Progress and Prospects in

Twistor String Theory

Marcus Spradlin

An Invitation to Twistor String Theory

Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.

An Invitation to Twistor String Theory

Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.

In December 2003, Witten uncovered several new layers of previously hidden mathematical richness in gluon scattering amplitudes and argued that the unexpected simplicity could be understood in terms of twistor string theory.

An Invitation to Twistor String Theory

Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.

In December 2003, Witten uncovered several new layers of previously hidden mathematical richness in gluon scattering amplitudes and argued that the unexpected simplicity could be understood in terms of twistor string theory.

Today, twistor string theory has blossomed into a very diverse and active community, which boasts an impressive array of results.

An Invitation to Twistor String Theory

Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.

In December 2003, Witten uncovered several new layers of previously hidden mathematical richness in gluon scattering amplitudes and argued that the unexpected simplicity could be understood in terms of twistor string theory.

Today, twistor string theory has blossomed into a very diverse and active community, which boasts an impressive array of results.

However, most of those results have little to do with twistors, and most have little to do with string theory!

An Invitation to Twistor String Theory

Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.

In December 2003, Witten uncovered several new layers of previously hidden mathematical richness in gluon scattering amplitudes and argued that the unexpected simplicity could be understood in terms of twistor string theory.

Today, twistor string theory has blossomed into a very diverse and active community, which boasts an impressive array of results.

However, most of those results have little to do with twistors, and most have little to do with string theory!

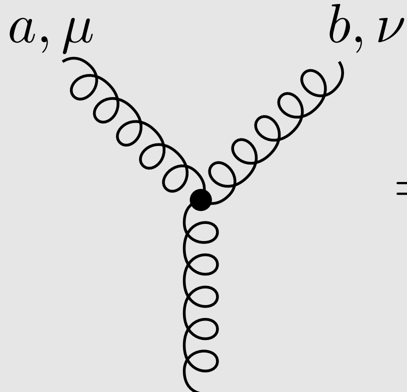
Nevertheless, the field is not yet ready for a new name because we don't really know yet where all of these developments are headed...

I. A Refresher Course

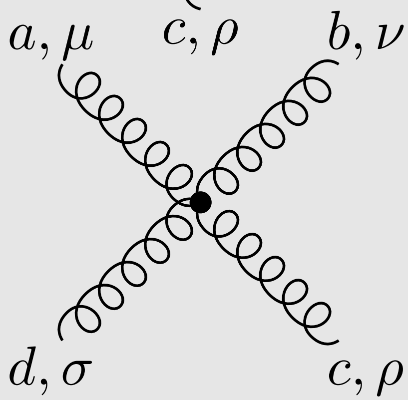
On Gluon Scattering Amplitudes

Gluon Amplitudes

We've known the rules for covariant perturbation theory for decades; they can be found in any textbook:



$$= g f^{abc} \left[g_{\mu\nu}(k-p)_\rho + g_{\nu\rho}(p-q)_\mu + g_{\rho\mu}(q-k)_\nu \right]$$



$$= -ig^2 f^{abc} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + \text{permutations}$$

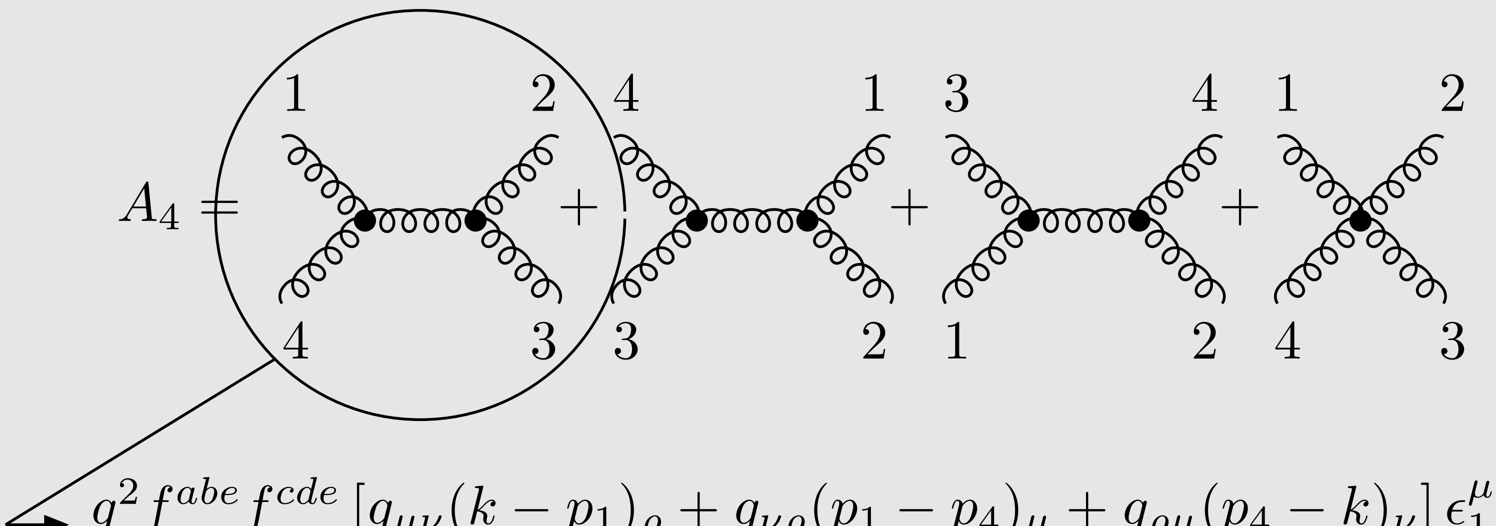
So what's the problem? To calculate any amplitude, simply write down all Feynman diagrams and sum them up!

