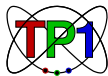


# The Electric Dipole Moment of the Neutron as a Probe of New Physics

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The results shown here were achieved in collaboration  
with Nikolay “Kolya” Uraltsev (arXiv:1202.6270 and arXiv:1205.0233)



Kolya deceased suddenly and unexpectedly  
in the morning of 13.2.2013  
This talk is also in commemoration of him ...

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# Introduction

- Electric Dipole moments (EDMs) are CP violating
- It tests flavour-diagonal CP violation
- This is small in the Standard Model (SM):
  - CKM CP violation is flavour changing at tree level
  - Strong CP is small for yet unknown reasons
- EDMs are important as test for new physics:
  - Many models predict new sources of CP violation
  - ... some of which are flavour diagonal at tree level
  - EDMs are the killer for many new physics models
- However, we need more CP violation for the universe

# Electric Dipole Moments: Generalities

- Electric dipole moments in classical physics

$$\vec{d} = \int d^3\vec{r} \rho(\vec{r})\vec{r} \quad \text{Energy: } U = \vec{d} \cdot \vec{E}$$

- Quantum Field theory:  
States are characterized by momentum  $\vec{p}$  and Spin  $\vec{J}$ :  
 $\vec{d}$  must be proportional to  $\vec{J}$

$$U = d\vec{J} \cdot \vec{E}$$

- $d$  must be parity odd:  
**P Violation (and also T Violation)**  $\rightarrow$  CP violation

# EDM's of particles

- Electromagnetic interaction with the EDM of a fermion:

$$\mathcal{L}_{\text{EDM}} = \frac{d}{2} \bar{\psi} i \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$$

- **This is flavour diagonal** and a static quantity
- This also holds for a composite particle such as a neutron
- Current status of measurements:
  - For the electron:  $d_e < 10.5 \times 10^{-28} \text{ e cm}$
  - For the neutron:  $d_N < 0.29 \times 10^{-25} \text{ e cm}$
- There are plans to improve this (in particular for the nucleon) by orders of magnitude.

# Only a few words about “Strong CP”

- The QCD Vacuum generates a (CP violating)  $\theta$  term:

$$\mathcal{L}_{\text{strong CP}} = \theta \frac{\alpha_s}{8\pi} G^{\mu\nu,a} \tilde{G}_{\mu\nu}^a$$

- Natural size would be  $\theta \sim 1$
- $\theta$  can be rotated away by an additional symmetry
- **Limit from Neutron EDM:**

$$d_N \sim \theta \times 10^{-15} \text{ e cm} \quad \text{thus} \quad \theta \leq 10^{-10}$$

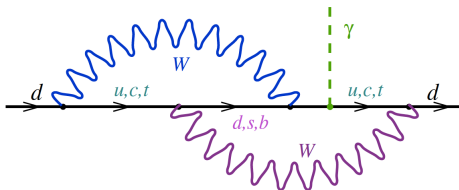
- **This is one of the puzzles of the SM**
- **We assume in what follows that  $\theta \equiv 0$ .**

# EDMs from the CKM Sector

- Any CP violation in the SM is proportional to

$$\Delta = \text{Im } V_{cs}^* V_{us} V_{cd} V_{ud}^*$$

- There is only a single 4<sup>th</sup> order rephasing invariant
- Standard Model without Strong CP:  
*d must be proportional to  $\Delta$ !*
- Thus we have two  $W$  exchanges.
- For an elementary fermion this is at least two loops:





- However, **sum of all the two-loop diagrams vanishes for quark edm's** → need another (gluon) loop Shabalin 78
- Result for  $d$  quark (similar for the up quark)

$$d_d = e \frac{m_d \alpha_s G_F^2 m_c^2 \Delta}{108 \pi^5} \left[ \ln^2 \frac{m_b^2}{m_c^2} \ln \frac{M_W^2}{m_b^2} + \dots \right] \sim -0.3 \times 10^{-34} e \text{ cm}$$

Khiplovich 86, Czarnecki, Krause 97

Naive composition of the Neutron edm:

$$d_N = \frac{4}{3} d_d - \frac{1}{3} d_u \sim 10^{-34} e \text{ cm}$$

**This is too small**, neutron is a composite object.

# Composite Objects: Neutron EDM

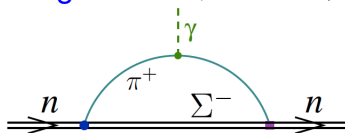
Well known fact: **There are long distance effects:**

(see reviews by Pospelov and Ritz)

- **Penguin Operators:**  $d \rightarrow s$  transitions (with CPV)

$$H_{\text{Pen}} \propto \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{3\pi} \sum_q (\bar{s} \Gamma_\mu T^a d) (\bar{q} \Gamma^\mu T^a q)$$

- **“Long distance strangeness”<sup>t</sup>:** (Gavela et al., Khriplovich et al, Pospelov et al.)



