ITS INSTITUTE
Intelligent Transportation Systems

Analysis of Single Frequency, Carrier Phase Based GPS Positioning Performance and Sensor Aiding Requirements

Final Report

Prepared by:

Demoz Gebre-Egziabher
Hamid Mokhtarzadeh

Department of Aerospace Engineering and Mechanics
University of Minnesota

CTS 13-01

Technical Report Documentation Page

<table>
<tr><td>1. Report No.
CTS 13-01</td><td>2.</td><td colspan="2">3. Recipients Accession No.</td></tr>
<tr><td colspan="2" rowspan="2">4. Title and Subtitle
Analysis of Single Frequency, Carrier Phase Based GPS Positioning Performance and Sensor Aiding Requirements</td><td colspan="2">5. Report Date
January 2013</td></tr>
<tr><td colspan="2">6.</td></tr>
<tr><td colspan="2">7. Author(s)
Demoz Gebre-Egziabher and Hamid Mokhtarzadeh</td><td colspan="2">8. Performing Organization Report No.</td></tr>
<tr><td colspan="2" rowspan="2">9. Performing Organization Name and Address
Department of Aerospace Engineering and Mechanics
University of Minnesota
110 Union St. SE
Minneapolis, MN 55455</td><td colspan="2">10. Project/Task/Work Unit No.
CTS Project #2010106</td></tr>
<tr><td colspan="2">11. Contract (C) or Grant (G) No.</td></tr>
<tr><td colspan="2" rowspan="2">12. Sponsoring Organization Name and Address
Intelligent Transportation Systems Institute
Center for Transportation Studies
University of Minnesota
200 Transportation and Safety Building
511 Washington Ave. SE
Minneapolis, MN 55455</td><td colspan="2">13. Type of Report and Period Covered
Final Report</td></tr>
<tr><td colspan="2">14. Sponsoring Agency Code</td></tr>
<tr><td colspan="4">15. Supplementary Notes
http://www.its.umn.edu/Publications/ResearchReports/</td></tr>
<tr><td colspan="4">16. Abstract (Limit: 250 words)
The work described in this report outlines the design and testing of a low-cost, single frequency, carrier phase positioning system. Furthermore, aiding sensor accuracy requirements are analyzed to improve the robustness of the carrier phase system after emerging from signal outages. The applications of interest are ones with safety-of-life implications such as driver assist systems.</td></tr>
<tr><td colspan="2">17. Document Analysis/Descriptors
Positioning, Integer ambiguity resolution, Sensors, Sensor aiding, Centimeter level positioning, Global Positioning System, GPS carrier phase</td><td colspan="2">18. Availability Statement
No restrictions. Document available from: National Technical Information Services, Alexandria, Virginia 22312</td></tr>
<tr><td>19. Security Class (this report)
Unclassified</td><td>20. Security Class (this page)
Unclassified</td><td>21. No. of Pages
50</td><td>22. Price</td></tr>
</table>

Analysis of Single Frequency, Carrier Phase Based GPS Positioning Performance and Sensor Aiding Requirements

Final Report

Prepared by:

Demoz Gebre-Egziabher
Hamid Mokhtarzadeh

Department of Aerospace Engineering and Mechanics
University of Minnesota

January 2013

Published by:

Intelligent Transportation Systems Institute
Center for Transportation Studies
University of Minnesota
200 Transportation and Safety Building
511 Washington Avenue SE
Minneapolis, Minnesota 55455

ACKNOWLEDGMENT

The authors wish to acknowledge those who made this research possible. The study was funded by the Intelligent Transportation Systems (ITS) Institute, a program of the University of Minnesota Center for Transportation Studies (CTS). Financial support was provided by the United States Department of Transportations Research and Innovative Technologies Administration (RITA).

The authors of this report would also like to acknowledge the cooperation and discussions with PhD student Adhika Pradipta Lie, as well as data collection assistance from researchers at CTS (Craig Shankwitz and Eddie Arpin).

Contents

List of Figures

Executive Summary

The work described in this report outlines the design and testing of a low-cost, single frequency, carrier phase positioning system. Furthermore, aiding sensor accuracy requirements are analyzed to improve the robustness of the carrier phase system after emerging from signal outages. The applications of interest are ones with safety-of-life implications such as driver assist systems for enhancing lane keeping performance in narrow lanes or during inclement weather when lane markings are obscured. In these applications, frequent GPS outages due to obstructions from buildings and highway overpasses often require solving for the integer ambiguities. The single frequency GPS receiver used for this work was the Hemisphere Crescent. Processing carrier phase data from this receiver was only intermittently successful. Even for data sets collected under ideal static and open-sky conditions, the integer ambiguity estimation process was often unsuccessful. Much effort was made to identify the source of the problem, but no conclusive reason was identified. Despite this setback, sensor accuracy requirements were studied analytically. and it was determined that aiding sensors capable of providing baseline vector position estimates with a standard deviation of less than $0.8\ m$ have potential to improve the integer ambiguity resolution time. This requirement, especially for signal outages lasting longer than several seconds, limits the candidate aiding sensors to higher-cost systems. Therefore low-cost, carrier phase based driver assist systems, using currently available receivers and sensors, continues to be a challenging problem. However, this may be overcome in the next decade as the second and third GPS civilian signal, L2C and L5, become fully avialable by 2019 [1].

CHAPTER 1

INTRODUCTION

Carrier Phase (CP) Differential Global Positioning Systems (DGPS) can provide position solutions at centimeter level accuracy. This precise measurement has historically been used for static applications like surveying and geodetics. Recent advances in receiver technology over the last two decades have allowed for CP measurements to be widely applied to dynamic applications like aircraft carrier landing, unmanned aerial vehicles, and farming machinery. Precise position information would be of interest to other land-based vehicles for applications like driver assist systems. The prospects of large-scale usage of CP measurements is increased by the recent availability of low-cost (<$300) GPS receivers which can output CP measurements. However, CP measurements require processing to resolve integer ambiguities before these measurements can be used.

The usefulness of CP measurements is limited by an initialization procedure which involves estimating an integer parameter, known as the integer ambiguity estimation problem [2]. Commonly used GPS measurements, based on code phase, do not include the integer ambiguity. This allows GPS receivers, like those common in car navigation GPS units, to provide position solutions seconds after the first satellite signals are received. In contrast, a CP based position solution may become available only after several minutes of continual lock onto satellite signals. But while code phase solutions are accurate to 1-5 meters, CP solutions can provide centimeter or sub centimeter level accuracy to the user. This relatively lengthy initialization procedure is required to estimate the ambiguous integer terms of the CP measurement. Unfortunately, this initialization procedure must be repeated for any momentary loss of satellite signals. This becomes very limiting for land-based applications where continual satellite lock is hard to maintain, like in urban environments or roadways with overpasses.

The goal of this work is to show the performance of a low-cost single frequency CP DGPS system, and to study the sensor requirements necessary to aid the integer ambiguity resolution process.

1.1. MOTIVATION

The carrier phase measurement model used for positioning applications is derived in Appendix A and can be written as

$$\Phi(t) = -LOS(t)\bar{x}(t) + \lambda N + \epsilon \tag{1.1}$$

where extra notation are dropped for simplicity. The challenges associated with estimating the integer ambiguity as well as the relative position vector from base to rover receiver can be better understood by looking at a simplified example. This example exemplifies why the integer ambiguity resolution can take several minutes in practice.

Equation 1.1 can be represented by a simplified scalar example:

$$y(t) = tp(t) + N + \epsilon(t)$$

The measurement $y(t)$ is the sum of the position multiplied by a time-varying coefficient, a constant N, and noise $\epsilon(t)$. Using measurements $y(t)$ the goal is to estimate $p(t)$ which necessitates also estimating the nuisance parameter N. To avoid the harder problem of estimating a time-varying position, we can assume the problem is static and $p(t) = p$. This is done for the same reason that most integer ambiguity resolution is done in static scenarios. The two unknowns require a minimum of two measurements to obtain a batch least-squares solution. Assuming the first measurement is at time t and the second measurement at time $t + \Delta$, and that $p(t)$ does not change over that interval, the least-squares problem can be formulated as:

$$\begin{aligned} \bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} t & 1 \\ t+\Delta & 1 \end{bmatrix} \begin{bmatrix} p \\ N \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \\ &= H\bar{x} + \bar{\epsilon} \end{aligned} \tag{1.2}$$

The least-squares solution to this problem is given as:

$$\begin{aligned} \hat{\bar{x}} = \begin{bmatrix} \hat{p} \\ \hat{N} \end{bmatrix} &= (H^T H)^{-1} H^T \bar{y} \\ &= \frac{1}{\Delta} \begin{bmatrix} -1 & 1 \\ t+\Delta & -t \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ &= \begin{bmatrix} p + \frac{\epsilon_2 - \epsilon_1}{\Delta} \\ N + t\frac{\epsilon_1 - \epsilon_2}{\Delta} + \epsilon_1 \end{bmatrix} \end{aligned} \tag{1.3}$$

Where $\hat{p}$ and $\hat{N}$ denote the least-squares estimate of the position and constant parameters, respectively. In the last expression, y_1 and y_2 have been replaced by the measurement model presented in Equation 1.2.

The absolute value of the error in the estimate of the constant N is given as:

$$|N - \hat{N}| = |t\frac{\epsilon_1 - \epsilon_2}{\Delta} + \epsilon_1|$$

from which two important observations can be made. Firstly, for a meaningful estimate of the constant N, the noise ϵ must be small in comparison to the desired accuracy of N since the estimate is corrupted additively with the noise. The second and more important observation is that the estimate error of N is highly dependent on the magnitude of the time interval Δ. The larger the time interval between the two measurements y_1 and y_2, the better the accuracy of the the estimate $\hat{N}$. This example can be expanded to show that even if the number of measurements are increased, but over a small time interval, there would be little improvement in the estimation accuracy of N.

This example presented above captures the challenges associated with integer ambiguity estimation. Returning to Equation 1.1, as with the example above, the position vector $\bar{x}(t)$ is multiplied by a time-dependent coefficient $LOS(t)$ and the constant N is additive with a fixed coefficient λ. The quality of the integer ambiguity estimate $\hat{N}$ will depend on the *time separation* between the subsequent measurements $\Phi(t)$ and $\Phi(t+\Delta)$. The time difference Δ must be large enough such that $LOS(t+\Delta)$ is sufficiently different than $LOS(t)$, and therefore carries new information used to improve the estimate of N.

Physically, the *line-of-sight* vector for satellite i is the unit vector from the receiver to satellite i. Because of the large separation between the satellite and a terrestrial receiver (on the order of 20,000 km), even the satellite's speed of $3.9\ km/s$ traveling around the earth causes very small changes to the LOS vector over tens of seconds. This explains why the integer ambiguity estimation process can take hundreds of seconds to achieve the desired accuracy - enough time must pass for the LOS vector to change and therefore new information becomes available for improving the estimate of N for each satellite.

The estimation accuracy required for integer ambiguity resolution is largely determined by the fact that the parameter should be an integer. The estimation process is two fold:

1. Estimate N for each satellite without any integer constraints, known as float solution $\hat{\hat{N}}$
2. Apply integer constraints to refine the float estimate and obtain the most likely set of integers $\check{\hat{N}}$

The estimate after the first step is commonly referred to as the *float-estimate* because the resulting vector of $\hat{\hat{N}}$ is a set of real numbers, not necessarily integer values. A simple method for applying step 2 is to simply round each entry of the float solution. This technique makes sense especially if the variance of the float estimate $\hat{\hat{N}}$ is small ($\sigma_{\hat{\hat{N}}} << 0.5$ cycles). As soon as the set of integers are of sufficient accuracy to be used for precise CP based positioning, they are declared as *fixed* and can be removed from the estimation process and subsequent CP measurements are used only to estimate the relative position vector $\bar{x}_{b,r}$. The time required for fixing the integers is called the Time to First Fix (TTFX).

Applications like driver assist systems for enhancing lane keeping performance impose two important and competing requirements on CP based positioning systems. First is that there should be a high level of integrity in the solution, since this application has safety-of-life implications. Secondly, the TTFX should be small, since the frequent GPS outages observed in urban environments requires solving for the integer ambiguities often. A step towards achieving both the time and integrity requirements is utilizing other sensor information on board modern vehicles to improve the integer ambiguity estimation process. Sensors like optical velocity meters can provide information to reduce the TTFX.

