximately 98,000 daily trips.

Table 1: *1990 Peak Hour Travel Conditions*

	a.m. Peak	p.m. Peak
Trips (1,000's)	518.1	678.1
Travel Time (1,000 hours)	152.0	164.2
Total "Gap" ($1,000)	1,049.5	949.6
Vehicle Miles (1,000)	5,071.6	5,590.2

Survey trip data were carefully expanded using cordon-line counts and Census of Population data to reflect private-passenger-vehicle travel in the entire metropolitan area on an average 1990 weekday. We limit the results reported here to those private-passenger-vehicle trips that were on the road at some point during the morning peak travel hour, 6:45-7:45 a.m. Each trip studied originated in one of the 1,200 traffic-analysis zones (TAZs) into which the metropolitan area is divided and terminated in one of the remaining 1,199 TAZs; congestion is generally not significant for intra-zonal trips. Table 1 contains data on 1990 peak-hour travel conditions.

The metropolitan area's road network is broken into 20,336 links which connect 7,363 nodes. These links include all roads on which more than 1,000 trips were taken on an average weekday. Examples of links include several-block stretches of an arterial or collector street and an expressway access ramp, HOV lane, or one-way segment between two interchanges. TBI surveys obtain origin and destination addresses and, hence, TAZs for each trip but not the route taken. The parts of Tranplan on which we relied most intensively rest on the assumption that each traveler selects that route -- a series of links -- for each trip which minimizes the trip's "impedance." Tranplan "loads" trips onto the network using a process that reaches an equilibrium in which no traveler is able to find a route with less impedance. *Average* travel time is Tranplan's default measure of impedance, but distance, a combination of time and distance, *marginal* travel time, or a variety of other measures could be used.

We assume that travel time is the only cost of trips that travelers bear directly; sensitivity analyses described in Appendix E indicate that incorporating vehicle operating costs into the analysis would have no appreciable effect on the results we report. Tranplan's default measure of travel time for some link, call it link i, is what its manual terms "the historic

standard Bureau of Public Roads capacity restraint formula." Using it, the time required to traverse link i, t_i, is

$$t_i = t_{i0}\{1 + 0.15[T_i/K_i]^4\} \tag{1}$$

where T_i is the rate at which vehicles travel on link i, K_i is a measure of capacity[9], and t_{i0} is the time required to traverse link i when no other trips are being taken on it.

If equation (1) gives travel time per trip on link i, the total travel time, τ_i expended by those who travel on it during an hour can be found by multiplying (1) through by T_i:

$$\tau_i = T_i\, t_i = T_i\, t_{i0}\, \{1 + 0.15[T_i/K_i]^4\} \tag{2}$$

Marginal travel time -- the change in total travel time when an additional traveler is added to the traffic stream -- can then be written

$$\partial\tau/\partial T_i = t_{i0}\, \{1 + 0.75[T_i/K_i]^4\} \tag{3}$$

The difference between equations (3) and (1) -- marginal travel time *minus* average travel time on link i -- multiplied by the average value of travel time is the cost an additional traveler imposes on all other travelers by adding to congestion on the link. Applying the Lisco income/travel-time relationship to the TBI menu of trips leads us to a morning peak-hour value of about \$12.50 per private-passenger-vehicle hour and what we term a "gap" between marginal and average-variable travel costs on link i of:

$$Gap = \$12.50 \cdot t_{i0} \cdot 0.6 \cdot [T_i/K_i]^4 = \$7.50 \cdot t_{i0} \cdot [T_i/K_i]^4 \tag{4}$$

If the number of trips taken were independent of the price charged for them the gap would be close to the appropriate congestion toll for link i. But imposing tolls would almost certainly lead to reduced travel and, hence, reduced congestion. For this reason, we use the word "toll" to refer to the difference between equations (3) and (1) only on road networks where (a) travelers have fully adjusted to the existence of a tolling system and (b) \$12.50 times the difference between equations (3) and (1) is the actual cost a traveler imposes on other link-i travelers by adding to the level of congestion. We use the word "gap" to refer to \$12.50 times the difference between equations (3) and (1) on road networks on which congestion tolls are not imposed. To emphasize, the *gap* on link i is the cost a link-i traveler imposes on other link-

Table 2: *Distribution of Current Gaps between Traveler-Borne and Marginal Costs of Travel During the Morning Peak Hour on the Twin Cities Road Network*

Gap/ Mile (Cents)	All Coded Road Links			Expressways Only		
	Miles of Road	Vehicle Miles (Dollars)	Aggregate Gaps (Dollars)	Miles of Expressway	Vehicle Miles	Aggregate Gaps (Dollars)
0-2.5	7,644	1,742,600	8,900	298	433,400	3,300
2.5-5.0	400	399,200	14,700	53	165,800	6,300
5.0-25	892	1,364,100	185,100	154	718,000	102,100
25-50	348	709,100	255,300	64	364,600	131,800
>50	392	939,700	992,700	74	473,700	376,700
Total	9,676	5,154,700	1,456,700	643	2,155,500	620,200

i travelers *when congestion tolls are not charged on link i.*

Table 2 summarizes these gaps during the morning peak hour. It indicates that, on 7,644 miles of the 9,676-mile coded road network and 298 of the 643 miles of expressway, the cost of the congestion that each vehicle imposes on all others amounts to less than 2.5¢ a vehicle mile--roughly the charge imposed by fuel taxes. At the opposite extreme, on 392 miles of all coded links and 74 miles of expressway, gaps between the directly borne and the marginal costs of a vehicle mile exceed 50¢. The aggregate gap is nearly $1.5 million on the entire road network and over $600,000 on its expressway component.

a) *When All Congested Roads Are Tolled:* Figure 6 summarizes on one very crowded page the remaining results of our work with Tranplan. If, during the morning peak hour, the occupants of each vehicle on the road network value their travel time at $12.50 an hour, the average time cost drivers incur directly is 37¢ a vehicle mile while the "gap" between these directly experienced costs and the full marginal costs of their trips averages 26¢ a vehicle mile. Thus, on average, the time cost drivers experience during the morning peak hour account for a bit less than 60% of the full costs of their trips to society. To emphasize, by adding to the level of congestion, the average vehicle on the road network during the morning peak imposes aggregate costs of 26¢ on all other vehicles for each mile that it travels. On the most congested 10-mile stretch of freeway, the gap is 62¢ a vehicle mile and on a few scattered road links, the

gap exceeds $5 a mile.

Suppose that the full-price elasticity of demand is - 1.0 -- *i.e.*, that a 1% increase in the full price of trips leads to a 1% reduction in the rate at which they are taken -- and that advanced electronics could collect tolls from vehicle operators without delays or, for the moment, capital or operating costs. It would then be efficient to impose marginal-cost tolls on all congested roads whether they be freeways, expressways, arterials, or collectors. With "congested" defined as involving congestion-delay costs greater than the 2¢ or so that fuel taxes impose on auto travelers, in the Twin Cities, about 2,000 of a total of about 9,700 miles of road are congested during the morning peak hour. Tolling all of these roads would reduce traffic volumes by about 12% on average and by about 25% on the most heavily congested stretches of freeway. On these stretches, congestion tolls would average about 21¢ a vehicle mile. On the average road, tolls would be about 9¢ a mile.

As Figure 6 suggests, the direct effect of congestion tolls would be to make the average road user worse off. On average, travelers would pay more for the trips they continue to take and would no longer take some trips that formerly yielded net benefits but now are worth less than their new, higher prices. While all travelers would benefit from faster trips, for most auto users, toll payments would exceed the value of these time savings. Only two groups with quite small memberships would gain from congestions pricing regardless of how toll revenues are used. These are current mass-transit users and auto travelers with very high incomes. Mass transit users would benefit from the more frequent service which toll-induced diversion of travelers from auto to bus would generate and, at least on expressways, from faster travel times. On the most congested roads, auto users with incomes greater than about $80,000 a year would gain more from travel-time savings than they would lose in the tolls they pay.

For just the morning peak hour, total toll collections would be about $390,000 per weekday. For the day as a whole, collections of around $1.5 million are about the right order of magnitude. Collecting tolls from morning peak-hour travelers would impose costs on remaining and tolled-off travelers of about $250,000 in the form of the tolls they pay less the
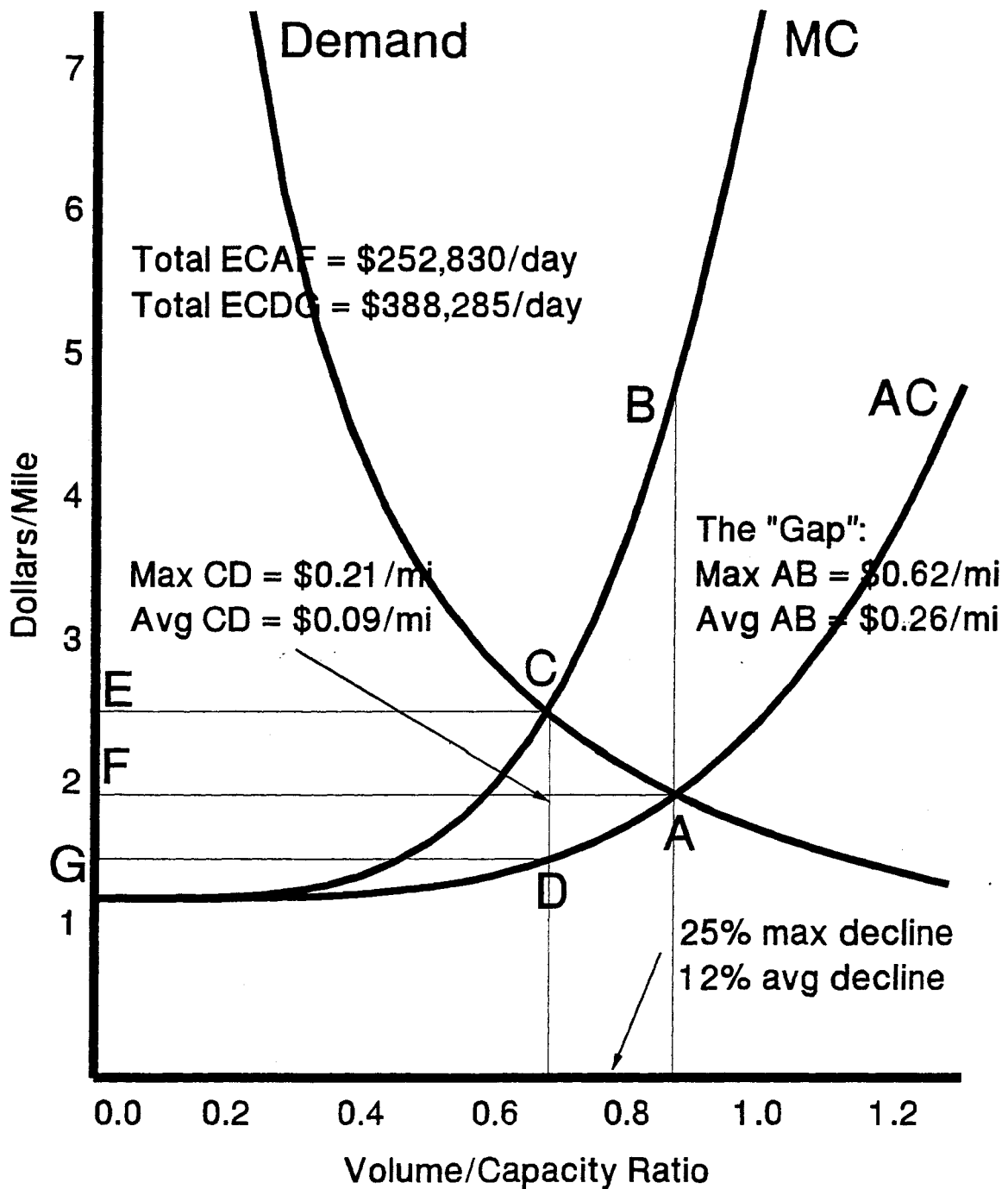
Figure 6

Morning Peak-Hour Congestion Costs, Tolls, and Traffic Reductions:
Full-Price Elasticity = -1, All Congested Roads Tolled,
Time Value = $12.50 per Vehicle Hour

Scale: 1 inch = 4.7 miles
▲ Minneapolis CBD
■ St. Paul CBD

━━━━━━━━━━ 10 Cents
━━━━━━━━━━ 20 Cents
━━━━━━━━━━ 30 Cents
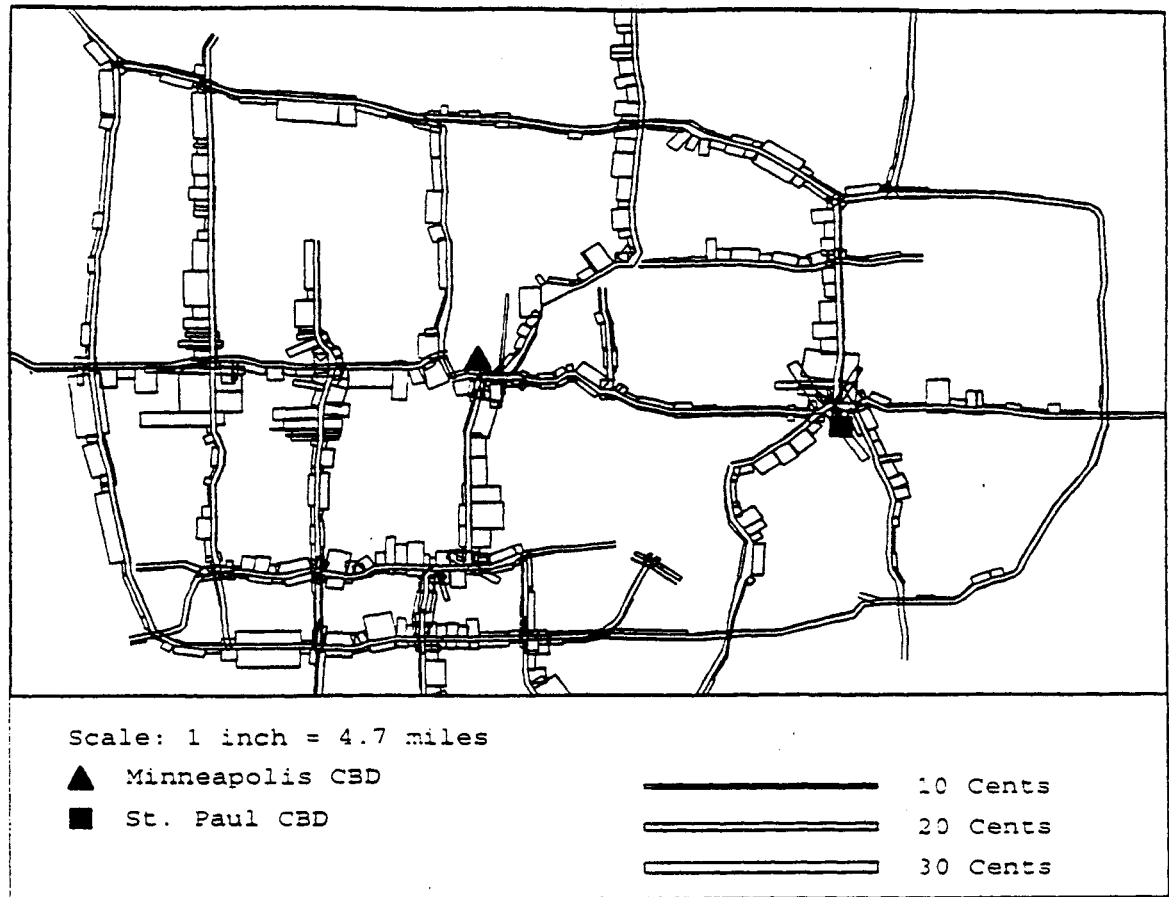
Figure 7: *Morning Peak "Gaps" per Mile on Twin Cities Area Limited-Access Roads*

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travel time savings they receive on their trips and the benefits they forego on trips that higher full prices induce them no longer to take. Thus, during the morning peak hour, congestion pricing would generate about $1.54 of revenue for each dollar of cost borne by travelers. Norman Foster tells us that, in a recent visit to California's State Route 91 tollway -- the USA's first private, congestion-priced toll road -- knowledgeable employees indicated its collection costs to be less than 10% of its revenues. At least in principle, therefore, it should be possible to come up with a redistribution of this loot that would make everyone better off. Unfortunately, we have yet to devise such a scheme -- see Chapter 6.

Without costly color prints, it is difficult to summarize graphically the effects of pricing on traffic flows in a large network. Examining its effects on limited access roads provides

Scale: 1 inch = 4.7 miles   Toll per mile

▲ Minneapolis CBD

■ St. Paul CBD

────────── 10 Cents

══════════ 20 Cents

▭▭▭▭▭▭▭▭▭ 30 Cents
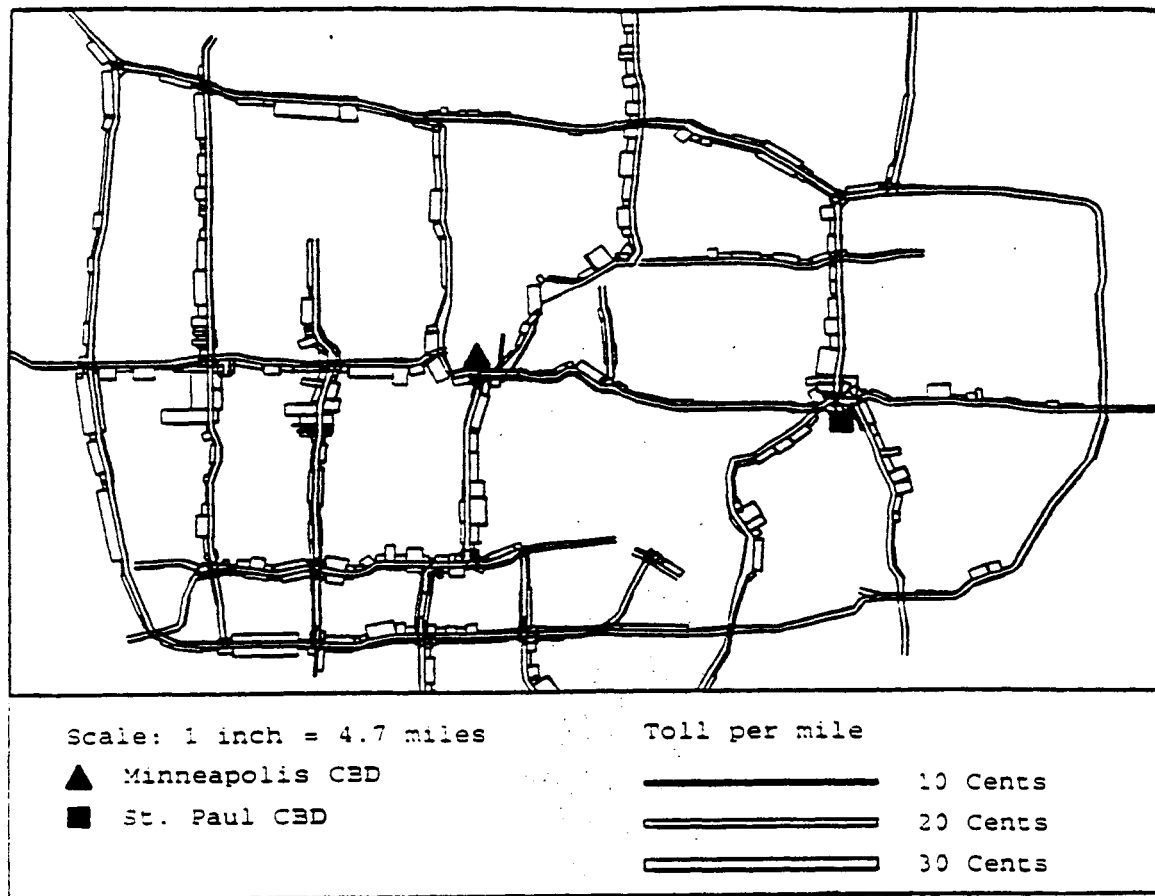
Figure 8: *Morning Peak Tolls per Mile on Twin Cities Area Limited-Access Roads:*
All Roads Tolled with Inelastic Trip Demand

some insights because their flows are generally high. Fortunately, the expressway network in the TCMA is small enough to make into readable plots. Figure 7 shows the "gap" per mile on TCMA freeways during the morning peak hour under current travel conditions. Gaps are particularly high, often over 50¢ a mile, on the roads leading to the Minneapolis and St. Paul central business districts (CBDs). Significant gaps occur on other freeways as well.