Complications

- The number of diagrams grows faster than factorially,

$$4 \to 4, 5 \to 25, 6 \to 220, 7 \to 2485, 8 \to 34300, 9 \to 559405, \ldots$$

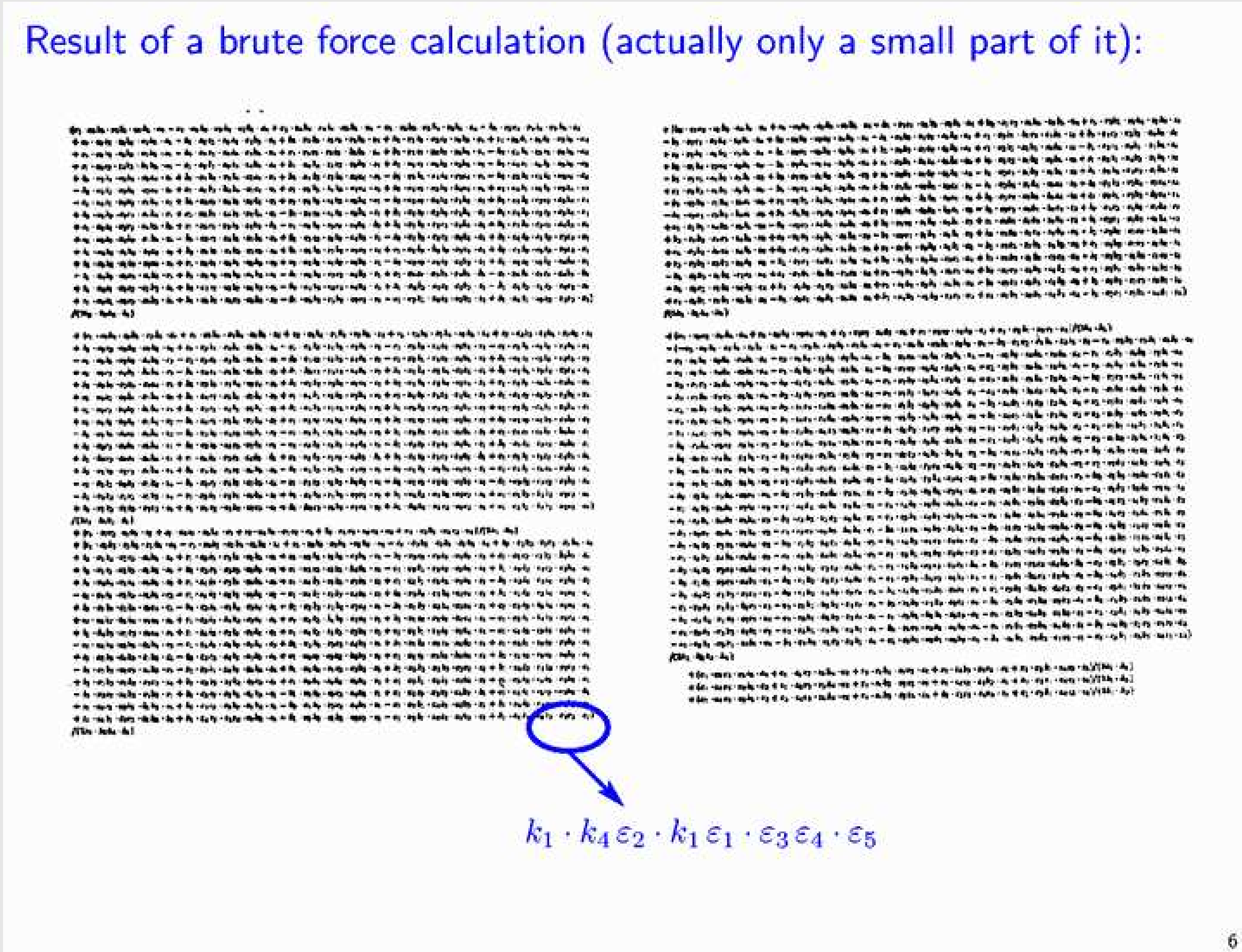
- Even a single diagram can be a complicated mess,



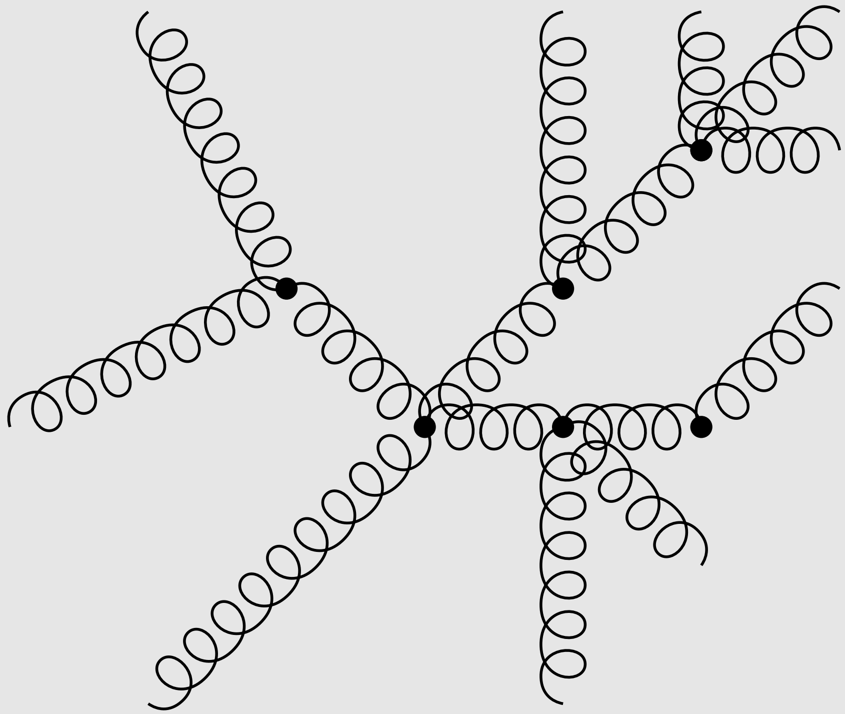
$$g^2 f^{abe} f^{cde} \left[g_{\mu\nu}(k-p_1)_\rho + g_{\nu\rho}(p_1-p_4)_\mu + g_{\rho\mu}(p_4-k)_\nu\right] \epsilon_1^\mu \epsilon_2^\lambda \epsilon_3^\sigma \epsilon_4^\nu$$
$$\times \frac{i}{k^2} \left[g_{\sigma\lambda}(k-p_2)^\rho + g_\lambda{}^\rho(p_2-p_3)_\sigma + g^\rho{}_\sigma(p_3-k)_\lambda\right],\ k = p_1 + p_4$$