- Much larger than the EDM's of the constituents
- **Still there is a loop suppression** in the penguins

# “Loop-less” EDM’s

Uraltsev, M

## Systematic Study:

- Start from the effective Hamiltonian  $H_W$  for weak interactions below  $M_W$ :
- $H_W$  is bi-linear in the CKM elements:  
CP violation will be second order in  $H_W$

$$\mathcal{L}_2 = i \frac{G_F^2}{4} \int d^4x \text{T} \{ H_W(x) H_W(0) \}$$

- In the nucleon top and bottom are irrelevant: At tree level this means to leave them out.

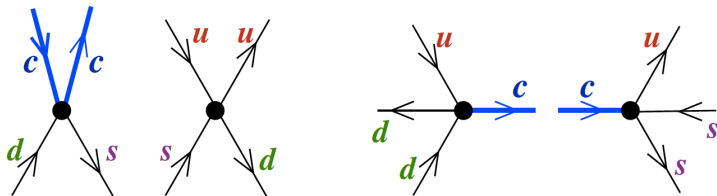
$$H_W = J_\mu^\dagger J^\mu, \quad J_\mu = V_{cs} \bar{c} \Gamma_\mu s + V_{cd} \bar{c} \Gamma_\mu d + V_{us} \bar{u} \Gamma_\mu s + V_{ud} \bar{u} \Gamma_\mu d$$

$$\text{with } \Gamma_\mu = \gamma_\mu (1 - \gamma_5)$$

- This looks almost like the two-generation case, however, the remaining  $2 \times 2$  matrix  $V_{ij}$  is **not unitary, not all phases can be removed.**
- In particular:

$$\Delta = \text{Im} V_{cs}^* V_{cd} V_{ud}^* V_{us} \neq 0$$

- $\mathcal{L}_2$  contains 256 terms, 64 are flavour neutral. Only two combinations proportional to  $\Delta$  (or its complex conjugate)
- These have  $q$  and  $\bar{q}$  for each flavor.



- Left: close the charm loop, **conventional penguin**
- Right diagram: **Integrate out highly virtual charm:**

$$\frac{iG_F^2}{2} V_{cs}^* V_{cd} V_{ud}^* V_{us} \times \int d^4x \text{T}\{(\bar{d}\Gamma_\mu c)(\bar{u}\Gamma^\mu d)(0) (\bar{c}\Gamma_\nu s)(\bar{s}\Gamma^\nu u)(x)\} + h.c.$$

- Charm Propagator

$$\underbrace{c(0)\bar{c}(x)} \rightarrow \left( \frac{1}{m_c - i\mathcal{D}} \right)_{0x}$$

- Expansion of the charm propagator:  $1/m_c$  expansion

$$\left( \frac{1}{m_c - i\not{D}} \right)_{0x} = \frac{1}{m_c} \delta^4(x) + \frac{1}{m_c^2} \delta^4(x) i\not{D} + \frac{1}{m_c^3} \delta^4(x) (i\not{D})^2 + \dots$$

- Left handed currents of the SM: only  $1/m_c^2$ ,  $1/m_c^4$  ...
- Thus in a  $1/m_c$  expansion:

$$\mathcal{L}_2^{CPV} = -i \frac{G_F^2 \Delta}{2m_c^2} \mathcal{O}_{uds}$$

$$\mathcal{O}_{uds} = (\bar{u} \Gamma_\mu d) (\bar{d} \Gamma^\mu i\not{D} \Gamma^\nu s) (\bar{s} \Gamma_\nu u) - h.c.$$

- No loops, no  $1/(16\pi^2)$  suppressions
- However, a local dim-10 operator appears ...

