HÖLDER CONTINUITY FOR SOLUTIONS OF CERTAIN DEGENERATE PARABOLIC SYSTEMS

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1. Introduction. We consider the parabolic system

$$u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + b(x,u,\nabla u) = 0 \tag{1}$$

where $u = u(x,t); \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^N, \nabla = \operatorname{grad}_x$ and x varies in an open set $\Omega \subset \mathbb{R}^N$. Naturally we assume $1 < p < \infty$. We assume that $b(x,t,u,Q) \in C^1(\mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^N \times M^{Nn} \to \mathbb{R}^N)$ satisfies the following controllable growth condition

$$|b_x(x,t,u,Q)| + |b_u(x,t,u,Q)|\ |Q| + |b_{Q^i_\alpha}(x,t,u,Q)Q^i_\alpha| \le c(1+|Q|^{p-1})$$

for all $x \in \mathbb{R}^n$, $t \in \mathbb{R}$, $u \in \mathbb{R}^N$ and $Q \in M^{Nn}$, and for some c.

Suppose that u is a solution to (1) which means

$$u \in C^0[0,T; L^2(\Omega \to \mathbb{R}^N)] \cap L^p[0,T; W^{1,p}(\Omega \to \mathbb{R}^N)]$$

and u satisfies

$$\int_{\Omega_T} -u\phi_t + |\nabla u|^{p-2}\nabla u \cdot \nabla\phi + b(z,u,\nabla u)\phi dz = 0$$

for all $\phi \in C_0^\infty(\Omega_T \to \mathbb{R}^N)$ where $z = (x,t)$ and $\Omega_T = \Omega \times (0,T)$.

The main result in this paper is the following theorem.

THEOREM 1. *Suppose that $u \in L^{r_0}_{\text{loc}}(\Omega_T), r_0 > \frac{n(2-p)}{p}$ then*

$$u \in C^{0,\alpha}_{\text{loc}}(\Omega_T)$$

for some $\alpha > 0$.

Note that from the definition of solution $u \in L^2_{\text{loc}}(\Omega_T)$. Thus the requirement that $r_0 > \frac{n(2-p)}{p}$ becomes restrictive only when $\frac{n(2-p)}{p} \ge 2$, i.e., when $p \le \frac{2n}{n+2}$.

When $p \ge 2$, E. DiBenedetto [5] proved that solutions for equations (1) with a natural growth condition on b are Hölder continuous. Essentially his proof lies on the truncation idea of DeGiorgi and a scaling and it seems not applicable to systems. In case $1 < p < 2$ E. DiBenedetto and C. Ya-zhe[10] proved Hölder continuity of solutions for equations. Also

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when $p > \frac{2n}{n+2}$, E. DiBenedetto and C. Ya-zhe[11] proved Hölder continuty for solutions of parabolic systems up to boundary.

On the other hand E. DiBenedetto and A. Friedman[6] proved that $\nabla u \in C^{\alpha}_{loc}$ for homogeneous systems when $p > \frac{2n}{n+2}$. Independently M. Wiegner[9] proved that $\nabla u \in C^{\alpha}_{loc}$ when $p \geq 2$. When $1 < p < 2$, H. Choe[1] proved that $\nabla u \in C^{\alpha}_{loc}$ for homogeneous systems.

Here we prove an inequality of Poincaré type. Once we have a Poincaré inequality, a Campanato type growth estimate for u follows from the L^∞ estimate of ∇u. Thus theorem 1 follows from the isomorphism theorem of Da Prato [4].

We assume $u, u_t, \nabla u, \nabla^2 u$ belong to a suitable L^q space. Justification for this appears in [6] when $p \geq 2$ and in [1] when $1 < p < 2$.

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2. L^∞ bound for ∇u. In this section we prove that u is bounded by following Moser iteration idea and we construct a weak Harnack inequality for $|u|$. In this section we define a cylinder Q_R by $Q_R = \{(x,t); |x - x_0| < R,\ \ t_0 - R^p < t < t_0\}$ where $(x_0, t_0) \in \mathbb{R}^n \times \mathbb{R}$ is a generic point. Also we define $\Lambda_R = \{t; t_0 - R^p < t < t_0\}$. A similar theorem appears in [1].

