Spin liquids on ladders and in 2d

MPA Fisher (with O. Motrunich)

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Interest: Quantum Spin liquid phases of 2d Mott insulators

Background: *Three classes of 2d Spin liquids*

a) Topological
b) Critical
c) “Quantum spin metals” - with singular “Bose” surfaces
   Example: d-wave Bose liquid (DBL)

Results:

• A new class of spin liquid phases on n-leg ladders,
  each a descendent of a 2d “Quantum spin metal”

• Existence of 2-leg ladder descendent of the DBL phase,
  (established using DMRG, ED, VMC, gauge theory)
Mott Insulators:

Insulating materials with an odd number of electrons/unit cell

Hubbard model with one electron per site on average:

\[
\mathcal{H} = -t \sum_{\langle ij \rangle} [c_{i\alpha}^\dagger c_{j\alpha} + h.c.] + U \sum_i n_{i\uparrow} n_{i\downarrow}
\]

For \( U \gg t \) electron gets self-localized

Residual spin physics:

\( s=1/2 \) operators on each site

Heisenberg Hamiltonian:

\[
H_{\text{spin}} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j
\]
Symmetry Breaking

Mott Insulator $\rightarrow$ Unit cell doubling ("Band Insulator")

Symmetry breaking

- Magnetic Long Ranged Order
  
  Ex: 2d square Lattice AFM

- Spin Peierls
  
  Valence Bond

\[ \begin{aligned}
\text{Valence Bond} &= \left| \uparrow \downarrow \rightangle - \left| \downarrow \uparrow \rightangle \\
\end{aligned} \]
Spin Liquid: Holy Grail

Theorem: Mott insulators with one electron/cell have low energy excitations above the ground state with \((E_1 - E_0) < \ln(L)/L\) for system on an L by L torus (Matt Hastings, 2005)

Remarkable implication - Exotic Quantum Ground States are guaranteed in a Mott insulator with no broken symmetries

Such quantum disordered ground states of a Mott insulator are generally referred to as “quantum spin liquids”
Spin Liquids: 3 Classes

1) Topological Spin Liquids

2) Critical (“algebraic”) Spin Liquids

3) “Quantum Spin Metals”
Topological Spin Liquids

- Ground state degeneracies on a torus
- Gapped “particle” excitations with fractional quantum numbers and long-ranged “statistical interactions”
- Fractional Statistics: Abelian and non-Abelian
- Simplest state is short-ranged RVB, $Z_2$ Gauge structure

Hamiltonians Exhibiting Topological Spin Liquids

- Exactly soluble Lattice gauge theories (loop models) - highly contrived, not built with spin operators
- Triangular lattice Dimer model at RK point (not a spin model per say)
- Kagome $J_1=J_2=J_3$ spin model in easy-axis limit (not SU(2) invariant)

No physically reasonable spin model (yet) shown to have a Topological spin liquid ground state
Critical Spin Liquids

- **Stable gapless phase** with no broken symmetries
- no free particle description
- Valence bonds on many length scales
- Power-law correlations at finite set of discrete momenta, $K_j, \ j=1,2,...N$

\[
G(r) \sim \sum_{j=1}^{N} \frac{\cos(K_j \cdot r + \phi_\alpha)}{|r|^{\eta_j}}
\]

Effective field theory is often “relativistic”:

1) Staggered flux phase of fermionic spinons; Compact QED3 with SU(4) Symmetry
2) Fermionized vortices - non-compact QED3 with flavor SU(N), N=4,6,8

\[
\mathcal{L}_{QED3} = \bar{\psi}_a \gamma^\mu (\partial_\mu - i\bar{\alpha}_\mu) \psi_a
\]

No physically reasonable spin model shown to have a Critical spin liquid ground state
“Quantum spin metals”

Gapless phase, no broken symmetries, no free particle description, valence bonds on all scales (as in the critical spin liquids)

Correlation functions singular along surfaces in momentum space

Effective field theory is not “relativistic”:
(eg. uniform RVB state with Fermi surface of spinons coupled to a compact U(1) gauge field)
Stability of such phases? Field theory is uncontrolled

No spin Hamiltonians known to have a “Quantum spin metal” ground state

Arguably the most complicated (intractable?) class of spin liquid
Experimental Candidates?