- The covariant derivative contains a photon:

$$O_{uds}^\alpha = (\bar{u}\Gamma_\mu d)(\bar{d}\Gamma^\mu i\gamma^\alpha \Gamma^\nu s)(\bar{s}\Gamma_\nu u) - (s \leftrightarrow d)$$

- from this we get the overall electromagnetic current relevant for EDM's

$$\mathcal{L}^\alpha = -ie\Delta \frac{G_F^2}{m_c^2} \left[ \frac{2}{3} O_{uds}^\alpha + i \int d^4x \text{T}\{O_{uds}(0)J_{\text{em}}^\alpha(x)\} \right]$$

- The matrix element between neutron states yields

$$\langle n(p+q) | \mathcal{L}^\alpha | n(p) \rangle \stackrel{q \rightarrow 0}{=} d_n q_\nu \bar{u}(p+q) i\sigma^{\alpha\nu} \gamma_5 u(p)$$

# How to estimate the matrix elements

- Local piece

$$\langle n(p+q) | O_{uds}^\alpha | n(p) \rangle = 2i \mathcal{K}_{uds} q_\nu \bar{n}(p+q) i\sigma^{\alpha\nu} \gamma_5 n(p)$$

- Estimating  $\mathcal{K}_{uds}$  is difficult:

- Naive estimate by dimensions:**  $\mathcal{K}_{uds} \approx \kappa \mu_{\text{had}}^5$
- $\kappa \sim 0.3$  for the suppression of strangeness
- $\mu \sim 0.5 \text{ GeV}$ , but this probably overestimates

$$\langle \bar{q}q \rangle \approx -(250 \text{ MeV})^3 \quad \text{yet} \quad \langle \bar{q}(iD)^2 q \rangle \approx -(650 \text{ MeV})^2 \langle \bar{q}q \rangle$$

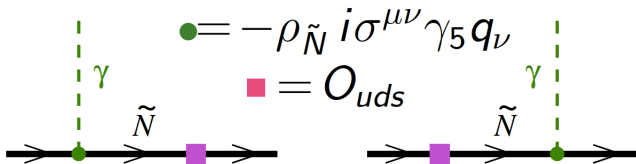
hence we write a factor of  $\mu_q^3 = (250 \text{ MeV})^3$  for each  $\bar{q}q$  pair, remaining dimensions from  $\mu_{\text{had}} \sim 500 \text{ MeV}$

- Thus from the local term we get

$$|d_n| = \frac{32}{3} e \frac{G_F^2 \Delta}{m_c^2} |\mathcal{K}_{uds}| = 3.3 \cdot 10^{-31} \text{ e cm} \times \kappa \left( \frac{\mu_q}{0.25 \text{ GeV}} \right)^6 \left( \frac{0.5 \text{ GeV}}{\mu_{\text{had}}} \right)$$



- Non-local term:  
 Estimate with a single intermediate  $\tilde{N}$  state



- Coupling  $\rho_{\tilde{N}}$  from  $\tilde{N} \rightarrow n\gamma$ :  $\rho_{\tilde{N}} \approx 0.34 \text{ GeV}^{-1}$

$$|d_n| \approx 32e \frac{G_F^2 \Delta}{m_c^2} \kappa \mu_q^6 \mu_{\text{hadr}} \frac{\rho_{\tilde{N}}}{M_{\tilde{N}} - M_n} \approx 1.4 \cdot 10^{-31} \text{ e cm} \times \kappa$$

- Consistent with other (as well crude) estimates

# Summary on the neutron EDM in the SM

- The loop-less estimate is (order of magnitude)

$$|d_n| = 10^{-31} \text{ e cm}$$

- Short distance loops will be parametrically small by loop factors  $1/(16\pi^2)$
- The EDM's of the constituents do not play any role
- **Strong CP remains a problem:**

$$|d_n| \approx 2.3 \cdot 10^{-16} \text{ e cm} \times \theta$$

- Given the current experimental bound:

$$|d_n| \leq 2.9 \cdot 10^{-26} \text{ e cm} \quad (90\%CL)$$

- $1/m_c$  suppression is present in the OPE, however, this is mild  $p/m_c \sim 0.5$
- Similar Effects in  $B$  decays: “Intrinsic charm”  
(Bigi et al. 2003, Zwicky et al. 2005, M. et al, 2010)
- $V - A$  Structure yields an additional factor of  $p/m_c$
- The loop-less contribution may as well be the dominant one!
- There are issues conceding the mass dependence, chiral limit, the limit  $m_s \rightarrow m_d$  etc.
- “SM minimizes the EDM of the neutron”