1.2. PROBLEM STATEMENT & SOLUTION

The objective of the research work described in this report was to design and test a low-cost single frequency carrier phase positioning system. Furthermore, aiding sensor accuracy requirements are analyzed to improve the robustness of the carrier phase system after emerging from signal outages. On a practical level, the work involved developing a low-cost CP DGPS positioning system and conducting experimental tests in urban driving scenarios, similar to that of driver assist applications. Secondly the aiding sensor performance requirements are analyzed. We determine a representative minimum sensor accuracy required to achieve faster integer ambiguity resolution. Lastly, recommendations on the utility of low-cost CP DGPS systems for ITS applications are made.

1.3. PRIOR WORK & CONTRIBUTIONS

There has been considerable work on CP DGPS positioning systems, both focusing on developing the data processing algorithms [3] as well as studying integer ambiguity resolution methods [4], but research on integrity as well as integer ambiguity validation is still lagging. Much effort has been focused on developing new integer ambiguity algorithms and showing the performance under several simulated and experimental test cases. The performance of the integer ambiguity resolution algorithm should be quantifiable and therefore the system should provide feedback to the user regarding the estimated accuracy of the position solution.

The Mathematical Geodesy and Positioning group at the Delft University of Technology has made significant contributions in developing integer ambiguity resolution algorithms. Most notable is their work extending the general least-squares problem to integer least-squares estimation [5, 6, 7]. Though the Least-squares AMBiguity Decorrelation Adjustment method (LAMBDA) in [7] is an optimal integer least-squares solution, the bootstrapping integer resolution algorithms presented in [8] has a much smaller computational burden, and more importantly, includes a theoretical framework to compute the exact probability of success in the integer fixing. The quantifiable performance is particularly attractive for applications like driver assist programs where performance requirements are important.

The work by researchers at the University of Calgary are related to this project as it studies the benefits of fusing measurements from Inertial Measurement Units (IMU) with the GPS integer ambiguity resolution algorithms [9], particularly during GPS outages. Three forms of IMU/GPS data fusion are compared and trade offs analyzed in terms of integer ambiguity resolution improvement. The work in [9] is similar to our objective of fusing data from external sensors to improve the integer ambiguity resolution, however [9] studies the performance of a high-quality dual frequency GPS receiver combined with the measurements of a tactical grade IMU. This set of equipment would cost in excess of several thousand dollars, which is prohibitively expensive for applications like driver assist systems to be widely implemented on a fleet of vehicles.

The work described here develops a low-cost (<$1.5k) carrier phase based GPS measurement system, using commercial OEM components parts thus demonstrating the performance of a low-cost precision positioning system. Finally, the effect of integrating external sensors for faster integer ambiguity resolution is analyzed.

1.4. REPORT ORGANIZATION

The remainder of this report is organized as follows: In Section 2 of the report, we discuss categories of satellite signal loss requiring integer ambiguity resolution. Next in Section 3 the procedure of *fixing* integers are described. In Section 4, the effect of using external sensors to aid the integer ambiguity resolution process is analyzed in terms of probability of successful fixing. In Section 5 the hardware and software developed as part of this work is described. The experimental GPS and aiding sensor data collected on an instrumented vehicle is presented Section 6. The report is ended with a summary and concluding remarks.

CHAPTER 2

SIGNAL LOSS AND CYCLE SLIPS

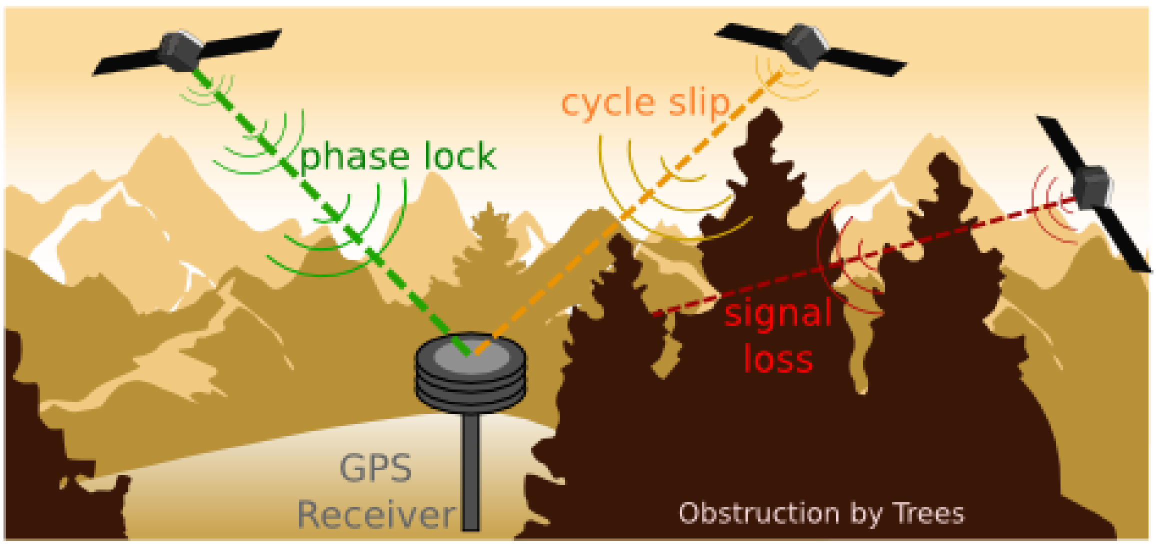


Figure 2.1: Scenario depicting a strong signal, cycle slip and signal loss due to obstructions

The integer ambiguity is constant during the time a GPS receiver maintains lock on the carrier tracking loop of the incoming satellite signal. This means that as soon as the integer ambiguity has been resolved, the satellite signal can be used to obtain a low-noise range measurement for precise relative positioning. This can continue for as long as the carrier tracking loop remains locked, which may be for as long as the satellite is within the receiver horizon.

Figure 2.1 depicts three satellites with varying signal quality at the receiver. The signal arriving at the receiver antenna without any obstructions usually can be maintained under lock by the receiver for long periods of time. This is in part due to advances in processing power and memory, and would not necessarily be the case with receivers from several decades ago [2]. A momentary loss of phase lock can result in a discontinuity in the integer cycle count, and is called a *cycle slip* [2]. In Figure 2.1 the center satellite signal is partially blocked by tree-tops and therefore the reduced signal strength causes occasional cycle slips to occur. As soon as phase lock is regained, the integer ambiguity has now changed and the prior fixed integer is invalid.

A second method for the integer ambiguity to become invalid is if the satellite signal is lost altogether. This is depicted by the satellite to the far right whose signal cannot pass the dense trees to reach the receiver. Satellites falling below the horizon also falls under this category. Effectively, it is the earth that obstructs the satellite signal in this case. In an urban setting, the obstruction may be caused by buildings, bridges, and even people and vehicles. If the receiver is moved to where no obstruction exists and the satellite signal is gained (or regained), then the integer ambiguity must be resolved before the carrier phase measurement based pseudorange can be used.

GPS receivers that output carrier phase measurements include a *Satellites Available* messages which indicates the satellites with available measurements. The measurements per satellite include:

- code-phase measurement: pseudorange measurement, commonly given in *meters*
- carrier phase measurement: ambiguous pseudorange, commonly given in *meters* or *cycles*
- cycle slip counter: cumulative number of cycle slips that have occurred
- SNR: signal to noise ratio

The *cycle slip counter* message must be monitored for each available satellite. A change in the counter value for any satellite means any previously known integer ambiguities are no longer valid. As part of this work, processing software was written which accounts for each of the listed messages.

CHAPTER 3

INTEGER FIXING

Integer fixing is part of the integer ambiguity estimation process. The two primary steps to integer ambiguity resolution were listed in Section 1.1. The integer ambiguity for each satellite does not change as long as there is continuous signal lock. The estimate of the set of integer ambiguities for the visible satellites are refined until the estimate uncertainty has sufficiently been reduced. At this point, the set of integer ambiguities are declared as *fixed*. This means the integer ambiguities are theoretically known, and since they are constant, there is no need to continue the estimation refinement. If there is a momentary loss of satellite signal, the estimation procedure should be repeated. Once the integers are fixed correctly, centimeter or sub centimeter level accurate position solutions are attainable. It is a challenging problem to decide when to declare the integers as fixed to ensure they are indeed correct. This is especially important for safety of life or liability critical applications. Incorrectly declaring the integers as fixed would cause the low-noise position solutions thereafter to effectively drift as the error caused by the incorrect integer would change as the satellite geometry changes. As previously demonstrated, time required for changing the satellite geometry is the primary means of refining integer ambiguity estimates. There is a trade off between availability and accuracy/integrity. Improving the availability of the precise positioning data requires shortening the TTFX. However the integer ambiguity estimate uncertainty is most readily reduced by changes in satellite geometry, which requires time. In some dynamic applications, too long a TTFX could mean satellite signal loss occurrences render the system entirely unavailable because signal loss occurs more frequently than is time for fixing the integer ambiguity set.

The measurement Equation A.3 derived in Appendix A needs to be solved. This will be rewritten with simplified notation for the i^{th} measurement:

$$\nabla\Delta\phi_i = \begin{bmatrix} -\lambda^{-1}\nabla\Delta LOS_i & 1 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \nabla\Delta N_i \end{bmatrix} + \nabla\Delta\epsilon_i, \tag{3.1}$$

Each mutual satellite between the base and rover receivers will add a new measurement and an unknown double-differenced integer ambiguity term. A reference satellite is used to form the double differences. Therefore for N satellites, there will be $(N-1)+3$ unknowns where the three additional unknowns are the relative position vector $\bar{x}$. This equation assumes that during the integer ambiguity resolution, the rover antenna is static. This reduces the TTFX, and may even be required for successful operation if only single frequency receivers are being used. If the rover antenna is dynamic, then a dynamic model is required to update the relative position vector $\bar{x}$.

Equation 3.1 can be expanded and written for a specific example where satellites 1 through 6 are available. Assuming satellite 1 is selected as the reference satellite, this can be written for a single time step:

$$\begin{bmatrix} \nabla\Delta\phi_2 \\ \nabla\Delta\phi_3 \\ \nabla\Delta\phi_4 \\ \nabla\Delta\phi_5 \\ \nabla\Delta\phi_6 \end{bmatrix} = \begin{bmatrix} -\lambda^{-1}\nabla\Delta LOS_2 & 1 & 0 & 0 & 0 & 0 \\ -\lambda^{-1}\nabla\Delta LOS_3 & 0 & 1 & 0 & 0 & 0 \\ -\lambda^{-1}\nabla\Delta LOS_4 & 0 & 0 & 1 & 0 & 0 \\ -\lambda^{-1}\nabla\Delta LOS_5 & 0 & 0 & 0 & 1 & 0 \\ -\lambda^{-1}\nabla\Delta LOS_6 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \nabla\Delta N_2 \\ \nabla\Delta N_3 \\ \nabla\Delta N_4 \\ \nabla\Delta N_5 \\ \nabla\Delta N_6 \end{bmatrix} + \begin{bmatrix} \nabla\Delta\epsilon_2 \\ \nabla\Delta\epsilon_3 \\ \nabla\Delta\epsilon_4 \\ \nabla\Delta\epsilon_5 \\ \nabla\Delta\epsilon_6 \end{bmatrix} \tag{3.2}$$

Equation 3.2 contains $(6-1)+3=8$ unknowns and has 5 measurements. As long as all six satellites remain available without cycle slip, each subsequent time step will add 5 measurements and the number of unknowns will be unchanged. However over short time intervals, there will be little changes to the measurement mapping matrix premultiplying the unknowns. Therefore additional measurements over short intervals have little effect on improving the estimates. Rather, measurements over epochs that allow the measurement mapping matrix to change will reduce the estimate uncertainties.