Even if aggregate demand is completely unresponsive to tolls, their equilibrium levels would be appreciably lower than current gaps because tolls induce drivers to use the road network more efficiently. Figure 8 shows differences per mile between the full and the directly experienced costs of trips when demand is inelastic and all roads are tolled optimally. The overall pattern of congestion declines somewhat, but the average difference between marginal and directly experienced costs declines substantially -- from an average of 23.6¢ with no tolls to
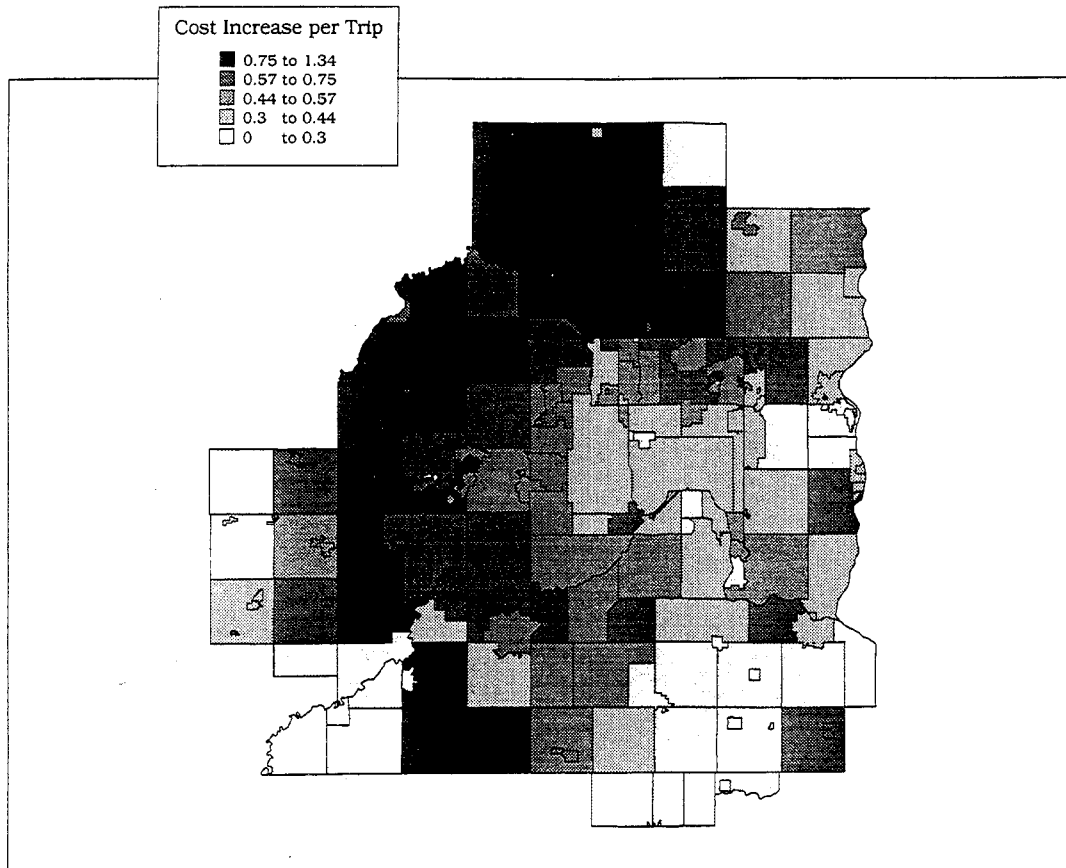
Figure 9: *Average Dollar Loss of Consumer Surplus per Trip in Twin Cities Minor Civil Divisions: All Roads Tolled with -0.5 Full-Price Elasticity of Demand*

15.9¢ per mile with tolls. Differences between marginal and directly experienced travel costs fall even more if demand is elastic. For example, if the elasticity of demand is -1.0, optimal tolls would reduce the marginal external cost of travel on freeways to an average of 8.0¢ per mile.

The effect of congestion pricing on drivers varies significantly with their locations. Figure 9 shows all minor civil divisions in the TCMA and the loss from foregone trips and from the higher costs of trips still taken if tolls are imposed on all congested roads and the elasticity of demand is -0.5. The losses are for trips currently taken and are assigned to each trip's origin in the morning and its destination in the afternoon. Losses range from trivial in some areas to over a dollar in some of the areas that are farthest from the CBDs. Other areas that are far from the CBDs show very small losses per trip, however. Travelers in those areas

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either do not use congested roads, or there may be sampling errors in the TBI -- few trips were taken to and from some outlying areas. Losses appear highest west of Minneapolis but are also high in a ring of suburbs north and west of the central cities. The same general pattern of losses occurs when other elasticities of demand are assumed, but their sizes vary greatly. For example, when demand is inelastic, losses of over $3.00 per trip occur in some parts of the metropolitan area.

b) When Only Limited Access Roads Are Tolled: Recognizing that even electronic toll collection is costly leads to the conclusion that it would probably be inefficient to toll lightly congested roads. Indeed, at present levels of congestion, when the transactions costs involved in collecting tolls are considered, it may be optimal to toll only limited access roads. On them, access limitations reduce substantially the number of points per mile at which vehicles must be monitored. Also, limited access roads are usually more heavily traveled than other roads; in the Twin Cities, 42% of peak-hour vehicle miles are driven on limited access roads which account for less than 7% of total roadway mileage. Since a large fraction of monitoring costs is independent of traffic levels, heavily traveled limited-access roads produce greater gross returns per dollar of monitoring costs.

Under the congestion-pricing system described in this subsection, each limited-access road is tolled at one-fourth of the gap -- again, the difference between marginal and directly borne travel costs. With inelastic demand, this fraction turns out to be optimal for the morning peak if tolls must be set at the same fraction of the gap on all limited-access roads and nearly optimal for the afternoon peak hour. A 25% fraction also produces large efficiency gains when the elasticity of demand is either -0.5 or -1.0. Such tolls may seem low, but larger tolls would divert so much traffic onto untolled roads that efficiency gains would decline. Note that this congestion pricing system is probably not optimal within the class of all systems which toll only limited access roads. For example, it would generally be better to charge lower tolls on freeways for which there are good, non-freeway substitutes and to charge higher tolls on freeways for which no good substitutes exist.

Table 3: *Present and Toll Equilibria for Morning Peak Hour*

	No Tolls	All Roads Tolled with Elasticity of		
		0.0	-0.5	-1.0
Trips (1,000s)	518.1	518.1	478.1	460.0
Travel Time (1,000 Hours)	152.0	146.7	128.0	119.9
Toll Revenue ($1,000s)		672.8	406.8	323.5
Lost Surplus ($1,000s)		620.3	313.1	218.6
Efficiency Gains ($1,000s)		52.4	93.6	104.9

	Tolls only on Expressways with Elasticity of		
	0.0	-0.5	-1.0
Trips (1,000s)	518.1	513.4	510.5
Travel Time (1,000 Hours)	150.6	147.7	146.0
Toll Revenue ($1,000s)	69.8	66.1	63.7
Lost Surplus ($1,000s)	55.5	39.4	31.7
Efficiency Gains ($1,000s)	14.3	26.7	32.0

Table 4: *Present and Toll Equilibria for Afternoon Peak Hour*

	No Tolls	All Roads Tolled with Elasticity of		
		0.0	-0.5	-1.0
Trips (1,000s)	678.1	678.1	629.7	605.9
Travel Time (1,000 Hours)	164.2	159.5	141.4	133.3
Toll Revenue ($1,000s)		599.6	383.4	310.2
Lost Surplus ($1,000s)		552.6	305.0	220.0
Efficiency Gains ($1,000s)		47.0	78.3	90.2

	Tolls only on Expressways with Elasticity of		
	0.0	-0.5	-1.0
Trips (1,000s)	678.1	673.1	670.0
Travel Time (1,000 Hours)	163.3	160.5	158.9
Toll Revenue ($1,000s)	61.0	57.4	55.4
Lost Surplus ($1,000s)	51.8	37.3	30.2
Efficiency Gains ($1,000s)	9.2	20.2	25.2

Tolling all roads at one-fourth of the marginal external cost of travel produces significant efficiency gains regardless of the elasticity of demand. When demand is totally inelastic, this system produces gains of approximately $25,000 per day. Efficiency gains increase as the elasticity of demand for travel increases. During the a.m. peak, gains are approximately 30% of the gains when all roads are tolled, regardless of the elasticity of demand. Gains from driv-

ers foregoing travel are smaller because few drivers actually decide not to travel. When the elasticity of demand is -0.5, less than 1% fewer trips are made if only freeways are tolled, but 7.7% fewer are made if all roads are tolled. Apparently, a reduction in a few trips with high marginal external costs can produce large efficiency gains. These results are summarized in the bottom halves of Tables 3 and 4. There may be a political advantage in tolling only limited access roads. Such a system yields consumer losses that are only 10-15% of those when all roads are tolled. Tolls are also much lower; the average driver would pay only 3.5¢ for each freeway mile driven. In addition, toll revenue is not very sensitive to the elasticity of demand. For elasticities in the 0 to -1 range, revenue ranges between $60,000 and $70,000 for the a.m. peak and between $55,000 and $62,000 for the p.m. peak period.

Chapter 5: *Estimating Congestion Costs and Congestion Tolls -- Emme/2*

Tranplan does not permit trips to disappear. For the morning peak, we simulated the effect of having elastic demands by providing each origin-destination pair with a dummy link to which we gave just enough capacity to attract the required number of trips. We then ignored travel on these dummy links in our final tabulations. We could not do this for the afternoon peak. Tranplan limits the number of links that can exit from any given Traffic Analysis Zone. The number of dummy links required from zones with heavy concentrations of employment often violates these limits.

Emme/2 is a transportation modeling package produced by Inro Consulting, a group of computer scientists associated with the University of Montreal. It allows a richer specification of traveler characteristics than Tranplan and also permits treating as elastic the travel demands of one group of travelers at a time. Cost posed an initial problem; Inro charges $24,000 for the version of Emme/2 that will handle the Twin Cities Metropolitan Area's 1,200 traffic analysis zones. Fortunately, the Minnesota Department of Transportation (Mn/DOT) once bought two 1,200-zone packages. Not having found a use for them, it agreed to allow us to borrow them. These negotiations completed, we quickly abandoned our (fortunately brief) attempts to do our own programming.

Emme/2 offers two additional significant advantages. First, it has a macro-programming language which allows us to coerce Emme/2 to recognize that people with different incomes and, hence, travel-time values will respond differently to any given menu of tolls. Inability to analyze these differential responses, we felt, severely limited our ability to deal realistically with the effects of imposing tolls only on portions of the road network. Therefore, we limited our work with Tranplan almost entirely to the admittedly unrealistic case of tolling all congested roads. Second, by allowing a maximum of only 24 iterations, our version of Tranplan did not allow us to refine our estimates of equilibrium to the degree we desired.

The iterative process involved in finding an equilibrium distribution of trips on the Twin Cities road network is time consuming. Although the 1990 Travel Behavior Inventory

Table 5: *Data on the Four Morning Peak Hour Travel Groups*

Income Bracket	Average Annual Household Income	Fraction of All Households	Travel-Time Value ($/hour)	Number of Trips (1,000s)	Time Cost ($1,000)
<$35,000	$25,900	36.8%	5.40	104.9	161.8
$35-55,000	$44,900	28.8%	11.25	213.4	724.5
$55-75,000	$65,000	16.2%	16.25	117.4	584.7
>$75,000	$87,520	18.2%	21.88	82.3	563.7
Totals		100.0%	12.88	518.0	2,034.7

distinguished eight income classes, we have thus far collapsed them into four groups. Even then, finding an equilibrium takes two-three days on a fast 486 DOS computer. Table 5 gives some details on these income groups and on their travel patterns.

Tables 6-9 give different perspectives on the effects on these four income groups of tolling all roads as well as tolling only expressways. Note from Table 6 that, if peak-hour travel were totally inelastic, low income travelers would have the worst of all worlds. Seeking uncongested routes to avoid tolls would result in their trips becoming so circuitous that they would be burdened not only by tolls, but also by spending more time on the road than they would in the absence of tolls; their time plus money costs of travel would almost double. In the inelastic-demand case, congestion pricing would increase their travel costs by 96% as opposed to 24% and 42% for the high-income group and all travelers respectively. With a -1.0 full-price elasticity of demand, their surplus loss would equal 34% of the pre-toll total costs of their trips. The corresponding fractions are 5% for the high-income group and 13% for all travelers. If congestion pricing is to be applied only to part of a congested road network, it would be inefficient to charge those who use its tolled portions close to the costs travelers impose on each other. Doing so would lead to inefficiently great diversion to the network's untolled portion. Indeed, on the Twin Cities network, setting expressway tolls equal to the costs each user imposes on other users would divert so much traffic to untolled arterials that the aggregate resource costs of travel on the network as a whole would be substantially greater than in the

Table 6: *Morning Peak-Hour Travel by Four Income Groups and Their Responses to Tolls*

Time-Value Group	Trips Taken	Time Cost	Tolls Paid	Total Time + Toll Costs
With No Tolls				
Low	104,994	$161,819		
Medium-Low	213,391	$724,475		
Medium-High	117,429	$584,690		
High	82,262	$563,737		
All	518,076	$2,034,721		
All Roads Tolled -- Inelastic Demand				
Low	104,994	$178,351	$138,395	$316,746
Medium-Low	213,391	$703,560	$384,710	$1,088,270
Medium-High	117,429	$545,124	$241,240	$786,364
High	82,262	$521,735	$175,551	$697,286
All	518,076	$1,944,770	$939,896	$2,884,666
All Roads Tolled -- -1 Full-Price Elasticity of Demand				
Low	80,375	$117,227	$44,956	$162,183
Medium-Low	185,544	$554,982	$171,162	$726,144
Medium-High	107,720	$463,613	$121,520	$585,133
High	78,511	$467,996	$96,203	$564,199
All	452,150	$1,063,818	$433,841	$1,497,659

absence of tolls. The literature on Ramsey-rule pricing provides the principles that would provide constrained-optimal tolls that are restricted to a subset of a network's congested roads.[10] Programming these principles for Emme/2 has been difficult. As we see it, doing so requires an equilibrium loading algorithm that stores not just link flows, as Emme/2 does, but also route flows. Even if we could augment Emme/2 with this capacity, the number of routes and the difficulty of imposing tolls on individual links that differ with the routes their users follow would make this a daunting task. Thus far, we have restricted attention to setting charges on each tolled link equal to the same fraction of the difference between that link's marginal and average congestion costs. In the Twin Cities, aggregate benefits do not vary greatly for fractions in the 20-40% range.

Table 7 indicates that travel-time savings partially compensate the three higher-income groups for the tolls they pay; for them, tolls paid are appreciably greater than the surpluses

Table 7: *Effects on the Four Income Classes during the Morning Peak of Tolling All Roads and Expressways Only*

Income Group	Elasticity: 0			Elasticity: -1		
	Time Cost	Toll	Lost Surplus	Time Cost	Toll	Lost Surplus
Aggregate Effects (in $1,000s) with All Roads Tolled						
Low	178.4	138.4	150.9	117.2	45.0	54.5
Medium-Low	703.6	384.7	363.8	555.0	171.2	124.0
Medium-High	545.1	241.2	201.7	463.6	121.5	58.4
High	521.7	175.6	133.5	468.0	96.2	27.2
Per Trip Effects (in $s) with All Roads Tolled						
Low		1.32	1.44		0.56	0.52
Medium-Low		1.80	1.70		0.92	0.58
Medium-High		2.05	1.72		1.13	0.50
High		2.13	1.62		1.23	0.33
Aggregate Effects (in $1,000s) with Only Expressways Tolled at 25% of "Gap"						
Low	168.5	8.1	14.7	156.7	7.3	9.3
Medium-Low	718.7	45.8	40.0	694.3	40.0	18.2
Medium-High	571.6	36.0	23.0	559.3	33.5	5.8
High	548.6	27.2	12.1	542.0	25.8	0.0
Per Trip Effects (in $s) with Only Expressways Tolled at 25% of "Gap"						
Low		0.08	0.14		0.07	0.09
Medium-Low		0.21	0.19		0.19	0.09
Medium-High		0.31	0.20		0.29	0.05
High		0.33	0.15		0.31	0.00

they lose. Not so for the lowest-income group, however. For them, tolls are less than lost surplus, even in the elastic-demand case; the increased circuity of the trips tolls induce them to take increases both the dollar *and* the time costs of their trips.

Obviously, tolling only expressways would shift traffic from them to the surface road network. Surprisingly, marginal-cost congestion pricing on all congested roads would also reduce traffic on expressways by more than on arterials. With all congested roads tolled, expressway and non-expressway vehicle miles would respectively decline by 19% and 8% in the elastic-demand case. With only expressways tolled at 25% of the difference between marginal and average congestion costs, expressway vehicle miles would decline by 8% while

Table 8: *Effect of Tolling Four Income Groups During the Morning Peak on the Time and Distance They Travel on Expressways and Other Roads and on Aggregate Differences between Marginal and Average Congestion Costs*

Time-Value Group	Time Spent Traveling (1,000 hours)		Vehicle Miles Traveled (1,000s)		Aggregate Marginal less Average Costs ($1,000s)	
	Expressway	Other	Expressway	Other	Expressway	Other
With No Tolls						
Low	10.7	19.5	423	572	118.1	135.5
Medium-Low	23.6	41.3	919	1,214	282.8	330.5
Medium-High	14.3	21.9	512	636	179.3	190.5
High	10.7	15.2	420	437	132.4	134.2
All	59.3	97.9	2,274	2,859	712.6	790.7
All Roads Tolled--Inelastic Demand						
Low	6.6	25.4	292	771	35.4	103.0
Medium-Low	19.3	43.4	830	1,331	143.8	240.9
Medium-High	13.4	20.5	565	626	112.0	129.2
High	10.2	13.9	436	421	84.3	91.3
All	49.5	103.2	2,123	3,149	375.5	564.4
All Roads Tolled -- -1 Full-Price Elasticity of Demand						
Low	4.4	17.3	208	552	12.4	32.6
Medium-Low	15.5	34.1	711	1,111	70.7	100.5
Medium-High	11.3	17.5	512	567	59.1	62.4
High	9.2	12.4	420	397	48.6	47.6
All	40.4	81.3	1,851	2,627	190.7	243.1
Only Expressways Tolled* -- Inelastic Demand						
Low	5.7	25.4	254	726	32.3	183.1
Medium-Low	20.1	43.7	854	1,261	183.2	367.6
Medium-High	14.0	21.2	583	606	144.1	196.7
High	10.8	14.3	456	406	108.8	131.8
All	50.6	104.6	2,147	2,999	468.3	879.2
Only Expressways Tolled* -- -1 Full-Price Elasticity of Demand						
Low	5.6	23.2	245	684	29.0	157.0
Medium-Low	19.6	42.4	819	1,245	160.0	342.9
Medium-High	14.1	20.6	576	600	134.0	187.9
High	11.2	13.9	460	402	103.3	125.5
All	50.5	100.1	2,100	2,931	426.3	812.3

*Tolls on each expressway link equal 25% of the difference between marginal and average trip costs.

Table 9: *Effect of Tolling Four Income Groups*
on the Distribution of Speeds at which They Travel:
Percentages of Trip Miles in Different Speed Categories

Speed Range (Miles/Hour)	Income Group			
	Low	Low Medium	High Medium	High

Speed Distribution with No Congestion Tolls

Speed Range (Miles/Hour)	Low	Low Medium	High Medium	High
0-15	1.7%	1.7%	1.9%	1.9%
15-30	26.8%	26.5%	26.4%	26.1%
30-45	40.0%	42.5%	43.3%	43.6%
45-60	31.5%	29.3%	28.4%	28.4%
Total	100.0%	100.0%	100.0%	100.0%

All Roads Tolled -- Inelastic Demand

Speed Range (Miles/Hour)	Low	Low Medium	High Medium	High
0-15	1.4%	1.1%	1.2%	1.1%
15-30	28.4%	21.9%	19.0%	18.2%
30-45	36.9%	42.4%	43.2%	43.9%
45-60	33.3%	34.6%	36.6%	36.8%
Total	100.0%	100.0%	100.0%	100.0%

All Roads Tolled -- -1 Full-Price Elasticity of Demand

Speed Range (Miles/Hour)	Low	Low Medium	High Medium	High
0-15	0.8%	0.6%	0.7%	0.8%
15-30	24.4%	17.8%	15.8%	15.1%
30-45	36.0%	39.0%	37.3%	38.5%
45-60	38.8%	42.6%	46.2%	45.6%
Total	100.0%	100.0%	100.0%	100.0%

Only Expressways Tolled* -- Inelastic Demand

Speed Range (Miles/Hour)	Low	Low Medium	High Medium	High
0-15	2.3%	1.9%	2.0%	1.9%
15-30	31.3%	24.9%	21.9%	20.6%
30-45	36.2%	40.6%	42.3%	42.6%
45-60	30.2%	32.6%	33.8%	34.9%
Total	100.0%	100.0%	100.0%	100.0%

Only Expressways Tolled* -- -1 Full-Price Elasticity of Demand

Speed Range (Miles/Hour)	Low	Low Medium	High Medium	High
0-15	2.0%	1.8%	2.0%	1.8%
15-30	29.9%	24.4%	21.3%	20.0%
30-45	36.8%	40.4%	42.0%	41.9%
45-60	31.3%	33.4%	34.7%	36.3%
Total	100.0%	100.0%	100.0%	100.0%

*Tolls on each expressway link equal 25% of the difference between marginal and average trip costs.

arterial travel would increase by 3%.

At present, the four income groups differ little in the distribution of their travel speeds.

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The low-income group spends about 2.5% more of its travel miles during the morning peak driving at 45-60 mph than do its upper-income counterparts and about 2.5% less per mile at 30-45 mph. Might this be a manifestation of reverse commuting? Congestion pricing of all roads would result in increased proportions of miles traveled at 45-60 mph for all groups. These increases would increase with income -- 6%, 18%, 29% and 30% for the low, medium-low, medium-high, and high groups, respectively.

Figures 10-14 further illustrate the differences between high- and low-income travelers in the effects congestion pricing on all congested roads would have on morning peak-hour travel patterns. Figures 10 and 11 deal with percentage changes in freeway and expressway travel when the full-price elasticity of demand for auto travel is, respectively, 0 and -1. Figures 12 and 13 cover absolute changes in Interstate 394-corridor travel patterns again when the demand elasticity is, respectively, 0 and -1. Finally, Figure 15 deals with absolute changes in and around the Minneapolis central business district with a -1 elasticity.

All of these figures reveal much the same pattern. Low income travelers would respond to congestion pricing with dramatic shifts away from expressways in main-flow directions and onto neighboring arterials but with modest shifts onto expressways in reverse flow directions. Reverse commuting? The resulting speed increases would attract modest numbers of high income travelers onto expressways. These shifts in travel patterns would significantly increase the average value of travel time on expressways (at least in the main direction of flow) and decrease it on arterials. Our computations take these second-order effects on tolls into account.