THEOREM 2. *Suppose that $u \in L^{r_0}_{\text{loc}}(\Omega_T)$, then $u \in L^{\infty}_{\text{loc}}(\Omega_T)$, where $r_0 > \frac{n(2-p)}{p}$ when $1 < p < 2$ and $r_0 = p$ when $p \geq 2$. Moreover we have that for all $Q_{2R_0} \subset \Omega_T$*

$$\sup_{Q_{R_0/2}} |u| \leq c \left[\left(\fint_{Q_{R_0}} |u|^{r_0} dz \right)^{\frac{1}{r_1}} + 1 \right] \tag{2}$$

for some constant c depending only on n, N and p, where $r_1 = r_0 - \frac{N}{p}(2-p)$ when $1 < p < 2$ and $r_1 = 2$ when $p \geq 2$.

Proof. Since $u \in L^2_{\text{loc}}(\Omega_T)$, we can assume $r_0 \geq 2$. Let $\rho < R$ and ψ be the standard cutoff function such that

$$\begin{aligned} &\psi = 1 \quad \text{in} \quad Q_\rho \\ &\psi = 0 \quad \text{in a neighborhood of parabolic boundary of } Q_R \\ &0 \leq \psi \leq 1, |\psi_t| \leq \frac{c}{(R-\rho)^p}, |\nabla \psi| \leq \frac{c}{R-\rho}. \end{aligned}$$

Let $\alpha_0 = r_0 - 2$. We apply $u|u|^\alpha \psi^p$ as a test function to (1) where $\alpha \geq \alpha_0 \geq 0$. So we have

$$\begin{aligned} &\sup_t \int |u|^{\alpha+2} \psi^p dx + (\alpha+1)^{2-p} \int |\nabla \left(\psi |u|^{\frac{\alpha+p}{p}} \right)|^p dz \\ &\leq c \int |u|^{\alpha+2} \psi^{p-1} |\psi_t| dz + c(\alpha+1)^{2-p} \int |u|^{\alpha+p} |\nabla \psi|^p dz \\ &+c \int (1 + |\nabla u|)^{p-1} |u|^{\alpha+1} \psi^p dz. \end{aligned} \tag{3}$$

Using Young's inequality on (3) we have

$$\tag{4} \begin{aligned} &\sup_t \int |u|^{\alpha+2}\psi^p dx + \int |\nabla(\psi|u|^{\frac{\alpha+p}{p}})|^p dz \\ &\leq \frac{c}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+2}\psi^{p-1} dz + \frac{c}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+p} dz + c|Q_R|. \end{aligned}$$

We assume $2 \leq p$. By Hölder's inequality and Sobolev's inequality on (4) we have

$$\tag{5} \begin{aligned} &\sup_{t_0-\rho^p \leq t \leq t_0} \int_{B_\rho} |u|^{\alpha+2} dx + \int_{\Lambda_\rho} \left[\int_{B_\rho} |u|^{(\alpha+p)\frac{n}{n-p}} dx \right]^{\frac{n-p}{n}} dt \\ &\leq \frac{c}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+p} dz + c\frac{|Q_R|}{(R-\rho)^p}. \end{aligned}$$

Hence using (5) and Hölder inequality we have

$$\tag{6} \begin{aligned} \int_{Q_\rho} |u|^{(\alpha+p)(1+\frac{p}{n}\frac{\alpha+2}{\alpha+p})} dz &\leq \left[\sup_t \int_{B_\rho} |u|^{\alpha+2} dx \right]^{\frac{p}{n}} \left[\int_{\Lambda_\rho} \left(\int_{B_\rho} |u|^{\frac{(\alpha+p)N}{N-p}} dx \right)^{\frac{n-p}{n}} dt \right] \\ &\leq c \left[\frac{1}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+p} dz + \frac{|Q_R|}{(R-\rho)^p} \right]^{1+\frac{p}{n}}. \end{aligned}$$

We define $\rho_\nu = \frac{R_0}{2}(1+2^{-\nu}), \nu = 0, 1, 2, \cdots$, and $Q_\nu = Q_{\rho_\nu}$. We now define α_ν by

$$\alpha_{\nu+1} = \left(1 + \frac{p}{n}\right)\alpha_\nu + \frac{2p}{n}, \quad \alpha_0 = 0.$$

Then we see that $\alpha_\nu = 2(\theta^\nu - 1)$ where $\theta = 1 + \frac{p}{n}$. Also note that

$$\lim_{\nu\to\infty} \frac{\theta^\nu}{\alpha_\nu + p} = \frac{1}{2}.$$

We define ϕ_ν by

$$\phi_\nu = \int_{Q_\nu} u^{\alpha_\nu + p} dz,$$

then we can write (6) as

$$\tag{7} \phi_{\nu+1} \leq c^\nu \phi_\nu^\theta + c^\nu$$

where c depends only on n, N and p. Iterating (7) we prove theorem 2 when $p \geq 2$.

