Properties of Hadrons from D4/D8-Brane System

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Refs) T. Sakai and S.S.

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① QCD & D4/D8 system

★ Construction of QCD

$$\boxed{\text{QCD = YM + quarks}}$$

- <u>Yang–Mills</u>

  fermion $\qquad S^1 \ni \tau \sim \tau + 2\pi M_{\mathrm{KK}}^{-1}$

  D4–brane on S^1 with $\lambda(x^\mu, \tau) = -\lambda(x^\mu, \tau + 2\pi M_{\mathrm{KK}}^{-1})$

  $$\boxed{\text{5 dim super YM on } S^1} \xrightarrow[E \ll M_{\mathrm{KK}}]{\cancel{\text{SUSY}}} \boxed{\text{4 dim pure YM}}$$

  Corresponding SUGRA solution is known. [Witten 1998]

- <u>quarks</u>

  Add probe D8–branes.

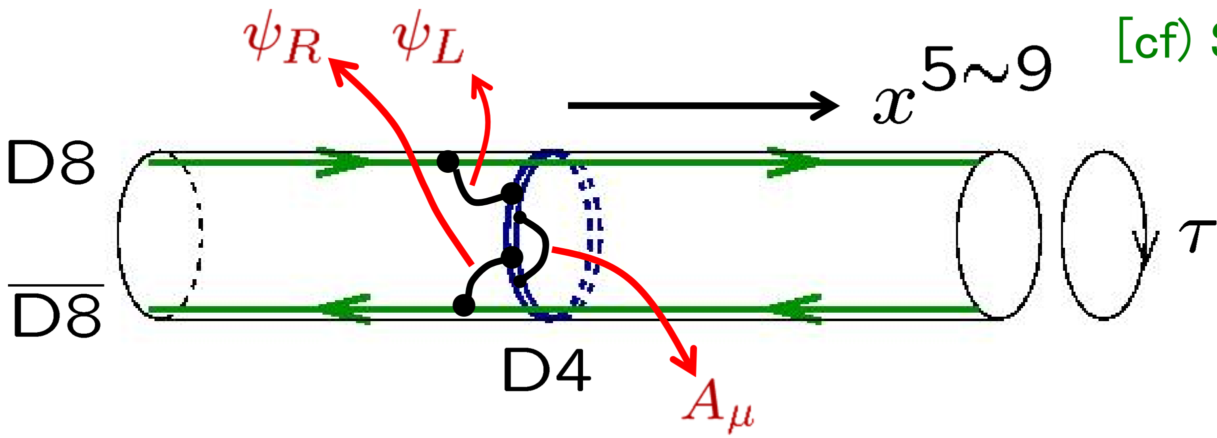
  cf) KMMW model : D4 + probe D6

  [Kruczenski–Mateos–Myers–Winter 2003]

★ the brane configuration

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	◯	◯	◯	◯	◯	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	◯	◯	◯	◯	–	◯	◯	◯	◯	◯

[cf) S.S.–K.Takahashi 2004]

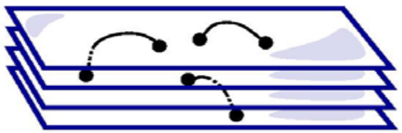


	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{\text{D8}}$ $U(N_f)_R$
A_μ	adjoint	1	1
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f

➡ 4 dim $U(N_c)$ QCD with N_f massless quarks

Outline

String theory



D4/D8 system

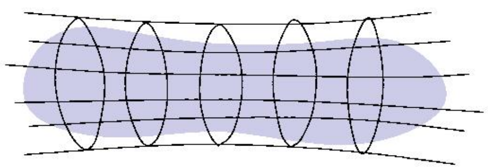
①

QCD

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\,F_{\mu\nu}^2 + i\bar{\psi}_i \not{D}\psi^i$$