3.1. BATCH AND RECURSIVE LEAST SQUARES

One method to solve for the unknowns is stacking the measurements over a period of time and using Batch Least Squares. In this case if the stacked measurement equation is denoted $y = Hx+v$ where y, x, and v respectively are, the measurements, unknown states, and zero-mean independent additive Gaussian noise with covariance $E\{vv^T\} = R$. The matrix H is the linear mapping between the unknown states and the measurements. Then the batch least squares estimate and associated covariance is given as [2]:

$$\begin{aligned} \hat{x} &= (H^TH)^{-1}H^Ty \\ P &= (H^TH)^{-1}H^TRH(H^TH)^{-1} \end{aligned}$$

The measurements over a length of time must be combined to reliably estimate the integer ambiguities and relative position vectors. It may be unreasonable to use batch least squares in a real-time implementation as the large matrix inversions may become computationally burdensome. An alternate approach is to update the estimates as new measurements become available. This can be done using recursive least squares. The state and covariance can be initialized as $\hat{x}_0$ and P_0 by using the unambiguous code-phase measurements, and subsequently updated at each time-step k [10]:

$$\begin{aligned} K_k &= P_{k-1}H_k^T(H_kP_{k-1}H_k^T + R_k)^{-1} \\ \hat{x}_k &= \hat{x}_{k-1} + K_k(y_k - H_k\hat{x}_{k-1}) \\ P_k &= (I - K_kH_k)P_{k-1} \end{aligned}$$

where K_k is an intermediate gain computed to update the current estimate and covariance, $\hat{x}_k$ and P_k, respectively.

3.1.1. Float vs Fixed Solution

Both batch and recursive least squares can be used to estimate the integer ambiguity terms for each satellite. Sufficient amounts of data can be used until eventually the associated covariance terms shrink below the desired threshold. What is missing from these estimators is a means to ensure the ambiguity estimates are indeed *integers*. Measurement noise and modeling imperfections will result in estimates that are not integers. These estimators as described do not constrain their results to *integer* estimates. This type of estimate is known as a *float-estimate*[2]. The unconstrained float-estimate can be used as an input to any number of *fixing algorithms*, which attempt to resolve the integer ambiguities to their true integer values. One fixing algorithm is to simply round the float estimates for each integer ambiguity. Even in this case, several minutes of data would still be necessary to shrink the float covariances to reliably round them. More sophisticated fixing algorithms have been designed, primarily to reduce the TTFX and to overcome the shortcomings of rounding.

3.2. Integer Least-Squares and LAMBDA Method

The output of a float estimator like Recursive Least-Squares or a Kalman filter includes a float-estimate of the ambiguities and an associated covariance, $\hat{N}$ and $\hat{P}_N$ respectively. The float-estimates are in general real-valued and not constrained to be integers. Integer Least-Squares is finding the set of integers N which minimize the cost function [2]

$$c(N) = (N - \hat{N})^T \hat{P}_N^{-1} (N - \hat{N}) \tag{3.3}$$

This is minimizing the distance between the float-estimate and the fixed solution, where the distances are weighted according to the float-estimate uncertainties. The procedure for finding the minimizing set of integers, denoted $\breve{N}$, involves an iterative search over candidate sets of integers and can be computationally challenging for real-time applications. Nonetheless, integer least-squares estimators maximize the probability of correct integer estimation [11] and therefore are of great interest.

The Least-Squares AMBiguity Decorrelation Adjustment (LAMBDA) method provides a relatively efficient means of searching for the minimizing set of integers. This is achieved by conducting the iterative search in a transformed search space. Once the minimizing set in the transformed space is determined, the transformation is undone and the integer set that minimizes the cost function 3.3 has been determined. Further details on the implementation of the LAMBDA method and the decorrelation transformation can be found in [6].

3.2.1. Validation

The LAMBDA method returns the minimizing integer set $\breve{N}$ and the associated cost $c(\breve{N})$. Additionally, the algorithm can return the second, third, and subsequent minimizing sets and associated costs as well. This can be done at each epoch and it is a nontrivial problem to decide which minimizing integer set is the correct set. This is the validation problem and is of great importance to real-time safety of life applications. Declaring an incorrect integer set as the fixed solution would cause positioning errors potentially larger than the specified centimeter level accuracy of carrier phase. Furthermore these errors would be time-varying as the changing satellite geometry varies the effect of the incorrect integers on the estimated baseline vector. Two steps are important for validation [12]: integer test and discrimination test.

The integer test quantifies the likelihood of an integer set being the correct set based on the weighted distance from the float-estimate. If the proposed integer set widely differs from the float-estimate, then the likelihood of the integer set being correct is reduced. The second test is a discrimination between two sets of proposed integer sets. For example, one discrimination test commonly used with the LAMBDA method is the *ratio test*. This test compares the ratio of the computed cost defined by Equation 3.3 for the minimizing set $c(\breve{N})$ and the second minimizing set $c(\breve{N}_s)$

$$\frac{c(\breve{N}_s)}{c(\breve{N})} > k \tag{3.4}$$

If the ratio is larger than a specified $k > 1$, then the minimizing set which has also passed the integer test is more likely to be the correct integer set. However if the first and second best integer sets have very similar cost functions, one is unable to discriminate between them in deciding which is the correct integer set. The choice of k is nontrivial, though empirically a value of 2 has worked well [12]. Both an integer test and a discrimination should be passed before an integer set is declared as fixed.

3.3. BOOTSTRAPPING

Integer bootstrapping is an alternative fixing algorithm which is both computationally attractive and also a has quantifiable metric associated with the estimated integers. The bootstrapped estimator is a generalization of integer rounding. The procedure begins with rounding the first unknown integer. The remaining float-estimates are updated based on their correlation with the first fixed integer. Next the second corrected float-estimate is rounded to become a fixed integer value. Subsequently the remaining float-estimates are again adjusted based on their correlation with the second integer. This process continues sequentially until all the float-estimates have been converted into fixed integer values. In the case that no correlation exists between the float estimates, the bootstrapping estimator reduces to integer rounding [13]. Unlike the LAMBDA algorithm or other Integer Least Squares search methods, Bootstrapping does not require any iterative searches.

The second attractive feature of the bootstrapping estimator is that the associated theoretical probability of successful fixing can be computed. Mathematically this is $Pr(\hat{N}_b = N)$, or the probability that the bootstrapping integer estimates $\hat{N}_b$ is equal to the true unknown integer set N. The success rate probability of fixing n integers correctly is

$$\begin{aligned} Pr(\hat{N}_b = N) &= \prod_{i=1}^{n}(2\Phi\left(\frac{1}{2\sigma_{i|I}}\right) - 1) &(3.5)\\ &\leq (2\Phi\left(\frac{1}{2ADOP}\right) - 1)^n &(3.6)\end{aligned}$$

where $ADOP = \sqrt{det(P_{\hat{N}})}^{1/n}$ is the ambiguity dilution of precision and $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} exp\{-\frac{1}{2}v^2\}dv$ is a normal random variable cumulative distribution function. The conditioned standard deviations $\sigma_{i|I}$ for $i = 1, 2, ...n$ are the entries of the diagonal matrix D in the triangular decomposition

$P_{\hat{N}} = LDL^T$. The detailed derivation of the bootstrapping estimator and these success rate probabilities can be found in [13, 8]. Equation 3.6 provides an upper bound on the success rate of the bootstrapping estimator and can easily be computed using the float-estimate covariance. If the computed upper bound is below the required success rate, then the bootstrapping estimator will not meet the problem requirements.

In this work we use the upper bound probability given by Equation 3.6 in an alternative manner. After a vehicle has recovered from an signal outage, the float estimator, which includes both baseline vector states and unknown integer ambiguities, is initialized by a secondary source of position information. The quality of the secondary source of position will affect the success rate probabilities for the bootstrapping estimator. Therefore, using Equation 3.6 we compute the upper bound success rate probability over a range of initial position qualities. These are specified by initial baseline vector covariance values. Additionally, the success rate probability will depend on the number of satellites $(n+1)$ and their particular geometry. In particular, this effects the evolution of the float estimate covariance $P_{\hat{N}}$ which changes the upper bound probabilities. In section 4.1 we will compute the upper bound success probability to understand the effect of sensor quality on the fixing process.

This section is ended with a comment on the relationship between the bootstrapping success rate and the probability of success for the integer least-squares LAMBDA algorithm. The bootstrapping success rate lower-bounds the success rate of the integer least-squares estimator [8]. Mathematically, if $P(\hat{N}_{LS} = N)$ is the integer least-squares success rate, then

$$Pr(\hat{N}_b = N) \leq Pr(\hat{N}_{LS} = N) \tag{3.7}$$

This is of great practical use because of the simplicity of computing the bootstrapping success rate probability. In contrast, computing the success rate of the integer least-squares is usually difficult and therefore having an available lower-bound is of great significance [8].

CHAPTER 4

AIDING WITH EXTERNAL SENSORS

Uninterrupted carrier phase differential GPS operation requires a minimum set of satellites with resolved integer ambiguities. Obstructions causing signal loss or cycle slip in the carrier phase measurements of the satellites invalidate the prior estimated integer ambiguity. Therefore the integer ambiguities will need to be estimated again after regaining the continuous lock on the satellite signals.

This would be common in urban applications, where buildings or underpasses can cause loss of several or even all the satellite signals. After passing the obstructions or emerging from under the underpass, the lost satellite carrier phase signals will be reacquired. That is when the estimation of the new unknown integer ambiguities must be commenced.

During the outage, external sensors can be used in two manners:

1. Provide an alternate source of position and/or velocity during the outage
2. Assist in the re-estimation of the new unknown integer ambiguities upon regaining signal lock

These functions can run in parallel and together may improve the system robustness to GPS signal outages.

4.1. SENSOR REQUIREMENTS

Here we conduct an analysis of the accuracy required by an aiding external sensor in order to benefit the estimation of the integer ambiguities. In this analysis, the sensor accuracy is compared against the GPS code-phase solution. As the vehicle emerges from the obstructions, soon the GPS receiver will reacquire satellite signals. Under most circumstances both code-phase and carrier phase measurements will become available. The code-phase measurement can be used to initialize the integer ambiguity estimation process. Therefore, the aiding sensor should provide a superior accuracy to that of the code-phase measurement in order to have gained substantial benefit by using the aiding sensor.

The baseline vector estimate in conjunction with the regained carrier phase measurement can be used to estimate the integer ambiguity. The quality of the resulting estimate is related to the baseline vector uncertainty by the relation derived in Appendix B:

$$\mathbf{P_N} = \mathbf{P_\Phi} + \mathbf{H}\mathbf{P_x}\mathbf{H}^T \tag{4.1}$$

The effect of the double differenced geometry matrix $\mathbf{H} = [LOS_b^i - LOS_b^j]$ is to scale the uncertainty contribution of the baseline vector, based on the number of satellites and their geometry. Data collected on December 14, 2011 as part of the data collection campaign was analyzed to find typical values. The diagonal entries of HH^T was approximately 0.6^2 for cases where $5 - 6$ satellites where being used.

If the aiding sensor for estimating the baseline vector is simply the code-phase measurement, then the $\mathbf{P_x} = \sigma^2_{code}(H^T H)^{-1}$ where $\sigma_{code} \approx 0.5m$ [2, pp. 207,178][1]. Therefore the standard deviation of the resulting integer ambiguity is

$$\begin{aligned}
\sigma_N &= \sqrt{\sigma^2_\Phi + \sigma^2_x} \\
&= \sqrt{(2 \times 0.005)^2 + (0.5)^2} m^2 \\
&\approx 0.25 m^2
\end{aligned} \tag{4.2}$$

where the error of the aiding sensor, in this case code-phase GPS measurements, dominates. If an aiding sensor is to improve the integer ambiguity estimate, then the contribution from the baseline vector uncertainty for the aiding sensor must be less than the same achievable by the code-phase measurement

$$\begin{aligned}
(HP_xH^T)_{sensor} &\leq (HP_xH^T)_{code} \\
\sigma^2_{sensor}(0.6^2) &\leq \sigma^2_{code} \\
\sigma_{sensor} &\leq 0.8m
\end{aligned} \tag{4.3}$$

The quality of the code-phase measurement provides a baseline, for which any additional aiding sensor should be selected to provide a superior baseline vector estimate. If the aiding sensor can estimate the baseline vector with a standard deviation $\sigma_{sensor} < 0.8\ m$, then the integer ambiguity resolution would be expected to improve in comparison to the case where only the code-phase solution is used. The actual GPS code-phase measurement accuracy will depend on factors like the multipath environment or satellite signal to noise ratio. However, the code-phase measurement errors are not of the same nature as dead reckoning errors and do not grow with time. A large category of aiding sensors, like odometers and inertial or velocity sensors, can be used to estimate the baseline vector during the GPS outage, however the dead reckoning nature of their estimate means the errors will grow with time. Only the highest quality of sensors, or very short signal outages, will have errors smaller than the accuracy of code-phase GPS. This makes a practical system relying on aiding sensors difficult to implement as the attainable benefits degrade quickly.