Figure 10a: Percentage Decrease in Low-Income-Group Expressway Travel During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = 0

Figure 10b: Percentage Increase in High-Income-Group Expressway Travel During Morning Peak-Hour: All Roads Tolled, Full-Price Demand Elasticity = 0
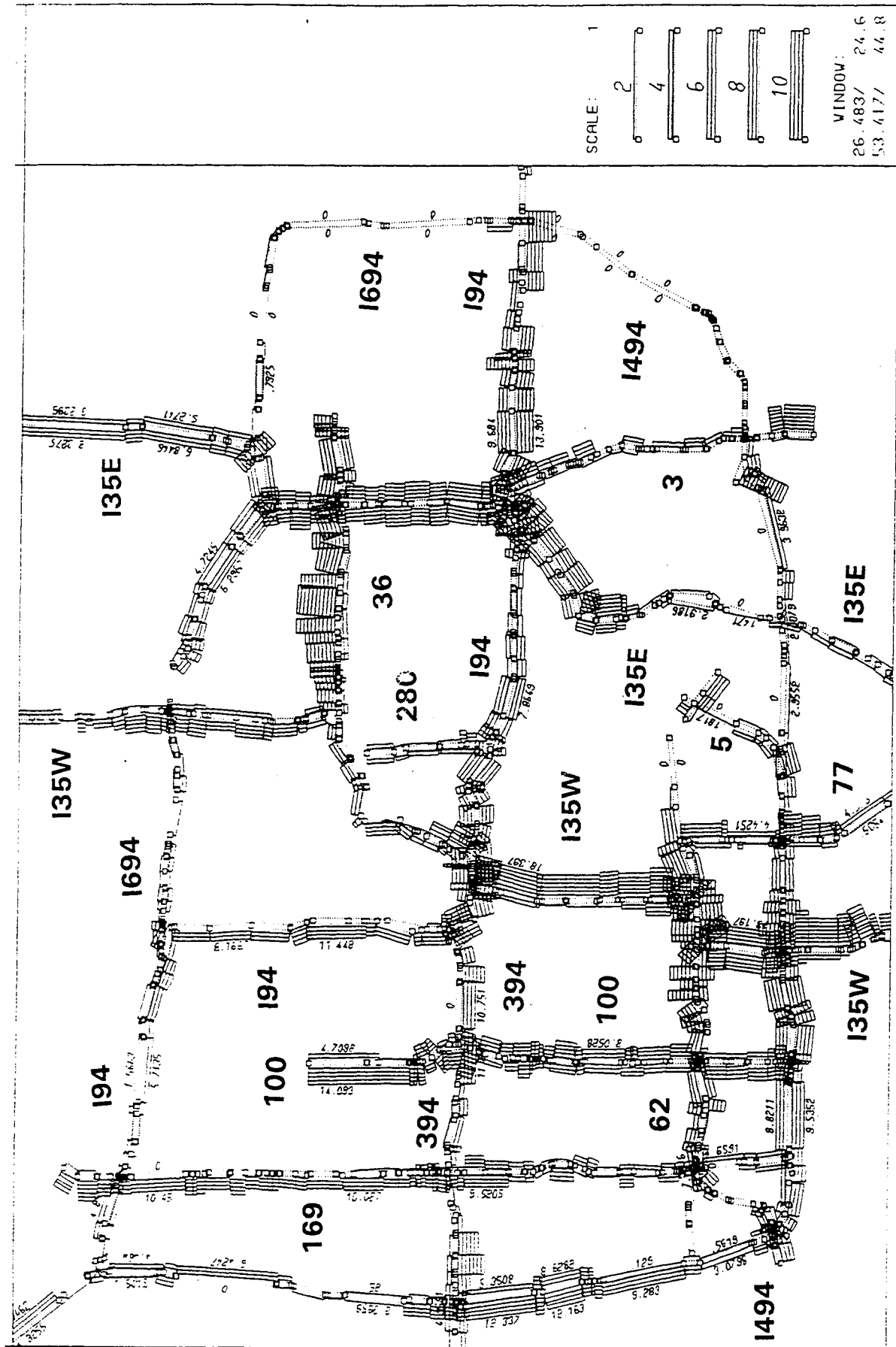
Figure 10c: Percentage Increase in Average Value of Travel Time on Expressways: All Roads Tolled, Full-Price Demand Elasticity = 0
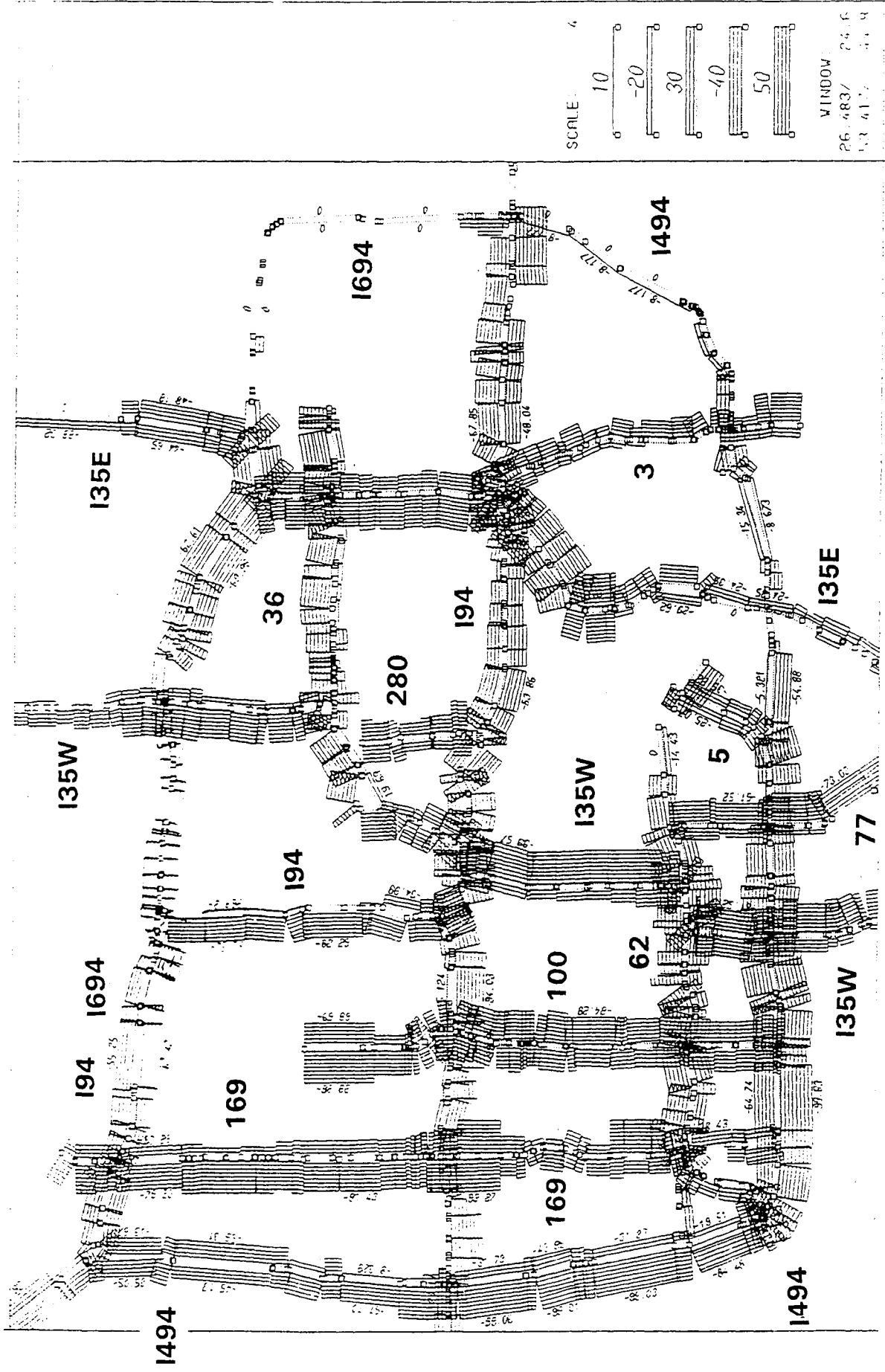
Figure 11a: Percentage Decrease in Low-Income-Group Expressway Travel During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1

Figure 11b: Percentage Increase in High-Income-Group Expressway Travel During
Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1

-48-
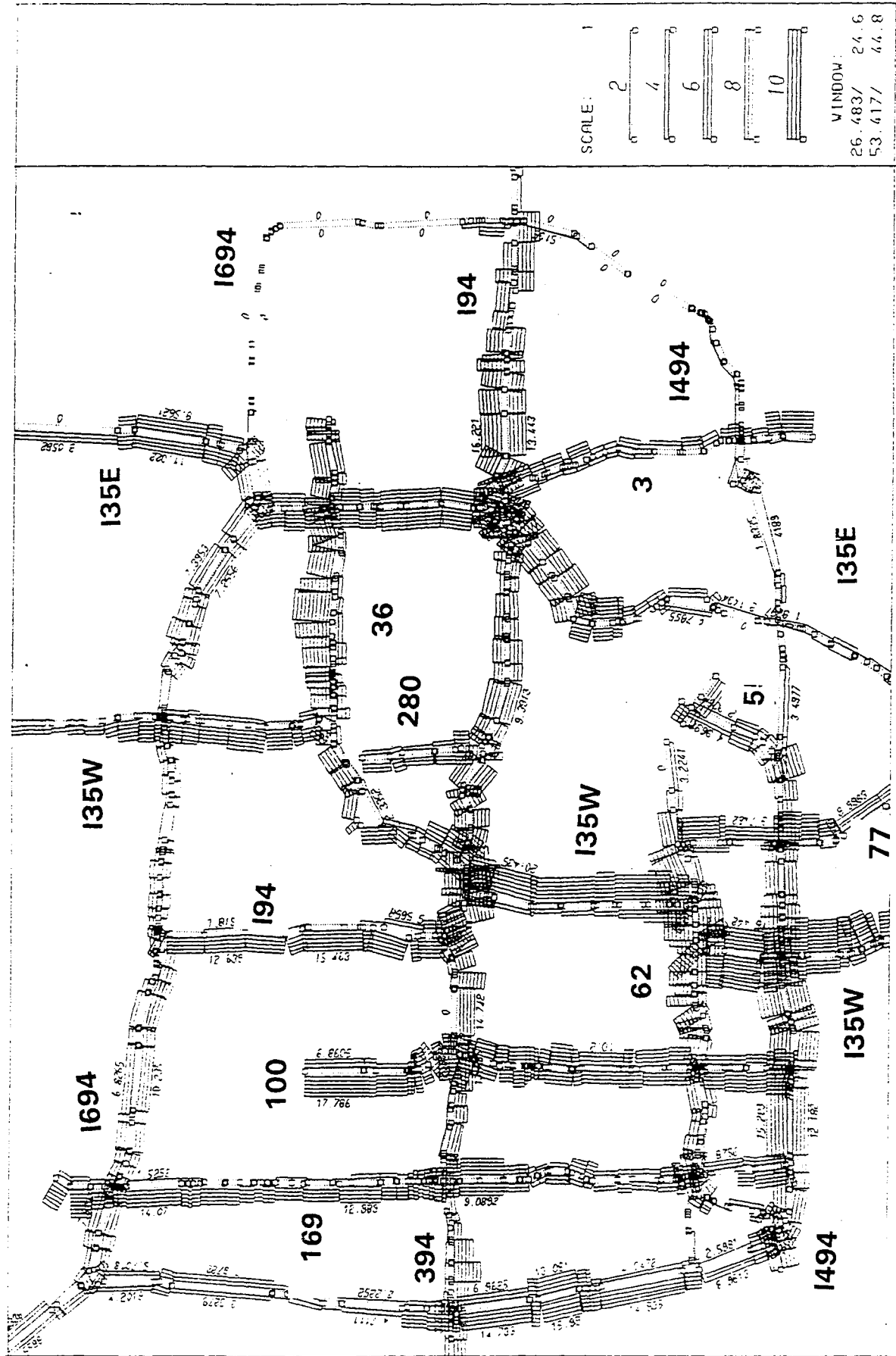
Figure 11c: Percentage Increase in Average Value of Travel Time on Expressways:
All Roads Tolled, Full-Price Demand Elasticity = -1

-49-
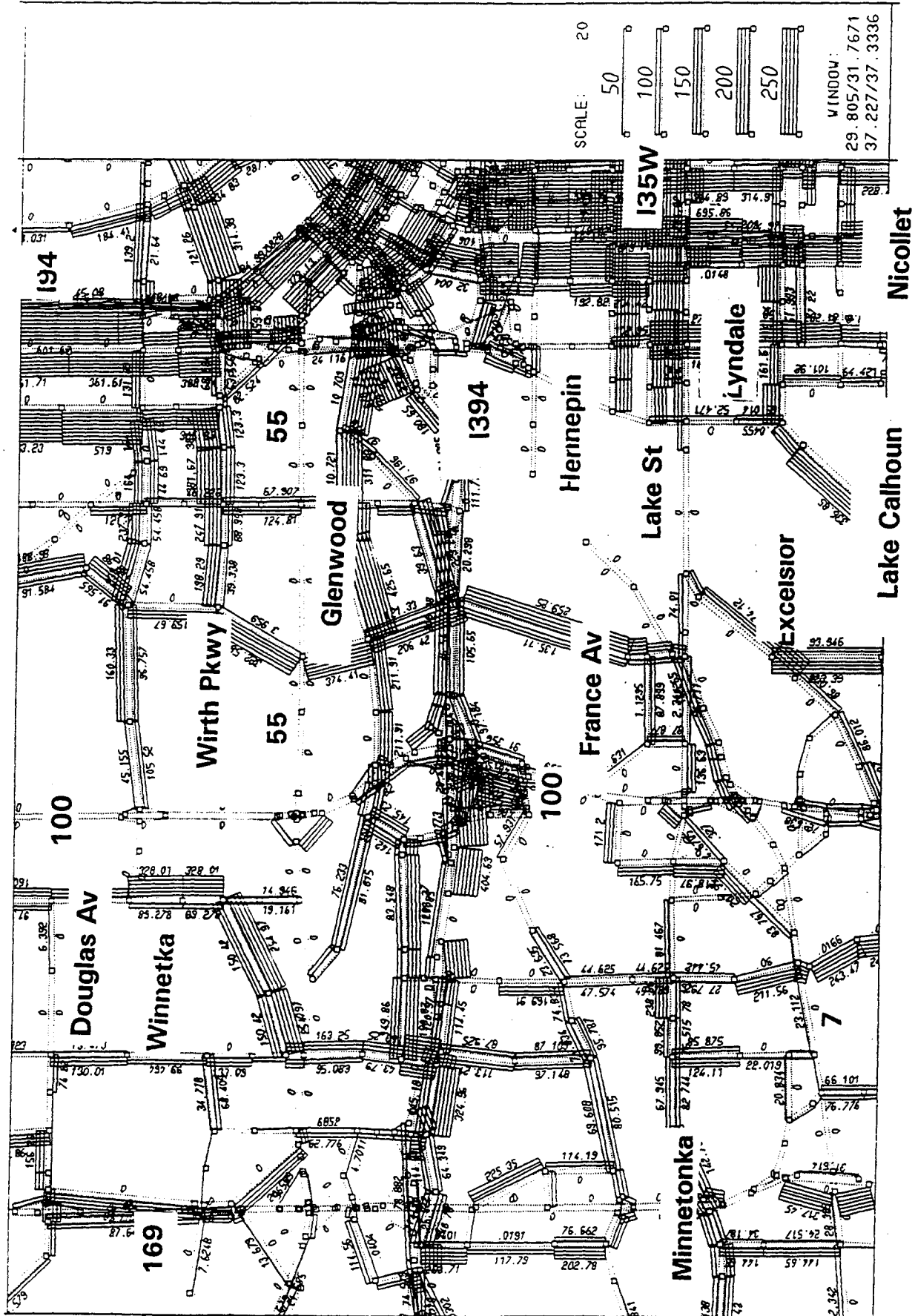
Figure 12a: Absolute Increase in Low-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = 0
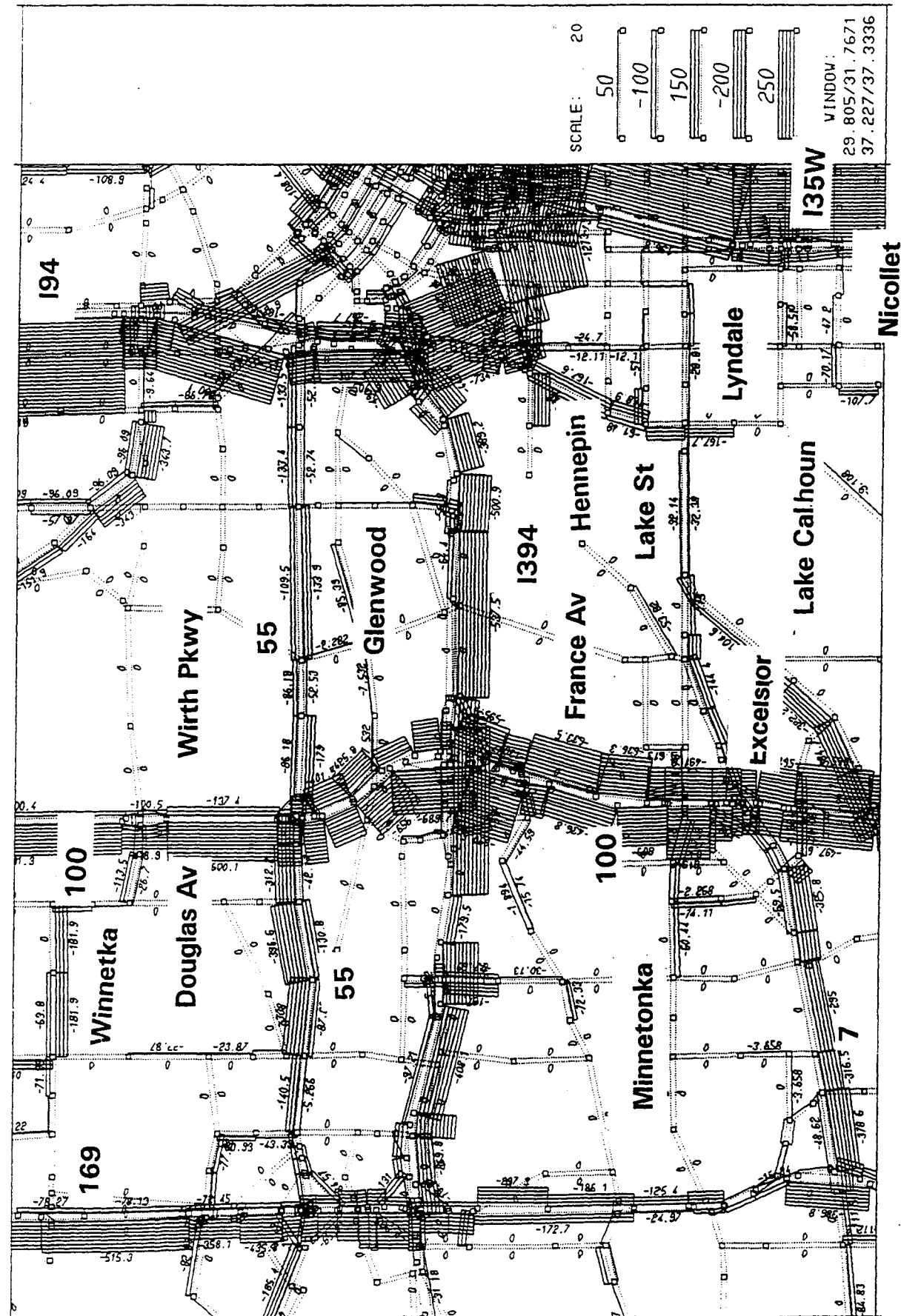
Figure 12b: Absolute Decrease in Low-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = 0
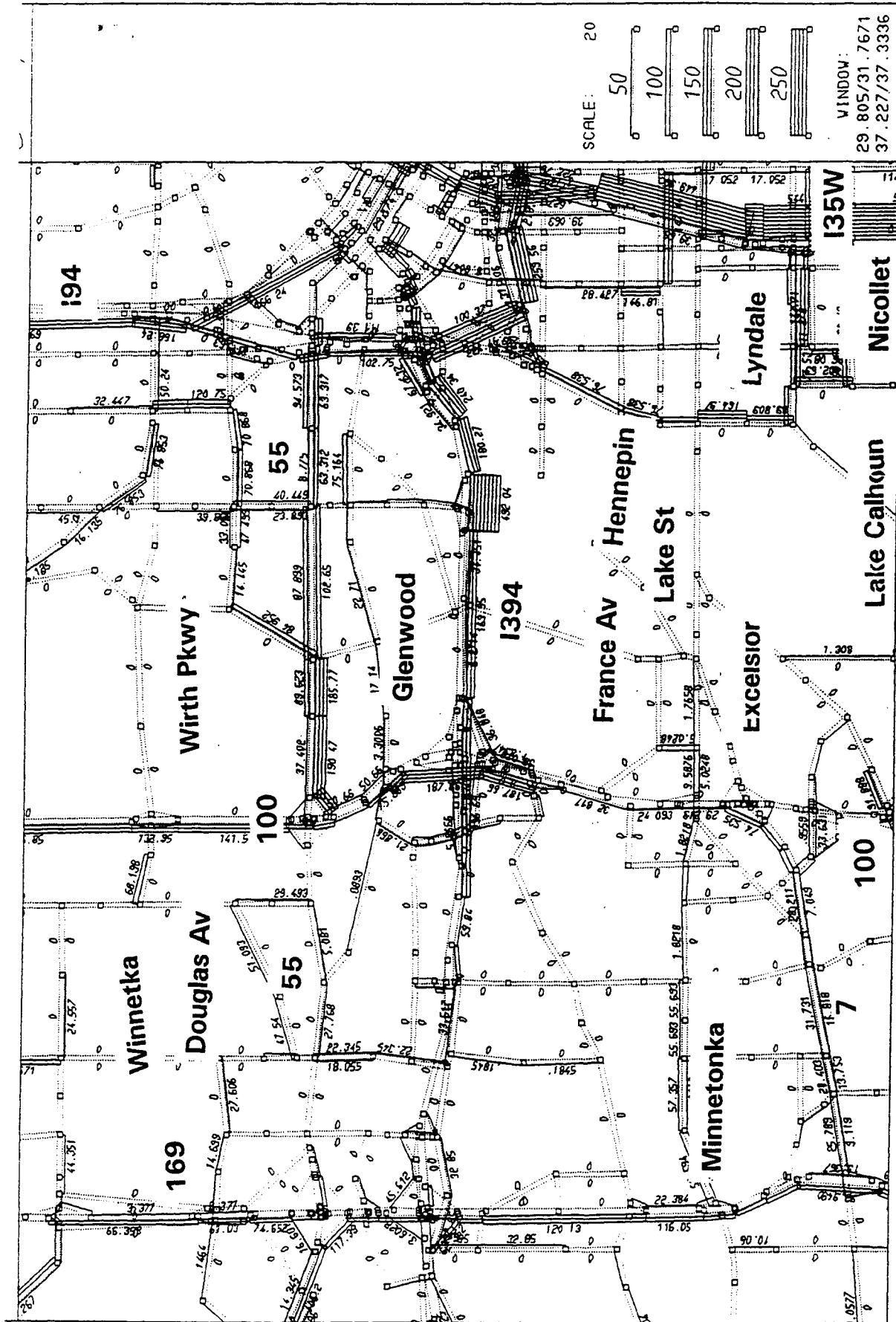
Figure 12c: Absolute Increase in High-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = 0
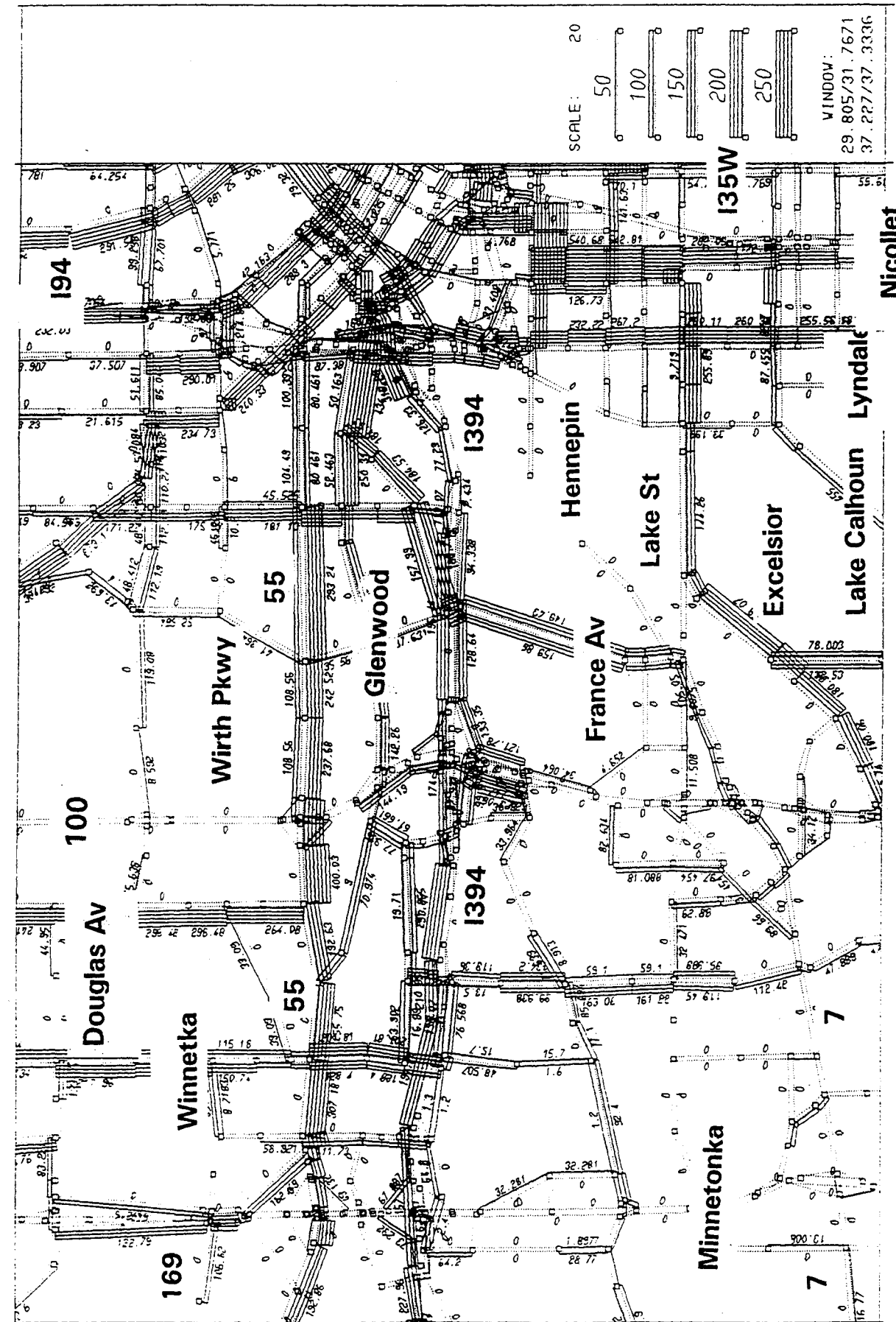
Figure 13a: Absolute Increase in Low-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1
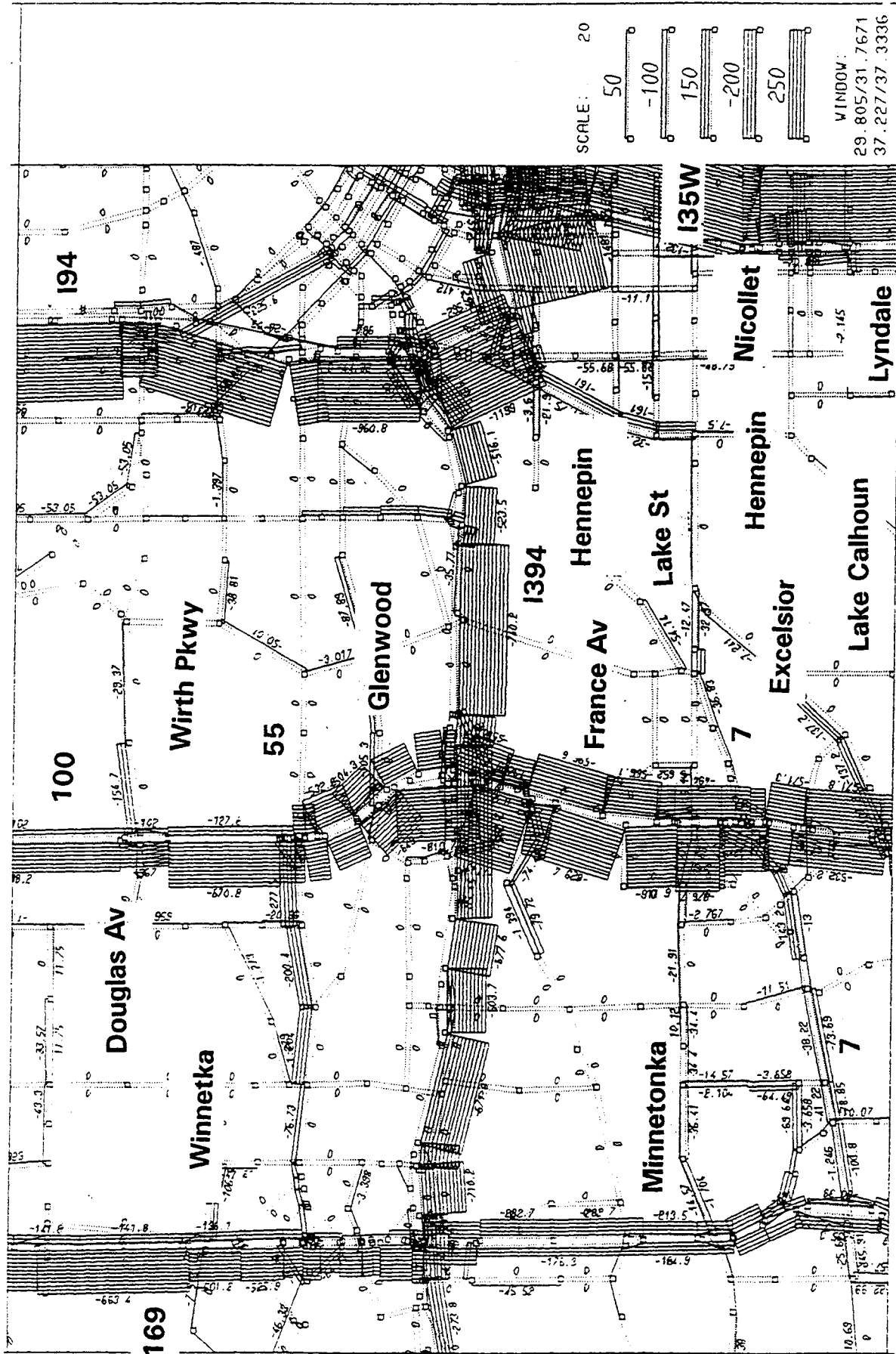
Figure 13b: Absolute Decrease in Low-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1