②

Supergravity



5 dim YM-CS theory

③

Hadron physics

$q\bar{q}$ $\qquad$ qqq

$\pi, \rho, a_1, \cdots$ $\qquad$ $p, n, \cdots$

② Holographic description of QCD

★ Probe approximation

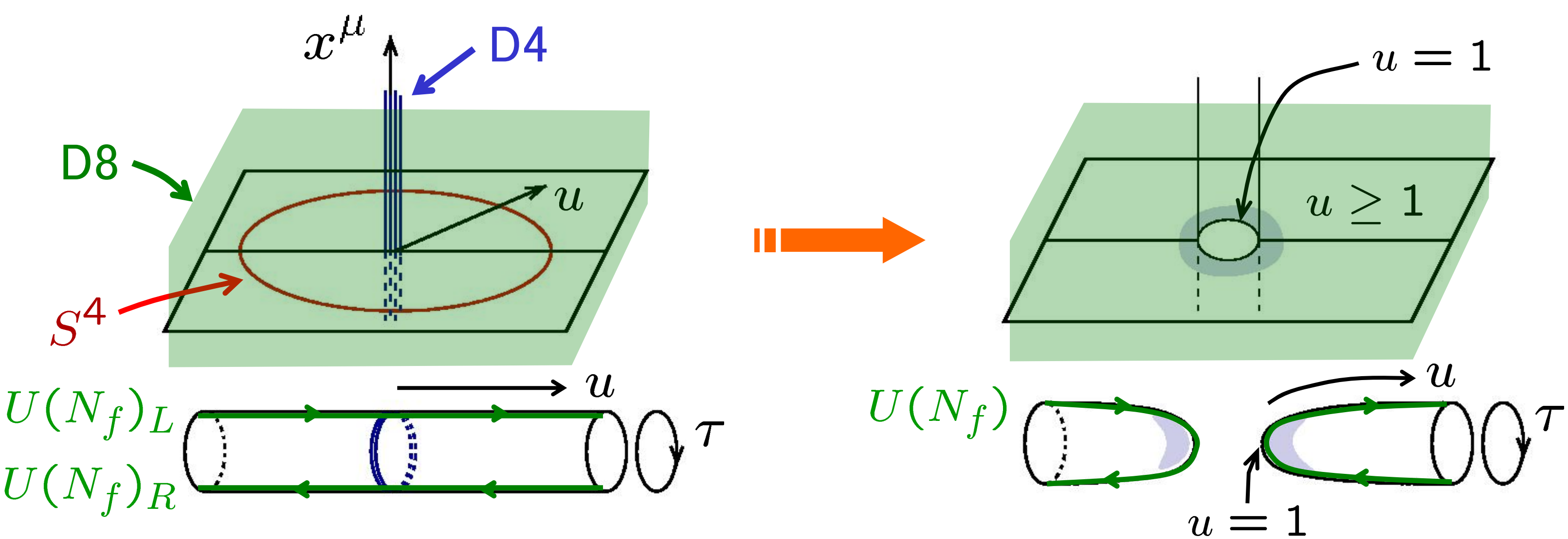
We assume $\boxed{N_c \gg N_f}$

[Karch–Katz 2002]

Only D4–brane is replaced with the SUGRA solution.
D8–$\overline{\text{D8}}$ pairs are treated as probes.

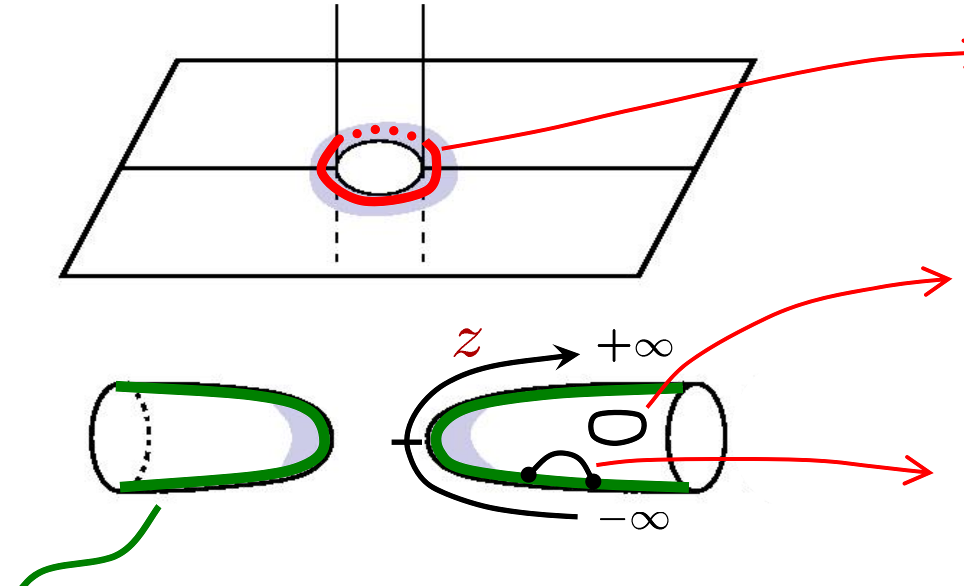
$$ds^2 = \frac{\lambda l_s^2}{3}\left[\frac{4}{9}M_{\text{KK}}^2\, u^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2) + u^{-3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)\right]$$

$f(u) = 1 - 1/u^3$



Chiral sym breaking !