Still, for $n=4$ the answer can be simplified by hand.

But already for $n = 5$ one finds a couple dozen pages of:



[Zvi Bern, KITP Colloquium]



$$= A_n(p_i^\mu, \epsilon_i^\mu)\mathrm{Tr}(T^{a_1}\cdots T^{a_n})$$

p_i = momenta

ϵ_i = polarizations

T^a = generators of the ‘color’ group

This collection of data is highly redundant, since

$$p_i \cdot p_i = 0, \qquad p_i \cdot \epsilon_i = 0 \qquad \text{for each } i.$$

There is an alternate choice of variables which magically simplifies many formulas.

Spinor Magic

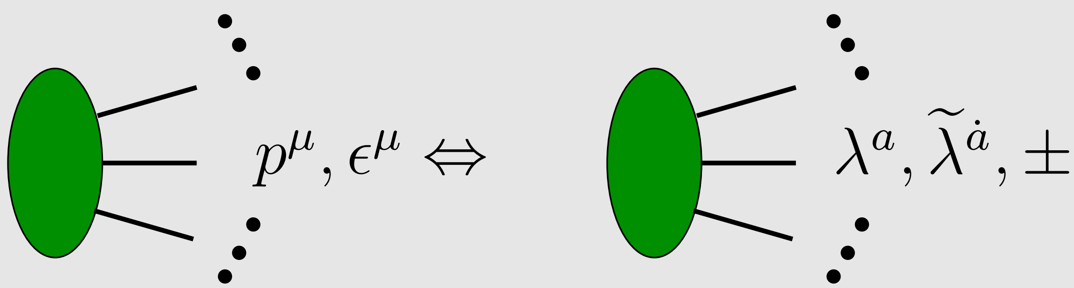
[Xu, Zhang, Chang (1984)]

Any null vector p^μ can be written as a 2×2 matrix with determinant zero, and hence can be decomposed into two commuting spinors of opposite chirality λ_a and $\widetilde{\lambda}_{\dot{a}}$.

$$p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} = \lambda_a \widetilde{\lambda}_{\dot{a}}$$

These spinors and a choice of helicity (+ or −) determines the polarization vector ϵ^μ.

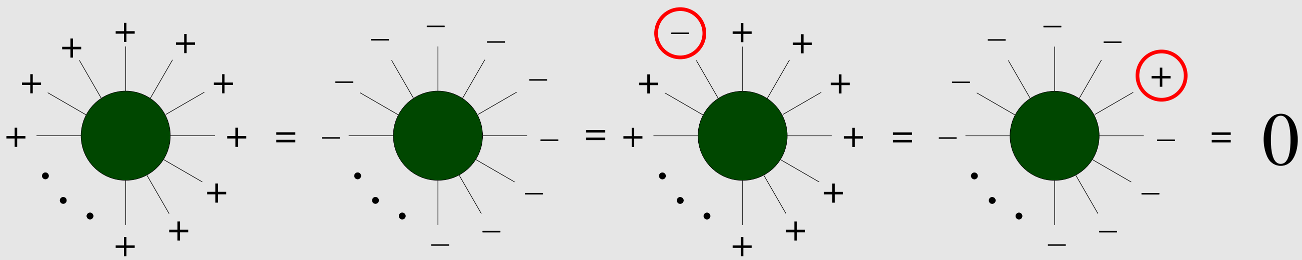
Instead of $A(p_i^\mu, \epsilon_i^\mu)$ amplitude is $A(\lambda_i^a, \widetilde{\lambda}_i^{\dot{a}}, \pm_i)$.



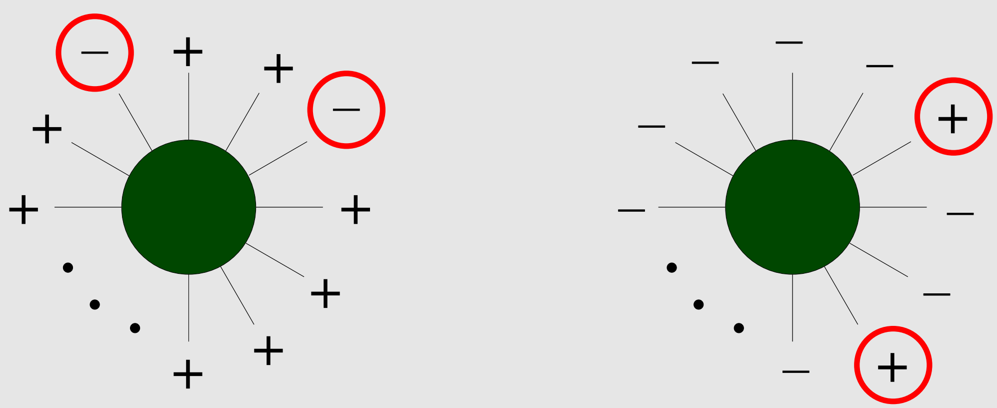
This notation allows compact expressions for gluon amplitudes.