1) Topological Spin Liquids: no candidates

2) Critical Spin Liquids: Kagome lattice compounds

- Iron Jarosite, $KFe_3(OH)_6(SO_4)_2$: $Fe^{3+} \ s=5/2, \ f = T_{cw}/T_N \sim 20$
- 2d “spinels” $SrCr_8Ga_4O_{19}$: $Cr^{3+} \ s=3/2, \ f \sim 100$
- Volborthite $Cu_3V_2O_7(OH)_2\ 2H_2O$: $Cu^{2+} \ s=1/2, \ f \sim 75$
- Herbertsmithite $ZnCu_3(OH)_6Cl_2$: $Cu^{2+} \ s=1/2, \ f > 600$

Key elements? Frustration, low spin, low coordination number

3) “Quantum Spin Metals”

- Triangular lattice Organic: $\kappa-(ET)_2Cu_2(CN)_3$: $s=1/2, f \sim 10^4$

Key elements? Mott insulator with small charge gap, large ring exchange

$$\mathcal{H} = J \sum_{<i,j>} S_i \cdot S_j + K \sum_{\text{rings}} (\mathcal{P}_4 + \mathcal{P}_4^{-1})$$  \quad J \sim K
Quantum Spin Metals: Ladders to the Rescue

n-leg Ladder:

Neel or Critical Spin liquid

“Quantum Spin Metal”

Few gapless 1d modes

Fingerprint of 2d singular surface - many gapless 1d modes, of order N

Expectation: New quasi-1d spin liquid phases on n-leg ladders, each a descendent of a 2d quantum spin metal
2d D-Wave Bose Liquid (DBL)  
(a putative “quantum spin metal”)  

O. Motrunich/MPAF

Hamiltonian:
- \( s=1/2 \) square lattice Heisenberg with 4-site ring exchange,  
- Zeeman field in easy-plane limit (maybe unnecessary)

\[
\mathcal{H}_{JK} = J \sum_{\langle ij \rangle} (S^+_i S^-_j + h.c.) + K \sum_{\text{rings}} (S^+_1 S^-_2 S^+_3 S^-_4 + h.c.) + h \sum_i S^z_i
\]

Phase diagram: \( K/J \) and \( <S^z> \)

- XY AFM (superfluid)  
- \( (K/J)_c \)  
- DBL ??  
- J-K Model has a sign problem - completely intractable
Theory of DBL phase

Slave Fermions: Mean field state with anisotropic Fermi surfaces for $d_1$ and $d_2$

$$S^+ = d_1^+ d_2^+$$

Gauge Theory: Fermions coupled to $U(1)$ gauge field

Gutzwiller wavefunction: product of Fermion determinants, elongated $x$ or $y$ direction

$$\Psi_{D_{xy}} (r_1, ..., r_N) = (det)_x \times (det)_y$$

d$_{xy}$ nodal structure in wavefunction
Singular “Bose” Surfaces in DBL

Boson Momentum distribution function:

\[ n_b(k) = \int G_b(r) e^{i \mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \]

Two singular lines in momentum space, Bose surfaces:

\[ \mathbf{k}_{F_1}(\mathbf{r}) \pm \mathbf{k}_{F_2}(\mathbf{r}) \]
DBL on the 2-Leg Ladder

Studied $H_{JK}$ on 2-leg ladder

- Exact Diagonalization (2 x 18)
- Variational Monte Carlo
- DMRG (2 x 50)
- quasi-1d gauge theory via bosonization

(E. Gull, D. Sheng, S. Trebst, O. Motrunich and MPAF)

Correlation Functions:

1) Momentum Distribution function
   \[ n(k_x, k_y) = \langle b_k^\dagger b_k \rangle; \quad k_y = 0, \pi \]