★ Hadrons in the model



D4 wrapped on S^4

→ ***baryons***

closed strings

→ ***glueballs***

open strings on D8

→ ***mesons***

D8−brane : $\{(x^0, x^1, x^2, x^3, z)\} \times \cancel{S^4}$

(Here we ignore KK−modes associated with the S^4 for simplicity.)

➡ Effective theory of mesons is

reduced to $\boxed{\text{5 dim } U(N_f) \text{ gauge theory} \quad A_\mu(x^\mu, z),\ \ A_z(x^\mu, z)}$

D8-brane action

- $$S_{\rm D8}^{\rm DBI} \simeq -T\int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN}+2\pi\alpha' F_{MN})}$$
$$\sim \int d^9x\, e^{-\phi}\sqrt{-g}\,g^{MN}g^{PQ}F_{MP}F_{NQ}+\cdots$$

Inserting the SUGRA solution,

$$K(z)\equiv 1+z^2$$

$$\Rightarrow \quad \boxed{S_{\rm D8}^{\rm DBI}\sim\kappa\int d^4x dz\,{\rm Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2+M_{\rm KK}^2K(z)F_{\mu z}^2\right)}$$

$$\kappa\equiv\frac{\lambda N_c}{108\pi^3}$$

- $$S_{\rm D8}^{\rm CS} \simeq \int_9 C\wedge {\rm Tr}e^{F/2\pi} \sim \int_9 dC_3\wedge\frac{1}{3!(2\pi)^3}\omega_5(A)+\cdots$$

CS 5-form
$$d\omega_5(A)={\rm Tr}\,F^3$$

D4 charge

$$\frac{1}{2\pi}\int_{S^4}dC_3=N_c \quad \Rightarrow \quad \boxed{S_{\rm D8}^{\rm CS}\sim\frac{N_c}{24\pi^2}\int_5\omega_5(A)}$$

This 5 dim YM-CS theory is considered as the effective theory of mesons.

[cf) Son-Stephanov 2003, Erlich-Katz-Son-Stephanov 2005, Da Rold-Pomarol 2005, Hirn-Sanz 2005, ...]

★ How realistic our "QCD" is ?

Our analysis is reliable when:

- large N_c
- large $\lambda = g_{\mathrm{YM}}^2 N_c$

$\Big\}$ … Supergravity approximation
(→ difficult to make M_{KK} large)

- $E \ll M_{\mathrm{KK}}$ ← scale of 5th dimension
- $N_c \gg N_f$ … probe approximation
- $m_q = 0$

We should not be too serious in the quantitative comparison with the experiments.

But, believe it or not, the agreement is quite impressive.

③ 5 dim gauge theory

→ 4 dim meson theory

★ mode expansion

$$A_\mu(x^\mu, z) = \sum_n B_\mu^{(n)}(x^\mu)\psi_n(z)$$
$$A_z(x^\mu, z) = \sum_n \varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

Chosen to diagonalize Kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$

$$\left[\begin{array}{ll} -K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n & \phi_n(z) = \partial_z\psi_n(z) \quad (n \geq 1) \\ \kappa\int dz\, K^{-1/3}\psi_n\psi_m = \delta_{nm} & \phi_0(z) = \dfrac{c}{K(z)} \end{array}\right]$$

- Using these, we obtain

$F_{\mu\nu}^{(n)} \equiv \partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)}$

eaten

massless scalar meson

$$S_{\mathrm{D8}}^{\mathrm{DBI}} \sim \sum_{n\geq 1}\int d^4x \mathrm{Tr}\left[\underbrace{\frac{1}{2}F_{\mu\nu}^{(n)2} + \lambda_n M_{\mathrm{KK}}^2\left(B_\mu^{(n)} - \partial_\mu\varphi^{(n)}\right)^2}_{\text{massive vector meson}}\right] + \int d^4x\, \mathrm{Tr}\,\partial_\mu\varphi^{(0)2}$$
$$+ \text{(interaction terms)}$$

- We interpret $\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$\pi, \rho, a_1, \cdots$ are ***unified*** in the ***5 dim gauge field !***

★ J^{PC} and mass spectrum

- We can show

	$\varphi^{(0)}$	$B_\mu^{(\text{odd})}$	$B_\mu^{(\text{even})}$
J^{PC}	0^{-+} pseudo-scalar	1^{--} vector	1^{++} axial-vector

→ consistent with our interpretation

$$\pi \sim \varphi^{(0)}, \quad \rho \sim B^{(1)}, \quad a_1 \sim B^{(2)}, \quad \text{etc.}$$