MHV Amplitudes

Amplitudes with all, or all but one, positive (or negative) helicity gluons vanish.



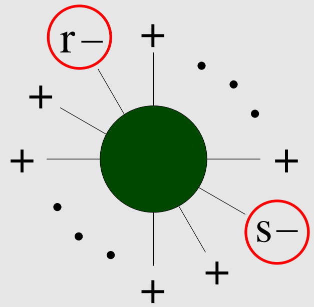
The simplest nonzero amplitudes are called maximally helicity violating (MHV).



mostly−plus MHV   mostly−minus MHV

Parke-Taylor Formula for MHV

For the MHV amplitude where gluons r and s have negative helicity, there is a very simple formula [Parke, Taylor (1986)]

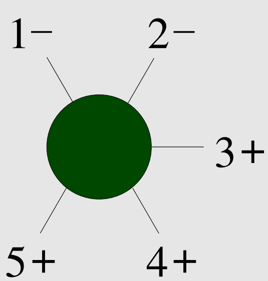


$$= ig^{n-2}\delta^4(p_1+\cdots+p_n)\left[\langle r,s\rangle^4 \prod_{i=1}^{n}\frac{1}{\langle i,i+1\rangle}\right],$$

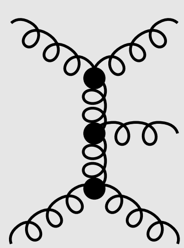
where we use the inner products

$$\langle i,j\rangle = \lambda_i^1\lambda_j^2 - \lambda_i^2\lambda_j^1, \quad [i,j] = \tilde\lambda_i^{\dot 1}\tilde\lambda_j^{\dot 2} - \tilde\lambda_i^{\dot 2}\tilde\lambda_j^{\dot 1}.$$

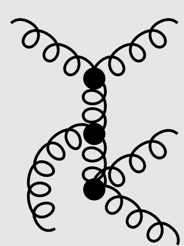
For example, the several dozen pages for $n=5$ collapses to...



$=$



$+$



$+$ many more $= \dfrac{\langle 1,2\rangle^3}{\langle 2,3\rangle\langle 3,4\rangle\langle 4,5\rangle\langle 5,1\rangle}$

Clearly, if one finds that a zillion Feynman diagrams add up to a simple expression which fits on one line, one becomes suspicious that something important might be going on...

2. Twistor String Theory

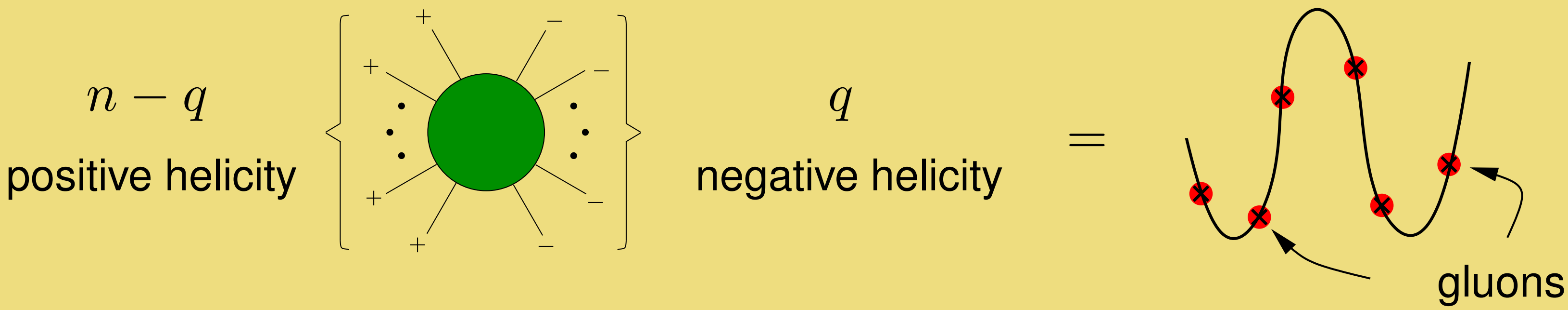
Amplitudes in Twistor Space

An amplitude can be expressed in (the naïvest version of) twistor space by "$\frac{1}{2}$-Fourier transform" with respect to $\widetilde{\lambda}$:

$$\widetilde{A}(\lambda_i, \mu_i, \pm_i) = \int d^{2n}\widetilde{\lambda}_i^a \; \exp\left[i\sum_{i=1}^{n}\mu_{\dot{a}i}\widetilde{\lambda}_i^{\dot{a}}\right] A(\lambda_i^a, \widetilde{\lambda}_i^{\dot{a}}, \pm_i).$$

Witten observed that the structure of gluon scattering amplitudes is very simple in twistor space.

Amplitudes and Curves



A tree-level n-point amplitude with q negative helicity gluons is zero unless it lies on a curve of

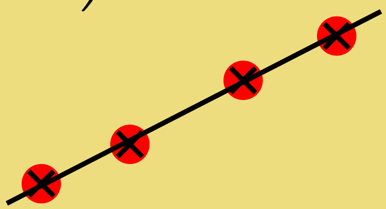
$$\text{degree} = q - 1.$$

In other words, the amplitude is nonzero only if there exists a curve in twistor space which passes through all n points specified by the external gluons! [Witten (2003)]

For example, MHV amplitudes are supported on lines $(d = q - 1 = 1)$

$$A(\lambda_i, \mu_i) = \int d^{2n}\lambda_i^{\dot a} \exp\left[i \sum_{j=1}^{n} \epsilon_{\dot a \dot b} \mu_j^{\dot a} \widetilde{\lambda}_j^{\dot b} \right] \delta^4\left(\sum_{i=1}^{4} \lambda_i^a \widetilde{\lambda}_i^{\dot a} \right) A_{\mathrm{MHV}}(\lambda)$$

$$= \int d^4x \prod_{i=1}^{n} \delta^2(\mu_{i\dot a} + x_{a\dot a}\lambda_i^{\dot a}) A_{\mathrm{MHV}}(\lambda)$$



This is zero unless μ_i and λ_i lie on the same line for each i!