2) Density-density structure factor
   \[ D(k) = \sum_r e^{i k \cdot r} \langle n_r n_0 \rangle \quad n_r = b_r^\dagger b_r \]

3) Pair-boson correlator
   \[ P(r_1, r_2; r_1', r_2') = \langle b_{r_1}^\dagger b_{r_2}^\dagger b_{r_1'} b_{r_2'} \rangle \]
Phase Diagrams

Four Phases:

1) XY AFM (Superfluid) - Bose “condensate”
2) D-Wave Bose Liquid - DBL
3) s-wave Pair Boson “condensate”
4) d-wave Pair Boson “condensate”
Superfluid

DMRG 2 x 48

- Boson correlator $n(q_x,q_y)$
- Density-density structure factor $D(q_x,q_y)$
- Pair correlation $P(x)$

Graphs showing the behavior of different parameters and correlations in a superfluid system.
Gauge fluctuations: Ampere’s Law

Gauge mean field theory predicts singularities in \( n(q) \) at:

\[
\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}
\]

Both DMRG and \( \text{det}_1 \times \text{det}_2 \) Wavefunction show singular cusps only at:

\[
\mathbf{k}_{F_1} - \mathbf{k}_{F_2}
\]

Why? Ampere’s Law - Parallel currents attract

d_1 and d_2 Fermions have opposite gauge charge, so right moving d_1 attracts left moving d_2 to form boson at momentum:

\[
\mathbf{k}_{F_1} - \mathbf{k}_{F_2}
\]
Summary & Outlook

• Three types of 2d spin liquids: Topological, critical and “quantum spin metal”
• “Quantum spin metal” phases with singular “Bose” surfaces have ladder descendents
• D-wave Bose liquid (DBL) predicted for 2d XY J-K ring Hamiltonian
• Established existence of the 2-leg ladder descendent of the DBL
  (combination of DMRG, ED, VMC, gauge theory plus bosonization)

Future generalizations (DMRG, VMC, gauge theory):

• n-leg ladders approaching 2d DBL
• Triangular strips: Heisenberg+ring/Hubbard model
  Quasi-1d descendents of “spinon fermi surface” phase?
• “D-Wave Metal” on the n-leg ladder?
S-wave Pair Boson “Condensate”

DMRG 2 x 48

\[ \rho = \frac{1}{3} \]

\[ \frac{K}{J} = 1.4 \]

\[ \frac{J_{\perp}}{J} = 0.1 \]
d-wave Pair Boson “Condensate”

DMRG 2 x 48

\[ \rho = \frac{1}{9} \]

\[ \frac{K}{J} = 3.0 \]

\[ \frac{J_{\perp}}{J} = 0.1 \]
D-Wave Metal

Itinerant non-Fermi liquid phase of 2d electrons

\[ c^\dagger_\alpha (\mathbf{r}) = b^\dagger (\mathbf{r}) f^\dagger_\alpha (\mathbf{r}) = d^\dagger_x (\mathbf{r}) d^\dagger_y (\mathbf{r}) f^\dagger_\alpha (\mathbf{r}) \]

Wavefunction:

\[ \Psi_{\text{electron}}(\uparrow, \downarrow) = [(\text{det})_x \times (\text{det})_y](\uparrow, \downarrow) \det(\uparrow) \det(\downarrow) \]

\( t \)-K Ring Hamiltonian (from strong coupling gauge theory)
(no double occupancy constraint)

\[ \mathcal{H}_t = -t \sum_{\langle ij \rangle} c^\dagger_{i\alpha} c_{j\alpha} + h.c. \]

\[ \mathcal{H}_K = K \sum_{\text{plaquettes}} [S^\dagger_{13} S^{}_{24} + h.c.] \]

\[ S^\dagger_{ij} = \frac{1}{\sqrt{2}} [c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} - c^\dagger_{i\downarrow} c^\dagger_{j\uparrow}] \]

t >> K Fermi liquid

t ~ K D-metal (?)