4.1.1. Aiding Sensor and Integer Ambiguity Success rate

An aiding sensor can provide an initial baseline vector estimate and covariance. It was determined in the previous section that a baseline vector position estimate with a standard deviation of less than $0.8\ m$ has potential to improve the integer ambiguity resolution time. Here we quantify the effect of the baseline vector initial uncertainty on the integer fixing success rate. This is done by plotting bootstrapping upper bound success rate for over a range of initial baseline vector estimate qualities. This is plotted for the case when 5 satellites are available in Figure 4.1. Having

[1] The errors in code-phase measurement can be much larger if multipath is present, as may be the case in an urban environment.

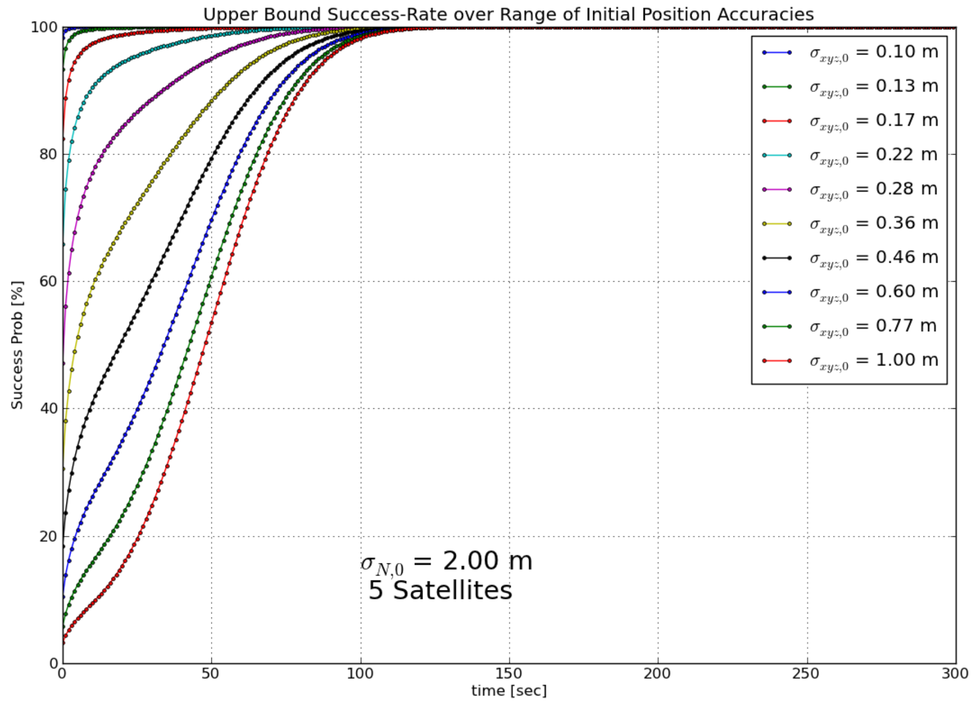


Figure 4.1: Bootstrapping upper bound success rate for various initial baseline vector qualities and 5 satellites available

5 satellites available is not uncommon in urban environments, since large portions of the sky may be blocked by buildings. It is clear that the initial baseline vector quality significantly affects the success rate probability. After 50 seconds of data collection, an estimator initialized with a baseline vector $\sigma = 1.0\ m$ has an upper bound success-probability of just over 40%. In contrast, if the baseline vector was initialized with $\sigma = 0.46\ m$, the upper bound success-probability can be as high as 70%. The success rate probabilities will also depend on the number of satellites available. A comparison of the upper bound success probabilities when 4, 5, or 6 satellites are used is given in Appendix D. By looking at all the success probabilities, what is clear is that the TTFX may be on the order of minutes if the required probability of success is high. This is a significant limitation if there are common obstructions causing signal loss and cycle slips.

CHAPTER 5

HARDWARE AND SOFTWARE

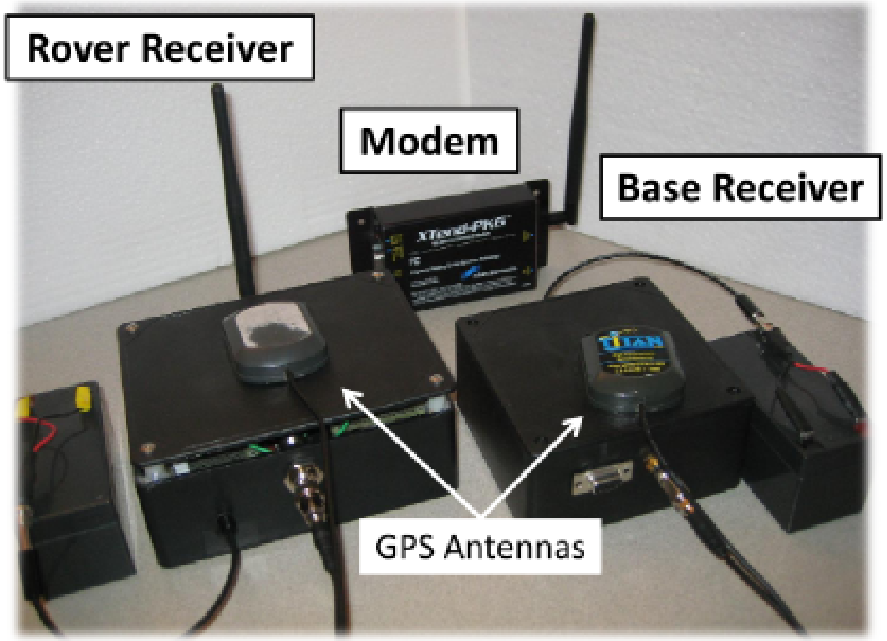
Hardware to facilitate data collection and algorithm testing were designed and built as part of this project. Two Hemisphere Crescent GPS Receivers were packaged in separate Base and Rover units, pictured in Figure 5.1. The units are mostly identical in function and each included:

- Hemisphere Crescent GPS Receiver
- Serial Data Logger
- Antenna Connector and Active Antenna
- Portable Power Supply and Voltage Regulator

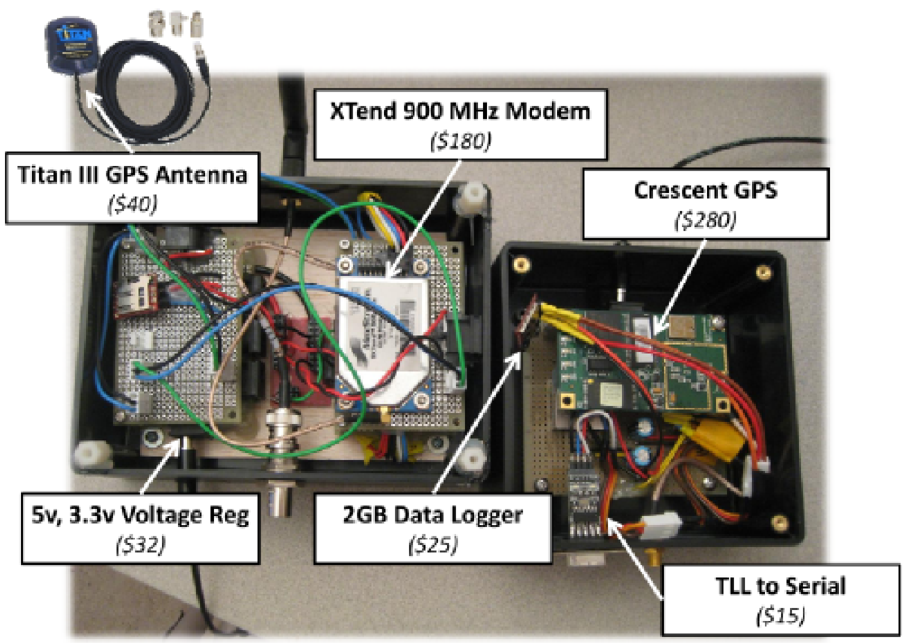
The rover unit also included a XTend 900 MHz modem for broadcasting data. The data is received by an external modem unit, which together with the base unit is connected to a laptop for real-time processing via serial ports. Additionally, the onboard data loggers serve as a backup to the modem and provided onboard logging for post-processing the data.

The hardware design went through several iterations, primarily to establish the required interconnections and to reach a design that was reasonably portable. The rover unit includes a vent for passive cooling whereas the base unit does not. For extended data collection in warm environments, the top of the base unit should be left partially open to allow for airflow.

Two sets of software were written for this project. First, a carrier phase single frequency real-time processing code was written in C/C++. The software was designed to run on a laptop and to take raw binary GPS measurements from two Hemisphere Crescent GPS receivers and generate centimeter-level accurate positioning solutions, logged to a text file. The second set of software was a library of functions in Python for visualizing the position solutions generated.



(a)



(b)

Figure 5.1: Base and rover GPS units

CHAPTER 6

DATA COLLECTED

6.1. PRELIMINARY TESTING

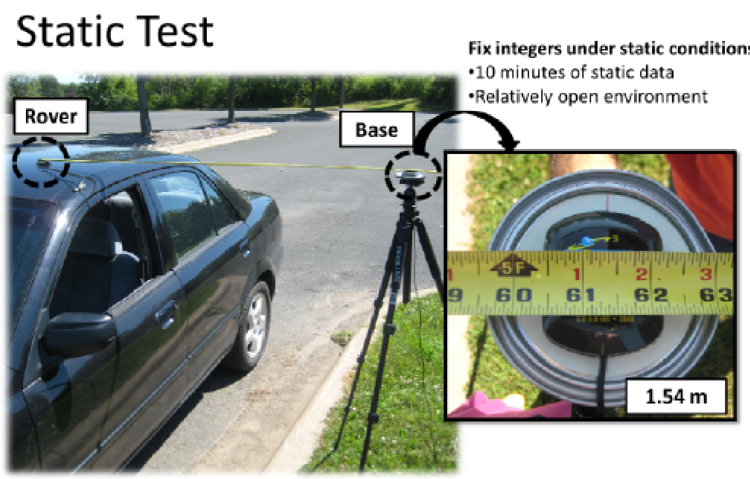


Figure 6.1: Static testing antenna separation

Preliminary data collection was completed in both static and dynamic testing. This allowed resolving software issues and to check the validity of the basic single frequency carrier phase differential GPS solution. This data was collected in July of 2010 in a parking lot of a community park in Maple Grove, MN. The base unit antenna was setup on a elevated tripod and the rover antenna was installed on the roof of a vehicle. Initially, the antennas were separated by 1.54 meters as shown in Figure 6.1. Ten minutes of data were collected in a static fashion. This was to allow for fixing the integer since integer fixing without additional sensors under dynamic situations is challenging.