-54-
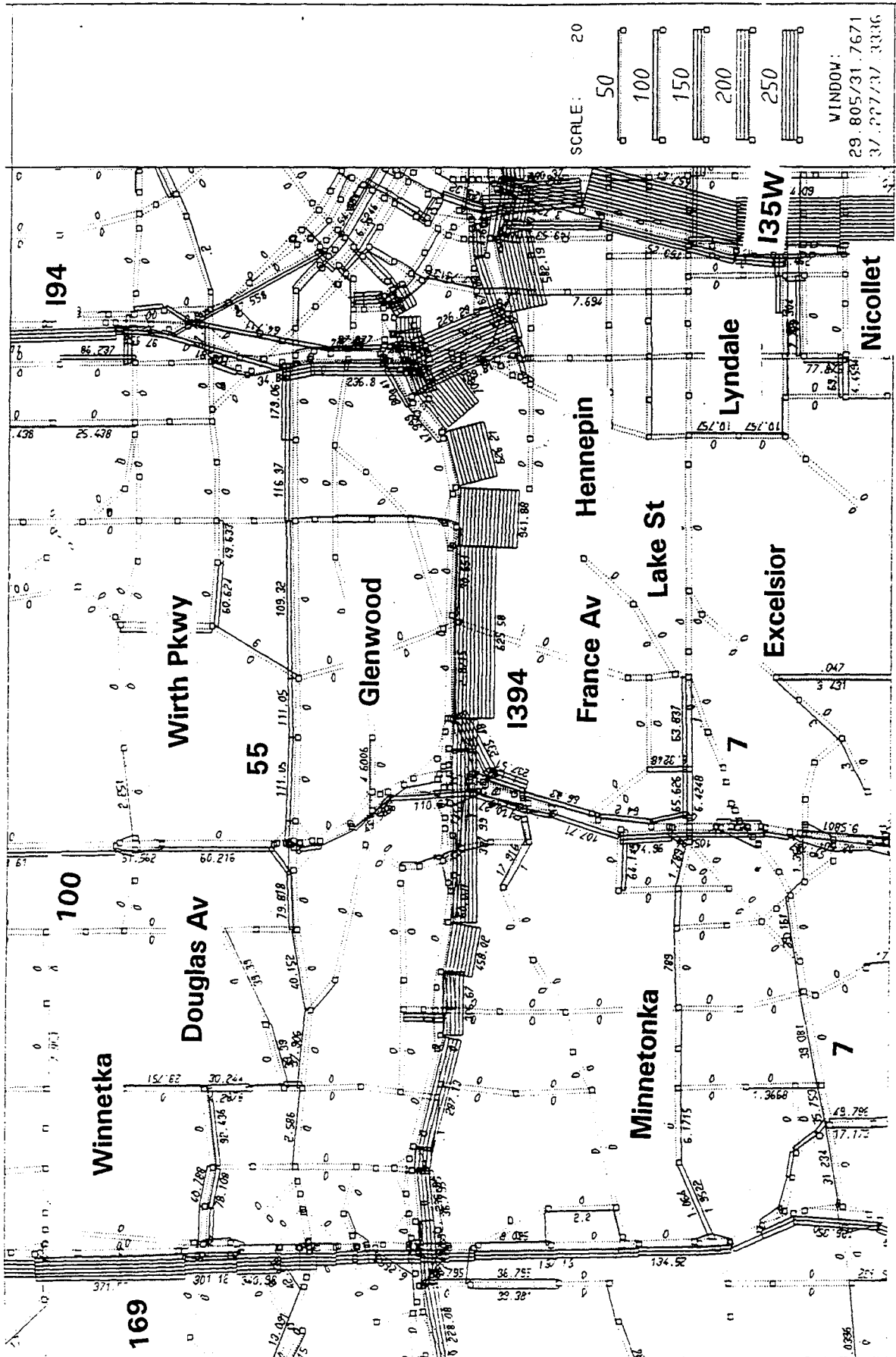
Figure 13c: Absolute Increase in High-Income-Group Travel on Roads in I-394 Corridor During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity =·-1
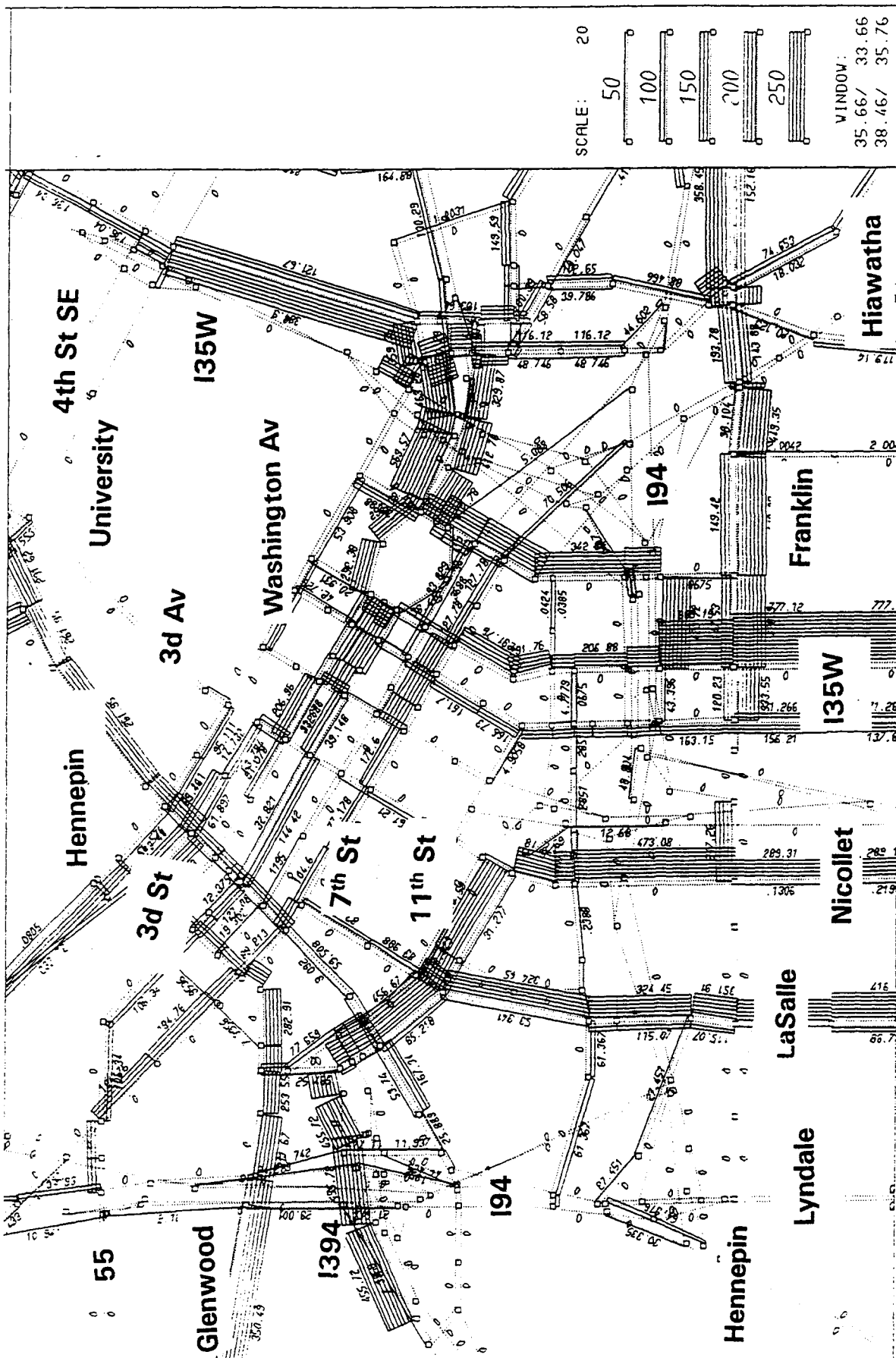
Figure 14a: Absolute Increase in Low-Income-Group Travel on Roads in Minneapolis Central Business District During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1

Figure 14b: Absolute Decrease in Low-Income-Group Travel on Roads in Minneapolis Central Business District During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1.
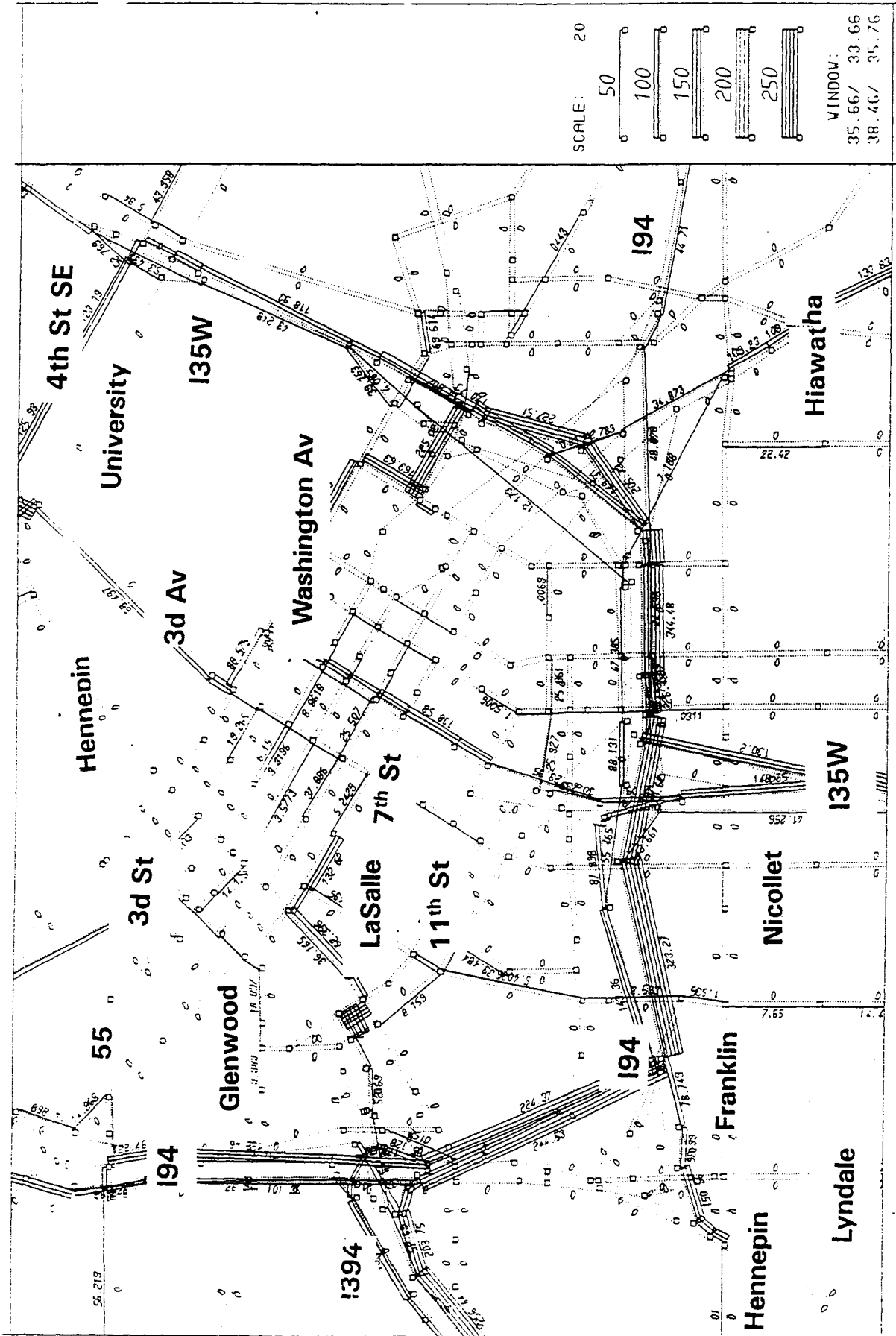
Figure 14c: Absolute Increase in High-Income-Group Travel on Roads in Minneapolis Central Business District During Morning Peak Hour: All Roads Tolled, Full-Price Demand Elasticity = -1

Chapter 6: *Distributing Toll Revenues to Compensate for the Losses Congestion Pricing Would Impose*

The first of the Intermodal Surface Transportation Efficiency Act's congestion-pricing demonstration grants went to the San Francisco Bay Area to help it institute time-of-day pricing on the Oakland Bay Bridge. After appreciable research and a substantial public relations effort that stressed the efficiency aspects of congestion pricing, the project foundered for want of a sponsor in the state legislature for the law that will be required to alter the bridge's toll structure. Apparently, no legislator was willing to cope with accusations of favoring tax increases just to increase the efficiency with which transportation facilities are used.

In May 1995, the State and Local Policy Program of the University of Minnesota's Humphrey Institute conducted a week-long "Citizens Jury." In it, a 24-member representative sample of the Twin Cities Metropolitan Area population was asked to render a verdict on presentations by advocates and opponents of congestion pricing. The proponents stressed the value of congestion pricing as a way of overcoming a shortage of funds to undertake worthy transportation improvements. The predominant view among the jurors: Congestion isn't a serious problem in the Twin Cities. If it becomes a problem, the most sensible way to finance worthy investments would be to raise the gas tax.

In no place of which we have heard has a spontaneous ground swell of demand for congestion pricing. In San Francisco and the Twin Cities, at least, claiming either enhanced efficiency or solving a funds shortage generated little enthusiasm for the concept.

Our modest proposal: Let's sell congestion pricing by emphasizing an important implication of its efficiency: getting something for nothing. Most taxes impose efficiency losses; the losses those taxed incur exceed the revenues governments receive through their imposition. Congestion pricing, though, results in efficiency gains. Our calculations suggest that the immediate effect of congestion pricing will be to make all but a small fraction of the population worse off. Our calculations also suggest, however, that tolling the entire road

network would generate $1.50-$1.75 in revenue for each dollar of surplus travelers would lose. This being the case, there ought to be a way to compensate all losers and still have a substantial pot left over to finance reduced real-estate, fuel, and other taxes as well as transportation projects. Thrill to the slogan,

Congestion Pricing, Everybody's Win-Win Proposition!

But there is a problem here. Givebacks should not be set up in a way that defeats the efficiency goal at which congestion pricing is aimed. To cite an extreme example of such a self-defeating procedure, suppose that all travelers know that they will each receive a check at the end of a month equal to their congestion-toll payments during that month. Such a scheme would certainly compensate all losers, but only at the price of destroying their incentives to base their travel decisions on the full costs of their trips.