For more complicated amplitudes one does not, in practice, evaluate the $\frac{1}{2}$-Fourier transform to twistor space. Rather, one probes the geometric structure of an amplitude by acting on it with certain differential operators.

These observations motivated Witten to try to construct some kind of string theory where the calculation of amplitudes would involve curves in twistor space, so that these geometric properties would be manifest.

Twistor String Theory: Ingredients

We want a theory whose spectrum is precisely that of (supersymmetric) Yang-Mills theory, without the infinite tower of massive string excitations that one usually has in string theory.

Twistor String Theory: Ingredients

We want a theory whose spectrum is precisely that of (supersymmetric) Yang-Mills theory, without the infinite tower of massive string excitations that one usually has in string theory.

This suggests that one should consider a topological string theory.

Twistor String Theory: Ingredients

We want a theory whose spectrum is precisely that of (supersymmetric) Yang-Mills theory, without the infinite tower of massive string excitations that one usually has in string theory.

This suggests that one should consider a topological string theory.

Supersymmetric twistor space, $\mathbb{CP}^{3|4}$, is actually a Calabi-Yau manifold, so it makes sense to consider the B-model on this space.

Twistor String Theory: Ingredients

We want a theory whose spectrum is precisely that of (supersymmetric) Yang-Mills theory, without the infinite tower of massive string excitations that one usually has in string theory.

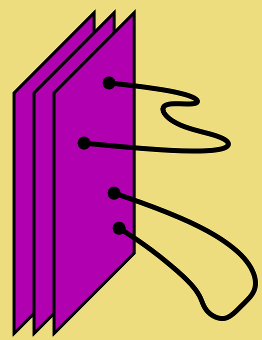
This suggests that one should consider a topological string theory.

Supersymmetric twistor space, $\mathbb{CP}^{3|4}$, is actually a Calabi-Yau manifold, so it makes sense to consider the B-model on this space.

Indeed the spectrum of open strings in this theory precisely corresponds to the field content of $\mathcal{N} = 4$ super-Yang-Mills theory.

Twistor String Theory: Ingredients

1. Open strings in the topological B-model on supertwistor space. These are the gluons in twistor string theory.



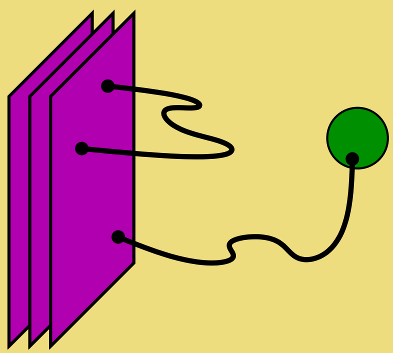
However, these ‘gluons’ are essentially free—their interactions constitute only self-dual Yang-Mills theory.

So we need additional ingredients which contribute to the effective action for the gluons, completing it to the full Yang-Mills theory.

Twistor String Theory: Ingredients

1. Open strings in the topological B-model on supertwistor space. These are the gluons in twistor string theory.

2. 'Instantonic' D-branes, which can wrap any curve inside supertwistor space. They are associated with new degrees of freedom. Integrating out these degrees of freedom produces an effective action for the gluons which, it turns out, is exactly $\mathcal{N} = 4$ Yang-Mills theory.

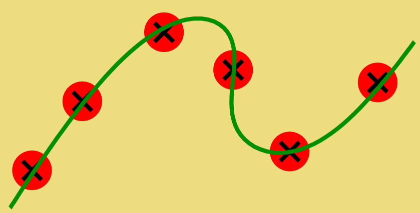


How do we know this?

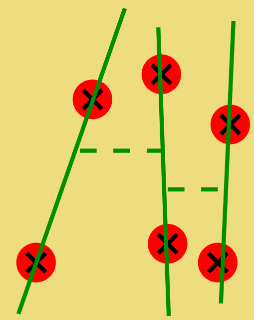
Twistor String Theory: Recipes

There actually exist two very distinct recipes for calculating the effective action in Witten's twistor string theory, depending on what kinds of curves one considers.

Specifically, one can consider either connected or disconnected curves,



cubic (degree 3) curve

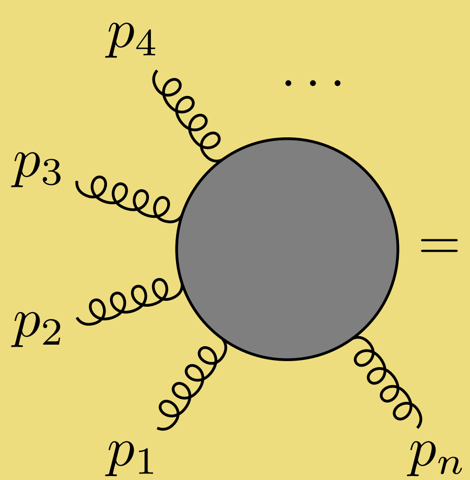


three disconnected lines

Calculations based on both kinds of curves separately reproduce the complete tree-level gluon S-matrix, as discussed in the talks by Roiban and Svrček respectively.