- (axial–) vector meson mass

$$m_n^2 = \lambda_n M_{\text{KK}}^2 \qquad -K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n$$

not established

	ρ	a_1	ρ'	(a_1')	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
our model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

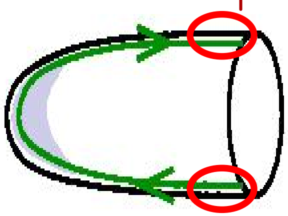
input $(M_{\text{KK}} \simeq 949 \text{ MeV})$

★ external $U(N_f)_L \times U(N_f)_R$ gauge field

- So far, we have implicitly assumed

$$A_\mu(x^\mu, z) \to 0 \ , \qquad (z \to \pm\infty)$$

Consider $\boxed{A_\mu(x^\mu, z) \to A_{\pm\mu}(x^\mu) \ , \qquad (z \to \pm\infty)}$



: interpreted as
the external $U(N_f)_L \times U(N_f)_R$ gauge fields

- This is consistent with the GKP–Witten prescription.

$$S_{\text{QCD}} \Longrightarrow S_{\text{QCD}} + \int d^4x \, A_{\pm\mu} J^\mu_\pm$$

- The ele–mag gauge field A_μ^{em} is introduced by

$$A_{+\mu} = A_{-\mu} = e\,Q\,A_\mu^{\text{em}}$$

charge matrix

$$Q = \frac{1}{3}\begin{pmatrix} 2 & \\ & -1 \end{pmatrix} \quad \text{for} \quad N_f = 2$$

★ How to obtain 4 dim effective action

The 4 dim effective action of the mesons
(including the interaction with the external gauge fields)
is obtained by inserting the mode expansion

$$\begin{aligned} A_z(x^\mu, z) &= \pi(x^\mu)\phi_0(z) \qquad \text{(higher modes are gauged away.)} \\ A_\mu(x^\mu, z) &= A_{+\mu}(x^\mu)\psi_+(z) + A_{-\mu}(x^\mu)\psi_-(z) + \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z) \end{aligned}$$

$$\psi_+(z) \to \begin{cases} 1 & z \to +\infty \\ 0 & z \to -\infty \end{cases} \quad \psi_-(z) \to \begin{cases} 0 & z \to +\infty \\ 1 & z \to -\infty \end{cases}$$ (appropriately chosen)

into the 5 dim YM–CS action and perform $\int dz$.

★ DBI part

$$S_{\rm D8}^{\rm DBI} \simeq \int d^4x \,{\rm Tr}\Big[\,\frac{1}{2e^2}\Big((F_{\mu\nu}^{+})^2 + (F_{\mu\nu}^{-})^2\Big)$$

$F_{\mu\nu}^{\pm}$: field strength of $A_{\pm\mu}$

$$e^{-2} \equiv \kappa \int dz\, K^{-1/3}\psi_{\pm}$$

$$+(\partial_\mu \pi)^2 + \left(\frac{1}{2}(\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 + \lambda_n (B_\mu^{(n)})^2\right)$$

$(M_{\rm KK} = 1 \text{ unit})$

$$-2\, f_\pi\, \partial_\mu \pi\, \mathcal{A}^\mu - 2 g_{\rho^n} B_\mu^{(2n-1)} \mathcal{V}^\mu - 2 g_{a^n} B_\mu^{(2n)} \mathcal{A}^\mu$$

$$+2\, g_{\rho^n\pi\pi}\, B_\mu^{(2n-1)} [\pi, \partial^\mu \pi] + \cdots \Big]$$

$$g_{\rho^n\pi\pi} \equiv \frac{1}{\pi}\int dz\, K^{-1}\psi_{2n-1}$$

$$\mathcal{V}_\mu \equiv \frac{1}{2}(A_{+\mu} + A_{-\mu})$$

$$\mathcal{A}_\mu \equiv \frac{1}{2}(A_{+\mu} - A_{-\mu})$$

$$g_{\rho^n} \equiv -2\,\kappa\, (K\partial_z \psi_{2n-1})\Big|_{z=+\infty}$$

$$g_{a^n} \equiv -2\,\kappa\, (K\partial_z \psi_{2n})\Big|_{z=+\infty}$$

- e^{-2} is divergent $\longleftrightarrow$ $U(N_f)_L \times U(N_f)_R$ is a global symmetry
- $A_{\pm\mu}$ couple only through the above terms.

➡ ***Vector meson dominance is naturally realized!***

★ Numerical results

$$\begin{cases} M_{\mathrm{KK}} \simeq 949 \text{ MeV} \\ \kappa \simeq 0.00745 \end{cases}$$

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{\mathrm{KK}}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{\mathrm{KK}}^2$	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{\mathrm{KK}}$	4.63 GeV	$2.8 \sim 4.2$ GeV

Most of the couplings agree with experiments within 20〜30 % error.