The processed relative positioning solution for the 10 minute static test is shown in Figure 6.2. The integer fixing algorithm used was LAMBDA and it required 450 seconds of data to fix the integers. The resulting carrier phase solution was accurate to the centimeter level, whereas the code-phase solution had errors closer to 0.5-1 meter. This demonstrates both the desirable high-accuracy position solution, and the long initialization time required to obtain it. The 8 minute initialization under static conditions would be unacceptable in certain applications. Nevertheless,

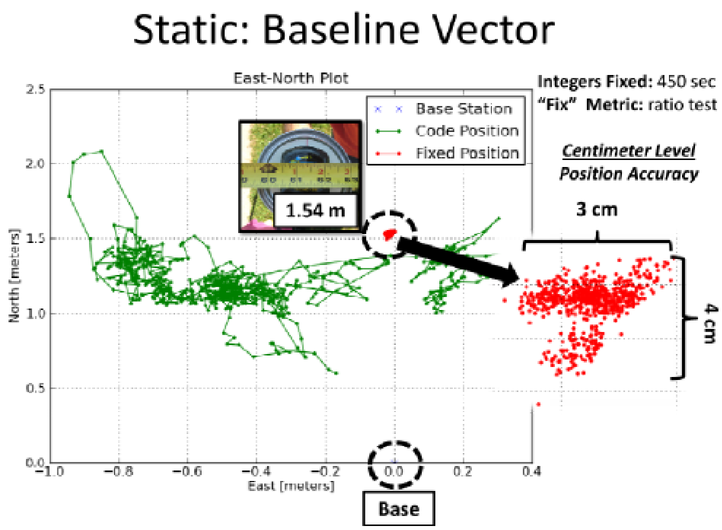


Figure 6.2: Processed static test solution

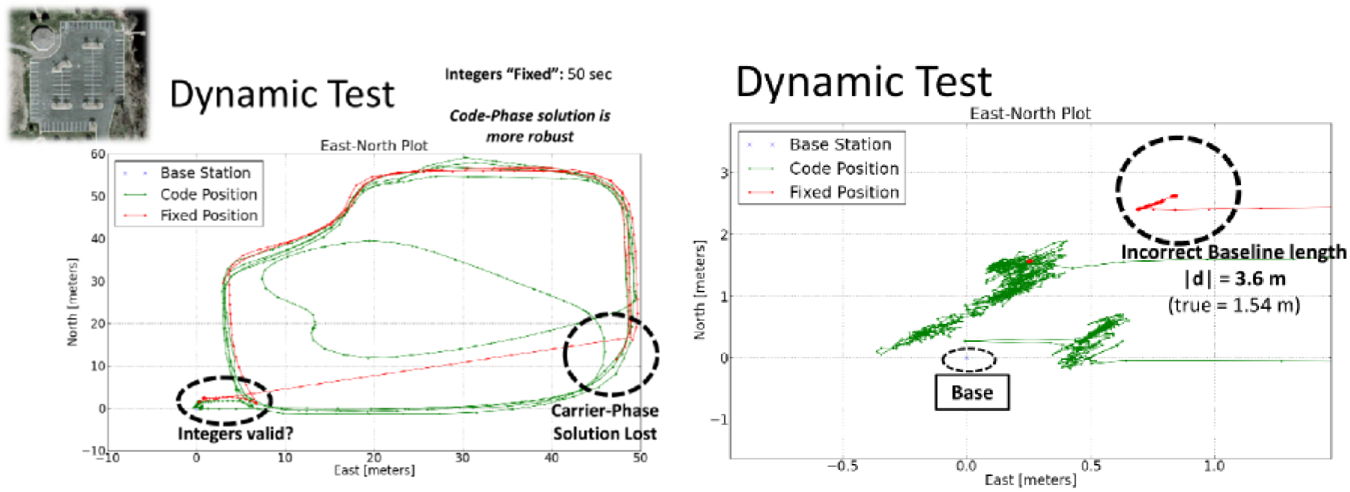
once the integer ambiguities have been resolved during static initialization, the vehicle can then proceed to travel all the while with accurate centimeter-level relative positioning. This is until signal obstructions cause the originally resolved integer ambiguities to become invalid.

A second test was conducted where the vehicle was static until integer ambiguities were resolved, and then the vehicle was driven around the parking lot. The resolved integers were used during the dynamic portion of the test. The path traveled as computed by the code-phase solution and the carrier phase solution are shown in Figures 6.3. Notice the robustness of the code-phase solution, whereas the carrier phase solution is lost due to a momentary signal outage. Thus the subsequent heart-shape path driven is entirely missed by the differential carrier phase solution. Once the vehicle returns to the original static parked position, the integers are eventually resolved, but the resolved integers are incorrect. Two methods to check the validity of the estimated fixed integers are checking the ratio test output, and to set the antennas apart at a known separation. Incorrect integers may initially compute a seemingly correct baseline length, but over several minutes of time that the satellites continue to change geometry, the baseline length will continue to change and an incorrect set of integers would cause the solution to drift. This is especially visible when the vehicle static, and is observable in Figure 6.3 (b).

6.2. Final Testing

A data collection campaign was designed and conducted to provide a reference data set for validation and future testing. The experiment was designed to collect carrier phase data on a road vehicle driving in an open environment. This way artificial signal outages may be simulated in software, making the data set more flexible. A well instrumented Chevrolet Impala, belonging to the Center of Transportation Studies and shown in Figure 6.6, was selected for this test. The vehicle was instrumented with:

- Hemisphere Crescent GPS Receiver



(a) (b)

Figure 6.3: (a) Dynamic carrier phase solution and signal loss are observed, and (b) zooms on the final carrier phase solution showing the incorrectly resolved integers after the outage
Satellite imagery courtesy of Google Maps

- Additional sensors
  - Inertial Measurement Units: Crossbow Hdx
  - Optical Velocity Sensors: S-350
- Reference Positioning
  - Dual Frequency Trimble GPS Receiver

A second equivalent Hemisphere Crescent GPS receiver was setup as a base station. The remaining discussion is for the nominal case where none of the aiding sensors are used. The reason for this limitation in experimental analysis is stated at the end of Section 6.2.2.

6.2.1. Location

The options for testing were restricted to locations within approximately 10 miles of the existing Center for Transportation Studies base stations. This was necessary to form an accurate truth reference solution based on the dual frequency Trimble GPS Receivers. The Apple Valley base station (ApVy) was selected for this purpose and a preliminary test data collection run was conducted on December 5, 2011. The preliminary testing used a single Hemisphere Crescent GPS receiver mounted on the roof of the vehicle. A combination of visual notes and the carrier phase signal lock data from the receiver were used to identify stretches of roadway where satellite visibility was unobstructed and cycle-slips were minimized. The satellite imagery was also beneficial but not always up to date. At several locations there were newly built homes where the satellite imagery showed open land.

The result of the preliminary testing was two proposed ”L” and ”S” routes shown in Figure 6.4. Both these routes had no overpasses and minimal obstructions like trees, buildings, and electric

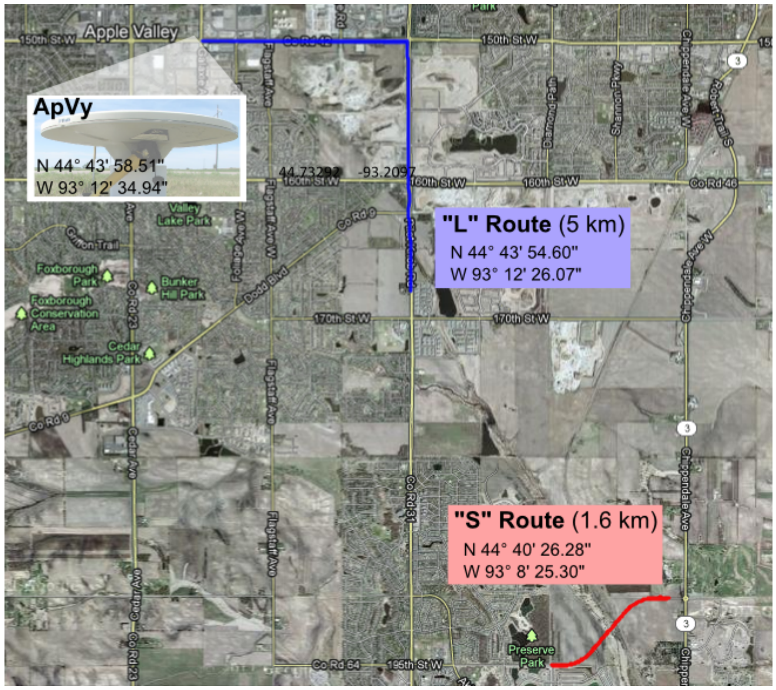


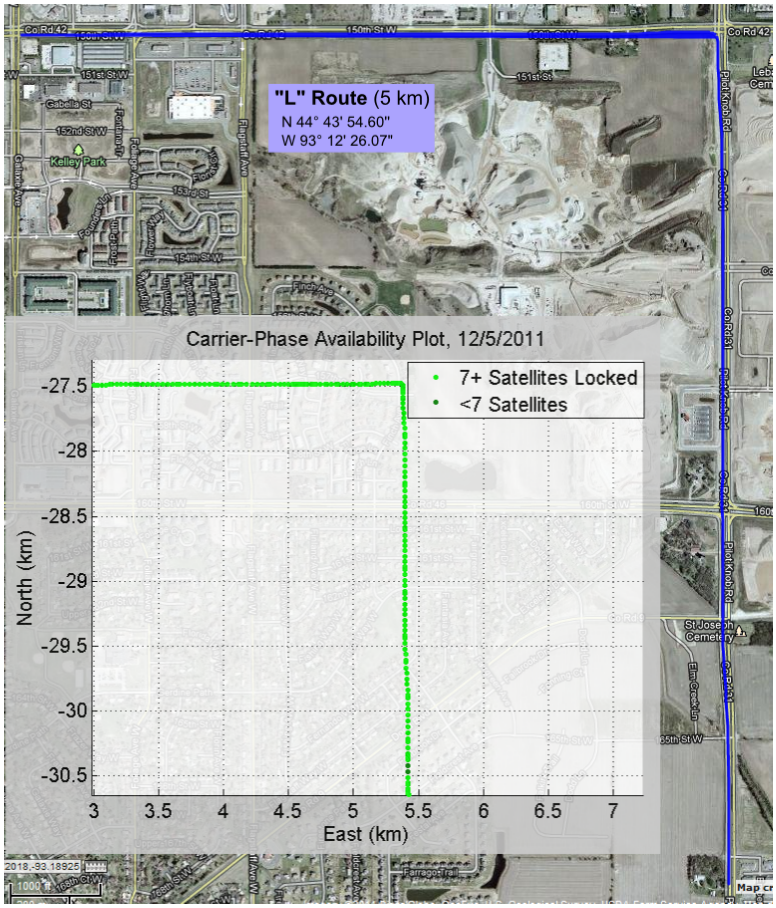
Figure 6.4: Two routes in Apple Valley identified for carrier phase testing
Satellite imagery courtesy of Google Maps

poles near the roadway. Processing the carrier phase data collected showed the number of satellites available with signal lock on these paths. These were plotted onto the respective paths to identify the sections of minimal cycle slip, as shown in Figure 6.5. It is clear that for the majority of both paths there are 7+ satellites with carrier phase signal lock. This preliminary testing guided the choice of locations where static obstructions did not exist. Nonetheless, other types of obstructions like passing large vehicles could not be excluded by preliminary testing.

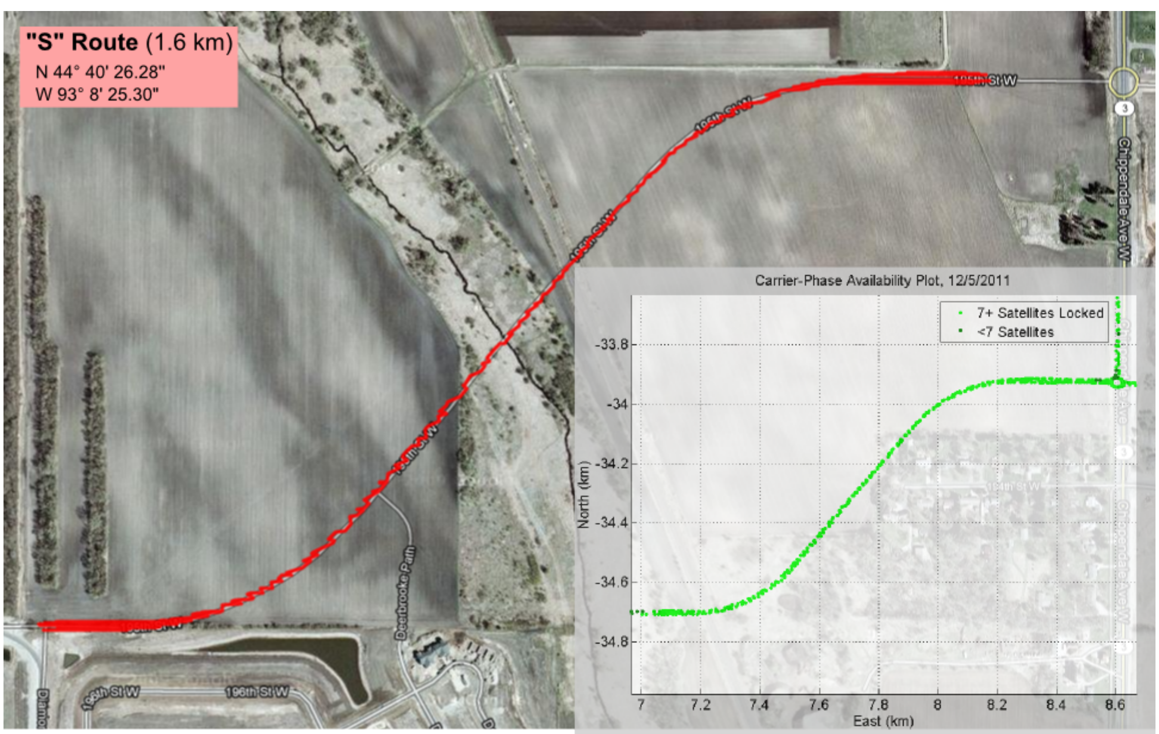
The final testing was conducted entirely on the ”S” route on December 14, 2011. The instrumented vehicle and the base station can be seen in Figure 6.6.