The Connected Prescription

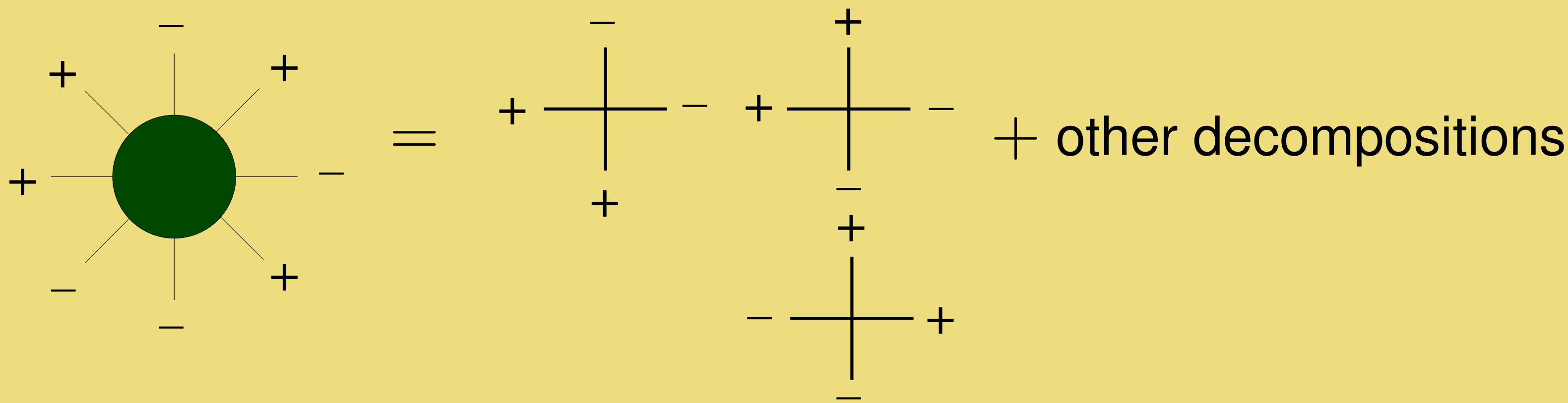
The former leads to a mysterious formula, derived from string theory, which recasts the problem of calculating *any* tree-level n-gluon scattering amplitude into the problem of solving some polynomial equations. [Roiban, M.S., Volovich (03/04)].



$$= i(2\pi)^4 \delta^4\left(\sum p_i\right) \sum_{x_j : f_i(x_j, p) = 0} \frac{1}{\det(\partial f_i / \partial x_j)}$$

The Disconnected Prescription

The disconnected prescription leads to a more computationally useful formula which expresses an arbitrary tree-level amplitude in terms of all possible decompositions into MHV subamplitudes (which must be continued off-shell in a suitable way).



[Cachazo, Svřcek, Witten (03/05)].

3. Applications of Twistor-Inspired Methods

Broad Goals of this Research Program

Explore the hidden mathematical structure in perturbative gauge theory, and

Exploit that structure to help make previously impossible calculations possible (in some cases, not just possible but trivial).

Generally, we begin with supersymmetric gauge theories, where the structure is simplest and new ideas are easiest to explore. Most of the techniques can be applied (see parallel talks), with some effort, to other theories, including honest QCD.

At tree-level there is no distinction: **tree-level gluon amplitudes in QCD are secretly supersymmetric**.

Tree Level

Why, in the 21st century, do we still find it useful to study tree amplitudes?

- Even just two years ago, **few** useful closed form expressions **were known**.
- **Compact explicit formulas** are better than having an algorithm can only be implemented numerically.
- Tree-level amplitudes form the basic building blocks of loop amplitudes through **unitarity** ,

$$\mathrm{Im}\, A^{1-\mathrm{loop}} \sim \sum \int A^{\mathrm{tree}} A^{\mathrm{tree}}.$$

  (and, more importantly, **generalized unitarity**).

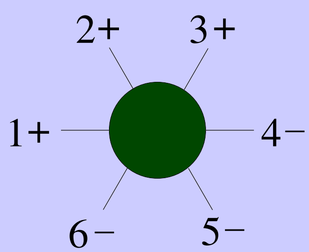
- A better understanding of the **mathematical structure** of tree-level amplitudes will guide us as we attack more complicated loop amplitudes.

Examples of Compact Forumlas

Consider the six-particle amplitude $A(1^+, 2^+, 3^+, 4^-, 5^-, 6^-)$, originally calculated by summing 220 Feynman diagrams.

[Berends & Giele (1987)], [Mangano, Parke, Xu (1988)],

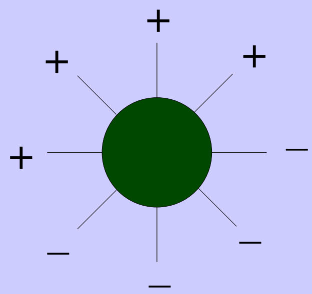
Today we know a very simple formula for this amplitude,



$$= \frac{\langle 1|2+3|4]^3}{(p_2+p_3+p_4)^2 [2\,3][3\,4]\langle 5\,6\rangle\langle 6\,1\rangle[2|3+4|5\rangle} + \frac{[6|1+2|3\rangle^3}{(p_6+p_1+p_2)^2[2\,1][1\,6]\langle 5\,4\rangle\langle 4\,3\rangle[2|1+6|5\rangle}.$$

From [Roiban, M.S., Volovich (12/04)], based on [Bern, Del Duca, Dixon, Kosower (10/04)].