Now we assume $1 < p < 2$. From (4) we get

$$
\begin{aligned}
&\sup_t \int_{B_\rho} |u|^{\alpha+2} dx + \int_{\Lambda_\rho} \left[\int_{B_\rho} |u|^{(\alpha+p)\frac{n}{n-p}} dx \right]^{\frac{n-p}{n}} dt \\
&\leq \frac{c}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+2} dz + c\frac{|Q_R)|}{(R-\rho)^p}
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
\int_{Q_\rho} |u|^{(\alpha+p)\left(1+\frac{p}{n}\frac{\alpha+2}{\alpha+p}\right)} dz &\leq \left[\sup_t \int_{B_\rho} |u|^{\alpha+2} dx \right]^{\frac{p}{n}} \left[\int_{\Lambda_\rho} \left(\int_{B_\rho} |u|^{(\alpha+p)\frac{n}{n-p}} dx \right)^{\frac{n-p}{n}} dt \right] \\
&\leq c \left[\frac{1}{(R-\rho)^p} \int_{Q_R} |u|^{\alpha+2} dz + \frac{|Q_R|}{(R-\rho)^p} \right]^{1+\frac{p}{n}}.
\end{aligned}
\tag{9}
$$

Defining α_ν by

$$
\alpha_{\nu+1} + 2 = \alpha_\nu \theta + p + \frac{2p}{n}, \quad \alpha_0 = r_0 - 2,
$$

we can have

$$
\phi_{\nu+1} \leq c^\nu \phi_\nu^\theta + c^\nu \tag{10}
$$

for some constant c depending only on n, N and p. We note that

$$
\lim_{\nu \to \infty} \frac{\alpha_\nu + 2}{\theta^\nu} = r_0 - \frac{n}{p}(2-p).
$$

Iterating (10) we prove theorem 2 when $1 < p < 2$. □

We define a new cylinder S_R by $S_R = B_R(x_0) \times (t_0 - R^2, t_0)$. We denote $\lambda = \sup_{S_R} |u|$. Now we recall the following theorem due to Choe (See Theorem 4 in [1]) which proves that $\nabla u \in L^\infty_{\text{loc}}$ for $1 < p < 2$.

THEOREM 3. *Suppose $S_{2R_0} \subset \Omega_T$ and $1 < p < 2$. Then there exists a constant c depending only on λ, p, q_0, N and n such that*

$$
\sup_{S_{R_0/2}} |\nabla u| \leq c \left(\left[\fint_{S_{R_0}} |\nabla u|^{q_0} dz \right]^{\frac{1}{q_1}} + 1 \right) \tag{11}
$$

where $q_0 > \frac{n}{2}(2-p)$ *and* $q_1 = \frac{2}{2q_0 - n(2-p)}$.

We note that theorem 3 implies Proposition 3.1$'$' in [11]. Suppose that

$$p > \frac{2n}{n+2}$$

then we see

$$p > \frac{n}{2}(2-p)$$

and we can take $q_0 = p$ and $q_1 = \frac{2}{2p+n(p-2)}$ in theorem 3. Consequently we have the following Collorary.

COLLORARY. *Suppose* $S_{2R_0} \subset \Omega_T$ *and* $\frac{2n}{n+2} < p < 2$. *Then there exists a constant* c *depending only on* n, N *and* p *such that*

$$\sup_{S_{R_0/2}} |\nabla u| \le c \left(\left[\fint_{S_{R_0}} |\nabla u|^p dz \right]^{\frac{2}{2p+n(p-2)}} + 1 \right). \tag{12}$$

Now we consider the case $p \ge 2$. Again by following Moser iteration we prove a weak Harnack inequality for ∇u. In this case we recall that E. DiBenedetto and C. Ya-zhe proved a similar theorem using a more complicated iteration(see Proposition 3.1 in [11]).