Sadly to say, we have thus far been unsuccessful in developing a scheme for distributing toll revenues among losers in a way that would eliminate their opposition to congestion pricing but would not appreciably diminish the efficiency of their responses to it. We are even further from concocting a scheme that would conform to generally accepted standards of equity and would leave an appreciable share of toll revenues for transportation improvements and other desirable projects.

Tables 10 and 11 describe one of our failed attempts at designing a toll-revenue distribution policy. Table 10 indicates that, in 1990, morning peak-hour auto travel was undertaken by 327,295 drivers. Most peak-hour drivers are members of multi-driver households; only 170,575 -- 19% -- of the metropolitan area's 875,000 households in 1990 contained peak-hour drivers. With all congested roads tolled, the aggregate daily losses to drivers from foregone trips and trips still taken are $849,000 or $264,100 if the full-price demand elasticity for auto trips is, respectively, 0 or - 1. Losses per driver (many take more than one peak hour trip) and per household average $2.60 and $4.98 respectively for a 0 price elasticity and $0.81 and $1.55 for a -1 elasticity. These losses are roughly evenly distributed among the lower three income groups but appreciably lower for the top group. If only

Table 10: *Losses from Congestion Tolls to*
Morning Peak-Hour Drivers and Households with Drivers

	7-County Household income Groups: 1990				
	≤ $35k	$35-55k	$55-75k	> $75	Totals
Morning peak-hour drivers	57,246	127,848	65,296	76,927	327,295
Households with morning peak-hour drivers	26,114	66,709	36,977	40,775	170,575

Daily Lost Surplus During Morning Peak Hour—All Roads Tolled

Full Price Elasticity = 0

	≤ $35k	$35-55k	$55-75k	> $75	Totals
Aggregate Loss	$150,900	$363,800	$201,700	$133,500	$849,900
Loss/driver	$2.64	$2.85	$3.09	$1.74	$2.60
Loss/driving household	$5.78	$5.45	$5.45	$3.27	$4.98

Full Price Elasticity = -1

	≤ $35k	$35-55k	$55-75k	> $75	Totals
Aggregate Loss	$54,500	$124,000	$58,400	$27,200	$264,100
Loss/driver	$0.95	$0.97	$0.89	$0.35	$0.81
Loss/driving household	$2.09	$1.86	$1.58	$0.67	$1.55

Daily Lost Surplus During Morning Peak Hour—Only Expressways Tolled

Full Price Elasticity = 0

	≤ $35k	$35-55k	$55-75k	> $75	Totals
Aggregate Loss	$12,800	$33,800	$17,500	$8,800	$72,900
Loss/driver	$0.22	$0.26	$0.27	$0.11	$0.22
Loss/driving household	$0.49	$0.51	$0.47	$0.22	$0.43

Full Price Elasticity = -1

	≤ $35k	$35-55k	$55-75k	> $75	Totals
Aggregate Loss	$9,300	$18,200	$5,800	---	$33,300
Loss/driver	$0.16	$0.14	$0.09	---	$0.10
Loss/driving household	$0.36	$0.27	$0.16	---	$0.20

expressways are tolled, losses are markedly lower, of course, but follow roughly the same relative pattern.

Again, only about 20% of metropolitan-area households have members who drive during the morning peak hour. Again, a partial or complete give-back of tolls just to them would reduce or eliminate their incentive to take the full costs of their trips into account in deciding when, where, and how to make them. This problem could be eliminated by providing

Table 11: *Suppose EACH Household in 7-County Metro Area Is Given the AVERAGE Loss to Driving Households in Its Income Group. How Much Would this Cost? How Much Would Be Left Over for Highway and Other Purposes?*

<u>7-County Household income Groups: 1990</u>

	<u>< $35k</u>	<u>$35-55k</u>	<u>$55-75k</u>	<u>> $75</u>	<u>Totals</u>
Number of Households	412,122	237,901	123,152	176,670	875,833

Daily Losses per Driving Household During Morning Peak Hour

All congested roads tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$5.78	$5.45	$5.45	$3.27	$4.98
Elasticity = -1	$2.09	$1.86	$1.58	$0.67	$1.55

Only expressways tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$0.49	$0.51	$0.47	$0.22	$0.43
Elasticity = -1	$0.36	$0.27	$0.16	---	$0.20

Daily Payments ($1,000) to Households Required to Leave **All** *Better Off*

All congested roads tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$2,382.1	$1,296.5	$671.1	$577.7	$4,361.6
Elasticity = -1	$861.3	$442.5	$191.6	$176.7	$1,357.5

Only expressways tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$201.9	$121.3	$57.9	---	$376.6
Elasticity = -1	$148.4	$64.2	$19.7	---	$175.2

Peak Hour Tolls Collected ($1,000)

All roads tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$138.4	$384.7	$241.2	$175.6	$939.9
Elasticity = -1	$45.1	$171.2	$121.5	$96.2	$433.8

Only express-ways tolled

	< $35k	$35-55k	$55-75k	> $75	Totals
Elasticity = 0	$6.2	$39.6	$30.6	$23.9	$100.3
Elasticity = -1	$5.5	$34.6	$28.4	$22.7	$91.3

givebacks to all households, not just those with peak hour drivers. Sadly to say, Table 11 reveals this distribution scheme to be a fiscal disaster. Depending on the assumed demand elasticity and whether congested roads or only expressways are tolled, financing it would cost

3-4 times toll revenue collections. We haven't yet computed the increase in fuel taxes that would be required to cover these costs.

Further study of congestion-toll rebate schemes is a high priority for our continuing congestion-pricing research. Among the schemes that we are currently examining are payments only to job holders and marketable "road scholarships" to low- and moderate-income households that diminish with increases in their income. On grounds that moving school opening hours out of peak periods is an administrative task considerably easier than instituting congestion pricing and that high-school students with after-school burger-flipping jobs do not contribute greatly to peak-hour road congestion, the former scheme would probably have some combination of income, age, and number-of-hours-worked limitations on payments to individual workers. We welcome suggestions for other schemes.

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Endnotes

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[1] That is, given a state of affairs in which a simultaneous doubling of the investment in a road and the traffic it carries would leave unchanged the costs to a user of a trip on it.

[2] We ignore here paying for the road damage that heavy axle-loads do. Newbery (1989) describes plausible circumstances under which marginal-cost damage charges would cover road-maintenance costs. Appendix A contains a simple proof of this and other assertions made in the text. That, given constant returns to scale, congestion tolls would exactly cover capital costs for a long-run optimal road does not necessarily mean that plowing all toll revenues back into road improvements would be efficient. Suppose that population and travel are no longer growing, that low-cost construction techniques have been developed to provide roads that will last forever without maintenance, and that a long-run-optimal road network has been created. Marginal-cost tolls would then function as a normal return on the capital society has invested in the road network that should be used in the same way as any other non-earmarked source of government revenue. Efficiency would not dictate either spending these revenues on highways or eliminating congestion tolls.

[3] In the road case, the full price -- the value of consumer-supplied inputs plus whatever toll may be charged -- is the relevant concept. See Section 2.

[4] The formulae for the solid and dashed average cost curves are, respectively, $N/K = 4(1 - t^*/t)t^*/t$ and $t = 1 + (N/K)^4$ where N/K denotes the ratio of vehicle volume to ideal capacity and t and t^* are respectively actual travel time per mile and travel time per mile at a zero N/K ratio.

[5] This example is adapted from Braess (1968).

[6] There is only one equilibrium assignment. To show this, let a, b, and c be the flows along routes A, B, and C respectively. Travel times along these routes will be $3(a + c)/2 + 50$ for A, $50 + 3(b + c)/2$ for B and $3(a + c)/2 + 15 + 3(b + c)/2$ for C. The time along route C is greater than that along route A only if $3(a + c)/2 + 15 + 3(b + c)/2 > 3(a + c)/2 + 50$. Rearranging terms shows that this inequality can hold only if $3(b + c)/2 > 35$, but this is impossible since $b + c \leqslant 20$. The same reasoning applies to the comparison of travel times between routes B and C. Thus, in equilibrium, all drivers use route C.

[7] If time per trip along one of the congested routes is $3n/2$ minutes, total travel time per hour is $3n^2/2$ minutes. Differentiating with respect to n yields $6n/2$ as marginal travel time per trip.

[8] For recent surveys, see Goodwin (1992) and Oum, *et al.* (1992).

[9] The measure of capacity implicit in equation (1) differs from that of Figure 4. In Figure 4, it takes twice as long to travel a mile at a volume/capacity ratio of one as at a volume/capacity ratio of zero. In equation (1), a volume/ capacity ratio of 1.6 is required for time per mile to be double that at a volume/capacity ratio of zero.

[10]This literature derives from a classic paper by Frank Ramsey, Ramsey (1928). His problem was to select a set of excise taxes that generate a desired level of government revenues while minimizing the costs of the distortions to economic activity that such taxes inevitably impose. In the simple case with which Ramsey dealt, optimality called for excises inversely proportional to the elasticity of demand for the taxed commodities with the proportionality factor increasing with increases in the revenue to be raised. Baumol and Bradford (1970) is an excellent survey of this literature. Mohring (1970) deals with its applications to the peak-load problem in general and to road pricing specifically.

Appendix A:

Proofs of Propositions about Traveler Behavior

and the Optimal Design and Pricing of Roads

(1) The Optimal Relationship Between Congestion Tolls and Capacity Costs: If the production trips on a highway involves constant returns to scale, travel time per trip can be written as a function of the volume-capacity ratio, N/K, and the hourly cost of capacity can be written as $P_K K$ where P_K is the hourly price of a unit of capacity -- depreciation, maintenance, and the interest that the funds invested in capacity could have earned if invested elsewhere. Ignoring vehicle operating costs for simplicity and denoting the average value to vehicle occupants of an hour's travel time by V, the total *variable* costs of the trips taken during an hour can be written

$$\textit{Variable Costs} = VNT = VNf(N/K) \qquad \text{(A-1)}$$

The short-run marginal cost of a trip is

$$\partial VC/\partial N = Vf(N/K) + VNf' [\partial(N/K)/\partial N] = Vf(N/K) + VN/Kf' \qquad \text{(A-2)}$$

The first term on the right of equation (A-2) is the average time cost per trip; the second is the difference between the trip's average and marginal time costs -- the cost each vehicle in the traffic stream imposes on the occupants of all other vehicles in the stream by slowing their trips. To set the price of a trip equal to its marginal cost, a toll equal to this latter amount must be charged each of the N travelers. If this were done, total toll collections would be $Vf'N^2/K$.

A plausible objective for a highway authority would be to select that level of capacity for a highway that would minimize the *total* costs of travel on it -- the time and other costs that users incur directly *plus* the cost to the authority of providing the highway's services, $P_K K$. More capacity means faster trips, but greater highway capital costs. The total cost of N trips an hour on a highway is

$$\textit{Total Costs} = VNf(N/K) + P_K K \qquad \text{(A-3)}$$

Differentiating with respect to K, setting the result equal to zero, and rearranging terms yields

$$- VN^2/K^2f' + P_K = 0 \qquad \text{(A-4)}$$

as the condition that must be satisfied by the cost-minimizing capacity level. In words, this equation says that the last dollar per hour spent to expand capacity should yield hourly user-cost savings of a dollar.

Multiplying equation (A-4) through by K and rearranging terms yields

$$Vf'N^2/K = P_K K \qquad \text{(A-5)}$$

The right-hand side of (A-5) is the hourly cost of the road's capacity. Again, from equation (A-2), the optimal congestion toll -- the cost each vehicle imposes on the costs of all other vehicle by adding to the level of congestion -- is $Vf'N/K$. If this toll were to be imposed on each vehicle, total toll collections would be $Vf'N^2/K$. But this is the left-hand side of (A-4) which equals the total hourly cost of the highway. In brief, then, given constant returns to scale, an optimally priced road that has been designed to minimize the sum of user and provider costs would generate toll revenues just sufficient to cover its provider's costs. Given constant returns to scale, optimally designed and priced roads would be exactly self-supporting.

(2) Traveler Behavior, the "Full Price" of Travel, and the Social Value of Travel Time: Considering a hypothetical bus line between Here and There is perhaps the simplest way to show how the disutility travelers incur from spending time in transit should be taken into account in optimizing and pricing transportation facilities. Suppose that N consumers utilize the services of this bus line. Consumer i ($i = 1,..., N$) derives utility from consuming s^i units per week of a general purpose commodity, stuff, conveniently priced at \$1 a unit. He also derives utility from what happens There during each of the b^i trips per week he takes from Here to There and back. However, he incurs disutility from the time, $\tau^i = b^i t$, he spends traveling where t is the number of hours required to take a trip. Travel time per trip is a function, $t(B, X)$, of the total number of trips taken, $B = \Sigma b^i$, and of X, the total number of bus hours of service provided on the route each week.

Consumer i's problem, then, is to maximize his utility, $u^i(s^i, b^i, \tau^i)$ subject to his budget constraint, $I = s^i + Fb^i$ where F is the fare per bus round trip. Setting up the Lagrangian expression

$$z^i = u^i(s^i, b^i, \tau^i) + \lambda^i(I^i - s^i - Fb^i)$$

and differentiating with respect to s^i and b^i yields

$$z_a^i = u_a^i - \lambda^i = 0 \qquad \text{(A-6)}$$

$$z_b^i = u_b^i + u_T^i t + u_T^i b \, t_B - \lambda^i F = 0 \qquad \text{(A-7)}$$

as first order conditions for utility maximization where subscripts refer to partial derivatives. It seems reasonable to suppose that consumer i does not take into account the effect his trips have on his own travel time. If so, $u_T^i \, b \, t_B$ can be ignored. Amending equation (A-7) to take this assumption into account and dividing by equation (A-6) would then yield

$$u_b^i/u_a^i + t \, u_T^i/u_a^i = F \qquad \text{(A-8)}$$

In equation (A-8), u_T^i/u_a^i, the ratio of the marginal *dis*utility of travel time to the marginal utility of dollars has the dimension dollars per hour. It therefore seems reasonable to substitute for this ratio $-V^i$, the money cost consumer i attaches to his travel time. Doing so changes equation (A-8) to

$$u_b^i/u_a^i = u_b^i/\lambda^i = F + V^i t \qquad \text{(A-9)}$$

This relationship says that consumer i will equate the ratio of the marginal utility of bus trips to that of dollars with the fare plus the time cost of a trip, an expression that is commonly referred to as the "full price" of a trip in the economics literature.

Suppose a public-spirited bus authority wishes to maximize a function, $W(u^1,\ldots,u^n)$, of the utility functions of bus users. In doing so, it is subject to the constraint $R = A + CX$ where R is the weekly flow of services available from the stock of resources at society's disposal. A is weekly consumption of dollars, Σa^i, and C is the number of units of resource services required to provide the services of a bus hour. Setting up the Lagrangian expression

$$Z = W(u^1,\ldots,u^n) + \eta(R - A - CX) \qquad \text{(A-10)}$$

and differentiating with respect to s^i and b^i yields:

$$W_i u_a^i - \eta = W_i \lambda^i - \eta = 0 \qquad \text{(A-11)}$$

$$W_i(u_b^i + u_T^i t) + \Sigma W_j u_T^j b^j t_B = 0 \qquad \text{(A-12)}$$

as first order conditions where W_i is $\partial W/\partial u^i$, the "marginal welfare weight" attached to individual i, the value society attaches to an increase in individual i's well-being. The second equality in (A-11) follows from equation (A-6)--that is, it follows from the fact that consumer i will adjust his consumption of dollars so that their marginal utility equals his marginal utility of

income, λ^i.

The authority responsible for providing bus service must take into account that consumers will act to maximize their utility levels subject to the budget constraints with which they are faced. This fact permits substitution of equations (A-8), (A-10), and (A-11) into equation (A-12). On doing so, equation (A-12) can be shown to reduce to

$$\eta(F - BVt_B) = 0 \qquad\qquad (A\text{-}13)$$

where V equals the weighted (by number of trips taken) average value of an hour of travel time, $\Sigma b^i V^i / \Sigma b^i$. The Lagrangian multiplier, η, can be interpreted as the welfare gain resulting from a one unit increase in available resource services. It is presumably positive. If travel time is valued at V dollars an hour, the total weekly time cost of trips would be $T = BV_t(B,X)$. Differentiating T with respect to B would yield an expression for the marginal time cost of a trip

$$\partial T/\partial B = Vt + BVt_B \qquad\qquad (A\text{-}14)$$

where, to repeat, subscripts refer to partial derivatives. The first term on the right of equation (A-14) can be interpreted as the average time cost of a trip. Hence BVt_B is the difference between average and marginal time costs--the costs an additional trip imposes on all other trip takers by increasing travel time per trip. Equation (A-13) can therefore be interpreted as saying that, if welfare is to be maximized, the fare per trip must equal the difference between the marginal and the average time costs of a trip; that is, the fare must equal the additional time costs resulting from an additional trip less those time costs incurred by the trip taker himself. This, of course, is essentially the same conclusion that we have asserted should be applied to auto travel.

Differentiating equation (A-10) with respect to X, the number of bus hours of service provided, and making substitutions similar to those which led from equation (A-12) to equation (A-13) yield

$$- \eta(VBt_x + C) = 0 \qquad\qquad (A\text{-}15)$$

This is the same result that would follow from selecting the value of X that would minimize

$VBt(B, X) + CX$, the total time and dollar costs of B trips if travel time is valued at V dollars an hour.

Appendix B:

EMME/2 Macros for Traffic Assignments

with Four Income Groups

This appendix describes and lists the programs and files needed by EMME/2 to determine equilibrium traffic flows when there are four types of commuters. The programs and files can easily be adapted to handle any number of commuter types. The idea is to use diagonalization to find equilibrium traffic flows. First, determine the roads which high income travelers will use if there are no other travelers. It makes sense to start with high income travelers, because this initial loading will generally assign them to faster roads, which is where you generally want them to end up. Second, determine the roads which the second highest income group would use taking the roads high income travelers use as given. Continue this process until all income groups are assigned to the network. Now one cycle has been completed. Perform additional cycles until the change in traveler behavior between cycles is small. We found what seemed like good approximations of equilibrium after five to seven cycles. As you perform more cycles it makes sense to perform more iterations of the equilibrium loading algorithm for each group of travelers. We started with seven or eight iterations and worked up to about twenty.

The main loading cycles are performed by the macro **load-fd2.m** (**load-ed2.m** in the case of elastic demand loadings). Its main job is to call two macros for each loading within the cycle: **preasgn.m** and **asgn-fd.m**. The macro **preasgn.m** calculates the amount of traffic of other types on each link. Once these calculations are made, the macro **asgn-fd.m** is called to assign the current driver type, given the amount of traffic caused by other types. More detailed descriptions of these macros are given below.

PROGRAMS AND FILES FOR THE INELASTIC-DEMAND CASE

Programs used to create EMME/2 data files:

D211-IN.BAS: This Basic program converts a TRANPLAN highway network into an EMME/2 highway network. The TRANPLAN network must be in ASCII format. The program uses the TRANPLAN network file **AM20-00.NET** as input and outputs the EMME/2 highway network to **D211.IN**. **D211.IN** can be input into EMME/2 using module **2.11**.

The format for each link in **D211.IN** is: ("a", node1, node2, free-flow travel time, "a",

link type, "1", volume-delay function, "0", "0", capacity). The first "a" tells EMME/2 to add the link. The free-flow travel time is in minutes. The second "a" says the link is used by the auto mode. The link types are: 1--metered freeway, 2--unmetered freeway, 3--metered ramp, 4--unmetered ramp, 5--divided arterial, 6--undivided arterial, 7--collector, 8--HOV, and 9--centroid connector. The volume-delay functions are: 1--freeway, 2--ramp, 3--arterial, collector, or HOV, and 4--centroid connector. The "1" and the "0"'s are place holders. The capacity is in autos per hour.

D311-IN.BAS: This QuickBasic program creates trips tables using data from the Travel Behavior Inventory (TBI). Each line of the TBI data must be in the format: origin node, destination node, number of trips and travel-time value. The data must be sorted by origin node, destination node, and travel-time value. The program uses the files **TBIFILE.1**, **TBIFILE.2**, **TBIFILE.3**, and **TBIFILE.4** for input and outputs to **D311.IN**. D311.IN contains one trip table for each of four ranges of travel time values. **D311.IN** can be input into EMME/2 using module 3.11.

Programs used to initialize the EMME/2 data bank:

DBANK-F.M: This macro creates the EMME/2 data bank in which all other macros run. It reads in the base network (**D211.IN**), reads in the trip tables **mf01** - **mf04** (**D311.IN**), and reads in the volume-delay functions (**D411.IN**). This program also creates additional matrices which EMME/2 needs. These matrices are: **mf05** (total trips), **mf06** (zero matrix), **mf07** (current travel time), **mf08** (travel cost with tolls and one time value), **mf09-mf10** (extra matrices), **mf11** - **mf14** (travel cost for commuter types 1 to 4). The program also initializes extra link attributes (**@vola1** - **@volau4** and sets up six scenarios: one for each commuter type, one for when there are no tolls, and one for when there are tolls and only one commuter type.

D201.IN: This file contains the mode table. The only mode of travel needed for the loading is the auto mode.

D411.IN: This file contains the volume-delay functions **fd01** - **fd04**. For functions 1 to 3, the

volume-delay function for type i is

$$fd01(volau) = fd02(volau) = fd03(volau)$$

$$= t_0 * (1+.15*((volau+ul1)/k)^4 + .6*(volau+ul2)*(volau+ul1)^3/k^4)$$

where t0 is free-flow travel time, k is capacity, volau is auto volume for type i, ul1 is the volume of other travelers, and ul2 is a weighted volume of other travelers. The term ul2 equals the sum of the volume of each of the other types of commuters times each commuter's value for travel time divided by type i's value for travel time. For example, suppose, for each type j, the volume is n_j and the value for travel time is v_j. Then, for type 1, ul2 will equal $(v_2 n_2 + v_3 n_3 + v_4 n_4) / v_1$.

The volume-delay function type 4 is for centroid connectors, which are not congestable so $fd04(volau) = t_0$. The volume delay functions used to compute equilibrium when there are four types of commuters are stored in **D411-1.IN**.

Files used to create equilibrium loadings:

LOAD-FD1.M: This macro performs the initial equilibrium loading for the network with four travel time values. It completely reloads the network so it should not be used if the network is already assigned and additional iterations are desired.