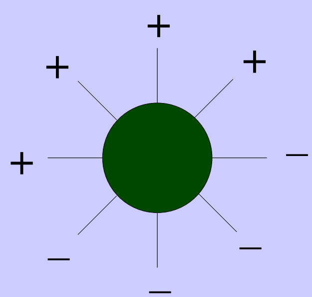
The eight-particle amplitude $A(1^-,2^-,3^-,4^-,5^+,6^+,7^+,8^+)$ would require 34,300 Feynman diagrams (probably never seriously attempted), or 44 MHV diagrams:



$$= \frac{[\eta\, 8]^3}{[8\,1][1\,2][2\,3][3\,\eta]} \frac{1}{(p_8+p_1+p_2+p_3)^2} \frac{\langle \eta\, 4\rangle^3}{\langle 4\,5\rangle\langle 5\,6\rangle\langle 6\,7\rangle\langle 7\,\eta\rangle}$$

$$+\, 43 \text{ similar terms}$$

Also in this case there is a simpler formula



$$= \frac{[5|4+3+2|1\rangle^3}{(p_2+p_3+p_4+p_5)^2[2\,3][3\,4][4\,5]\langle 6\,7\rangle\langle 7\,8\rangle\langle 8\,1\rangle[2|3+4+5|6\rangle}$$

$$+\, 5 \text{ similar terms}$$

[Roiban, M.S., Volovich (12/04)].

On-Shell Recursion

Where do these simple formulas come from? Their discoveries were ‘accidents’, but in hindsight we can observe that these compact formulas all seem to come out naturally from the on-shell recursion

$$A_n = \sum_{r=2}^{n-2} A_{r+1} \frac{1}{p_r^2} A_{n+1-r} \tag{1}$$

[Britto, Cachazo, Feng (12/04) & with Witten (01/05)].

The recursion relations admit closed form, analytic solutions for ‘split helicity’ amplitudes [Britto, Feng, Roiban, M.S., Volovich (03/05)].

$\Longrightarrow$ Amplitudes which were previously impossible to compute, or could only be evaluated numerically, can now be written down in closed form with no effort.

Tree Level Summary

As promised, the tree-level techniques have been widely applied:

MHV rules:

- for gluons with fermions and scalars [Georgiou, Khoze 04/04], [Wu, Zhu 06/04],
- for amplitudes with quarks [Georgiou, Glover, Khoze 07/04], [Su, Wu 07/04],
- for Higgs plus partons [Dixon, Glover, Khoze 11/04], [Badger, Glover, Khoze 12/04],
- and for electroweak vector boson currents [Bern, Forde, Kosower, Mastrolia 12/04].

On-shell recursion relations:

- for amplitudes with gluons and fermions [Luo, Wen 01/05, 02/05],
- and for massive particles [Badger, Glover, Khoze, Svrček 04/05],
- and for graviton amplitudes [Bedford, Brandhuber, Spence, Travaglini 02/05], [Cachazo, Svrček 02/05].

One Loop

Let me, very briefly, review some of the progress that has been accomplished by Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder with important contributions also from several people whose names don't begin with B, including but not limited to Cachazo, Dixon, Dunbar, Feng Forde, Perkins, Quigley, Rozali, Spence, Travaglini.

We follow the strategy I advertised at the beginning of this section:

One Loop

1. Begin with maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills theory, where the structure is simplest.

- The tree-level MHV rules of Cachazo, Svřcek and Witten can be sewn together to evaluate one-loop amplitudes [Brandhuber, Spence, Travaglini (2004–2006)]. This is like having a 'disconnected prescription' at one loop, so in some sense this is the closest we have to a 'twistor string' construction which works at one loop.
- Use generalized unitarity (in twistor space—this is essential; more on this later) to determine any one-loop amplitude in $\mathcal{N} = 4$ [Britto, Cachazo, Feng (2004)].

One Loop

1. Begin with maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills theory, where the structure is simplest.

2. Extend the results, with some effort, to less symmetric cases such as $\mathcal{N} = 1$ Yang-Mills.

- All $\mathcal{N} = 1$ MHV amplitudes from MHV diagrams. [Quigley, Rozali; Bedford, Brandhuber, Spence, Travaglini (10/04)].
- All $\mathcal{N} = 1$ NMHV amplitudes from quadruple cuts [Bidder, Bjerrum-Bohr, Dunbar, Perkins (02/05)]
- A new basis of boxes and triangles allowing for all $\mathcal{N} = 1$ amplitudes to be computed from generalized unitarity [Britto, Buchbinder, Cachazo, Feng (03/05)]

One Loop

1. Begin with maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills theory, where the structure is simplest.

2. Extend the results, with some effort, to less symmetric cases such as $\mathcal{N} = 1$ Yang-Mills.

3. Finally, take it all the way to QCD.

The on-shell bootstrap of Bern, Dixon, Kosower takes advantage of 'generalized analyticity' (more on this later) and has been succesully used to derive analytic formulas for several new one-loop multi-parton amplitudes in QCD, including several all-multiplicity formulas for particular helicity configurations [$\{$Berger, Bern, Dixon, Forde, Kosower$\}$ and some subsets thereof].

4. Prospects

Does Twistor String Theory Still Need ‘Twistor’?

These developments seem to have become rather disconnected from twistor theory. So, it is natural to ask, who needs twistors?

Does Twistor String Theory Still Need 'Twistor'?

In Minkowski signature, the positive- and negative-chirality spinors λ and $\widetilde{\lambda}$ appearing in the spinor helicity decomposition $p = \lambda\widetilde{\lambda}$ are related by complex conjugation $\overline{\lambda} = \widetilde{\lambda}$ for real momentum p.

Does Twistor String Theory Still Need 'Twistor'?

In Minkowski signature, the positive- and negative-chirality spinors λ and $\widetilde{\lambda}$ appearing in the spinor helicity decomposition $p = \lambda\widetilde{\lambda}$ are related by complex conjugation $\overline{\lambda} = \widetilde{\lambda}$ for real momentum p.

Of course, we've long known that it is very profitable to relax the requirement that p should be real, and to study the analytic structure of amplitudes as functions of complex momenta.

Does Twistor String Theory Still Need 'Twistor'?