THEOREM 4. *Suppose* $S_{2R_0} \subset \Omega_T$ *and* $p \ge 2$. *Then there exists a constant* c *depending only on* n, N *and* p *such that*

$$\sup_{S_{R_0/2}} |\nabla u| \le c \left(\left[\fint_{S_{R_0}} |\nabla u|^p dz \right]^{\frac{1}{2}} + 1 \right). \tag{13}$$

Proof. Differentiating (1) with respect to x_γ we have

$$\begin{aligned}(u^i_{x_\gamma})_t - (a^{ij}_{\alpha\beta}|\nabla u|^{p-2}u^j_{x_\gamma x_\alpha})_{x_\beta} + b^i_{x_\gamma}(x,t,u,\nabla u)\\ + b^i_{u^j}(z,u,\nabla u)u^j_{x_\gamma} + b^i_{Q^j_\alpha}(x,t,u,\nabla u)u^j_{x_\alpha \alpha_\gamma} = 0,\end{aligned} \tag{14}$$

where

$$a^{ij}_{\alpha\beta} = \delta^{ij}\delta_{\alpha\beta} + (p-2)\frac{u^i_{x_\beta}u^j_{x_\alpha}}{|\nabla u|^2}$$

and δ is the Kronecker delta function. We introduce a new cutoff function ψ such that

$$\psi = 1 \text{ in } S_\rho$$
$$\psi = 0 \text{ in a neighborhood of parabolic boundary of } S_R$$
$$0 \le \psi \le 1, |\psi_t| \le \frac{c}{(R-\rho)^2}, |\nabla\psi| \le \frac{c}{R-\rho}.$$

Taking $\phi = u_{x_\gamma}|\nabla u|^\alpha \psi^2$, $\alpha \geq 0$ as a test function to (14) we get

$$\begin{aligned} &\sup_t \int |\nabla u|^{\alpha+2}\psi^2 dx + \int |\nabla(|\nabla u|^{\frac{\alpha+p}{2}}\psi)|^2 dz \\ (15) \qquad &\leq c\int |\nabla u|^{\alpha+2}\psi|\psi_t| dz + c\int |\nabla u|^{\alpha+p}|\nabla\psi|^2 dz. \\ &+c\int (1+|\nabla u|^{p+\alpha})\psi^2 dz. \end{aligned}$$

By Hölder's inequality and Sobolev's inequality we have

$$\begin{aligned} &\int_{S_\rho} |\nabla u|^{(\alpha+p)(1+\frac{2}{n}\frac{\alpha+2}{\alpha+p})} dz \\ (16) \qquad &\leq c\left[\sup_t \int |\nabla u|^{\alpha+2}\psi^2 dx\right]^{\frac{2}{n}} \left[\int |\nabla(|\nabla u|^{\frac{\alpha+p}{2}}\psi)|^2 dz\right] \\ &\leq c\left[\frac{1}{(R-\rho)^2}\int_{S_R} |\nabla u|^{\alpha+p} dz + \frac{|S_R|}{(R-\rho)^2}\right]^{1+\frac{2}{n}}. \end{aligned}$$

We set $\rho_\nu = (1+2^{-\nu})\frac{R_0}{2}$, $\rho = \rho_{\nu+1}$ and $R = \rho_\nu$. We define $\mu = 1 + \frac{2}{n}$. Also we define α_ν by

$$\alpha_{\nu+1} + p = \mu\alpha_\nu + p + \frac{4}{n}, \quad \alpha_0 = 0.$$

If we define

$$\Phi(\nu) = \int_{S_{\rho_\nu}} |\nabla u|^{\alpha_\nu + p} dz,$$

then (16) can be written as follows

$$(17) \qquad \Phi(\nu+1) \leq c^\nu \Phi(\nu)^\mu + c^\nu$$

for some c depending only on n and p. We note that

$$\lim_{\nu\to\infty} \frac{\mu^\nu}{\alpha_\nu + p} = \frac{1}{2}.$$

So iterating (17), we prove theorem 4. □

3. Hölder continuity of u. In this section we define $\Lambda_R^- = (t_0 - 2R^p, t_0 - R^p)$ and $Q_R^- = B_R \times \Lambda_R^-$. We introduce a cutoff function $\eta \in C_0^\infty(B_R)$ such that

$$\begin{aligned} &\eta = 1 \text{ in } B_{\frac{R}{2}} \\ &0 \leq \eta \leq 1, |\nabla\eta| \leq \frac{c}{R}. \end{aligned}$$

Also we define

$$u_{R,t} = \frac{1}{|B_R|} \int\limits_{B_R} u(x,t)dx$$

and

$$u_R = \frac{1}{|Q_R|} \int\limits_{Q_R} udz.$$

First we prove a lemma which is essential for a Poincaré inequality for solutions of a degenerate parabolic system.