This macro calls the macros **asgn-ini.m**, **asgn-fd.m** and **preasgn.m**. First, **asgn-ini.m** is called to set up the scenario for each commuter type for loading. Then **preasg.m** and **asgn-fd.m** are called four times each. **Preasg.m** is called to calculate ul1 and ul2 for the commuter type to be loaded next. Then **asgn-fd.m** is called to load that commuter type.

This macro produces two reports. The output from the equilibrium loading module is saved as **load-1.out**. In addition, the macro **rptcost1.m** is called. It produces a summary of the time cost and the toll cost each commuter type experiences. Its output is saved as **ucost-1.out**.

LOAD-FD2.M: This macro performs additional iterations to refine the equilibrium obtained with **load-fd1.m** or previous uses of **load-fd2.m**. It takes one parameter for each cycle to be performed and one extra parameter. The extra parameter is the number of cycles which have

already been performed and it must be the first parameter. It produces the same reports for each cycle which **load-fd1.m** produces.

ASGN-FD.M: This macro carries out a fixed demand equilibrium traffic assignment. It takes two parameters, the commuter type and the number of iterations to perform. Travel costs for commuter type i are stored in the matrix **mf1i**.

PREASGN.M: This macro calculates weighted and unweighted additional volumes. These calculations make drivers of one type account for congestion caused by drivers of other types. The macro takes 8 parameters, four pairs of numbers. Each pair corresponds to one type of commuter and the commuter's corresponding valuation for travel time. The first pair is for the commuter type to be assigned next. The weighted and unweighted additional volumes are stored in **ul1** and **ul2**, respectively.

Files used to produce reports:

RPTCOST1.M: This macro produces a summary of the time cost and toll cost of travel for each commuter type. The time cost and the toll cost are given in hours and in dollars. This macro takes 4 parameters: the valuation of travel time for types 1, 2, 3, and 4, respectively.

Program Listings: Here are listings of the programs and files EMME/2 needs to perform an equilibrium assignment with four commuter types and completely inelastic demand.

The program D211.IN:

```
10 '
20 '   Translate Tranplan Network into EMME/2 Network
30 '
40 INPUT, "AM20-00.NET" FOR INPUT AS #1
50 OPEN "D211.IN" FOR OUTPUT AS #2
100 '
110 '   SKIP HEADERS
120 '
130 FOR I = 1 TO 5
140   LINE INPUT #1, H$
150 NEXT I
200 '
210 '   OUTPUT NODE DATA
220 '
230 PRINT #2, "t nodes init   / (a, centroid, number, coord1, coord2)"
240 CTR = 0
250 LINE INPUT #1, L$
```

The program D211.IN: (Continued)

```
260  IF LEFT$(L$, 1) < > "N" THEN GOTO 400
270  CTR = CTR + 1
280  IF CTR < = 1200 THEN PRINT #2, "a* ";  ELSE PRINT #2, "a  ";
290  PRINT #2, MID$(L$, 3, 5); MID$(L$, 10, 8); MID$(L$, 21, 8)
300 GOTO 250
400 '
410 '   OUTPUT LINK DATA
420 '
430 PRINT #2, " "
440 PRINT #2, "t links init   / (a, n1, n2, t0, mode; type, lanes; fn, ul1;
ul2, k)"
450 GOTO 480
460 IF EOF(1) THEN GOTO 600
470 LINE INPUT #1, L$
480  IF MID$(L$, 11, 1) < > "0" THEN A$ = MID$(L$, 11, 1) ELSE A$ = "8"
490  PRINT #2, "a "; MID$(L$, 2, 9);
491  PRINT #2, USING " #.##"; VAL(MID$(L$, 18, 3)) / 100;
492  PRINT #2, " 1 a ";
500  PRINT #2, A$; " 1 ";
510  IF VAL(A$) < = 2 THEN B$ = "1" ELSE IF VAL(A$) < = 4 THEN B$ = "2" ELSE
IF VAL(A$) < = 8 THEN B$ = "3" ELSE B$ = "4"
520  PRINT #2, B$; " 0 0 ";
525  IF MID$(L$, 34, 5) = "   0" THEN PRINT #2, "  99" ELSE PRINT #2,
MID$(L$, 34, 5)
530 GOTO 460
600 '
610 '   FINISH
620 '
630 CLOSE
640 BEEP: SYSTEM
```

The file D311-IN.BAS:

```
REM
REM   Translate TBI Data into EMME/2 Trip tables
REM

F$ = "TBIFILE"

REM   Loop To Strip Data

FOR I = 1 TO 4
  NUM$ = RIGHT$(STR$(I), 1)
  OPEN "D:\EMME2\WSTOLLS\D311\" + F$ + "." + NUM$ FOR INPUT AS #1
  OPEN "D:\EMME2\WSTOLLS\D311\TEMP." + NUM$ FOR OUTPUT AS #2

  DO
    LINE INPUT #1, L$
    GOSUB ACCEPT
    IF AC = 1 THEN PRINT #2, L$
  LOOP UNTIL EOF(1)
```

The file D311-IN.BAS: (Continued)

```
   CLOSE
NEXT I

REM   Main Loop

OPEN "D:\EMME2\WSTOLLS\D311\" + F$ + ".IN" FOR OUTPUT AS #2
PRINT #2, "t matrices / (add, id, descr, default, descr)"
FOR I = 1 TO 4
  NUM$ = RIGHT$(STR$(I), 1)
  OPEN "D:\EMME2\WSTOLLS\D311\TEMP." + NUM$ FOR INPUT AS #1
  PRINT #2, "a matrix=mf0" + NUM$ + " TYPE-" + NUM$ + " 0 " + F$ + "-Trips"
  OLDOR = 0: OLDDEST = 0: TRIPTOT = 0

  DO
    INPUT #1, NEWOR, NEWDEST, TRIPFAC
    TRIPS = TRIPFAC / 100
    IF (NEWOR = OLDOR AND NEWDEST = OLDDEST) THEN GOSUB SAMEPAIR ELSE GOSUB
NEWPAIR
  LOOP UNTIL EOF(1)

  CLOSE #1
NEXT I
END

ACCEPT:
AC = 1
IF (VAL(MID$(L$, 10, 10)) = 0 OR MID$(L$, 1, 2) < > " ") THEN AC = 0
IF (VAL(MID$(L$, 2, 5)) = 0 OR VAL(MID$(L$, 13, 6)) = 0) THEN AC = 0
RETURN

SAMEPAIR:
TRIPTOT = TRIPTOT + TRIPS
IF EOF(1) THEN GOSUB PRINTOLD

RETURN

NEWPAIR:
IF OLDOR > 0 THEN GOSUB PRINTOLD
OLDOR = NEWOR: OLDDEST = NEWDEST: TRIPTOT = TRIPS
IF EOF(1) THEN GOSUB PRINTOLD
RETURN

PRINTOLD:
PRINT #2, USING " ####"; OLDOR; OLDDEST;
PRINT #2, " :";
PRINT #2, USING " #####.##"; TRIPTOT
RETURN
```

The file DBANK-F.M:

```
6          / scenarios
1200       / centroids
7800       / nodes
20400      / links
10         / turns
10         / transit vehicles
10         / transit lines
100        / line segments
20         / matrices
10         / functions per class
100        / operators per class
100        / log book size
200        / demarcation size
100000     / extra network attributes
no         / node labels
no         / user data on transit line segments
'AM Network with 4 Types (Fixed Demand Loadings)' / title
yes        / confirm dimensions
3          / terminal type (here Non Graphic)
4          / printer type
5          / plot file type (here GPL/GPR plotfile, color)
da         / initials
1          / scenario to be created
'Type 1 Commuters' / scenario title
2.01       /##### read in mode table
1          / auto mode
2          / report to print file
q
2.11       /##### read in base network
2          / report to print file
2.41       /##### network calculations
3          / read in with batch entry
ul2        / read ul2
1,10       / all link types
n          / no report
y          / save result
q
3.11       /##### read in trip tables
2          / report to print file
4.11       /##### read in functions
2          / report in print file
3.12       /##### enter matrices interactively
1          / initialize matrices
mf05
TRIP-A
Complete Trip Table
0          / default value
1          / initialize matrices
mf06
TRIP-0
```

Zero Trip Table
0   / default value
1   / initialize matrices
mf07
TRVTIM
Current Travel Cost
0   / default value
1   / initialize matrices
mf08
COST-1
Travel Cost With One Type
0   / default value
1   / initialize matrices
mf09
EX-1

0   / default value
1   / initialize matrices
mf10
EX-2

0   / default value
1   / initialize matrices
mf11
.COST-1
Travel Cost for Type 1
0
1   / initialize matrices
mf12
COST-2
Travel Cost for Type 2
0
1   / initialize matrices
mf13
COST-3
Travel Cost for Type 3
0

1   / initialize matrices
mf14
COST-4
Travel Cost for Type 4
0

q
3.21 /##### matrix calculations
1   / matrix calculations
y   / save result
mf05
n   / don't change header
mf01 + mf02 + mf03 + mf04

The file DBANK-F.M: (continued)

```
n          / no submatrix
2          / send to printer
q
4.11 /##### read in functions
2          / report to print file
2.42 / ##### Extra Attribute Manipulations
2          / create attribute
2          / link
@vola1    / name
Volume of other type # autos
0          / default value
2          / create attribute
2          / link
@vola2    / name
Volume of other type # autos
0          / default value
2          / create attribute
2          / link
@vola3    / name
Volume of other type # autos
0          / default value
2          / create attribute
2          / link
@vola4    / name
Old equilibrium auto volume
0          / default value
q          / end
1.22     / ##### Scenario Manipulations
3          / copy scenario
1          / scenario to copy
2          / scenario to hold copy
'Type 2 Commuters' / scenario title
n          / new scenario current?
3          / copy scenario
1          / scenario to copy
3          / scenario to hold copy
'Type 3 Commuters' / scenario title
n          / new scenario current?
3          / copy scenario
1          / scenario to copy
4          / scenario to hold copy
'Type 4 Commuters' / scenario title
n          / new scenario current?
3          / copy scenario
1          / scenario to copy
5          / scenario to hold copy
'Minimum Travel Time Loading' / scenario title
n          / new scenario current?
3          / copy scenario
1          / scenario to copy
6          / scenario to hold copy
'Loading With Tolls and One Type' / scenario title
```

The file DBANK-F.M: (continued)

```
n          / new scenario current?
q          / quit
off=15     / echo mode off
on=2       / print main menu
```

The file D201.IN:

```
t modes init   / (add, id, desc, type, plot, user (4) )
a a auto      1 1  0.00  0.00  0.00  0.00
```

The file D411-IN:

```
t  functions  init   / (add, name, expr)
a fd01 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd02 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4△
a fd03 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a  fd04 = length
```

The file LOAD-FD1.M:

```
~x=0
~:loop
~x+1
s=%x%
5.11 / ##### Assign Commuters
1          / fixed demand
~?q=2
2          / new assignment
1          / single class
1          / no added volumes
mf06       / zero matrix
           / no vehicle occupancy matrix
           / no additional demand matrix
mf1%x%     / travel time matrix for type x
n
1          / number of iterations
0:r        / relative gap stopping criteria
0:r        / normalized gap stopping criteria
5.21 / ###### Auto Assignment
2          / send to printer
~?!x=4
~$loop
~<preasgn.m 4 21.875 1 5.395 2 11.245 3 16.25
~<asgn-fd.m 4 10
~<preasgn.m 3 16.25 1 5.395 2 11.245 4 21.875
~<asgn-fd.m 3 10
```

The file LOAD-FD1.M: (continued)

```
˜<preasgn.m 2 11.245 1 5.395 3 16.25 4 21.875
˜<asgn-fd.m 2 10
˜<preasgn.m 1 5.395 2 11.245 3 16.25 4 21.875
˜<asgn-fd.m 1 10
˜!move reports load-1.out
˜<rptcost1.m 5.395 11.245 16.25 21.875 .25
˜!move reports ucost-1.out
```

The files LOAD-FD2.M:

```
˜x=%1%
˜y=%0%
˜x-1
˜y-%x%
˜y-1
˜%
˜:loop
˜x+1
˜<preasgn.m 4 21.875 1 5.395 2 11.245 3 16.25
˜<asgn-fd.m 4 %1%
˜<preasgn.m 3 16.25 1 5.395 2 11.245 4 21.875
˜<asgn-fd.m 3 %1%
˜<preasgn.m 2 11.245 1 5.395 3 16.25 4 21.875
˜<asgn-fd.m 2 %1%
˜<preasgn.m 1 5.395 2 11.245 3 16.25 4 21.875
˜<asgn-fd.m 1 %1%
˜!move reports load-%x%.out
˜<rptcost1.m 5.395 11.245 16.25 21.875 .45
˜!move reports ucost-%x%.out
˜%
˜?!x=%y%
˜$loop
˜:end
q
```


```
˜/
˜/           PERFORM A FIXED-DEMAND LOADING
˜/
˜/           #1--Scenario
˜/               #2--Additional Iterations
˜/
s=%1%
5.11 / ##### Assign Commuters
1          / fixed demand
˜?q=2
˜$reasgn
1          / single class
1          / no added volumes
mf0%1%        / trip matrix
```

The file ASGN-FD.M: (Continued)

```
                / no vehicle occupancy matrix
                / no additional demand matrix
                / no travel time matrix
%2%         / number of iterations
0:r         / relative gap stopping criteria
0:r         / normalized gap stopping criteria
~$start
~:reasgn
1           / more iterations of old assignment
~p=1041
~x=%p%
~x+%2%
%x%         / maximum number of iterations
0:r         / relative gap stopping criteria
0:r         / normalized gap stopping criteria
~:start
5.21 / ###### Auto Assignment
2           / send to printer
```

The file PREASGN.M:

```
s=%1%
2.41 / ##### Network Calculations
1           / network calculations
y           / save result
@vola4    / save in @vola4
n           / don't alter description
volau

1,10

4           / no report
2           / copy  attribute from another scenario
%3%         / copy from scenario #
volau     / use auto volume
@vola1    / save in @vola1
n           / don't alter description
1,10      / range of links to copy

2           / copy  attribute from another scenario
%5%         / copy from scenario #
volau     / use auto volume
@vola2    / save in @vola2
n           / don't alter description
1,10      / range of links to copy

2           / copy  attribute from another scenario
%7%         / copy from scenario #
volau     / use auto volume
@vola3    / save in @vola3
```

The file PREASGN.M: (continued)

```
n        / don't alter description
1,10     / range of links to copy

1        / network calculation
y        / save result
ul1      / save as ul1
@vola1 + @vola2 + @vola3

1,10     / range of links

4        / no report
1        / network calculation
y        / save result
ul2      / save as ul2
(%4% * @vola1 + %6% * @vola2 + %8% * @vola3)/%2%

1,10     / range of links

4        / no report
q
```

The file RPTCOST1.M:

```
s=5
~x=0
2.41 / ##### Network Calculations
~:loop1
~x+1
2        / copy  attribute from another scenario
%x%        / copy from scenario 1
volau    / use auto volume
@vola%x%   / save in @vola#
n        / don't alter description
1,10     / range of links to copy

~?!x=4
~$loop1
1        / network calculation
y        / save result
ul1      / save as ul1
@vola1 + @vola2 + @vola3 + @vola4

1,10     / range of links

3        / punch
1        / network calculation
y        / save result
ul2      / save as ul2
%1% * @vola1 + %2% * @vola2 + %3% * @vola3 + %4% * @vola4
```

The file RPTCOST1.M: (continued)

```
1,10        / range of links to copy

4           / no report
1           / do calculations
n           / don't save result
%1% * @vola1 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%2% * @vola2 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%3% * @vola3 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%4% * @vola4 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
~x=0
~:loop3
~x+1
1           / do calculations
n           / don't save result
@vola%x% * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
~?!x=4
~$loop3
~x=0
~:loop4
~x+1
1           / do calculations
```

The file RPTCOST1.M: (continued)

```
n            / don't save result
@vola%x% * (vdf ! = 4) * .6 * length * ul2 * ul1^3 / (ul3^4 * 60)

1,10

2            / summary
2            / send to printer
~?!x=4
~$loop4
q
```

PROGRAMS AND FILES FOR THE ELASTIC-DEMAND CASE

This document describes and lists the programs and files needed by EMME/2 to determine equilibrium traffic flows when there are four types of commuters. The programs and files can easily be adapted to handle any number of commuter types.

Programs used to create EMME/2 data files:

D211-IN.BAS and D311-IN.BAS : These Basic programs are the same as the files used when demand was fixed.

Programs used to initialize the EMME/2 data bank:

DBANK-E.M: This macro creates the EMME/2 data bank in which all other macros are run. It performs all the tasks the corresponding macro (**dbank-fd.m**) does when demand is inelastic. In addition it initializes extra matrices for storage of inverse demand function values (mf21-mf24). This macro also reads in the same mode table (D201.IN), base network (D211.IN), and trip tables (D311.IN) that dbank-f.m does. The file d411.in is also read in. Now d411.in must contain data on the demand functions and well as data on volume delay functions.

D201.IN: This file is the same as the file used when demand was fixed.

D411-5.IN and D411-1.IN: The volume-delay functions are the same as the functions when demand is inelastic. Now that demand is elastic, the auto demand functions are also needed. The file **d411-5.in** is used when the elasticity of demand for travel is -0.5 and the file **d411-1.in** is used when the elasticity of demand for travel is -1.0. Demand functions are

fa01 = mat1 * mat2/(upqau.max..001)^b where **b** is the elasticity of demand. **Mat1** is the current travel time and **mat2** is the number of trips currently taken by travelers of the type to be loaded. The travel cost is **upqau**. To ensure division by zero does not occur the denominator is the maximum of **upqau** and .001.

Files used to create equilibrium loadings:

LOAD-ED1.M : This macro performs the initial loading for the network with four travel time values. It completely reloads the network so it should not be used if the network is already assigned and additional iterations are desired.

This macro calls the macros **asgn-fd.m**, **asgn-vd.m** and **preasgn.m**. First **asgn-fd.m** is called to set up the scenario of each commuter type for loading. Second **preasgn.m** and **asgn-fd.m** are called to obtain starting values for the variable demand loading. Finally, **preasgn.m** and **asgn-vd.m** are called four times each to do the first cycle of the variable demand equilibrium loading. Note that **preasgn.m** and **asgn-fd.m** are the same macros that were used when demand was fixed.

This macro produces two reports. The output from the fixed demand and variable demand equilibrium loading modules is saved as **load-1.out**. In addition, the macro **rptcost12.m** is called. It produces a summary of the time cost, the toll cost, the loss to consumer surplus, and the number of trips for each commuter type. Its output is saved as **ucost-1.out**.

LOAD-ED2.M: This macro performs additional iterations to refine the equilibrium obtained with **load-ed1.m** or previous uses of **load-ed2.m**. It takes one parameter for each cycle to be performed and one extra parameter. The extra parameter is the number of cycles which have already been performed and it must be the first parameter. It produces the same reports for each cycle which **load-ed1.m** produces.

ASGN-FD.M and PREASGN.M: These macros are the same as they were when demand was fixed.

ASGN-VD.M: This macro carries out a variable demand equilibrium traffic assignment. It

takes two parameters: the commuter type to assign and the number of additional iterations to perform. This macro stores demand for type i commuters in matrix **mf1i** and inverse function values for type i commuters in **mf2i**.

Files used to produce reports:

RPTCOST2.M: This macro produces a summary of the time cost and toll cost of travel for each commuter type. This macro takes 4 parameters: the valuation of travel time for types 1, 2, 3, and 4, respectively. This macro also reports the number of trips each commuter type takes and the loss to commuter surplus they experience.

Program Listings: Here are listings of the programs and files EMME/2 needs to perform an equilibrium assignment with four commuter types and elastic demand.

D211-IN.BAS and **D311-IN.BAS:**

These files are the same as when demand is fixed.