In Minkowski signature, the positive- and negative-chirality spinors λ and $\widetilde{\lambda}$ appearing in the spinor helicity decomposition $p = \lambda\widetilde{\lambda}$ are related by complex conjugation $\overline{\lambda} = \widetilde{\lambda}$ for real momentum p.

Of course, we've long known that it is very profitable to relax the requirement that p should be real, and to study the analytic structure of amplitudes as functions of complex momenta.

Twistor theory naturally suggests a richer notion of complexification—namely, that λ and $\widetilde{\lambda}$ should be considered as independent complex variables.

Does Twistor String Theory Still Need 'Twistor'?

In Minkowski signature, the positive- and negative-chirality spinors λ and $\widetilde{\lambda}$ appearing in the spinor helicity decomposition $p = \lambda\widetilde{\lambda}$ are related by complex conjugation $\overline{\lambda} = \widetilde{\lambda}$ for real momentum p.

Of course, we've long known that it is very profitable to relax the requirement that p should be real, and to study the analytic structure of amplitudes as functions of complex momenta.

Twistor theory naturally suggests a richer notion of complexification—namely, that λ and $\widetilde{\lambda}$ should be considered as independent complex variables.

Many recent developments have been made possible by properly appreciating the implications of this generalized analytic structure. This has also been emphasized, and spectacularly applied, in the talks of Berger and Forde.

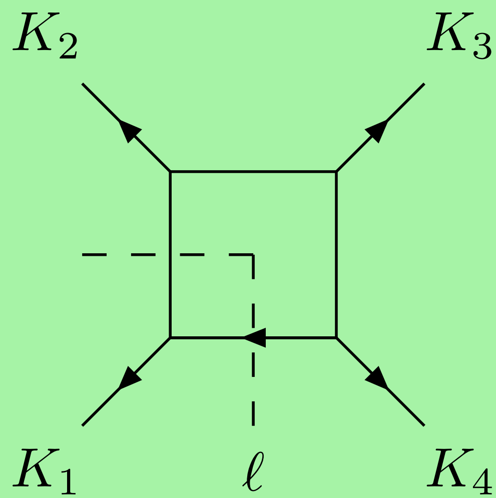
1. The ‘connected instanton prescription’ (see Roiban’s talk) fails if one tries to impose the naive reality condition.

2. The derivation and application of on-shell recursion relations at tree-level and one-loop relies crucially on the ability to shift λ and $\widetilde{\lambda}$ independently.

3. Generalized unitarity: It is often very fruitful to study the poles and branch cuts of loop amplitudes. However, many singularities which vanish accidentally for any complex p, and appear only when one allows for independent complex λ and $\widetilde{\lambda}$.

In particular, the three-gluon amplitude vanishes on-shell for complex p, but it is nonzero for generic λ and $\widetilde{\lambda}$.

The following double-cut would vanish when K_1 is a single-gluon state:



Therefore, we would get no information about the coefficient of the pole

$$\frac{1}{\ell^2}\frac{1}{(\ell - K_1)^2}$$

in the amplitude if we restricted our attention to complex p.

Does Twistor String Theory Still Need 'Twistor'?

In summary, consideration of generalized analyticity, where one allows λ and $\widetilde{\lambda}$ to be independent complex variables, exposes a much richer analytic structure than ordinary, complex-p analyticity.

The full implications of this have yet to be appreciated.

Does Twistor String Theory Still Need ‘String’?

Recent developments seem to have become rather disconnected from string theory. So, it is natural to ask, who needs strings?

Does Twistor String Theory Still Need 'String'?

Recent developments seem to have become rather disconnected from string theory. So, it is natural to ask, who needs strings?

This is actually less clear. So far there has been no one-loop string theory calculation which might shed some light on these developments—Witten's twistor string theory apparently differs from $\mathcal{N} = 4$ super-Yang-Mills theory at loop level.

Does Twistor String Theory Still Need 'String'?

Recent developments seem to have become rather disconnected from string theory. So, it is natural to ask, who needs strings?

This is actually less clear. So far there has been no one-loop string theory calculation which might shed some light on these developments—Witten's twistor string theory apparently differs from $\mathcal{N} = 4$ super-Yang-Mills theory at loop level.

Moreover there are other string theories one can construct which reproduce the same tree-level amplitudes but apparently differ at loop level. [Berkovits, Witten (2004)].

Does Twistor String Theory Still Need ‘String’?

Recent developments seem to have become rather disconnected from string theory. So, it is natural to ask, who needs strings?

This is actually less clear. So far there has been no one-loop string theory calculation which might shed some light on these developments—Witten’s twistor string theory apparently differs from $\mathcal{N} = 4$ super-Yang-Mills theory at loop level.

Moreover there are other string theories one can construct which reproduce the same tree-level amplitudes but apparently differ at loop level. [Berkovits, Witten (2004)].

Finally, there are several different ways to understand MHV diagrams and the on-shell recursion relation purely within the context of Lagrangian field theory, including [Gorsky; Mansfield; Mason, Skinner; Vaman, Yao].

Does Twistor String Theory Still Need 'String'?

In summary, at this point it is not at all clear that string theory is the unifying framework behind these developments.

It is clear that something is going on, but we're far from writing the final chapter in this book.

Summary

- Formulas for scattering amplitudes in gauge theory exhibit simplicity that is completely obscure in the underlying Feynman diagrams.
- Some of this simplicity can be made manifest by thinking about the structure of amplitudes expressed in twistor space, and can be explained (at least at tree level) in terms of a corresponding twistor string theory.
- New insights into the structure of amplitudes (in particular, generalized analyticity) have led to great progress in our ability to calculate amplitudes which were previously out of reach.
- Prospects are great for continued progress, both in supersymmetric gauge theories as well as QCD.