LEMMA 1. *Suppose $Q_{2R} \subset \Omega_T$, then u satisfies the following inequality*

$$\begin{aligned} &\sup_{t\in\Lambda_R} \int\limits_{\Lambda_R^-} ds \int\limits_{B_R} \eta^2 |u(x,t) - u_{R,s}|^2 dx \\ &\leq cR^{s_1} \int\limits_{Q_{2R}} |\nabla u|^p dz + c \int\limits_{Q_R^-} |u - u_{R,t}|^2 dz + cR^{s_2}. \end{aligned} \tag{18}$$

for all $R < R_0$ where c depends only on n, N and p, and

$$s_1 = p \text{ and } s_2 = n + 2p \text{ when } p \geq 2$$
$$s_1 = p(p-1) \text{ and } s_2 = n + p + p(p-1) \text{ when } 1 < p < 2.$$

Proof. Since $u \in C^0[0,T; L^2(\Omega)]$, there exists $\bar{t}(s) \in \Lambda_R$ for all $s \in \Lambda_R^-$ such that

$$\int_{B_R} \eta^p |u(x,\bar{t}) - u_{R,s}|^2 dx = \sup_{0\geq t\geq s} \int\limits_{B_R} \eta^p |u(x,t) - u_{R,s}|^2 dx.$$

We take $(u - u_{R,s})\eta^p \chi_{[s,\bar{t}]}$ as a test function to (1) where $\chi_{[x,\bar{t}]} = \mathbb{R} \to \mathbb{R}$ is the characteristic function such that $\chi_{[s,\bar{t}]}(s) = 1$ for all $s \in [s,\bar{t}]$ and $\chi_{[s,\bar{t}]}(s) = 0$ for all $s \notin [s,\bar{t}]$. Hence we have

$$\begin{aligned} &\int u_t \cdot (u - u_{R,s})\eta^p(x)\chi_{[s,\bar{t}]} dz \\ &+ \int |\nabla u|^{p-2} \nabla u \cdot \nabla((u - u_{R,s})\eta^p)\chi_{[s,\bar{t}]} dz \\ &+ \int b(x,u,\nabla u)(u - u_{R,s})\eta^p \chi_{[s,\bar{t}]} dz = 0. \end{aligned} \tag{19}$$

Now we assume $p \in [2, \infty)$. Using Young's inequality and the structure condition on b we have

$$\begin{aligned} & \int |u - u_{R,s}|^2 \eta^p(x,\bar{t})dx + \int\limits_{B_R \times [s,\bar{t}]} |\nabla u|^p \eta^p dz \\ (20) \qquad & \le c \int |u - u_{R,s}|^2 \eta^p(x,s)dx + \frac{\varepsilon}{R^p} \int\limits_{B_R \times [s,\bar{t}]} |u - u_{R,s}|^p \eta^p dz \\ & + c \int\limits_{B_R \times [s,\bar{t}]} |\nabla u|^p dz + cR^{n+p} \end{aligned}$$

for some c independent of R. Integrating (20) with respect to s from $t_0 - 2R^p$ to $t_0 - R^p$, we have

$$\begin{aligned} & \int\limits_{\Lambda_R^-} ds \int\limits_{B_R} |u - u_{R,s}|^2 \eta^p(x,\bar{t})dx \\ (21) \qquad & \le c \int\limits_{Q_R^-} |u - u_{R,t}|^2 \eta^p dz + \frac{c\varepsilon}{R^p} \int\limits_{\Lambda_R^-} ds \int\limits_{B_R \times [s,\bar{t}]} |u - u_{R,s}|^2 \eta^p dz \\ & + cR^p \int\limits_{Q_{2R}} |\nabla u|^p dz + cR^{n+2p}. \end{aligned}$$

By the choice of $\bar{t}$ we have that for small ε

$$\begin{aligned} & \int\limits_{\Lambda_R^-} ds \int\limits_{B_R} |u - u_{R,s}|^2 \eta^p(x,\bar{t})dx \\ (22) \qquad & \le c \int\limits_{Q_R^-} |u - u_{R,t}|^2 \eta^p dz + cR^p \int\limits_{Q_{2R}} |\nabla u|^p dz + cR^{n+2p}. \end{aligned}$$

So we have lemma 2 when $p \geq 2$.