The file DBANK-E.M:

```
6          / scenarios
1200       / centroids
7800       / nodes
20400      / links
10         / turns
10         / transit vehicles
10         / transit lines
100        / line segments
24         / matrices
10         / functions per class
100        / operators per class
100        / log book size
200        / demarcation size
100000     / extra network attributes
no         / node labels
no         / user data on transit line segments
'AM Peak Network with 4 Commuter Types' / title
yes        / confirm dimensions
3          / terminal type (here Non Graphic)
4          / printer type
5          / plot file type (here GPL/GPR plotfile, color)
da         / initials
1          / scenario to be created
'Type 1 Commuters' / scenario title
2.01 /##### read in mode table
```

The file DBANK-E.M: (continued)

```
1        / auto mode
2          / report to print file
q
2.11 /##### read in base network
2          / report to print file
3.11 /##### read in matrices
2          / report to print file
3.12 /##### enter matrices interactively
1
mf05
TRIP-A
Complete Trip Table
0        / default value
1
mf06

TRIP-0
Zero Trip Table
0        / default value
1
mf07
TIME-0
Current Travel Time
0        / default value
1
mf08
COST-1
Travel Cost--One Commuter Type
0        / default value
1
mf09
TRVTIM
Travel Time Values-Temp
0        / default value
1
mf10
DFNVAL
Demand Fn Values-Temp
0        / default value
1
mf11
DMND-1
Variable Demand-1
0
1
mf12
DMND-2
Variable Demand-2
0
1
mf13
DMND-3
```

The file DBANK-E.M: (continued)

```
Variable Demand-3
0
1
mf14
DMND-4
Variable Demand-4
0
1
mf15
EXTRA1
Extra Storage Matrix
0
1
mf16
EXTRA2
Extra Storage Matrix

0
1
mf21
IFNV-1
Inverse Fn Values-1
0
1
mf22
IFNV-2
Inverse Fn Values-2
0
1
mf23
IFNV-3
Inverse Fn Values-3
0
1
mf24
IFNV-4
Inverse Fn Values-4
0
q
3.21 /##### matrix calculations
1         / matrix calculations
y         / save result
mf05
n         / don't change header
mf01 + mf02 + mf03 + mf04


n         / no submatrix
2         / send to printer
q
4.11 /##### read in functions
2         / report to print file
```

The file DBANK-E.M: (continued)

```
2.42 / ##### Extra Attribute Manipulations
2        / create attribute
2        / link
@vola1   / name
Volume of other type # autos
0        / default value
2        / create attribute
2        / link
@vola2   / name
Volume of other type # autos
0        / default value
2        / create attribute
2        / link
@vola3   / name
Volume of other type # autos
0        / default value
2        / create attribute
2        / link
@vola4   / name
Old equilibrium auto volume
0        / default value
q        / end
1.22     / ##### Scenario Manipulations
3           / copy scenario
.1          / scenario to copy
2           / scenario to hold copy
'Type 2 Commuters' / scenario title
n           / new scenario current?
3           / copy scenario
1           / scenario to copy
3           / scenario to hold copy
'Type 3 Commuters' / scenario title
n           / new scenario current?
3           / copy scenario
1           / scenario to copy
4           / scenario to hold copy
'Type 4 Commuters' / scenario title
n           / new scenario current?
3           / copy scenario
1           / scenario to copy
5           / scenario to hold copy
'Minimum Travel Time Loading' / scenario title
n           / new scenario current?
3           / copy scenario
1           / scenario to copy
6           / scenario to hold copy
'Loading With Tolls and One Type' / scenario title
n           / new scenario current?
q           / quit
off=15   / echo mode off
on=2     / print main menu
```

The file D411-5.IN:

```
t  functions  init   / (add, name, expr)
a fd01 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd02 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd03 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd04 = length
a fa01 = mat1 * mat2 / (upqau.max..001)**.5
```

The file D411-1.IN:

```
t  functions  init   / (add, name, expr)
a fd01 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd02 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd03 = length * (1 + .15 * ((volau+ul1)/ul3)^4) +
   .6 * length * (volau+ul2) * (volau+ul1)^3 / ul3^4
a fd04 = length
a fa01 = mat1 * mat2 / (upqau.max..001)
```

The file LOAD-ED1.M:

```
~x=0
~:loop
~x+1
s=%x%
5.11 / ##### Assign Commuters
1          / fixed demand
~?q=2
2          / new assignment
1          / single class
1          / no added volumes
mf06       / zero matrix
           / no vehicle occupancy matrix
           / no additional demand matrix
mf1%x%     / travel time matrix for type x
n
1          / number of iterations
0:r        / relative gap stopping criteria
0:r        / normalized gap stopping criteria
5.21 / ###### Auto Assignment
2          / send to printer
~?!x=4
~$loop
~ <preasgn.m 4 21.875 1 5.395 2 11.245 3 16.25
~ <asgn-fd.m 4 10
~ <preasgn.m 3 16.25 1 5.395 2 11.245 4 21.875
~ <asgn-fd.m 3 10
```

The file LOAD-ED1.M: (continued)

```
~<preasgn.m 2 11.245 1 5.395 3 16.25 4 21.875
~<asgn-fd.m 2 10
~<preasgn.m 1 5.395 2 11.245 3 16.25 4 21.875
~<asgn-fd.m 1 10
~<preasgn.m 4 21.875 1 5.395 2 11.245 3 16.25
~<asgn-vd.m 4 10
~<preasgn.m 3 16.25 1 5.395 2 11.245 4 21.875
~<asgn-vd.m 3 10
~<preasgn.m 2 11.245 1 5.395 3 16.25 4 21.875
~<asgn-vd.m 2 10
~<preasgn.m 1 5.395 2 11.245 3 16.25 4 21.875
~<asgn-vd.m 1 10
~!move reports load-1.out
~<rptcost2.m 5.395 11.245 16.25 21.875 .25
~!move reports ucost-1.out
```

The file LOAD-ED2.M:

```
~!del reports
~x=%1%
~y=%0%
~x-1
~y-%x%
~y-1
~%
~:loop
~x+1
~<preasgn.m 4 21.875 1 5.395 2 11.245 3 16.25
~<asgn-vd.m 4 %1%
~<preasgn.m 3 16.25 1 5.395 2 11.245 4 21.875
~<asgn-vd.m 3 %1%
~<preasgn.m 2 11.245 1 5.395 3 16.25 4 21.875
~<asgn-vd.m 2 %1%
~<preasgn.m 1 5.395 2 11.245 3 16.25 4 21.875
~<asgn-vd.m 1 %1%
~!move reports load-%x%.out
~<rptcost2.m 5.395 11.245 16.25 21.875 .25
~!move reports ucost-%x%.out
~%
~?!x=%y%
~$loop
~:end
q
```

The file ASGN-VD.M:

```
s=%1%
5.11 / ##### Assign Commuters
 3          / variable demand
~?q=2
~$reasgn
 1          / single class
 1          / no additional volumes
mf1%1%      / matrix to hold variable demand
 n          / don't change header


 1          / same function for all pairs
01          / index of function
mf0%1%      / MAT1
mf07        / MAT2


mf09        / matrix to hold travel time
 n          / don't change header
mf10        / matrix to hold demand fn values
 n          / don't change header
mf2%1%      / matrix to hold inverse fn values
 n          / don't change header
.%2%        / maximum number of iterations
0:r         / stopping criteria for normalized gap
5.21 / ###### Auto Assignment
 2          / send to printer
~$end
~:reasgn
 1          / more iterations of old assignment
~?q=1
 y          / continue anyway
 y          / continue anyway
~p=1041
~x=%p%
~x+%2%
%x%         / maximum number of iterations
0:r         / stopping criteria for normalized gap
5.21 / ###### Auto Assignment
 2          / send to printer
~:end
```

The file RPTCOST2.M:

```
s=5
~x=0
2.41 / ##### Network Calculations
~:loop1
~x+1
 2          / copy attribute from another scenario
```

The file RPTCOST2.M: (Continued)

```
%x%         / copy from scenario 1
volau       / use auto volume
@vola%x%    / save in @vola#
n           / don't alter description
1,10        / range of links to copy

~?!x=4
~$loop1
1           / network calculation
y           / save result
ul1         / save as ul1
@vola1 + @vola2 + @vola3 + @vola4

1,10        / range of links

4           / no report
1           / network calculation
y           / save result
ul2         / save as ul2
%1% * @vola1 + %2% * @vola2 + %3% * @vola3 + %4% * @vola4

1,10        / range of links to copy

4           / no report
1           / do calculations
n           / don't save result
%1% * @vola1 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%2% * @vola2 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10
2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%3% * @vola3 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10

2           / summary
2           / send to printer
1           / do calculations
n           / don't save result
%4% * @vola4 * length * (1 + (vdf != 4) * .15 * (ul1 / ul3)^4) / 60

1,10
```

```
2           / summary
2           / send to printer
~x=0
~:loop2
~x+1
1           / do calculations
n           / don't save result
@vola%x% * (vdf != 4) * .6 * length * ul2 * ul1^3 / (ul3^4 * 60)


1,10


2           / summary
2           / send to printer
~?!x=4
~$loop2
q
~x=0
~:loop3
~x+1
3.21
1
n
mf0%x% * mf07 * (ln((p==q) + mf2%x%) - ln((p==q) + mf07)) / 60


n


2
1
n
mf1%x%


n


2
q
~?!x=4
~$loop3
```

Appendix C:

The Traffic Assignment Model

In this appendix we describe a standard model of equilibrium traffic flows similar to, for example, LeBlanc, Morlok, and Pierskalla [1975]. In the first part of this appendix we assume all travelers place the same value on travel time, i.e., each traveler would be willing to pay the same price to save one hour of travel time. We assume traffic flows are constant throughout a period. The traffic flow along a section of road determines the time it takes to traverse the section. Each driver attempts to minimize the full cost of the trip he or she is making. The full cost of travel equals the resource plus toll costs of travel. Demand for each trip is a function of its full cost. A traffic assignment specifies the number of drivers using each route. An equilibrium traffic assignment is one in which (*i*) taking other drivers' route choices as given, no driver can change routes and lower the cost of his or her trip; and (*ii*) for each trip, the total number of drivers on all routes who make the trip equals the demand for the trip. Equilibrium traffic flows exist and are unique if resource costs on each link increase strictly with the volume of traffic using the link and if the demand function for each trip is strictly decreasing. Table C-1 contains a summary of the symbols which are used in Appendices C and D.

(1) **The Transportation Technology**: The transportation technology is specified by a finite set of nodes, $\mathcal{N}$, a finite set of links, $\mathcal{L}$, and a set of routes, $\mathcal{R}$. Let $\mathcal{N} = \{N_1, N_2, \ldots, N_P\}$. A *node* represents a point at which either trips or links can originate or terminate. Let $\mathcal{L} = \{1, 2, \ldots, L\}$. A *link* is a one-way section of road connecting two nodes. Associated with each link, l, is a travel-time function, t_l. A *travel-time function* (congestion function) maps the volume of autos using a link during a period into the time it takes to traverse the link. Thus, if the number of autos using link l during a period is $v_l \geq 0$ then the time it takes each auto to traverse l is $t_l(v_l)$.

An *origin-destination pair* is an ordered pair of nodes. Let $\mathcal{W} = \mathcal{N} \times \mathcal{N}$ be the set of all possible origin-destination pairs. A trip must be made between the nodes of an origin-destination pair. We will sometimes refer to an origin-destination pair as a trip. A trip must be made along a route. A *route* is a sequence of distinct links along which travel is feasible. The natural restriction on a sequence of links, $(l_1, l_2, \ldots, l_m)$, is that, for each

Symbol	Definition
$\mathcal{N}$	the set of nodes
$\mathcal{L}$	the set of links
l	a link
t_l	the travel-time function for link l
$\mathcal{W} = \mathcal{N} \times \mathcal{N}$	the set of all origin-destination pairs
$w = (w_1, w_2)$	an origin-destination pair or a trip
$\mathcal{R}$	the set of all routes
$\mathcal{R}_w$	the set of routes along which trip w can be made
v	a link-flow vector
a	a route-flow vector
n	a trip-flow vector
p	a congestion-pricing system
$C(r, v, p)$	the cost of using route r given v and p
$C_w^*(v, p)$	the minimum cost of making trip w given v and p
D_w	the demand function for trip w
h_w	the inverse demand function for trip w
q	the value commuters place on travel time
$f(a)$	the link-flow vector given the route-flow vector, a
$e(p)$	the equilibrium link-flow vector given tolls, p
$S(p)$	the welfare benefits of imposing tolls, p
$\mathcal{K}$	a subset of the set of all links
F_l	the fixed cost of tolling link l
R_+^K	the nonnegative orthont of R^k

Table C-1: Symbols used in Appendices C and D.

$k \in \{1, 2, \ldots, m - 1\}$, link l_k terminates at the node at which l_{k+1} originates. For each trip, $w = (w_1, w_2)$, let $\mathcal{R}_w$ be the set of all routes which originate at node w_1 and terminate at node w_2. For any w, $\mathcal{R}_w$ is finite because the sequence of links in a route must be distinct. Since a route must consist of distinct links, a route can contain at most L links. There are $n!/(n - k)!$ ways in which sequences of k distinct elements can be formed from a set of n distinct objects. This means that there can be no more than $\sum_{i=1}^{L} L!/(L - i)!$ routes joining any two nodes. Assume travel is possible between any two nodes, so that if $w \in \mathcal{W}$ then $\mathcal{R}_w \neq \emptyset$. Let $\mathcal{R} = \{r_1, r_2, \ldots, r_M\}$ be the set of routes between all pairs of nodes.

(2) Travelers: The *full cost of travel* along a route equals the sum of the resource costs plus the toll costs of using all links in the route. The demand for trips between any origin-destination pair is a function of the full cost of travel. For each $w \in \mathcal{W}$, let $D_w(c)$ be the number of commuters who will travel from w_1 to w_2 if the full cost of travel is c. In general, travel between a pair of nodes can be made along a number of routes, and the full cost of travel along each route may differ. We assume that commuters view all routes which make the same trip as perfect substitutes, so they will only use routes which have the same, minimum, full cost of travel. Because of this equivalence, it makes sense to speak of *the* full cost of making a trip.

Assume time is the only resource used in travel. A commuter's *valuation of travel time* is the maximum amount the commuter would be willing to pay to save one unit of travel time. Assume the occupant(s) of each vehicle value travel time at q dollars per unit. Each driver making a trip chooses a route which minimizes the full cost of travel. The choice set of a commuter traveling from w_1 to w_2 is $\mathcal{R}_w$.

Let R_+^K be the nonnegative orthont of R^K. A *link-assignment vector*, v, is an element of R_+^L such that, for each $l \in \{1, 2, \ldots, L\}$, v_l is the volume of travelers using l. We will sometimes refer to a link-assignment vector as a link-flow vector or simply as link flows. Similarly, we will sometimes substitute route-flow vector or route flows for route-assignment vector and trip-flow vector or trip flows for trip-assignment vector. A *congestion-pricing system*, p, is an element of R_+^L such that, for each link, l, p_l is the toll on l. Note that the

toll on a link may not be negative. Assume that the effect of any one commuter on travel time is negligible. We will assume that variables involving flows or prices can equal any non-negative real number. The time frames generally used in analyzing equilibrium flows make this assumption reasonable. During peak hours in the TCMA, for example, the flow of travelers making any particular trip or traveling along any particular link is often over 100 vehicles per hour. A commuter making trip w, when the congestion pricing system is p and the link-assignment vector is v, solves the problem

$$\min_{r \in \mathcal{R}_w} \sum_{l \in r} (q\, t_l(v_l) + p_l). \tag{C-1}$$

(3) Equilibrium: A route-assignment vector specifies the volume of travelers flowing along all routes, as opposed to the volume flowing along all links. A *route-assignment vector*, a, is an element of R_+^M such that, for each $i \in \{1, 2, \ldots, M\}$, a_i is the volume of travelers using route, r_i. To each route-assignment vector there corresponds a unique link-flow vector. Let f map route-assignment vectors into link-flow vectors so that, for each $l \in \{1, 2, \ldots, L\}$ and each route-assignment vector, a, $f_l(a)$ is the flow of traffic along l. Formally,

$$f_l(a) = \sum_{r \in \mathcal{R}} \chi_l(r) a_r \tag{C-2}$$

where $\chi_l(r)$ equals one if link l is in route r and zero if l is not in r.

Define the function C so that $C(r, v, p)$ is the full cost of travel on route r when the link-assignment vector is v and the congestion-pricing system is p. Again, the full cost of travel along route r is the sum of the resource plus toll costs of using all links in r. For each origin-destination pair, w, define the function C_w^* so that $C_w^*(v, p)$ is the minimum cost of making trip w when the link-flow vector is v and the congestion-pricing system is p. Note that $C_w^*(v, p)$ is the minimum found by solving (C-1). This minimum exists because the set of routes is finite so the problem is one of finding the minimum of a finite set of real numbers.

An *equilibrium traffic assignment for the congestion-pricing system, p,* is defined as a route-assignment vector, a, such that, for all trips, w,

(i) if $r \in \mathcal{R}_w$ and $a_r > 0$ then $C(r, f(a), p) = C_w^*(f(a), p)$, and

(ii) $\displaystyle\sum_{r \in \mathcal{R}_w} a_r = D_w(C_w^*(f(a), p)).$

Part (i) of the definition states that, in equilibrium, traffic flows only along minimum cost routes. Part (ii) states that, in equilibrium, the number of vehicles traveling along routes between any pair of nodes equals the demand for travel between those nodes, given the full cost of travel between them. A link-flow vector, v, is defined to be an equilibrium link-flow vector if and only if there exists an equilibrium route-assignment vector, a, such that $v = f(a)$.

Beckmann [1956] showed that the problem of finding an equilibrium traffic-assignment vector can be transformed into a constrained optimization problem. Assume that, for every trip $w \in \mathcal{W}$, the demand function, D_w, is continuous, strictly decreasing, and differentiable. Let $W = N^2$ be the number of origin-destination pairs. For each trip, w, let h_w be the inverse demand function for w. Also, assume that the travel-time function, t_l, for each link l is continuous, strictly increasing, and differentiable. Let $\bar{n}$ be some positive real number. Beckmann showed that under these conditions, a link-flow vector, v^*, is an equilibrium link-flow vector if and only if there exists $n^* \in R_+^W$, such that (v^*, n^*) solves the problem:

$$\min_{(v,n) \in R_+^L \times R_+^W} \sum_{l \in \mathcal{L}} \int_0^{v_l} (q t_l(y) + p_l)\, dy - \sum_{w \in \mathcal{W}} \int_{\bar{n}}^{n_w} h_w(y)\, dy \tag{C-3}$$

$$\text{subject to} \quad n_w = \sum_{r \in \mathcal{R}_w} a_r \text{ for all } w \in \mathcal{W}, \tag{C-4}$$

$$v_l = f_l(a) \text{ for all } l \in \mathcal{L}, \text{ and} \tag{C-5}$$

$$a_r \geq 0 \text{ for all } r \in \mathcal{R}. \tag{C-6}$$

It is not apparent that the two problems are equivalent. To gain some insight into why they are, suppose there exists a solution to (C-3)-(C-6) which is interior, i.e., a solution in which $v_l > 0$ for all l, $n_w > 0$ for all w, and $a_r > 0$ for all r. Let α, β, and γ be the Lagrange multipliers for the constraints described by (C-4), (C-5), and (C-6), respectively. The dimensions of the multiplier vectors are W, L, and M, respectively. The first order condition for achieving an optimum for each $w \in \mathcal{W}$ with respect to n_w is $h_w(n_w) = \alpha_w$. The first order condition for each $l \in \mathcal{L}$ with respect to v_l is $q t_l(v_l) + p_l = \beta_l$. The first order condition for each $r \in \mathcal{R}$ with respect to a_r is $\alpha_w = \sum_{l \in r} \beta_l$, where w is the origin-destination

pair between which route r travels. Substituting the first two conditions into the last we get

$$h_w(n_w) = \sum_{l \in r} q t_l(v_l) + p_l \qquad \text{(C-7)}$$

when route r makes trip w. This means that the number of travelers making trip w is consistent with the cost of travel on all of the routes along which the trip is made.

Kojima [1975] proves that a solution to the constrained optimization problem will exist under quite general conditions. Specifically, Kojima shows that if (i) every ordered pair of nodes is connected by a path; (ii) each link travel-time function is positive and continuous; and (iii) all demand functions are non-negative, continuous, and bounded from above then a solution exists. If all travel-time functions are strictly increasing and differentiable and all demand functions are strictly decreasing and differentiable, the programming problem has a strictly convex objective function. For any function $g(x)$ let $g'(x)$ denote the derivative of g and for any matrix M let M^T denote the transpose of M. The matrix of second partial derivatives of the objective function specified in (C-3) with respect to v is $q I_L (t_1'(v_1), t_2'(v_2), \ldots, t_L'(v_L))^T$ where I_L is the identity matrix of dimension L. The matrix of second partials with respect to n is $-I_W (h_1'(n_1), h_2'(n_2), \ldots, h_W'(n_W))^T$ where I_W is the identity matrix of dimension W. Since each travel-time function is strictly increasing and each inverse demand function is strictly decreasing, the matrices of second partial derivatives are positive definite. Because the matrices are positive definite, the objective function must be strictly convex. Since the problem also has linear constraints, if a solution exists, it is unique. In addition to Beckmann's conditions, assume that all demand functions are bounded from above. Then Kojima's conditions are satisfied, too. Under Beckmann's conditions and the boundedness condition, a solution to the problem exists and the solution is unique. The equilibrium *link*-flow vector, v^*, is unique, but equilibrium *route*-flow vectors are not generally unique. The equilibrium trip-flow vector, n^*, is also unique. Also, in equilibrium the full cost of travel along each route is uniquely determined. This follows because the cost of travel along a route depends only on the congestion-pricing system and the link-flow vector. Because the cost of using every route is uniquely determined, in equilibrium the minimum cost of travel between every origin-destination pair is also uniquely determined.

We work with some demand functions that are not bounded above. Specifically, we assume that $(i)'$ every ordered pair of nodes in a network is connected by a path; $(ii)'$ each link travel-time function is continuous, strictly increasing, differentiable, and bounded below by some $\epsilon > 0$; and $(iii)'$ all demand functions are non-negative, continuous, strictly decreasing, and differentiable. Assumption $(ii)'$ includes the restriction that every travel-time function is bounded below by some positive number. This assumption means that the time it takes to traverse any link is positive, even if there are no travelers using the link. This assumption seems reasonable. It is violated in the example of the Braess paradox network only to simplify the problem. We use Kojima's result to show that equilibrium link flows and trip flows exist and are unique under conditions $(i)'$-$(iii)'$. See Appendix A of Anderson [1996].

Reframing the problem of finding an equilibrium link-flow vector as one of finding the solution to a convex programming problem provides insights into the properties of equilibria. It also provides a potential method for solving the problem numerically. The programming problem appears daunting, however, because the number of constraints is greater than the number of routes. Fortunately, LeBlanc, Morlok, and Pierskalla [1975] developed an algorithm which solves the constrained optimization problem efficiently, even on large networks. The problem is solved iteratively. First, start with an initial vector, (v^i, n^i). Second, given v^i, find a minimum cost route between every pair of nodes. This is a time-consuming step. Calculating the least costly paths on a network with approximately 8,000 nodes and 20,000 links takes EMME/2 almost three minutes, on a DX2 running at 66MHz. Third, calculate the trip-flow vector, $\hat{n}$, which would result if the cost of travel were equal to the costs travelers would incur along the routes obtained in the second step. Then calculate the link-flow vector, $\hat{v}$, which would result if the trips, $\hat{n}$, were made on the routes obtained in step 2. Finally, obtain a new vector $(v^\theta, n^\theta) = (\theta v^i + (1 - \theta)\hat{v}, \theta n^i + (1 - \theta)\hat{n})$ by choosing $\theta \in [0, 1]$ to solve the problem

$$\min_\theta \sum_{l \in \mathcal{L}} \int_0^{v_l^\theta} (q t_l(y) + p_l)\, dy - \sum_{w \in W} \int_{\bar{n}}^{n_w^\theta} h_w(y)\, dy. \tag{C-8}$$

If the new vector is sufficiently close to the initial vector, then stop. Otherwise, repeat the

steps using (v^θ, n^θ) as the initial vector.