# New Physics Effects in the Neutron EDM

- Motivated by the LHCb measurements of

$$\Delta a_{\text{CP}} = \mathcal{A}_{\text{CP}}(D^0 \rightarrow K^+ K^-) - \mathcal{A}_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^-)$$

we consider CP violating operators for  $\Delta C = \pm 1$

- **What is the effect of such operators on the neutron EDM?**

# CKM CP Violation in Charm Decays

- Effective (four fermion) weak interaction

$$\begin{aligned} H_{\text{eff}} &= \frac{G_F}{\sqrt{2}} \left\{ V_{cs}^* V_{us} [(\bar{c}u)(\bar{s}s) - (\bar{c}u)(\bar{d}d)] \right. \\ &\quad \left. - V_{cb}^* V_{ub} [(\bar{c}u)(\bar{d}d) - (\bar{c}u)(\bar{b}b)] \right\} \\ &= \frac{G_F}{\sqrt{2}} \sin \theta_C \cos \theta_C [o_1 + r_{\text{SM}} e^{-i\gamma} o_2] \end{aligned}$$

- Strong CKM Suppr.:  $r_{\text{SM}} = \left| \frac{V_{cb}^* V_{ub}}{V_{cs}^* V_{us}} \right| \sim 7.5 \times 10^{-4}$
- Matrix Elements:  $m_f^{(i)} = \langle D^0 | o_i | f \rangle$

- Amplitude

$$A(D^0 \rightarrow f) = \frac{G_F}{\sqrt{2}} \sin \theta_C \cos \theta_C \left[ m_f^{(1)} + r_{\text{SM}} e^{-i\gamma} m_f^{(2)} \right]$$

- CP Asymmetry: interference between the two contributions in the amplitude:

$$\mathcal{A}_{\text{CP}}(D^0 \rightarrow f) = 2r_{\text{SM}} \left| \frac{m_f^{(2)}}{m_f^{(1)}} \right| \sin \delta_f \sin \gamma$$

- Assuming U Spin:

$$\mathcal{A}_{\text{CP}}(D^0 \rightarrow K^+ K^-) = -\mathcal{A}_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^-)$$

$$\Delta a_{\text{CP}} \sim 2\mathcal{A}_{\text{CP}}(D^0 \rightarrow K^+ K^-) = 4r_{\text{SM}} \left| \frac{m_f^{(2)}}{m_f^{(1)}} \right| \sin \delta_f \sin \gamma$$

- Let:  $\sin \delta_f \sin \gamma \sim 1$

$$\Delta a_{\text{CP}} = 3 \times 10^{-4} \left| \frac{m_f^{(2)}}{m_f^{(1)}} \right|$$

- Expectation:

$$\left| \frac{m_f^{(2)}}{m_f^{(1)}} \right| \sim \left| \frac{\langle D^0 | (\bar{c}d)(\bar{d}u) | K^+ K^- \rangle}{\langle D^0 | (\bar{c}s)(\bar{s}u) | K^+ K^- \rangle} \right|$$

- should be (significantly?) less than unity ...
- 2012 LHCb data were on the largish side, effect is now smaller and looks pretty normal.

# New Physics

- Assume: **There is some new physics in  $\Delta C = \pm 1$**
- Assume: The SM contribution to is small  
**For simplicity we completely neglect it**
- Pick some dim-6 operators with complex couplings,  
**which induce additional CP violation**

$$O_1 = em_c \bar{c} i \sigma_{\alpha\beta} F^{\alpha\beta} \gamma_5 u, \quad O_3 = [\bar{c} \Gamma_\mu u] ([\bar{s} \Gamma^\mu s] + [\bar{d} \Gamma^\mu d]),$$
$$O_2 = g_s m_c \bar{c} i \sigma_{\alpha\beta} G^{\alpha\beta} \gamma_5 u, \quad O_4 = (\bar{c} \gamma_\mu (1 + \gamma_5) u) (\bar{d} \gamma^\mu (1 - \gamma_5) d)$$

and define

$$\mathcal{L}_{\text{np}} = -\frac{G_F}{\sqrt{2}} \sin \theta_C \cos \theta_C \sum_k c_k O_k,$$



# Estimate the matrix elements

- $O_3$  is the SM operator, **with an arbitrary coefficient**

$$\langle \pi^+ \pi^- | O_3 | D \rangle \approx -f_\pi f_+^{D \rightarrow \pi}(0) M_D^2$$

- $O_4$  can be estimated in naive factorization

$$\langle \pi^+ \pi^- | O_4 | D \rangle \approx f_\pi f_+^{D \rightarrow \pi}(0) M_D^2 \frac{1}{N_c} \frac{2m_\pi^2}{(m_u + m_d)m_c}.$$

