The Nucleon Electron Dipole Moment in Light-Front QCD

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Relating the electric dipole moments of leptons and baryons to fundamental CP-violating parameters is crucial to determining the nature of CP violation.

Estimates of hadronic electric dipole moments depend on the hadron’s non-perturbative structure.

For example, in the SM (CKM mechanism of CP violation), long-distance effects ($\pi$-loop) give for the neutron

$$d_{n}^{KM} \approx 10^{-32} \text{e-cm}$$

[Gavela et al., PLB 1982; Khriplovich & Zhitnitsky, PLB 1982]

whereas a LL computation in three-loops yields

$$d_{d}^{KM} \approx 10^{-34} \text{e-cm}.$$  

[Czarnecki & Krause, PRL 1997]

Here we analyze the nucleon electric dipole moment in the light-front formalism of QCD.

Evaluating $d_{n}$ and $d_{p}$ is also important to interpreting the $^{2}H$ EDM.

[Lebedev et al., PRD 2004]
We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle =$$

$$\bar{U}(P', \lambda') \left[ F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu\alpha} q_\alpha + F_3(q^2) \frac{-1}{2M} \sigma^{\mu\alpha} q_5 q_\alpha \right] U(P, \lambda)$$

We ignore the anapole form factor and define

$$\kappa = \frac{e}{2M} [F_2(0)] , \quad d = \frac{e}{M} [F_3(0)]$$

We will find a close connection between $\kappa$ and $d$, as long recognized. [Bigi, Uralstev, NPB 1991]
We work in the Drell \((q^+ = 0)\) frame:

\[
q = (q^+, q^-, q_\perp) = (0, -q^2 / P^+, q_\perp)
\]

\[
P = (P^+, P^-, P_\perp) = (P^+, M^2 / P^+, 0_\perp)
\]

\[
F_1(q^2) = \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \uparrow \right\rangle = \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \downarrow \right\rangle
\]

\[
\frac{F_2(q^2)}{2M} = \frac{1}{2} \left[ -\frac{1}{q^L} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \downarrow \right\rangle + \frac{1}{q^R} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \uparrow \right\rangle \right]
\]

\[
\frac{F_3(q^2)}{2M} = \frac{i}{2} \left[ -\frac{1}{q^L} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \downarrow \right\rangle - \frac{1}{q^R} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right\rangle P, \uparrow \right\rangle \right]
\]

\(q^{R/L} \equiv q^1 \pm iq^2\).

Both \(F_2(q^2)\) and \(F_3(q^2)\) are helicity-flip form factors.
Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$ and assumed simple vacuum imply:

$$
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \left[ -\frac{1}{q^L} \psi_{a}^{\dagger}\star(x_i, k_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_{a}^{\dagger}\star(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) \right],
$$

$$
\frac{F_3(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{i}{2} \times \left[ -\frac{1}{q^L} \psi_{a}^{\dagger}\star(x_i, k_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_{a}^{\dagger}\star(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) \right],
$$

$k'_{\perp j} = k_{\perp j} + (1 - x_j)q_\perp$

for the struck constituent $j$ and

$k'_{\perp i} = k_{\perp i} - x_iq_\perp$

for each spectator ($i \neq j$). $q^+ = 0 \implies$ only $n' = n$. 
Particles remain on their energy shell in analogy to the on-mass-shell condition of the equal-time formalism. Consider transformations on $k_\perp$; thus $|k_\perp|^2$, $k^+$, $k^-$ are unchanged.

**Parity $P_\perp$:**

A vector $d^\mu$ transforms as $d^R \rightarrow -d^L$, $d^L \rightarrow -d^R$, $d^\pm \rightarrow d^\pm$.

$$
P_\perp a^\lambda_{p^L,p^R} P_\perp^\dagger = \eta_a a^{-\lambda}_{-p^R,-p^L},
$$

$$
P_\perp b^\lambda_{p^L,p^R} P_\perp^\dagger = \eta_b b^{-\lambda}_{-p^R,-p^L},
$$

$$
-\frac{1}{q^L} \langle P + q, \uparrow | J^+(0) | P, \downarrow \rangle \xrightleftharpoons[\mathcal{P}]{\mathcal{P}} \frac{1}{q^R} \langle P + q, \downarrow | J^+(0) | P, \uparrow \rangle
$$

$$
\psi^\uparrow_a(k_\perp i, x_i, \lambda_i) \xrightleftharpoons[\mathcal{P}]{\mathcal{P}} \psi^\downarrow_a(k'_\perp i, x_i, -\lambda_i) \quad \text{with} \quad k'_\perp i = (-k^1_i, k^2_i)
$$

$F_2(q^2)$ is even and $F_3(q^2)$ is odd under $P_\perp$. 
Discrete Symmetries on the Light Front

Time Reversal $T_\perp$:

Momentum $q^\mu$ transforms as $q^R \rightarrow -q^L$, $q^L \rightarrow -q^R$, $q^\pm \rightarrow q^\pm$.
Thus $x^\mu = (x^+, x^-, x^L, x^R) \rightarrow (-x^+, -x^-, x^R, -x^L)$.