In case $p \in (1, 2)$ we estimate the second term of (19) as follows

$$\begin{aligned} & \int |\nabla u|^{p-2} \nabla u \cdot \nabla((u - u_{R,s})\eta^p) \chi_{[s,\bar{t}]} dz \\ (23) \qquad & = \int |\nabla u|^p \eta^p \chi_{[s,\bar{t}]} dz + p \int |\nabla u|^{p-2} u^i_{x_\alpha} \eta_{x_\alpha} (u^i - u^i_{R,s}) \eta^{p-1} \chi_{[s,\bar{t}]} dz \end{aligned}$$

and using tha fact that $p(p-1) < p$ and $\frac{p}{p-1} > 2$, we have

$$
\begin{aligned}
&\left|\int |\nabla u|^{p-2} u^i_{x_\alpha} \eta_{x_\alpha}(u^i - u^i_{R,s})\eta^{p-1} \chi_{[s,\bar{t}]} dz\right| \\
&\leq \int |\nabla u|^{p-1} |\nabla \eta| \eta^{p-1} |u - u_{R,s}| \chi_{[s,\bar{t}]} dz \\
&\leq \frac{\varepsilon}{R^p} \int |u - u_{R,s}|^{\frac{p}{p-1}} \eta^p \chi_{[s,\bar{t}]} dz + \frac{c}{R^{p(2-p)}} \int |\nabla u|^{p(p-1)} dz \\
&\leq \frac{c\varepsilon}{R^p} \int |u - u_{R,s}|^2 \eta^p \chi_{[s,\bar{t}]} dz + \frac{c}{R^{p(2-p)}} \int\limits_{B_R \times [s,\bar{t}]} |\nabla u|^p dz + cR^{N+p(p-1)}.
\end{aligned}
\tag{24}
$$

Applying Young's inequality and the structure condition of b on (19) and combining (19), (23) and (24) we have

$$
\begin{aligned}
&\int\limits_{B_R} |u - u_{R,s}|^2 \eta^p(x, \bar{t}) dz \\
&\leq \frac{\varepsilon}{R^p} \int |u - u_{R,s}|^2 \eta^p \chi_{[s,\bar{t}]} dz + \frac{c}{R^{p(2-p)}} \int\limits_{B_R \times [s,\bar{t}]} |\nabla u|^p dz \\
&+ cR^{n+p(p-1)} + c \int_{B_R} |u - u_{R,s}|^2 \eta^p(x, s) dx.
\end{aligned}
\tag{25}
$$

Integrating (25) with respect to s and using the choice of $\bar{t}$ we prove lemma 2 when $1 < p < 2$. □

Now we prove a Poincaré's inequality.

THEOREM 4. *Suppose $Q_{2R} \subset \Omega_T$, then we have*

$$
\begin{aligned}
\int\limits_{Q_{\frac{R}{2}}} |u - u_{\frac{R}{2}}|^2 dz \leq cR^2 \int\limits_{Q_{2R}} |\nabla u|^2 dz \\
+ cR^{s_1} \int\limits_{Q_{2R}} |\nabla u|^p dz + cR^{s_2},
\end{aligned}
\tag{26}
$$

where c is independent of R.

Proof. Using lemma 2 we have

$$
\begin{aligned}
\int_{Q_{\frac{R}{2}}} |u-u_{\frac{R}{2}}|^2 dz \le{}& c \int_{Q_R} |u-u_{R,s}|^2 \eta^p dz \\
\le{}& \frac{c}{R^p} \int_{\Lambda_R^-} ds \int_{Q_R} |u-u_{R,s}|^2 \eta^p dz \\
\le{}& c \sup_{t\in\Lambda_R} \int_{\Lambda'_{-R}} ds \int_{B_R} |u-u_{R,s}|^2 \eta^p(x,\bar{t}) dx \qquad (27)\\
\le{}& c \int_{Q_R^-} |u-u_{R,t}|^2 \eta^p dz + cR^{s_1} \int_{Q_{2R}} |\nabla u|^p dz + cR^{s_2} \\
\le{}& cR^2 \int_{Q_{2R}} |\nabla u|^2 dz + cR^{s_1} \int_{Q_{2R}} |\nabla u|^p dz + R^{s_2}
\end{aligned}
$$

where we used a Poincaré type inequality for x variables only as follows

$$\int_{B_R} |u-u_{R,t}|^2 \eta^p dx \le cR^2 \int_{B_R} |\nabla u|^2 dx. \quad \square$$

Proof of Theorem 1. Since ∇u is bounded, we have from theorem 4

$$
\begin{aligned}
\int_{Q_R} |u-u_R|^2 dz &\le cR^{n+p+2} && \text{when } p\in[2,\infty) \\
&\le cR^{n+p+p(p-1)} && \text{when } p\in(1,2)
\end{aligned}
$$

where c is independent of R. Hence by isomorphism theorem of Da Prato[4] we prove theorem 1. $\square$

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