6.2.2. RESULTS

The basic carrier phase solution using the Hemisphere Crescent GPS receiver on the vehicle and the base receiver on the tripod was attempted. The float estimate was computed using a recursive least-squares algorithm, described in Section 3.1. The float estimate was then fixed using the LAMBDA algorithm described in Section 3.2. The first 15 minutes of data were collected with the vehicle static and the antennas separated by a measured 6.74 meters. The float and fixed estimates at each epoch were used to compute the estimated baseline length between the antennas. The fixing validation metric of the ratio test was also computed. The plot in Figure 6.7 shows the float and fixed antenna separation estimate along with the ratio test output overlaid with the measured antenna separation. Despite the open sky and long data set, neither does the baseline length estimate converge correctly, nor does the ratio test display any trend of growing confidence. This sort of performance was intermittently observed with other data sets as well, and we were unable to determine the source. A cross examiniation with the fixed solution of the reference receivers may provide insight into the poor performance attained from the carrier phase measurements of the Hemisphere Crescent receivers. However, further investigation into this matter was terminated due to project time constraints. This prevented the possibility of experimentally testing the fixing process when aiding with the optical or inertial sensor measurements.



(a)



(b)

Figure 6.5: Two routes in Apple Valley identified for carrier phase testing
Satellite imagery courtesy of Google Maps

Figure 6.6: Instrumented testing vehicle at northeast starting point of ”S” route. The base receiver antenna is on the tripod behind the vehicle.

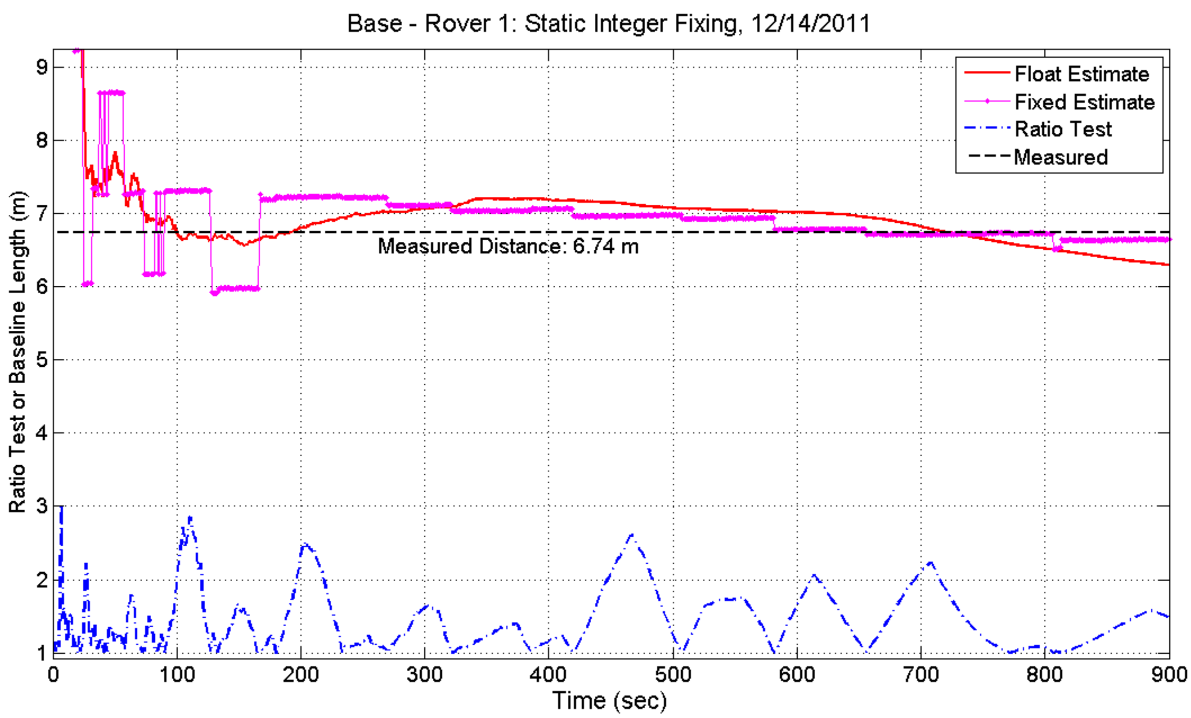


Figure 6.7: Float and fixed estimates of baseline length overlayed with the measured antenna separation

CHAPTER 7

SUMMARY & CONCLUSION

The work described in this report outlines the design and testing of a low-cost, single frequency, carrier phase positioning system. Furthermore, the aiding sensor accuracy requirements to improve the robustness of the carrier phase system after emerging from signal outages were analyzed. The single frequency GPS receiver used for this work was the Hemisphere Crescent. Carrier phase data collected with this receiver were processed using C/C++ software written for this purpose, which was only intermittently successful. Despite data sets commonly being collected under ideal static and open-sky conditions, the integer ambiguity estimation process was often unsuccessful. Much effort was spent to identify the source of the problem, but no conclusive reason was identified. This was unfortunate as the final reference data set also was plagued with the same inconclusive integer fixing problem. Despite this setback, sensor accuracy requirements were studied analytically. As part of this work it was determined that aiding sensors capable of providing baseline vector position estimates with a standard deviation of less than $0.8\ m$ have potential to improve the integer ambiguity resolution time. This is a tight requirement, especially for signal outages lasting longer than several seconds, and limits the candidate sensors to more accurate higher-cost sensors. This presents a challenge, as the objective is to design a low-cost system.

This conflicting requirement limits the application of current low-cost, single frequency, receivers to applications where the environment is controlled in favor of carrier phase measurements. This work highlighted the current challenges of using low-cost single frequency systems for accurate positioning. However, this problem will continue to be of interest with the continued implementation of GPS modernization. A second and third GPS civilian signal, L2C and L5, will become fully available by 2019 [1]. There is reason to believe low-cost dual and triple frequency receivers will become common within the next decade. Therefore, low-cost carrier phase differential positioning will be of greater interest in the coming years.

References

[1] Official U.S. Government information about GPS and related topics, "GPS Modernization: New Civilian Signals." http://www.gps.gov/systems/gps/modernization/civilsignals/. Accessed: 4/1/2012.

[2] P. Misra and P. Enge, *Global Positioning System, Signals, Measurements, and Performance*. Lincoln, MA: Ganga-Jamuna Press, 2001.

[3] S. Comstock, "Development of a Low-Latency, High Data Rate, Differential GPS Relative Positioning System for UAV Formation Flight control," Master's thesis, Air Force Institute of Technology, Wright-Patterson AFB, Ohio, September 2006.

[4] G. Zheng, *Methods for Enhancing Carrier Phase GNSS Positioning And Attitude Determination Performance*. PhD thesis, University of Minnesota, Minneapolis, Minnesota, May 2010.

[5] P. J. G. Teunissen, "Least-Squares Estimation of the Integer GPS Ambiguities," Invited lecture, Section IV Theory and Methodology, IAG General Meeting, August, 1993.

[6] P. J. G. Teunissen, "The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation," *Journal of Geodesy*, vol. 70, pp. 65–82, 1995.

[7] J. de P.J. and C. Tiberius, "The LAMBDA method for integer ambiguity estimation: implementation aspects," Tech. Rep. 12, Delft University of Technology, 1996.

[8] P. J. G. Teunissen, "GNSS ambiguity bootstrapping: theory and applications," pp. 246–254, In Proceedings KIS2001.

[9] M. G. Petovello, "Benefits of Using a Tactical-Grade IMU for High-Accuracy Positioning," *Journal of The Institute of Navigation*, vol. 51, no. 1, 2004.

[10] D. Simon, *Optimal State Estimation*. Hoboken, NJ: John Wiley & Sons, Inc, 2006.

[11] P. J. G. Teunissen, "An optimality property of the integer least-squares estimator," *Journal of Geodesy*, vol. 73, pp. 587–593, 1999.

[12] S. Verhagen, "Integer ambiguity validation: an open problem," *GPS Solutions*, vol. 8, pp. 36–43, 2004.

[13] P. J. G. Teunissen, "Integer aperture bootstrapping: a new GNSS ambiguity estimator with controllable fail-rate," *Journal of Geodesy*, vol. 79, pp. 389–397, 2005.

APPENDIX A

CARRIER PHASE MEASUREMENT MODEL

APPENDIX A

CARRIER PHASE MEASUREMENT MODEL

The carrier phase measurement is the difference between the phase of the signal generated in the receiver and the phase of the received signal sent by the satellite, and is given by:

$$\Phi_b^i = r_b^i + \lambda N_b^i + c(\delta t_b - \delta t^i) - I_b^i + T_b^i + \epsilon_{b,\Phi}^i$$

Where:

Φ_b^i carrier phase measurement (m)

r_b^i range between base receiver to ith satellite (m)

λ signal wavelength (m)

N_b^i integer ambiguity (cycles)

$c, \delta t_b, \delta t^i$ speed of light (m/s), receiver and satellite clock bias (sec)

I_b^i, T_b^i ionosphere and troposphere errors (m)

$\epsilon_{b,\Phi}^i$ carrier phase measurement noise (m)

The additive measurement noise can be modeled as zero-mean white Gaussian noise with a standard deviation $\sigma(\epsilon_\Phi) \approx 0.025\ m$ [2]. For positioning applications, the range measurement between the user and each satellite is of interest. All other terms must be known, modeled, estimated, or canceled by measurement differencing in order to extract the desired range from the carrier phase measurement.

Using two receivers, base (b) and rover (r), mutual error terms exist for measurements to the same satellite. The ionosphere, troposphere, and satellite clock bias errors may be canceled by differencing the base and rover receiver measurements. This is known as single differencing between receivers and is given by:

$$\begin{aligned} \Delta\Phi_{b,r}^i &= \Phi_b^i - \Phi_r^i \\ &= \Delta r_{b,r}^i + \lambda \Delta N_{b,r}^i + c(\Delta\delta t_{b,r}) + \Delta\epsilon_{br,\Phi}^i, \end{aligned}$$

The measurement differencing technique can again be applied to cancel the receiver clock bias error ($\Delta\delta t_{b,r}$) by differencing the measurements between two satellites, i and j. This is known as double differencing (DD) and is given by:

$$\nabla\Delta\Phi^{i,j}_{b,r} = \Delta\Phi^{i}_{b,r} - \Delta\Phi^{j}_{b,r} \tag{A.1}$$

$$= \nabla\Delta r^{i,j}_{b,r} + \lambda\nabla\Delta N^{i,j}_{b,r} + \nabla\Delta\epsilon^{i,j}_{br,\Phi}, \tag{A.2}$$

The double difference CP measurements are the sum of DD ranges and DD integer ambiguities with additive noise. The integer ambiguity estimation problem involves estimating the value of the DD integers ($\nabla\Delta N^{i,j}_{b,r}$). The double differencing for all single difference measurements is done with respect to a single reference satellite j. The cost paid for canceling mutual error sources is that DD magnifies the measurement noise and introduces correlation between each DD satellite measurement.