- $O_2$  estimated from the ratio of partonic rates

$$\langle \pi^+ \pi^- | O_2 | D \rangle \approx 4\pi g_s \sqrt{3} f_\pi f_+^{D \rightarrow \pi}(0) M_D^2.$$

- $O_1$  estimated from the ratio of partonic rates

$$\langle \pi^+ \pi^- | O_1 | D \rangle \approx 8\sqrt{2}\pi e q_d f_\pi f_+^{D \rightarrow \pi}(0) M_D^2$$

# Summary on the Coefficients

- Taking these operators as the single source of CPV, we can estimate the imaginary parts of the coefficients, up to the state strong phase  $\delta_{\text{FSI}}$

$$\begin{aligned} |\text{Im } c_1| &\approx \frac{5.2 \cdot 10^{-2}}{|\sin \delta_{\text{FSI}}|}, & |\text{Im } c_2| &\approx \frac{0.10 \cdot 10^{-3}}{|\sin \delta_{\text{FSI}}|}, \\ |\text{Im } c_3| &\approx \frac{2 \cdot 10^{-3}}{|\sin \delta_{\text{FSI}}|}, & |\text{Im } c_4| &\approx \frac{4.6 \cdot 10^{-3}}{|\sin \delta_{\text{FSI}}|}, \end{aligned}$$

- Fixing these numbers and operators, one may investigate the impact on the neutron EDM

# New Physics Effects in the Neutron EDM

- Insert the new physics operators discussed in the part on charm CPV
- **These will generate contributions to the neutron EDM**
- Some of the operators contain right handed quarks:  
**This can lift the helicity suppression**

	$-i\langle\pi^+\pi^- O_k D^0\rangle$	$ \sin\delta_{\text{FSI}}\text{Im}c_k $	$d_n, e\cdot\text{cm}$
$O_1$	$8\sqrt{2}\pi\alpha q_d f_\pi f_+^{D\rightarrow\pi}(0)M_D^2$	$5.2\cdot 10^{-2}$	$2\cdot 10^{-27}$
$O_2$	$4\pi g_s\sqrt{3}f_\pi f_+^{D\rightarrow\pi}(0)M_D^2$	$1.0\cdot 10^{-4}$	$8\cdot 10^{-30} \mid 3\cdot 10^{-30}$
$O_3$	$-f_\pi f_+^{D\rightarrow\pi}(0)M_D^2$	$2\cdot 10^{-3}$	$10^{-30}$
$O_4$	$f_\pi f_+^{D\rightarrow\pi}(0)M_D^2 \frac{1}{N_c} \frac{2m_\pi^2}{(m_u+m_d)m_c}$	$4.6\cdot 10^{-3}$	$5\cdot 10^{-30}$

## Estimates for the additional effects:

- $O_1 = e m_c \bar{c} i(\sigma F) \gamma_5 u$ :  $d_n \sim 10^4 d_n^{(SM)}$
- $O_2 = g_s m_c \bar{c} i(\sigma G) \gamma_5 u$ :  $d_n \sim 30 d_n^{(SM)}$  (right handed c)
- $O_3 = [\bar{c} \Gamma_\mu u] ([\bar{s} \Gamma^\mu s] + [\bar{d} \Gamma^\mu d])$ :  $d_n \sim 10 d_n^{(SM)}$
- $O_4 = (\bar{c} \gamma_\mu (1 + \gamma_5) u) (\bar{d} \gamma^\mu (1 - \gamma_5) d)$ :  $d_n \sim 50 d_n^{(SM)}$

The current experimental limits are safe w/r to charm CPV

# Some comments on other work

One may try to accommodate the observed charm CPV in a model

- Effective TH analysis and impact on charm CPV on  $\epsilon'/\epsilon$  (Isidori et al. 2012)
- Supersymmetric models (Giudice et al. 2012)
- **Models (like SUSY) imply effects in other places.**

# Outlook

- The neutron EDM remains one of the most stringent constraints on flavor diagonal CPV from New Physics
- ... although a precise prediction in the SM remains difficult due to the unknown hadronic matrix elements
- The “loop-less” contribution could turn out to be the most important one
- ... although it is difficult to estimate.
- The experimental limit is still several orders of magnitude away from the SM prediction
- ... despite of the uncertainties.
- There is a good motivation to improve the limits on the neutron EDM