$T_\perp$ is antiunitarity but

$$T_\perp a_{\rho L, \rho R}^\lambda T_\perp^\dagger = \tilde{\eta}_a a_{\rho R, -\rho L}^\lambda ,$$
$$T_\perp b_{\rho L, \rho R}^\lambda T_\perp^\dagger = \tilde{\eta}_b b_{\rho R, -\rho L}^\lambda ,$$

$$\langle P + q, \uparrow | J^+(0) | P, \downarrow \rangle \xrightarrow{T_\perp} (\langle P + \tilde{q}, \uparrow | J^+(0) | P, \downarrow \rangle)^*$$
$$= -\langle P + q, \uparrow | J^+(0) | P, \downarrow \rangle ,$$
$$\langle P + q, \downarrow | J^+(0) | P, \uparrow \rangle \xrightarrow{T_\perp} (\langle P + \tilde{q}, \downarrow | J^+(0) | P, \uparrow \rangle)^*$$
$$= -\langle P + q, \downarrow | J^+(0) | P, \uparrow \rangle ,$$

with $\tilde{q} = (q^+, q^-, \tilde{q}_\perp)$ and $\tilde{q}_\perp = (-q^1, q^2)$.
Re($F_2$) and Im($F_3$) are even and Re($F_3$) and Im($F_2$) are odd under $T_\perp$.  

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\( \beta_a \) violates \( \mathcal{P}_\perp \) and \( \mathcal{T}_\perp \).

\[
\psi_a^{\uparrow}(x_i, k_{\perp i}, \lambda_i) = \phi_a^{\uparrow}(x_i, k_{\perp i}, \lambda_i)e^{i\beta_a/2}, \quad \psi_a^{\downarrow}(x_i, k_{\perp i}, \lambda_i) = \phi_a^{\downarrow}(x_i, k_{\perp i}, \lambda_i)e^{-i\beta_a/2}
\]

\[
\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a)\Xi_a
\]

\[
\frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a)\Xi_a,
\]

\[
\Xi_a = \int \frac{d^2k_{\perp}d x}{16\pi^3} \sum_j \frac{1}{-q^1 + iq^2} \left[ \phi_a^{\ast}(x_i, k_{\perp j}, \lambda_i) \phi_a^{\downarrow}(x_i, k_{\perp j}, \lambda_i) \right].
\]

For Fock component \( a \):

\[
[F_3(q^2)]_a = (\tan \beta_a)[F_2(q^2)]_a
\]

\[
d_a = (\tan \beta_a)2\kappa_a \quad \text{or} \quad d_a = 2\kappa_a\beta_a \quad \text{as} \quad q^2 \to 0
\]
Implications for Models of CP Violation

CP violation via a QCD $\theta$-term.
In a $q(qq)_0$ model of the nucleon

$$d^n \approx e^{\beta_n} \kappa^n (2 \cdot 10^{-14} \text{ cm}), \quad d^p \approx e^{\beta_p} \kappa^p (2 \cdot 10^{-14} \text{ cm}),$$

Since $\delta L_{CP}$ is isoscalar, $\beta^n = \beta^p$ and

$$(d^n + d^p)/(d^p - d^n) = (\kappa^n + \kappa^p)/(\kappa^p - \kappa^n) \approx -0.12/3.70 \approx -0.03.$$  
Smaller than leading-order QCD sum rule estimate. \cite{Pospelov and Ritz, PRL 1999}

In a chiral Lagrangian framework \cite{Baluni, PRD 1979; Crewther et al., PLB 1979; Pich & de Rafael, NPB 1991}

\begin{align*}
\text{(a)} & \quad \pi^- \quad n \quad p \quad n \\
\text{(b)} & \quad \pi^- \quad n \quad p \quad n
\end{align*}

$d^n$ (and $d^p$) determined from b) as $\pi$-loop is logarithmically enhanced. Here $d^n = -d^p$; no isoscalar component.
Can we estimate $d^n$?

Assume Fock state sum saturated by $uudd\bar{u}$ Fock component:

$$|\beta_a| \approx 2 \left| \frac{\bar{g}_{\pi NN}}{g_{\pi NN}} \right| \log \left( \frac{M_N}{M_\pi} \right) \approx 4 \left( \frac{0.027}{13.4} \right) |\bar{\theta}|$$

$ar{g}_{\pi NN}$ is the CP-violating $g_{\pi NN}$ coupling constant

$$d^n \sim \bar{\theta} e(3 \cdot 10^{-16}\text{cm})$$

Comparable to existing estimates.
It has long been recognized that the hadronic matrix elements in the anomalous magnetic and electric dipole moments must be related, up to CP-violating effects. [Bigi, Uralstev, 1991]

Here, however, using the light-front formalism we find a general equality, based on first principles.

- Relation holds for spin-1/2 systems, in general, not only neutron and is independent of the mechanism of CP violation.

- Both the EDM and anomalous magnetic moment should be calculated in a given model, to test for consistency.

- We argue that $d^n$ and $d^p$ (in the SM) should echo the isospin structure of the anomalous magnetic moments.
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Summary and Outlook

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