The algorithm developed by LeBlanc, *et al.* is an adaptation of an algorithm developed by Frank and Wolfe [1956]. The Frank and Wolfe algorithm was developed to solve constrained optimization problems in which the number of constraints is too large to evaluate explicitly. Frank and Wolfe show that their algorithm converges at least as fast as $1/n$. No general results on the rate of convergence of the adapted algorithm are available, however. At each iteration of the Frank and Wolfe algorithm, a "good" direction in which to search is found. This, combined with the fact that their objective function is quadratic, enable them to obtain an explicit bound on the rate of convergence of their algorithm. It is not possible to obtain a bound in the same way for the adapted algorithm. This is because, at each iteration, the search for a new link flow vector is conducted in the direction of the link flows which would occur if all traffic were assigned to the least costly routes, given the last approximation of link flows. This is a "good" direction to search in that, unless the old link flow vector was an equilibrium, the search yields a vector which provides a better solution to the programming problem. There may be much better directions to search, however. In particular, there is no guarantee that the algorithm is searching in a direction which is "close to" the direction of the gradient of the objective function. No attempt is made to find a better direction to search, because the programming problem has so many constraints that determining a better direction would generally be extremely time-consuming. In practice, the algorithm converges fairly rapidly. In the cases where demand is completely inelastic, the algorithm converged within 100 iterations to a value of the objective function which was within 0.2% of a lower bound on the problem's solution. When demand is elastic it is not possible to calculate a lower bound on the problem's solution in the same way, but convergence appears to occur at a similar rate.

(4) **Extending to Drivers with Differing Valuations of Time:** In this section we describe how to relax the assumption that all drivers have the same valuation for travel time and assume that each commuter is one of a finite set of types. All commuters of the same type place the same value on travel time. Each commuter still attempts to minimize the

direct cost of his or her trip, but now the direct cost depends on the commuter's type. Given a congestion pricing system, there still exists a unique equilibrium. Not all equilibrium in which each driver pays the marginal external cost of travel are optimal, however. We numerically approximate equilibrium traffic flows in the TCMA when all roads are tolled and when only limited access roads are tolled. We then examine the effects of pricing on travelers from different income groups.

Each commuter is one of I types. Let $\mathcal{I} = \{1, 2, \ldots, I\}$ be the set of types. For each $i \in \mathcal{I}$, let q_i be the valuation of travel time for a commuter of type i. Commuters now differ in the values they place on travel time and in the origin and destination of the trips they demand. Each commuter causes the same amount of congestion on a road, regardless of type. Suppose that, for each link l, the congestion function is t_l, link volume is v_l, and the toll is p_l. A commuter of type i, making trip $w \in \mathcal{W}$, solves the problem

$$\min_{r \in \mathcal{R}_w} \sum_{l \in r} (q_i\, t_l(v_l) + p_l). \tag{C-9}$$

Demand for a trip is a function of the trip's full cost. For each trip, w, and for each $i \in \mathcal{I}$, let D_{iw} be the demand for w by commuters of type i.

Equilibrium occurs when (i) each commuter making a trip does so in the least costly way given the route choices of all other drivers, the congestion pricing system, and the commuter's valuation of travel time and (ii) the number of commuters of each type making each trip equals the demand for the trip, given its cost. A route assignment is now an i-tuple of vectors, $a = (a^1, a^2, \ldots, a^I)$, where each a^i is an element of R^M such that a_j^i is the flow of type i travelers along route j. Similarly, a link assignment is now an i-tuple of vectors, $v = (v^1, v^2, \ldots, v^I)$, where each v^i is an element of R^L such that v_l^i is the flow of type i travelers along link l. Define $\hat{f}$ to be the function which associates a link assignment with each route assignment. This mean $\hat{f}(a) = (f(a^1), f(a^2), \ldots, f(a^I))$ where the function f is the same one defined in the previous section. For each $i \in \mathcal{I}$, define $C_i(r, v, p)$ to be the full cost of travel for a commuter of type i, on route r, when the link assignment is v and the congestion-pricing system is p. Also, for each $i \in \mathcal{I}$, define $C_{iw}^*(v, p)$ to be the minimum cost of making trip w, for a commuter of type i, when the link assignment is v and the

congestion-pricing system is p.

For each trip w and for each commuter type i, let D_{iw} be the demand function for w by commuters of type i. An equilibrium traffic assignment for the congestion-pricing system p is a route assignment, a, such that, for all $w \in \mathcal{W}$, and for all $i \in \mathcal{I}$,

(i) if $r \in \mathcal{R}_w$ and $a_r^i > 0$ then $C_i(r, \hat{f}(a), p) = C_{iw}^*(\hat{f}(a), p)$, and

(ii) $\displaystyle\sum_{r \in \mathcal{R}_w} a_r^i = D_{iw}(C_{iw}^*(\hat{f}(a), p))$.

As before, part (i) of the definition states that only minimum cost routes are used and part (ii) states that, given the cost of making a trip, the number of travelers using routes which complete the trip equals the number of those trips demanded. Now these conditions must hold for each type of commuter, however.

The problem of finding an equilibrium link-flow vector can be formulated as a concave programming problem subject to linear constraints. For any congestion-pricing system there will exist a unique vector of link-flows and trip-flows if conditions $(i)'$-$(iii)'$ hold for each type of commuter (see Fernandez and Friesz [1983]). The programming problem is solved by using diagonalization and the method of LeBlanc, *et al.* Start with an initial vector of link flows, $v_i = (v_i^1, v_i^2, \ldots, v_i^I)$. Then determine equilibrium flows for commuters of type 1, holding the link flows of all other types constant. After these equilibrium flows v_{i+1}^1 are found, hold them fixed while equilibrium flows for type 2 commuters are found. Continue until new equilibrium link flows are calculated for all types. If the new flows are sufficiently close to the initial flows, then stop. Otherwise, repeat the process, replacing the initial vector of flows with the new vector of flows.

Appendix D:

Optimal Pricing

As in Appendix C, we first assume that all commuters place the same value on travel time. An optimal congestion-pricing system maximizes consumer surplus plus toll revenue minus toll collection costs. We consider two cases. In the first, each link can be tolled costlessly. In this case, an optimal congestion pricing system exists and the link-flow vector resulting from the system is unique. The optimal congestion-pricing system and the optimal link-flow vector can be solved for numerically using the technique developed by LeBlanc, *et al.* In the second case, there is a fixed cost of tolling a link. This cost represents the cost of installing the equipment needed to monitor traffic flows. In this case, we do not know of an efficient method of searching for an optimal pricing system.

(1) Pricing When Tolling is Costless: If all roads are costless to toll, an optimal congestion-pricing system maximizes the sum of toll revenue plus the surplus that consumers derive from making trips. Equivalently, an optimal system maximizes the sum of the value each consumer making a trip places on it minus the aggregate value of time spent on travel. Consider the problem of finding optimal link-flow and trip-flow vectors. Using the same notation as in Appendix C, the social planner's problem is

$$\max_{(v,n) \in R_+^L \times R_+^W} \sum_{w \in \mathcal{W}} \int_{\bar{n}}^{n_w} h_w(y)\, dy - \sum_{l \in \mathcal{L}} q\, v_l\, t_l(v_l) \tag{D-10}$$

subject to (C-4) - (C-6).

Assume that $(i)'$-$(iii)'$ hold, so that each demand function (and hence each inverse demand function) is continuous, strictly decreasing, and differentiable, and each travel-time function is continuous, strictly increasing, differentiable, and bounded below by some $\epsilon > 0$. Note that for every link l, $d/dv_l(q\, v_l\, t_l(v_l))$ equals $(t_l(v_l) + v_l\, t_l'(v_l))$. Thus

$$\sum_{l \in \mathcal{L}} q\, v_l\, t_l(v_l) = \sum_{l \in \mathcal{L}} \int_0^{v_l} q\, (t_l(y) + v_l\, t_l'(y))\, dy. \tag{D-11}$$

This means the social planner's problem can be rewritten as

$$\min_{(v,n) \in R_+^L \times R_+^W} \sum_{l \in \mathcal{L}} \int_0^{v_l} (t_l(y) + y\, t_l'(y))\, dy - \sum_{w \in \mathcal{W}} \int_{\bar{n}}^{n_w} h_w(y)\, dy \tag{D-12}$$

subject to (C-4) - (C-6).

D-2

The problem is now in the same form as Beckmann's problem. Note that since, for every $l \in \mathcal{L}$, the congestion function, t_l, is continuous, strictly increasing, and bounded below by some $\epsilon > 0$, the function mapping each v_l into $q(t_l(v_l) + v_l t_l'(v_l))$ also has these properties. Using the result from the last appendix, this means that the social planner's problem has a unique solution. Call this solution (v^*, n^*).

Pricing can be used to internalize the costs of congestion externalities. Tolls can be set so that the equilibrium link and trip flow vectors are optimal. The total time cost that travelers on a link, l, experience is $q\, v_l\, t_l(v_l)$, so the marginal social cost of travel on l is $q\,(t_l(v_l) + v_l\, t_l'(v_l))$. The externality is internalized when all drivers pay the marginal social cost of travel. A driver using link l already pays a time cost equal to $q\, t_l(v_l)$, so the toll must equal $q\, v_l\, t_l'(v_l)$. This quantity, the marginal external cost of travel, is the increase in travel time an additional driver causes others to experience. Define the congestion-pricing system p^* so that $p_l^* = q\, v_l^*\, t_l'(v_l^*)$ for each link l. If the travel-time function on each link l were $t_l(v_l) + v_l\, t_l'(v_l)$, then v^* would be an equilibrium link-flow vector. Note that v^* is the first component of the solution to the problem

$$\min_{(v,n)\in R_+^L \times R_+^W} \sum_{l\in\mathcal{L}} \int_0^{v_l} (t_l(y) + p_l^*)\, dy - \sum_{w\in\mathcal{W}} \int_{\bar{n}}^{n_w} h_w(y)\, dy \qquad \text{(D-13)}$$

subject to (C-4) - (C-6).

because the same marginal conditions are satisfied by the objective function in (D-3) and the objective function in (D-2) with respect to link and trip flows. This means that v^* is an equilibrium link-flow vector for the congestion-pricing system p^*. Optimal link flows can be found by using the method of LeBlanc, *et al.* to solve the problem given in (D-2). The algorithm for approximating equilibrium generally converges more slowly in this case, because the new congestion functions increase more quickly. Then the optimal congestion-pricing system can be found by applying the rule $p_l^* = q\, v_l^*\, t_l'(v_l^*)$.

(2) Pricing with Fixed Costs: The fixed costs of the equipment needed to monitor traffic are sufficiently large that it may not be desirable to toll all roads. Assume that, for any link, l, all users of the link must be charged the same toll, p_l. If all links are *not* tolled, relaxing this

constraint may allow the social planner to increase surplus. Technologically, the constraint seems reasonable. Allowing prices to depend on the route a driver uses would mean it is possible to monitor the driver's use of links which are not tolled. If this were technologically feasible, however, each driver could be charged the full cost of his or her trip and the situation would be the same as that of the previous section. Assume that conditions $(i)'$-$(iii)'$ hold so that, for each trip w, the demand function, D_w, is continuous, strictly decreasing, and differentiable, and for each link, l, the travel-time function, t_l, is continuous, strictly increasing, differentiable, and bounded below by some $\epsilon > 0$. Finally, assume it is only feasible to place tolls on a subset, $\mathcal{K}$, of $\mathcal{L}$. Define $A_\mathcal{K} \equiv \{p \in R_+^L \mid p_i = 0 \text{ if } l_i \text{ is not in } \mathcal{K}\}$. If only links in $\mathcal{K}$ can be tolled, then the set of feasible congestion pricing systems is $A_\mathcal{K}$. From Appendix C, we know there exists a unique equilibrium link-flow vector, v, given any congestion-pricing system, p. Let e map congestion-pricing systems into equilibrium link-flow vectors. Let $S(p)$ be defined by

$$S(p) \equiv \sum_{l \in \mathcal{L}} p_l \, e_l(p) - \sum_{w \in \mathcal{W}_e} \int_{C_w^*(e(0),0)}^{C_w^*(e(p),p)} D_w(c). \tag{D-14}$$

The first sum in $S(p)$ is the toll revenue raised by the pricing system p. The second term is the difference between the consumer surplus when tolls are p and the consumer surplus when there are no tolls. Generally, imposing higher tolls will increase both toll revenue and the full cost of travel. The latter will decrease the surplus that consumers derive from travel.

The social planner's problem is to choose p in $A_\mathcal{K}$ so that $S(p)$ is maximized. Note that e is a continuous function of p and, for each trip w, C_w^* is a continuous function of p. See Section 2 of Appendix A in Anderson [1996]. This means that S is a continuous function of p. Now assume that there exists some real number, m, such that, for each demand function, D_w, if $c > m$ then $D_w(c) = 0$. Then raising the price on any link above m will not change toll revenue or traffic flows, because no travelers will use a link if the toll on it is larger than m. This means optimal prices are bounded above by m. The planner's problem is now one of maximizing a continuous function over a compact set, so it has a solution.

Now assume that for each link $l \in \mathcal{L}$ there is a fixed cost, F_l, of tolling l. The social

planner's problem then becomes

$$\max_{\mathcal{K} \subseteq \mathcal{L}, p \in A_{\mathcal{K}}} S(p) - \sum_{l \in \mathcal{K}} F_l. \tag{D-15}$$

A solution to this problem exists because the set of all subsets of $\mathcal{L}$ is finite. We do not know of an efficient algorithm for solving this problem, however. Attention cannot be restricted to imposing tolls that are less than the marginal external cost of travel. In the Braess paradox network, if the fixed costs of tolling all links other than l_3 are sufficiently high, then only link l_3 should be tolled, and it should be tolled above the marginal external cost of travel on it. The marginal external cost of travel on l_3 is zero, but the toll on it should be set at p where $p \geq \$20$ so that no travelers use the link. When all links are tolled the optimization problem can be decentralized: optimal tolls are those that equate the full cost of travel on each link with the marginal social cost of travel on that link. If some links can not be tolled, a negative externality occurs when tolling a link diverts drivers onto untolled links. Therefore, to find the optimal toll on a link, the planner must know the marginal external cost of travel on all routes which use that link and on all routes to which traffic will be diverted if the toll is raised. Determining these effects may be difficult computationally. One problem is that the method LeBlanc, *et al.* use to find equilibrium flows does not store route-assignment vectors. The method only stores link-assignment vectors because storing route-assignment vectors would take much more memory. In addition, the problem of finding the optimal subset of links to toll adds significantly to the complexity of the problem because the set of all subsets of $\mathcal{L}$ has 2^L elements.

The problem of finding an optimal congestion pricing system is more complicated than before because, when marginal cost pricing is used, congestion (actually cost) functions are no longer symmetric with respect to commuter type. This is because commuters with low valuations of time do not increase the marginal social cost of travel as much as commuters with high valuations of time. Each commuter causes the same delay on a link for other drivers, but marginal cost tolls depend on the types of drivers on a link. There may be multiple equilibria in which the toll on each link equals the marginal cost of travel on that link. The cost of making the same trip may not be the same in the different equilibria. See

the next section for an example. In simple examples we have examined, the algorithm we use to find an equilibrium in which all travelers pay marginal cost prices converges to the optimal link and trip flows. We am not sure, however, if the algorithm converges to the optimal link and trip flows when we use it to approximate equilibria for the whole TCMA.

(3) An Example of Multiple Equilibria: In this section we will present an example which shows that there may be multiple equilibria with marginal cost pricing when there is more than one commuter type. Suppose that there are only two nodes and two links in the network. Let both links 1 and 2 go from node N_1 to node N_2. Assume that both links have the same congestion function, $t(v) = 10 + v$, which gives the travel time in minutes on each link as a function of the number of autos using the link.

Assume that there are two commuter types and that commuters of type 1 value travel time at \$1 per minute and commuters of type 2 value travel time at \$2 per minute. Also assume that each type of commuter demands 40 trips from node N_1 to node N_2. Let v_j^i be the number of commuters of type i on link j. The total cost of travel on link j is then

$$v_j^1(10 + v_j^1 + v_j^2) + 2v_j^2(10 + v_j^1 + v_j^2). \tag{D-16}$$

This is the sum of the number of commuters of each type using link j multiplied by the value that type places on travel time multiplied by the travel time on link j.

The marginal social cost of travel on link j for a commuter type i is

$$i\,(10 + v_j^1 + v_j^2) + (v_j^1 + 2v_j^2). \tag{D-17}$$

The first term is the value a commuter of type i places on the time it takes to traverse link j. The second term is the marginal external cost of travel along link j, the additional travel time that the commuter causes others to experience.

There is an equilibrium with marginal cost pricing where $v_1^1 = v_2^1 = v_1^2 = v_2^2 = 20$. This is an equilibrium because each commuter pays \$60 in tolls and it takes a commuter 50 time units to traverse either link. Every commuter is indifferent between using link 1 and using link 2.

There is another equilibrium with marginal cost pricing where $v_1^1 = 40$, $v_2^1 = 0$, $v_1^2 = 5$, and $v_2^2 = 35$. In this case, travel time on link 1 is 55 units and the toll is \$50. The

travel time on link 2 is 45 units and the toll is $70. Travelers of type 1 prefer link 1 to link 2 because it takes only 10 extra units of time, which each type 1 commuter values at $10, and saves them $20 in tolls. Travelers of type 2 are indifferent between the two links because link 2 saves them 10 units of time, which they value at $20, but costs them an extra $20 in tolls. All travelers are behaving optimally, given the choices of other travelers, so this link flow vector is also an equilibrium with marginal cost pricing.

Appendix E:

Robustness Testing

In this appendix we present data on the robustness of our previous results with respect to three parameters of the model. The parameters are the cost per mile of operating a vehicle, the number of trips drivers make, and the form of the congestion function. Previously, we had assumed that time was the only resource used in travel. In this appendix, we assume that drivers must pay either five or ten cents per mile to operate their vehicles in addition to the time cost they incur in travel. Previously, we used the TBI to determine the number of trips drivers make between each origin-destination pair. In this appendix we determine the efficiency gains which would result if drivers made either 90% or 110% of the trips they reported in the TBI. The congestion functions we used previously were Standard Bureau of Public Roads congestion functions. These functions have the form $t(v) = t_0(1 + .15(v/k)^4)$ where v is traffic volume, t_0 is free-flow travel time, and k is the capacity of the link. The exponent in the function determines how congestable the link is. In this appendix we examine the effects of assuming the exponent equals either 3.5 or 4.5.

The results presented in this appendix are for the a.m. peak hour and we assume that demand is completely inelastic. We also assume that there is only one type of commuter and each commuter values travel time at $10 per hour. Given a parameterization, we determine equilibrium traffic flows when there are no tolls. Then we compare these flows to equilibrium flows when marginal cost pricing is used on all roads. Since there is only one type of commuter, equilibrium flows are optimal if marginal cost pricing is used. Using marginal cost pricing requires knowing the form of the congestion function for each link. Marginal cost pricing will result in optimal flows reguardless of the number of trips being made on the network and vehicles' operating costs. The road authority can set optimal tolls on a link by adjusting them to reflect the number of travelers on the link. If the road authority doesn't know the form of the congestion function, however, they may not be able to set tolls optimally. Because of this, we examine two cases for each congestion function we consider. In the first case, the road authority incorrectly thinks the exponent is 4. In the second case, the road authority knows the value of the exponent and sets tolls accordingly. This case is examined because the road authority may eventually gather the data to determine the value

Parameters	No Tolls		Tolls			
rt y op	Time	Oper.	Time	Toll	Oper.	Gain
1.0, 4.0, $.05	1523.1	252.7	1471.4	683.0	256.0	48.4
1.0, 4.0, $.10	1523.4	504.3	1471.4	687.9	510.9	45.4
1.0, 4.0, $.00	1520.0	0	1467.2	672.8	0	52.4
0.9, 4.0, $.00	1305.9	0	1268.8	438.5	0	37.1
1.1, 4.0, $.00	1765.0	0	1699.4	1018.8	0	65.6
1.0, 3.5, $.00	1492.3	0	1448.7	554.3	0	43.6
1.0, 4.5, $.00	1555.3	0	1490.3	823.3	0	65.0
1.0, 3.5, $.00	1492.3	0	1450.5	678.8	0	41.8
1.0, 4.5, $.00	1555.3	0	1491.6	666.6	0	63.7

Table E-1: The aggregate effects of tolls for alternative parameterizations: The table is for the a.m. peak hour when demand is completely inelastic, there is one type of commuter, and each commuter values time at $10 per hour. All values are in thousands of dollars. The parameters are the ratio of trips to the base case (rt), the exponent in the congestion function (y), and the operating cost per vehicle mile (op). When the parameterization is $\hat{y}$, the congestion function has exponent y, but tolls are set as if the exponent is 4.0.

of the exponent. If the congestion function is $t(v) = t_0(1 + .15(v/k)^y)$ then the marginal cost of travel on the link is $t(v) = t_0(1 + .15(1 + y)(v/k)^y)$. The optimal toll equals the difference between marginal cost and average cost. If the exponent is y then the toll should equal the value of $t_0 y(v/k)^y$ units of time.

Table E-1 contains a summary of the results of the robustness testing. The vehicles' operating costs have little effect on the efficiency gains of congestion pricing. Tolls result in almost the same savings in travel time that they would result in if there were no operating costs. Efficiency gains are slightly smaller because travelers drive further when there are

tolls. This is consistent with many models of urban form which predict that congestion pricing will increase vehicle miles traveled because drivers will take more circuitous routes to avoid congestion in the CBD. The effect in the TCMA is small, however, increasing vehicle miles driven by only one or two percent.

For the parameterizations studied, the largest effects on efficiency gains were caused by the changes in the number of trips taken. When the number of trips taken declined by 10%, efficiency gains fell by 29%; when they increased by 10%, efficiency gains increased by 25%. It appears that, without expansions in the road network, relatively small increases in travel demand may make congestion pricing significantly more attractive. The 10% increase in trips caused aggregate travel time to increase by 16%. These effects may be overstated, however. Increasing all trips in the sample by 10% may magnify congestion problems because the origins and destinations of the trips actually taken are spread more evenly over the TCMA than those in the sample. This bias may be compensated for, however, if increases in travel demand do not take place evenly throughout the metropolitan area.

The exponent in the congestion function has a significant effect on efficiency gains. Efficiency gains would fall by 17% if the exponent were 3.5, and they would increase by 25% if the exponent were 4.5. Fortunately, it appears that setting tolls based on an exponent of 4.0 when the actual exponent is 3.5 or 4.5 makes little difference. Pricing incorrectly leads to efficiency gains which differ from the maximum attainable gains by only two to four percent. Significant changes do occur in toll revenue, however when different exponents are used to set prices. The sensitivity of revenues, but not efficiency gains, to the level of tolls means it may be possible to charge lower (and presumably more politically popular) tolls and still bring about large efficiency gains.