The DD range measurements for each satellite, $\nabla\Delta r^{i,j}_{b,r}$ can be related to the relative position vector from the base receiver to the rover receiver. This is given as the dot product of the relative position vector onto the differenced line of sight vector between satellites i and j. Plugging this into Equation A.2, we have:

$$\nabla\Delta\Phi^{i,j}_{b,r} \approx -[LOS^{i}_{b} - LOS^{j}_{b}]\bar{x}_{br} + \lambda\nabla\Delta N^{i,j}_{b,r} + \nabla\Delta\epsilon^{i,j}_{br,\Phi}, \tag{A.3}$$

Where LOS^{i}_{b} and LOS^{j}_{b} stand for the Line Of Sight (LOS) vectors from the base receiver to satellites i and j, respectively. This is obtained by a Taylor Series expansion of a non-linear range measurement equation, and applying the single and double differences. The large distance between the receiver and the satellites minimizes the errors introduced by linearization. The absolute position of the rover receiver can be obtained only if the absolute position of the base receiver is known. Otherwise only a relative position solution will be available. This linear measurement equation can be used to estimate the relative position vector from base receiver to rover receiver $\bar{x}_{br}$. If the DD integer ambiguities are unknown, then both the relative position vector and the DD integer ambiguity must be estimated together.

APPENDIX B

AIDING SENSOR ACCURACY REQUIREMENT

APPENDIX B

AIDING SENSOR ACCURACY REQUIREMENT

The required sensor accuracy is derived starting with the double differenced carrier phase measurement model, Equation A.3. The equation is rewritten below, where subscripts and double-difference symbols are dropped for simplicity. The integer ambiguity N is assumed to be converted to units of distance.

$$\mathbf{\Phi} = \mathbf{H}\bar{\mathbf{x}} + \mathbf{N} \tag{B.1}$$

Equation B.2 represents the true carrier phase measurement. The measured carrier phase measurement, denoted by $\hat{\mathbf{\Phi}}$ is assumed to be corrupted by additive zero-mean white Gaussian noise, which after double differencing has a variance of $\mathbf{P_\Phi}$.

$$\hat{\mathbf{\Phi}} = \mathbf{\Phi} + \epsilon \tag{B.2}$$

The errors in the carrier phase measurements $\mathbf{\Phi}$, the baseline vector $\mathbf{x}$, and the integer ambiguities $\mathbf{N}$ are defined as

$$\begin{aligned} \mathbf{e_x} &= \mathbf{x} - \hat{\mathbf{x}} && \text{(B.3)} \\ \mathbf{e_\Phi} &= \mathbf{\Phi} - \hat{\mathbf{\Phi}} && \text{(B.4)} \\ \mathbf{e_N} &= \mathbf{N} - \hat{\mathbf{N}} && \text{(B.5)} \end{aligned}$$

Equations B.1 is used to relate the relationship between the error in the integer ambiguity and the errors in the baseline vector and carrier phase measurement.

$$\begin{aligned} \mathbf{e_N} &= \mathbf{N} - \hat{\mathbf{N}} \\ &= (\mathbf{\Phi} - \mathbf{Hx}) - (\hat{\mathbf{\Phi}} - \mathbf{H}\hat{\mathbf{x}}) \\ &= (\mathbf{\Phi} - \hat{\mathbf{\Phi}}) - \mathbf{H}(\mathbf{x} - \hat{\mathbf{x}}) \\ &= \mathbf{e_\Phi} - \mathbf{He_x} \end{aligned} \tag{B.6}$$

The covariance of the integer ambiguity can be computed using the error expression

$$\begin{aligned} \mathbf{P_N} = E\{\mathbf{e_N}\mathbf{e_N}^T\} &= (\mathbf{e_\Phi} - \mathbf{He_x})(\mathbf{e_\Phi} - \mathbf{He_x})^T \\ &= E\{\mathbf{e_\Phi}\mathbf{e_\Phi}^T\} - E\{\mathbf{e_\Phi}\mathbf{e_x}^T\}\mathbf{H}^T - \mathbf{H}E\{\mathbf{e_x}\mathbf{e_\Phi}^T\} + \mathbf{H}E\{\mathbf{e_x}\mathbf{e_x}^T\}\mathbf{H}^T \\ &= \mathbf{P_\Phi} - \mathbf{P_{\Phi,x}}\mathbf{H}^T - \mathbf{HP_{\Phi,x}} + \mathbf{HP_x}\mathbf{H}^T \end{aligned} \tag{B.7}$$

During a signal outage, it is reasonable to assume the errors accumulated by the aiding sensors estimate of the baseline vector $\hat{\mathbf{x}}$ would be uncorrelated with the errors in the carrier phase measurement $\hat{\boldsymbol{\Phi}}$ available after reacquiring GPS signal. Therefore the cross correlation errors, $\mathbf{P}_{\boldsymbol{\Phi},\mathbf{x}}$ are zero. The resulting expression of the covariance of the integer ambiguity is

$$\mathbf{P_N} = \mathbf{P}_{\boldsymbol{\Phi}} + \mathbf{H}\mathbf{P}_{\mathbf{x}}\mathbf{H}^T \tag{B.8}$$

The standard deviation of the double-differenced carrier phase measurements is [2, pp. 178,268]:

$$\sigma_{\Phi} = 2 * \sigma(\epsilon_{\Phi}) = 2 \times 0.5cm \tag{B.9}$$

The uncertainty in the integer ambiguities ($\sigma_{\mathbf{N}}$) after emerging from a signal outage will depend on both σ_{Φ} and the errors in the baseline vector estimate $\sigma_{\mathbf{x}}$ attained by the aiding sensor. Additionally the double differenced geometry matrix $\mathbf{H}$ will affect the scaling of the baseline vector errors. The baseline vector estimate along with the current carrier phase measurements can form a float estimate of the integer ambiguities.

One simple but practical method of resolving integer ambiguities is to round the float solution one at a time. This method is attractive when the covariance of the integer ambiguity float estimate, $\mathbf{P_N}$, is sufficiently small. For example, if $\sigma_{\mathbf{N}} < 0.25\ cycles$, then fixed integer ambiguities obtained by rounding would theoretically be correct 95% of the time. This assumes that the original float estimate was consistent (i.e. zero mean).

APPENDIX C

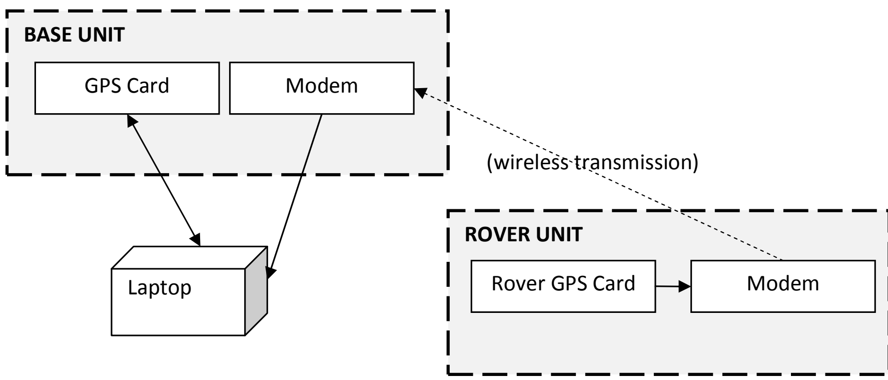
HARDWARE DESIGN

APPENDIX C

HARDWARE DESIGN

Last Revised: March 29, 2012

The *Base* unit is designed to communicate with a Laptop using a cable whereas the *Rover* is designed to communicate to the Laptop wirelessly. This is best depicted in a figure:



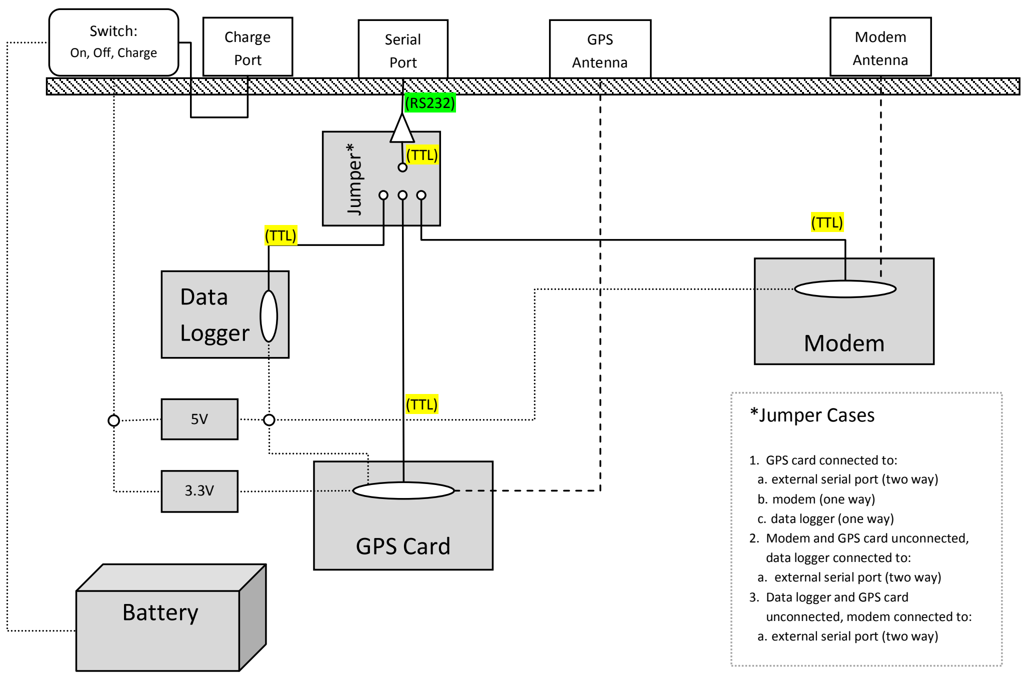
Therefore the modem inside the *Base* unit does NOT connect with its own GPS receiver, but instead only serves as a link for the *Rover* data. Optionally the modem can be packaged separately from the *Base* unit altogether and then the *Base* unit would not contain any modem.

List of Components and Associated Cost per DGPS Box:

Item	Description	Cost
Hemisphere Crescent GPS OEM Board	GPS receiver with carrier phase output capability	$280
Titan III GPS Antenna		$40
Digi 9XTend Modem	modem for data transfer	$180
Modem Antenna		$20
OpenLog Data Logger	serial data logger	$25
TTL to RS-232 Converter	two necessary, $12.5 each	$25
Battery		$30
3.3v and 5v Voltage Regulators	two voltage levels required	$10
Connectors, and Misc hardware	header pins, wires, project box	$40
	Total	$650