★ CS-term

Inserting the mode exp. into the CS–term, we obtain

$$S_{\mathrm{D8}}^{\mathrm{CS}} \simeq -\frac{N_c\, i}{4\pi^2 f_\pi^2}\int_4 \mathrm{Tr}\left[\pi\, d\rho^n d\rho^m c_{\rho^n\rho^m} + \cdots\right]$$

$$c_{\rho^n\rho^m} = \frac{1}{\pi}\int dz\, K^{-1}\psi_{2n-1}\psi_{2m-1}$$

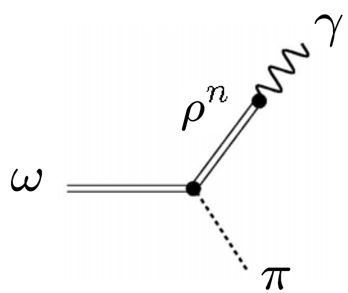
Moreover, one can show

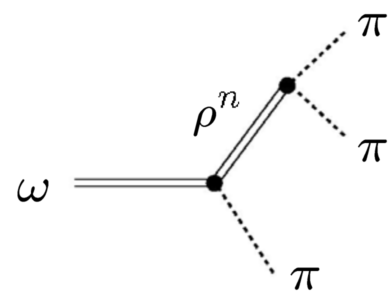
- All the terms including $A_\pm$ are cancelled.
  (complete vector meson dominance)
- Terms with more than one pion field vanish.

★ $\omega \to \pi^0 \gamma$ **and** $\omega \to \pi^0 \pi^+ \pi^-$

$$\rho_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0_\mu + \omega_\mu) & \rho^+_\mu \\ \rho^-_\mu & \frac{1}{\sqrt{2}}(-\rho^0_\mu + \omega_\mu) \end{pmatrix}$$

- The relevant diagrams for these decay are





Exactly the same as the ***GSW model*** !

[Gell-Mann –Sharp–Wagner 1962]

- Furthermore, we find

$$\Gamma(\omega \to \pi^0 \gamma) = \frac{N_c^2}{3} \frac{\alpha}{64\pi^4 f_\pi^2} \left(\sum_{n=1}^{\infty} \frac{c_{\rho^1 \rho^n} g_{\rho^n}}{m^2_{\rho^n}} \right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3} \frac{\alpha}{64\pi^4 f_\pi^2} {g_{\rho\pi\pi}}^2 |\mathbf{p}_\pi|^3$$

⟹ reproduces the proposal given by Fujiwara et al !

[Fujiwara–Kugo–Terao–Uehara–Yamawaki 1985]

★ Other topics

- Baryon

$\boxed{\text{Skyrmion}} \simeq \boxed{\text{Instanton in 5 dim}} \simeq \boxed{\text{D4 wrapped on } S^4}$

- Chiral anomaly, WZW term
- axial $U(1)_A$ anomaly
- η' meson mass

Witten–Veneziano formula $\boxed{m_{\eta'}^2 = \frac{2N_f}{f_\pi^2}\chi_g}$

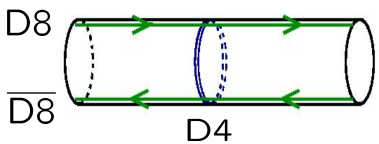
$$\left(\chi_g \equiv \frac{1}{(16\pi^2)^2}\int d^4x \langle \mathrm{Tr}F\widetilde{F}(x)\mathrm{Tr}F\widetilde{F}(0)\rangle \right)$$

Topological susceptibility

④ Summary

Summary

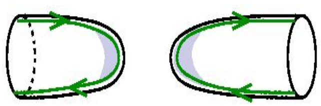
String theory



QCD

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\, F_{\mu\nu}^2 + i\bar{\psi}_i \not{D} \psi^i$$

Sugra + probe D8



$$S_{\mathrm{D8}} \sim \kappa \int d^4x dz \,\mathrm{Tr}\left(\frac{1}{2}K^{-1/3}F_{\mu\nu}^2 + KF_{\mu z}^2\right) + \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

Hadron physics

$q\bar{q}$ $\qquad$ qqq

$\pi, \rho, a_1, \cdots$ $\qquad$ $p, n, \cdots$

<u>Conclusion</u>

- Though the approximation is still very crude, our model catches various qualitative features of QCD and provides new insights in the low energy hadron physics.

- The numerical results are also encouraging.

“ ***much better than expected !*** ”