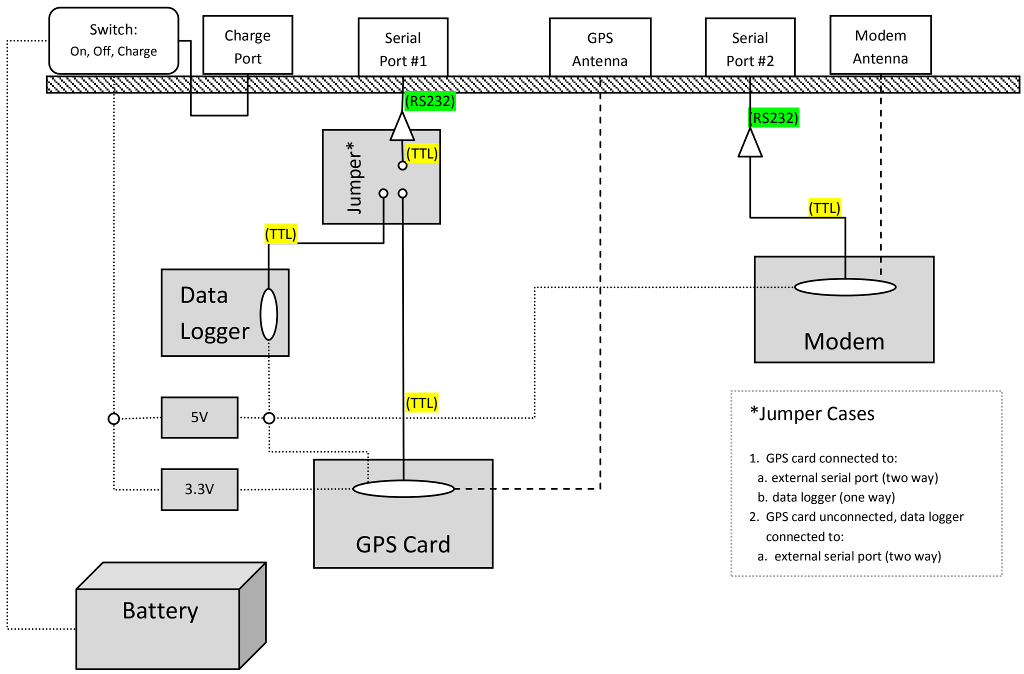
ROVER INTERCONNECTION

Rover Unit Hardware Interconnection



Base Interconnection

Base Unit Hardware Interconnection



APPENDIX D

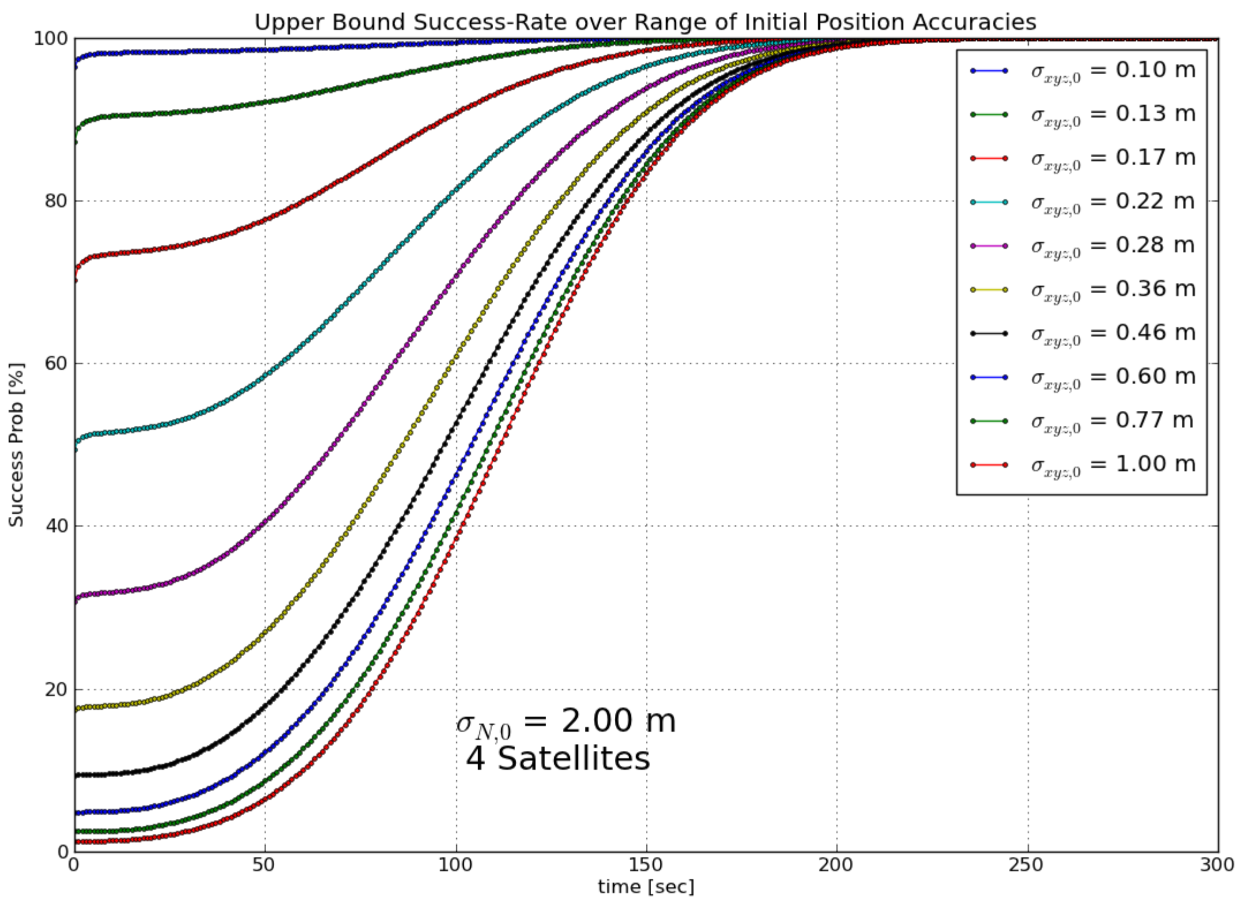
SUCCESS RATE AND NUMBER OF SATELLITES

APPENDIX D

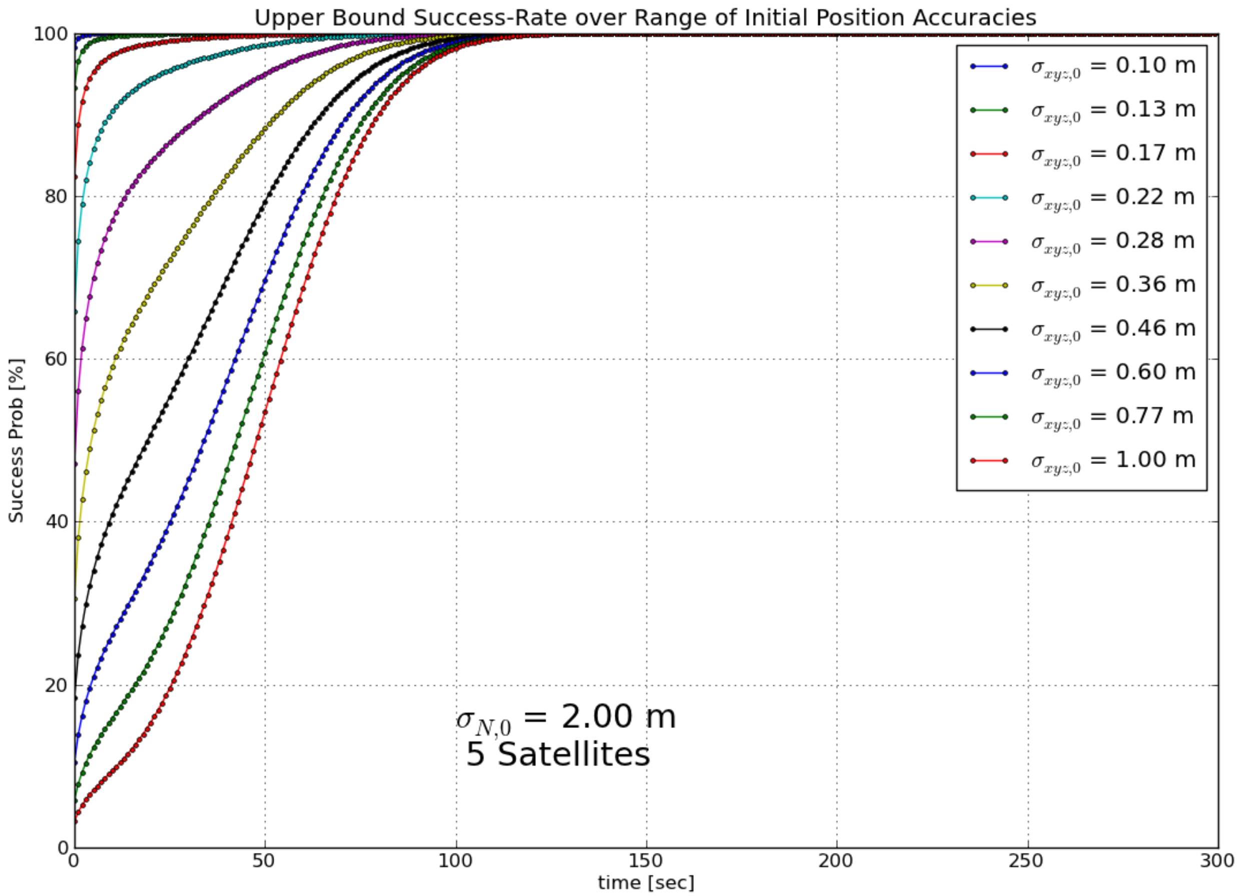
SUCCESS RATE AND NUMBER OF SATELLITES

The probability of fixing the integer ambiguities is affected by the number of satellites available. The below three plots show changes in success rate when there are 4, 5, and 6 satellites available. The plots are all 300 seconds in duration and were collected under static conditions on December 14, 2011. The three three plots have overlapping time intervals and therefore are subject to the same satellite geometries.

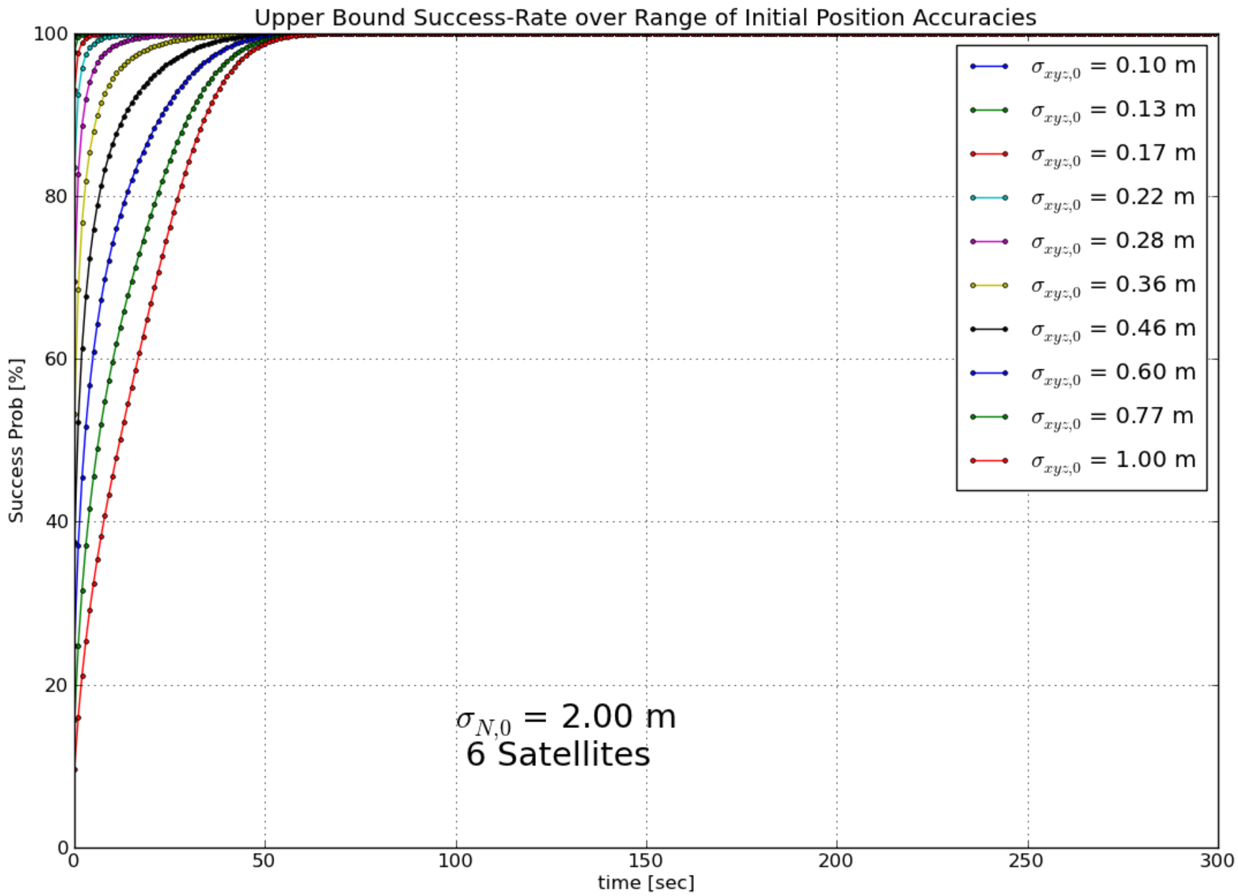
4 Satellites:



5 Satellites:



6 Satellites:



The last plot showing the upper bound success rate when using 6 satellites is dramatically improved in comparison to when 4 or 5 satellites are available. For example, starting with baseline vector accurate to 0.5 meters (1-σ), the probability of correctly fixing the integers can be as high as 90 percent after 25 seconds. In order to compute the exact success rate probability under the bootstrapping estimator, Equation 3.5 should be used. As was described in section 3.3, this exact probability gives a lower-bound on any integer least-squares estimator.